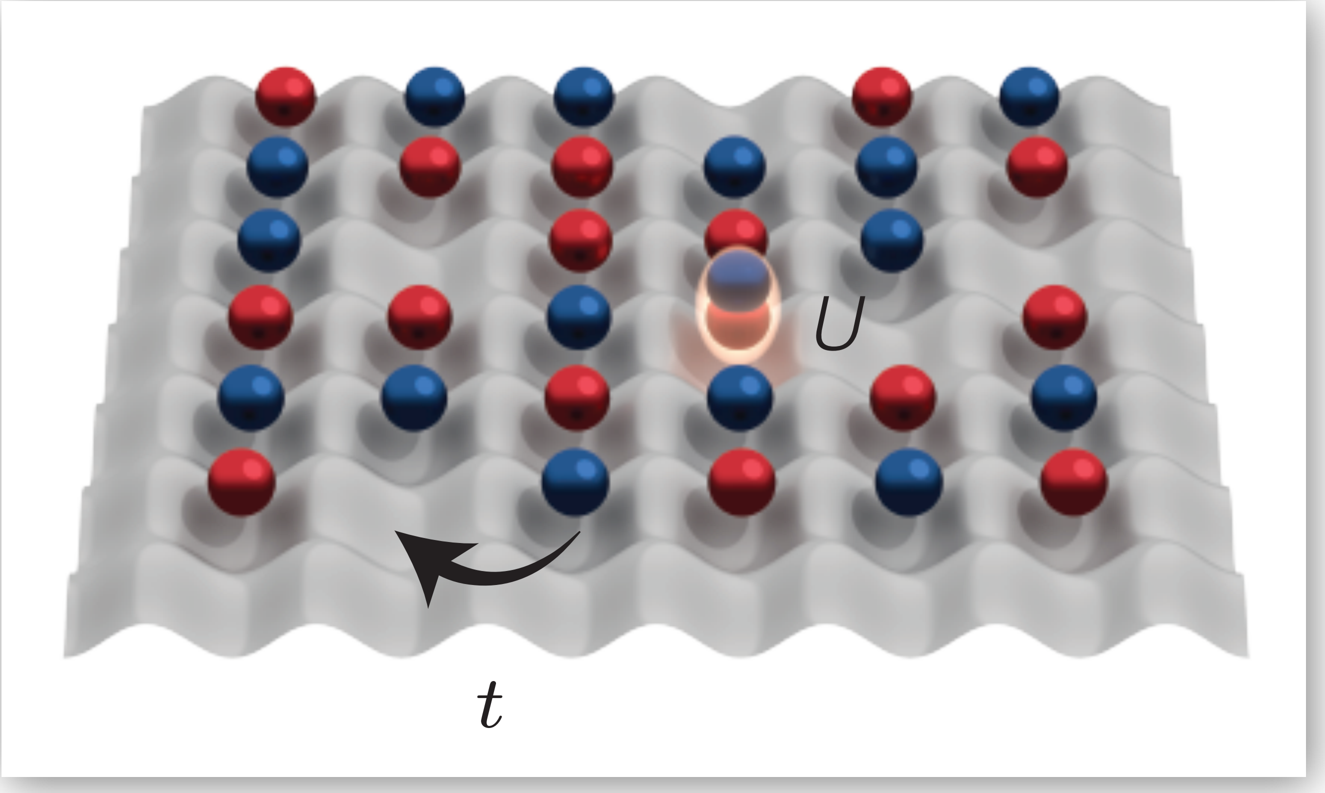
Benasque, Feb 05 2026

Quantum simulation of the doped Hubbard model

Fabian Grusdt
Ludwigs-Maximilians University Munich
Munich Center for Quantum Science and Technology



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www.quantummanybody.de

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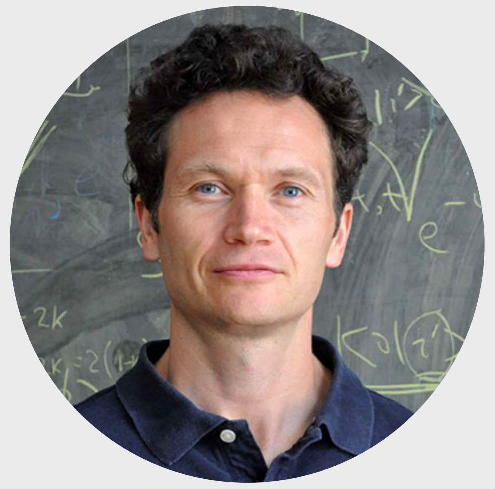

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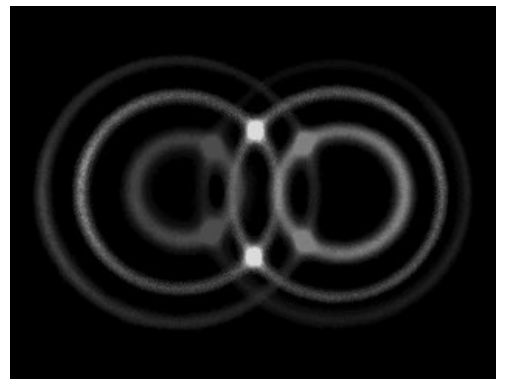
FOR 2414 DFG-Research Unit

Motivation & Introduction

Quantum Optics / AMO physics

Individual atoms and photons



Frontiers of Quantum Simulation

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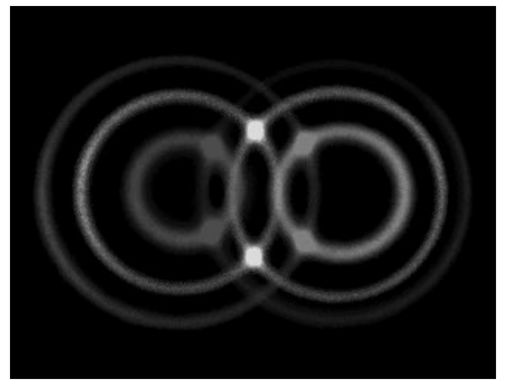
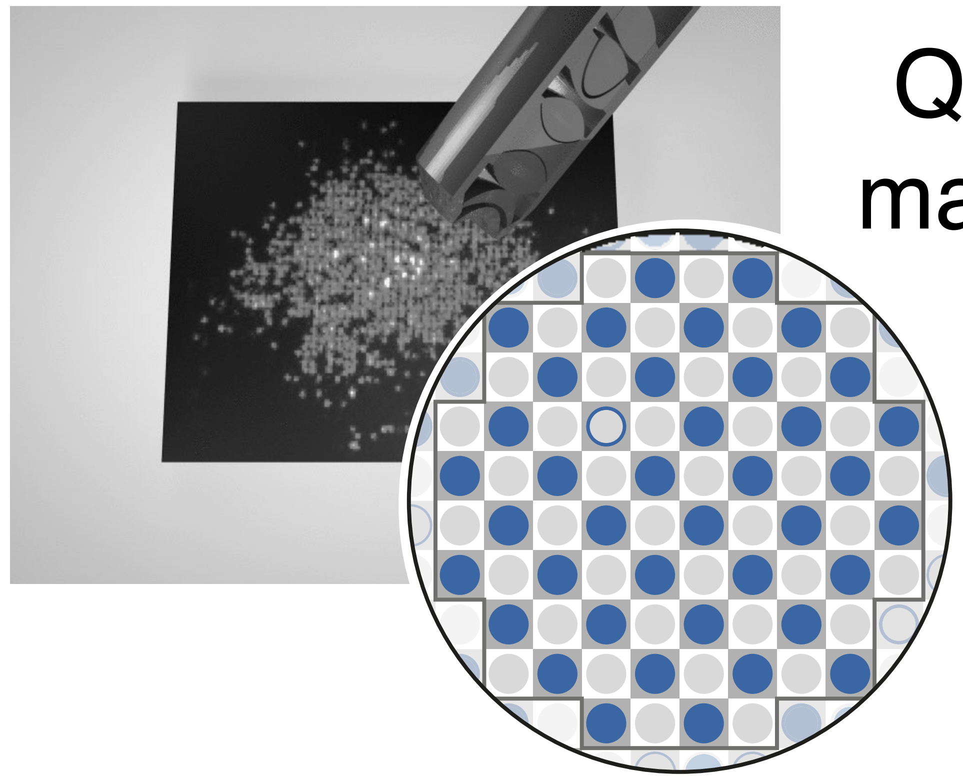


Frontiers of Quantum Simulation

Many-body effects: Quantum Matter

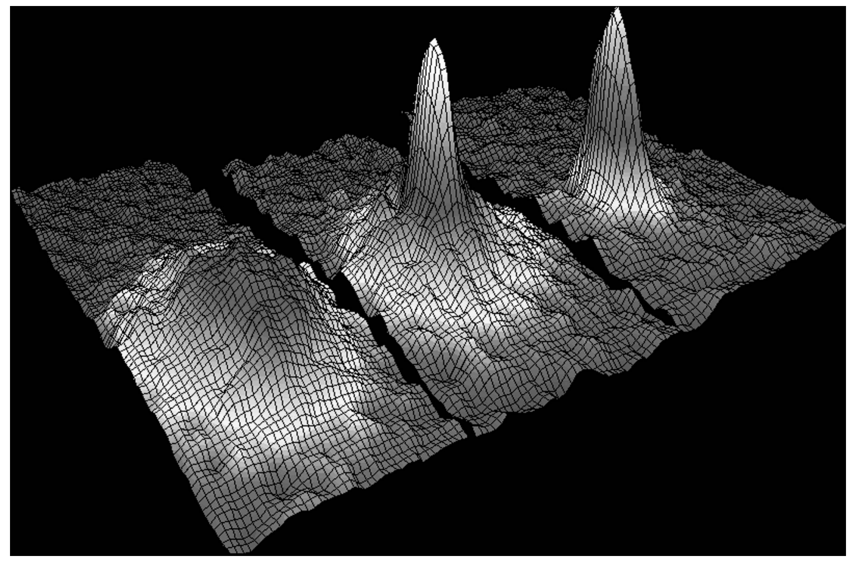


Quantum magnetism

Hart et al., Nature 519 (2015)

Mazurenko et al., Nature 545 (2017)

Interacting Bose-gases



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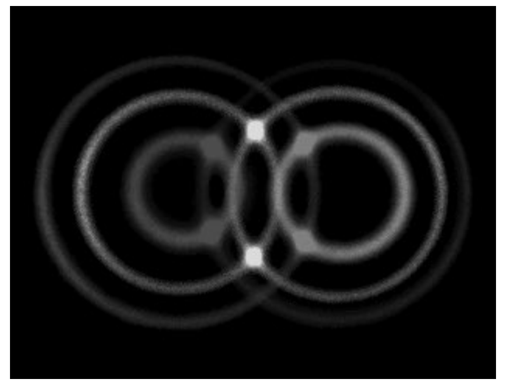

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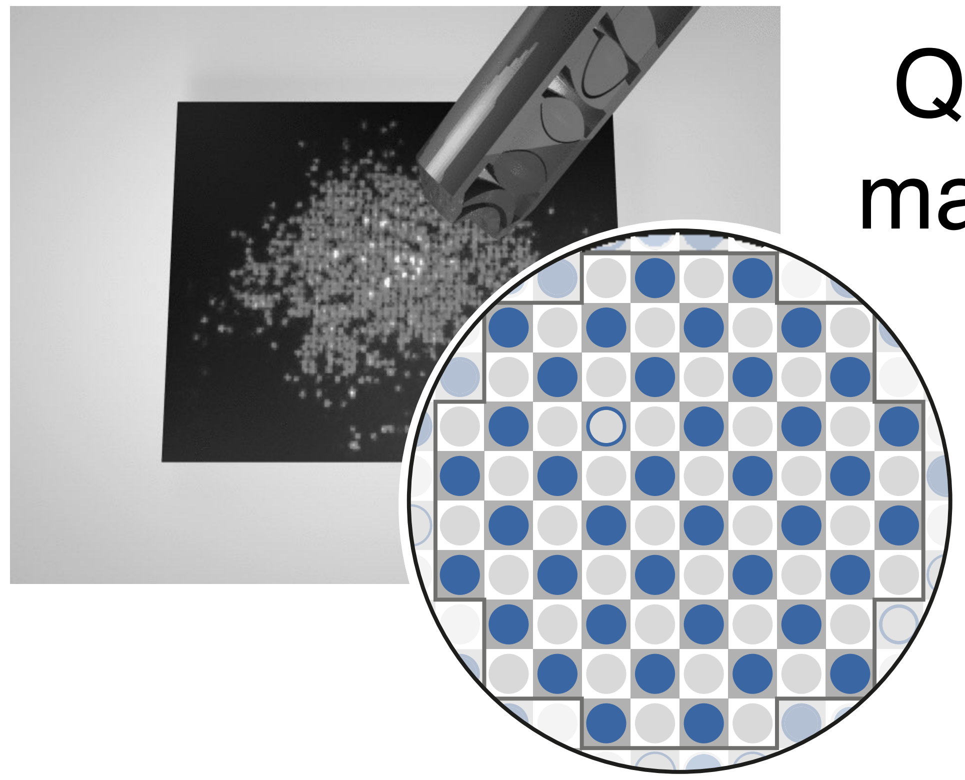

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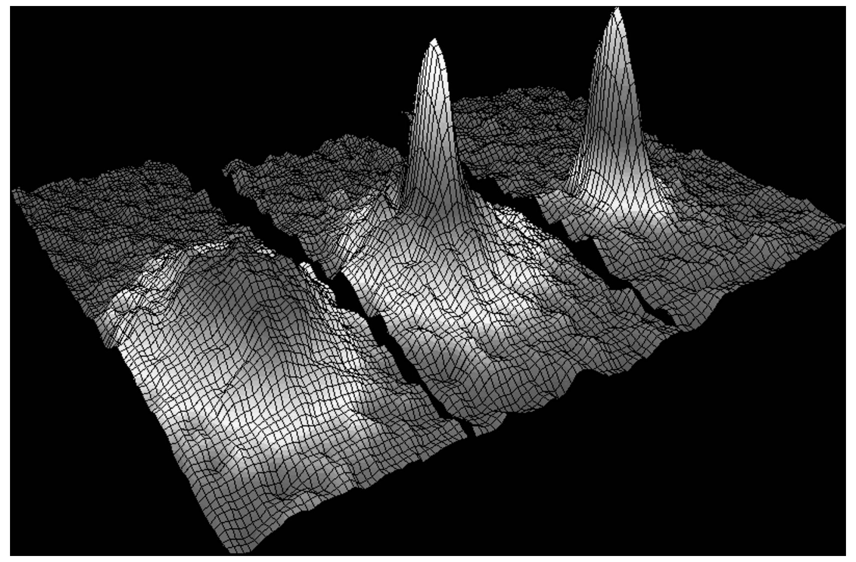

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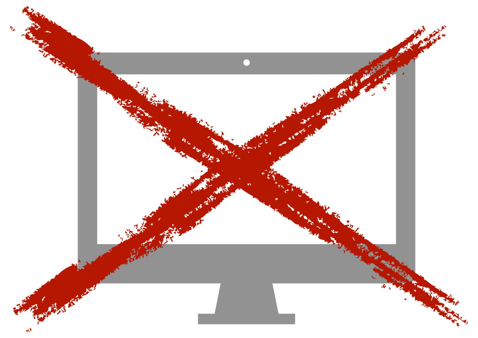

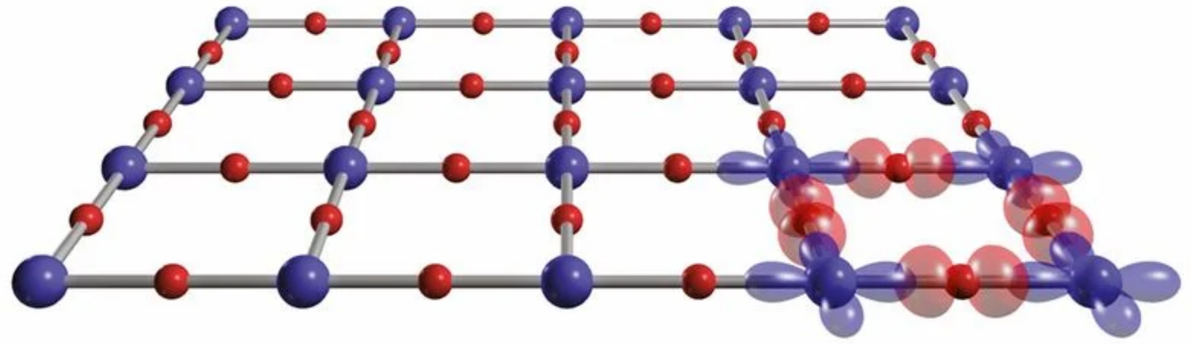

Barisic et al., PNAS 110 (2013)

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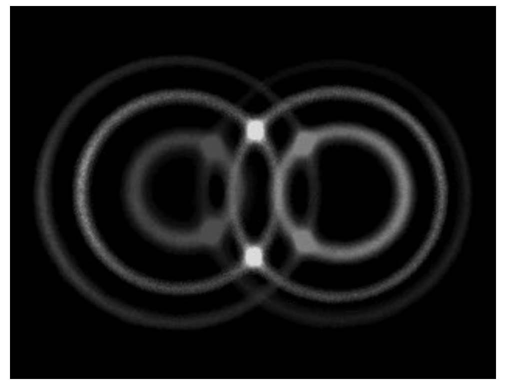

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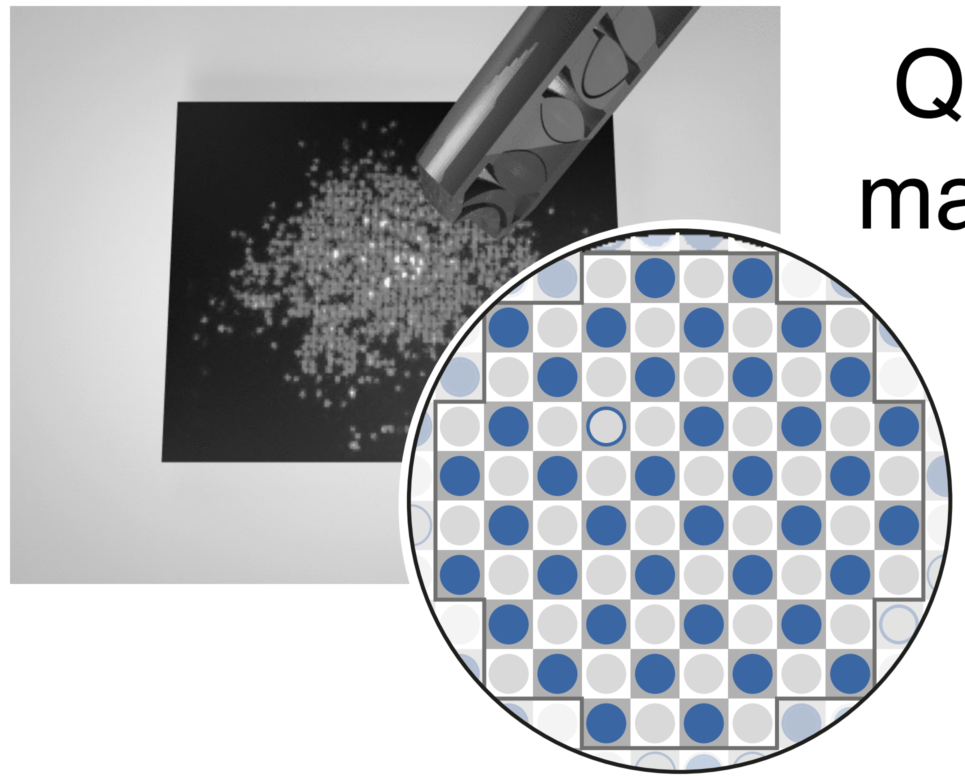

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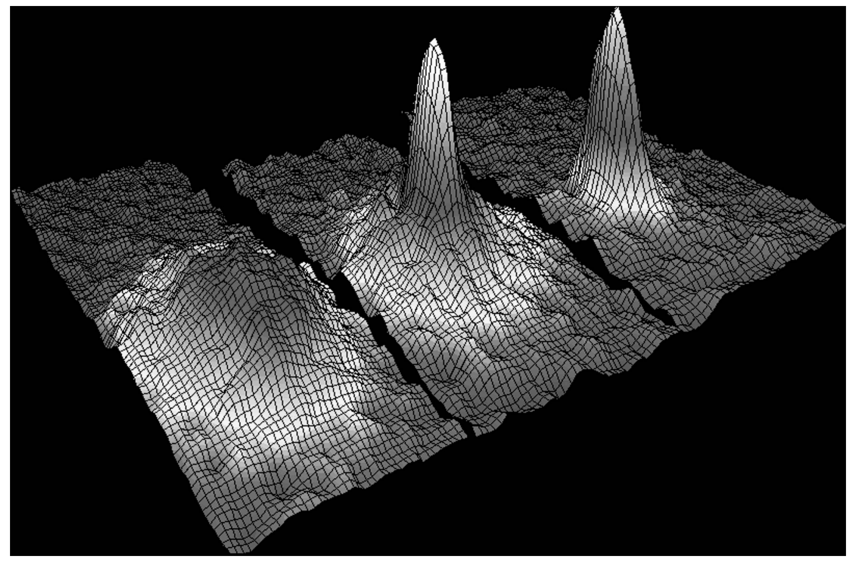

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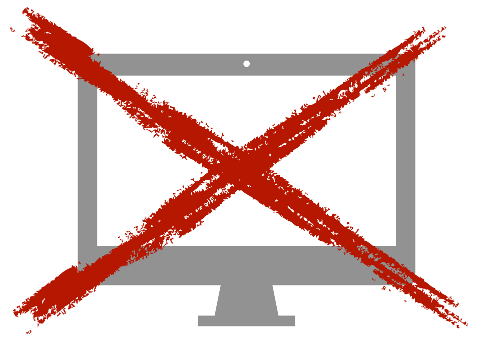

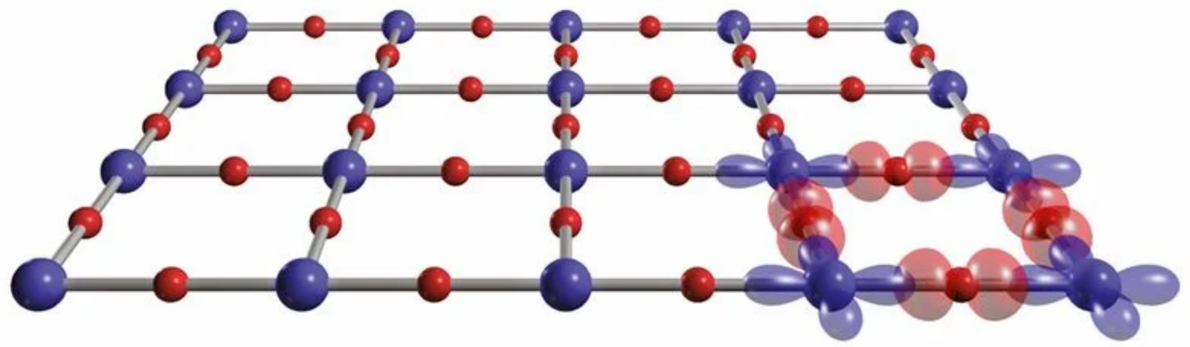

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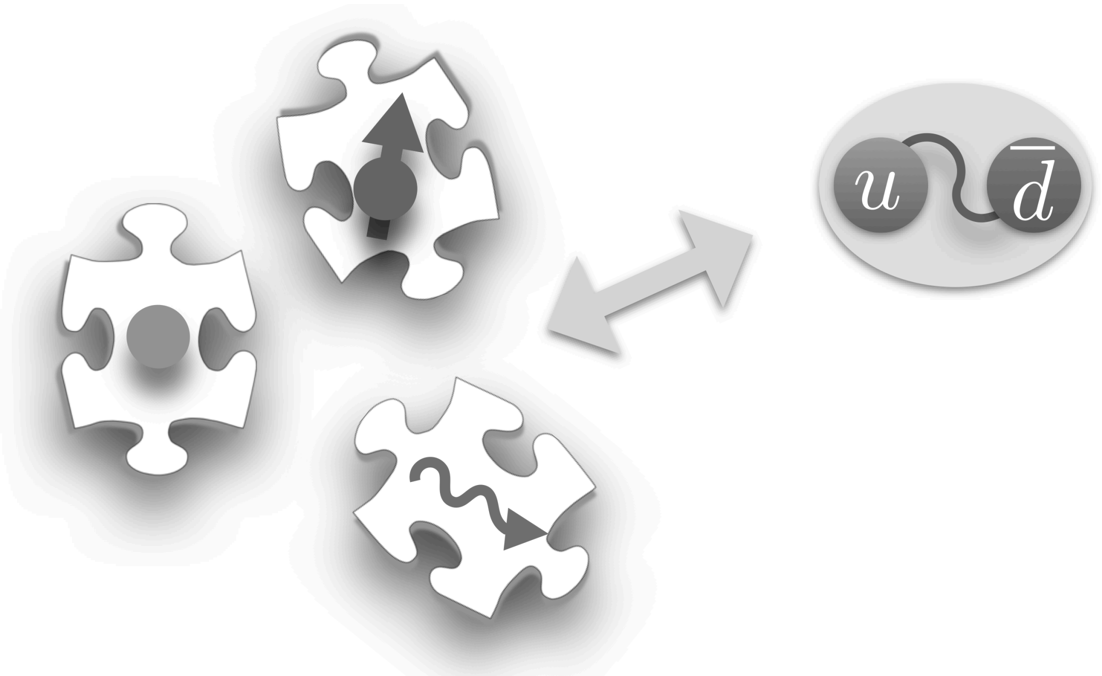
Emergent constituents



Motivation & Introduction

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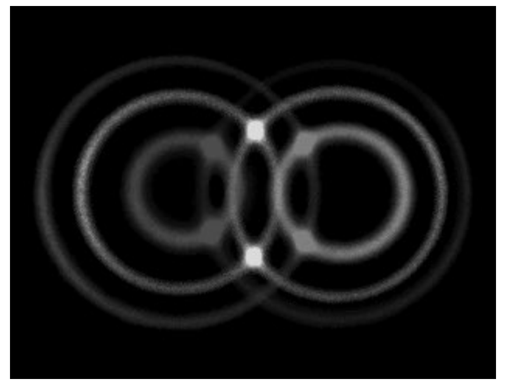
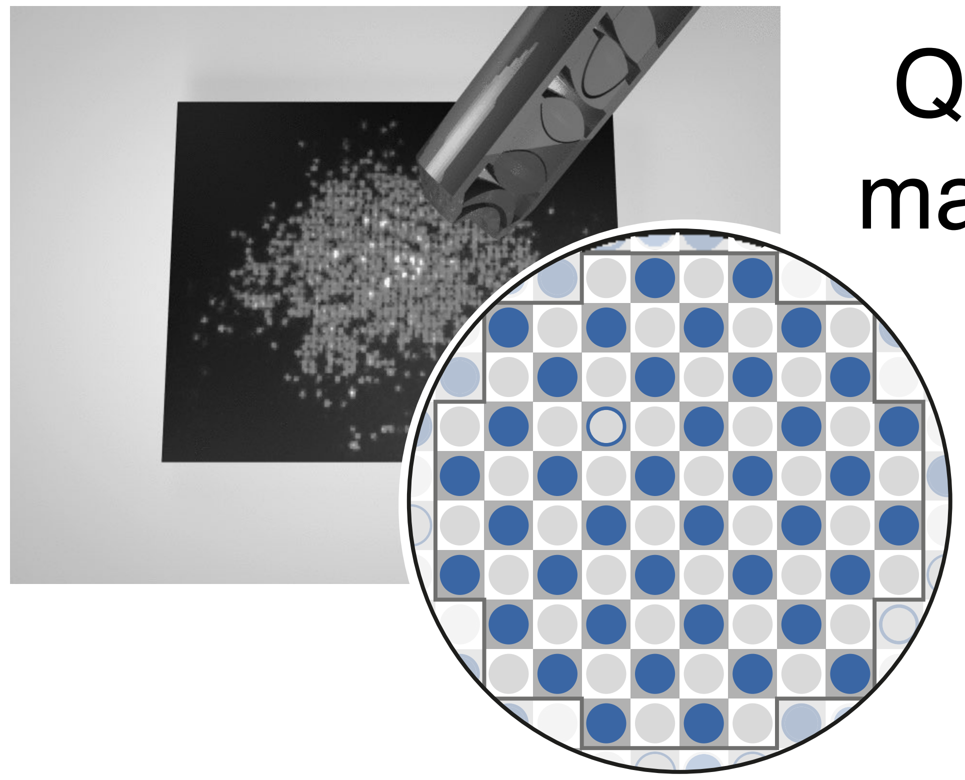


Frontiers of Quantum Simulation

Many-body effects: Quantum Matter



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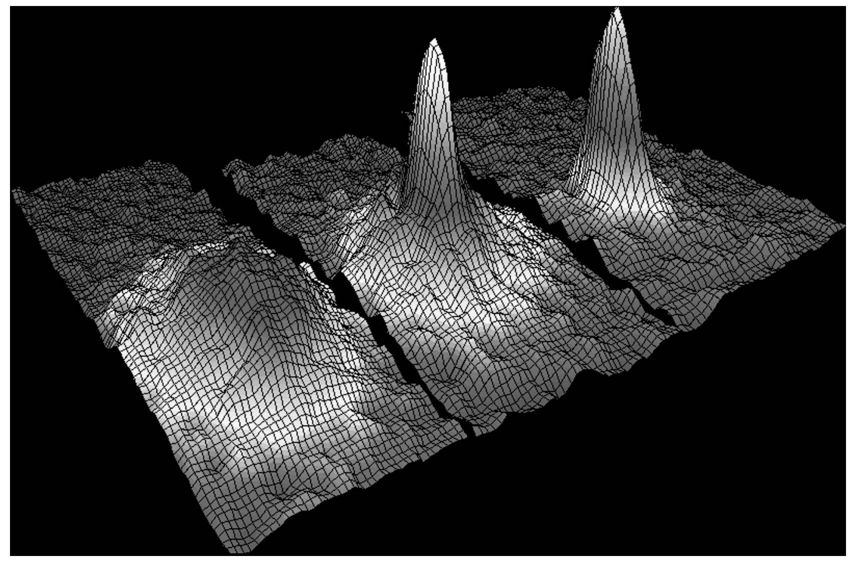

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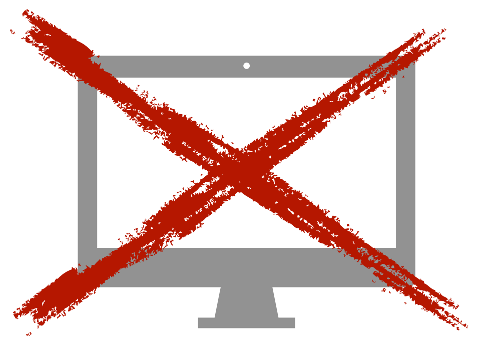

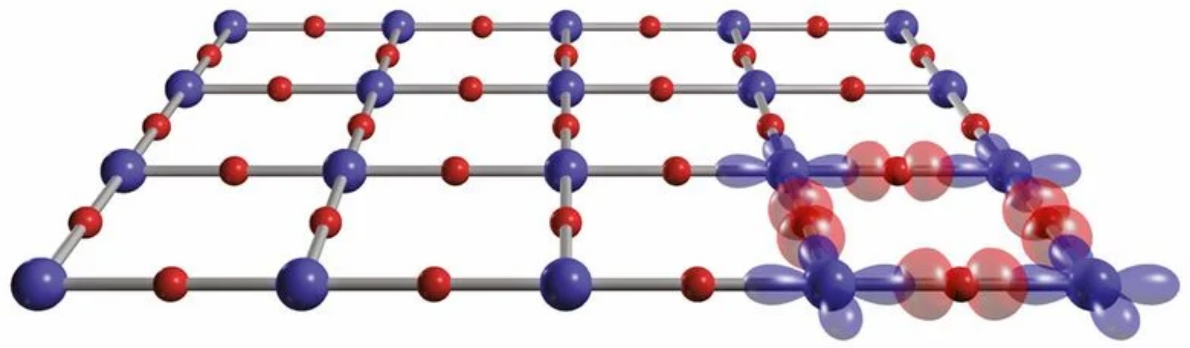

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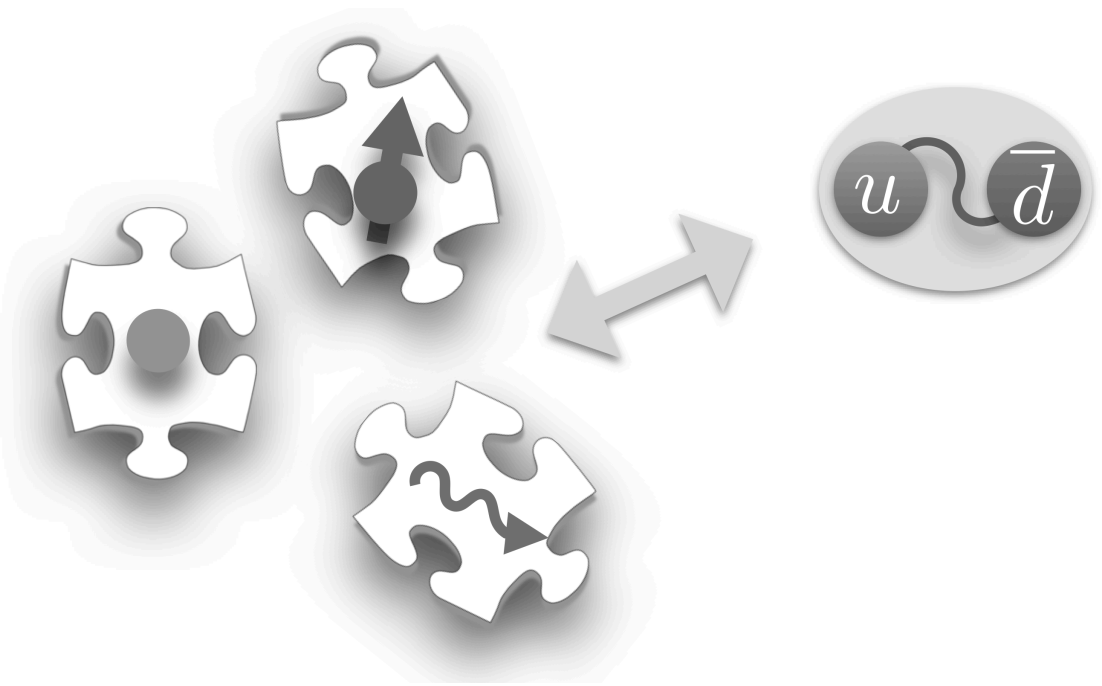
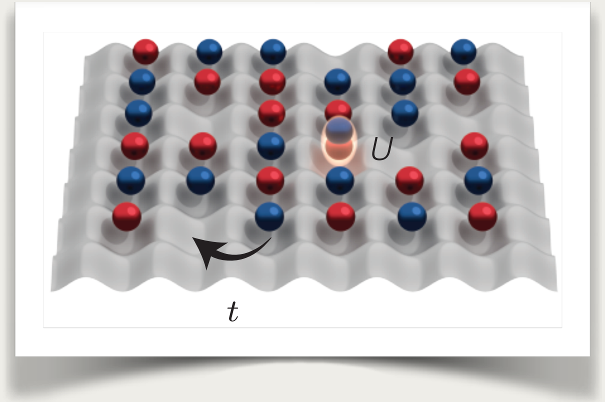


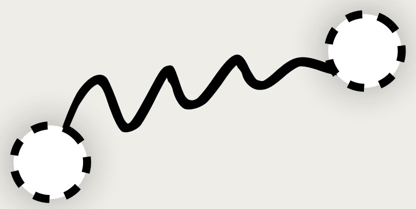
Effective field theory

$$\mathcal{L} = |(\partial_\mu - 2ia_\mu)\Psi|^2 + \dots$$

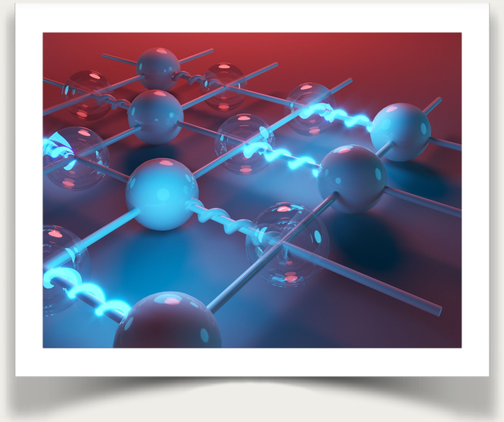
Outline



Phase diagram of high-Tc superconductors

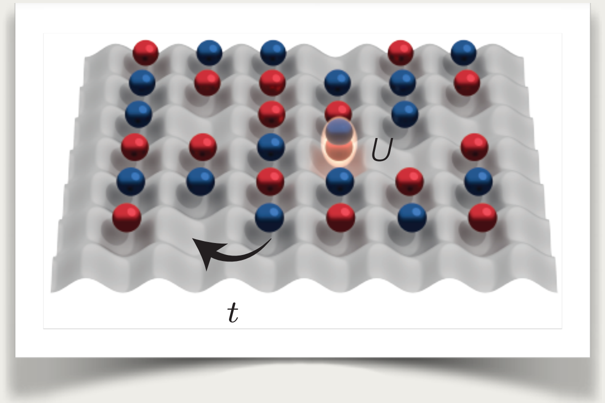


Strong coupling theory



Hidden orders

Outline



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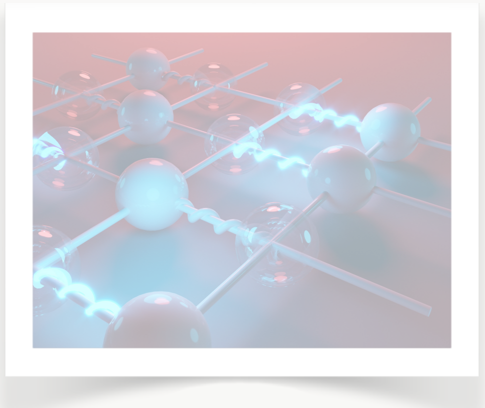
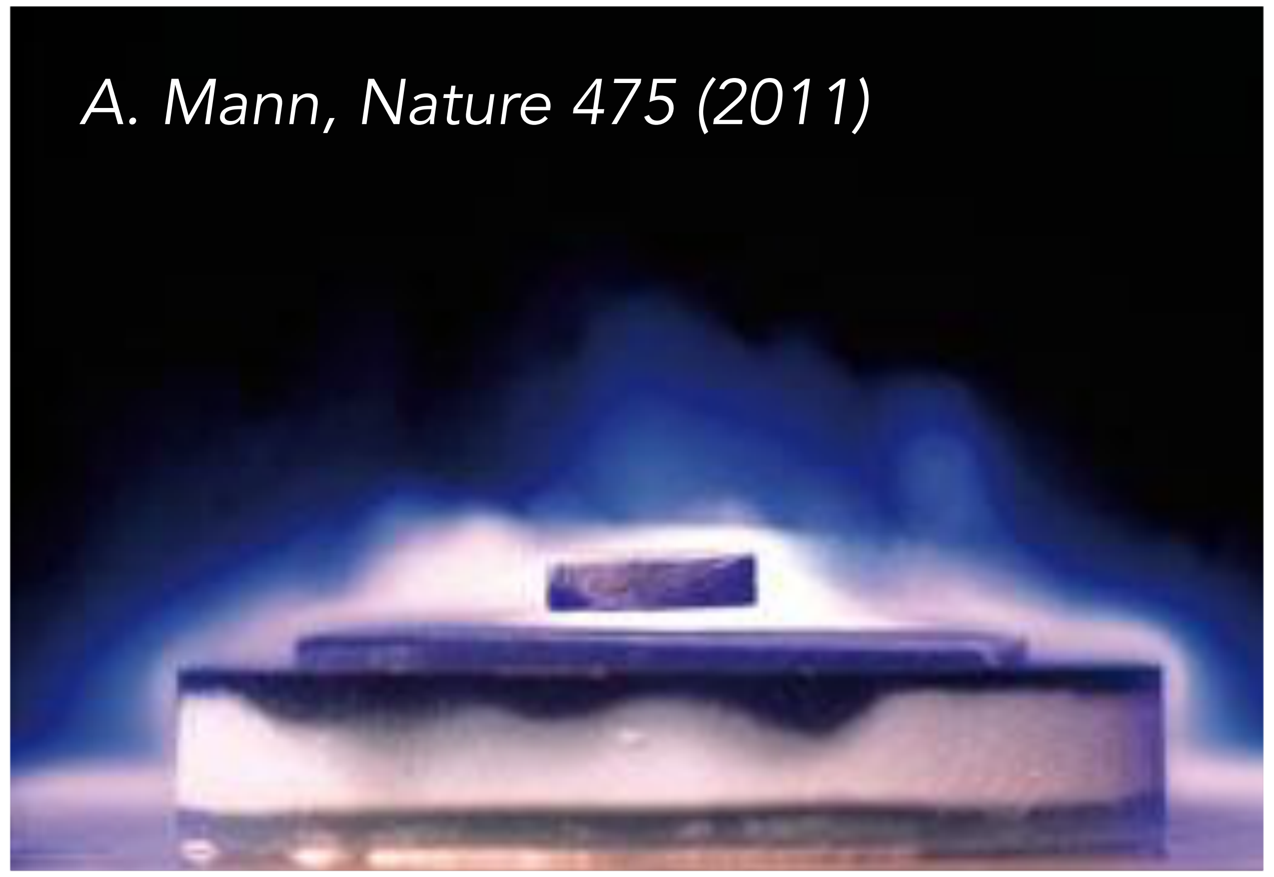


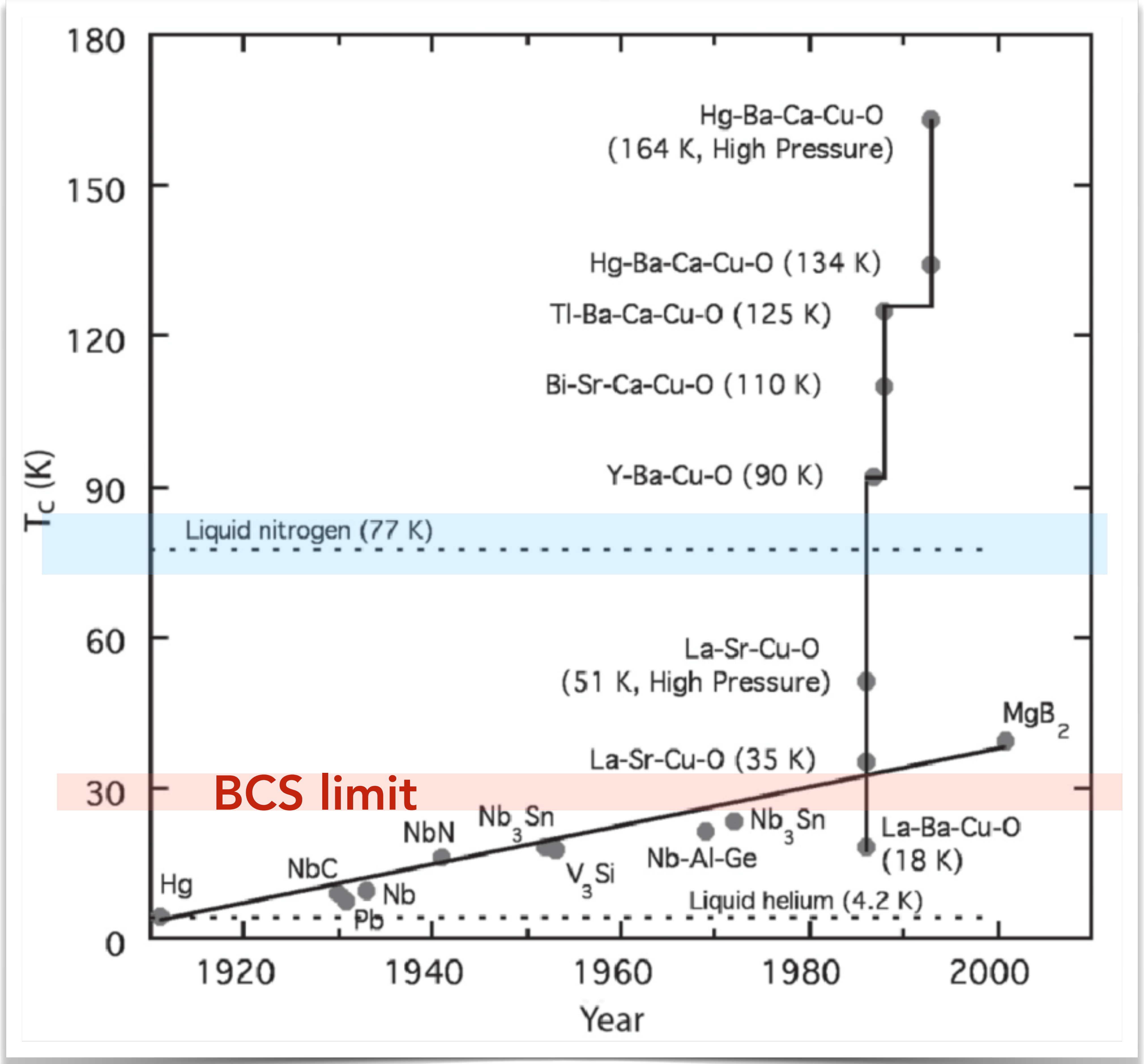
Hidden orders

Part 1.I: Background
— high-Tc superconductivity



Background

High-temperature superconductivity:



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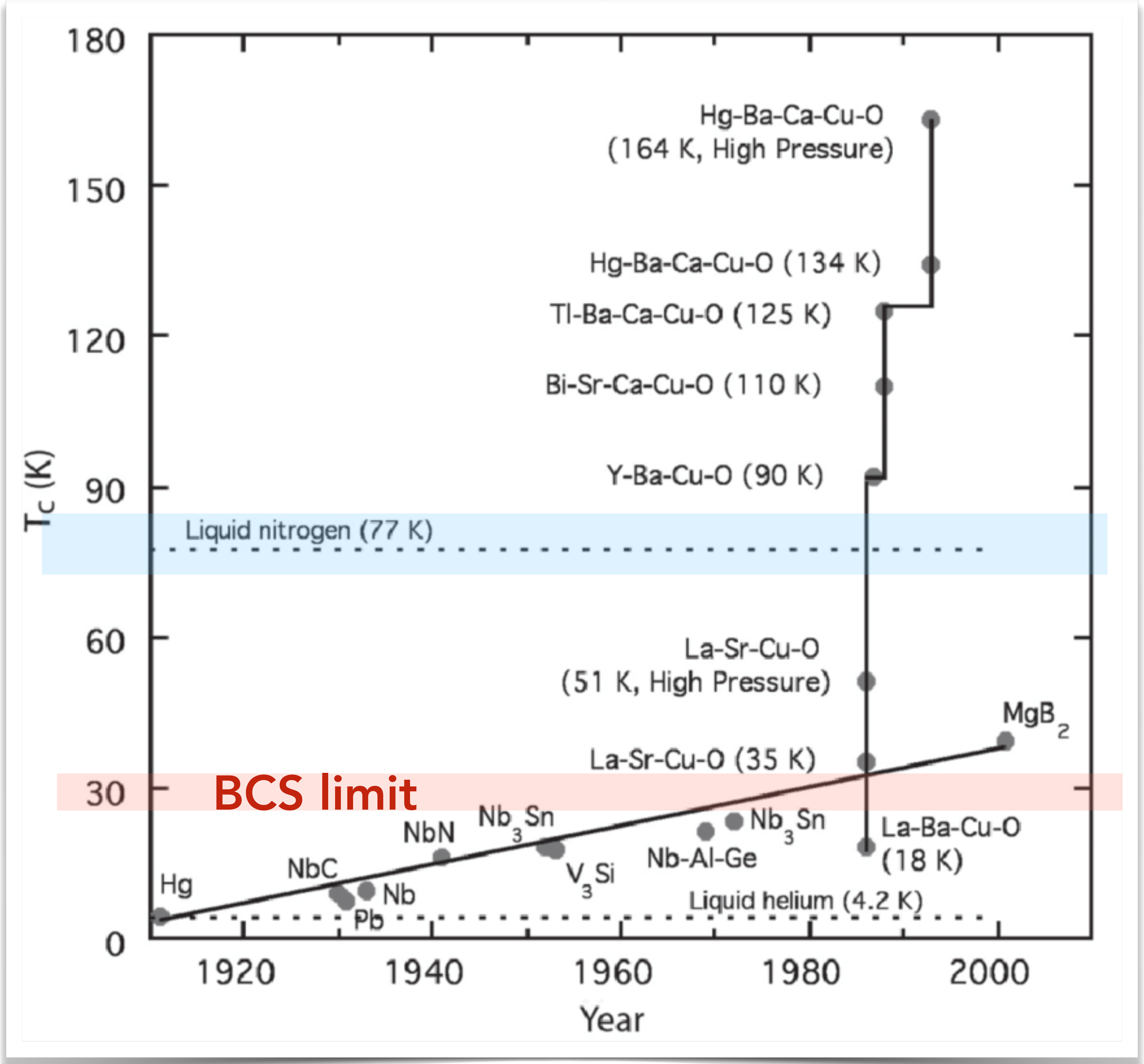
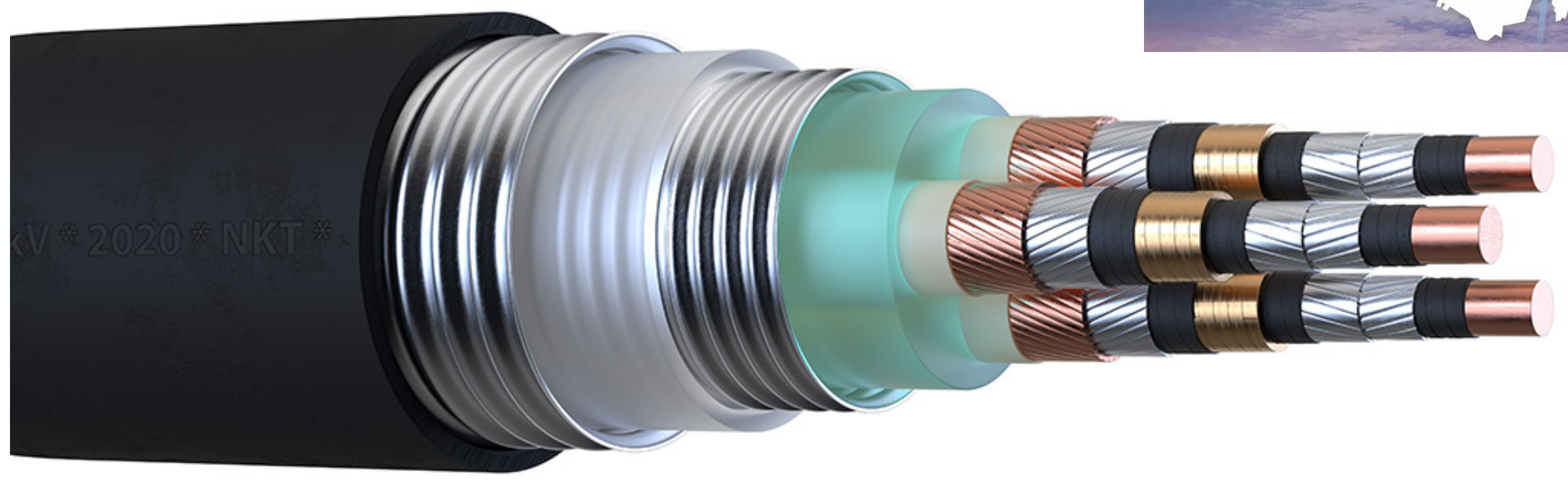


* ***Latest applications:***

Munich's SuperLink
SC cable (~12 km)



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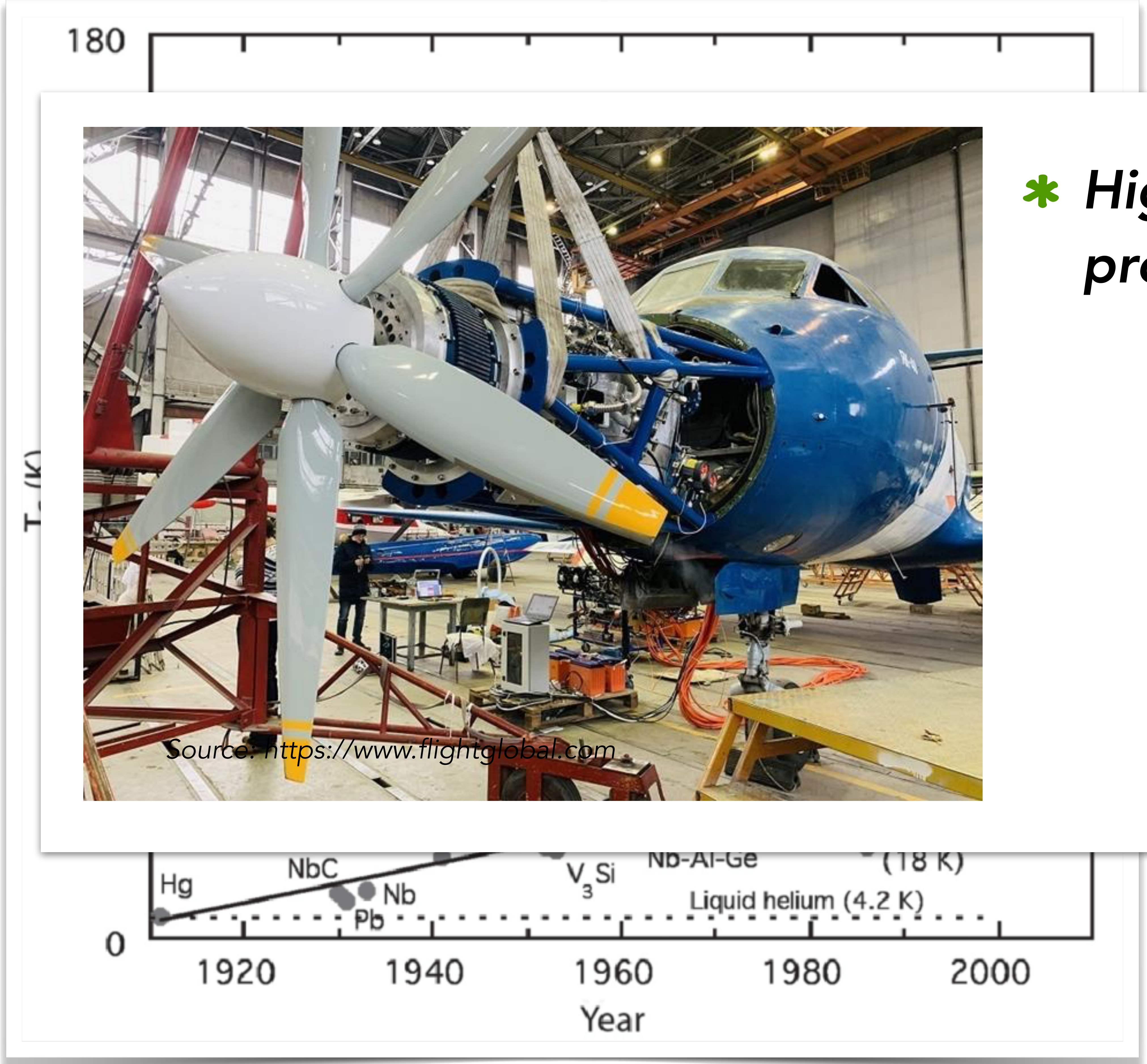

- *High-Tc propeller!*

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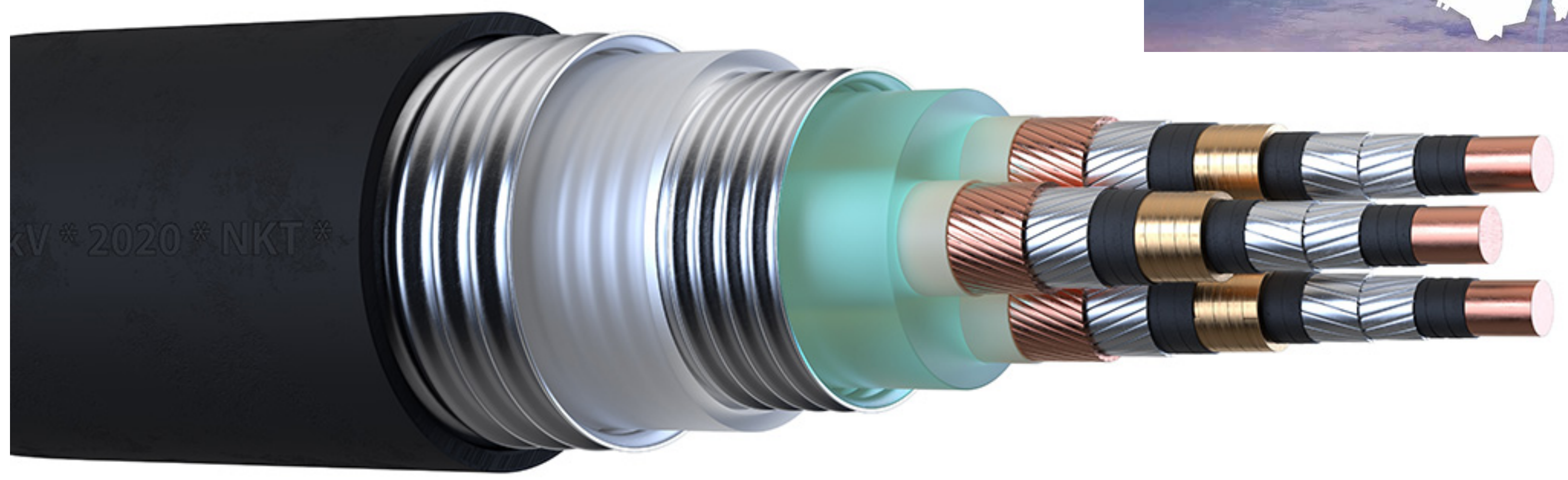

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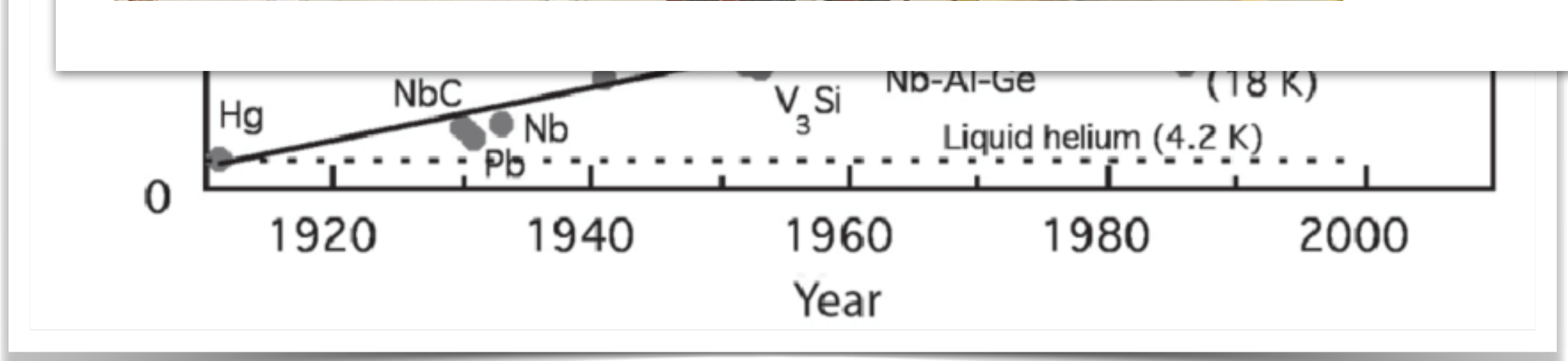

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* ***High-Tc wind turbine!***

A little history of superconductivity

Ohm's law

resistivity

$$I = U/R$$

electrical current

A little history of superconductivity

Ohm's law

$$I = U/R$$

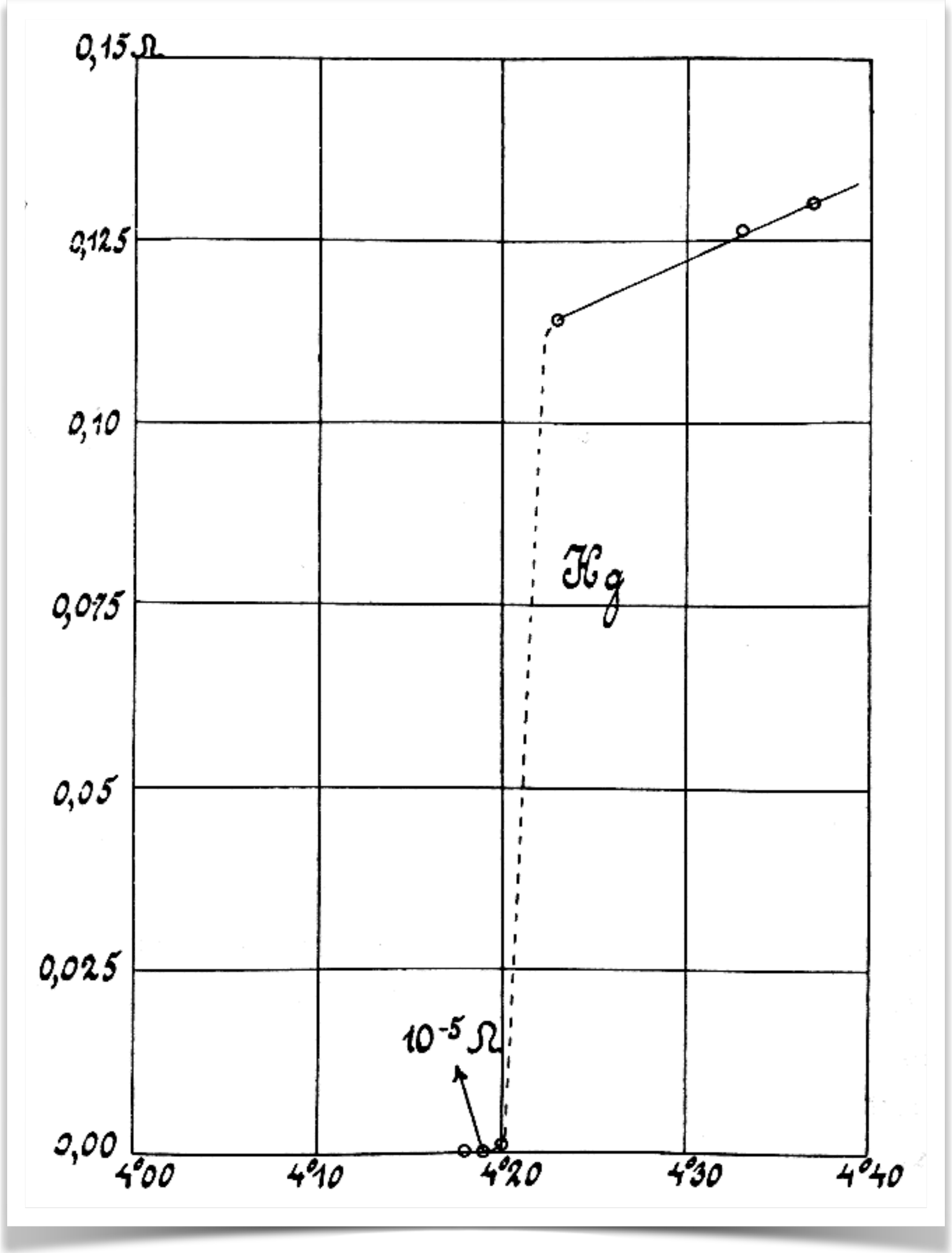
electrical current

resistivity

Superconductivity: *Heike Kamerlingh Onnes, 1911*

➡ Liquid Helium: T = 4.22 K (−268,93 °C)

1913: Physics Nobel Prize



A little history of superconductivity

Superconductivity is a quantum effect

- 1911: Discovery of superconductivity
- 1913: H. Kamerlingh Onnes
- ~1926: Development of quantum mechanics
- 1933: Meissner effect

Two classes of elementary particles:

➡ Bosons ➡ Fermions

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- 1937/38: Explanation of **boson** superfluidity, London theory of superconductivity

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Pairing is essential:

fermion + fermion = boson

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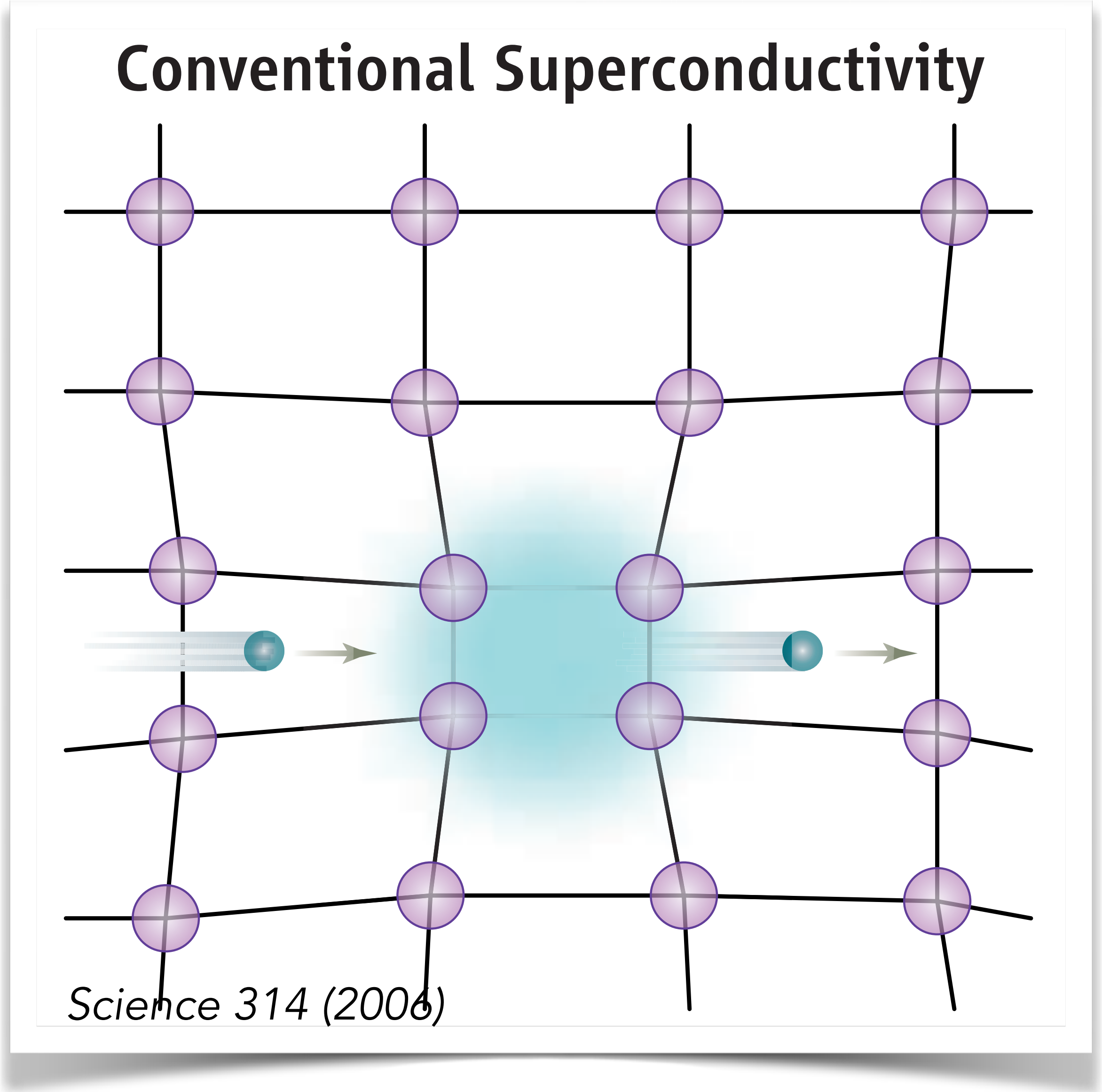
Superconductivity is a quantum effect

- 1911: Discovery of superconductivity
- 1950: Ginzburg-Landau theory;
  isotope-effect implies electron-phonon interactions
- 1955: effective interaction (Jellium);
  gap hypothesis

Fermion + Fermion = Boson

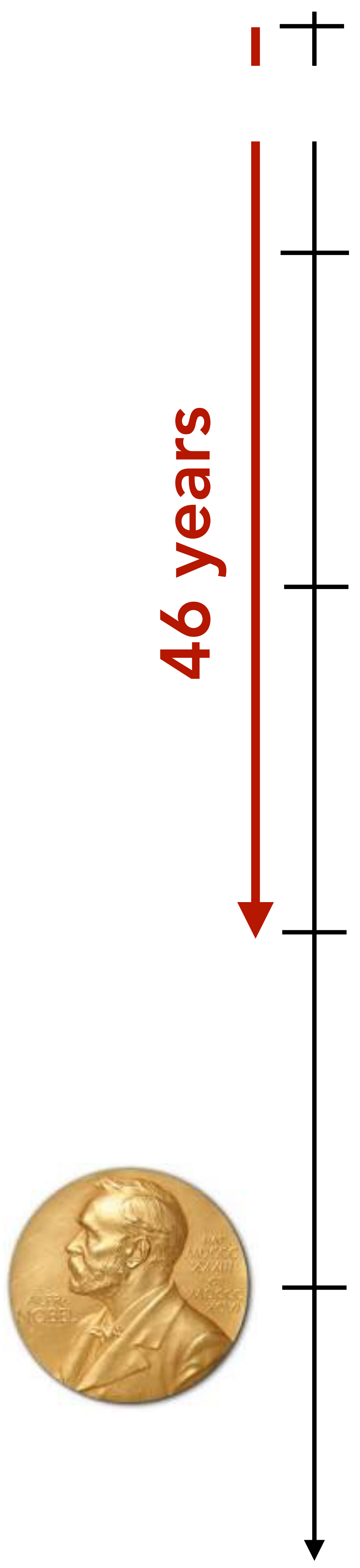


➡ Electrons form pairs

➡ Pairs condense;
gap out Fermi sea

A little history of superconductivity

Superconductivity is a quantum effect



1911: Discovery of superconductivity

1950: Ginzburg-Landau theory;
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1955: effective interaction (Jellium);
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1957: Quantum theory of
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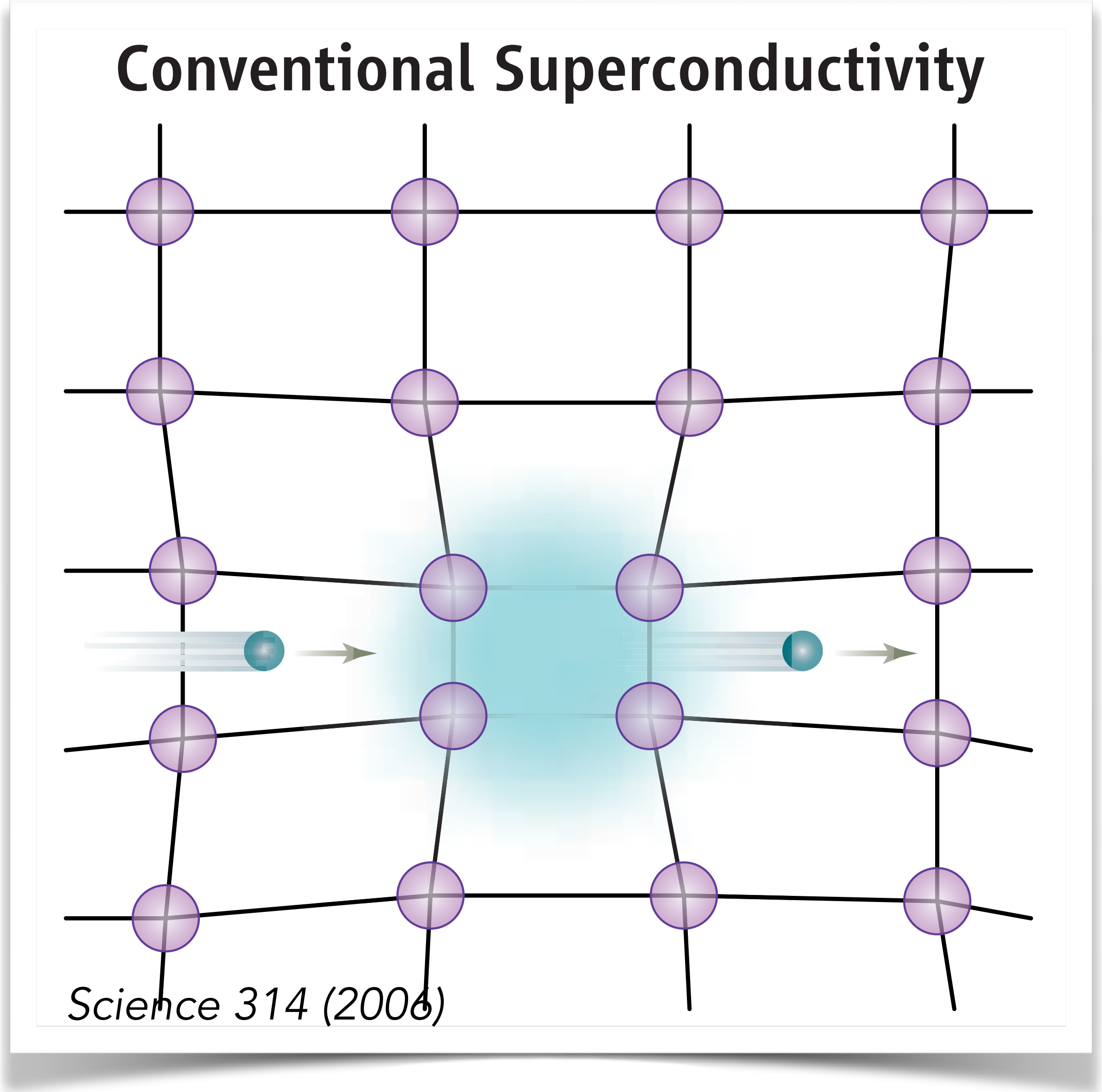
1972: Bardeen, Cooper & Schrieffer (BCS)

Fermion + Fermion = Boson



Science 314 (2006)

➡ Electrons form pairs

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- 1911: Discovery of superconductivity
- 1913: H. Kamerlingh Onnes

- 1957: Quantum theory of superconductivity (BCS)

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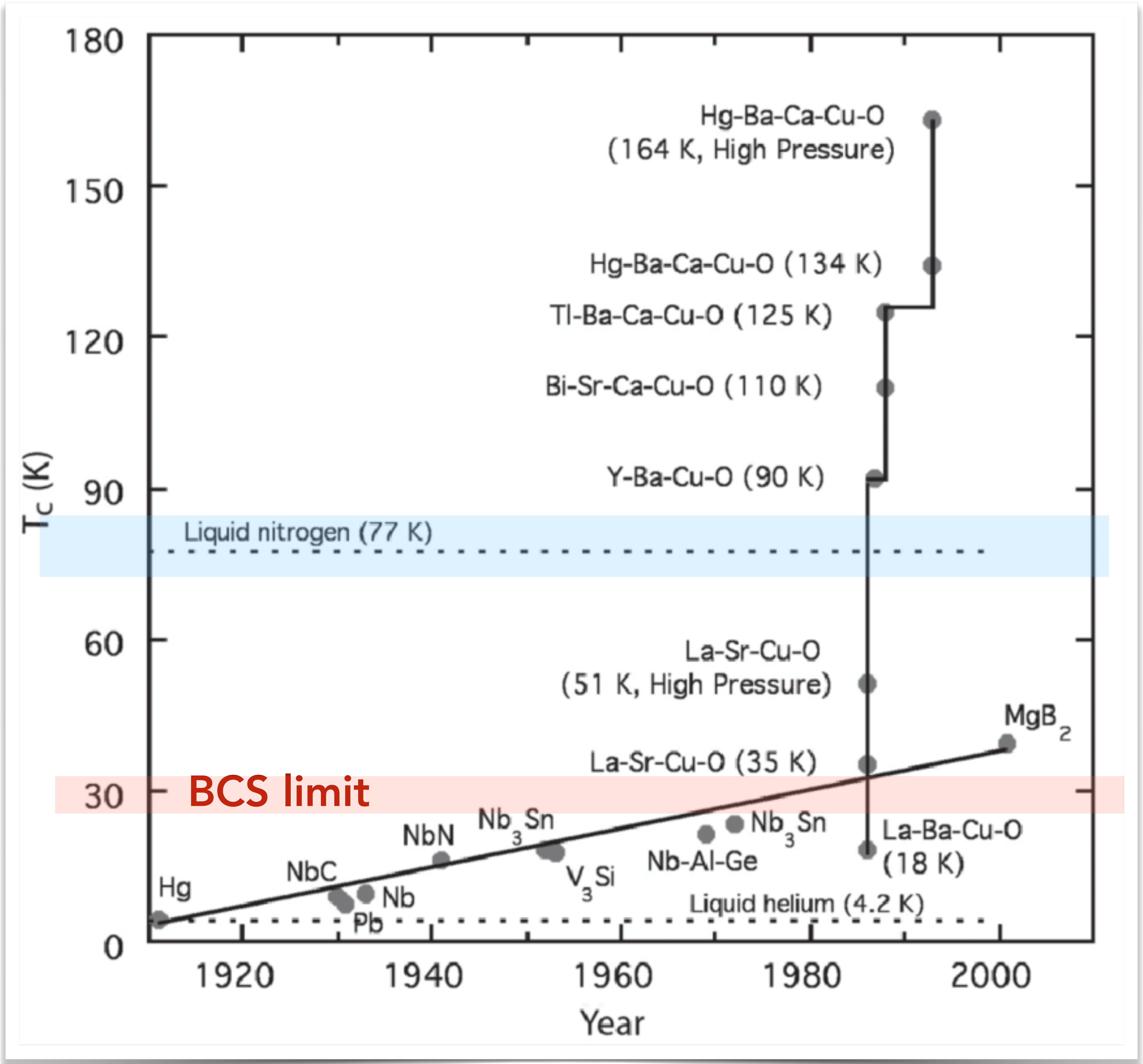
- 1986: Discovery of high-Tc superconductivity
- 1987: Bednorz & Müller

~40 years

…
2026
…



A little history of superconductivity

High-Tc superconductivity

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46 years

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High-Tc superconductivity

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1955: ~~effective interaction~~ (Jellium); **strong correlations**
gap hypothesis

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1987/88: Hubbard & t-J models
Anderson / Zhang & Rice

A little history of superconductivity

High-Tc superconductivity

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No isotope effect

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2022: Seamus Davis' group:
superexchange-mediated interactions

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Pseudogap?!

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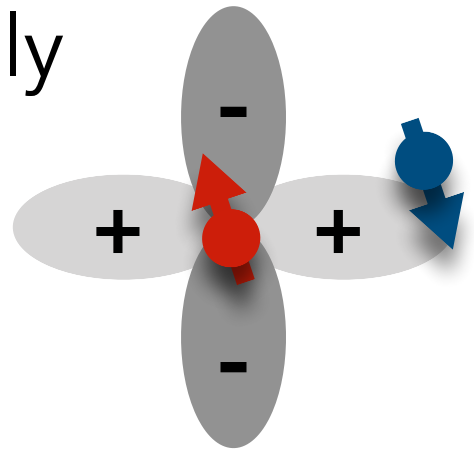
1987/88: Hubbard & t-J models
Anderson / Zhang & Rice

1993: **d-wave pairing** conclusively established: Wollman et al.



2022: Seamus Davis' group:
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A little history of superconductivity

High-Tc superconductivity

Key open problems:

- theory of maximum Tc
- relation to collective phases: pseudogap, stripes,...
- universal pairing mechanism?

⇒ **microscopic understanding?**

A little history of superconductivity

High-Tc superconductivity

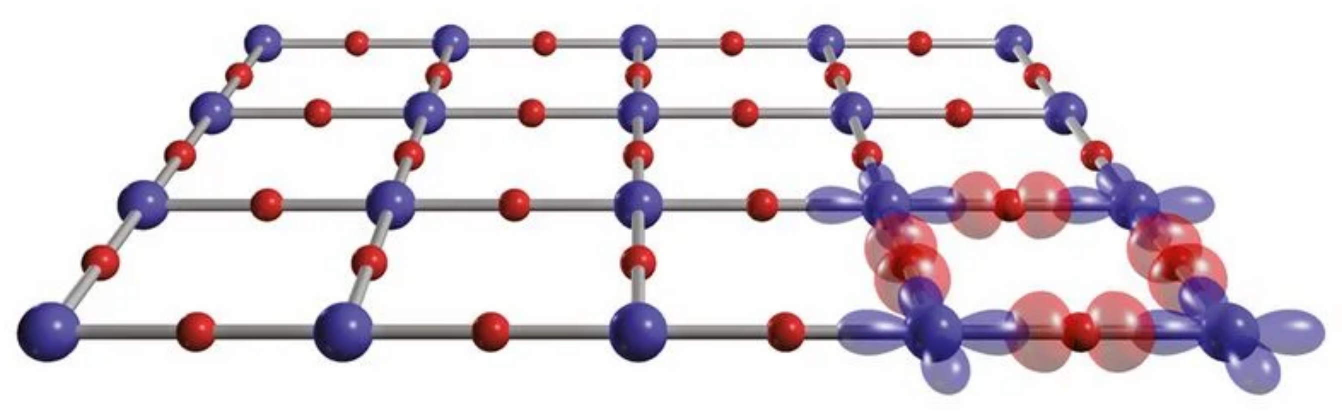
Key open problems:

- theory of maximum Tc
- relation to collective phases: pseudogap, stripes,...
- universal pairing mechanism?

⇒ **microscopic understanding?**

Some agreement, at least:

- *Hubbard / t-J model physics in 2D planes*



Anderson, Emery, Zhang & Rice, …

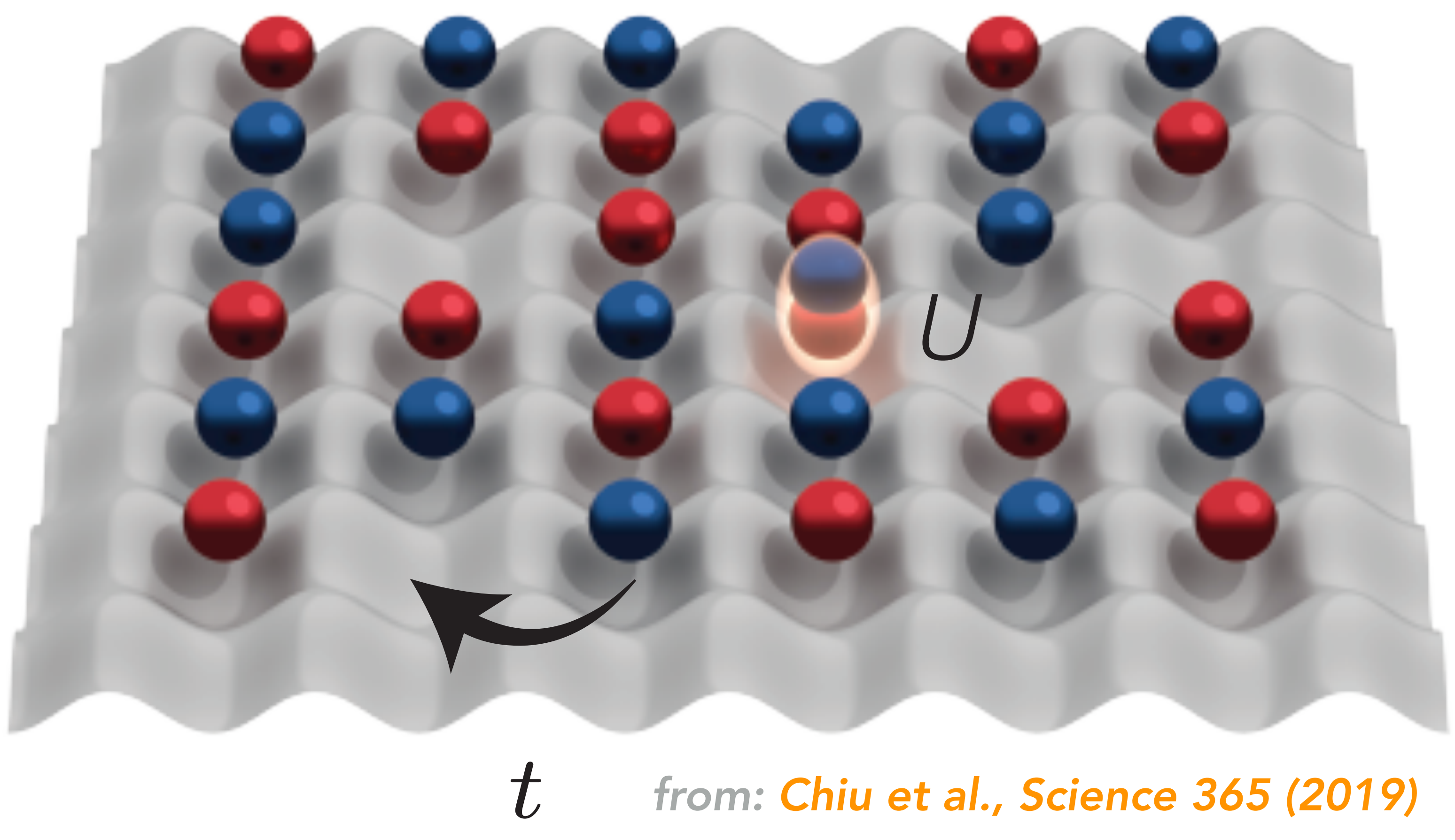
⇒ **microscopic model(s)!**

A little history of superconductivity

Fermi-Hubbard and *t-J* models

$$\hat{\mathcal{H}}_{\mathrm{FH}} = -t \sum_{\langle \mathbf{i},\mathbf{j} \rangle,\sigma} (\hat{c}^\dagger_{\mathbf{i},\sigma} \hat{c}_{\mathbf{j},\sigma} + \mathrm{h.c.}) + U \sum_{\mathbf{j}} \hat{n}_{\mathbf{j},\uparrow} \hat{n}_{\mathbf{j},\downarrow}$$

$$+t' \sum_{\langle\langle \mathbf{i},\mathbf{j} \rangle\rangle} \ldots$$



from: Chiu et al., Science 365 (2019)

A little history of superconductivity

Fermi-Hubbard and *t-J* models

repulsive *U>0* — beyond Kohn-Luttinger?

Kohn & Luttinger, PRL 15 (1965)

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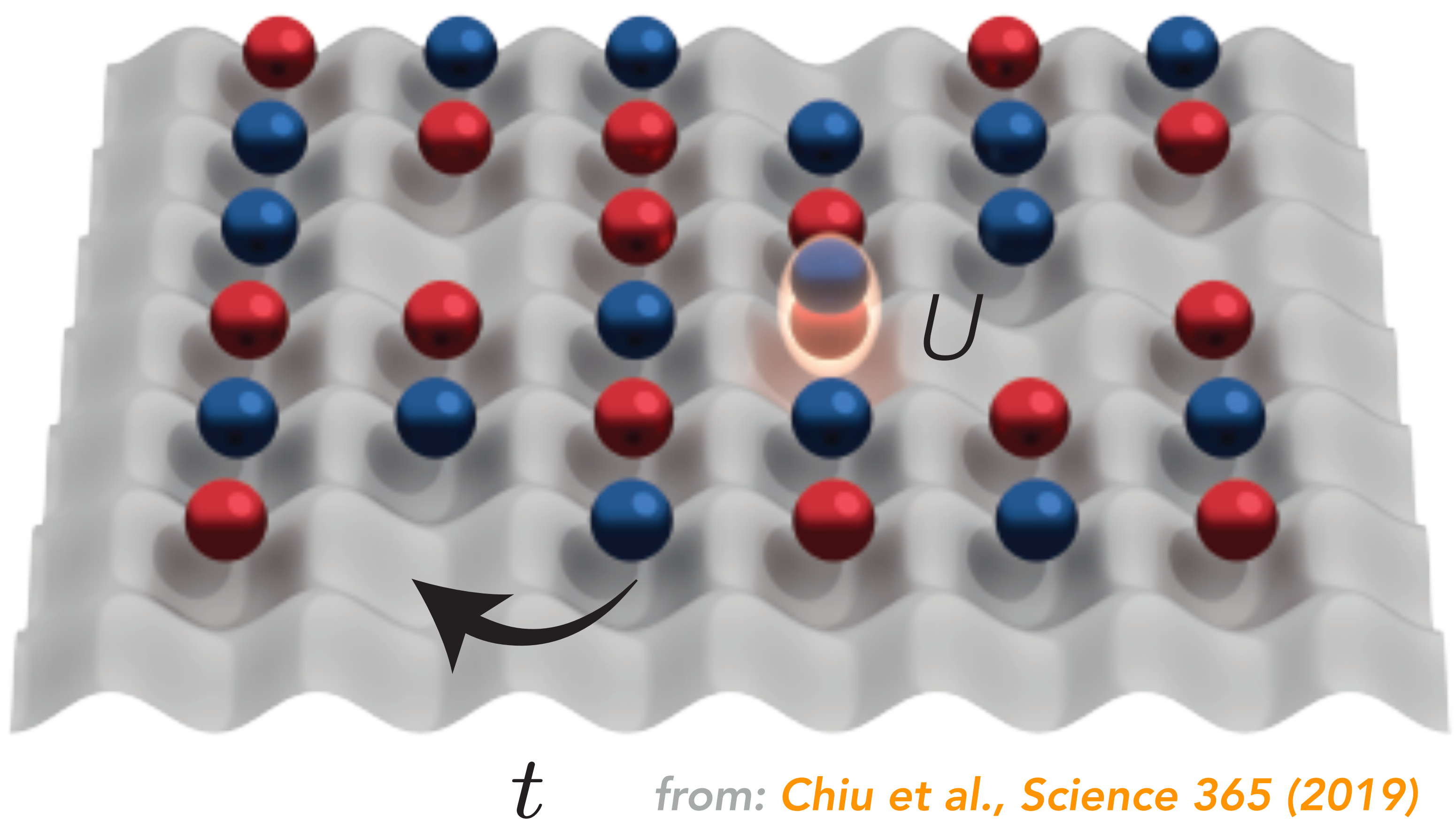

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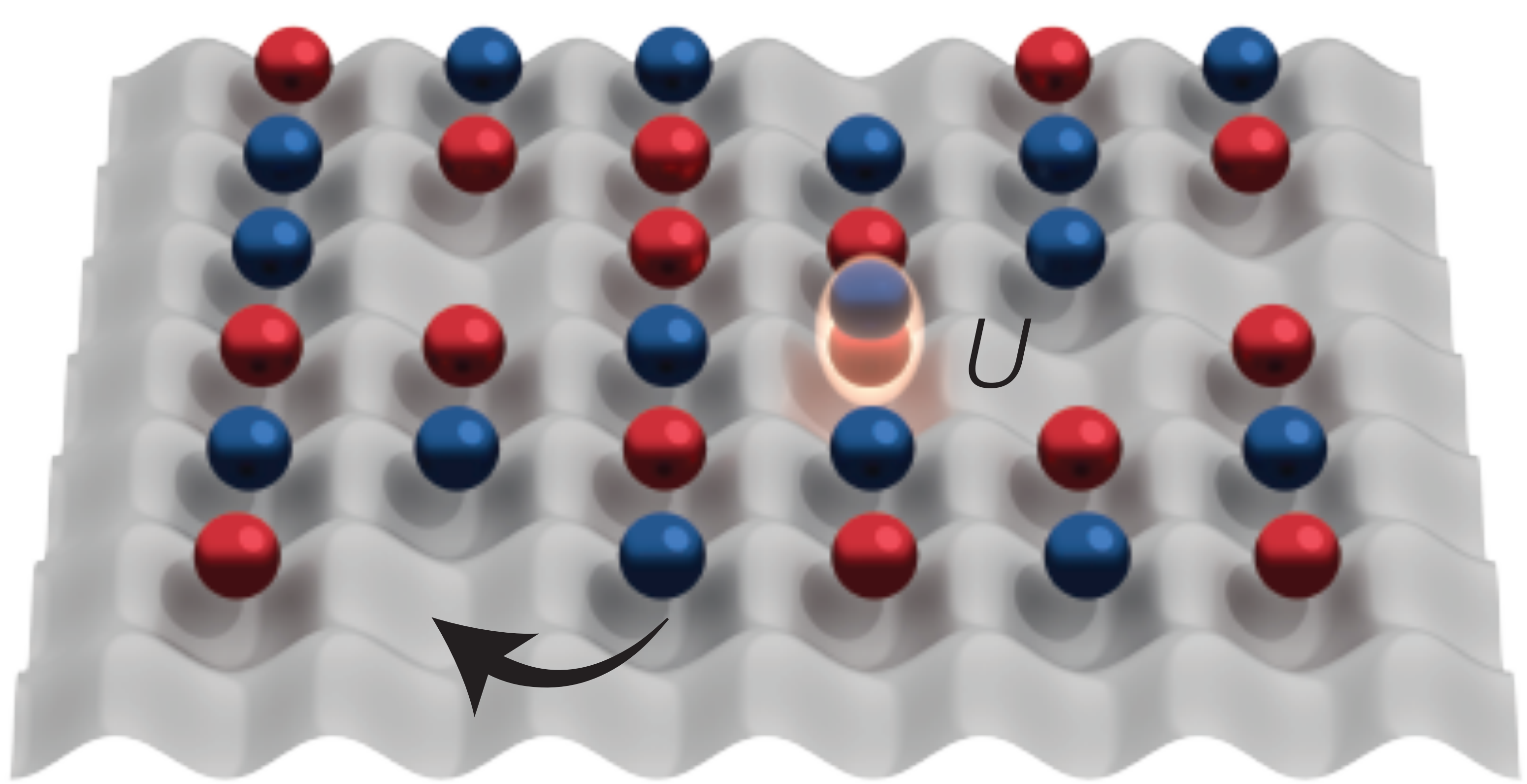


t from: *Chiu et al., Science 365 (2019)*

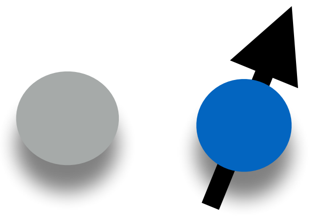
- *Large-U limit: t-J model*

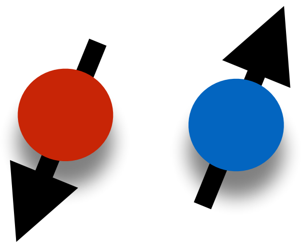
no double-occupancy!

$$\hat{\mathcal{H}}_{t-J} = -t\,\hat{\mathcal{P}}\Big[\sum_{\langle \mathbf{i},\mathbf{j}\rangle,\sigma} \hat{c}^\dagger_{\mathbf{i},\sigma}\hat{c}_{\mathbf{j},\sigma} + \text{h.c.}\Big]\hat{\mathcal{P}} + J\sum_{\langle \mathbf{i},\mathbf{j}\rangle}\left(\hat{\mathbf{S}}_{\mathbf{i}}\cdot\hat{\mathbf{S}}_{\mathbf{j}} - \frac{1}{4}\hat{n}_{\mathbf{i}}\hat{n}_{\mathbf{j}}\right)$$

+ NNN hopping term O(*J*)

$$+t' \sum_{\langle\langle \mathbf{i},\mathbf{j}\rangle\rangle} \dots$$





A little history of superconductivity

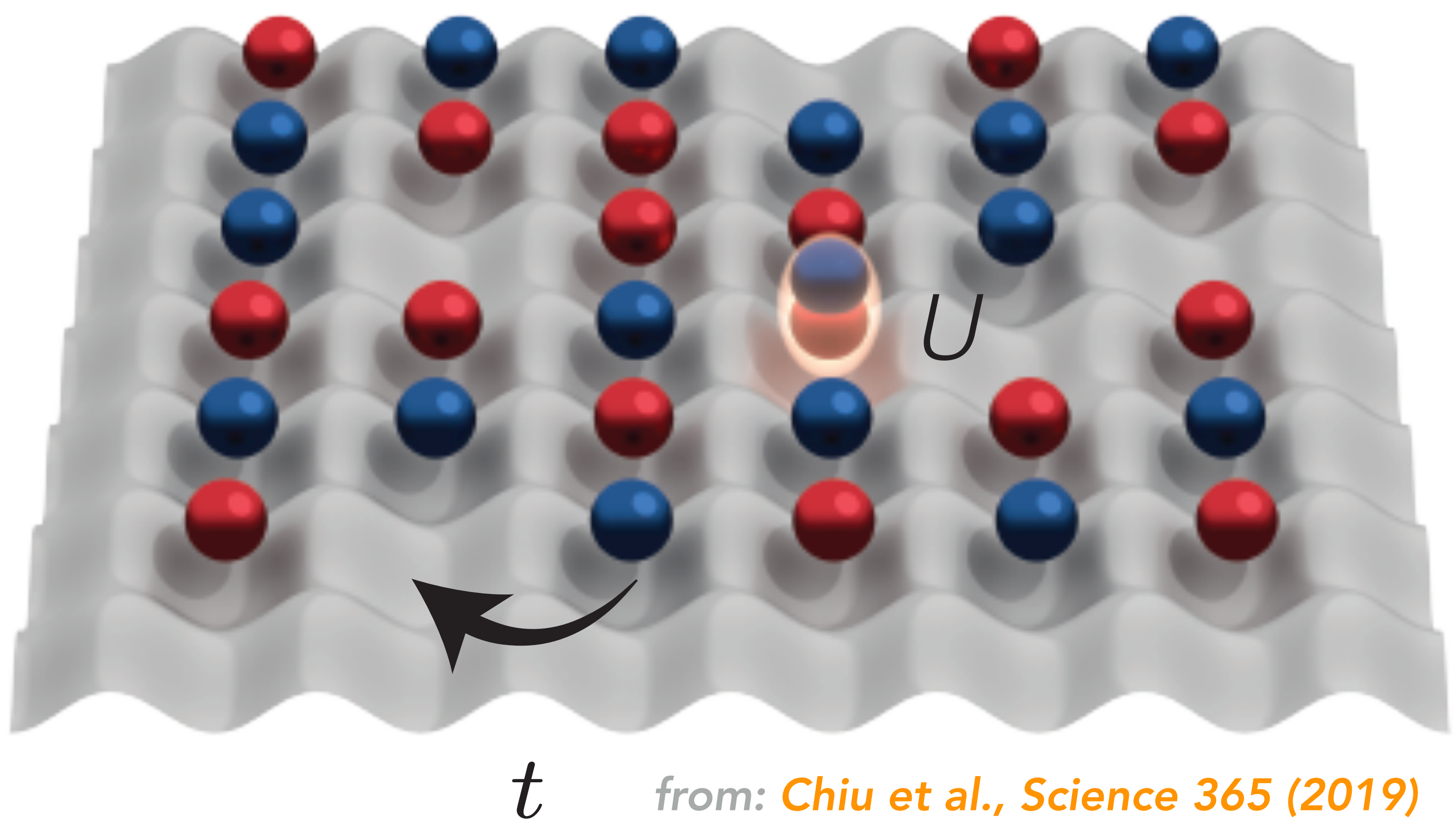
Fermi-Hubbard and *t-J* models

repulsive *U>0* — *beyond Kohn-Luttinger?*

Kohn & Luttinger, PRL 15 (1965)

$$\hat{\mathcal{H}}_{\rm FH} = -t \sum_{\langle \mathbf{i},\mathbf{j}\rangle,\sigma} \left(\hat{c}^\dagger_{\mathbf{i},\sigma} \hat{c}_{\mathbf{j},\sigma} + \text{h.c.} \right) + U \sum_{\mathbf{j}} \hat{n}_{\mathbf{j},\uparrow} \hat{n}_{\mathbf{j},\downarrow}$$

$$+ t' \sum_{\langle\langle \mathbf{i},\mathbf{j}\rangle\rangle} \dots$$



from: Chiu et al., Science 365 (2019)

* *Large-U limit: t-J model*

no double-occupancy!

$$\hat{\mathcal{H}}_{t-J} = -t\, \hat{\mathcal{P}} \Big[\sum_{\langle \mathbf{i},\mathbf{j}\rangle,\sigma} \hat{c}^\dagger_{\mathbf{i},\sigma} \hat{c}_{\mathbf{j},\sigma} + \text{h.c.} \Big] \hat{\mathcal{P}} + J \sum_{\langle \mathbf{i},\mathbf{j}\rangle} \left(\hat{\mathbf{S}}_{\mathbf{i}} \cdot \hat{\mathbf{S}}_{\mathbf{j}} - \frac{1}{4} \hat{n}_{\mathbf{i}} \hat{n}_{\mathbf{j}} \right)$$

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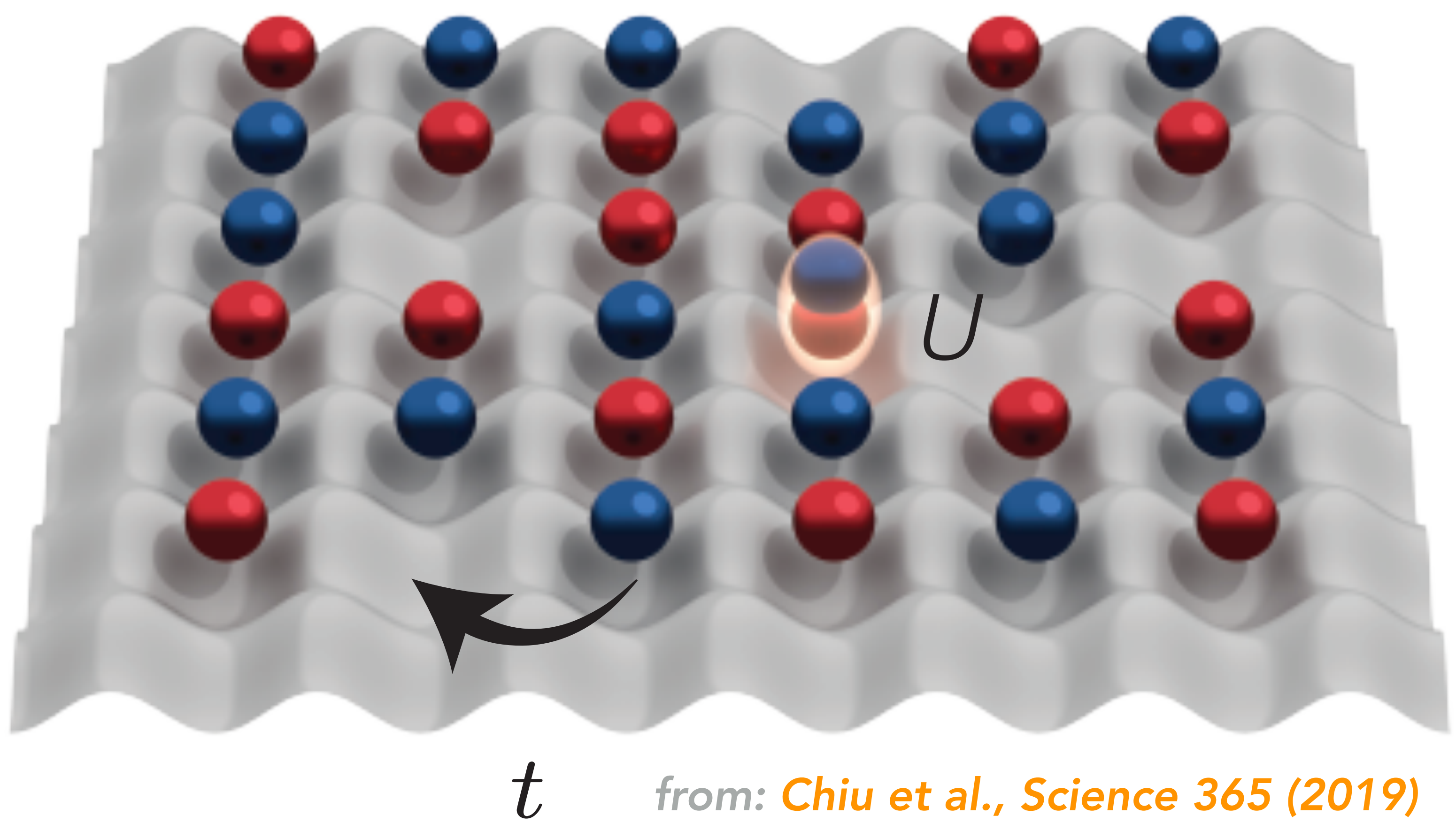

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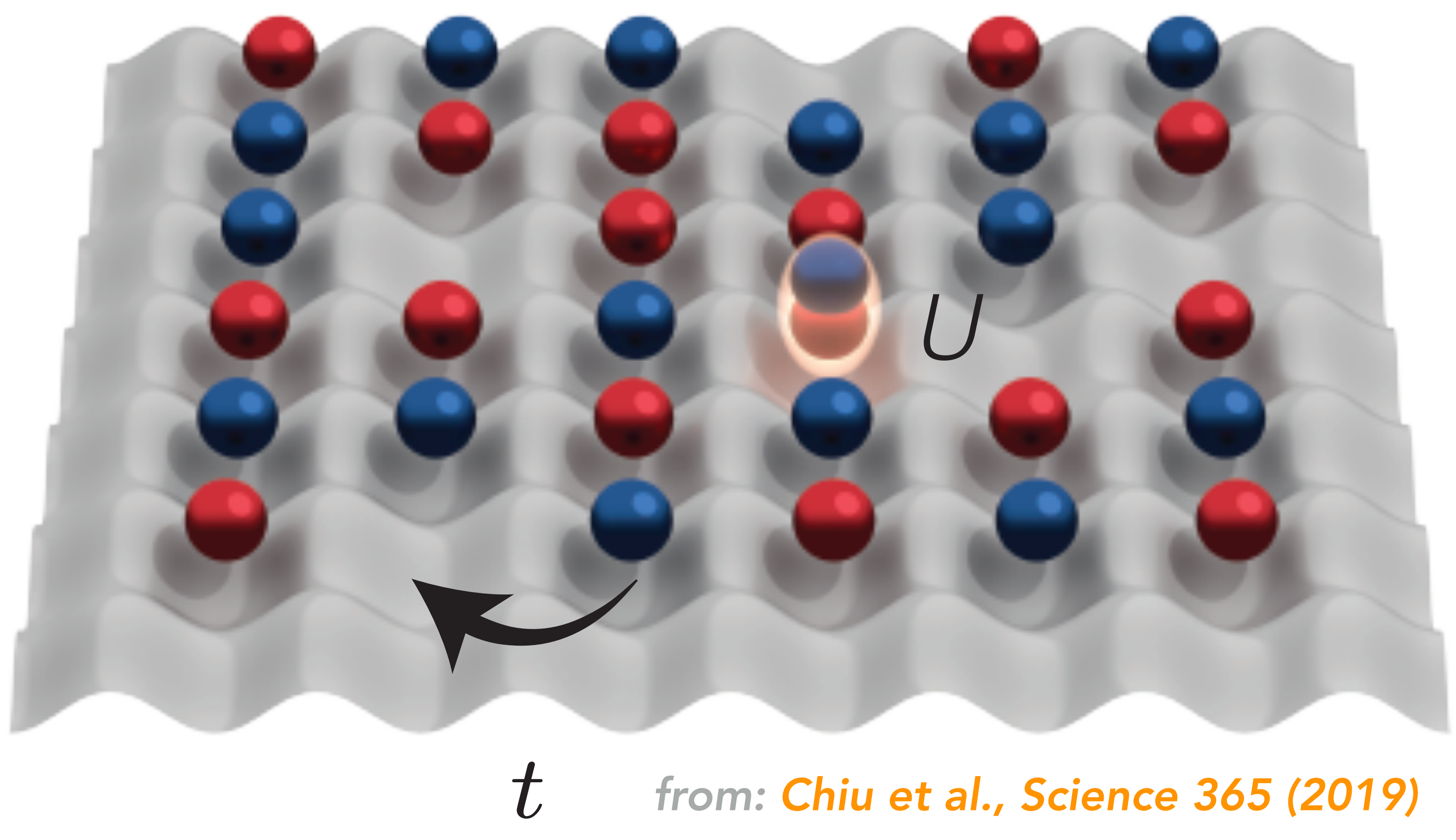

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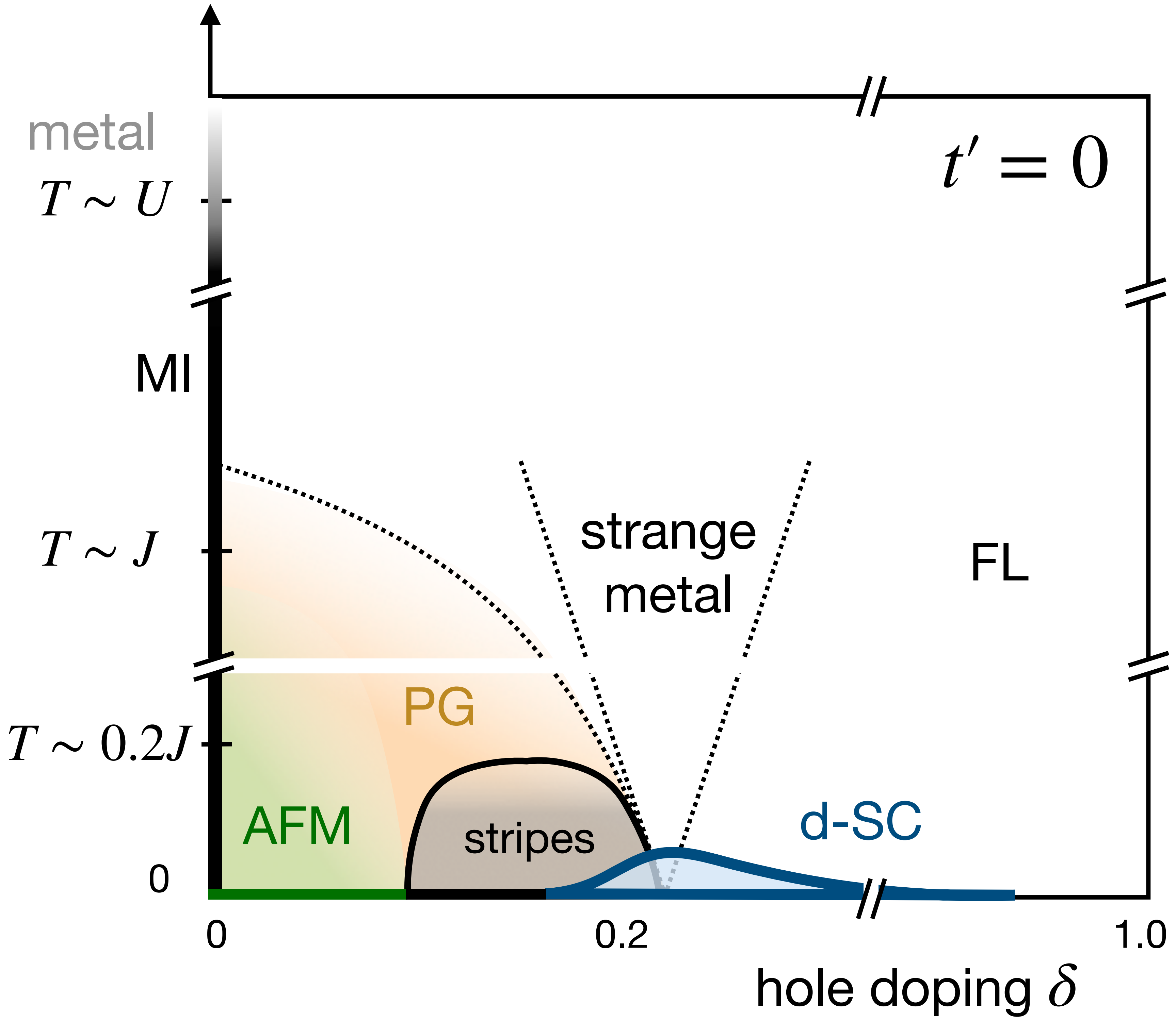
~~+ NNN hopping term O(J)~~

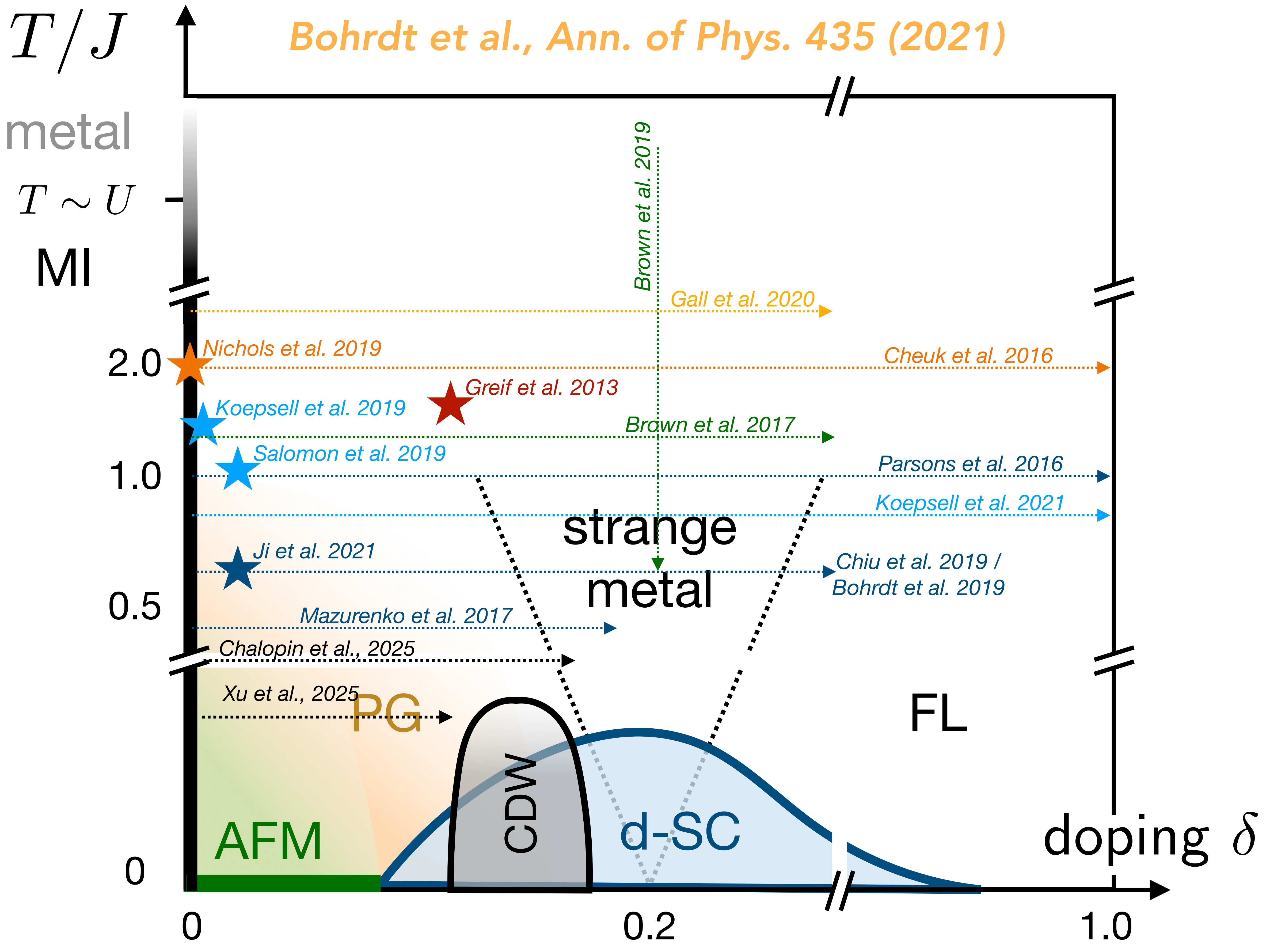
~~$+t' \sum_{\langle\langle \mathbf{i},\mathbf{j}\rangle\rangle} \ldots$~~

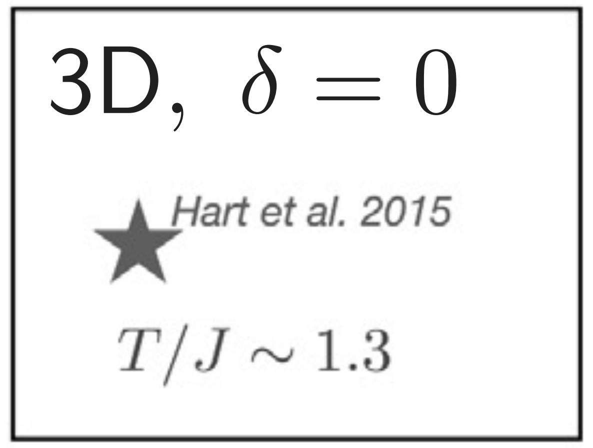
neglected in *t-J*

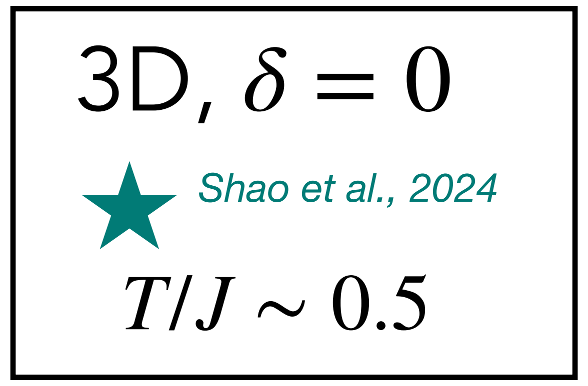
Part 1.II: Overview of the phase diagram



Conjectured phase diagram of cuprates/ Hubbard model

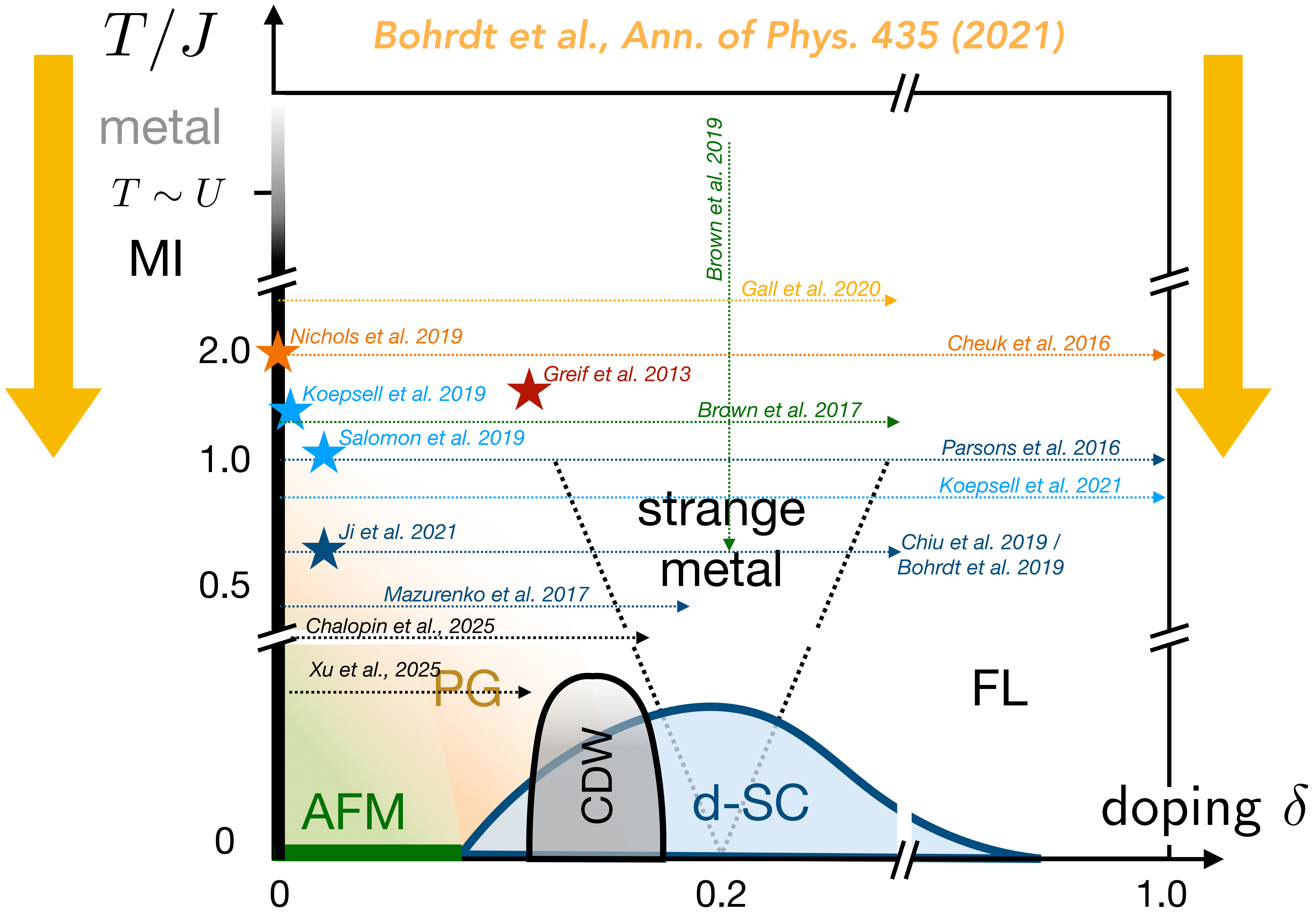




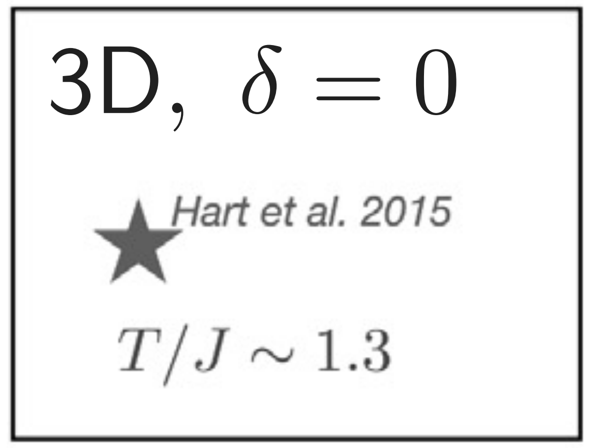


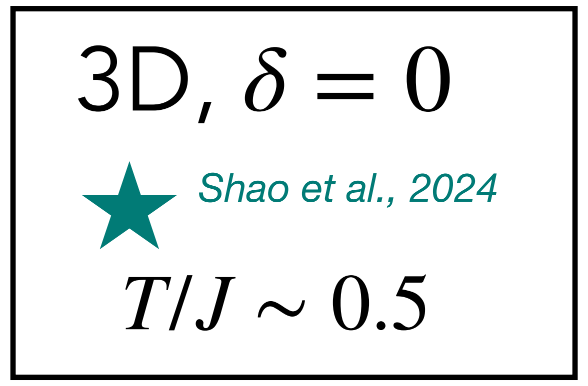
Solution strategies

Conjectured phase diagram of cuprates/ Hubbard model



Quantum Monte-Carlo simulation: hot due to sign problem





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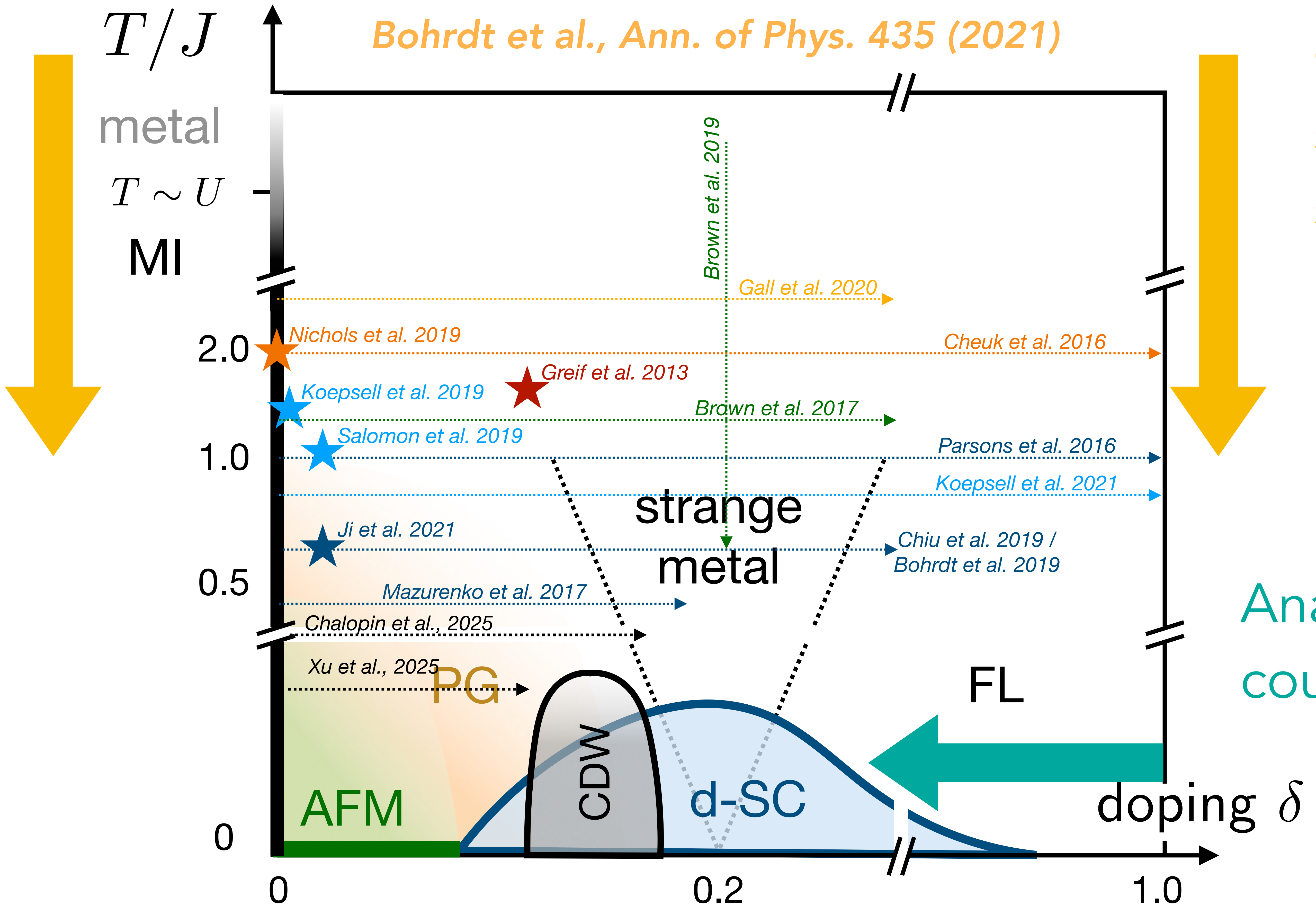

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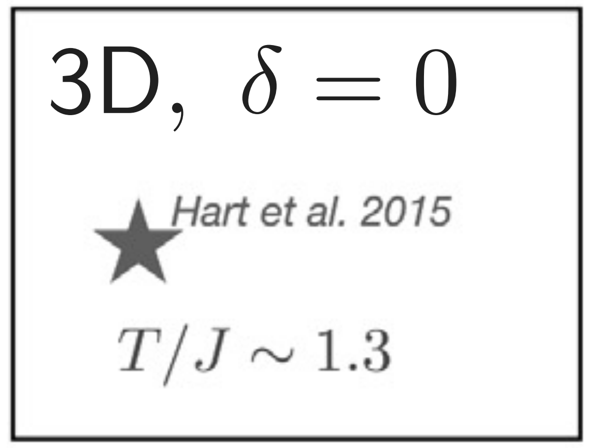

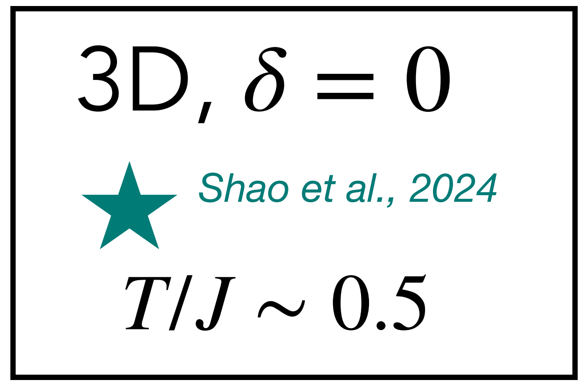

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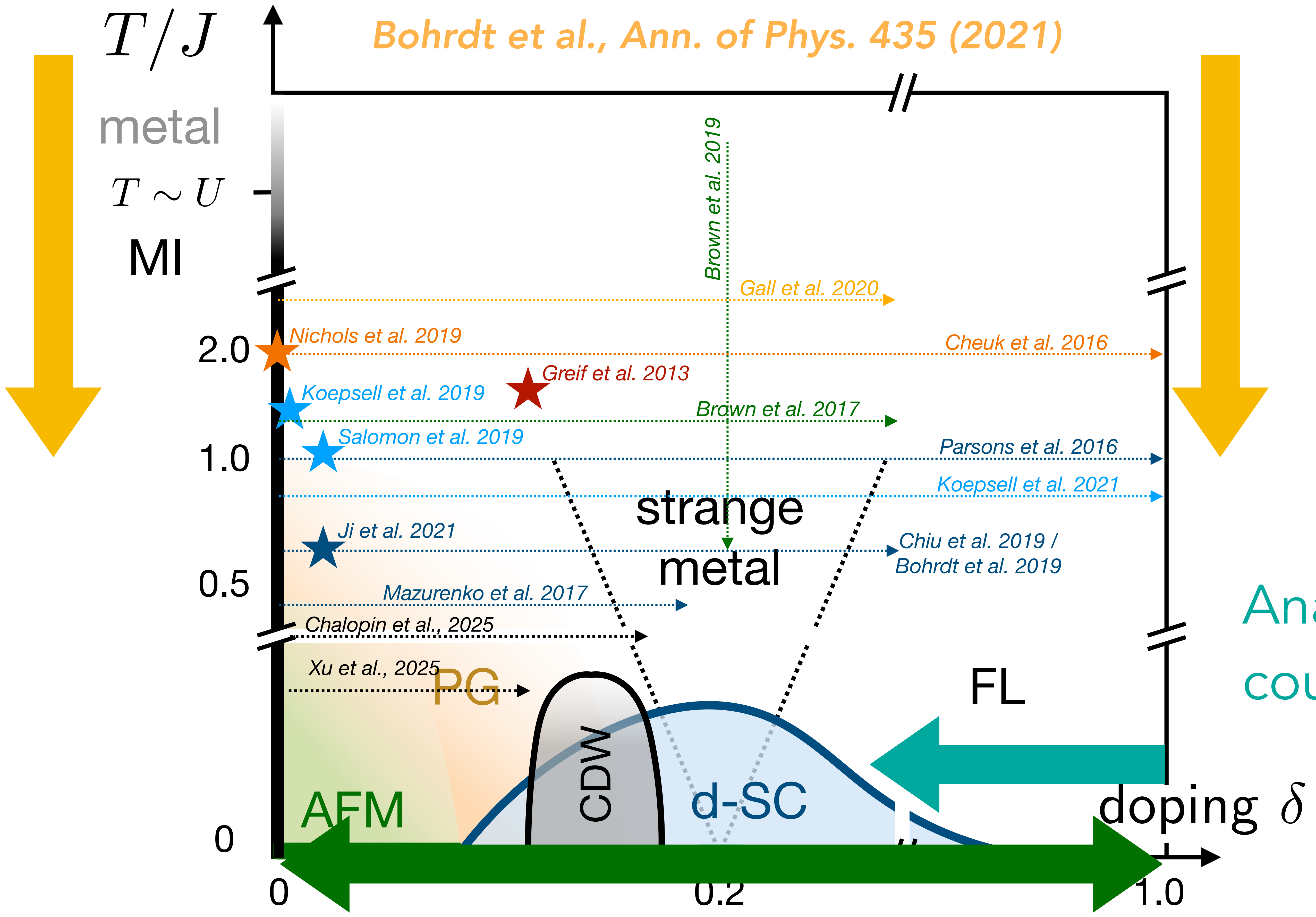

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3D, $\delta = 0$

Hart et al. 2015

$T/J \sim 1.3$

3D, $\delta = 0$

Shao et al., 2024

$T/J \sim 0.5$

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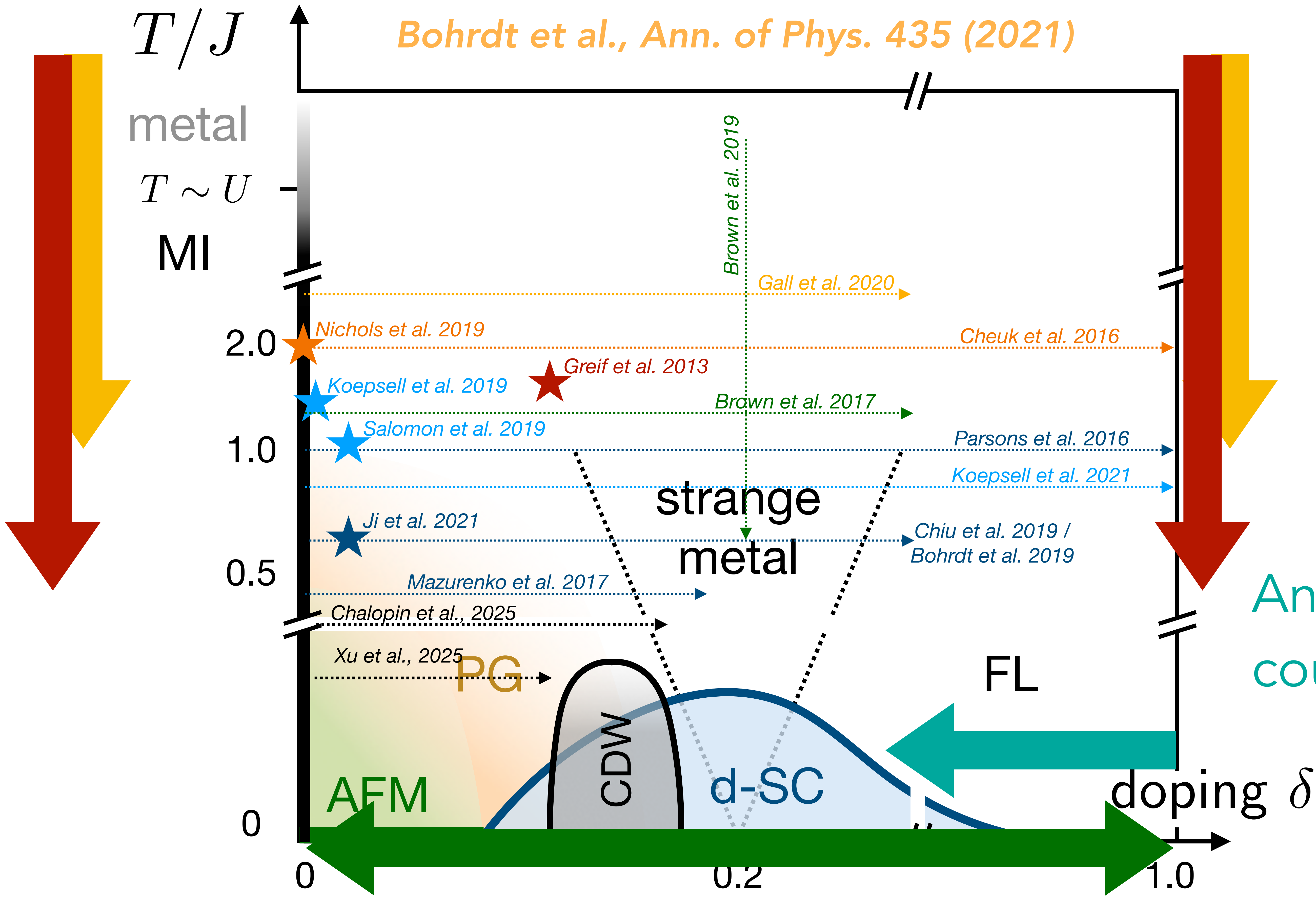
DMRG / tensor networks: *T=0*

Solution strategies

Conjectured phase diagram of cuprates/ Hubbard model

Cold atoms: relatively *hot* in units of *J*!



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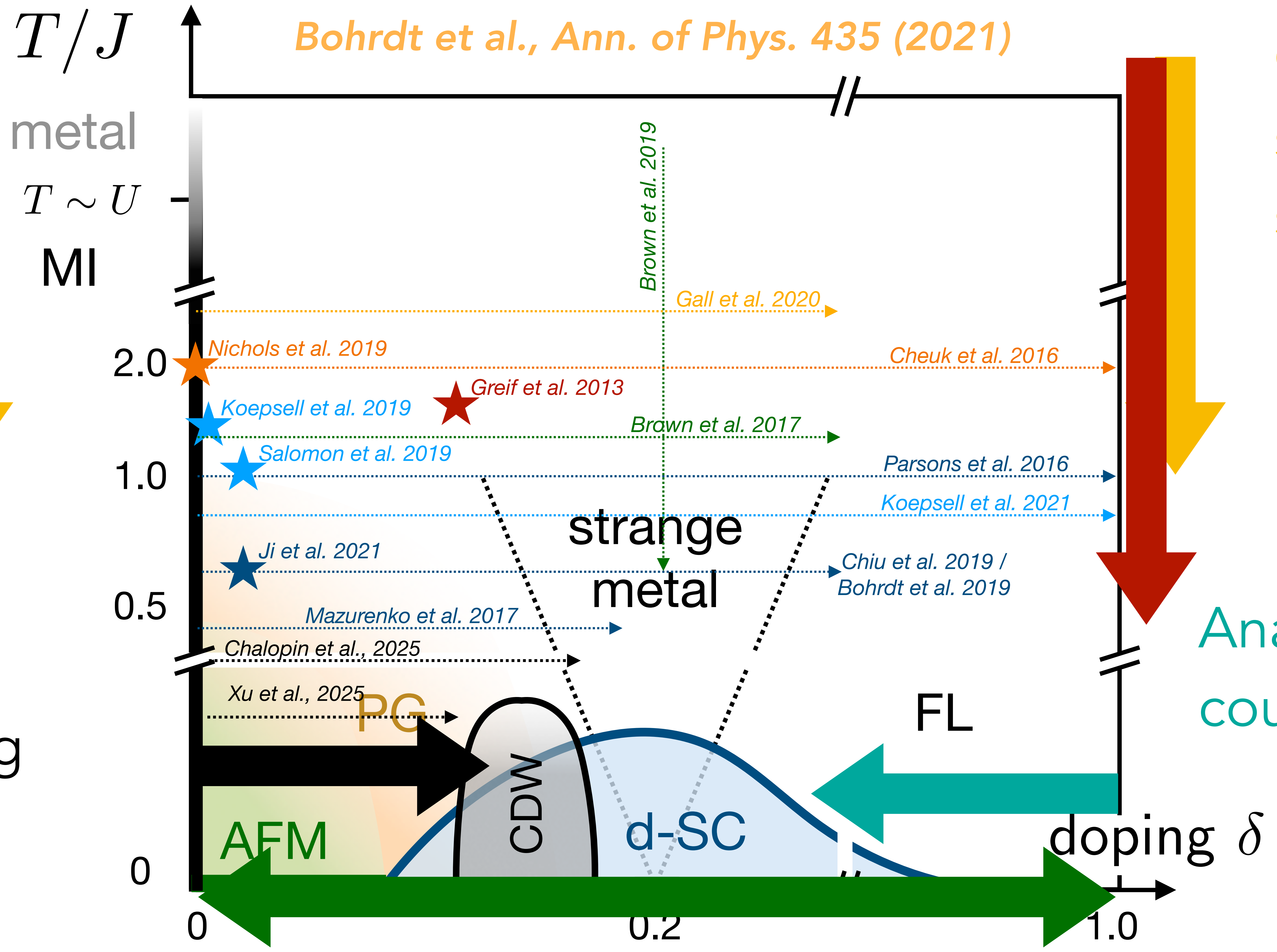

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Strong coupling approach

DMRG / tensor networks: *T=0*

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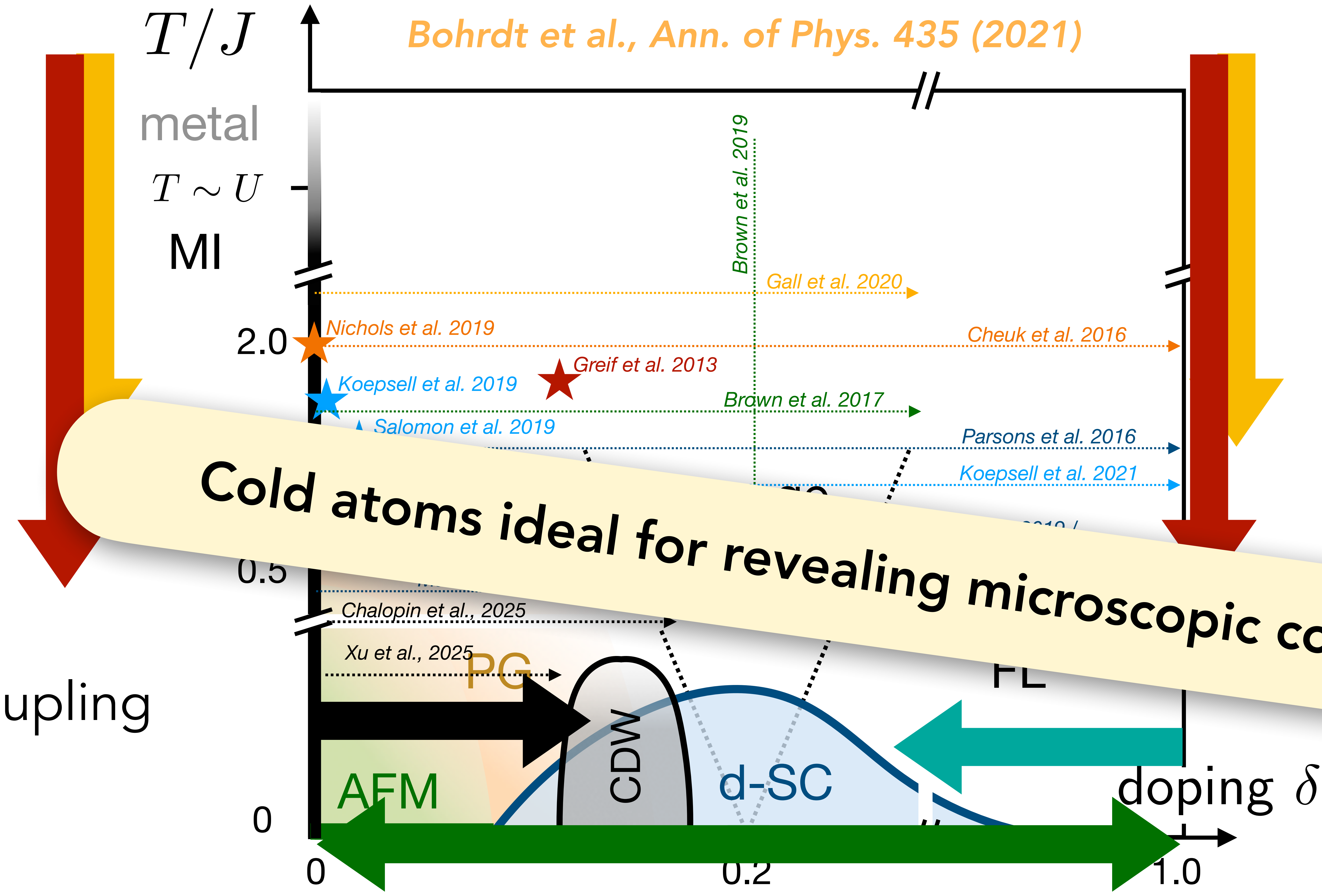

Cold atoms ideal for revealing microscopic constituents!

Strong coupling approach

Mainly weak …ing

DMRG / tensor networks: *T=0*

3D, $\delta = 0$

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$T/J \sim 1.3$

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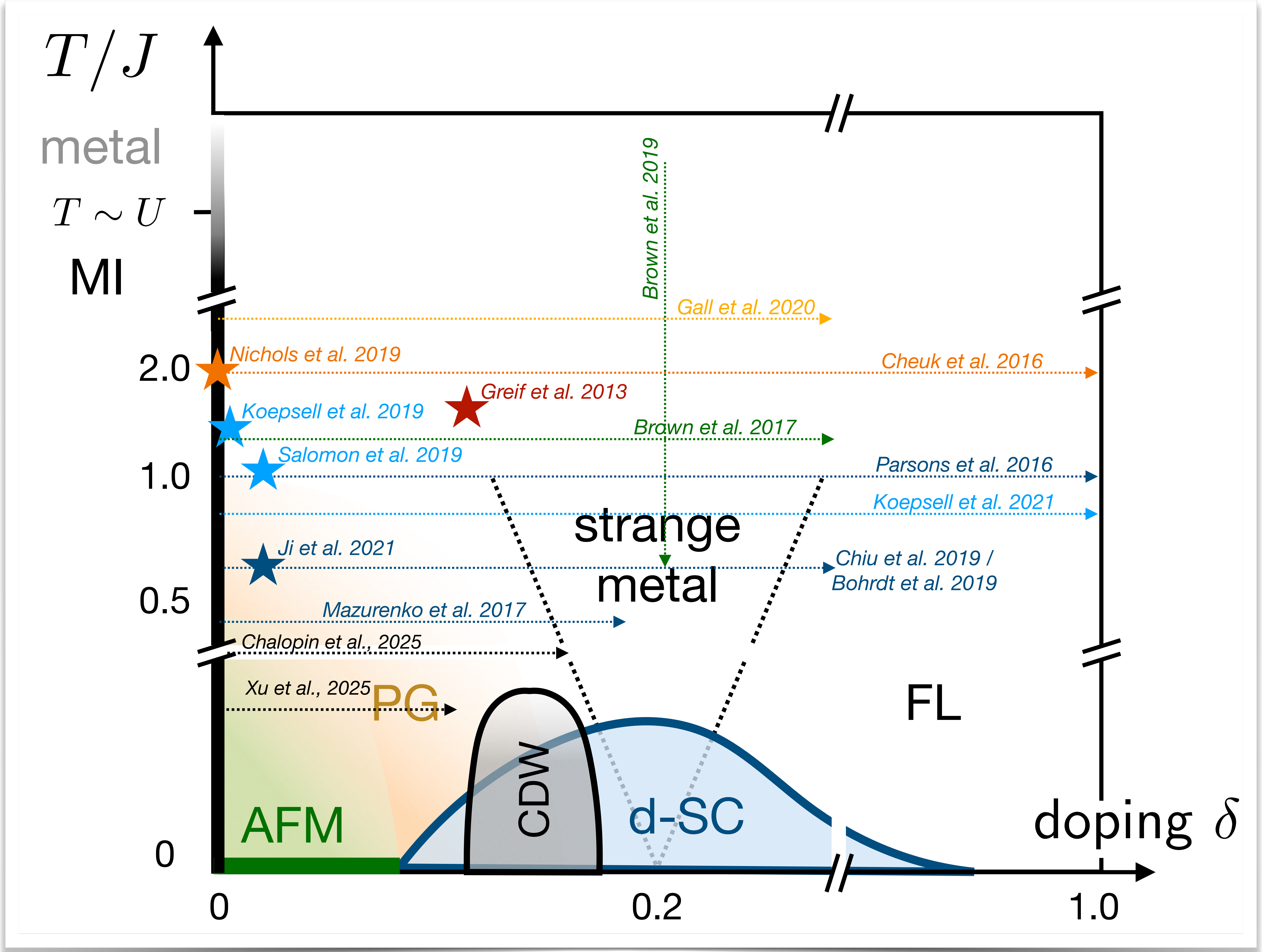
Hubbard phase diagram

Undoped 'parent' state: Antiferromagnetism



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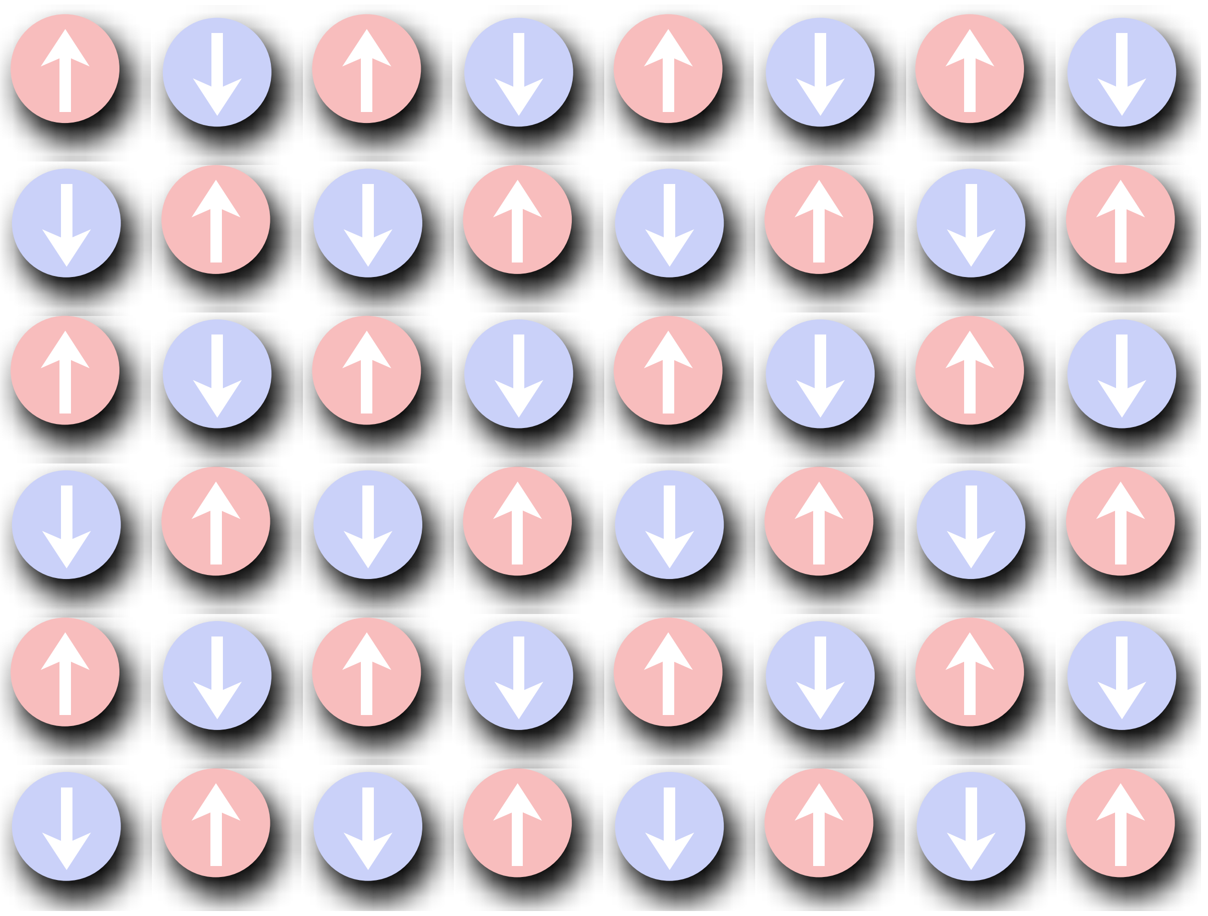
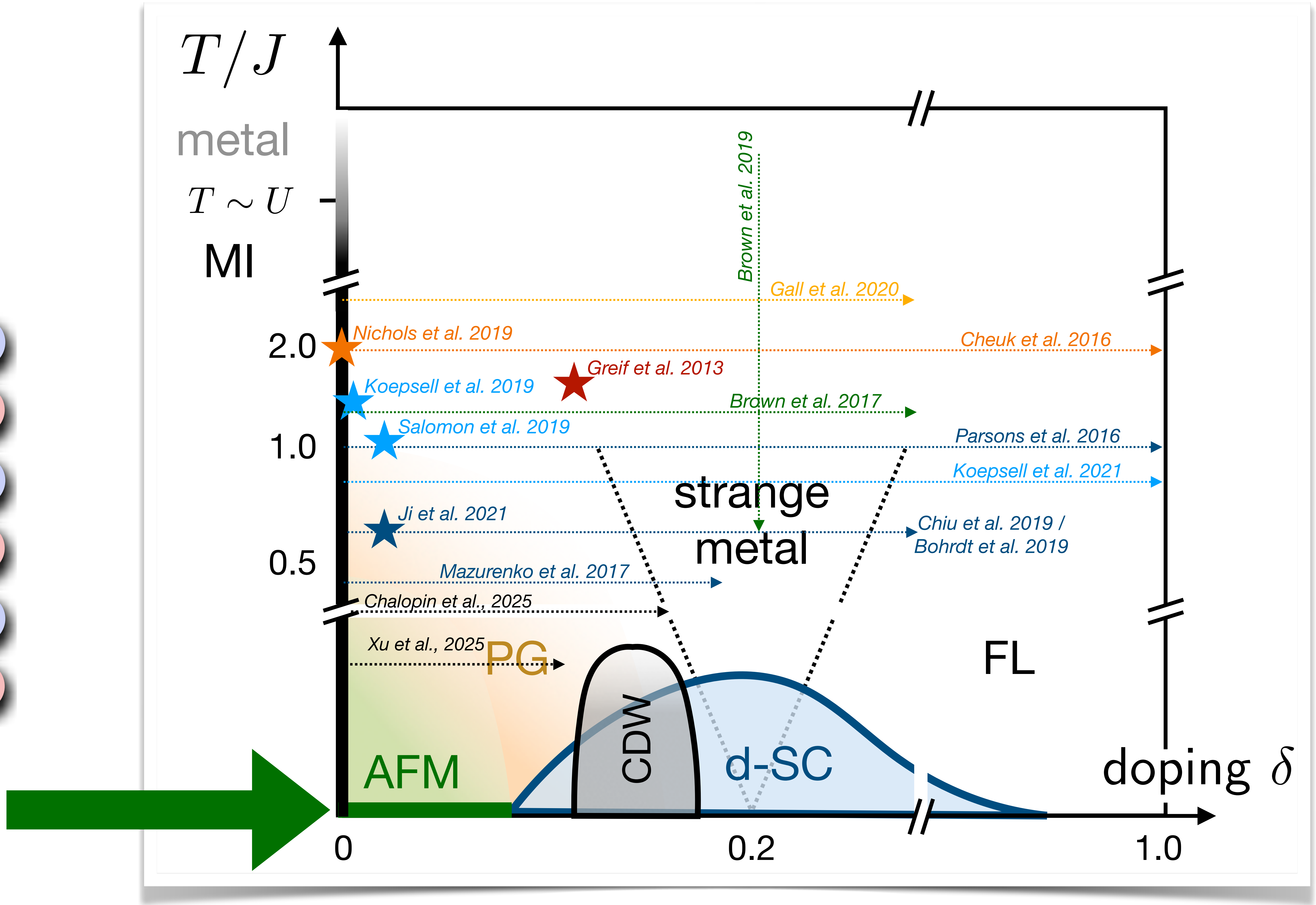


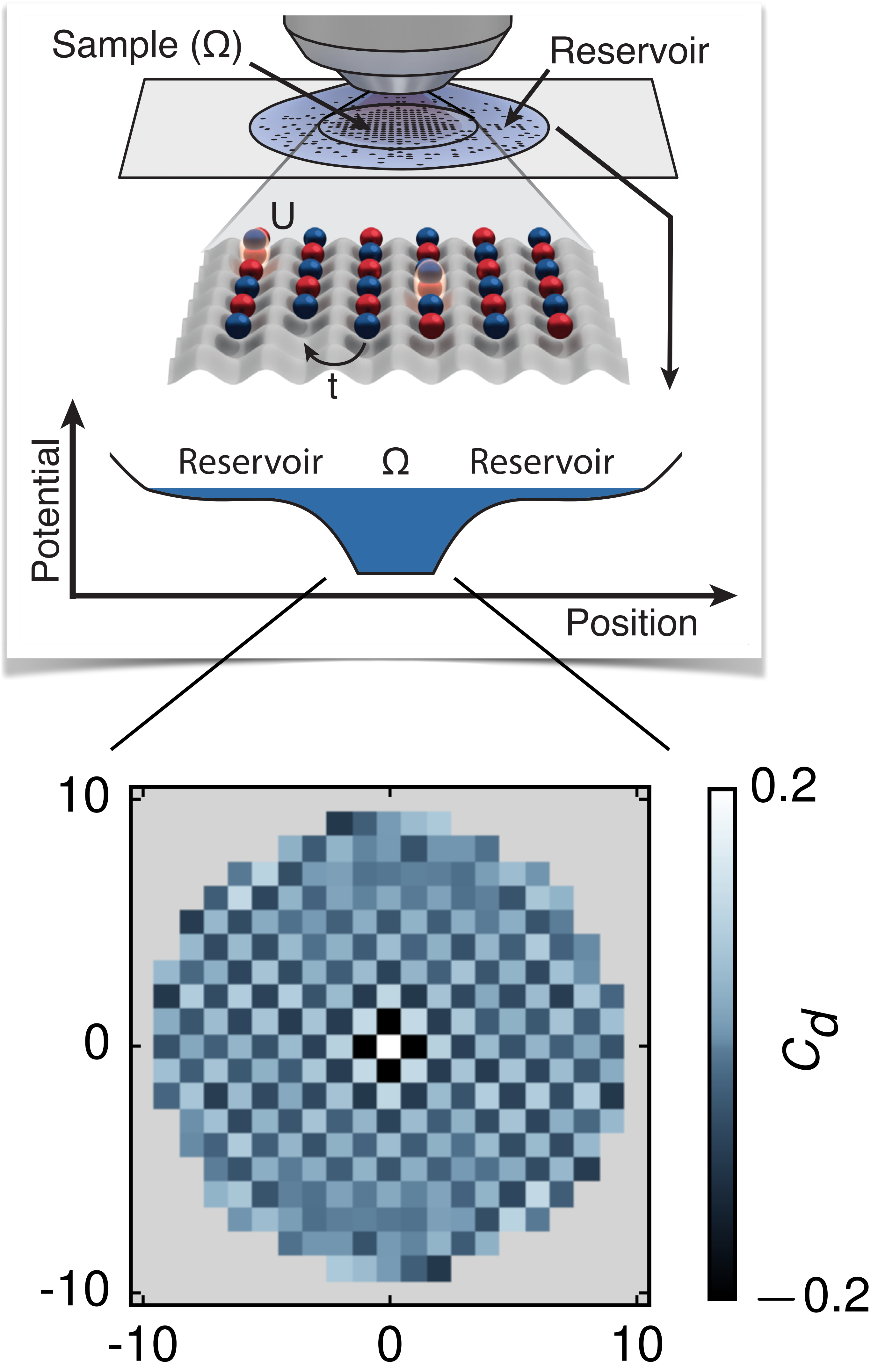


Hubbard phase diagram

Long-range anti-ferromagnetism in 2D:

Experiment Greiner group, Harvard

Mazurenko et al., Nature 545 (2017)



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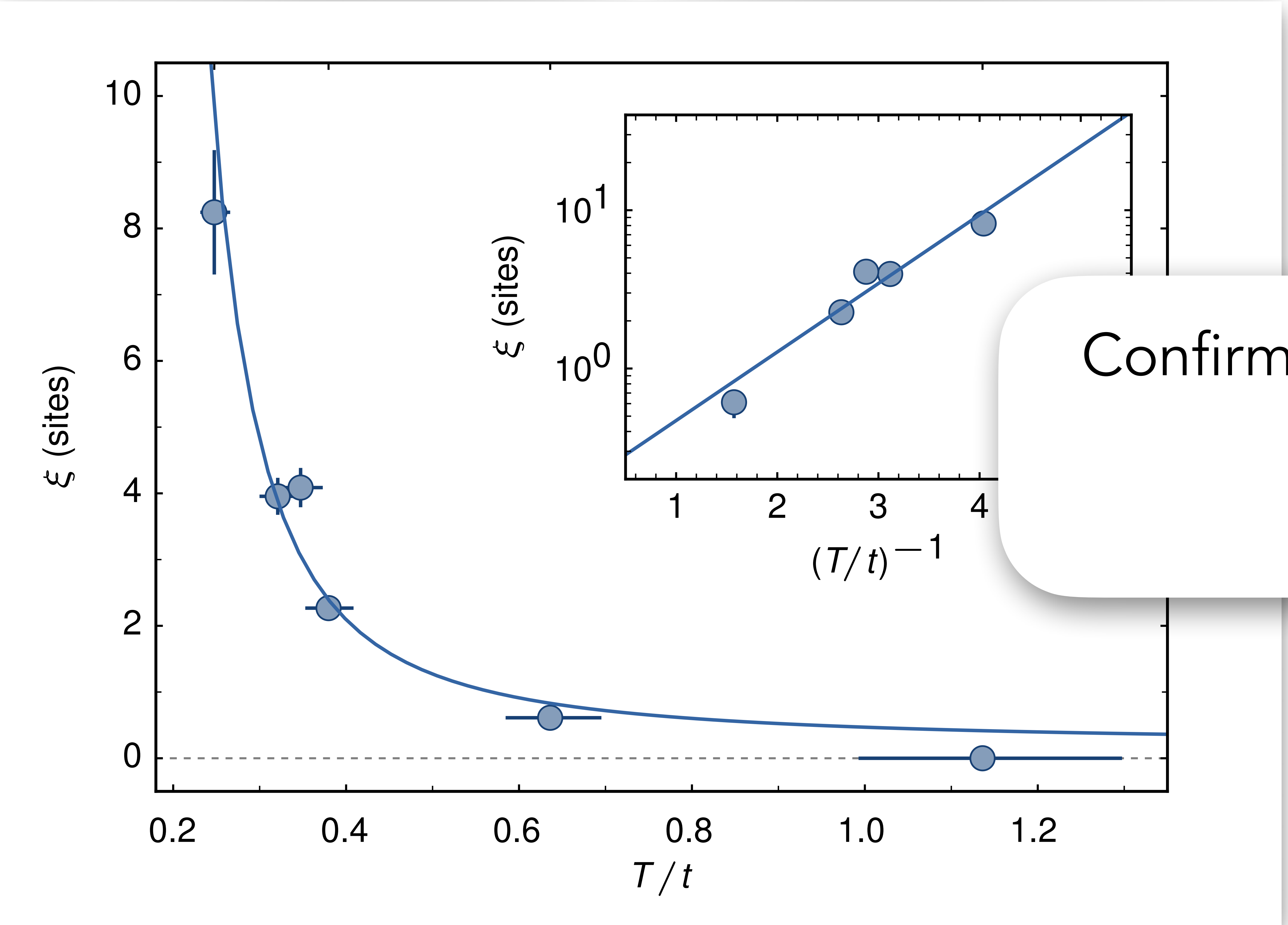

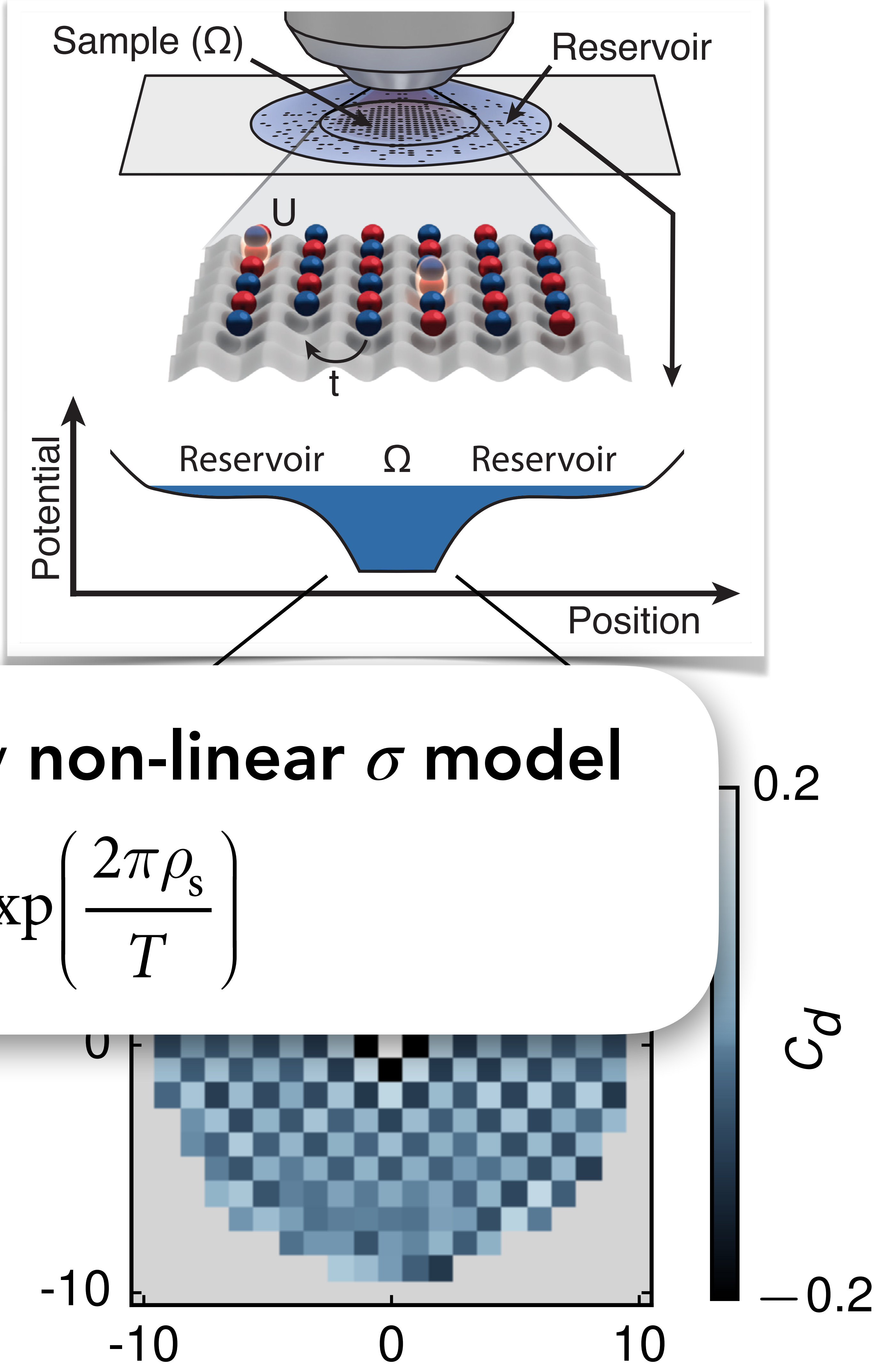

Confirms prediction by **non-linear σ model**

$$\xi(T) = C_\xi \exp\left(\frac{2\pi\rho_s}{T}\right)$$

Hubbard phase diagram

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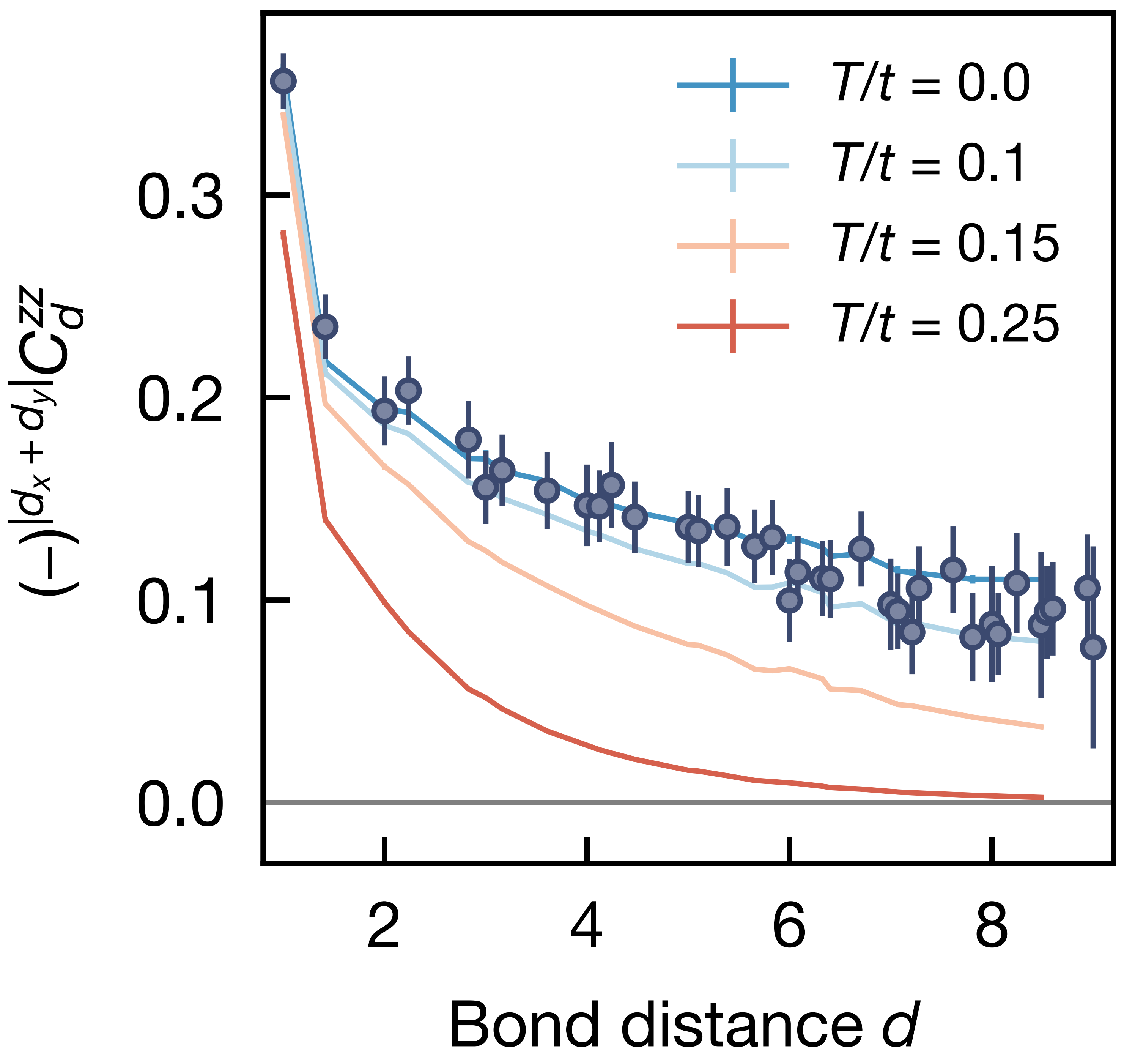
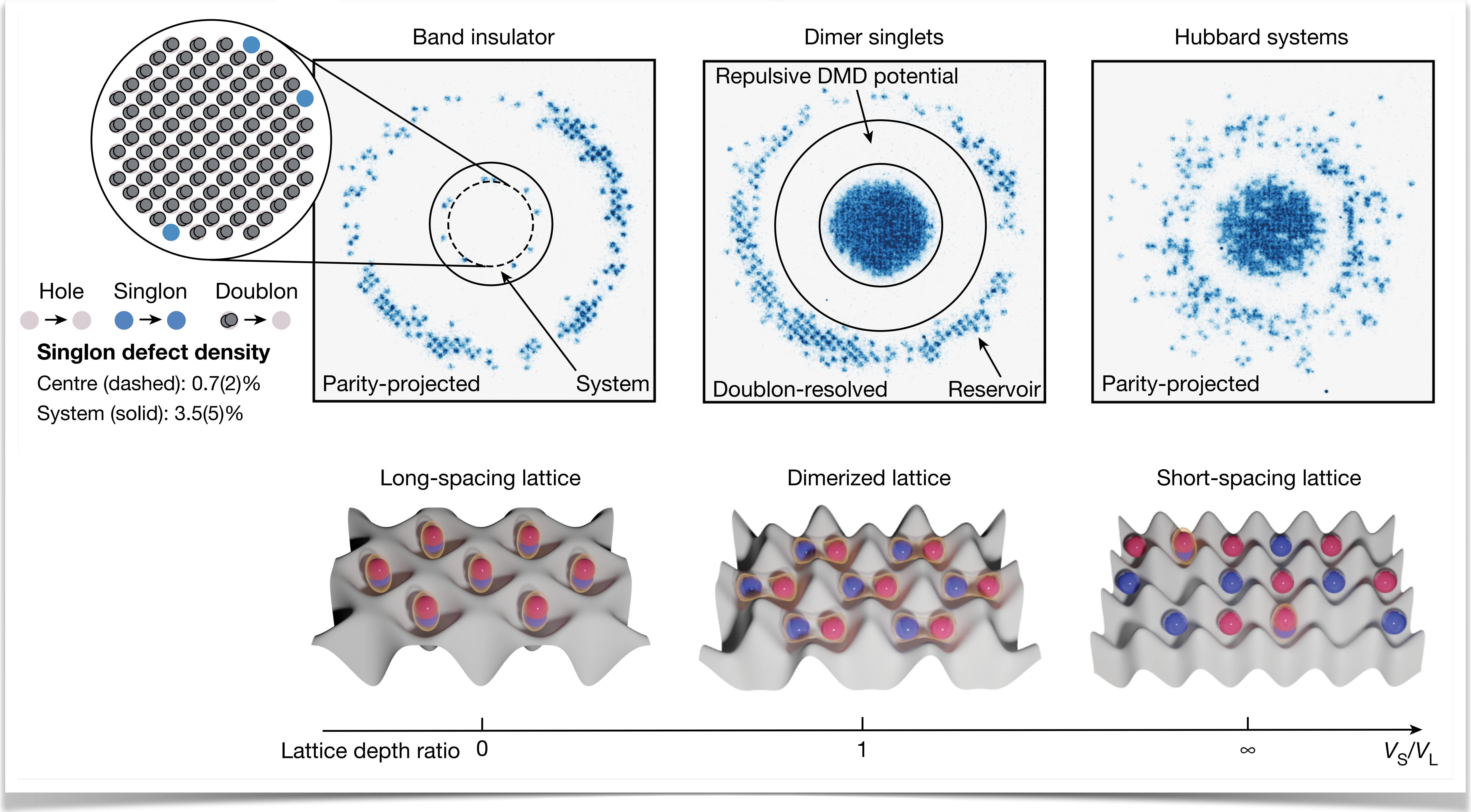
Xu et al., Nature 642 (2025)





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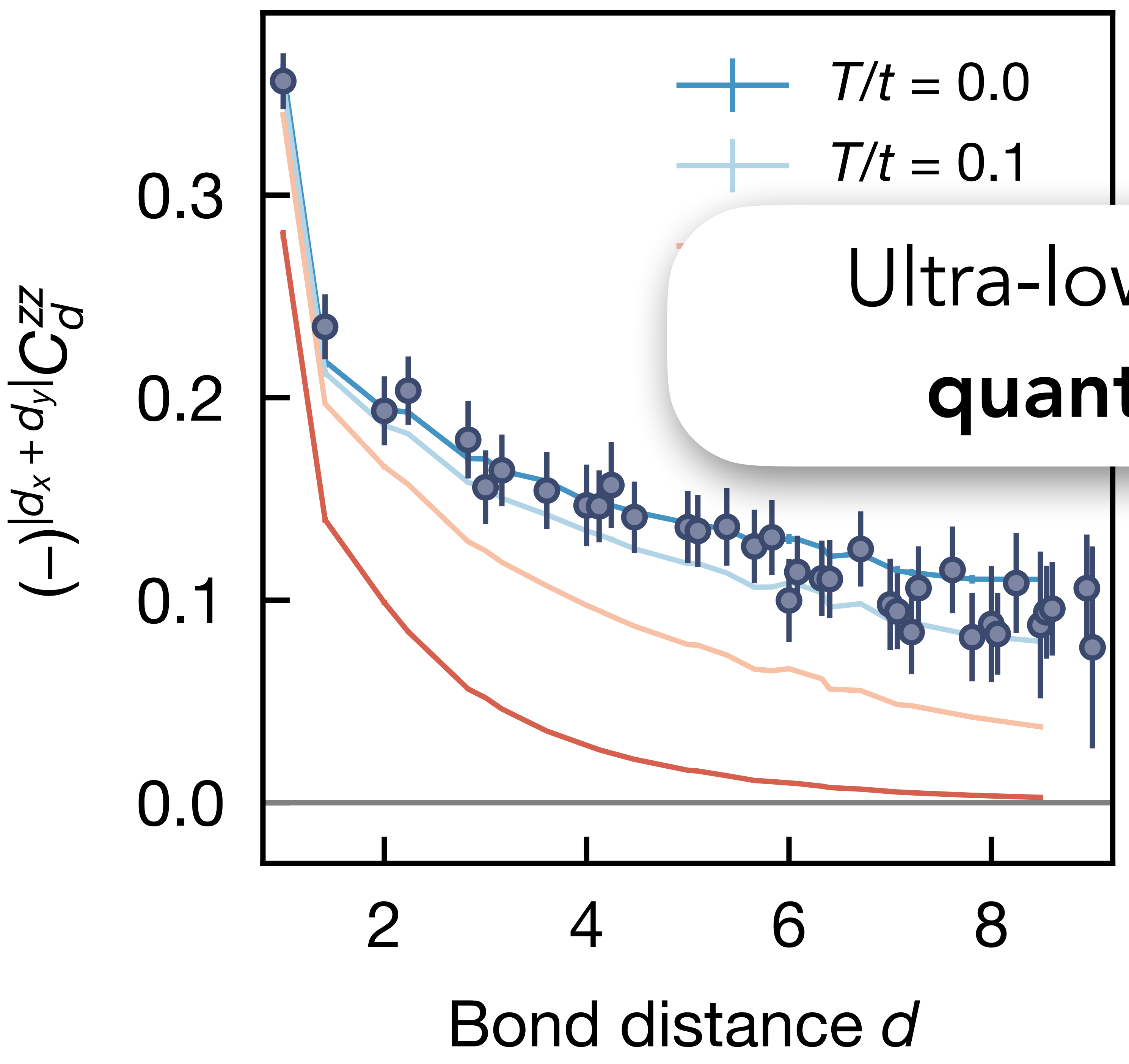
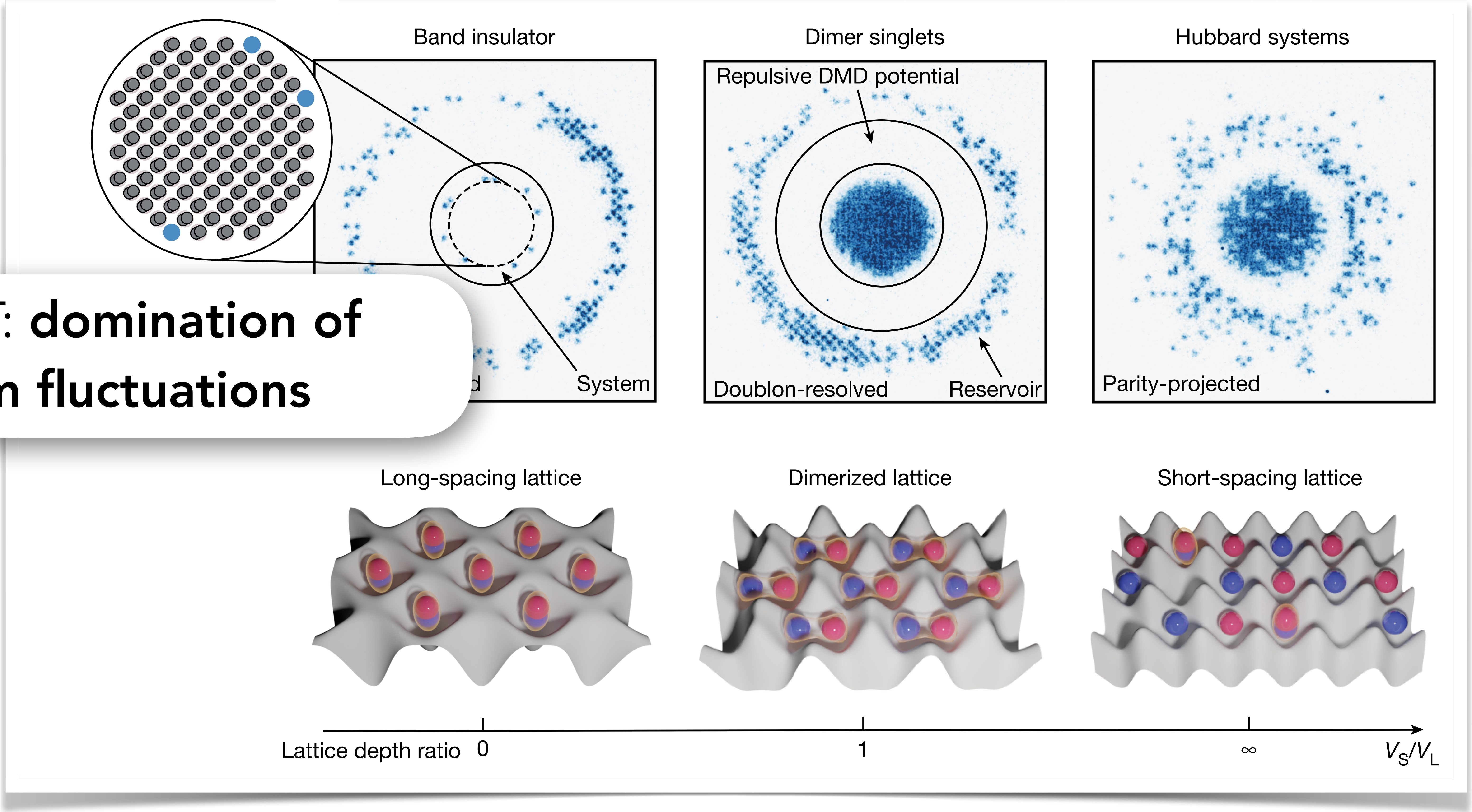


Ultra-low T: **domination of quantum fluctuations**

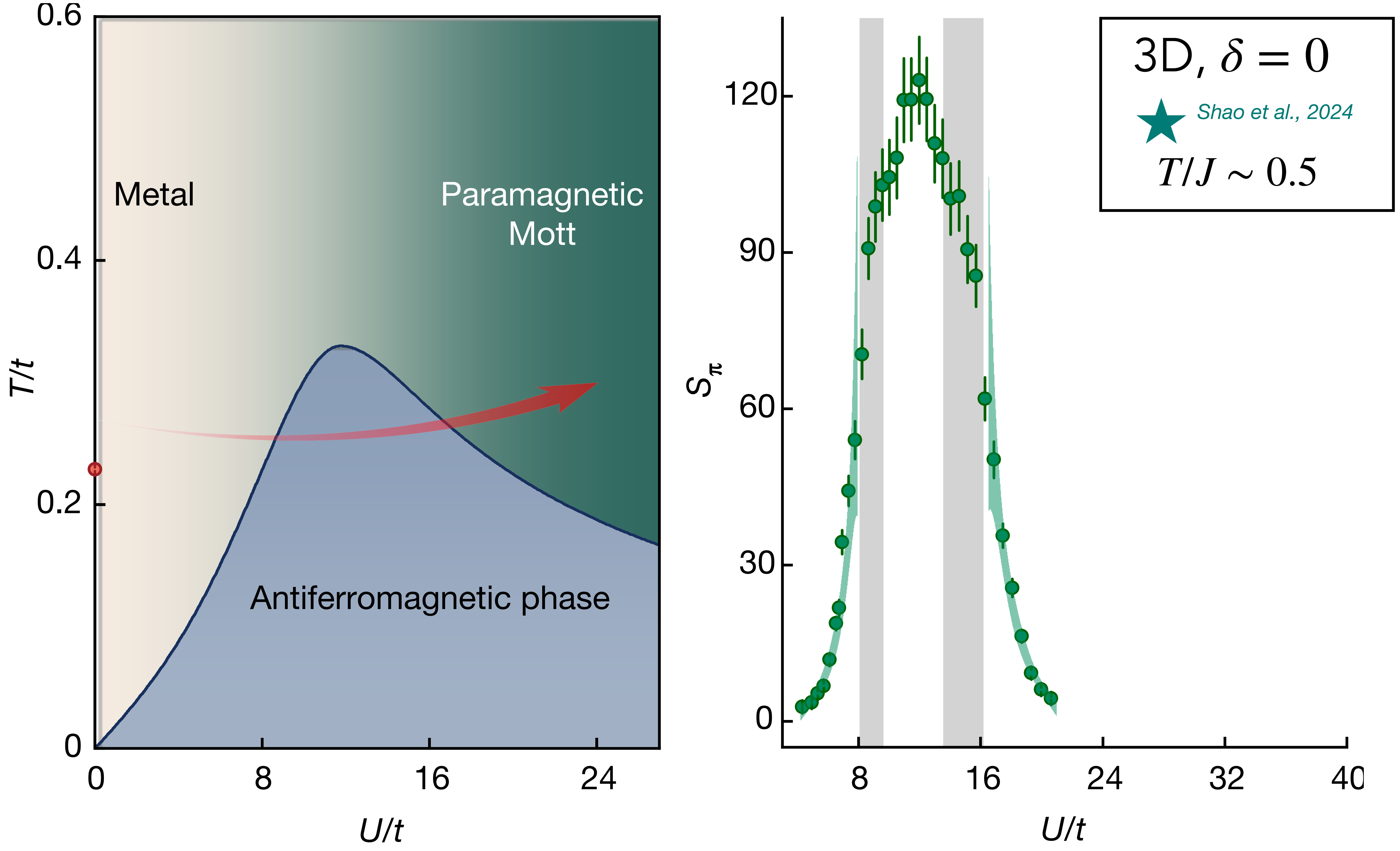


Hubbard phase diagram

Long-range anti-ferromagnetism in 3D:

Experiment Pan group, China

Shao et al., Nature 632 (2024)



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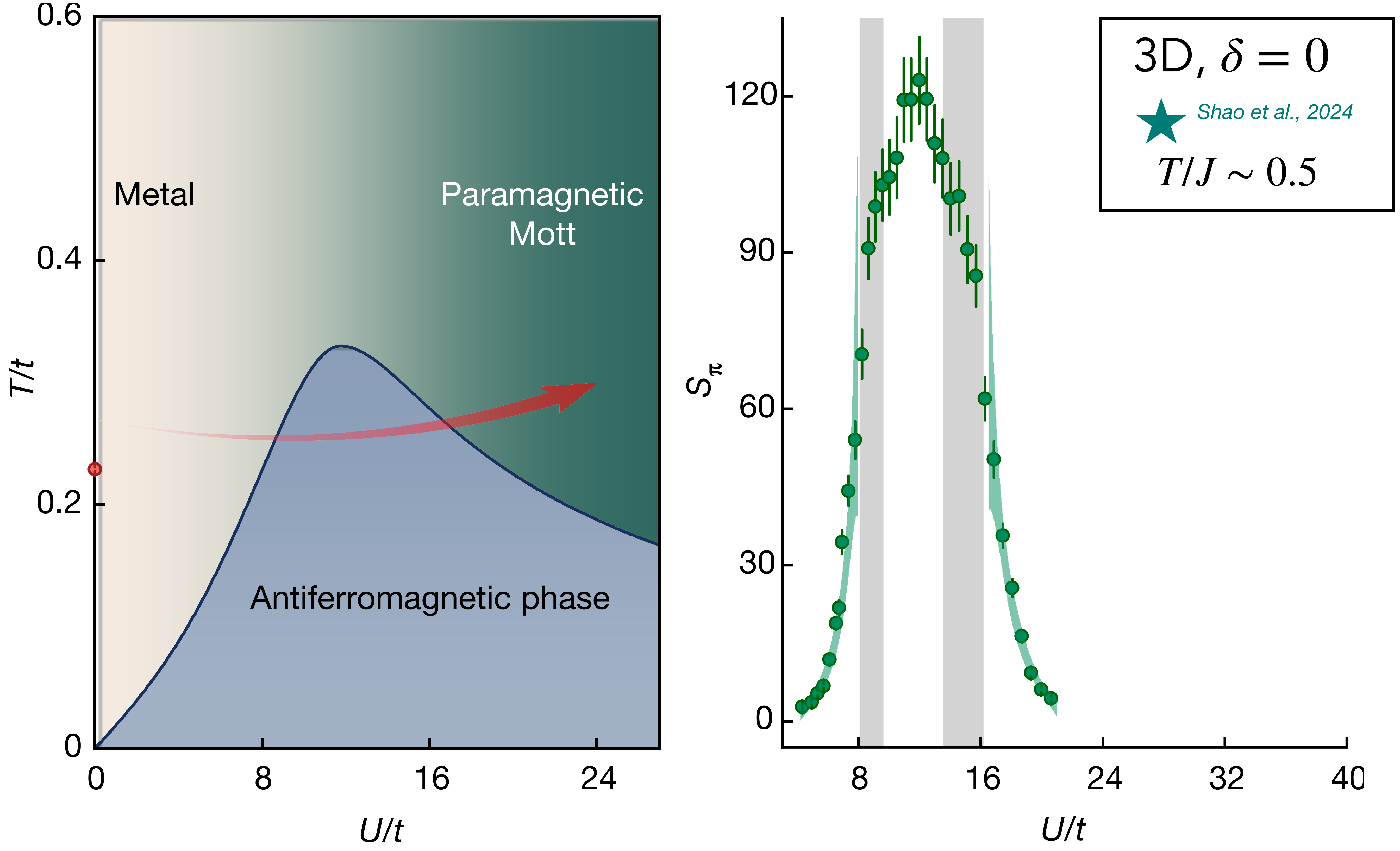
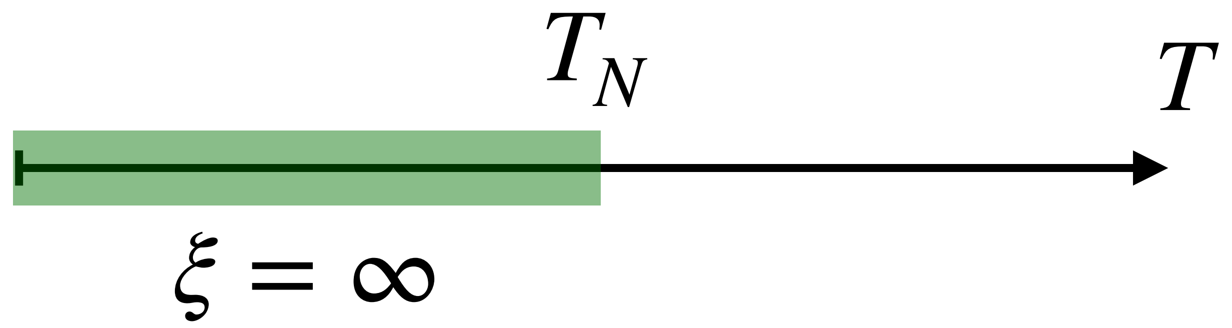
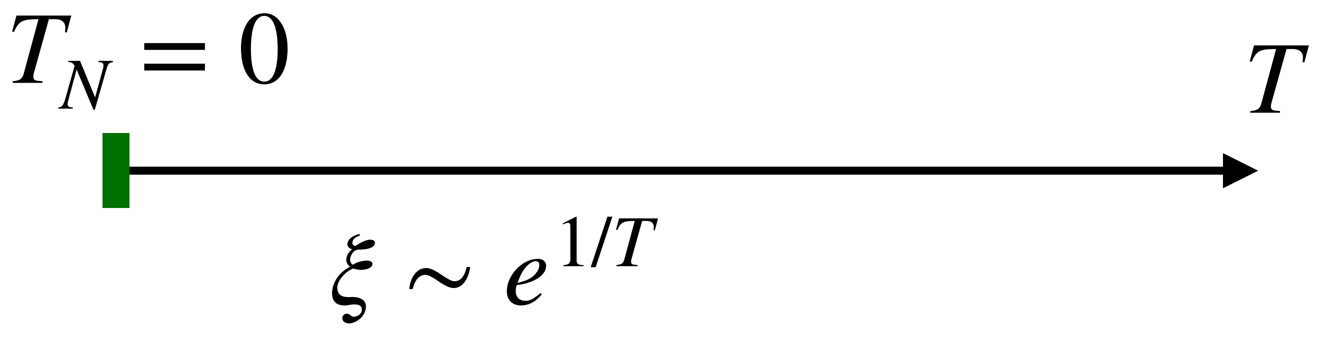


- 3D: true long-range order at T>0:



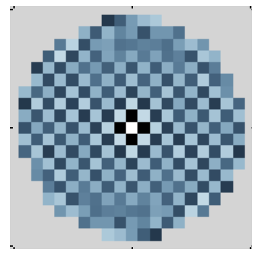
- 2D: no long-range order at T>0 *(Mermin-Wagner):*



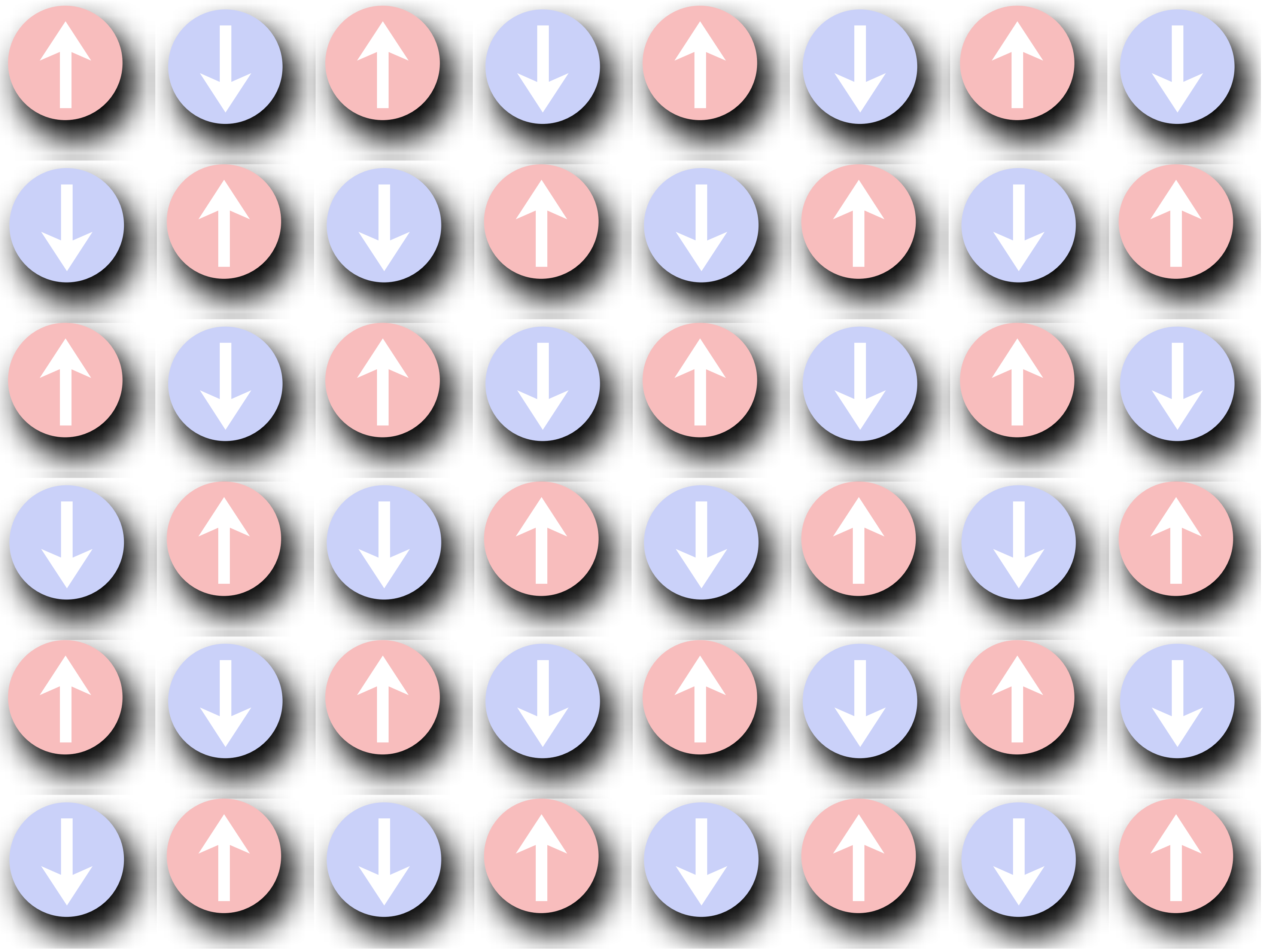
Hubbard phase diagram

Beyond the ‘parent’ state

- Interesting ‘vacuum’ state:
  AFM insulator

$|$  $\rangle = |0\rangle$

- Doping δ: add free charge carriers



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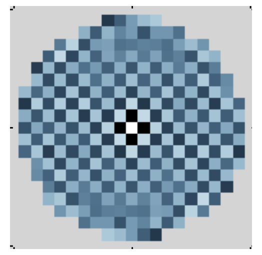
$|$  $\rangle = |0\rangle$

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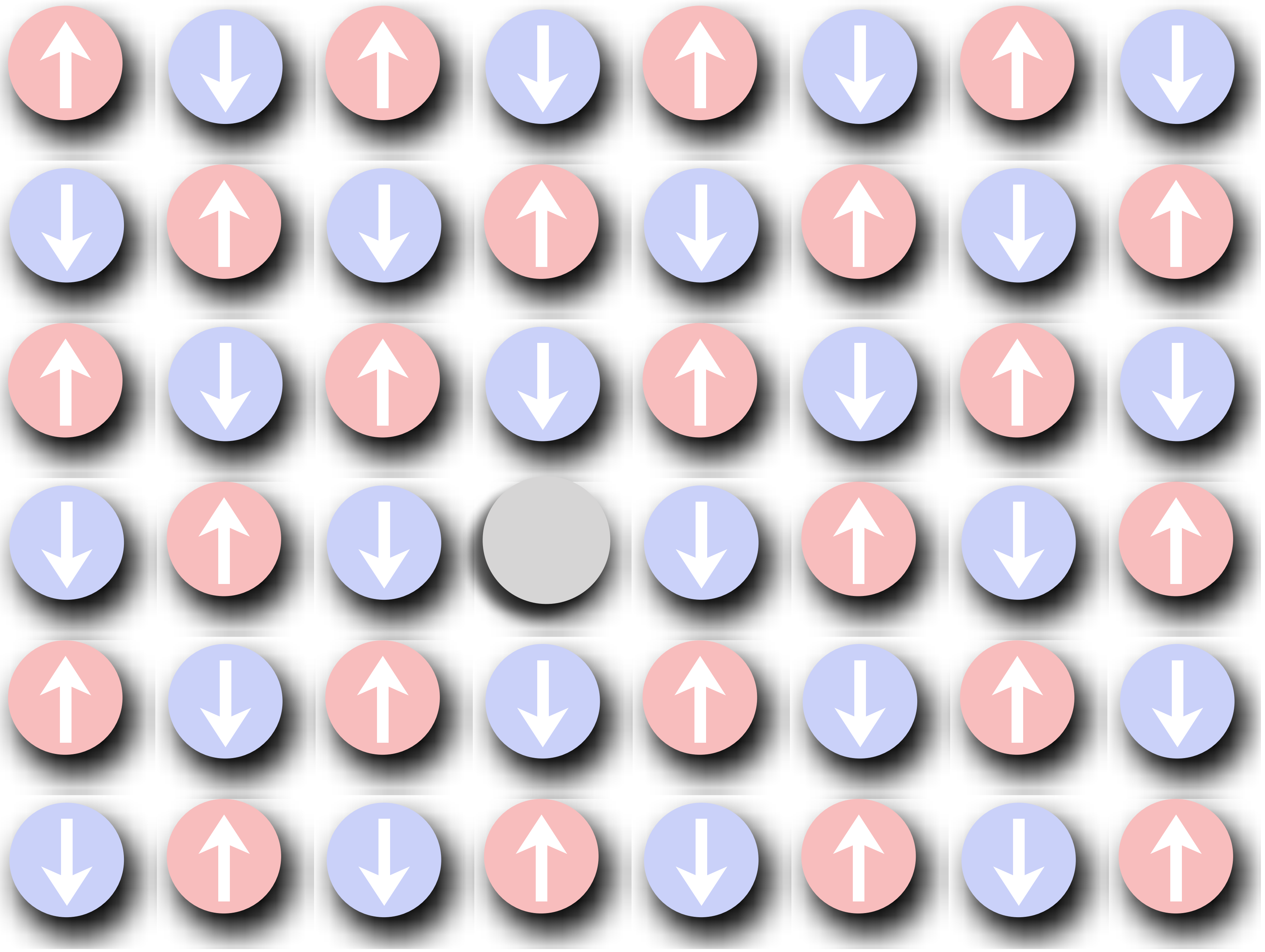

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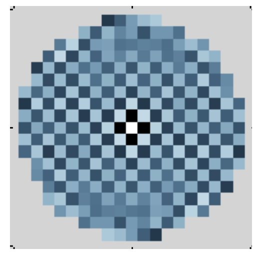
$|$  $\rangle = |0\rangle$

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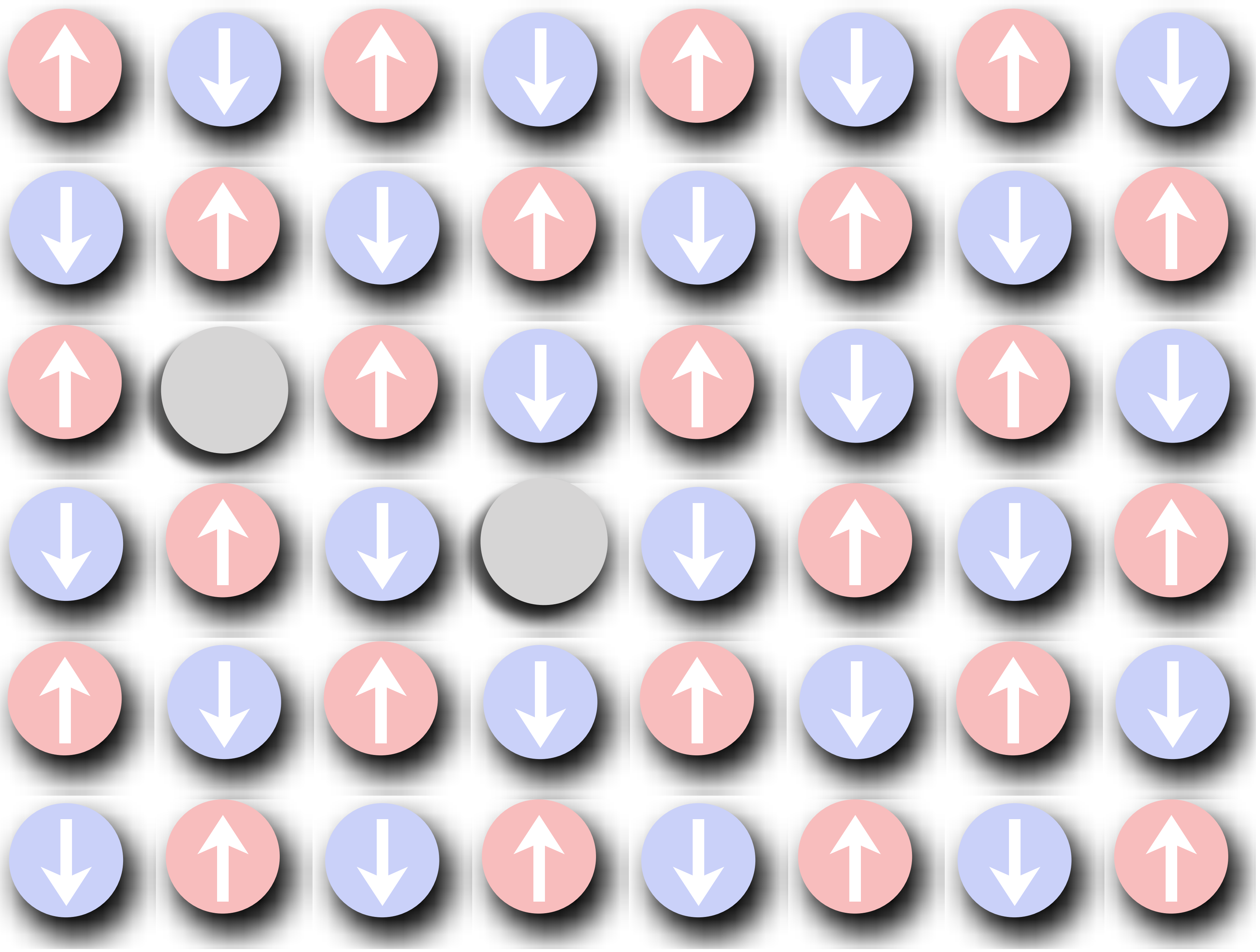

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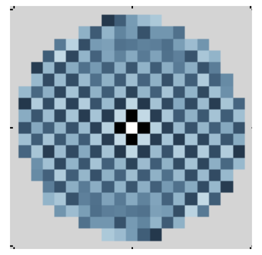
$|$  $\rangle = |0\rangle$

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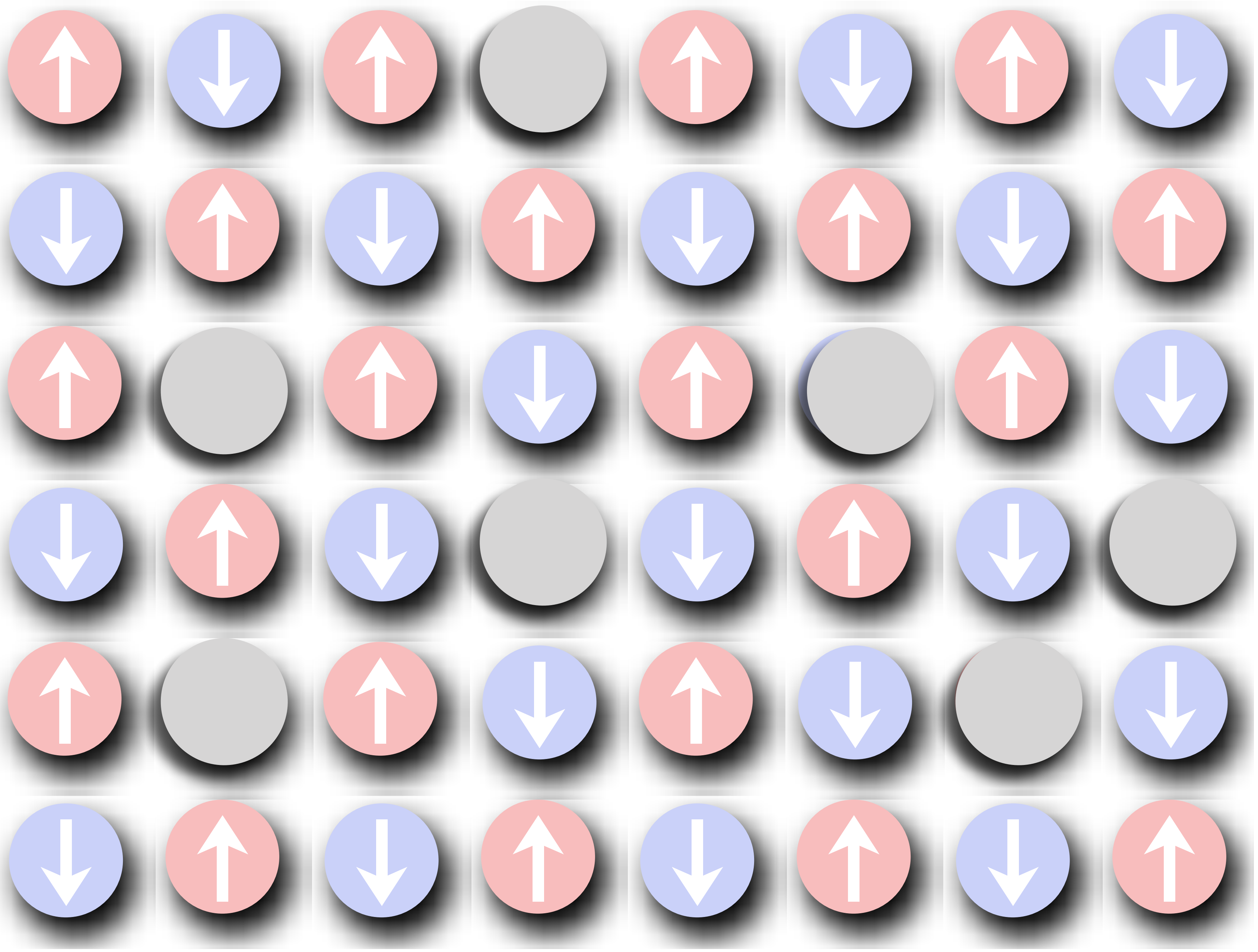

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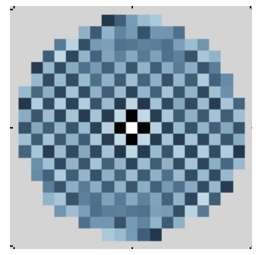
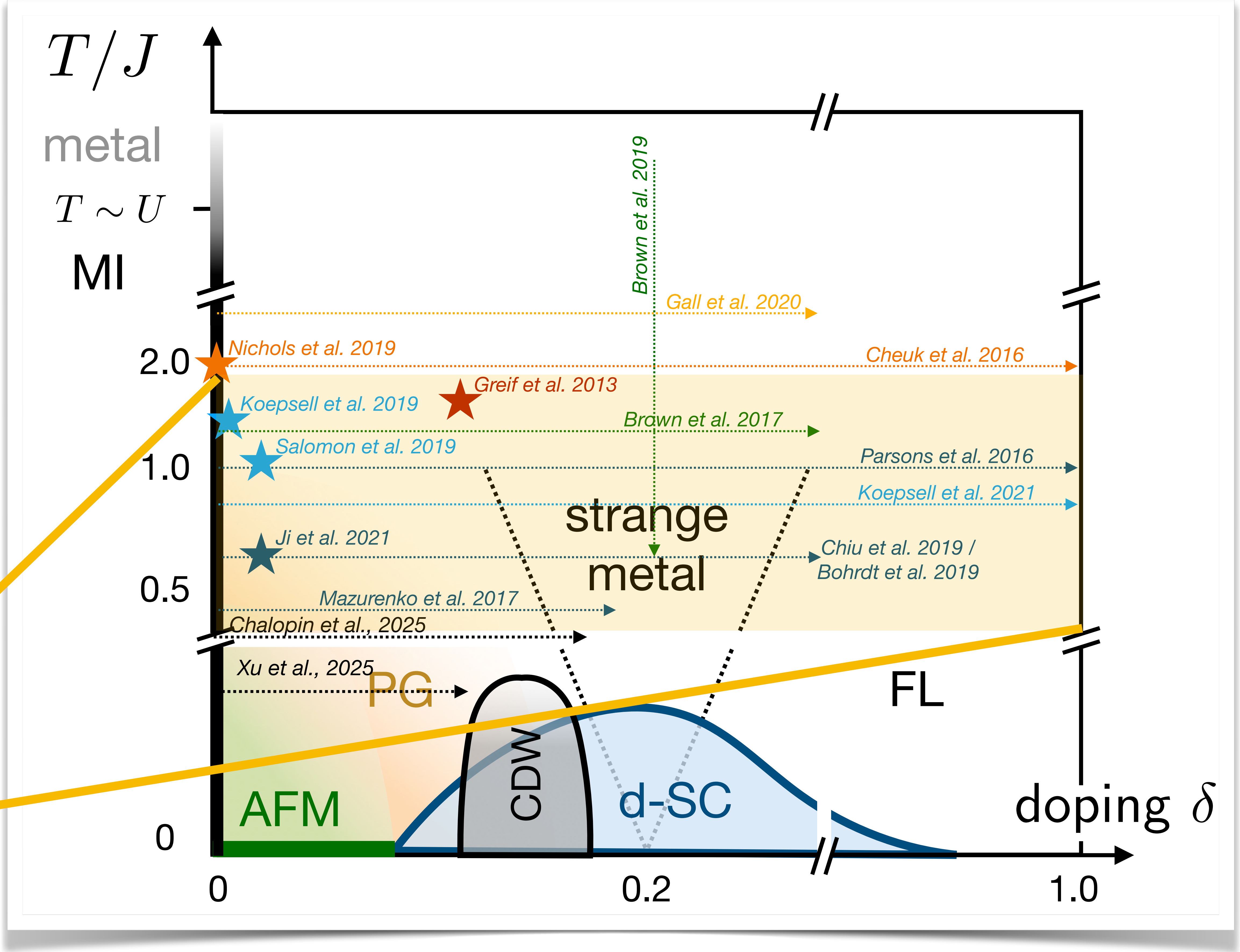
$|$  $\rangle = |0\rangle$

* Doping δ: add free charge carriers

Search for emergent constituents!



Review: Bohrdt et al., Ann. Phys. 435 (2021)

Hubbard phase diagram

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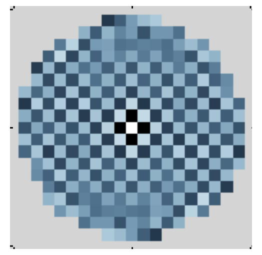
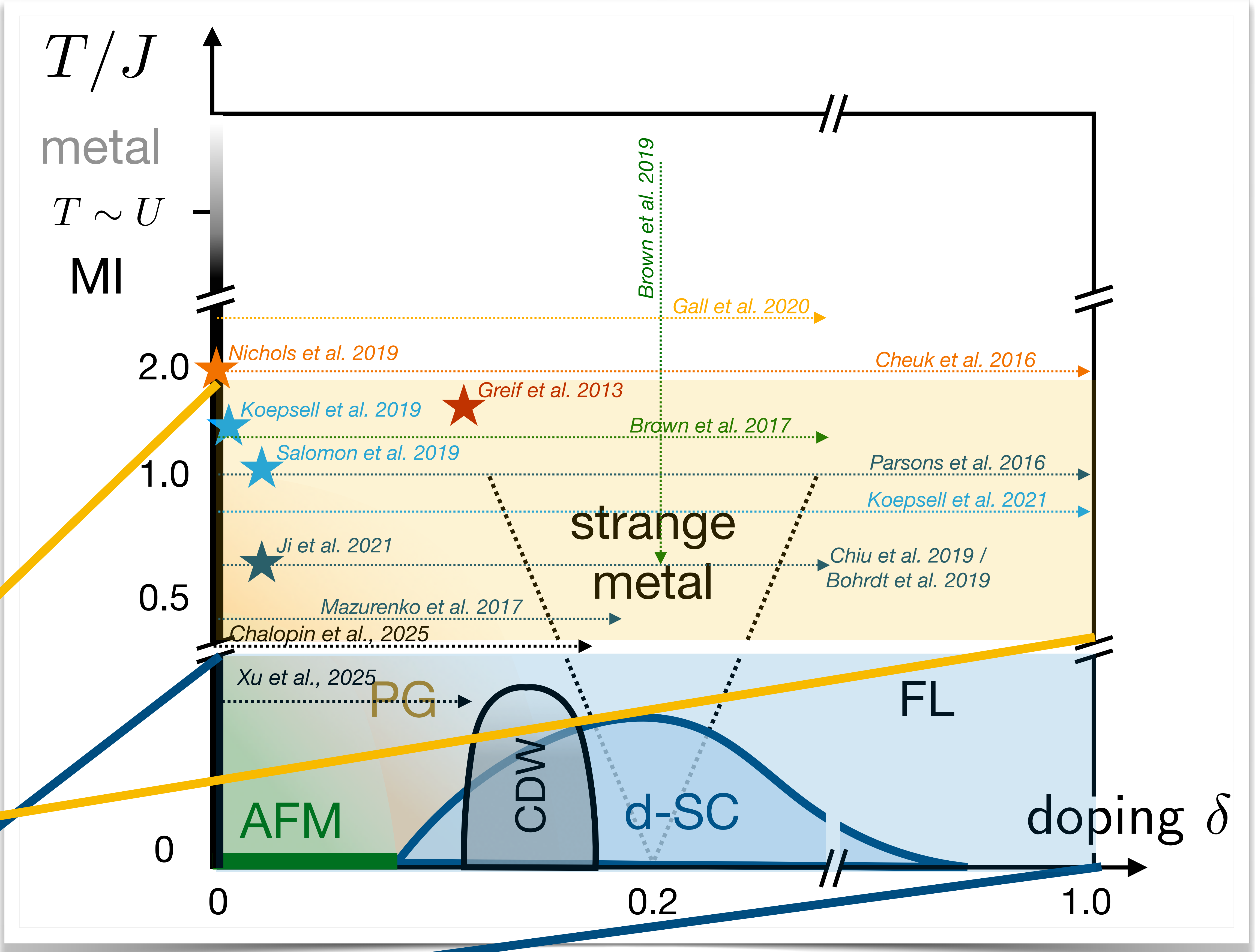
$|$  $\rangle = |0\rangle$

- Doping δ: add free charge carriers

Search for emergent constituents!

Collective phases:
Cold atoms still too hot…



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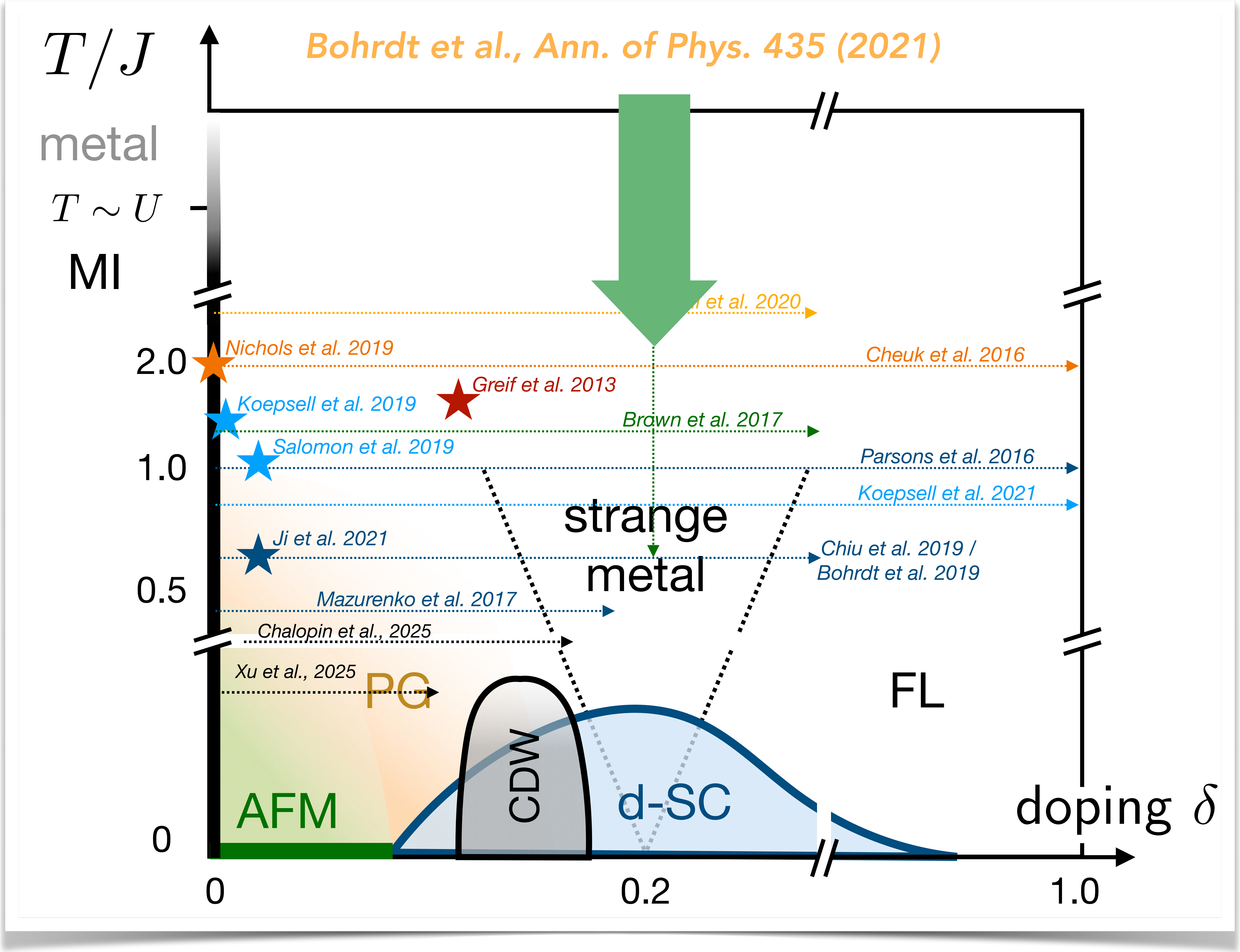

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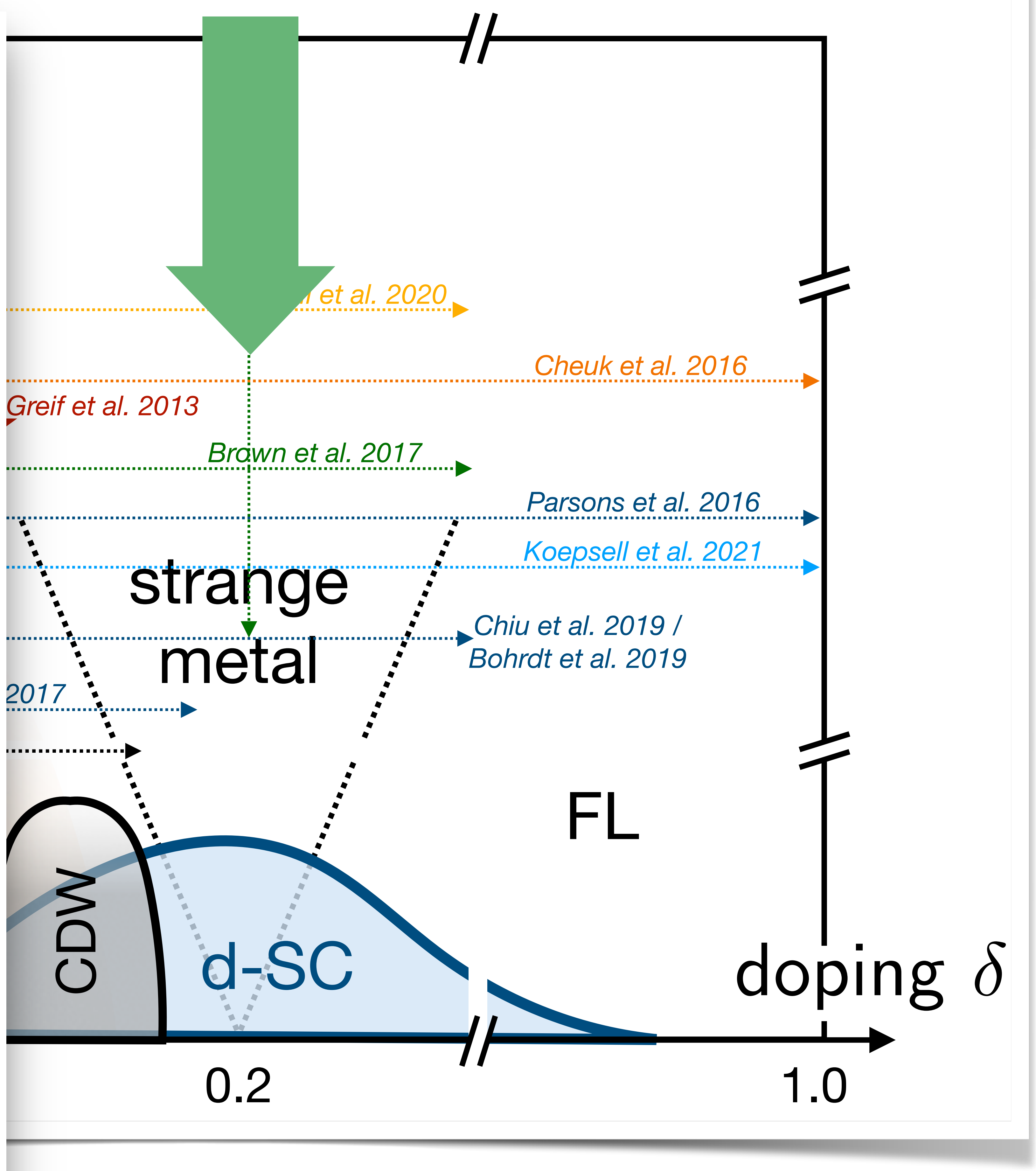
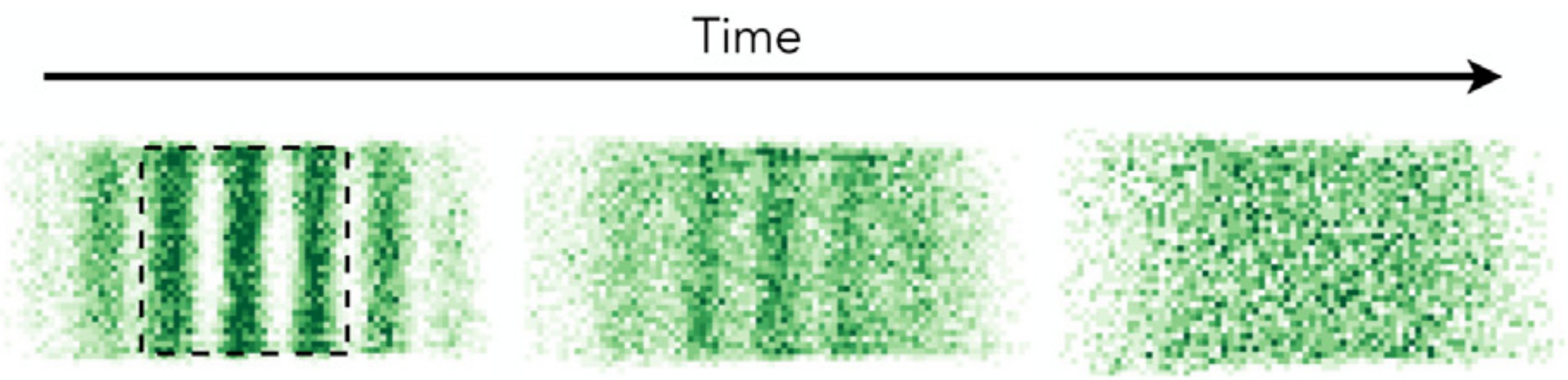
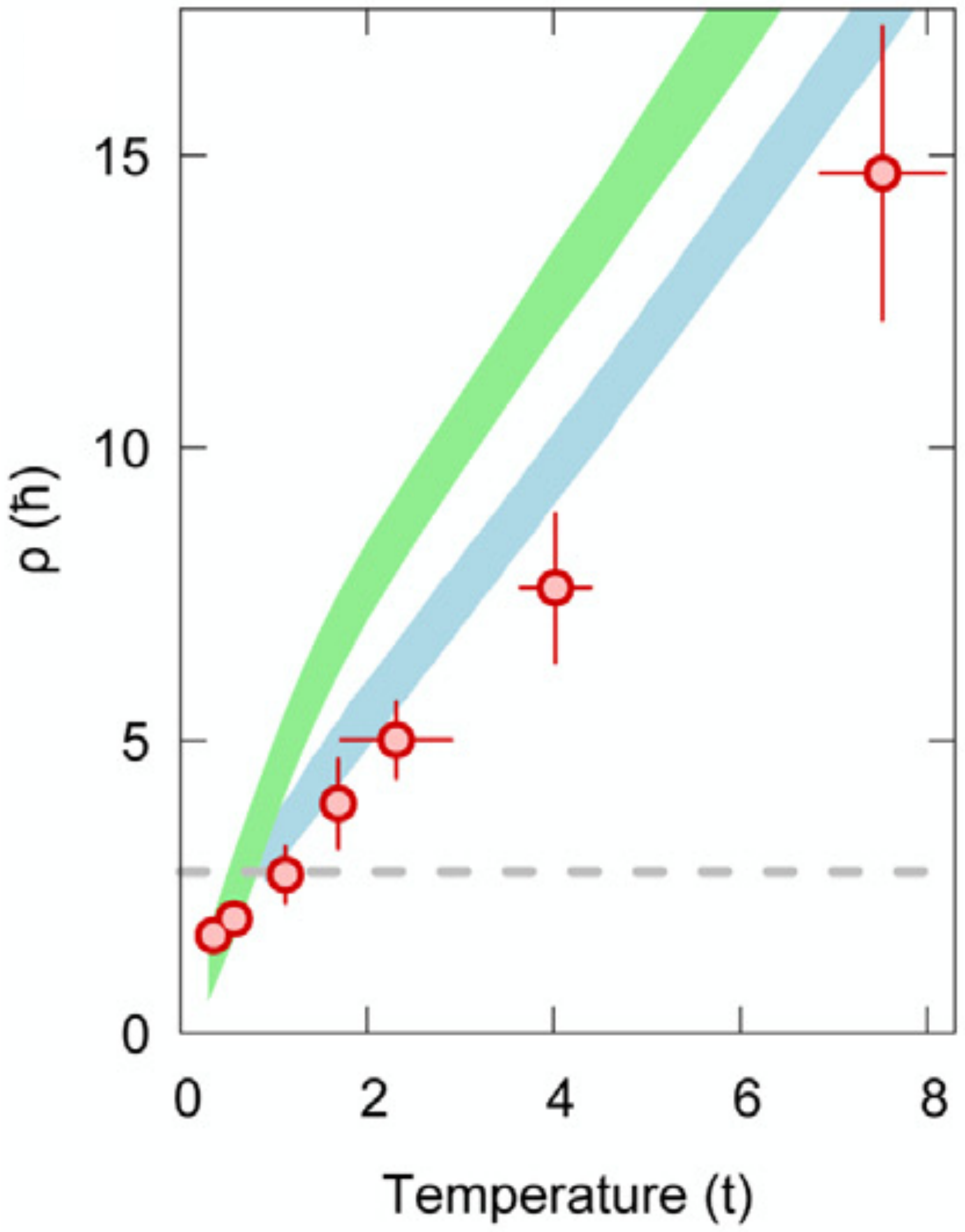


Strange metal

Brown et al., Science 363 (2019) — Bakr lab



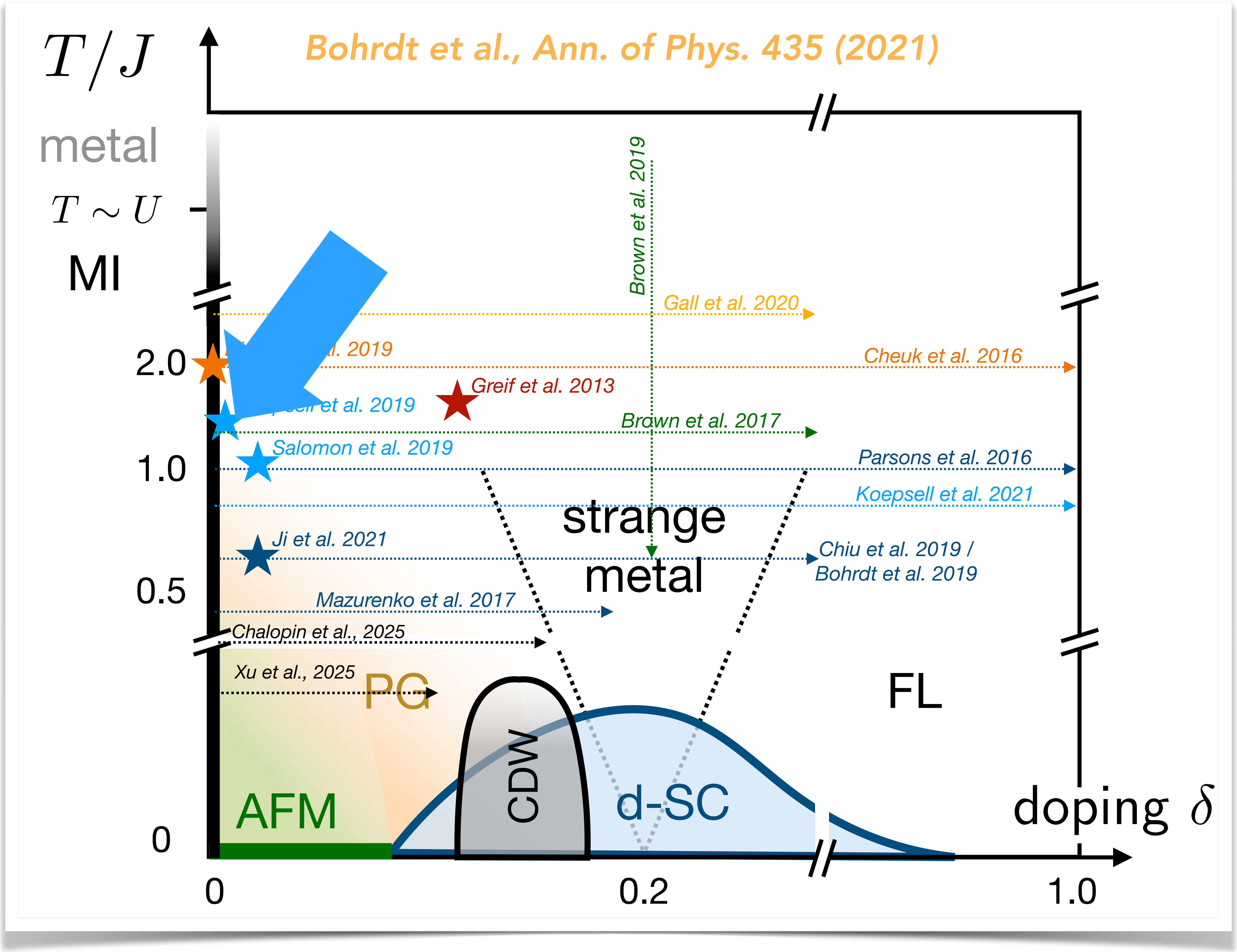


- *Measured: diffusion, compressibility — Nernst-Einstein equation:*

$$\sigma = \chi_C D$$

- *Linear in T resistivity*

Hubbard phase diagram



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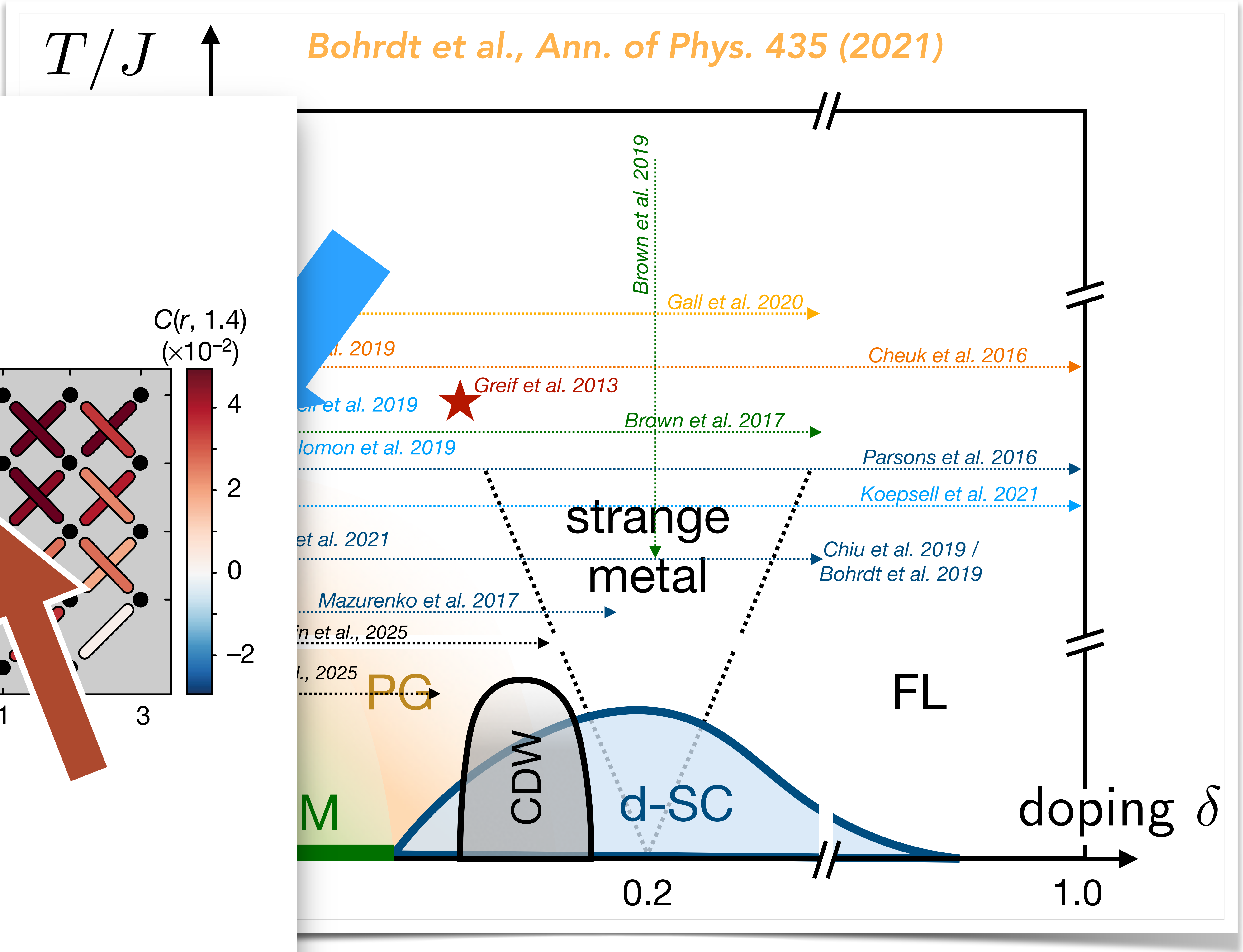
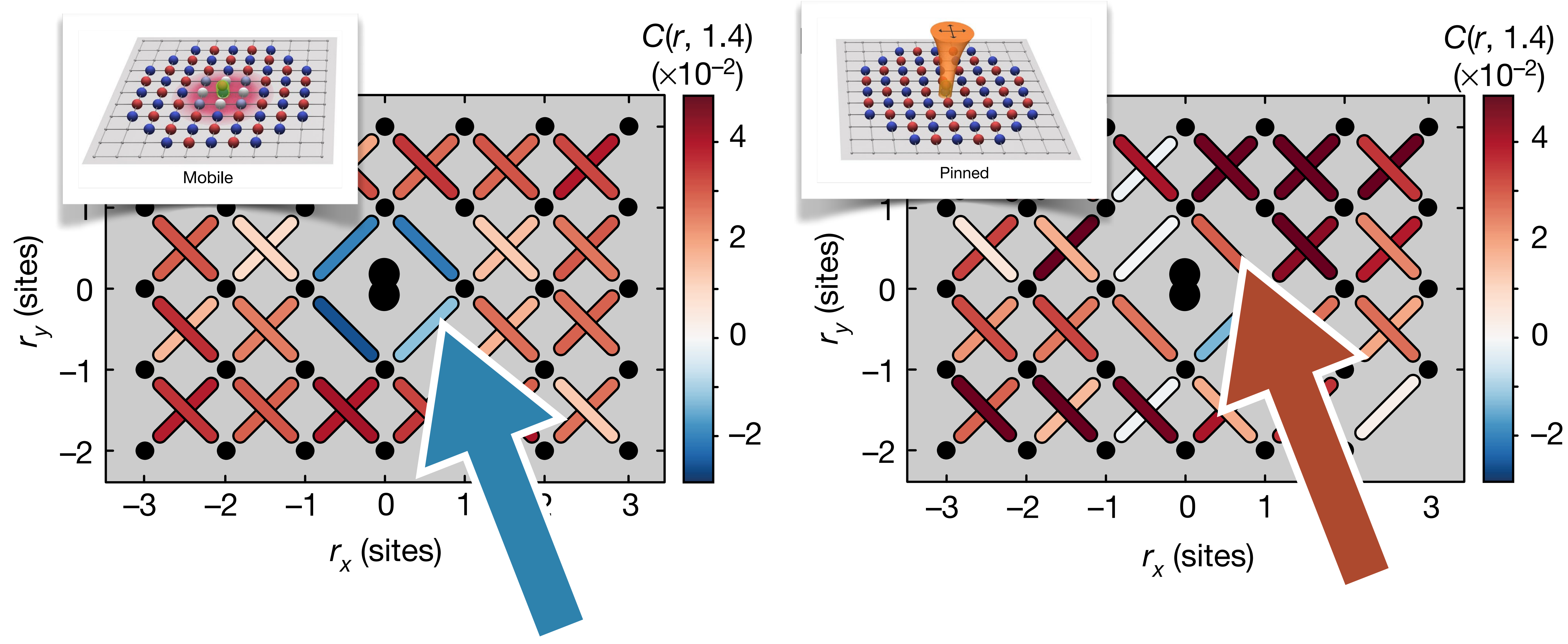


Magnetic polarons in 2D

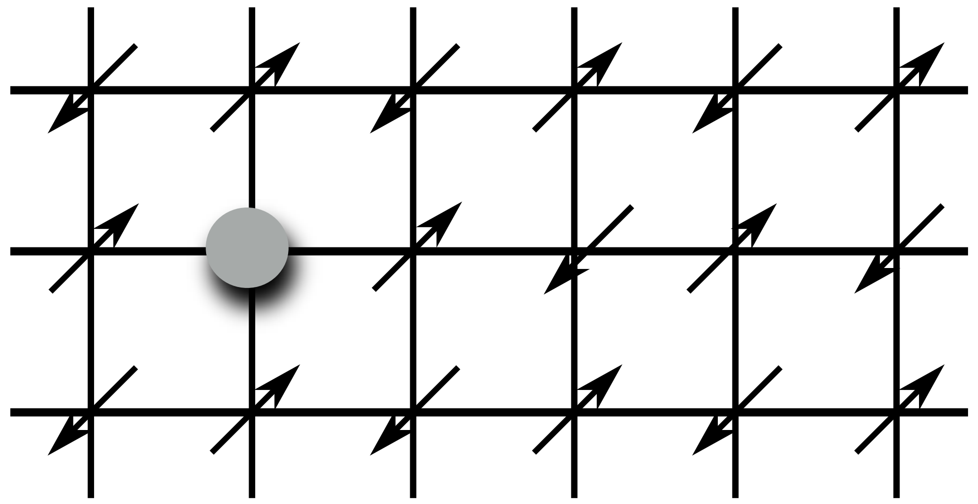
Koepsell et al., Nature 572 (2019) — Bloch lab



* Magnetic dressing cloud of mobile hole

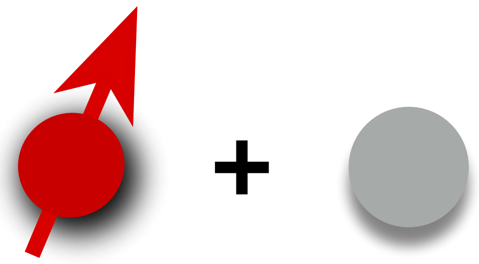
Hubbard phase diagram

Linear confinement force in doped antiferromagnets



* **Fractional spin** excitation!

$$S = 1/2$$



Bulaevskii et al., JETP 27 (1968), Trugman, PRB 37 (1988)
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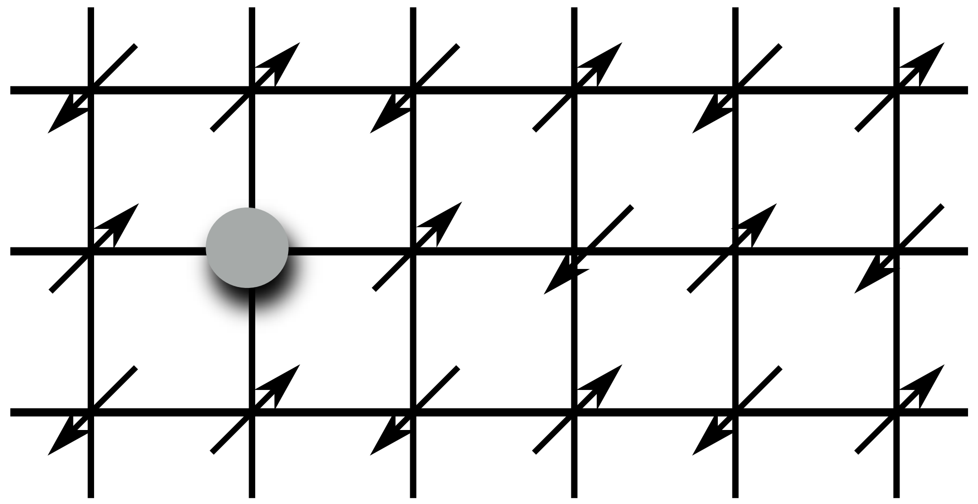

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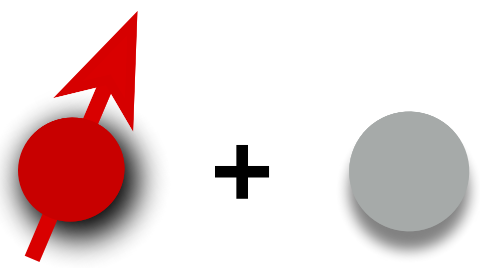
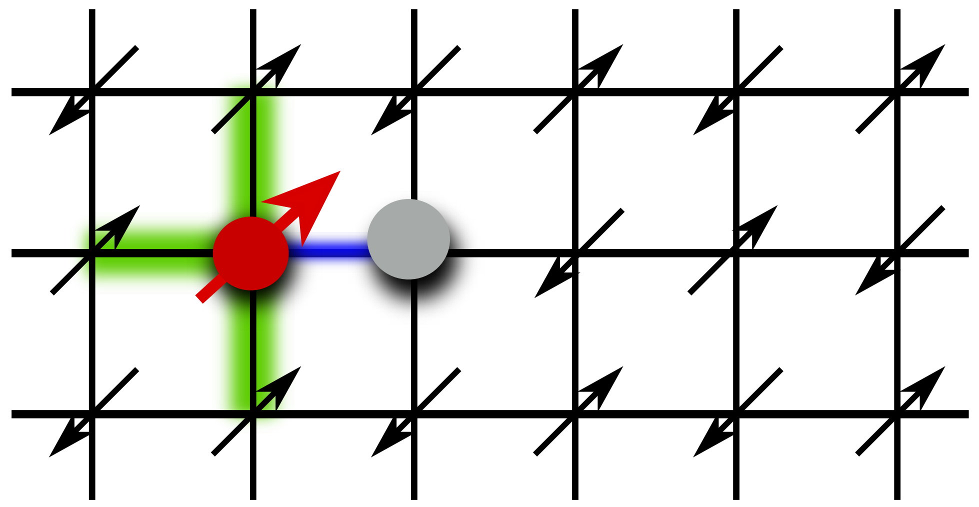
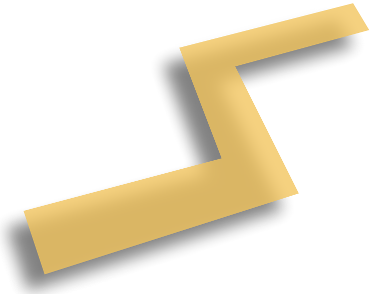




- Movement of the hole distorts the Neel state — **spin is carried by distortion!**



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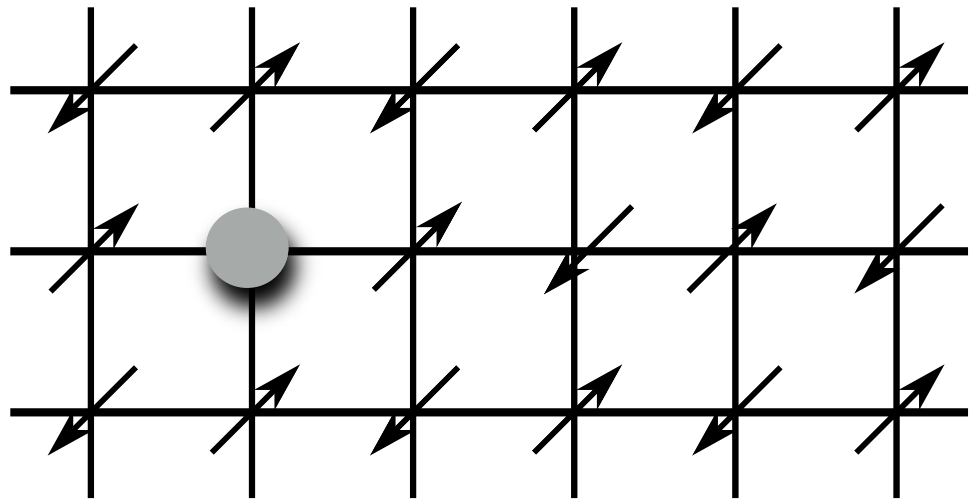

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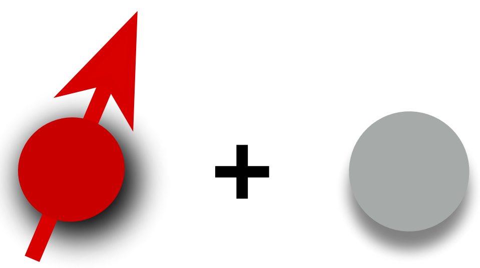

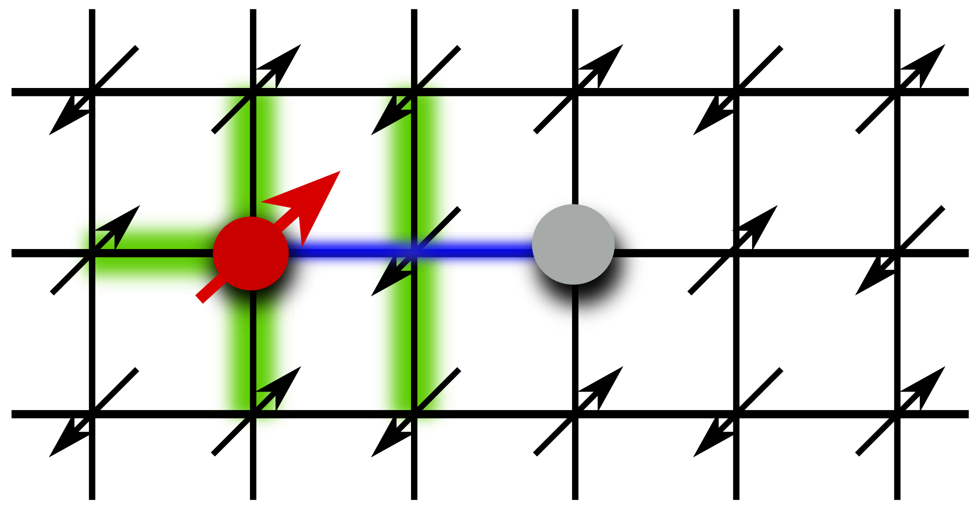

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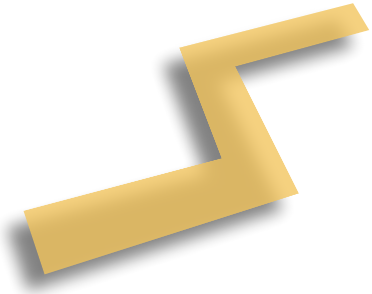

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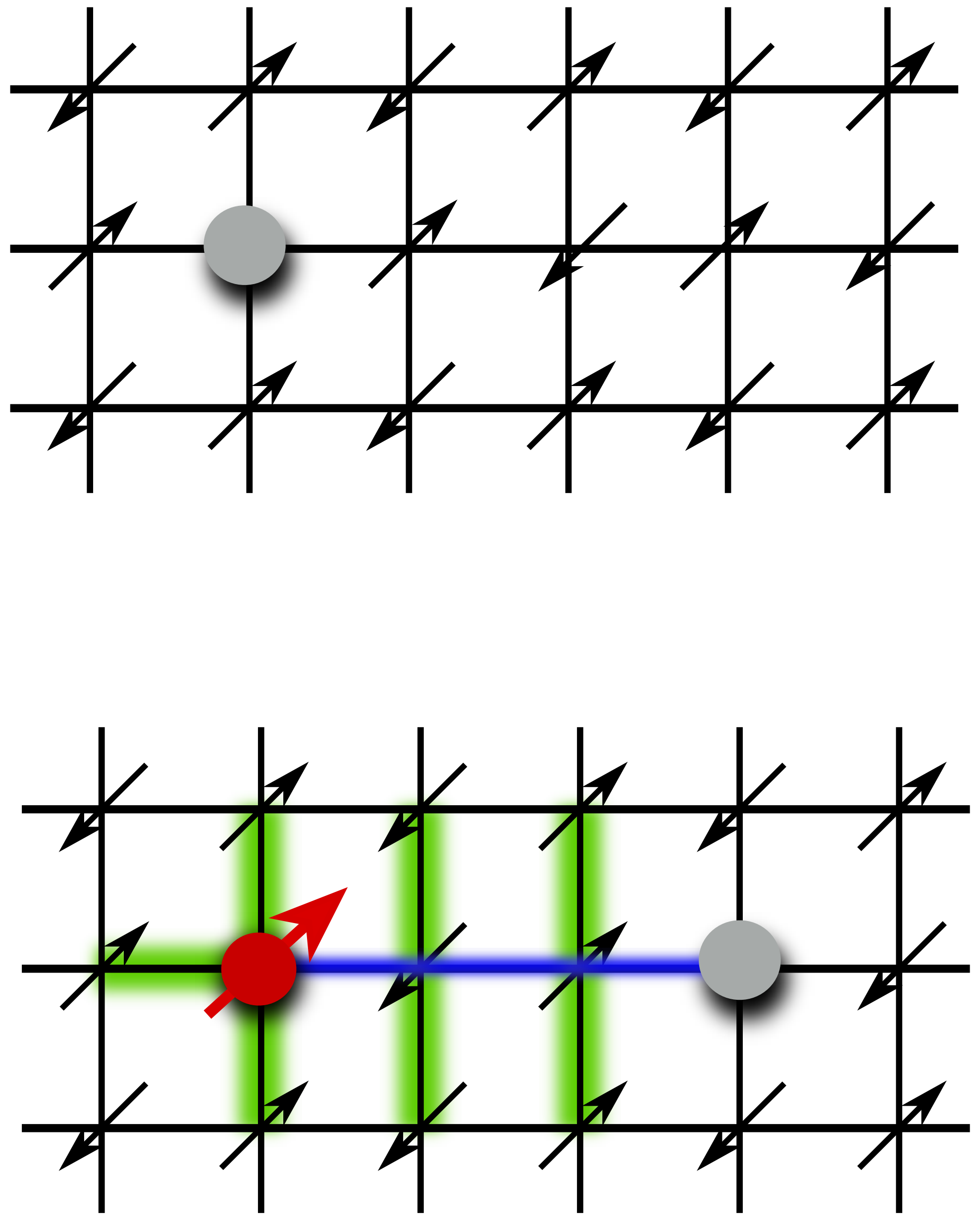

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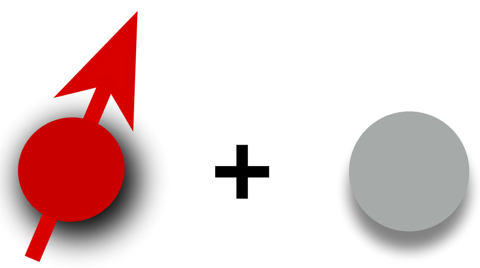

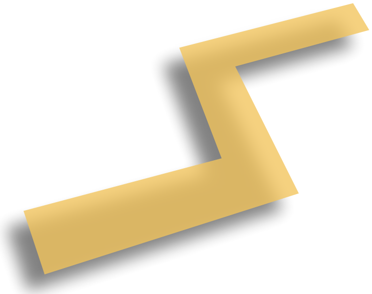

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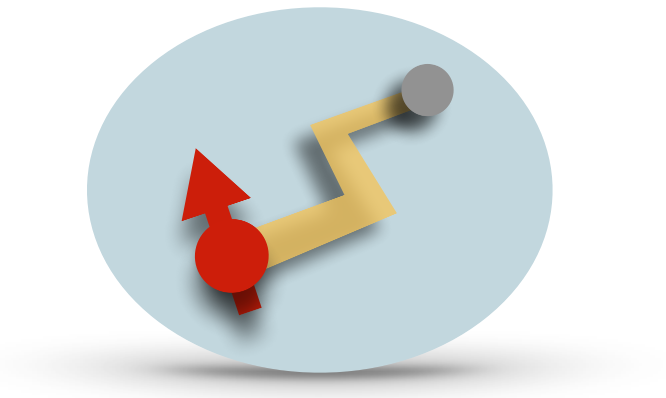
$$E \propto \ell$$



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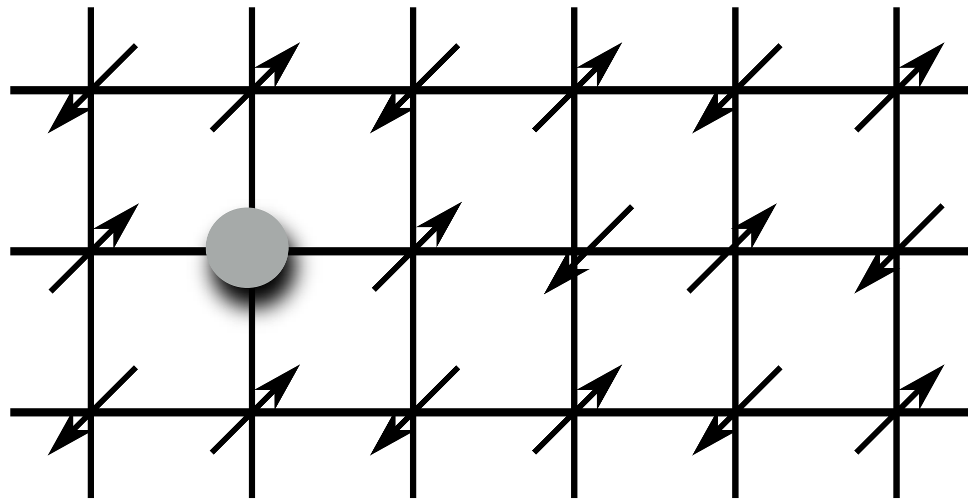

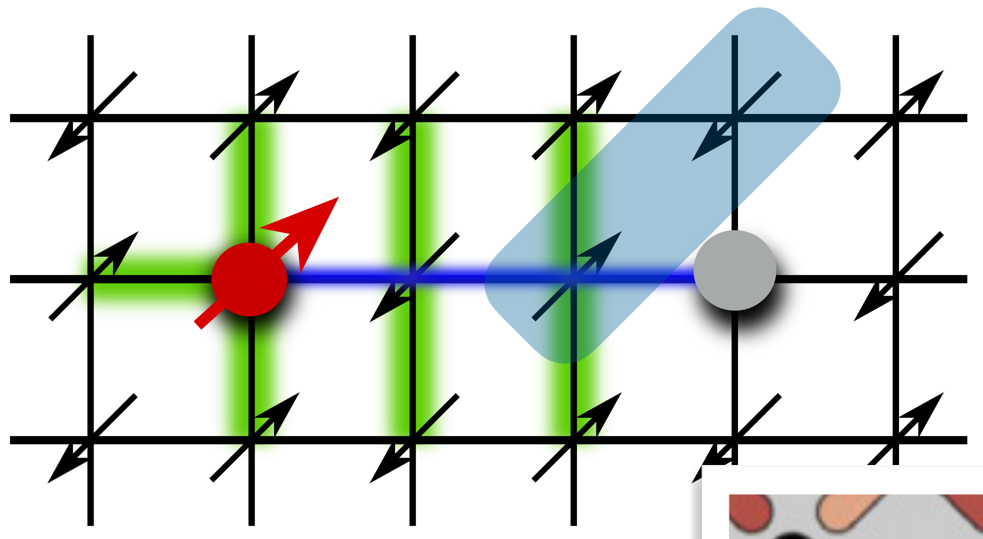

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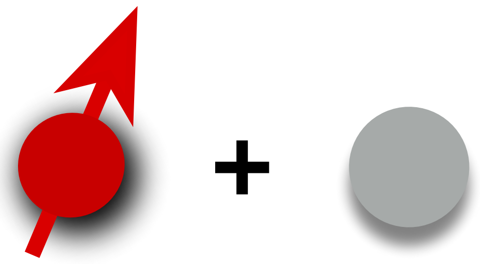

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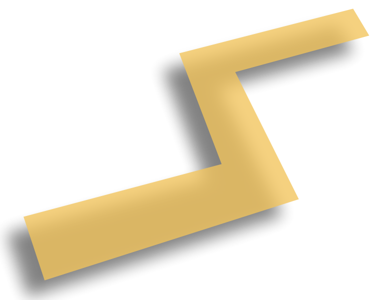

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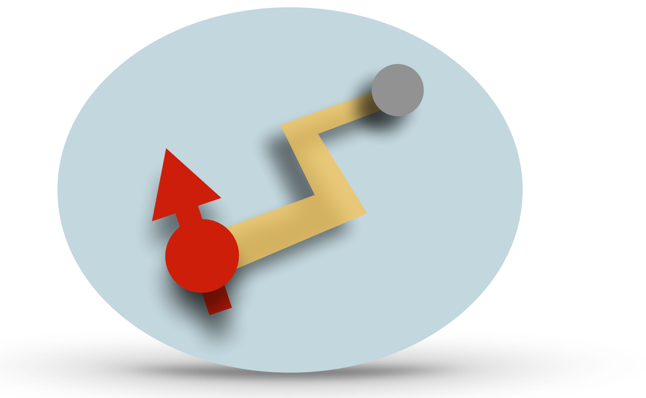

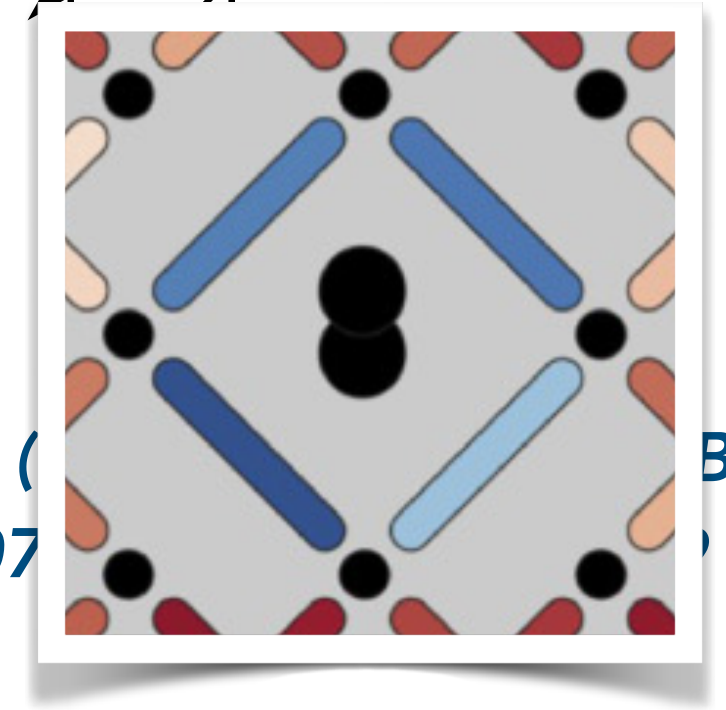

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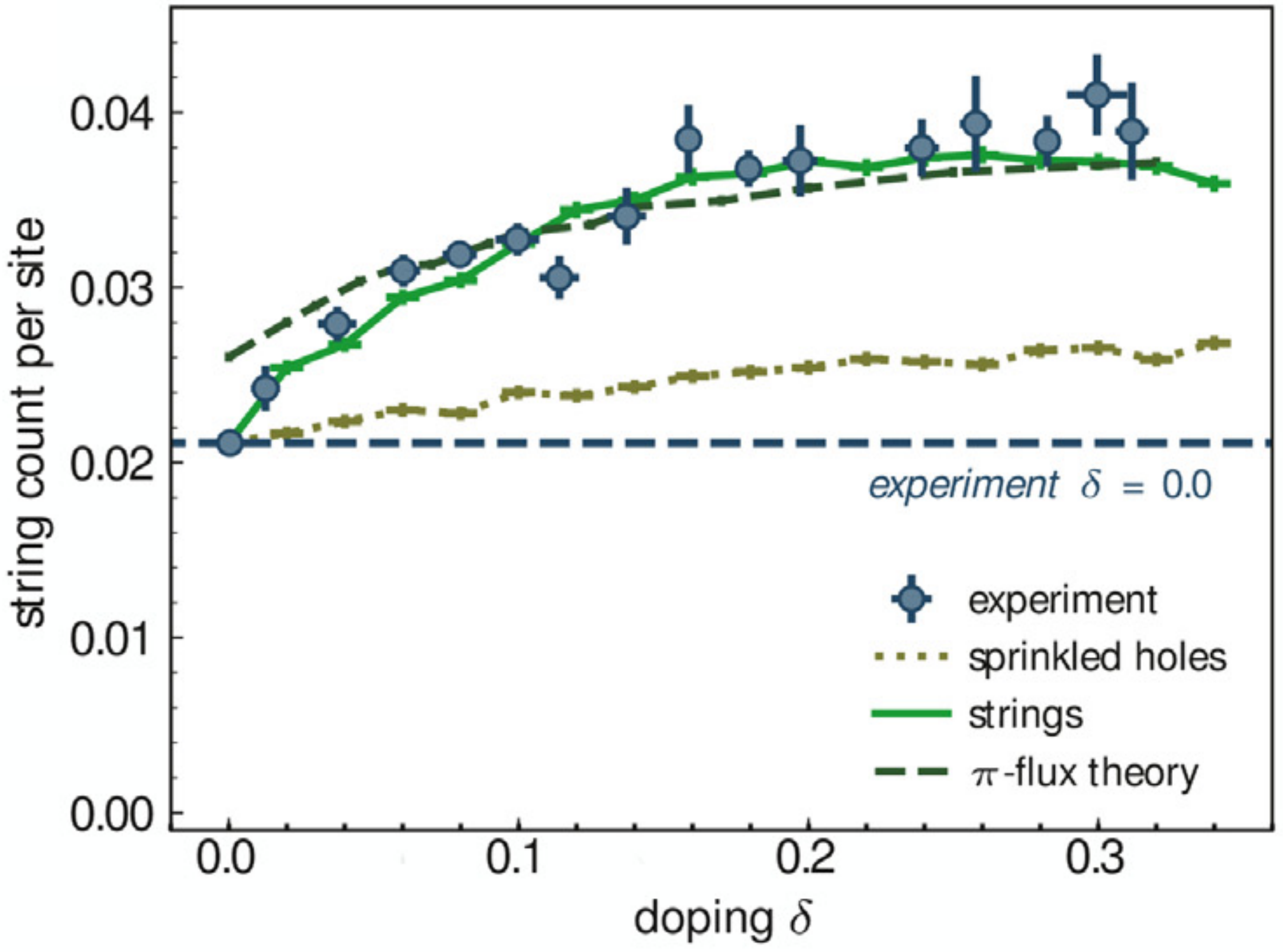
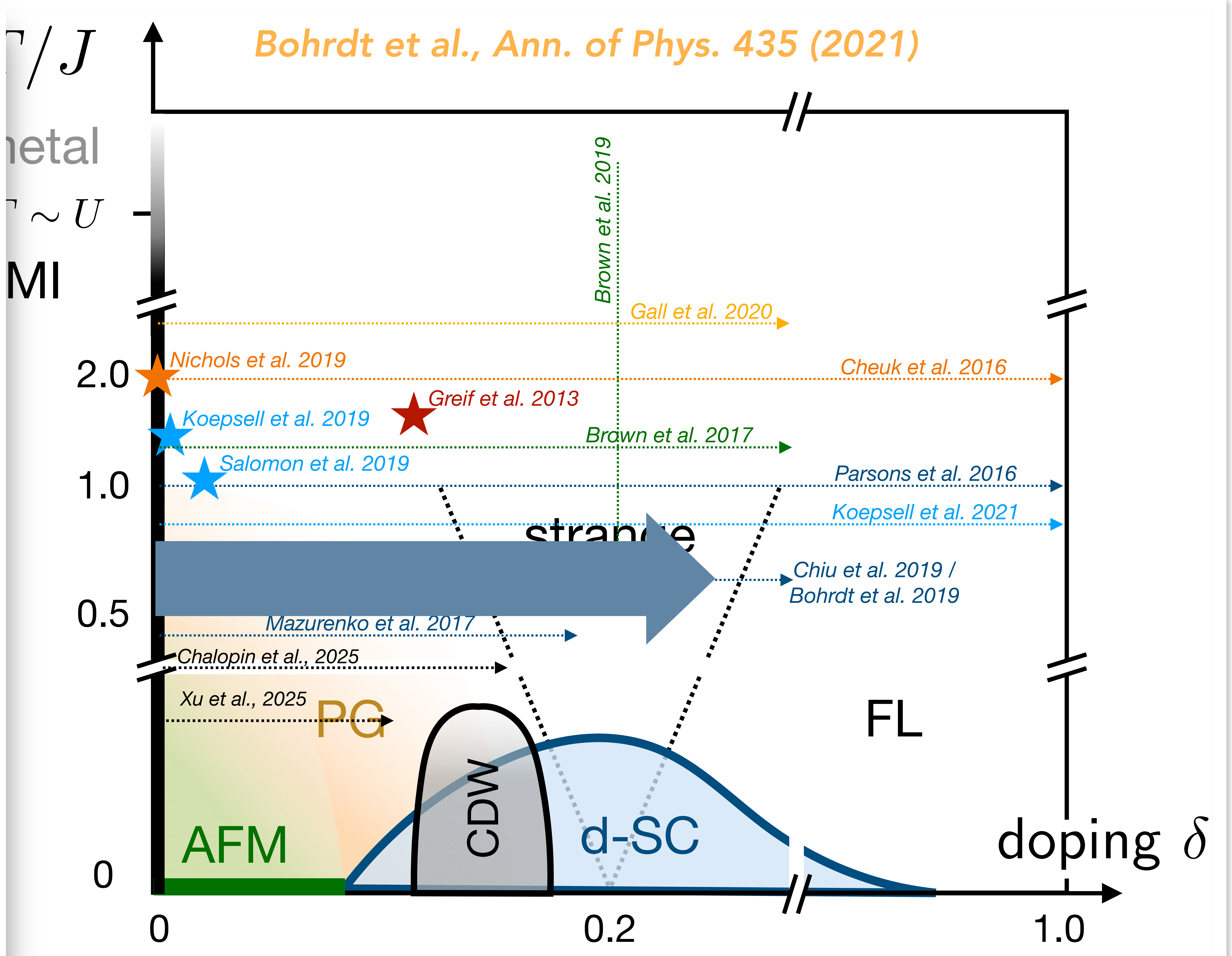
Hubbard phase diagram

Finite doping: string patterns

Chiu et al., Science 365 (2019) — Greiner lab



* *Ensemble of individual magnetic polarons*



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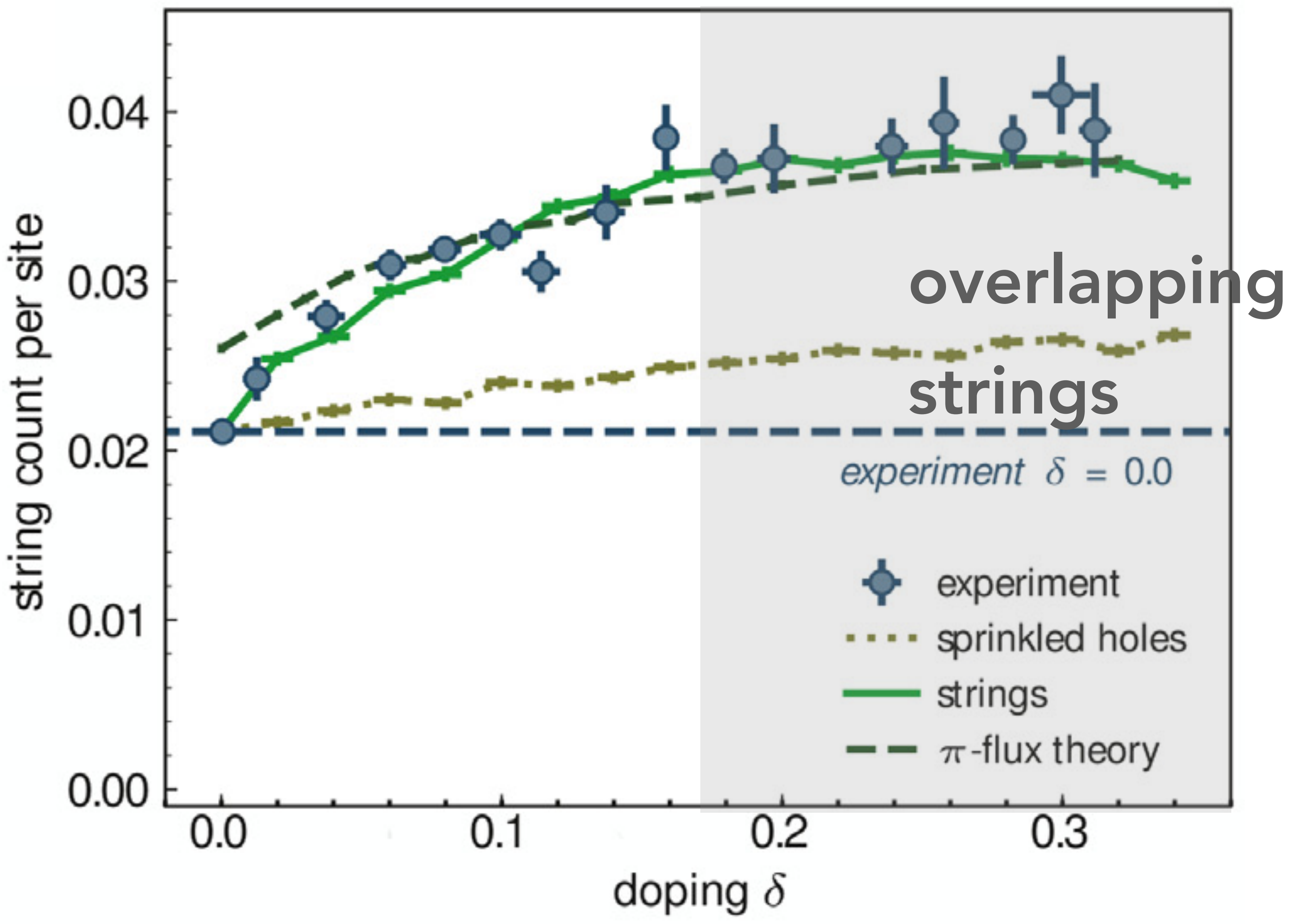

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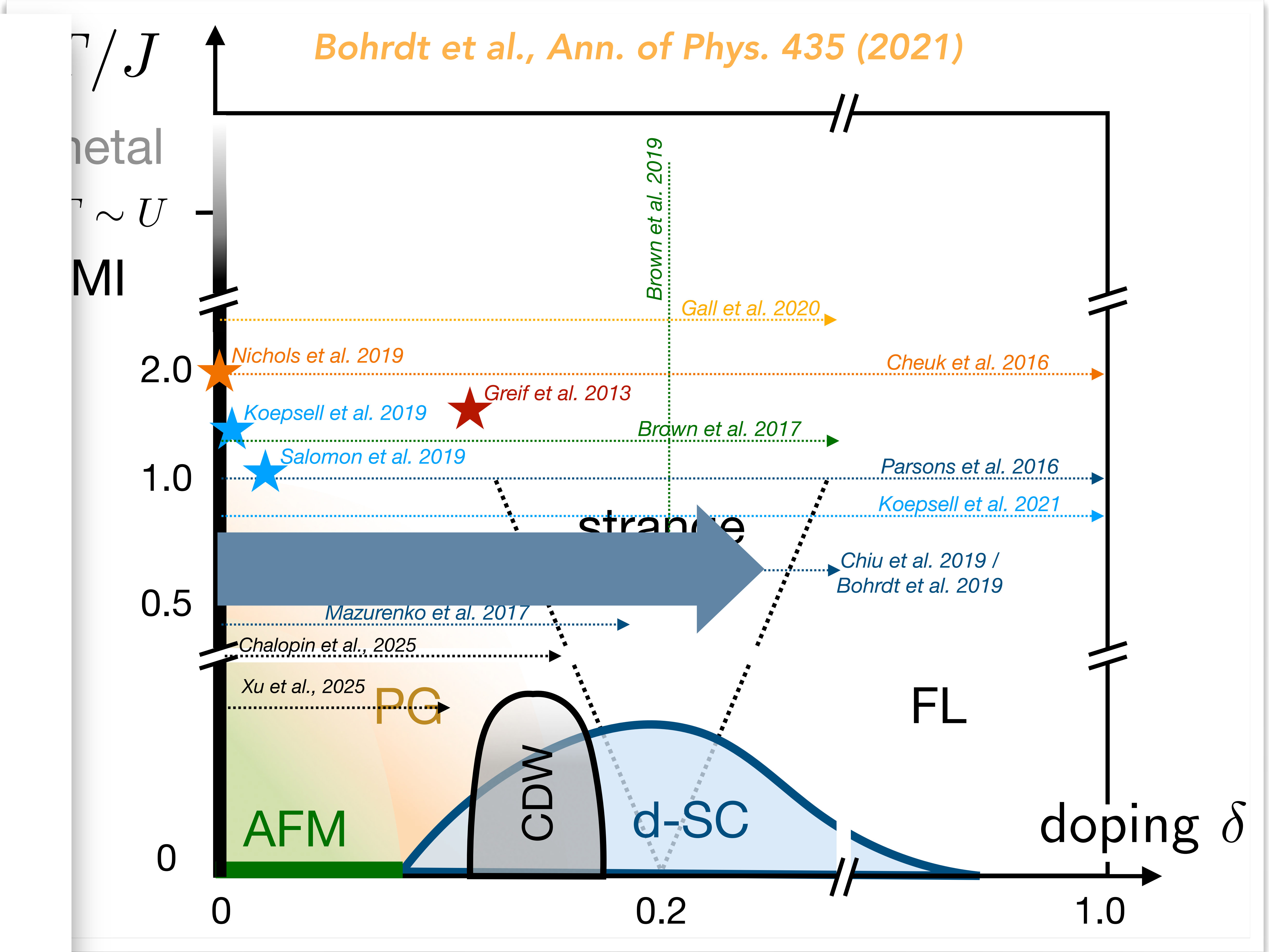

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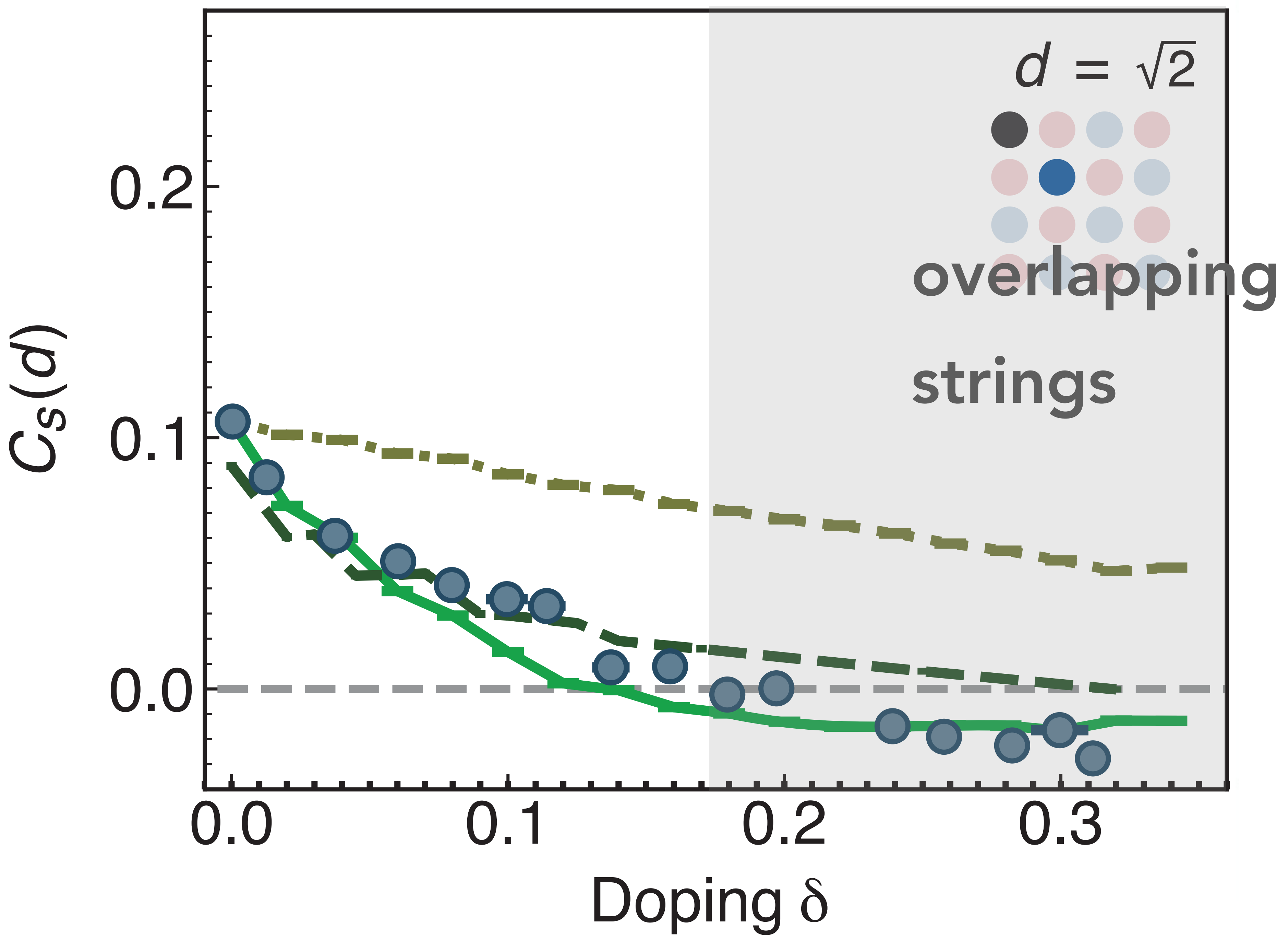

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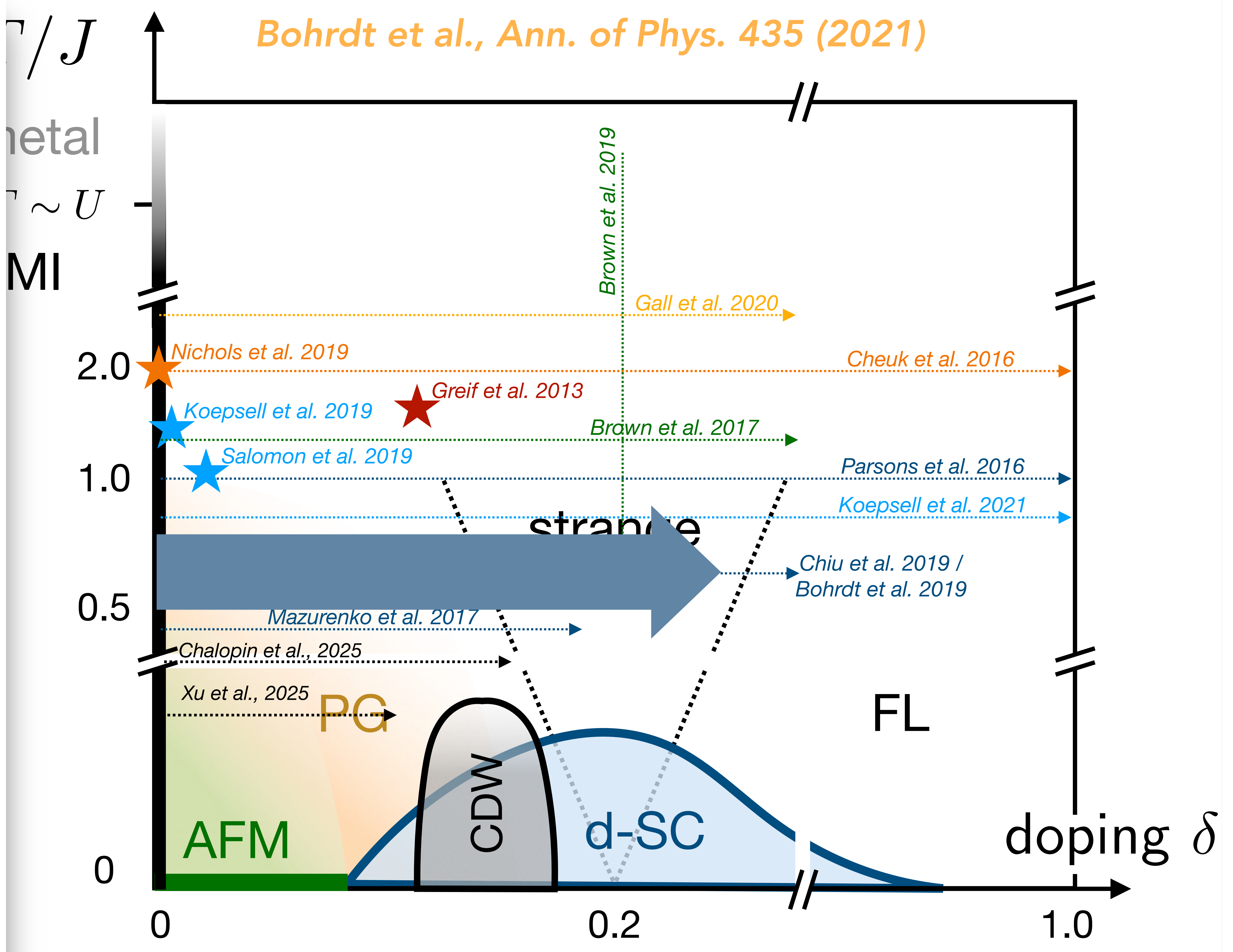
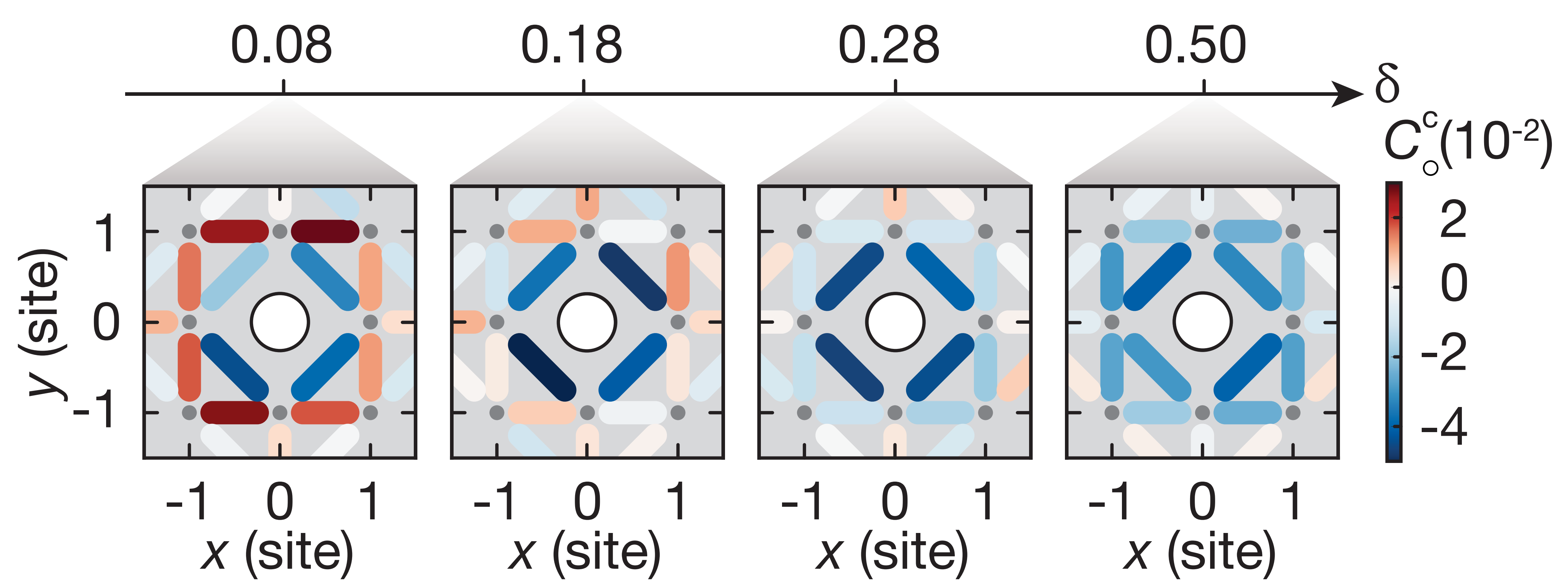


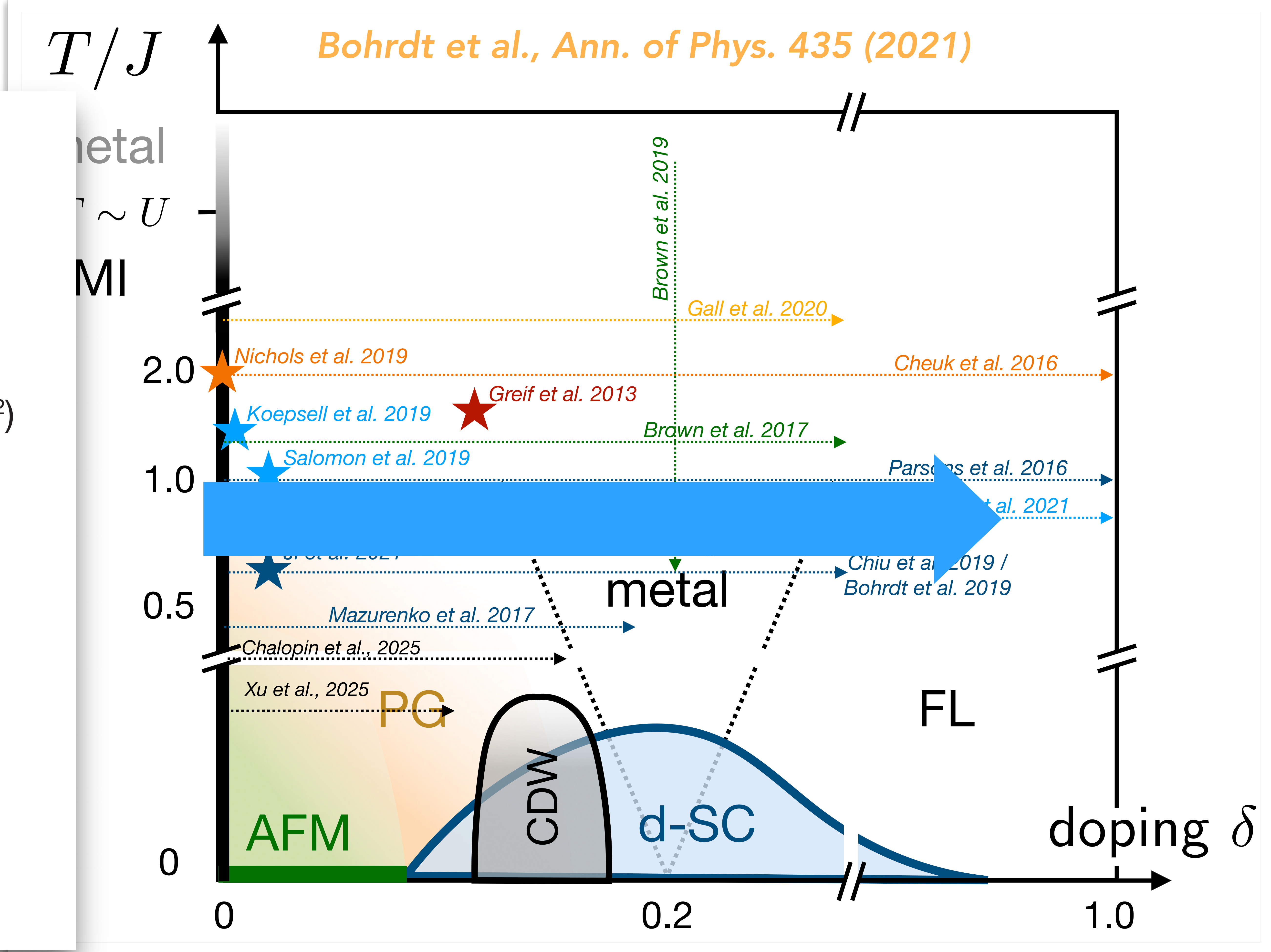
Hubbard phase diagram

Finite doping crossover to FL

Koepsell et al., Science 374 (2021) — Bloch lab



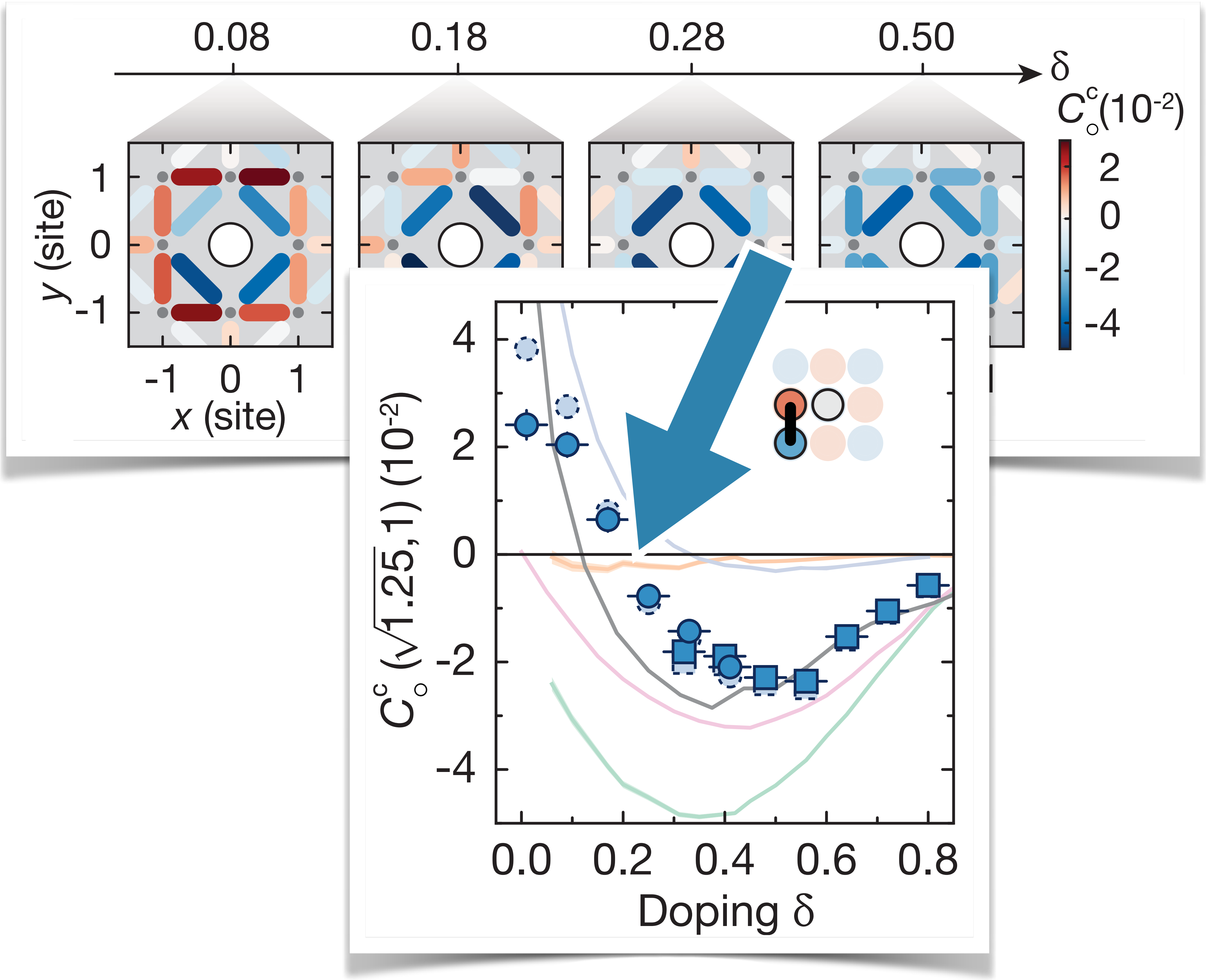
- *Constituting charge carriers change structure!*



Hubbard phase diagram

Changing nature of charge carriers: Hubbard / cuprates

- *Cold atoms:* Koepsell et al. Science 374 (2021)



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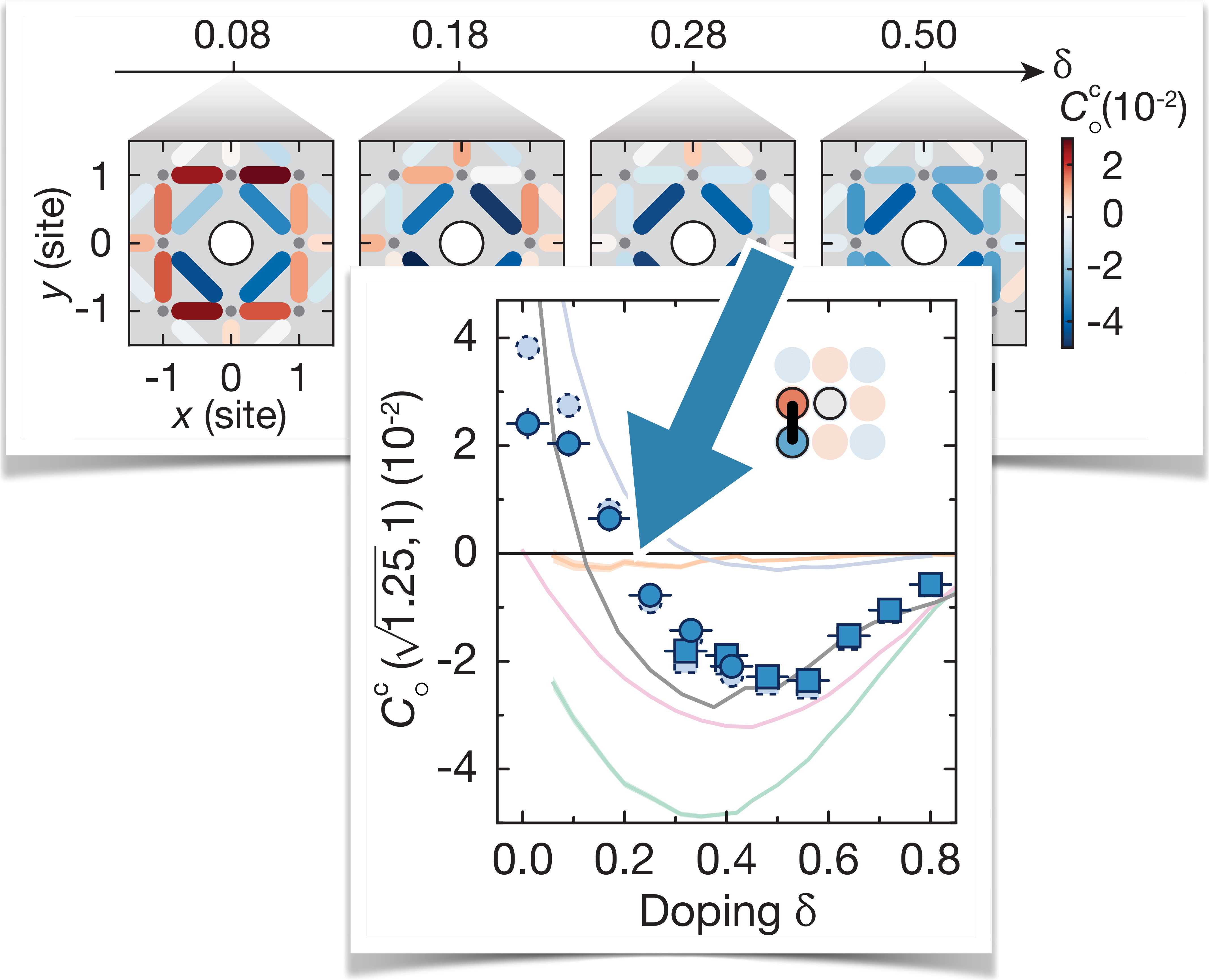

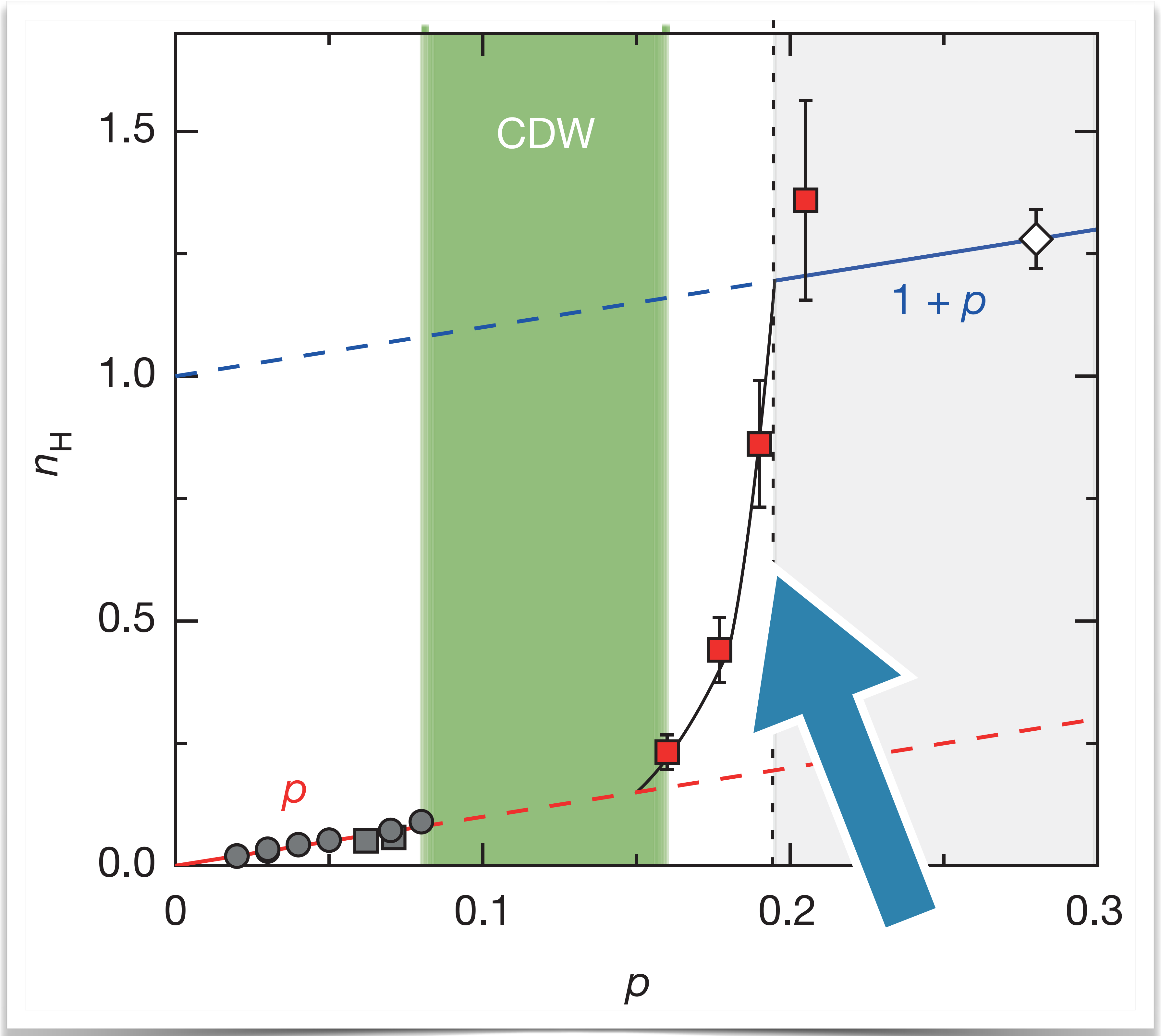

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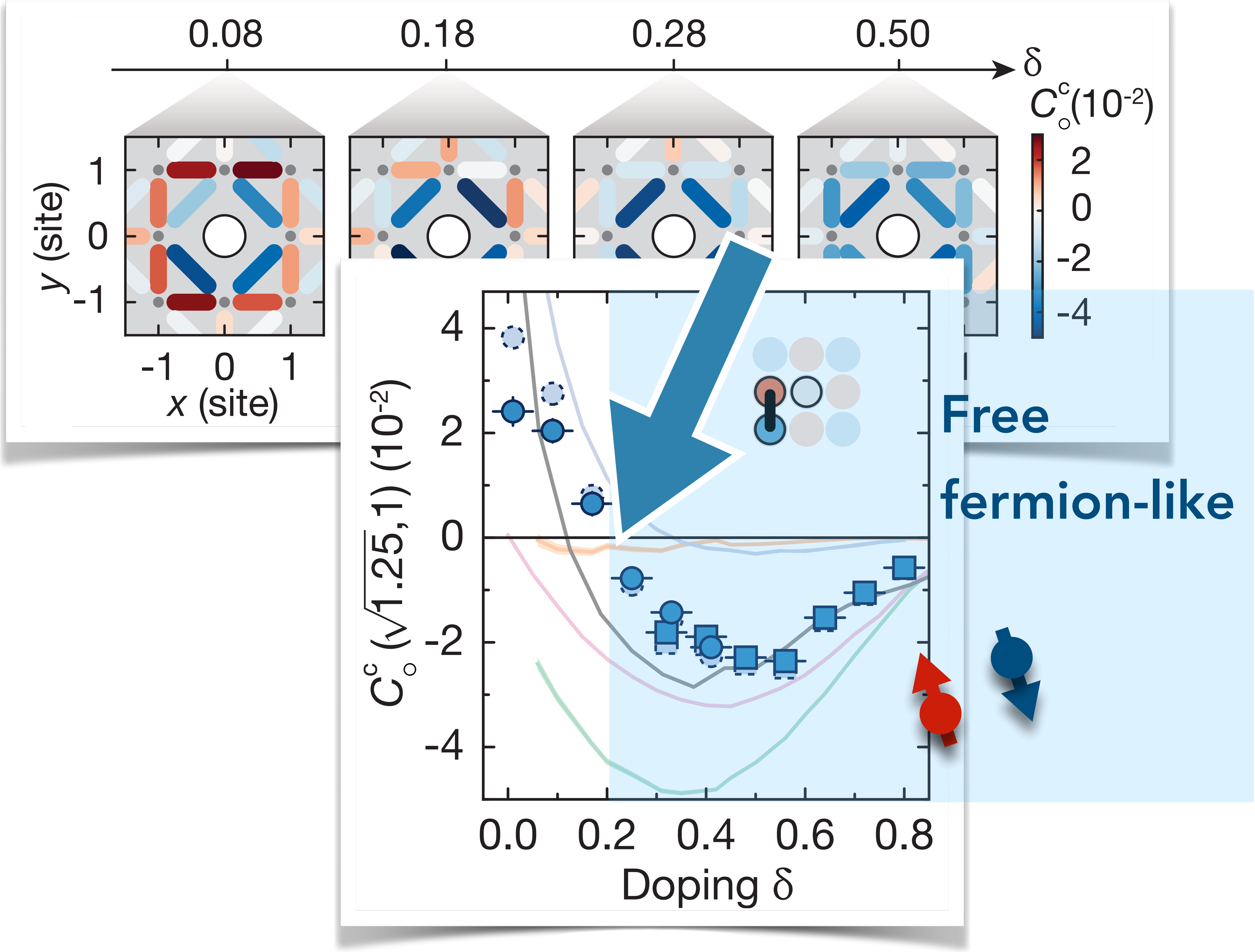

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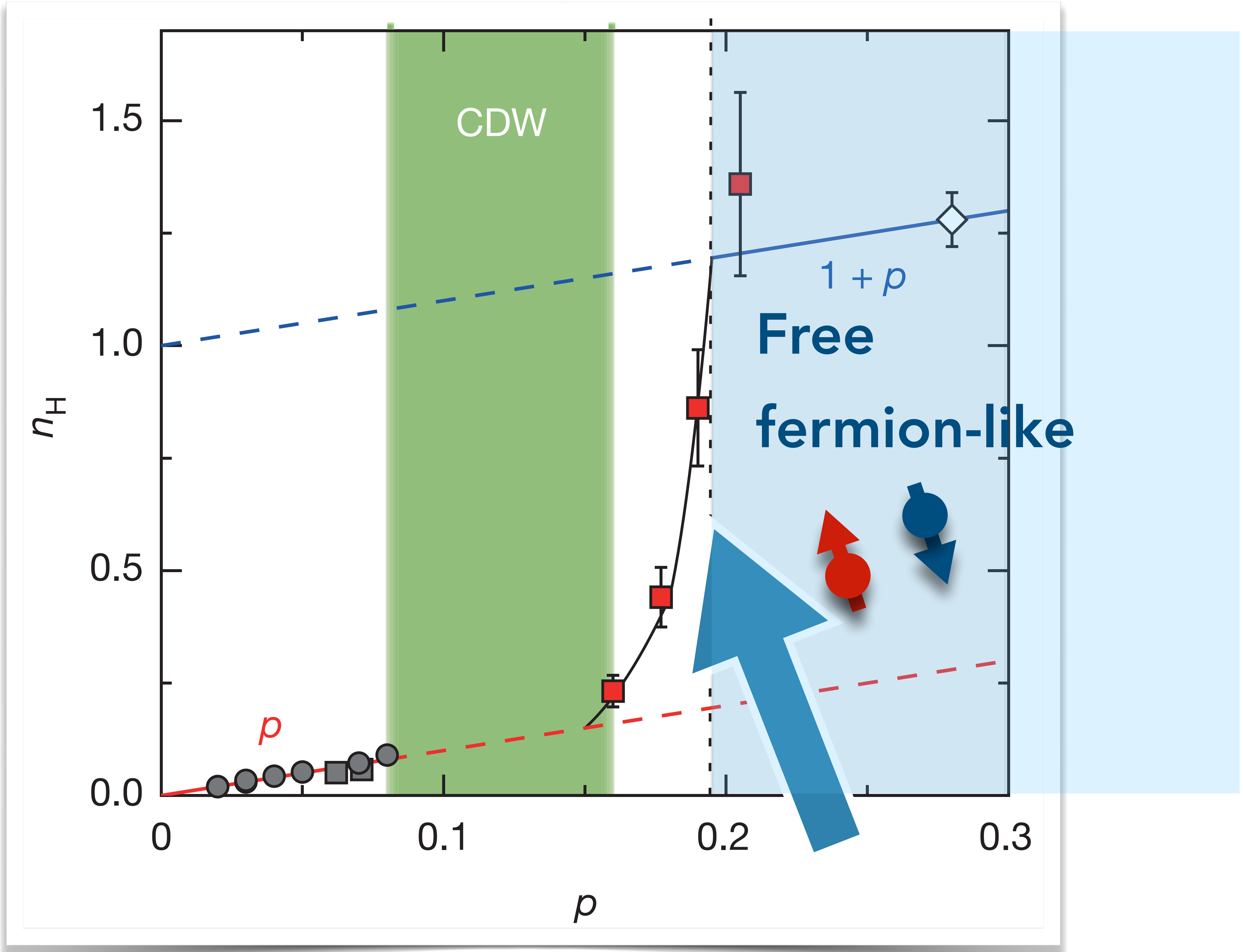

Hubbard phase diagram

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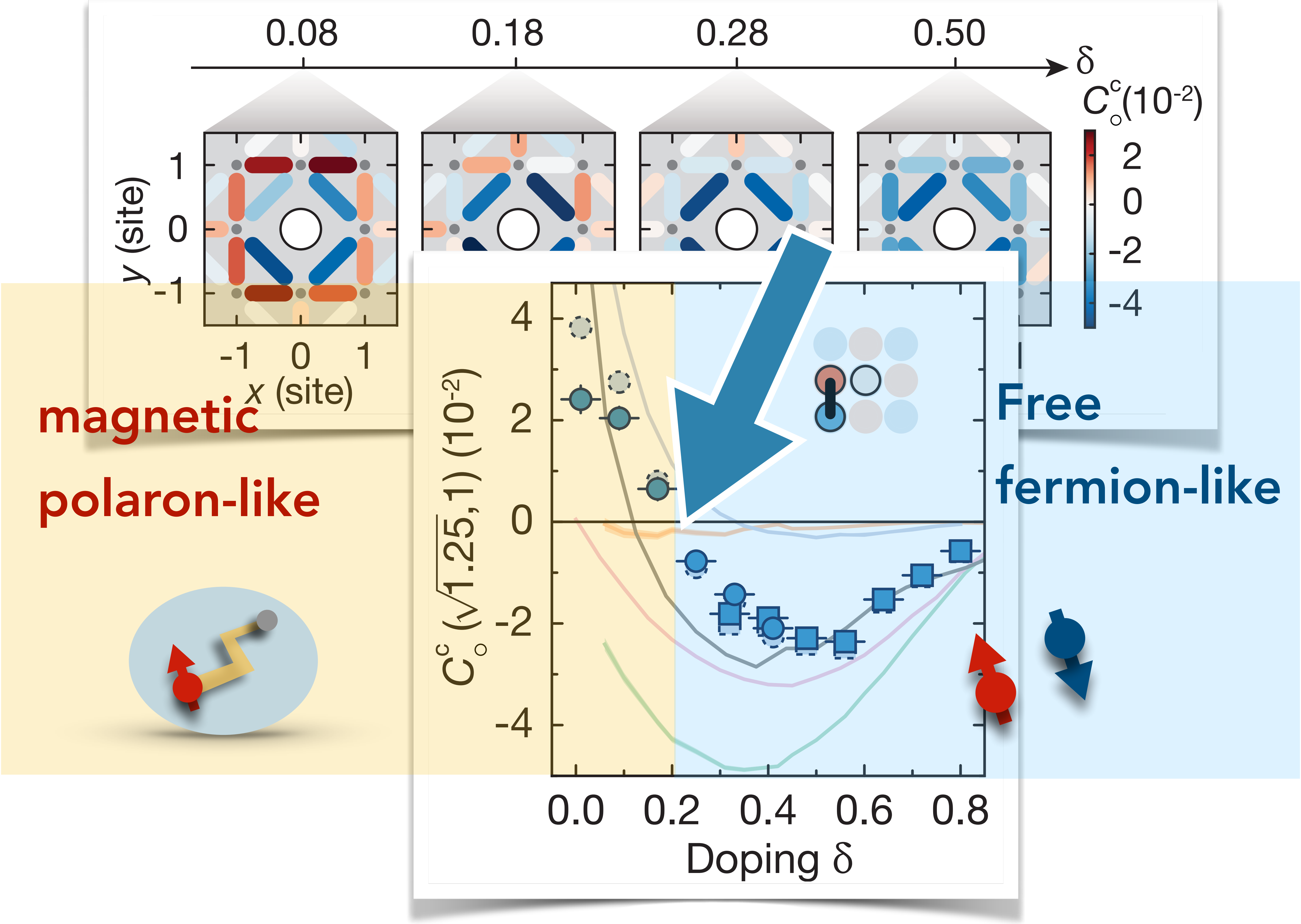

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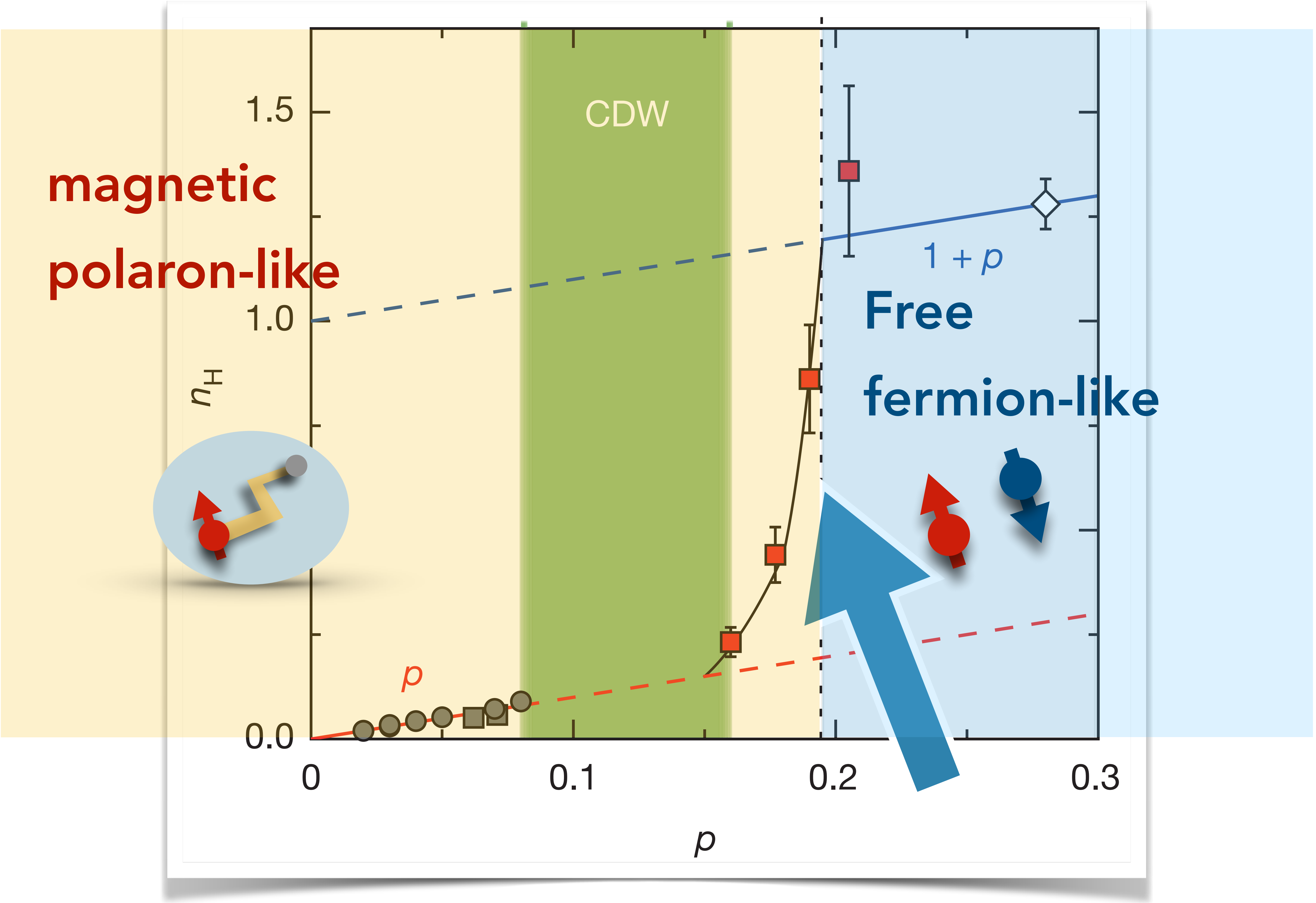
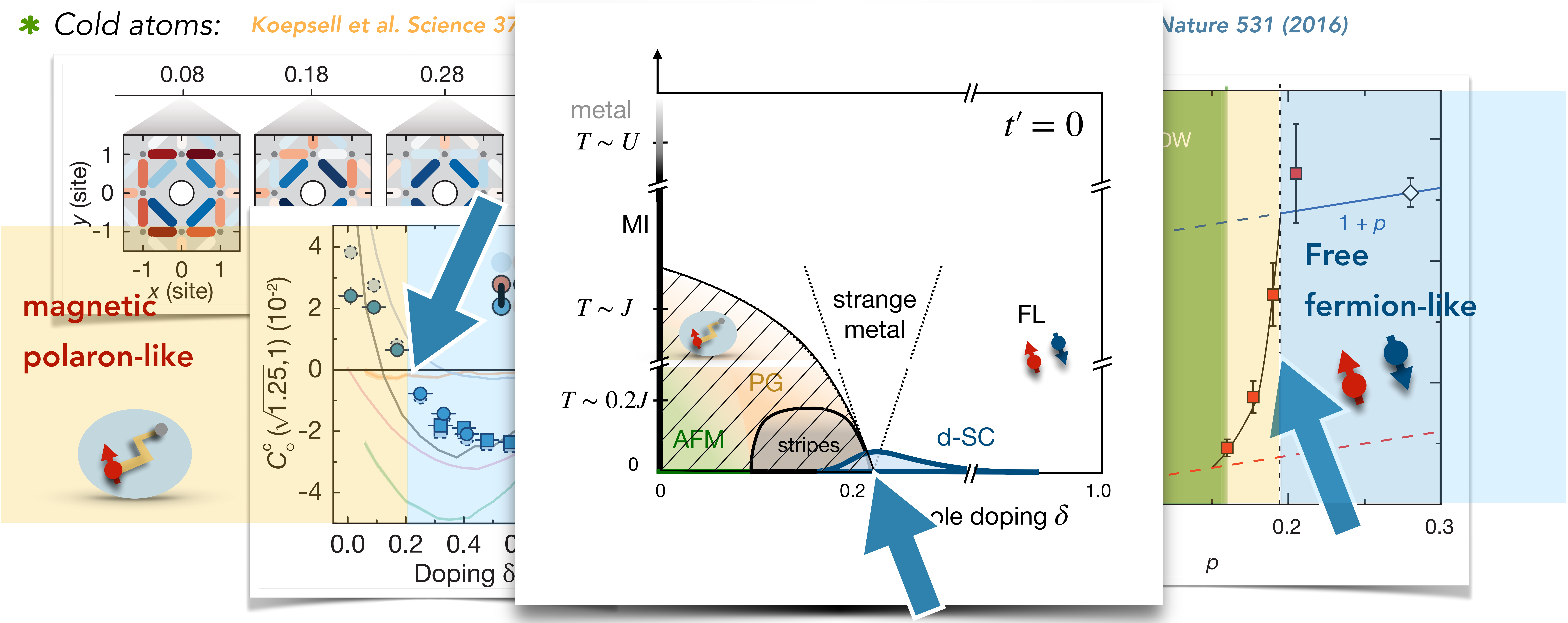


Hubbard phase diagram

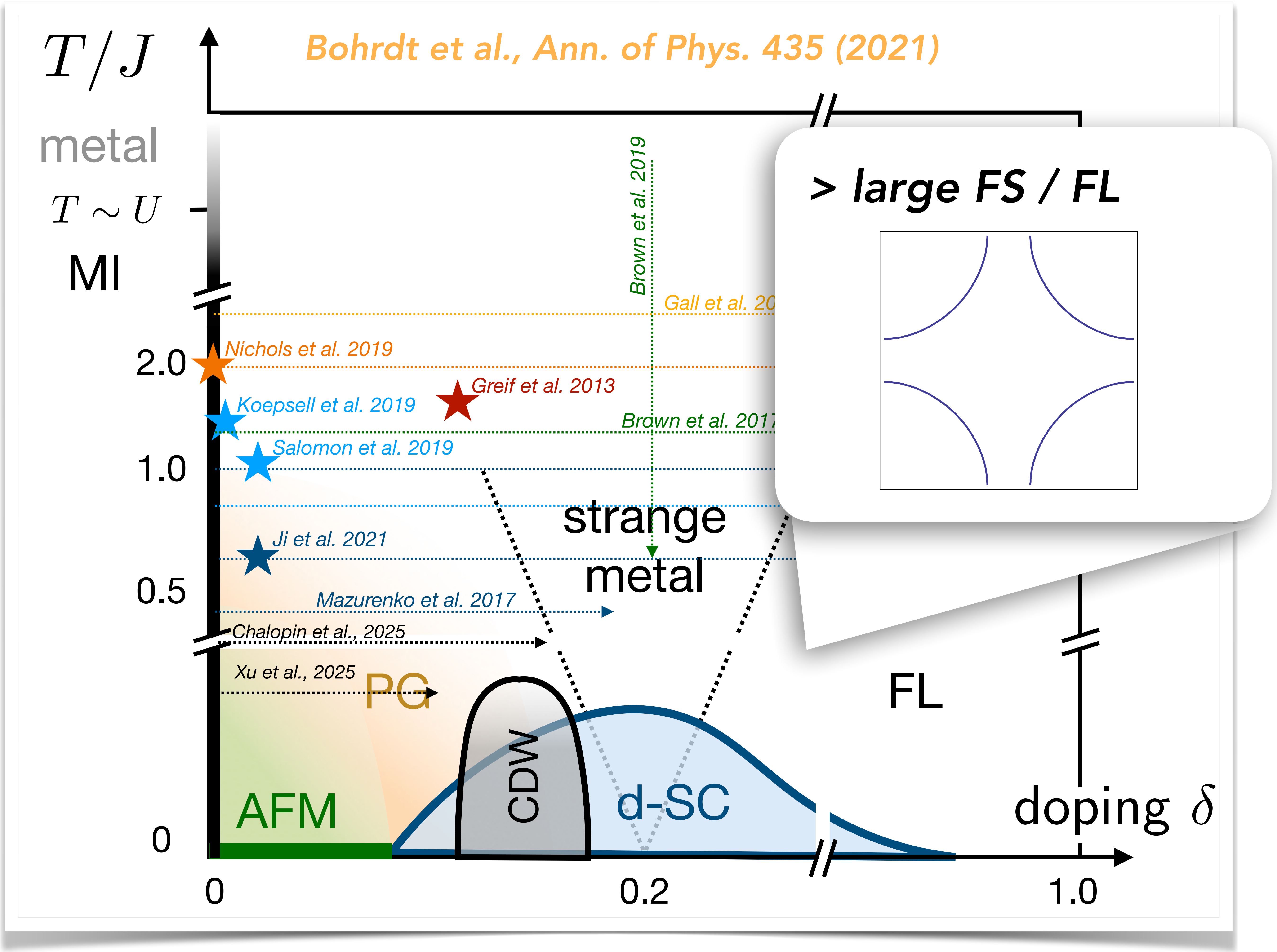
Changing nature of charge carriers: Hubbard / cuprates

* Cold atoms: Koepsell et al. Science 37 … Nature 531 (2016)



Hubbard phase diagram

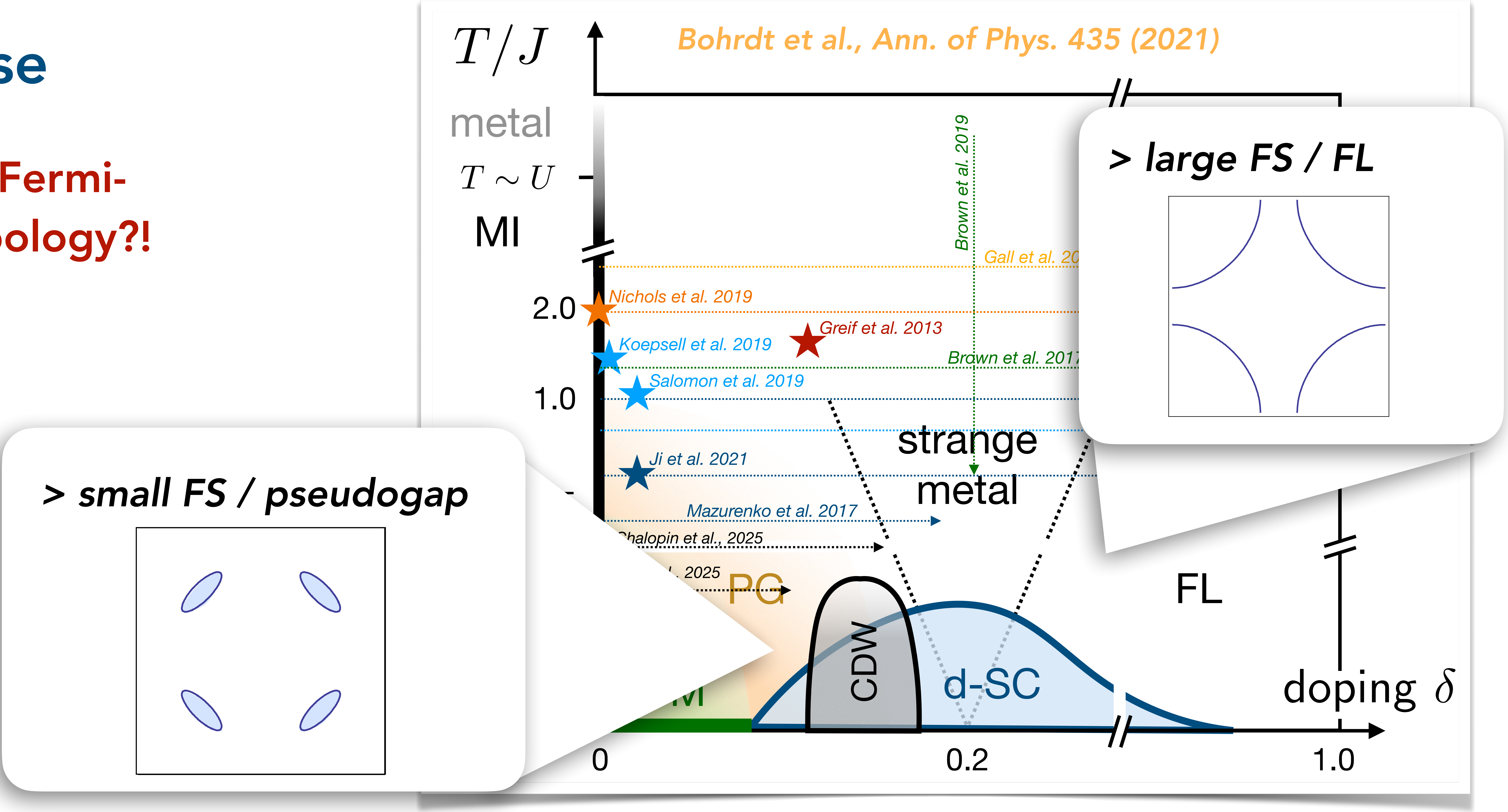
Pseudogap phase



Hubbard phase diagram

Pseudogap phase

Change of Fermi-surface topology?!



Hubbard phase diagram

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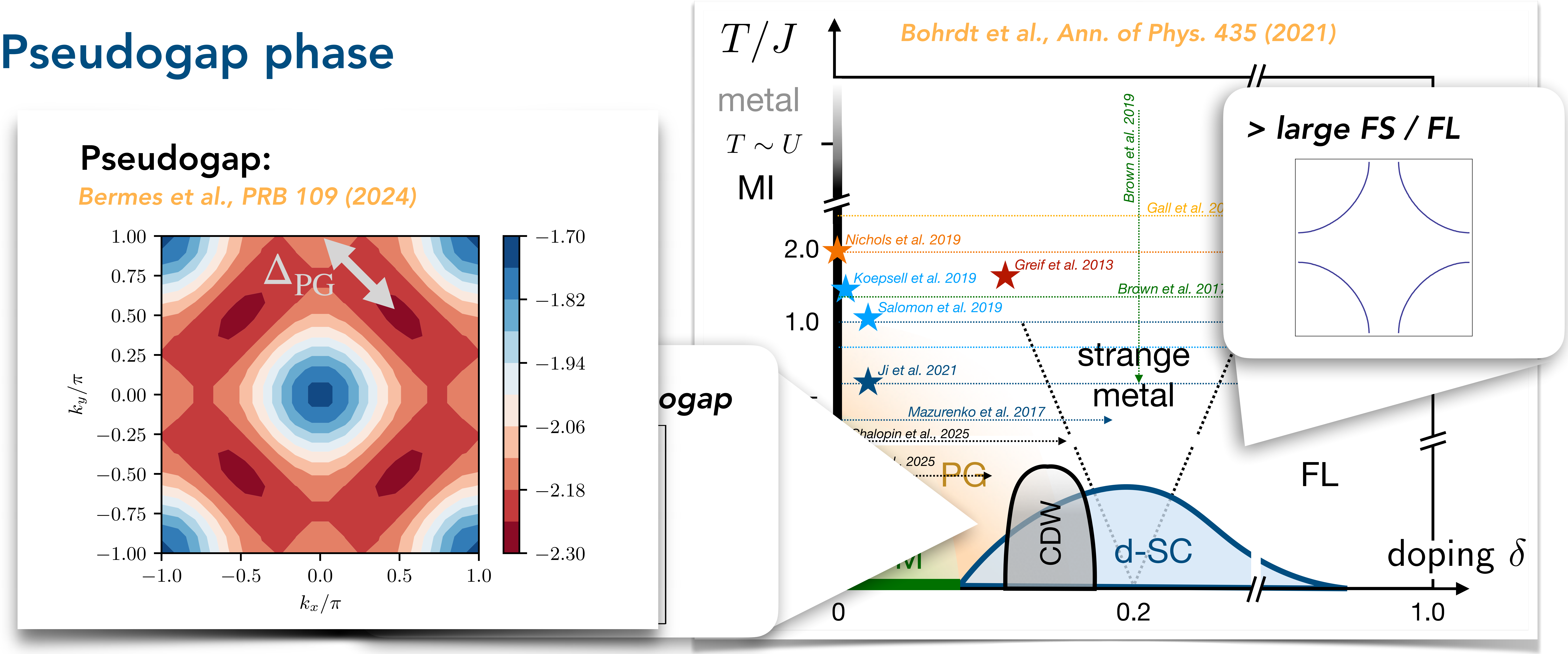
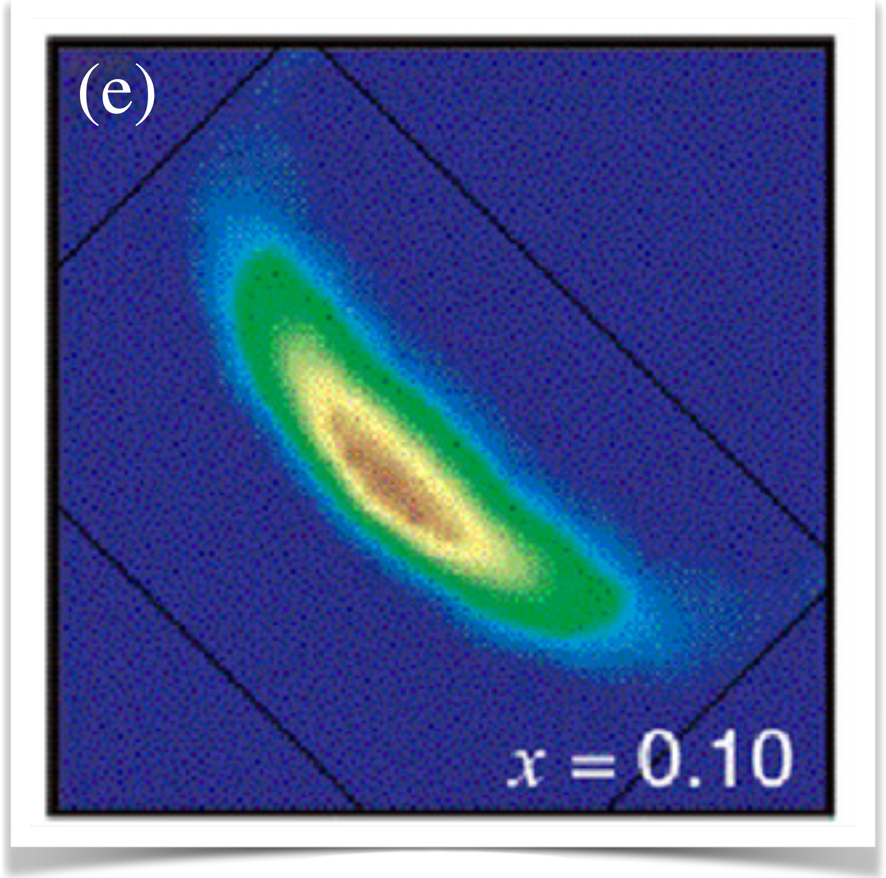


Hubbard phase diagram

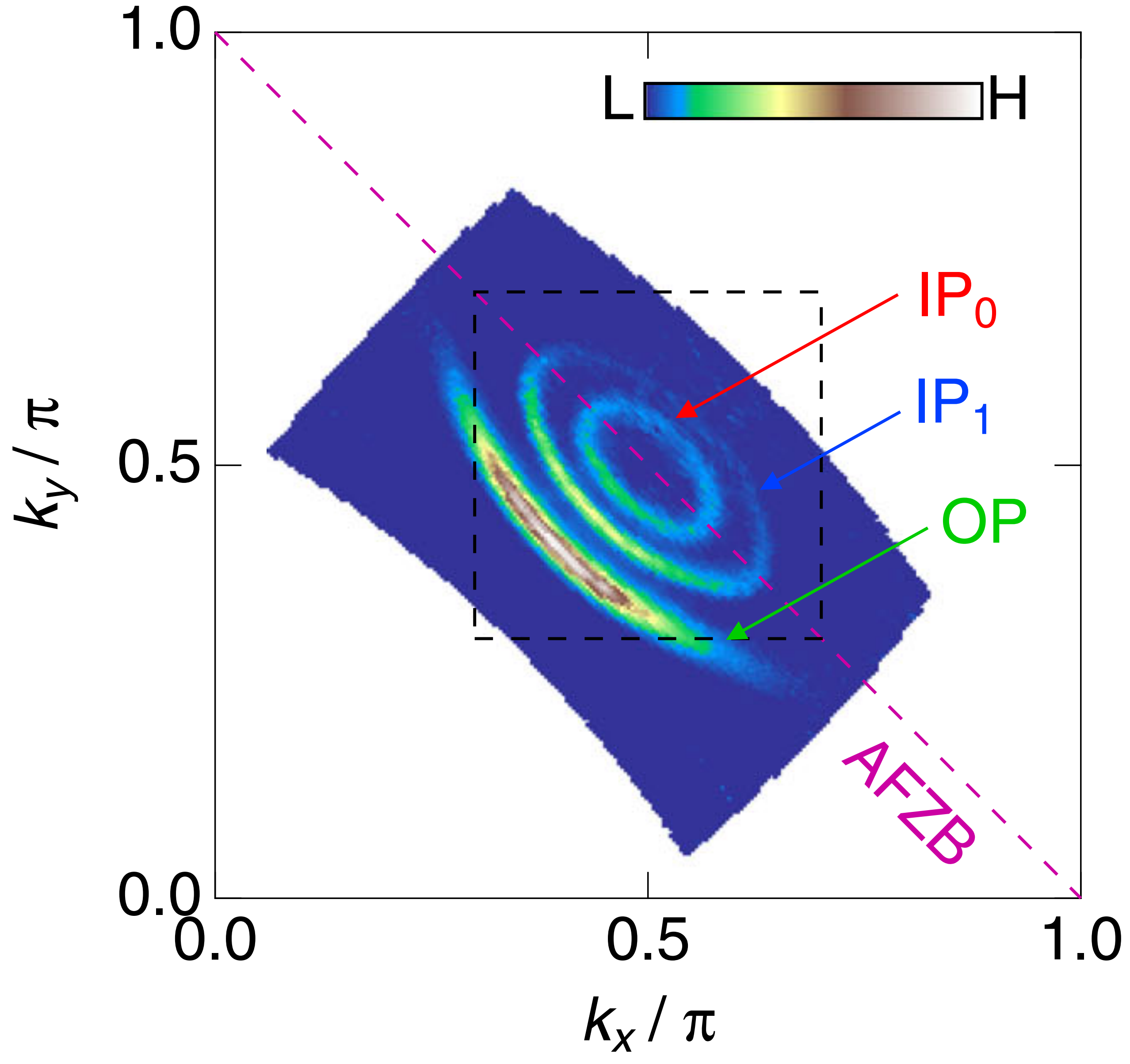
Pseudogap phase

Pseudogap:

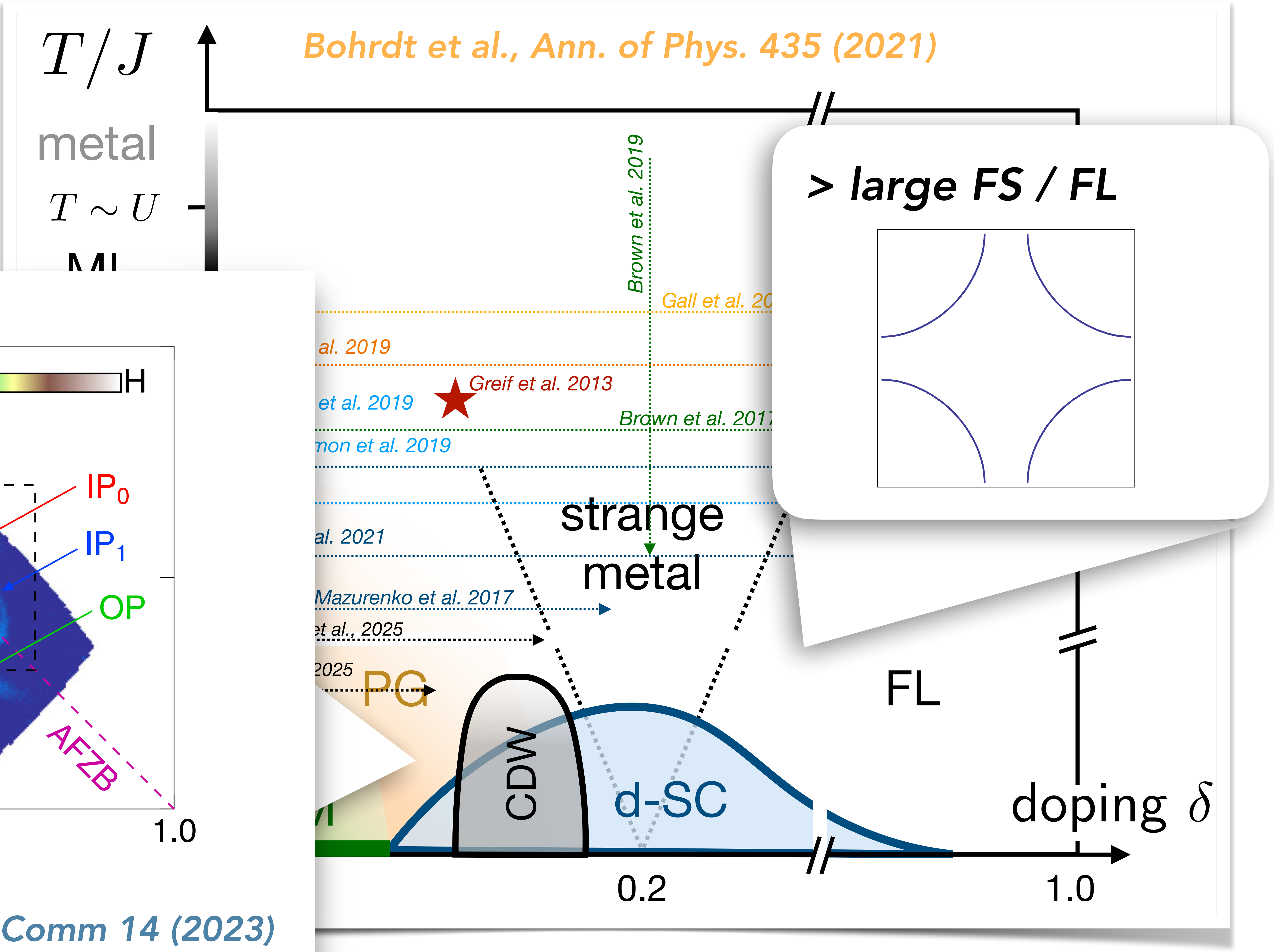
Fermi arcs:



Shen et al., Science (2005)



Kurokawa et al., Nat. Comm 14 (2023)

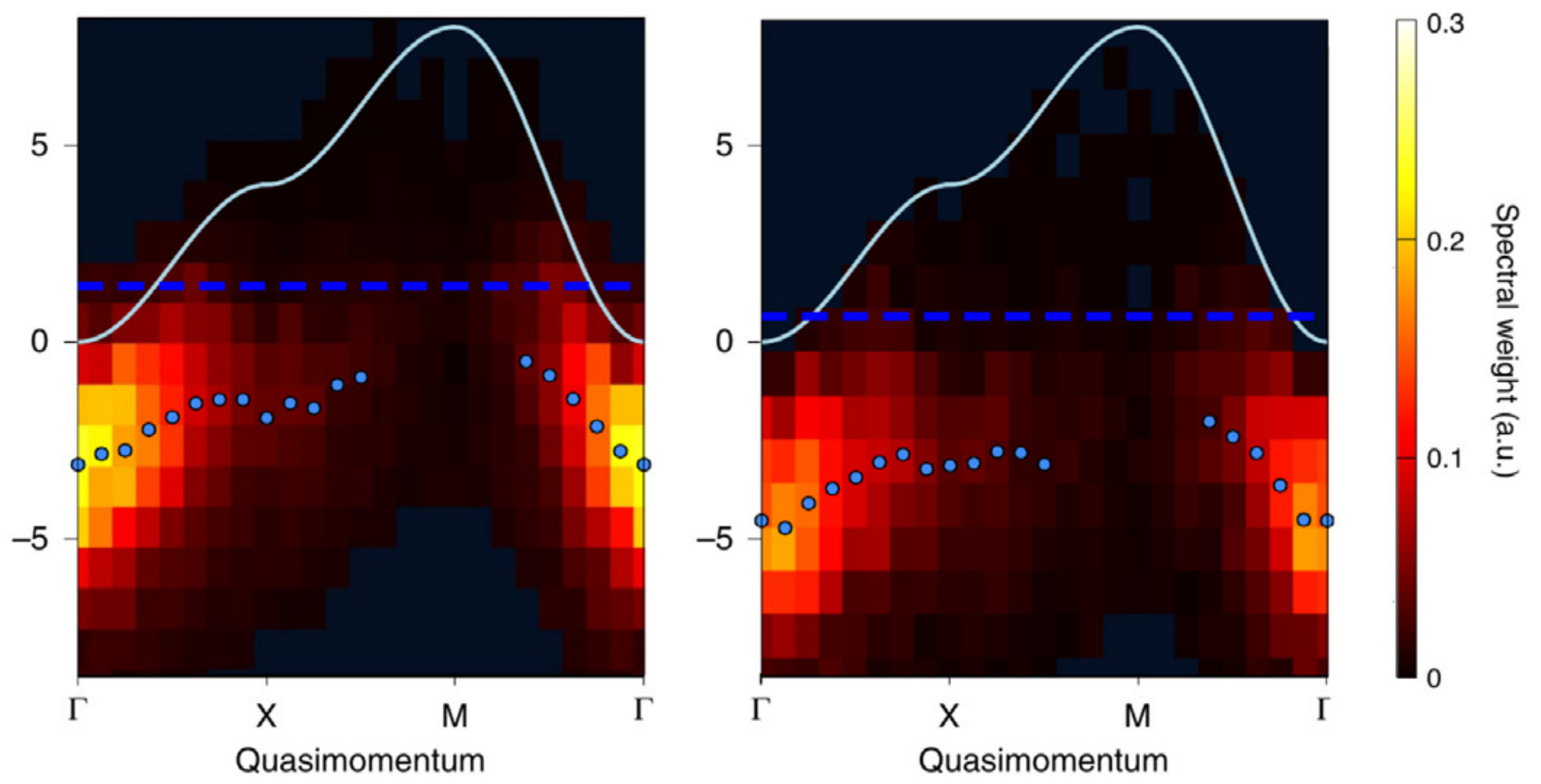


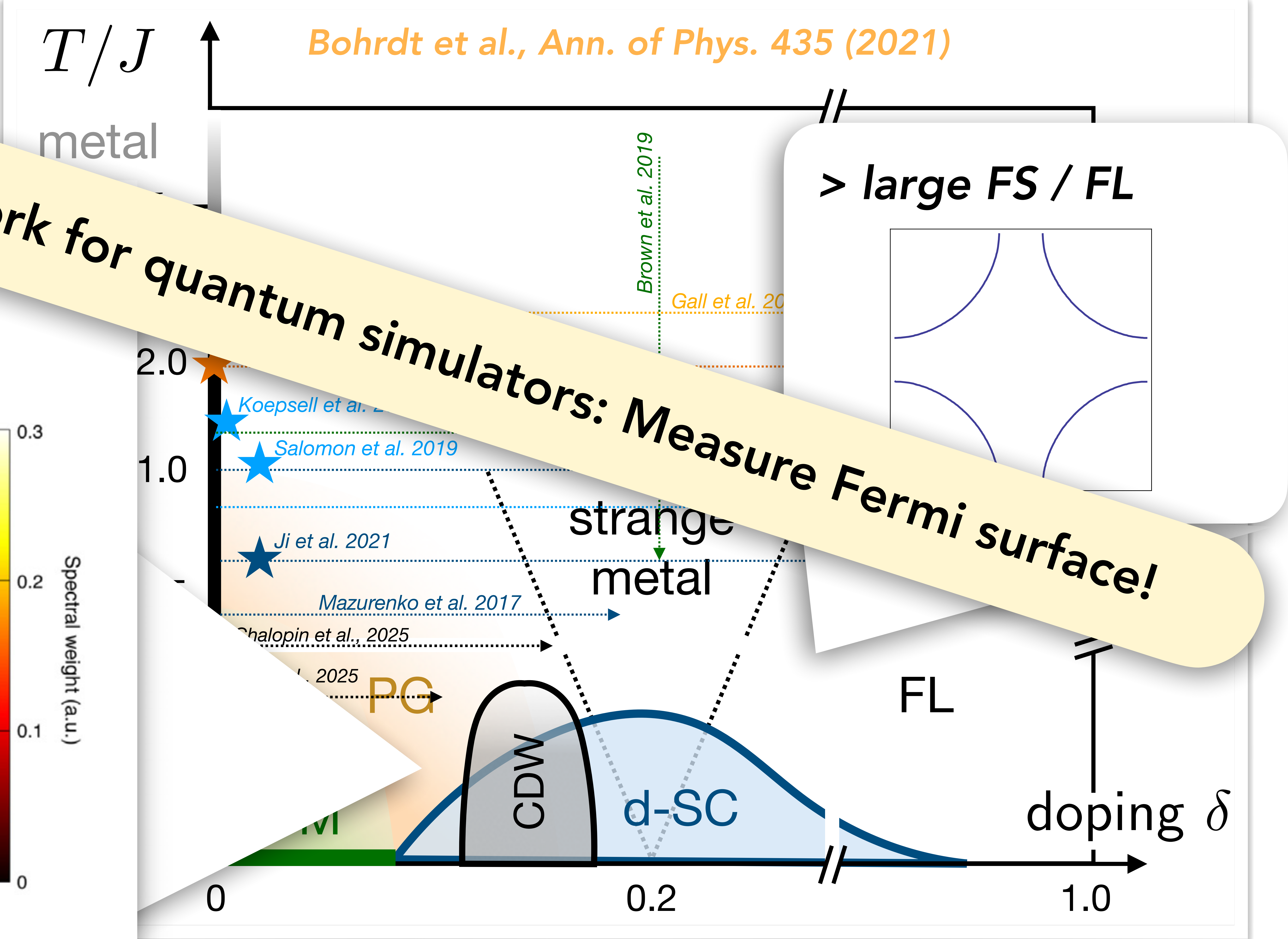
Hubbard phase diagram

Pseudogap phase

Cold-atom ARPES

Brown et al., Science 363 (2019) — Bakr lab





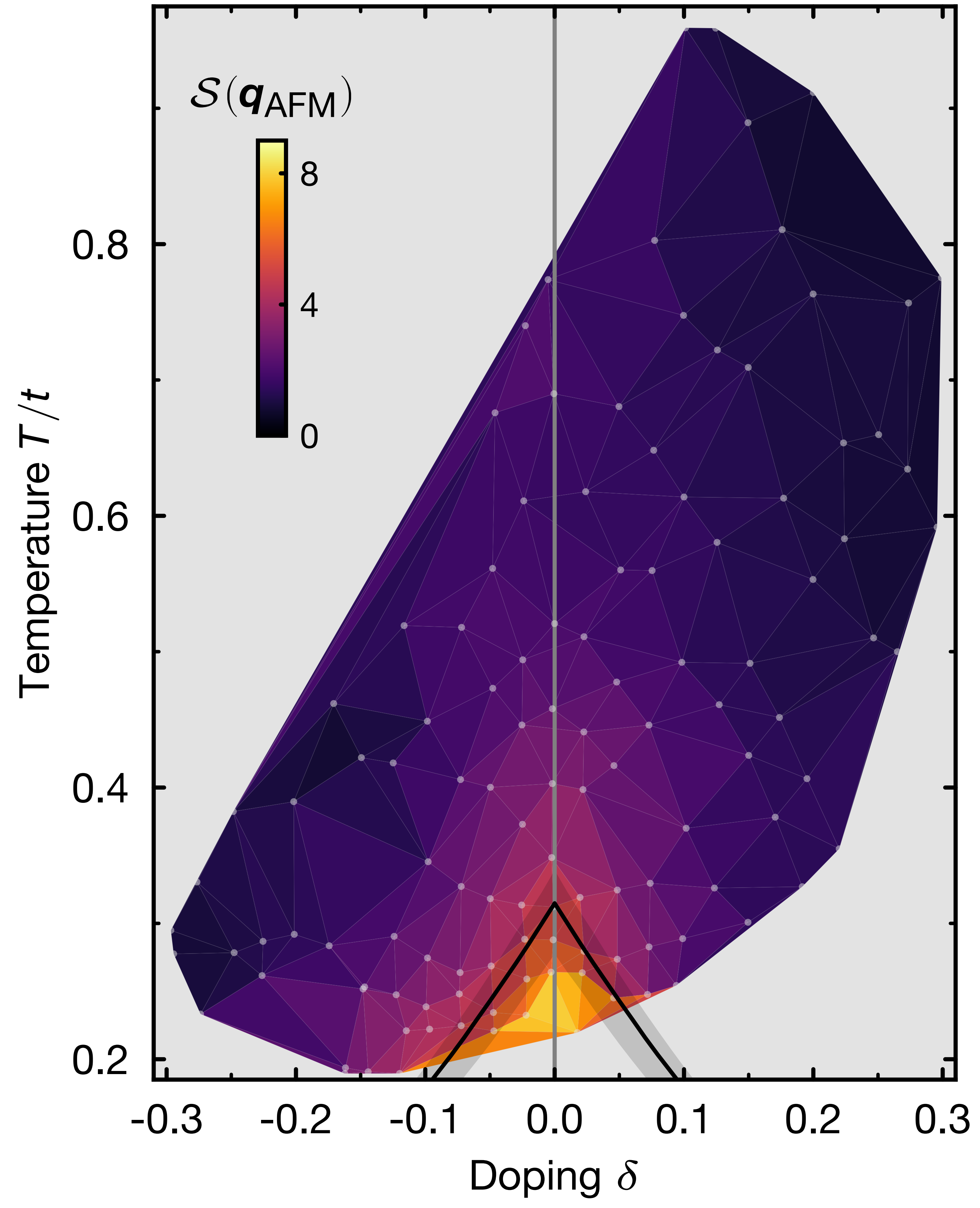
Homework for quantum simulators: Measure Fermi surface!

Hubbard phase diagram

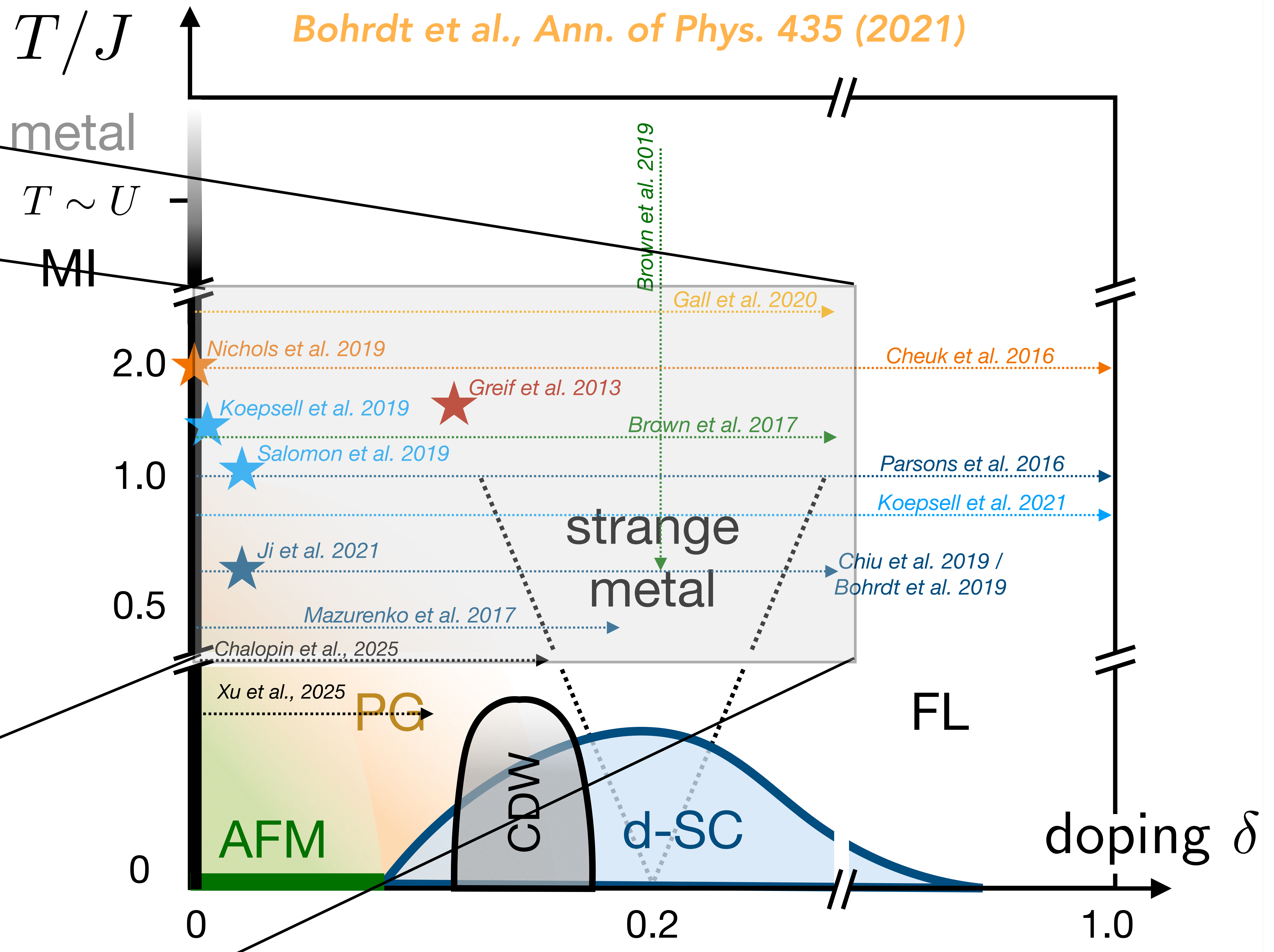
Pseudogap phase

Spin structure factor - universal scaling

Chalopin et al., PNAS 123 (2026) — Bloch lab



Bohrdt et al., Ann. of Phys. 435 (2021)



Hubbard phase diagram

Pseudogap phase

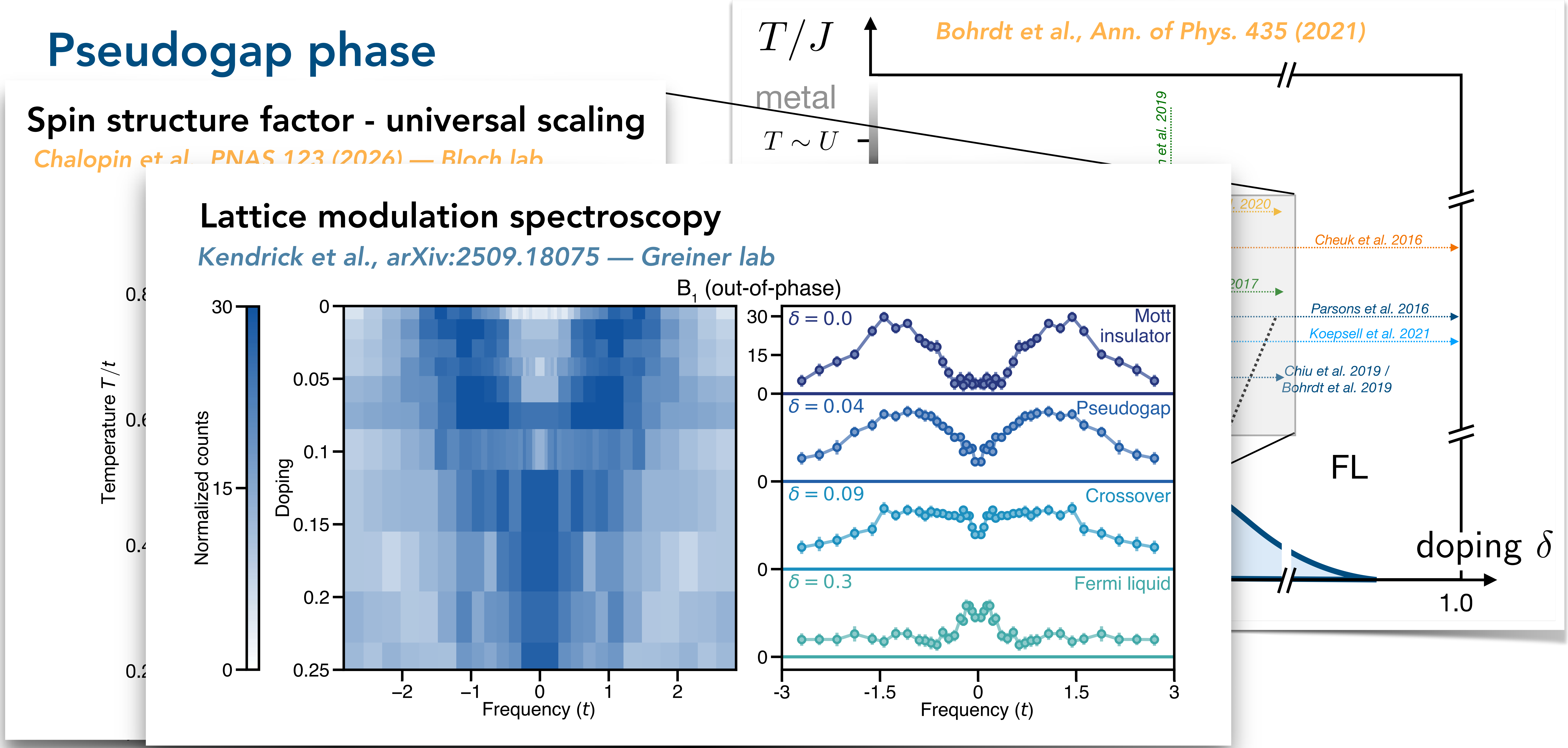
Spin structure factor - universal scaling

Chalopin et al., PNAS 123 (2026) — Bloch lab

Lattice modulation spectroscopy

Kendrick et al., arXiv:2509.18075 — Greiner lab

Bohrdt et al., Ann. of Phys. 435 (2021)



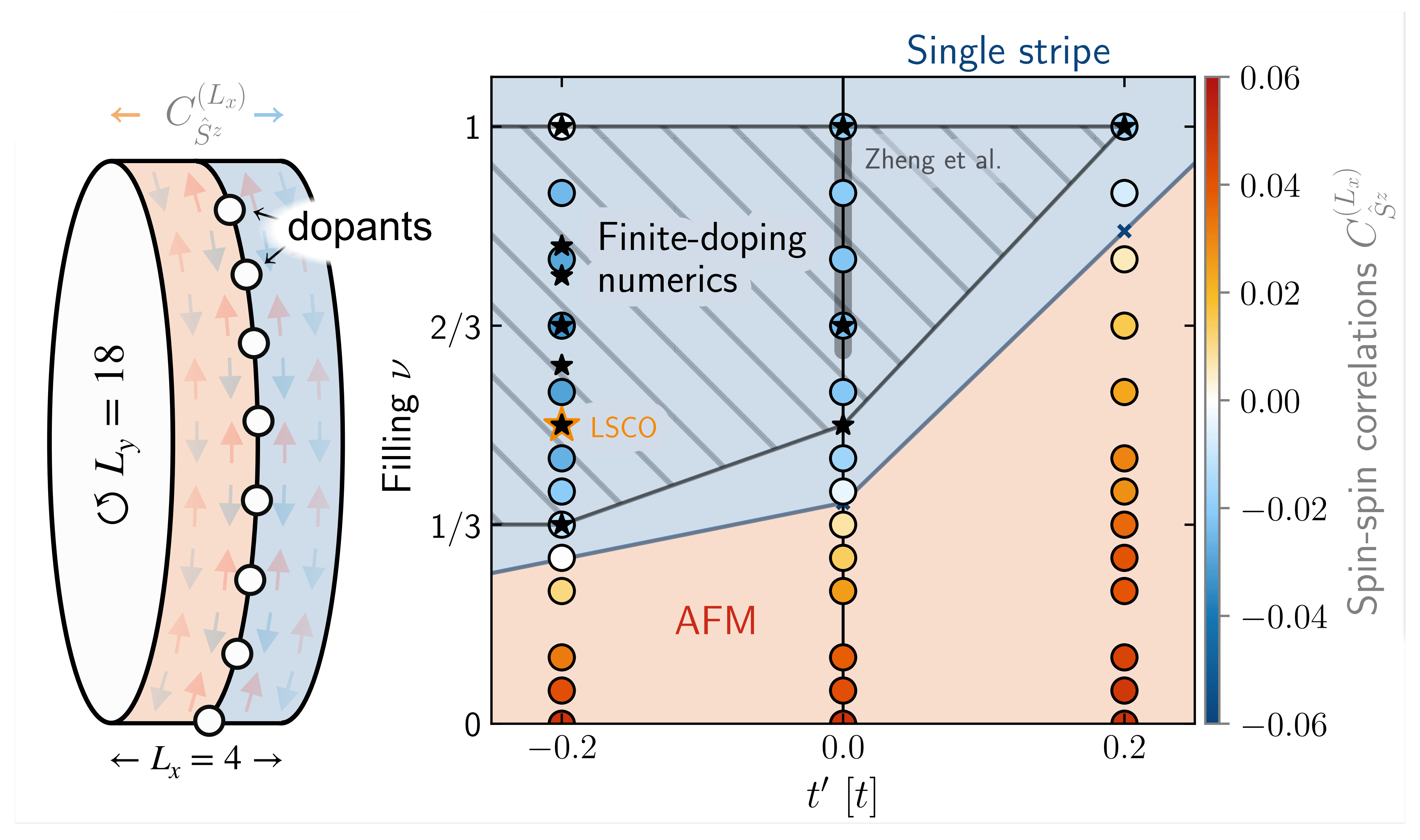
Hubbard phase diagram

Stripes in Hubbard models

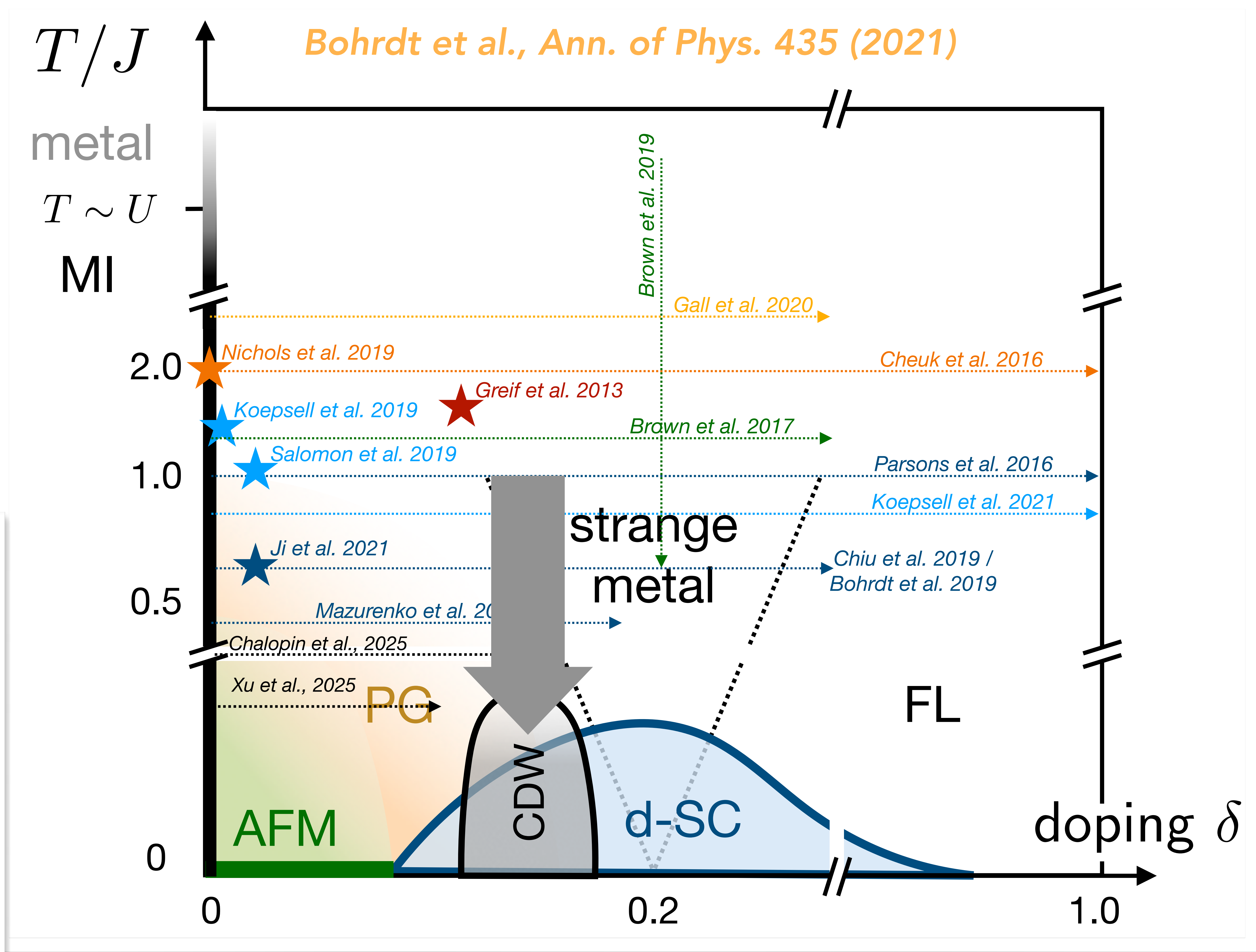
- *Not yet seen in vanilla Hubbard simulators*
- *Strong numerical evidence at T=0*

Corboz et al., PRB 84 (2011); Huang et al., Science 358 (2017); Zheng et al., Science 358 (2017); Xu et al., PRR 4 (2022)

Blatz et al., arXiv:2512:15714 (2025)



Bohrdt et al., Ann. of Phys. 435 (2021)

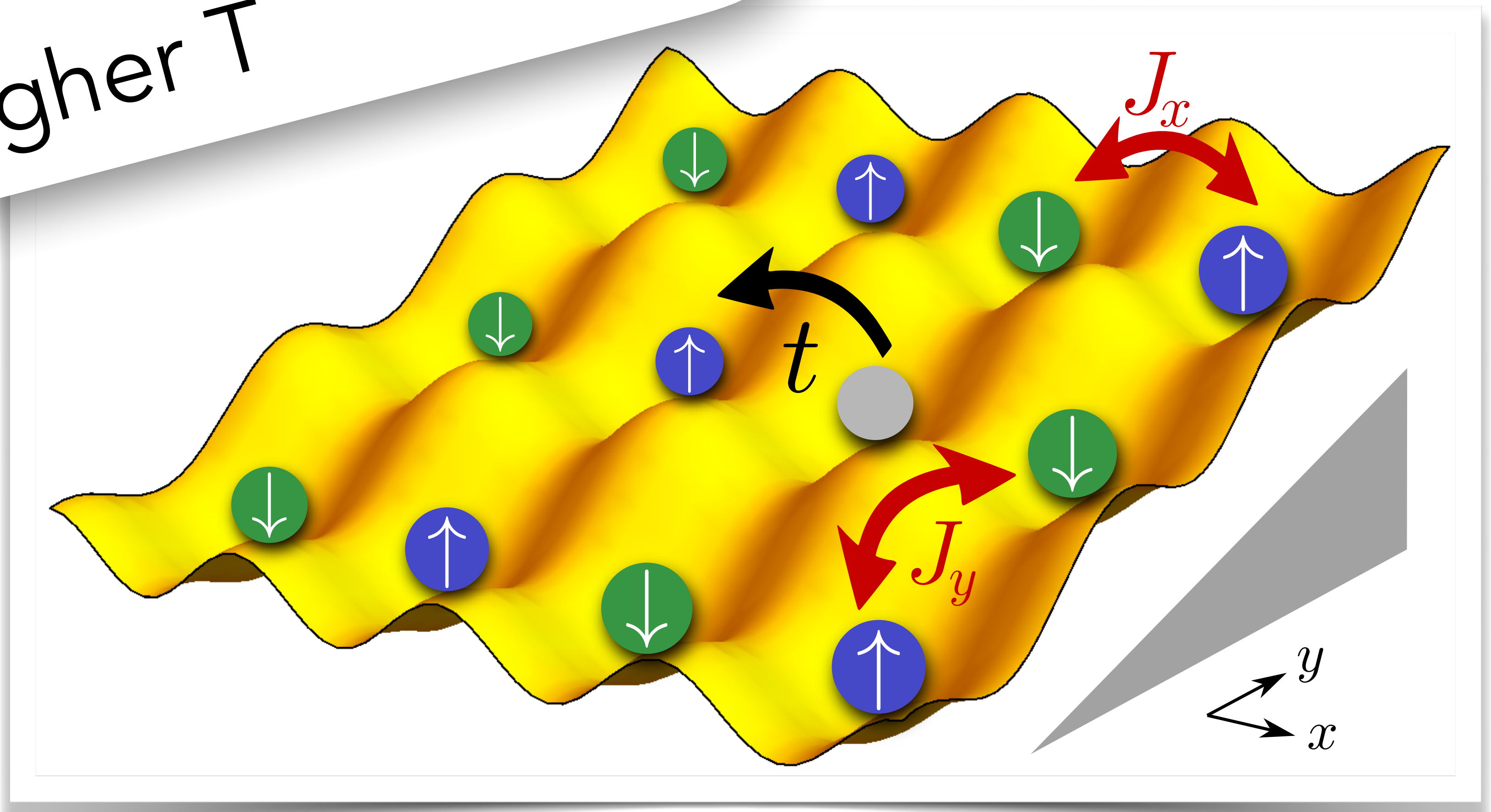


Hubbard phase diagram

Stripes in mixed-dimensional (mixD) Hubbard models

Trick: Less frustration, stable up to higher T

* **Suppressed** charge motion along *y*!



Grusdt et al., SciPost Phys. 5 (2018)

Hubbard phase diagram

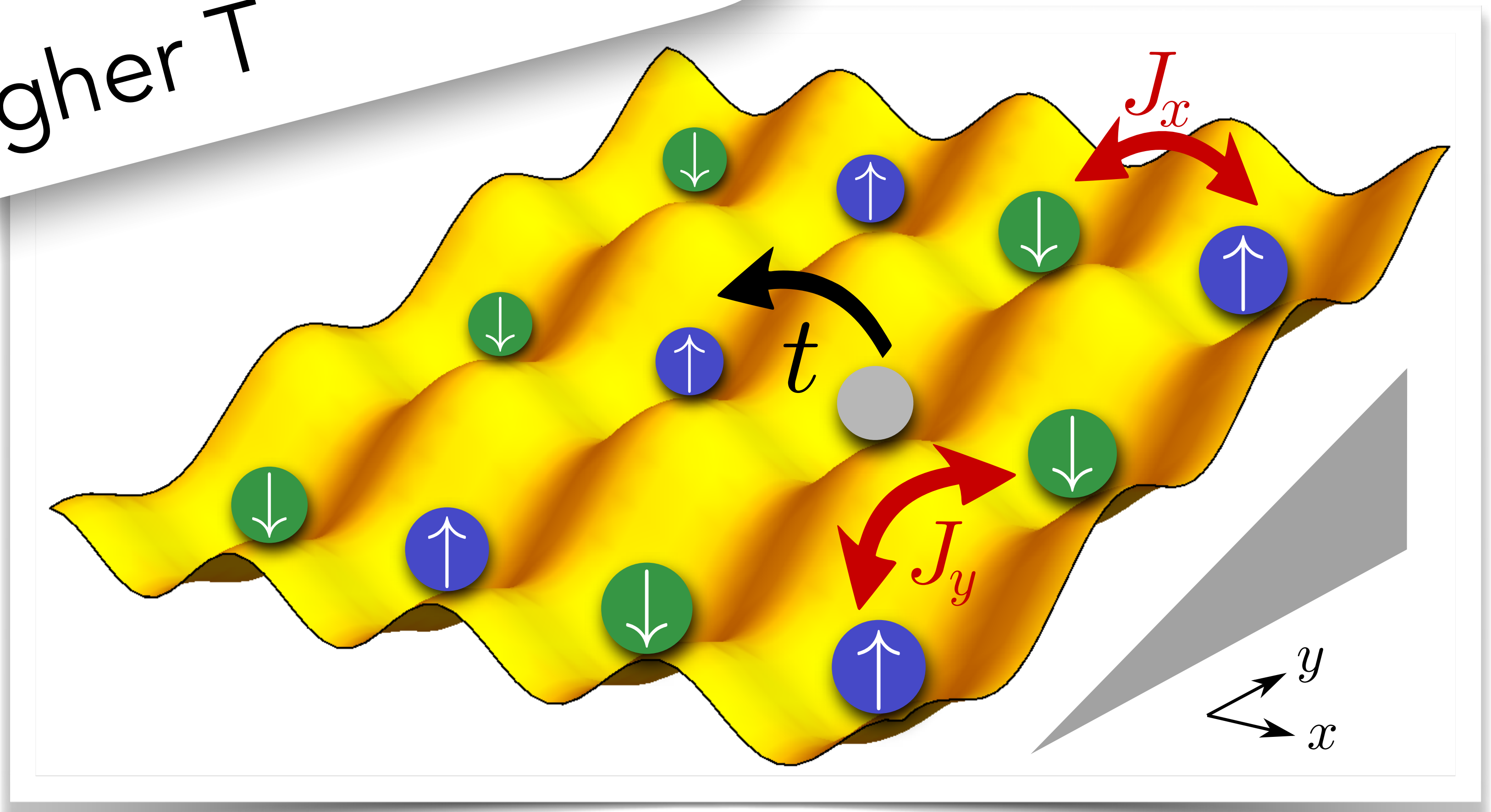
Stripes in mixed-dimensional (mixD) Hubbard models

Trick: Less frustration, stable up to higher T

* **Suppressed** charge motion along *y*!
* **Tunable super-exchange** interactions along *y*!

$$J_x = \frac{4t^2}{U}, \qquad J_y = \frac{2t^2}{U+\Delta} + \frac{2t^2}{U-\Delta}$$

see also: Dimitrova et al., PRL 124 (2020)
Trotzky et al., Science 319 (2008)



Grusdt et al., SciPost Phys. 5 (2018)

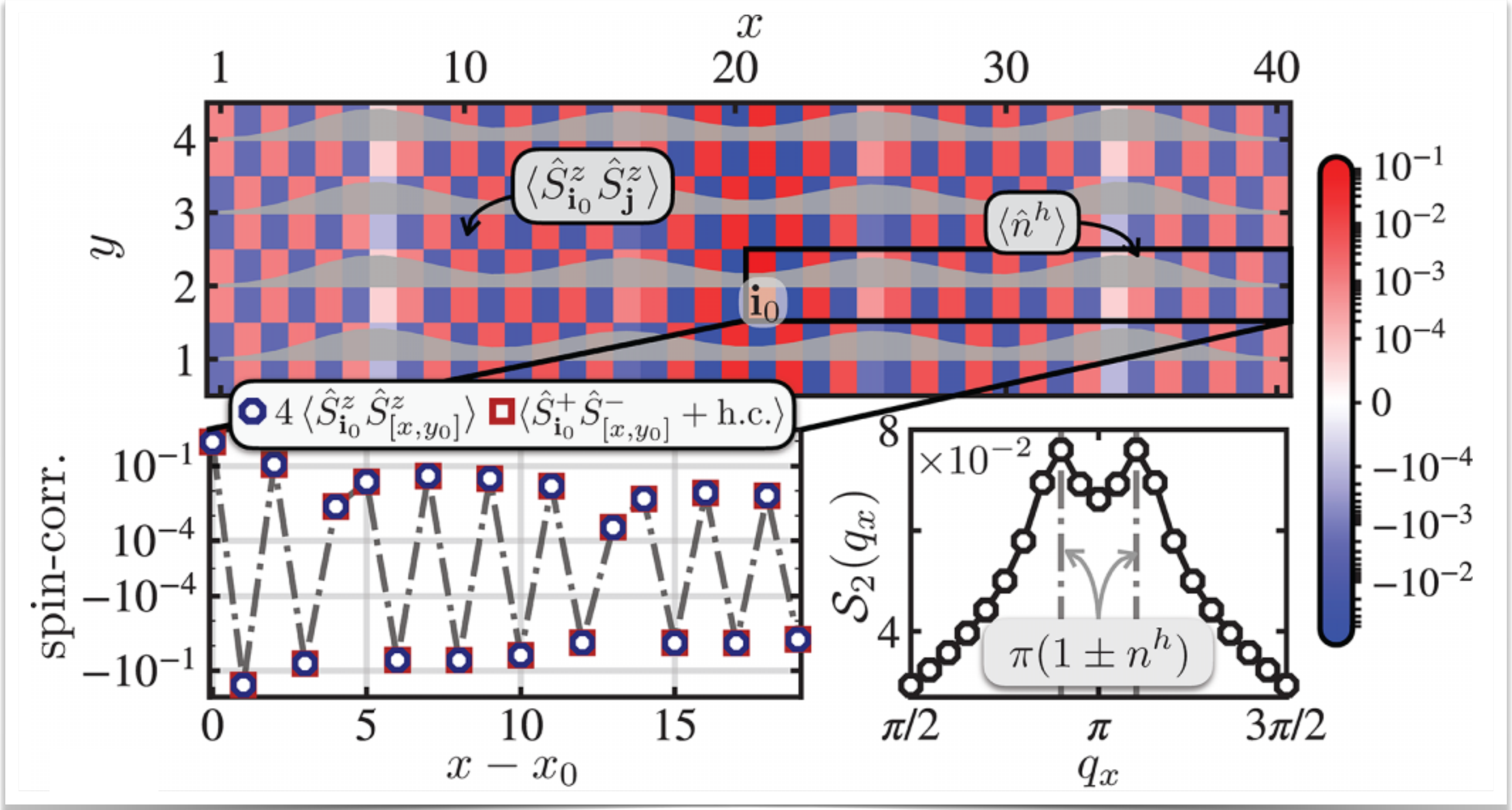
* Effective mixD *t*-*J* Hamiltonian:

$$\hat{\mathcal{H}} = \sum_{\langle \boldsymbol{i},\boldsymbol{j}\rangle_x} \left[-t \sum_{\sigma} \hat{\mathcal{P}}_{\mathrm{GW}} \left(\hat{c}^{\dagger}_{\boldsymbol{i},\sigma} \hat{c}_{\boldsymbol{j},\sigma} + \mathrm{h.c.} \right) \hat{\mathcal{P}}_{\mathrm{GW}} + J_x \left(\hat{\boldsymbol{S}}_{\boldsymbol{i}} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{j}} - \frac{\hat{n}_{\boldsymbol{i}} \hat{n}_{\boldsymbol{j}}}{4} \right) \right] + J_y \sum_{\langle \boldsymbol{i},\boldsymbol{j}\rangle_y} \left(\hat{\boldsymbol{S}}_{\boldsymbol{i}} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{j}} - \frac{\hat{n}_{\boldsymbol{i}} \hat{n}_{\boldsymbol{j}}}{4} \right)$$

Hubbard phase diagram

Stripes in mixed-dimensional (mixD) Hubbard models

Schlömer et al., PRR 5, L022027 (2023)



- Ground state DMRG: incommensurate AFM

Hubbard phase diagram

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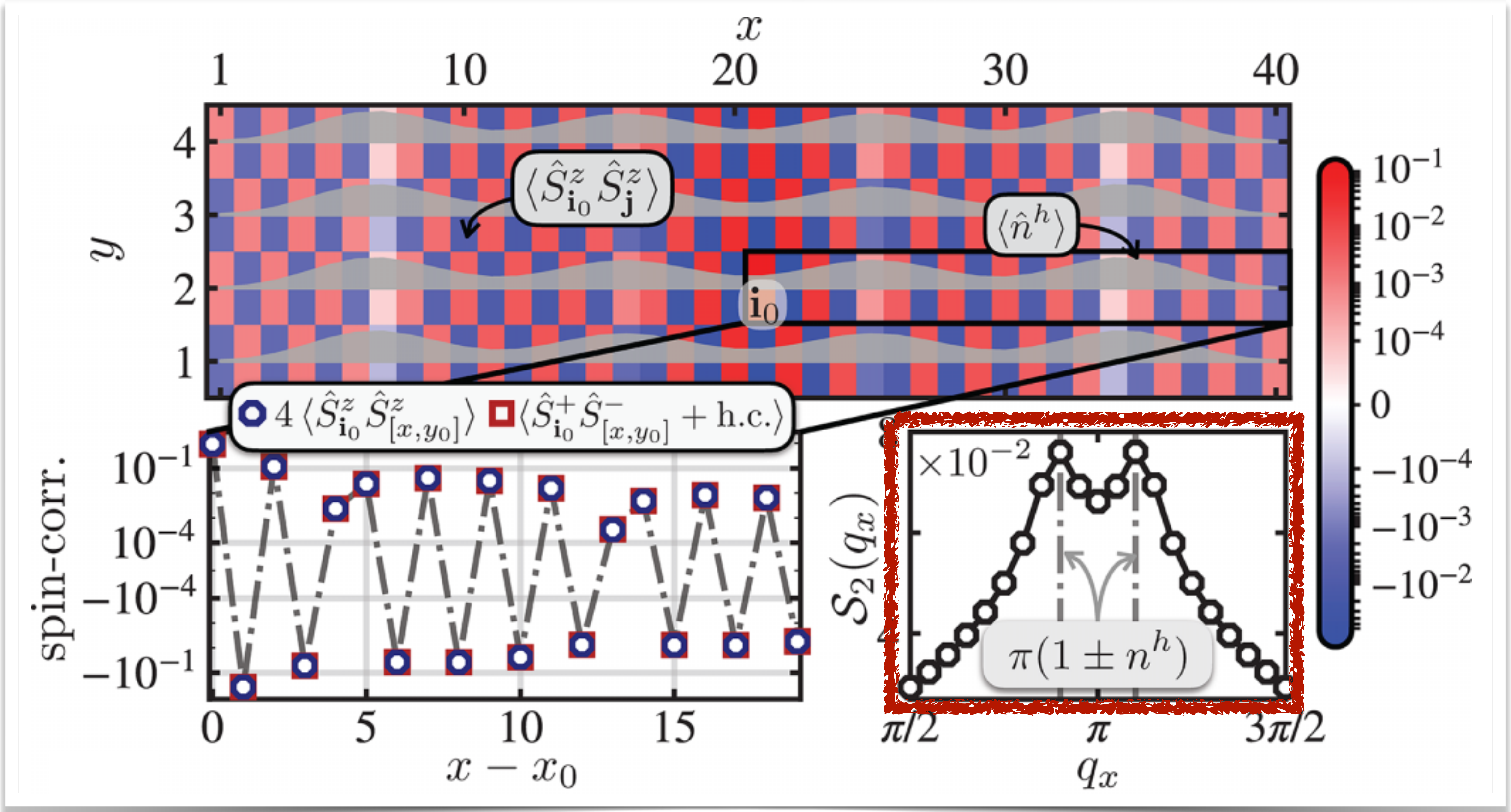

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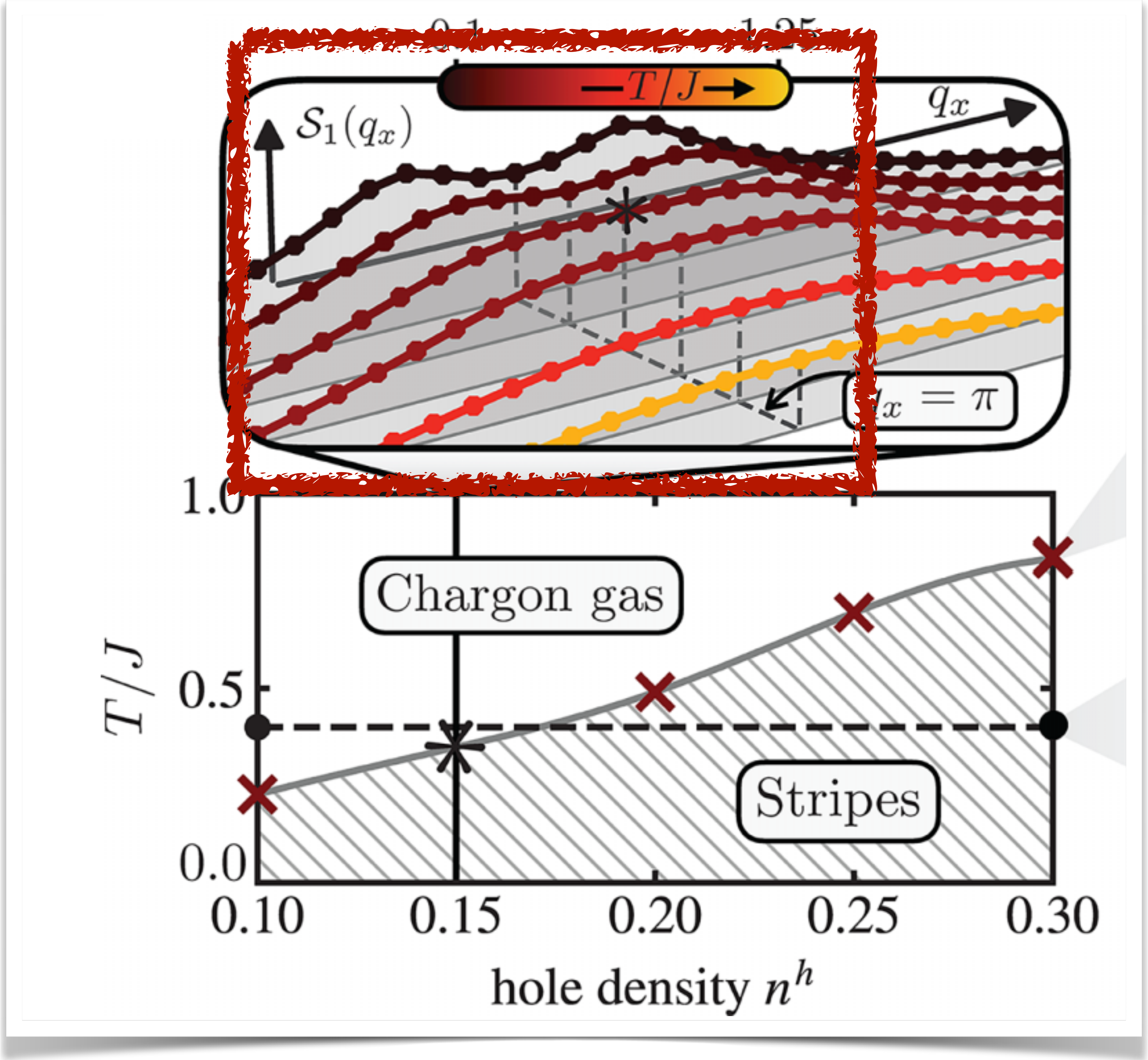


- Ground state DMRG: incommensurate AFM
- Finite-temperature DMRG simulations:

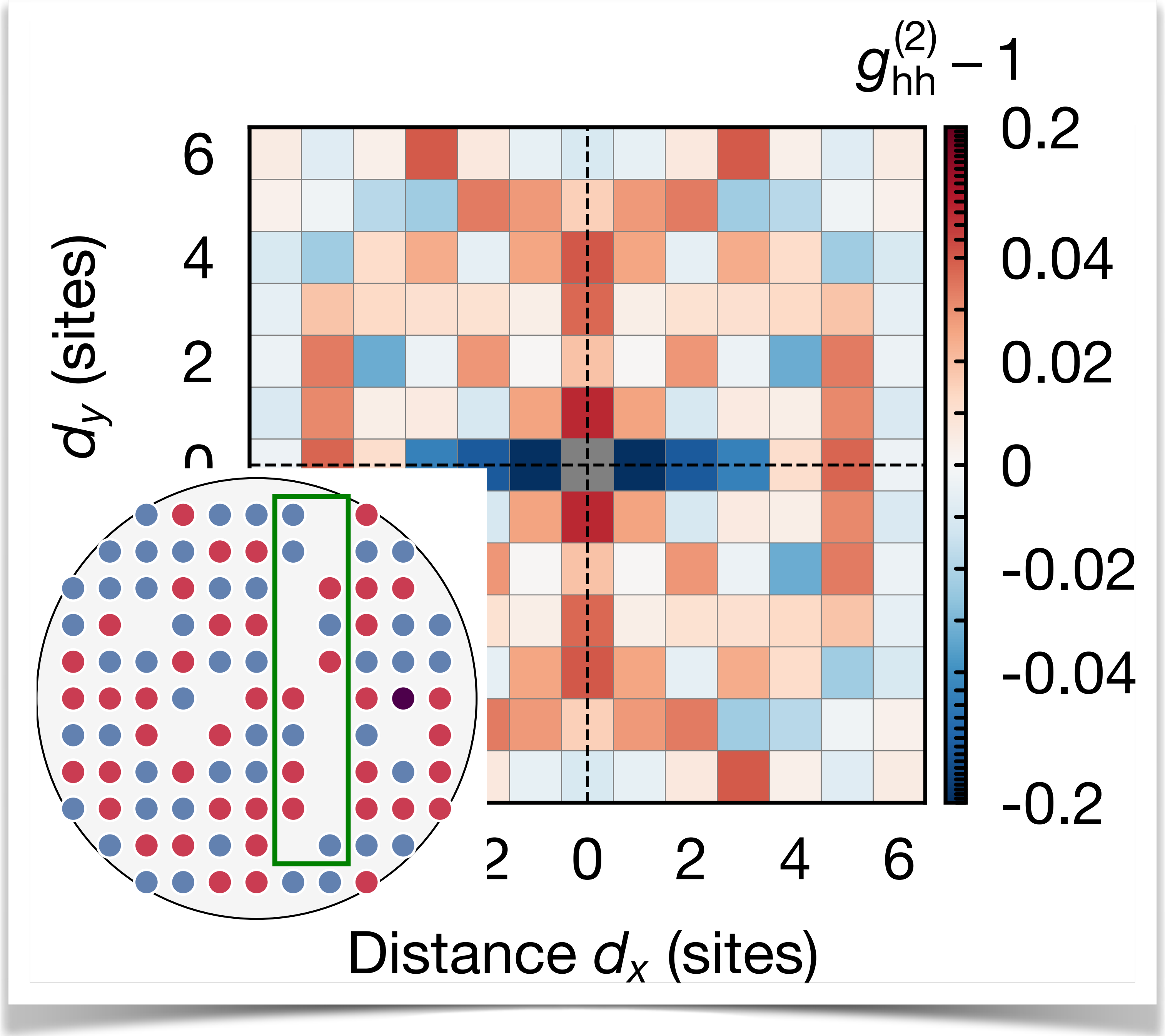
High critical temp:

$$T_c \simeq 0.8J$$

Hubbard phase diagram

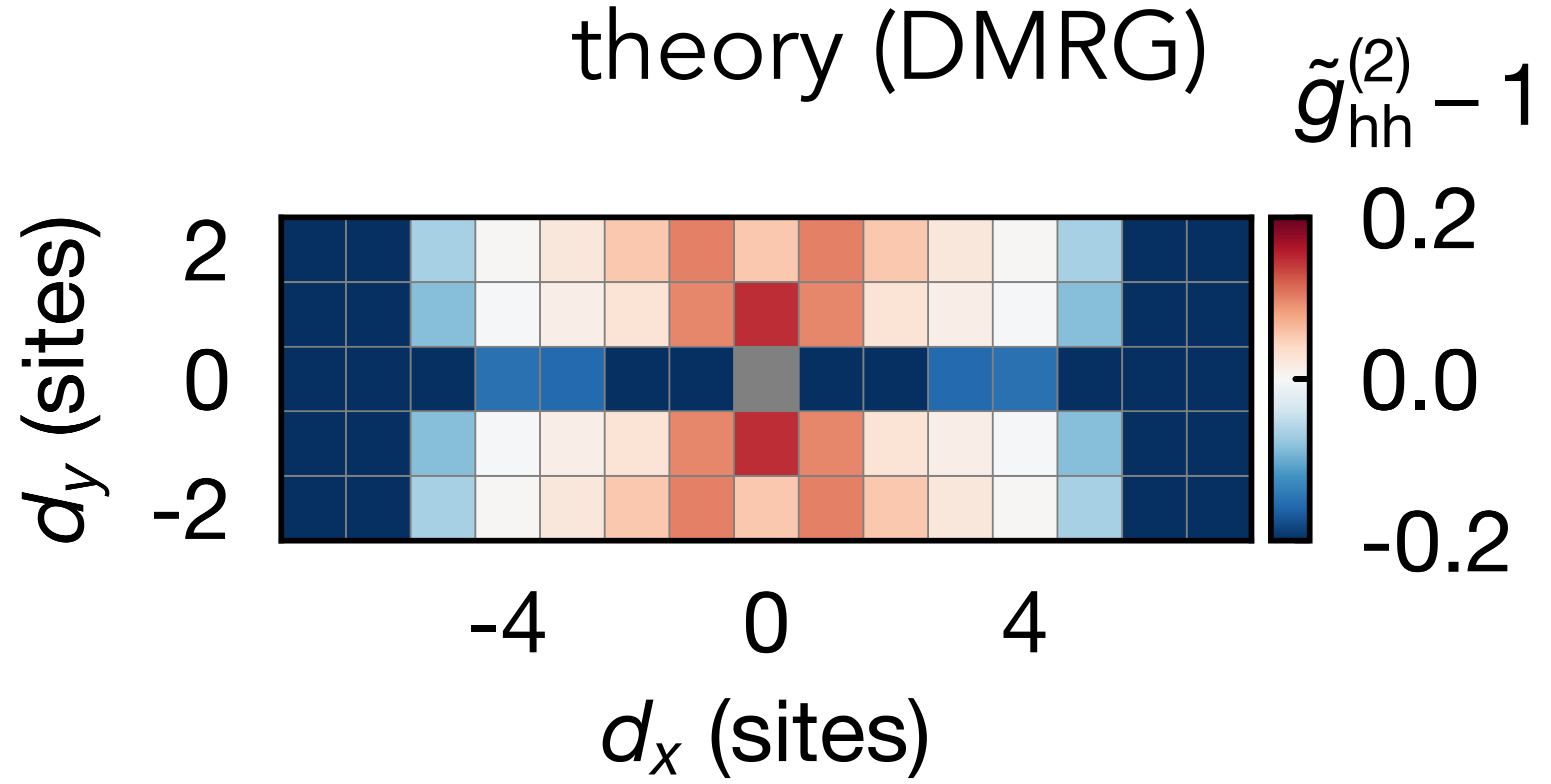
Stripes in mixed-dimensional (mixD) Hubbard models

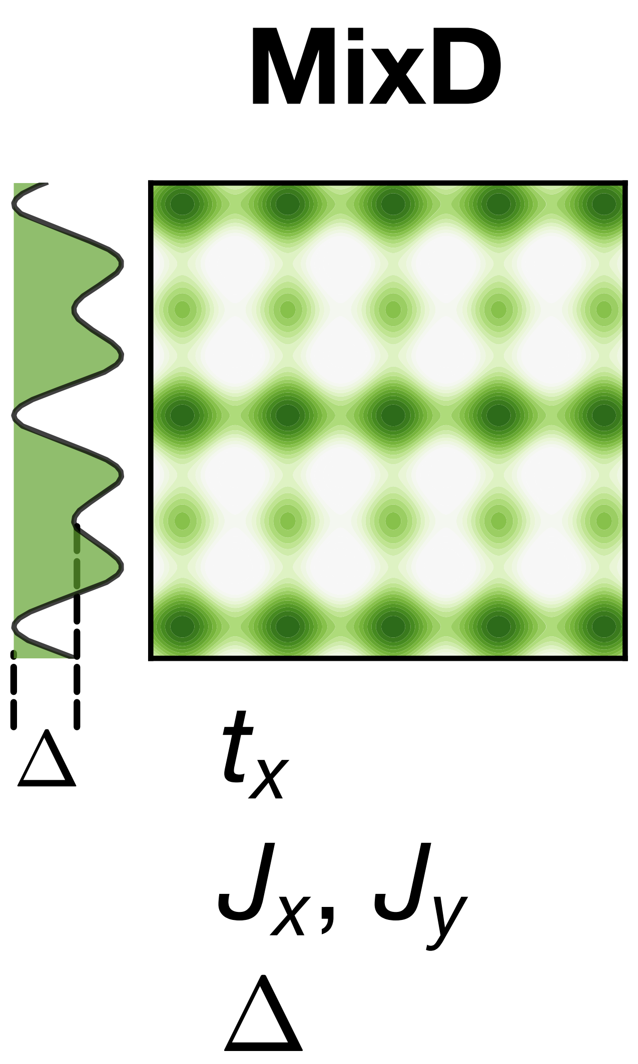
Bourgund et al., Nature 637 (2025) — Bloch lab



- Hole bunching across chains

theory (DMRG)

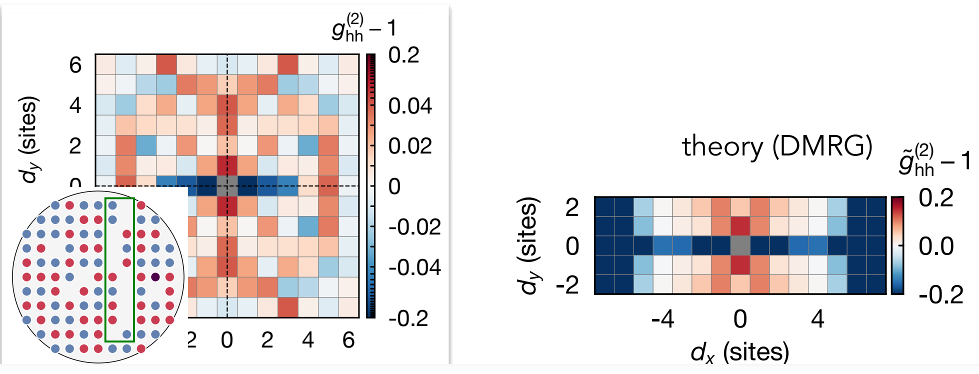


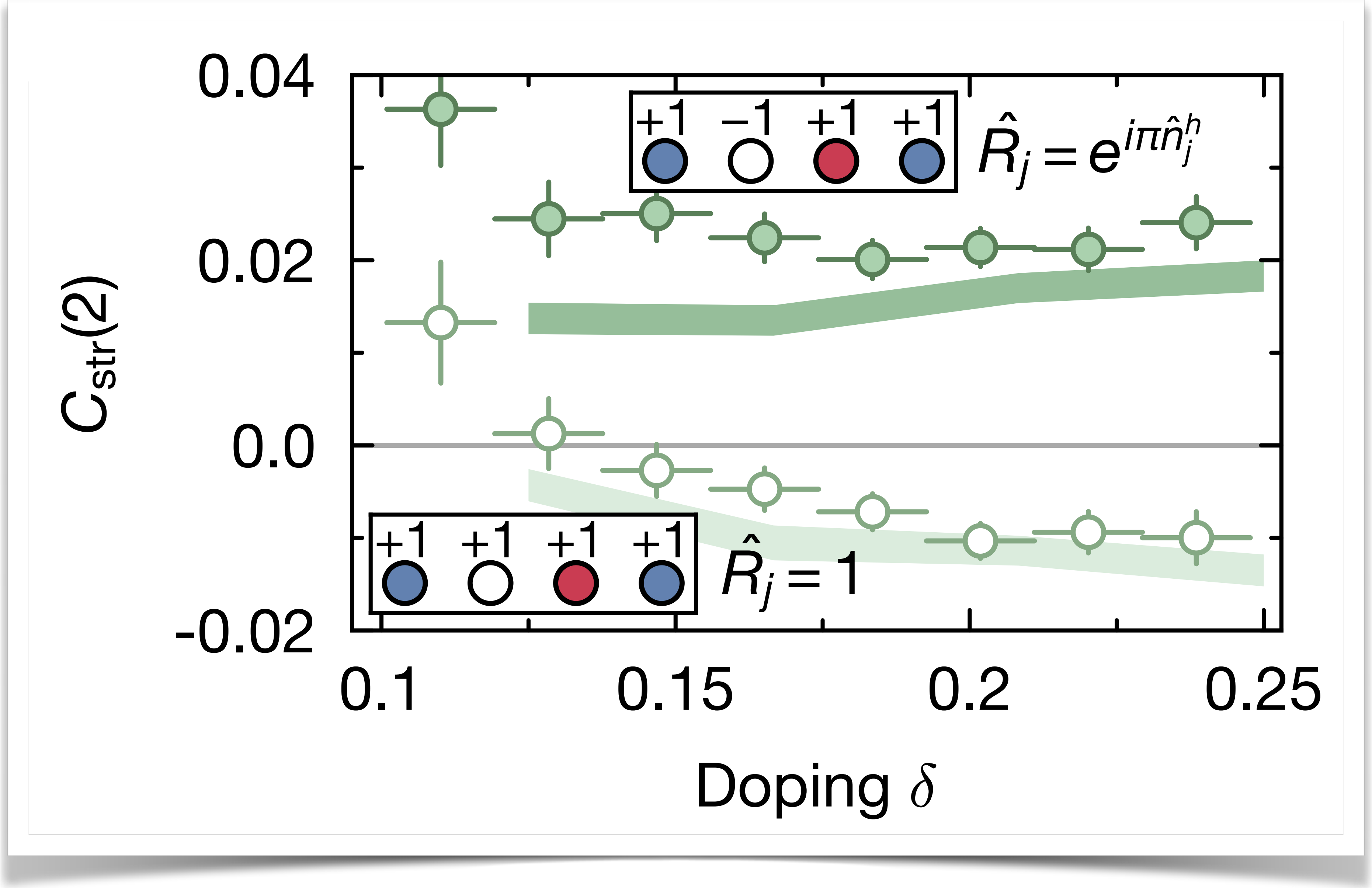


Hubbard phase diagram

Stripes in mixed-dimensional (mixD) Hubbard models

Bourgund et al., Nature 637 (2025) — Bloch lab



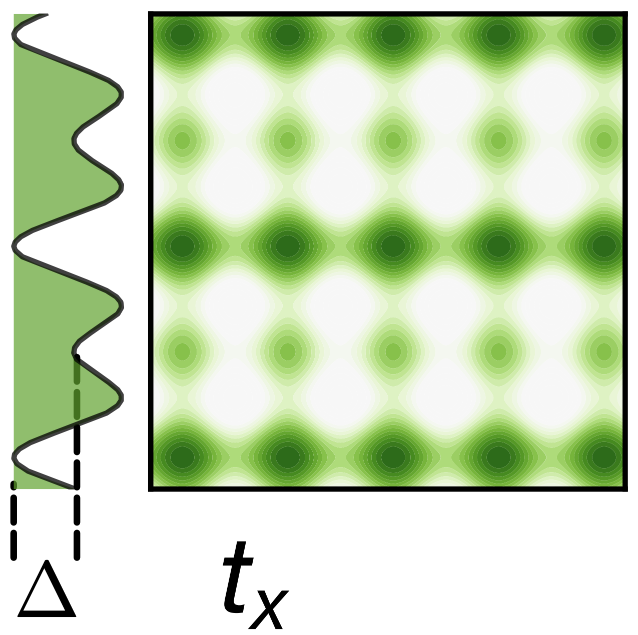


✱ Hole bunching across chains

✱ Hidden correlations

$$C_{\mathrm{str}}(d) = \frac{1}{\mathcal{N}_d} \sum_i \frac{\langle \hat{S}^z_i \left(\prod_{j=1}^{d-1} \hat{R}_{i+j} \right) \hat{S}^z_{i+d} \rangle - \langle \hat{S}^z_i \rangle \langle \hat{S}^z_{i+d} \rangle}{\sigma(\hat{S}^z_i)\sigma(\hat{S}^z_{i+d})}$$

MixD



t_x

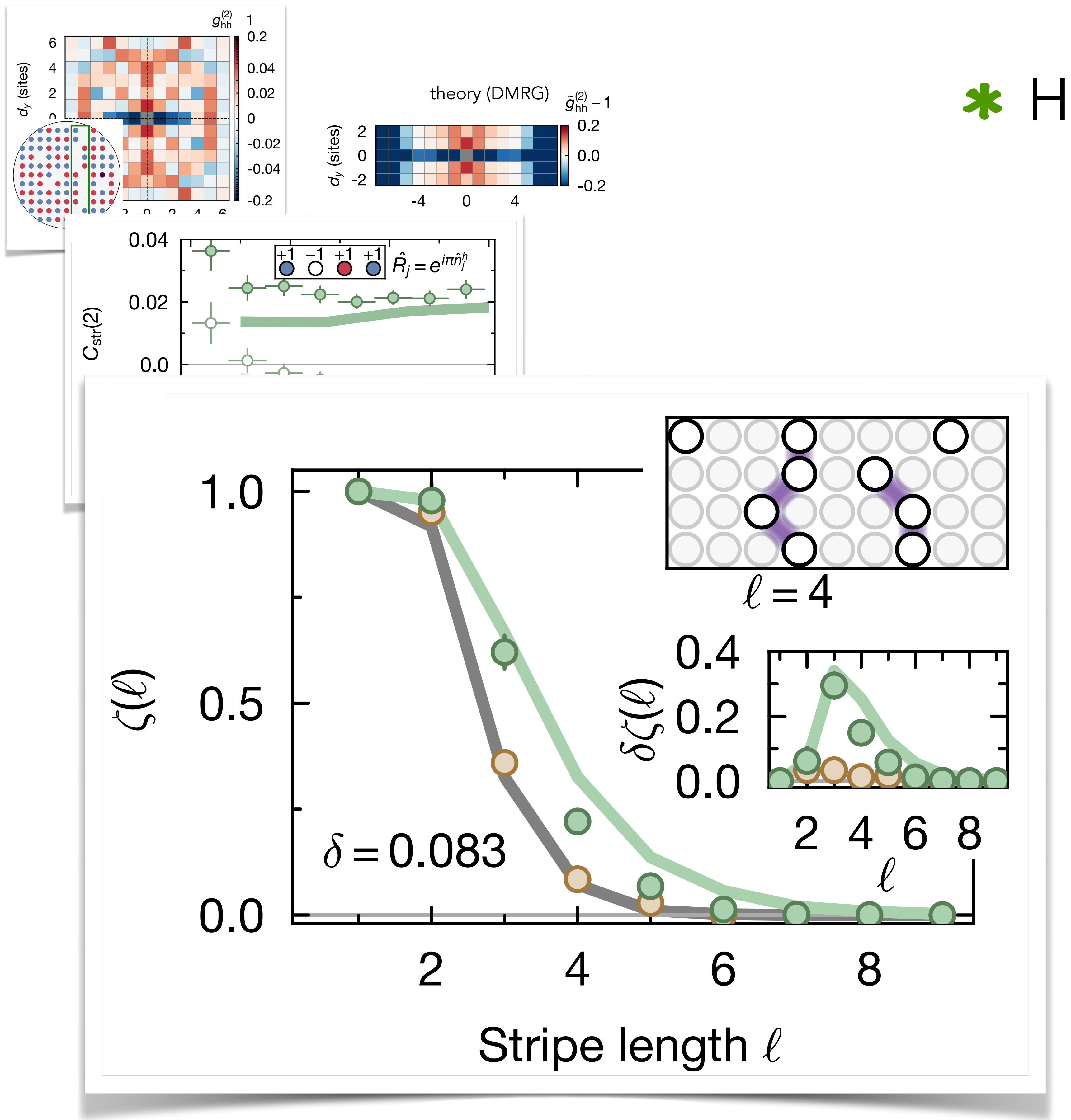
J_x, J_y

Δ

Hubbard phase diagram

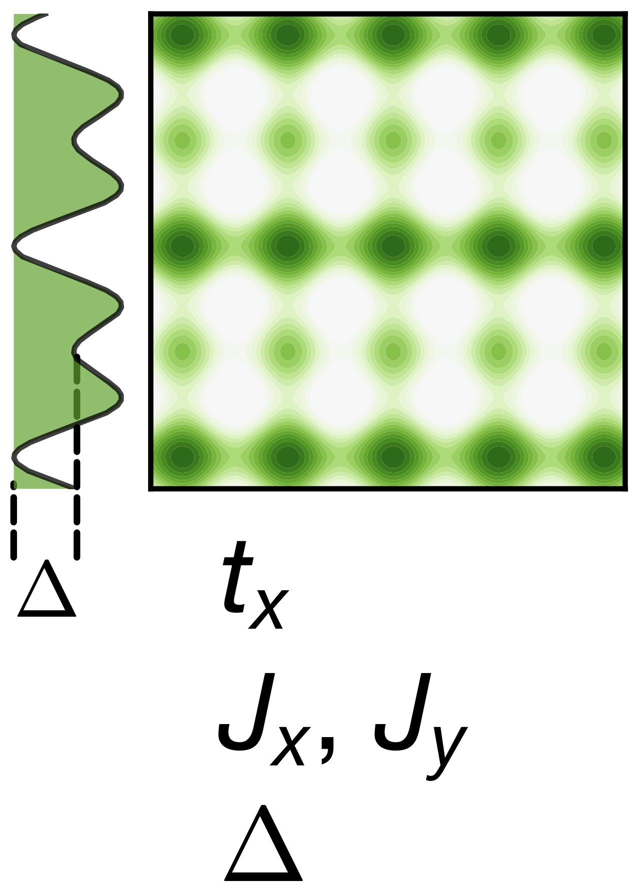
Stripes in mixed-dimensional (mixD) Hubbard models

Bourgund et al., Nature 637 (2025) — Bloch lab



- Hole bunching across chains
- Hidden correlations
- Extended stripe patterns

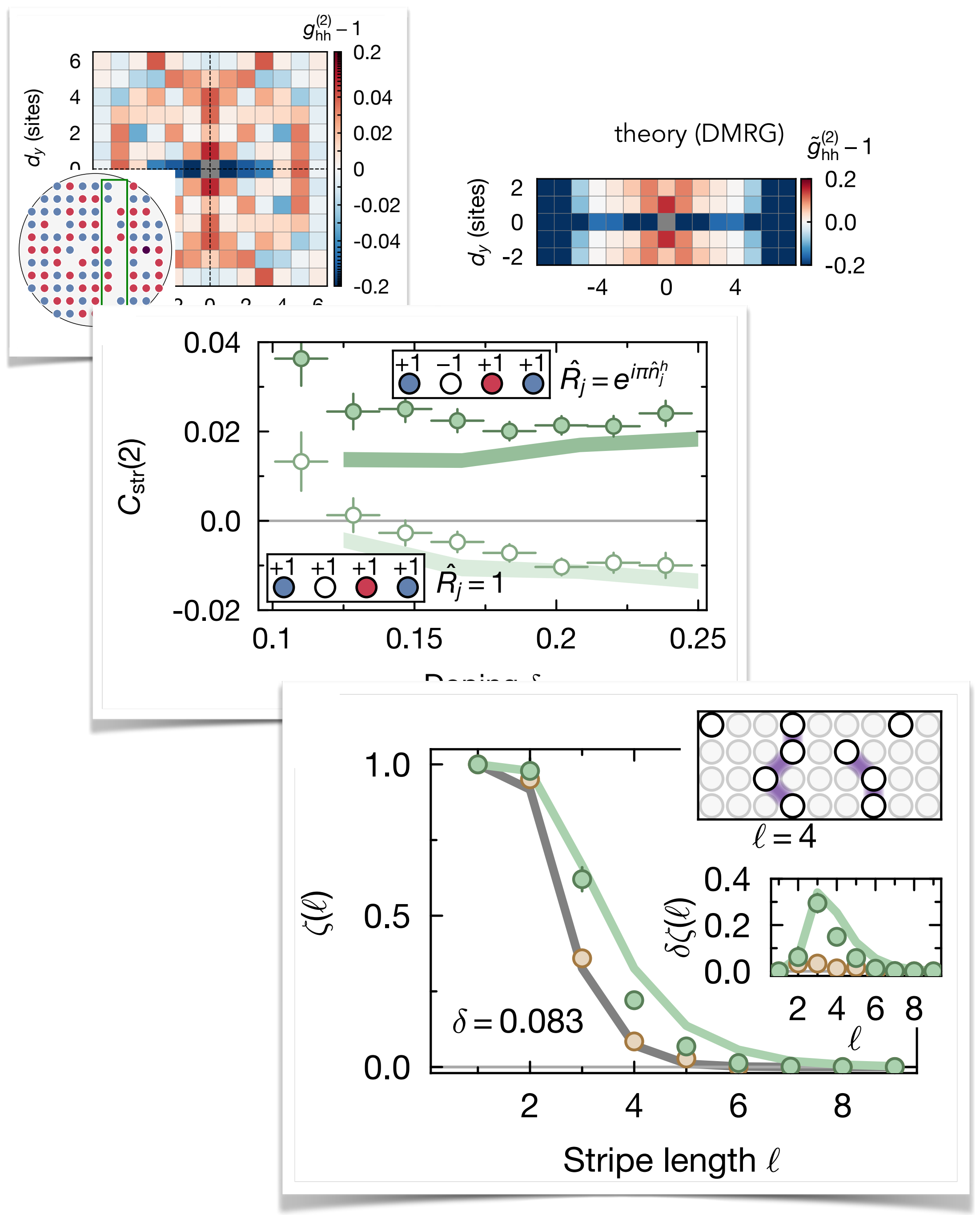
MixD



Hubbard phase diagram

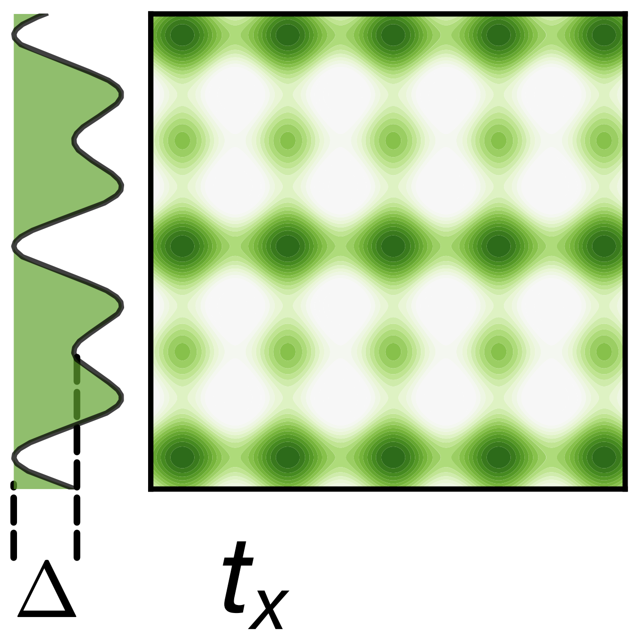
Stripes in mixed-dimensional (mixD) Hubbard models

Bourgund et al., Nature 637 (2025) — Bloch lab



- Hole bunching across chains
- Hidden correlations
- Extended stripe patterns
- Two energy scales: individual stripes vs. stripe-stripe interaction

MixD



t_x

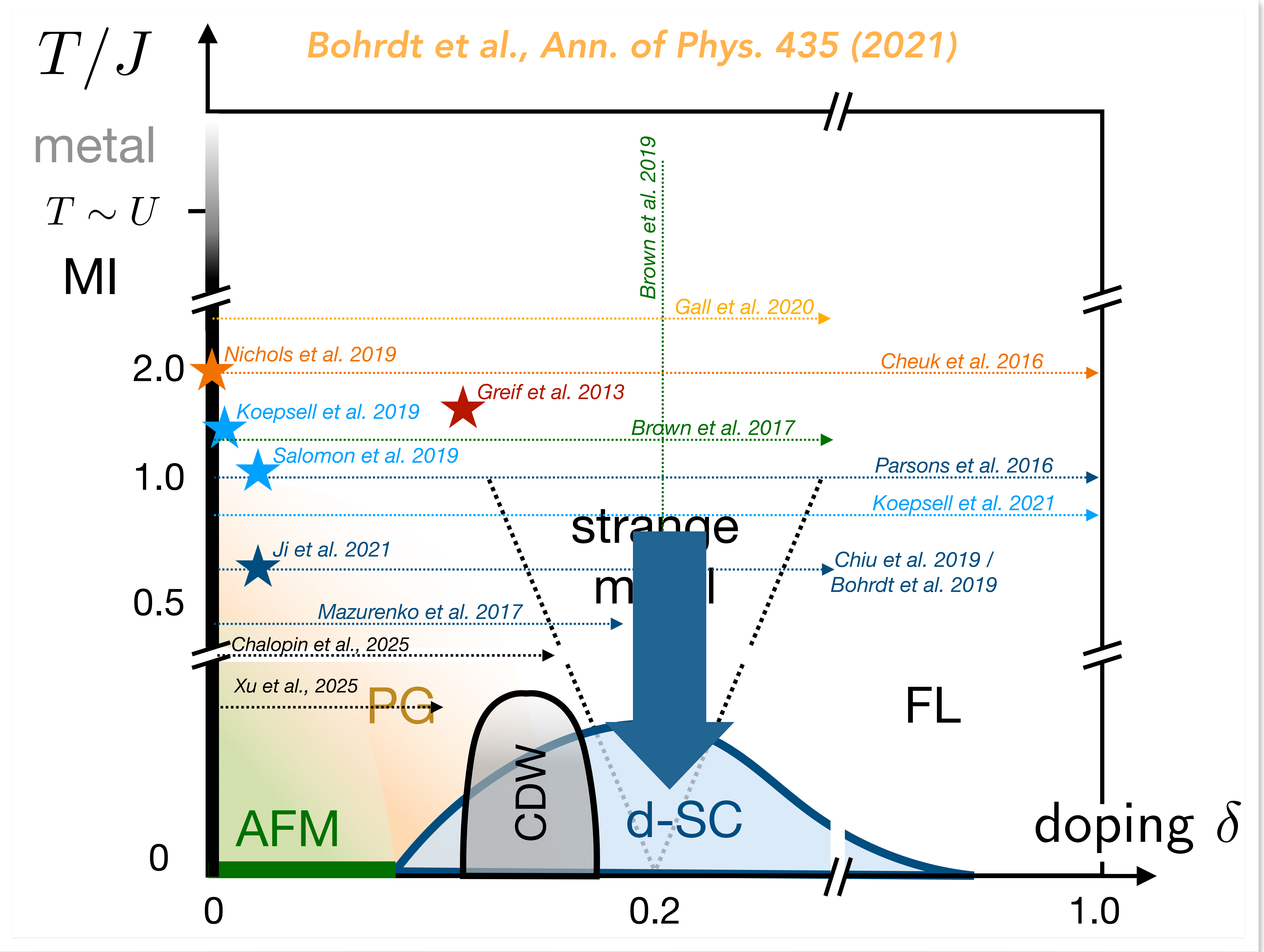
J_x, J_y

Δ

Hubbard phase diagram

Superconductivity & pairing in Hubbard models

- *Not yet seen in vanilla Hubbard simulators*
- *Strong numerical evidence at T=0 upon including t'*
  Corboz et al. PRL 113 (2014); Xu et al., Science 384 (2024); Roth et al., arXiv:2511.07566 (2025)



Hubbard phase diagram

Superconductivity & pairing in Hubbard models

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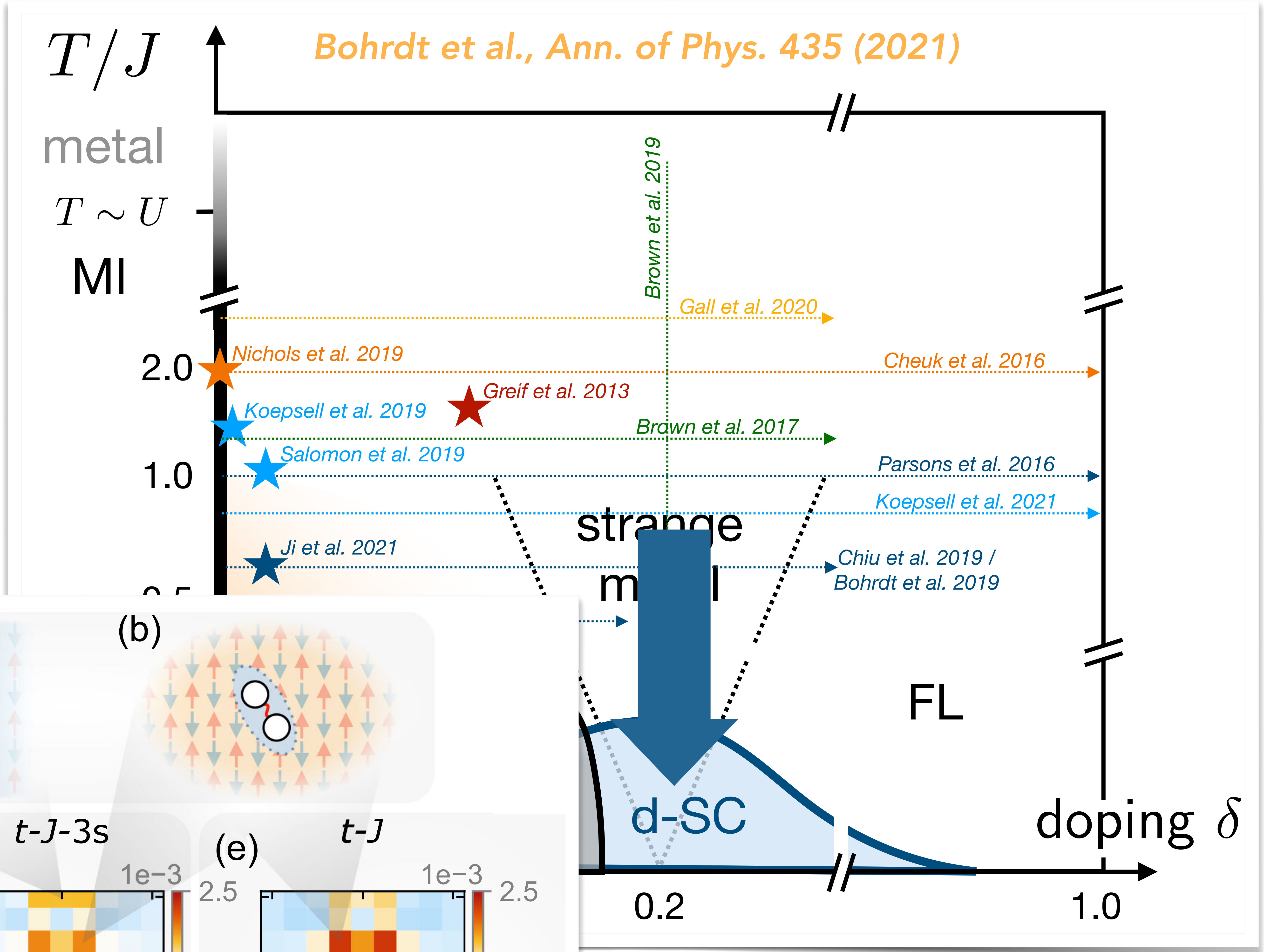
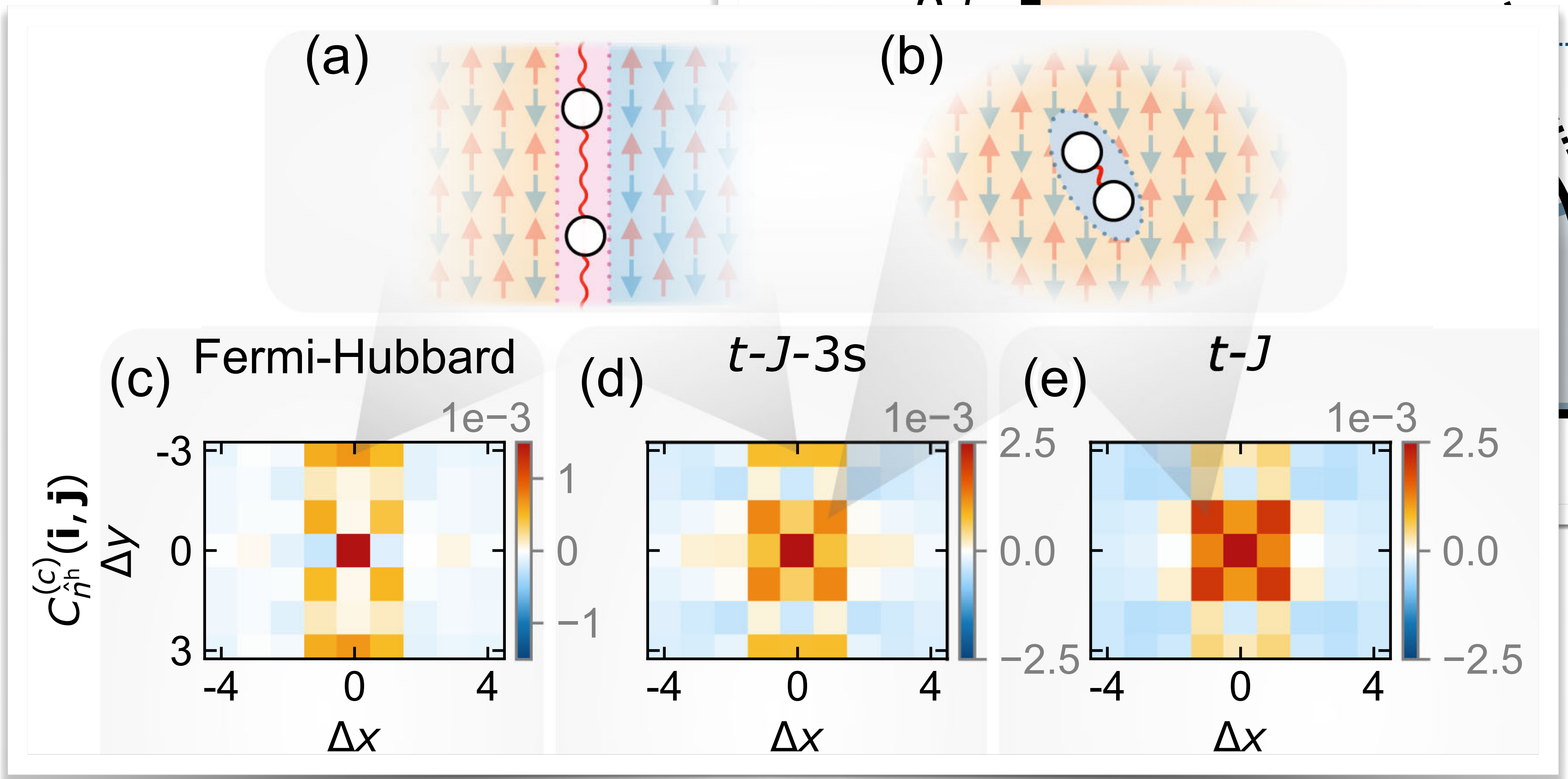
- *Strong numerical evidence at T=0 upon including t'*
  Corboz et al. PRL 113 (2014); Xu et al., Science 384 (2024); Roth et al., arXiv:2511.07566 (2025)
- *Two types of pairs*
  Blatz et al., PRX 15 (2025)



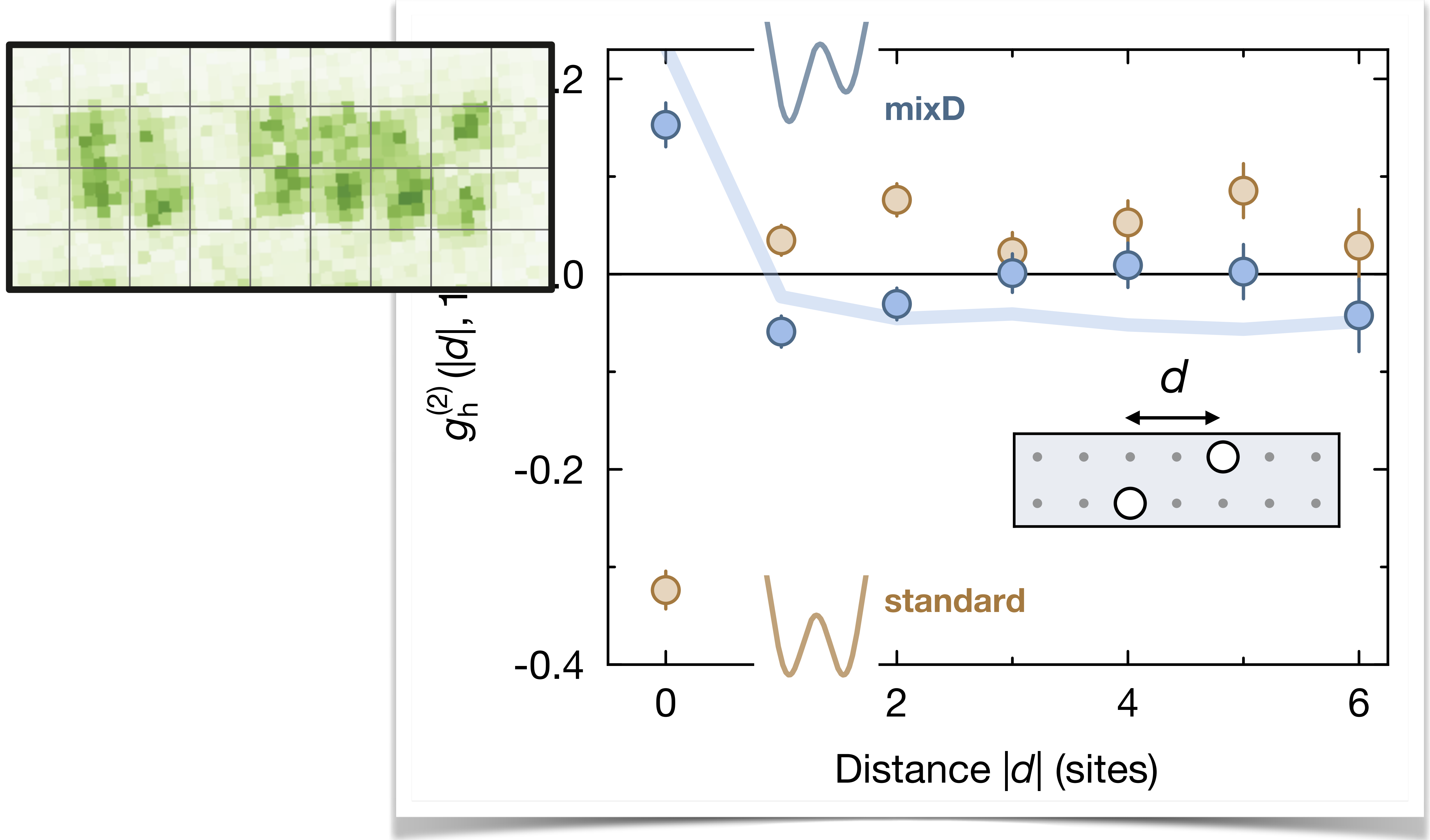


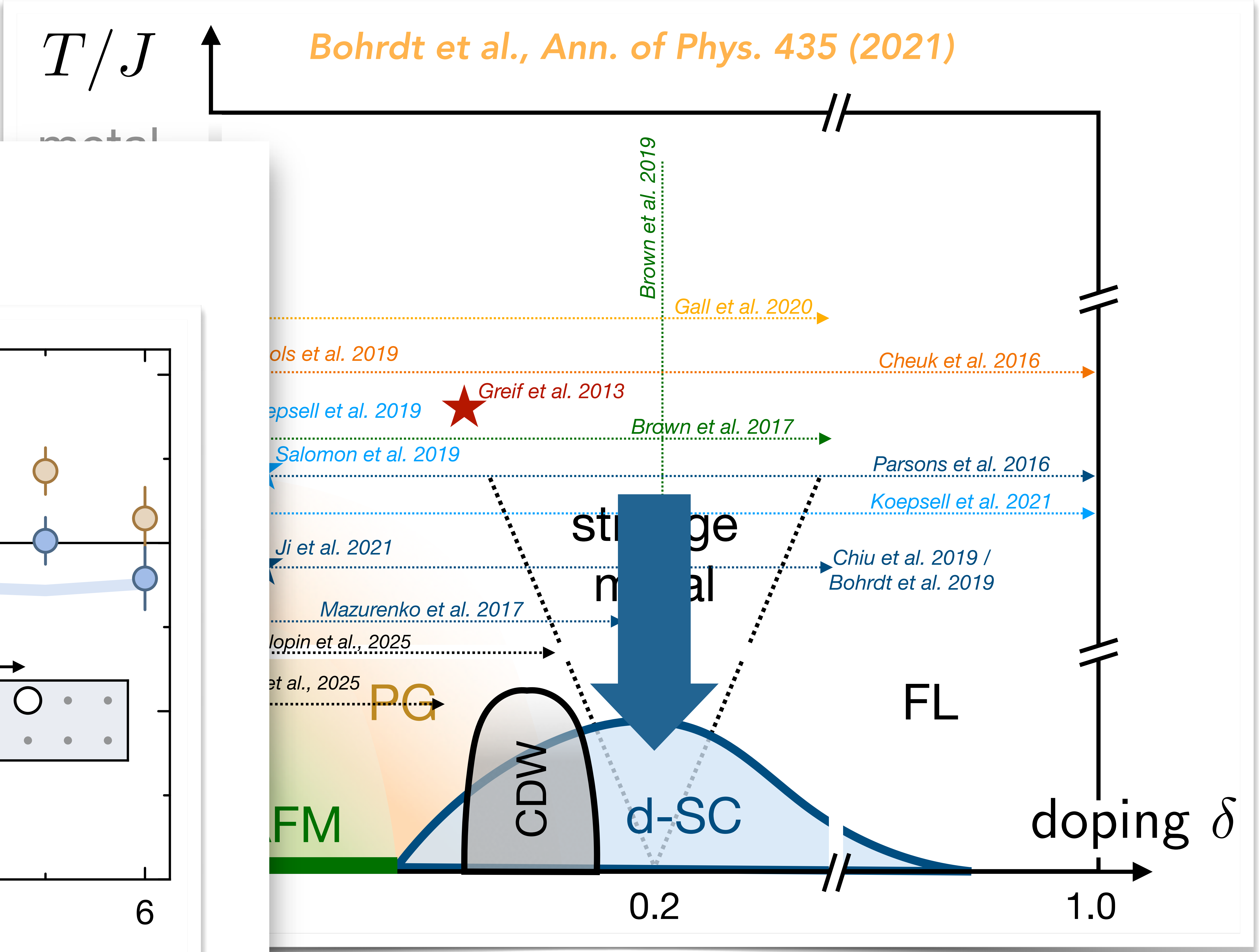
Hubbard phase diagram

Strong pairing in mixD

MixD ladders:

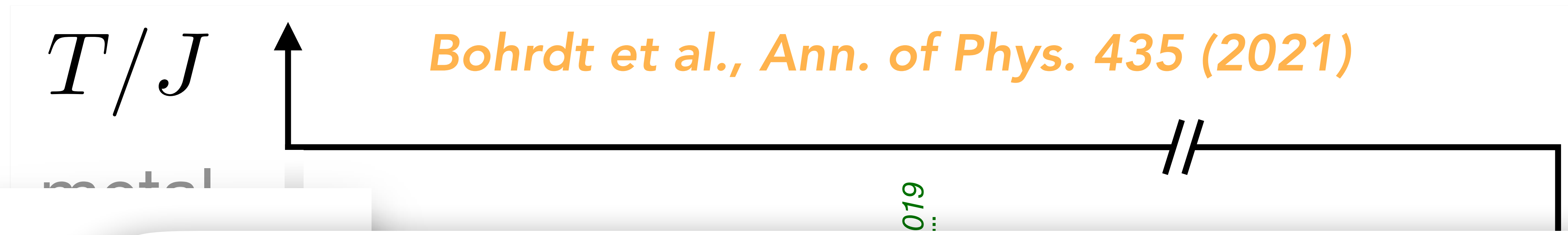
Hirthe et al., Nature 613 (2023) — Bloch lab





Hubbard phase diagram

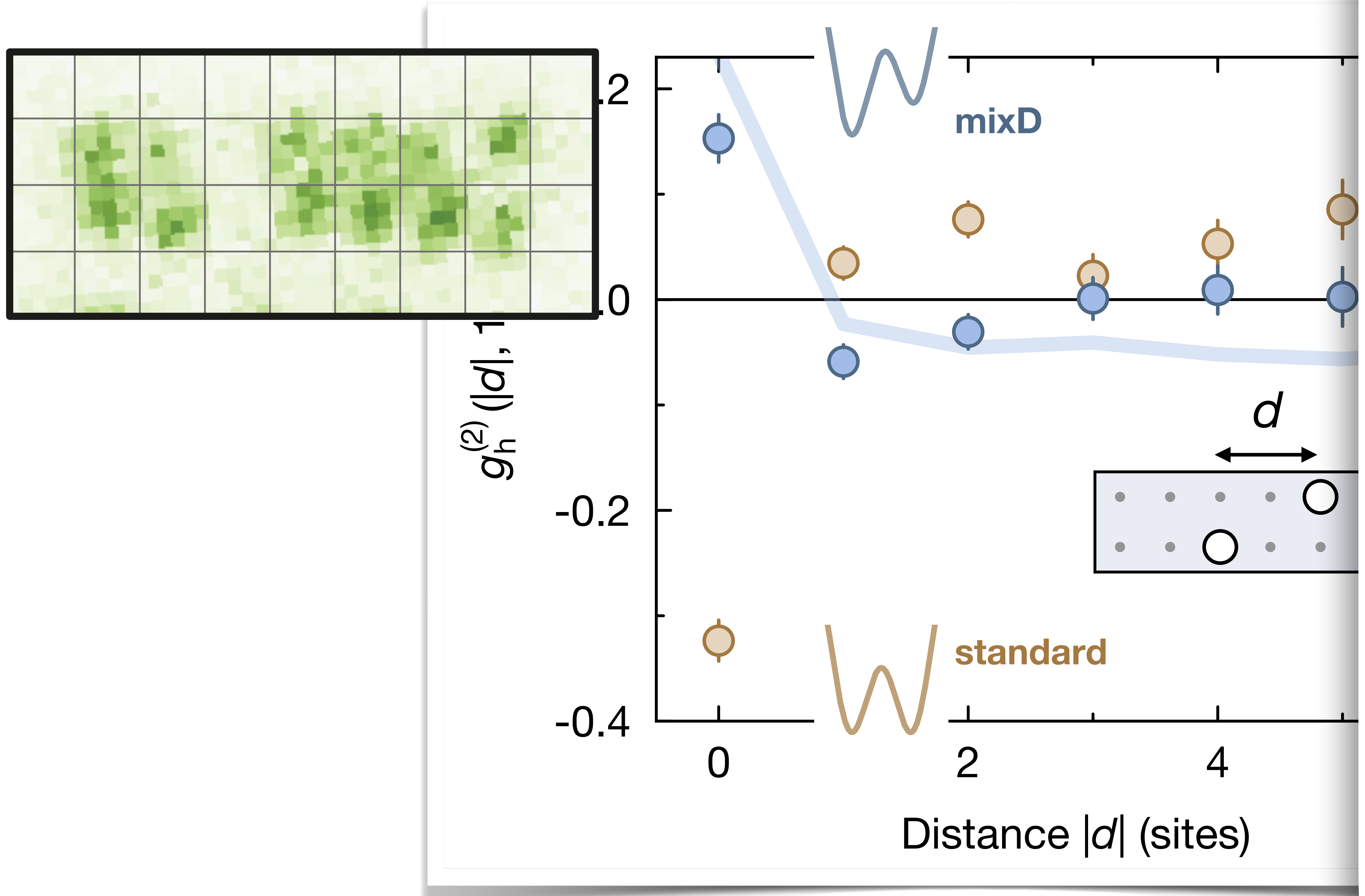
Strong pairing in mixD



Bohrdt et al., Ann. of Phys. 435 (2021)

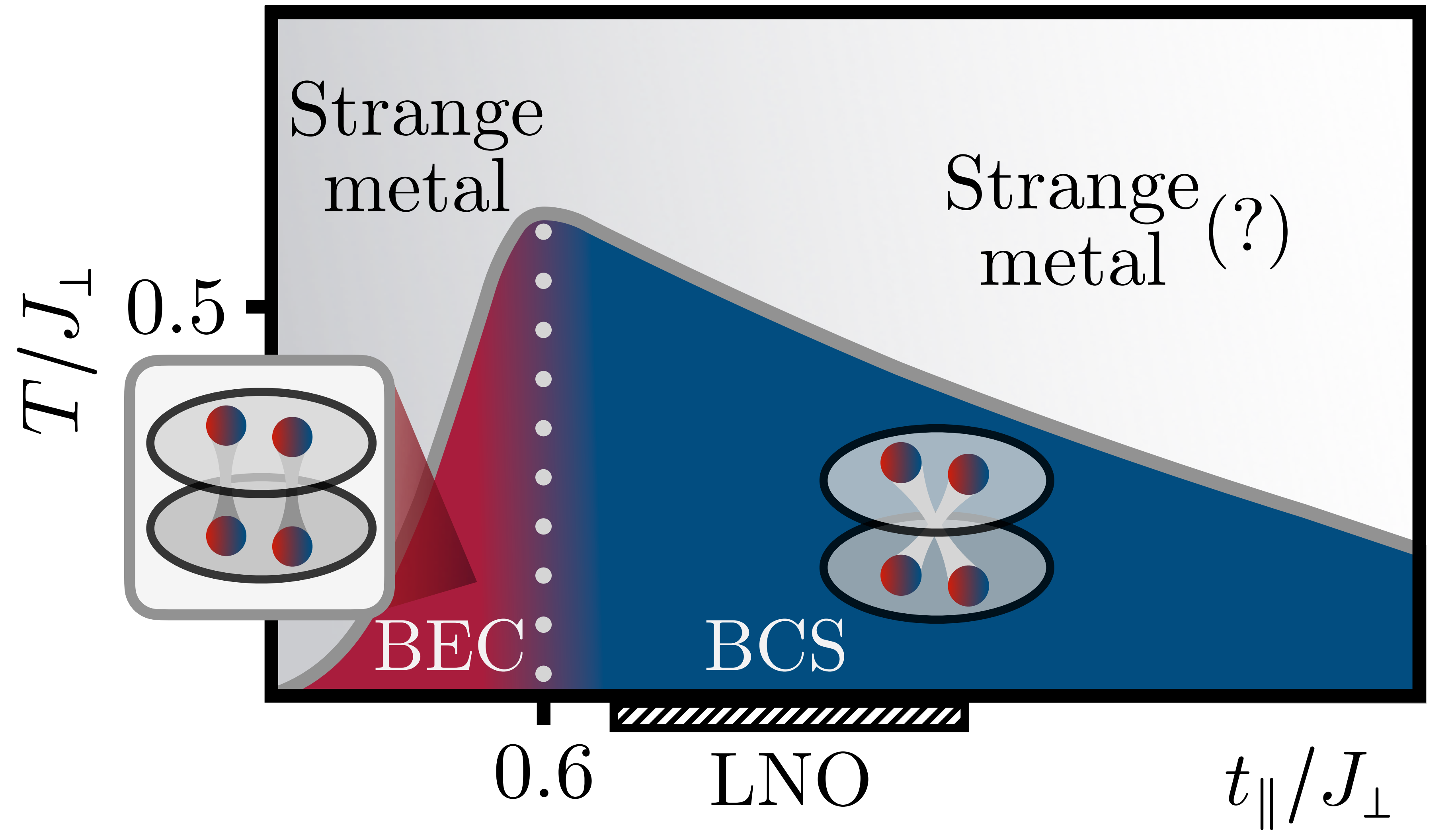
MixD ladders:

Hirthe et al., Nature 613 (2023) — Bloch lab



MixD bilayer - enables large critical SC Tc:

Schlömer et al., Comm. Phys. 7 (2024)
Schlömer et al., PRX Quant. 5 (2024)

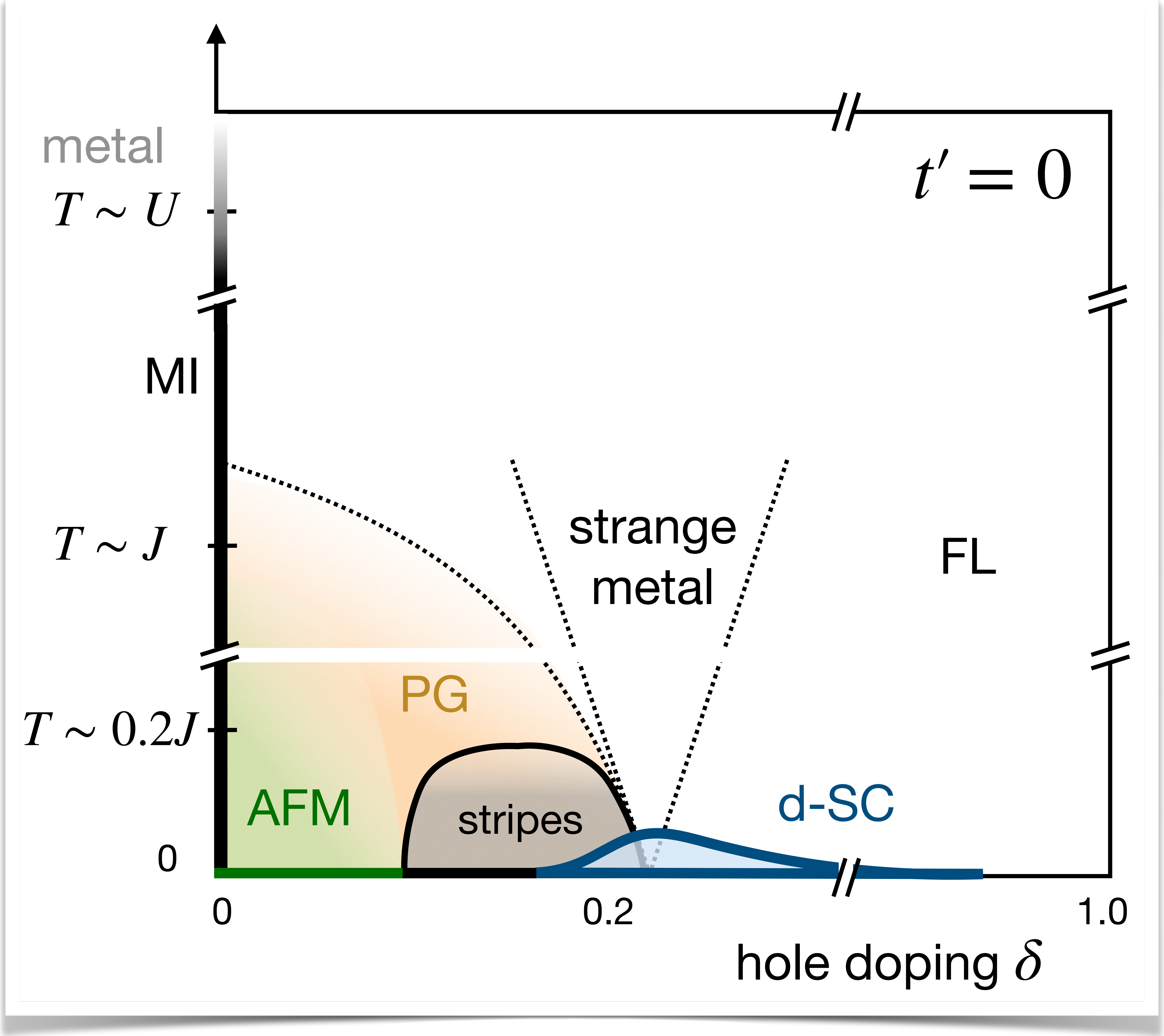


$T_c \simeq 0.5 J_\perp$ — achievable in cold atoms!

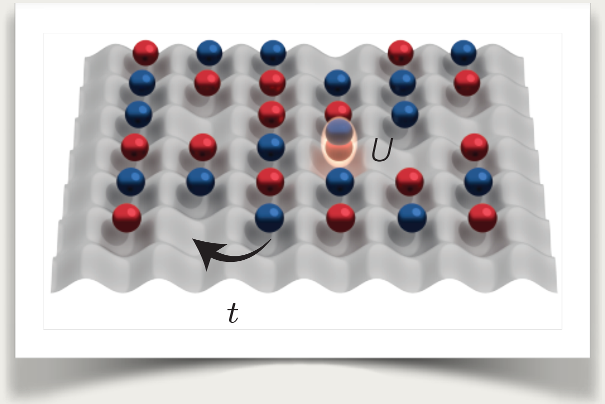
Summary Part 1

The rich Hubbard model phase diagram

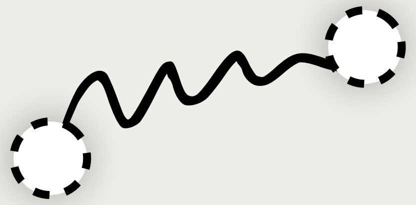
- unified understanding of collective phases
- key challenge for cold atoms: colder temperatures



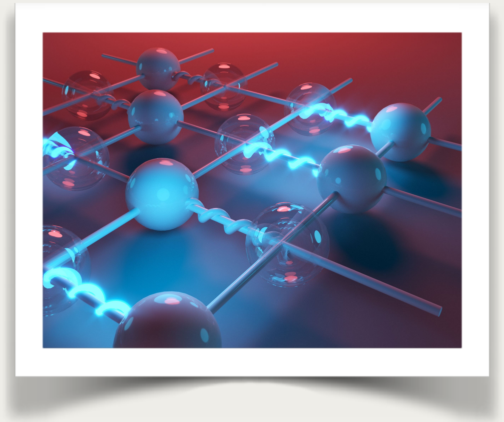
Outline



Phase diagram of high-Tc superconductors

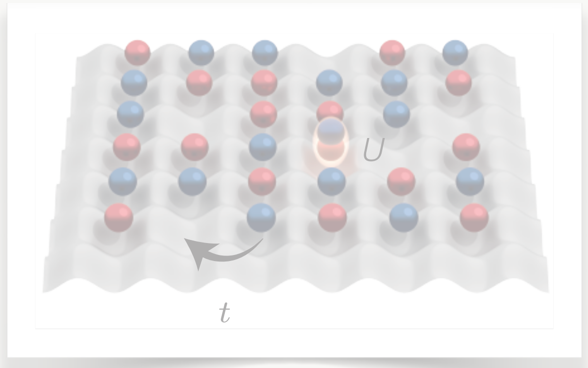


Strong coupling theory



Hidden orders

Outline



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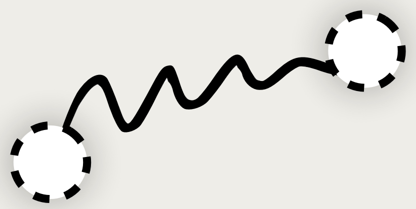

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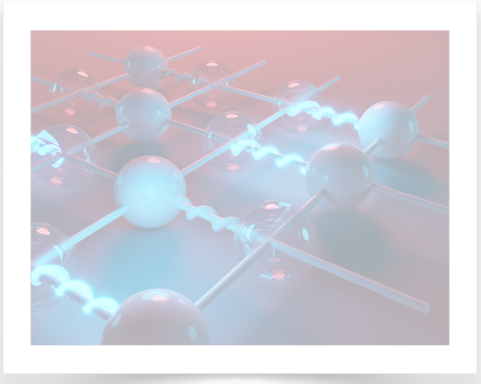
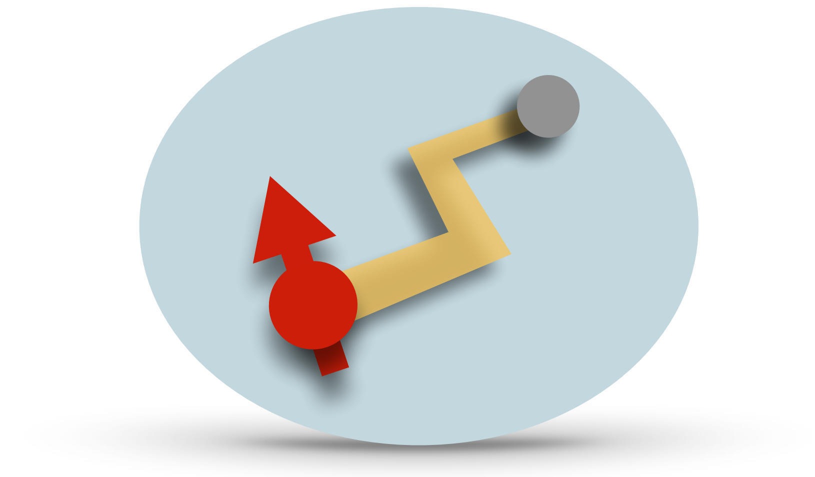


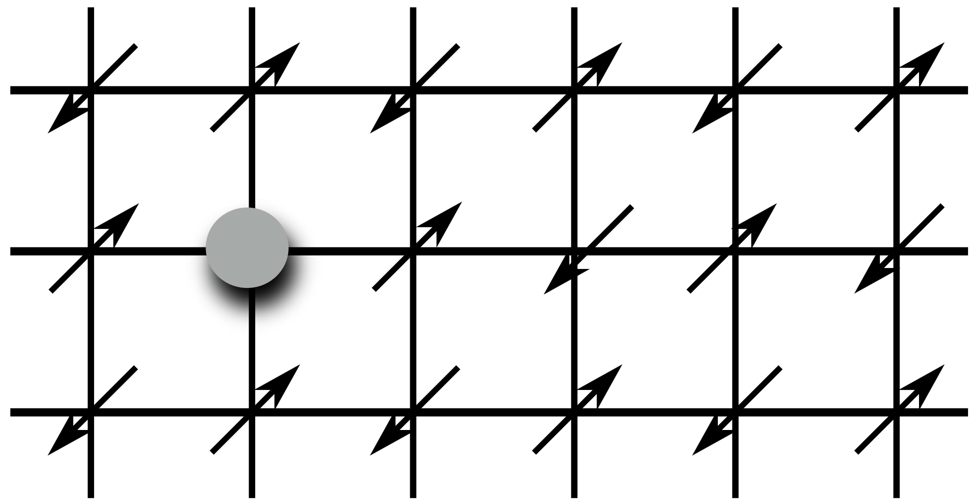
Hidden orders

PART 2.I: One-hole excitations & Parton picture



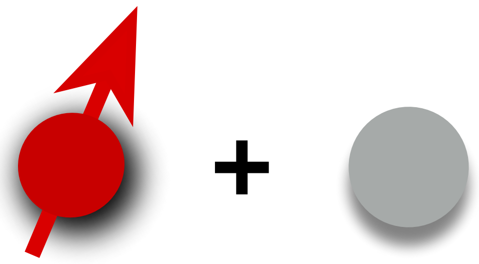
Strong coupling theory

Parton picture of doped antiferromagnets



* **Fractional spin** excitation!

$$S = 1/2$$



Bulaevskii et al., JETP 27 (1968), Trugman, PRB 37 (1988)
Manousakis, PRB 75 (2007, Kane et al., PRB 39 (1989)

Grusdt et al., PRX 8 (2018)
Grusdt et al., PRB 99 (2019)

Bohrdt et al., PRB 102 (2020)
Bohrdt et al., PRL 127 (2021)

Strong coupling theory

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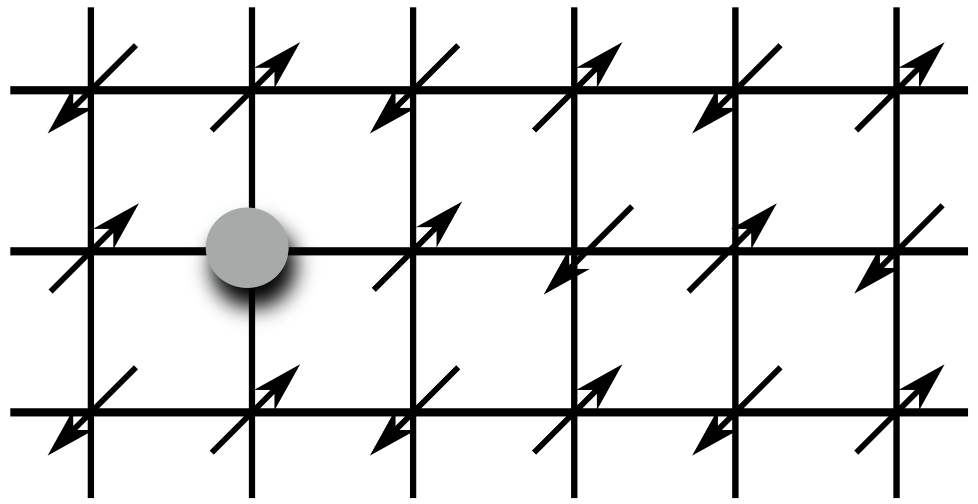

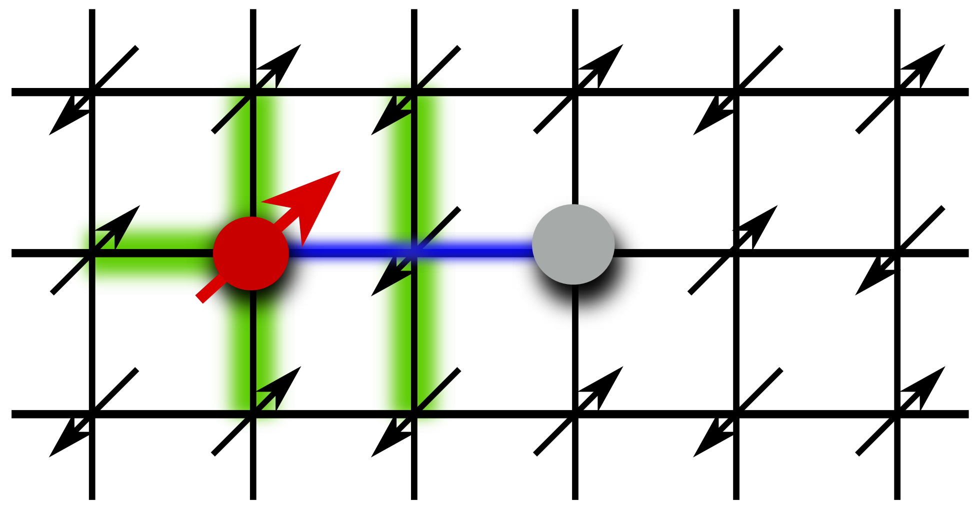

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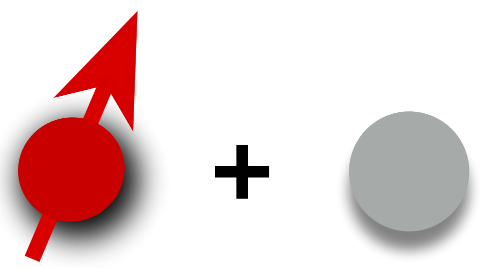
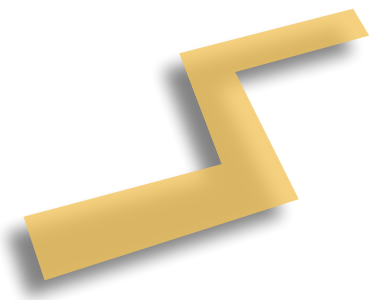


- Movement of the hole distorts the Neel state — **spin is carried by distortion!**



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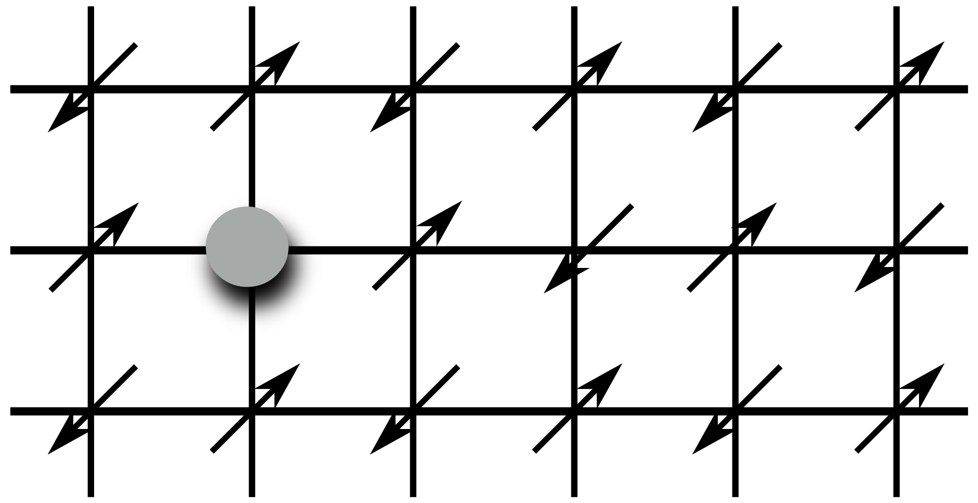

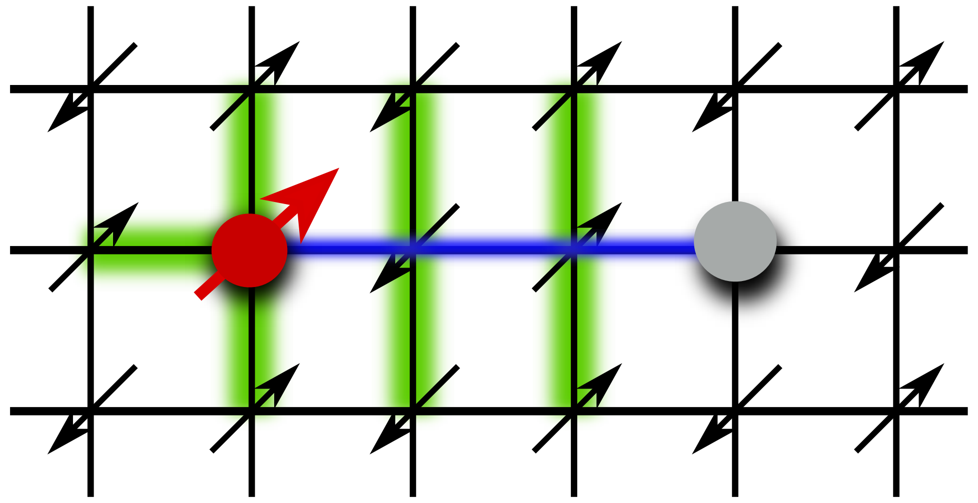

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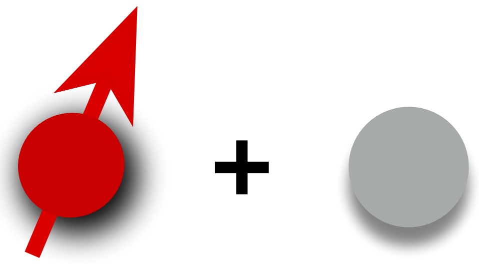

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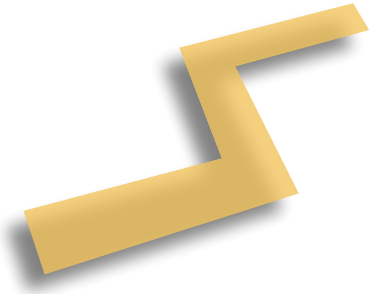
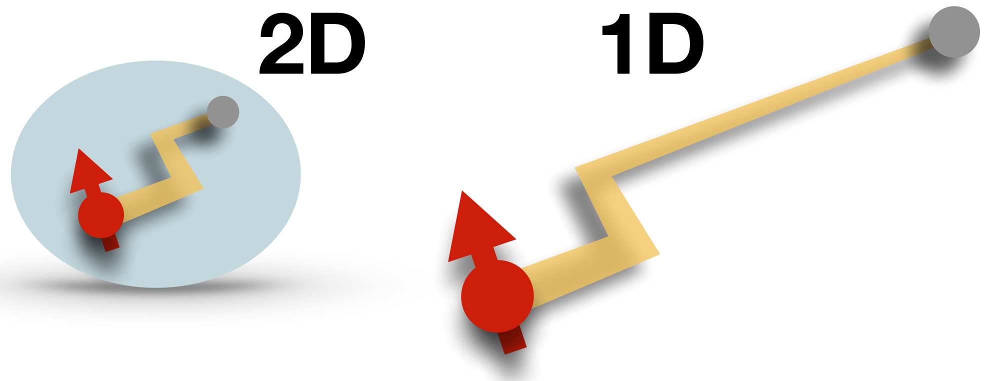


- Chargon is **bound (unbound) to the spinon** at end of string in 2D (1D)!

$$E \propto \ell \qquad (E \simeq \textbf{const.})$$



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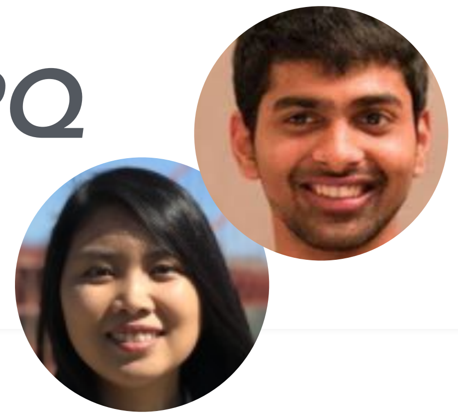
Strong coupling theory

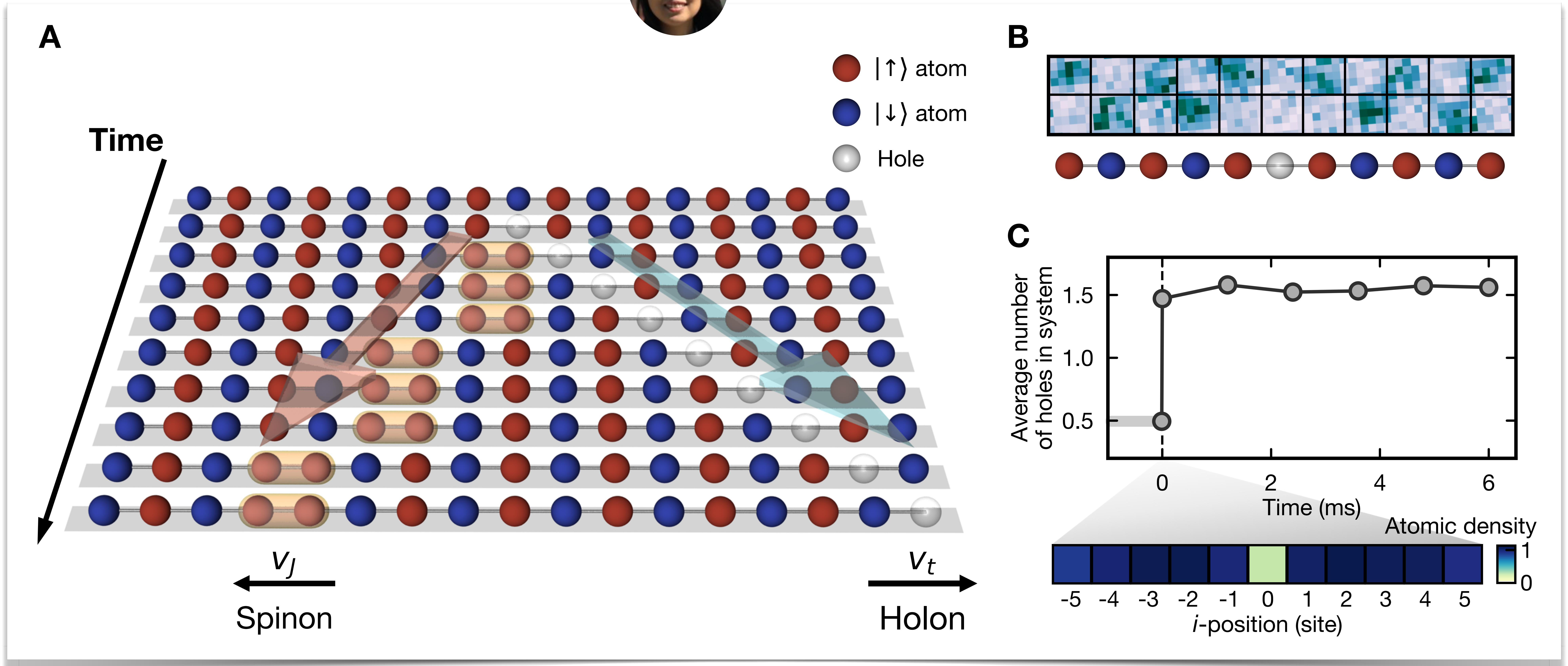
Spin-charge separation in 1D:

Experiment Gross/Bloch group, MPQ

Vijayan et al., Science 367 (2020)



* Suddenly creating a hole in 1D



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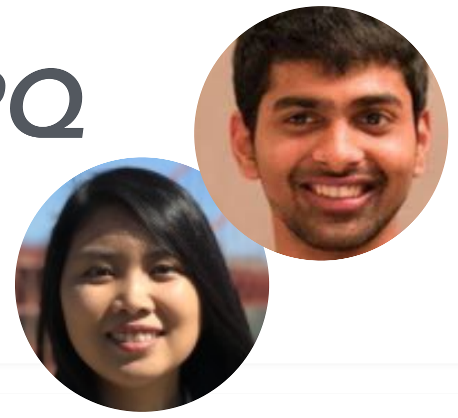

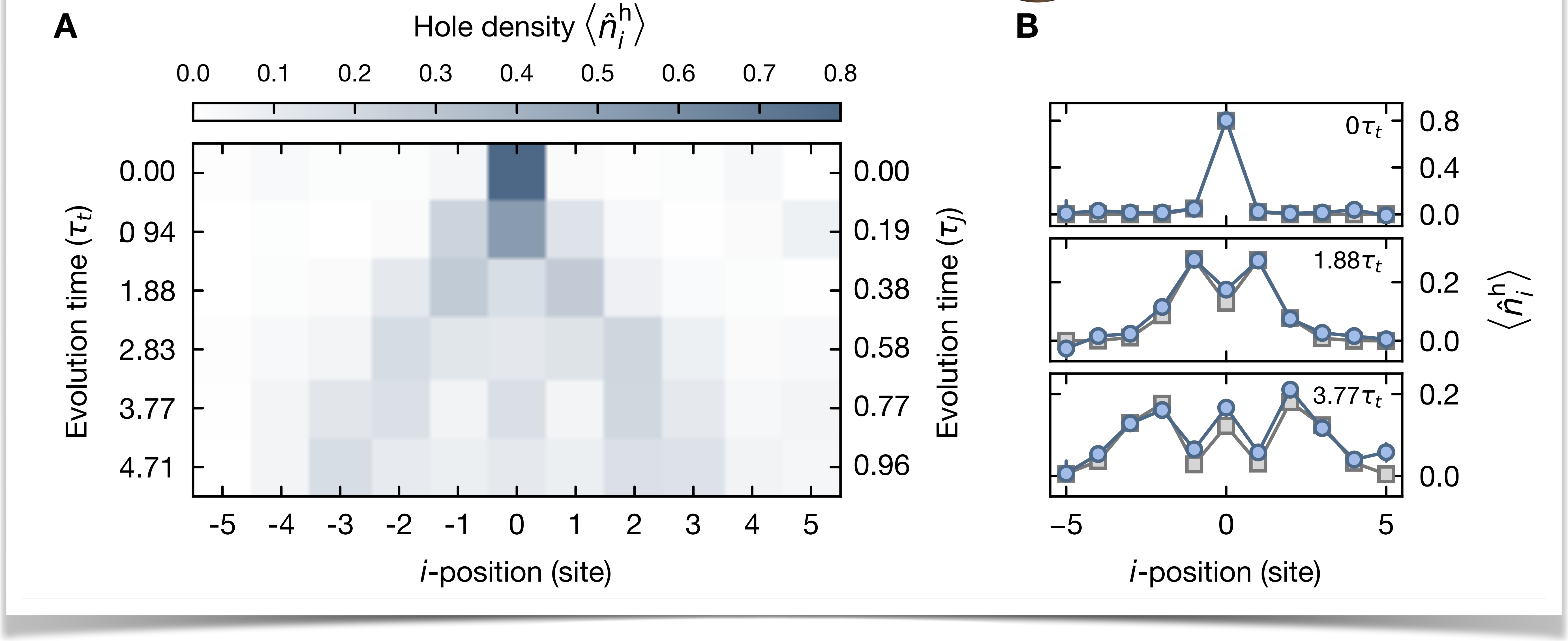

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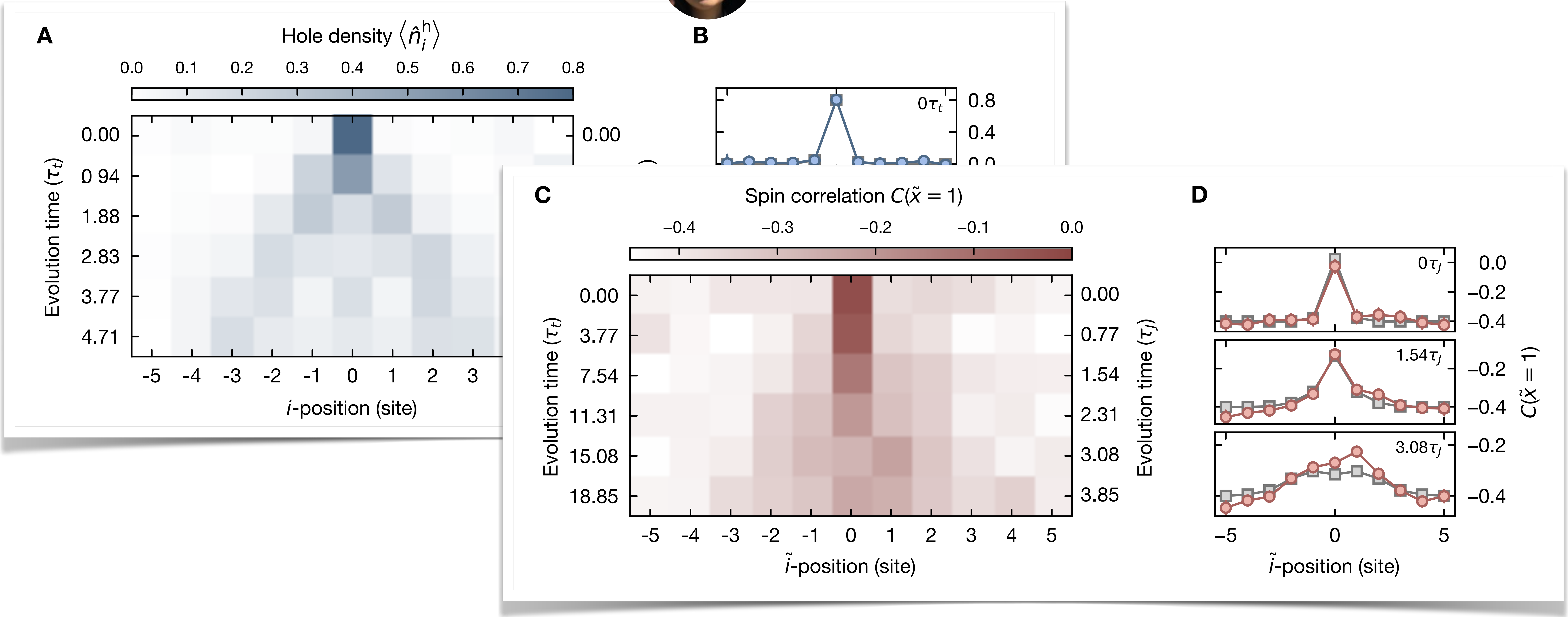

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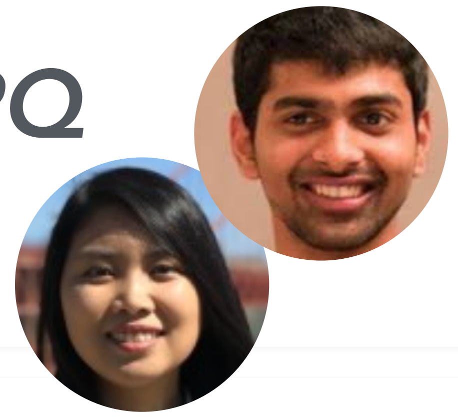

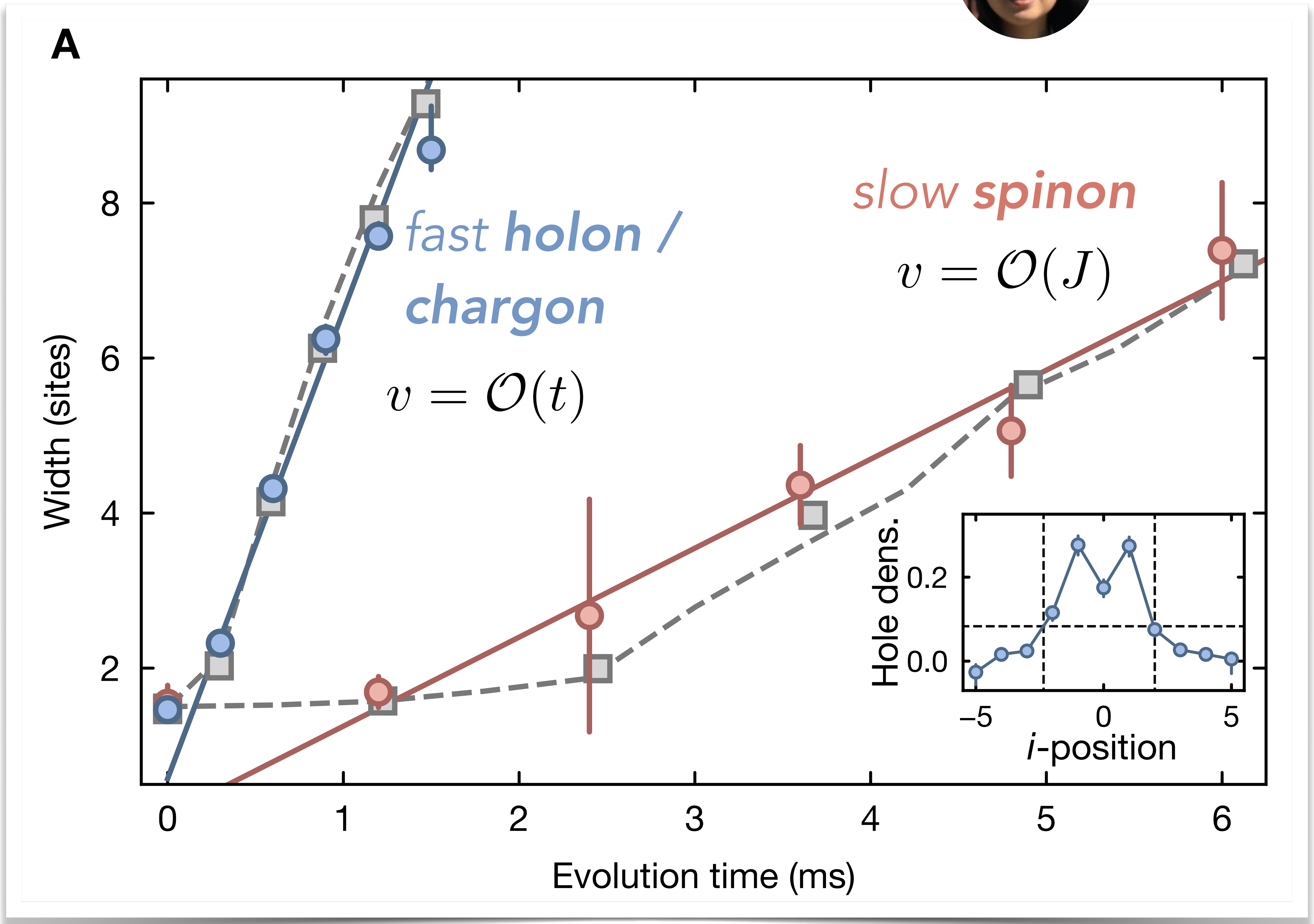

- Weakly interacting, de-confined partons!

Strong coupling theory

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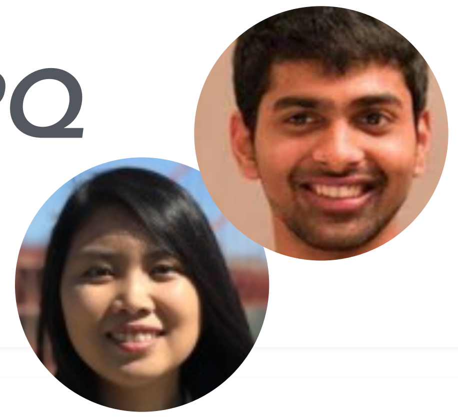
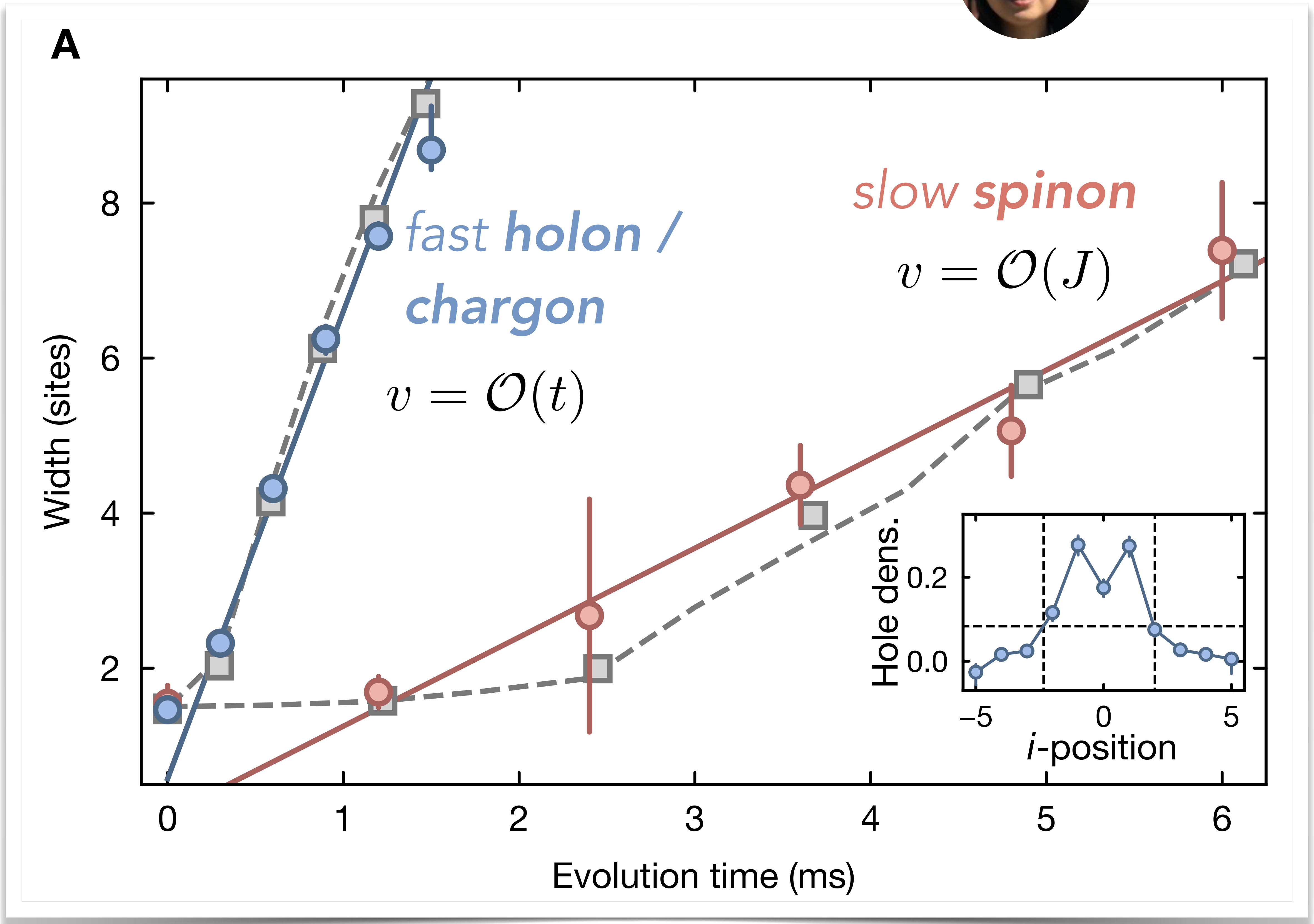
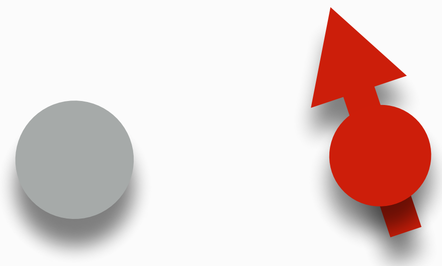




Formally:



$$\hat{c}_{\boldsymbol{j},\sigma} = \hat{h}^\dagger_{\boldsymbol{j}} \hat{f}_{\boldsymbol{j},\sigma}$$

see e.g.: *Baskaran et al., Sol. St. Comm. 63 (1987)*

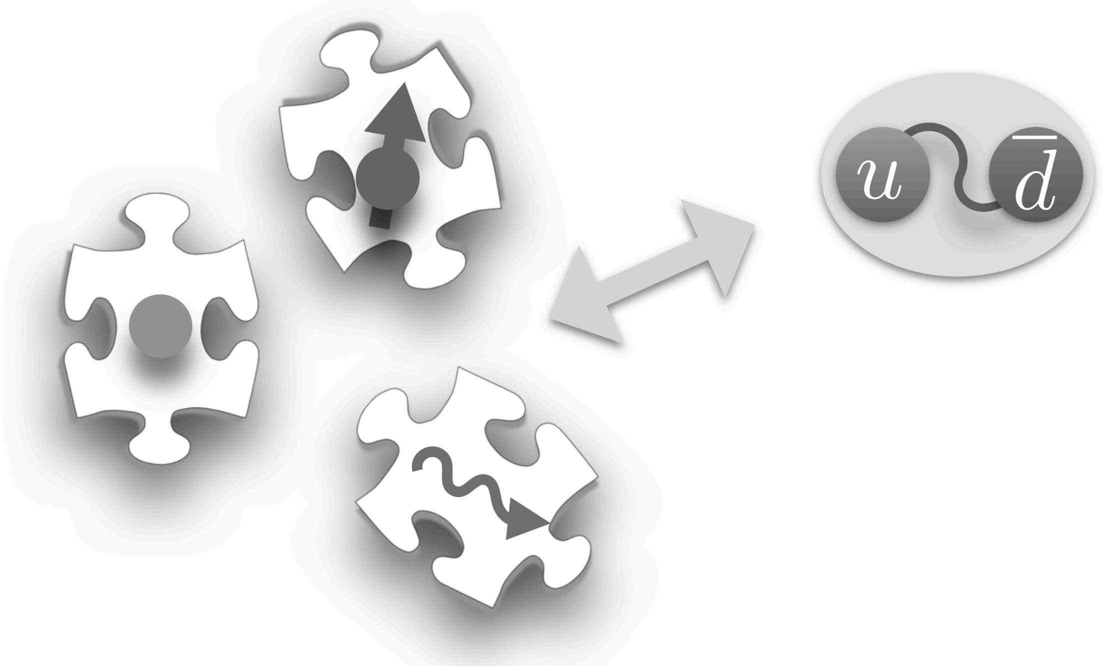
$$\hat{\mathcal{H}} \simeq \hat{\mathcal{H}}_{\rm h}[h] + \hat{\mathcal{H}}_{\rm s}[f]$$

$$|\Psi(t)\rangle \simeq |\Psi_{\rm h}(t)\rangle \otimes |\Psi_{\rm s}(t)\rangle$$

- Weakly interacting, de-confined partons!

Strong coupling theory

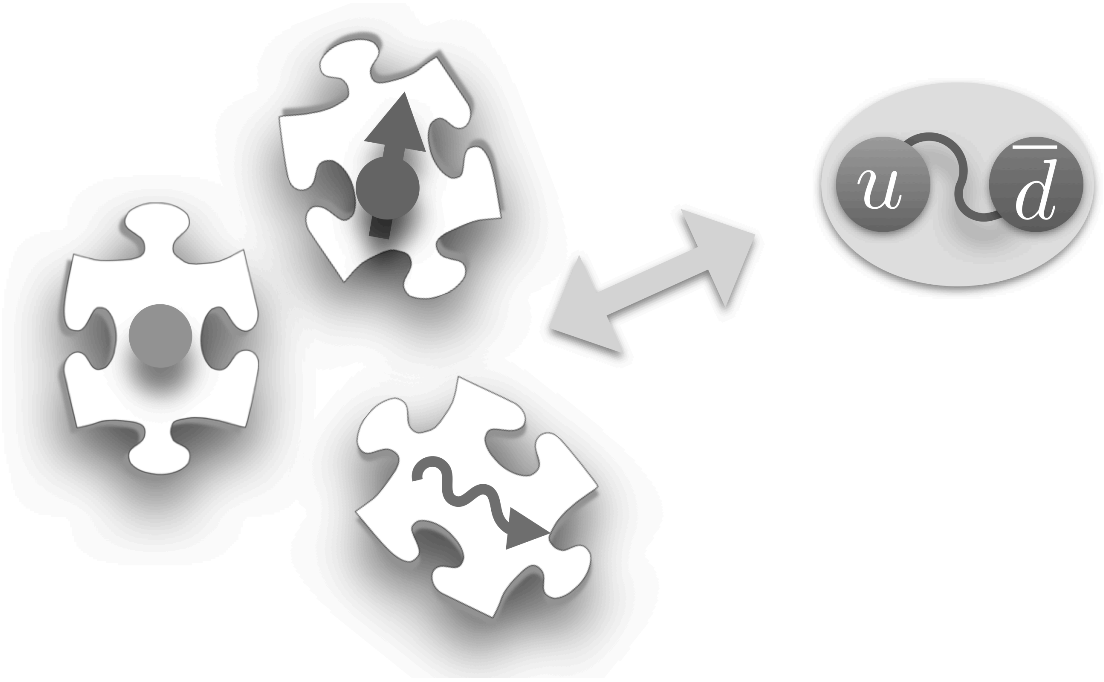
A decades old dream…



Could the charge carriers in high-Tc have an internal, quark-like structure?

Strong coupling theory

A decades old dream...



Could the charge carriers in high-Tc have an internal, quark-like structure?

Evidence for composite nature of quasiparticles in the 2D t–J model

P. Béran [a,b], D. Poilblanc [b,c], R.B. Laughlin [c]
hâtel, CH-2000 Neuchâtel, Switzerland
é Paul Sabatier, F-31062 Toulouse, France
versity, Stanford, CA 94305, USA

rch 1996; accepted 5 April 1996

VOLUME 79, NUMBER 9 PHYSICAL REVIEW LETTERS 1 SEPTEMBER 1997

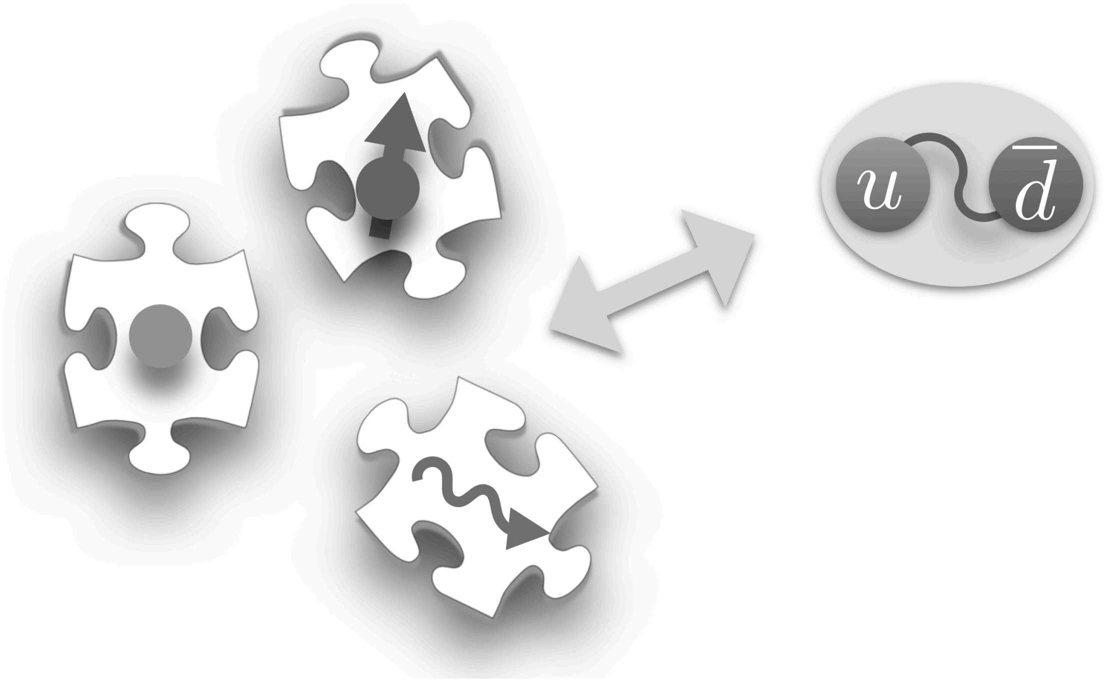
Evidence for Quasiparticle Decay in Photoemission from Underdoped Cuprates

R. B. Laughlin
Department of Physics, Stanford University, Stanford, California 94305

(Received 14 August 1996; revised manuscript received 12 June 1997)

Strong coupling theory

A decades old dream…



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Evidence for composite nature of quasiparticles in the 2D $t-J$ model

P. B

PHYSICAL REVIEW LETTERS **127,** 197004 (2021)

Rotational Resonances and Regge-like Trajectories in Lightly Doped

A. Bohrdt,[1,2,3,4,*] E. Demler,[4] and F. Grusdt[5,2]

VOLUME 79, NUMBER 9 PHYSICAL REVIEW LETTERS

Evidence for Quasiparticle Decay in

PRL **98,** 067004 (2007) PH

…rgy Anomaly in the Angle-Resolved Photoemission Spectra of High …erconductors: Possible Evidence of Spinon and Holon Branches

PNAS

Quantum dimer mod…

Matthias Punk[a,b,c], Andrea Allais[d], and Subir S…

PHYSICAL REVIEW B **102**, 035139 (2020)

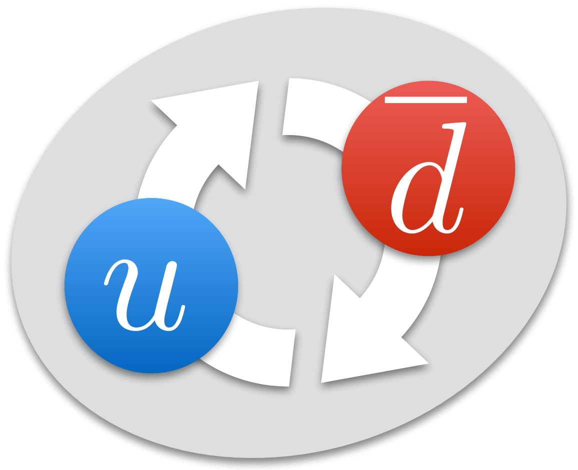
Parton theory of angle-resolved photoemission spectroscopy spectra in antiferromagnetic Mott insulators

Annabelle Bohrdt,[1,2,*] Eugene Demler,[3] Frank Pollmann,[1,2] Michael Knap,[1,2] and Fabian Grusdt[4,1,2]

Strong coupling theory

The discovery of quarks:

M. Riordan, "The discovery of quarks", Science 256 (1992)



Just ~~twenty~~ some fifty years ago, physicists were beginning to realize that the protons and neutrons at the heart of the atomic nucleus are not elementary particles, after all. Instead, they appeared to be composed of curious pointlike objects called "quarks," a name borrowed from a line in James Joyce's novel, *Finnegans Wake*. First proposed in 1964 by Gell-Mann *(1)* and Zweig *(2)*, these

LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Strong coupling theory

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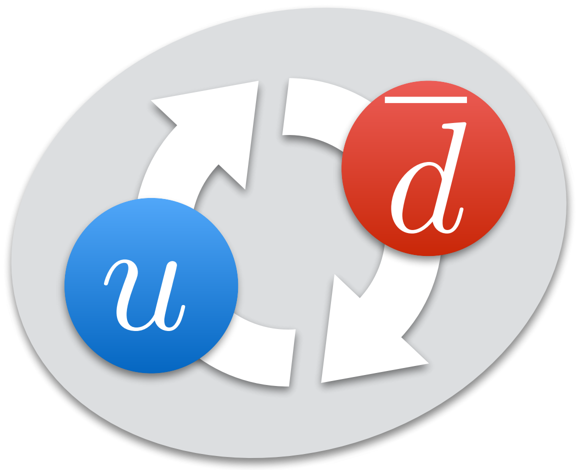
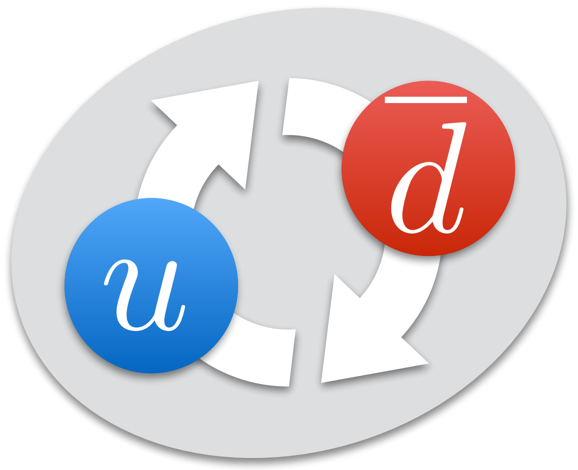


By the beginning of the 1960s, physicists had shown that protons and neutrons (known collectively as "nucleons") had a finite size of about 10^{-13} cm, as indicated by elastic electron-nucleon scattering experiments of Hofstadter and his Stanford coworkers *(5)*, but the great majority considered these particles to be "soft" objects with only a diffuse internal structure. Along

Strong coupling theory

The discovery of quarks:

M. Riordan, "The discovery of quarks", Science 256 (1992)



Just ~~twenty~~ some fifty years ago, physicists were beginning to realize that the protons and neutrons at the heart of the atomic nucleus are not elementary particles, after all. Instead, they appeared to be composed of curious pointlike objects called "quarks," a name borrowed from a line in James Joyce's novel, *Finnegans Wake*. First proposed in 1964 by Gell-Mann *(1)* and Zweig *(2)*, these

By the beginning of the 1960s, physicists had shown that protons and neutrons (known collectively as "nucleons") had a finite size of about 10^{-13} cm, as indicated by elastic electron-nucleon scattering experiments of Hofstadter and his Stanford coworkers *(5)*, but the great majority considered these particles to be "soft" objects with only a diffuse internal structure. Along

The same is true in high-Tc cuprate materials today — doped holes are widely considered as "soft objects" with only a diffuse internal structure!

Strong coupling theory

The discovery of quarks:

M. Riordan, "The discovery of quarks", Science 256 (1992)

In seeking a deeper explanation for the regularities of the SU3 classification scheme, Gell-Mann and Zweig invented quarks *(1,2)*.

But the idea of fractional charges was fairly repulsive to physicists of the day; in his original paper, Gell-Mann even wrote that "a search for stable quarks of charge −1/3 or +2/3 at the highest energy accelerators would help to reassure us of the nonexistence of real quarks." After several years of fruitless searches *(3)*, most particle physicists agreed that although quarks might be useful mathematical constructs, they had no innate physical reality as objects of experience.

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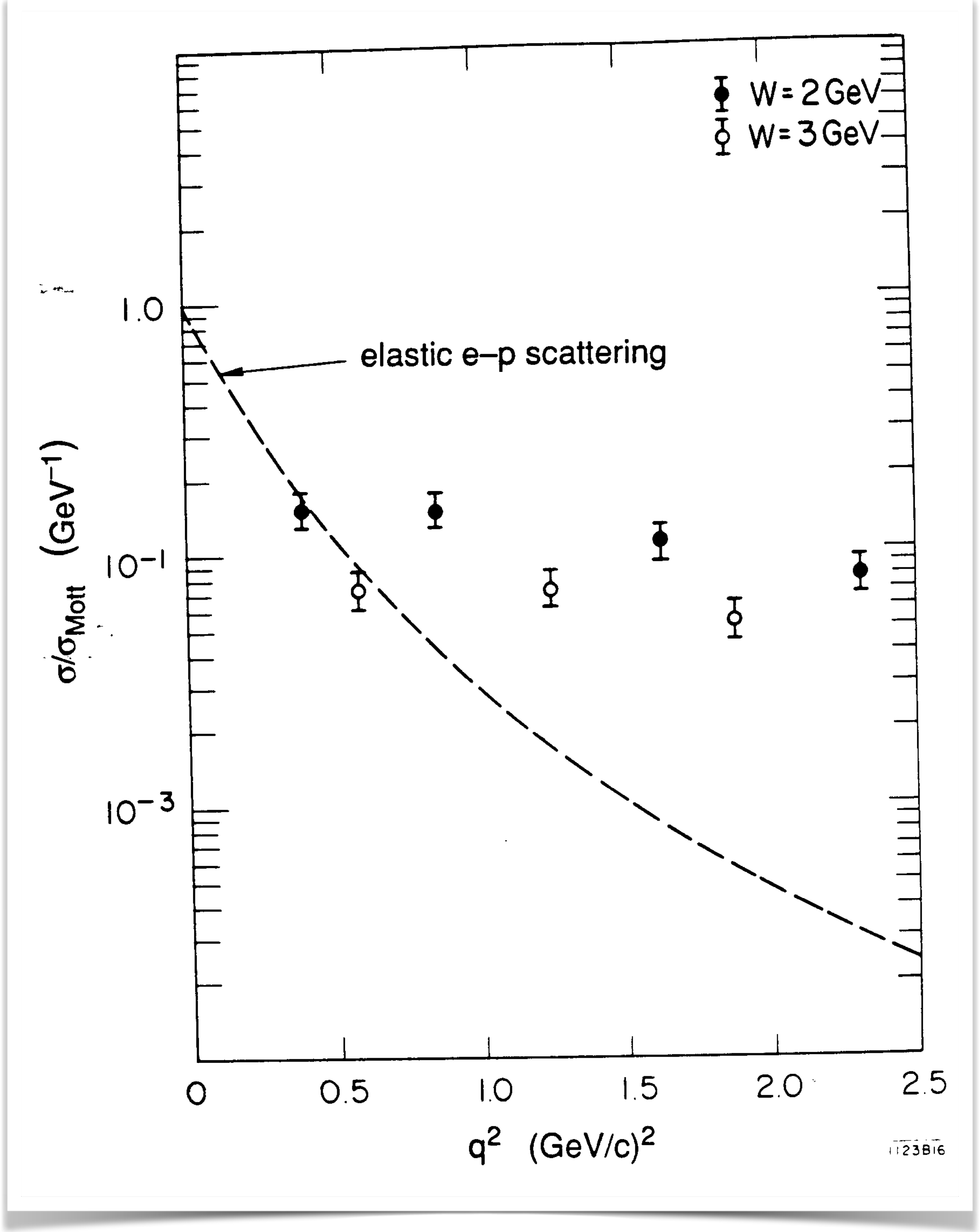

MIT — SLAC collaboration

Strong coupling theory

- ***Scattering experiments** à la Rutherford* Riordan, Science 256 (1992)

  > full spatial & temporal resolution in quantum gas microscopes! see e.g. Chiu et al., Science 365 (2019)

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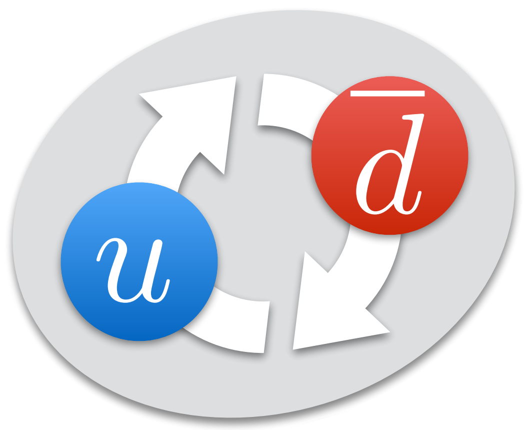
* ***Regge trajectories:*** *rotational excitations* Greensite, Prog. Part. Nuc. Phys. 51 (2003)

$n^{2s+1}\ell_J$	$u\bar{d}$	$u\bar{s}$
1^1S_0	$\boldsymbol{\pi}$	$\boldsymbol{K}$
1^1P_1	$\boldsymbol{b}_1(1235)$	$\boldsymbol{K}_{1B}$
1^1D_2	$\boldsymbol{\pi}_2(1670)$	$\boldsymbol{K}_2(1770)$
1^3F_4	$\boldsymbol{a}_4(2040)$	$\boldsymbol{K}_4^*(2045)$
2^1S_0	$\boldsymbol{\pi}(1300)$	$\boldsymbol{K}(1460)$



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Strong coupling theory

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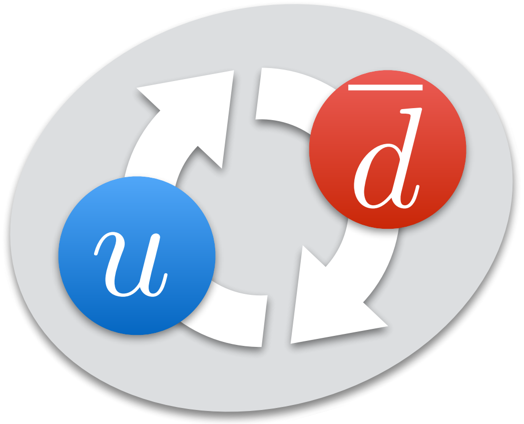
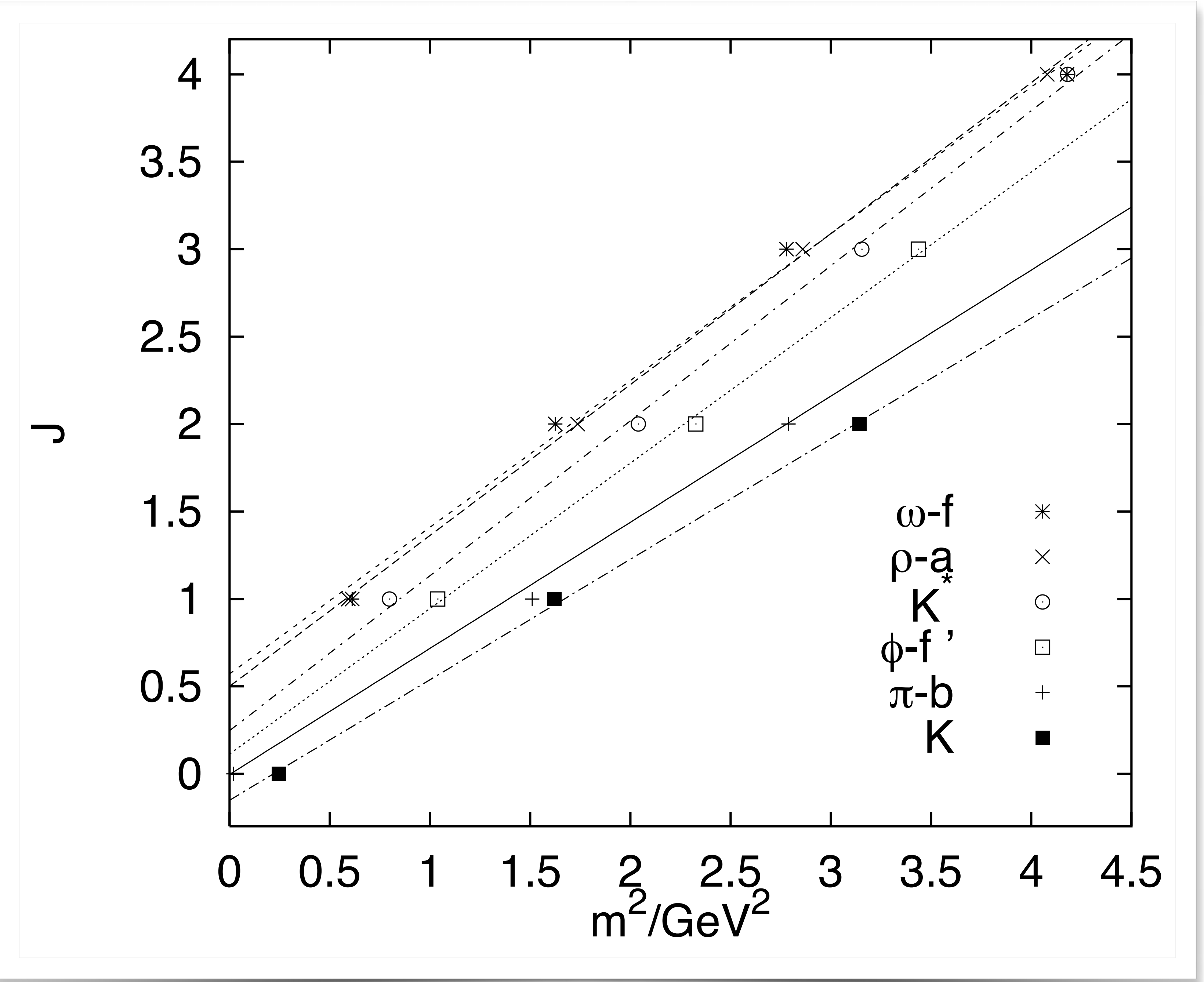




> rigid rotor model: $J = \dfrac{m^2}{2\pi\sigma}$

Strong coupling theory

Discovery of rotational excitations of dopants

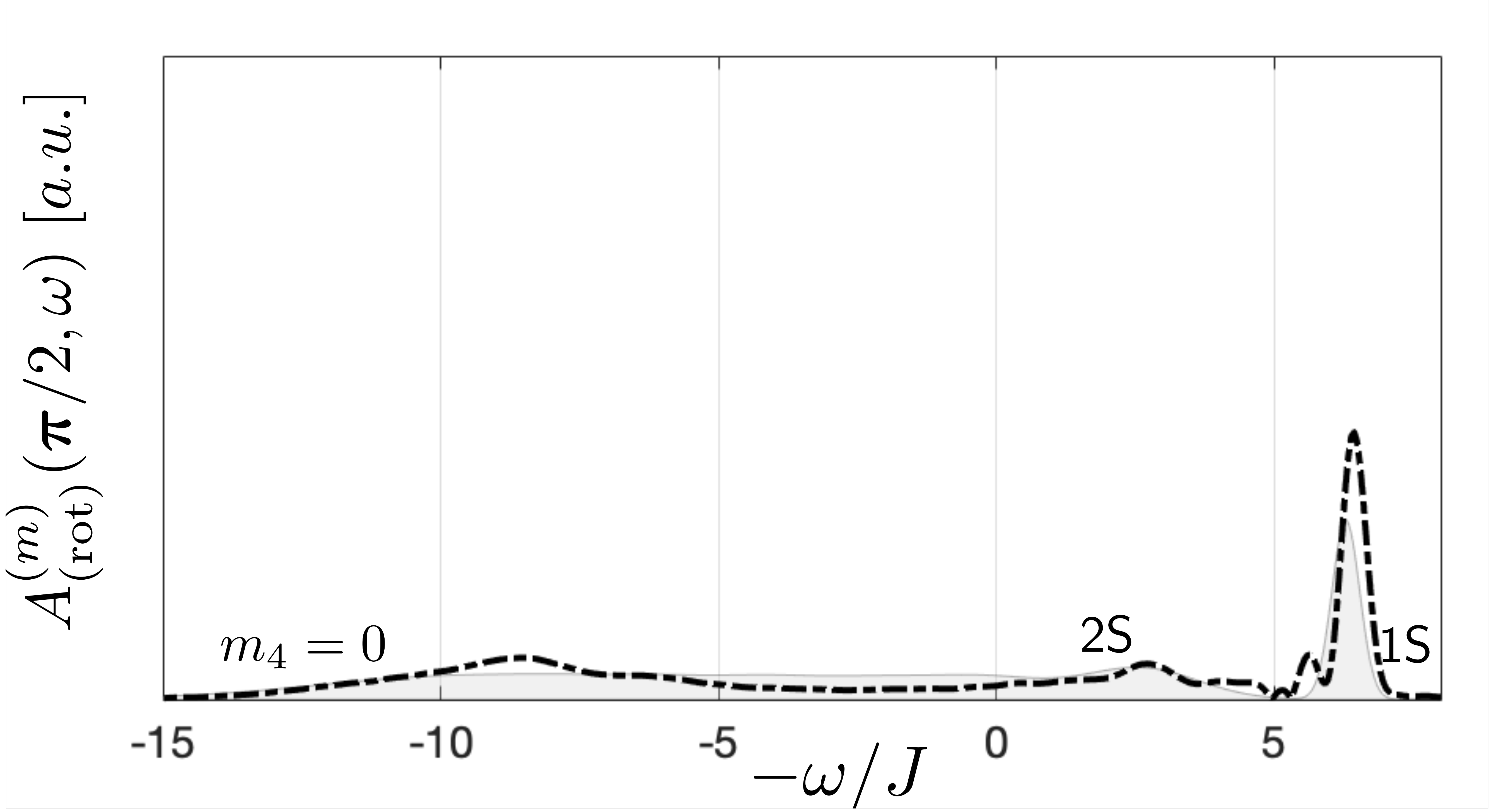
> Proof of meson-nature!

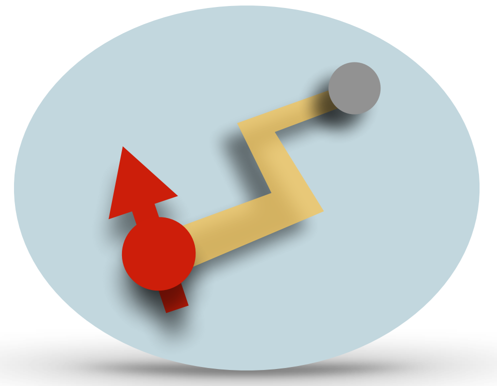
PHYSICAL REVIEW LETTERS **127**, 197004 (2021)

Bohrdt et al., PRL 127 (2021)

Rotational Resonances and Regge-like Trajectories in Lightly Doped Antiferromagnets

A. Bohrdt,[1,2,3,4,*] E. Demler,[4] and F. Grusdt[5,2]





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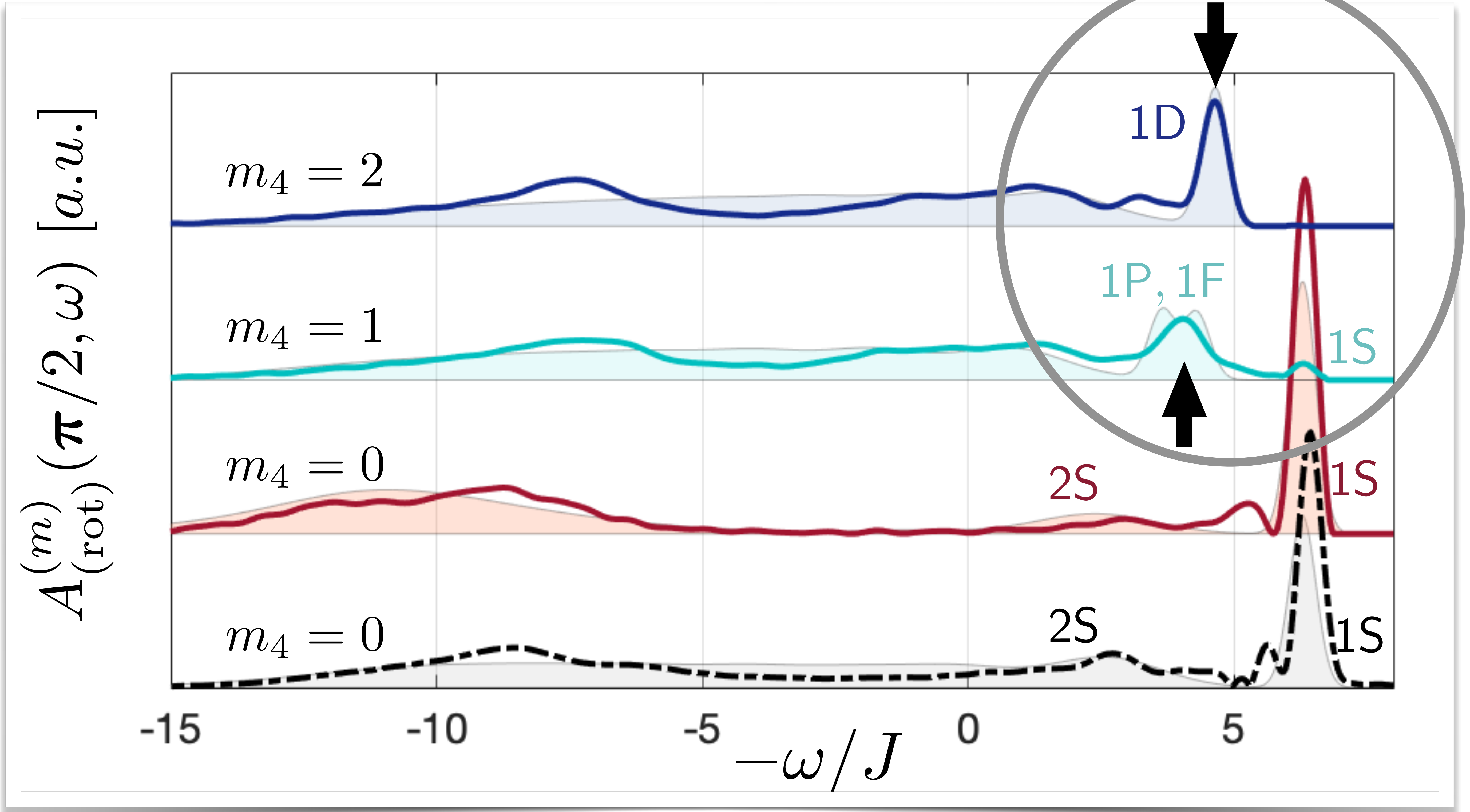

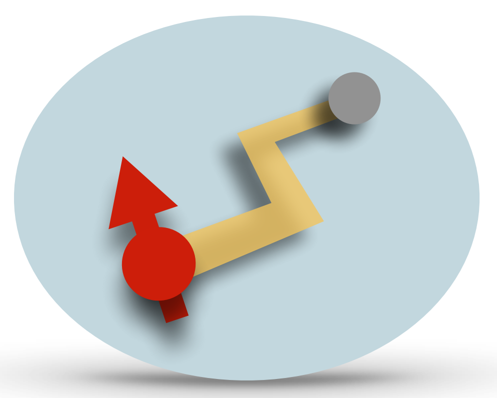

— rotational ARPES

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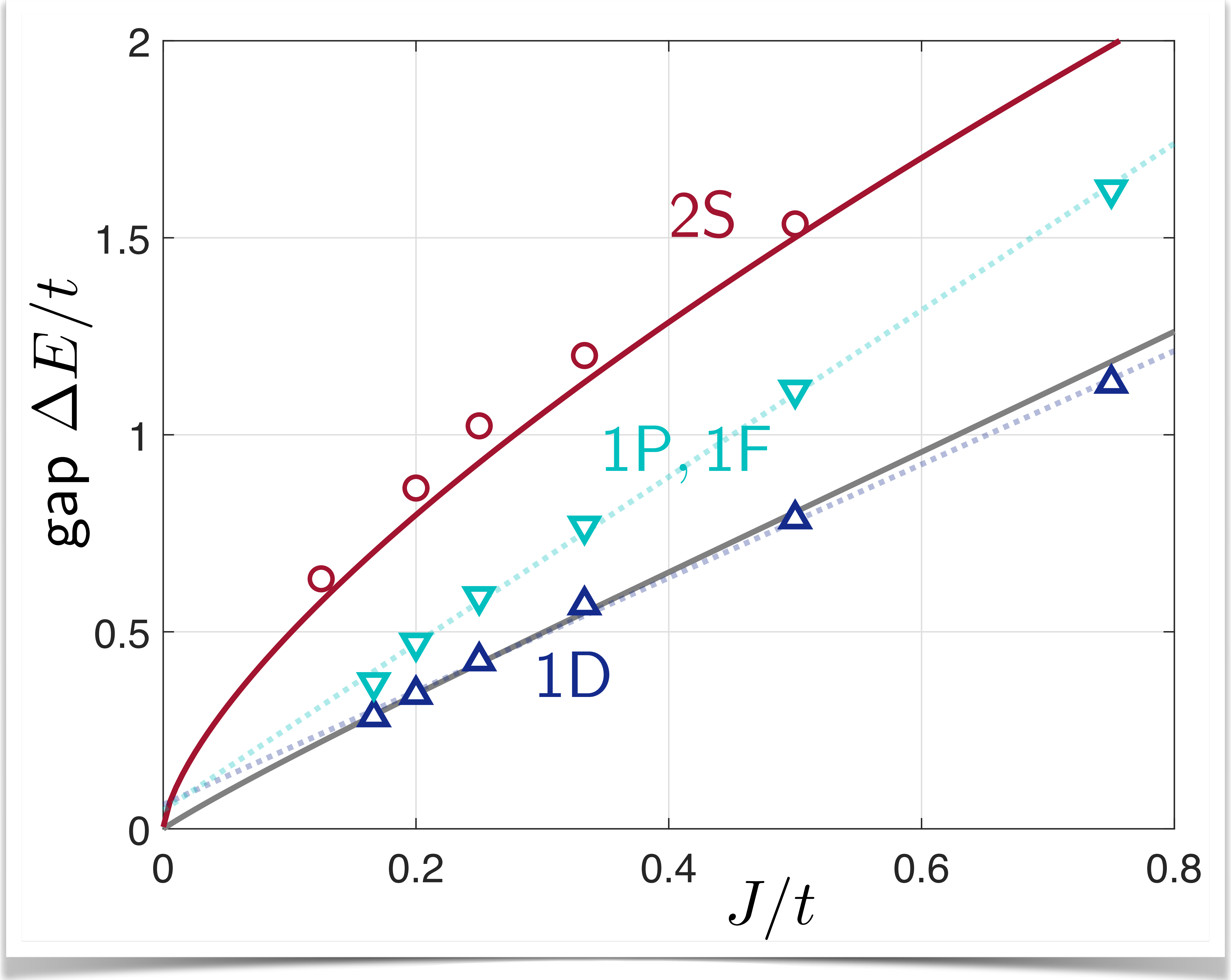
Strong coupling theory

Regge-like trajectories:

energy



- Vibrational states:

$$\Delta E_{\mathrm{vib}} \simeq t^{1/3} J^{2/3}$$

- Rotational states:

$$\Delta E_{\mathrm{rot}} \simeq J$$

As predicted by string model!

Grusdt et al., PRX 8 (2018)
Bohrdt et al., PRL 127 (2021)
Simons & Gunn, PRB 41 (1990)

Strong coupling theory

Regge-like trajectories:

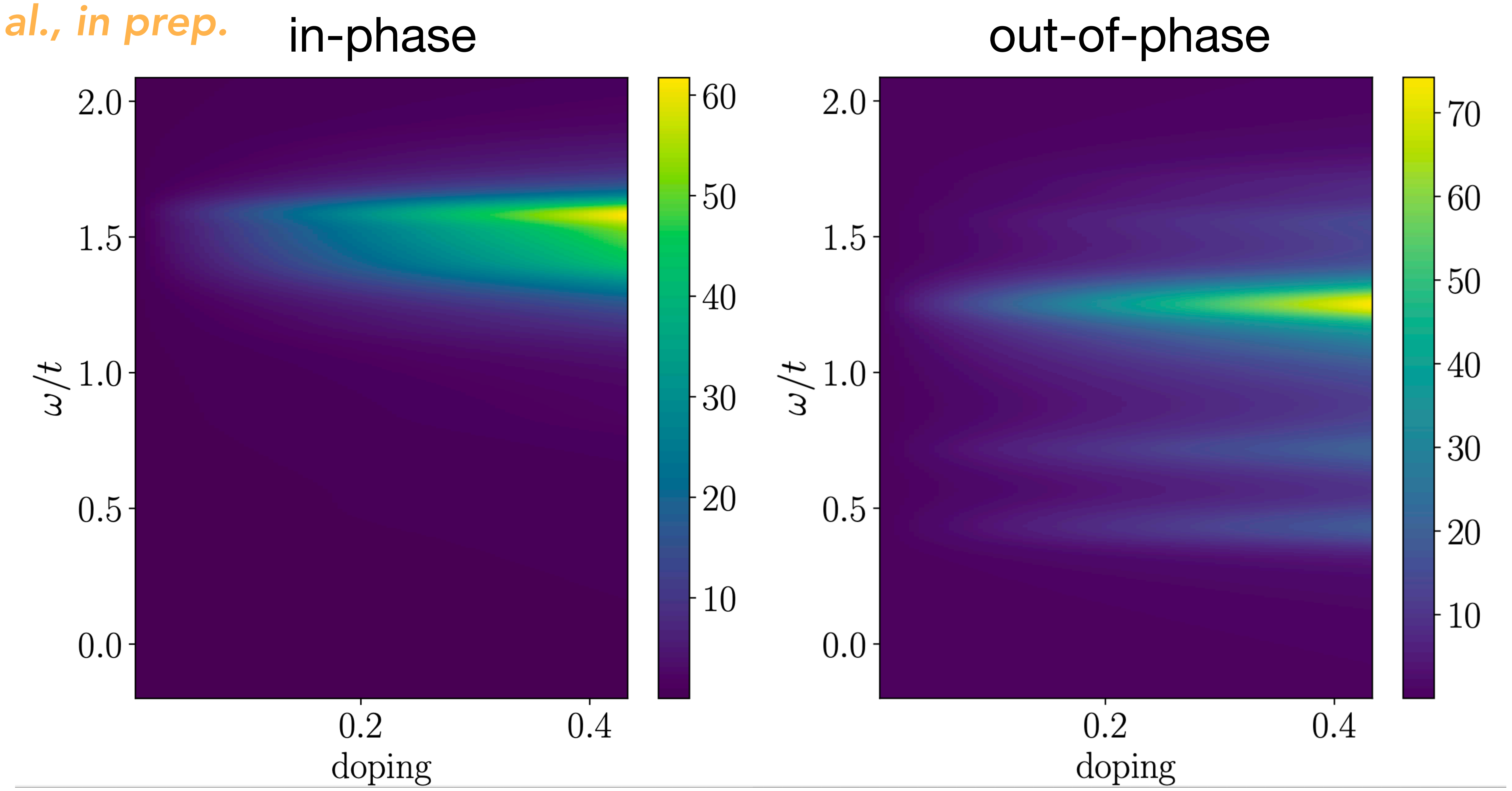
Lattice modulation spectroscopy

$$t_\mu(t) = t\ [1 + A\cos(\omega t + \phi_\mu)]$$

Kendrick et al., arXiv:2509.18075 — Greiner lab

Bermes et al., in prep.

Theory proposal



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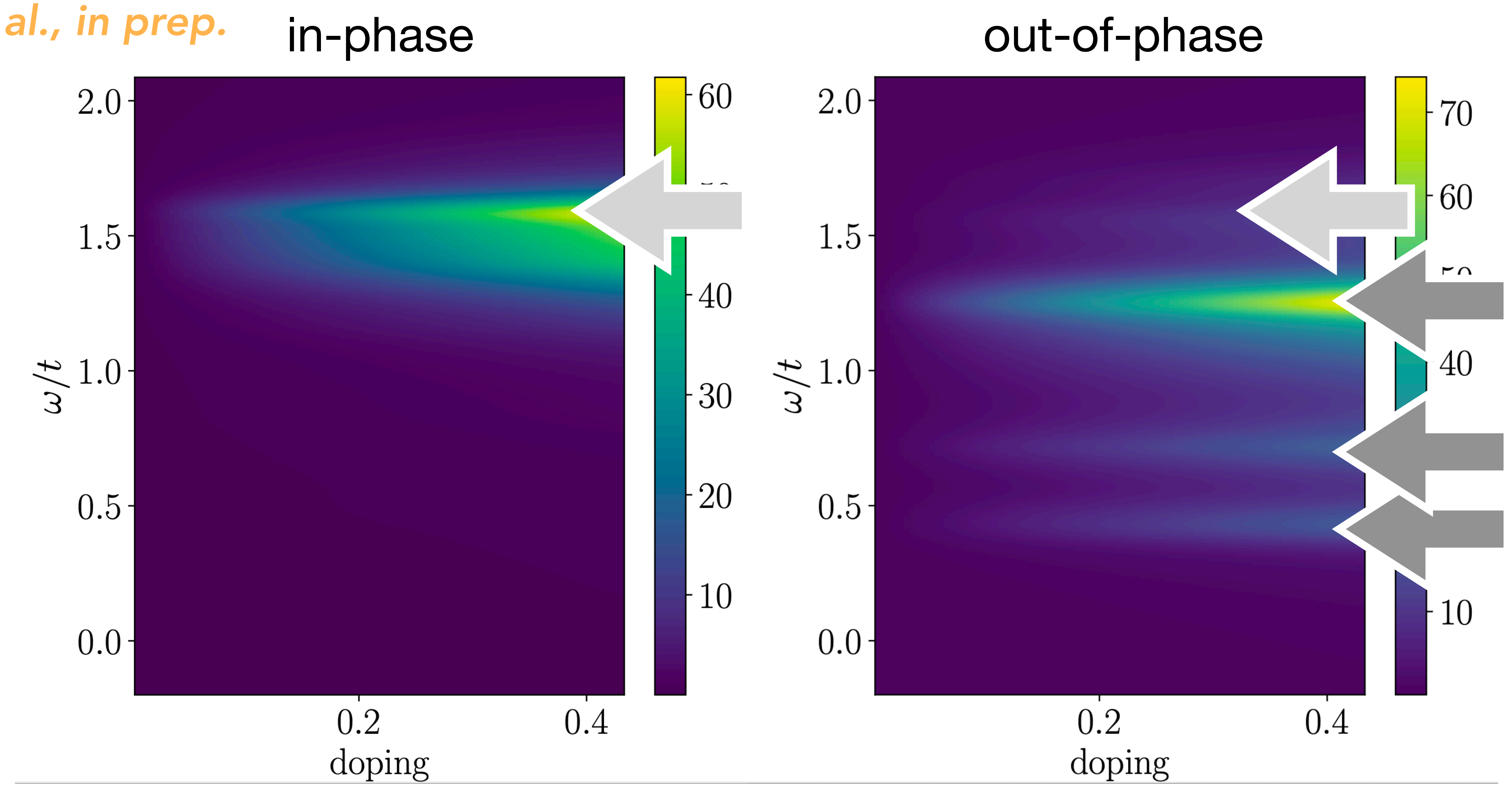

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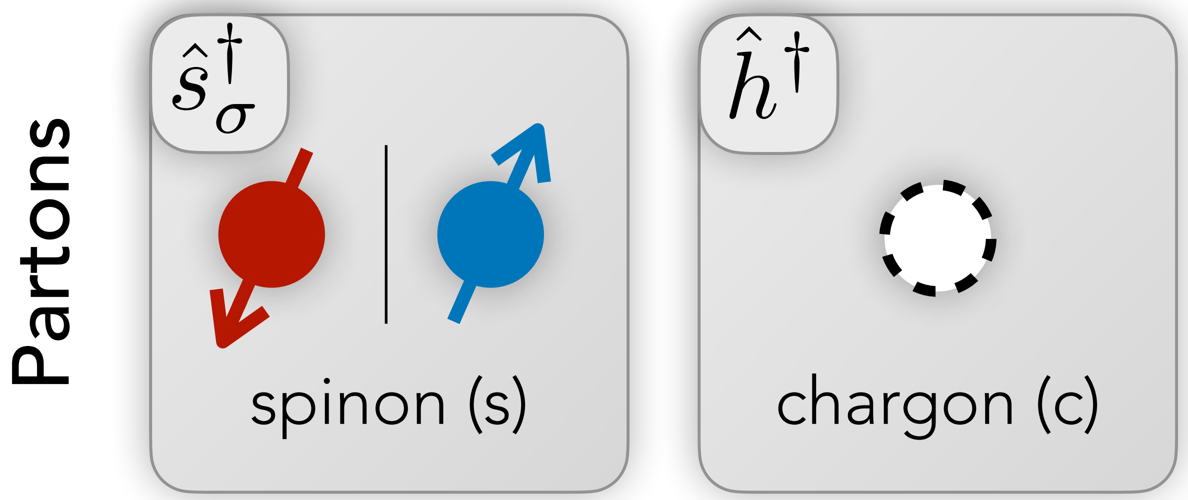
From microscopic…

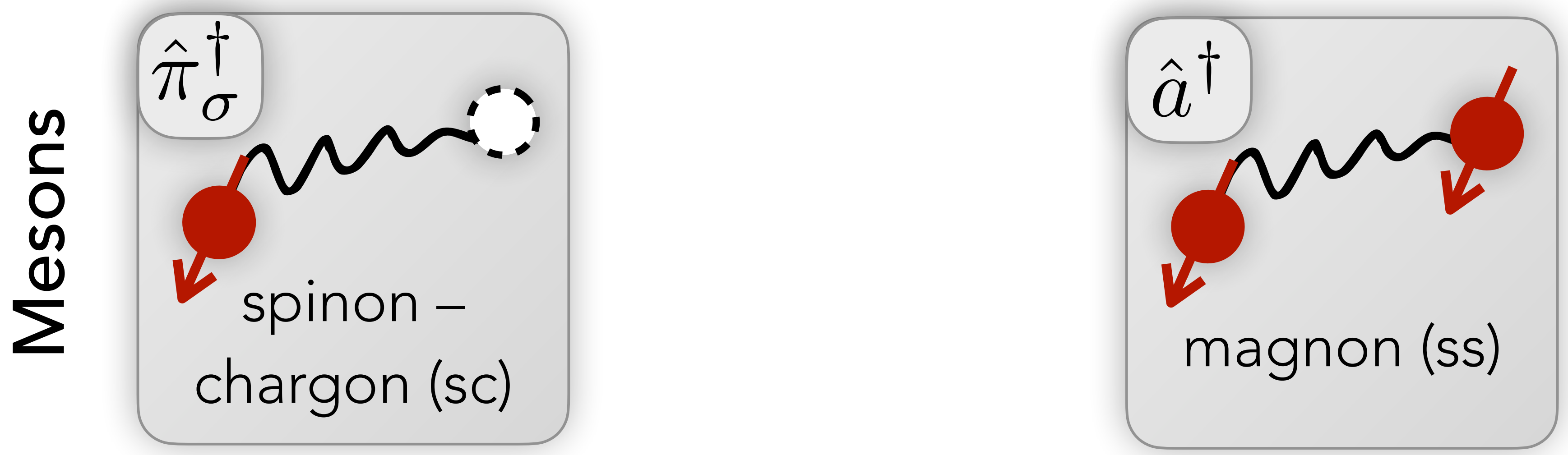


- Complicated Hamiltonian

$$\hat{\mathcal{H}}_{t-J} = \dots$$

- Theories of partons: Anderson's RVB picture (doubted)

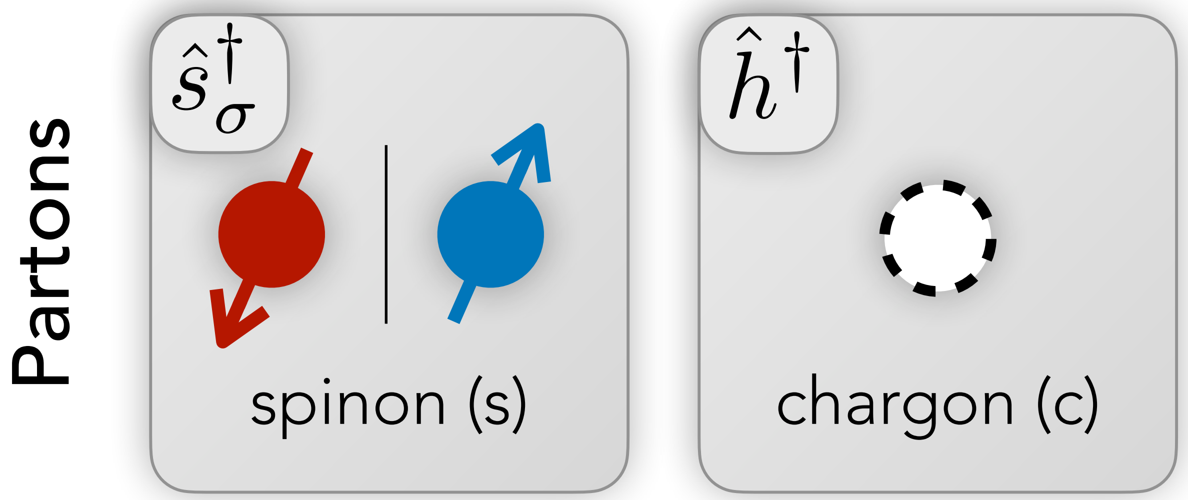
… to hadronic constituents



- Long-lived quasiparticle excitations

Strong coupling theory

From microscopic…



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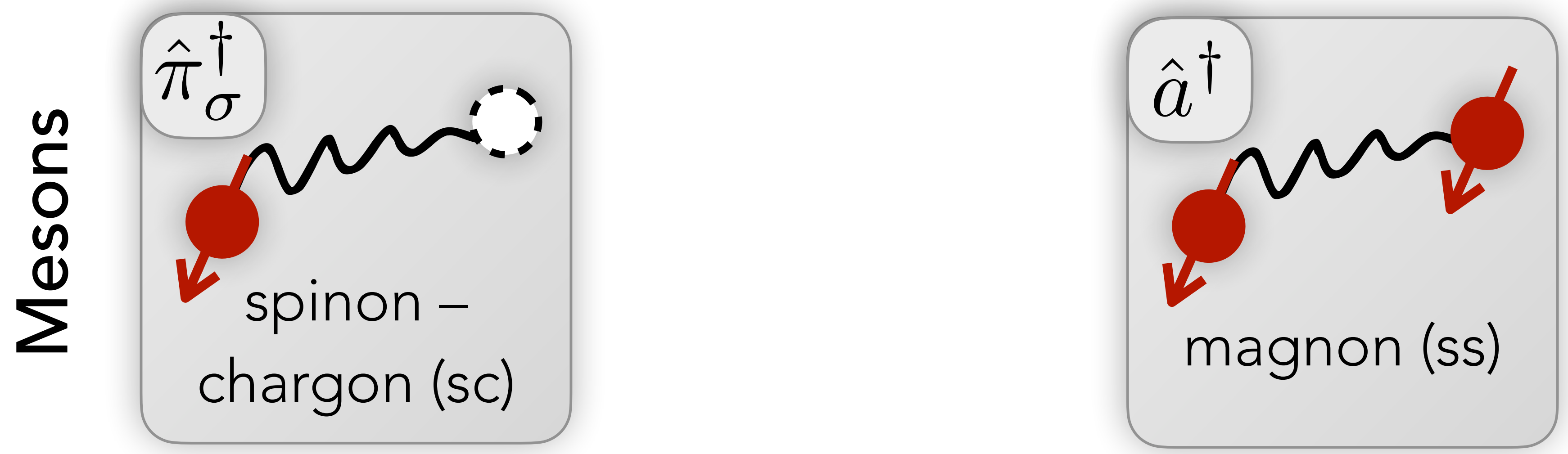
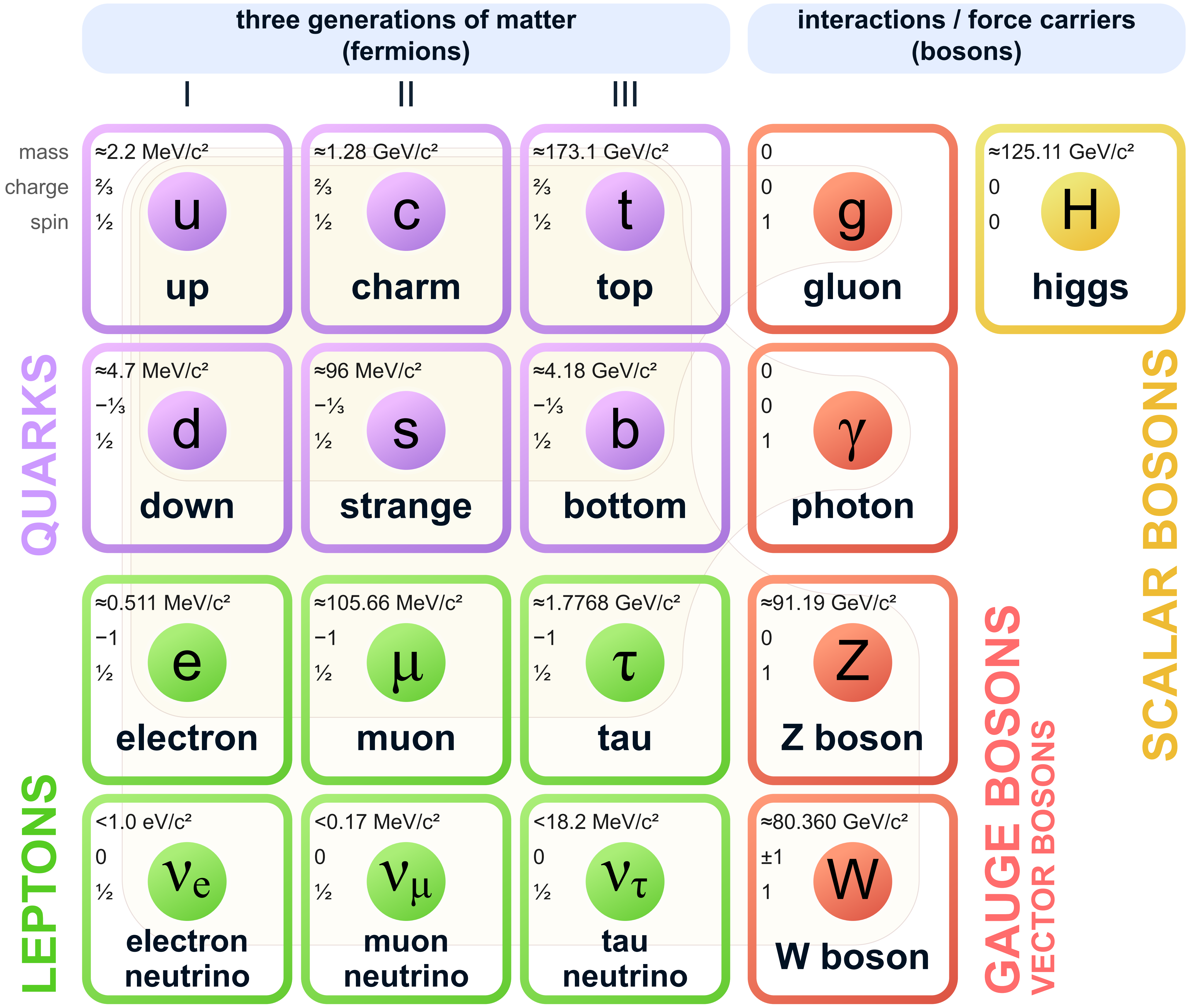


- Long-lived quasiparticle excitations

Striking signatures of confinement in weakly doped models of high-Tc superconductors!

Strong coupling theory

From the standard model...



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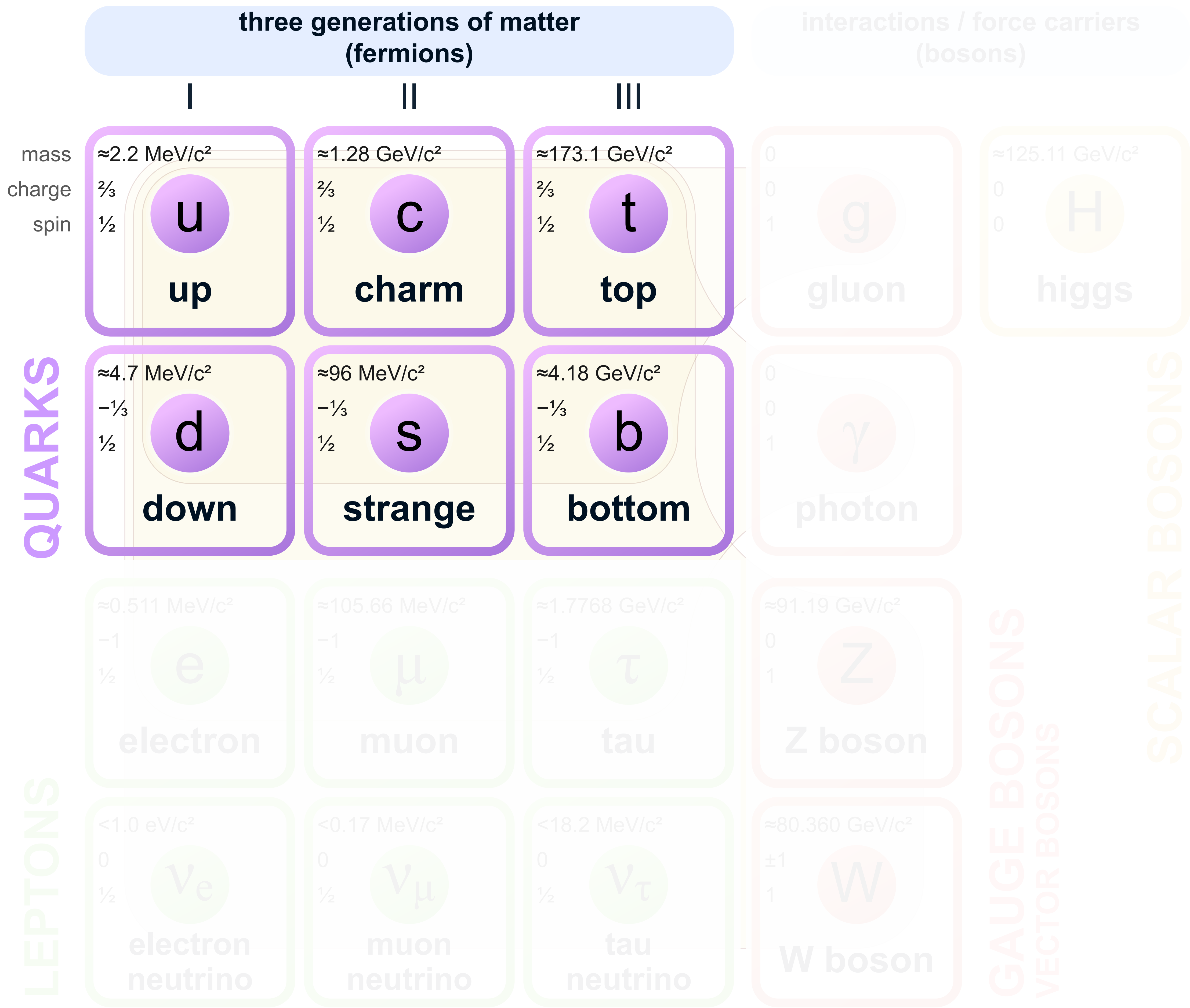
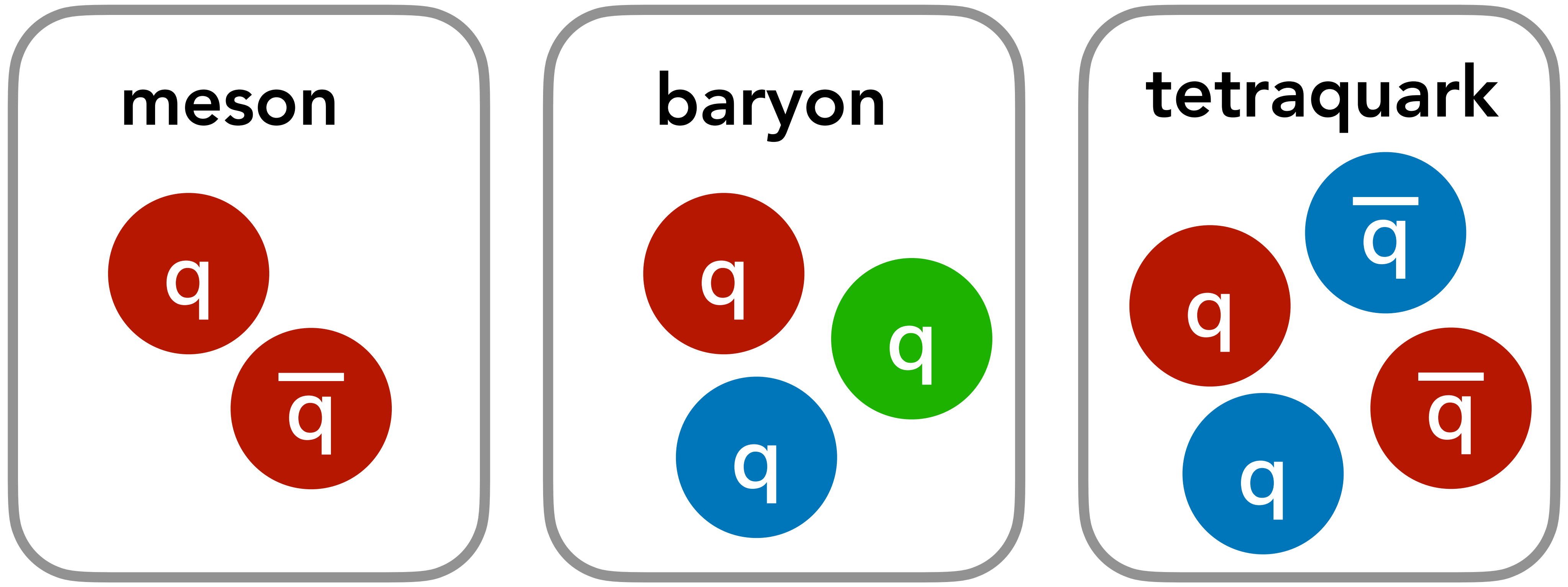
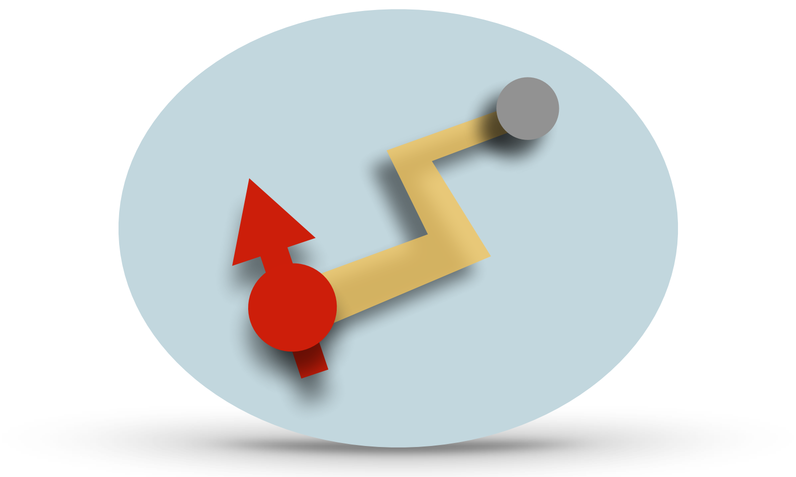


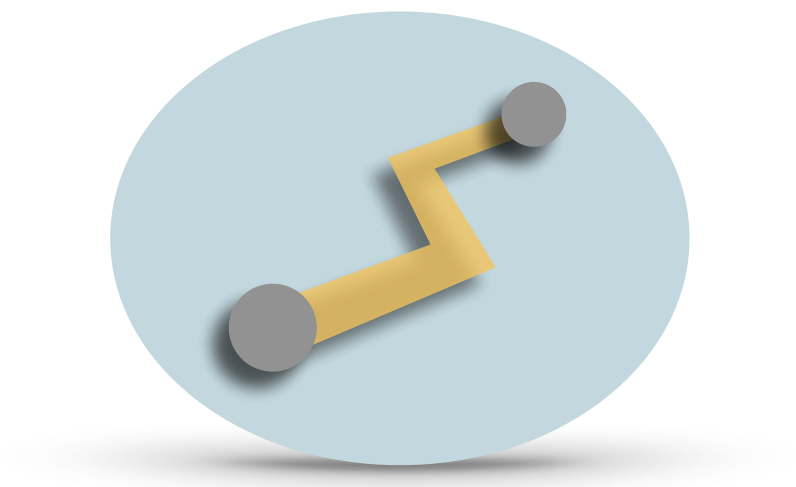
… to hadrons



… to nucleons … to …

PART 2.II: Two-hole excitations — Pairing & Feshbach hypothesis



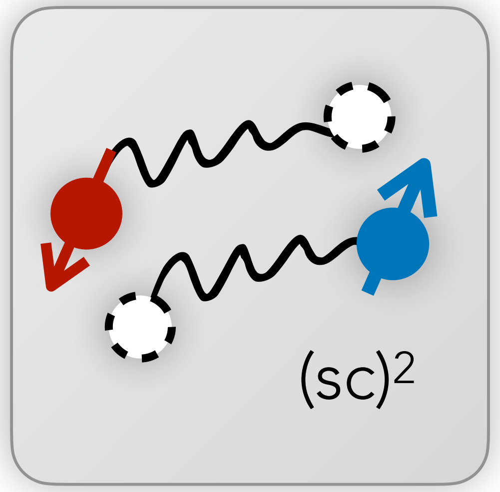


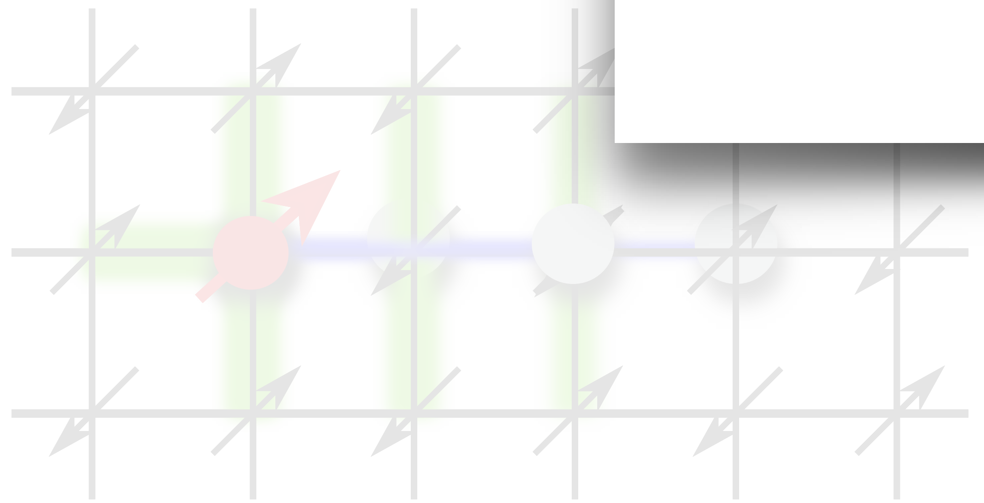
Strong coupling theory

Meson structure of magnetic polarons:

Spinon-chargon (sc)^2 Cooper pair

Pair of type I =





...el state

* Hole is **bound to the fractional spin (spinon)** at end of string (2D)!

$$E \propto \ell$$

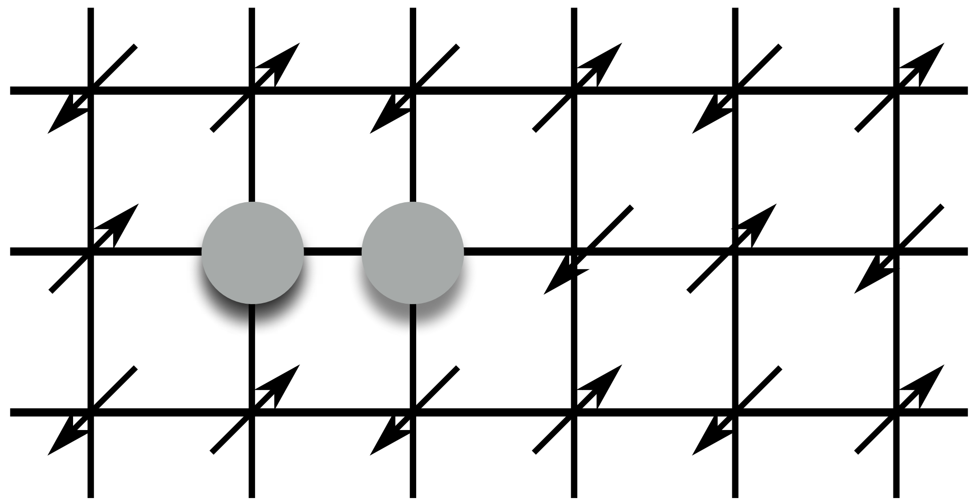
Bulaevskii et al., JETP 27 (1968), Trugman, PRB 37 (1988)
Manousakis, PRB 75 (2007, Kane et al., PRB 39 (1989)

Grusdt et al., PRX 8 (2018)
Grusdt et al., PRB 99 (2019)

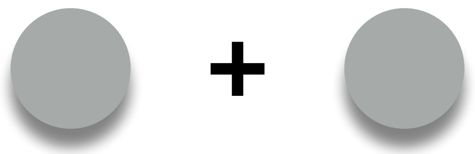
Bohrdt et al., PRB 102 (2020)
Bohrdt et al., PRL 127 (2021)

Strong coupling theory

Meson structure of pairs:



* **Two** hole excitations



Shraiman & Siggia, PRL 60 (1988)
Bohrdt et al., Nat. Phys. 18 (2022)
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Strong coupling theory

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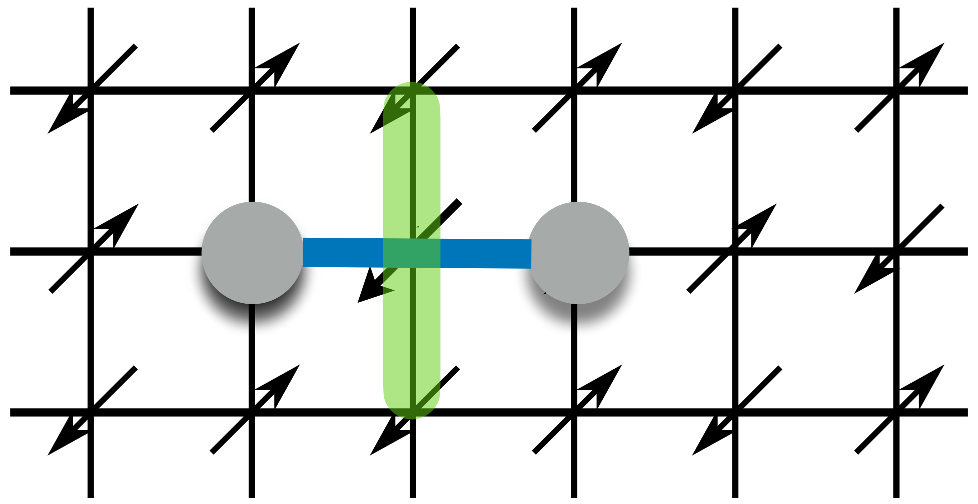

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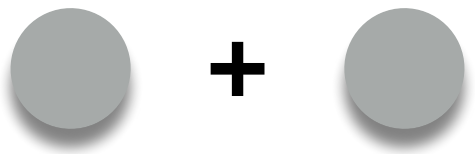
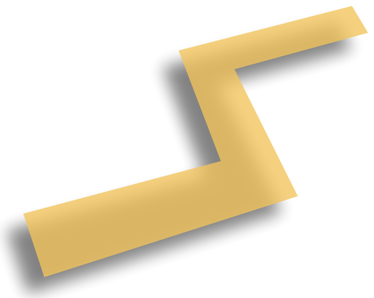


- Hole motion generates **string**



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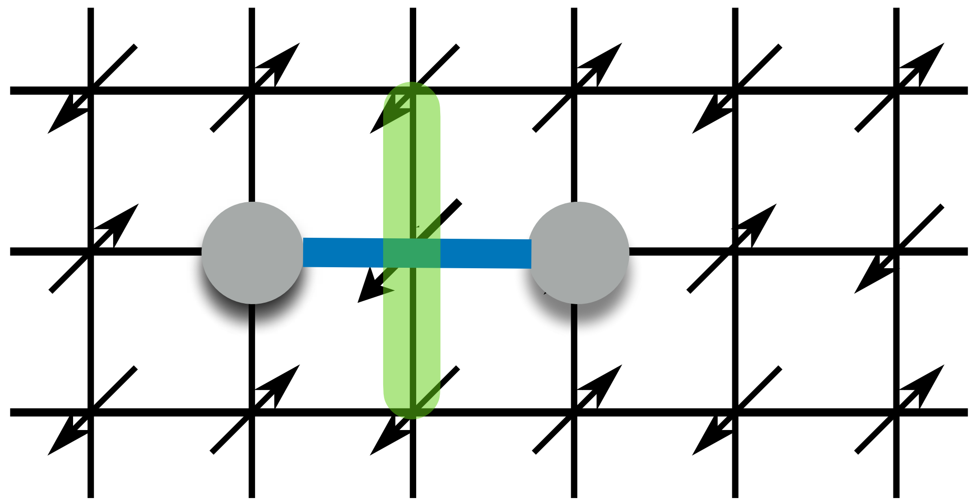

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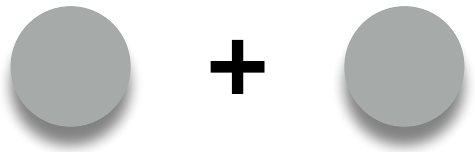

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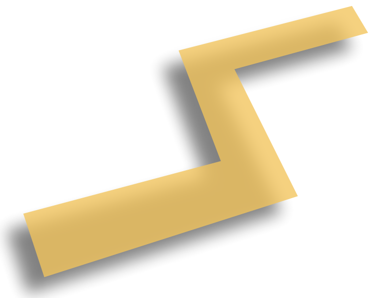
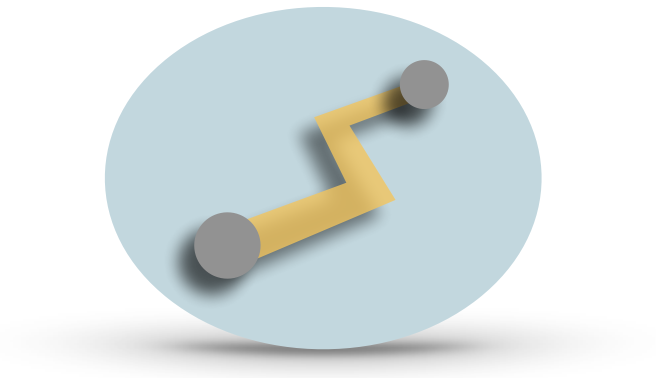


* One hole is **bound to another hole** at end of string (2D)

$$E \propto \ell$$



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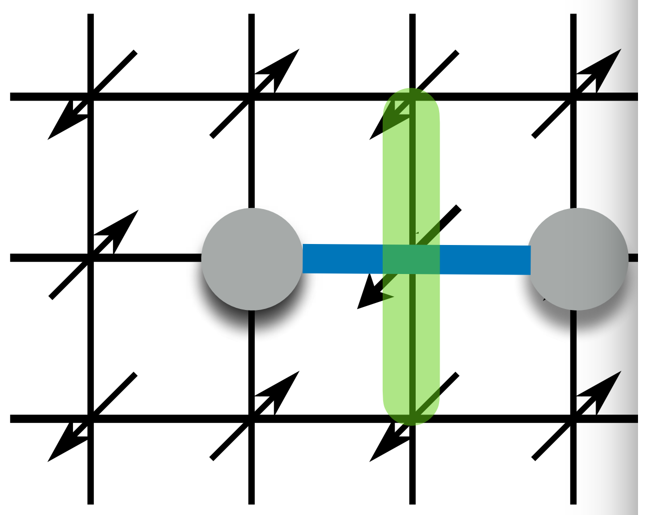
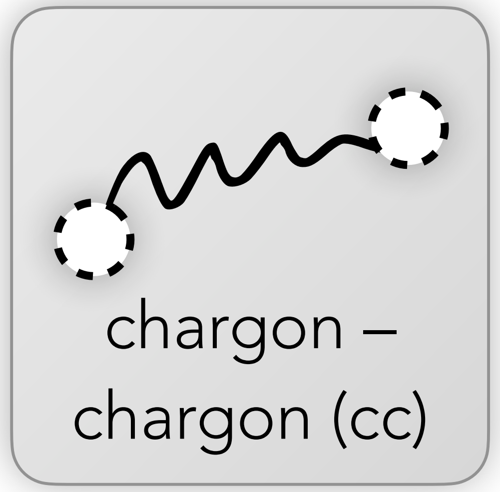
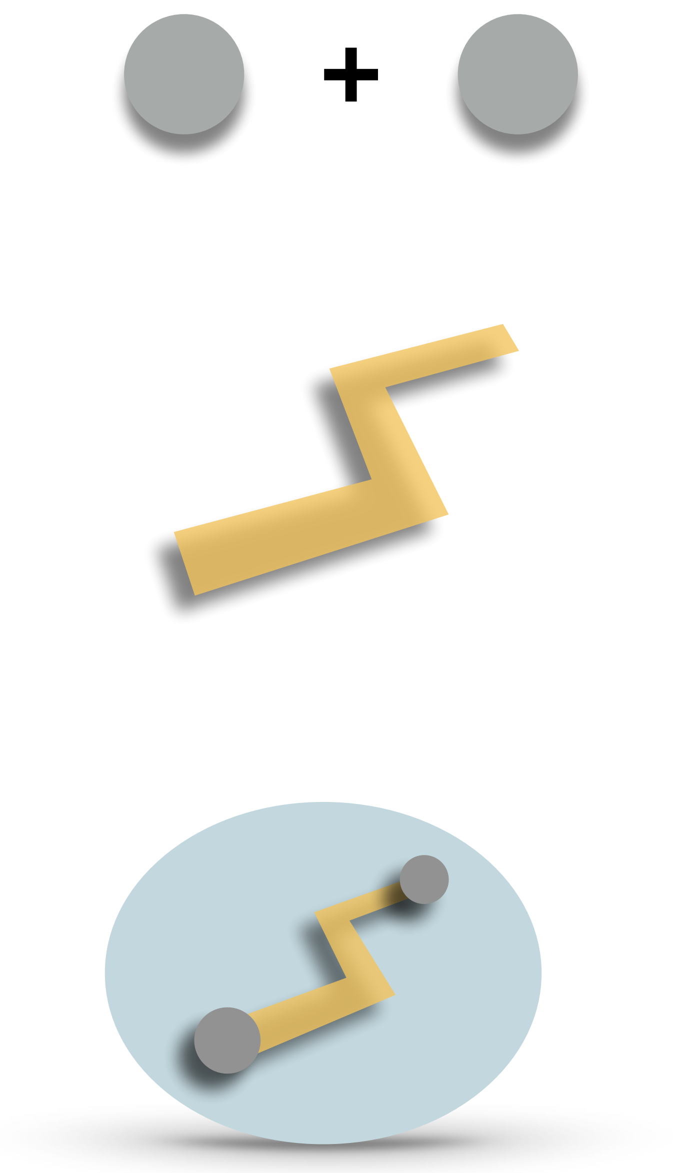


Chargon-chargon (cc) meson

Pair of type II =





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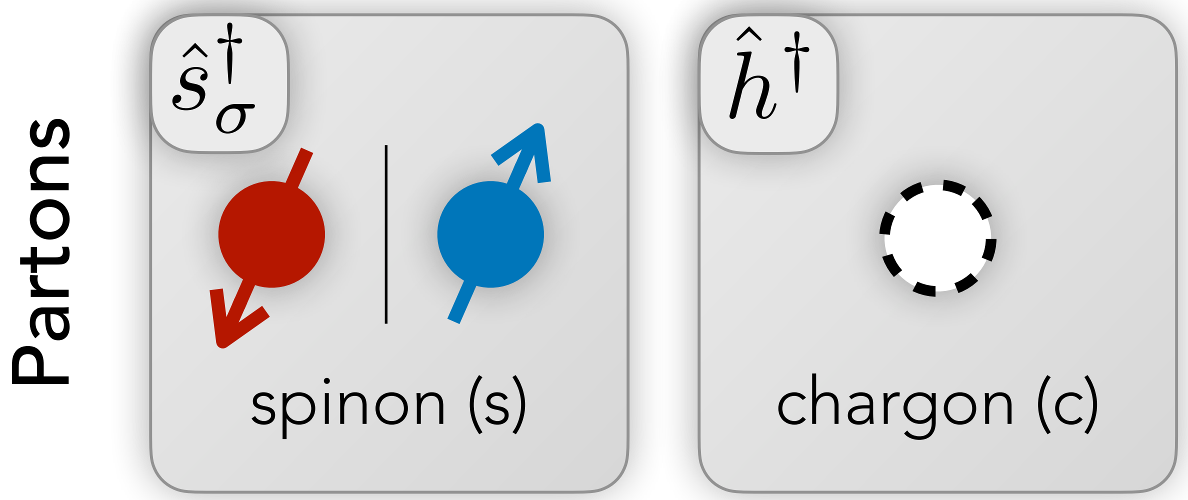
Strong coupling theory

From microscopic…



- Complicated Hamiltonian

$$\hat{\mathcal{H}}_{t-J} = \dots$$

- Theories of partons: Anderson's RVB picture (doubted)

… to hadronic constituents

- Long-lived quasiparticle excitations

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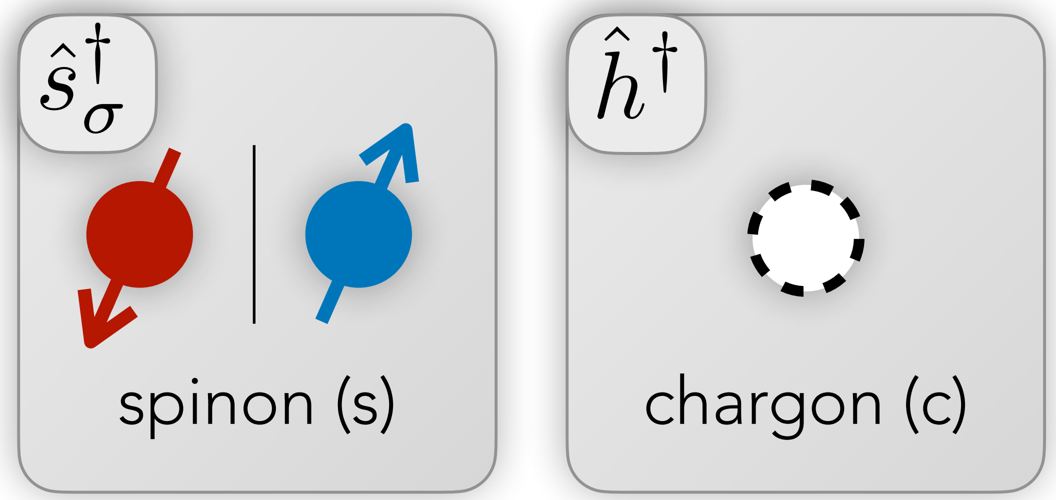
Strong coupling theory

From microscopic…

Partons



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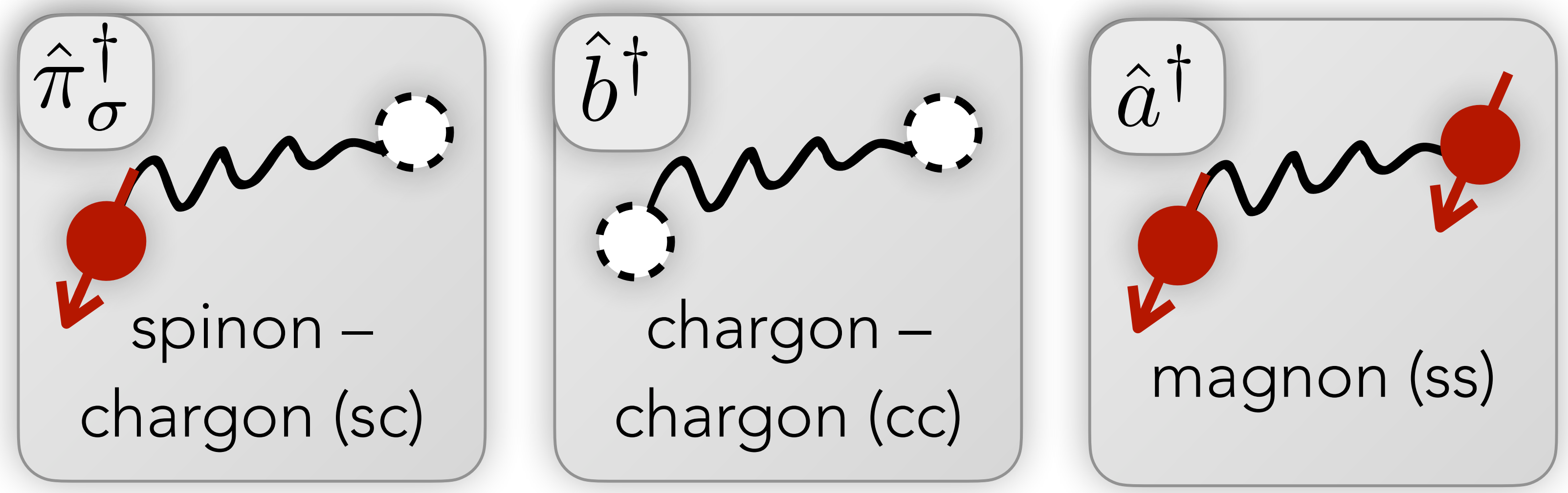
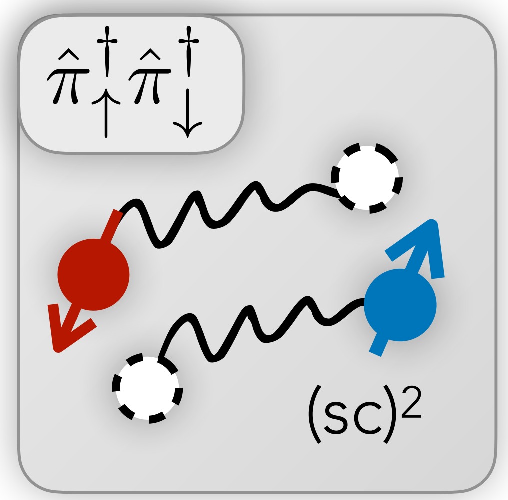


Tetra-parton



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Strong coupling theory

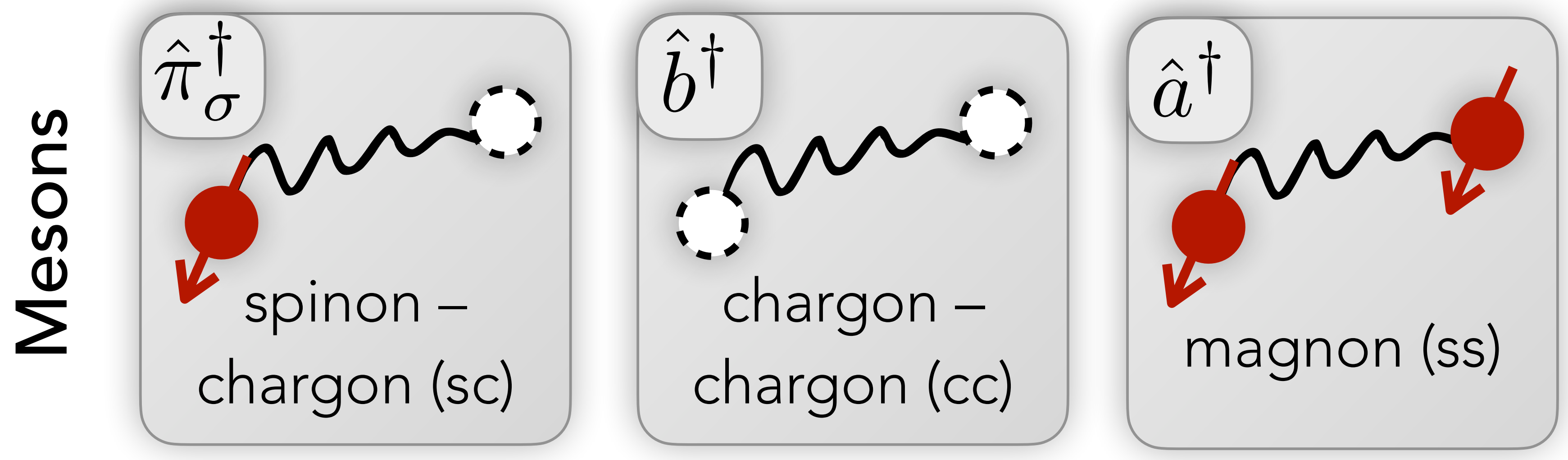
Validity of the parton picture

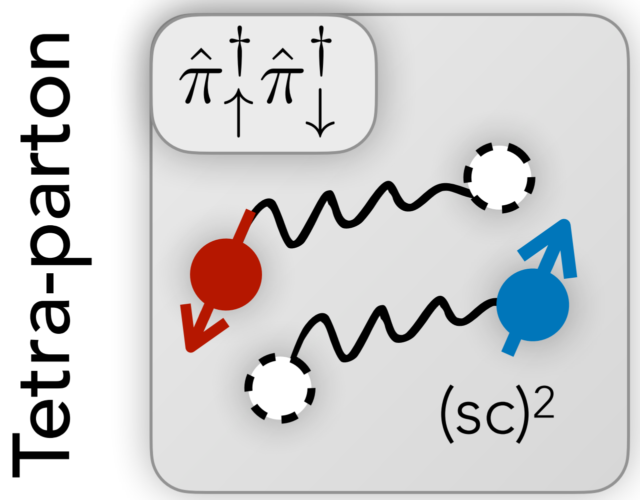
Emergent hadronic constituents

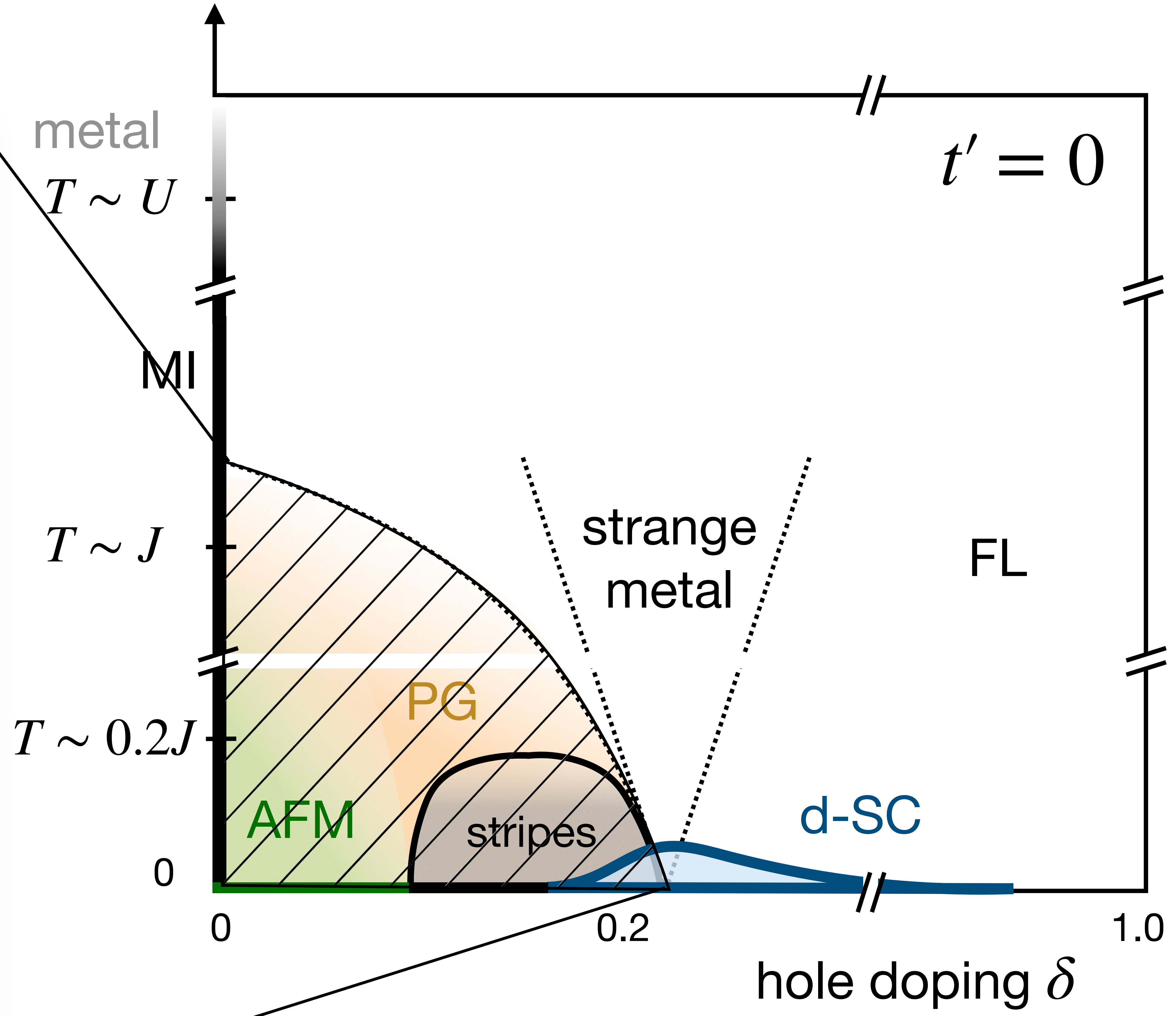
Béran et al., Nuc. Phys. B 473 (1996)

Bohrdt et al., PRB 102 (2020)

Homeier et al., arXiv:2312.02982







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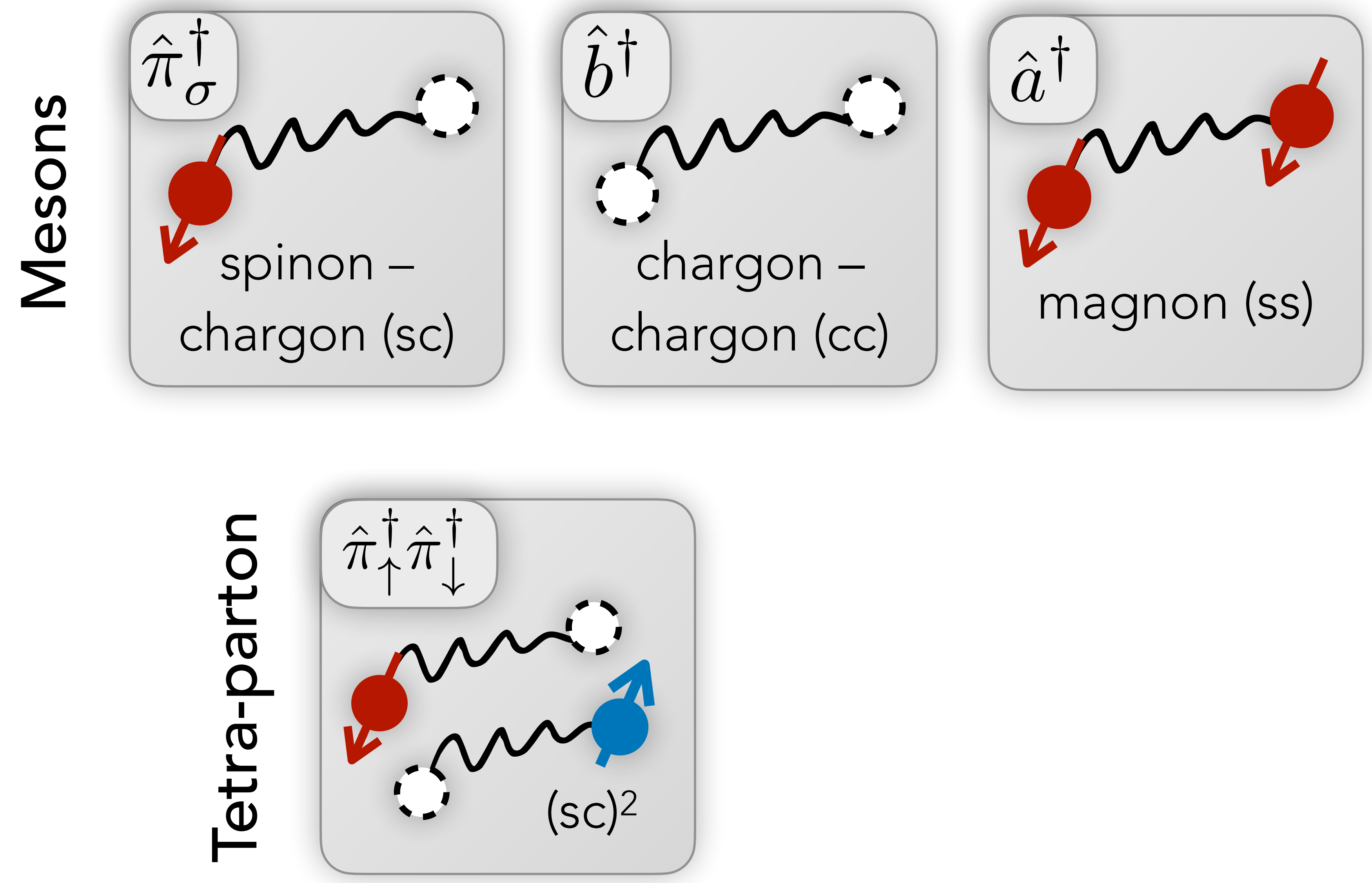
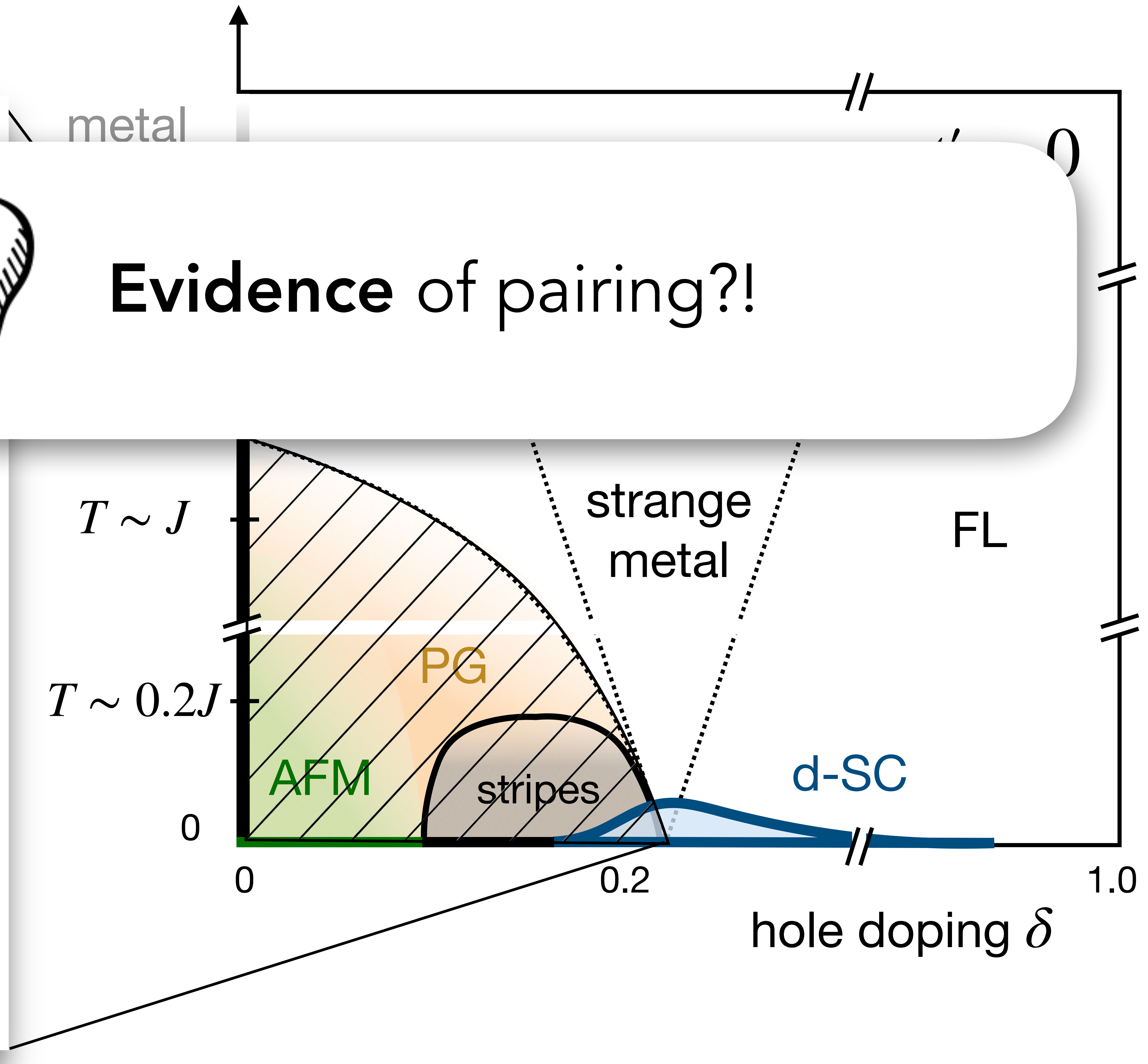


Evidence of pairing?!



Strong coupling theory

Two-hole spectroscopy:

- Pair Green's function:

$$\mathcal{G}_{\text{rot}}^{(m_4)}(\boldsymbol{k},t) = \theta(t)\left\langle \Psi_0 \middle| \hat{\Delta}_{m_4}^{(s)\dagger}(\boldsymbol{k},t)\hat{\Delta}_{m_4}^{(s)}(\boldsymbol{k},0) \middle| \Psi_0 \right\rangle$$

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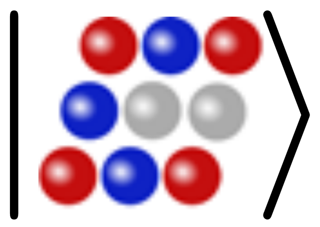
— *pair creation:*

$$\hat{\Delta}_{m_4}(\boldsymbol{j},\sigma,\sigma') = \sum_{\boldsymbol{i}:\langle \boldsymbol{i},\boldsymbol{j}\rangle} e^{i m_4 \varphi_{\boldsymbol{i}-\boldsymbol{j}}} \hat{c}_{\boldsymbol{i},\sigma'} \hat{c}_{\boldsymbol{j},\sigma}$$



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$$\hat{\Delta}_{m_4}(\boldsymbol{j},\sigma,\sigma') = \sum_{\boldsymbol{i}:\langle \boldsymbol{i},\boldsymbol{j}\rangle} e^{im_4\varphi_{\boldsymbol{i}-\boldsymbol{j}}}\hat{c}_{\boldsymbol{i},\sigma'}\hat{c}_{\boldsymbol{j},\sigma}$$

* Rotational excitations:

|  ⟩ $+e^{im_4\pi/2}$ |  ⟩ $+e^{im_4\pi}$ |  ⟩ $+e^{im_4 3\pi/2}$ |  ⟩

Strong coupling theory

Two-hole spectroscopy:

- Pair Green's function:

$$\mathcal{G}^{(m_4)}_{\mathrm{rot}}(\boldsymbol{k},t)=\theta(t)\langle\Psi_0|\hat{\Delta}^{(s)\dagger}_{m_4}(\boldsymbol{k},t)\hat{\Delta}^{(s)}_{m_4}(\boldsymbol{k},0)|\Psi_0\rangle$$

— *pair creation:*

$$\hat{\Delta}_{m_4}(\boldsymbol{j},\sigma,\sigma')=\sum_{\boldsymbol{i}:\langle\boldsymbol{i},\boldsymbol{j}\rangle} e^{im_4\varphi_{\boldsymbol{i}-\boldsymbol{j}}}\hat{c}_{\boldsymbol{i},\sigma'}\hat{c}_{\boldsymbol{j},\sigma}$$

- The *t-XXZ* model:

$$\hat{\mathcal{H}}_{t-\mathrm{XXZ}}=\sum_{\langle\boldsymbol{i},\boldsymbol{j}\rangle}\left(J_\perp\left(\hat{S}^x_{\boldsymbol{i}}\hat{S}^x_{\boldsymbol{j}}+\hat{S}^y_{\boldsymbol{i}}\hat{S}^y_{\boldsymbol{j}}\right)+J_z\hat{S}^z_{\boldsymbol{i}}\hat{S}^z_{\boldsymbol{j}}\right)-$$

$$-t\,\hat{\mathcal{P}}\sum_{\langle\boldsymbol{i},\boldsymbol{j}\rangle}\sum_{\sigma}\left(\hat{c}^\dagger_{\boldsymbol{i},\sigma}\hat{c}_{\boldsymbol{j},\sigma}+\mathrm{h.c.}\right)\hat{\mathcal{P}}-\frac{J_z}{4}\sum_{\langle\boldsymbol{i},\boldsymbol{j}\rangle}\hat{n}_{\boldsymbol{i}}\hat{n}_{\boldsymbol{j}}.$$

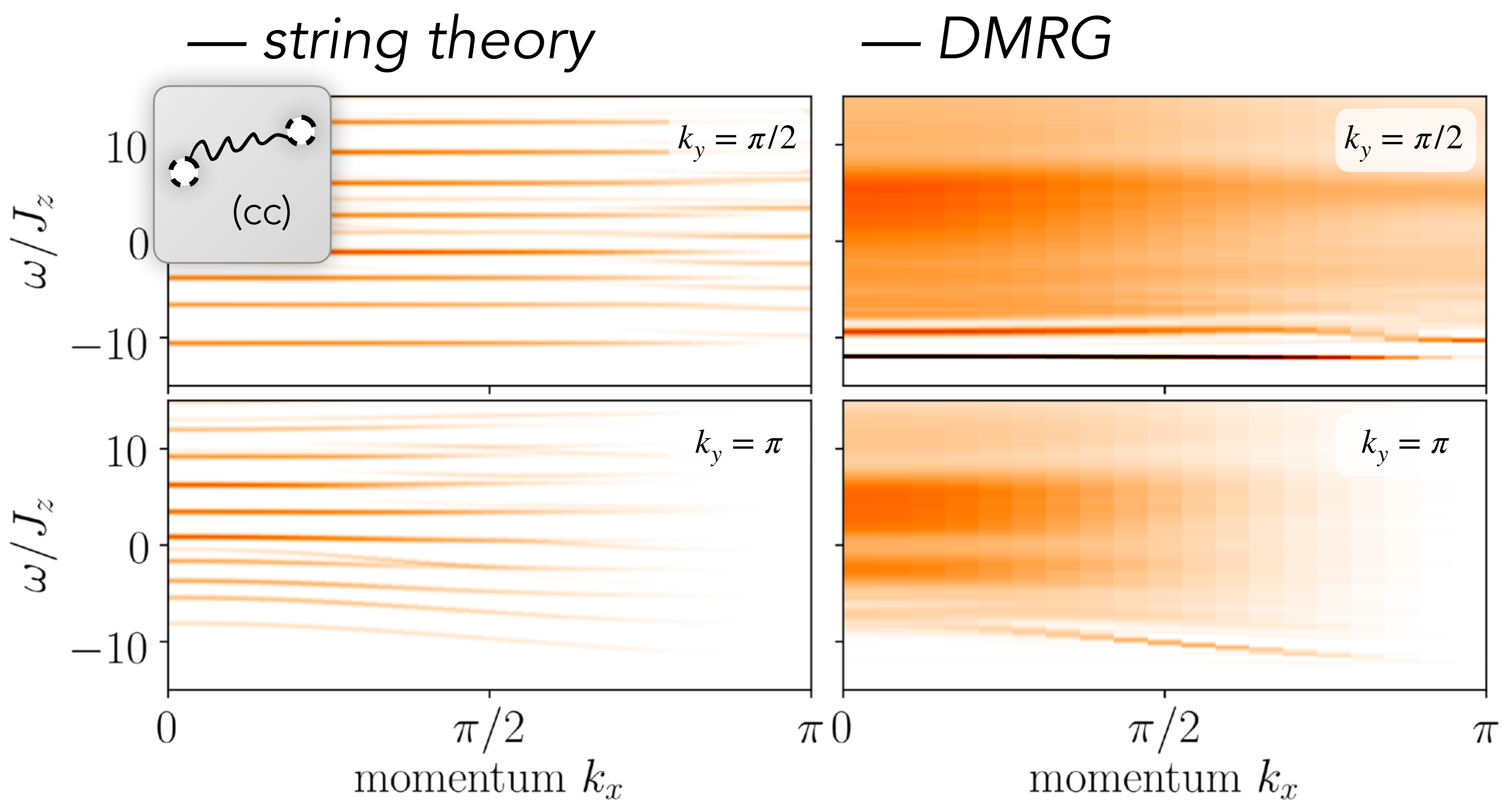
- Rotational excitations:

$|$  $\rangle + e^{im_4\pi/2}|$  $\rangle + e^{im_4\pi}|$  $\rangle + e^{im_4 3\pi/2}|$  $\rangle$

Strong coupling theory

Signatures of cc pairs:

Bohrdt et al., Nat. Comm. 14 (2023)

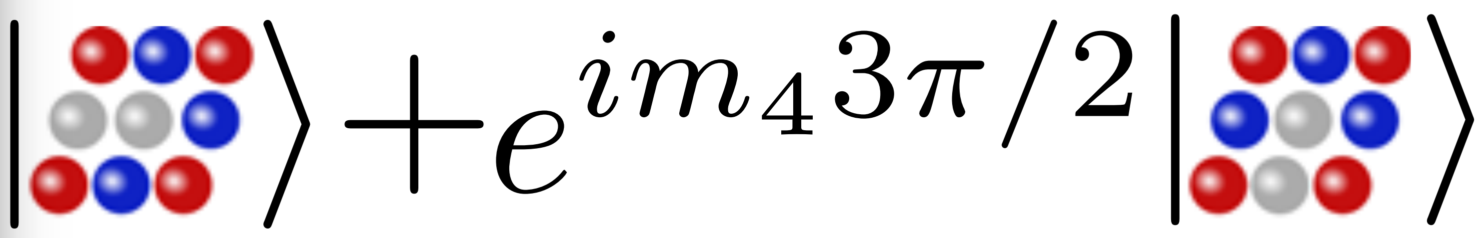


t-Jz model, d-wave pairing

* The *t-XXZ* model:

$$\hat{\mathcal{H}}_{t-\mathrm{XXZ}} = \sum_{\langle \boldsymbol{i},\boldsymbol{j}\rangle} \left(J_\perp \left(\hat{S}^x_{\boldsymbol{i}} \hat{S}^x_{\boldsymbol{j}} + \hat{S}^y_{\boldsymbol{i}} \hat{S}^y_{\boldsymbol{j}} \right) + J_z \hat{S}^z_{\boldsymbol{i}} \hat{S}^z_{\boldsymbol{j}} \right) - $$

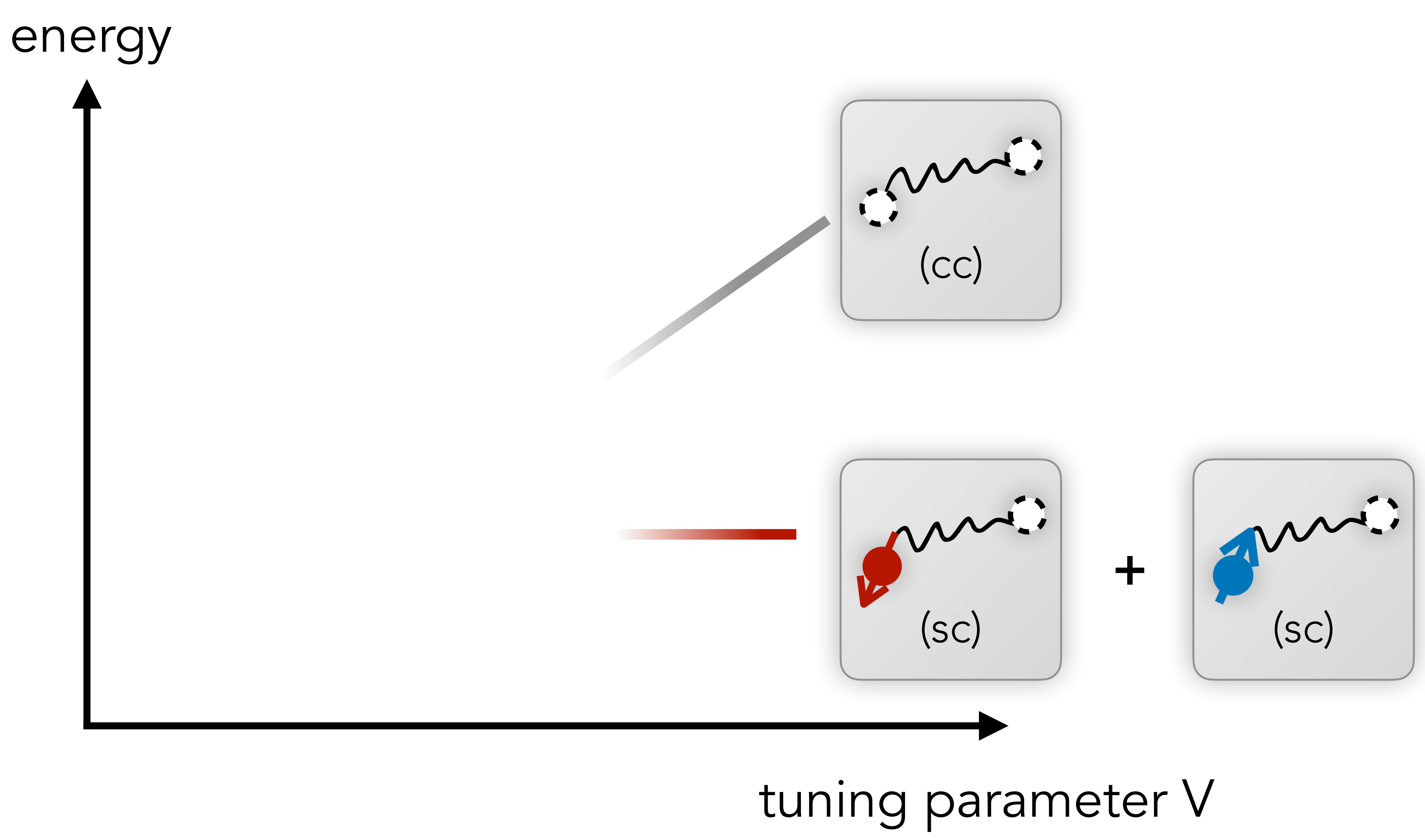
$$- t\, \hat{\mathcal{P}} \sum_{\langle \boldsymbol{i},\boldsymbol{j}\rangle} \sum_{\sigma} \left(\hat{c}^\dagger_{\boldsymbol{i},\sigma} \hat{c}_{\boldsymbol{j},\sigma} + \mathrm{h.c.} \right) \hat{\mathcal{P}} - \frac{J_z}{4} \sum_{\langle \boldsymbol{i},\boldsymbol{j}\rangle} \hat{n}_{\boldsymbol{i}} \hat{n}_{\boldsymbol{j}}.$$



Feshbach hypothesis

From one meson to another

Homeier et al., Nat. Comm. 16 (2025)

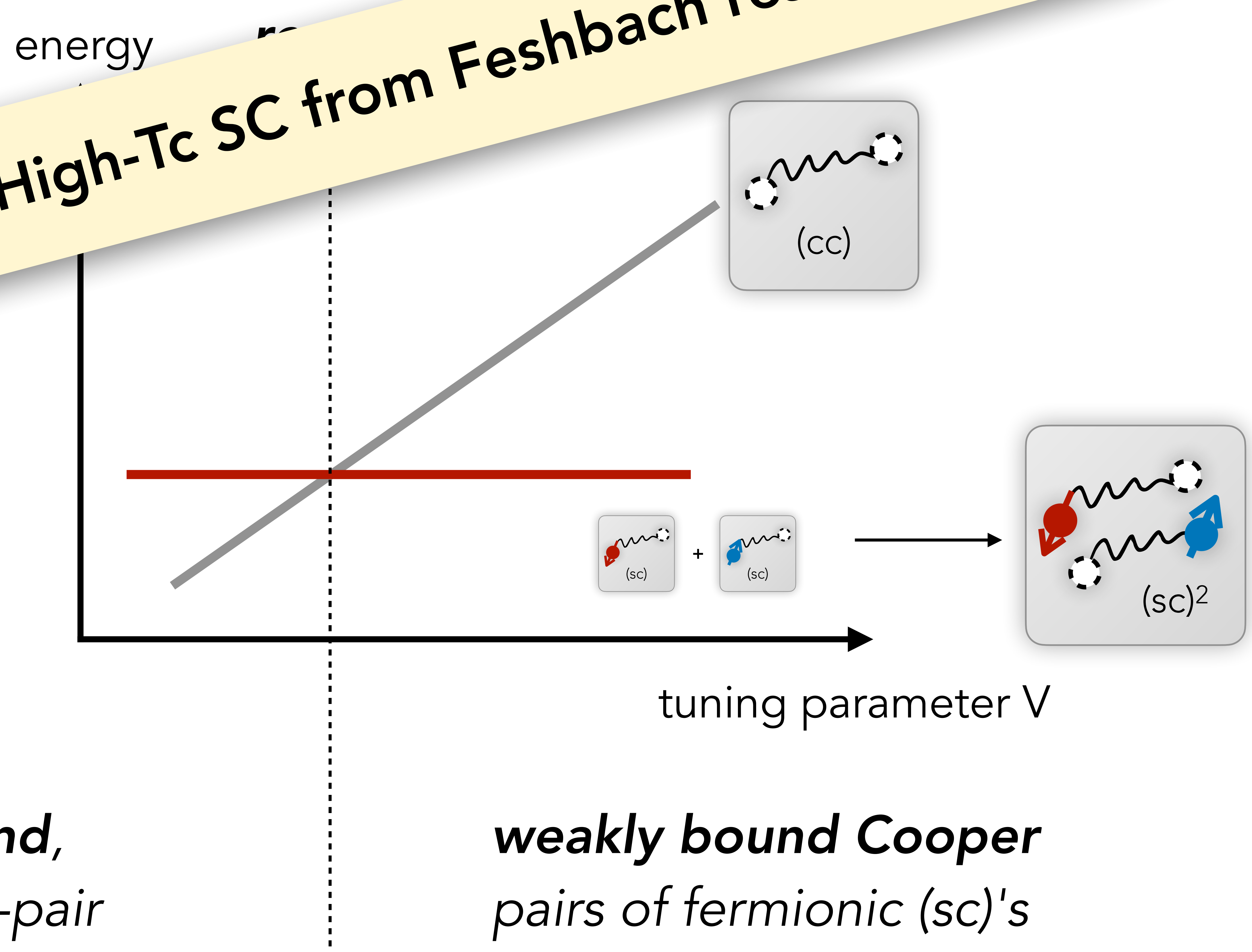


Feshbach hypothesis

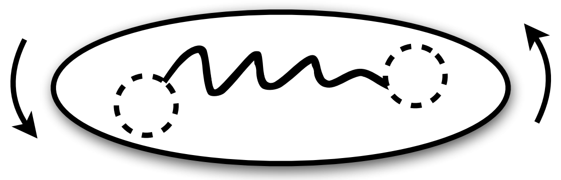
From one meson to another

Homeier et al., Nat. Comm. 16 (2025)

Hypothesis: High-Tc SC from Feshbach resonance!



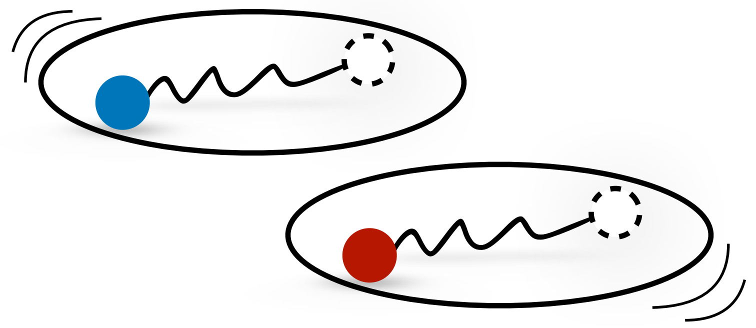
BEC-like superconductivity



tightly bound,
bosonic (cc)-pair

weakly bound Cooper
pairs of fermionic (sc)'s

BCS-like superconductivity



Feshbach hypothesis

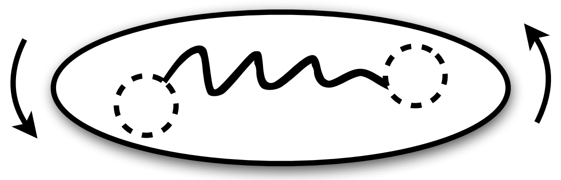
From one meson to another

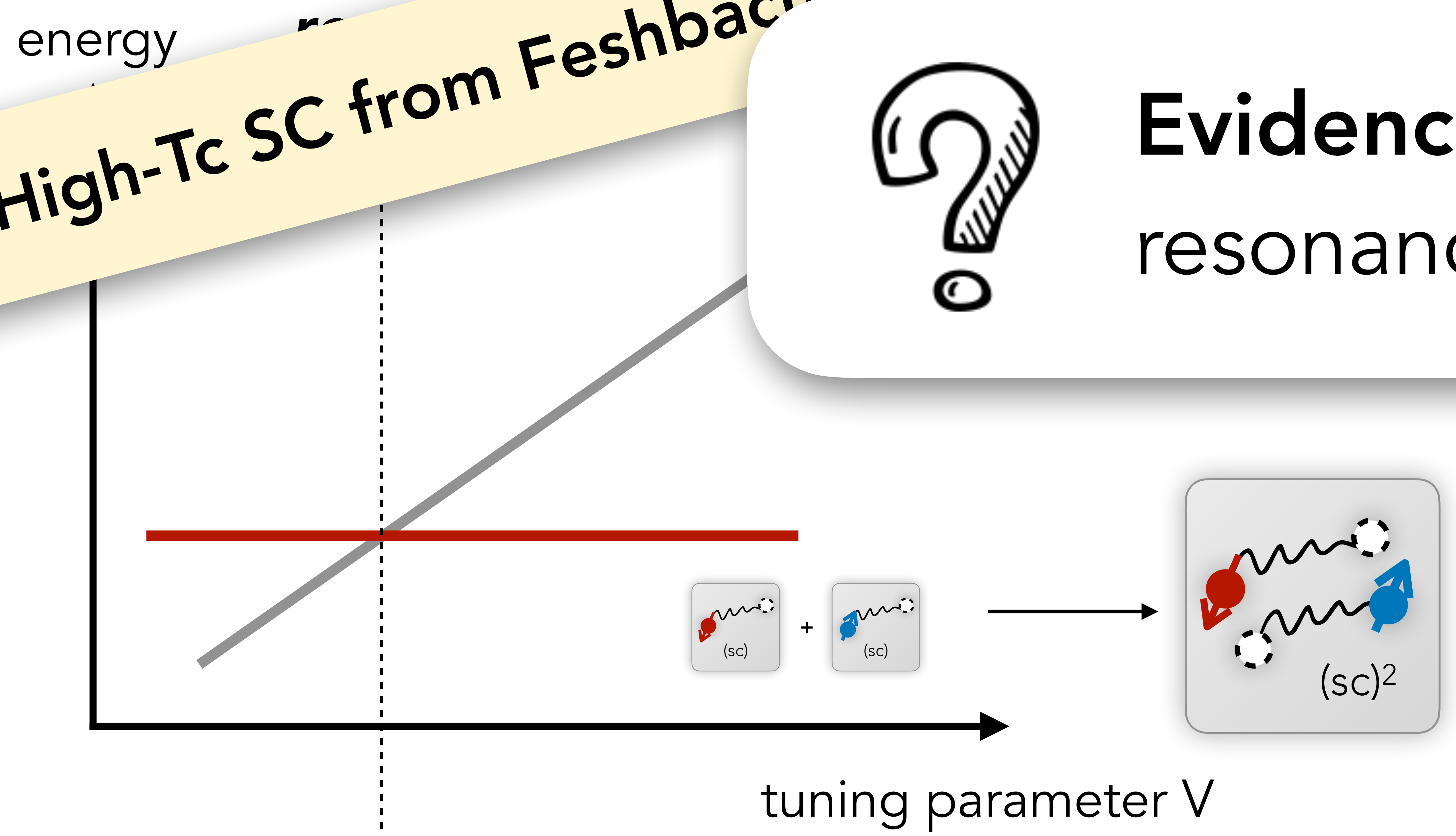
Homeier et al., Nat. Comm. 16 (2025)

Hypothesis: High-Tc SC from Feshbach resonance!

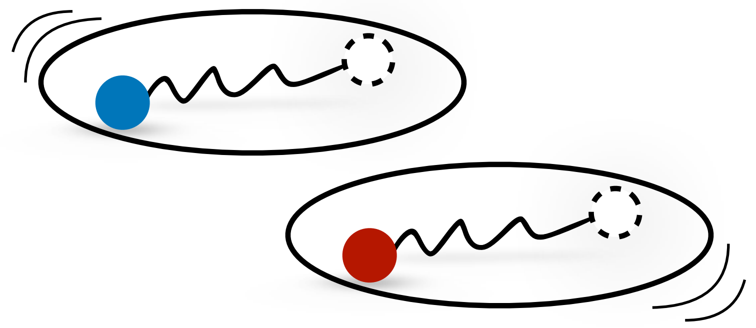
Evidence of Feshbach resonance?!

BEC-like superconductivity





BCS-like superconductivity



tightly bound,
bosonic (cc)-pair

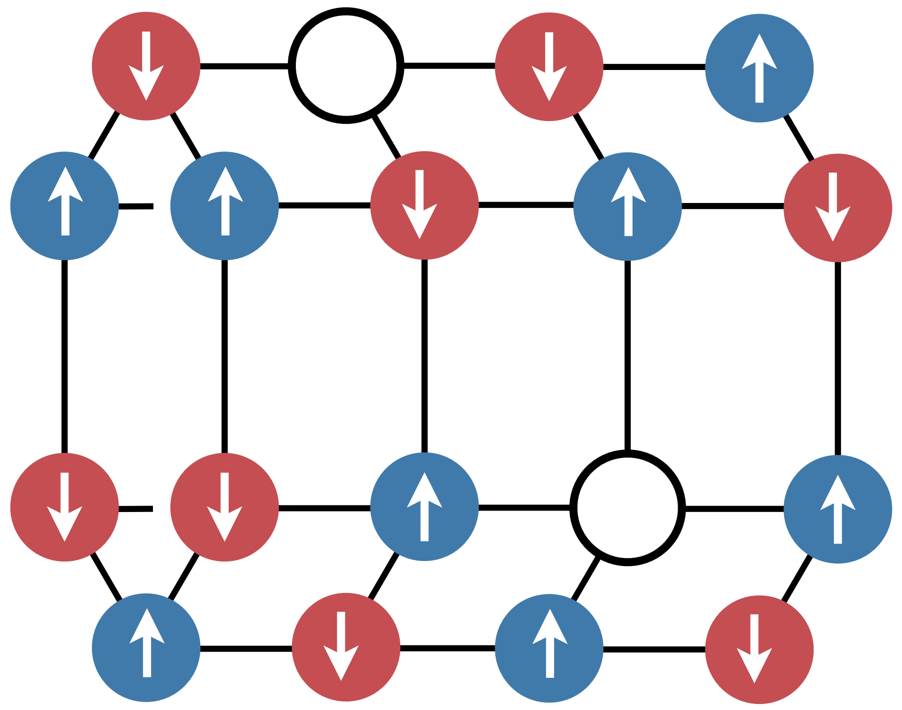
weakly bound Cooper
pairs of fermionic (sc)'s

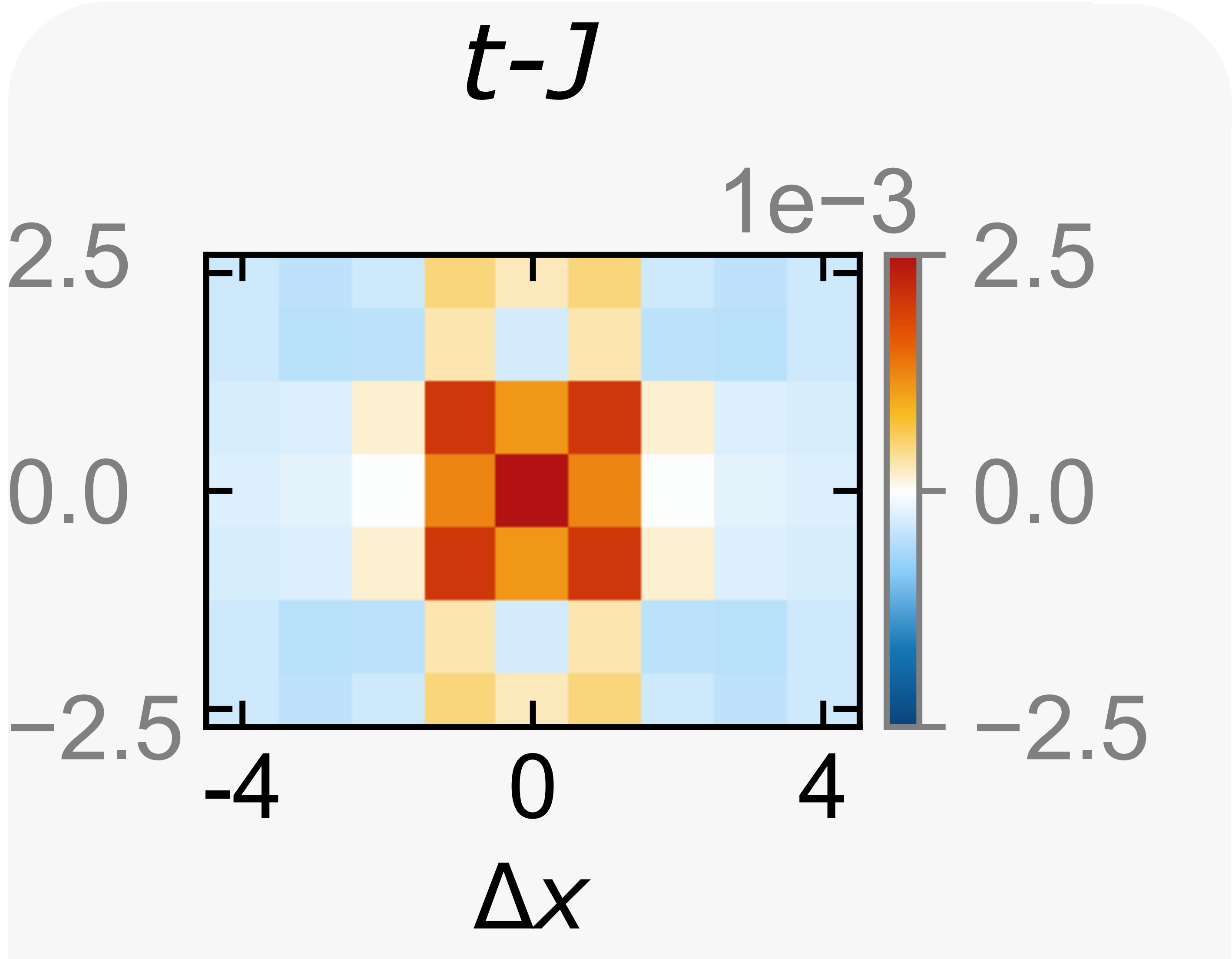
Feshbach hypothesis

Two types of pairs:

Blatz et al., PRX 15 (2025)

* Hole-hole correlations: $C^{(c)} = \langle \hat{n}^h_{\mathbf{i}} \hat{n}^h_{\mathbf{j}} \rangle$, $\mathbf{i} = \mathbf{j} + (\Delta x, \Delta y)$ $U/t = 8, t/J = 2, t' = 0$





Feshbach hypothesis

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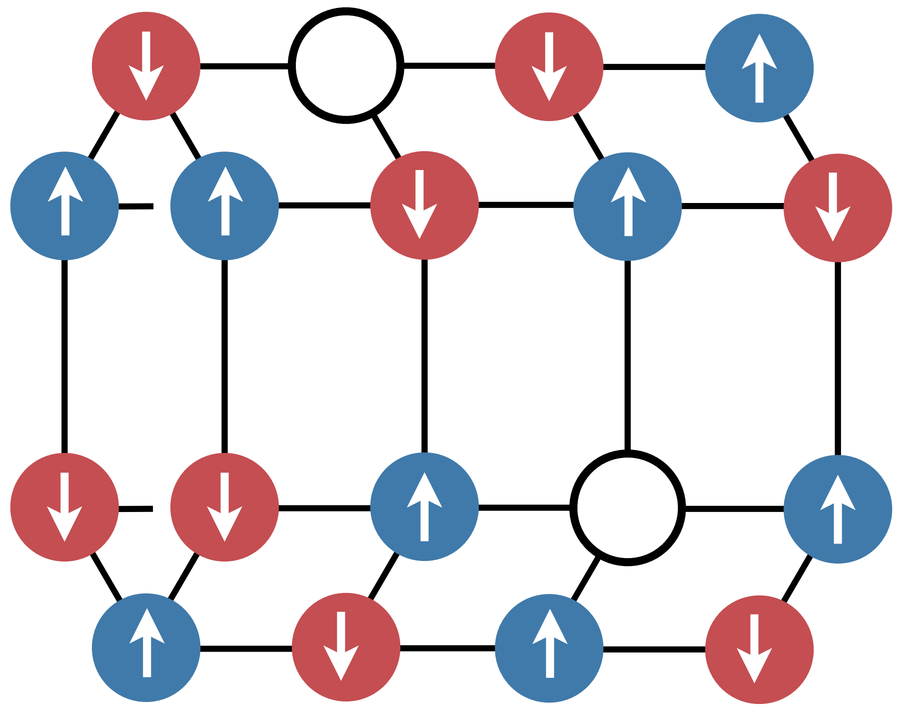
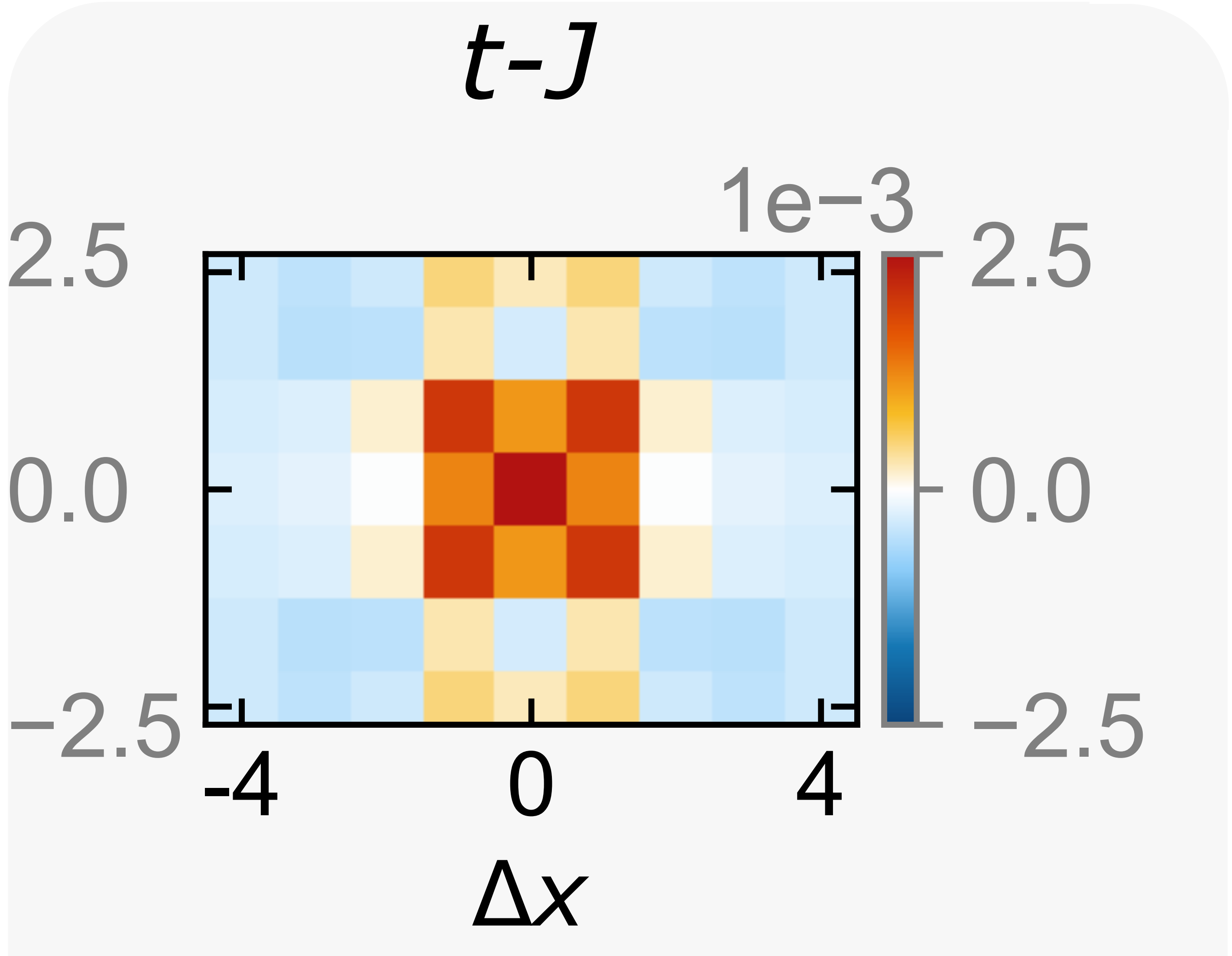
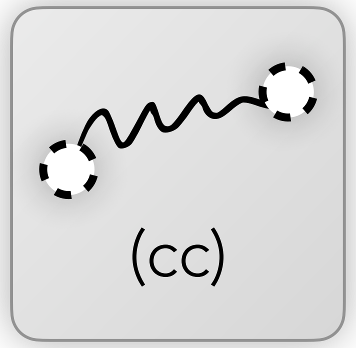




tightly bound



Feshbach hypothesis

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Blatz et al., PRX 15 (2025)

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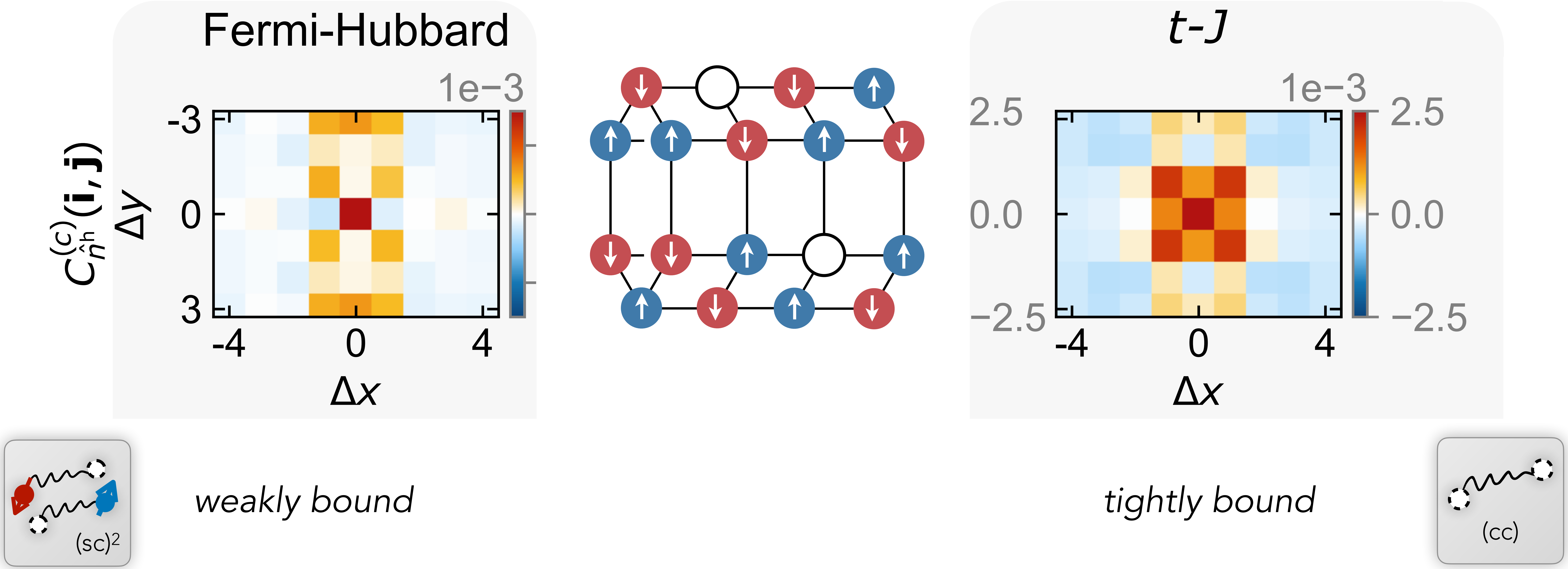


weakly bound

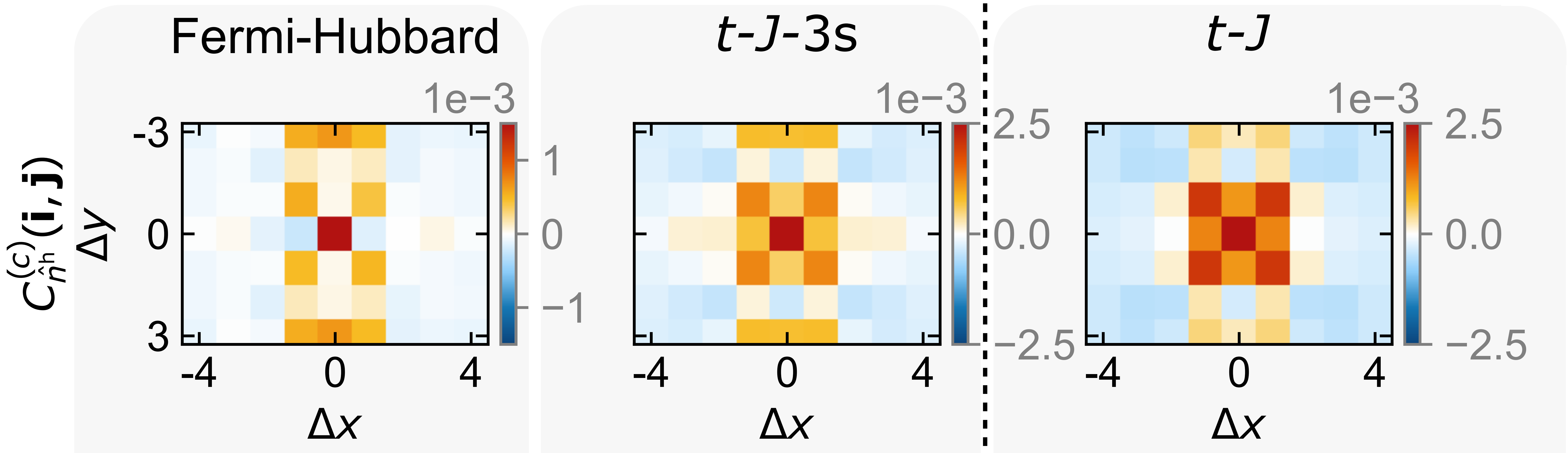
tightly bound

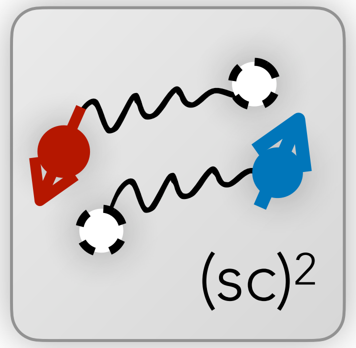
Feshbach hypothesis

Two types of pairs:

Blatz et al., PRX 15 (2025)

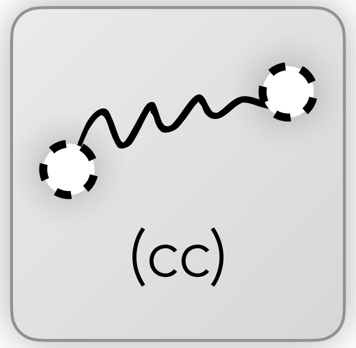
* Hole-hole correlations: $C^{(c)} = \langle \hat{n}^h_{\mathbf{i}} \hat{n}^h_{\mathbf{j}} \rangle$, $\mathbf{i} = \mathbf{j} + (\Delta x, \Delta y)$ $U/t = 8, t/J = 2, t' = 0$





weakly bound

tightly bound



Feshbach hypothesis

Two-hole spectroscopy

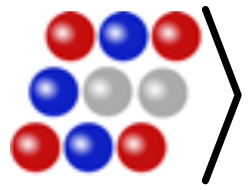
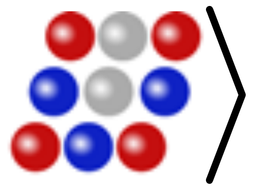
Bohrdt et al., Nat. Comm. 14 (2023)   Bermes et al., in prep.

- The *t-XXZ* model:

$$\hat{\mathcal{H}}_{t-\mathrm{XXZ}} = \sum_{\langle \boldsymbol{i},\boldsymbol{j}\rangle} \left(\cancel{J_\perp \left(\hat{S}^x_{\boldsymbol{i}} \hat{S}^x_{\boldsymbol{j}} + \hat{S}^y_{\boldsymbol{i}} \hat{S}^y_{\boldsymbol{j}} \right)} + J_z \hat{S}^z_{\boldsymbol{i}} \hat{S}^z_{\boldsymbol{j}} \right) - $$

$$- t\, \hat{\mathcal{P}} \sum_{\langle \boldsymbol{i},\boldsymbol{j}\rangle} \sum_{\sigma} \left(\hat{c}^\dagger_{\boldsymbol{i},\sigma} \hat{c}_{\boldsymbol{j},\sigma} + \mathrm{h.c.} \right) \hat{\mathcal{P}} - \frac{J_z}{4} \sum_{\langle \boldsymbol{i},\boldsymbol{j}\rangle} \hat{n}_{\boldsymbol{i}} \hat{n}_{\boldsymbol{j}}.$$

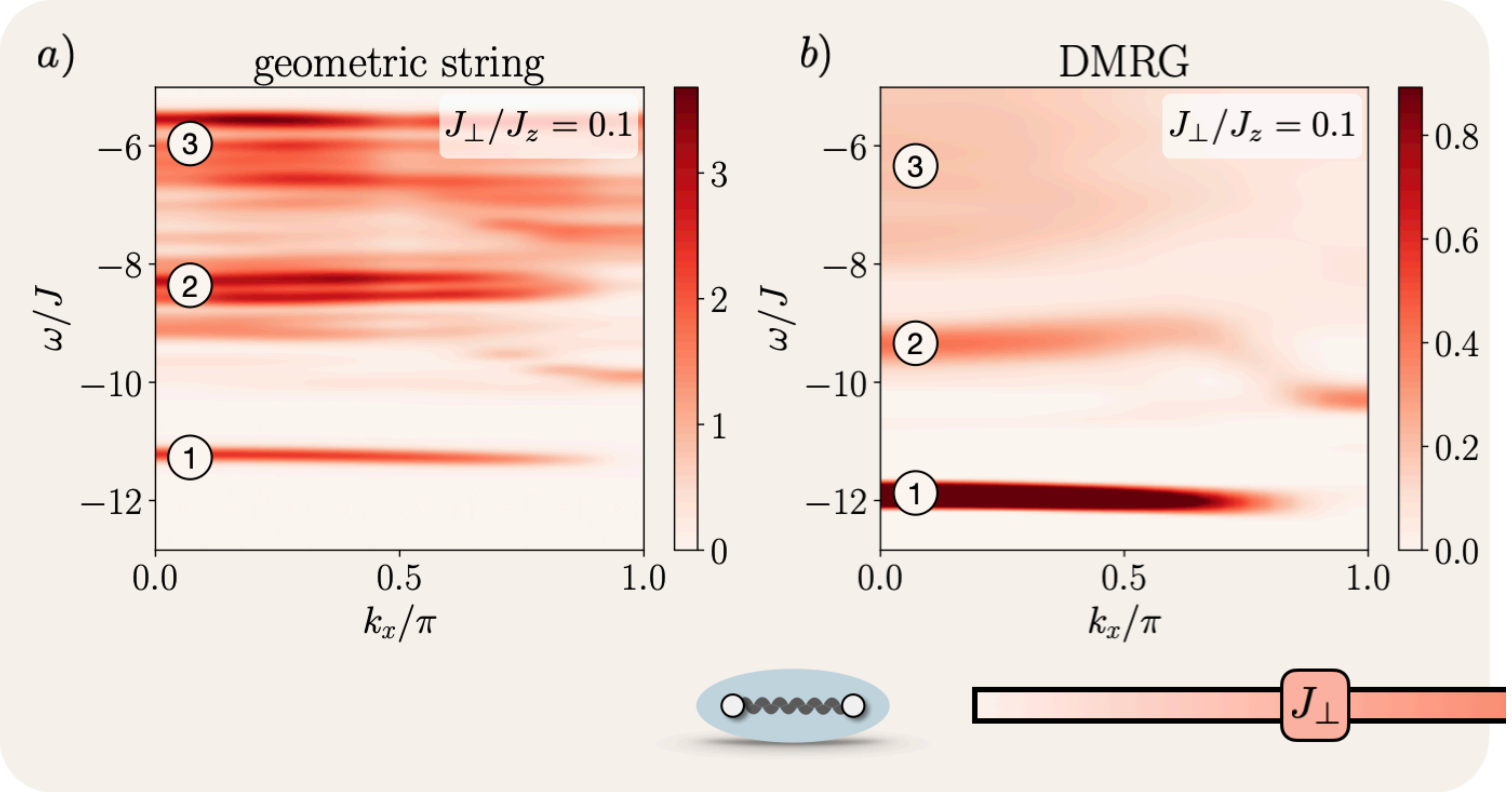
- Pair spectrum

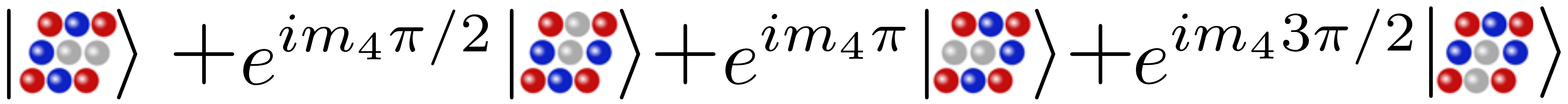
$|$  $\rangle + e^{i m_4 \pi/2} |$  $\rangle + e^{i m_4 \pi} |$  $\rangle + e^{i m_4 3\pi/2} |$  $\rangle$

Feshbach hypothesis

Two-hole spectroscopy

Bohrdt et al., Nat. Comm. 14 (2023) *Bermes et al., in prep.*



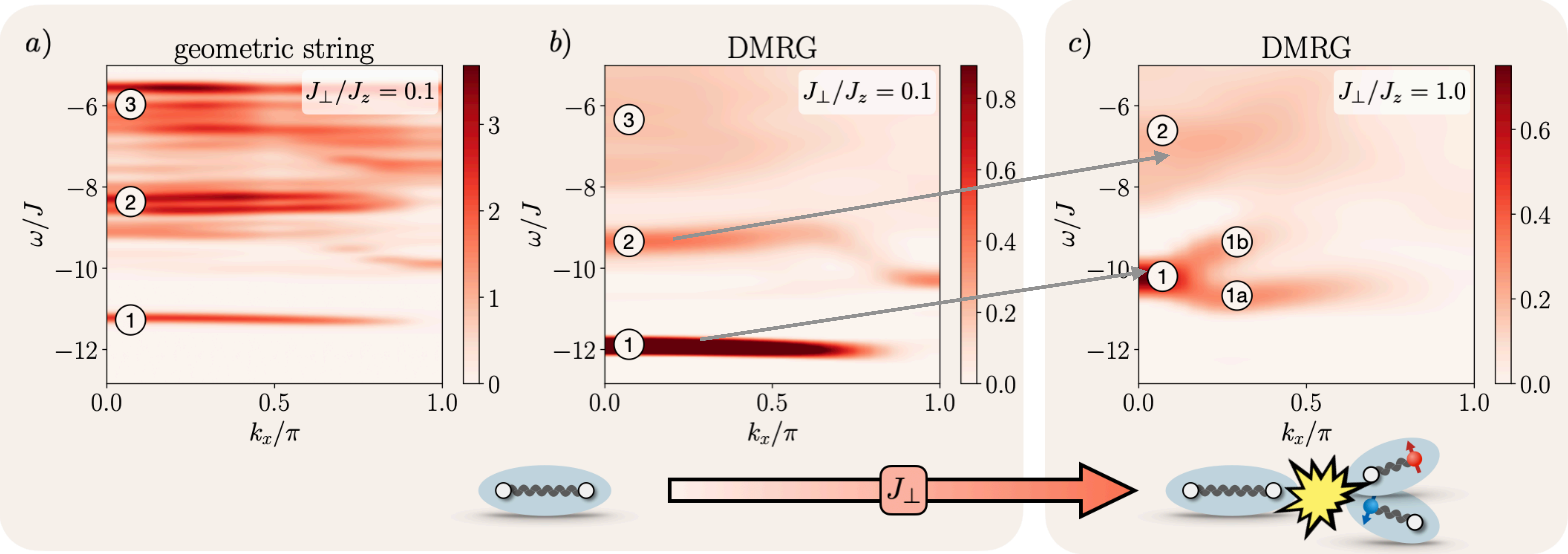


Feshbach hypothesis

Two-hole spectroscopy

Bohrdt et al., Nat. Comm. 14 (2023)

Bermes et al., in prep.

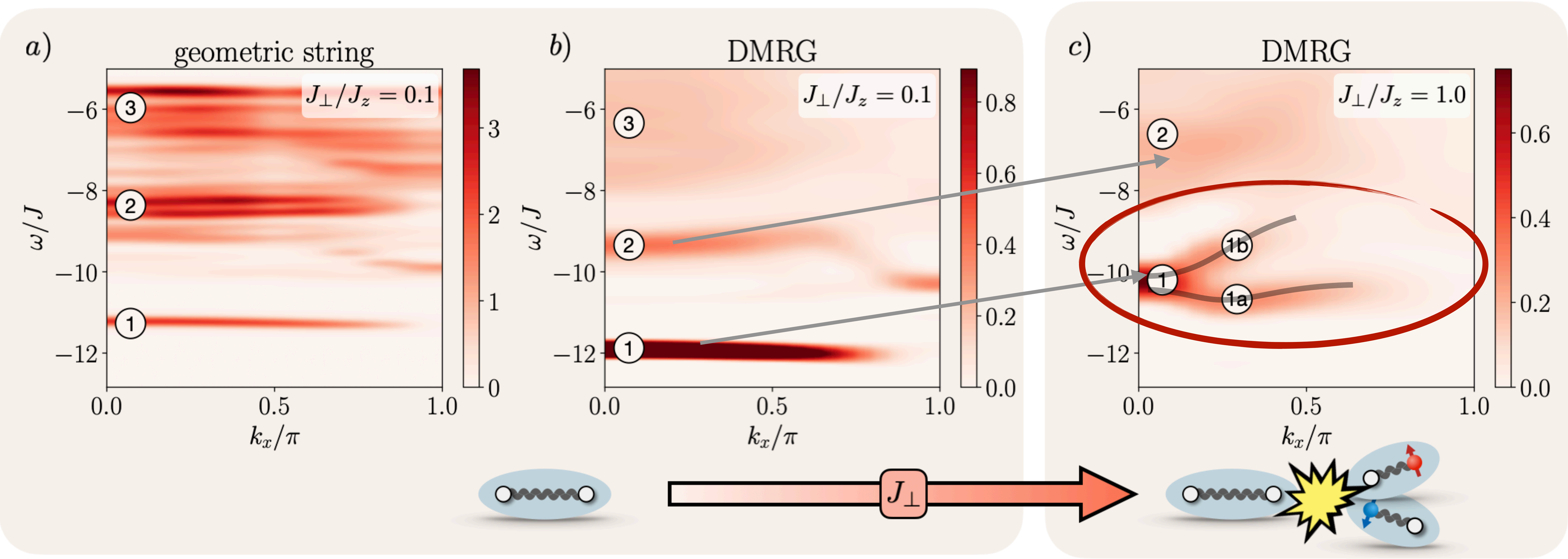


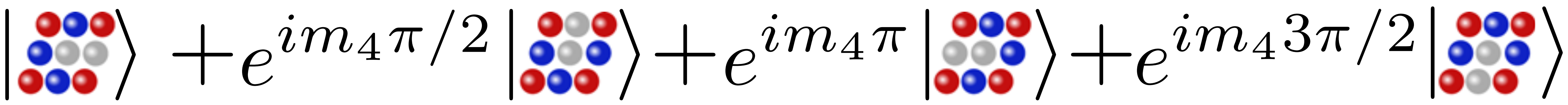
$$\left|\,\cdot\,\right\rangle + e^{im_4\pi/2}\left|\,\cdot\,\right\rangle + e^{im_4\pi}\left|\,\cdot\,\right\rangle + e^{im_4 3\pi/2}\left|\,\cdot\,\right\rangle$$

Feshbach hypothesis

Two-hole spectroscopy

Bohrdt et al., Nat. Comm. 14 (2023) *Bermes et al., in prep.*





Feshbach hypothesis

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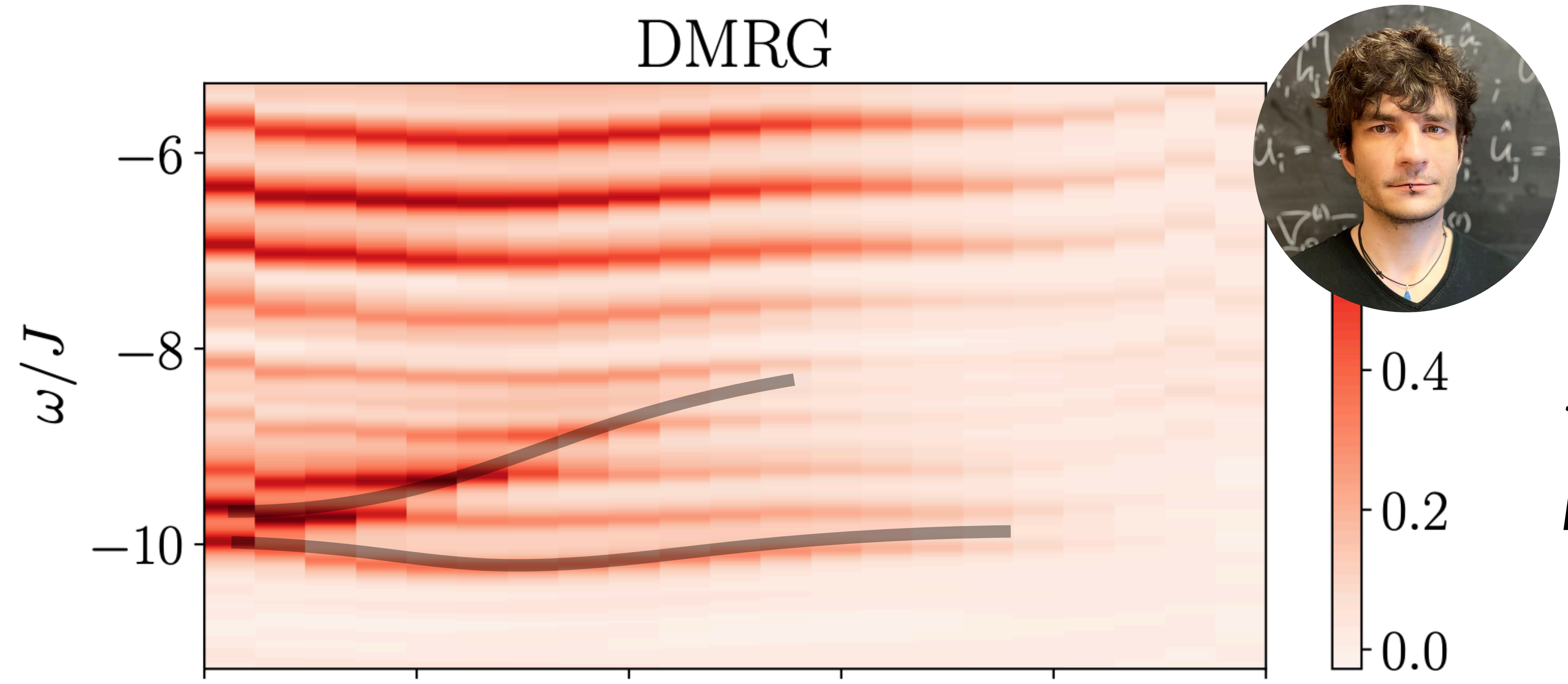


$J_\perp = J_z = J$

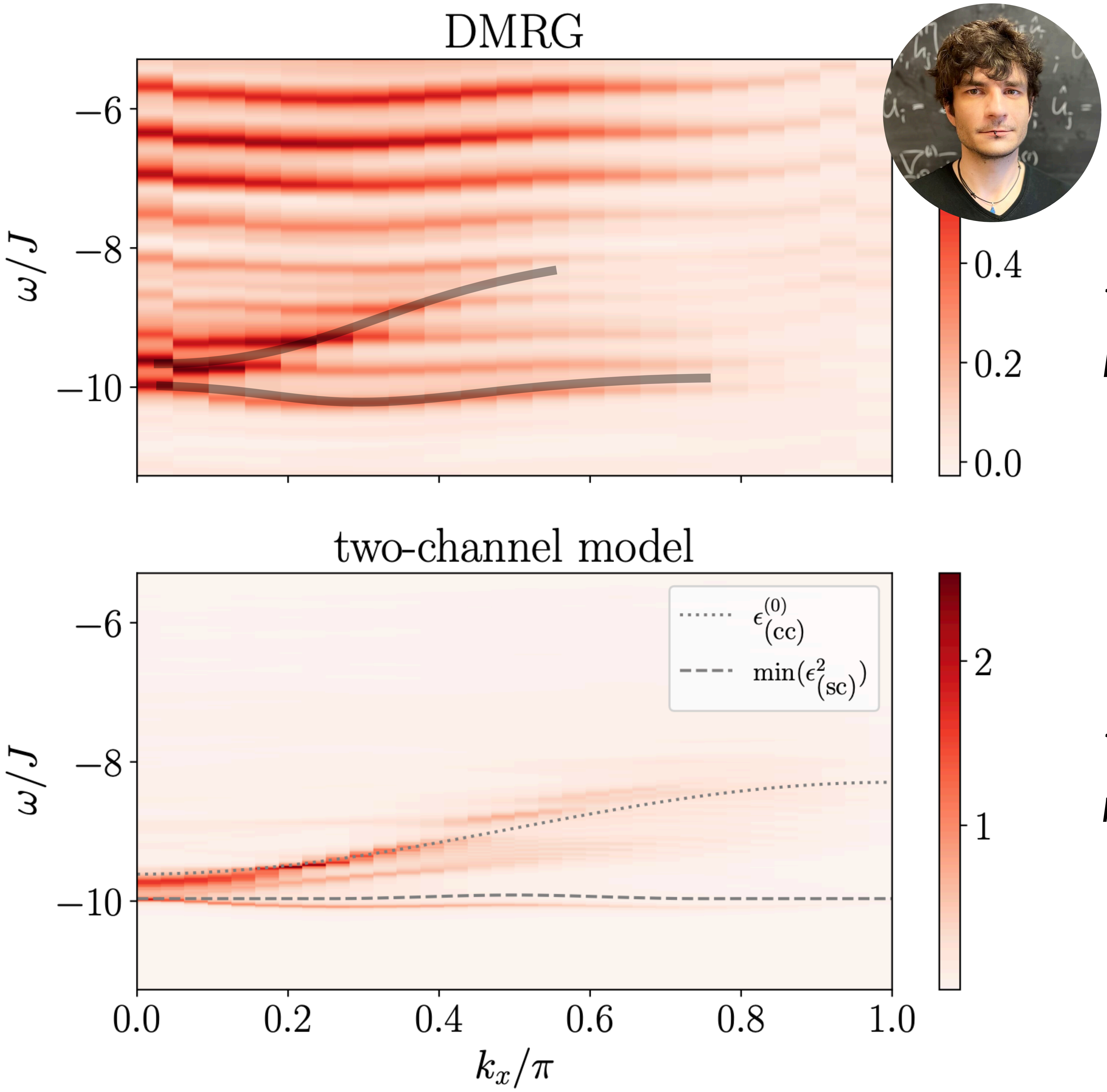
< *High-resolution DMRG study*

Feshbach hypothesis

Two-hole spectroscopy

Bohrdt et al., Nat. Comm. 14 (2023)

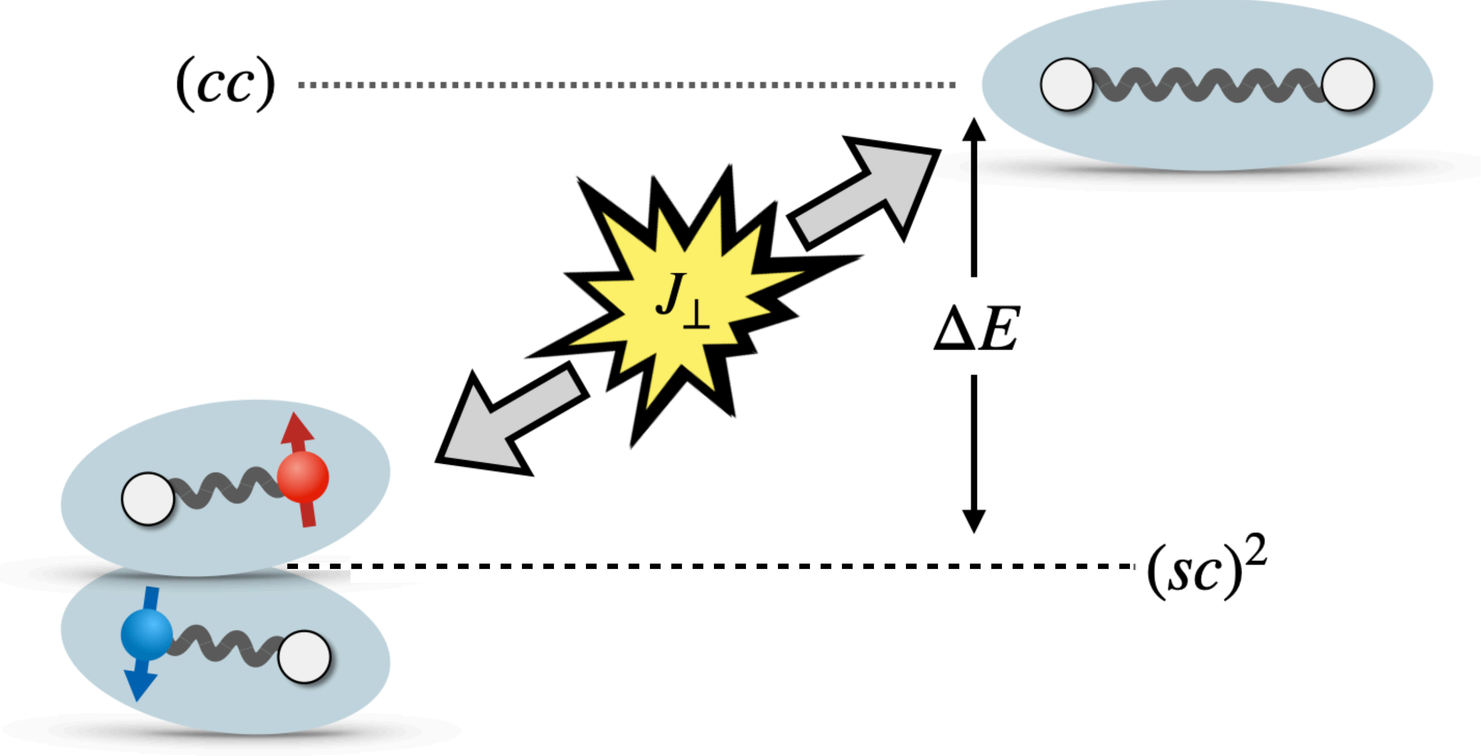
Bermes et al., in prep.



$$J_\perp = J_z = J$$

< High-resolution DMRG study

< Two-channel meson model

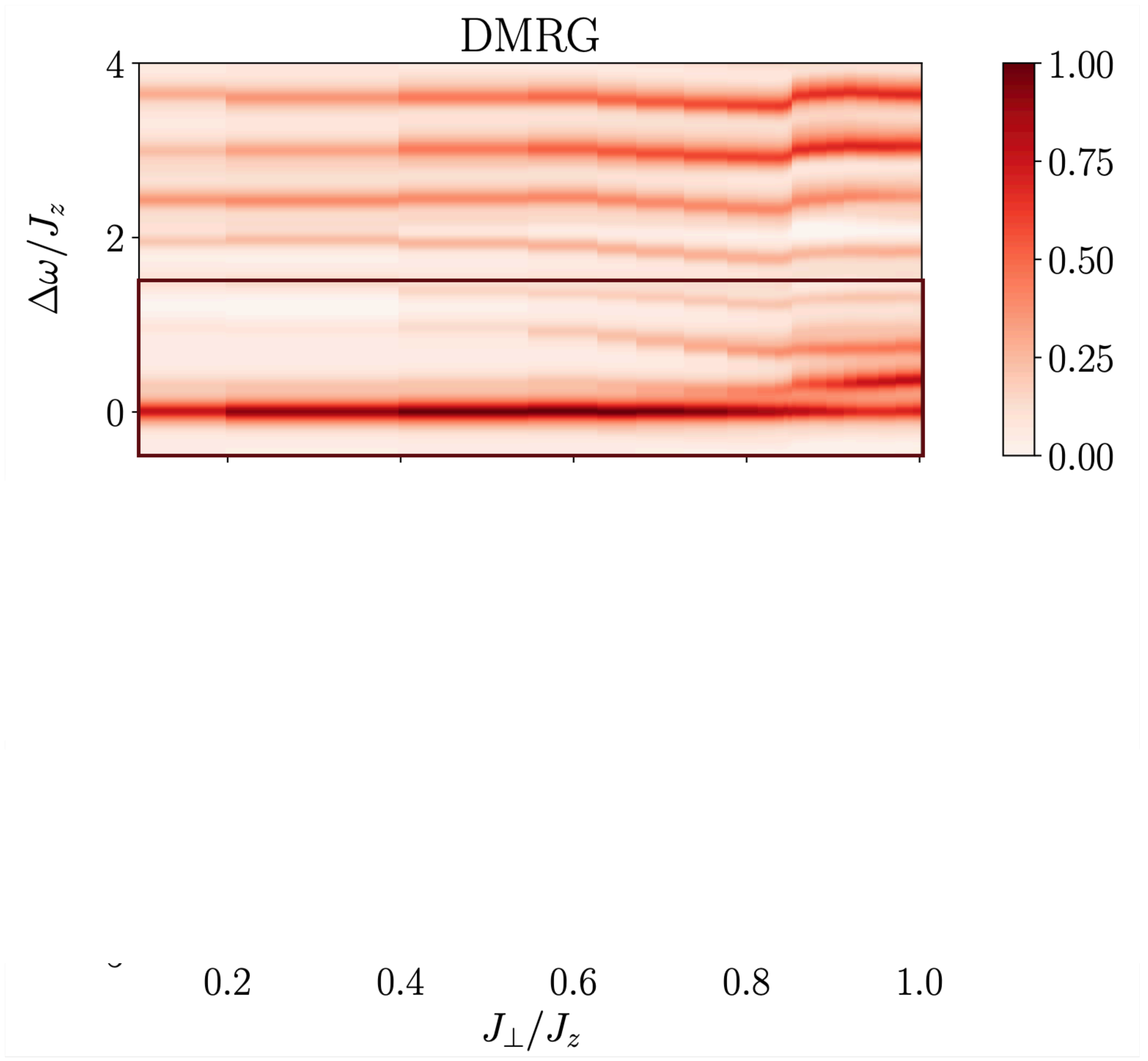


Feshbach hypothesis

Two-hole spectroscopy

Bermes et al., in prep.

> *> Avoided crossing*



Feshbach hypothesis

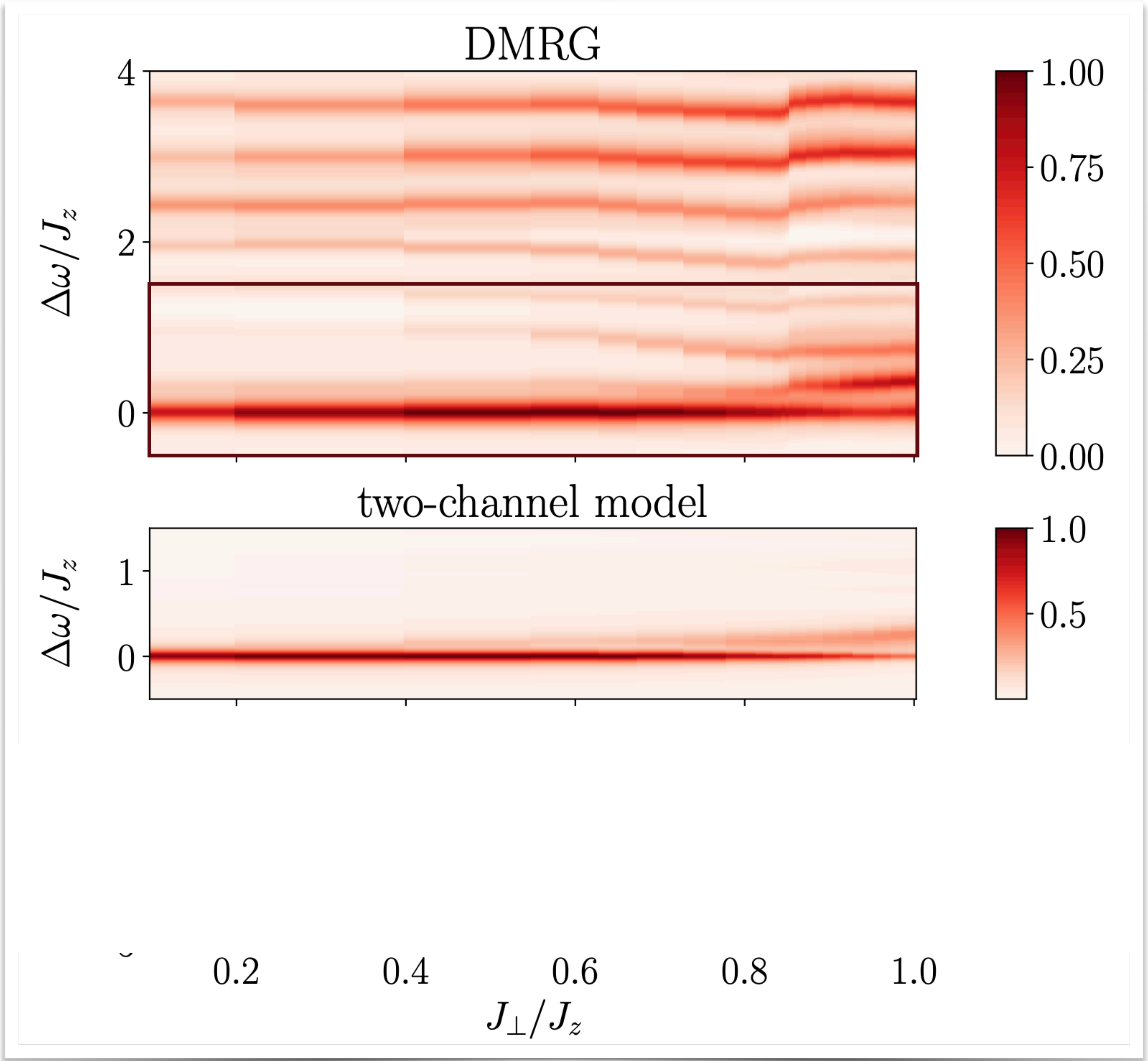
Two-hole spectroscopy

Bermes et al., in prep.

> *Avoided crossing*

> *Fit* ΔE

$$\begin{pmatrix} 0 & \mathcal{O}(J_\perp) \\ \mathcal{O}(J_\perp) & \Delta E \end{pmatrix}$$



Feshbach hypothesis

Two-hole spectroscopy

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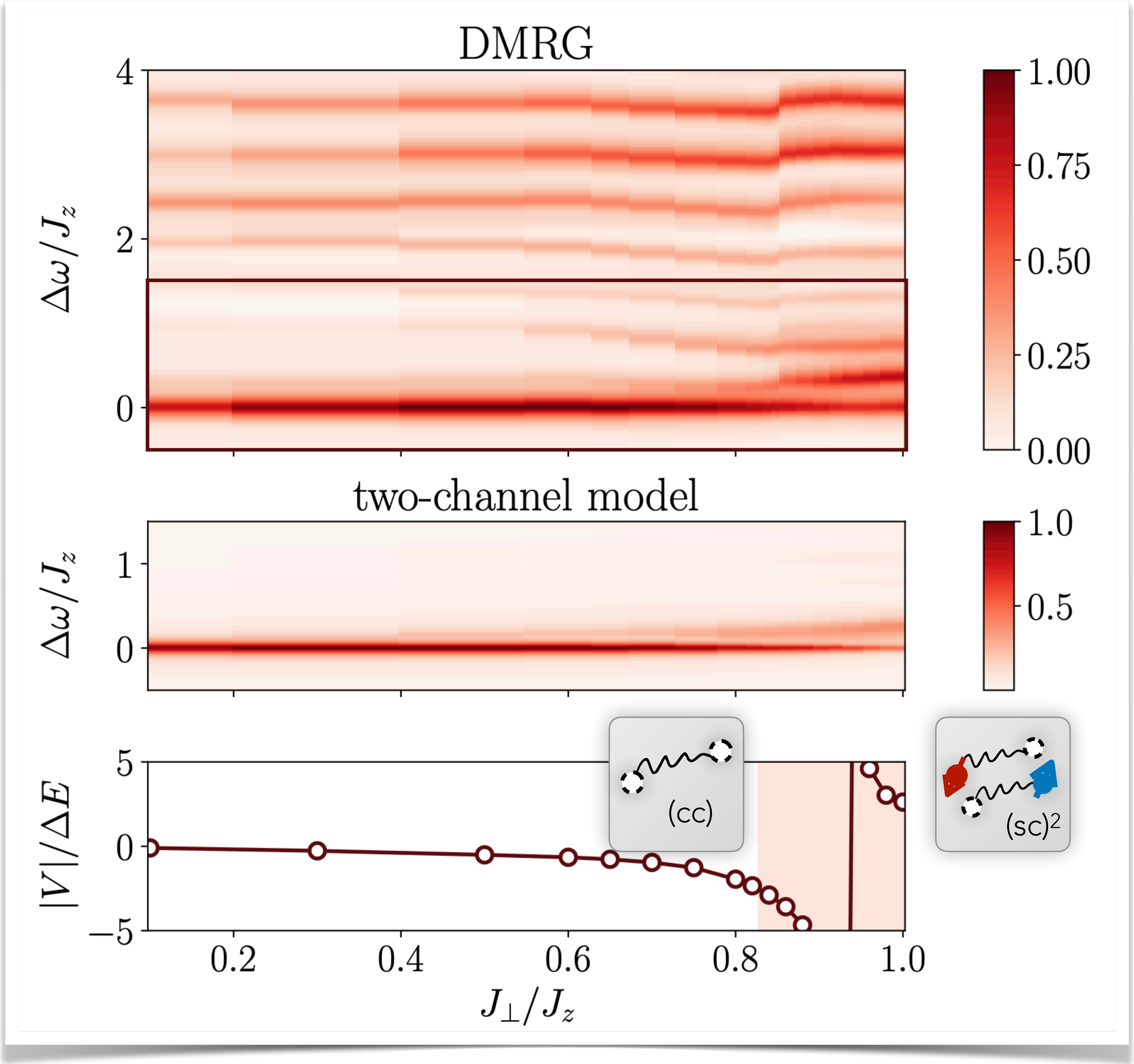
> *Emergent Feshbach resonance*



Feshbach hypothesis

Two-hole spectroscopy

Bermes et al., in prep.

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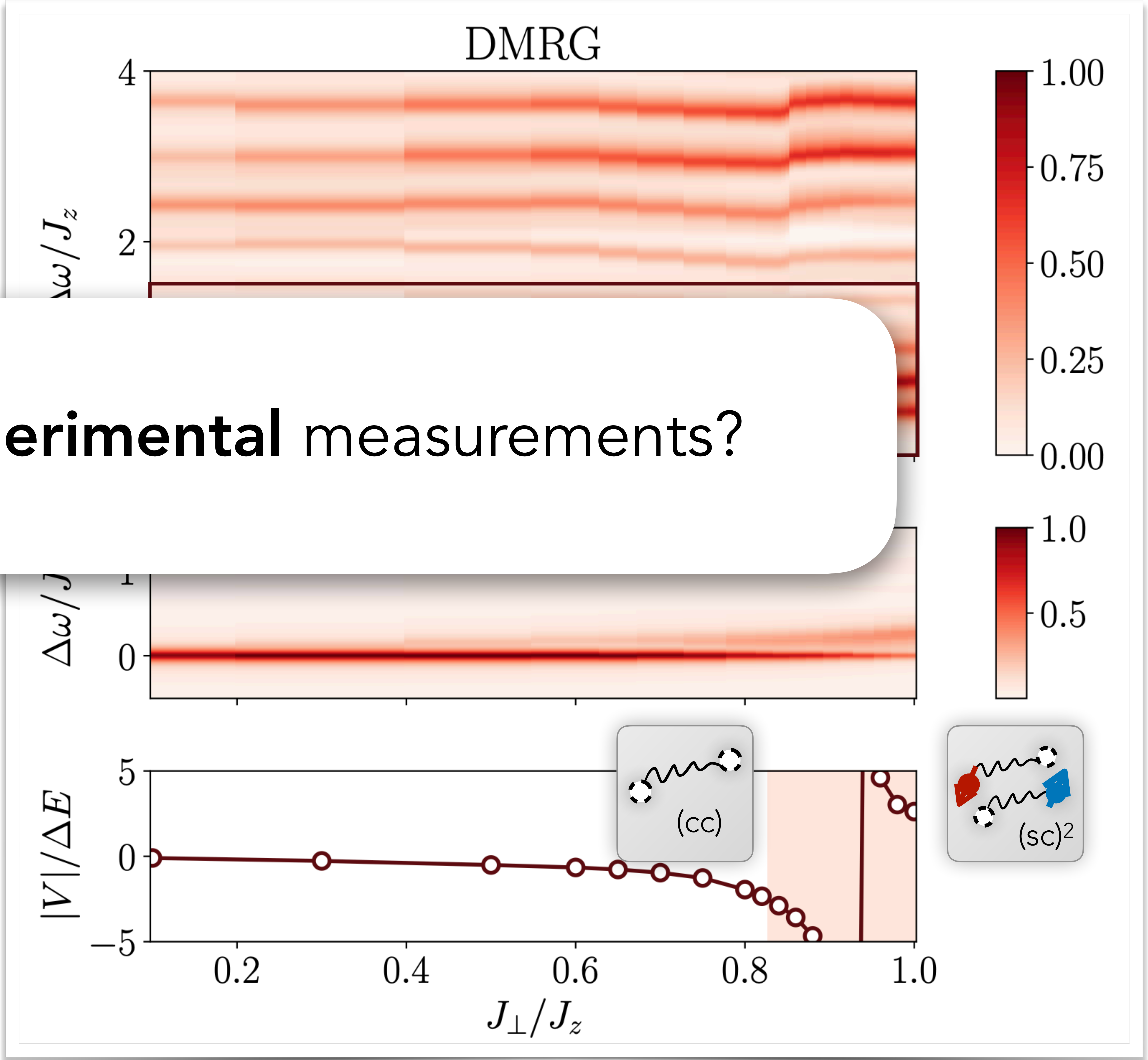

Experimental measurements?

Experimental realization: Two-hole spectroscopy

Bermes et al., in prep.

$$\hat{\Delta}_{m_4}^{(s)}(\mathbf{j}) = \sum_{\mathbf{i}:\langle \mathbf{i},\mathbf{j}\rangle} e^{i m_4 \varphi_{\mathbf{i}-\mathbf{j}}} \left(\hat{c}_{\mathbf{i},\uparrow} \hat{c}_{\mathbf{j},\downarrow} - \hat{c}_{\mathbf{i},\downarrow} \hat{c}_{\mathbf{j},\uparrow} \right)$$

Feshbach hypothesis

Experimental realization: Two-hole spectroscopy

Bermes et al., in prep.

- The attractive route: $U \to -U$

  Ho et al., PRA 79 (2005)

$$\hat{c}_{\mathbf{j},\downarrow} \to \hat{c}^\dagger_{\mathbf{j},\downarrow}(-1)^{j_x+j_y}$$

$$\hat{c}_{\mathbf{j},\uparrow} \to \hat{c}_{\mathbf{j},\uparrow}$$

$$\hat{\Delta}^{(s)}_{m_4}(\mathbf{j}) = \sum_{\mathbf{i}:\langle \mathbf{i},\mathbf{j}\rangle} e^{i m_4 \varphi_{\mathbf{i}-\mathbf{j}}} \left(\hat{c}_{\mathbf{i},\uparrow}\hat{c}_{\mathbf{j},\downarrow} - \hat{c}_{\mathbf{i},\downarrow}\hat{c}_{\mathbf{j},\uparrow} \right)$$

Feshbach hypothesis

Experimental realization: Two-hole spectroscopy

Bermes et al., in prep.

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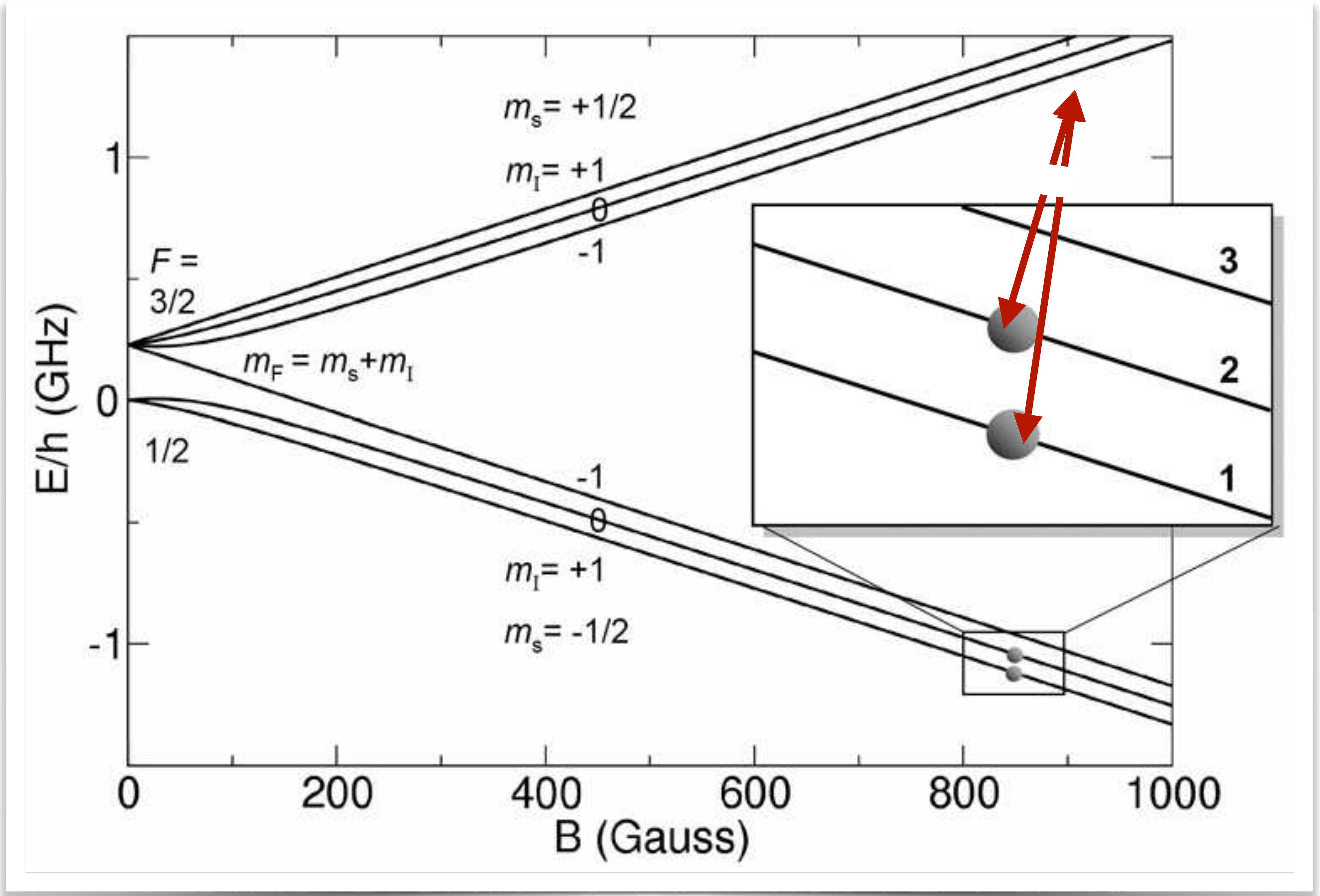
$$\hat{c}_{\mathbf{j},\uparrow} \to \hat{c}_{\mathbf{j},\uparrow}$$

- **Raman spectroscopy:**

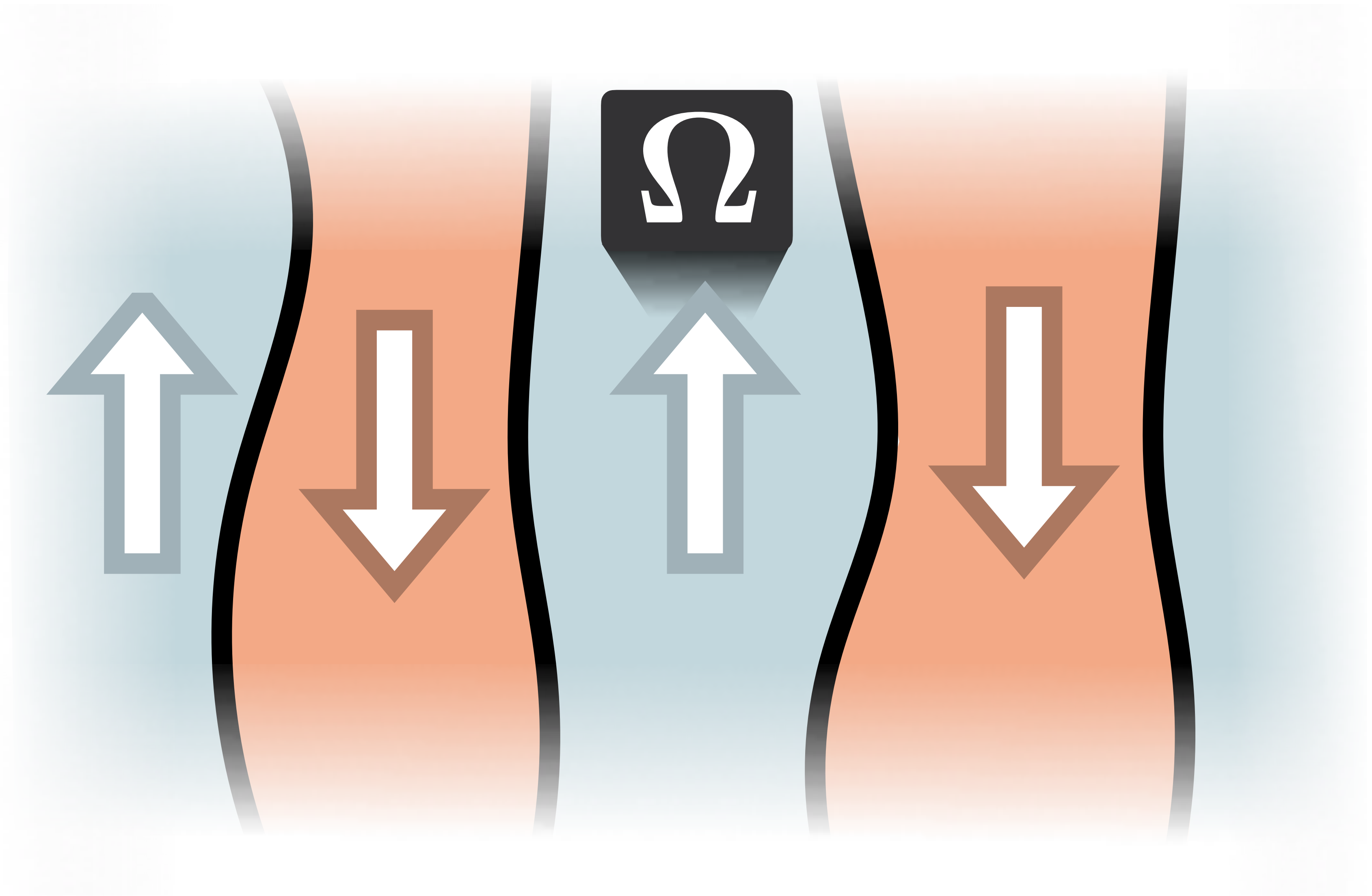
$$\hat{\Delta}^{(s)}_{m_4}(\mathbf{j}) = \sum_{\mathbf{i}:\langle\mathbf{i},\mathbf{j}\rangle} e^{im_4\varphi_{\mathbf{i}-\mathbf{j}}}\left(\hat{c}_{\mathbf{i},\uparrow}\hat{c}_{\mathbf{j},\downarrow} - \hat{c}_{\mathbf{i},\downarrow}\hat{c}_{\mathbf{j},\uparrow}\right)$$

$$\longrightarrow \sum_{\mathbf{i}:\langle\mathbf{i},\mathbf{j}\rangle} e^{im_4\varphi_{\mathbf{i}-\mathbf{j}}}\left((-1)^{\mathbf{j}}\hat{c}_{\mathbf{i},\uparrow}\hat{c}^\dagger_{\mathbf{j},\downarrow} - (-1)^{\mathbf{i}}\hat{c}^\dagger_{\mathbf{i},\downarrow}\hat{c}_{\mathbf{j},\uparrow}\right)$$



Grimm, Proc. School Enrico Fermi, Vol 164 (2007)

PART 2.III: Relation to stripes



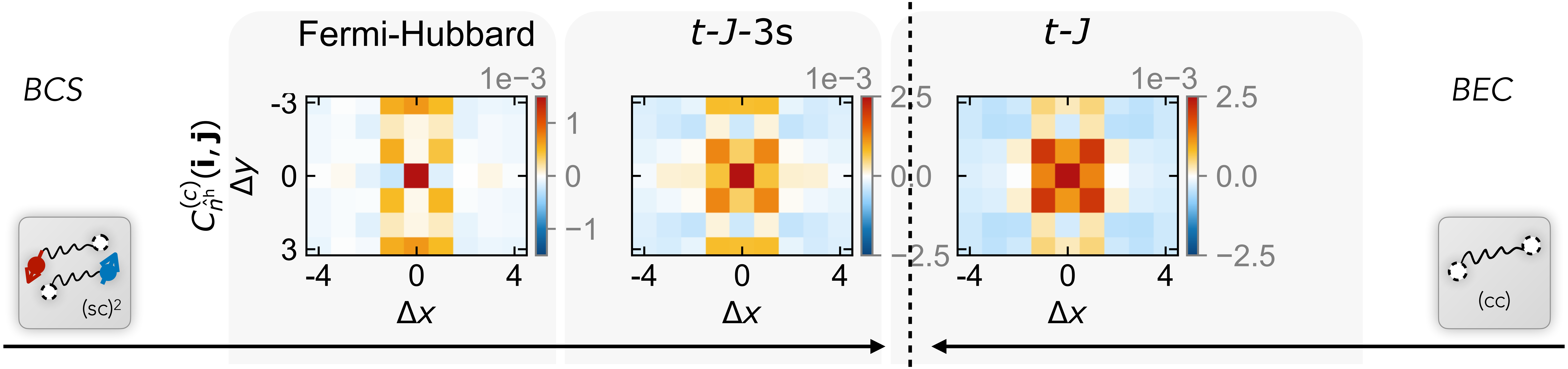
Feshbach hypothesis

Connection to stripe formation

Blatz et al., PRX 15 (2025)

* Hole-hole correlations: $C^{(c)} = \langle \hat{n}^h_{\mathbf{i}} \hat{n}^h_{\mathbf{j}} \rangle$

$U/t = 8, t/J = 2, t' = 0$



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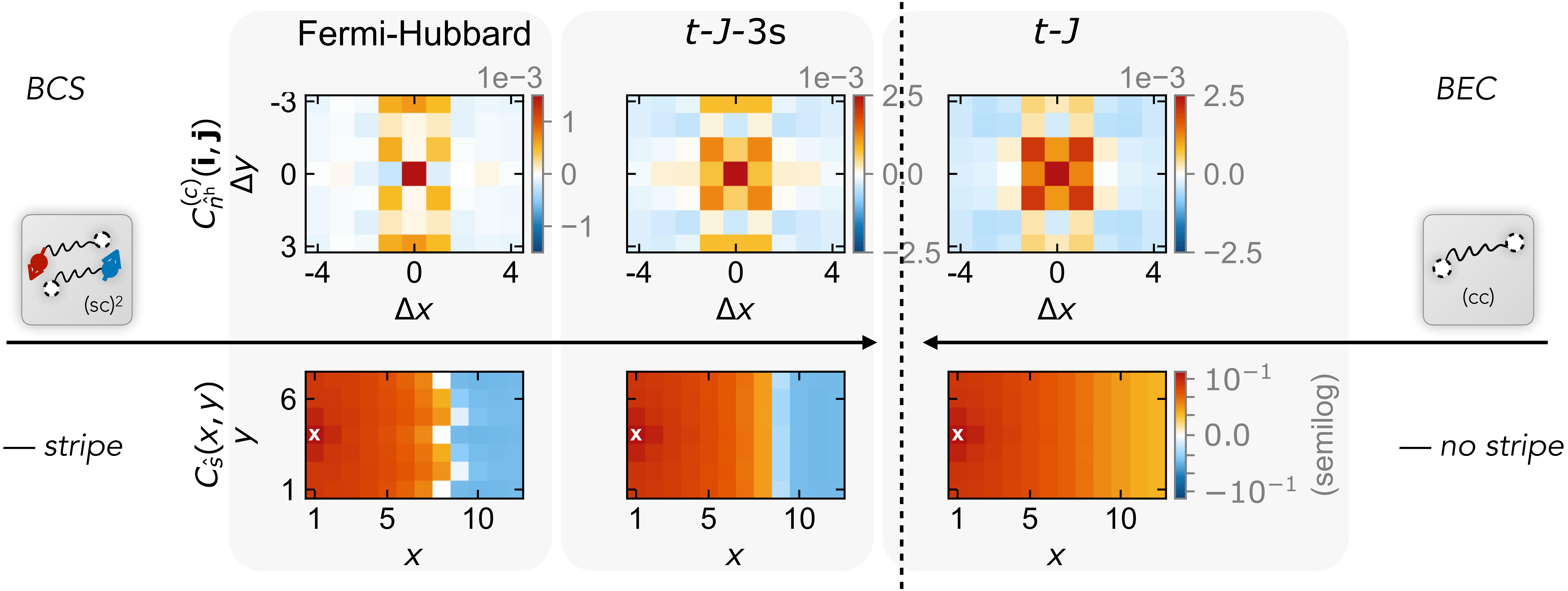


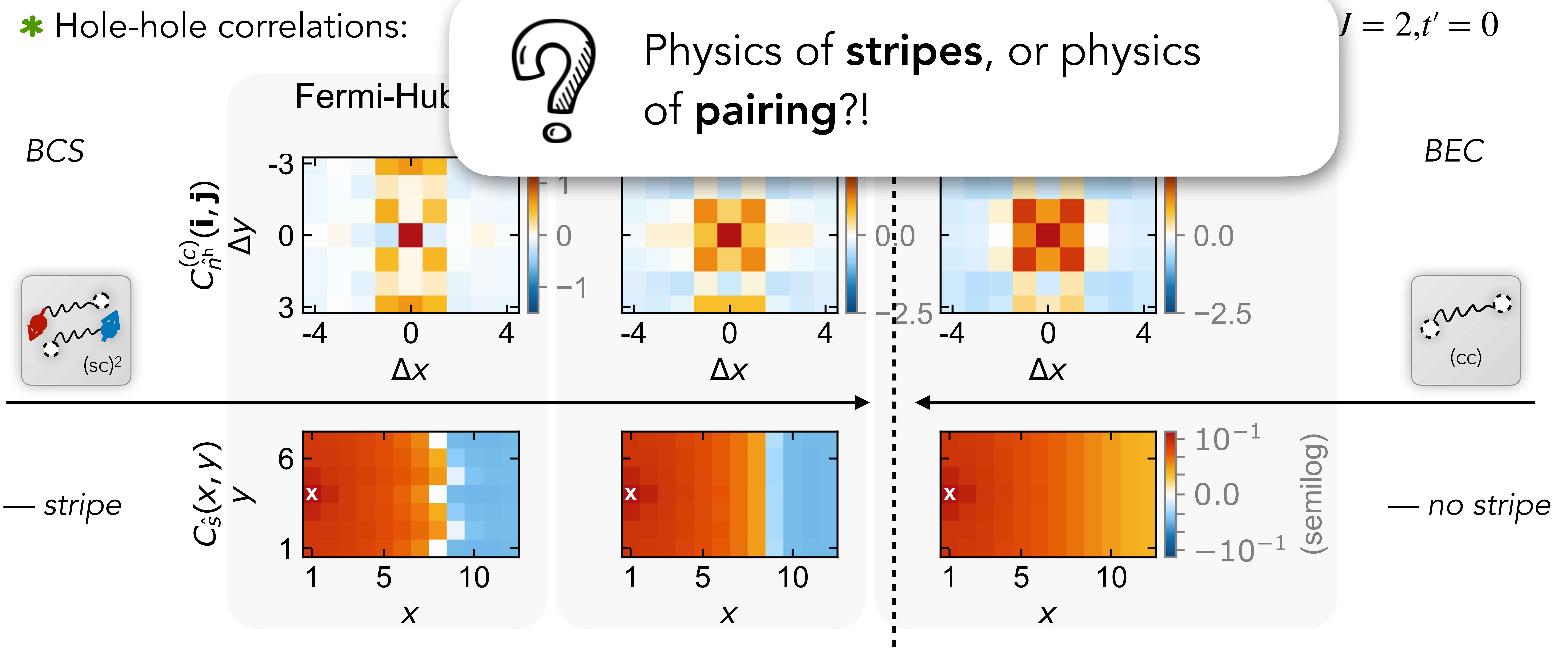
Feshbach hypothesis

Connection to stripe formation

Blatz et al., PRX 15 (2025)

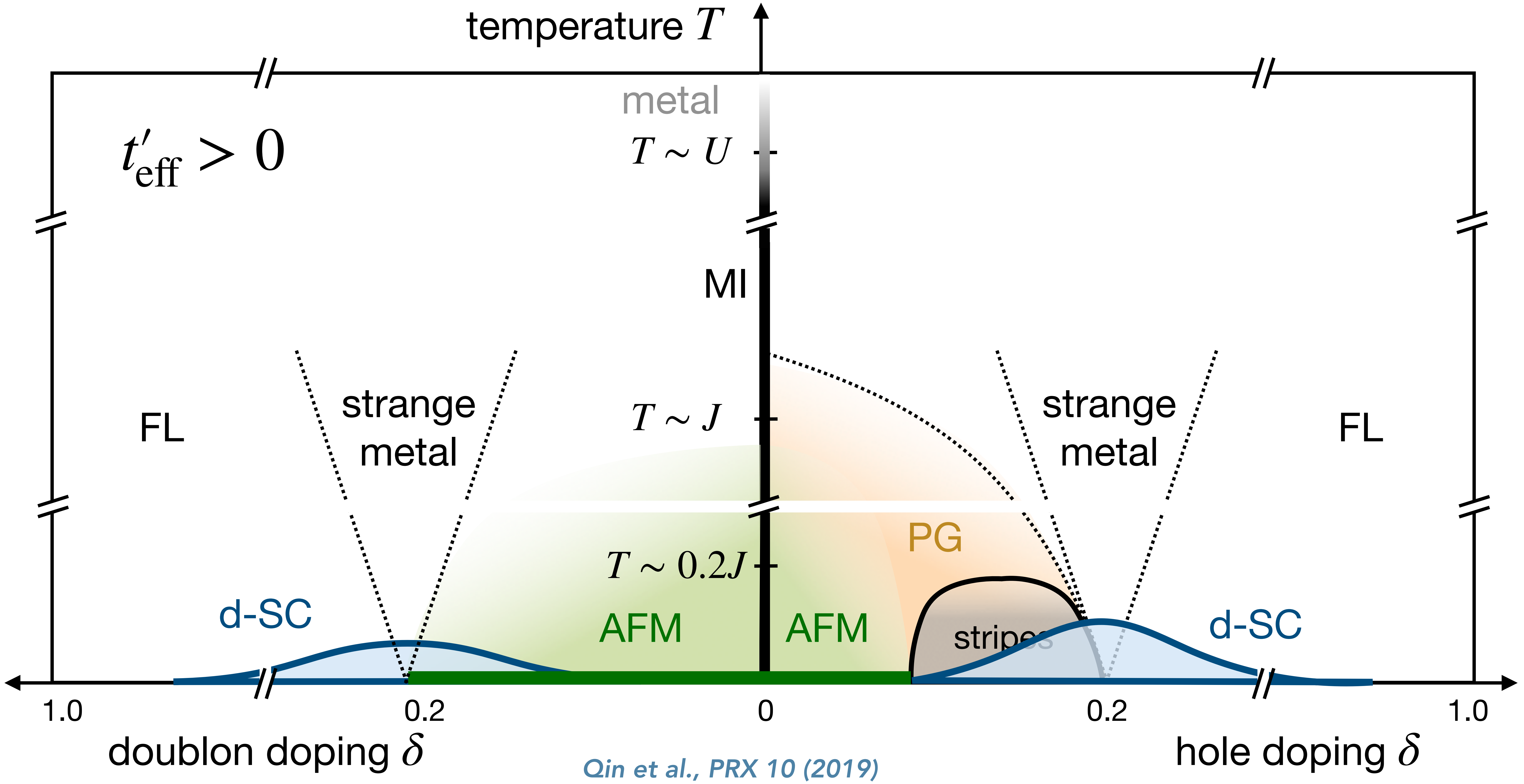
* Hole-hole correlations:

Physics of **stripes**, or physics of **pairing**?!



Feshbach hypothesis

Phase diagram: Large-*U*, *t'* Hubbard



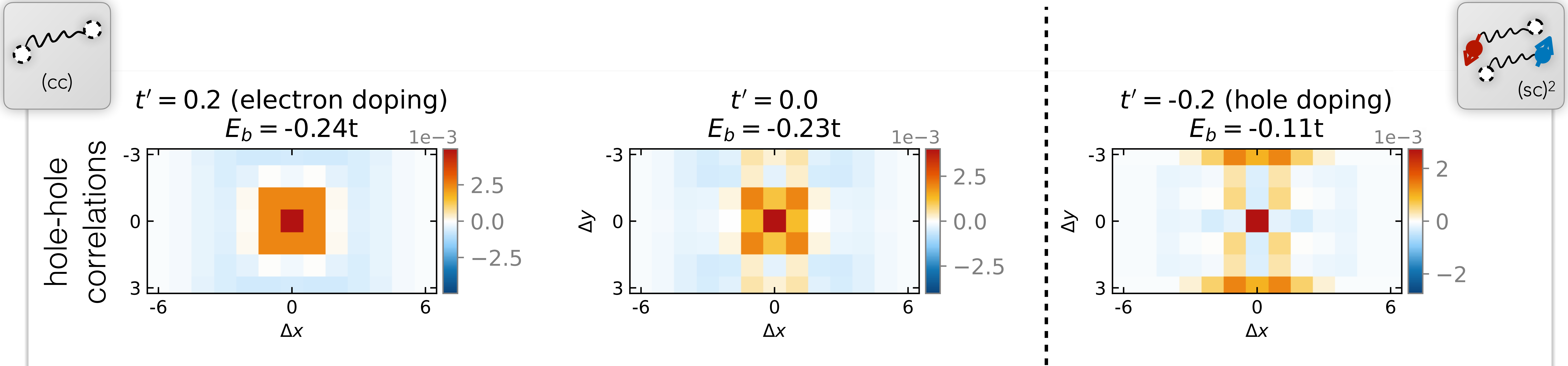
Qin et al., PRX 10 (2019)
Xu et al., Science 384 (2024)

Feshbach hypothesis

Two types of pairs:

t-J model on 6-leg cylinder

Blatz et al., PRX 15 (2025)



Feshbach hypothesis

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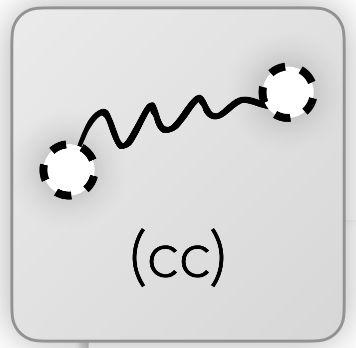
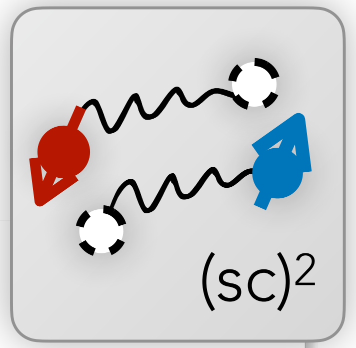
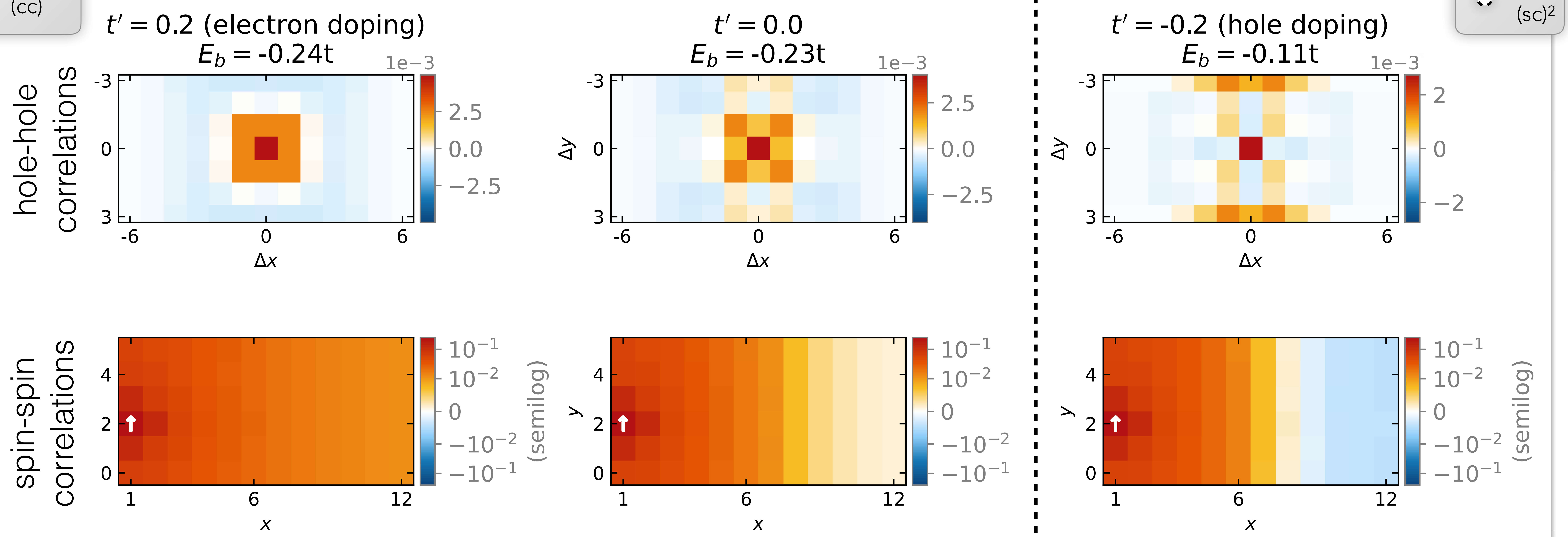
Blatz et al., PRX 15 (2025)



— *no stripe*

— *stripe*

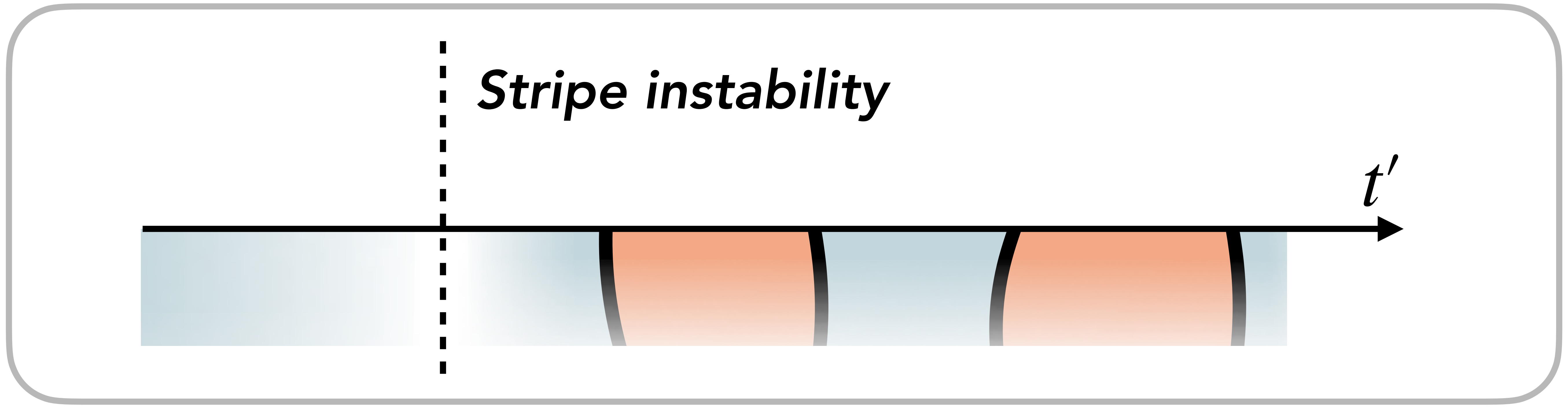




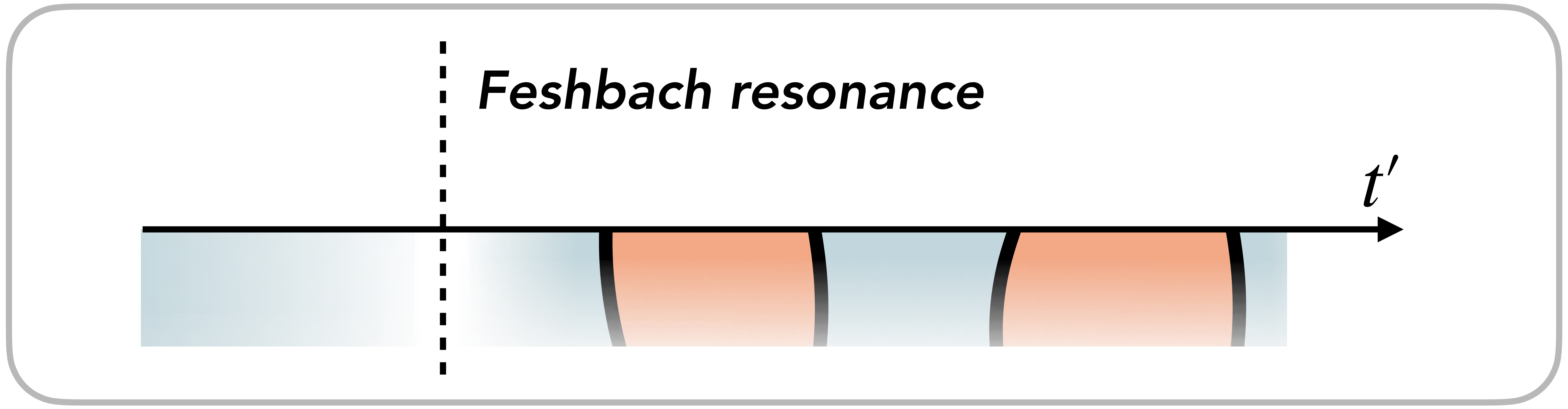
Feshbach hypothesis

Stripe or pairing instability?

- Scenario 1:



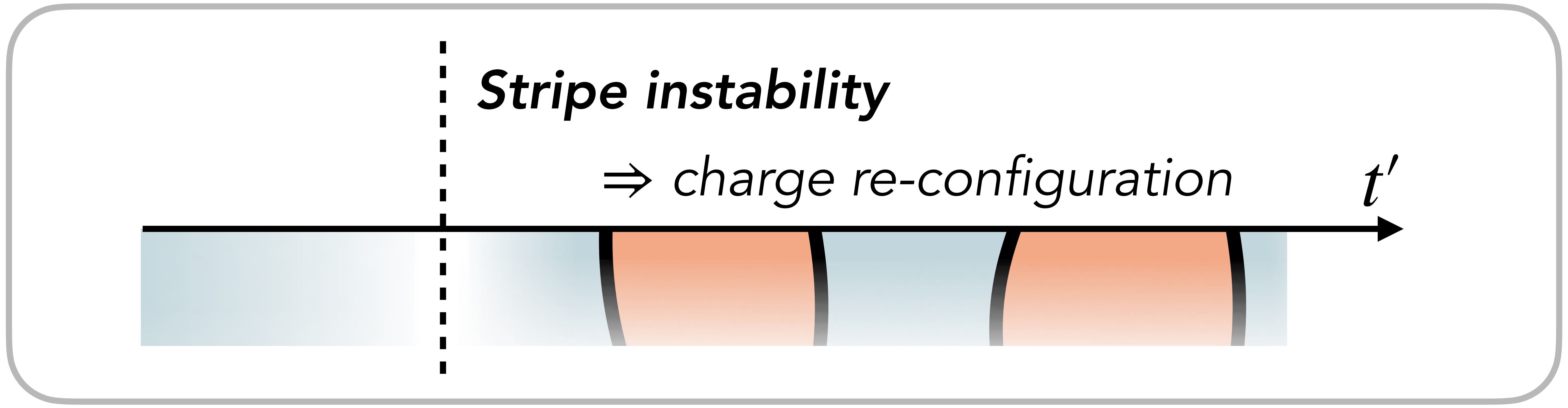
- Scenario 2:



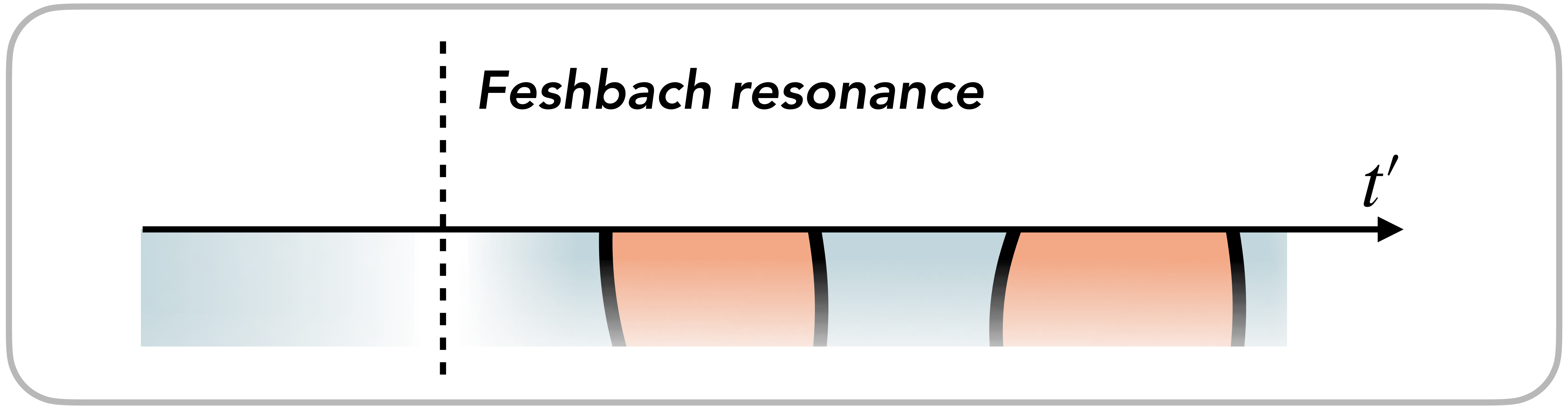
Feshbach hypothesis

Stripe or pairing instability?

- Scenario 1:



- Scenario 2:



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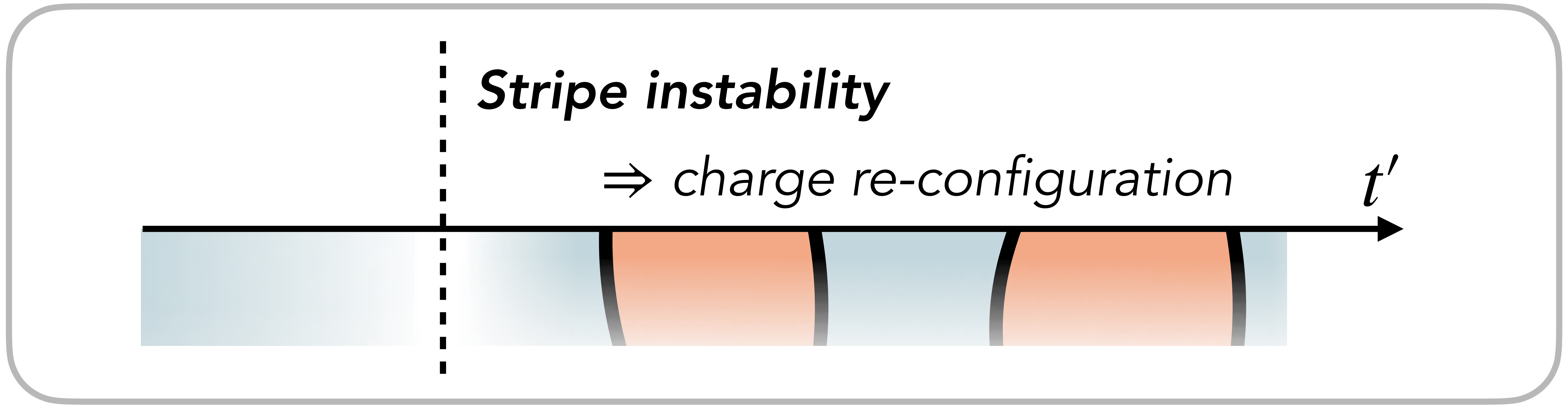
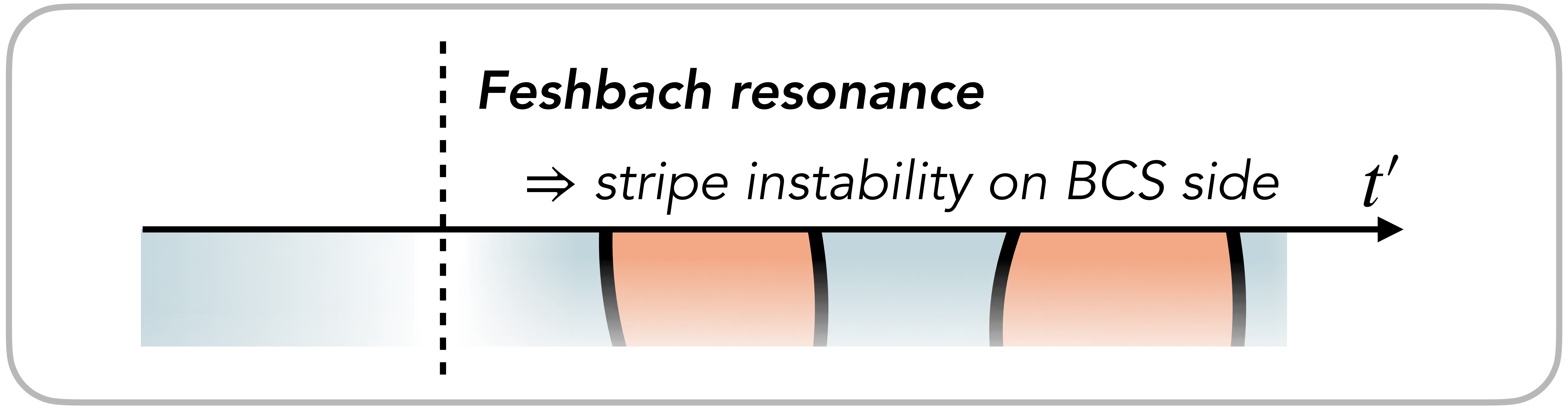


- Scenario 2:

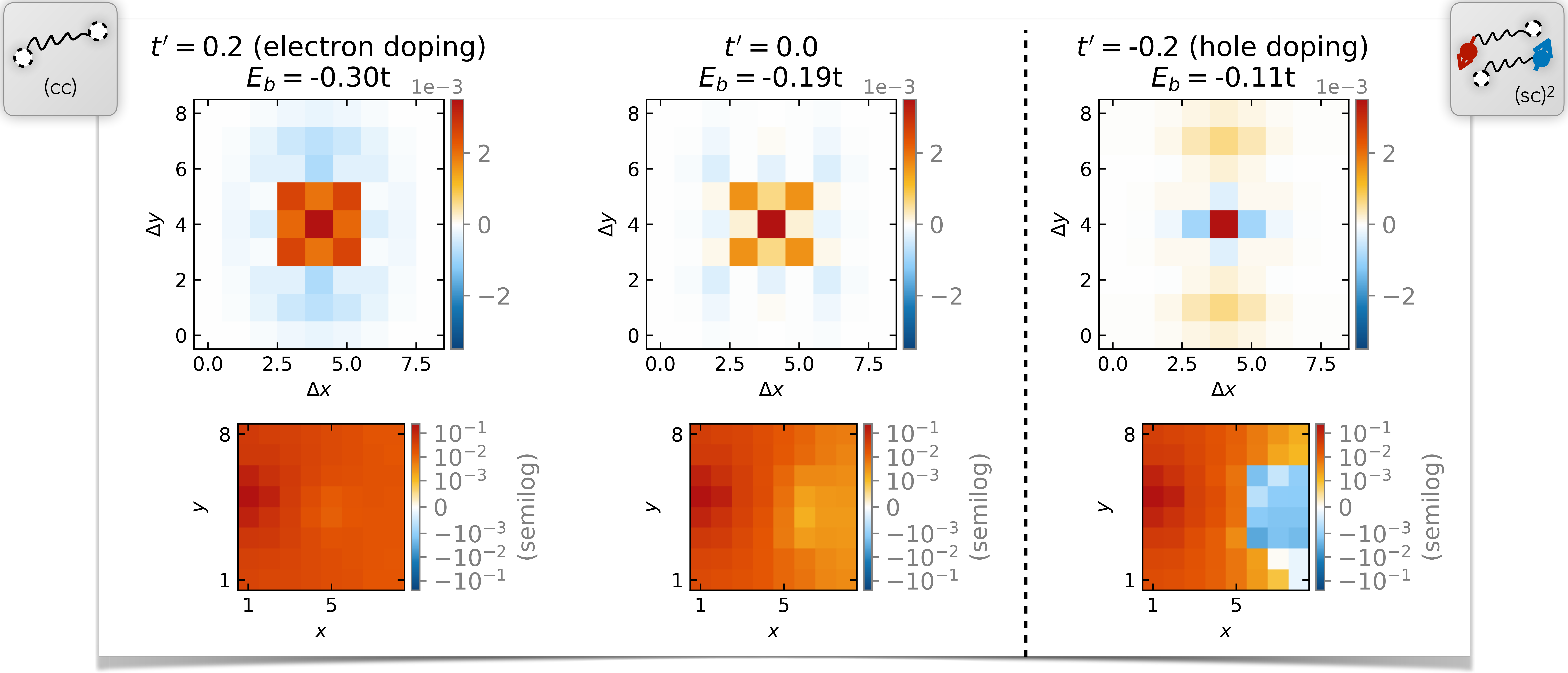


Feshbach hypothesis

Stripe or pairing instability?

t-J model on open ***8x8 patch***

Blatz et al., in prep.

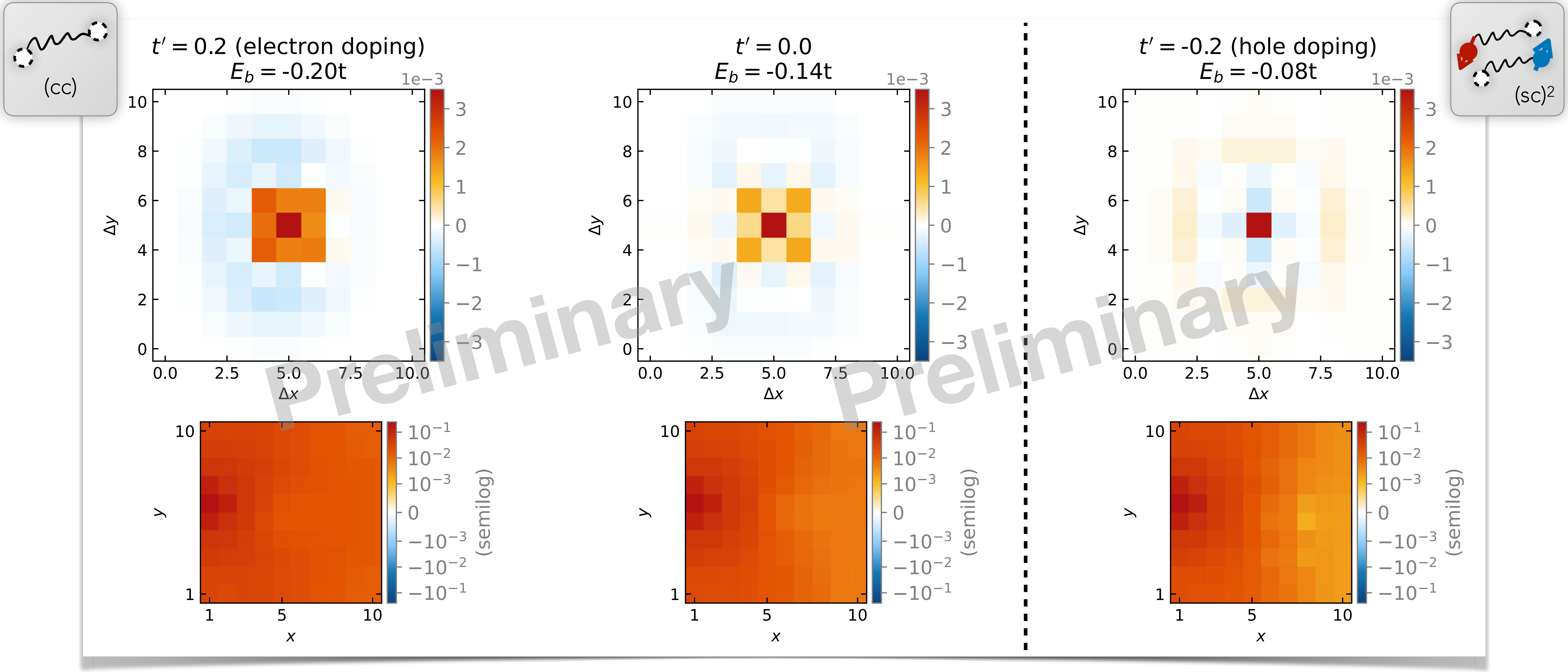


Feshbach hypothesis

Stripe or pairing instability?

*t-J model on open **10x10 patch***

Blatz et al., in prep.



Feshbach hypothesis

Stripe or pairing instability?

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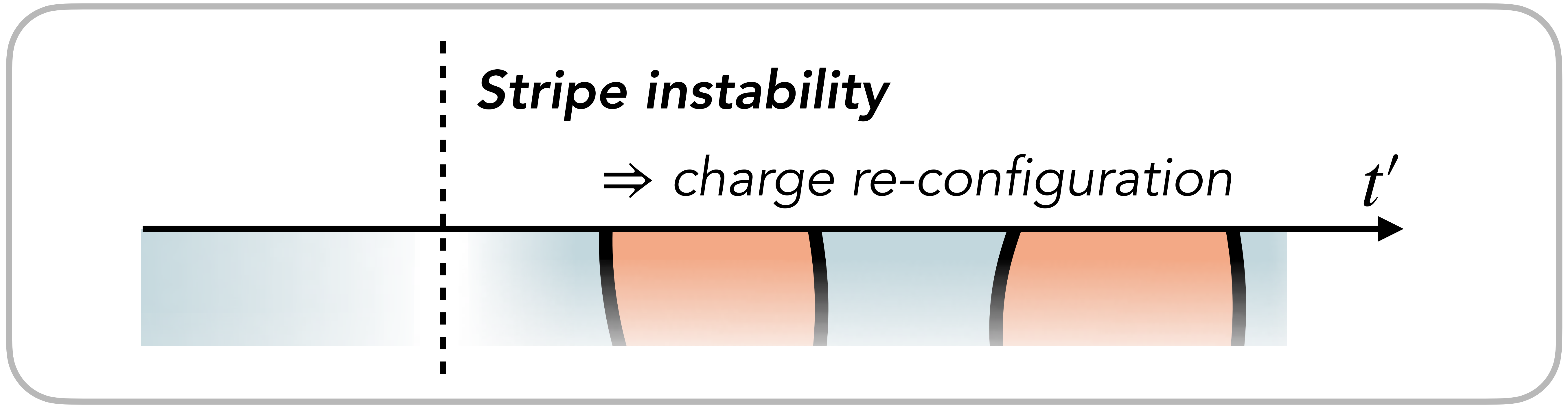

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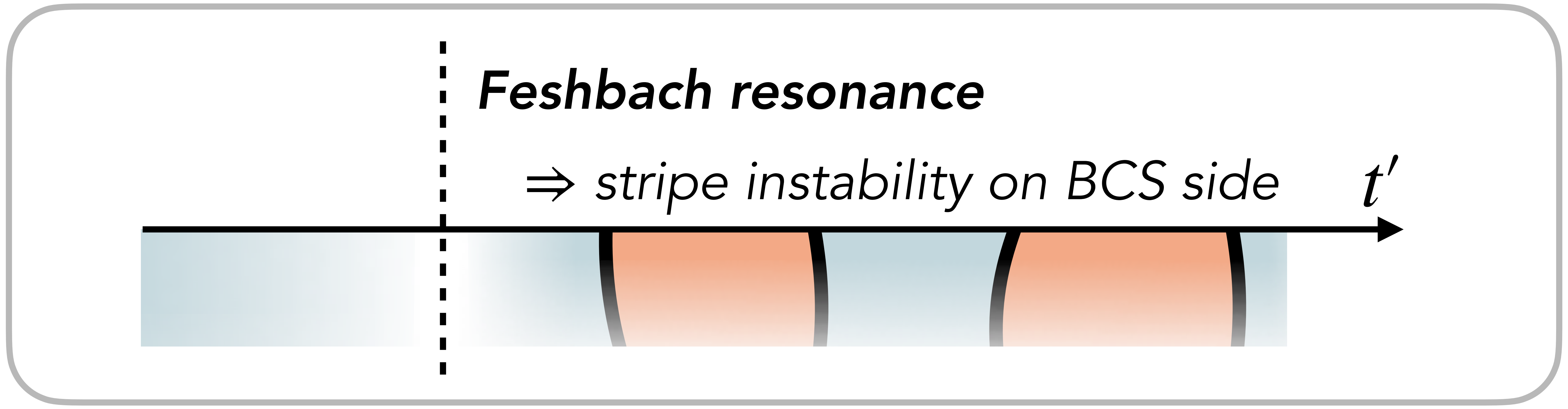
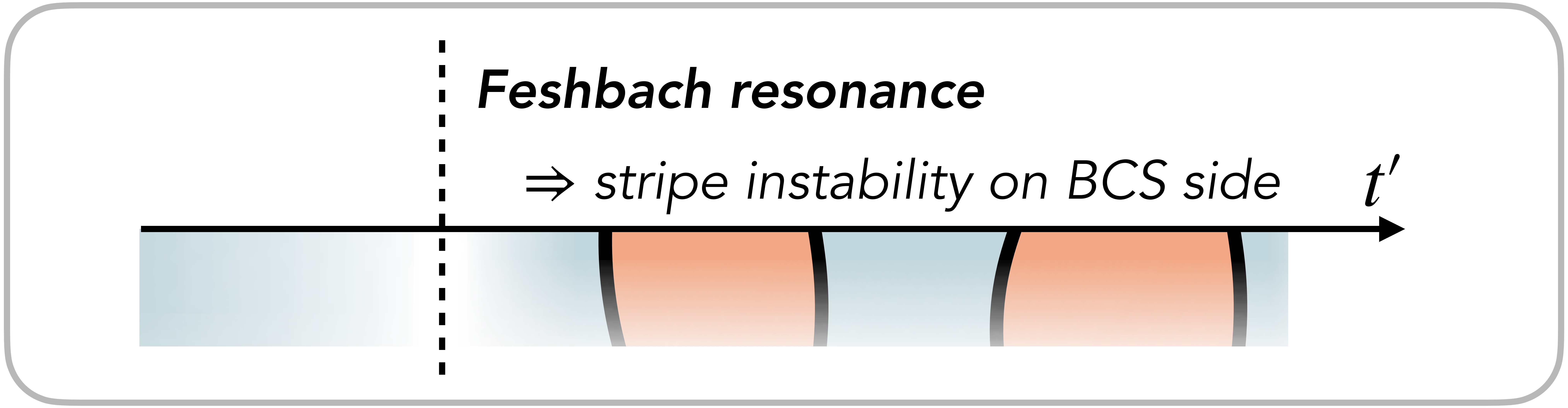


Feshbach hypothesis

Stripe or pairing instability?

- Scenario 2:



Summary Part 2

Rich physics from emergent constituents:

* **Emergent constituents:**
  two kinds of mesons

  — *confined @ strong coupling*

Summary Part 2

Rich physics from emergent constituents:

* **Emergent constituents:**
  two kinds of mesons

  — *confined @ strong coupling*

* **Feshbach hypothesis:**

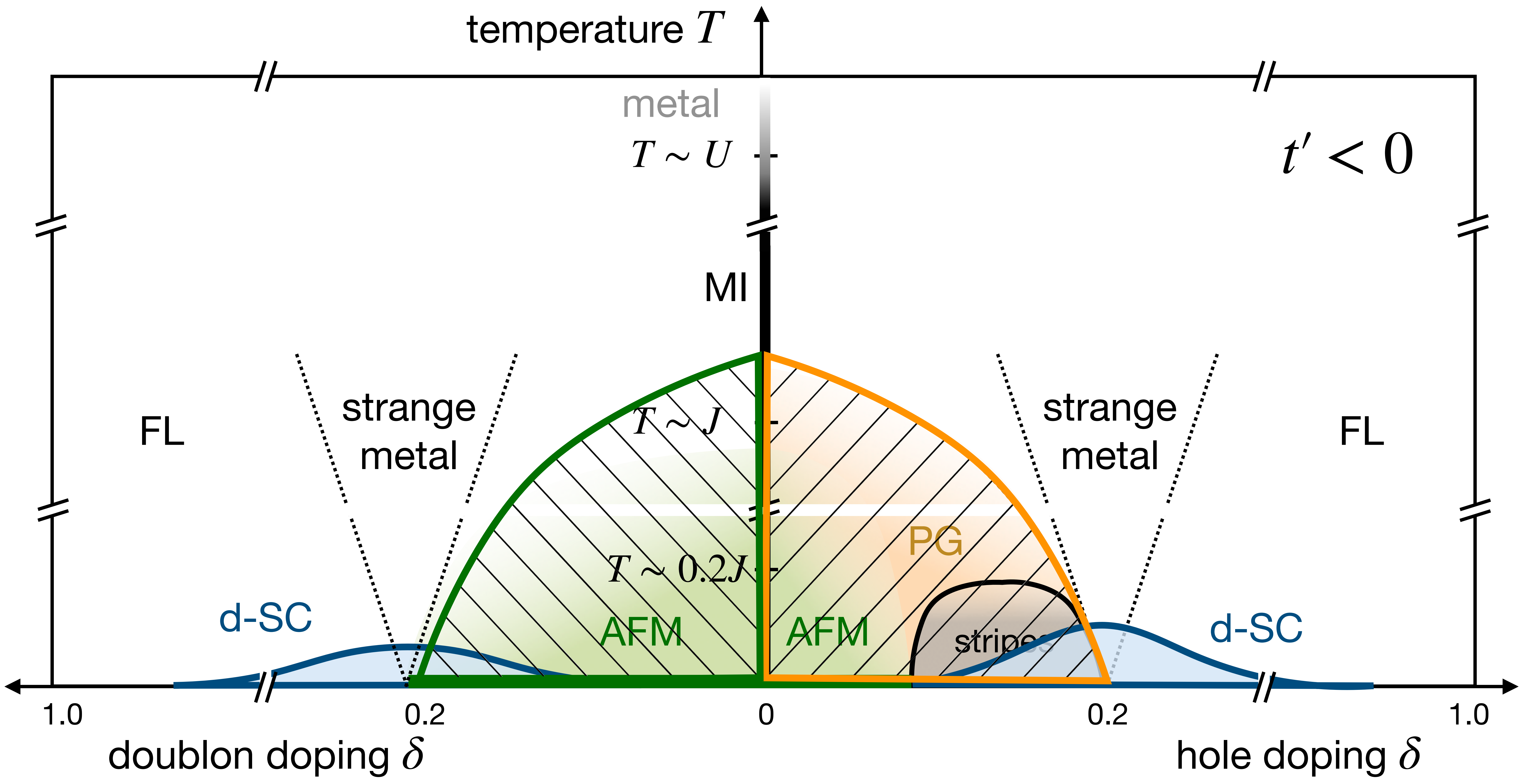
  — *mediated interactions*

  — *d-wave resonance*

Summary Part 2

Feshbach Hypothesis of high-Tc superconductivity

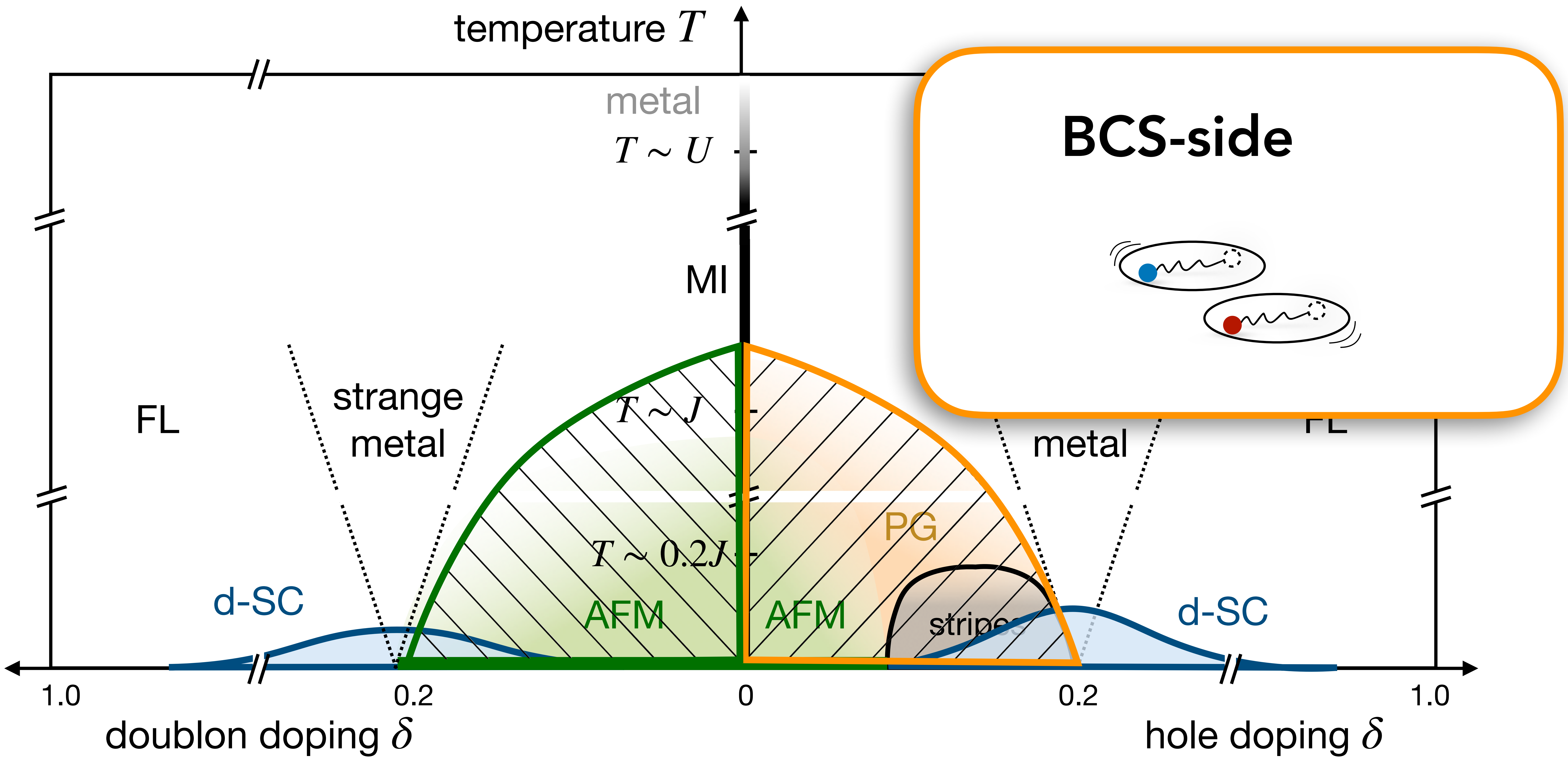
Blatz et al., in prep.



Summary Part 2

Feshbach Hypothesis of high-Tc superconductivity

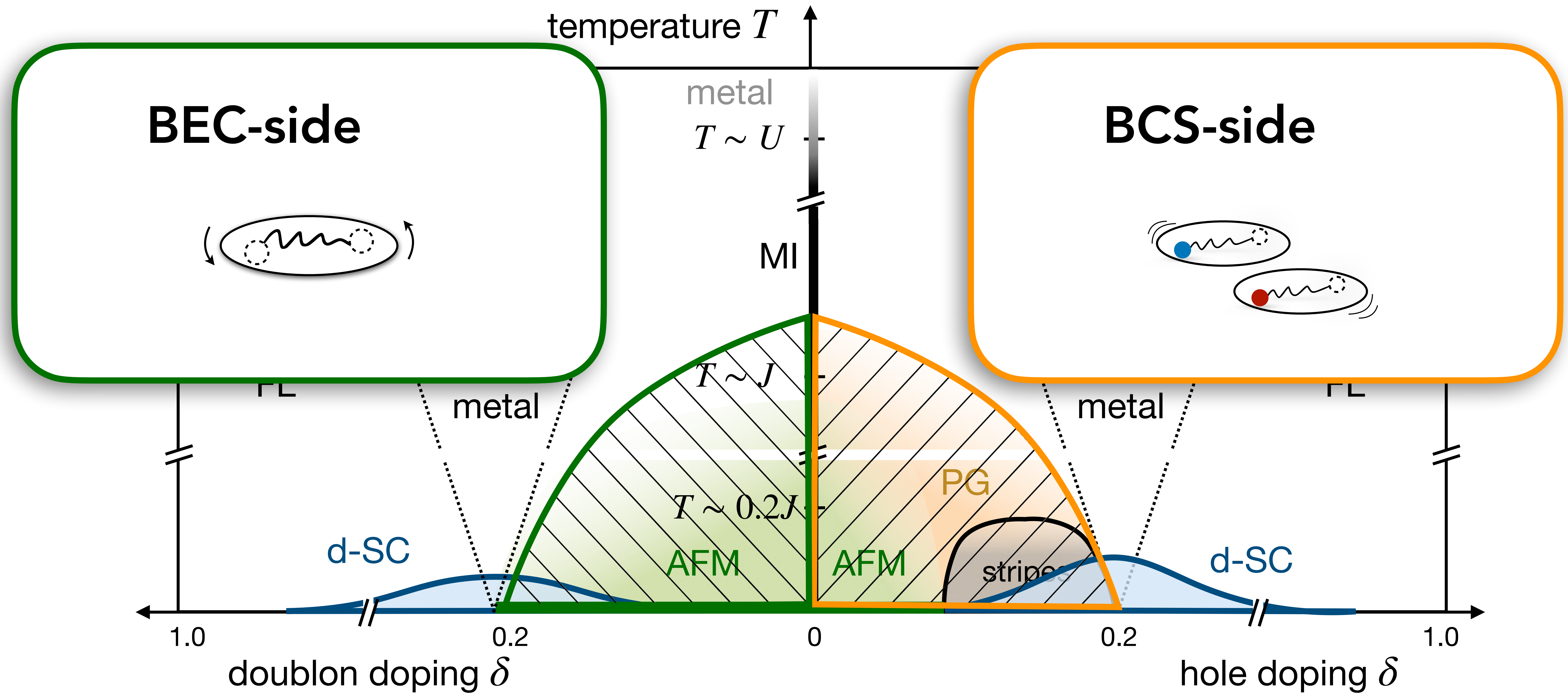
Blatz et al., in prep.



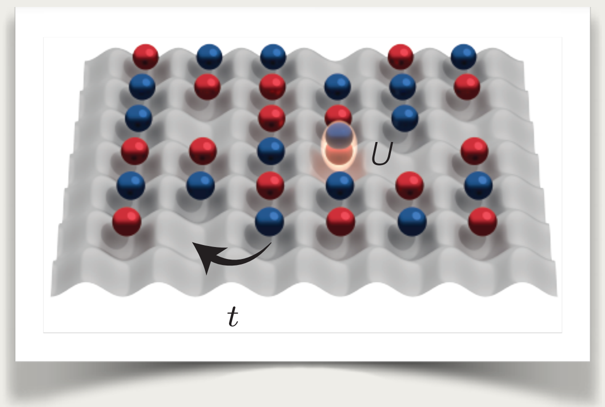
Summary Part 2

Feshbach Hypothesis of high-Tc superconductivity

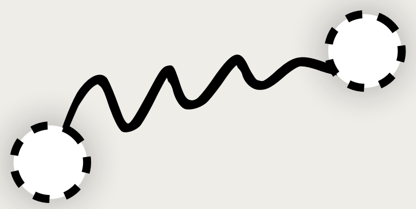
Blatz et al., in prep.



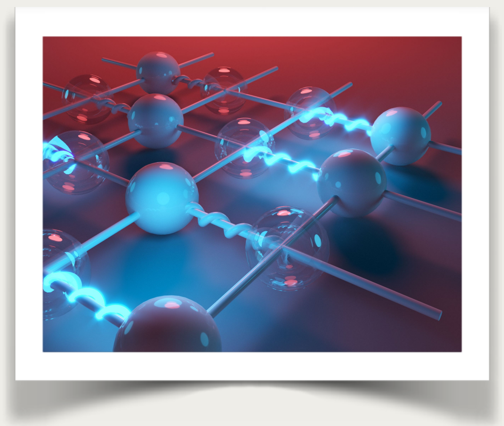
Outline



Phase diagram of high-Tc superconductors

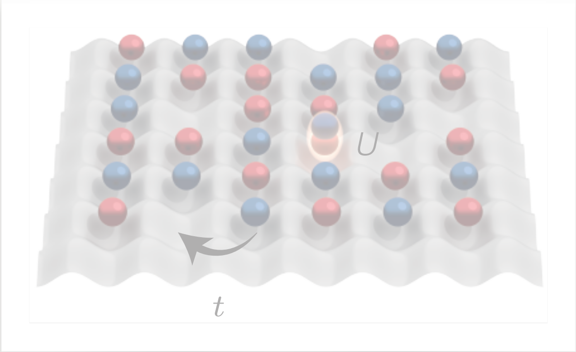


Strong coupling theory



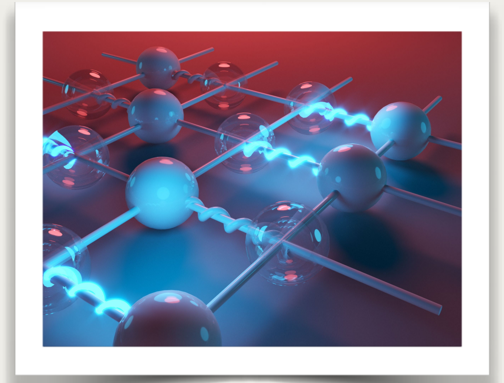
Hidden orders

Outline



Phase diagram of high-Tc superconductors

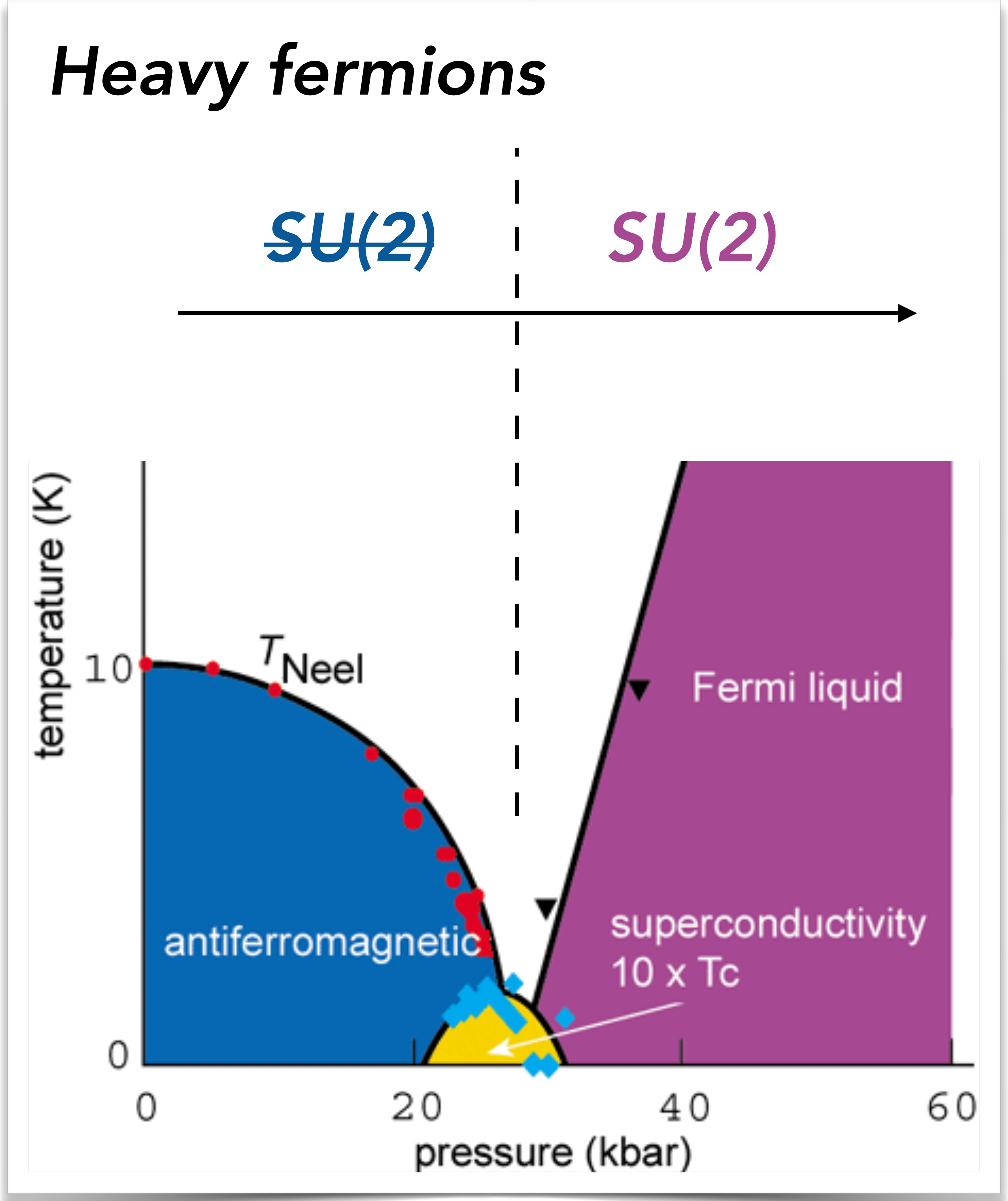
Strong coupling theory



Hidden orders

Motivation

Breaking of the SU(2) symmetry: correlated electrons



https://physicsworld.com/a/ferromagnetic-superconductors/

Motivation

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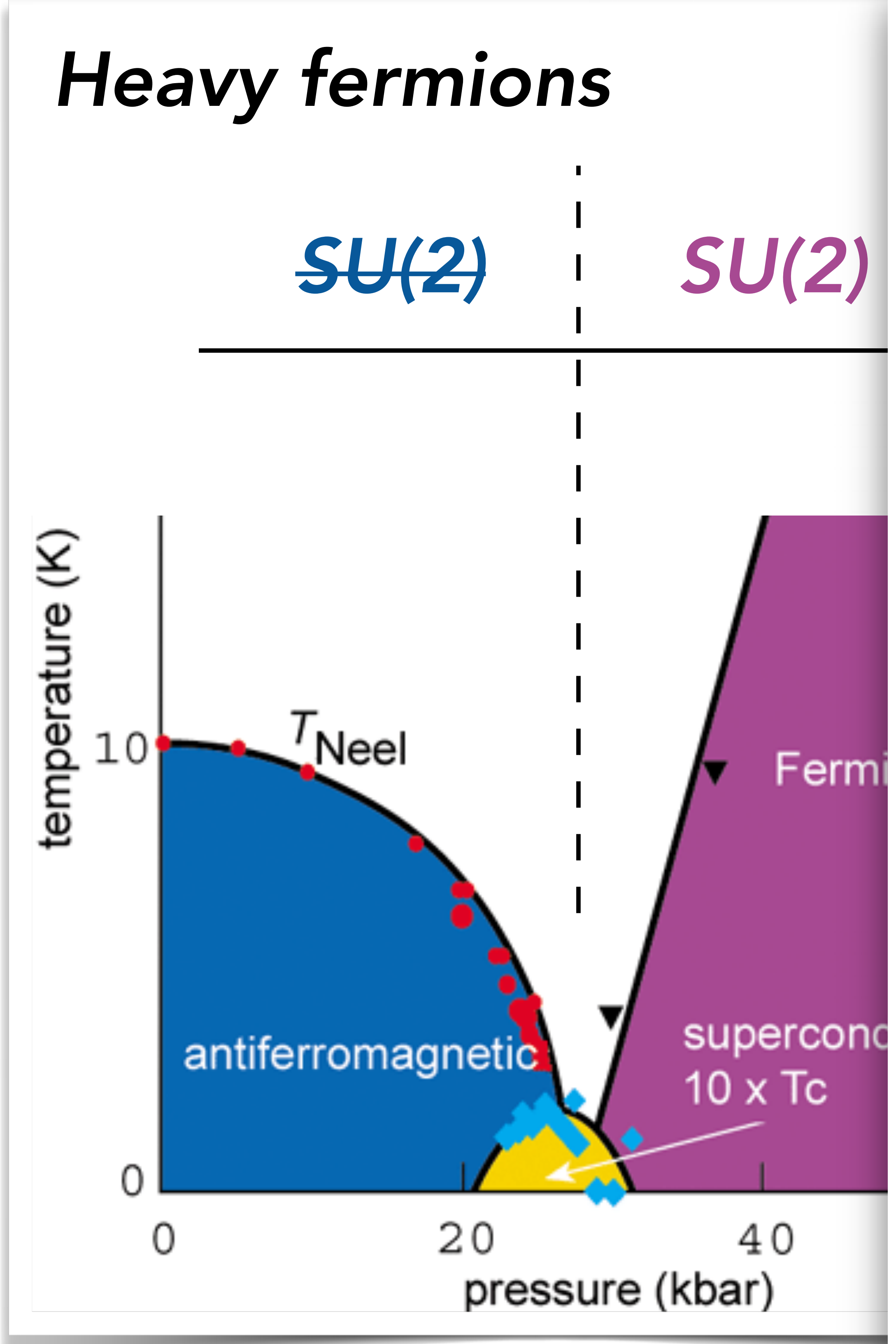

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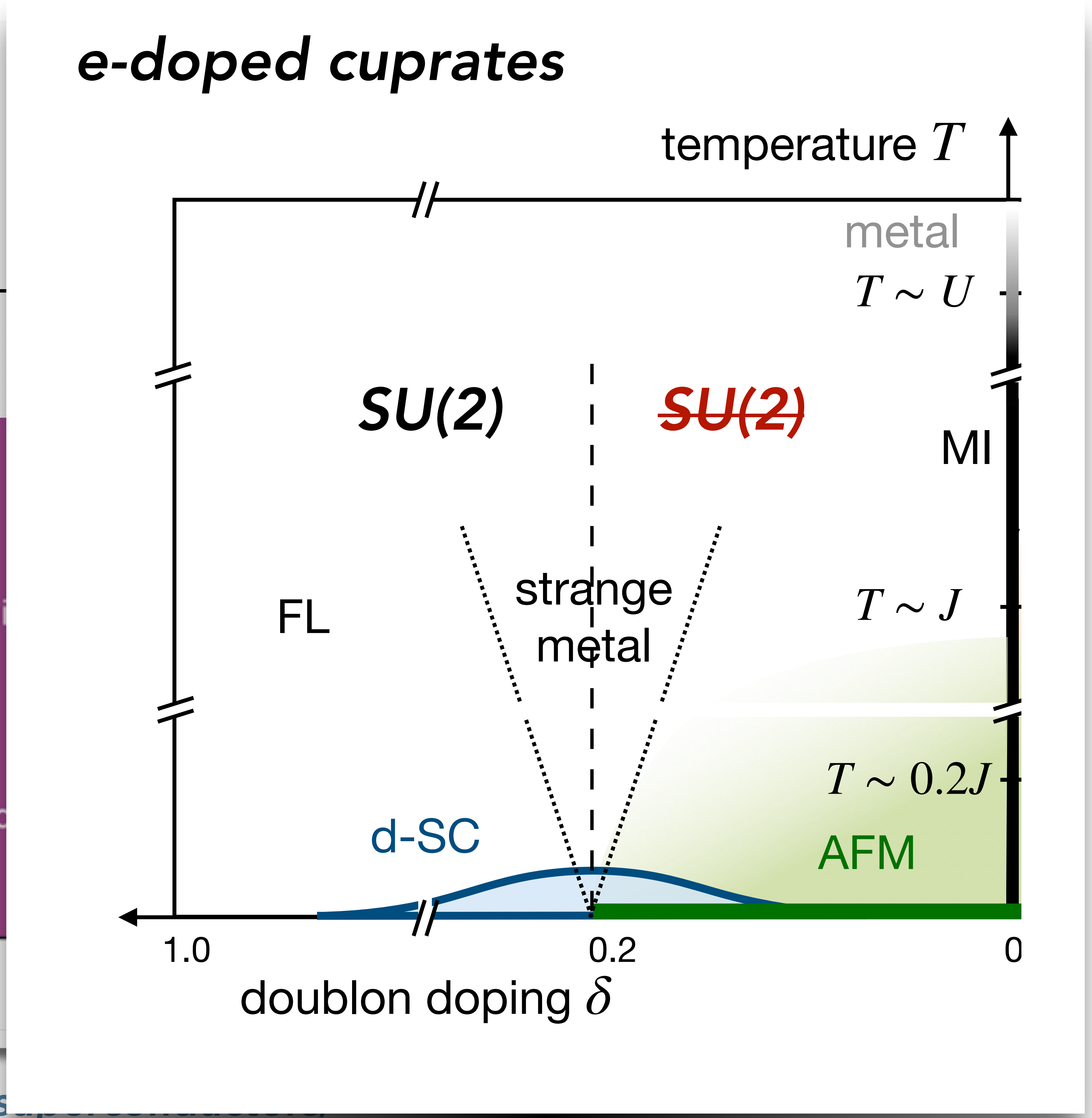
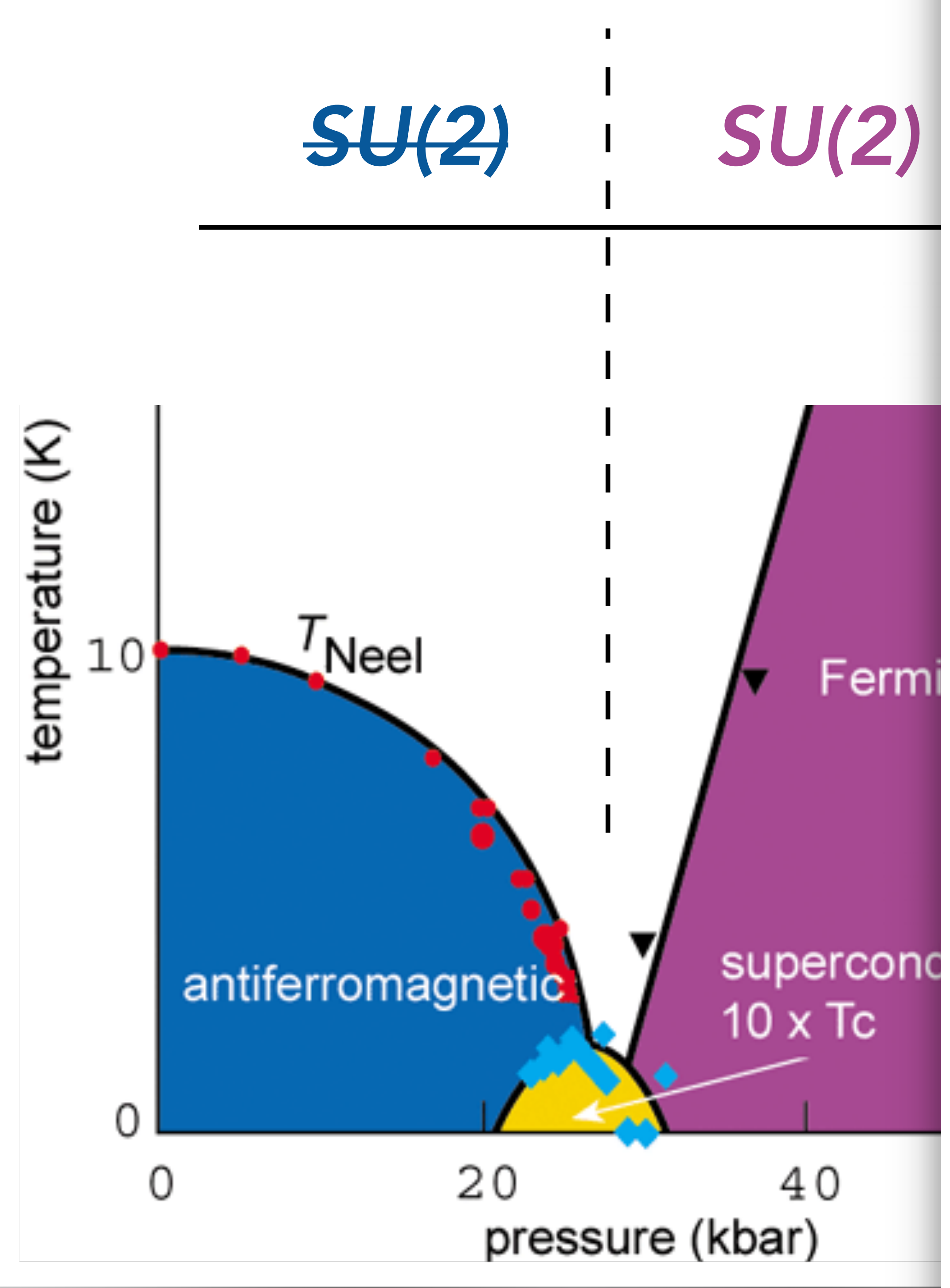


Motivation

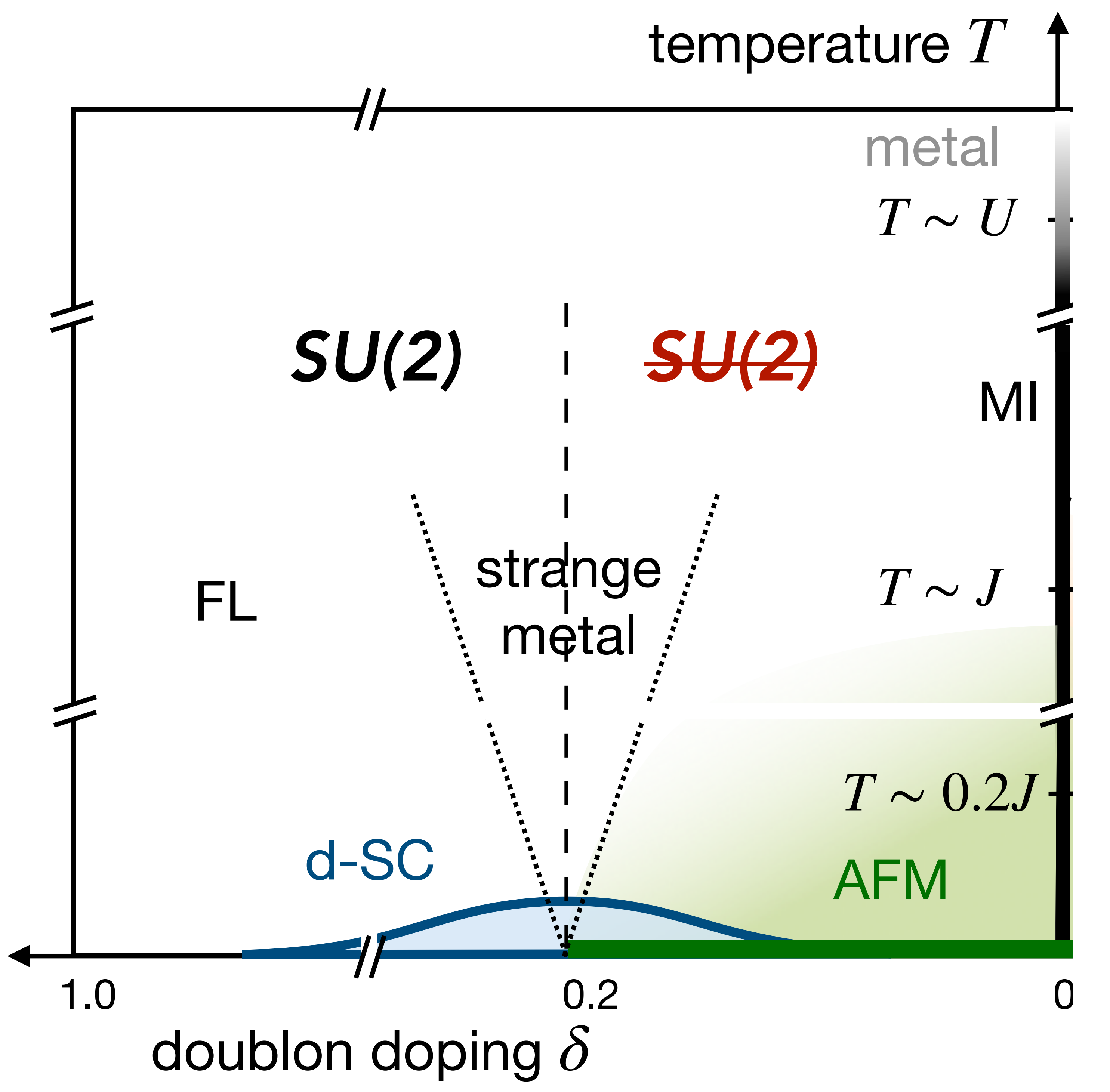
Breaking of the SU(2) symmetry: correlated electrons

Heavy fermions

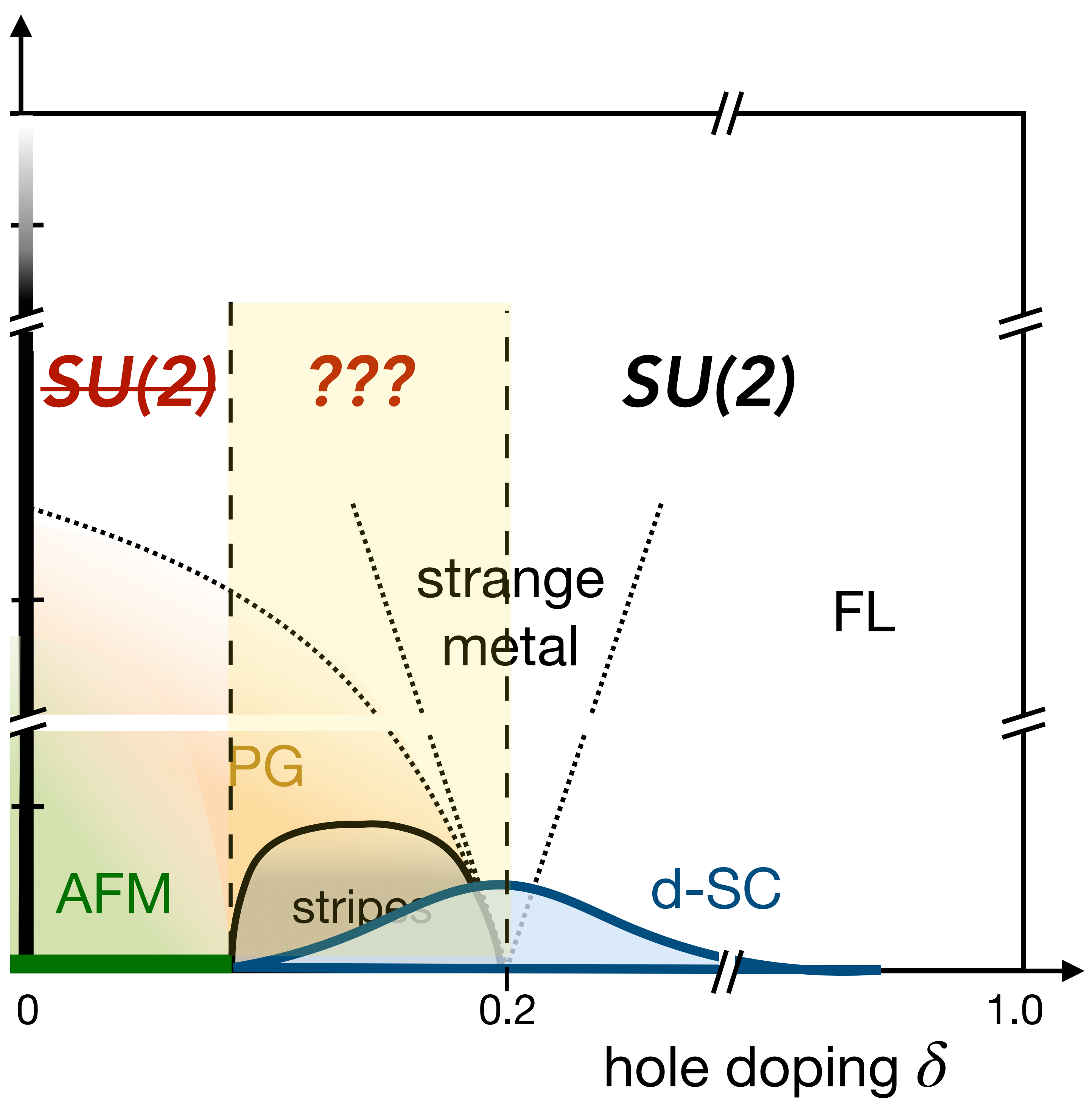


https://physicsworld.com/a/ferromagnetic-s

e-doped cuprates



h-doped cuprates



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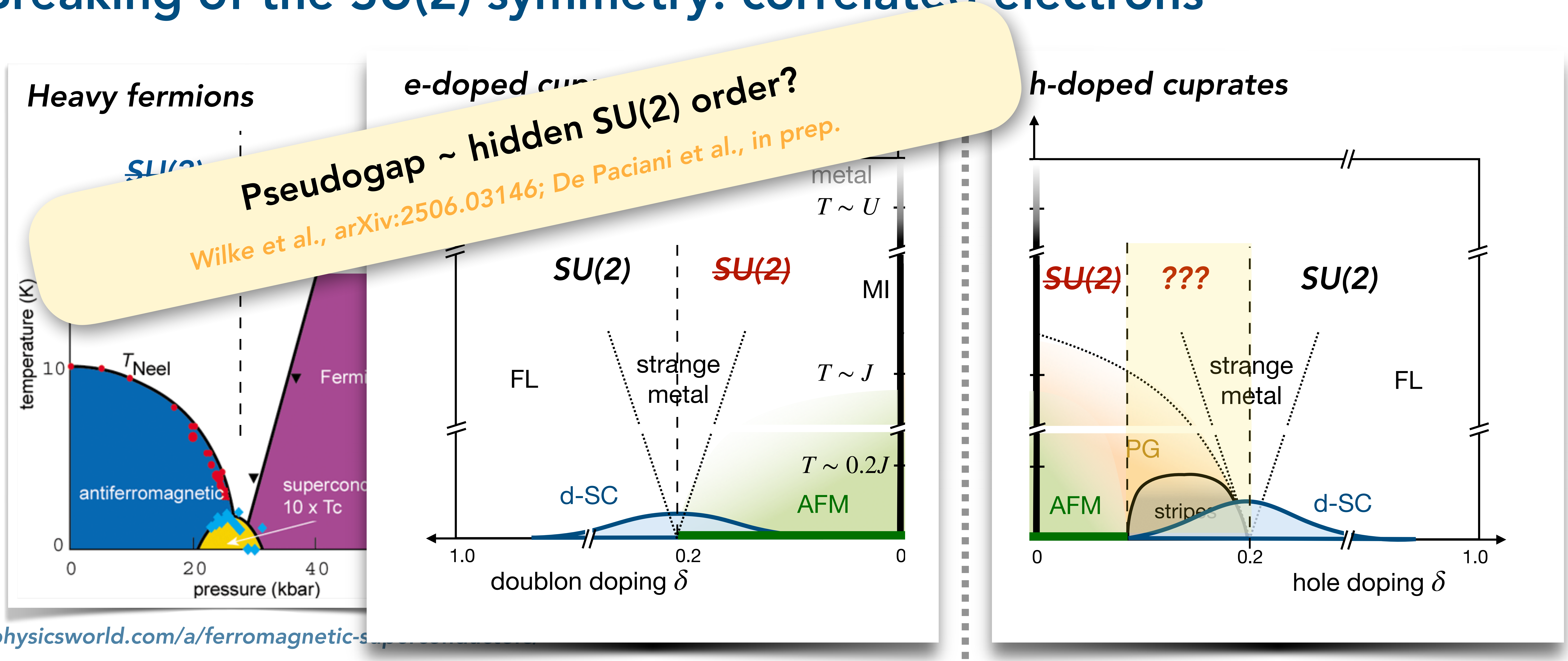

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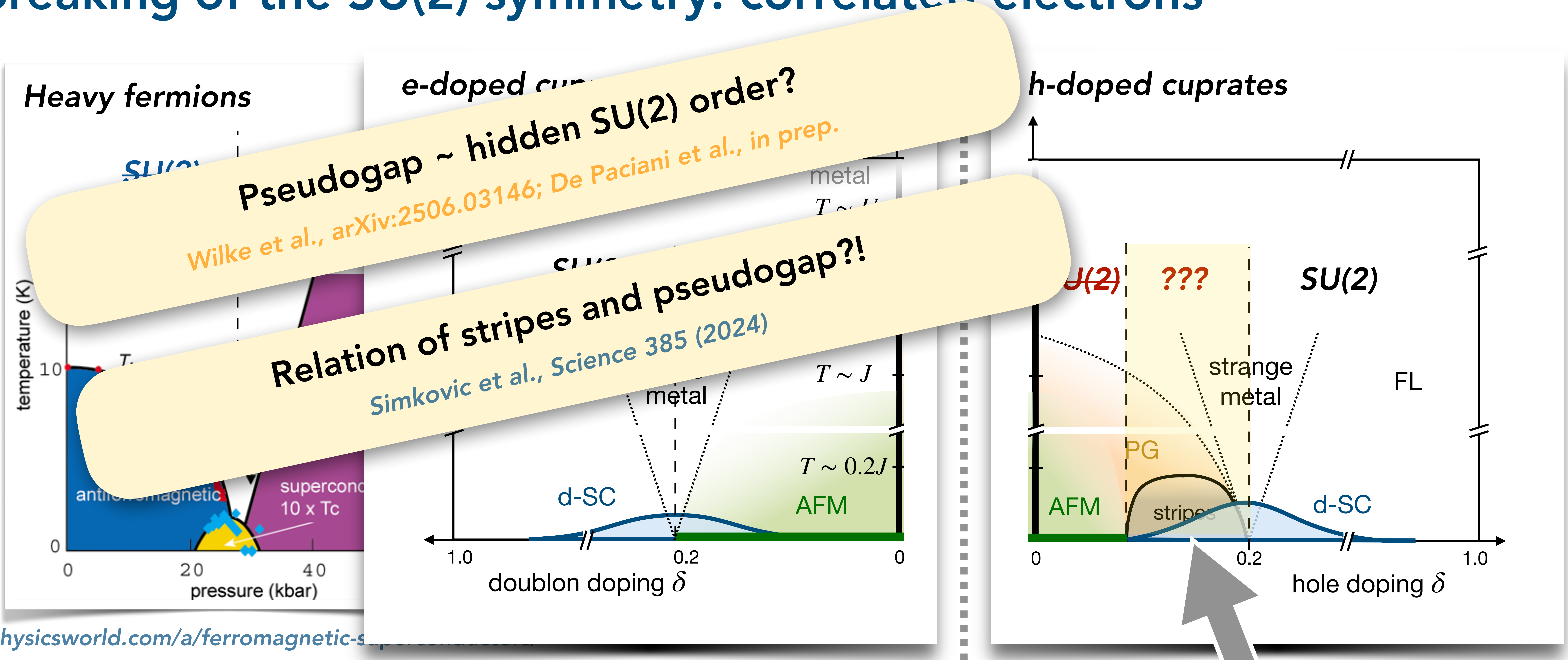
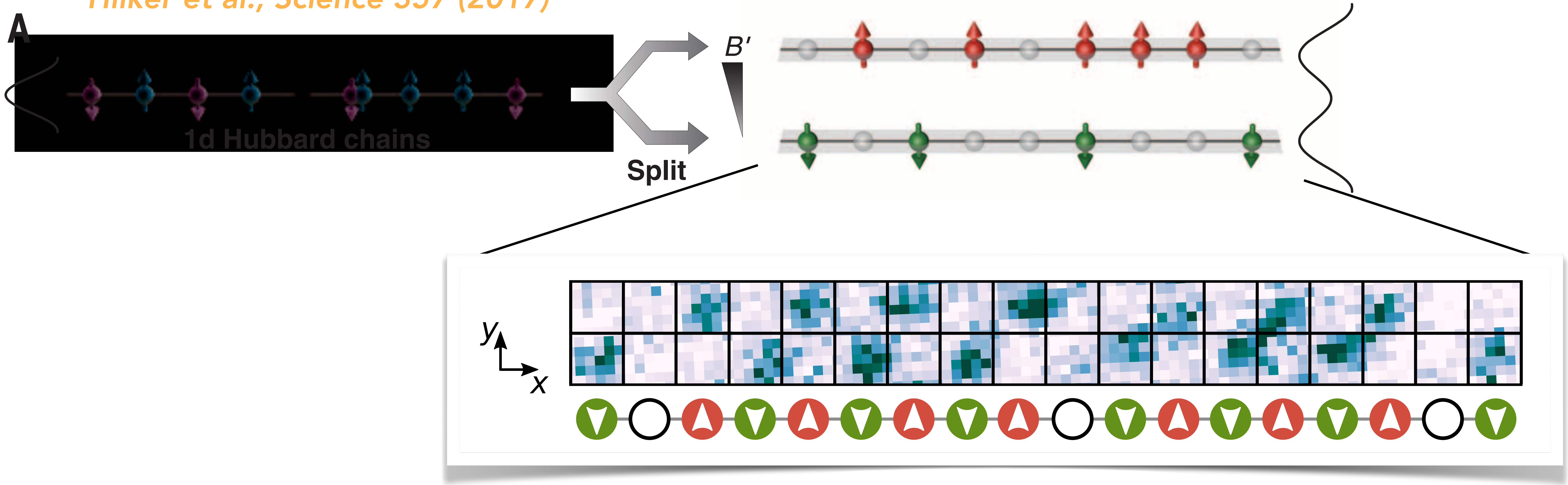


https://physicsworld.com/a/ferromagnetic-s...

no stripes *stripes*

PART 3.I: Hidden antiferro-magnetism in 1D

Boll et al., Science 353 (2016)
Hilker et al., Science 357 (2017)
— *Bloch/Gross lab*

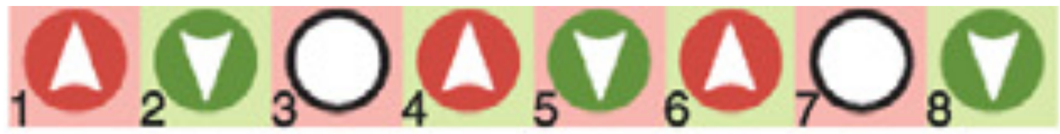


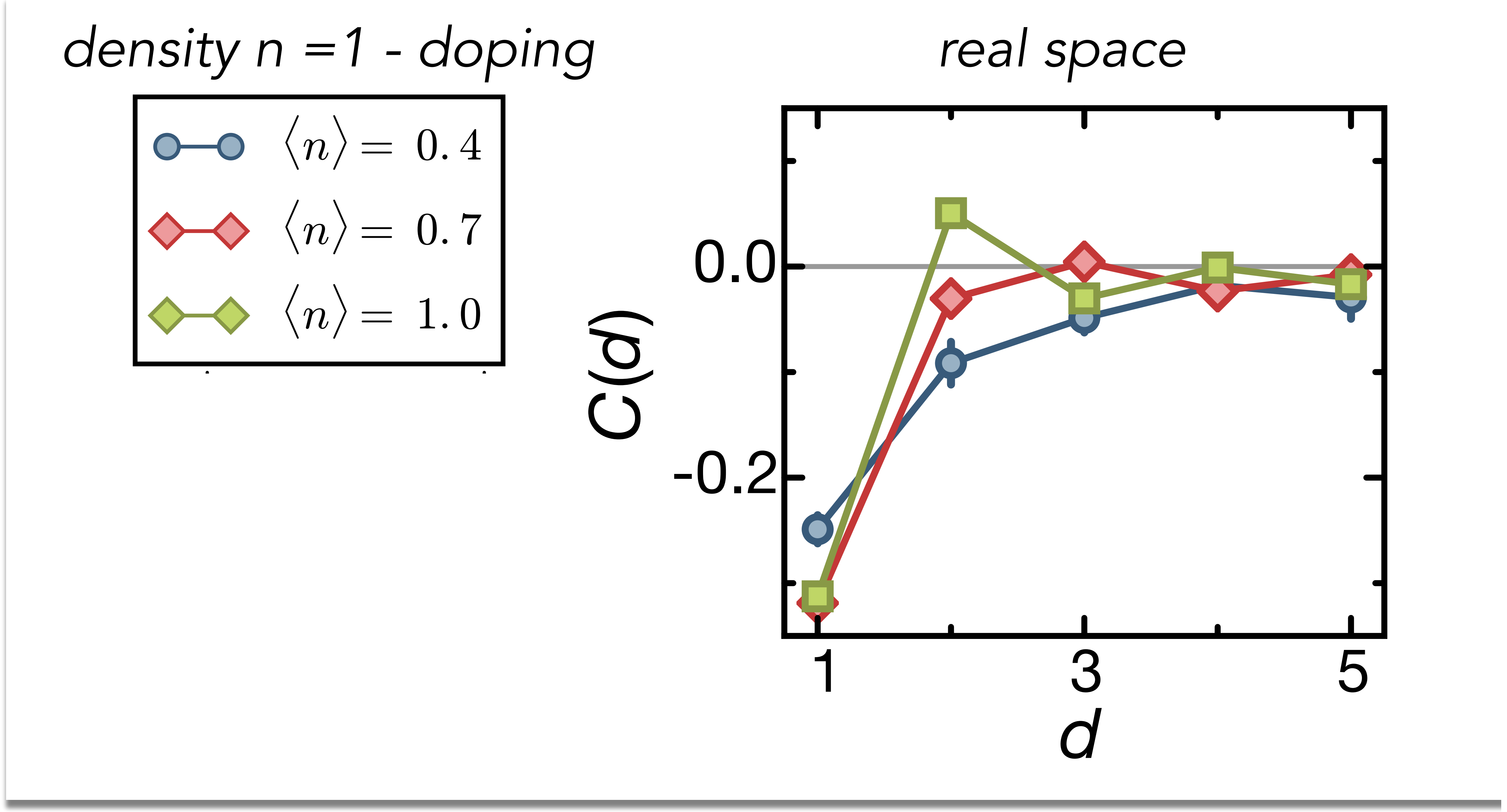
Hidden order

Hidden AFM correlations in 1D

Hilker et al., Science 357 (2017) — Bloch/Gross lab

$$C \quad (d) = 4 \left\langle \hat{S}^z_{j+d} \qquad\qquad \hat{S}^z_j \right\rangle$$



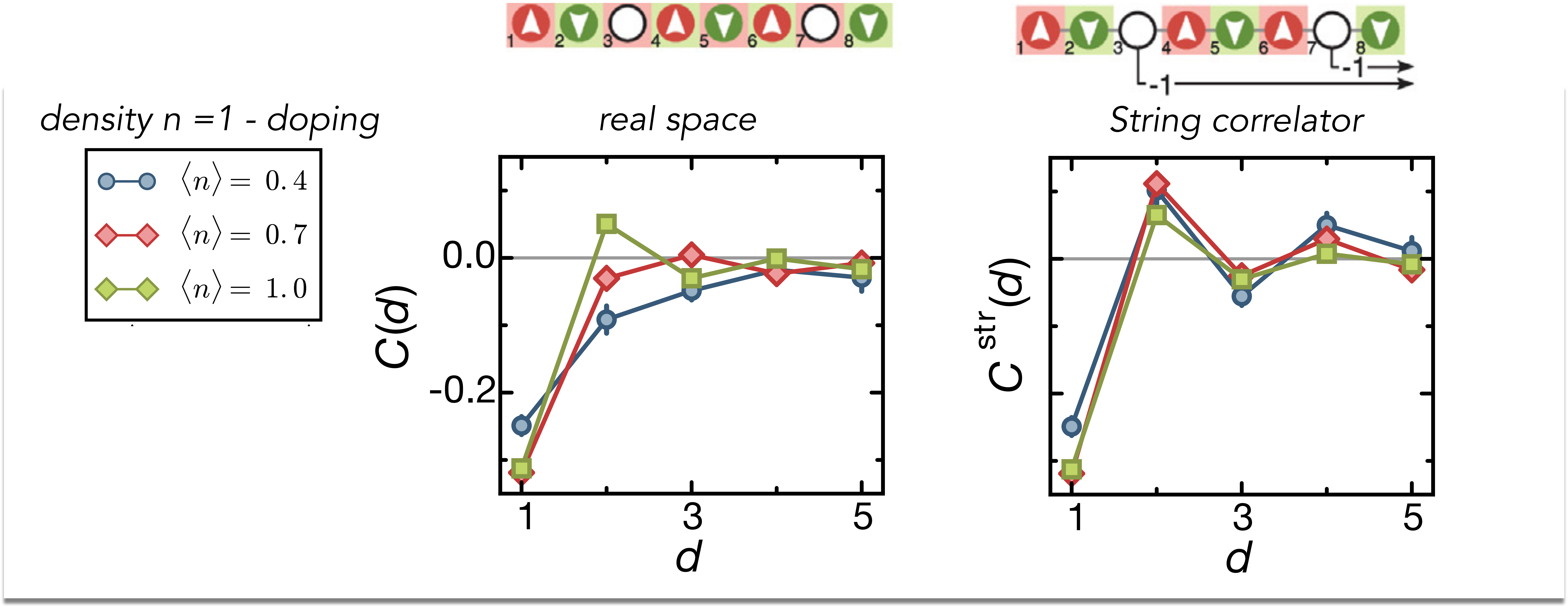


Hidden order

Hidden AFM correlations in 1D

Hilker et al., Science 357 (2017) — Bloch/Gross lab

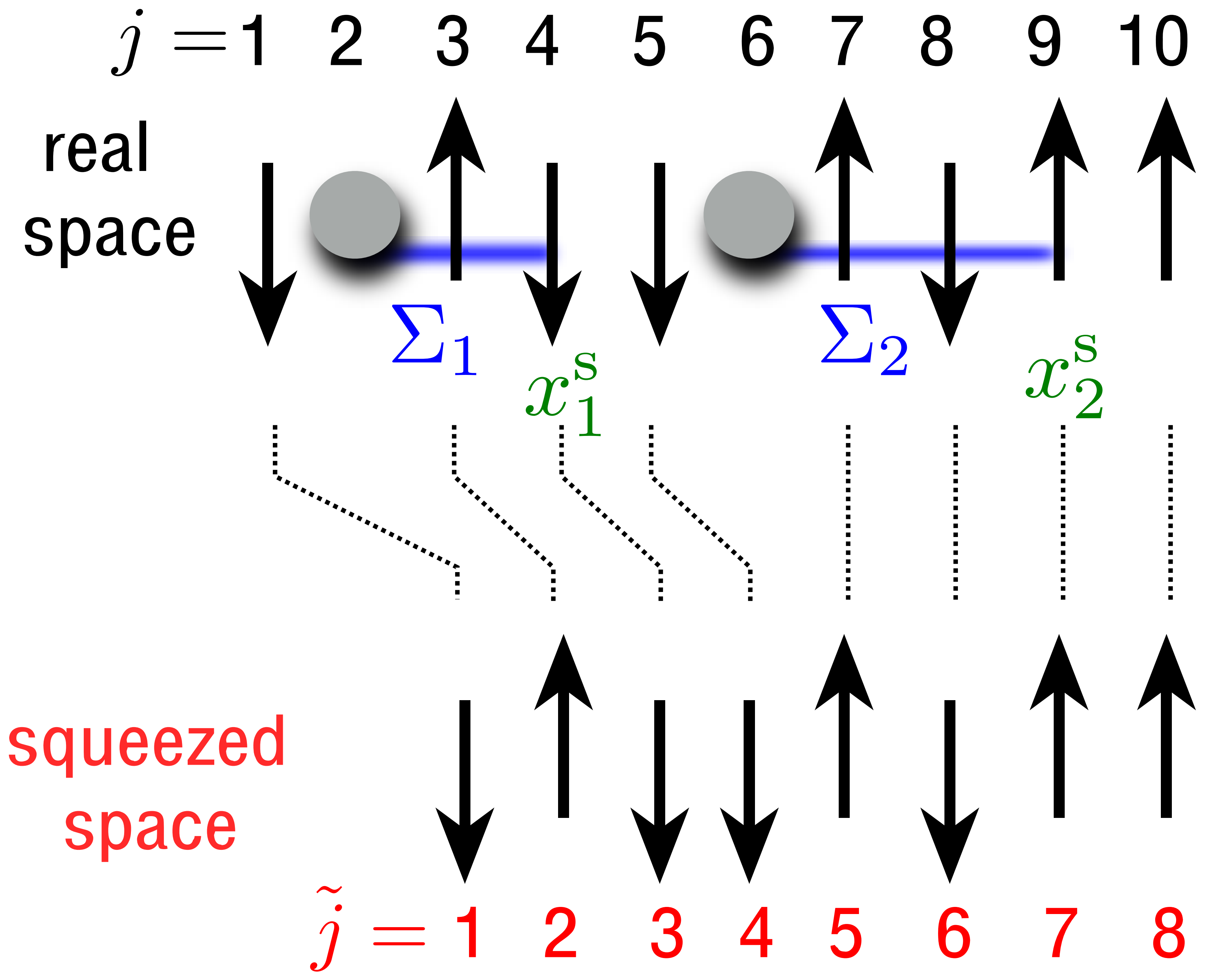
$$C_{\mathrm{str}}(d) = 4 \left\langle \hat{S}^z_{j+d} \prod_{i=1}^{d-1} e^{i\pi \hat{n}^h_{j+i}} \; \hat{S}^z_j \right\rangle$$

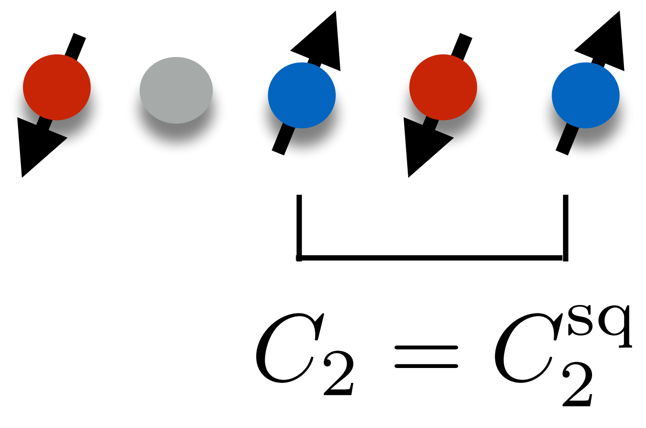


Hidden AFM correlations in 1D

Squeezed space

Ogata & Shiba, PRB 41 (1990); Kruis et al., PRB 70 (2004)





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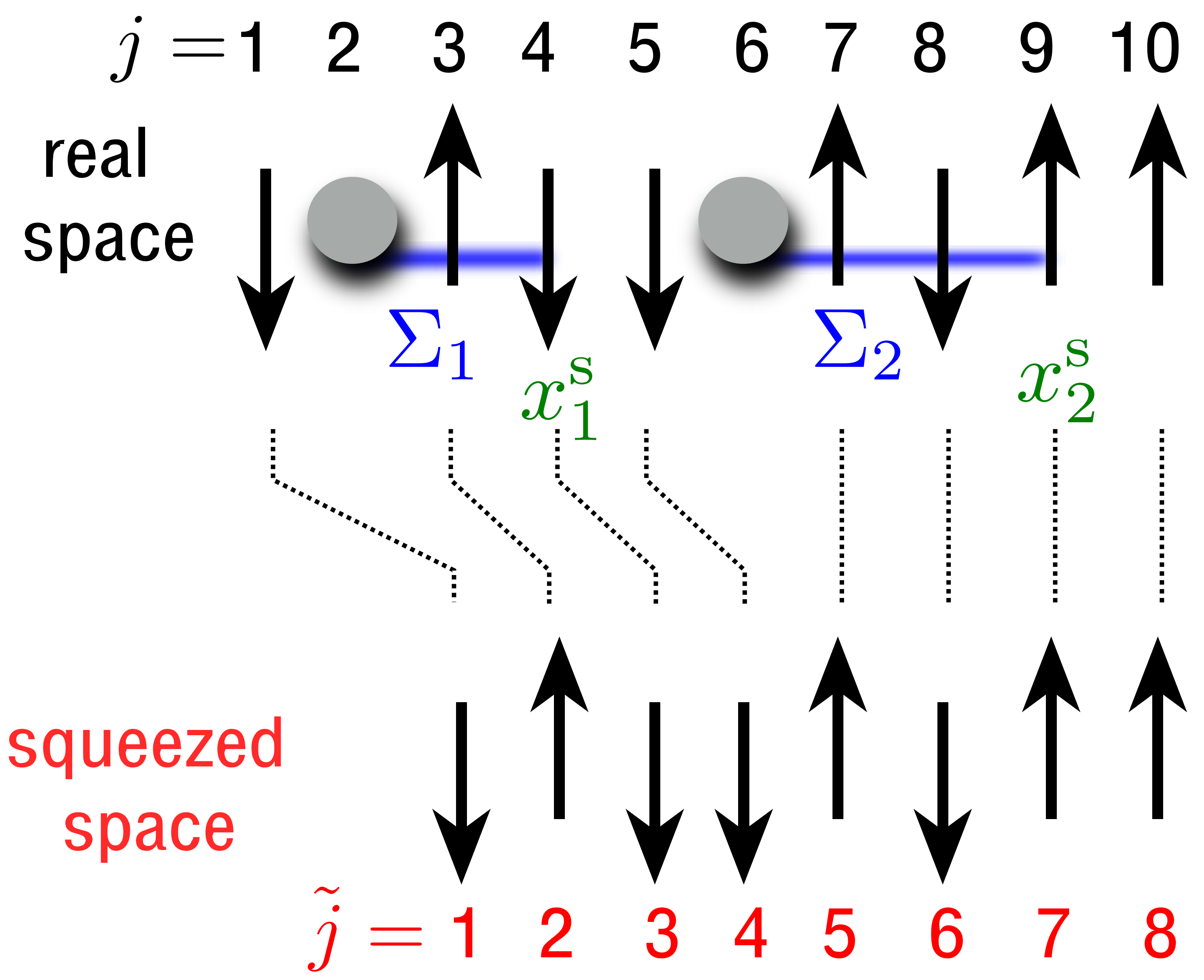

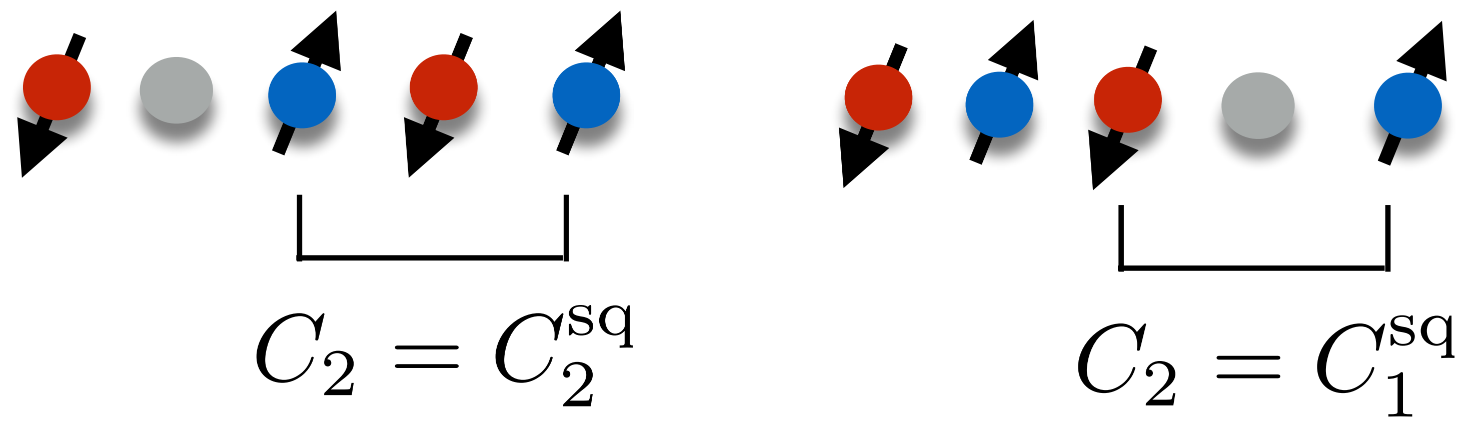

Hidden order

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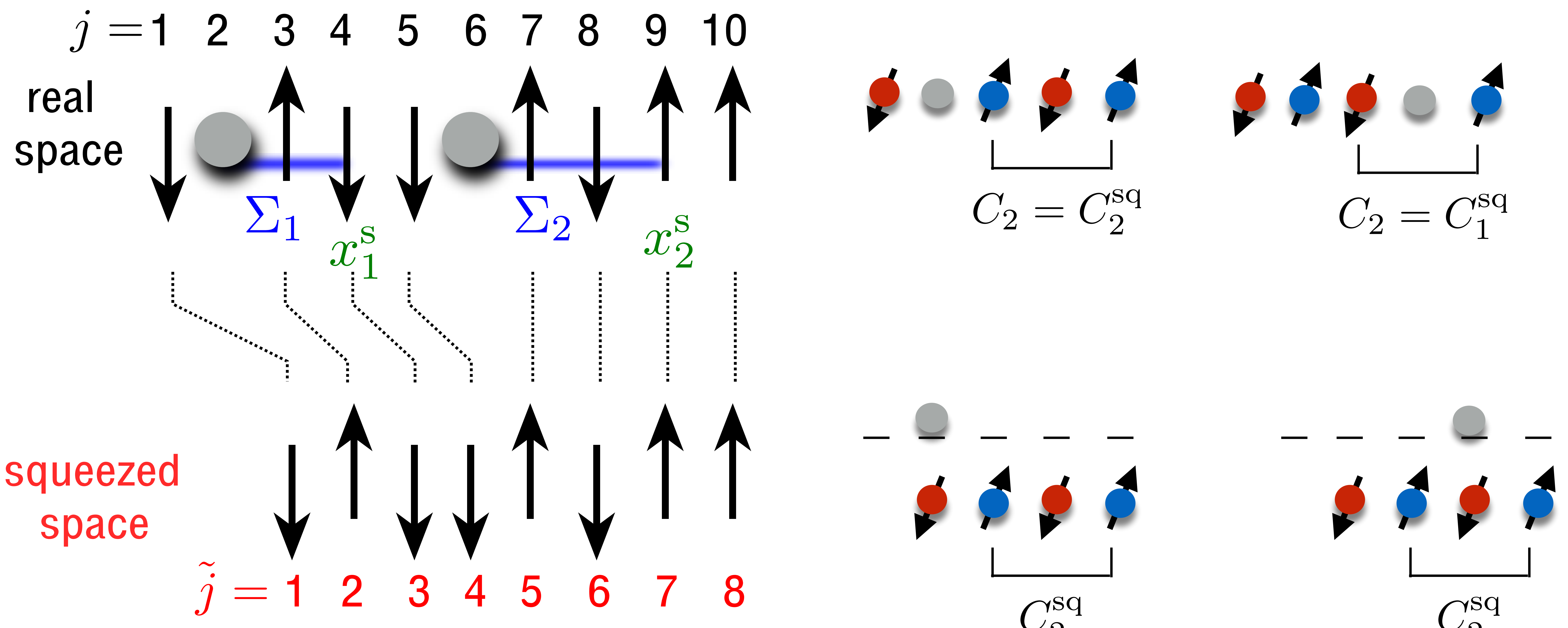

Hidden order

Hidden AFM correlations in 1D

Squeezed space

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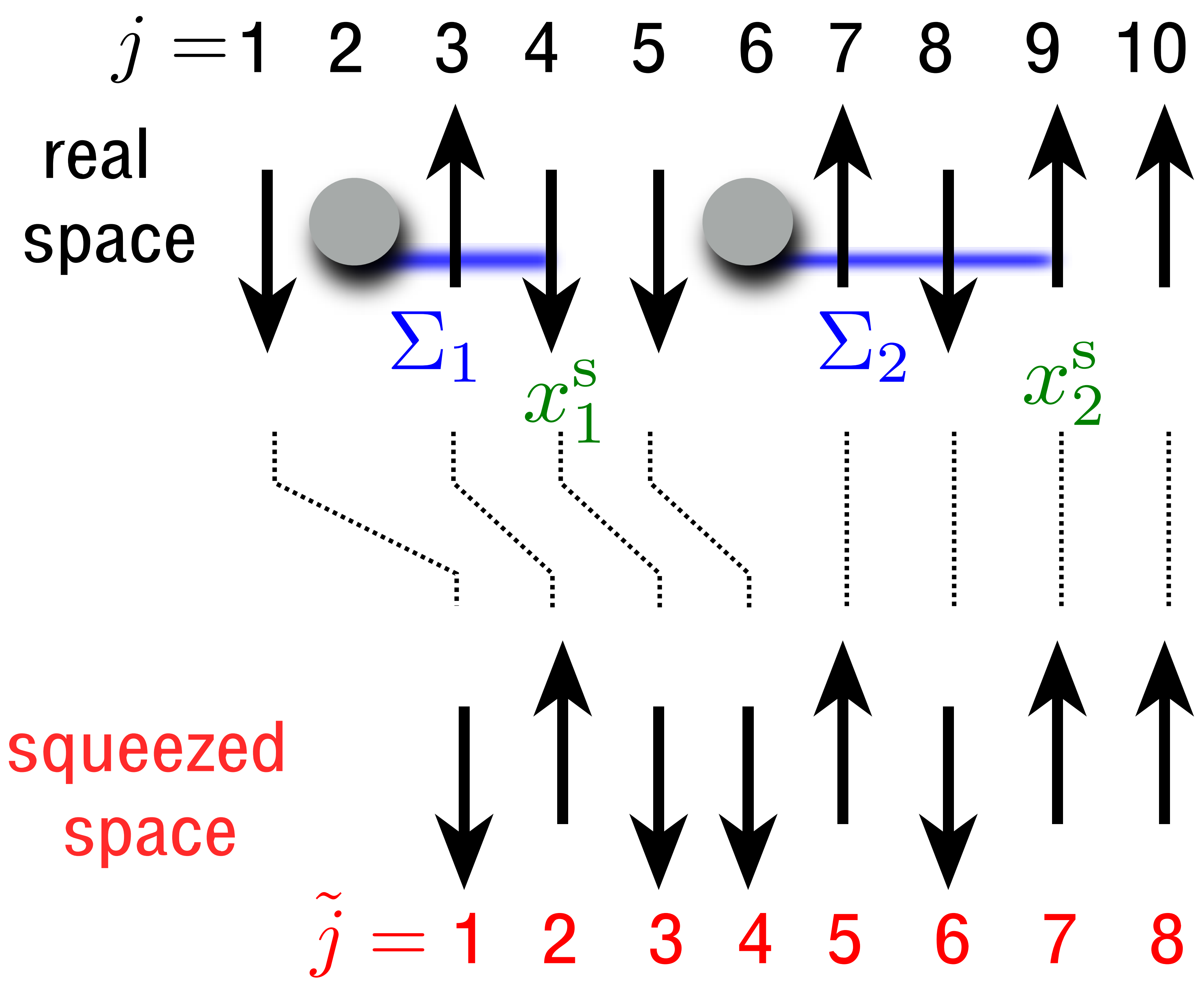


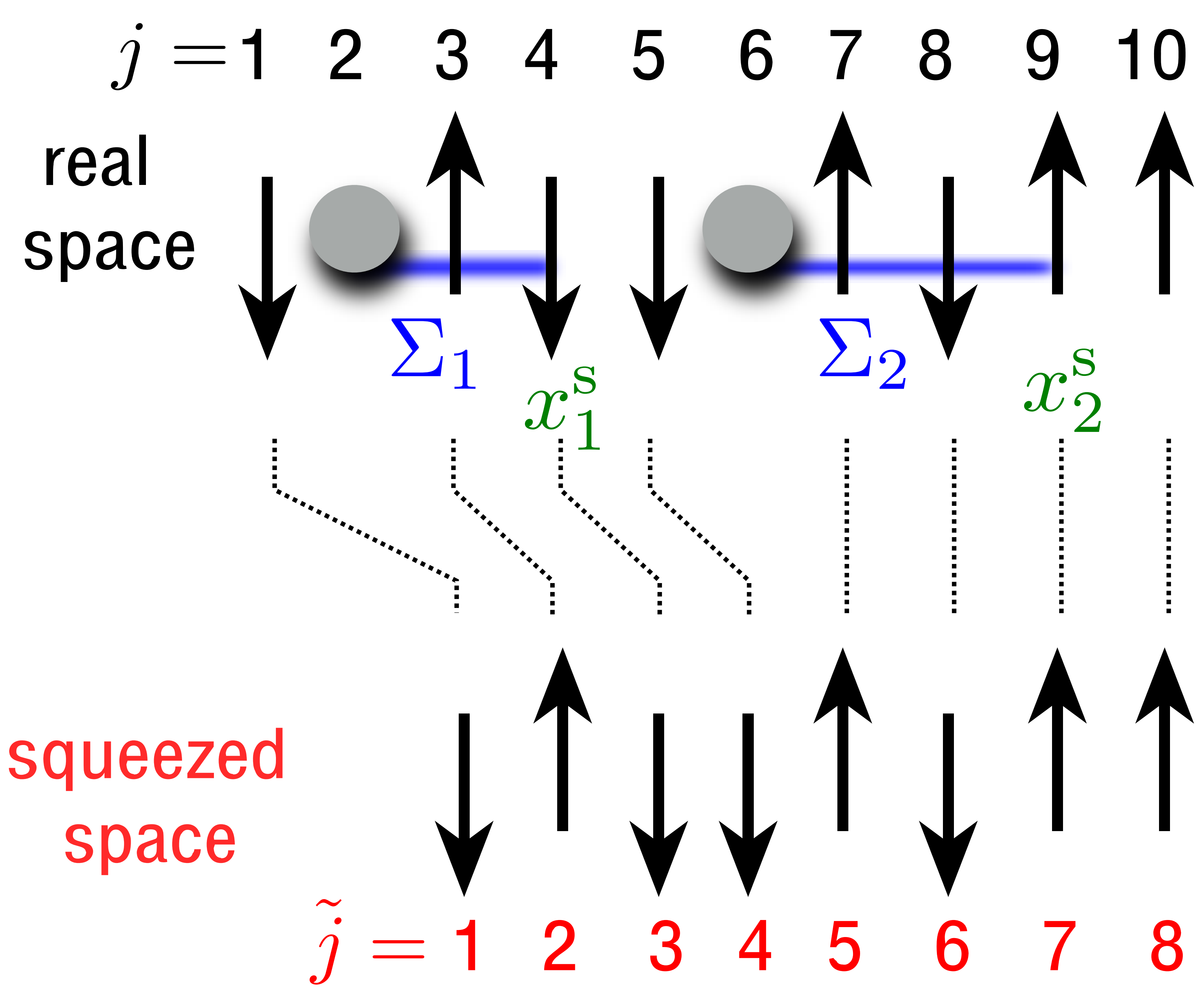
- Ground state wavefunction: $t \gg J$

$$|\psi\rangle \approx |\psi_\mathrm{h}\rangle_\mathrm{bonds} \otimes |\psi_\mathrm{s}\rangle_\mathrm{squeezed}$$

Hidden AFM correlations in 1D

Squeezed space

Ogata & Shiba, PRB 41 (1990); Kruis et al., PRB 70 (2004)



* Ground state wavefunction: $t \gg J$

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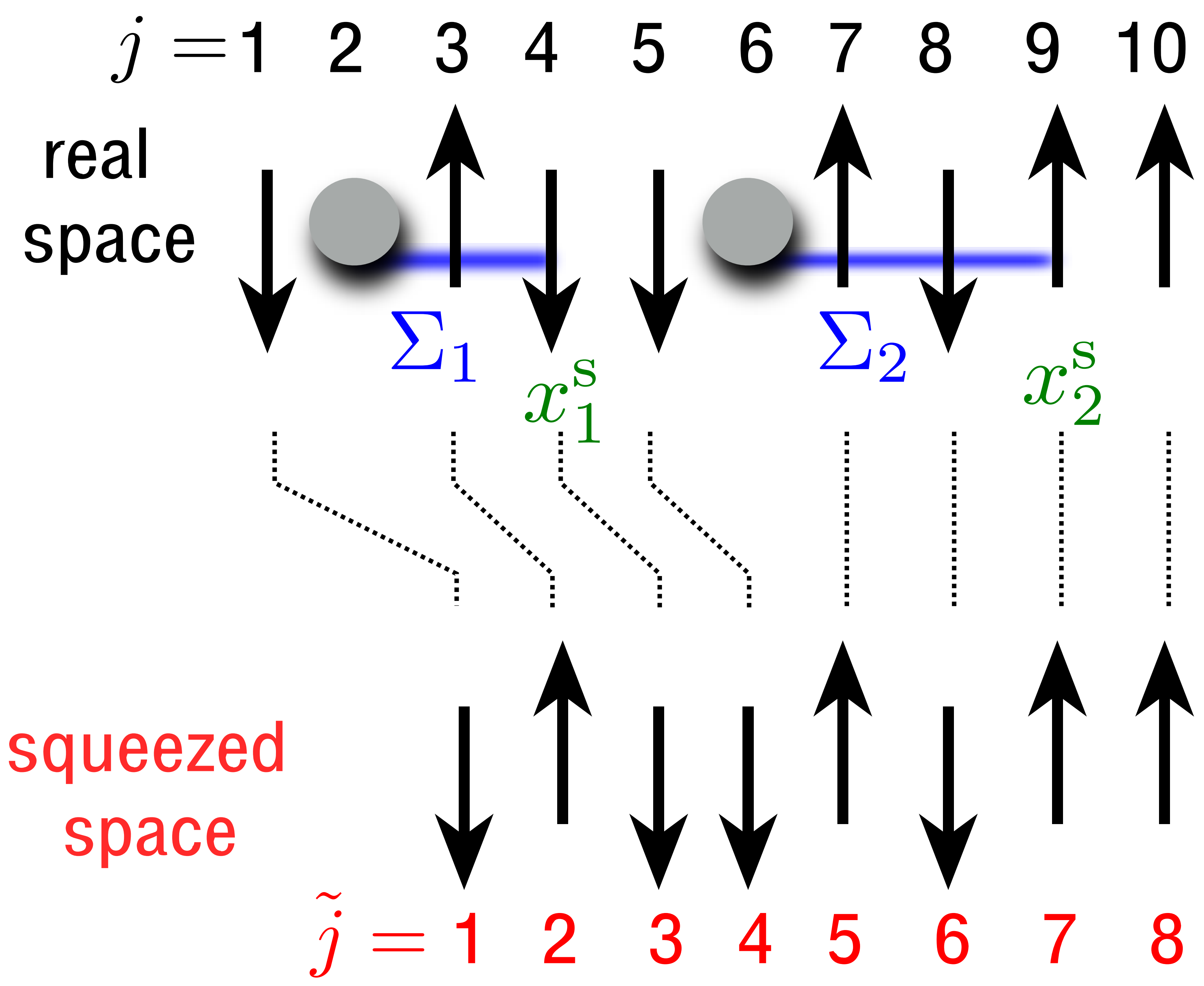
free, spin-less fermions

Hidden AFM correlations in 1D

Squeezed space

Ogata & Shiba, PRB 41 (1990); Kruis et al., PRB 70 (2004)



* Ground state wavefunction: $t \gg J$

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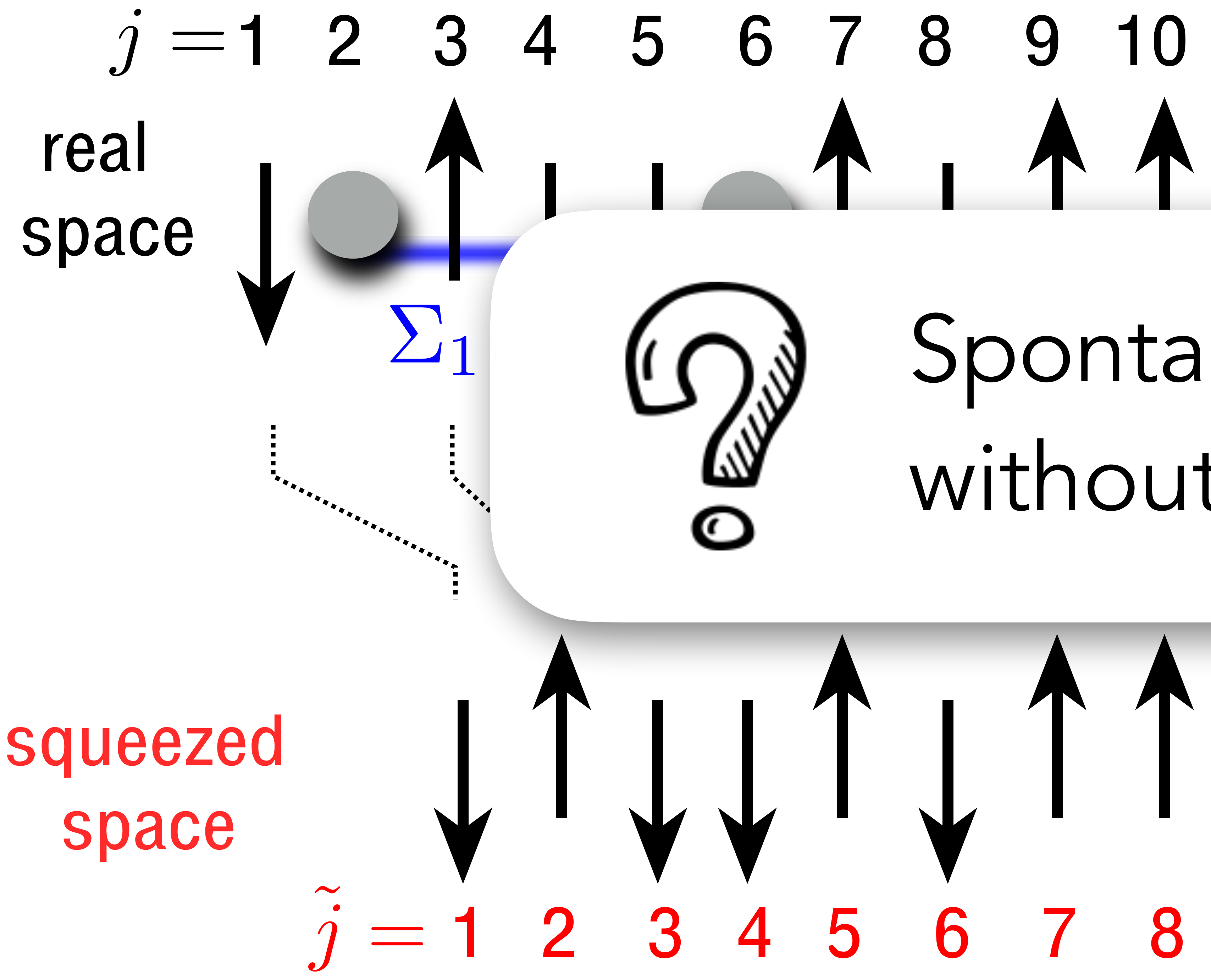
free, spin-less fermions

quantum Heisenberg AFM

Hidden AFM correlations in 1D

Squeezed space

Ogata & Shiba, PRB 41 (1990); Kruis et al., PRB 70 (2004)



* Ground state wavefunction: $t \gg J$

squeezed

fermions

quantum Heisenberg AFM

Spontaneous symmetry-breaking without long-range order?!

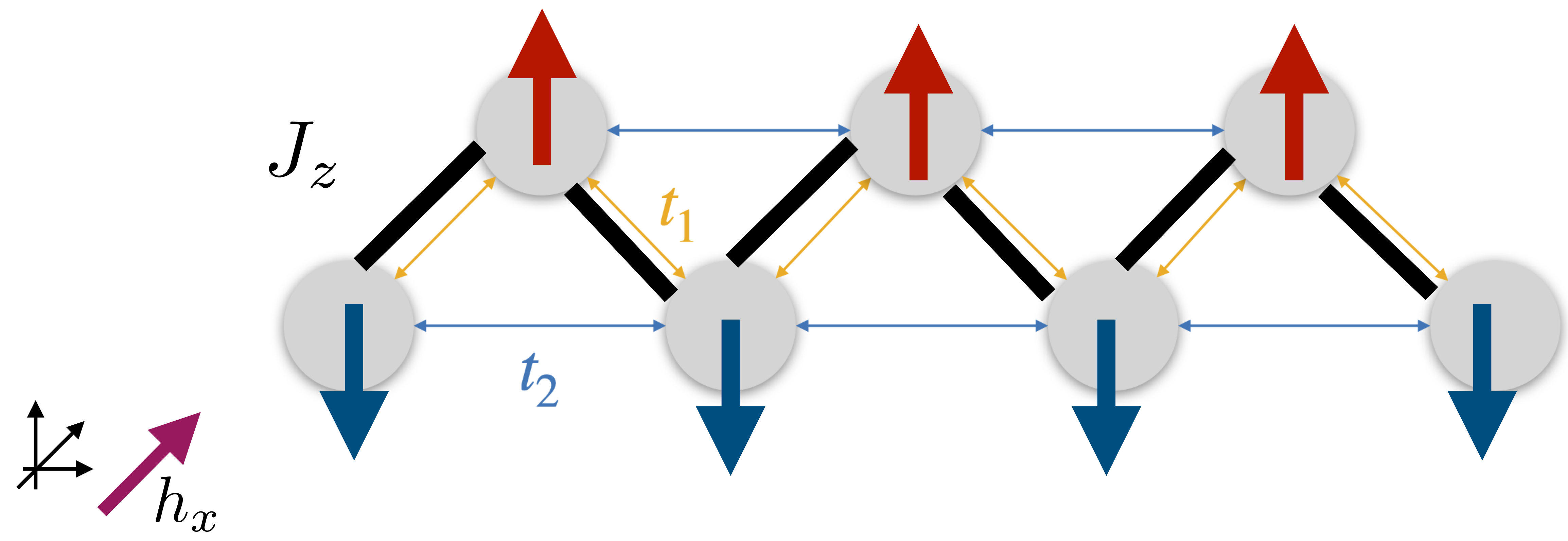
Hidden order

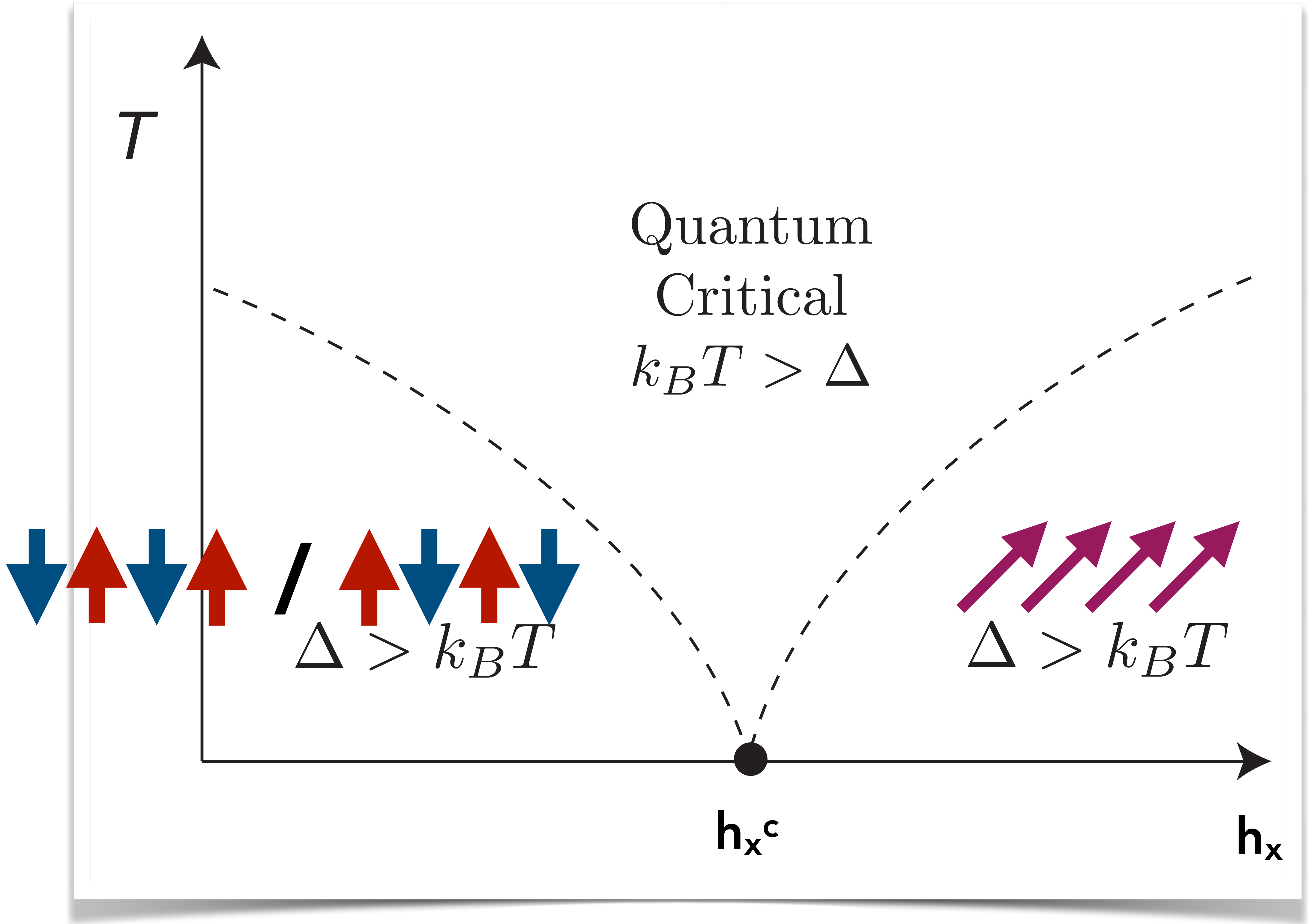
Doped transverse-field Ising (t-TFI) model

see also: *Thorngren et al., PRB 104 (2021)*

Wilke et al., in prep.

$$\hat{\mathcal{H}} = J_z \sum_{\langle i,j \rangle} \hat{S}_i^z \hat{S}_j^z + h_x \sum_j \hat{S}_j^x$$





Sachdev, "Quantum phase transitions"

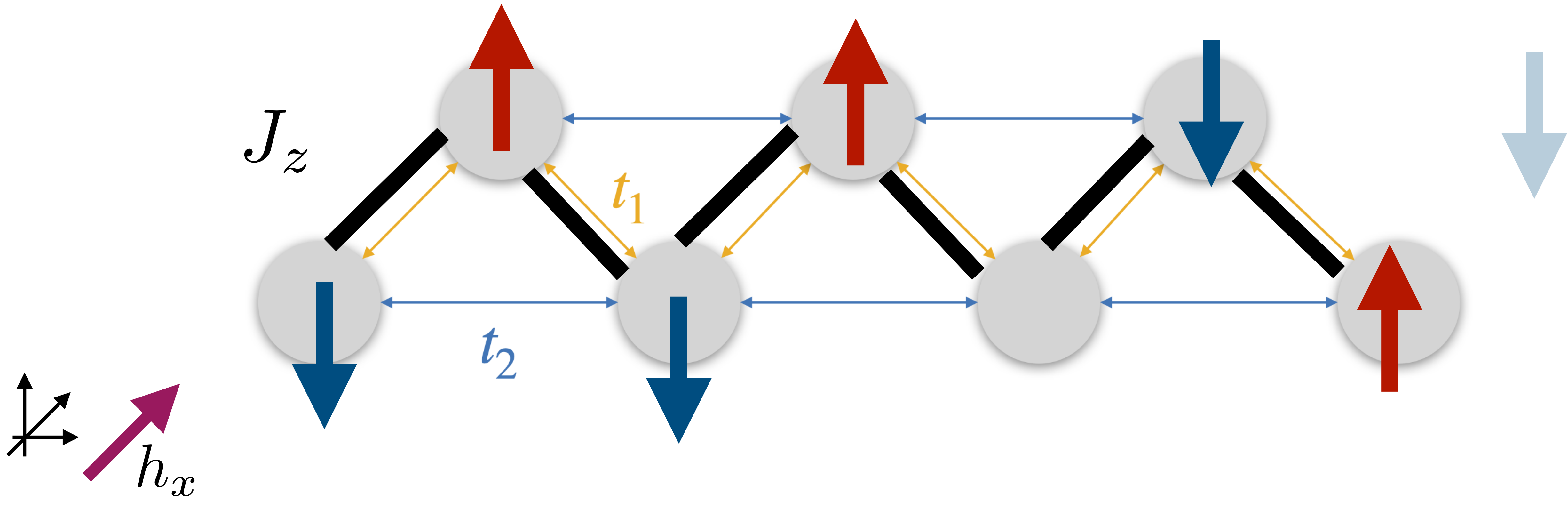
Hidden order

Doped transverse-field Ising (t-TFI) model

see also: Thorngren et al., PRB 104 (2021)

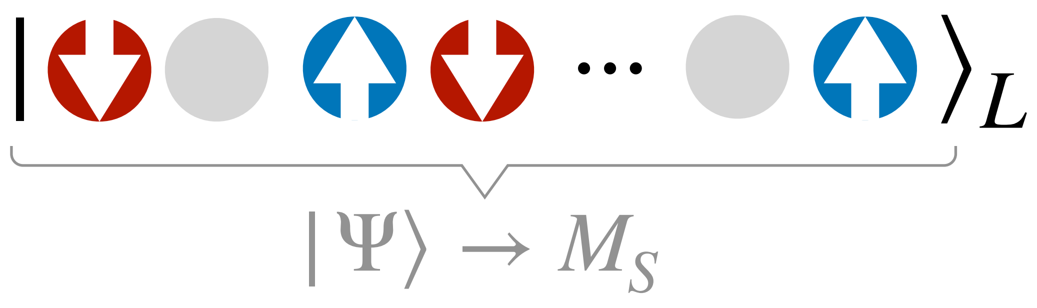
Wilke et al., in prep.

$$\hat{\mathcal{H}} = J_z \sum_{\langle i,j \rangle} \hat{S}_i^z \hat{S}_j^z + h_x \sum_j \hat{S}_j^x - t_1 \sum_{\langle i,j \rangle, \sigma} \hat{\mathcal{P}} \left(\hat{c}_{j,\sigma}^\dagger \hat{c}_{i,\sigma} + \text{h.c.} \right) \hat{\mathcal{P}}$$

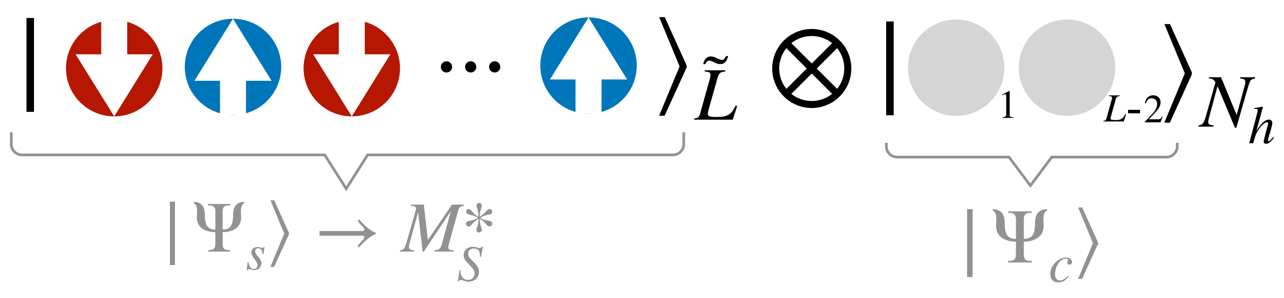


- Hidden AFM order:

Real Space



Squeezed Space



Hidden order

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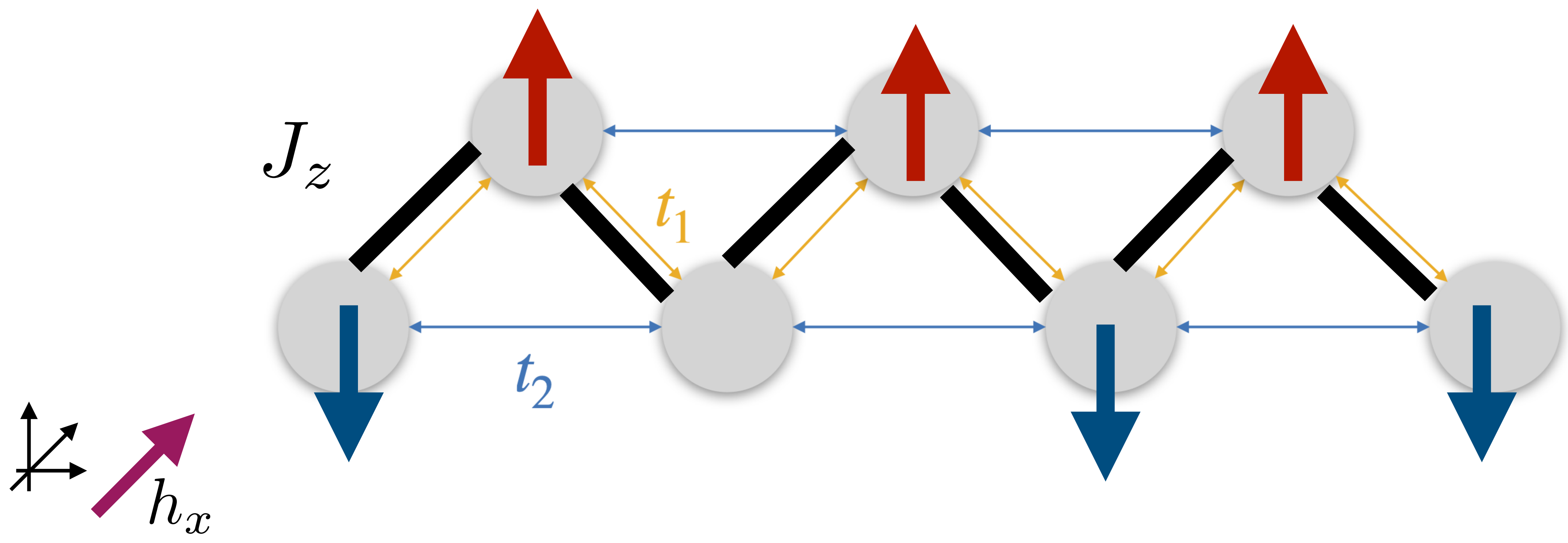
$$- t_2 \sum_{\langle\langle i,j \rangle\rangle, \sigma} \hat{\mathcal{P}} \left(\hat{c}^\dagger_{j,\sigma} \hat{c}_{i,\sigma} + \mathrm{h.c.} \right) \hat{\mathcal{P}}$$



Hidden order

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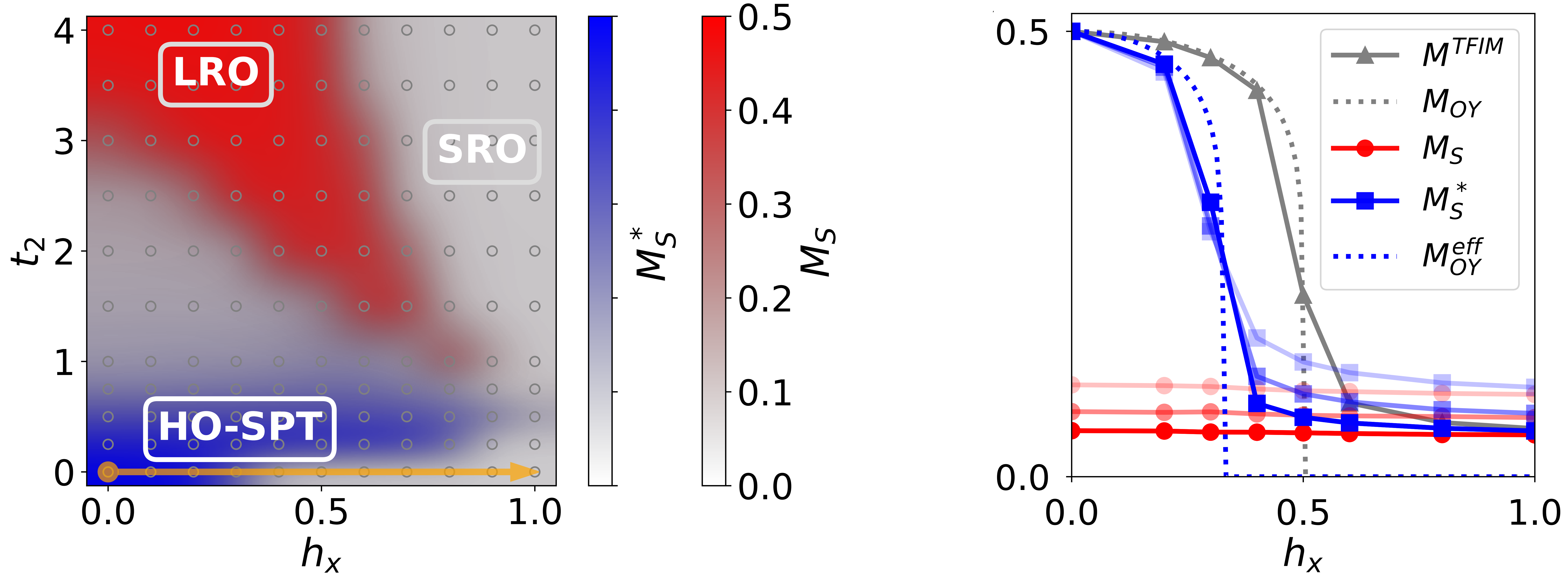

Hidden order

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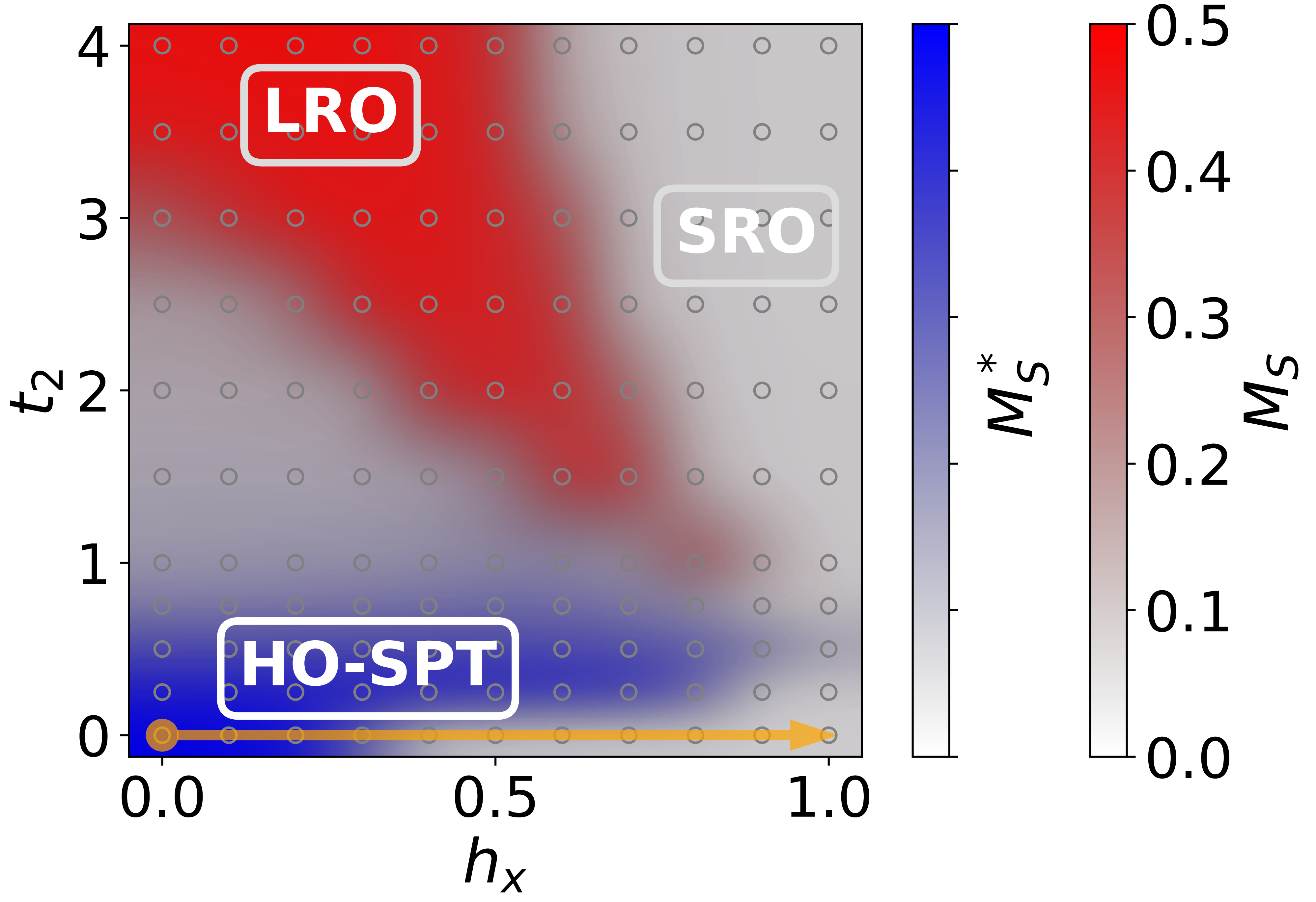

HO-SPT Phase:

- Power-law instead of long-range correlations!
- **Symmetry-breaking without long-range order!**
- QCP without local order parameter!

Hidden order

Doped transverse-field Ising (TFI) model

see also: Thorngren et al., PRB 104 (2021)

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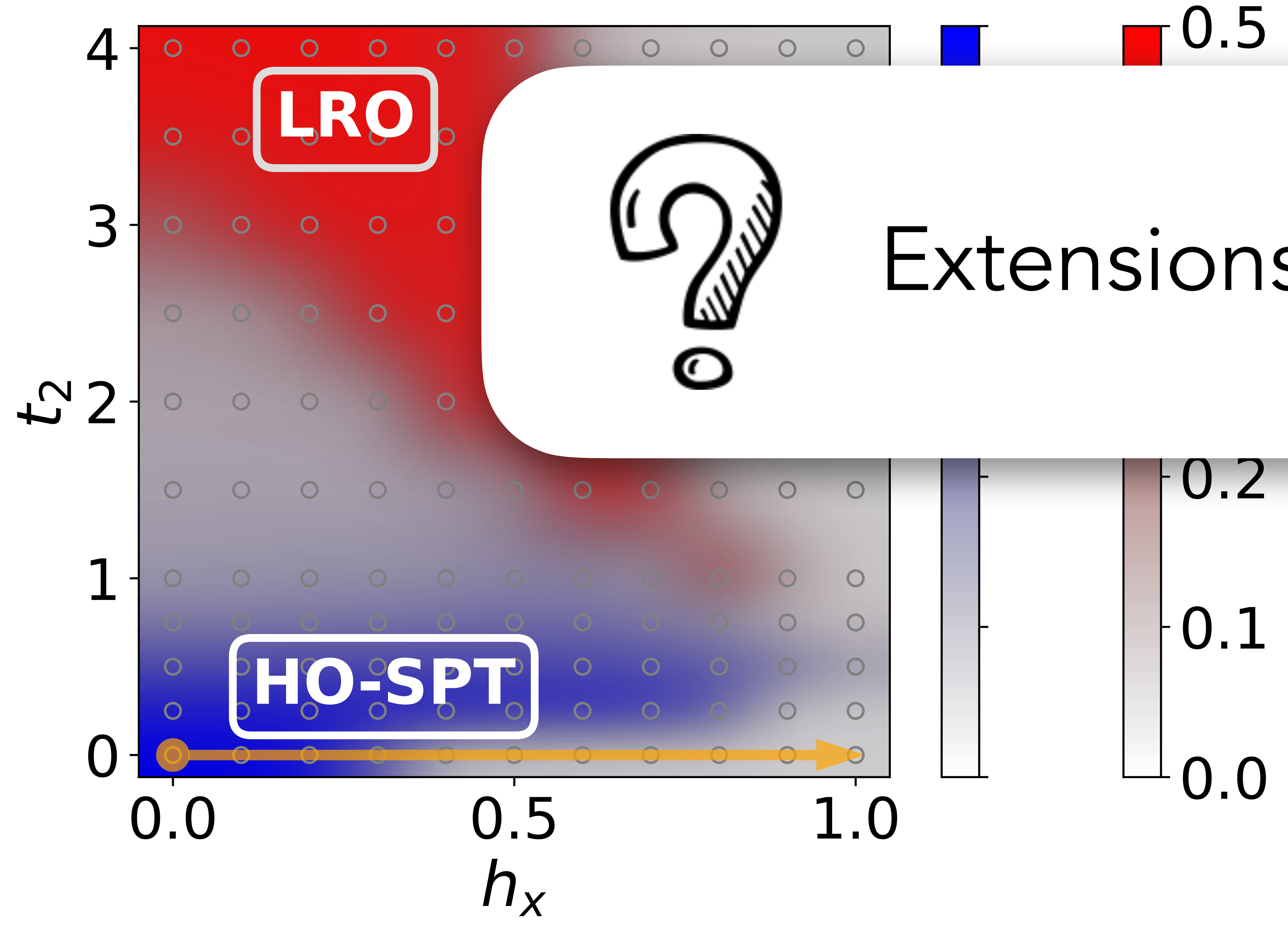
$$\hat{\mathcal{H}} = J_z \sum_{\langle i,j \rangle} \hat{S}_i^z \hat{S}_j^z + h_x \sum_j \hat{S}_j^x - t_1 \sum_{\langle i,j \rangle,\sigma} \hat{\mathcal{P}} \left(\hat{c}_{j,\sigma}^\dagger \hat{c}_{i,\sigma} + \text{h.c.} \right) \hat{\mathcal{P}}$$



HO-SPT Phase:

…of

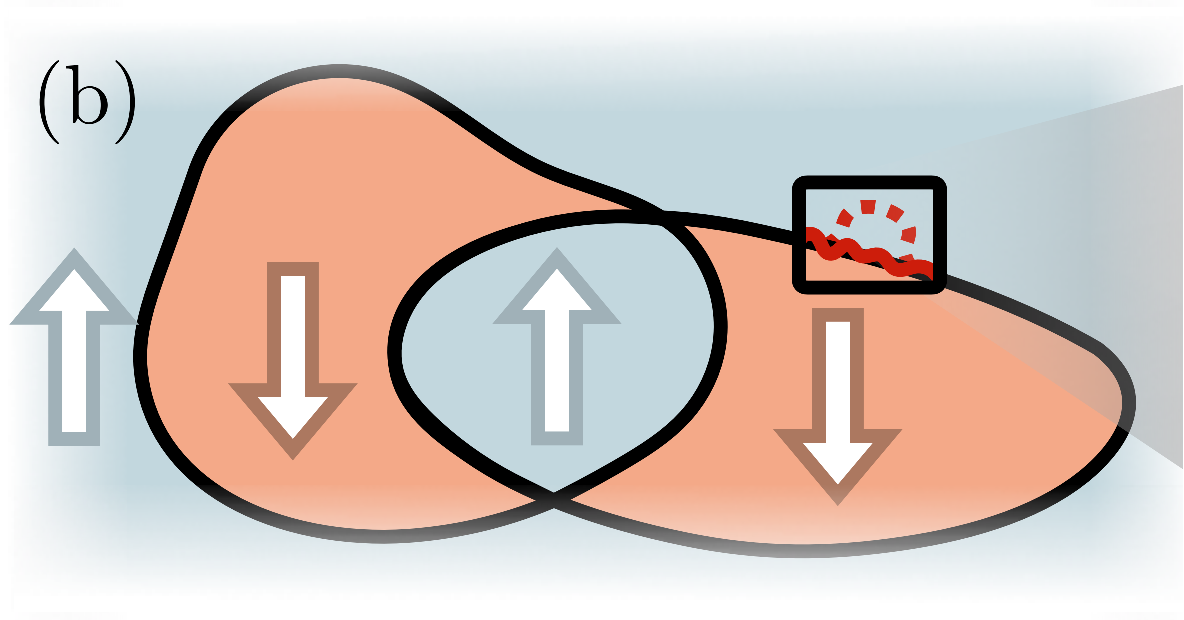
…ions!

…ng without long-range order!

* QCP without local order parameter!

Extensions to higher **D>1**?

PART 3.II: Hidden antiferro-magnetism in 2D & Ising gauge theory



stripes > link variables $\hat{\tau}^z_{\langle \mathbf{i},\mathbf{j} \rangle}$

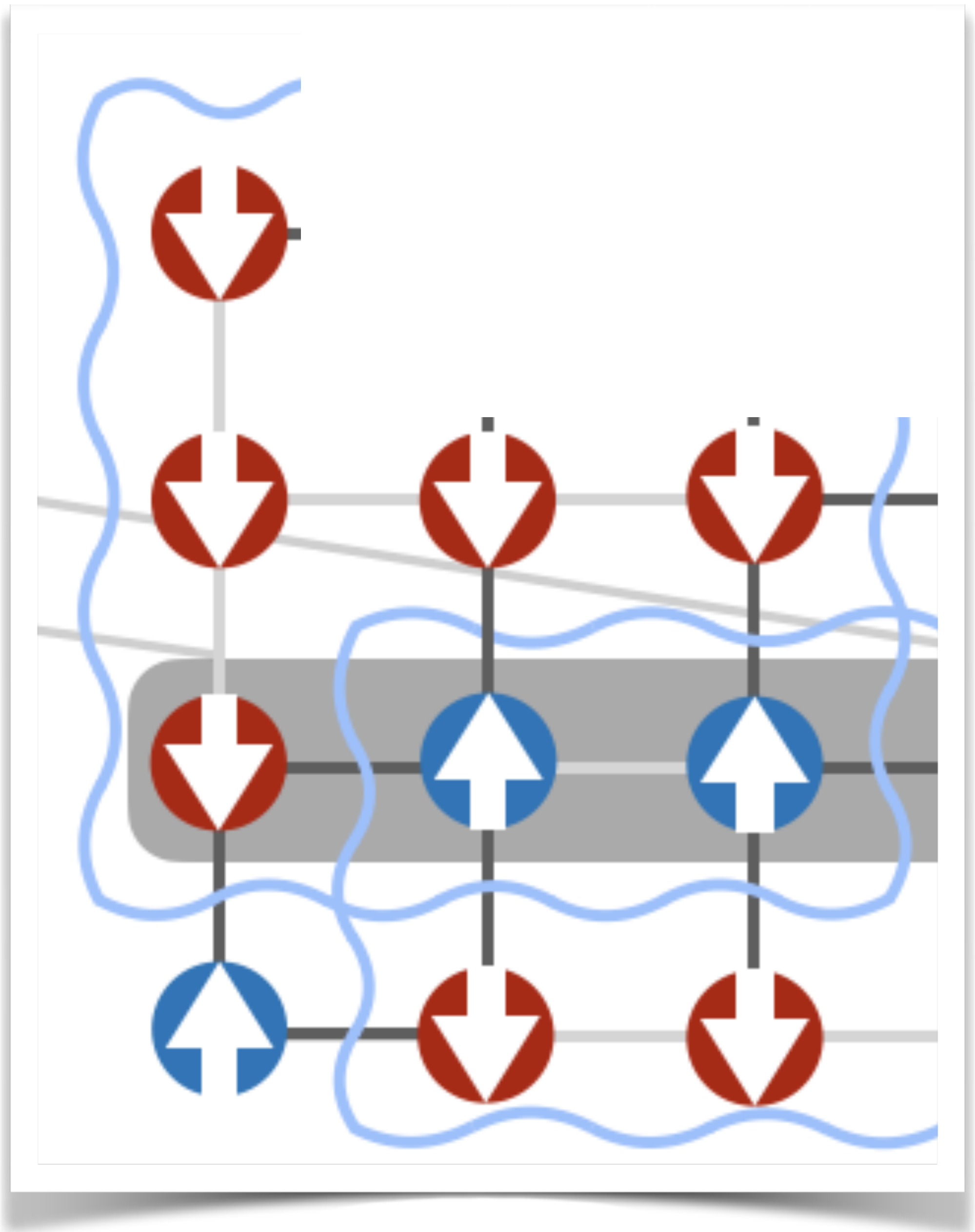
Néel order > Ising spins $\hat{S}^z_{\mathbf{j}}$

Hidden order

The 2D Hidden Ising order (HIO) model

Wilke et al., arXiv:2506.03146

$$\hat{H} = -J_S \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \hat{S}^z_{\mathbf{i}} \hat{S}^z_{\mathbf{j}} \hat{\tau}^z_{\langle \mathbf{i},\mathbf{j} \rangle} + h_S \sum_{\mathbf{j}} \hat{S}^x_{\mathbf{j}}$$



Hidden order

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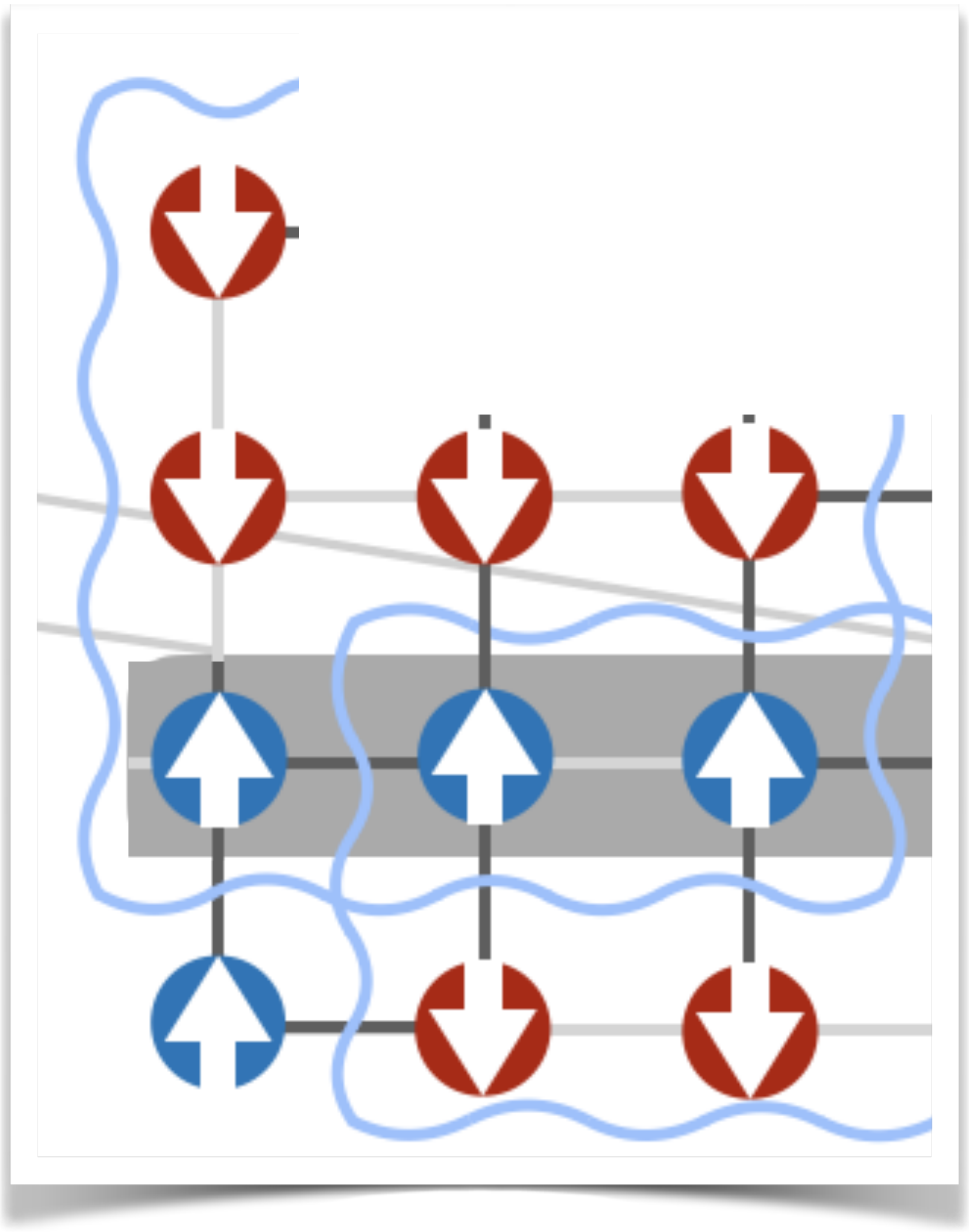

Hidden order

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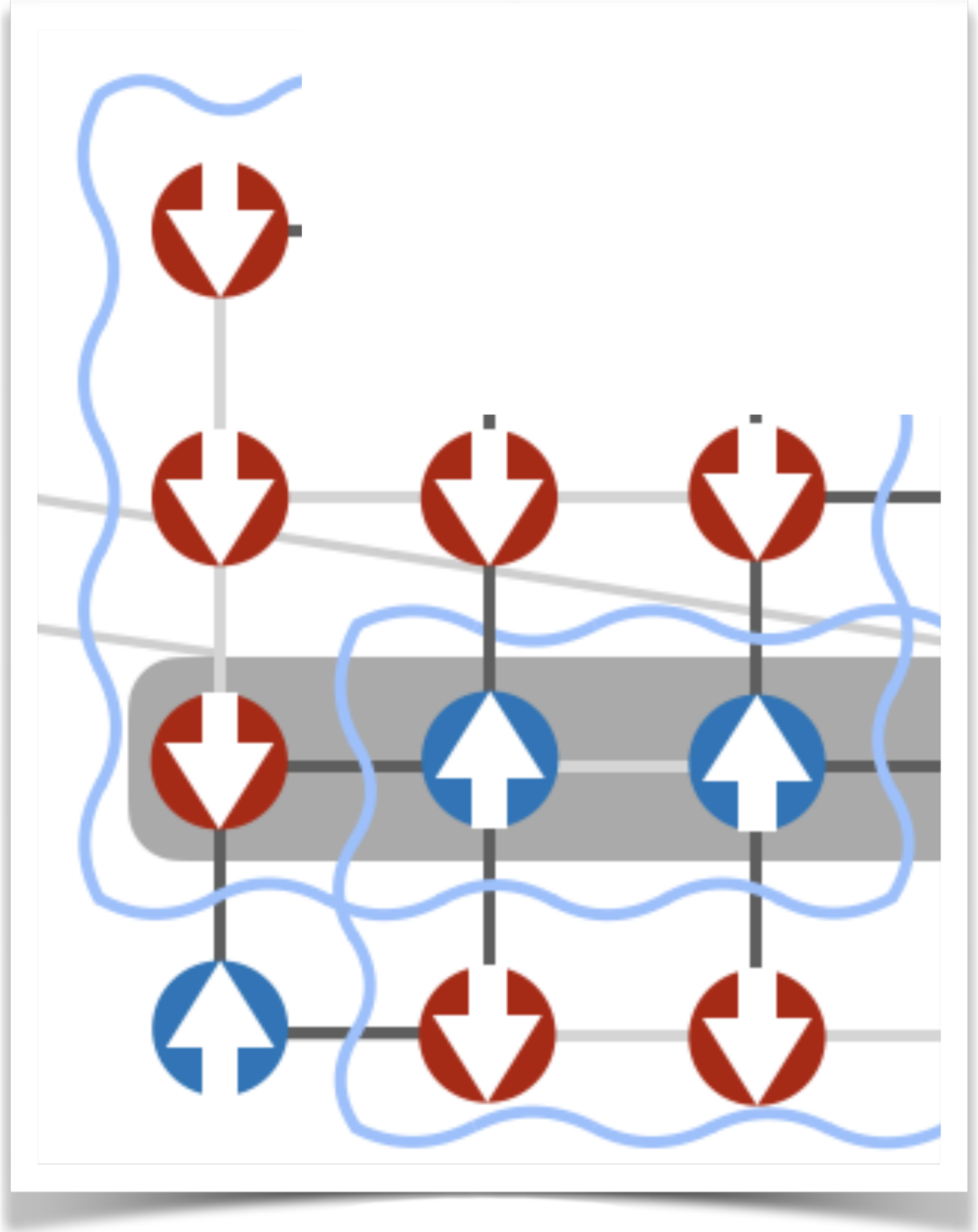
Wilke et al., arXiv:2506.03146

$$\hat{H} = -J_S \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \hat{S}^z_{\mathbf{i}} \hat{S}^z_{\mathbf{j}} \hat{\tau}^z_{\langle \mathbf{i},\mathbf{j} \rangle} + h_S \sum_{\mathbf{j}} \hat{S}^x_{\mathbf{j}}$$



Hidden order

The 2D Hidden Ising order (HIO) model

Wilke et al., arXiv:2506.03146

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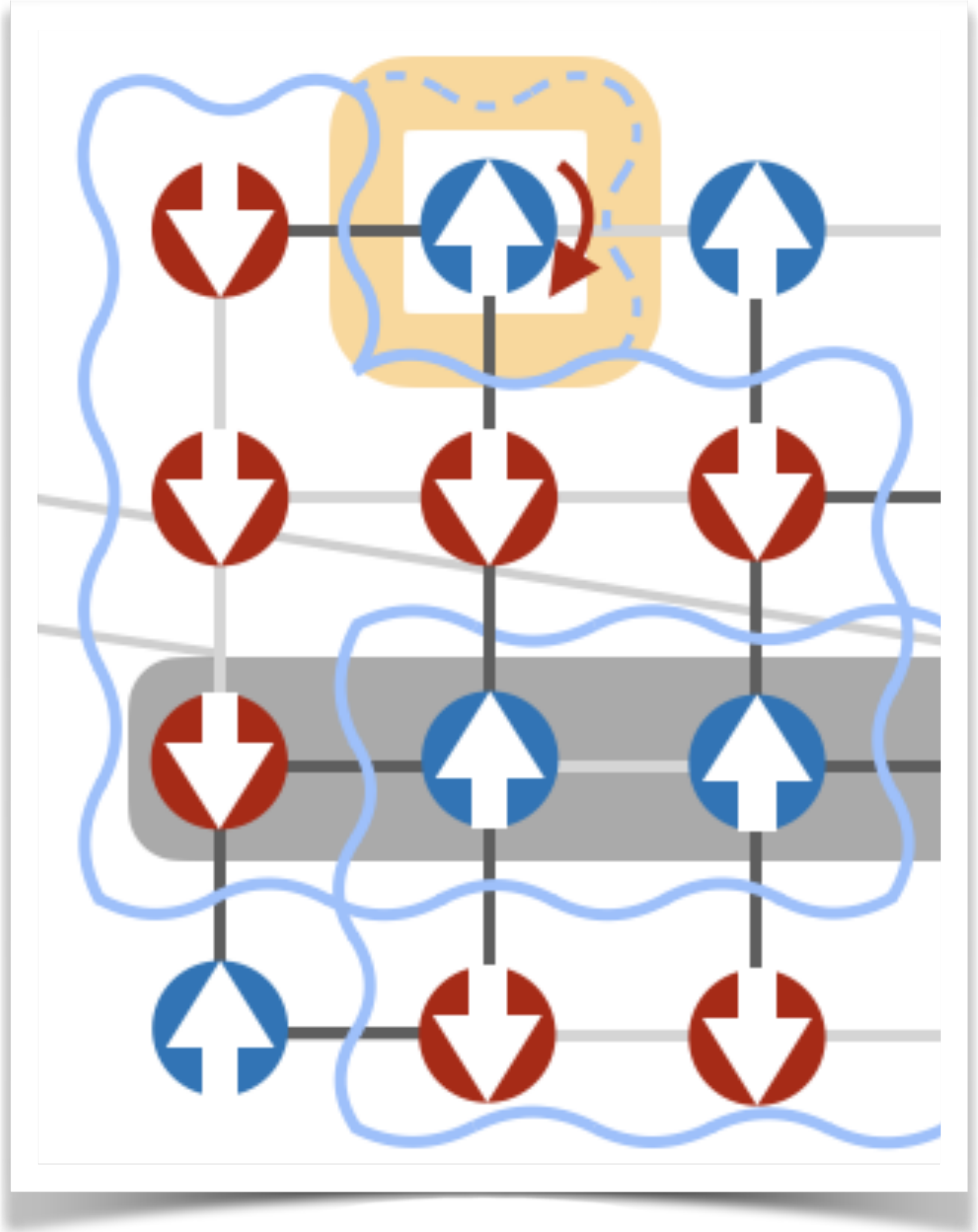

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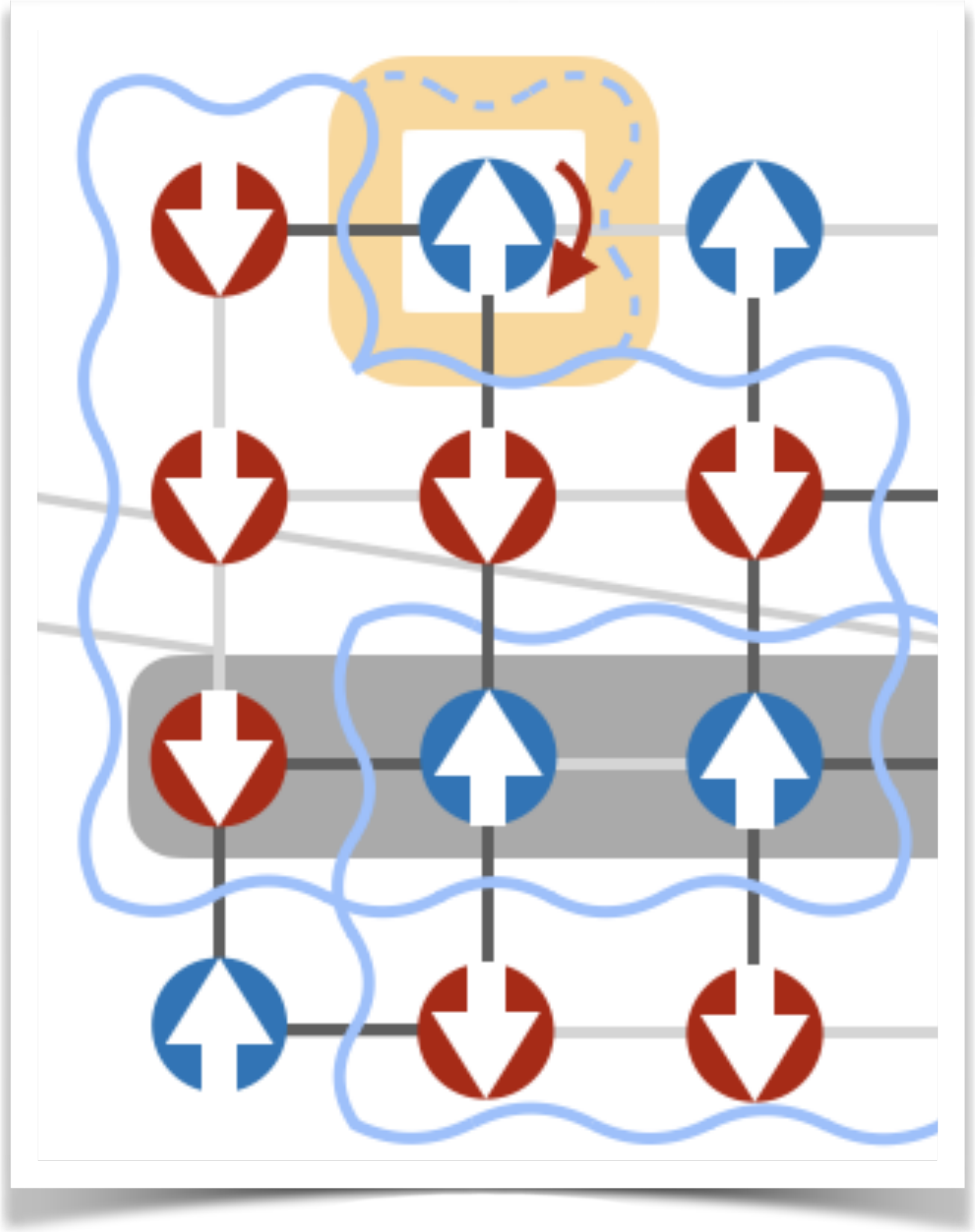

Closed-loop subspace: $\mu_\tau \to \infty$

$$\prod_{l \in \square} \hat{\tau}^z_l \left|\psi\right\rangle \equiv \hat{B}_\square \left|\psi\right\rangle = \left|\psi\right\rangle$$

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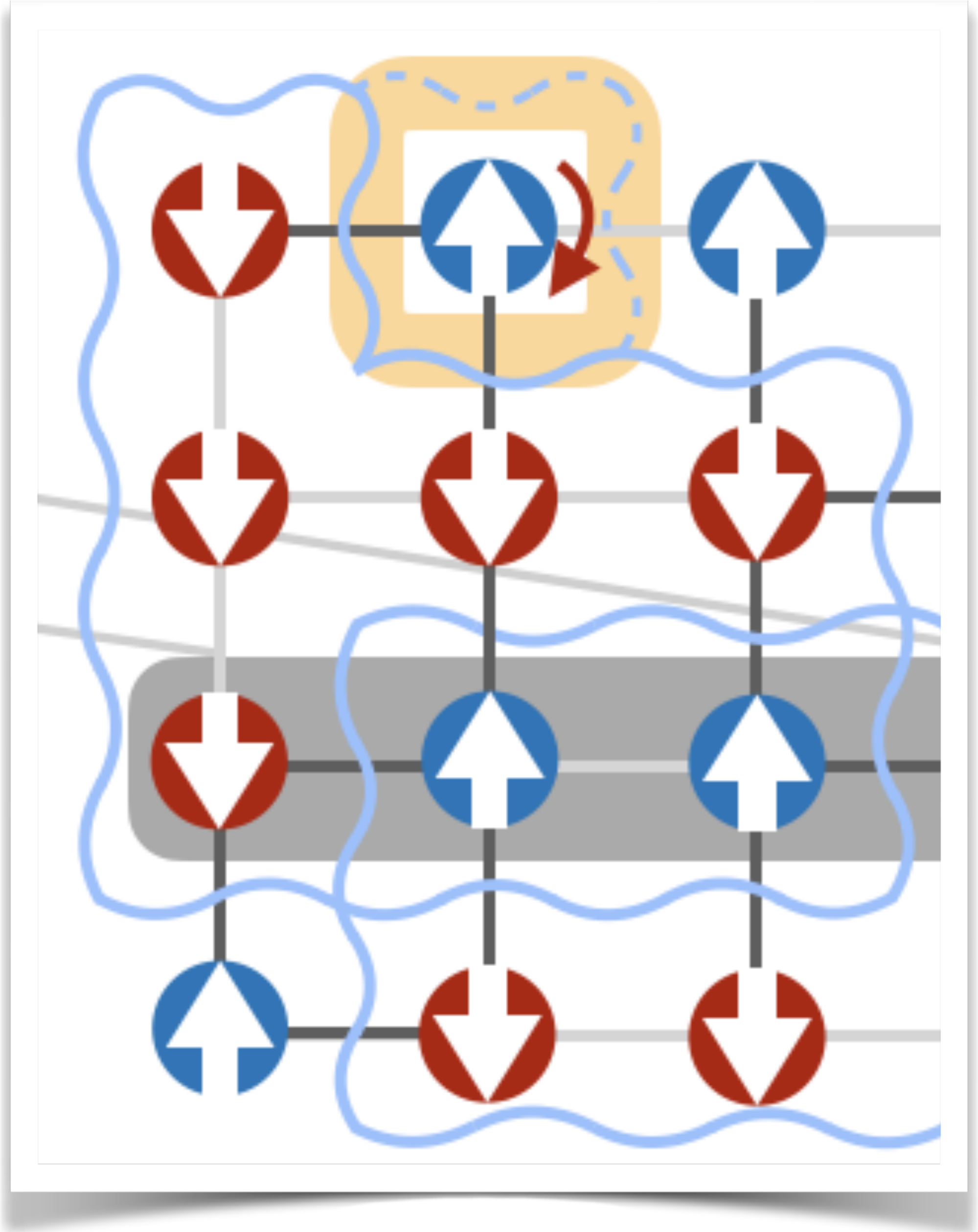
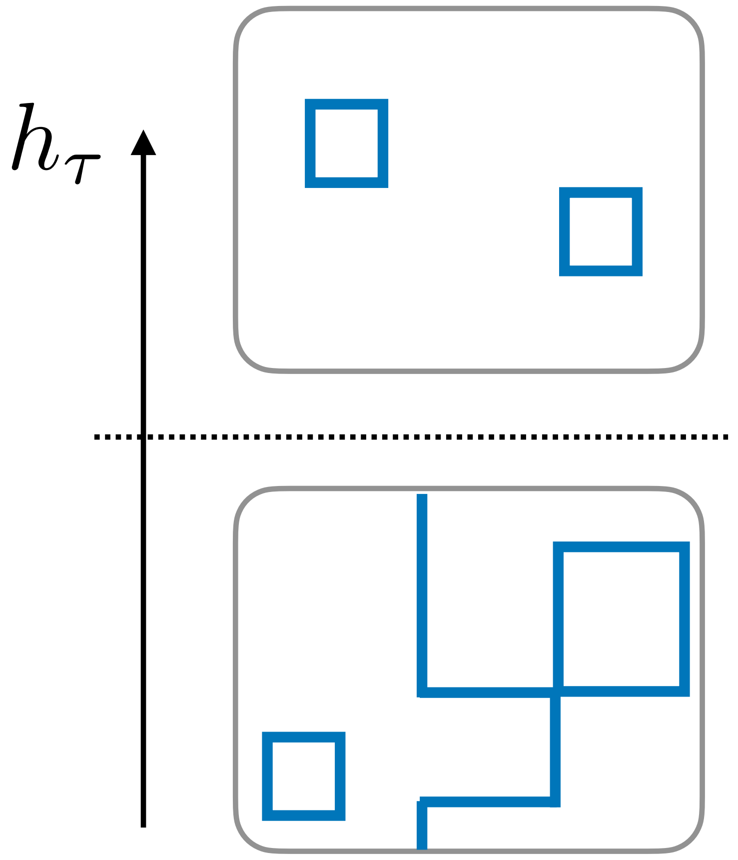


- String tension:



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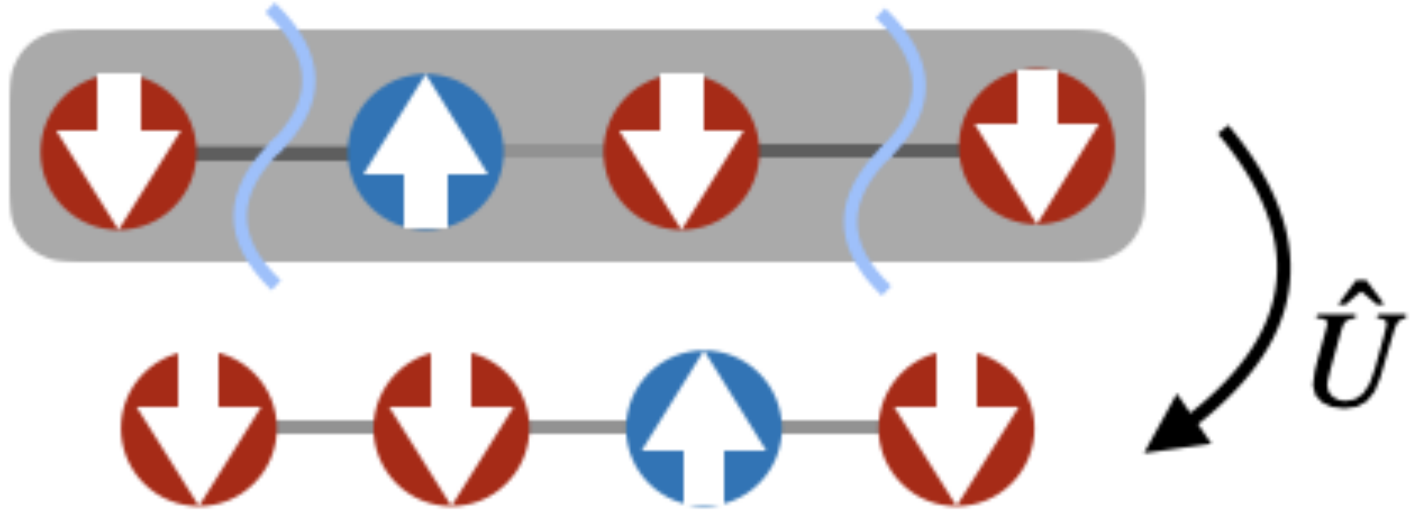
Hidden order

The 2D Hidden Ising order (HIO) model

Wilke et al., arXiv:2506.03146

Exact decoupling in *squeezed space* ($\mu_\tau \to \infty$):



$$\hat{U} = \prod \hat{U}_j = \prod_j (2\hat{S}^x_j)^{\hat{p}_j}$$

$$\hat{p}_j = \frac{1}{2}\left(1 - \prod_{\ell \in \mathcal{L}_j} \hat{\tau}^z_\ell\right)$$

Hidden order

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Wilke et al., arXiv:2506.03146

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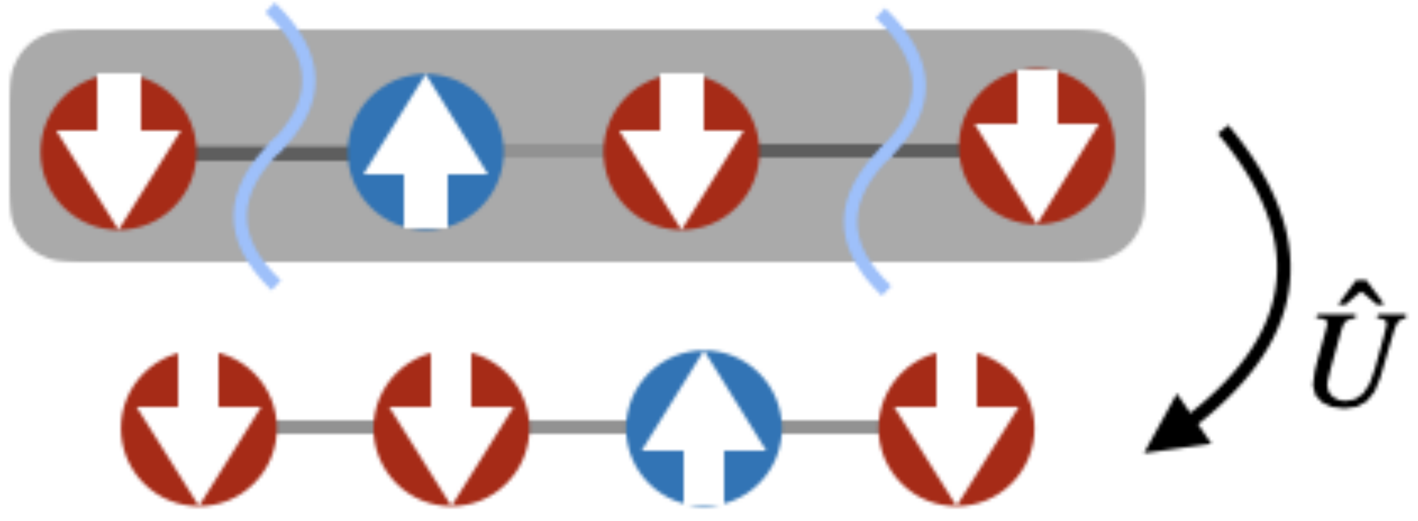
$$+ J_\tau \sum_{\mathbf{j}} \prod_{l \in +_{\mathbf{j}}} \hat{\tau}^x_l - h_\tau \sum_l \hat{\tau}^z_l$$



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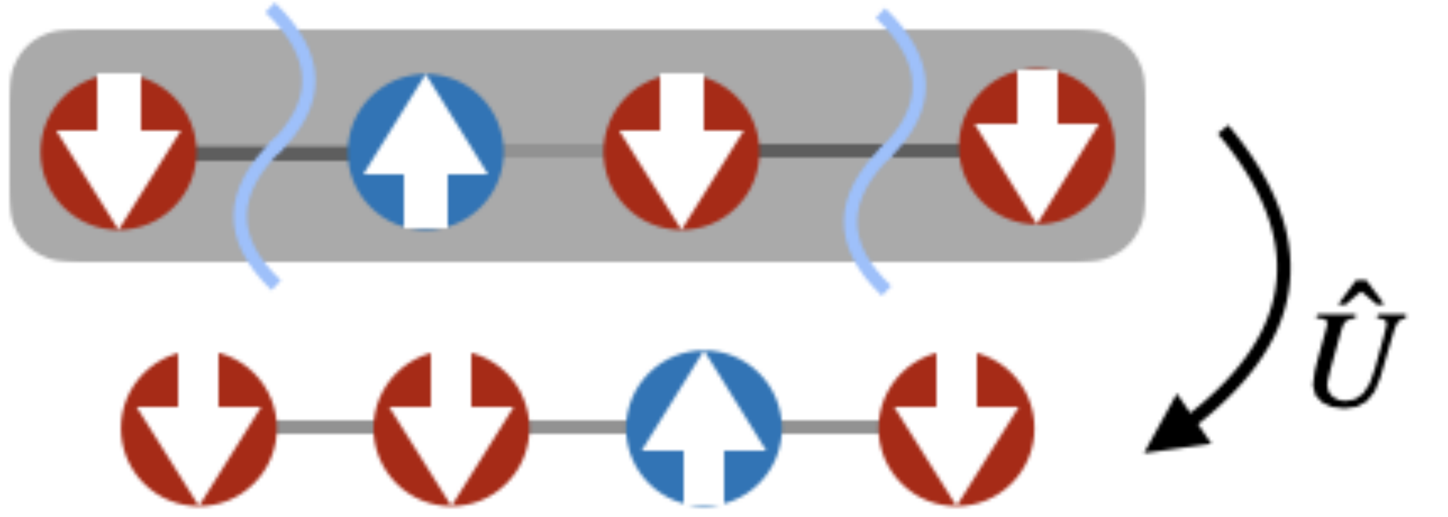
$$\underbrace{+ J_\tau \sum_{\mathbf{j}} \prod_{l \in +_{\mathbf{j}}} \hat\tau^x_l - h_\tau \sum_l \hat\tau^z_l}_{\hat H_{\mathrm{TC-F}}}$$



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Hidden order

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Wilke et al., arXiv:2506.03146

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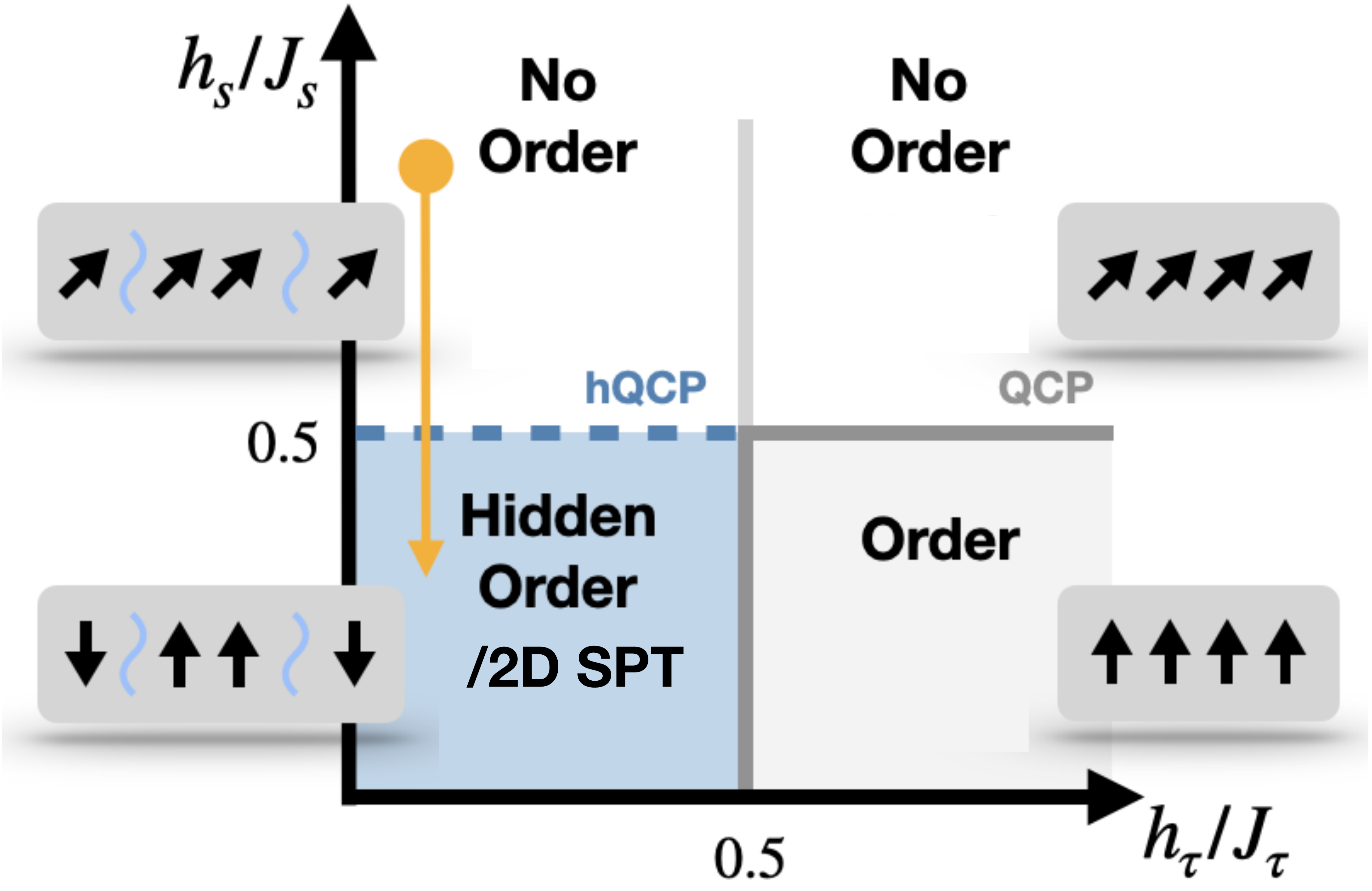
Zero-temperature phase diagram:



NO / O phases:

- GL paradigm

Hidden order

The 2D Hidden Ising order (HIO) model

Wilke et al., arXiv:2506.03146

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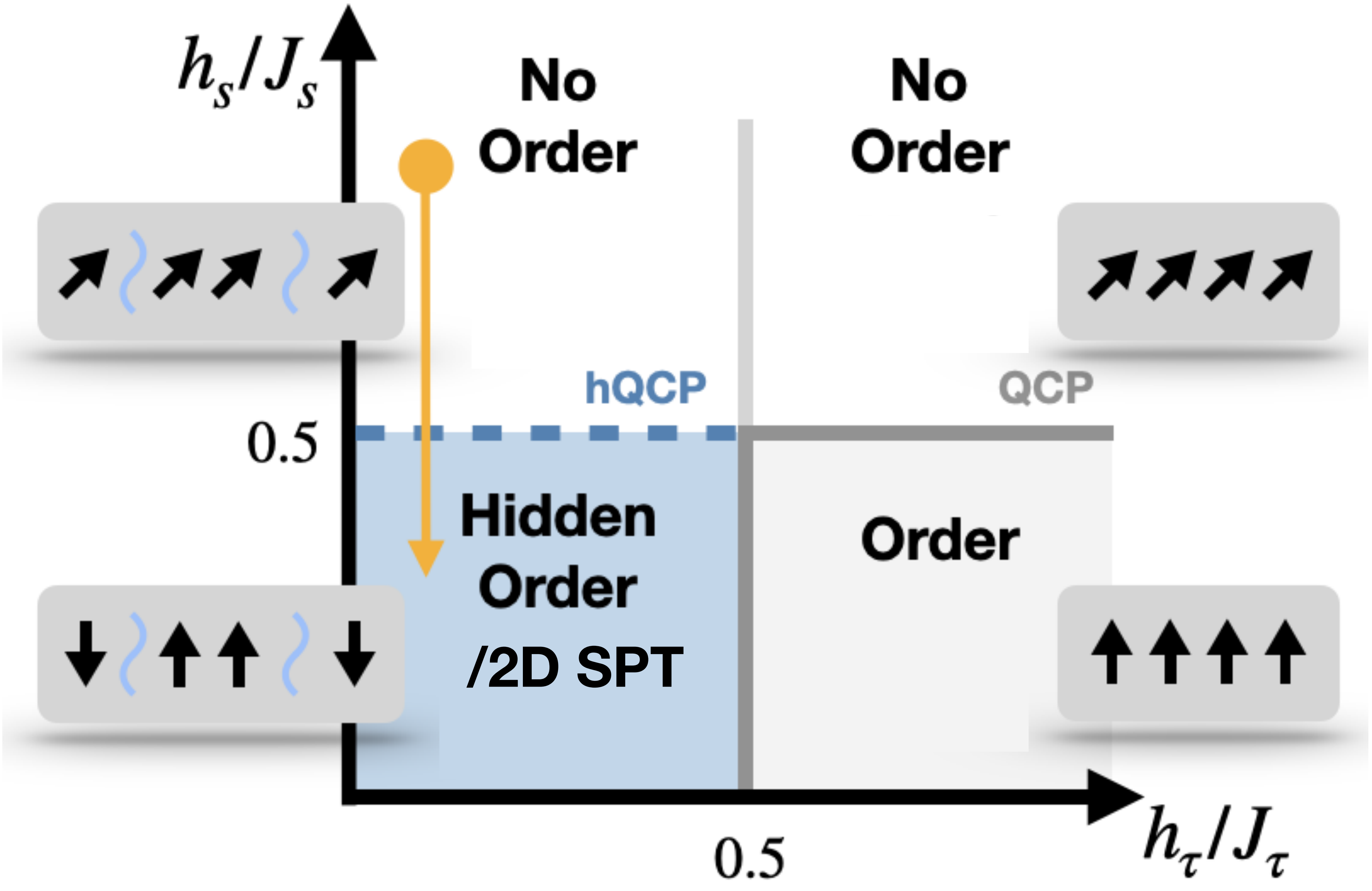

NO / O phases:

- GL paradigm

HO = 2D SPT phase:

- String order

$$C^*(\mathbf{j}) = \left\langle \hat{S}^z_{\mathbf{r}} \left(\prod_{l \in \mathcal{L}_{\mathbf{j}}} \hat{\tau}^z_l \right) \hat{S}^z_{\mathbf{j}} \right\rangle$$

- SSB in squeezed space
- Edge LRO

Hidden order

The 2D Hidden Ising order (HIO) model

Wilke et al., arXiv:2506.03146

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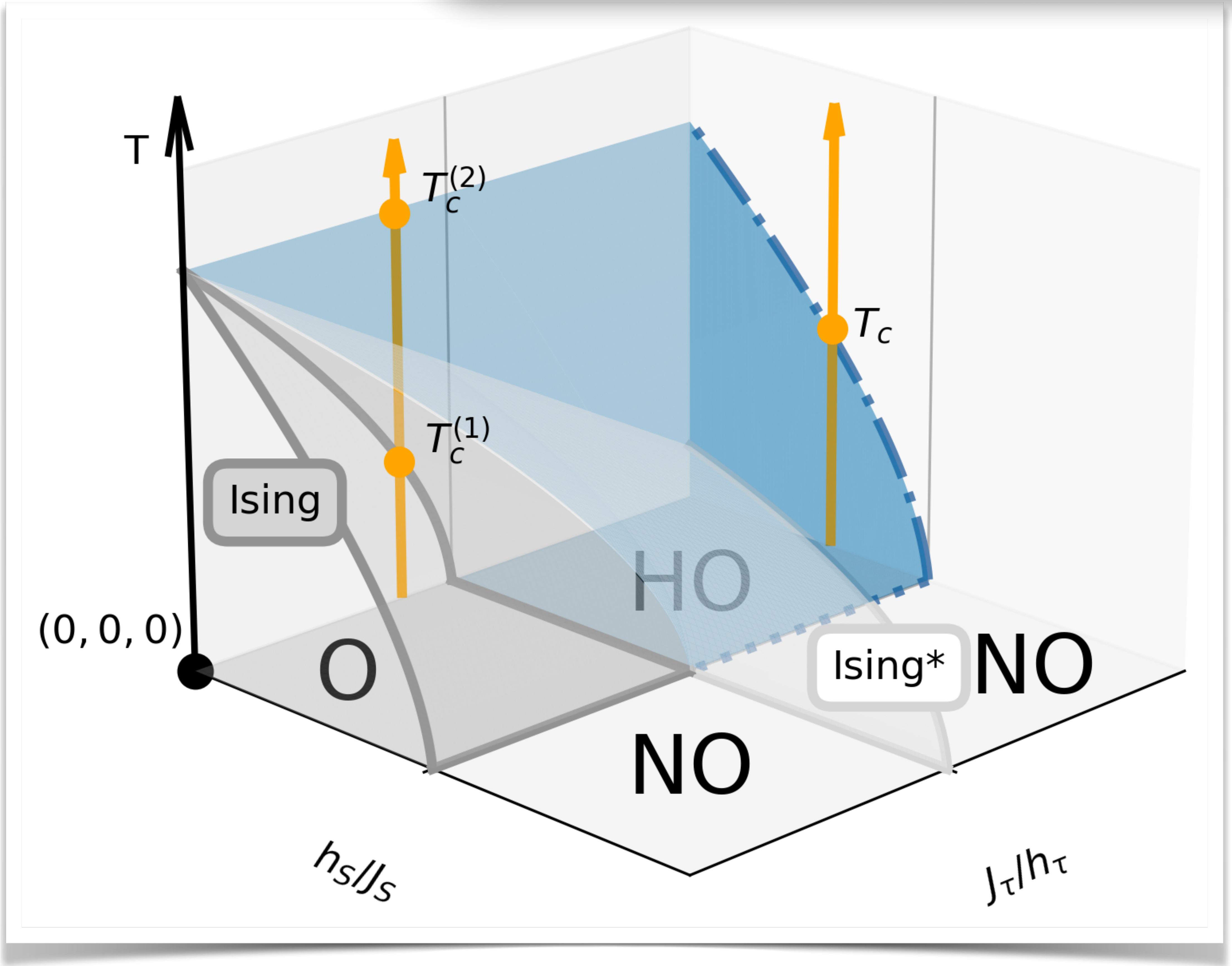
Finite-temperature phase diagram:

- Finite-T SPT transition

$$T_c|_{\mathrm{SPT}} > 0$$



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Wilke et al., arXiv:2506.03146

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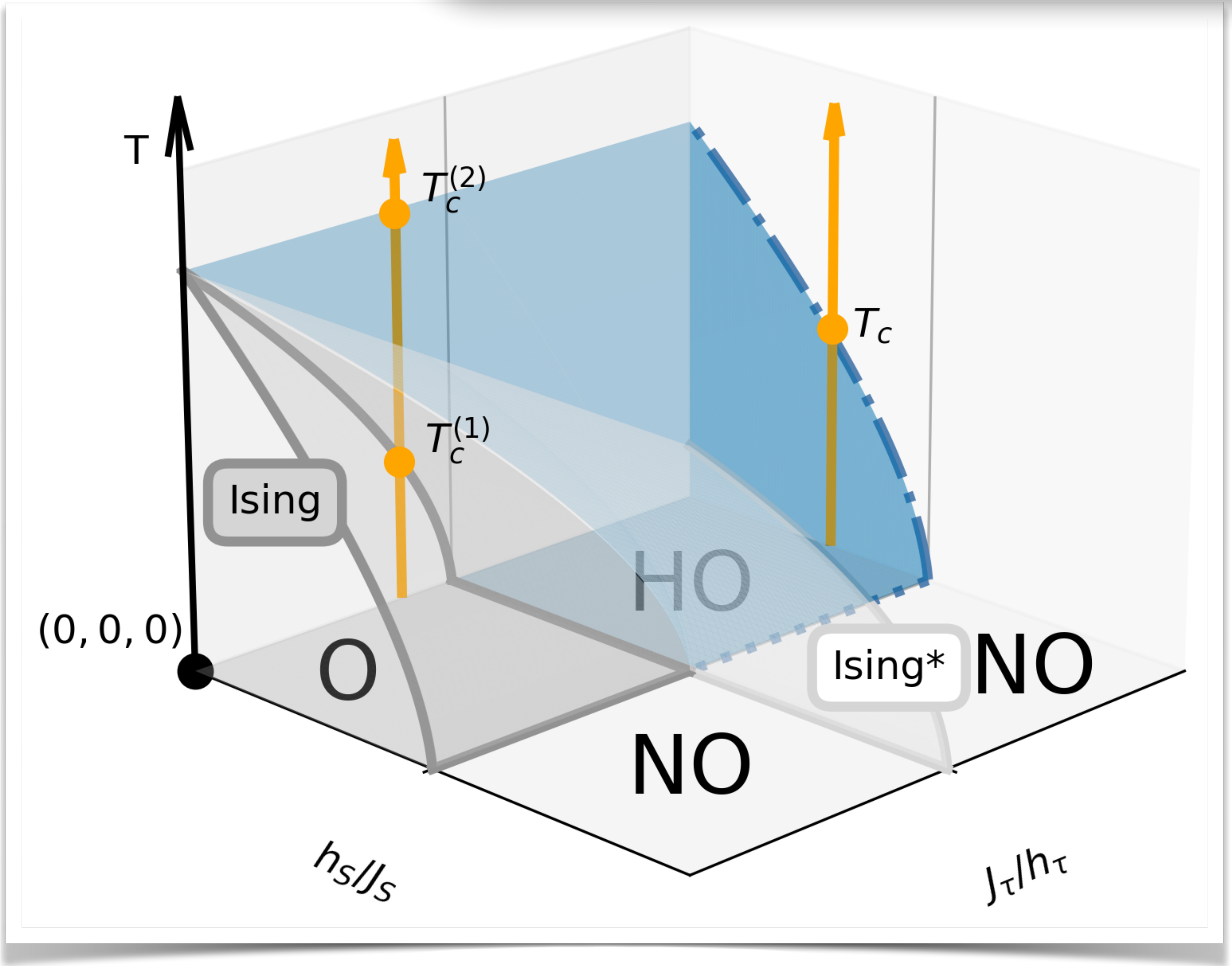
$$0 < T_c^{(1)}|_{\mathrm{Ising}} < T_c^{(2)}|_{\mathrm{SPT}}$$



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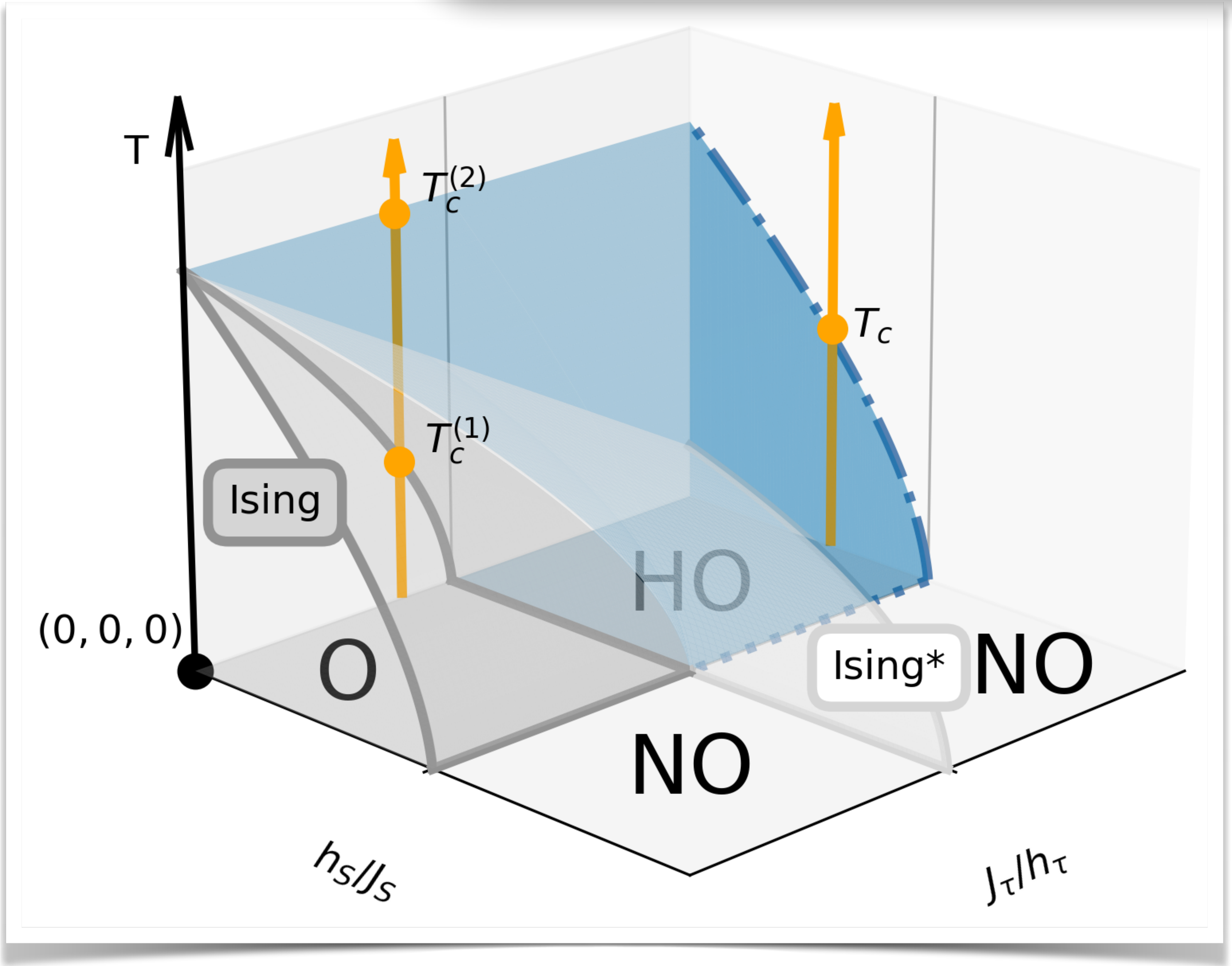
- LRO: 1D edge, T>0

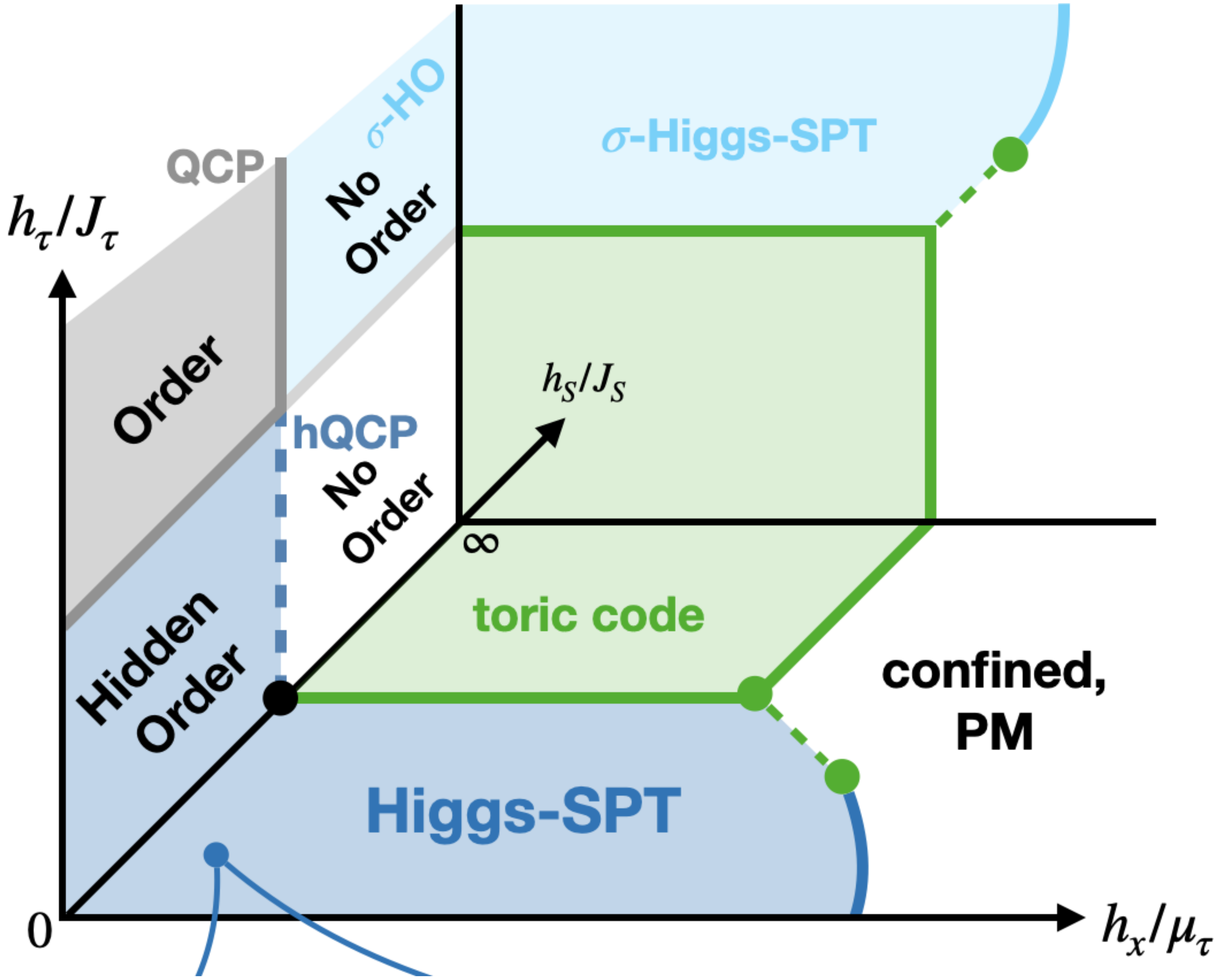


Hidden order

Dual-Higgs Ising-gauge theory

Wilke et al., arXiv:2506.03146

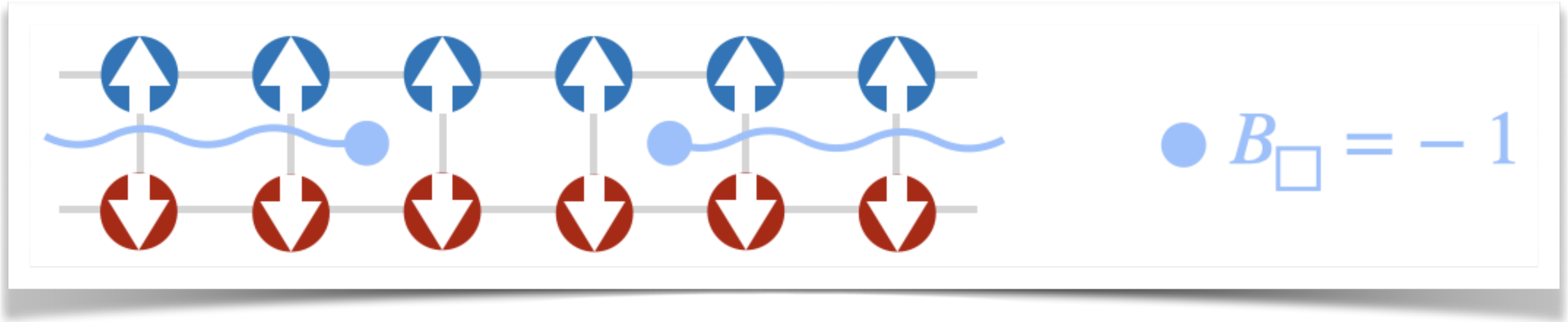
HO = Higgs-SPT   Higgs=SPT: Verresen et al., arXiv:2211.01376



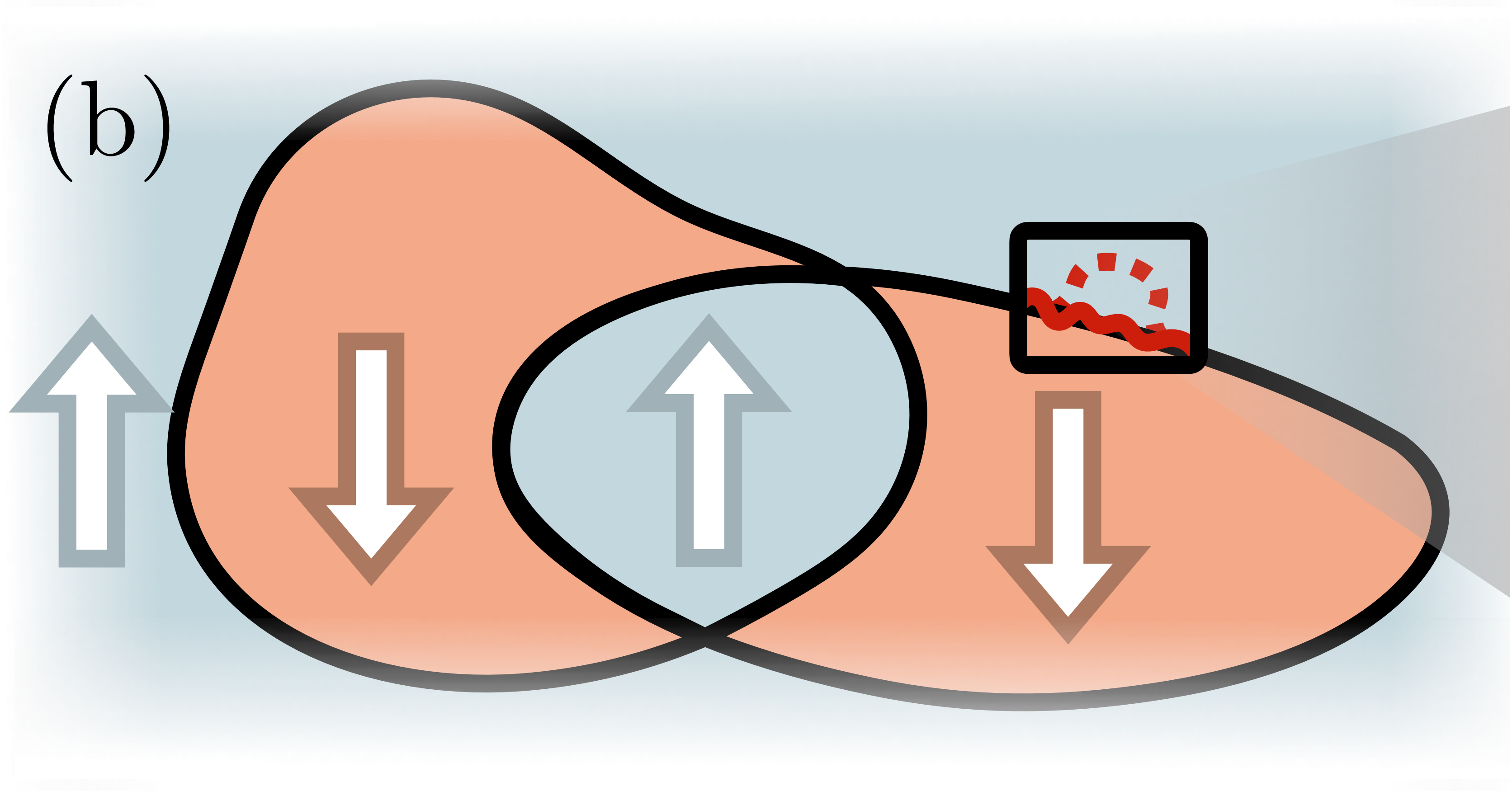
Open strings:

$$\hat{H} \to \hat{H} + h_X \sum_l \hat{\tau}_l^x$$

- Ising domain wall: confining force



PART 3.III: Phenomenology of fluctuating stripes & Pseudogap

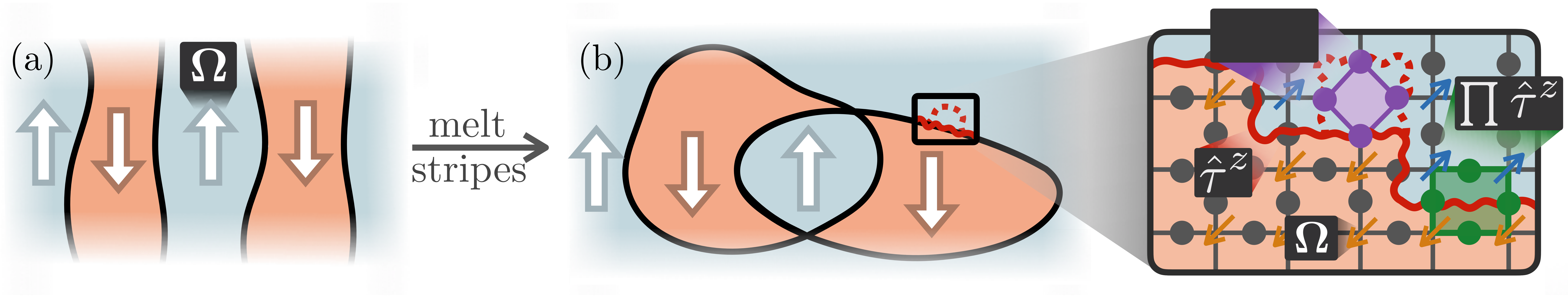


Hidden order

Melting stripes: Hidden AFM order

Schlömer et al., PRX Quantum 6 (2025)

- High-T: AFM*



- Finite-T: classical loop-gas model

$$\hat{\mathcal{H}}_{\mathrm{cl}} = -K_{\square} \sum_{\square} \prod_{\boldsymbol{\ell} \in \square} \hat{\tau}^z_{\boldsymbol{\ell}} - h \sum_{\boldsymbol{\ell}} \hat{\tau}^z_{\boldsymbol{\ell}}$$

Dual to 2D Ising model!

Wegner, J. Math. Phys. 12 (1971)

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Schlömer et al., PRX Quantum 6 (2025)

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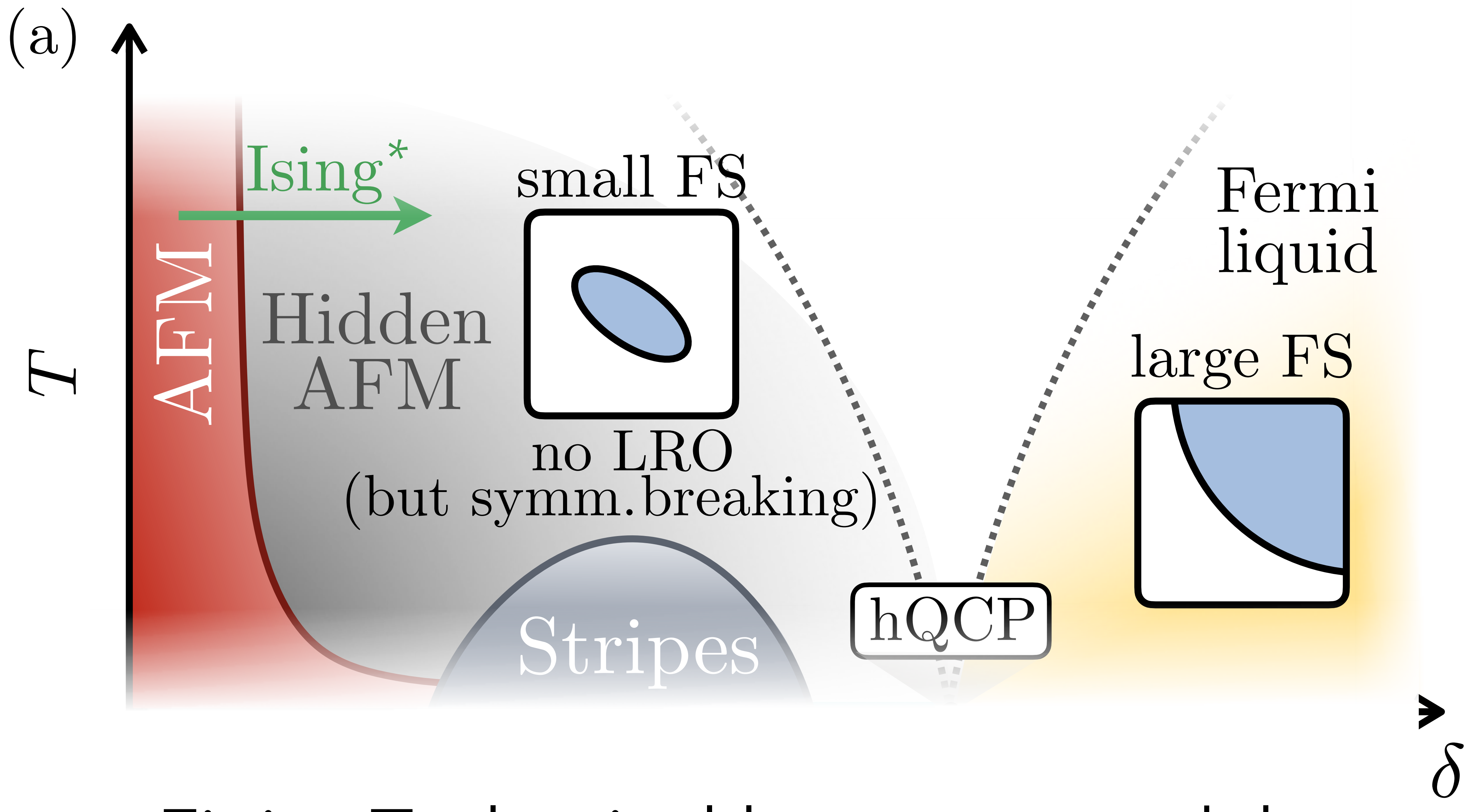

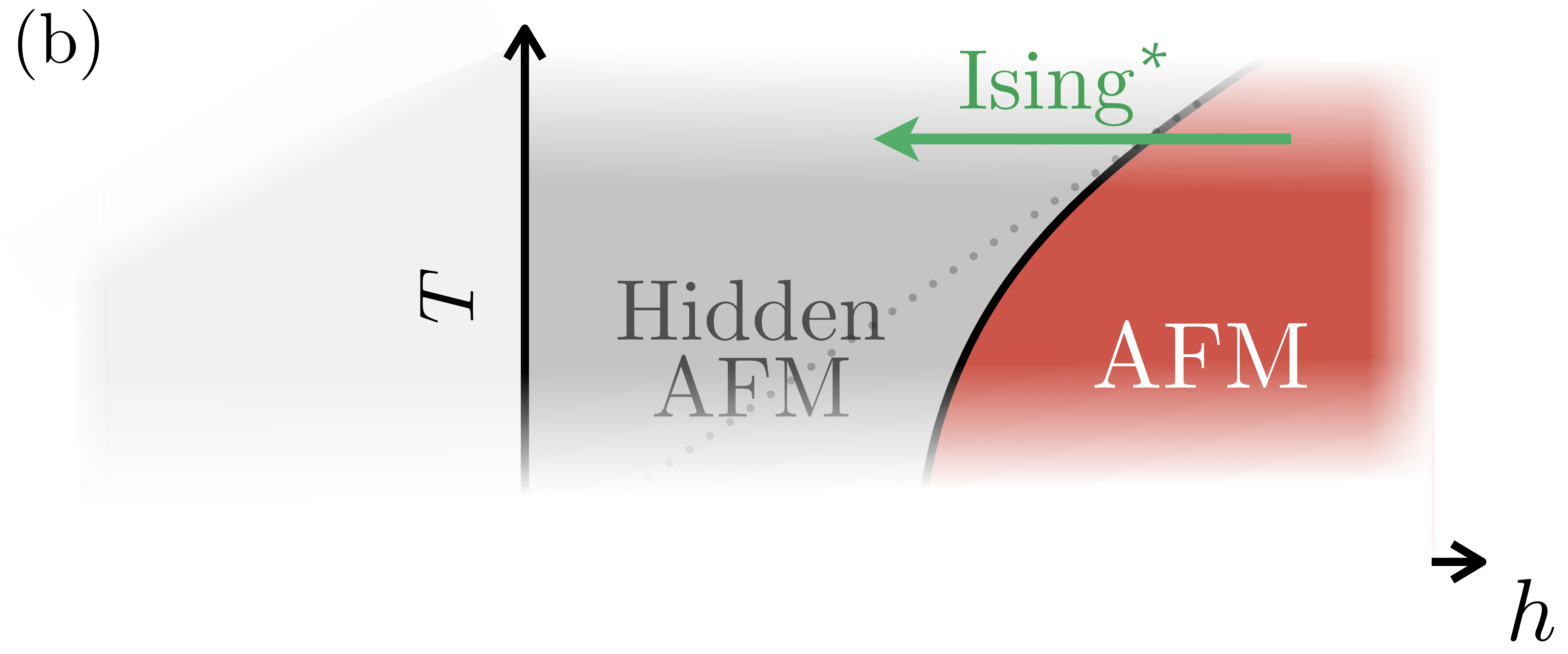

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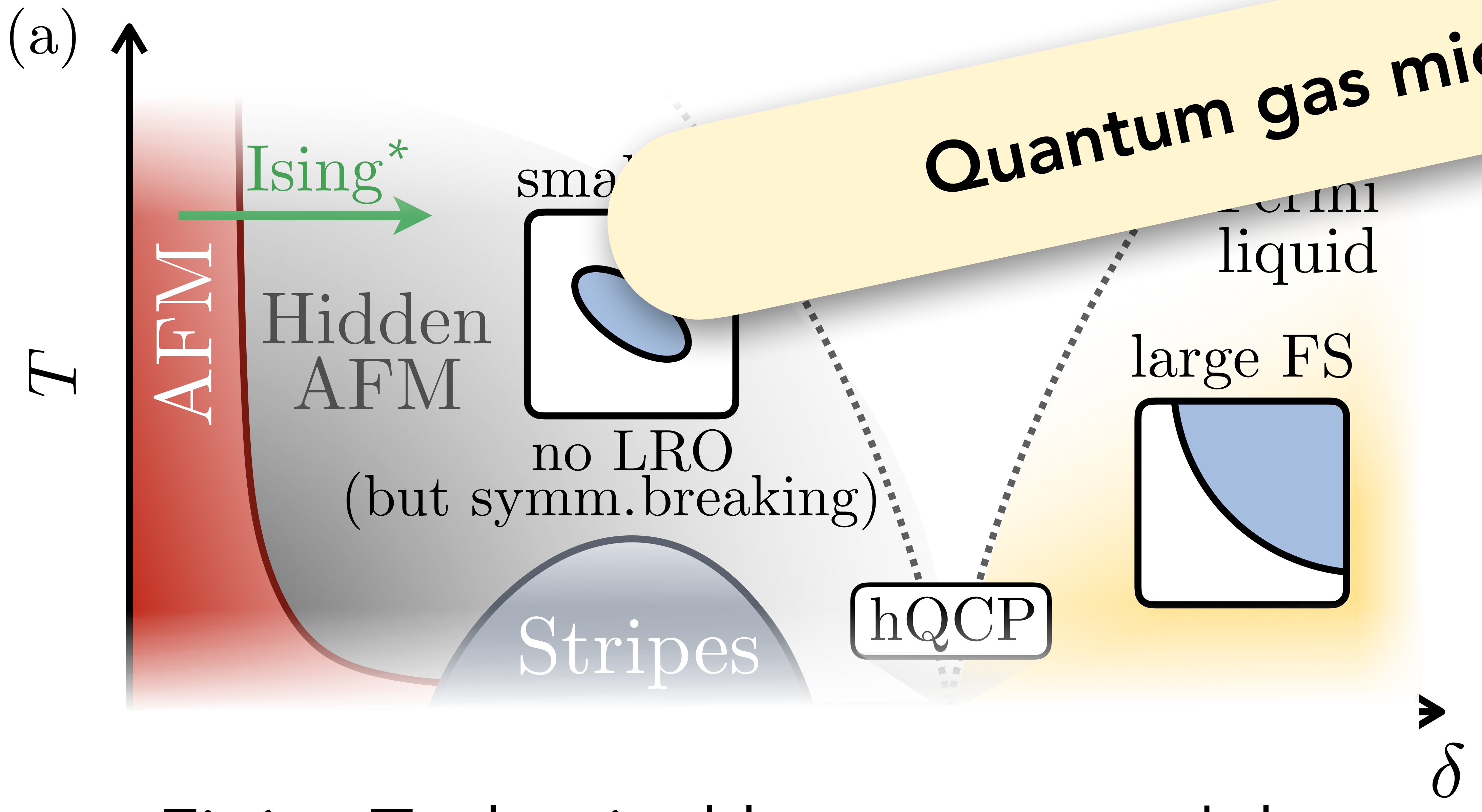
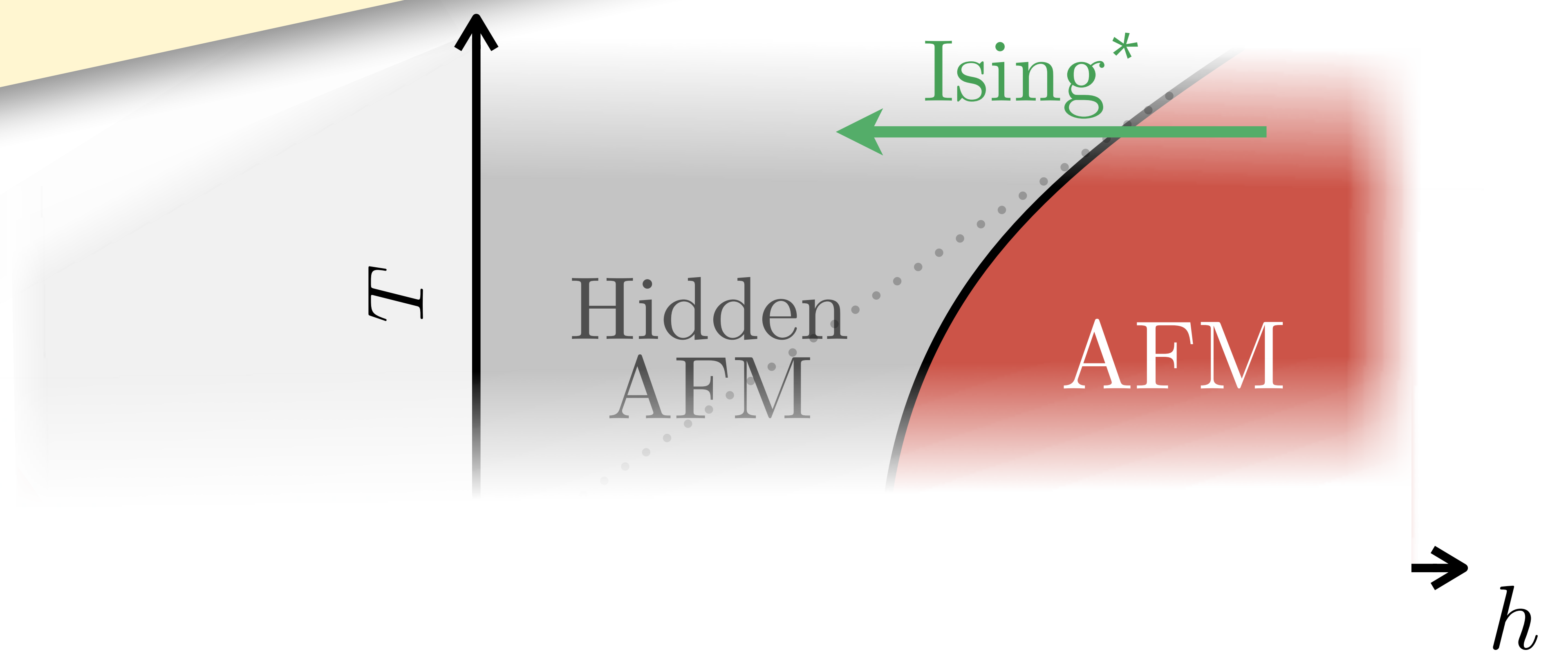


Quantum gas microscopes can check!



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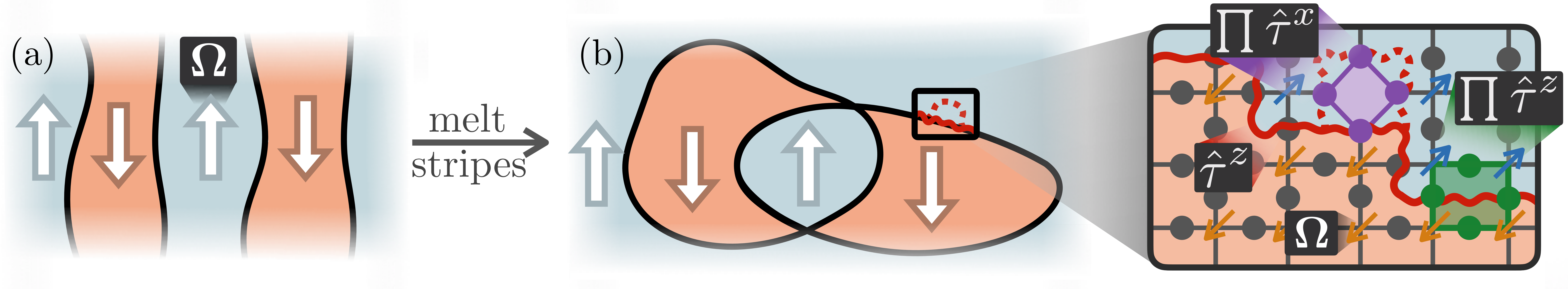
Melting stripes: Hidden AFM order

Schlömer et al., PRX Quantum 6 (2025)

- Low-T: AFM*



- Low-T: quantum fluctuations

$$\hat{\mathcal{H}}_{\mathrm{TC}} = -K_{\square} \sum_{\square} \prod_{\ell \in \square} \hat{\tau}^z_{\ell} - h \sum_{\ell} \hat{\tau}^z_{\ell} + K_{+} \sum_{+} \prod_{\ell \in +} \hat{\tau}^x_{\ell}$$

Kitaev's toric code

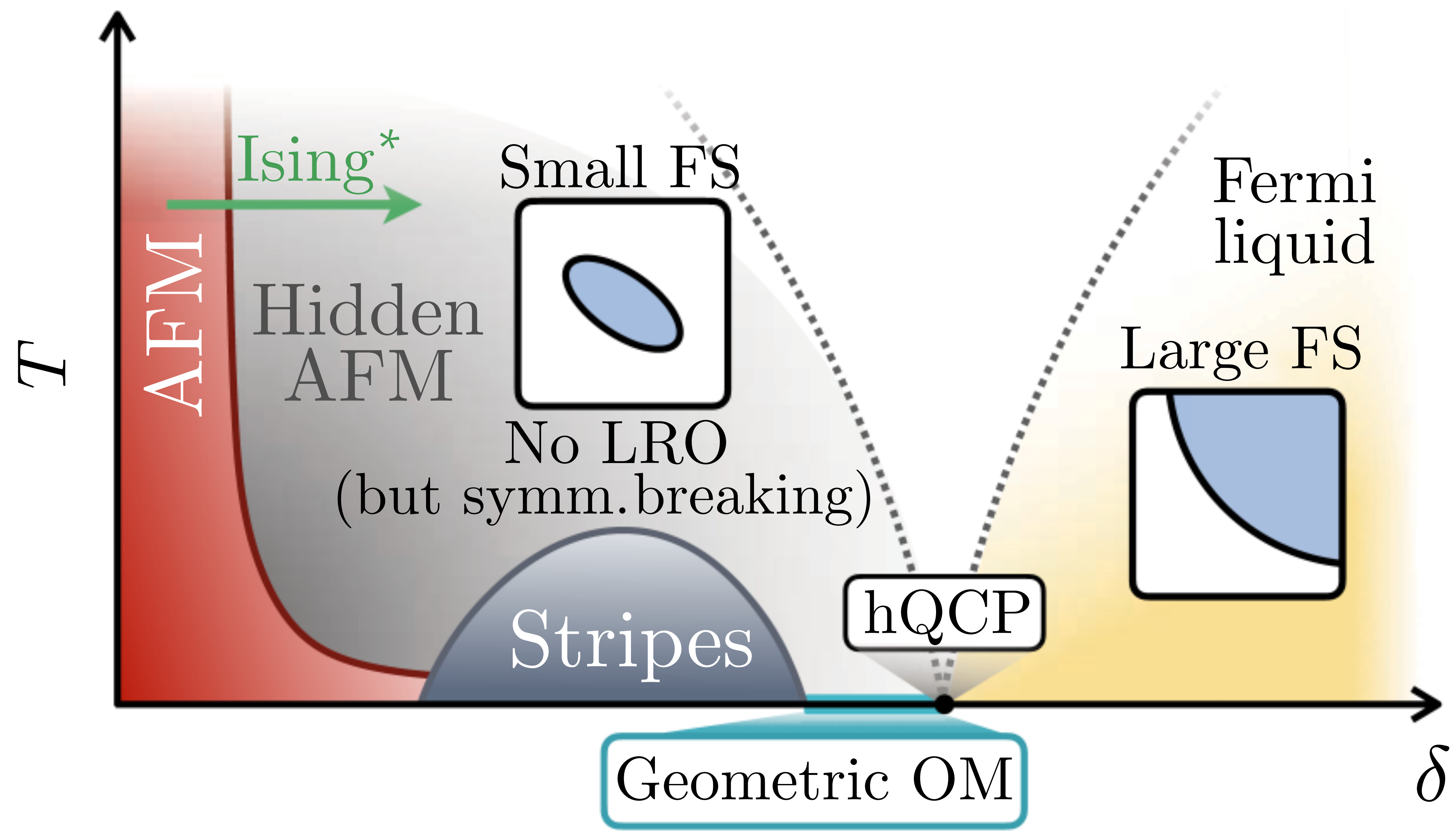
Kitaev, Ann. Phys. 303 (2003)

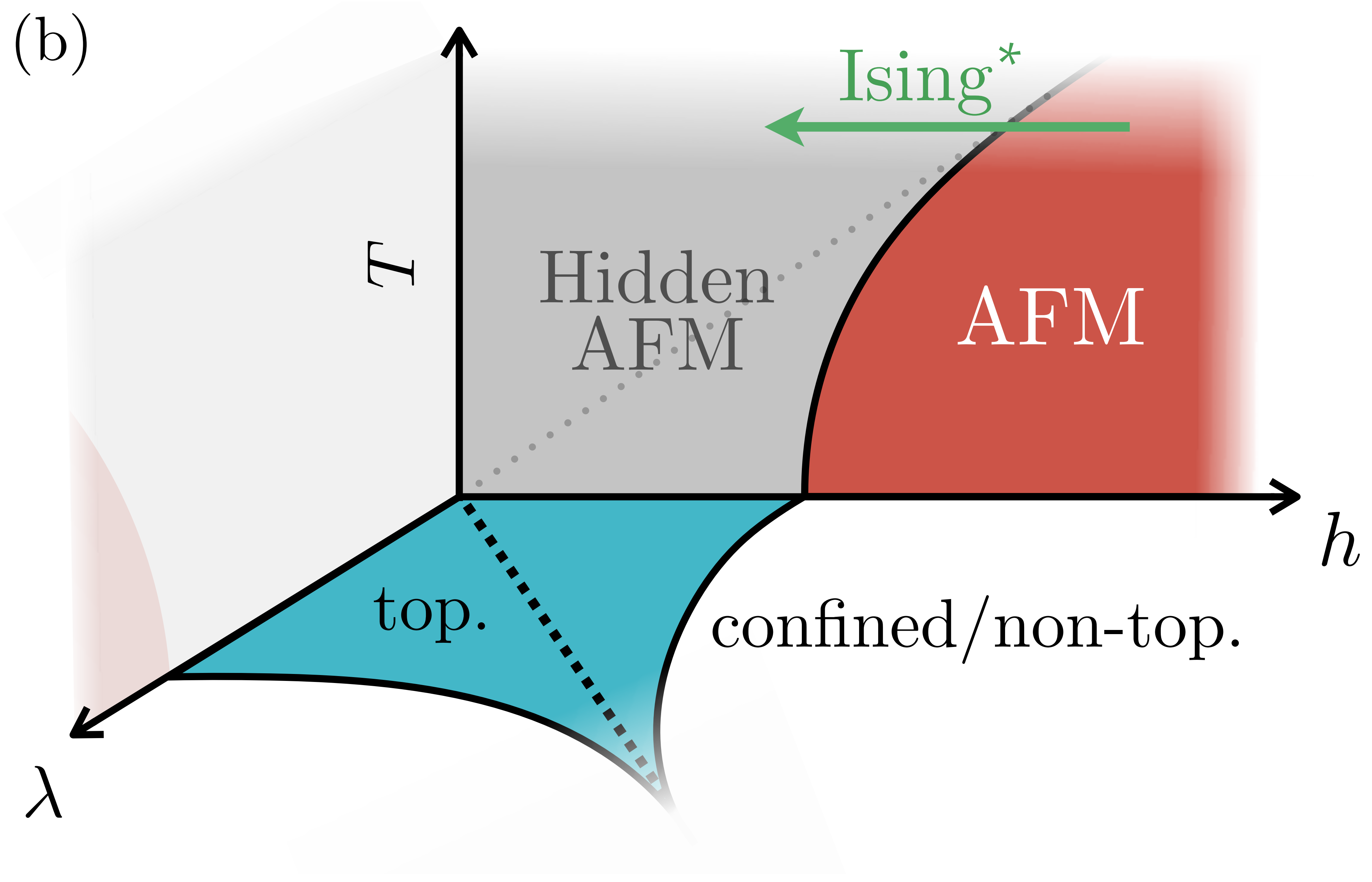
Hidden order

Melting stripes: Hidden AFM order

Schlömer et al., PRX Quantum 6 (2025)

- Low-T: AFM* and topological order





Kitaev's toric code

Pseudogap from stripes

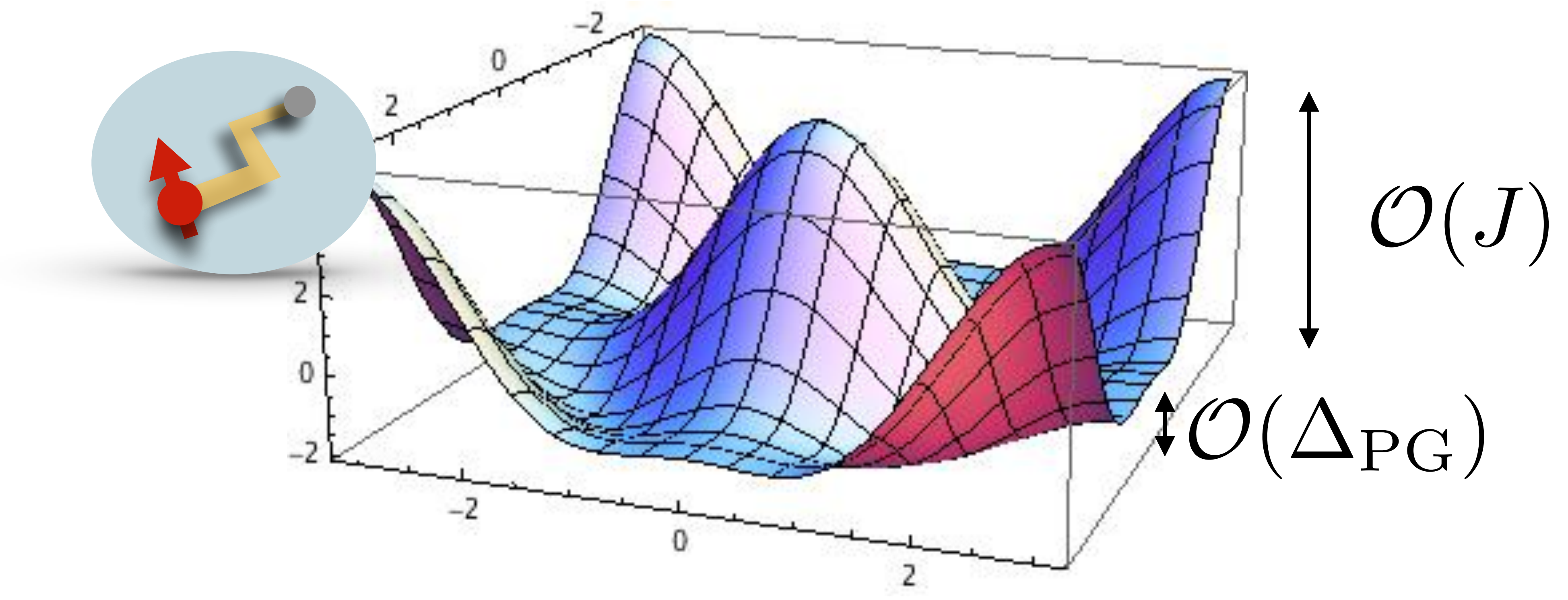
Geometric Orthogonal Metal (GOM): Luttinger's theorem

Schlömer et al., PRX Quantum 6 (2025)

- Doping an AFM: Magnetic polarons

  Kane et al., PRB 39 (1989); Sachdev et al., PRB 39 (1989)

  Bermes et al., PRB 109 (2024)



Pseudogap from stripes

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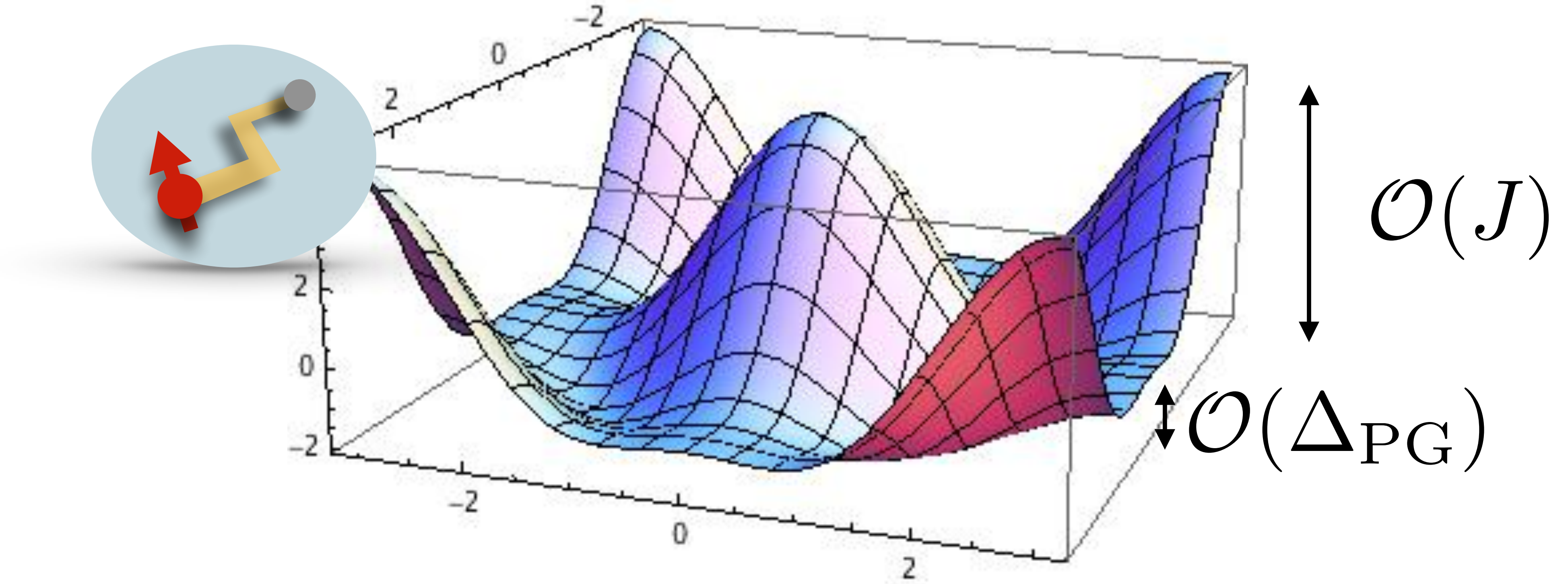
Schlömer et al., PRX Quantum 6 (2025)



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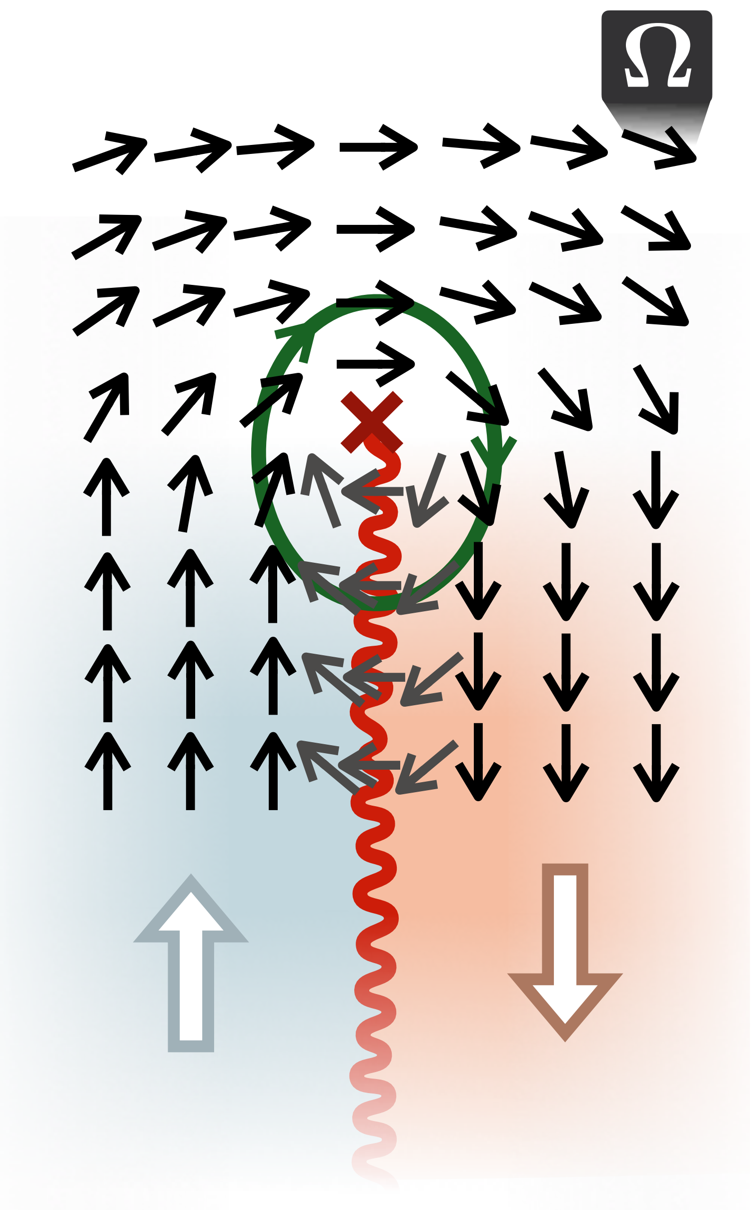
- Z2 gauge-charge :

$$\gamma_{\mathrm{Berry}} = \pi \equiv \gamma_{e-m}$$



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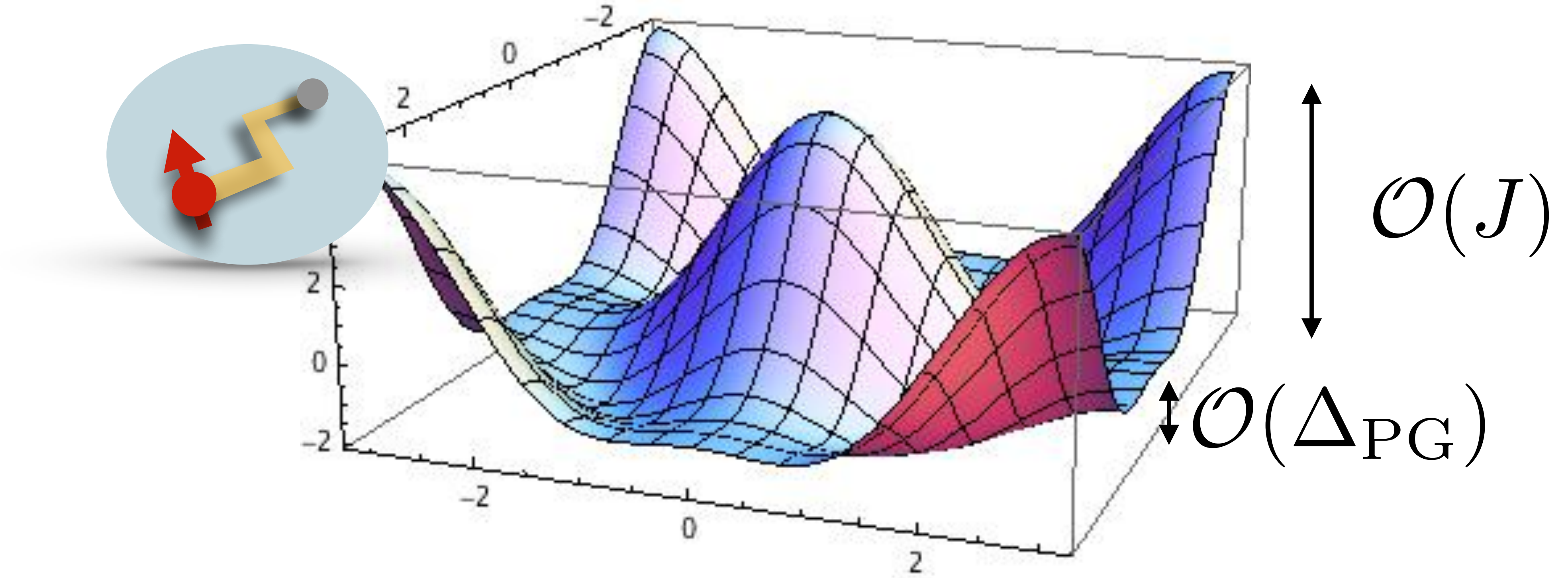

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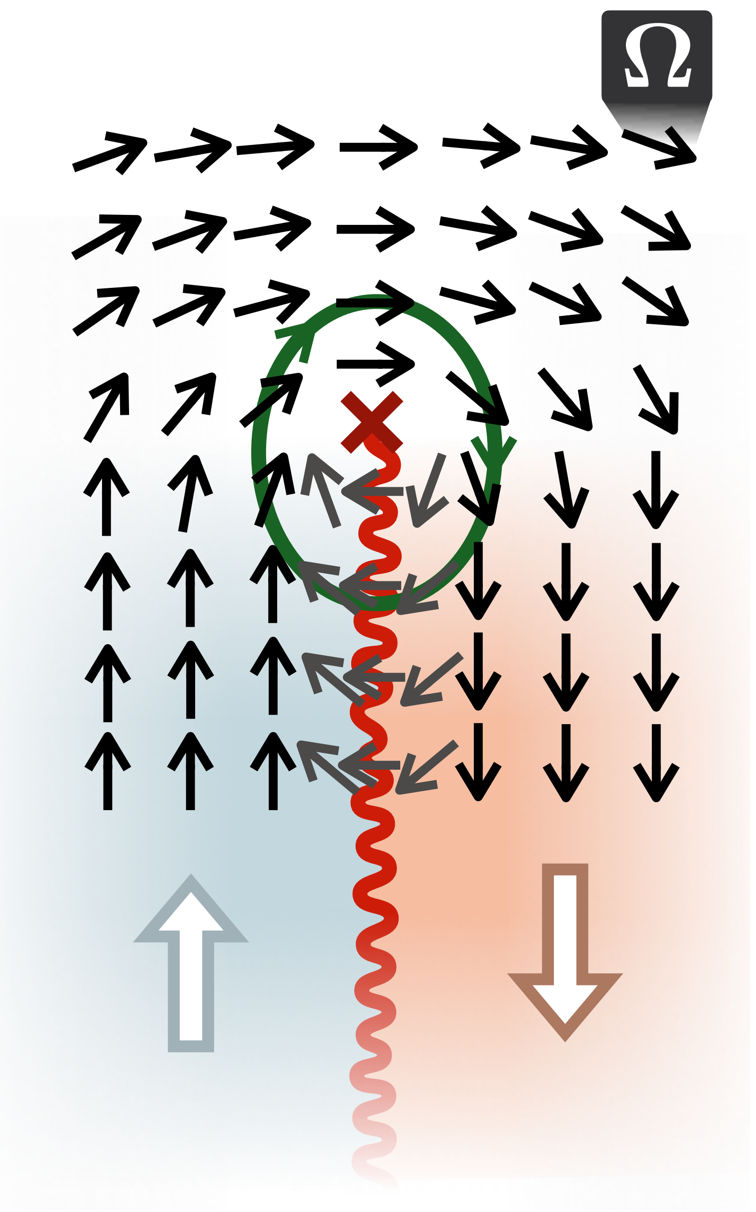
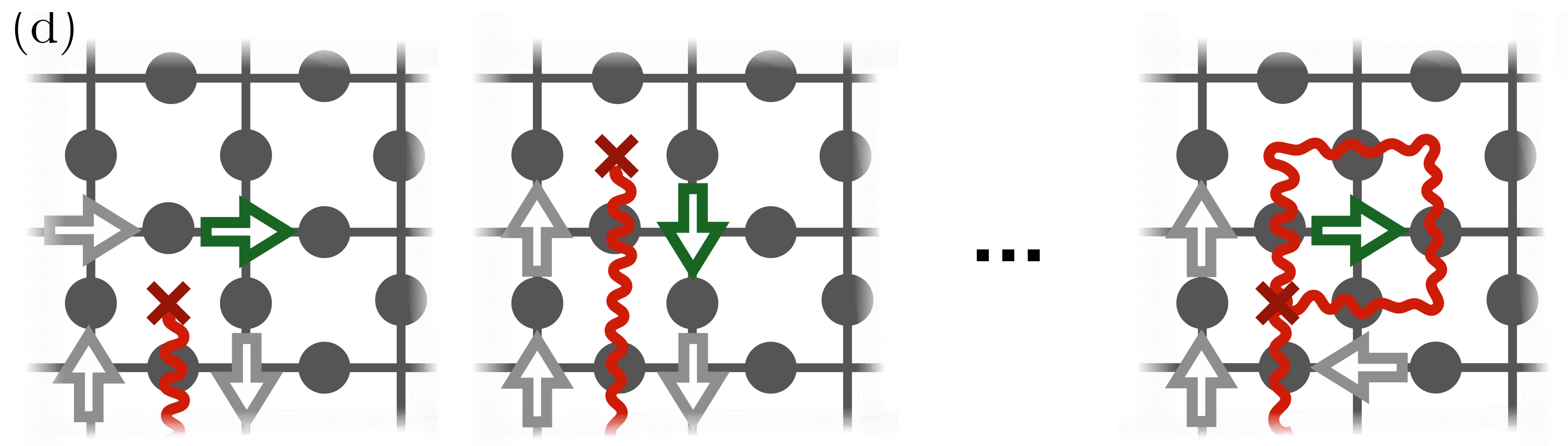


- Odd Z2 spin-liquid:




Quantum
simulation


Quantum simulation
Stripes
Pseudogap
Hidden order


Quantum
simulation
mixD t-J
Stripes
Pseudogap
fermionic
Hubbard/ t-J
bosonic t-J
Hidden order

Acknowledgements

www.quantummanybody.de

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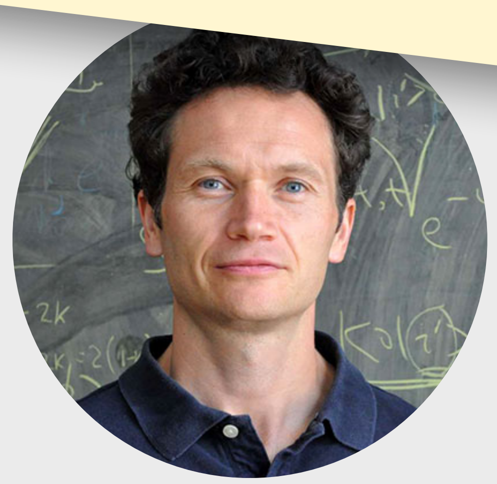
THANK YOU FOR YOUR ATTENTION!

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