Many-body open quantum systems: Dynamics and Mixing

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Outline

Monday (23.02.2026):

Dynamics:

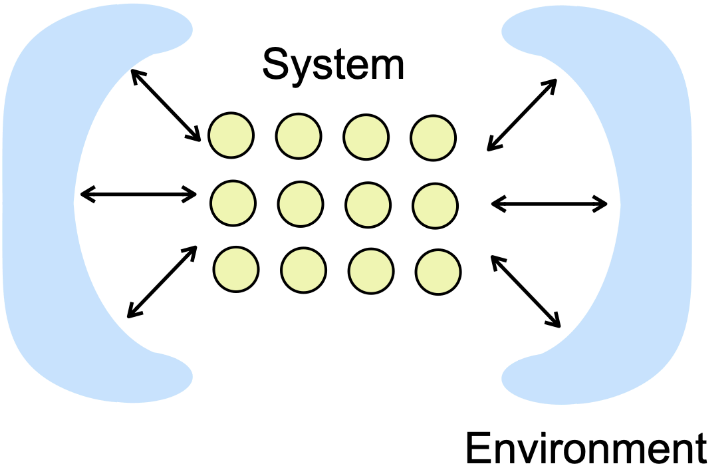
- Basics: Models, Lindbladians, connection to classical probability.
- Lattice-models, light cones and Lieb-Robinson bounds.
- Entanglement/separability.

Tuesday (24.02.2026):

Steady states:

- Spectral Properties, Irreducibility.
- Mixing times:
  - Spectral gaps.
  - Modified Log-Sobolev Inequality.
- Rapid mixing and consequences.

Open systems



Accurate description of experimental systems

Noisy Quantum information processing platforms.

Quantum optics e.g. quantum emitters coupled to photonic fields.

Many-body physics

Thermalization, driven-dissipative phases …

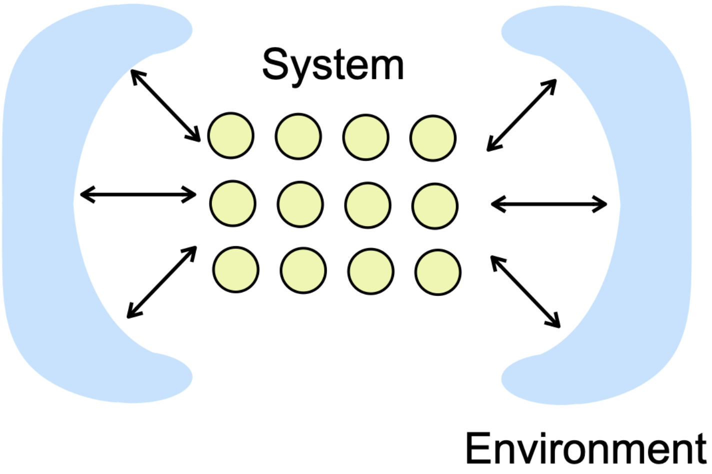
Generalization of classical probabilistic models

Probability distributions → Density matrices.

Classical Langevin equations → Quantum Open system models.

Model

$$H_{SE}(t) = \underbrace{H_S}_{\text{System Operators}} + \sum_{\alpha} \underbrace{X_\alpha} \underbrace{B_\alpha(t)}_{\text{Environment Operators}}$$



System dynamics:

$$\rho_S(t) = \mathrm{Tr}_E[U_{SE}(t,0)(\rho_S(0) \otimes \rho_E(0))U_{SE}(0,t)]$$

Environment can be specified via $n-$point correlation functions:

$$K_{\alpha_1,\alpha_2\ldots\alpha_n}(t_1, t_2\ldots t_n) = \mathrm{Tr}_E[B_{\alpha_1}(t_1)B_{\alpha_2}(t_2)\ldots B_{\alpha_n}(t_n)\rho_E(0)]$$

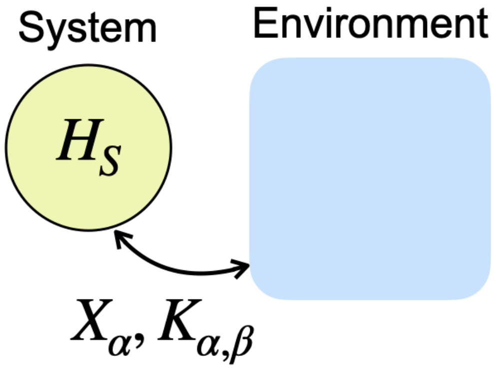
Models: Stationary and Gaussian environments

Stationarity: $K_{\alpha_1,\alpha_2\ldots\alpha_n}(t_1,t_2\ldots t_n) = K_{\alpha_1,\alpha_2\ldots\alpha_n}(t_1+\tau,t_2+\tau\ldots t_n+\tau)$

Gaussianity: $K_{\alpha_1,\alpha_2\ldots\alpha_n}(t_1,t_2\ldots t_n)$ satisfies Wick's/Isserlis's theorem:

$$K_{\alpha_1,\alpha_2\ldots\alpha_n}(t_1,t_2\ldots t_n) = K_{\alpha_1,\alpha_2}(t_1,t_2)K_{\alpha_3,\alpha_4}(t_3,t_4)\ldots + \text{all other pairings}$$

Open systems with stationary Gaussian environments specified by:
System Hamiltonian H_S, Jump operators X_α and memory kernels $K_{\alpha,\beta}(t_1,t_2) \sim K_{\alpha,\beta}(t_1-t_2)$.



Markovian models

- $K_{\alpha,\beta}(t_1, t_2) \sim k_{\alpha,\beta}\delta(t_1 - t_2)$, system dynamics described by Lindblad master equation

$$\frac{d}{dt}\rho(t) = -i[H_S, \rho(t)] + \sum_{\alpha,\beta} k_{\alpha,\beta}\left(X_\alpha \rho(t) X_\beta - \frac{1}{2}\{X_\beta X_\alpha, \rho(t)\}\right)$$

- **Positive semi-definiteness requirement:**
  - For any set $\{f_\alpha(t)\}_\alpha$

$$K_{\alpha,\beta}(t_1, t_2) = \mathrm{Tr}[B_\alpha(t_1)B_\beta(t_2)\rho_E(0)] \implies \sum_{\alpha,\beta}\int_{t_1,t_2} f^*_\alpha(t_1) f_\beta(t_2) K_{\alpha,\beta}(t_1, t_2) dt_1 dt_2 \geq 0$$

  - For Markovian case: The matrix $\mathrm{K} = [k_{\alpha,\beta}]$ is positive semi-definite.

$$\mathrm{K} = \sum_n \lambda_n v_n v_n^\dagger \implies \frac{d}{dt}\rho(t) = -i[H_S, \rho(t)] + \sum_n \mathscr{D}_{L_n}\rho(t)$$

where $\mathscr{D}_L(X) = LXL^\dagger - \frac{1}{2}\{L^\dagger L, X\}$ and $L_n = \sqrt{\lambda_n}\sum_\alpha v_{n,\alpha} X_\alpha$

GKSL theorem

- For master equations:

$$\frac{d}{dt}\rho(t) = \mathscr{L}\rho(t) \text{ with } \mathscr{L}X = -i[H_S, X] + \sum_n L_n X L_n^\dagger - \frac{1}{2}\{L_n^\dagger L_n, X\}$$

and any $t \geq 0$, $e^{\mathscr{L}t}$ is a completely-positive-trace-preserving map.

- **GKSL theorem:** If $e^{\mathscr{L}t}$ is a completely positive trace-preserving map $\forall t \geq 0$ then,

$$\mathscr{L}X = -i[H_S, X] + \sum_n L_n X L_n^\dagger - \frac{1}{2}\{L_n^\dagger L_n, X\}$$

for some H_S, $\{L_n\}_n$.

Lindblad (1976), Gorini, Koassakowski, Sudarshan (1976)

A connection to classical probabilistic dynamics

Quantum Systems

State-space: $\mathbb{C}^D$

State: $\rho(t) = [\rho_{i,j}(t)]_{i,j \in \{1,2\ldots D\}}$

Dynamics: $\dot{\rho}(t) = \mathscr{L}\rho(t)$,
where $\mathscr{L}$ is of Lindblad form.

Classical Systems

State-space: $\{1,2\ldots D\}$

State: $p(t) = [p_1(t), p_2(t)\ldots p_D(t)]$

Dynamics: $\dot{p}(t) = Ap(t)$,
where $A_{i,j}$ = Rate of state $j \to i$,
and $A_{i,i} = -\Sigma_{j\neq i} A_{j,i}$

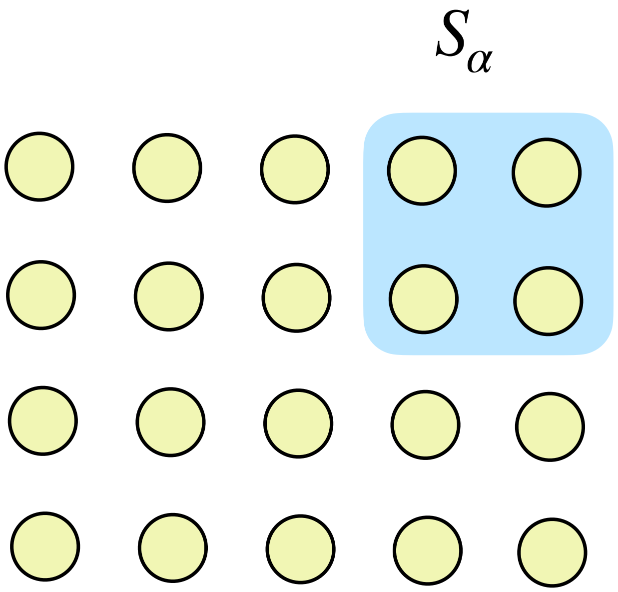
- **Classical dynamics as a special case of quantum master equation:**

$$\dot{\rho}(t) = \sum_{i\neq j} \mathscr{D}_{L_{i,j}}\rho(t) \text{ where } L_{i,j} = A_{i,j}^{1/2}|i\rangle\langle j|$$

More explicitly: $\dot{\rho}_{i,i}(t) = \sum_j A_{i,j}\rho_{j,j}(t)$ and $\dot{\rho}_{i,j}(t) = \frac{1}{2}(A_{i,i} + A_{j,j})\rho_{i,j}(t)$

i.e., *On-diagonal* elements follow classical Markov evolution, and *off-diagonal* elements decay exponentially.

Lattice models for open systems



$$\mathscr{L} = \sum_\alpha \mathscr{L}_\alpha$$

$\mathscr{L}_\alpha$ acts only on spins in geometrically local region S_α

$$\mathscr{L}_\alpha = -i[h_\alpha, \cdot\,] + \sum_k \mathscr{D}_{L_\alpha^{(k)}}$$

where h_α and $L_\alpha^{(k)}$ are supported on S_α.

$L_\alpha^{(k)}$ **act on single site**

Simplest model of noise/decoherence.

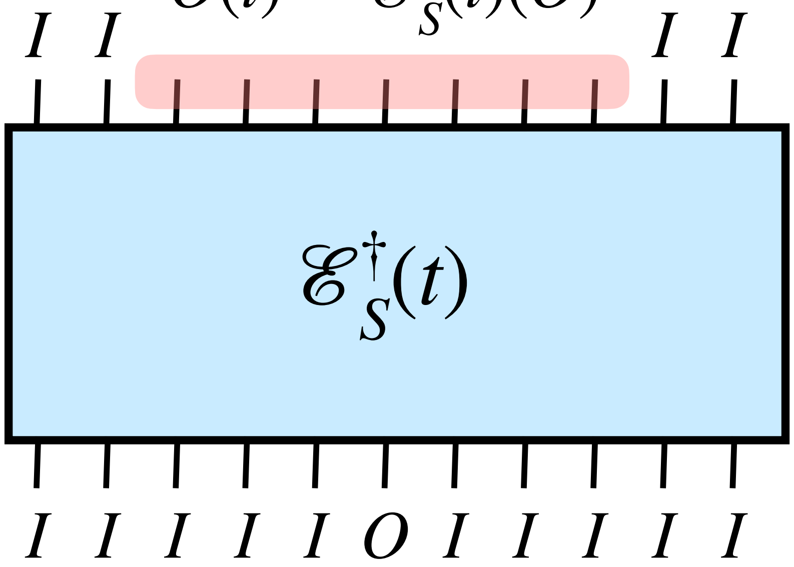
$L_\alpha^{(k)}$ **act on multiple sites**

Collective dissipation processes e.g. stimulated photon emission in AMO systems.

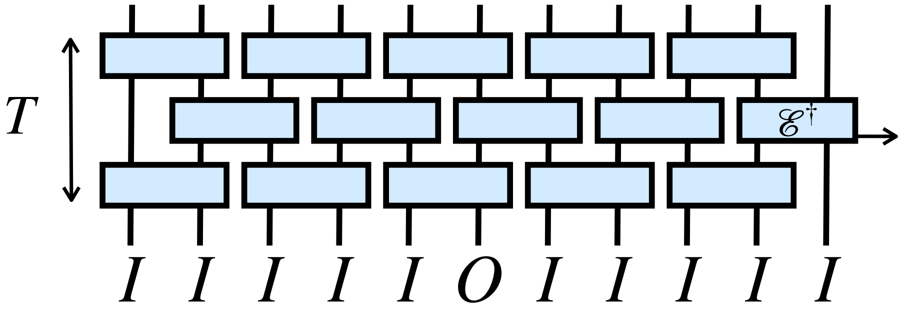
Light-cones in dissipative dynamics

How do local observables spread during dynamics?

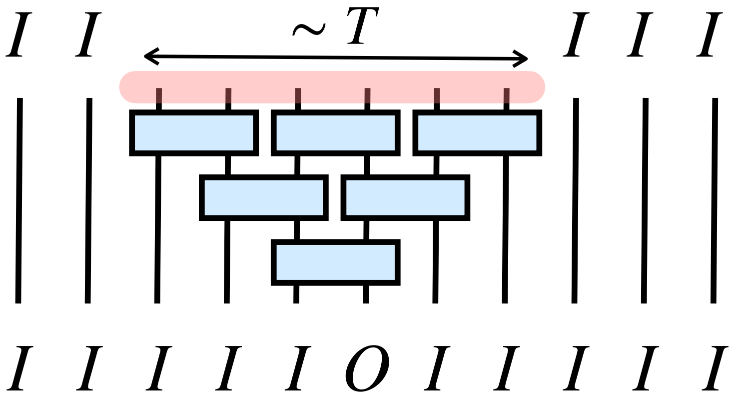
$$O(t) = \mathscr{E}_S^{\dagger}(t)(O)$$



Discrete-time model:



Since $\mathscr{E}^{\dagger}(I \otimes I) = \sum_i K_i^{\dagger} K_i = I \otimes I$

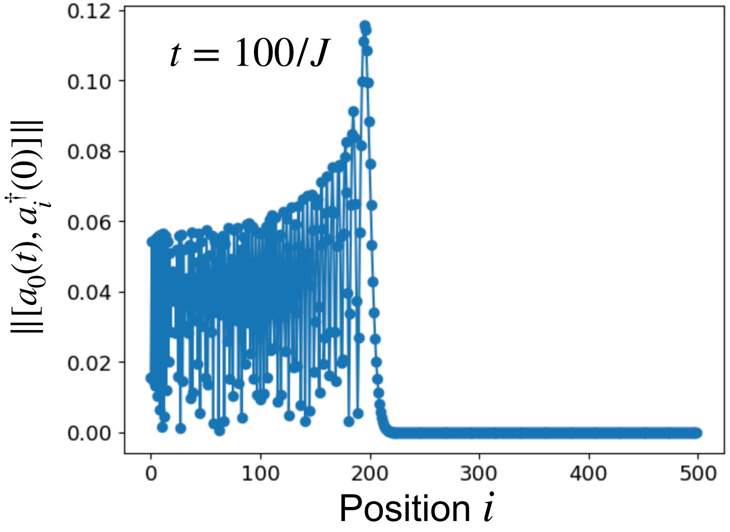


Light-cones in dissipative dynamics

No strict light-cone in continuous-time lattice models.

Example: Tight-binding boson chain.

$$H = J \sum_i a_i^\dagger a_{i+1} + \text{h.c.:}$$



Formalizing approximate light-cones:

Lieb, Robinson (1972).

. . . .

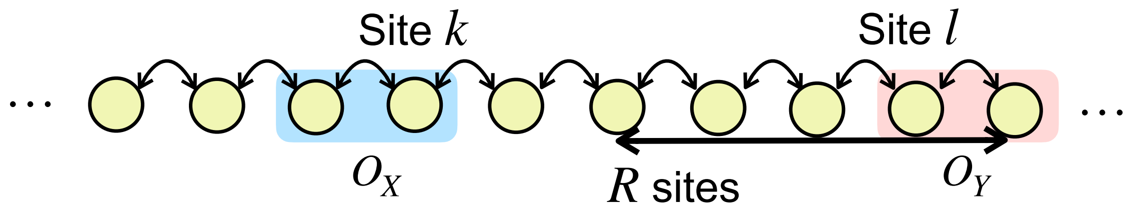
Hastings, Koma (2004), Schuch et al (2011), Poulin (2010), MC Tran et al (2021), Kuwahara, Saito et al (2021), C. Yin et al (2022)

Applications:

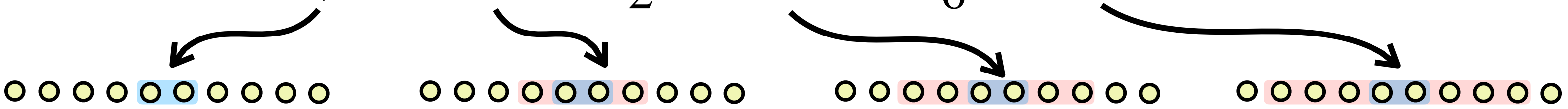
- Exponential decay of correlations and area laws in gapped Hamiltonians ground states. Hastings, Koma (2005), Hastings (2007)
- Stability of gap of Hamiltonians with Topological Quantum Order. Bravyi, Hastings (2010), Spyridos, Pytel (2011)
- Stability of local observables in rapid mixing Lindbladians. Cubitt et al (2013)
- Optimal algorithms for Hamiltonian Simulation. Hastings, Haah, Kothari, Low (2018)
- Hamiltonian and Lindbladian learning with near-optimal sample complexity. Franca et al (2024), Lopez et al RT (2025)

Lieb-Robinson bounds: Informal derivation

Nearest Neighbor Lindbladian
$\mathscr{L} = \Sigma_i \mathscr{L}_{i,i+1}$ with $\|\mathscr{L}_{i,i+1}\| \leq J$, and
local operators O_X, O_Y



$$e^{\mathscr{L}^\dagger t}(O_X) = \underbrace{O_X} + t\underbrace{\mathscr{L}^\dagger(O)} + \frac{t^2}{2}\underbrace{(\mathscr{L}^\dagger)^2(O_X)} + \frac{1}{6}\underbrace{(\mathscr{L}^\dagger)^3(O_X)} + \dots$$

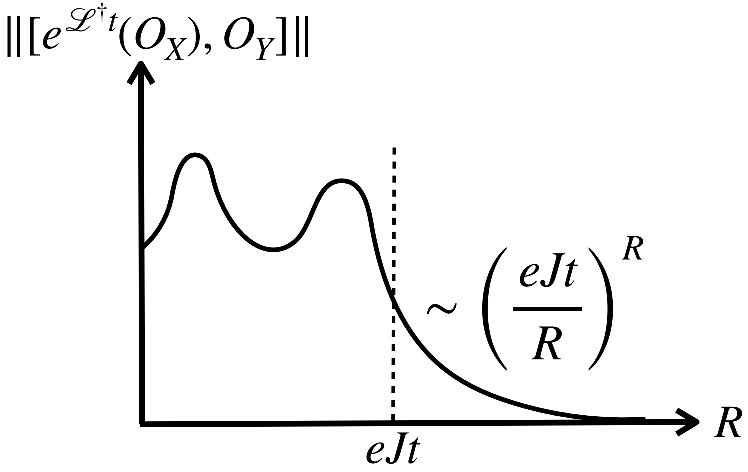


To leading order:

$$\|[e^{\mathscr{L}^\dagger t}(O_X), O_Y]\| \approx \frac{t^R}{R!}\|[(\mathscr{L}^\dagger)^R(O_X), O_Y]\|$$

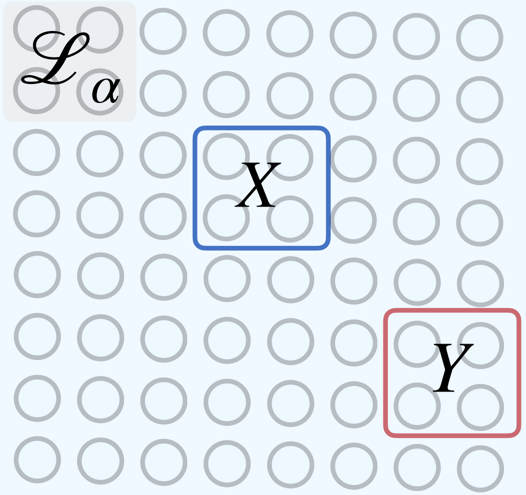
$$= \frac{t^R}{R!}\|[\mathscr{L}_{l-1,l}\mathscr{L}_{l-2,l-1}\dots\mathscr{L}_{k,k+1}(O_X), O_Y]\|$$

$$\leq 2\frac{J^R t^R}{R!}\|O_X\|\|O_Y\| \sim \left(\frac{eJt}{R}\right)^R$$



Lieb-Robinson bounds: Formal statement

Lattice of qudits: $\mathscr{L} = \Sigma_\alpha \mathscr{L}_\alpha$, with $\mathscr{L}_\alpha$ being geometrically local.



For any operator O_X on X, operator O_Y on Y

$$\|[e^{\mathscr{L}^\dagger t}(O_X), O_Y]\| \leq c\|O_X\|\|O_Y\||X|\exp\left(\frac{vt - d(X, Y)}{R}\right)$$

Lieb-Robinson velocity: $v = e\mathscr{Z}RJ,$

$\|\mathscr{L}_\alpha\|_\diamond \leq J,$ $|\text{diam}(\mathscr{L}_\alpha)| \leq R,$

$\mathscr{Z}$ = Coord. Number of $\mathscr{L}_\alpha$

Poulin (2010)

Lieb-Robinson bounds for non-Markovian models:

- Similar bounds for environments with short-memories.
- Environments with long-memories can violate linear light cones.

RT, Yu, Rudner (2024)

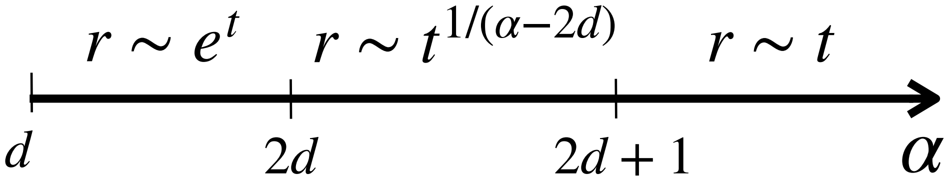
Lieb-Robinson bounds: Long-range models

- Lindbladians with interactions between all possible pairs of sites which fall off with distance (example in Rydberg systems, Coloumb interactions)

$$\mathscr{L} = \sum_{i,j} \mathscr{L}_{i,j}, \text{ where } \|\mathscr{L}_{i,j}\| \leq \frac{J}{r_{i,j}^{\alpha}}$$

- The shape of the light-cone depends on α and lattice dimensionality d

For closed systems



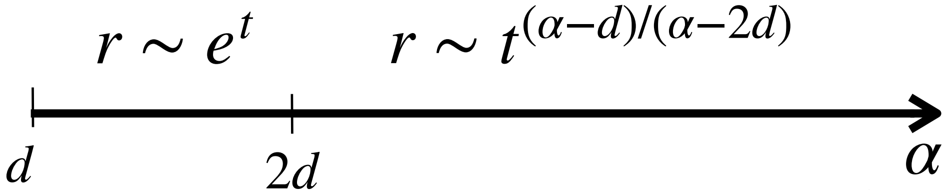
$r \sim e^t$ $\quad r \sim t^{1/(\alpha-2d)}$ $\quad r \sim t$

$d \quad 2d \quad 2d+1 \quad \alpha$

Saturating examples are also known.

Hastings, Koma (2004), MC Tran et al (2021),
Kuwahara, Saito et al (2020)

For open systems



$r \sim e^t$ $\quad r \sim t^{(\alpha-d)/(\alpha-2d)}$

$d \quad 2d \quad \alpha$

Linear light-cone only for
$\alpha > 3,\ d = 1$

Sweke et al (2019), Andrew Guo et al (2021),

Typical operator spreading: Overview

Out-of-time ordered correlators and operator weight distributions studying single-site dissipation competing with Hamiltonian dynamics.

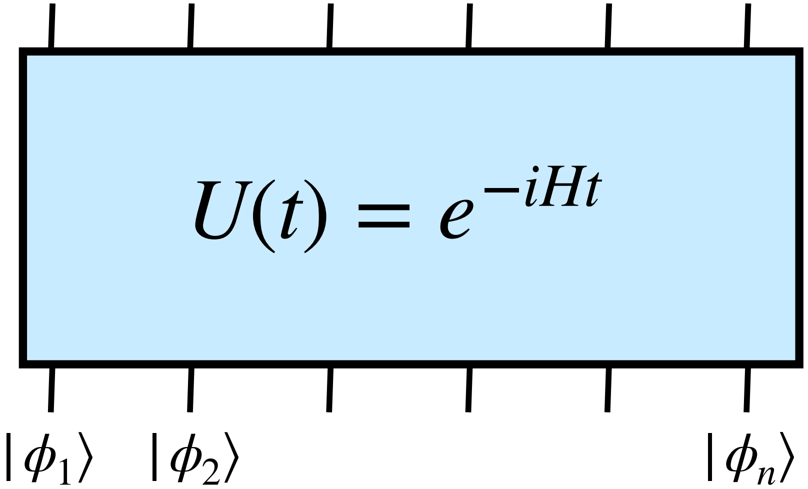
Schuster and Yao (2023)

Studying operator growth via Krylov complexity measures.

Liu, Tang, Zhai (2022), Bhattacharya, Nandy et al (2022, 2023), Bhattacharjee, Cao et al (2023), Srivatsa, Keyserlingk (2024),

Entanglement transitions

If $t \geq n$, then state at time t is volume-law entangled.



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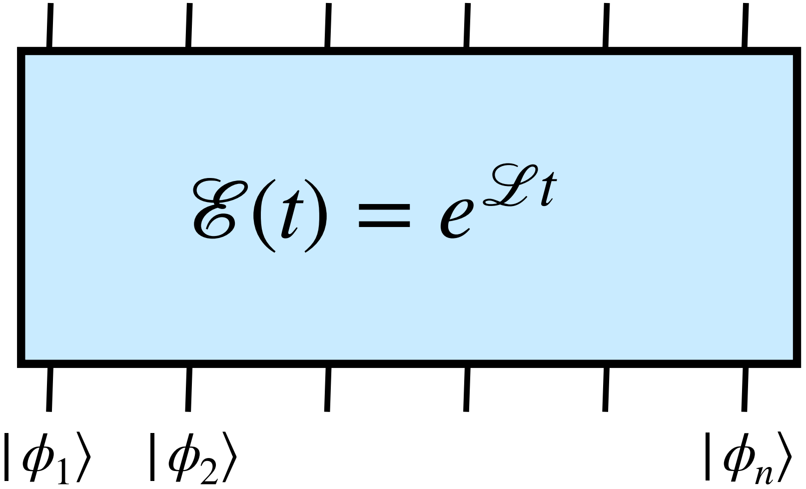
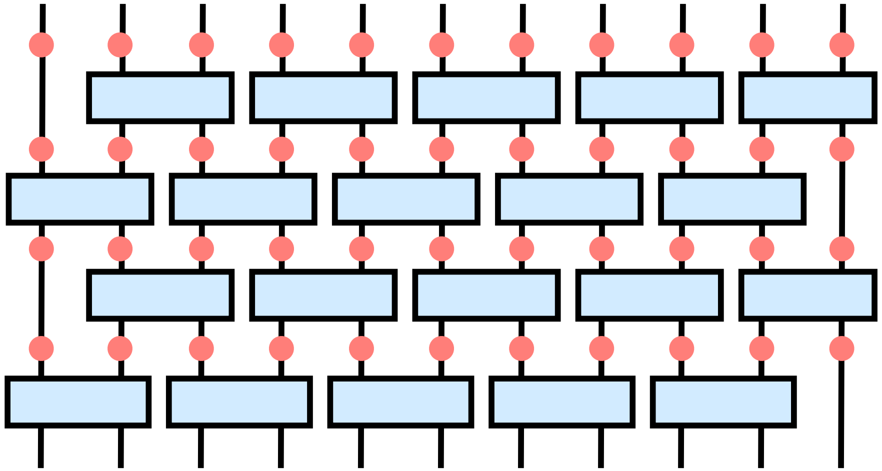


Discrete-time model



▭ = Unitary U

● $= (1-p)\,\Big|\, + p$ ●

where p is noise-rate,

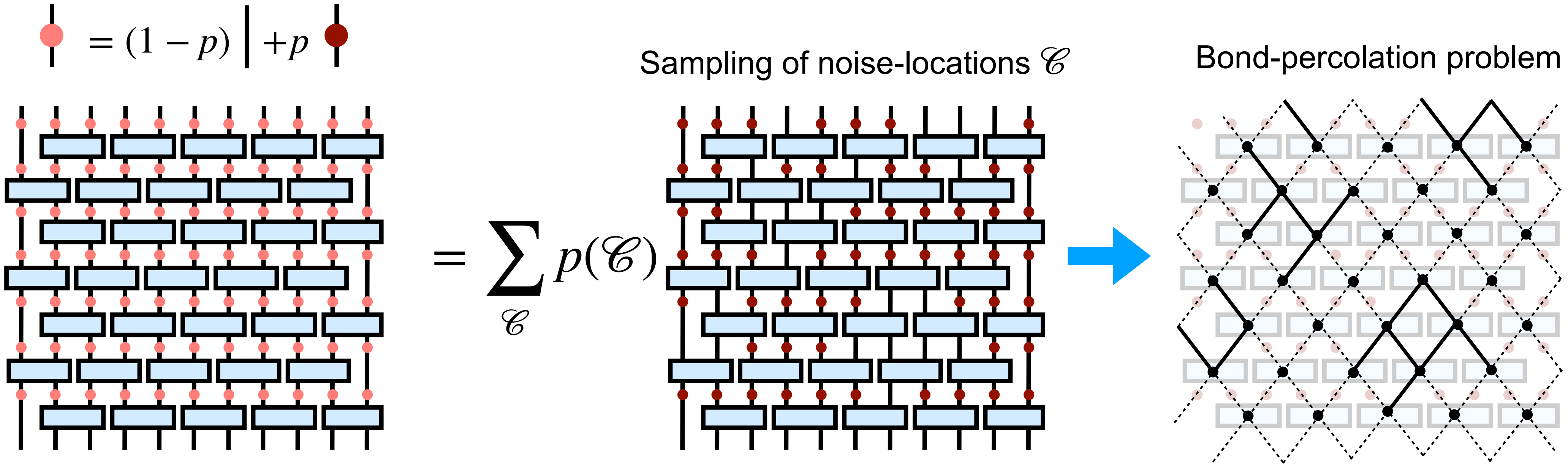
● = Entanglement breaking channel $\mathcal{N}$

$$\mathcal{N}(\rho) = \sum_i \sigma_i \mathrm{Tr}(P_i \rho)$$

$p = 0$: Volume law entanglement and
$p = 1$: Completely separable states.

Percolation transitions

Aharonov (1999),
Nahum et al (2017).



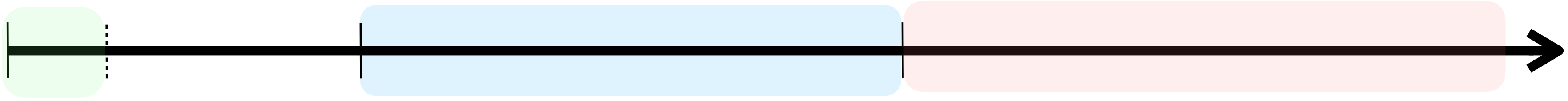
Gate ≅ Site
Possible Noise location ≅ Bond
Bond is broken with probability p

For $p > p_c \approx 0.45$, the size of largest connected cluster $\sim \log(n)$

$\implies$ At-most $\mathcal{O}(\log n)$ sites can be entangled.

Entanglement breaking transitions

Discrete-time model



$p = 0$

Only $\log(n)$ qubits are entangled.

All the sites are separable

Possible volume-law entanglement:
Can perform fault-tolerant QC
2D: Ben-Or et al (2013)
1D: Oles Shtanko et al (2024)

$p = p_c$

Aharonov et al (1999)

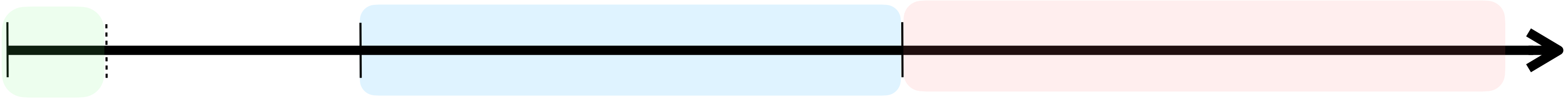
$p = p'_c$

If $\mathscr{N}$ is strongly entanglement breaking (e.g. depolarizing)

Continuous-time model

$$\frac{d}{dt}\rho(t) = -i[H, \rho(t)] + \kappa \sum_{\alpha,k} \mathscr{D}_{L_\alpha^{(k)}}\rho(t)$$

where for each site α, $\{L_\alpha^{(k)}\}_k$ span all single-site operators.



$\kappa = 0$

Only $\log(n)$ qubits are entangled

All the sites are separable

Possible volume-law entanglement:
Can perform fault-tolerant QC
2D: RT, J. I. Cirac et al (2022)

$\kappa = \kappa_c$

RT, J.I. Cirac (2022)

$\kappa = \kappa'_c$

Gonzalez et al RT et al (2025)

Summary

- Basics: Models, Lindbladians, connection to classical probability.

- Lattice-models, light cones and Lieb-Robinson bounds.

- Entanglement/separability transitions at high noise rate.