arXiv:2511.17370

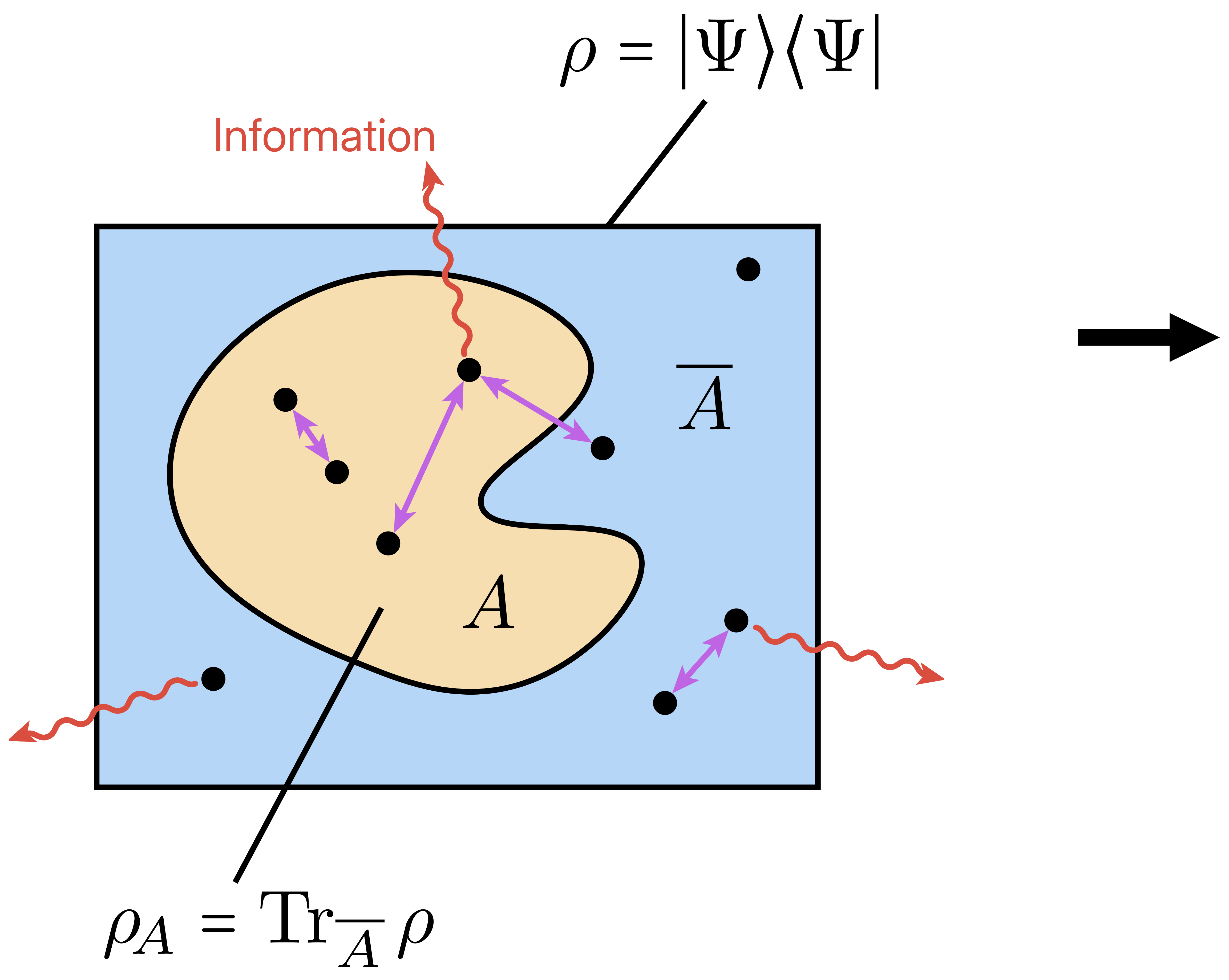
Robust detection of an entanglement transition in the projective transverse field Ising model

Felix Roser, 20.02.2026

Universität Stuttgart

Institute for Theoretical Physics

Can quantum properties survive decoherence?



Can quantum entanglement survive under monitoring?

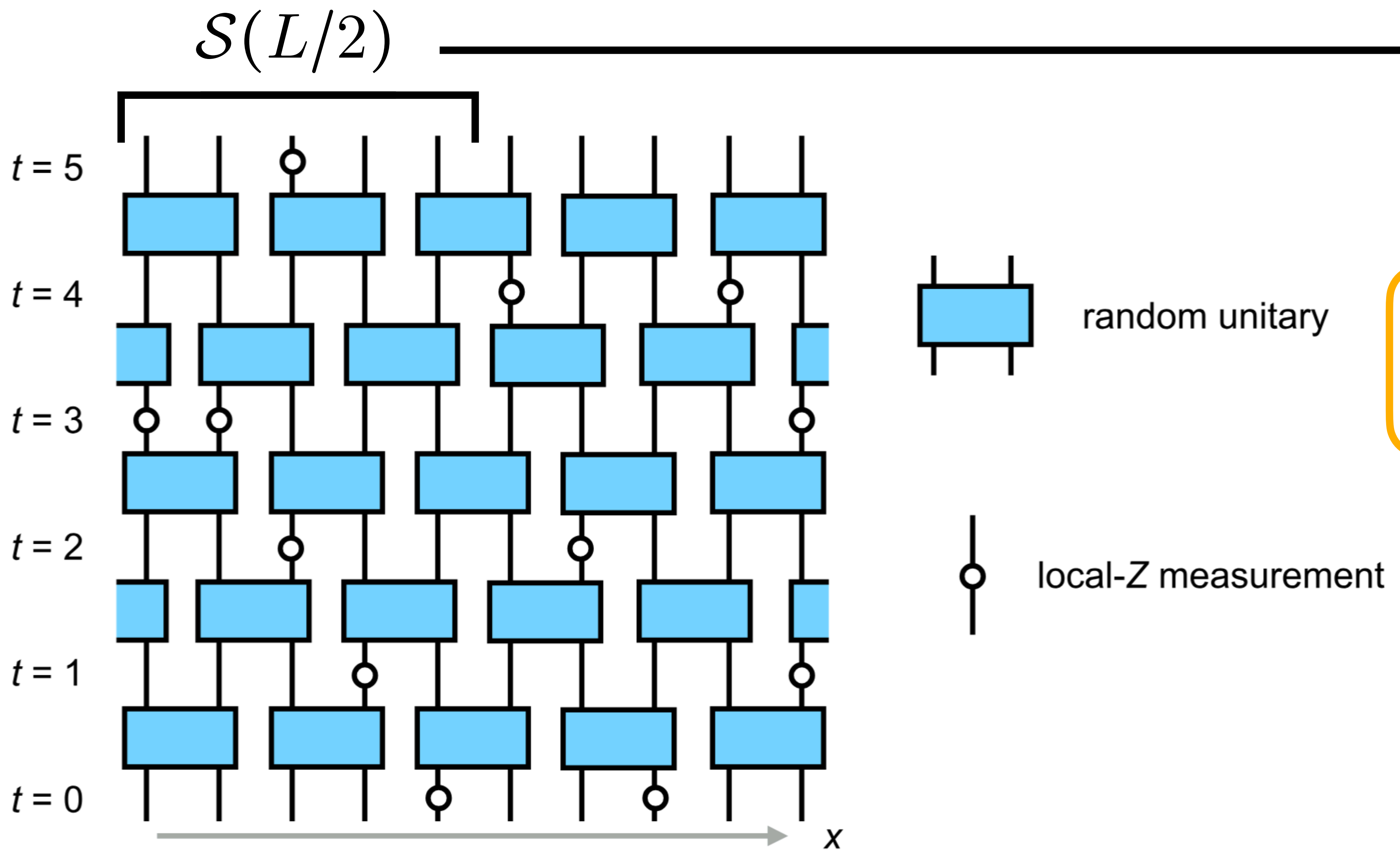
Von Neumann entanglement entropy

$$\mathcal{S}_A = -\,\mathrm{Tr}\left[\rho_A \log_2 \rho_A\right]$$

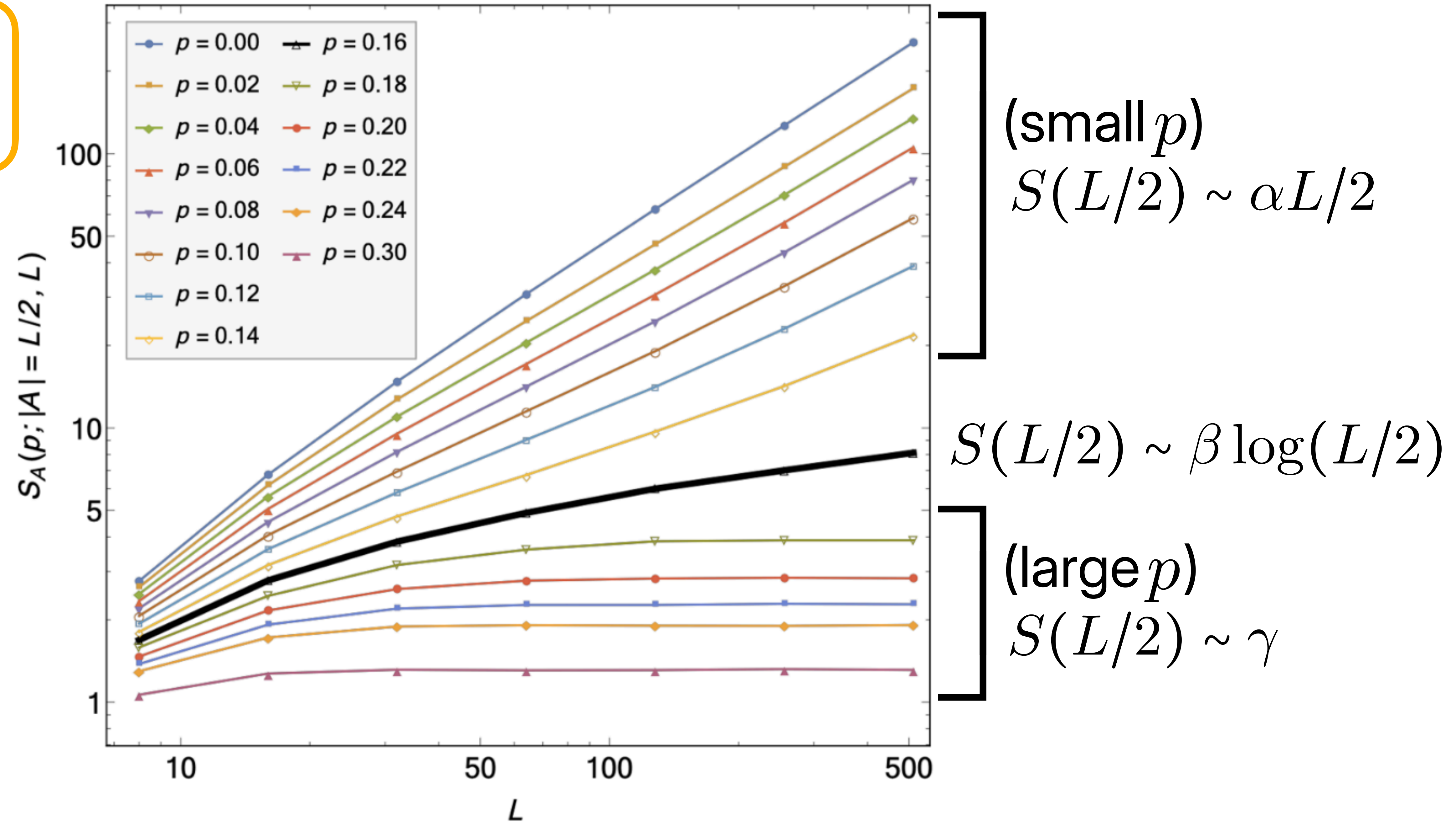
Li, Chen & Fisher, PRB **98**, 205136 (2018)
Li, Chen & Fisher, PRB **100**, 134306 (2019)

Entanglement transitions

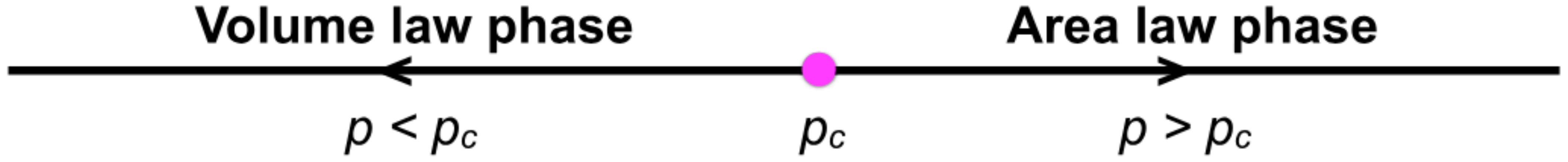
Hybrid random quantum circuits



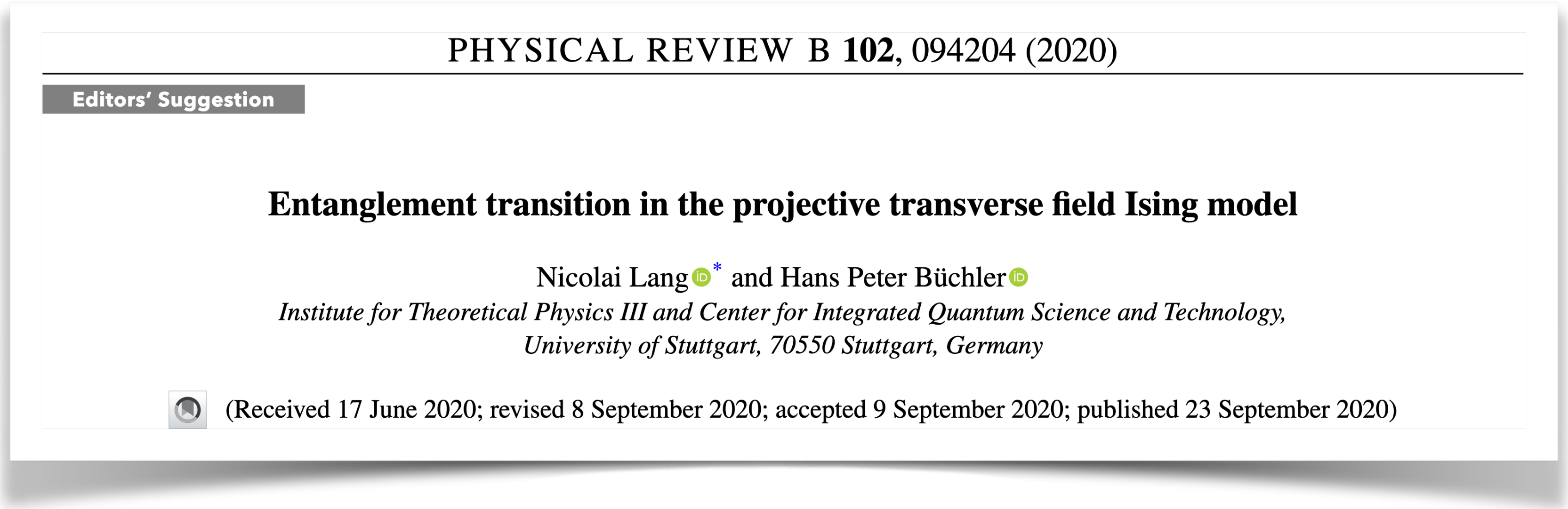
Sample average
$S_A = \langle\langle \mathcal{S}_A \rangle\rangle$



- Random unitaries
  → entangle multiple qubits
- Measurements (with probability p)
  → disentangle single qubits



Projective transverse field Ising model (PTIM)


PHYSICAL REVIEW B **102**, 094204 (2020)

Editors' Suggestion

Entanglement transition in the projective transverse field Ising model

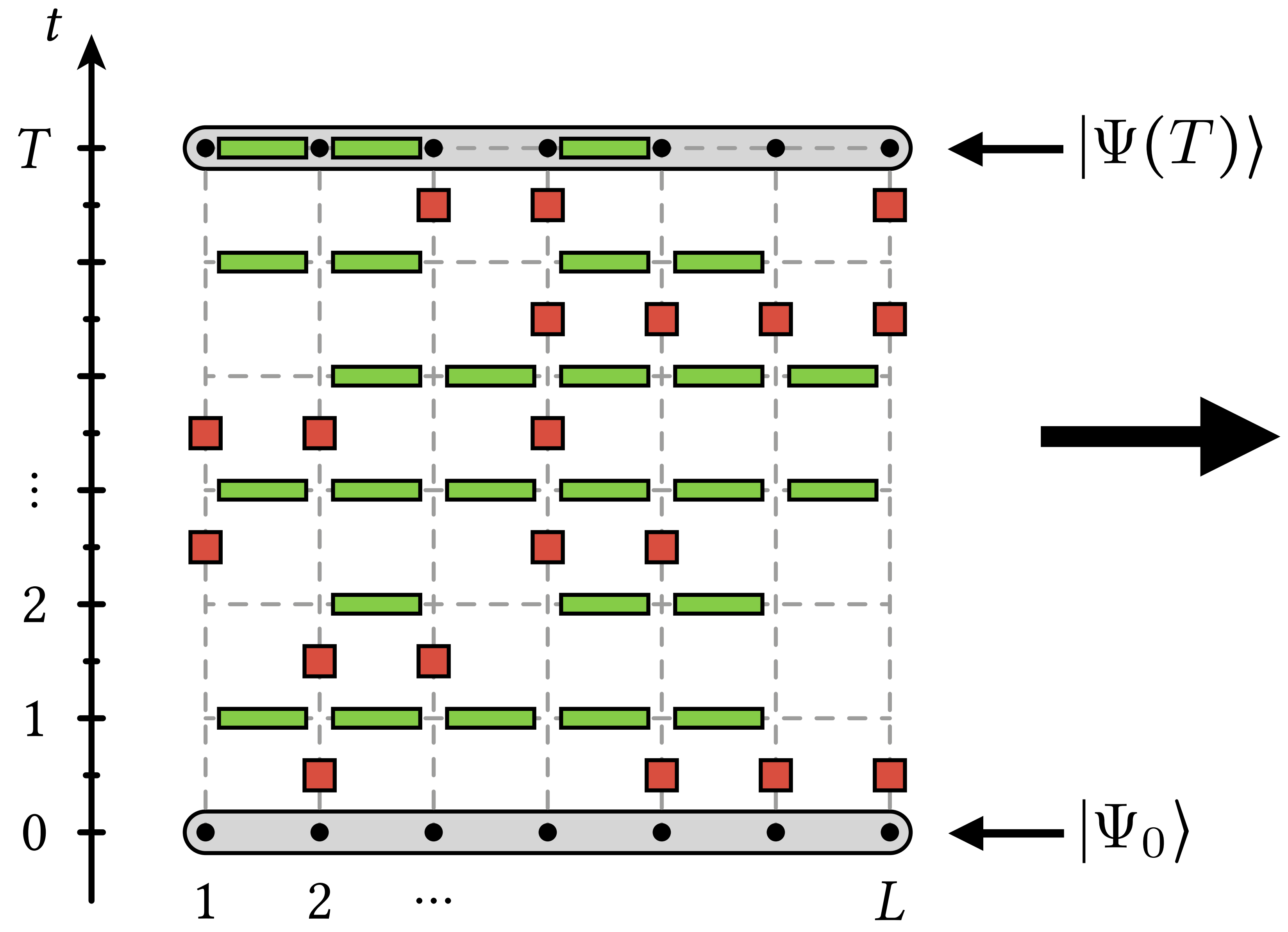
Nicolai Lang* and Hans Peter Büchler
Institute for Theoretical Physics III and Center for Integrated Quantum Science and Technology, University of Stuttgart, 70550 Stuttgart, Germany

(Received 17 June 2020; revised 8 September 2020; accepted 9 September 2020; published 23 September 2020)

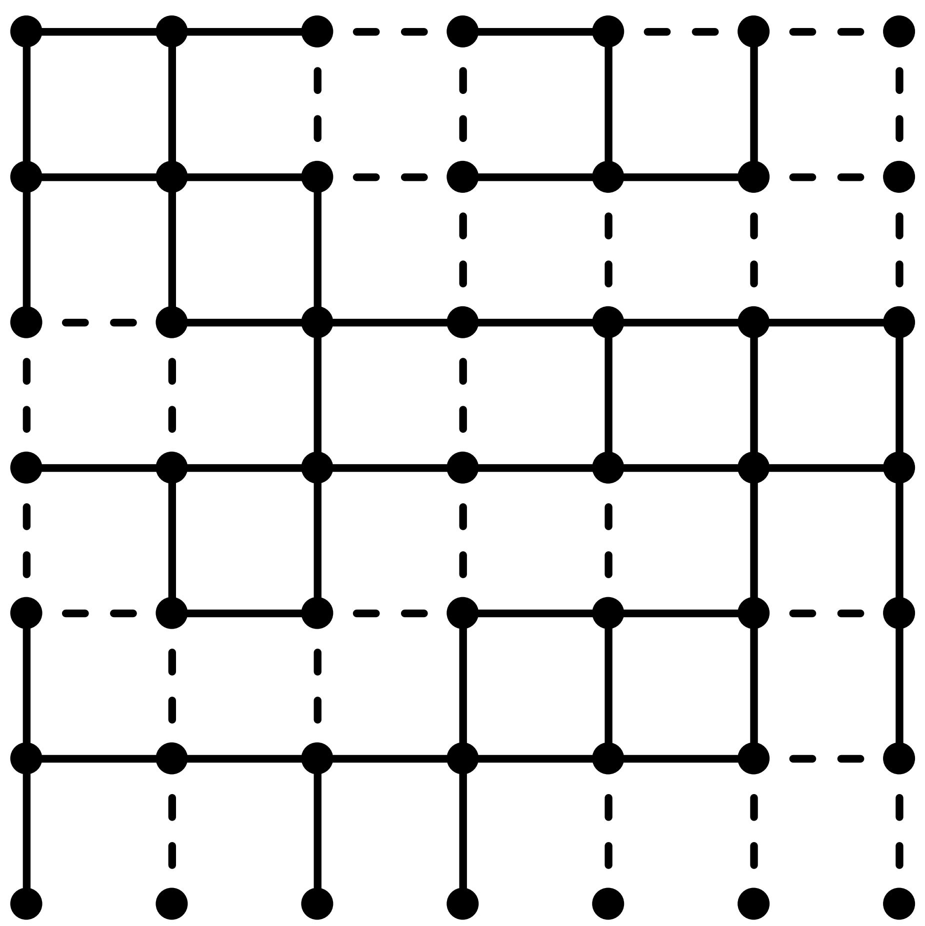
Nahum & Skinner, PRR **2**, 023288 (2020)

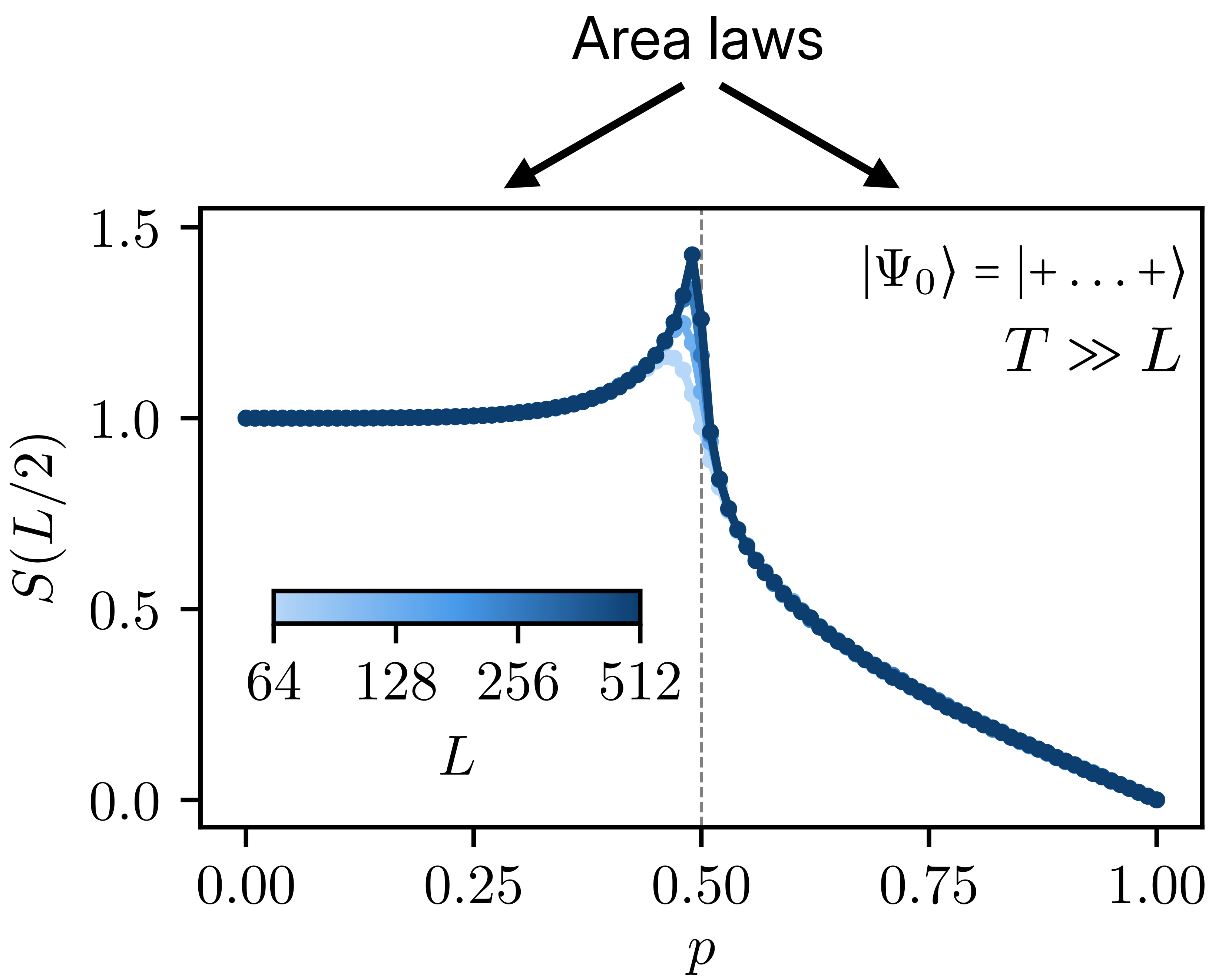
■ : X_i measurement (probability p)

▬ : $Z_i Z_{i+1}$ measurement (probability $1-p$)



Bond percolation



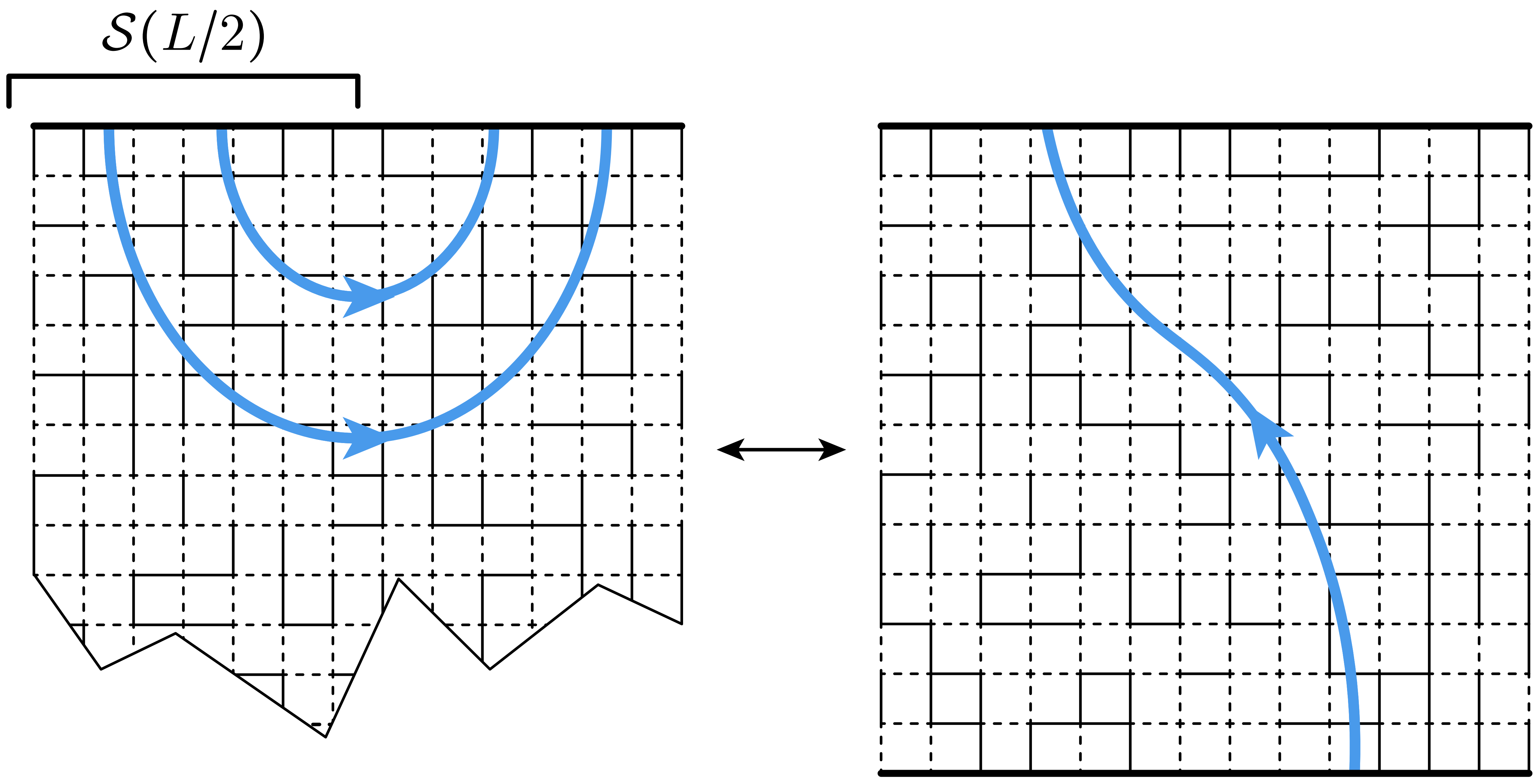


Examples:

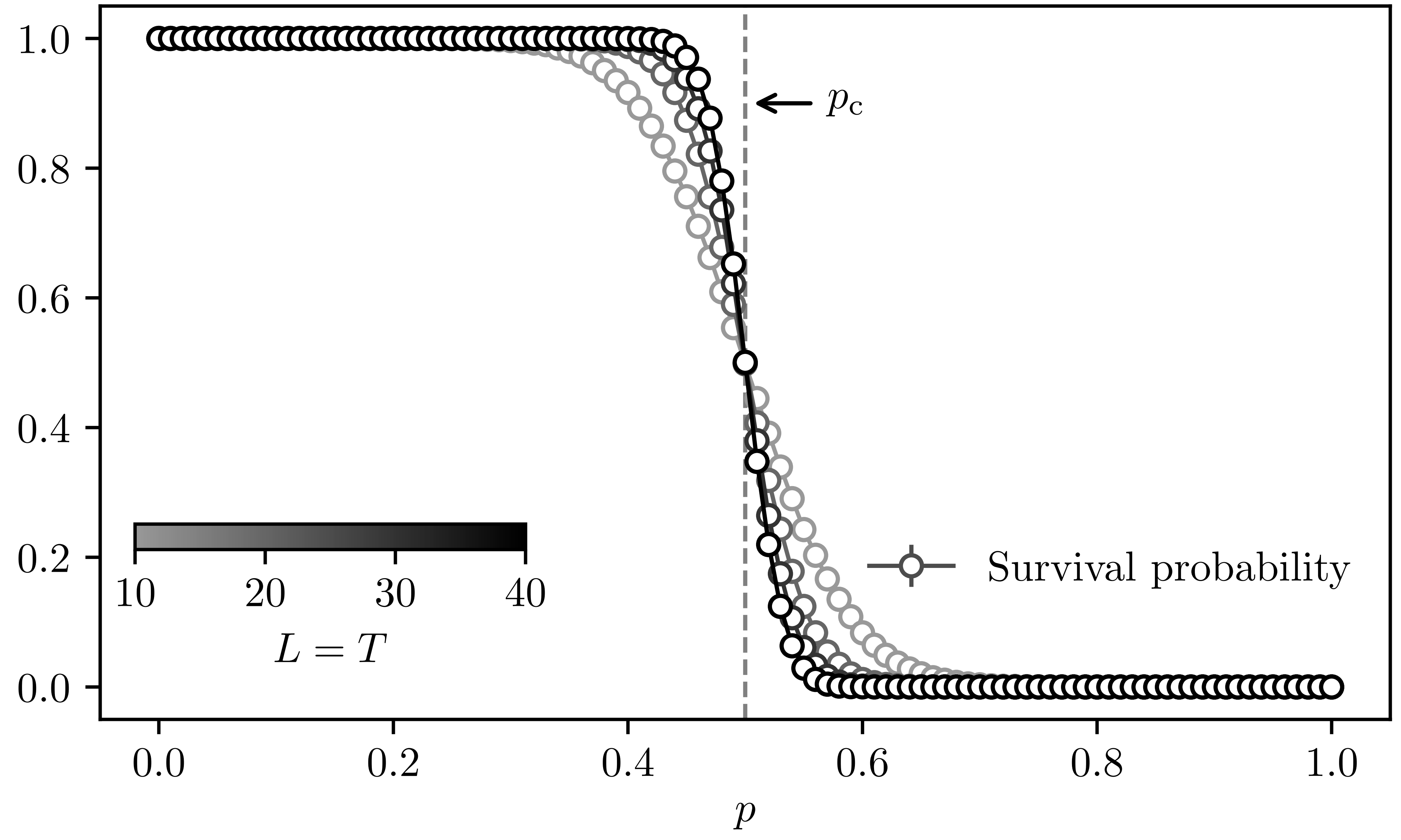
$$\frac{|00\rangle + |11\rangle}{\sqrt{2}} \longrightarrow \boxed{X_1} \rightarrow |++\rangle \text{ or } |--\rangle$$

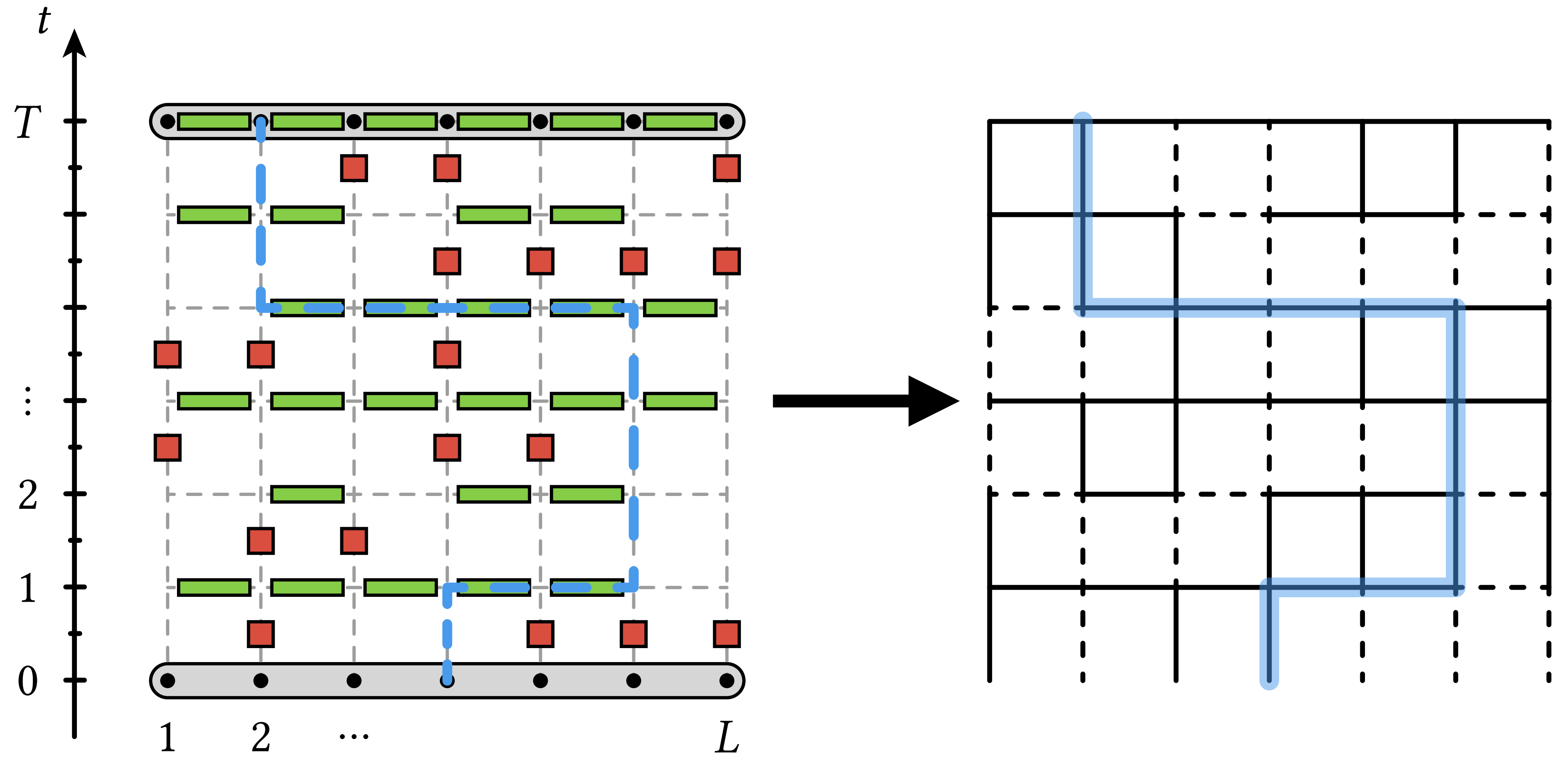
$$|++\rangle \longrightarrow \boxed{Z_1 Z_2} \rightarrow \frac{|00\rangle + |11\rangle}{\sqrt{2}} \text{ or } \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

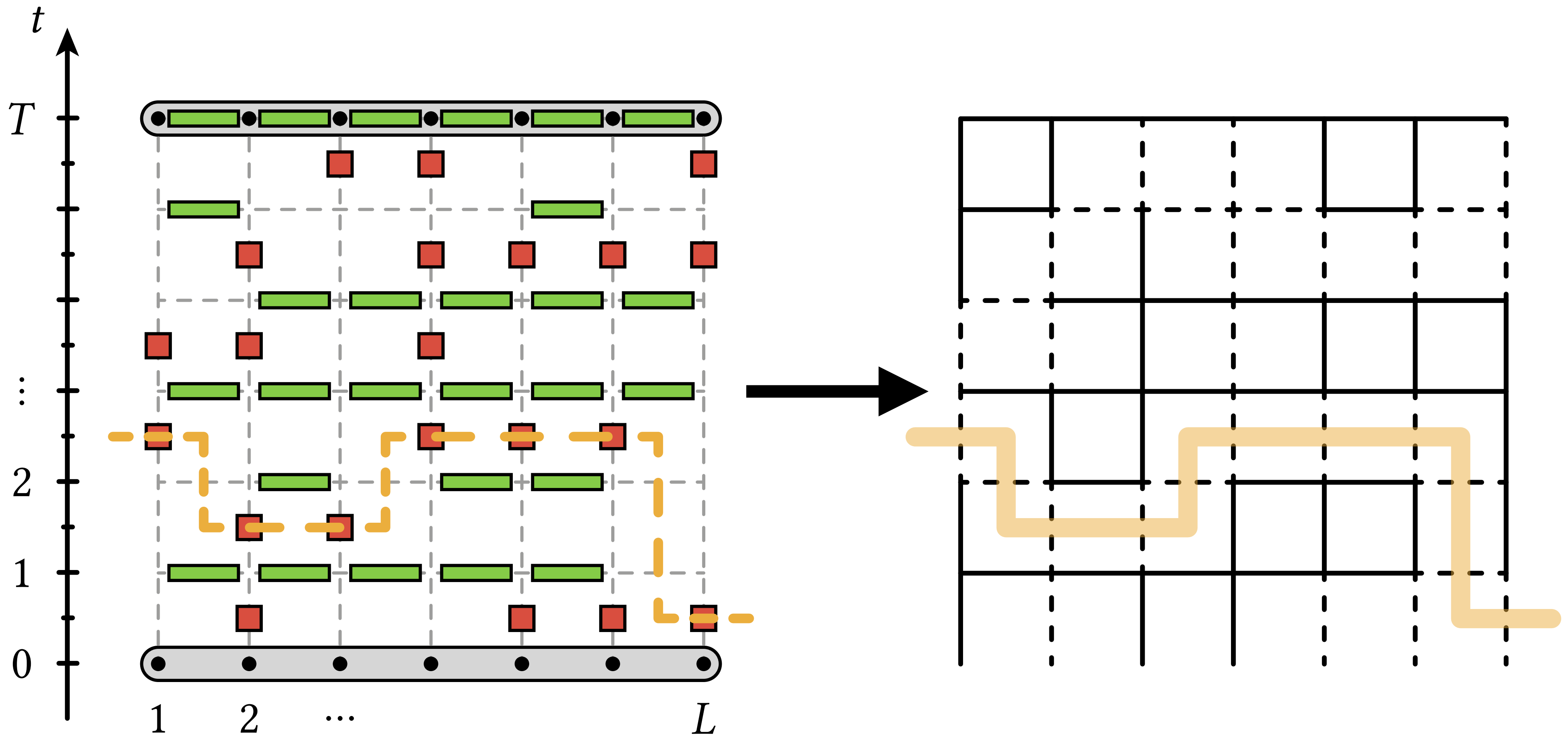
Distinguishing PTIM phases



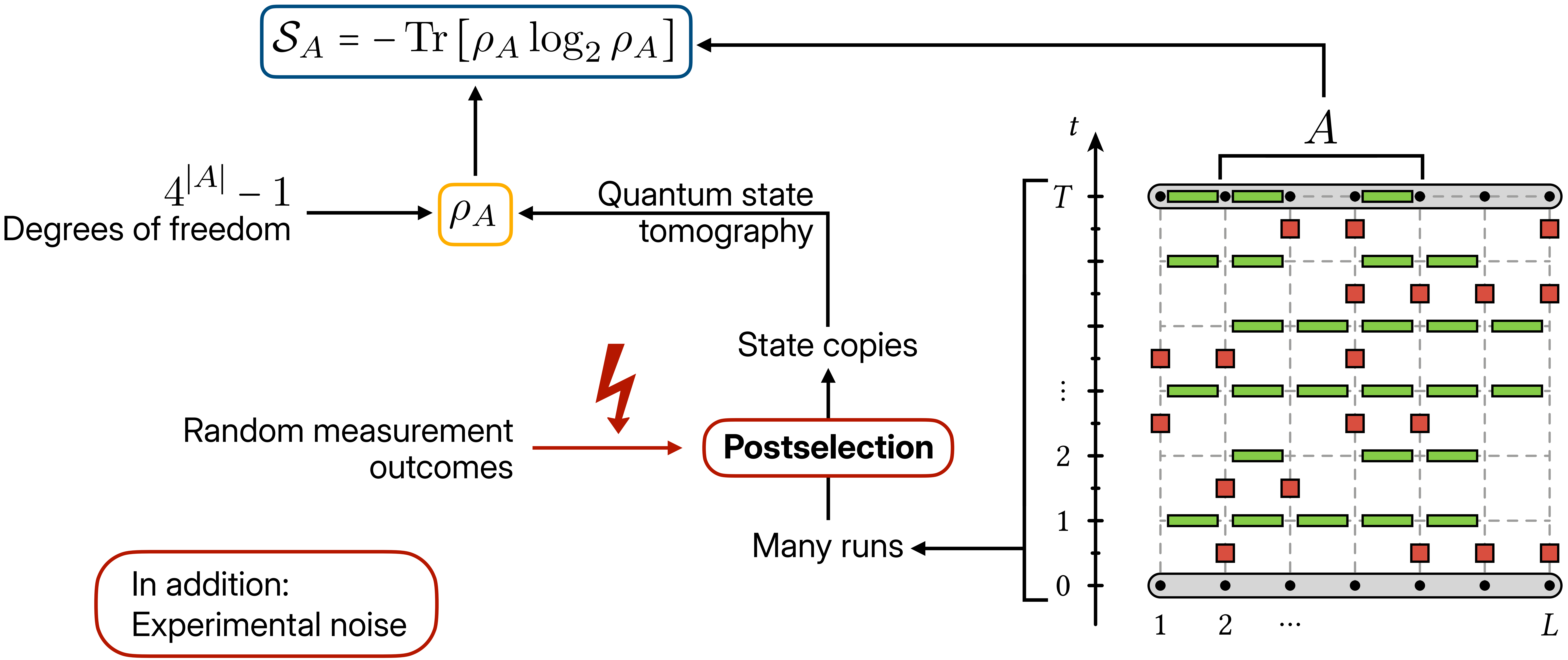
"Does the system remember its initial state?"





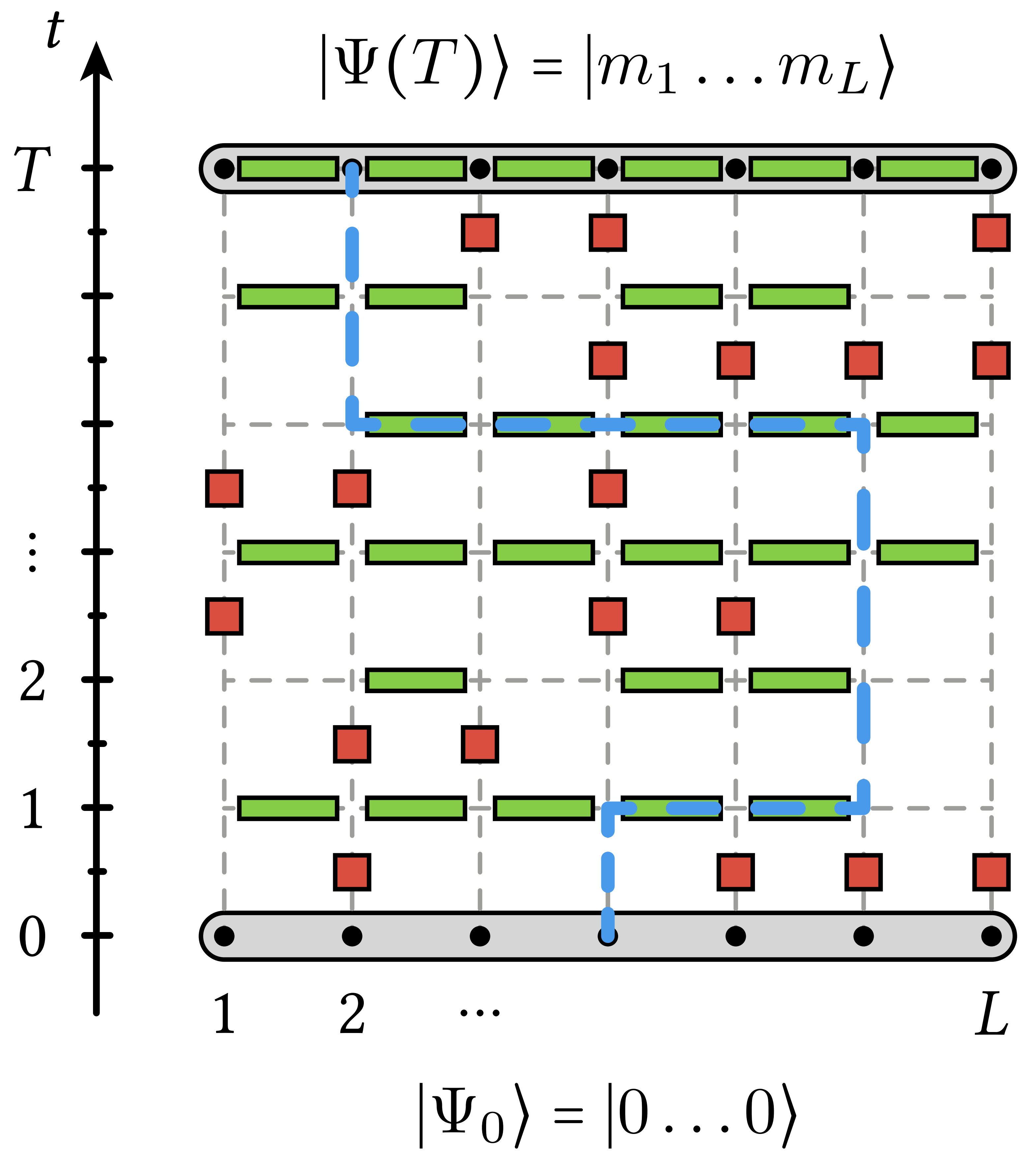


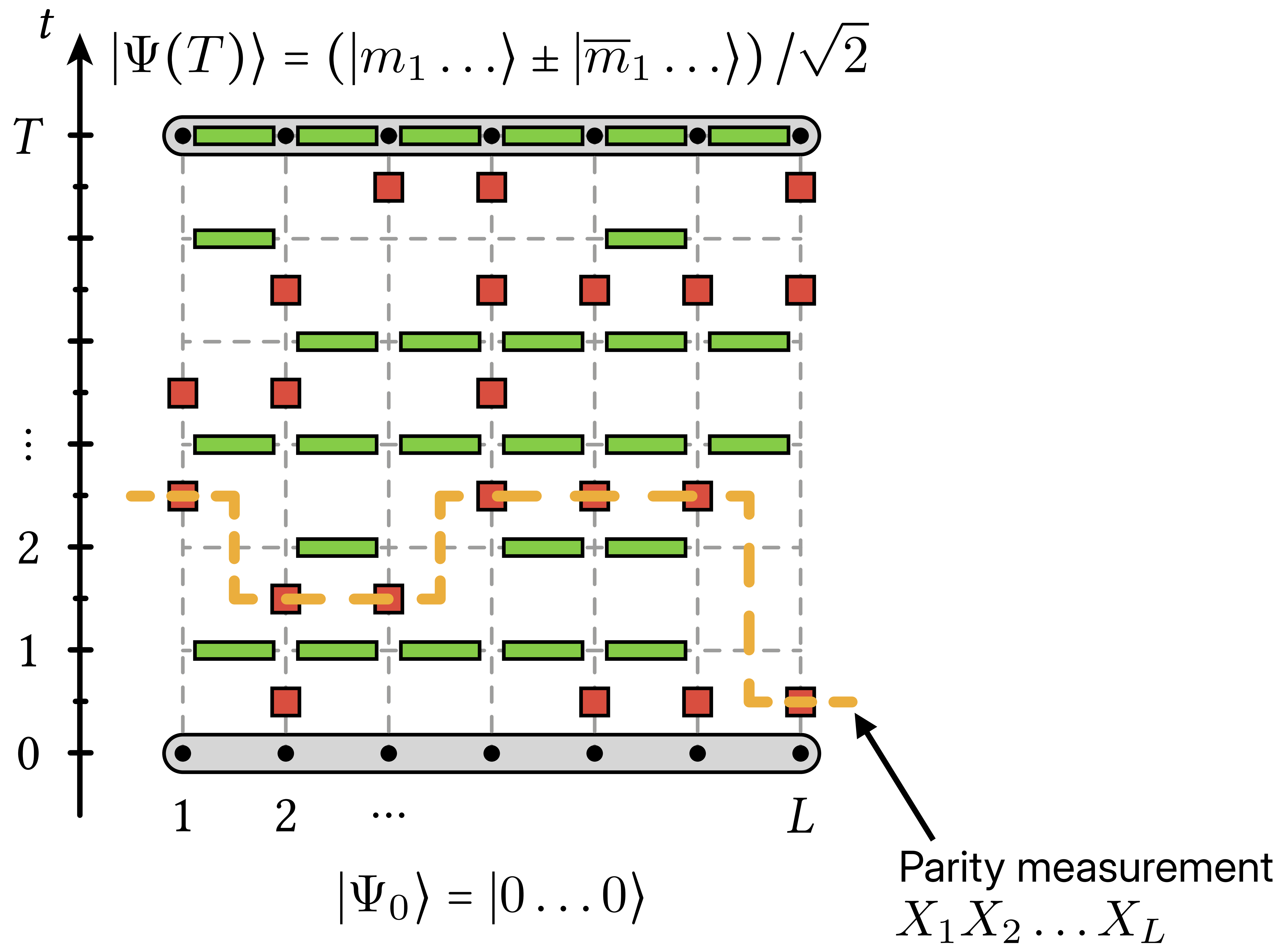
Measuring entanglement is difficult!



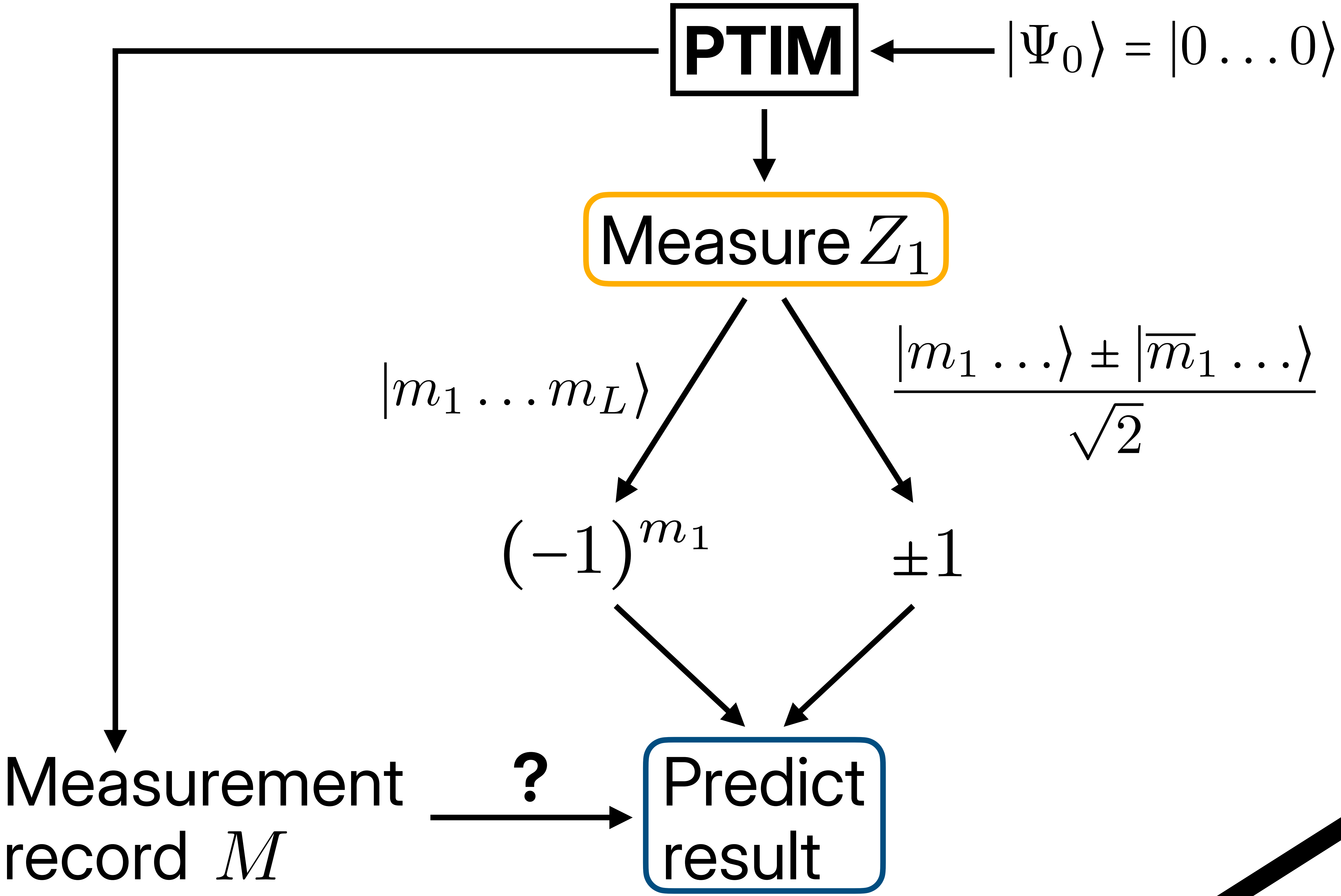
PTIM as quantum memory

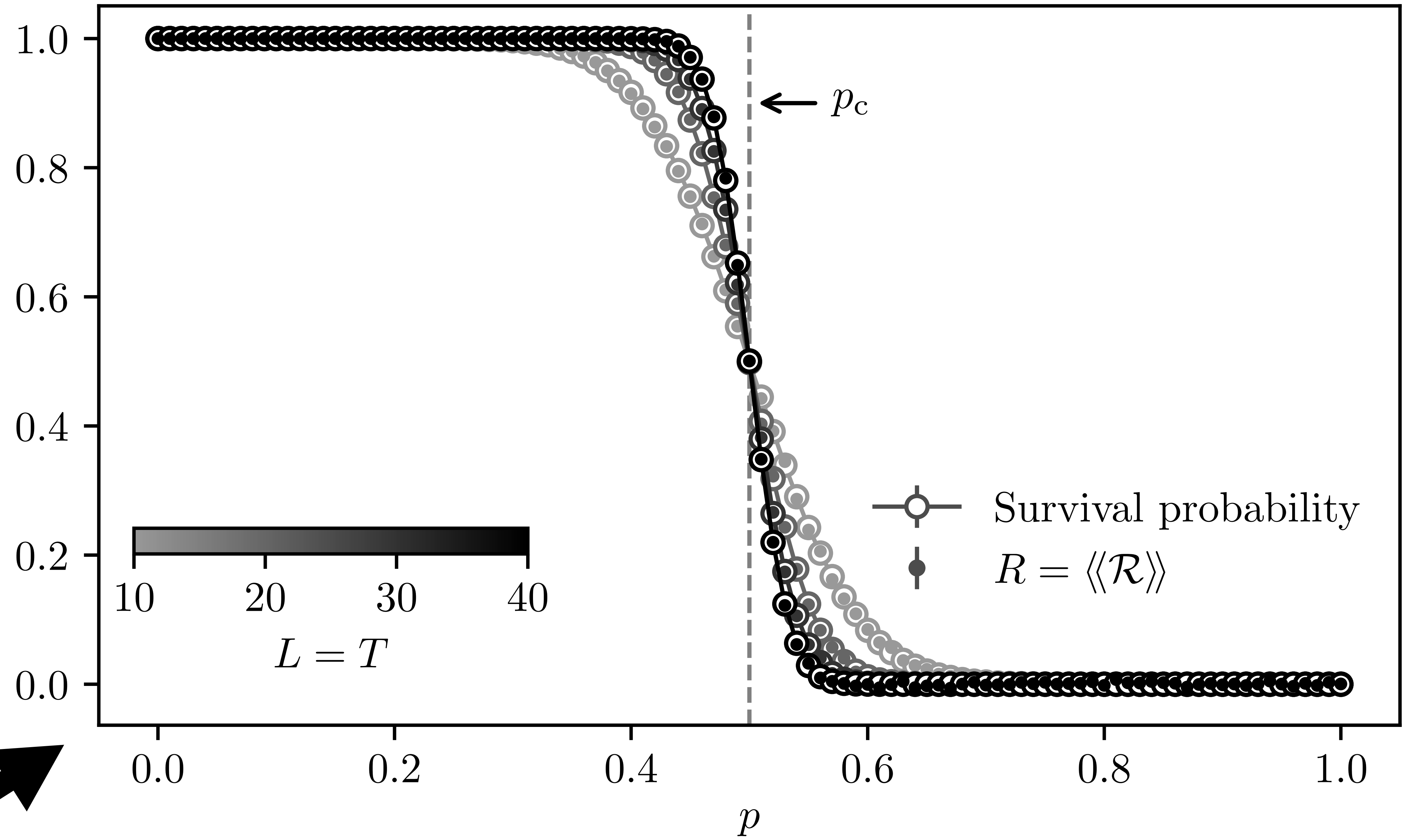
Gullans & Huse, PRL **125**, 070606 (2019)
Gullans & Huse, PRX **10**, 041020 (2020)
Choi, Bao, Qi & Altman, PRL **125**, 030505 (2020)
Roser, Büchler, Lang, PRB **107**, 214201 (2023)





Detecting the PTIM transition





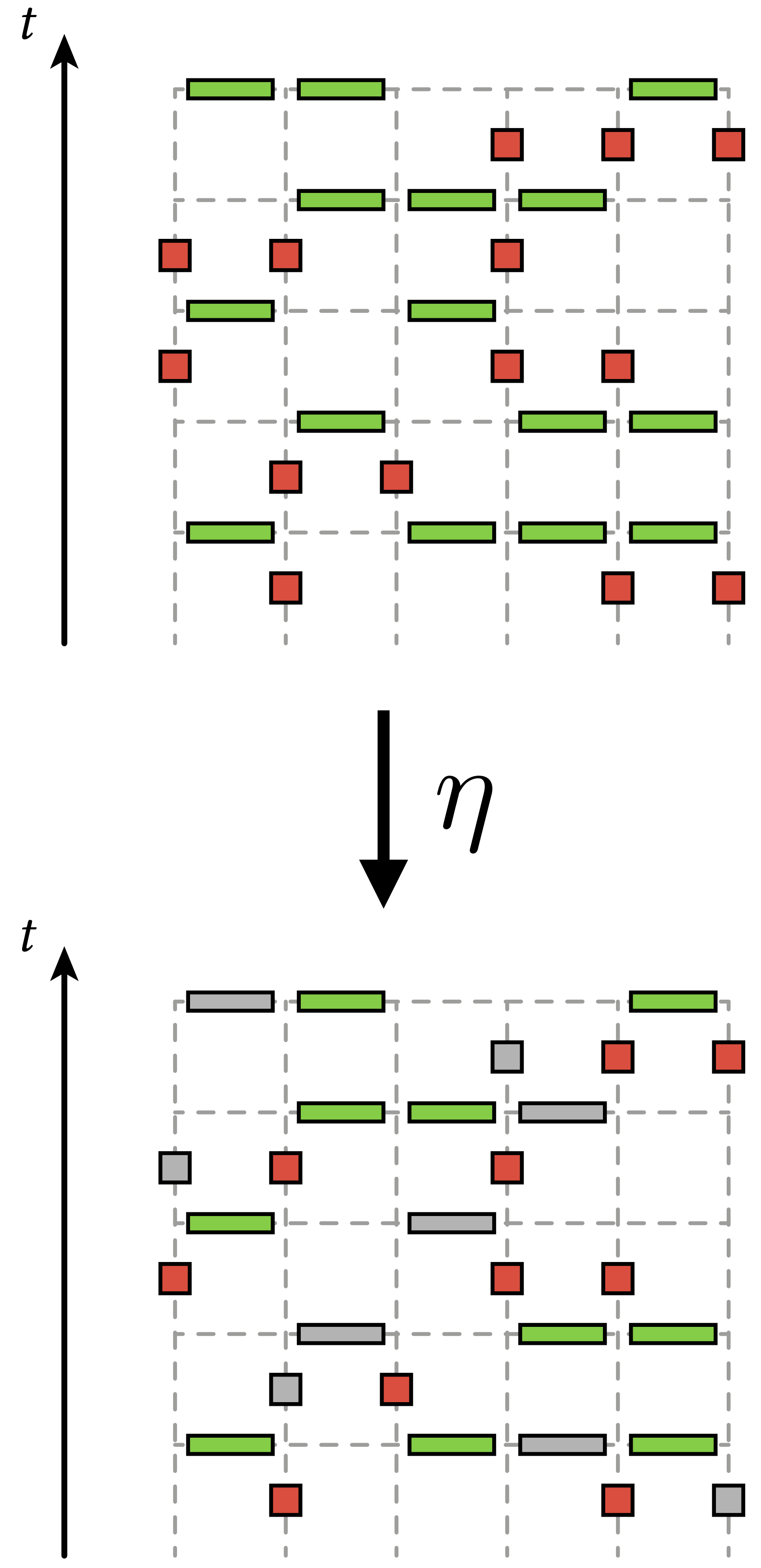
Correlation function: $\mathcal{R} = \begin{cases} +1 & \text{correct prediction} \\ -1 & \text{incorrect prediction} \end{cases}$

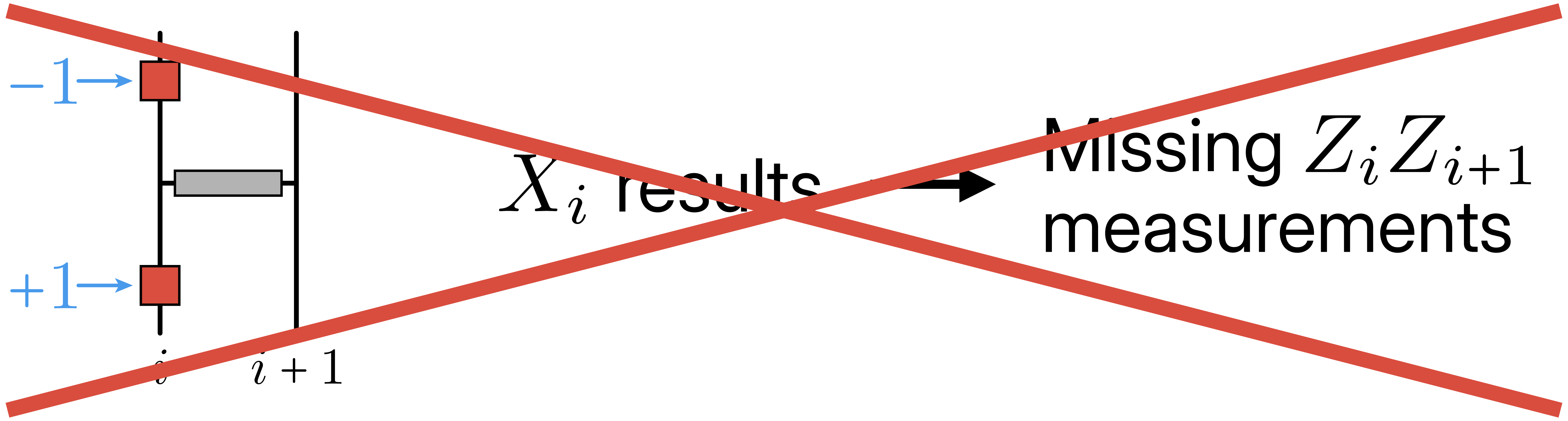
- Can this be measured? ✓
- Can this be scaled? ✓
- Is this robust under noise? ✗

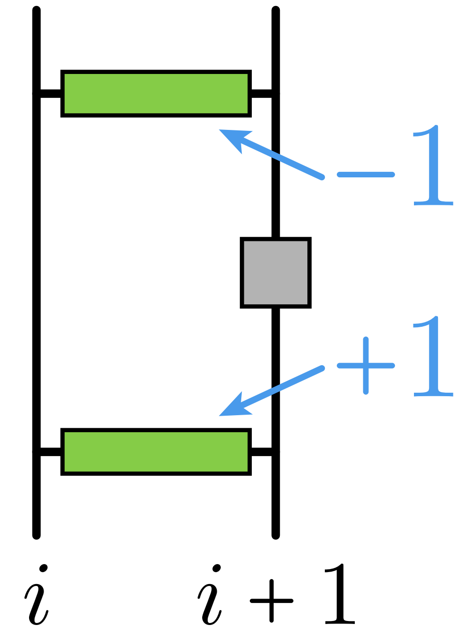
PTIM with noise

Error correction

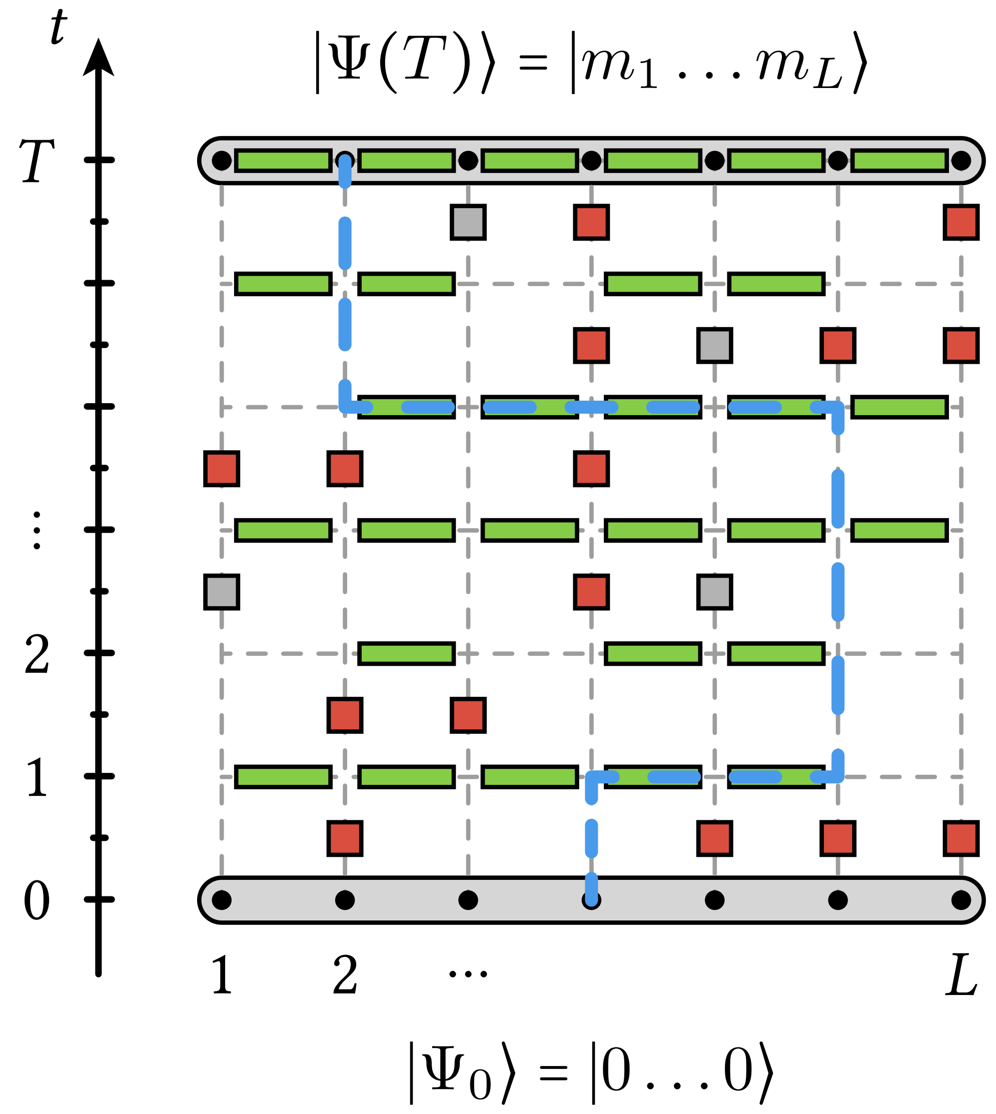
Noise model:







$Z_i Z_{i+1}$ results → Missing X_i measurements

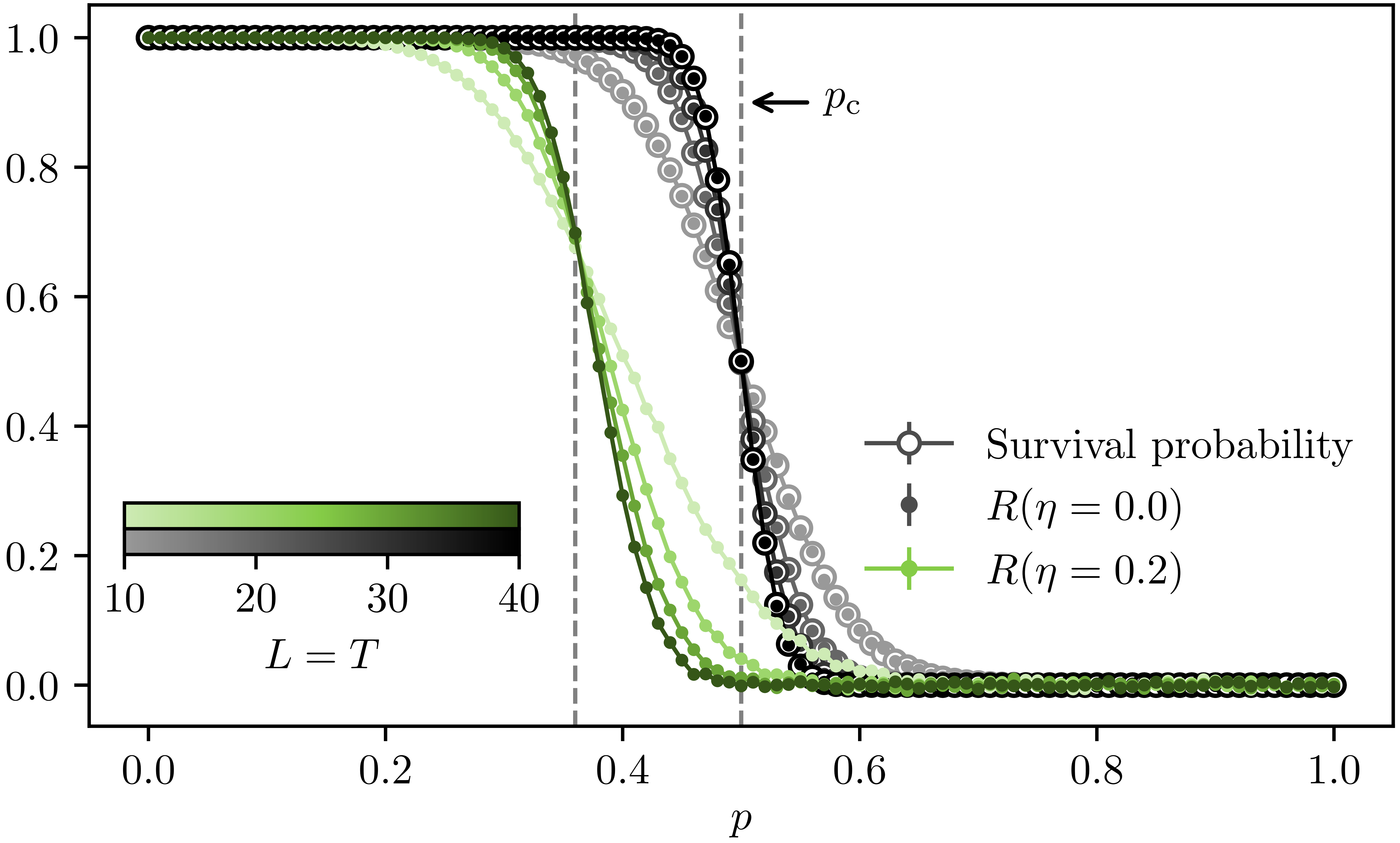


Augment measurement record

→ Find least number of additional measurements

Dennis, Kitaev, Landahl & Preskill, arXiv: quant-ph/0110143
Bravyi, Suchara & Vargo, PRA **90**, 032326 (2014)
Roser, Büchler & Lang, PRB **107**, 214201 (2023)

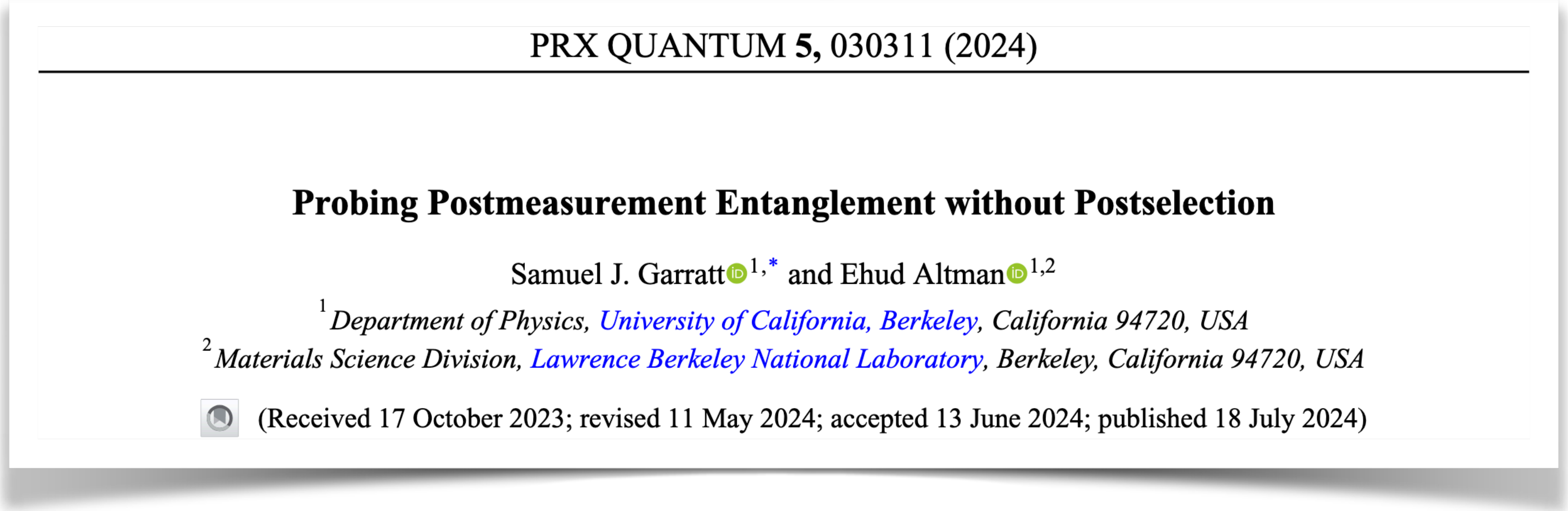
Accessible lower bound for the entanglement transition



Can we also measure an upper bound?

Upper-bounding entanglement

What if a single run only yields limited information?


PRX QUANTUM 5, 030311 (2024)

Probing Postmeasurement Entanglement without Postselection

Samuel J. Garratt[1,*] and Ehud Altman[1,2]

[1] *Department of Physics, University of California, Berkeley, California 94720, USA*
[2] *Materials Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*

(Received 17 October 2023; revised 11 May 2024; accepted 13 June 2024; published 18 July 2024)

Cannot be measured → $S_A = \langle\langle -\mathrm{Tr}\left[\rho_A \log_2 \rho_A\right]\rangle\rangle \leq \langle\langle -\mathrm{Tr}\left[\rho_A^{\mathrm{S}} \log_2 \rho_A^{\mathrm{C}}\right]\rangle\rangle = S_A^{\mathrm{SC}}$ ← Can be measured

Shadow $\langle\langle \rho_A^{\mathrm{S}} \rangle\rangle \to \rho_A$

$$\rho_i^{\mathrm{S}} = \frac{3}{2}\left(\mathbb{1} + O_i \cdot o\right) - \mathbb{1}$$

Random Pauli measurement
$O_i \in \{X_i, Y_i, Z_i\}$

Measurement result
$o \in \{\pm 1\}$

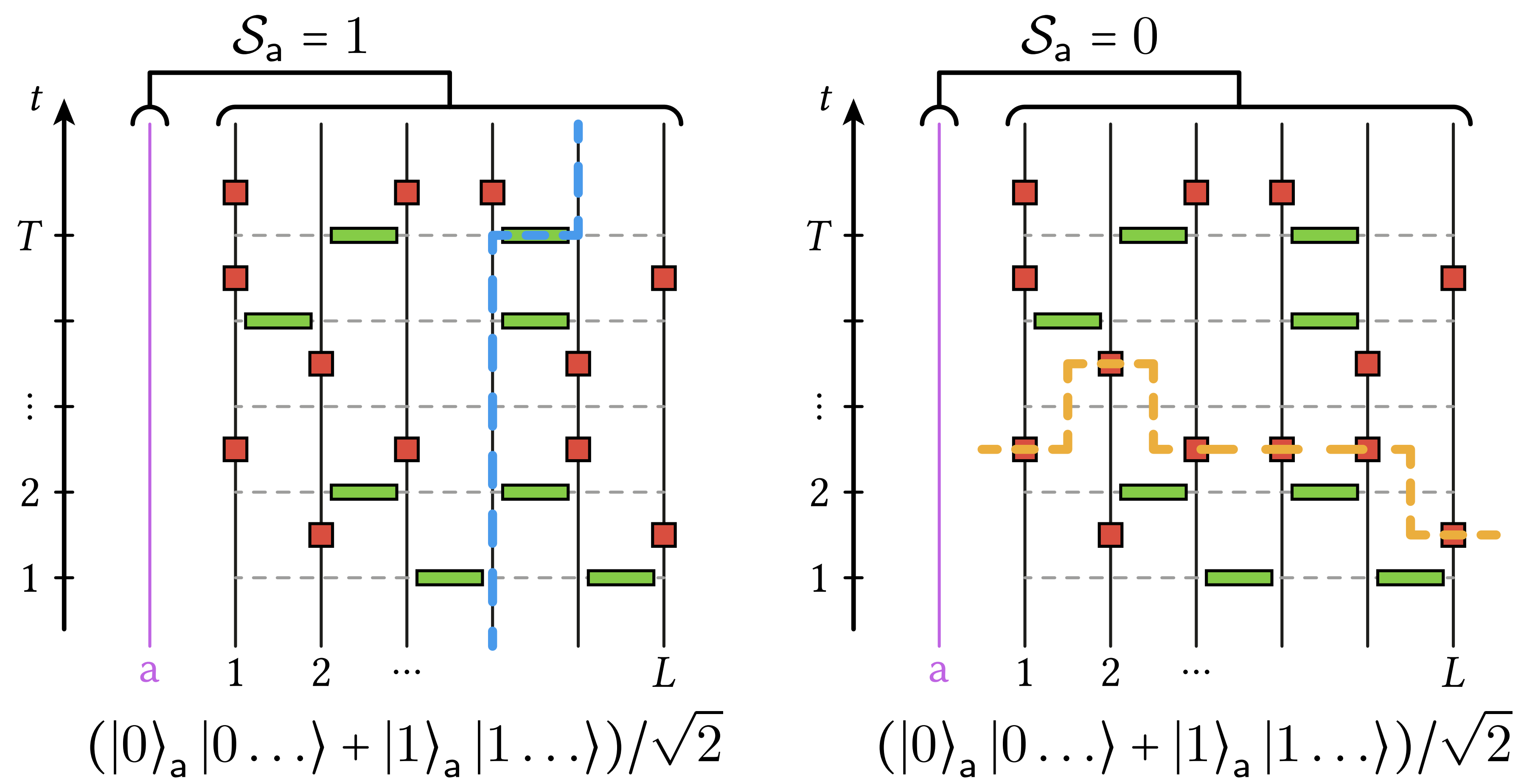
Classical state prediction $\rho_A^{\mathrm{C}}(M)$

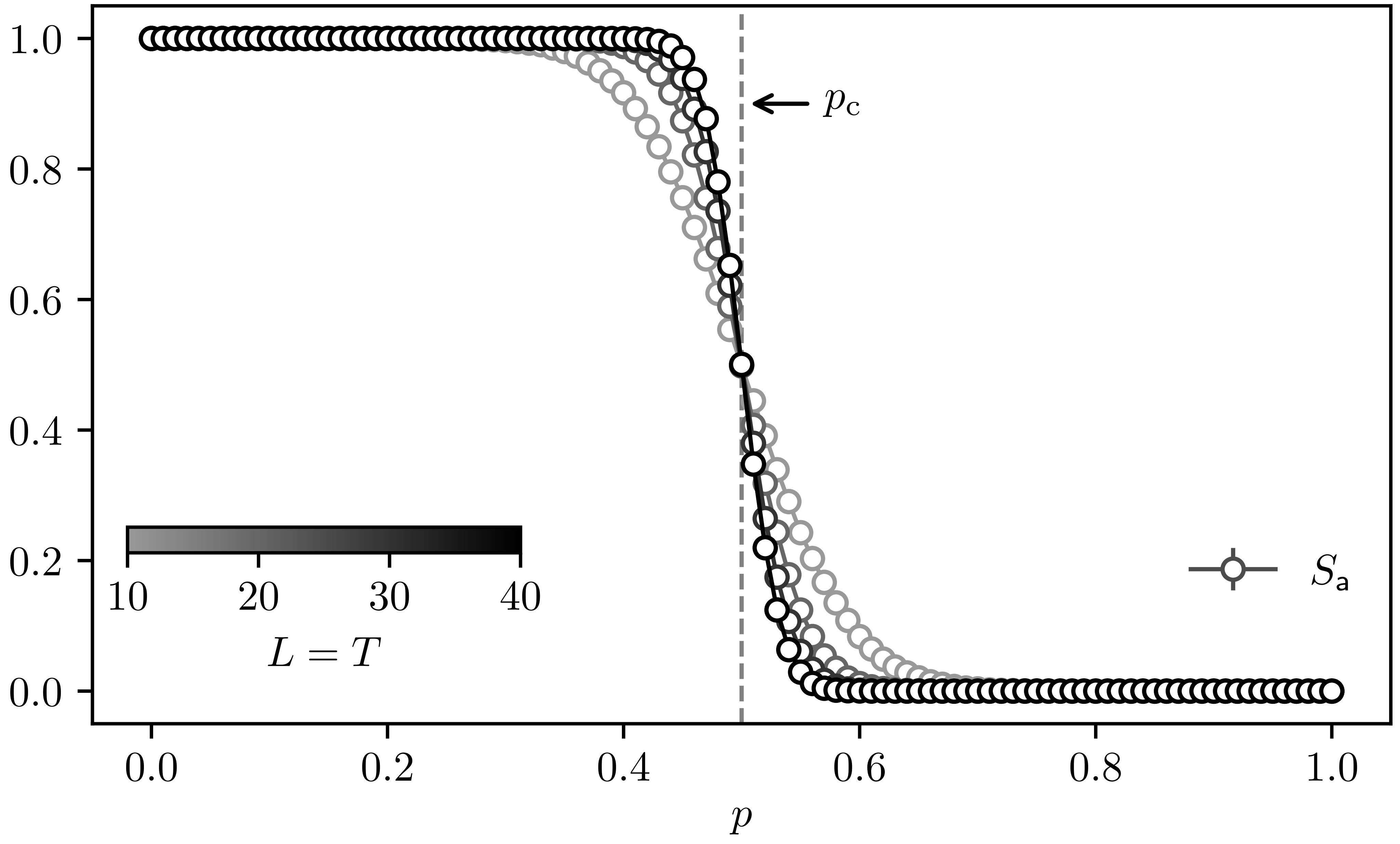
Depolarizing:

$$\rho_A^{\mathrm{C}}(M, \varepsilon) = (1 - \varepsilon)\, \rho_A^{\mathrm{C}}(M) + \frac{\varepsilon}{2^{|A|}} \mathbb{1}$$

- Errors diverge for large subsystems A!
- Not robust under noise!
- Entanglement entropy $\neq$ entanglement transition!

Upper-bounding the entanglement transition





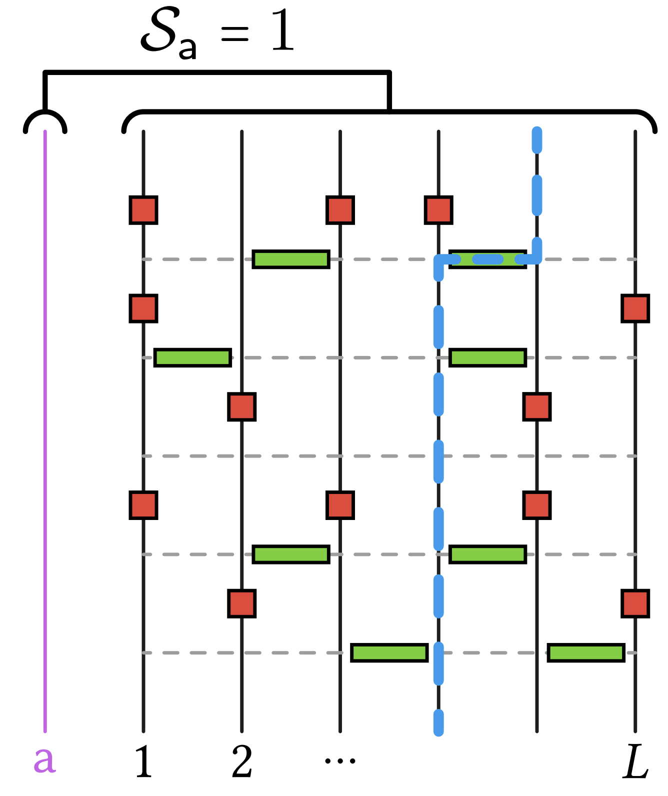
Upper bound

$$S_a \leq \langle\!\langle -\operatorname{Tr}\left[\rho_a^S \log_2 \rho_a^C\right]\rangle\!\rangle$$

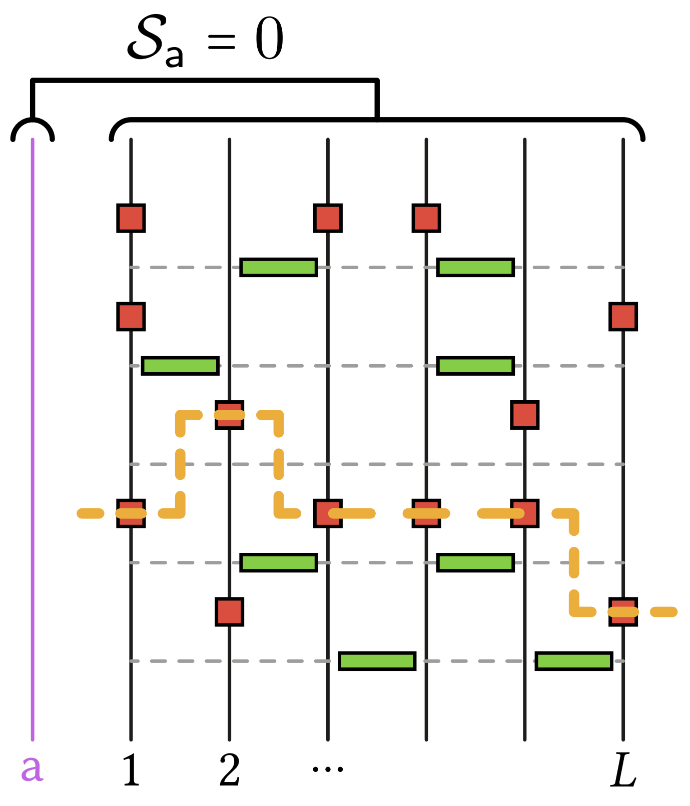
Random shadow measurement
$O_a \in \{X_a, Y_a, Z_a\}$

Robust state prediction
$\rho_a^C(M, \varepsilon)$

Robust state prediction



$\longrightarrow \rho_a^C = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$



$\longrightarrow \rho_a^C$ pure

$|\Psi(T)\rangle$

Eigenstate: $X_a X_1 \dots X_L$

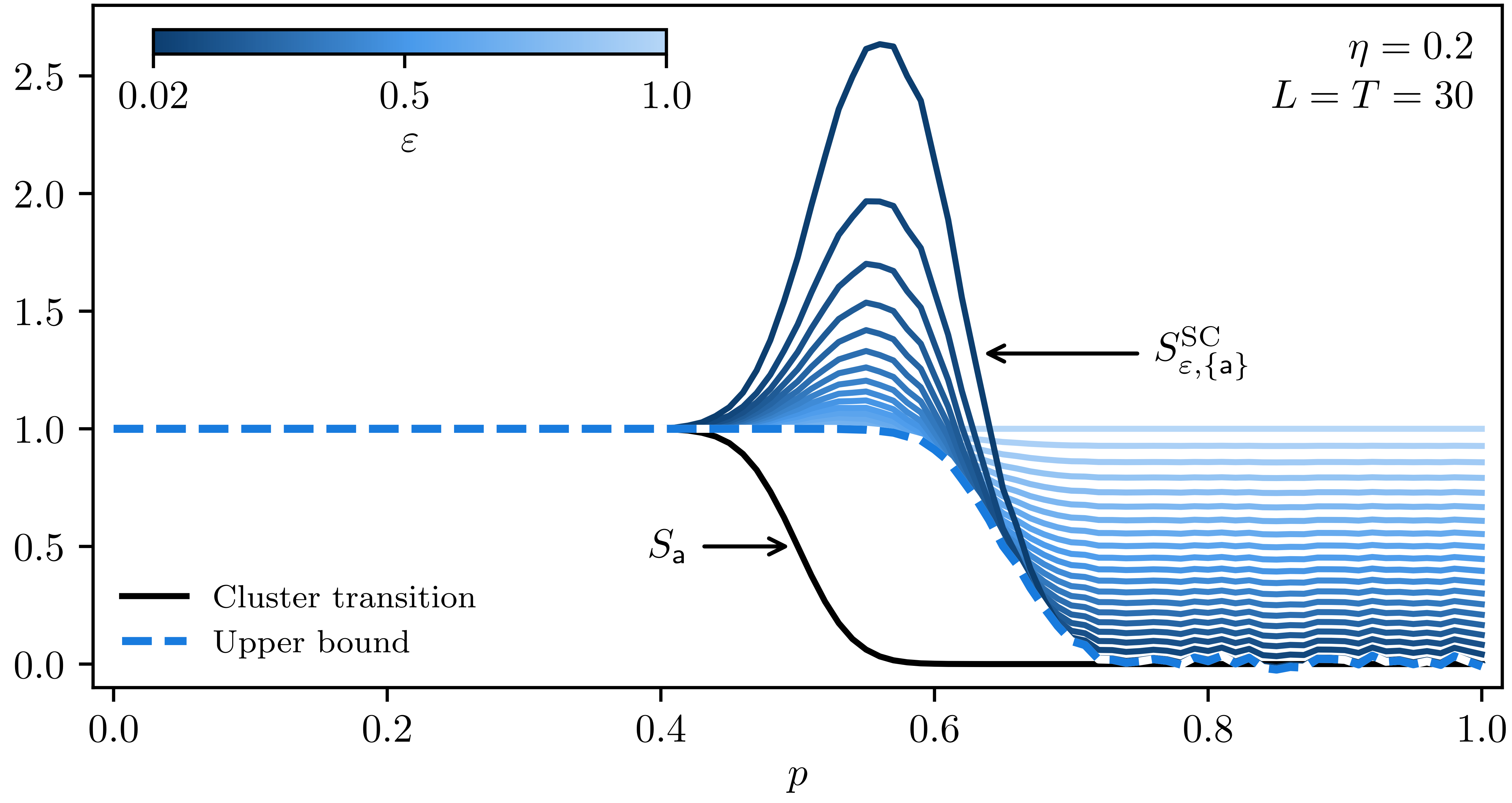
Eigenstate: $X_1 \dots X_L$

$\eta > 0$

Error correction:

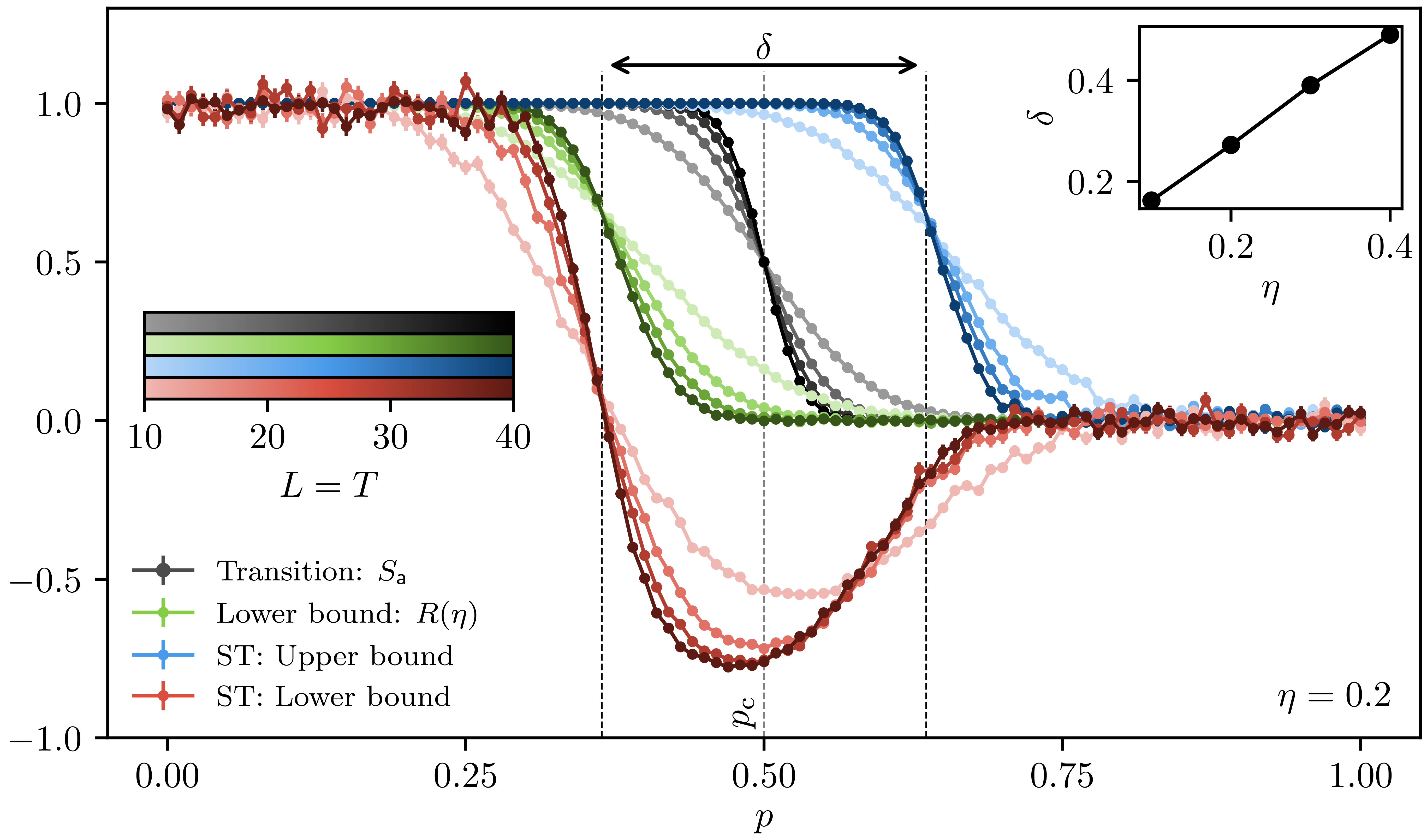
X_i-results $\longrightarrow$ Predicted $Z_i Z_{i+1}$ measurements

$\longrightarrow$ Classical prediction $\rho_a^C(M, \varepsilon)$



Summary

Experimental detection of the PTIM's entanglement transition



- **Accessible lower bound** via decoding
- **Accessible upper bound** via shadow tomography with error correction
- Sharpness δ of the bounds measures the noise rate η

Outlook

- This procedure should work for different noise models

  ➡ If they are symmetric under $X_1 X_2 \cdots X_L$

  ➡ Excluding faulty measurements (incorrect outcomes)

- The protocol should work on other models

  ➡ If they can be simulated efficiently (for state predictions)

  ➡ If efficient error correction algorithms are known (for robustness)