中国科学院理论物理研究所
Institute of Theoretical Physics, Chinese Academy of Sciences

CENTRO DE CIENCIAS DE BENASQUE PEDRO PASCUAL

Quantum Supercritical Regime with Universal Magnetocaloric Scaling in Ising model

Enze Lv, Institute of Theoretical Physics (ITP), China

E. Lv, N. Xi, Y. Jin, WL, Nature Commun. 16, 10646 (2025)

J. Wang, **E. Lv**, X. Li, Y. Jin, WL, PRB, 112, 195120 (2025)

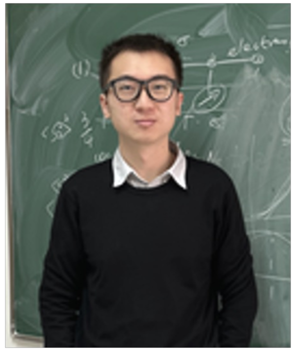
X. Liu, **E. Lv**, J. Xiang, WL, arXiv:2601.07810 (2026)

Collaborators

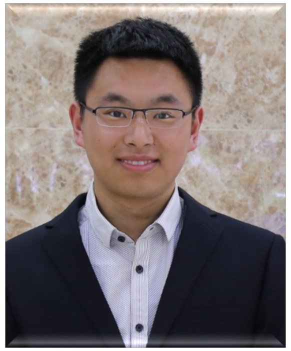
Condensed Matter Physics

ITP

Prof. Wei Li



Junsen Wang
Postdoc



Ning Xi
Postdoc

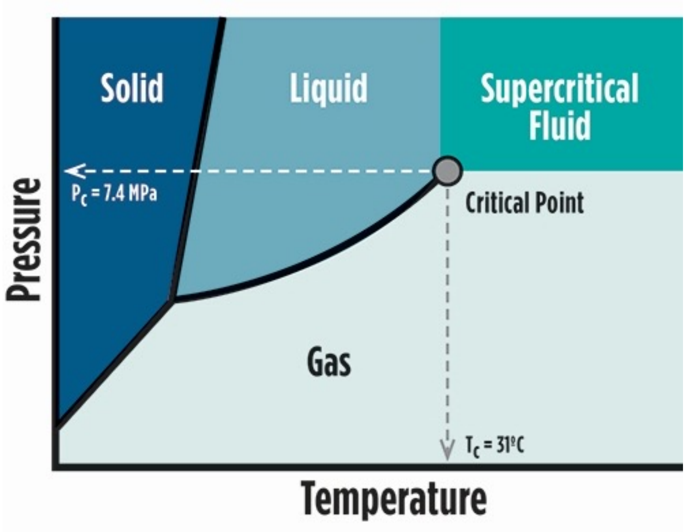
Statistical Physics

ITP

Prof. Yuliang Jin

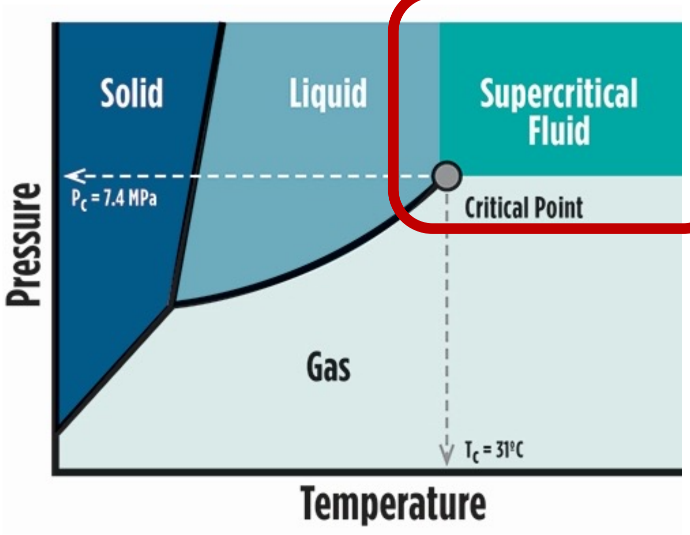
Xinyang Li
Postdoc

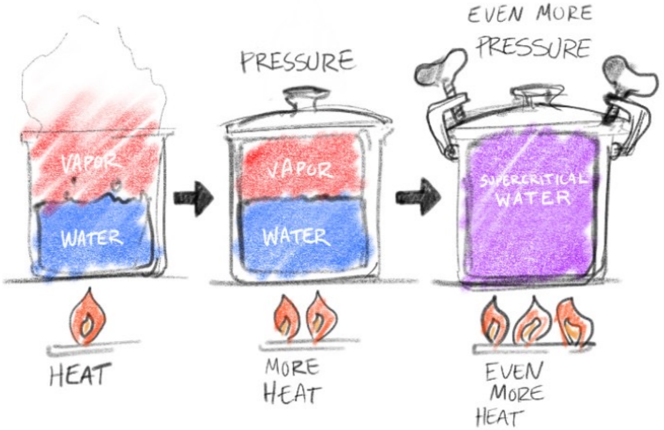
- **Supercriticality in Liquid-Gas System**



Supercritical Fluid in Liquid-Gas Phase Diagram

- Supercriticality: beyond the liquid-gas thermal critical point





- Supercritical fluid: intriguing state of matter

> Strong thermal fluctuations
> Sensitive to the external field (e.g., pressure)
> Scaling law and universal behavior

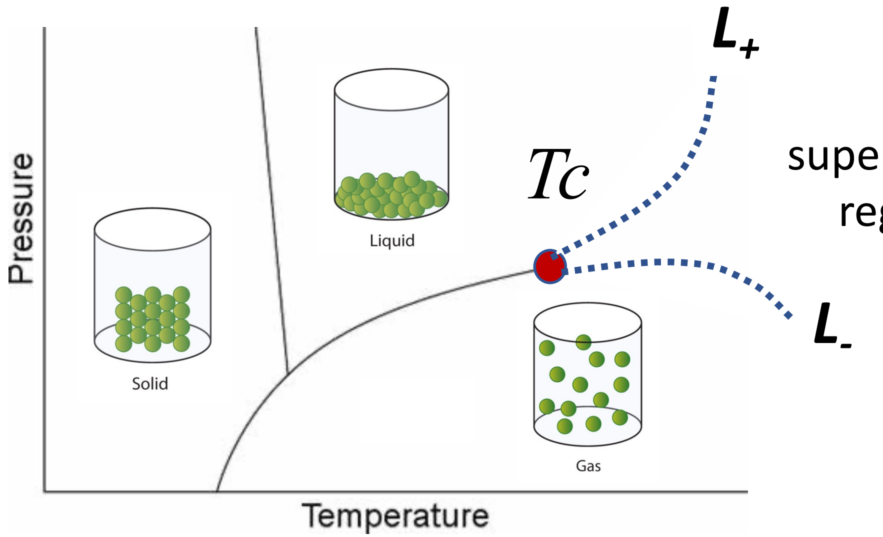
and extremly useful ...

> Chemical engineering
> Oil exploration
> Refrigeration
> Engine and generator

Supercritical Regime with Crossover Scaling

■ Liquid-gas transition

- Two crossover lines $|\delta P| \sim (T - T_c)^{\beta+\gamma}$

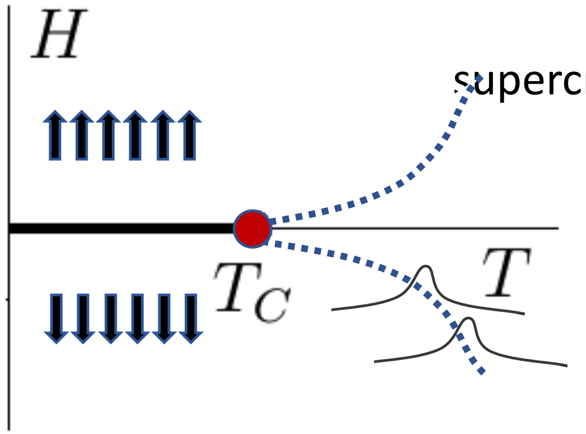


✓ ***response functions:*** *susceptibility*

Analogy

■ Ferromagnetic Ising transition

- Two crossover lines $h \sim (T - T_c)^{\beta+\gamma}$



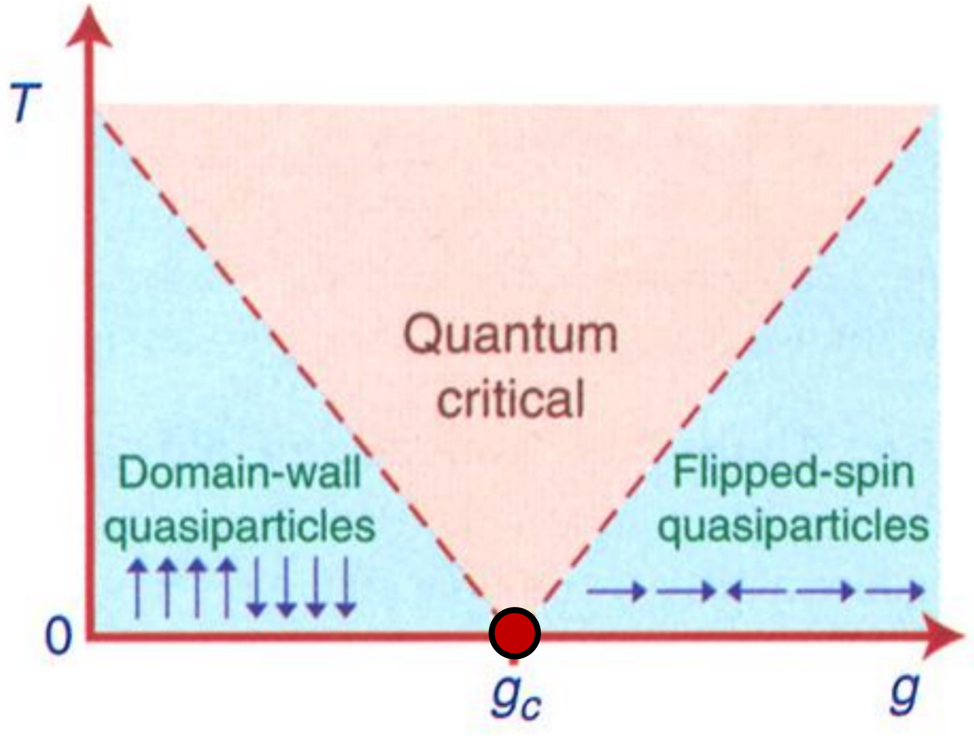
✓ ***response functions:*** *specific heat, magentic susceptibility, ...*

Li & Jin, PNAS 2024

β is critical exponent of order parameter
γ is critical exponent of susceptibility

Quantum Critical Regime and Crossover Scaling

Two crossover lines $(g - g_c) \sim T^{1/z\nu}$

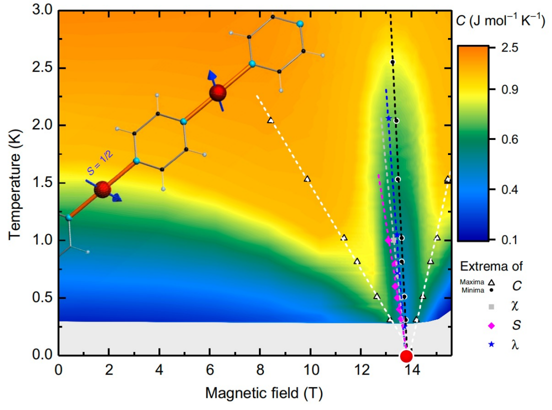


z is dynamic critical exponent
ν is correlation–length critical exponent

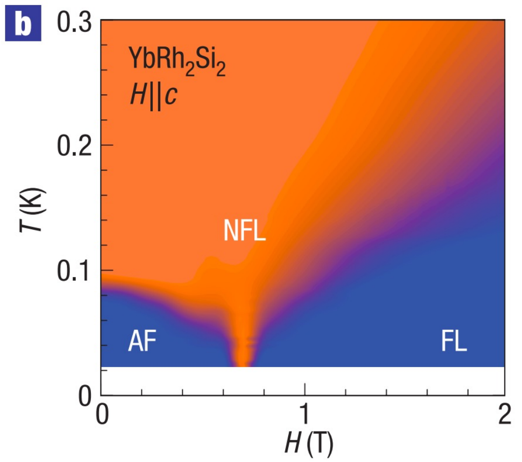
Sachdev Science (2000)

■ **Quantum criticaity**: intriging state in many systems.

- **O**riginate from the ***quantum critical point***
- **O**bey distinct ***quantum scaling***
- **O**bserved in many ***strongly correlated systems***



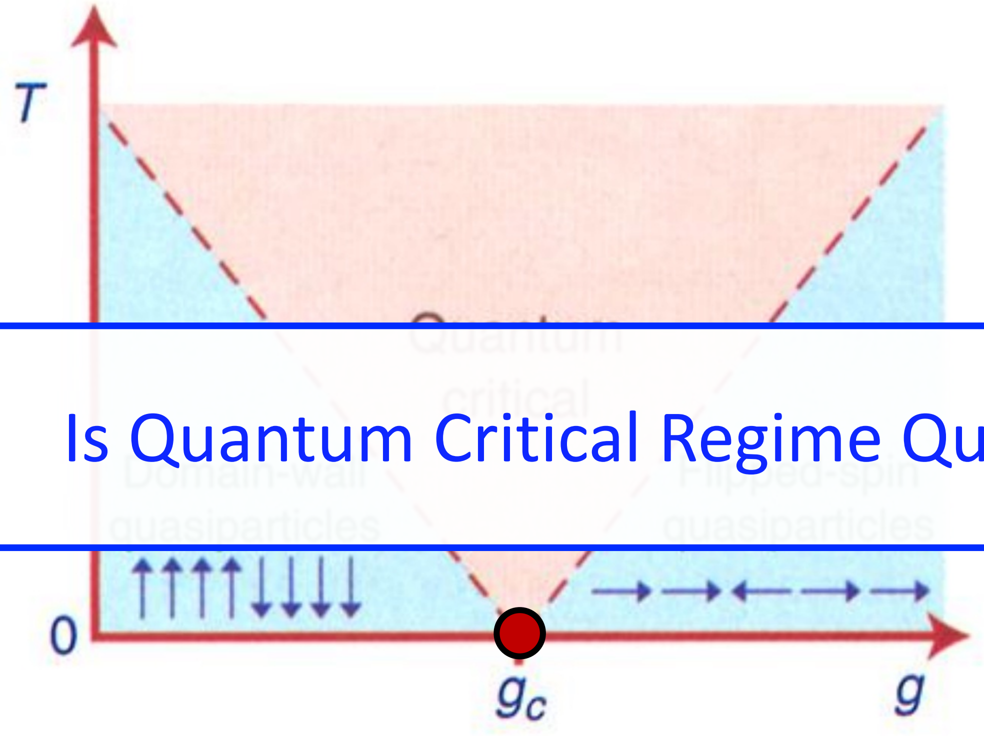
quantum magnet
(Breunig SA 2017)



heavy-fermion system
(Gegenwart NP 2008)

Quantum Critical Regime and Crossover Scaling

Two crossover lines $(g - g_c) \sim T^{1/z\nu}$



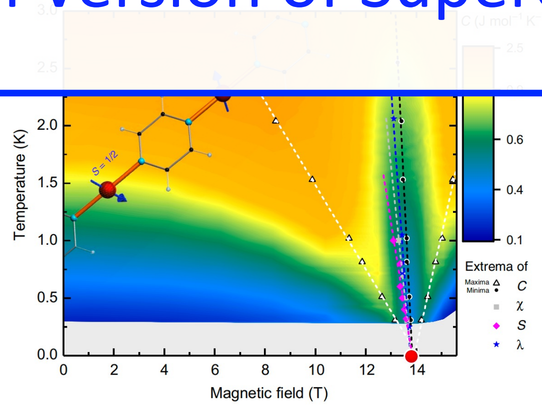
z is dynamic critical exponent
ν is correlation-length critical exponent

Sachdev Science (2000)

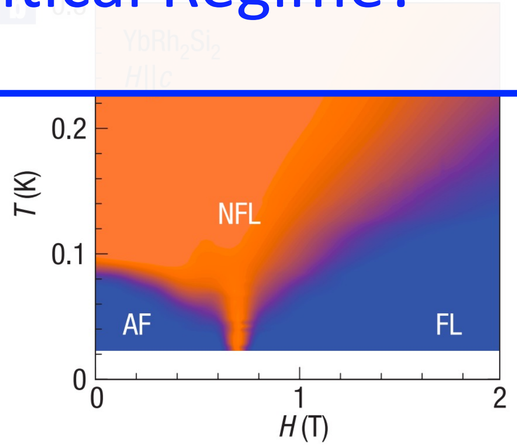
- **Quantum criticaity**: intriguing state in many systems.

> **O**riginate from the ***quantum critical point***
> **O**bey distinct ***quantum scaling***
> **O**bserved in many ***strongly correlated systems***

Is Quantum Critical Regime Quantum Version of Supercritical Regime?



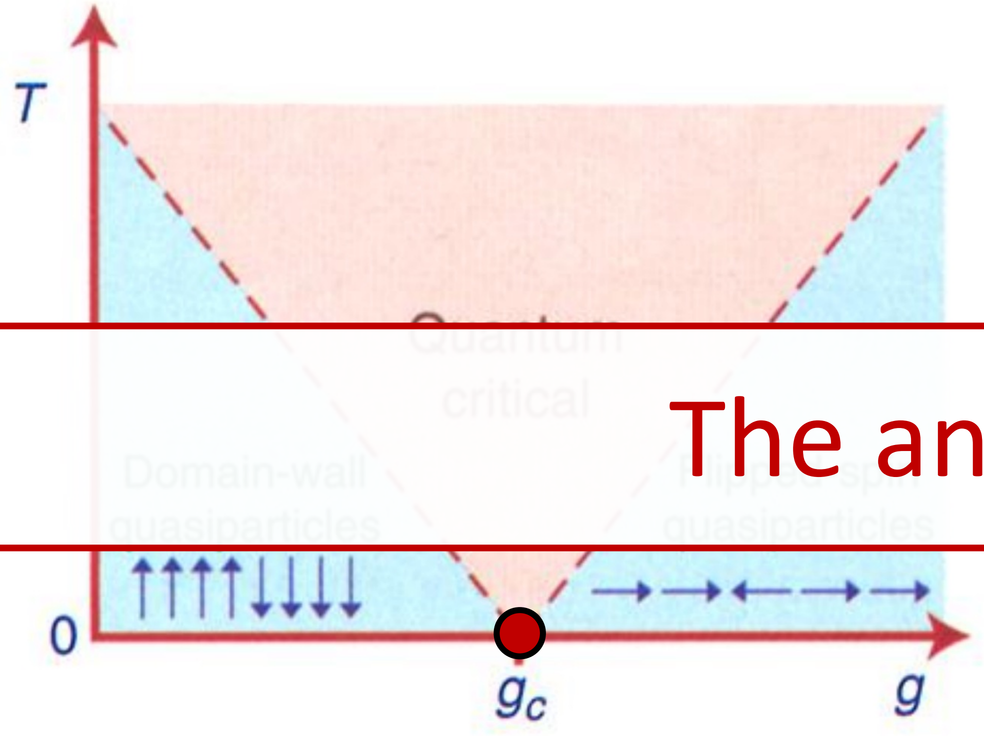
quantum magnet
(Breunig SA 2017)



heavy-fermion system
(Gegenwart NP 2008)

Quantum Critical Regime and Crossover Scaling

Two crossover lines $(g - g_c) \sim T^{1/z\nu}$



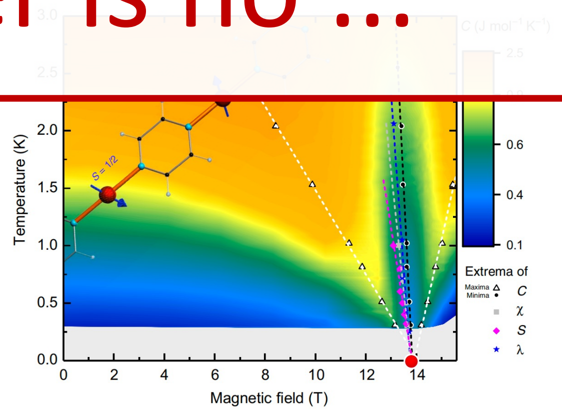
z is dynamic critical exponent
ν is correlation–length critical exponent

Sachdev Science (2000)

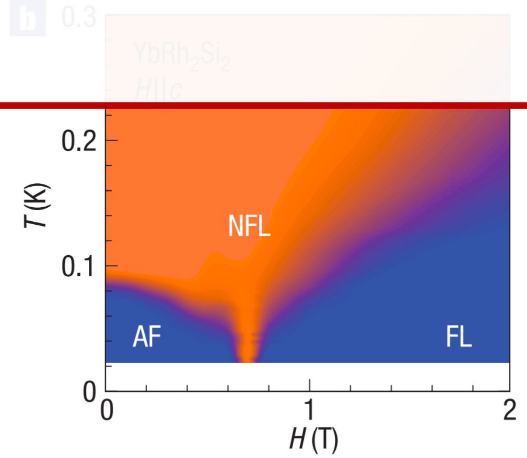
- **Quantum criticaity**: intriguing state in many systems.

> Originate from the ***quantum critical point***
> Obey distinct ***quantum scaling***
> Observed in many ***strongly correlated systems***

The answer is no …



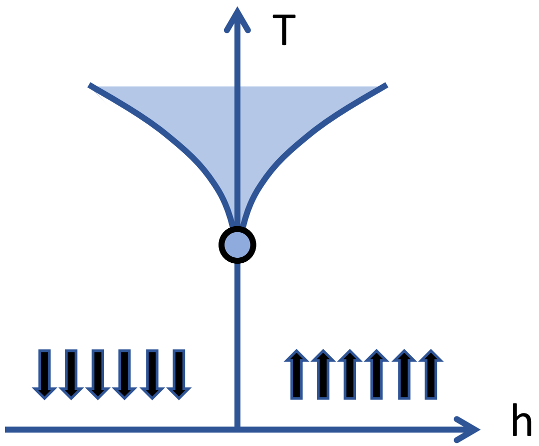
quantum magnet
(Breunig SA 2017)



heavy-fermion system
(Gegenwart NP 2008)

Supercritical Regime vs. Quantum Critical Regime

Supercritical regime

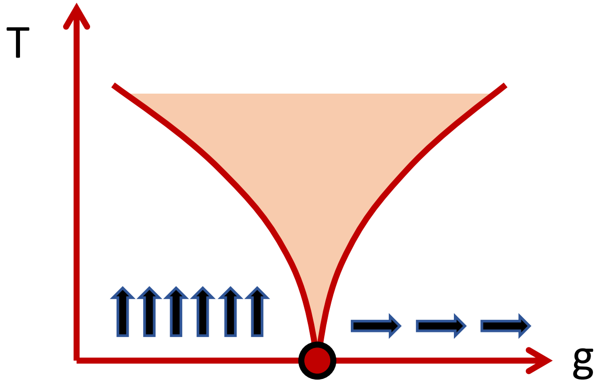


$$H = -\sum_{<i,j>} S_i^z S_j^z - h\sum_i S_i^z$$

h field ***couples to order parameter***

Supercritical crossover $|\delta P| \sim (T - T_c)^{\beta+\gamma}$

Quantum critical regime



$$H = -\sum_{<i,j>} S_i^z S_j^z - g\sum_i S_i^x$$

g field ***introduces quantum fluctuation***

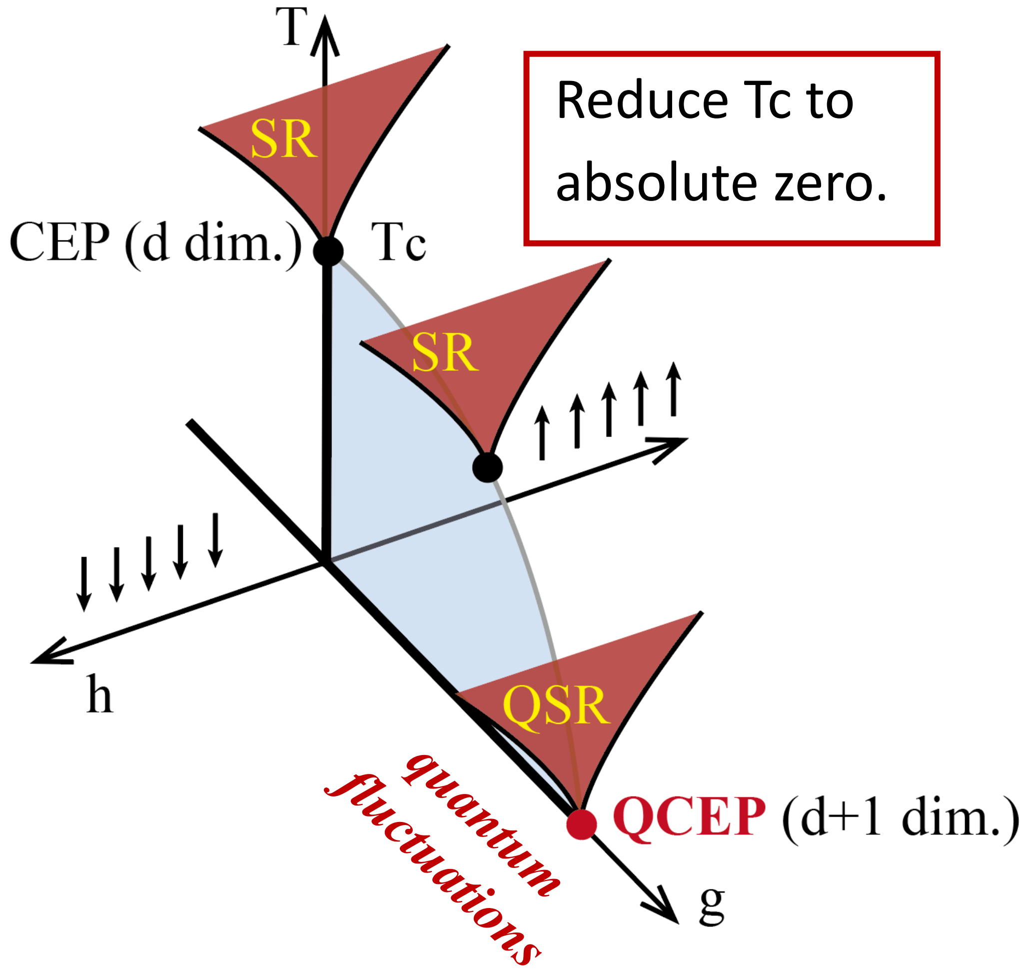
Quantum critical crossover $g - g_c \sim T^{1/z\nu}$

Does There Exist Quantum Supercritical Regime?

From Supercritical Regime to Quantum Supercritical Regime

- Introducing quantum fluctuations

$$H = -\sum_{<i,j>} S_i^z S_j^z - g\sum_i S_i^x - h\sum_i S_i^z$$



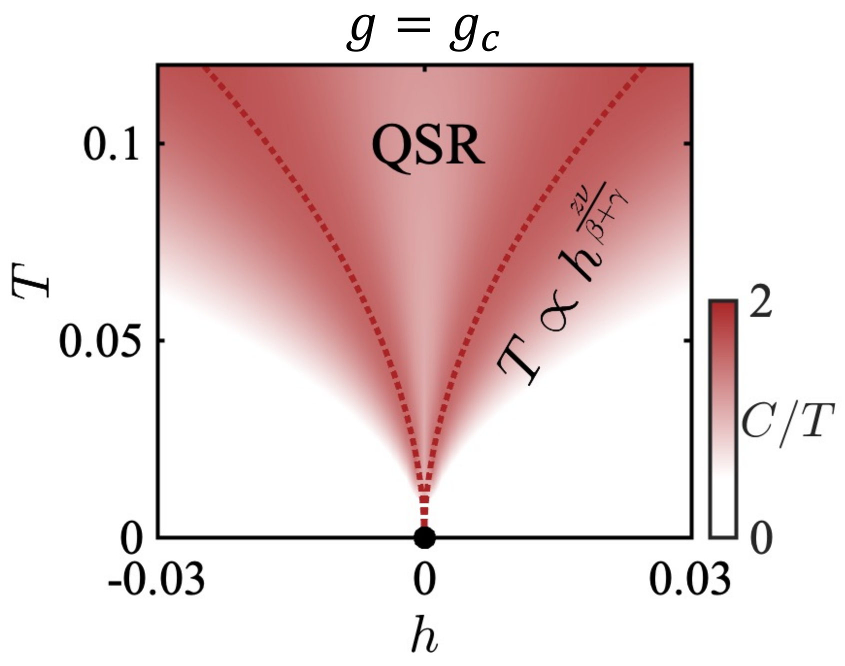
Suppressed

Critical endpoint (CEP) → Quantum critical endpoint (QCEP)

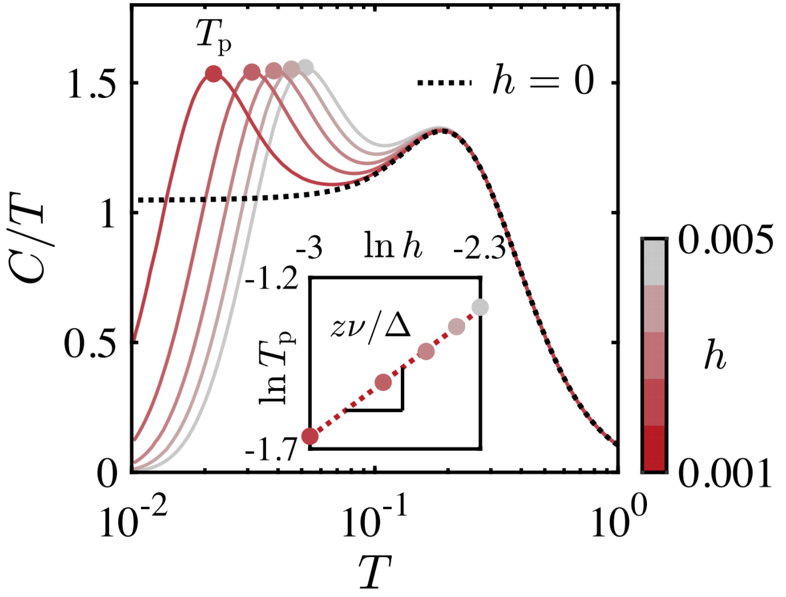
Supercritical regime (SR) → Quantum supercritical regime (QSR)

Quantum Supercritical Crossover — Specific Heat

- **1D Quantum Ising Model:** $H = -\sum_{<i,j>} S_i^z S_j^z - g\sum_i S_i^x - h\sum_i S_i^z$



Quantum Supercritical Crossovers



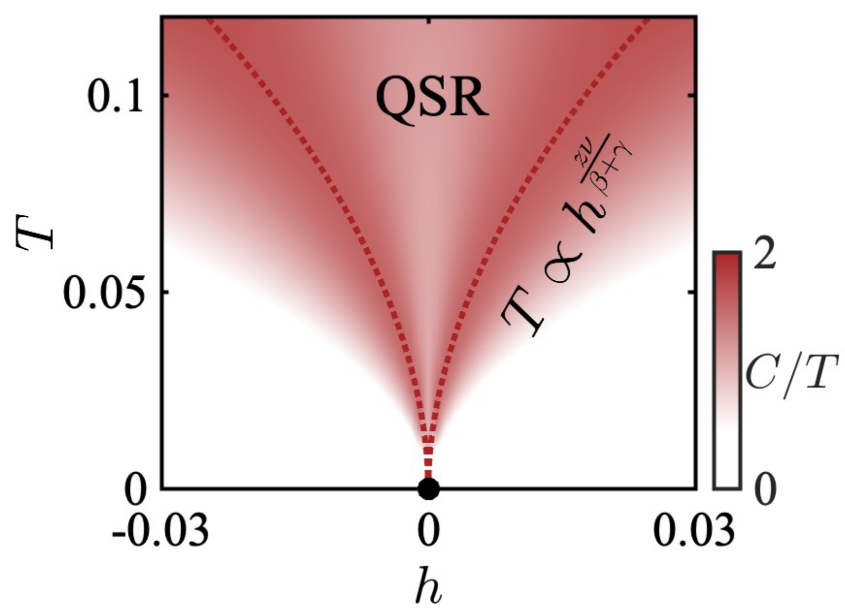
Specific Heat: Quantum Scalings

Peak location follows $T_p \propto h^{\frac{z\nu}{\Delta}}$, with $\Delta = \beta + \gamma$

$C \propto T^{\frac{d}{z}} \propto T$

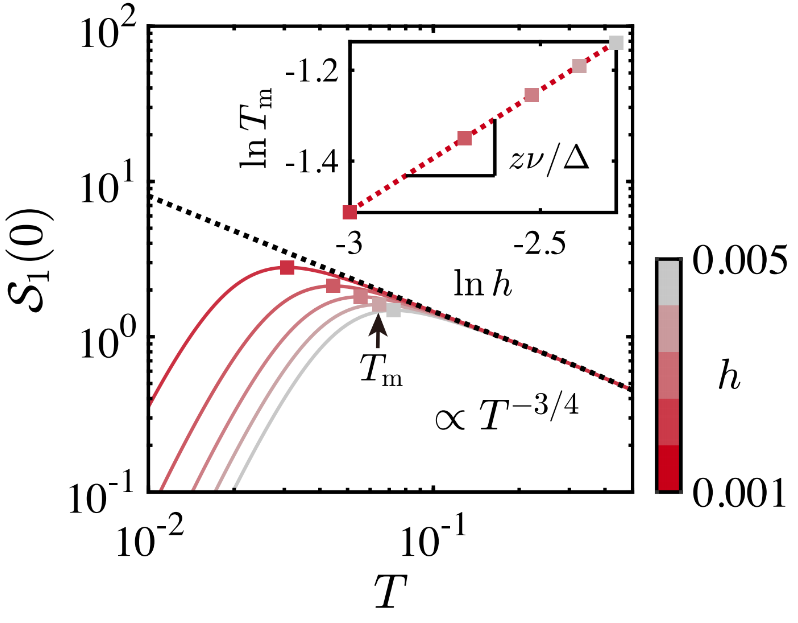
Quantum Supercritical Crossover — Spin Dynamics

- **Spin-lattice relaxation** $\frac{1}{T_1} \simeq \lim_{\omega\to 0^+} T \sum \frac{\chi''_{\alpha\alpha}(\omega)}{\omega}$, $\chi''_{\alpha\alpha}(\omega)$ local dynamical susceptibility



Quantum supercritical crossovers

Peaks of $S_1(0)$ follow $T \propto h^{\frac{z\nu}{\Delta}}$, with $\Delta = \beta + \gamma$

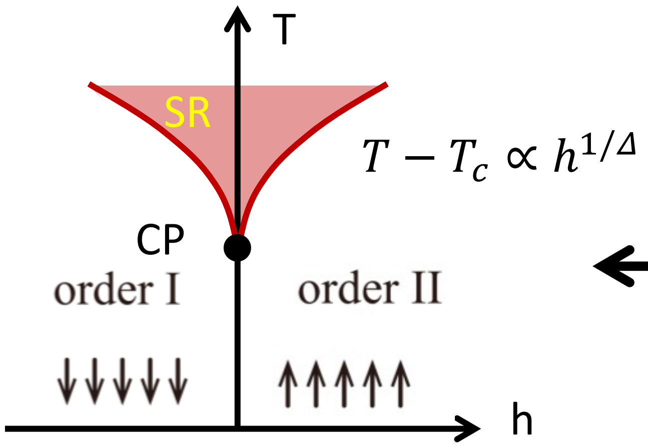


Quantum scalings

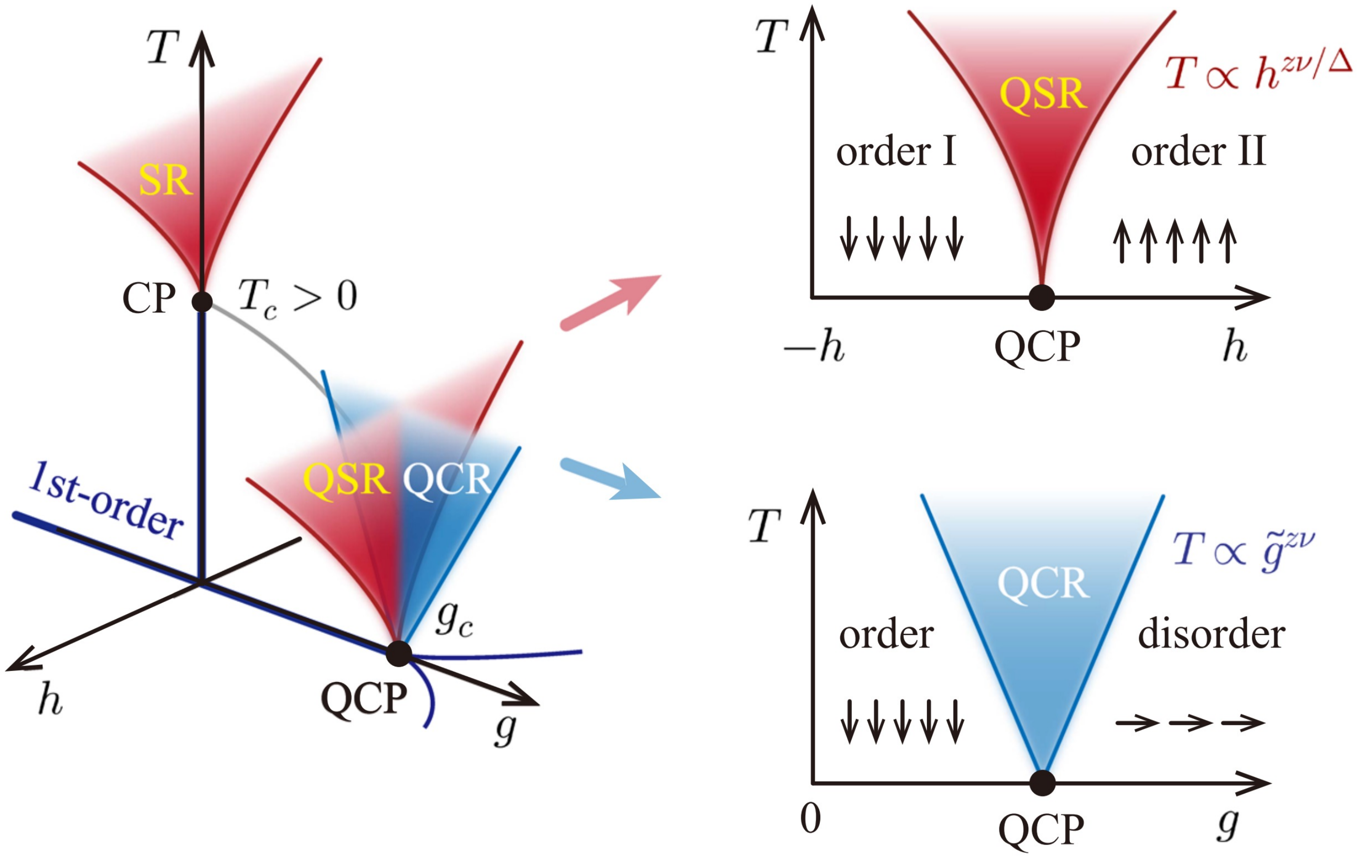
$1/T_1 \propto T^{\frac{\eta+d-2}{z}} \propto T^{-3/4}$

N. Xi, Y. Gao et al, arXiv:2403.11895 (2024)

Difference Between SR, QCR and QSR

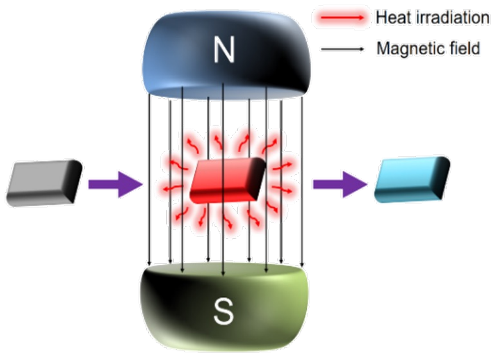


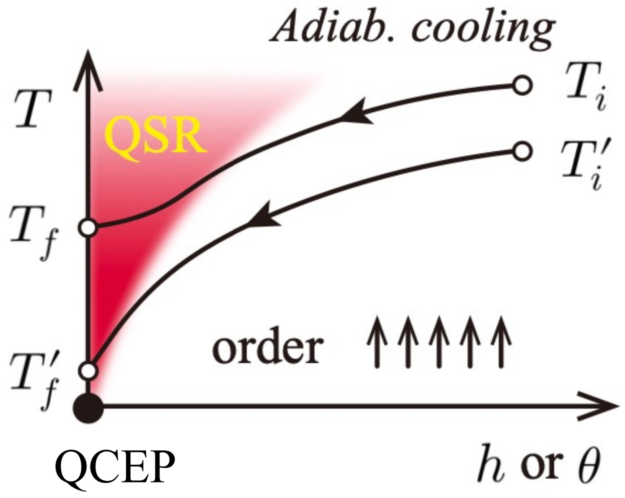
$\Delta = \beta + \gamma$



Is there significant effect for quantum supercritical regime?

❑ Quantum Supercritical Cooling

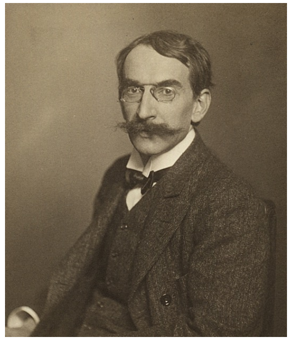




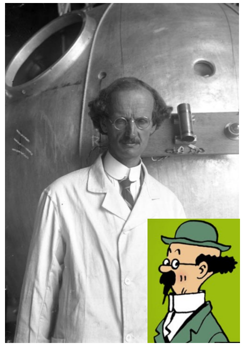
Weiss-Piccard Scenario: Magnetocaloric Effect near Curie Point

Le phénomène magnétocalorique *1917*

Pierre Weiss, Auguste Piccard



Weiss



Piccard

Above Curie point (634.9 K):

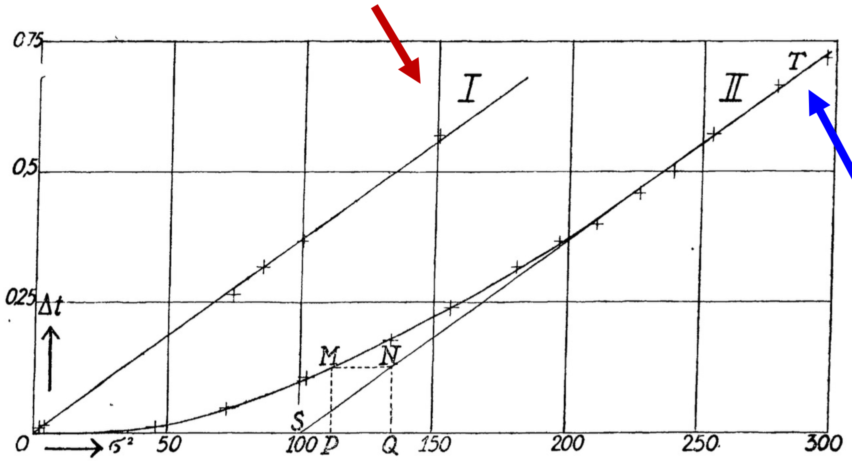


Fig. 1.

- ***Nickel***
  T_c=629.6 K

Below Curie point (627.2 K)

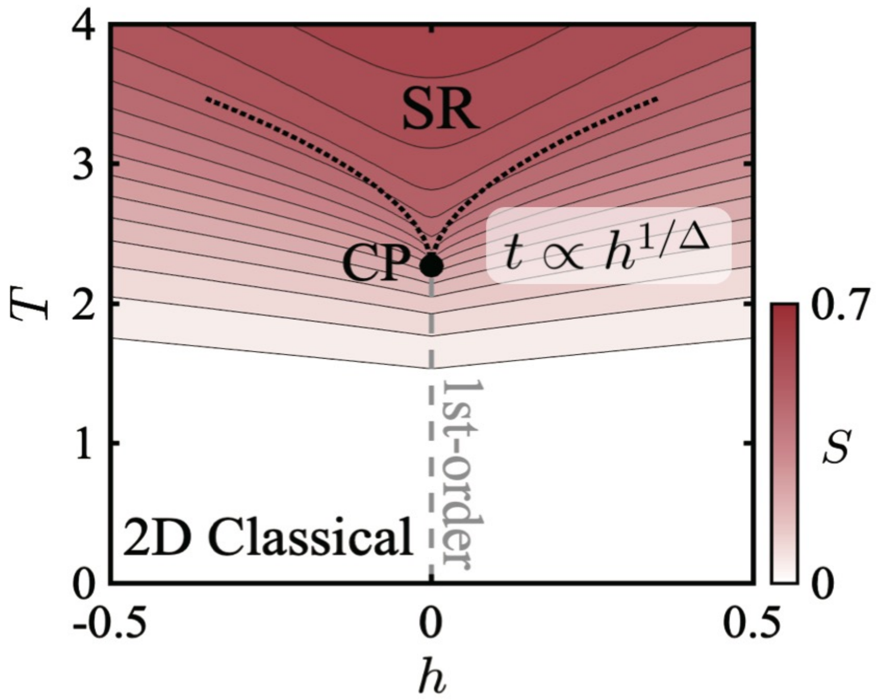
- "In the ***vicinity of the Curie point***, we observed ***very sensitive temperature variations***, accompanying the magnetic field establishes or suppresses."
- "***Spontaneous magnetization*** causes only a ***small*** thermal phenomenon"

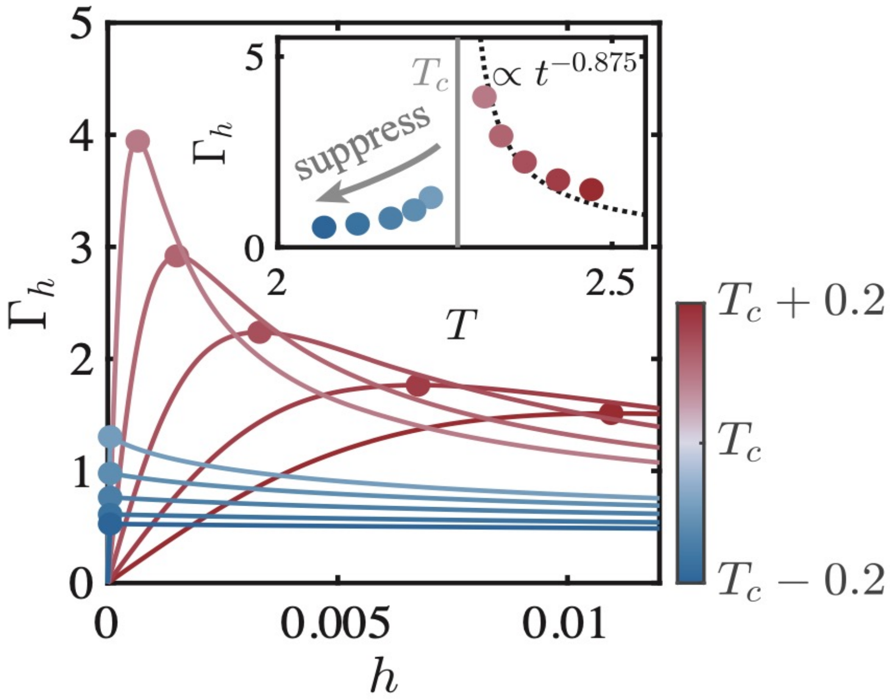
J. Phys. Theor. Appl. 1, 7 (1917)

Classical Ising Model: Diverging MCE in Supercritical Regime

2D FM classical Ising model

TRG method





- Grüneisen ratio:

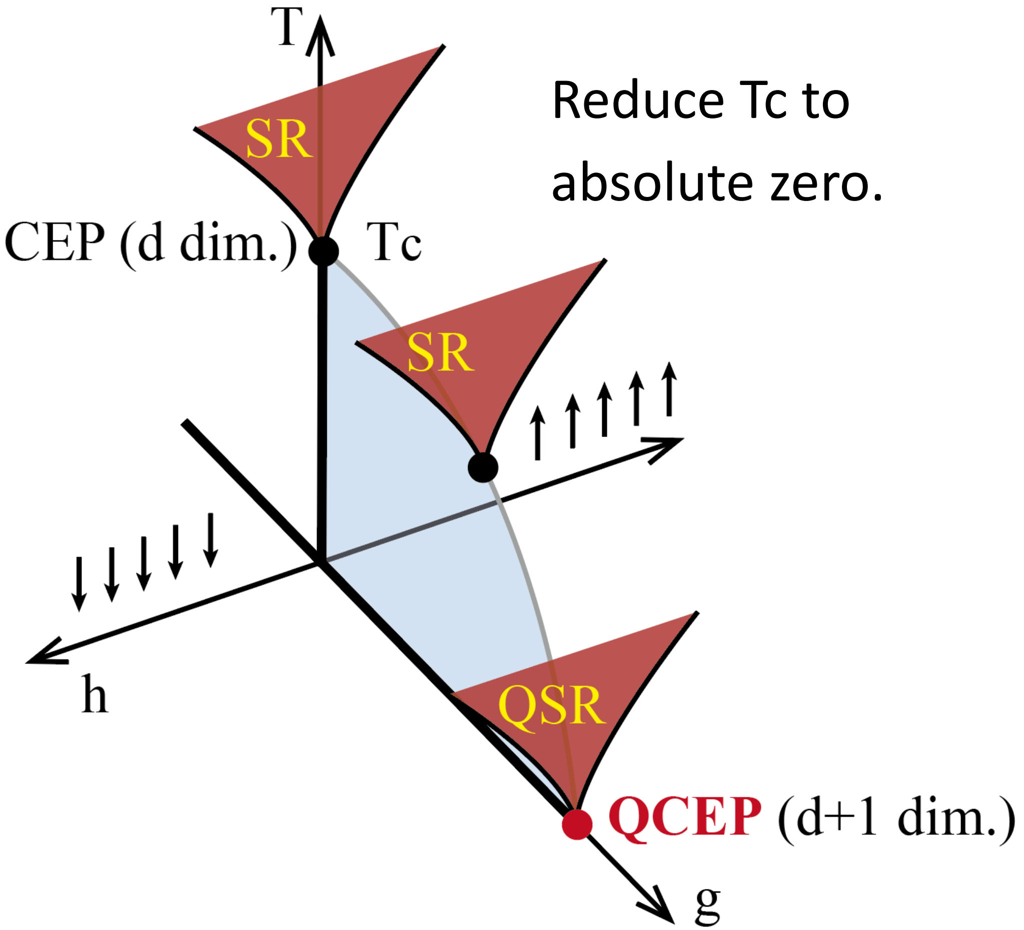
$$\Gamma_h \equiv \frac{1}{T}\left(\frac{\partial T}{\partial h}\right)_S$$

- Universal divergence (2D Ising):

$$\Gamma_h \propto \left(\frac{T - T_c}{T_c}\right)^{-\beta-\gamma+1} \propto \left(\frac{T - T_c}{T_c}\right)^{-7/8}$$

How to break through the limitations of Tc?

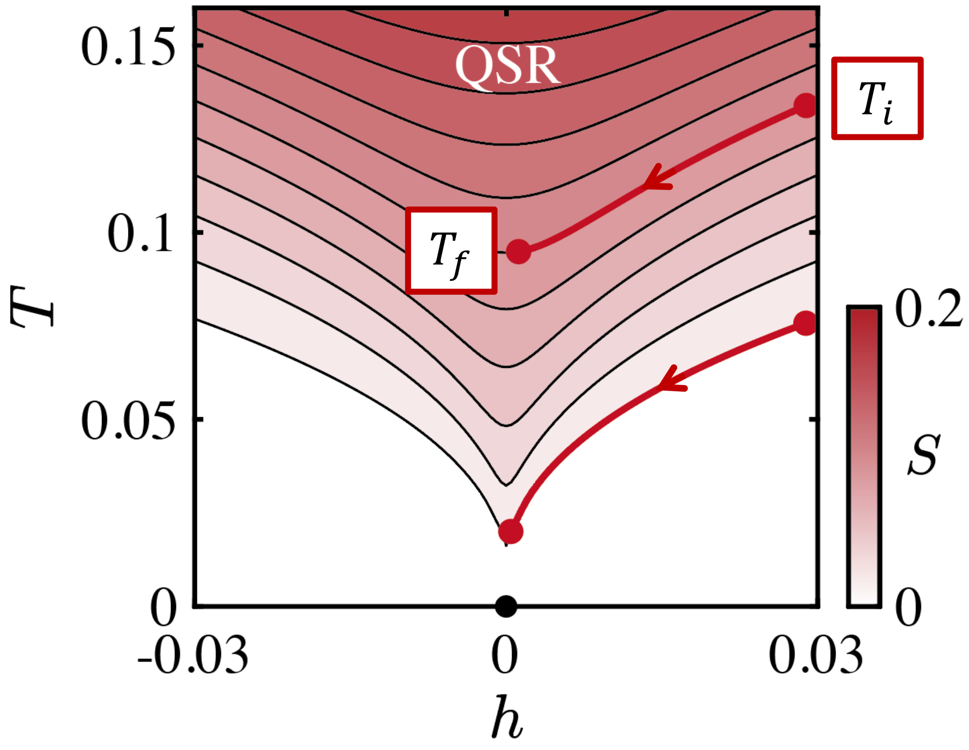
Quantum Supercritical Regime Can Do It



Reduce Tc to absolute zero.

Quantum Supercritical Cooling

Isentropic lines of 1D quantum Ising model



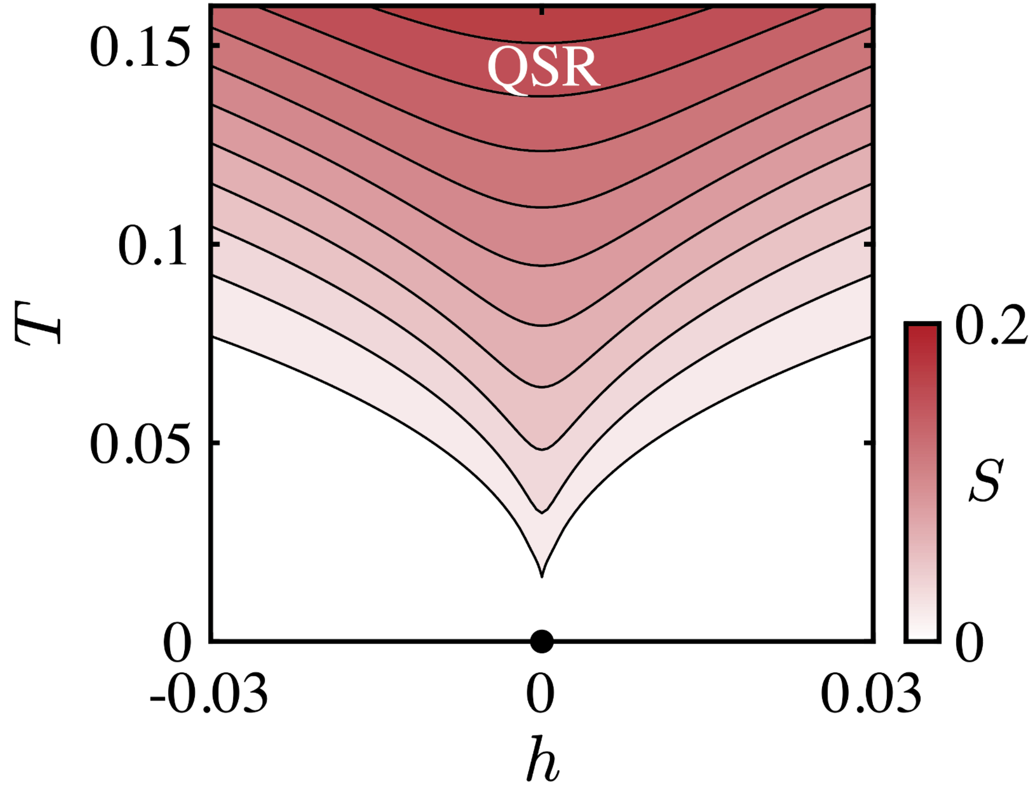
Grüneisen ratio: $$\Gamma_h \equiv \frac{1}{T}\left(\frac{\partial T}{\partial h}\right)_S$$

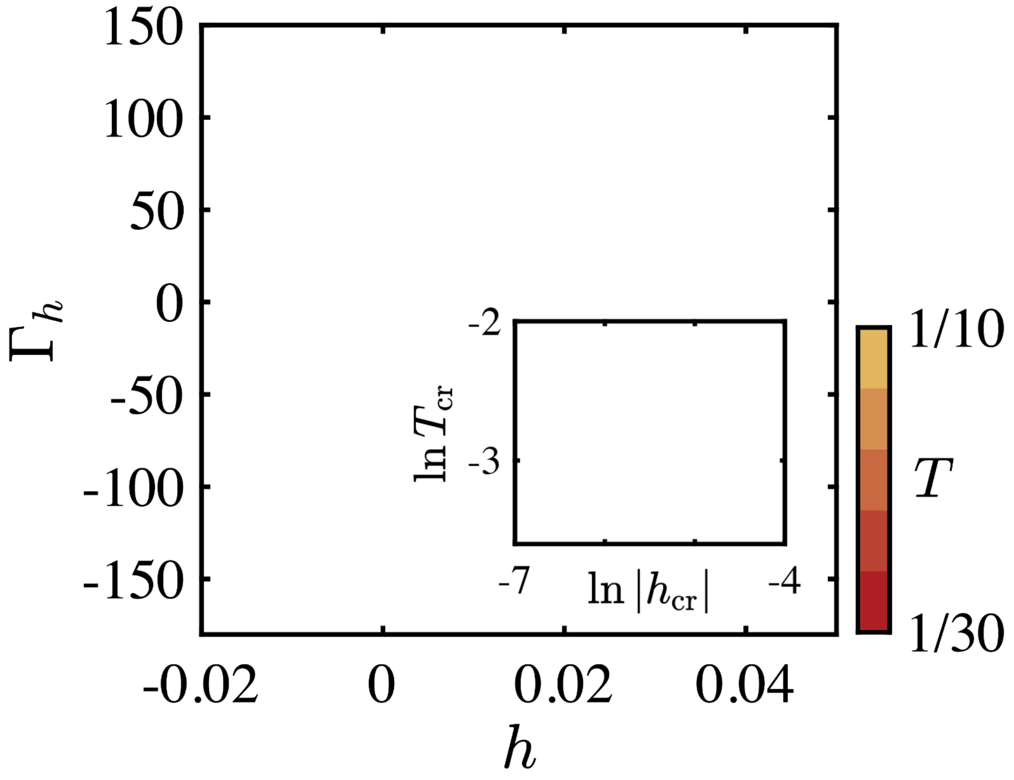
Prominent MCE near the QCP, within the QSR

The lowest temperature appears at $h = 0$

Quantum Supercritical Regime

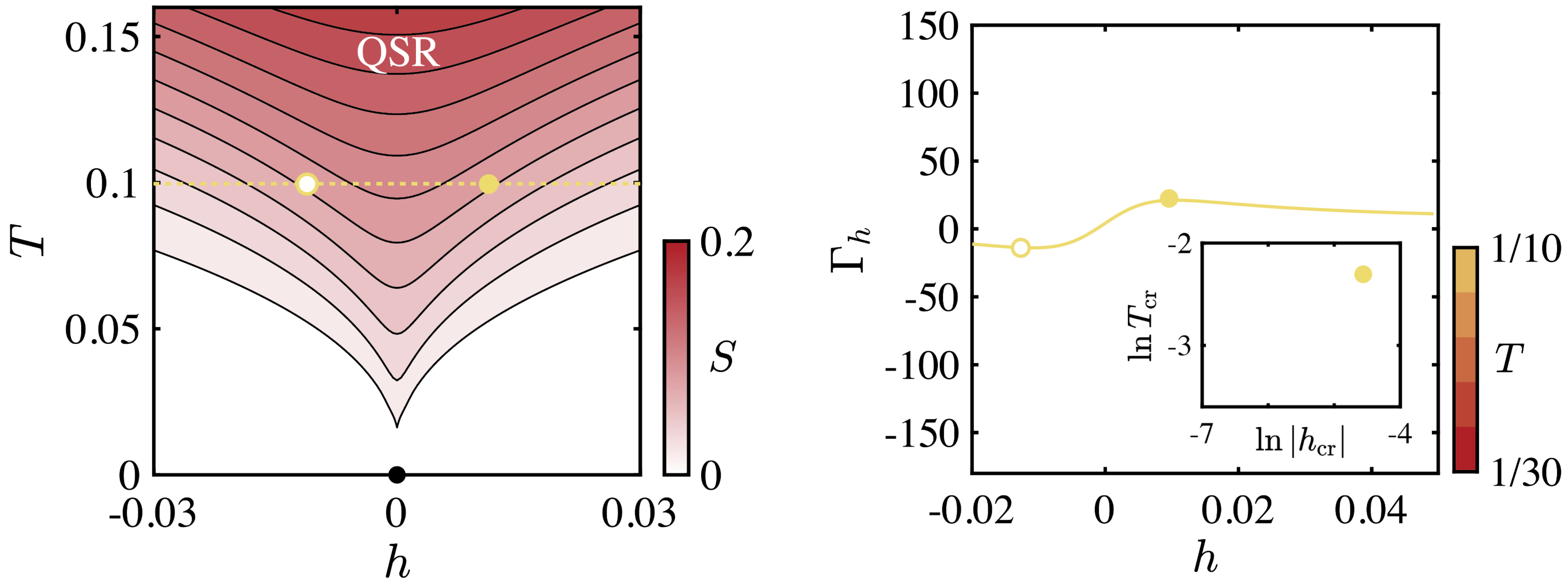
$$\Gamma_h \equiv \frac{1}{T}\left(\frac{\partial T}{\partial h}\right)_S$$





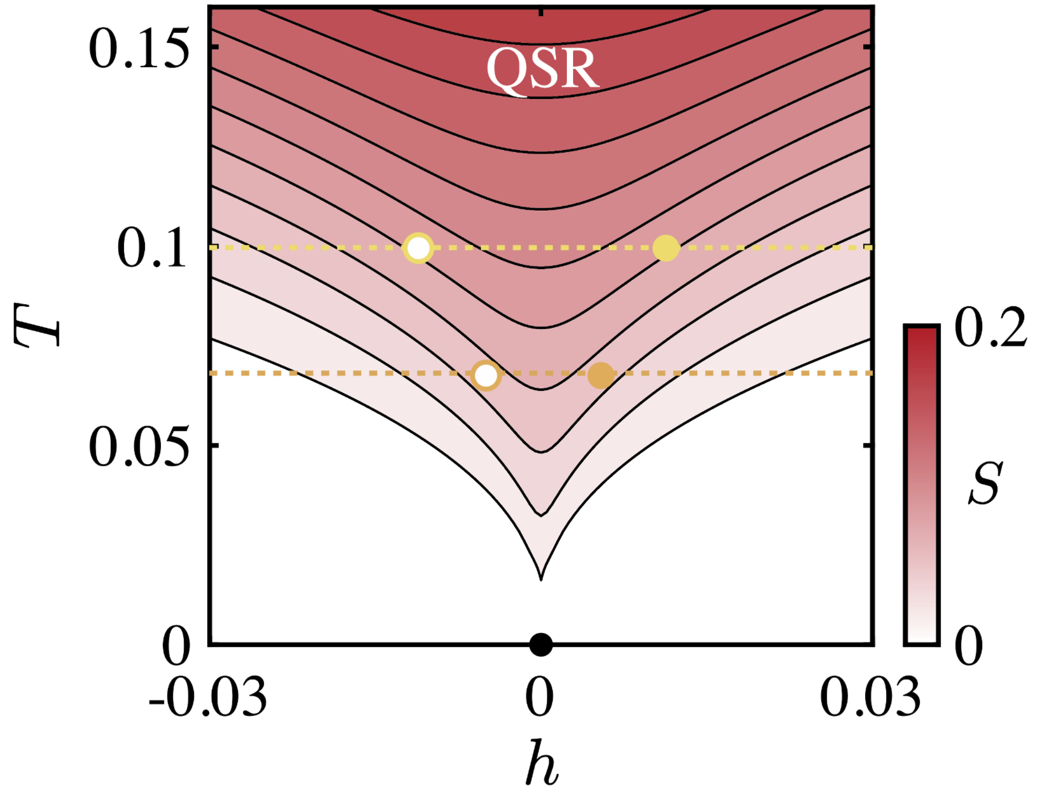
Quantum Supercritical Regime

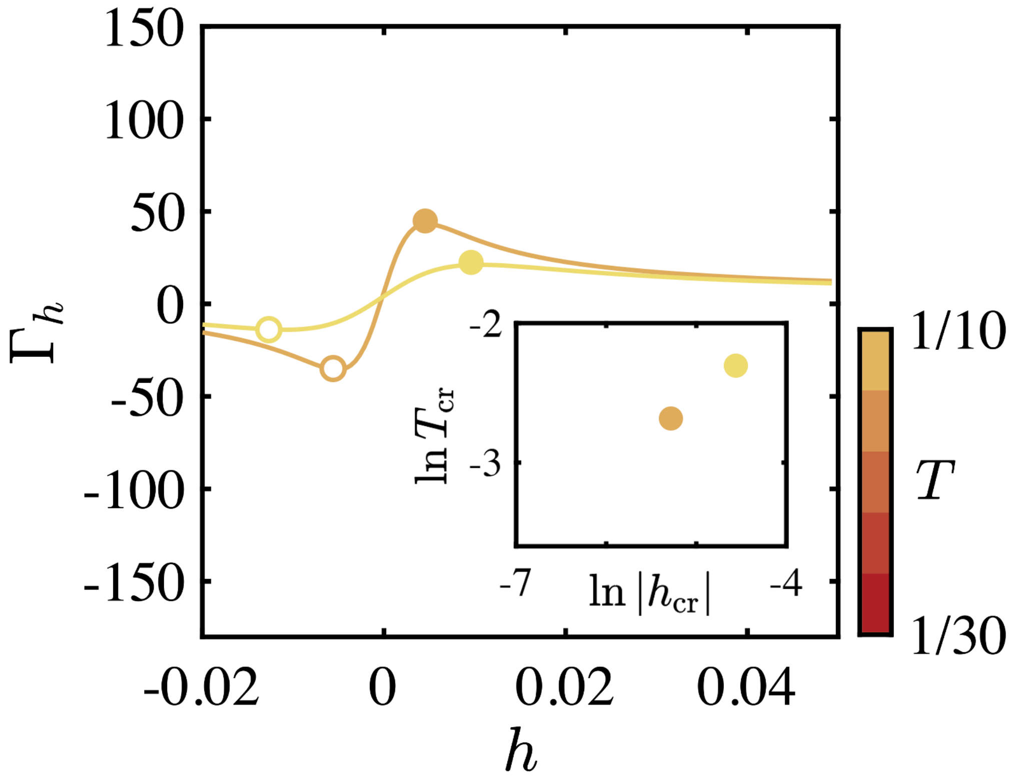
$$\Gamma_h \equiv \frac{1}{T}\left(\frac{\partial T}{\partial h}\right)_S$$



Quantum Supercritical Regime

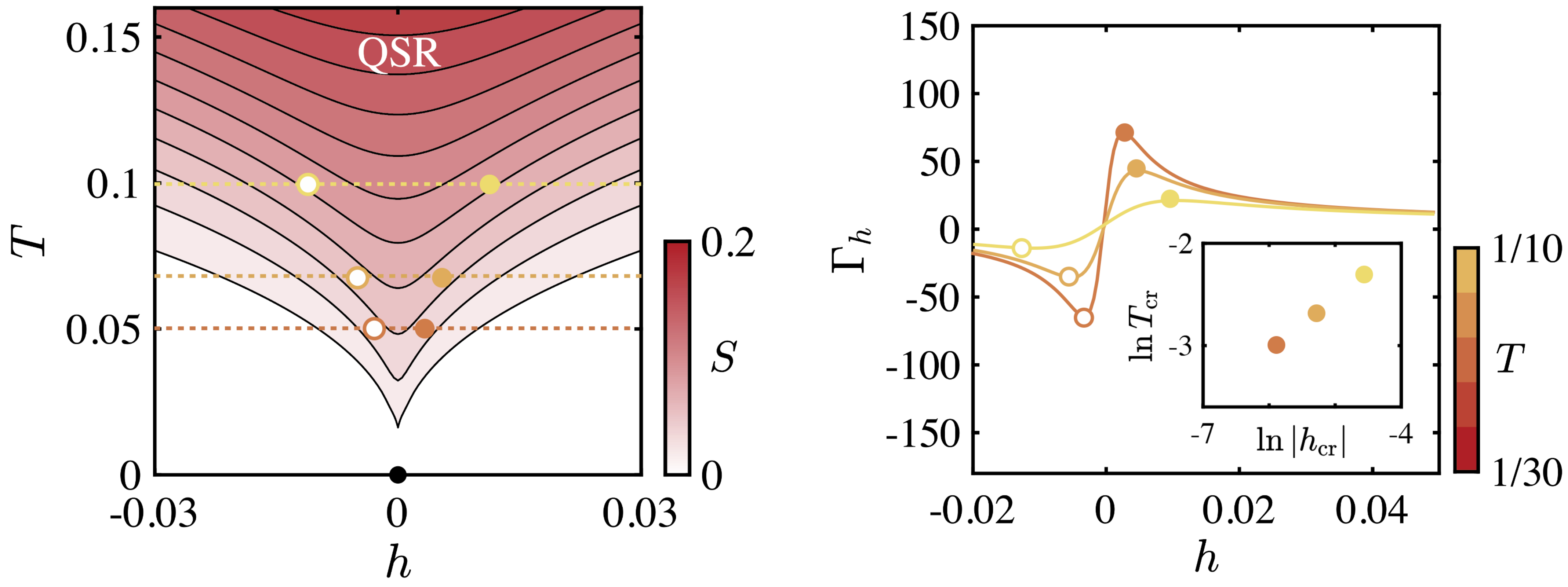
$$\Gamma_h \equiv \frac{1}{T}\left(\frac{\partial T}{\partial h}\right)_S$$





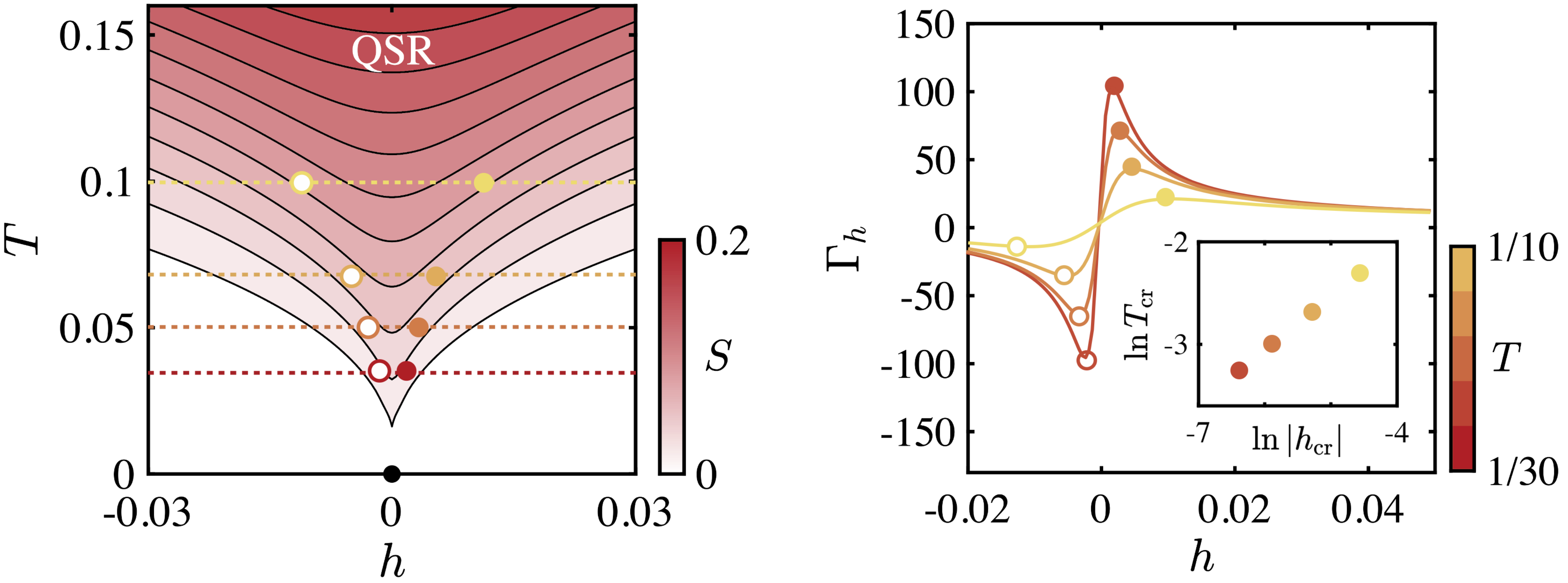
Quantum Supercritical Regime

$$\mathit{\Gamma}_h \equiv \frac{1}{T}\left(\frac{\partial T}{\partial h}\right)_S$$



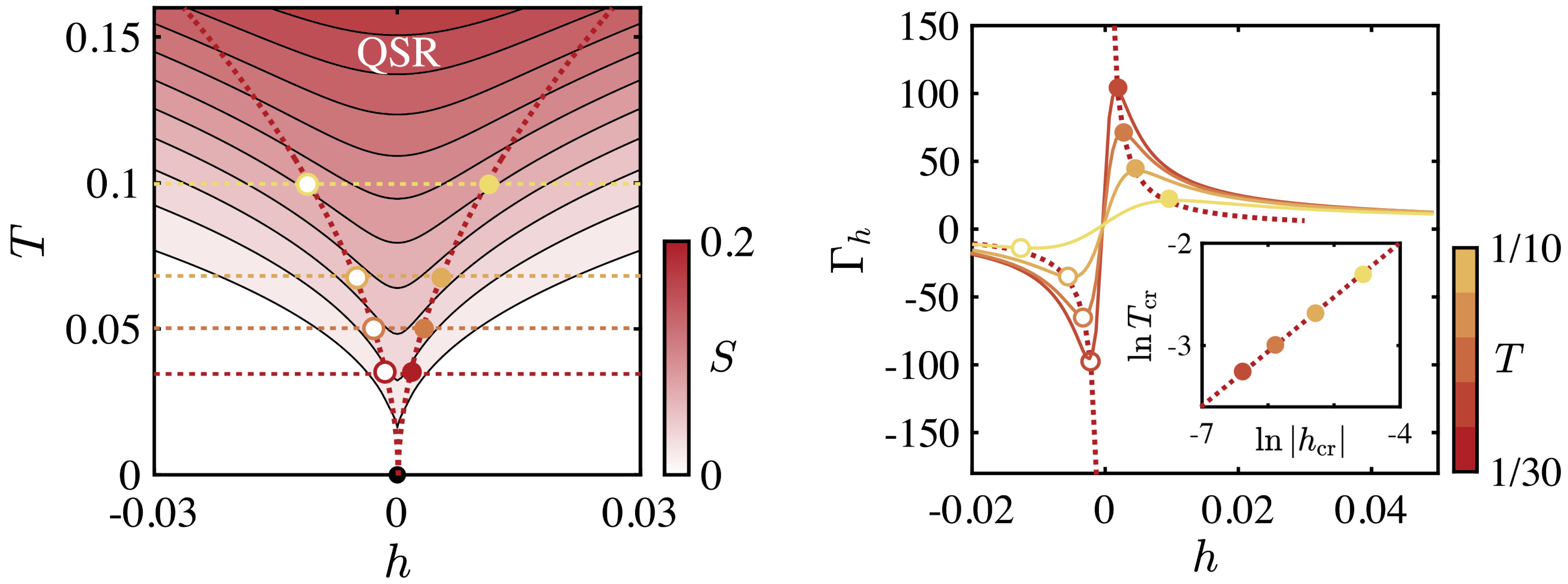
Quantum Supercritical Regime

$$\Gamma_h \equiv \frac{1}{T}\left(\frac{\partial T}{\partial h}\right)_S$$



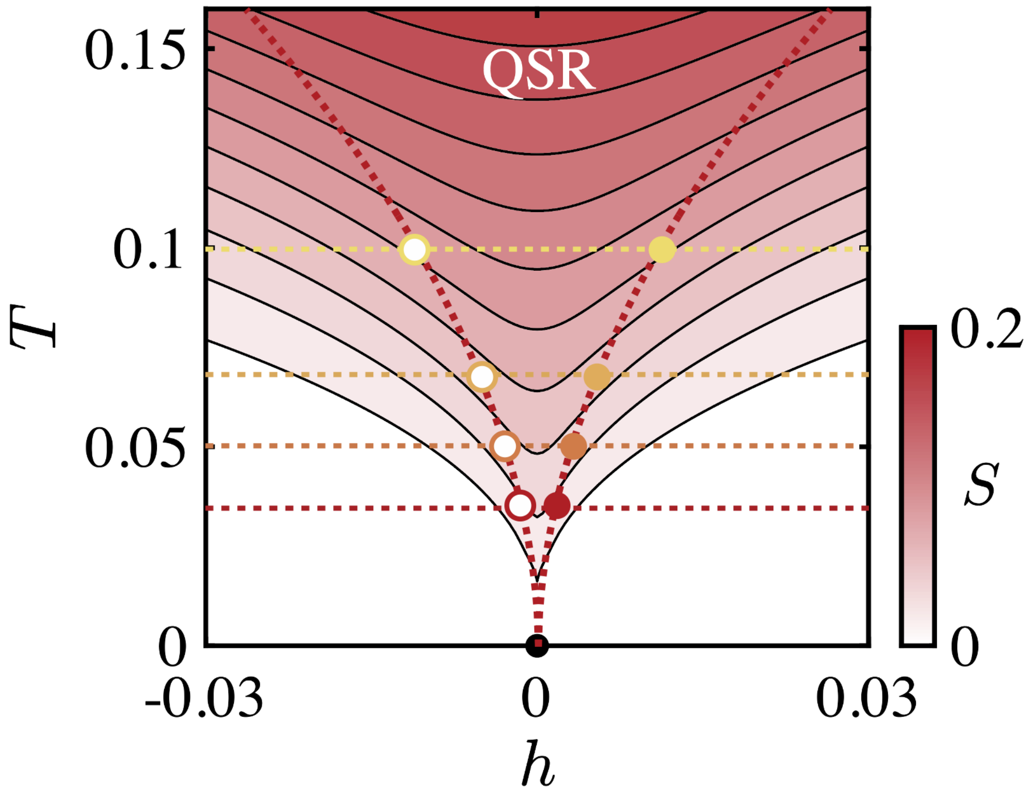
Quantum Supercritical Regime

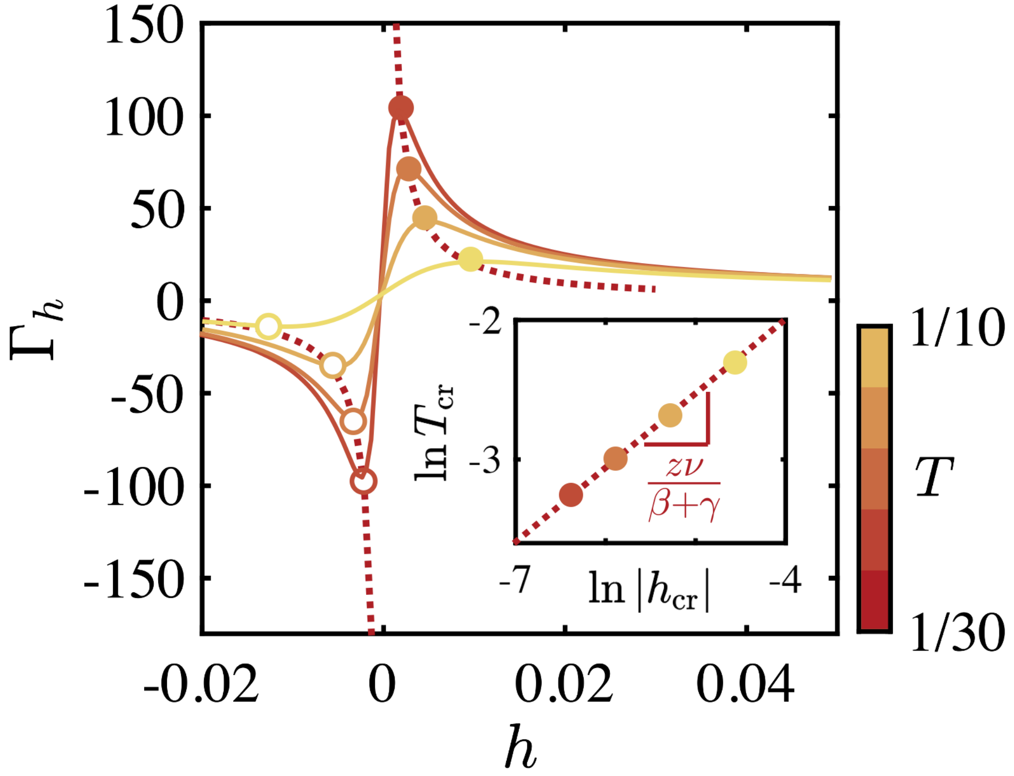
$$\varGamma_h \equiv \frac{1}{T}\left(\frac{\partial T}{\partial h}\right)_S$$



Quantum Supercritical Regime

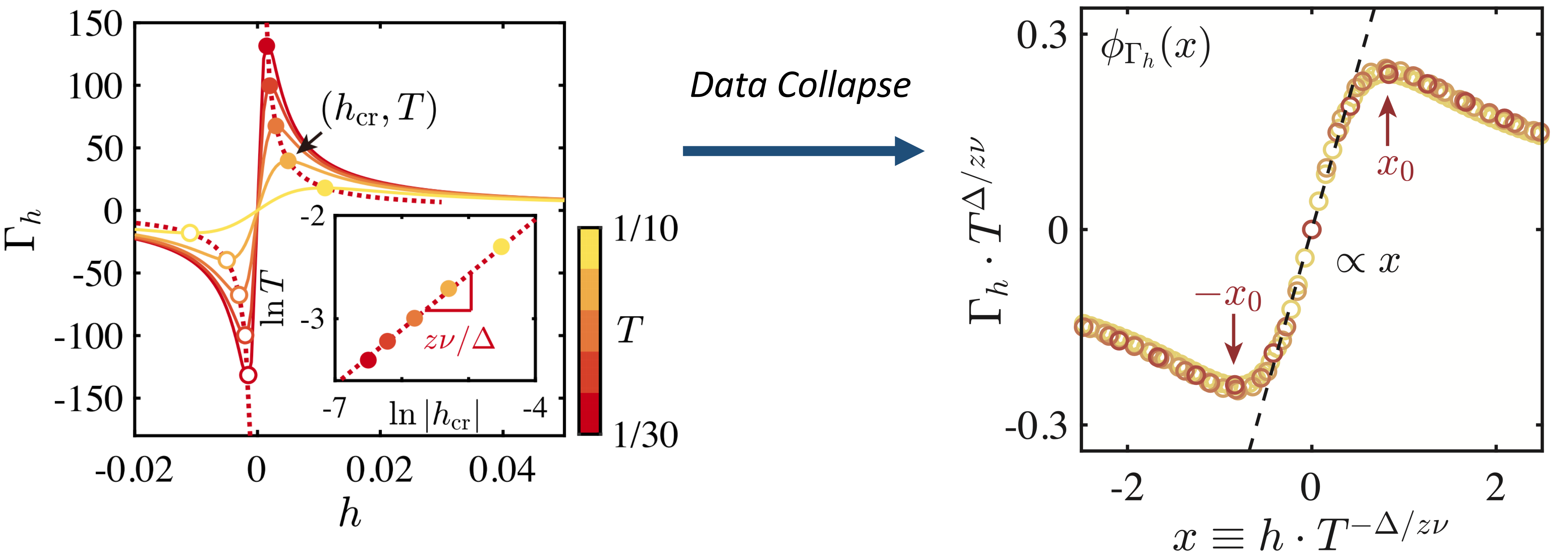
$$\Gamma_h \equiv \frac{1}{T}\left(\frac{\partial T}{\partial h}\right)_S$$





Diverging Grüneisen ratio: $\Gamma_h \propto T^{-\frac{\beta+\gamma}{z\nu}}$

Universal Quantum Supercritical Cooling



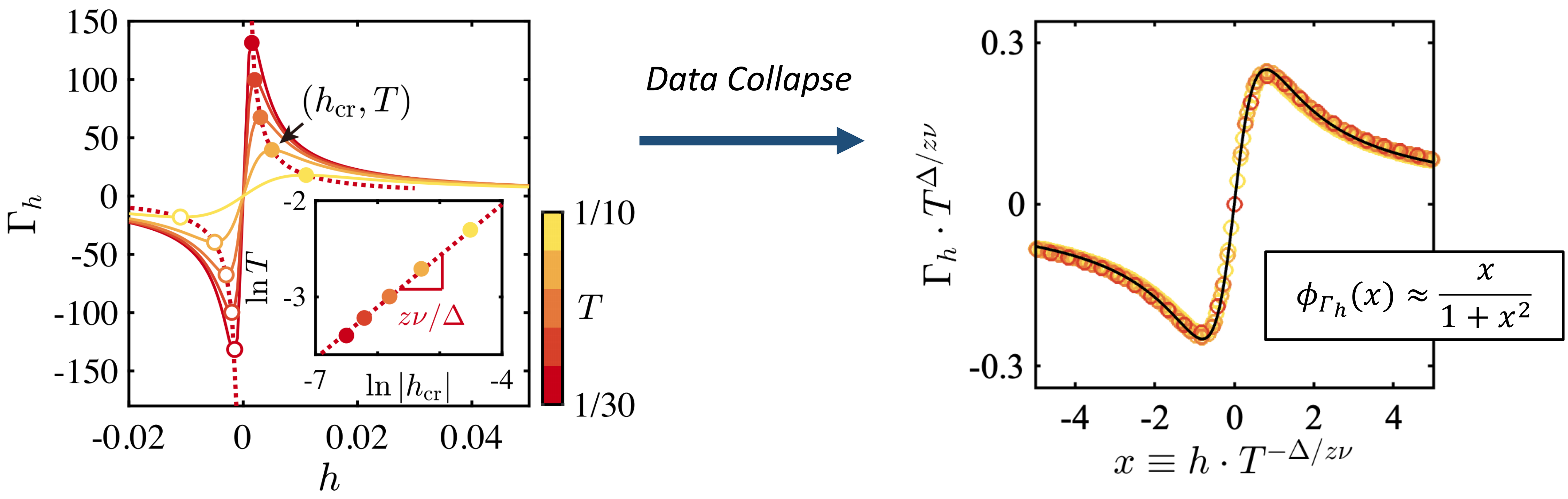
Scaling Form: $\Gamma_h = T^{-\frac{\Delta}{z\nu}} \phi_{\Gamma_h}\left(h \cdot T^{-\frac{\Delta}{z\nu}}\right)$

$\Delta = \beta + \gamma$

Crossovers: $h \cdot T^{-\frac{\Delta}{z\nu}} = \pm x_0$ → $h \propto T^{\frac{\Delta}{z\nu}}$

Divergence: $\Gamma_h = T^{-\frac{\Delta}{z\nu}} \phi_{\Gamma_h}(x_0)$ → $\Gamma_h \propto T^{-\frac{\Delta}{z\nu}}$

Universal Quantum Supercritical Cooling



universal form of entropy $S = T^{d/z}\phi_S(h \cdot T^{-\frac{\Delta}{z\nu}})$

even scaling function $\phi_S(x) = \phi_0 + \phi_2 x^2 + \cdots$

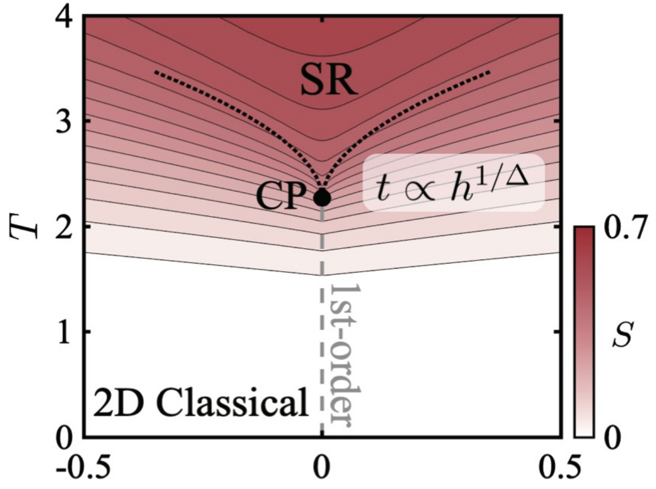
Gruneisen ratio $\Gamma_h = -\frac{(\partial S/\partial h)}{T(\partial S/\partial T)}$

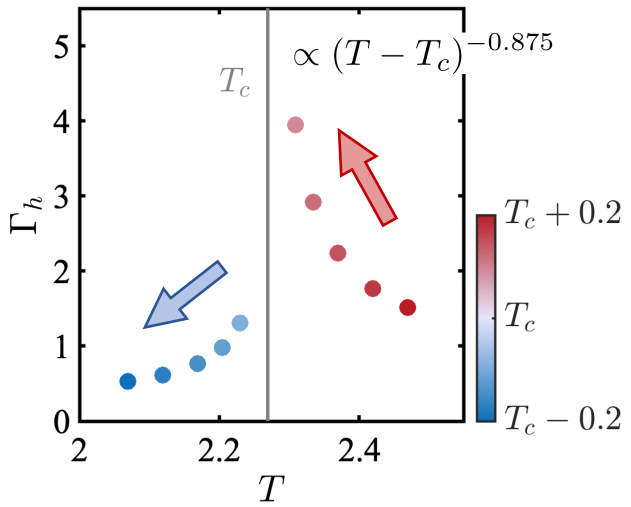
$$\phi_{\Gamma_h}(x) \approx \frac{x}{1+x^2}$$

suitable for other universality class

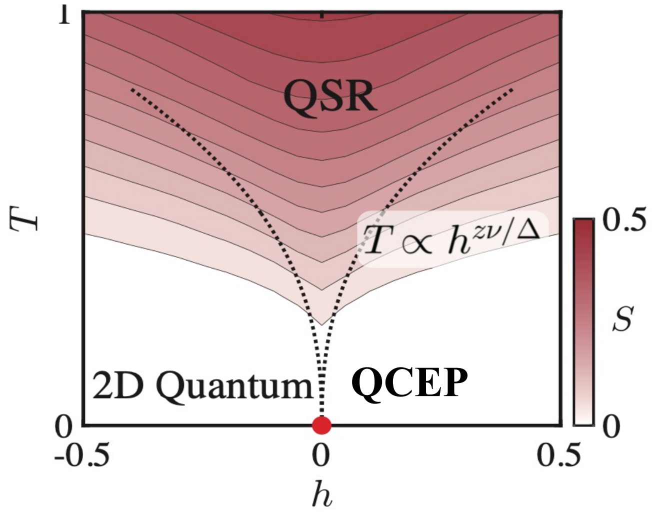
Classical vs. Quantum

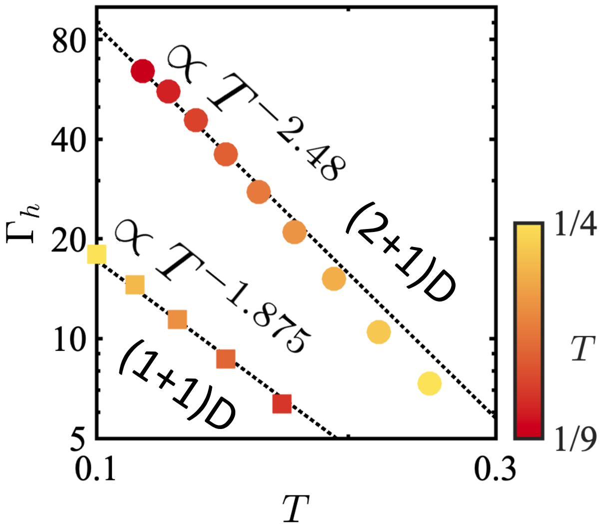
- 2D Classical Ising





- (2+1)D Quantum Ising





- **Classical:** diverging towards T_c

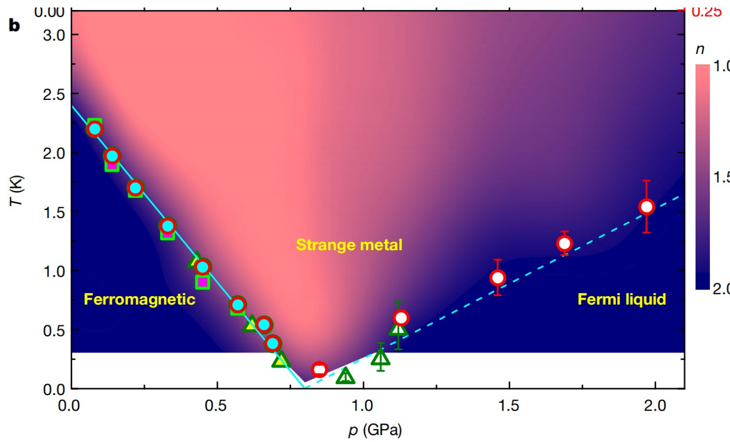
$$\Gamma_h \sim \left(\frac{T-T_c}{T_c}\right)^{-\Delta+1},\ \Delta \equiv \beta+\gamma = 15/8$$

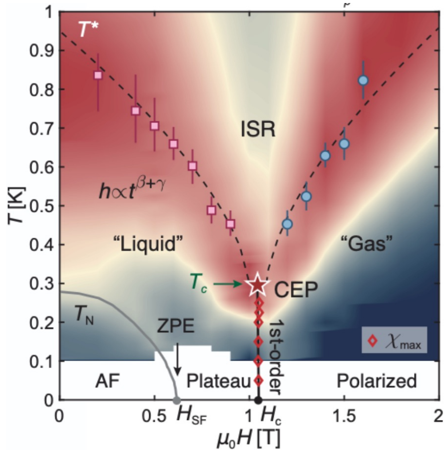
- **Quantum:** diverging towards *T=0*

$$\Gamma_h \sim T^{-\Delta/z\nu},\ \Delta/z\nu \simeq 2.482$$

Universality Class	Δ	$z\nu$	$\Delta - 1$	$\Delta/z\nu$
(1+1)D Ising	15/8	1	7/8	15/8
(2+1)D Ising	1.564	0.630	0.564	2.482
(2+1)D XY	1.667	0.672	0.667	2.481
(2+1)D O(3)	1.765	0.711	0.765	2.482
(1+1)D 3-Potts	14/9	5/6	5/9	28/15
(1+1)D 4-Potts	5/4	2/3	1/4	15/8
Mean Field	3/2	1	1/2	3/2

❑ Material Realization of (Quantum) Supercriticality



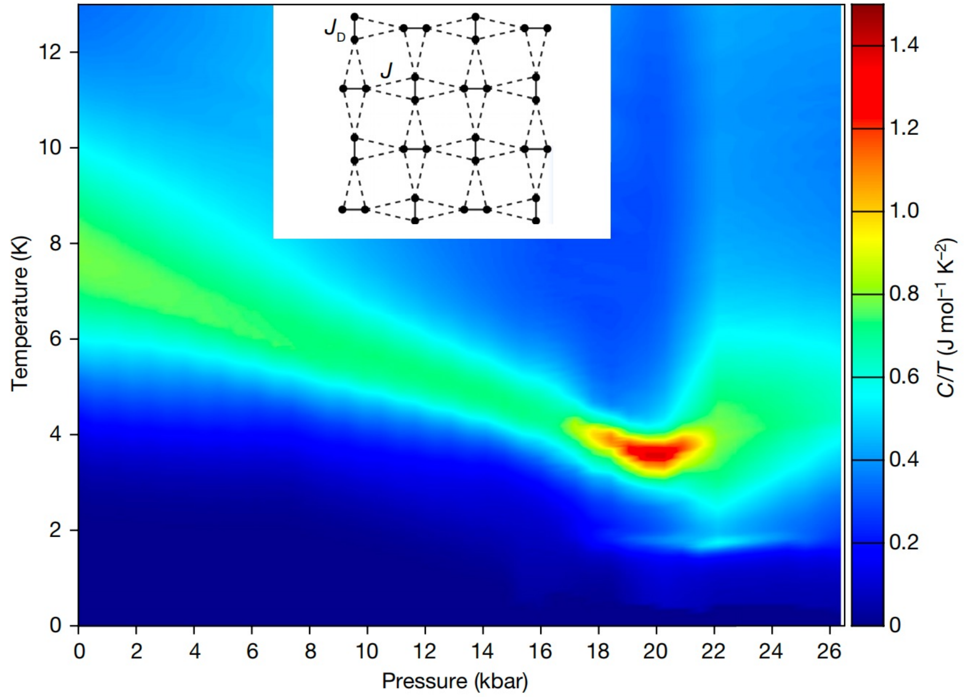


S. Bin, et al, Nature **579,** 51 (2020)

X. Liu, **E. Lv**, J. Xiang, WL, arXiv:2601.07810 (2026)

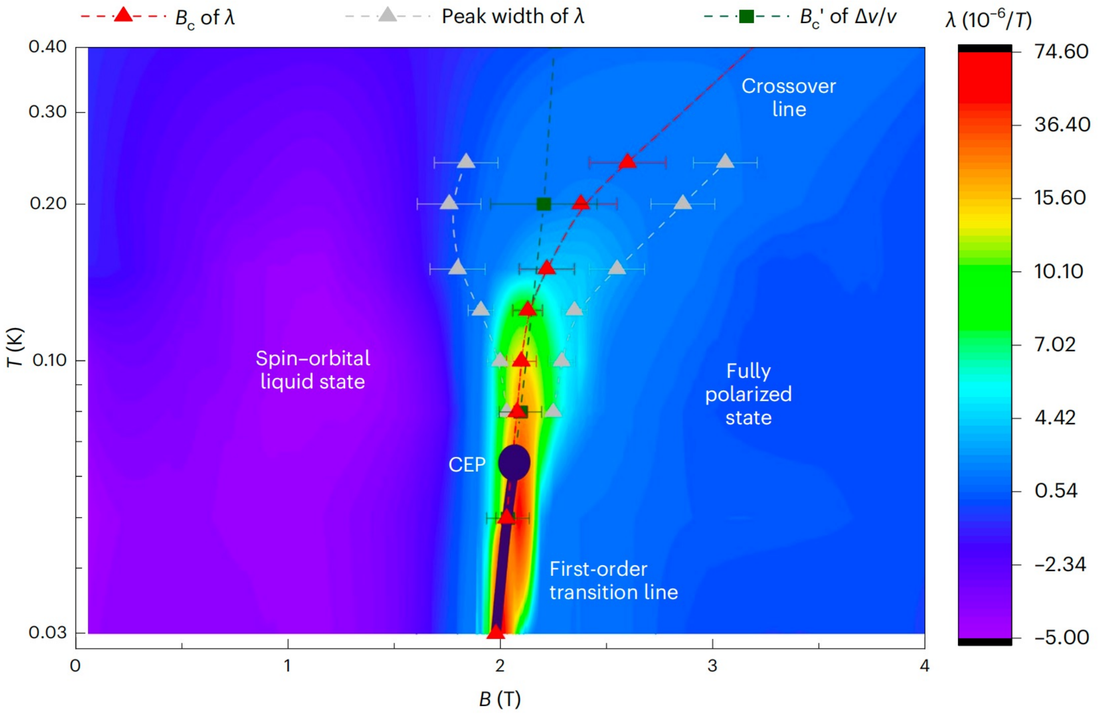
Material Realization: Classical Supercriticality

- $SrCu_2(BO_3)_2$: Shastry-Sutherland lattice



Nature **592**, 370 (2021)

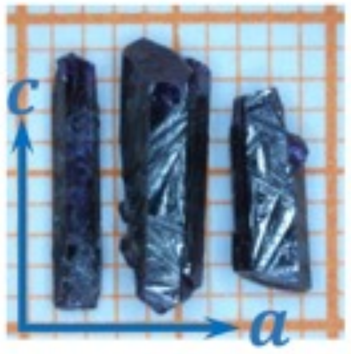
- PrZr2O7: quantum spin-ice material

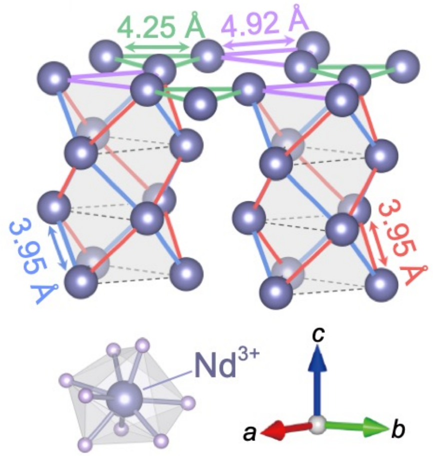


Nature Physics **19**, 92 (2023)

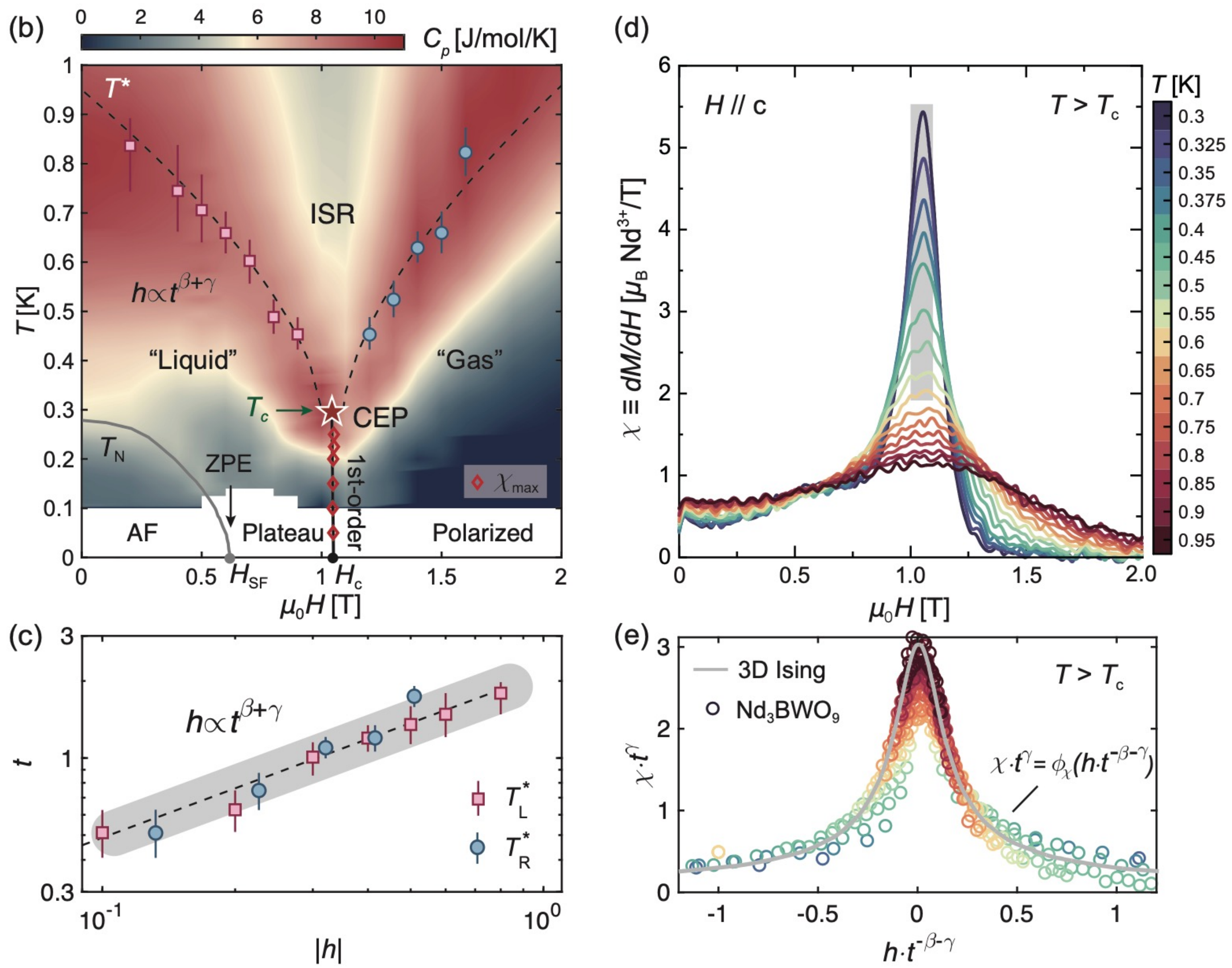
Supercriticality in Spiral Ising Antiferromagnet Nd_3BWO_9

X. Y. Liu





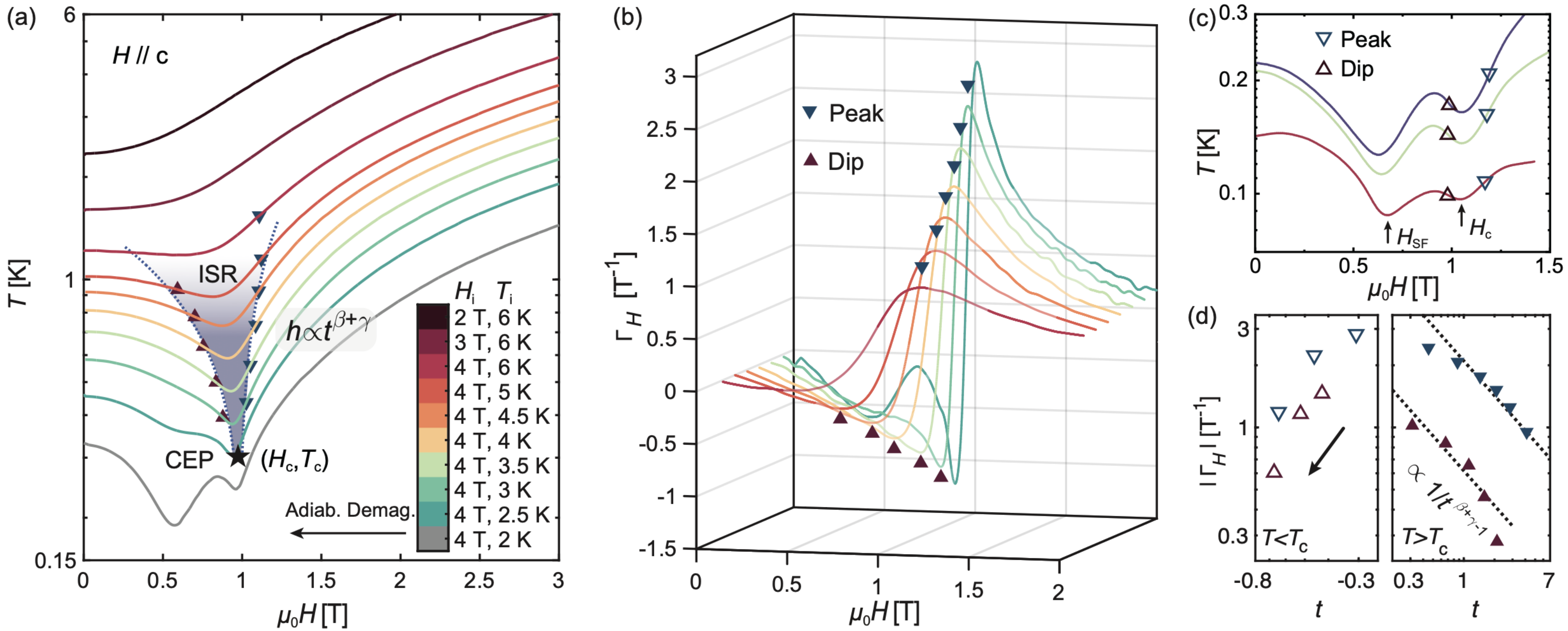
General "Spin Ice"



- ➢ Supercritical crossover line follows **supercritical scaling** $h \propto t^{\beta+\gamma}$
- ➢ Magnetic susceptibility data collapses onto the **3D Ising** scaling function

$$h \equiv (H - H_c)/H_c$$
$$t \equiv (T - T_c)/T_c$$

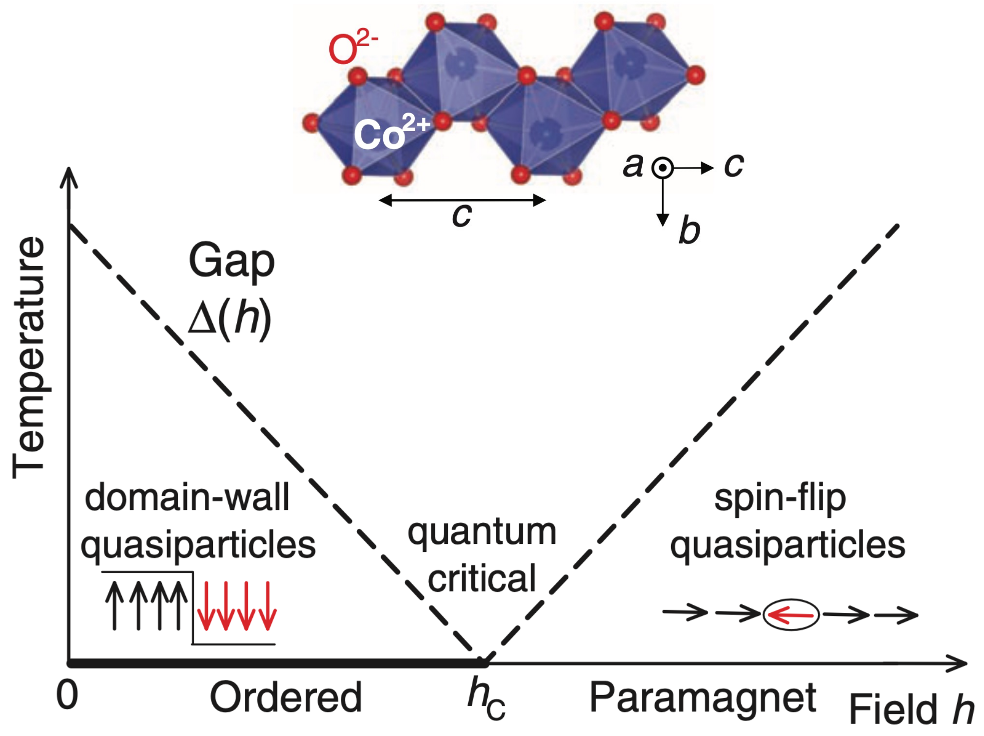
Supercritical Magnetocaloric effect



- Strong cooling effect near the **CEP** and within the **Ising supercritical regime**
- **Diverging Gruneisen ratio** $\Gamma_h \propto 1/t^{\beta+\gamma-1}$
- Maybe Supercritical phenomena can be observed in other spin ice materials

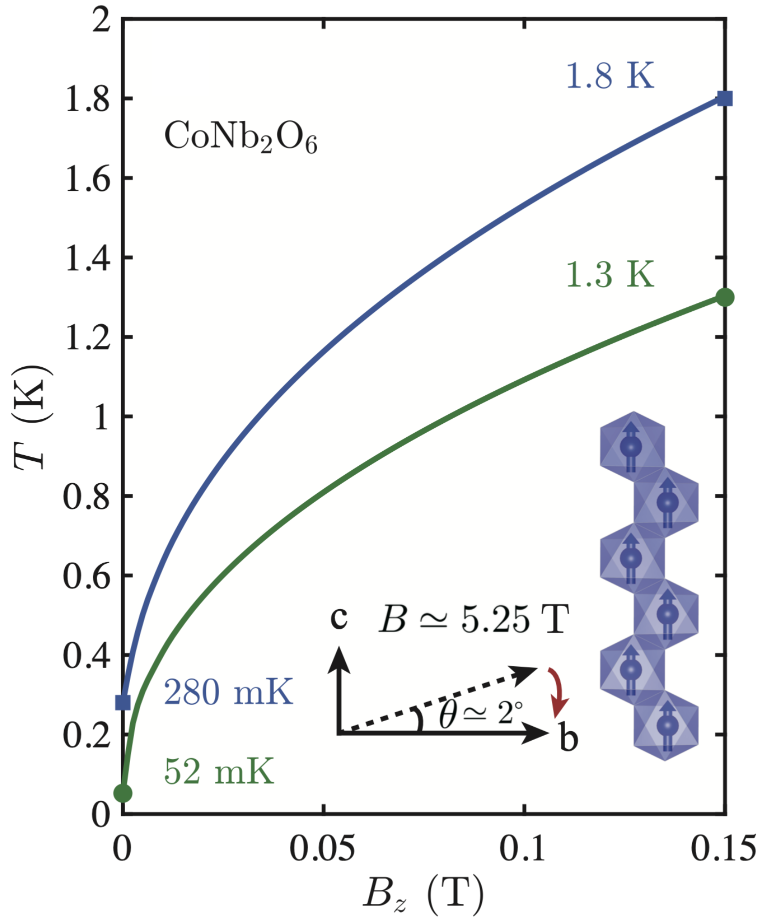
Material Realization: Quantum Supercriticality

$CoNb_2O_6$: FM Ising-chain compound



Coldea *et al.*, **Science** 2010

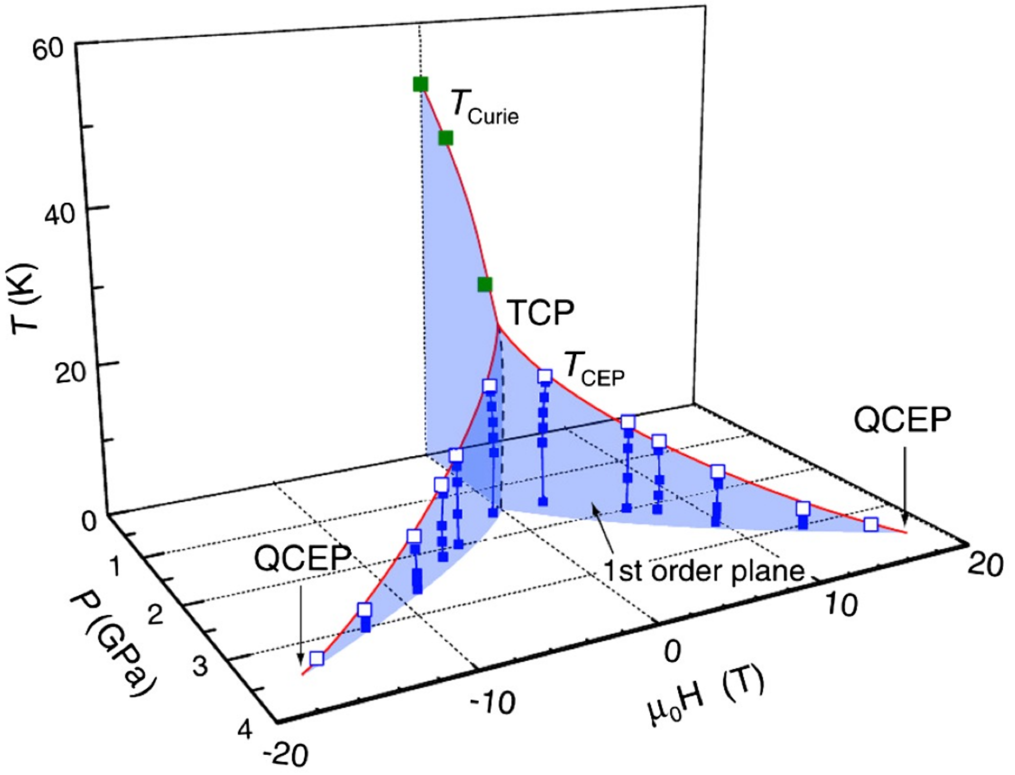
QSR Cooling: small h induces large ΔT



- Twist the field by a very small angle (2 degree)

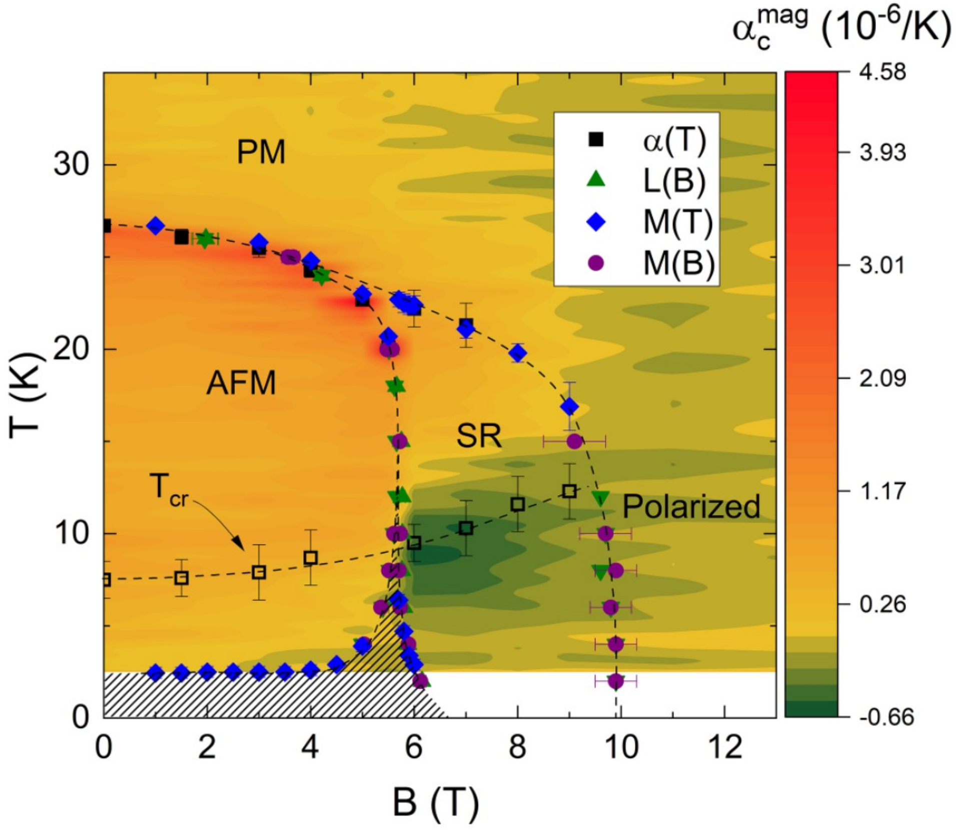
Quantum Critical Endpoint

Metallic Quantum Ferromagnet



UGe2, UCoAl, etc
quantum-critical end points (QCEP)
Rev. Mod. Phys. 88, 025006 (2016)

Frustrated Magnets



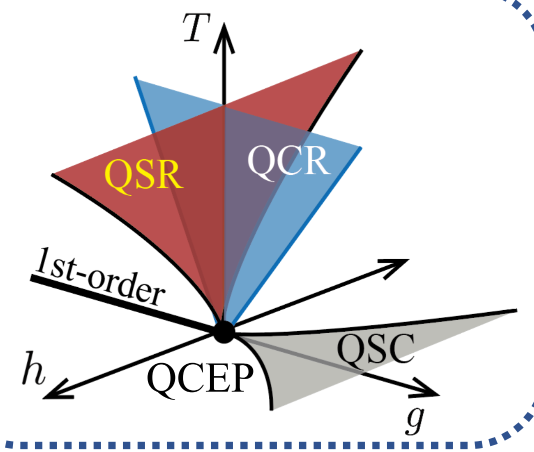
Signatures of a ***quantum critical endpoint*** in the **Kitaev candidate Na2Co2TeO6**, PRB 110, L140402 (2024)

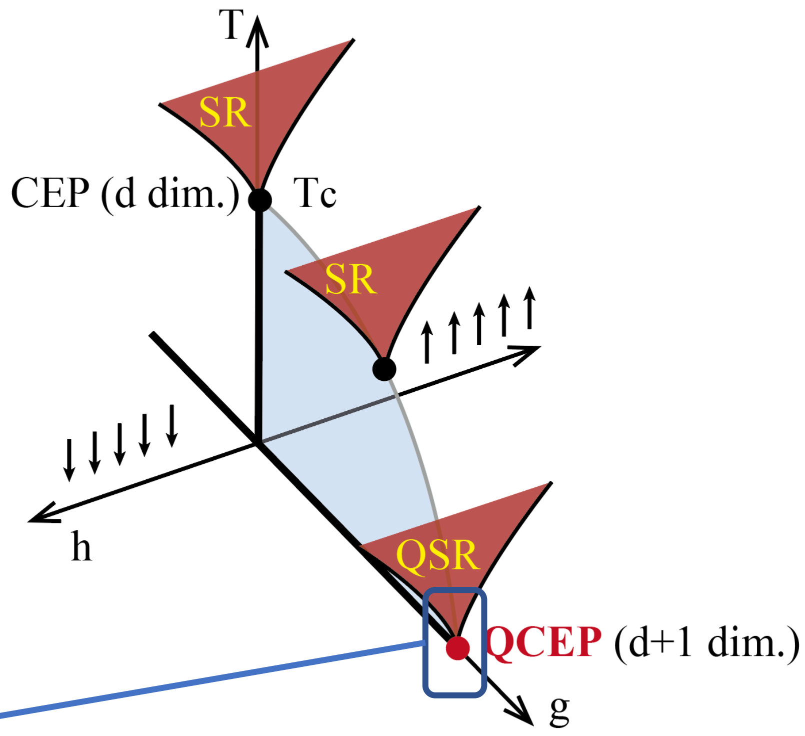
Summary

- **Propose the new concept of quantum supercritical regime (QSR)**

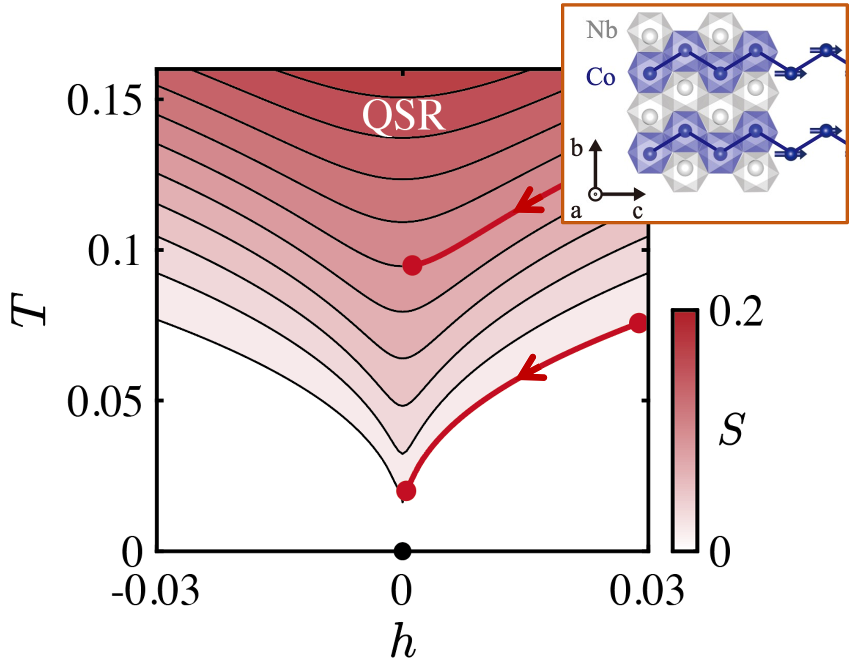
➢ **Crossovers:**

$$h \propto T^{\frac{\Delta}{z\nu}}$$





- **Discover a universally boosted magnetocaloric effect**



➢ **Universal cooling:**

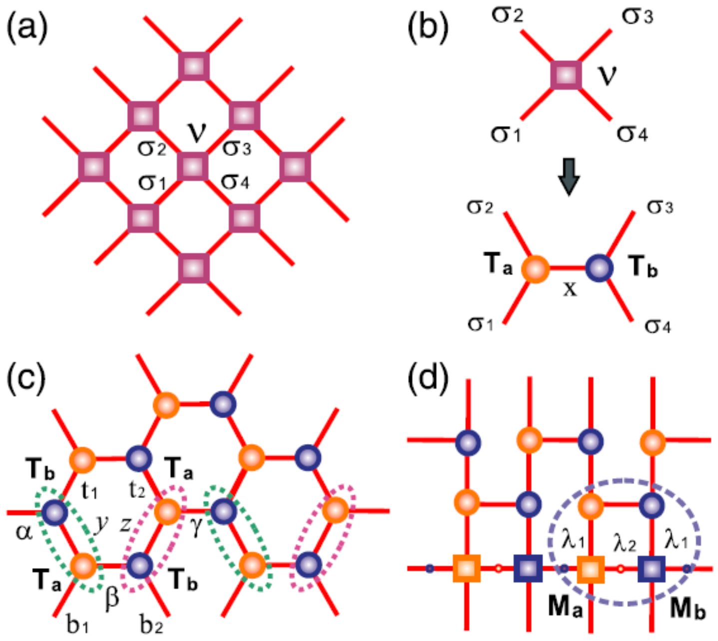
$$\Gamma_h \propto T^{-\frac{\Delta}{z\nu}}, \Delta \equiv \beta + \gamma$$

Thanks for your attention!

backup

Accurate Finite-T Method: Thermal Tensor Networks

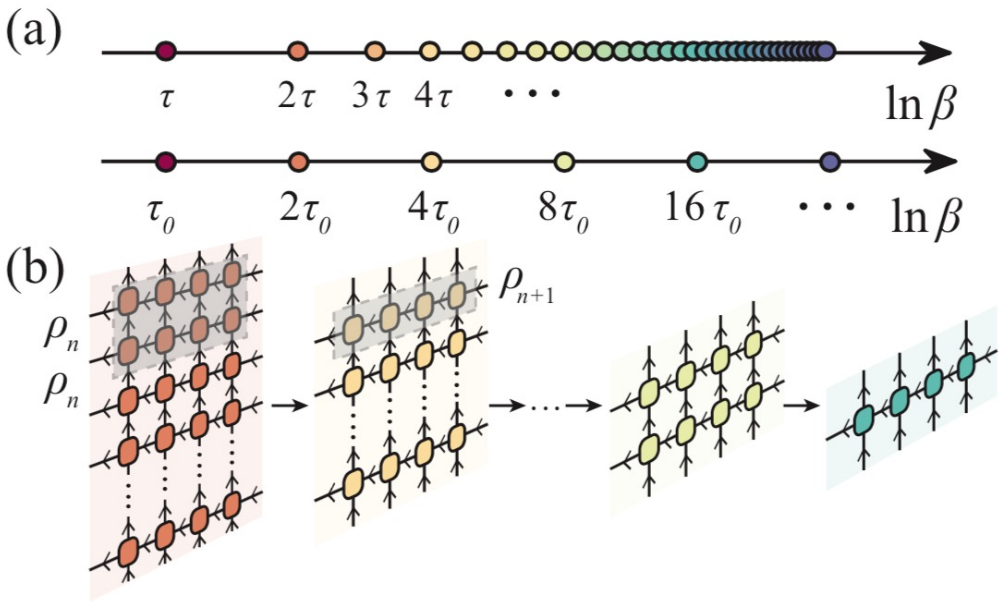
LTRG



$$Z_N \simeq \mathrm{Tr}[e^{-\beta H_1/K} e^{-\beta H_2/K}]^K$$
$$= \sum_{\{\sigma_i^j\}} \prod_{j=1}^{K} \langle \sigma_1^{2j-1} \ldots \sigma_N^{2j-1} | e^{-\beta H_1/K} | \sigma_1^{2j} \ldots \sigma_N^{2j} \rangle$$
$$\times \langle \sigma_1^{2j} \ldots \sigma_N^{2j} | e^{-\beta H_2/K} | \sigma_1^{2j+1} \ldots \sigma_N^{2j+1} \rangle,$$

- ***Linearized TRG:*** PRL 106, 207202 (2011)

XTRG

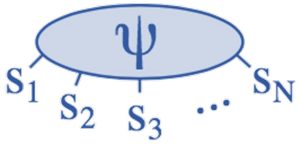
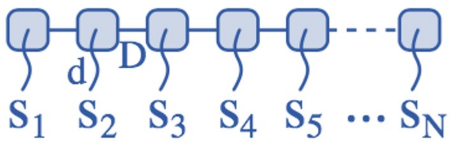
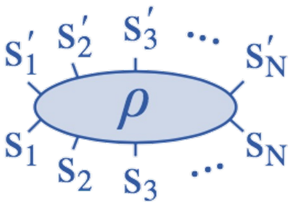
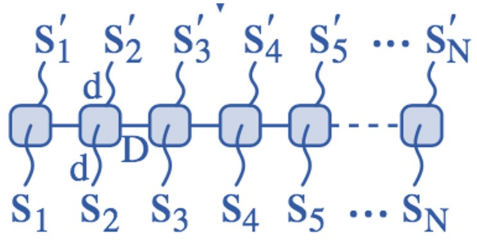


$$\rho_n \times \rho_n \to \rho_{n+1}$$

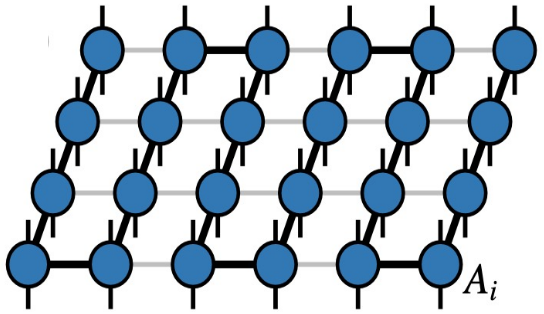
✓ **Inspired by logarithmic entanglement scaling**

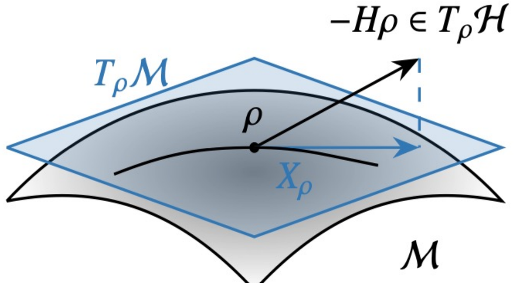
- ***Exponential TRG:*** PRX 8, 031082 (2018)

Matrix Product Operator Approach for Finite-T Calculations

Many-body Problem		Tensor Networks	Reduced Para.	
 many-body w.f.	d^N →		$N \times (dD^2)$	Matrix Product States
 density matrix	d^{2N} →		$N \times (d^2 D^2)$	Matrix Product Operators

- 2D lattice: tangent-space TRG





$$\frac{dA_i}{d\beta} = -H_i^{(1)}A_i + A_i^L H_i^{(0)} S_i,$$

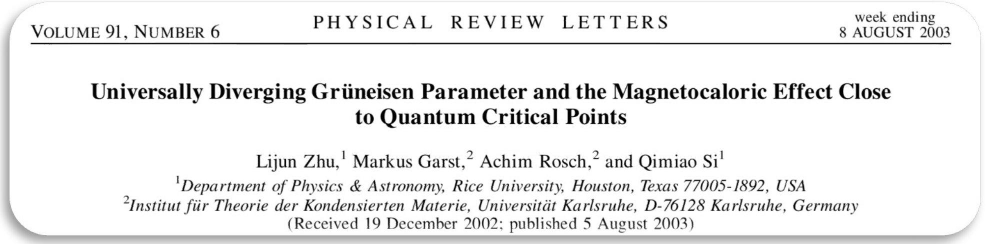
➢ Benchmark: Square-lattice Hubbard model

Method	Cylinder	Temperature	Refs.
METTS	Width 4	T/t=0.05	PRX2021
tanTRG	**Width 8**	**T/t<0.05**	**Our work***

* PRL 130, 226502 (2023) ***Editors' Suggestion***

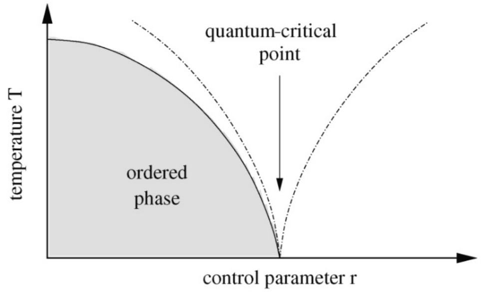
Reminder: Quantum Critical Cooling

■ Quantum criticality MCE


VOLUME 91, NUMBER 6 PHYSICAL REVIEW LETTERS week ending 8 AUGUST 2003

Universally Diverging Grüneisen Parameter and the Magnetocaloric Effect Close to Quantum Critical Points

Lijun Zhu,[1] Markus Garst,[2] Achim Rosch,[2] and Qimiao Si[1]
[1]Department of Physics & Astronomy, Rice University, Houston, Texas 77005-1892, USA
[2]Institut für Theorie der Kondensierten Materie, Universität Karlsruhe, D-76128 Karlsruhe, Germany
(Received 19 December 2002; published 5 August 2003)



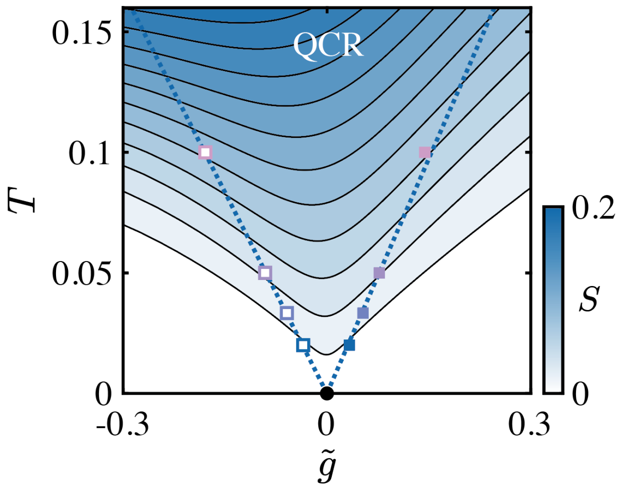
Qimiao Si, **Rice University**

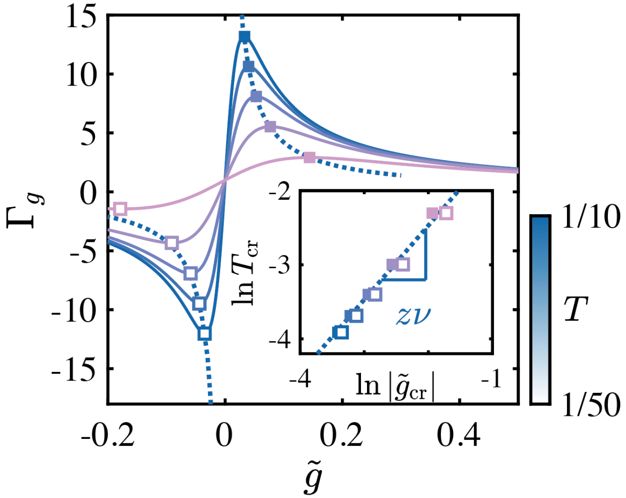
- ➢ Gruneisen ratio and the magnetocaloric effect are divergent.

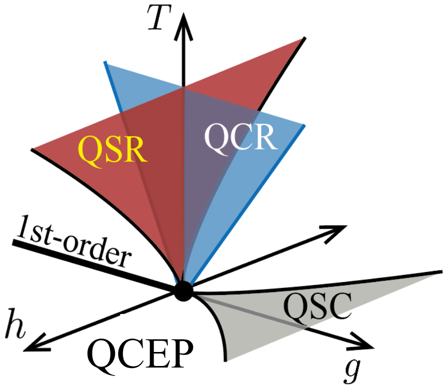
- ➢ Gruneisen ratio and the magnetocaloric effect are divergent.

- ▪ Universally diverging $\Gamma \sim T^{-1/z\nu}$

■ QCR cooling also exhibits scaling law







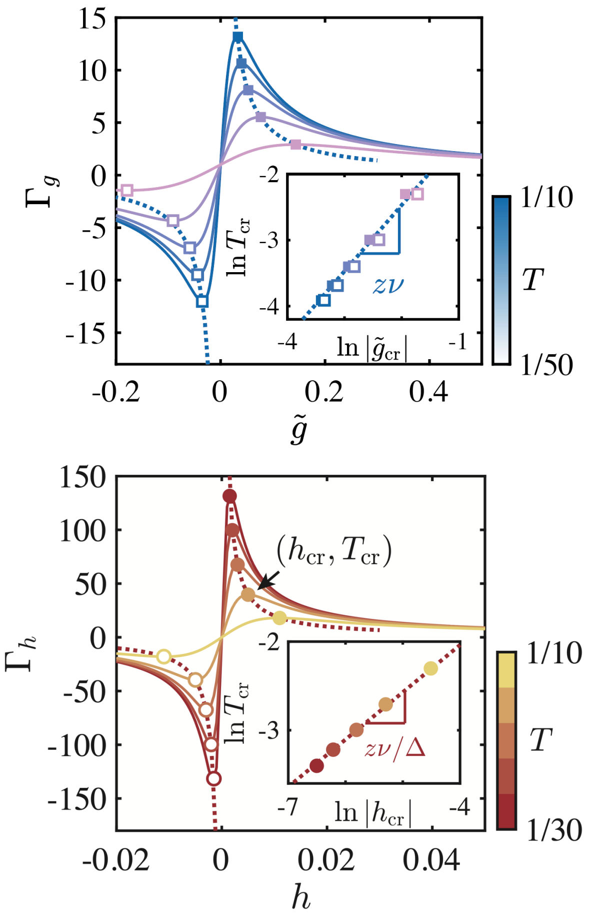
- ➢ **The QCR and QSR cooling have distinct scalings!**

■ ***How do they compare?***

Quantum Supercritical vs. Quantum Critical Cooling

Different fields

(fluctuation *vs.* symmetry-breaking)

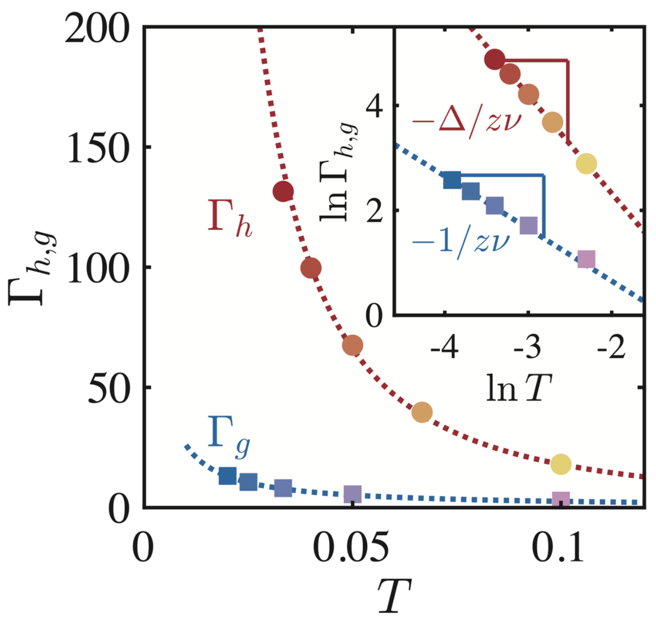


Distinct scaling law

$(\frac{\Delta}{z\nu}$ *vs.* $\frac{1}{z\nu})$

QSR cooling: $\Gamma_h \propto T^{-\frac{\Delta}{z\nu}}$

QCR cooling: $\Gamma_g \propto T^{-\frac{1}{z\nu}}$



MCE boost!

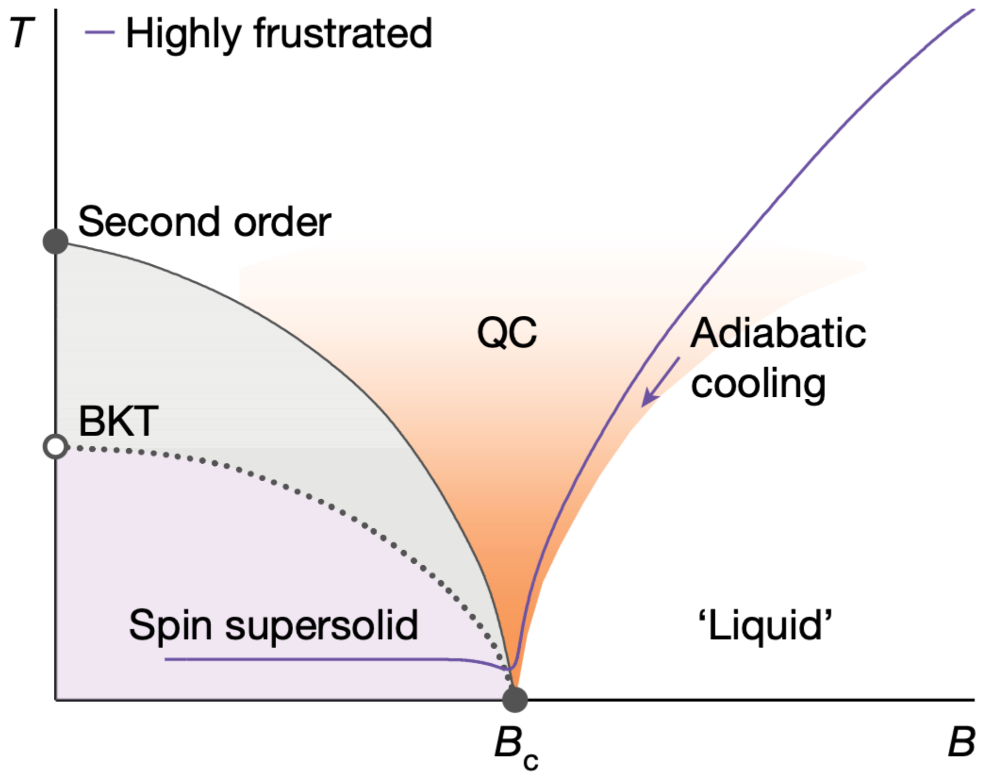
(exponential amplification)

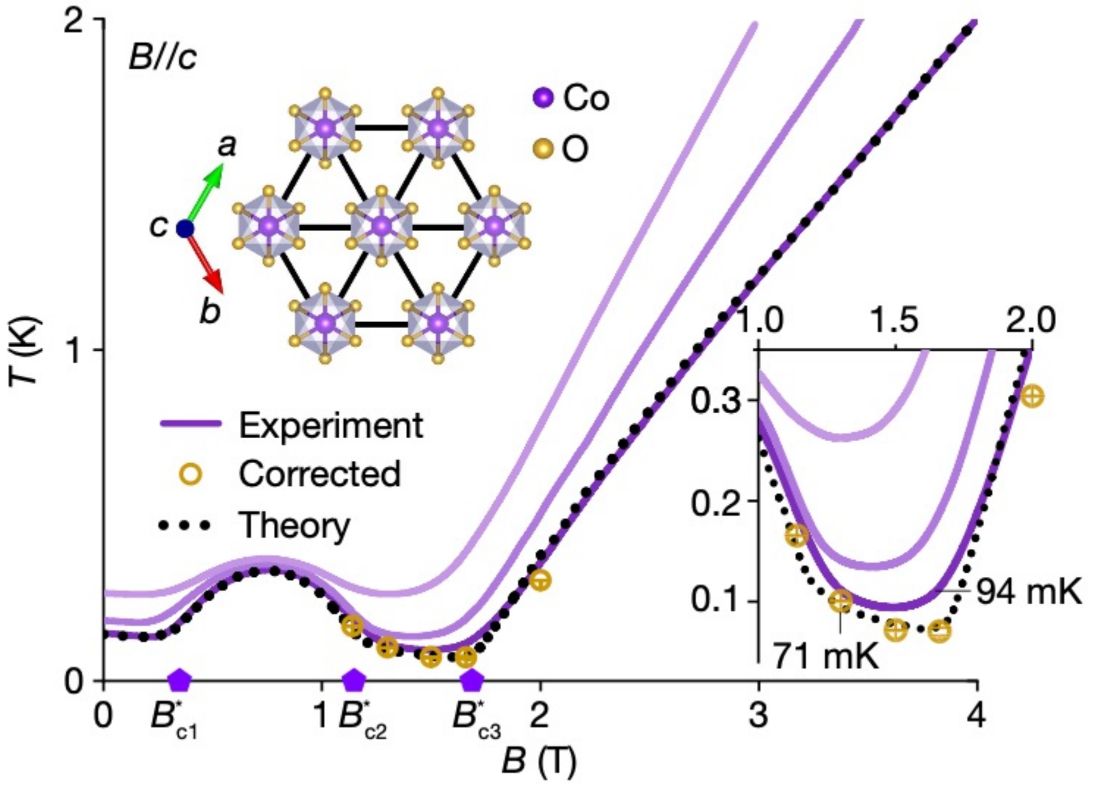
$$\Gamma_h \propto (\Gamma_g)^{\Delta}$$

Univ. Class	Δ	$z\nu$	$\Delta/z\nu$
(1+1)D Ising	15/8	1	15/8
(2+1)D Ising	1.564	0.630	2.482
(2+1)D XY	1.667	0.672	2.481
(2+1)D O(3)	1.765	0.711	2.482
(1+1)D 3-Potts	14/9	5/6	28/15
(1+1)D 4-Potts	5/4	2/3	15/8
Mean Field	3/2	1	3/2

Experimental Observation of QCR Cooling

❑ Spin Supersolid Transition: Giant Magnetocaloric Effect



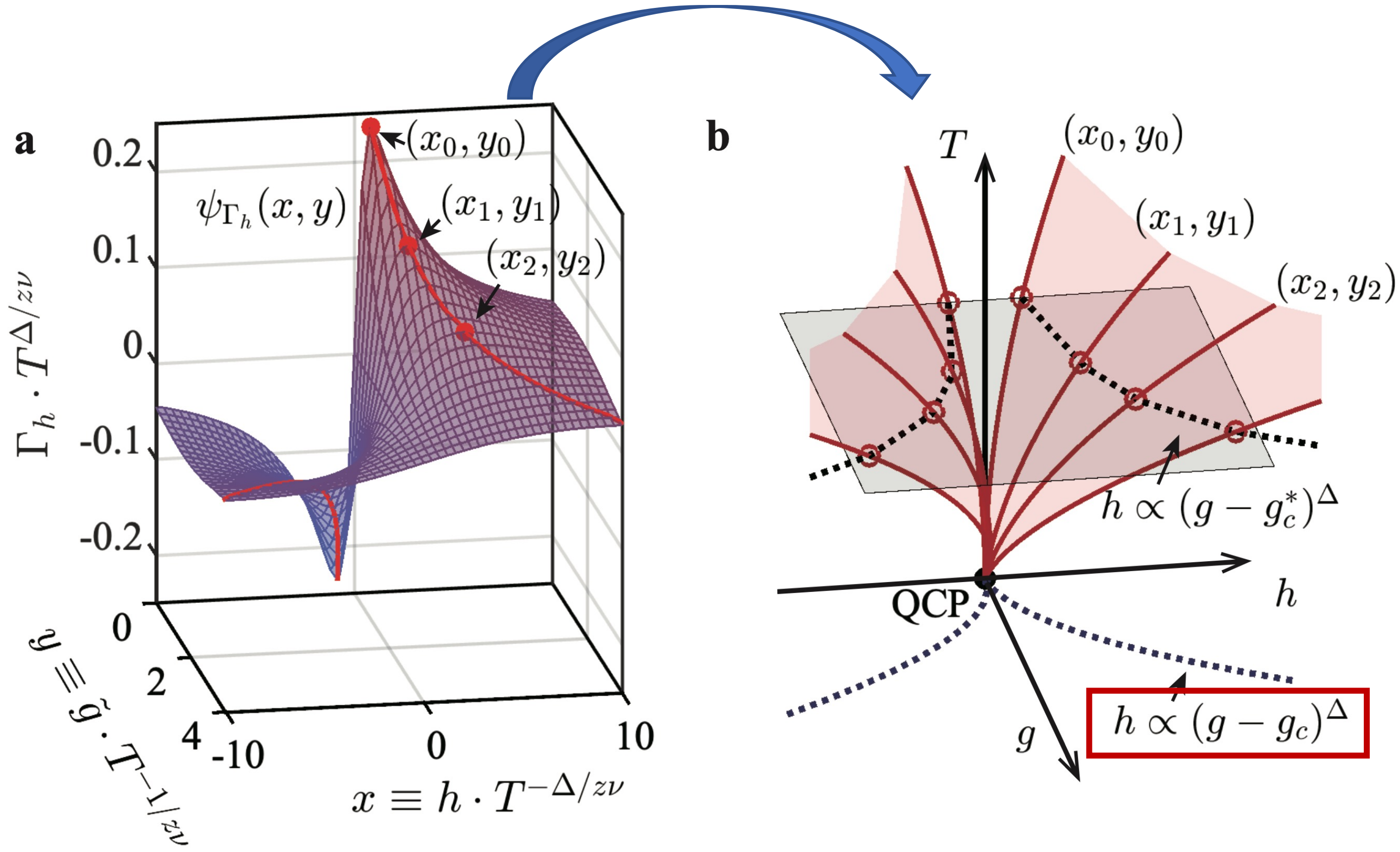


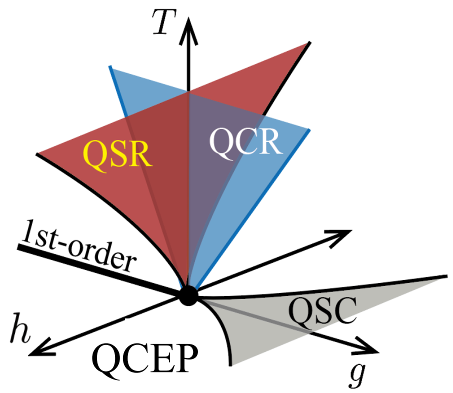
Nature 625, 270–275 (2024)

2024年度中国科学十大进展

Hyperscaling Function and Crossover Surface

- ❑ **From crossover lines to crossover surface**





Hyperscaling function

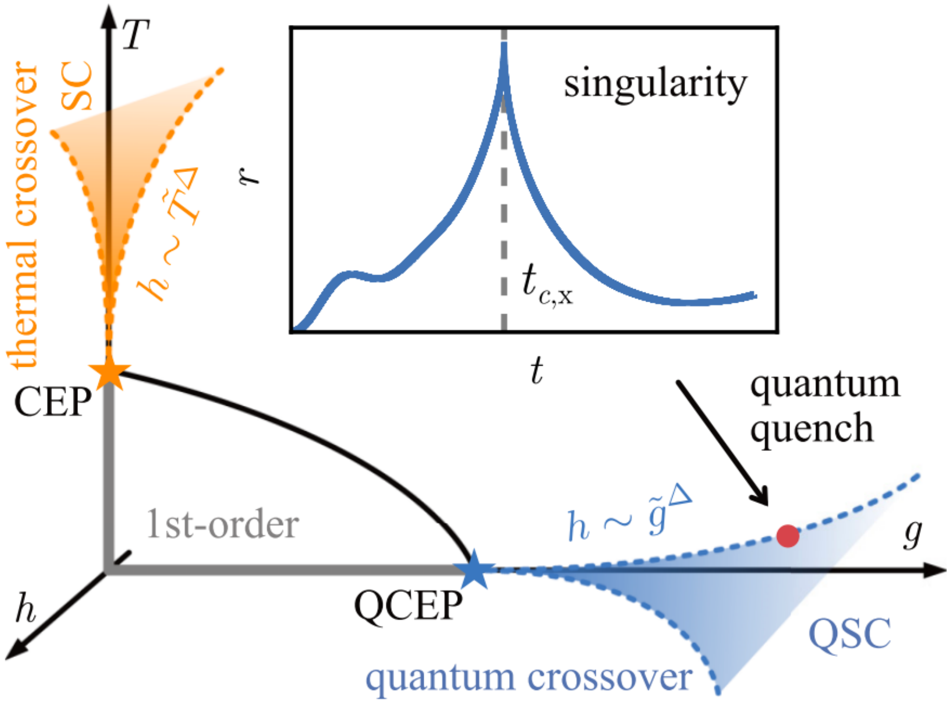
$$\Gamma_h = T^{-\frac{\Delta}{z\nu}} \psi_{\Gamma_h}(x,y)$$

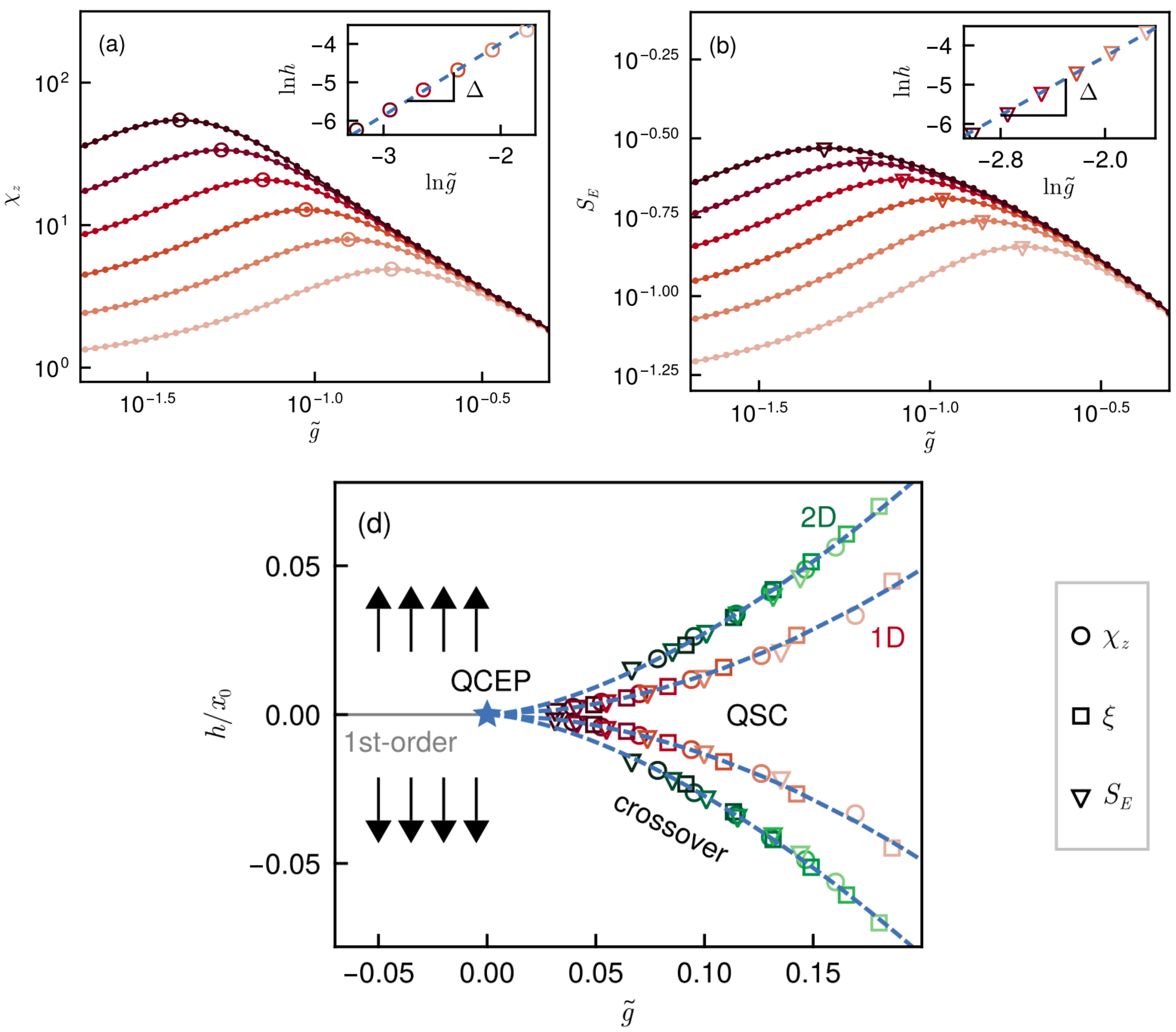
$$x = h \cdot T^{-\frac{\Delta}{z\nu}}, \; y = \tilde{g} \cdot T^{-\frac{1}{z\nu}}$$

In the zero-temperature limit, arriving at $h \propto (g-g_c)^\Delta$

Ground-State Crossover

$$H = -\sum_{<i,j>} S_i^z S_j^z - g\sum_i S_i^x - h\sum_i S_i^z$$





Ground-state crossover scaling: $h \propto \tilde{g}^{\Delta}$

similar to the classical scaling: $h \propto \tilde{T}^{\Delta}$

J. Wang, **E. Lv**, X. Li, Y. Jin, WL, PRB, 112, 195120 (2025)

Dynamical Singularity

Loschmidt echo:

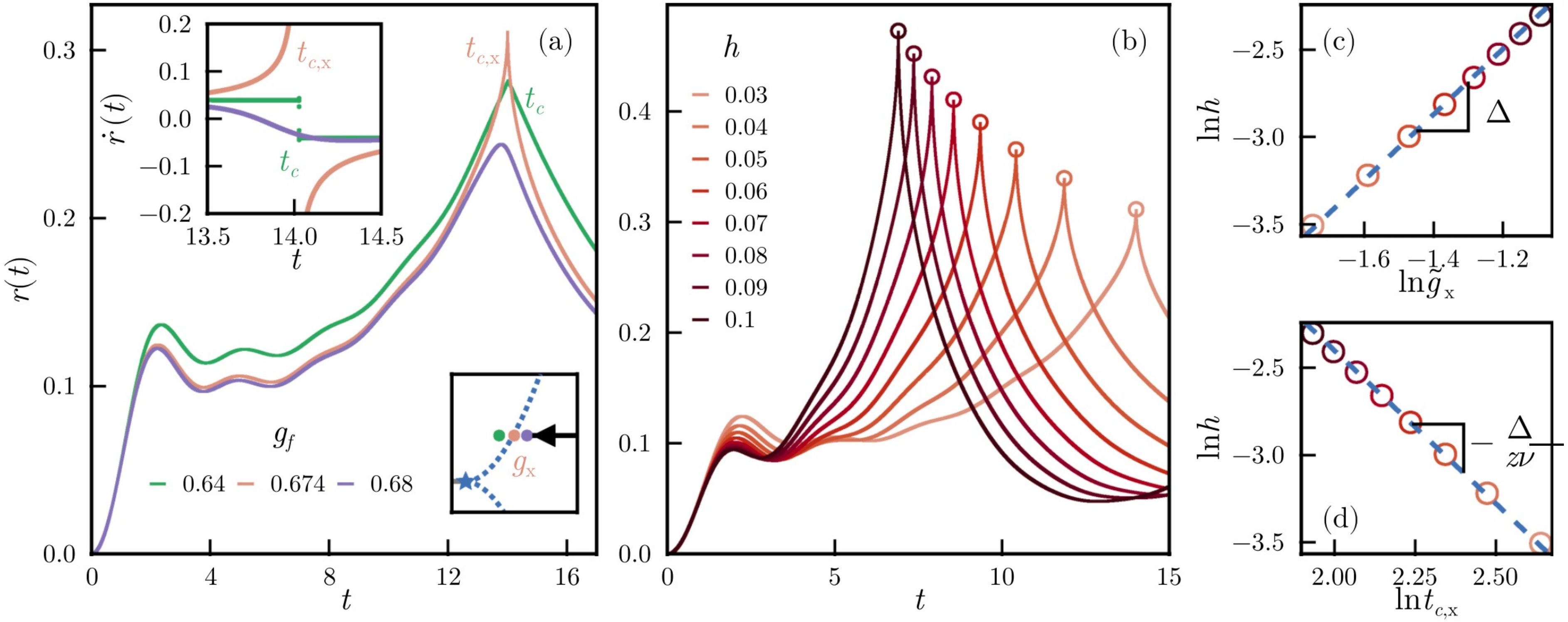
$$G(t) = \langle \psi_i | e^{-iH_f t} | \psi_i \rangle$$

Loschmidt rate:

$$r(t) = -\lim_{L\to\infty} \frac{1}{L} \ln |G(t)|^2$$

initinal state:

$$|\psi_i\rangle = |\rightarrow, \rightarrow, \ldots \rightarrow\rangle$$



$$h \sim \tilde{g}_x^{\Delta}$$

$$h \sim (1/t_{c,x})^{\Delta/z\nu}$$

Loschmidt rate can identify ground-state supercritical crossovers by the 1/2-cusp