Measuring temporal entropies in experiments

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Motivation

Spatial entanglement

Same time correlations between different spatial region.

Essential role in:

- Quantum Phase Transitions
- Quantum Information Theory
- Proxy for ergodicity in out-of-equilibrium dynamics

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Temporal Entanglement

Measures correlations of constituents across ***different temporal regions***.

Connected to:

- Equilibration and thermalization
- Geometric information (holographic theories)

Motivation

Q1: Can we measure temporal entanglement in experiments?

Motivation

Q1: Can we measure temporal entanglement in experiments?

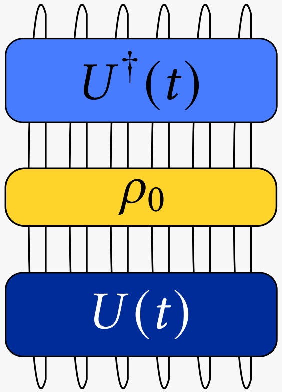
Q2: What information about the system can we learn from it?

Spatial and Temporal Entanglement

Diagrams

Most quantities can be found through the trace of the initial density matrix ρ_0, the unitary evolution operator $U(t)$ and some combination of operators $\{O_j\}$.

$$1 = \mathrm{tr}(\rho(t)) = \mathrm{tr}\big(U(t)\rho_0 U^\dagger(t)\big) =$$



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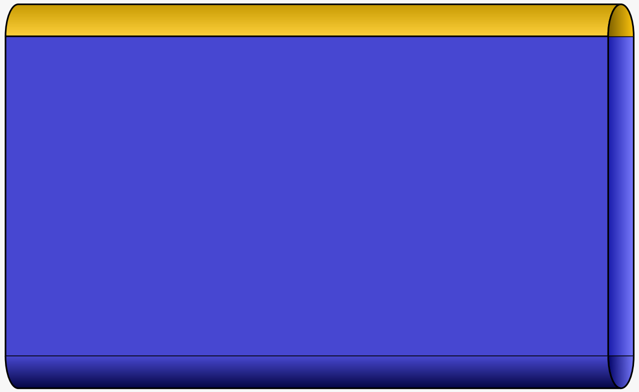
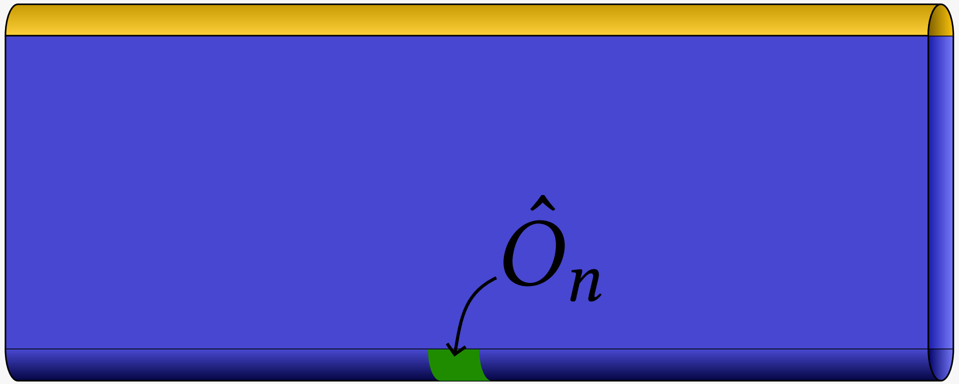


Diagrams

Most quantities can be found through the trace of the initial density matrix ρ_0, the unitary evolution operator $U(t)$ and some combination of operators $\{O_j\}$.

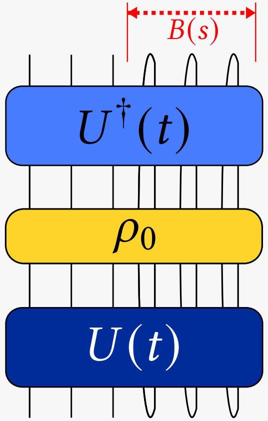
$$\left\langle \hat{O}_n(t) \right\rangle = \mathrm{tr}\left(\rho(t)\hat{O}_n\right) =$$



Spatial entanglement

It is characterised by *space-like cut* in the partition function dividing the space in subsystem A and B.

$$\rho_A(t) = \mathrm{tr}_{B(s)}\left(U^\dagger(t)\rho_0 U(t)\right) =$$



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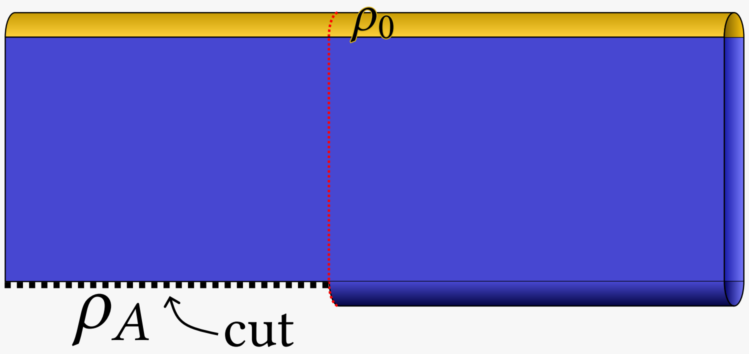


Renyi entropies quantify the entanglement of a system,

$$S_\alpha(t) = \frac{1}{1-\alpha}\log(\mathrm{Tr}(\rho_A(t)^\alpha))$$

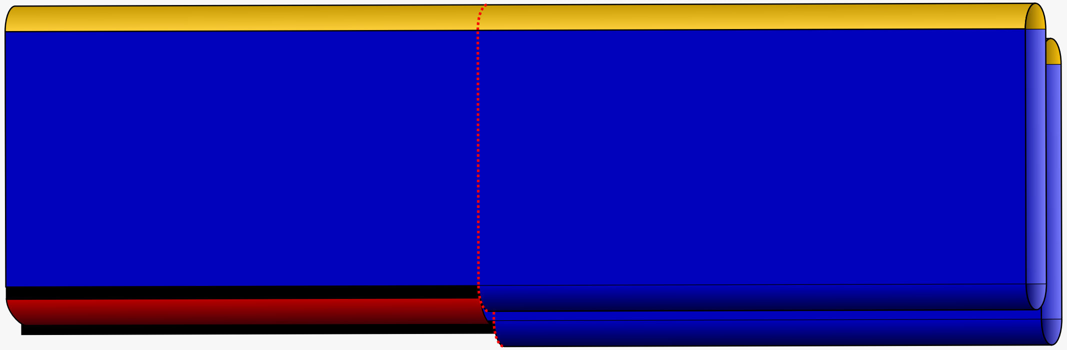
Spatial entanglement

Renyi entropies are computed as,

$$S_\alpha(t) = \frac{1}{1-\alpha}\log(\mathrm{Tr}(\rho_A(t)^\alpha))$$

for the case $\alpha = 2$, $\rho^2_{A(t)}$ is depicted as

$$\rho^2_A(t) =$$



ρ^2_A

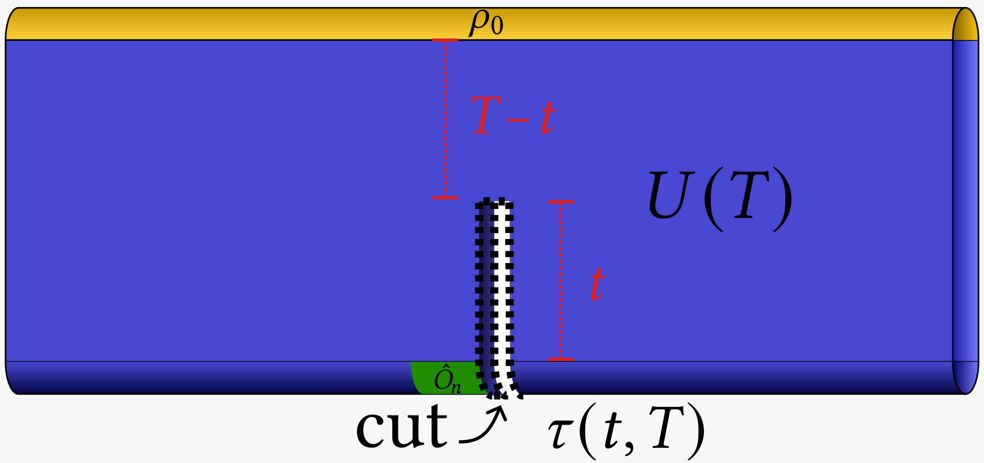
To calculate $S_2(t)$ we only need to trace over the open contours.

Temporal entanglement

It is characterised by *time-like cut* in the partition function dividing the time in subsystem $(0, T-t)$ and $(T-t, T)$.

The reduced transition matrix $\tau(t,T)$ acts similarly to the reduced desity matrix $\rho_A(t)$.

$$\tau(t,T) =$$



Analogous quantity with a 90 degree rotation.

Measurement protocol

$$\mathcal{T}^{\alpha}(t)_{O_j} = \mathrm{tr}\left[\left(\tau_{\hat{O}}(t,T)\right)^{\alpha}\right]$$ trace of reduced transition matrix

$$\mathcal{S}^{\alpha}(t)_{O_j} = \frac{1}{1-\alpha}\log\left[\mathcal{T}^{\alpha}(t)_{O_j}\right]$$ generalized temporal entropies

Measurement protocol

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for $\alpha \in \mathbb{N}$, $\mathcal{S}^{\alpha}$ accessible in experiment $\Rightarrow$ replica trick:

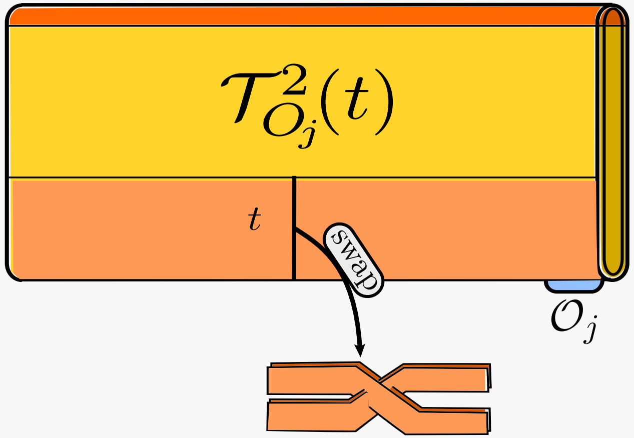
two ($\alpha = 2$) copies of the same system

$$\left[\tau_{\hat{O}}(t,T)^2\right]_{a,b} = \sum_c \left[\tau_{\hat{O}}(t,T)\right]_{a,c}\left[\tau_{\hat{O}}(t,T)\right]_{c,b}$$

$$\mathrm{tr}\left[\tau_{\hat{O}}(t,T)^2\right] = \sum_{d,c} \left[\tau_{\hat{O}}(t,T)\right]_{d,c}\left[\tau_{\hat{O}}(t,T)\right]_{c,d}$$

Measurement protocol

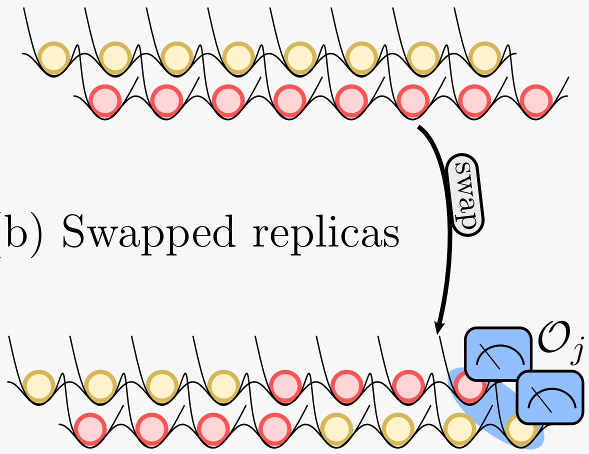
Generalized temporal entropies



Physical Experiments

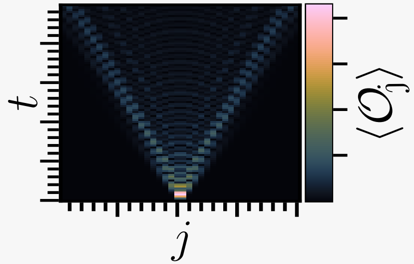
(a) Replicated system

(b) Swapped replicas

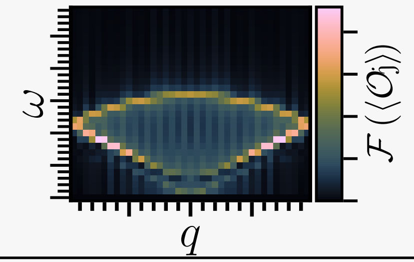


Characterization

(c) Temporal evolution: replicated operator



(d) Fourier Transform

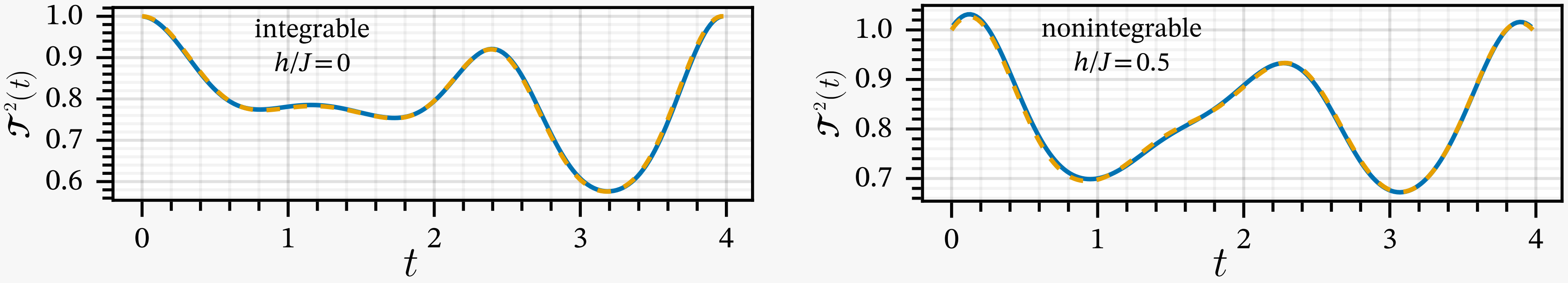


Results

1: Generalized temporal entropies (GTE) are real valued

In general, the reduced transition matrix are *complex-valued*, and if they are diagonalizable, so is their spectrum.

Identidying GTE with measurements implies they are *real-valued*.

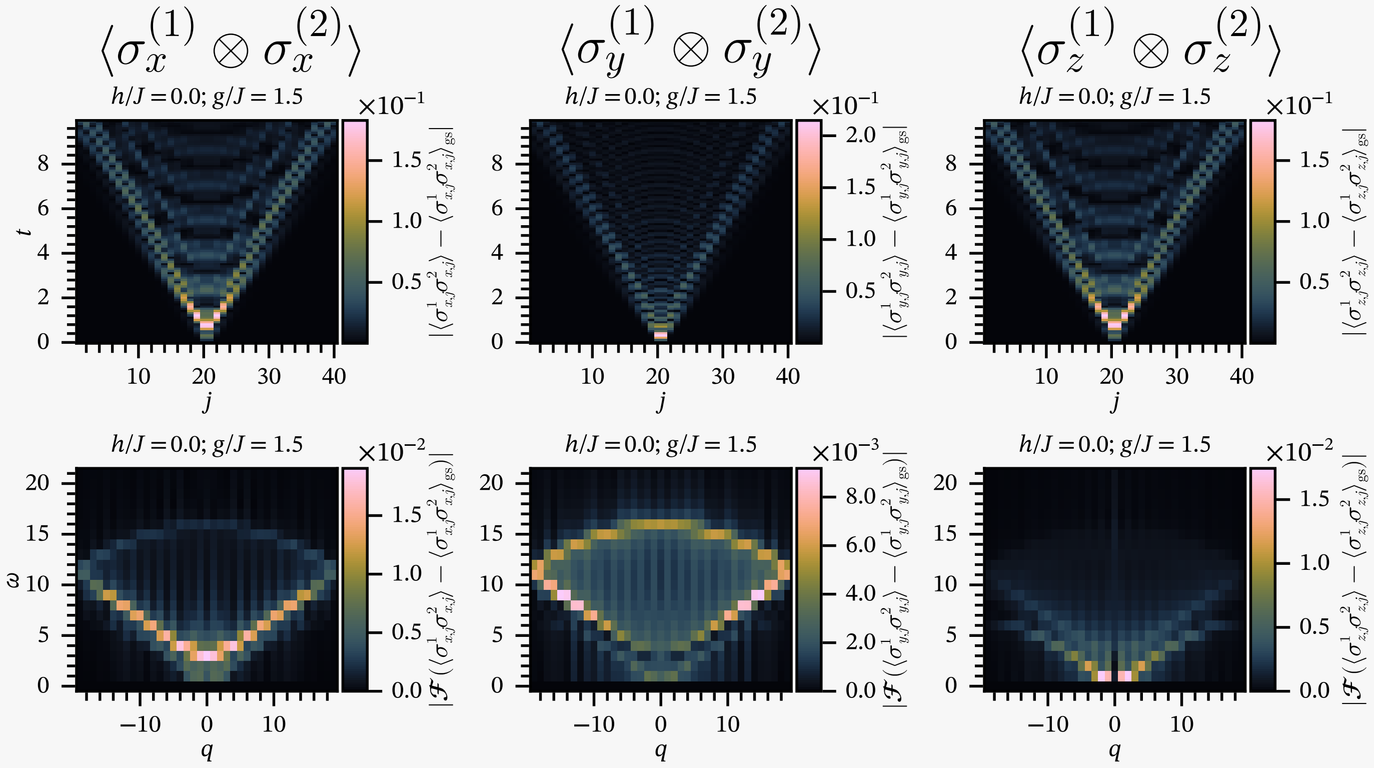


2: Generalized temporal entropies can be interpreted as dynamical probes of geometric quenches

Similar to pump-prove experiments our system is first evolved for some time $T - t$, then our system is excited by a perturbation and it is finally measured to characterize the evolution.

Few operators are sufficient to characterize the system.

2: Generalized temporal entropies can be interpreted as dynamical probes of geometric quenches



3: Generalized temporal entropies can be measured in state-of-the-art quantum simulators.

To measure GTE in quantum simulators we require three capabilities:

1. The ability to initialize two identical replicas of the same system.
2. Temporal control over inter-replica coupling.
3. Access to local observables in different spatial regions (or momentum-resolved observables).

These capabilities are available in platforms like cold atoms in optical lattices, Rydberg arrays, trapped ions, and superconducting qubits.

Dynamical Information from GTEs

Ising model

We are interested in knowing what information we can learn from GTEs.

To do so, we study the Transverse Field Ising model (TFI).

$$H = -J \sum_{n=1}^{N-1} \sigma_n^x \sigma_{n+1}^x + \sum_{n=1}^{N} (g\sigma_n^z + h\sigma_n^x),$$

This model is integrable when $h = 0$, we will study it at different points of its phase diagram.

Ising model

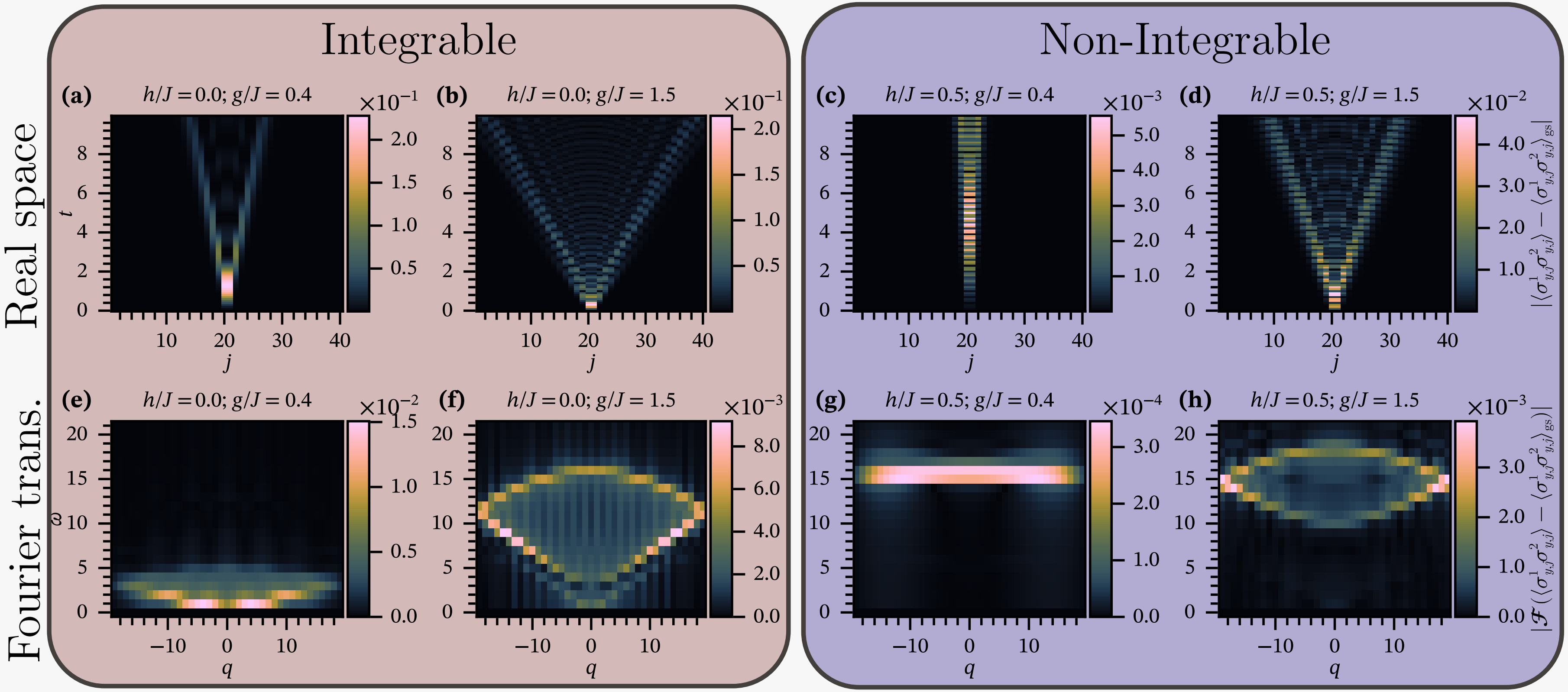
The setup is the to study what we can learn from GTEs is

1. We prepare two independent systems 1,2 at the ground state of the initial Hamiltonian $H^{(1)}$ and $H^{(2)}$.
2. After time $T - t$ we quench the system by changing the Hamiltonian to $H^{(1,2)}$ and $H^{(2,1)}$ and they evolve until time T.
3. At T we measure in both systems with $O_j^{(1)} \otimes O_j^{(2)}$

We are interested in Generalizied Purities,

$$\mathcal{T}^2(t,T)_{\hat{O}_j} = \frac{\operatorname{tr}\left[\hat{O}_j^{(1)} \otimes \hat{O}_j^{(2)} \tau^{(1,2)}(t,T)\right]}{\left\langle \hat{O}_j(T) \right\rangle^2}$$

The systems simulated have $N = 40$ qubits, and the operator studied is $\hat{O}_n = \sigma_n^y$:



Decay of the soft mode

The Fourier transformation informs us about the integrability breaking (or the symmetry breaking) between $h = 0$ and $h \neq 0$.

- **Integrable case:** There is a clear signal at $q = 0$ and around $\omega \approx 0$.
- **Non-Integrable case:** The signal around $q = 0$ and $\omega \approx 0$ disappears and a gap opens.

This property can be of **experimental importance** to assess whether a prepared system is in the integrable phase or not.

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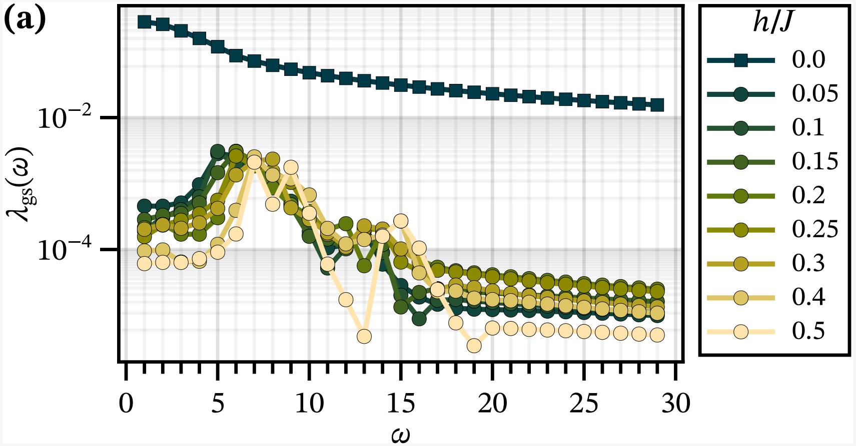
This property can be of **experimental importance** to assess whether a prepared system is in the integrable phase or not.

How robust is this property under small perturbations on the parameters or temperature?

Decay of the soft mode

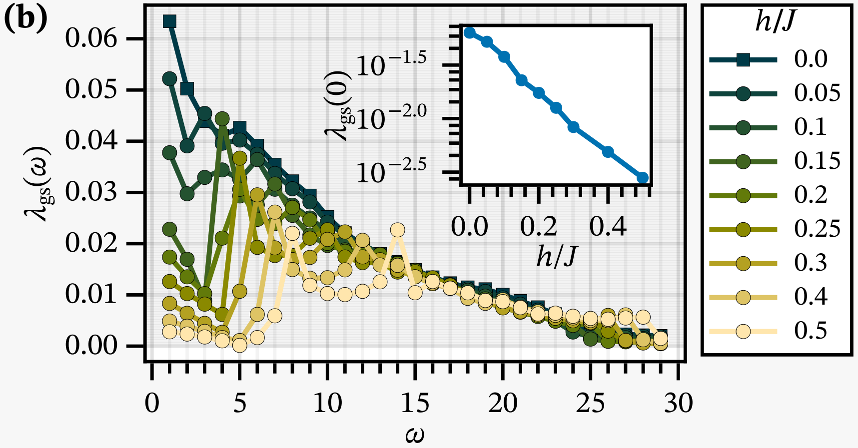
We study F.T. of the G. Purity along $q = 0$ ($\lambda_{gs}(\omega)$) to assess how the gap opens under a small integrability-breaking.

Ferromagnetic phase $\frac{g}{J} = 0.4$



Integrabilty ⇔ gap

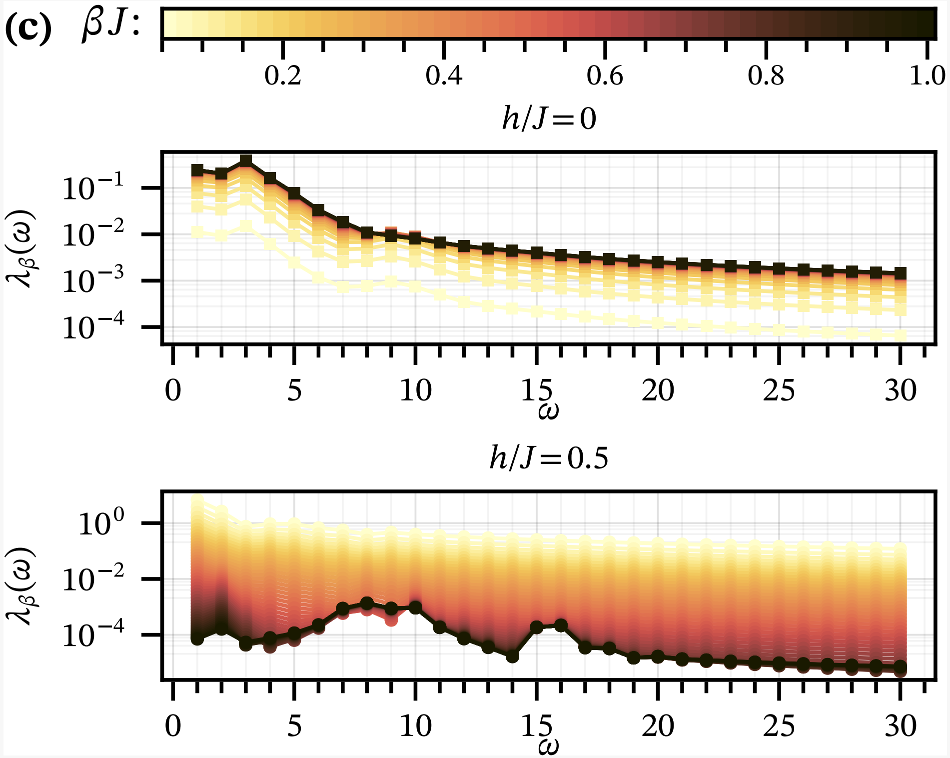
Paramagnetic phase $\frac{g}{J} = 1.5$



Integrabilty ⇔ exponential decay of gap

Decay of the soft mode

In this case we strart with the infinite temperature density matrix $\rho_0 = \frac{1}{2}\mathbb{I}$. And apply some time-steps of Euclidean evolution. We can see that the gap opens bast and at $\beta J \approx 1$, it is fully developed.

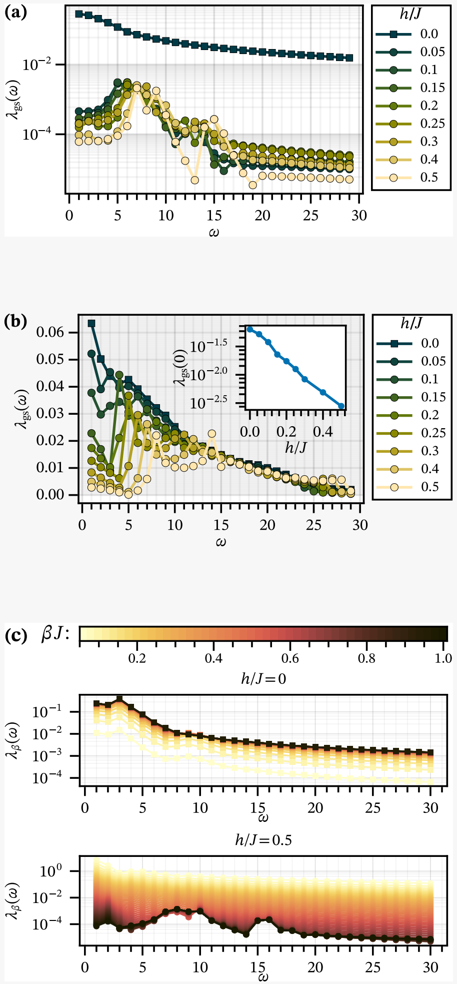


Outlook

We propose a quench protocol to experimentally access to GTEs, which requies at least two copies of a system.

We observe that GTE are sensitive to dynamical properties of the system.

There are still many **open questions** such as the relation between *GTE* and *spatial entanglement*, its connection to other measures of temporal entanglement, or witnesses of entanglement.



Thanks for your attention!

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Normalization problem

We treat the reduced transfer matrix $\tau_O(t, T)$ as if it were a reduced density matrix but there are crutial differences:

- $\tau_O(t, T)$ is not normalized.
- $\tau_O(t, T)$ is not Hermitian.

The non-Hermicity can be adressed by defining the GTE as the expected value of a quench protocol (always real).

The normalization only matters because dependeing on the definition, the generalized purity can be above one thus the GTE is smaller than 0!

Normalization problem

There are two main avenues to normalize the reduced density matrix:

Calabrese and Cardy (2004):

$$\tau(t,T) = \frac{|L_O\rangle\langle R|}{\sqrt{\langle L_O|L_O\rangle \cdot \langle R|R\rangle}}$$

- $\mathcal{T}_O^2 \leqslant 1$
- *Temporal product states* $\mathcal{T}_O^2 \neq 1$.
- *The normalization is not tight and often* $\mathcal{T}_O^2 \ll 1$.

Mollabashi, Shiba, Takayanagi et al. (2021):

$$\tau(t,T) = \frac{|L_O\rangle\langle R|}{\langle L_O|R\rangle}$$

- $\mathcal{T}_O^2$ *bounds unknown, but can be above 1.*
- *Temporal product states* $\mathcal{T}_O^2 = 1$.

Normalization problem

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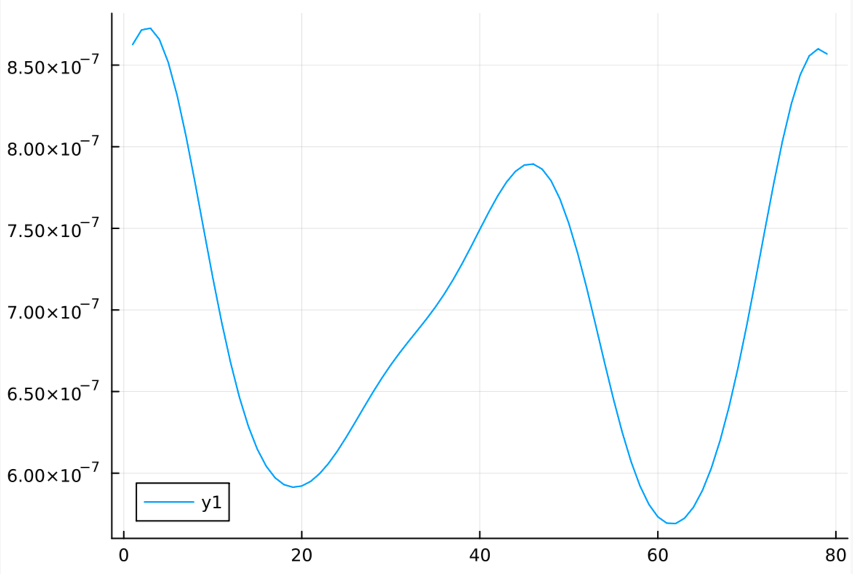
Calabrese and Cardy (2004):



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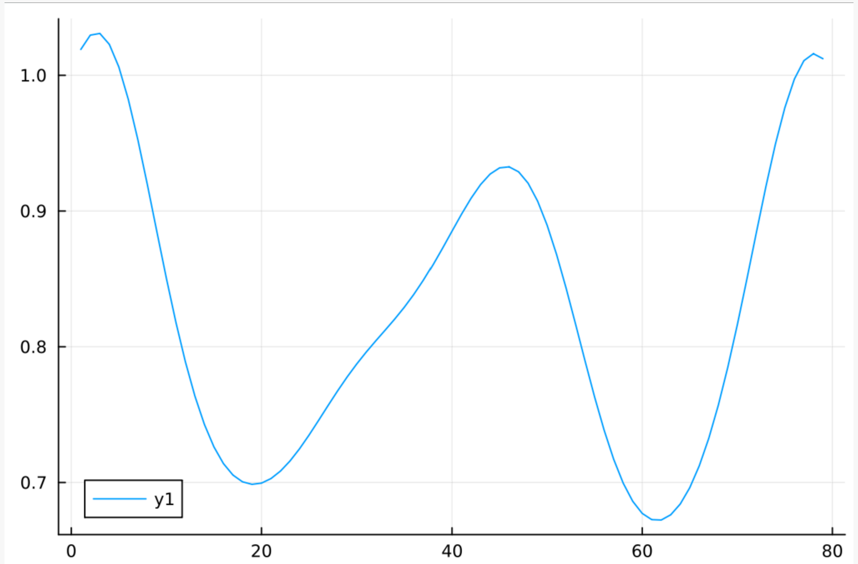
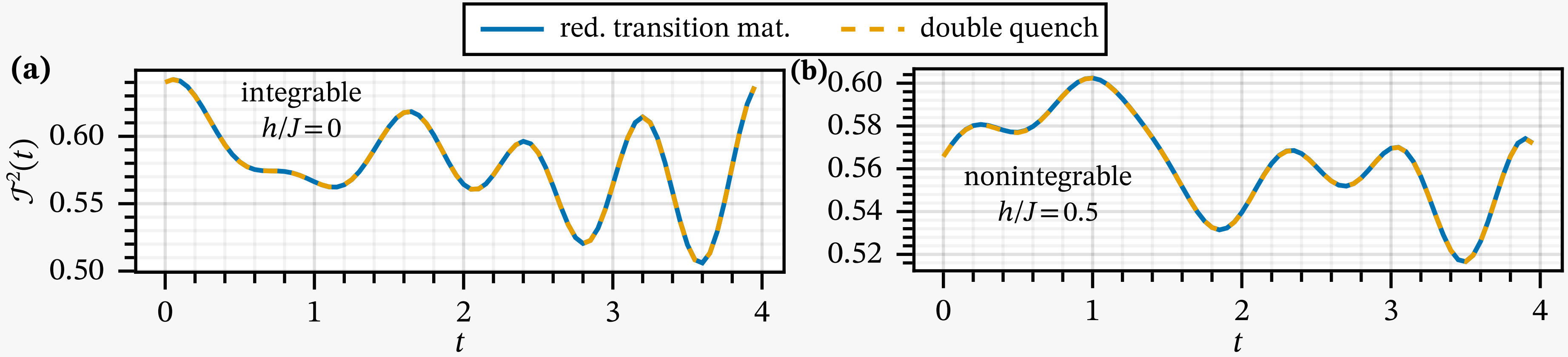
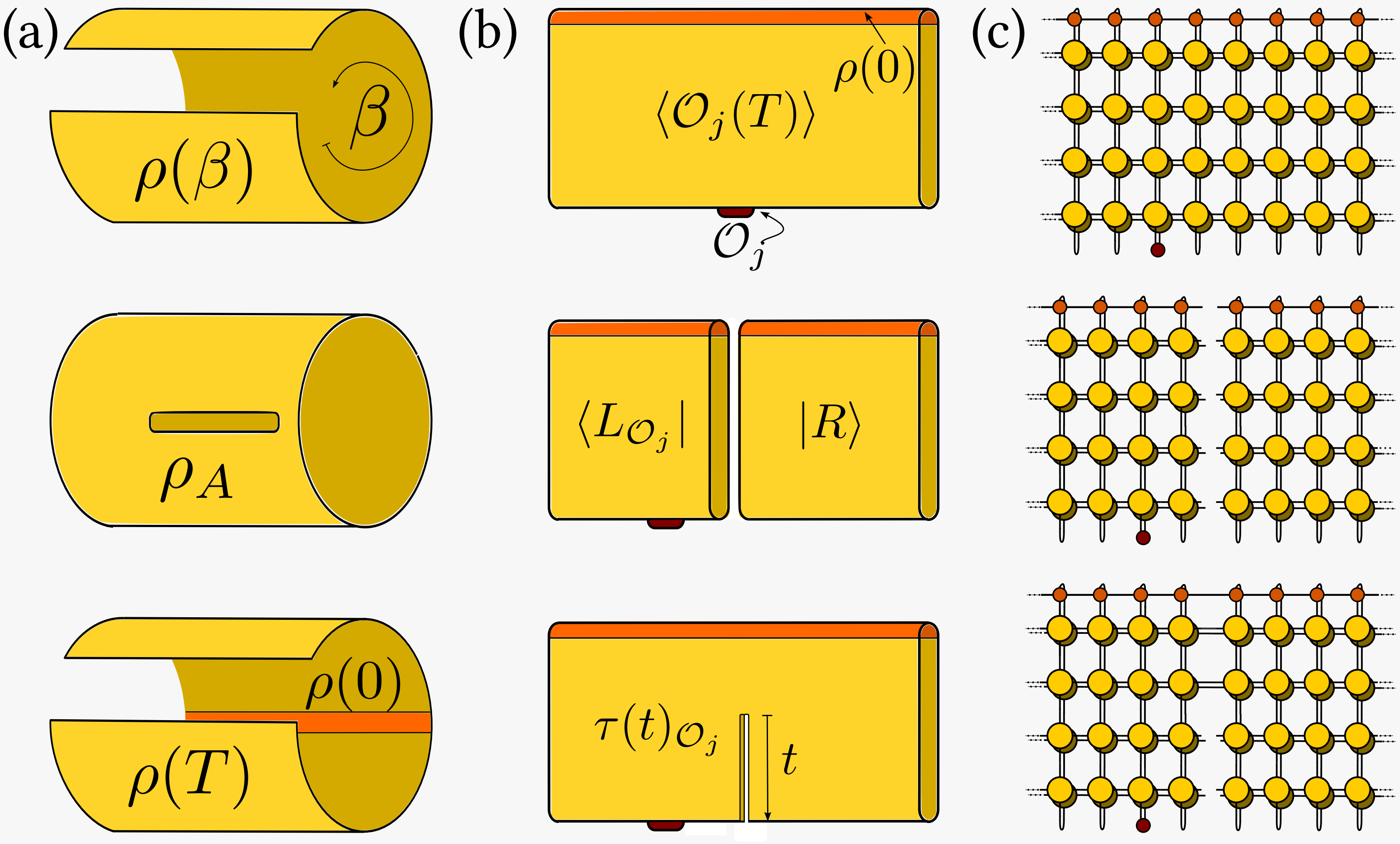


Normalization problem

The normalization problem can be solved looking at twist operators between system 1 and system 2. This way, we would access to the **to a mixed spatio-temporal purity, of a single site**.

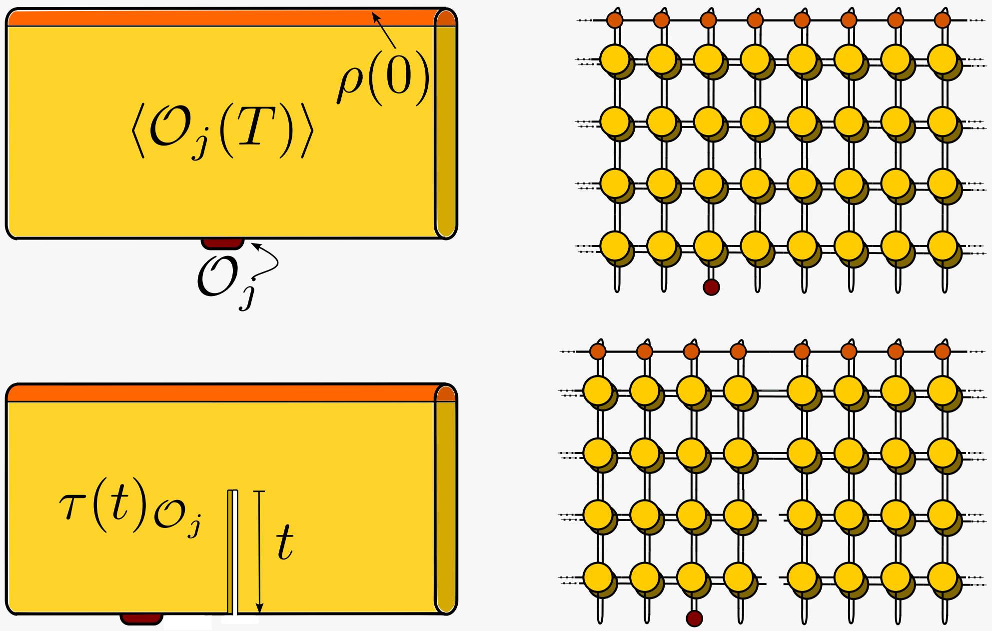




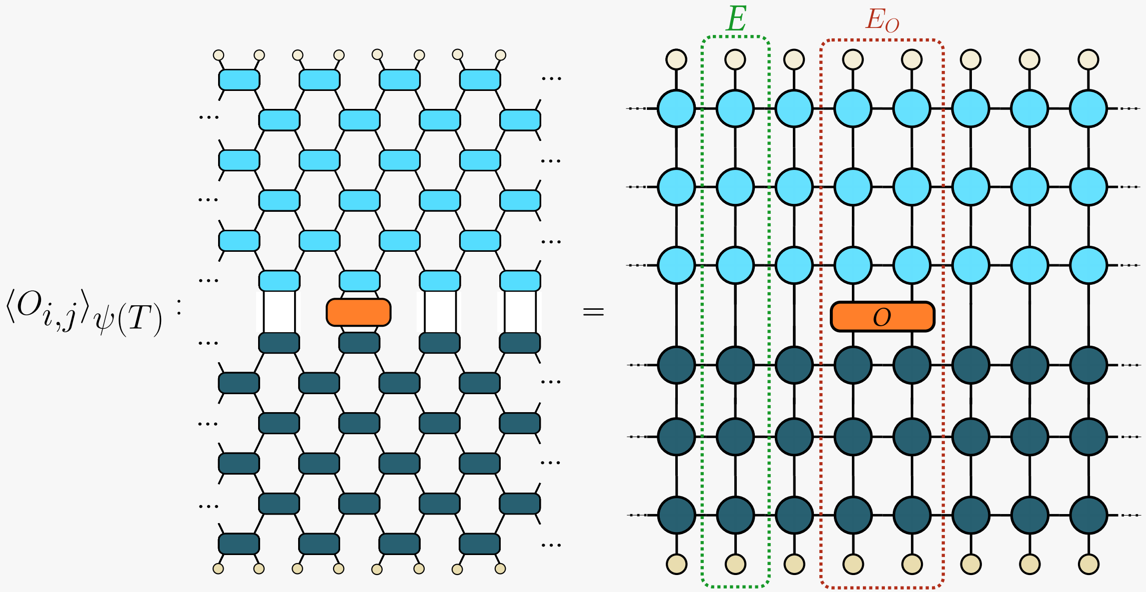
(a)
β
$\rho(\beta)$
ρ_A
$\rho(0)$
$\rho(T)$
(b)
$\rho(0)$
$\langle \mathcal{O}_j(T) \rangle$
$\mathcal{O}_j$
$\langle L_{\mathcal{O}_j} |$
$|R\rangle$
$\tau(t)_{\mathcal{O}_j}$
t
(c)

Tensor network representation

This set up can be mapped to a tensor network, it is similar to perform time evolution, therefore we can use time evolution block decimation (TEBD) to simulate the set up.



Instead of contracting the network from top to bottom, one could try to contract the nerwork horizontally, which sometimes is more efficient and one can access to arbitrarily large systems.



After iterating this process, one would obtain the *left* and *right* eigenvectors and the computing the generalized temporal entropy is trivial.