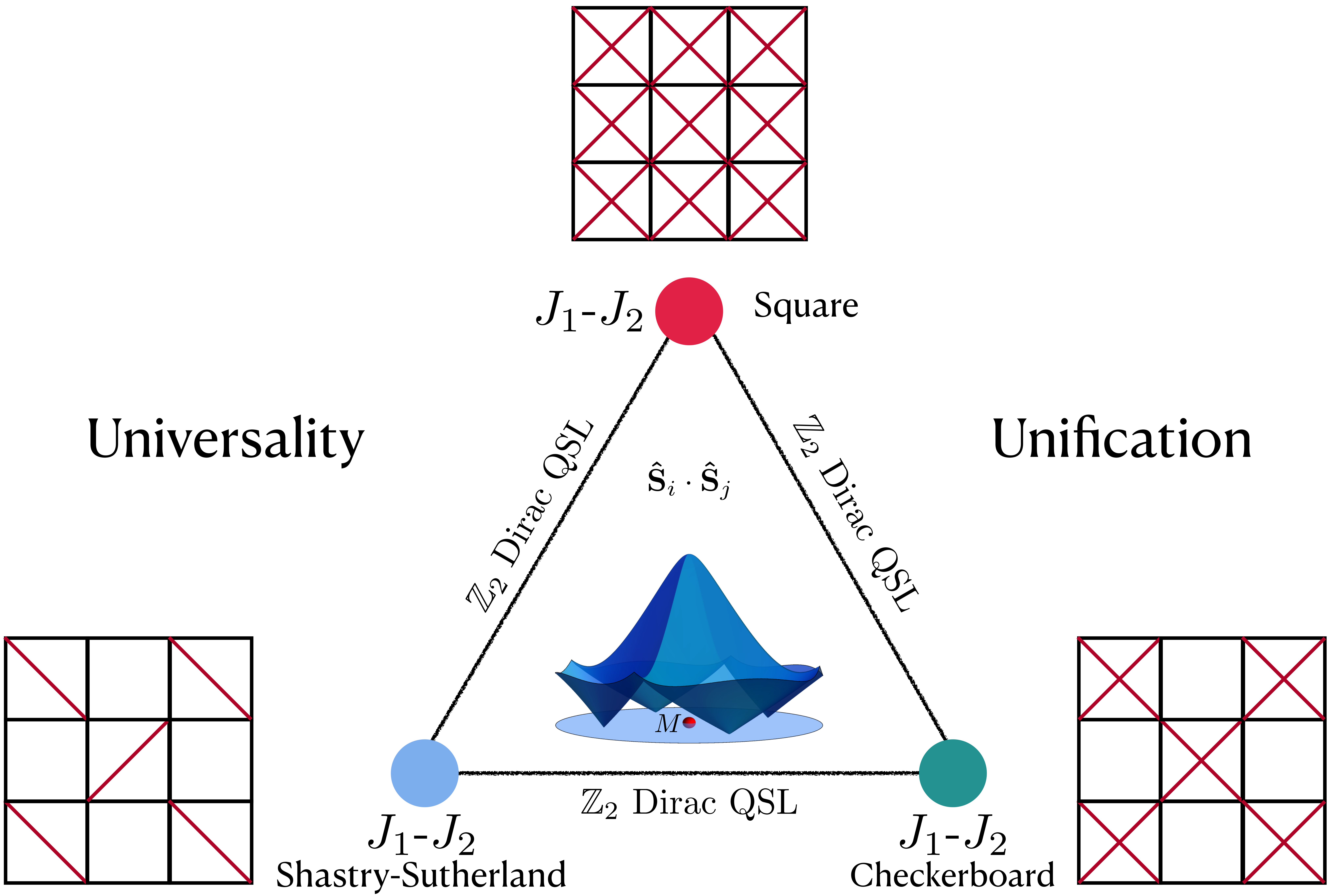
Distinct lattices, one Quantum Spin Liquid

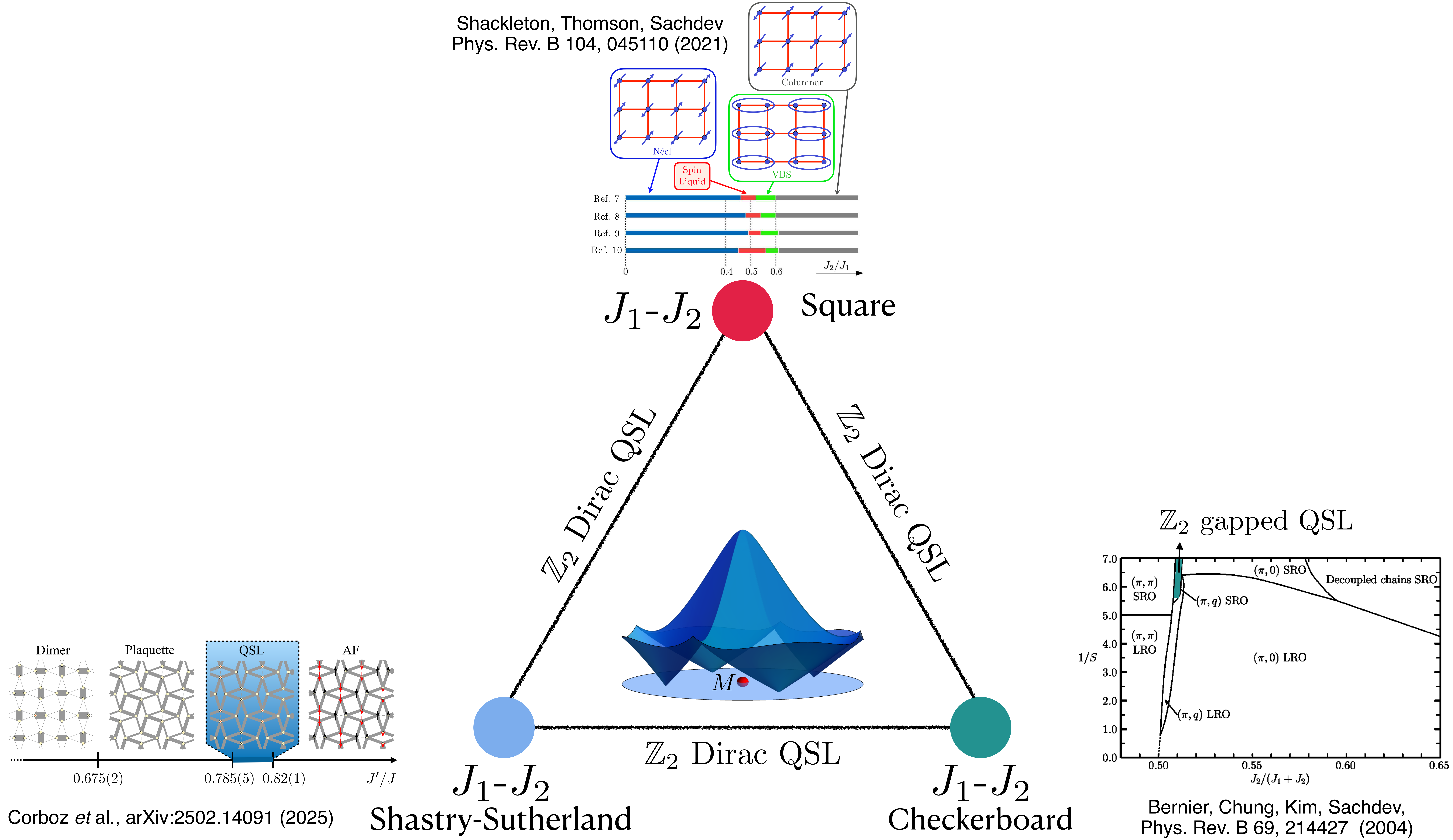
Entanglement in Strongly Correlated Electron Systems

15-28 February 2026, Benasque, Spain

Yasir Iqbal, Indian Institute of Technology Madras, Chennai, India



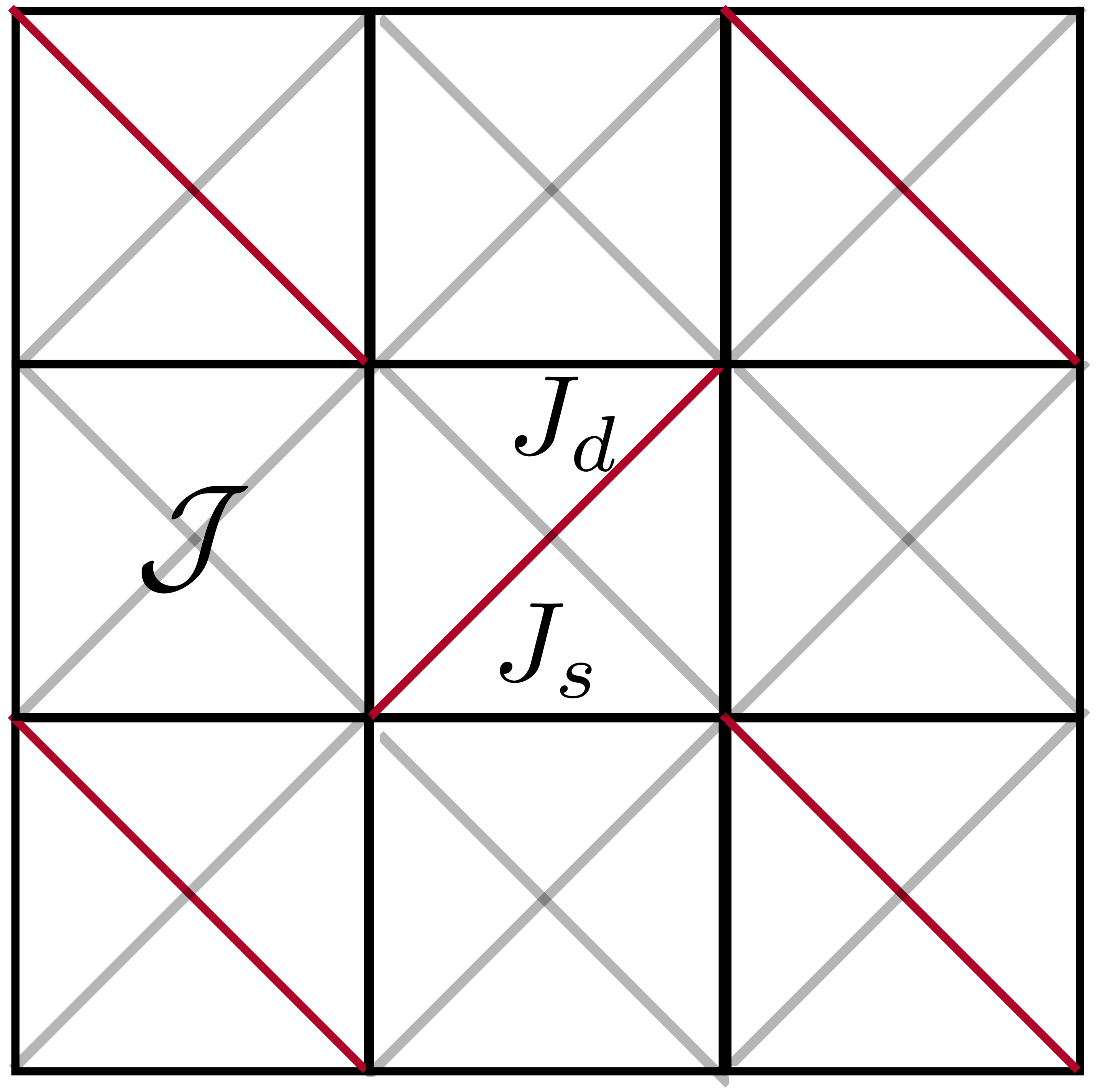
J_1-J_2
Square
Universality
Unification
$\mathbb{Z}_2$ Dirac QSL
$\mathbb{Z}_2$ Dirac QSL
$\hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$
M
$\mathbb{Z}_2$ Dirac QSL
J_1-J_2
Shastry-Sutherland
J_1-J_2
Checkerboard

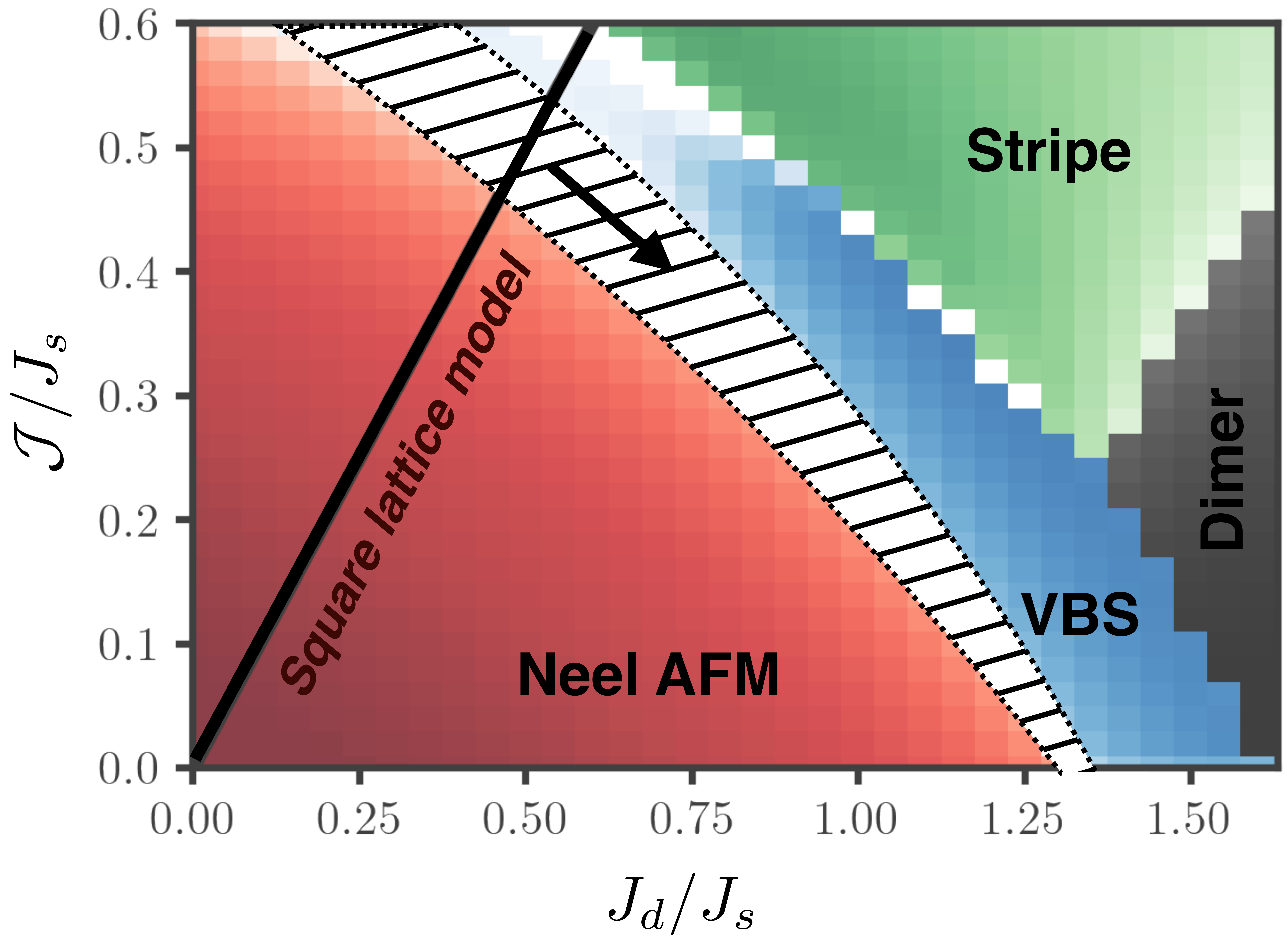


Shackleton, Thomson, Sachdev
Phys. Rev. B 104, 045110 (2021)
Néel
Columnar
VBS
Spin Liquid
Ref. 7
Ref. 8
Ref. 9
Ref. 10
0
0.4
0.5
0.6
J_2/J_1
J_1-J_2
Square
$\mathbb{Z}_2$ Dirac QSL
$\mathbb{Z}_2$ Dirac QSL
$\mathbb{Z}_2$ Dirac QSL
M
J_1-J_2
Shastry-Sutherland
J_1-J_2
Checkerboard
Dimer
Plaquette
QSL
AF
0.675(2)
0.785(5)
0.82(1)
J'/J
Corboz et al., arXiv:2502.14091 (2025)
$\mathbb{Z}_2$ gapped QSL
(π,π) SRO
$(\pi,0)$ SRO
Decoupled chains SRO
(π,q) SRO
(π,π) LRO
$(\pi,0)$ LRO
(π,q) LRO
$1/S$
$J_2/(J_1+J_2)$
Bernier, Chung, Kim, Sachdev,
Phys. Rev. B 69, 214427 (2004)

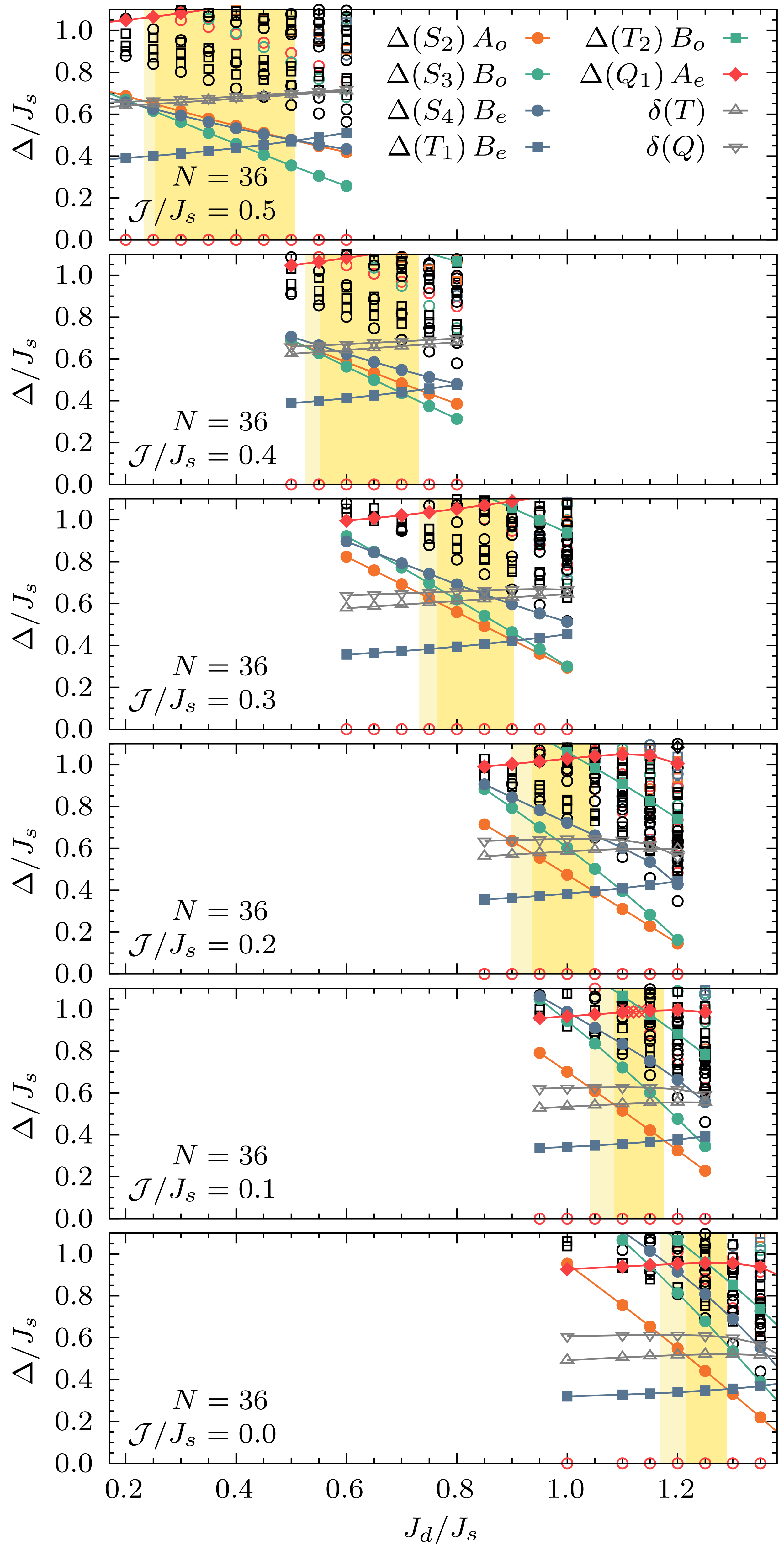
River of liquidity: Shastry-Sutherland to square

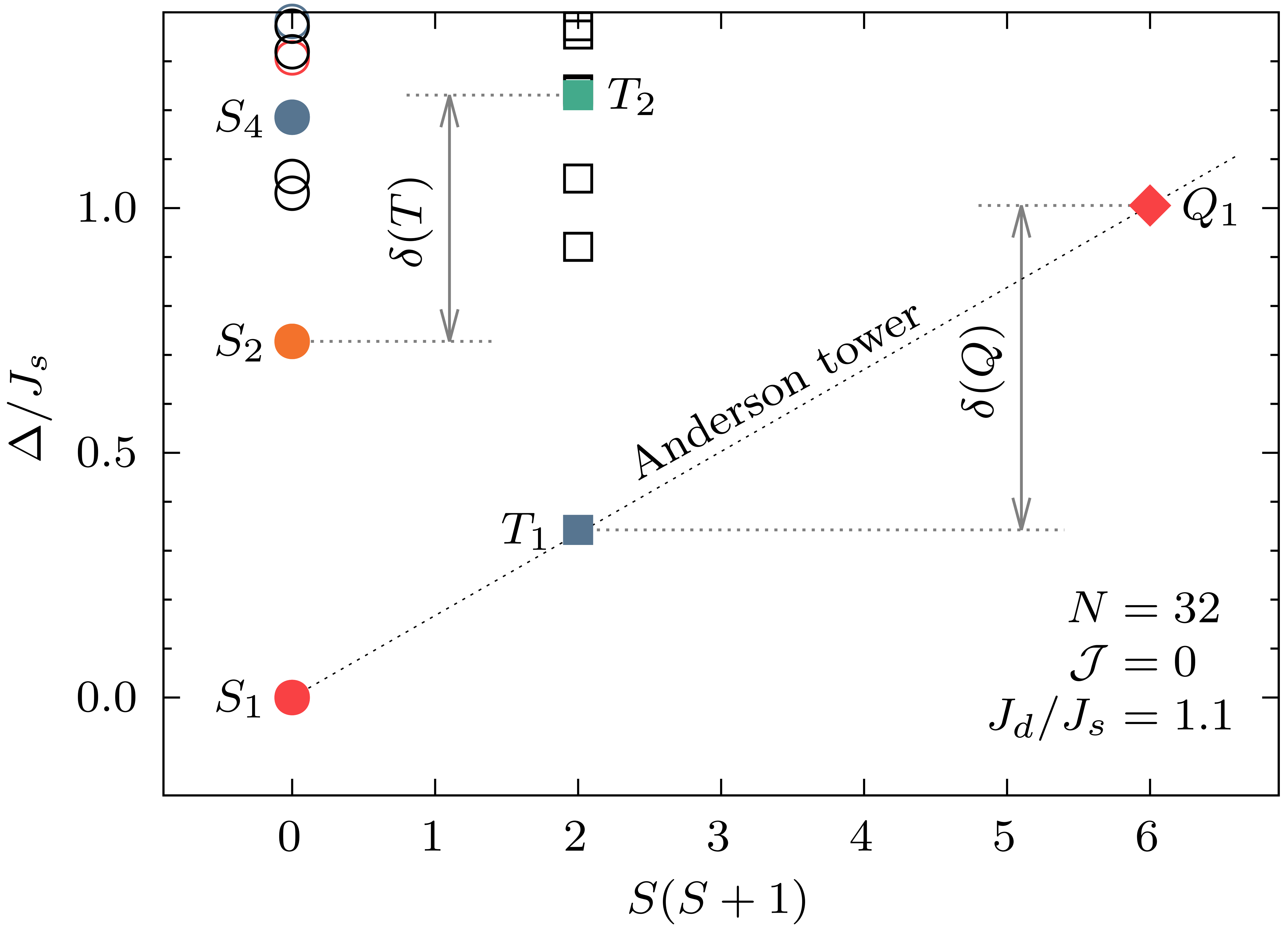
S=1/2 Quantum Phase Diagram

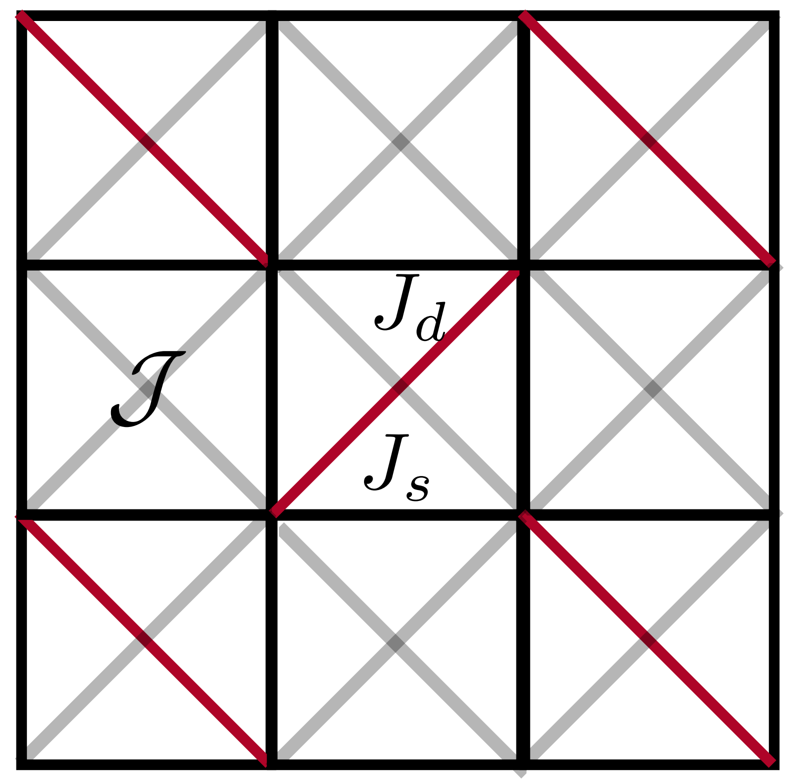


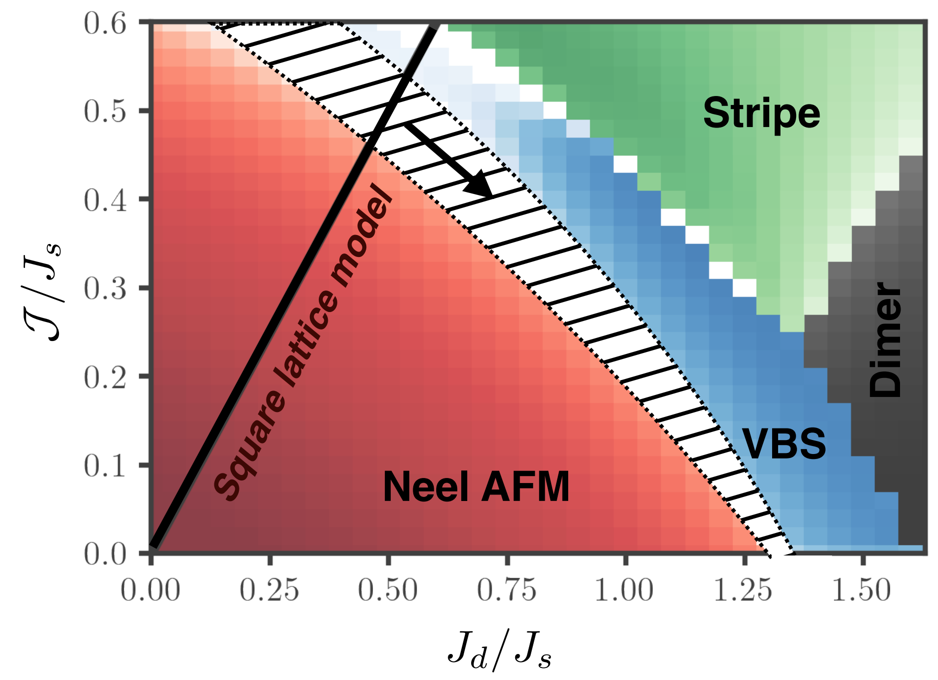


Evidence from Exact Diagonalization

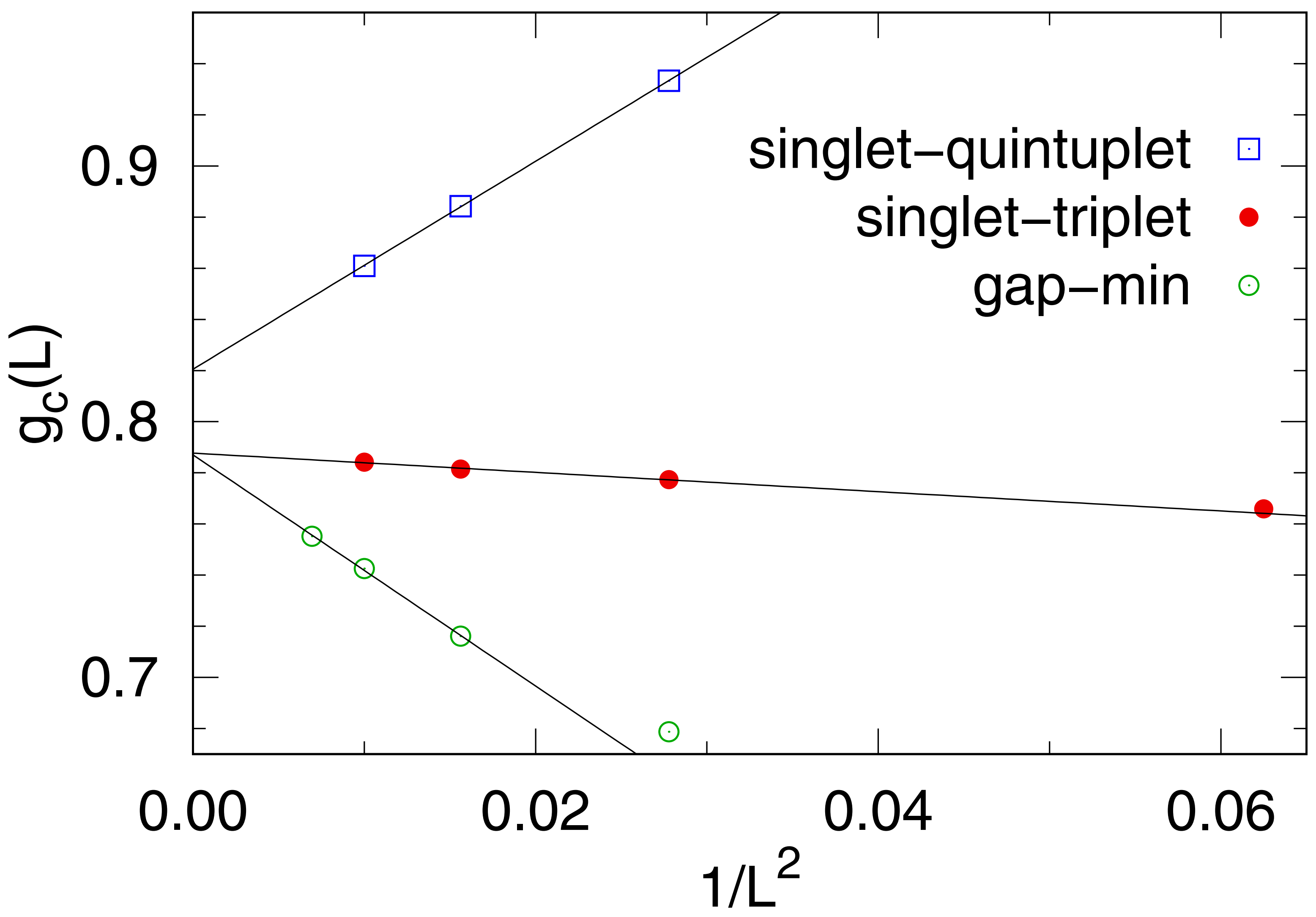


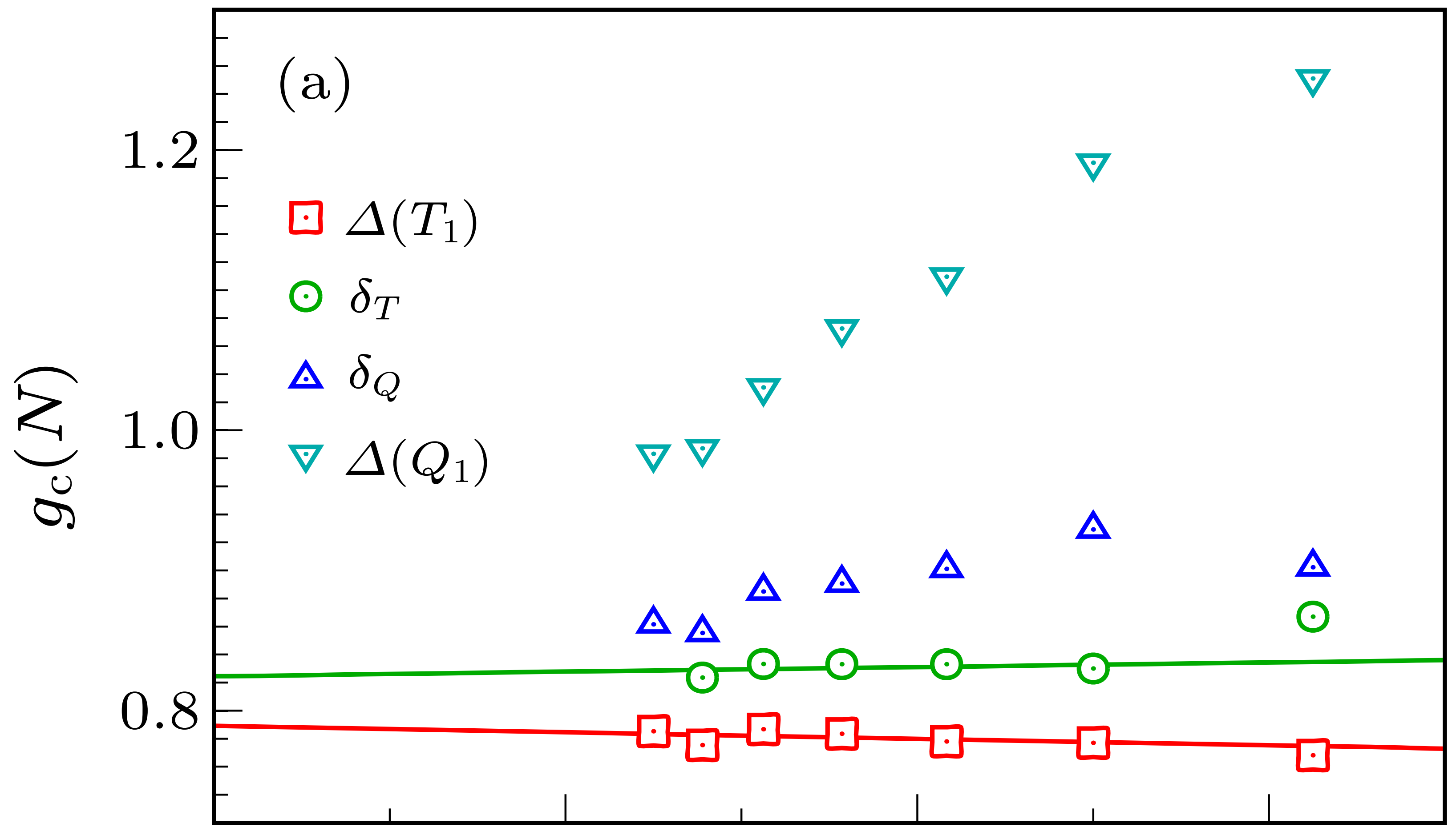




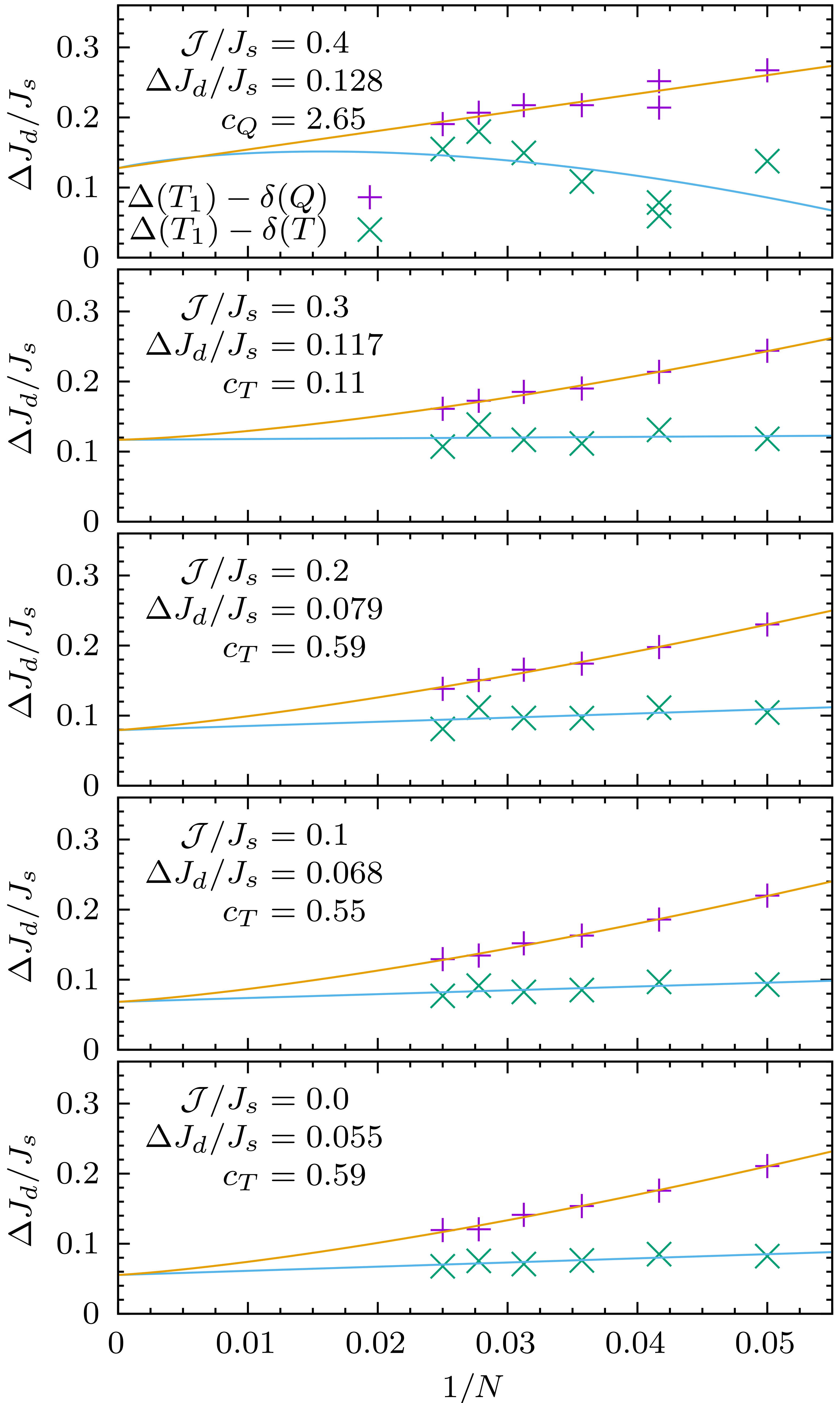


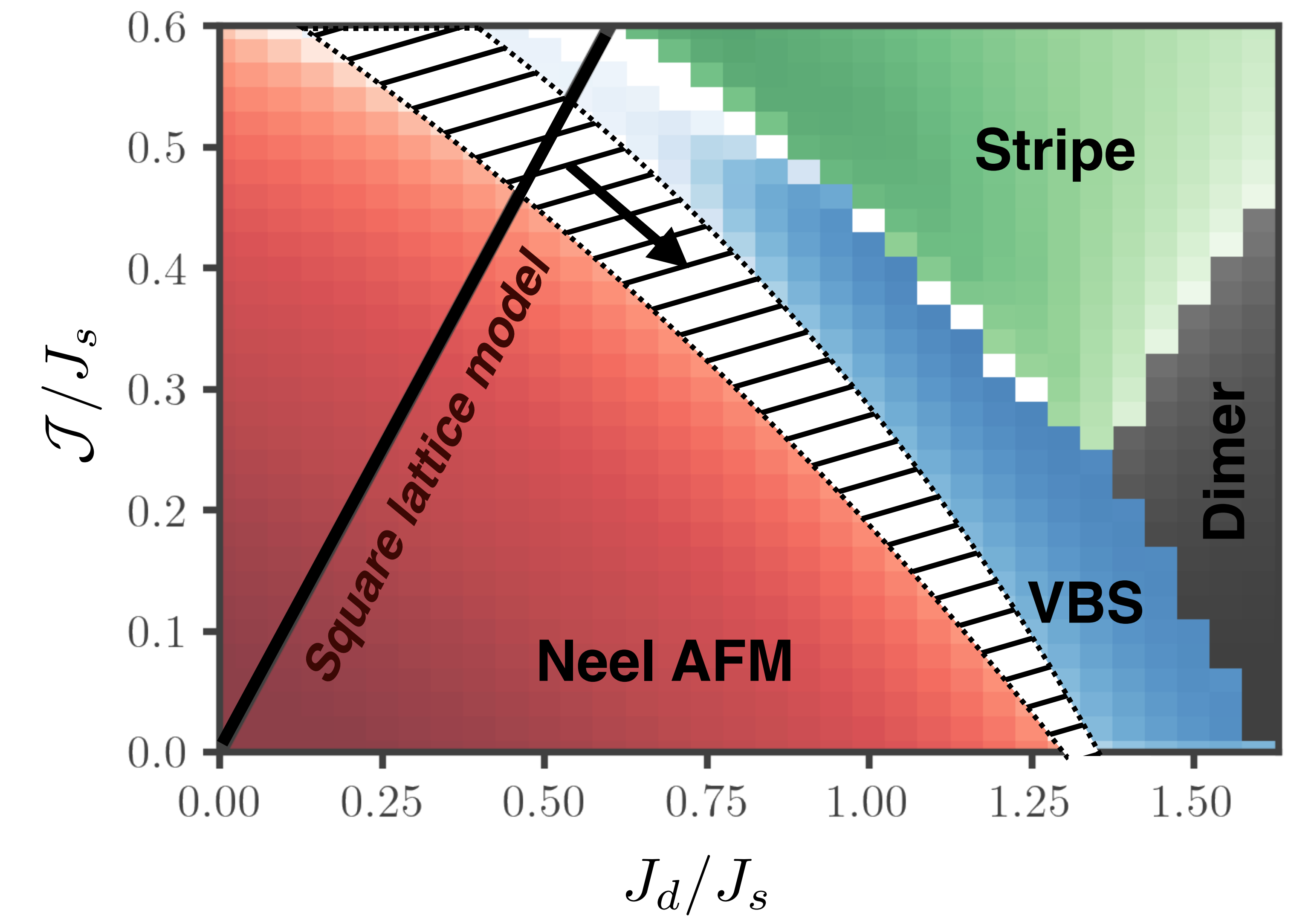
Finite-Size Scaling of the width of the QSL river





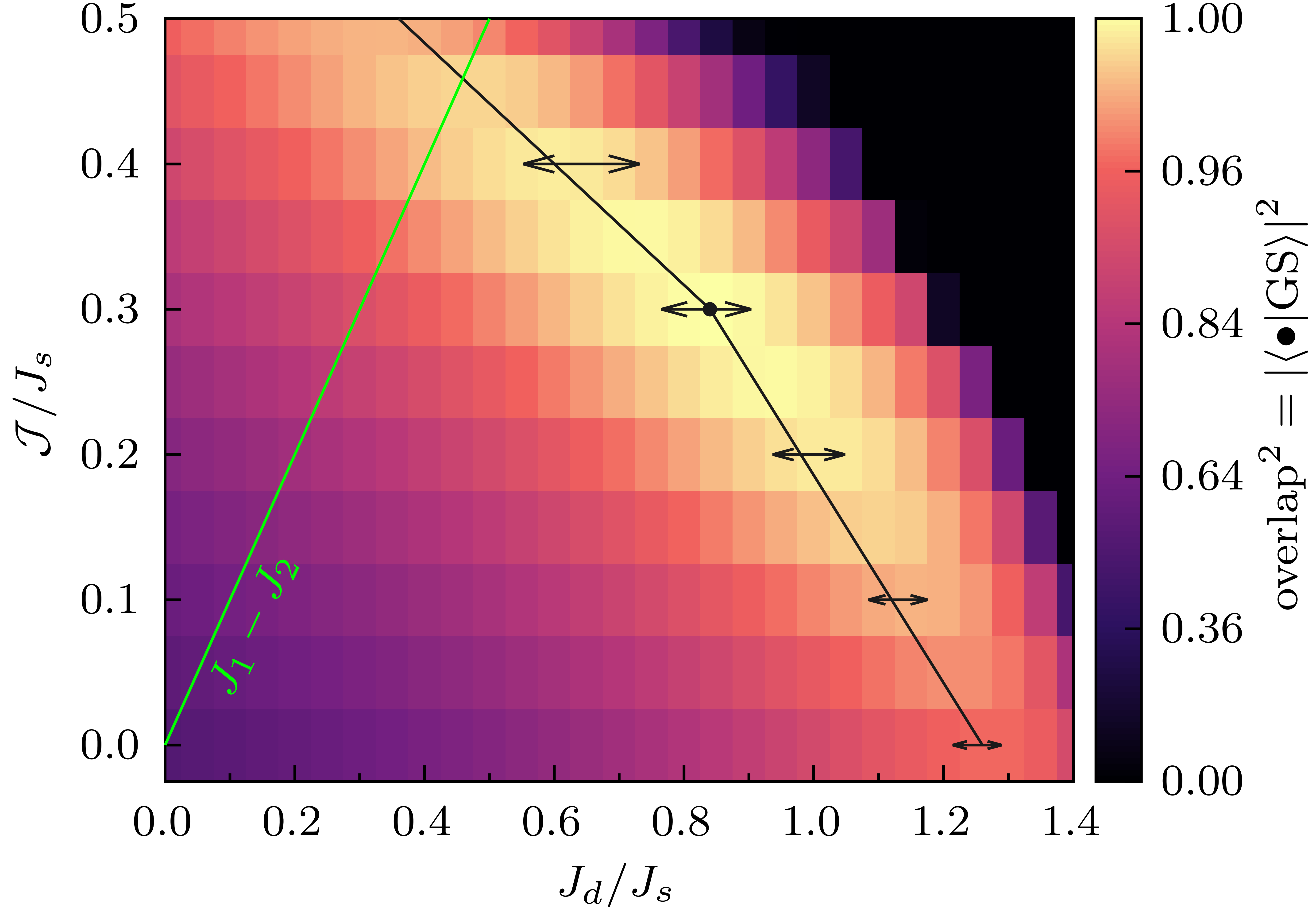
Yang, Sandvik, Wang, Phys. Rev. B 105, L060409 (2022)

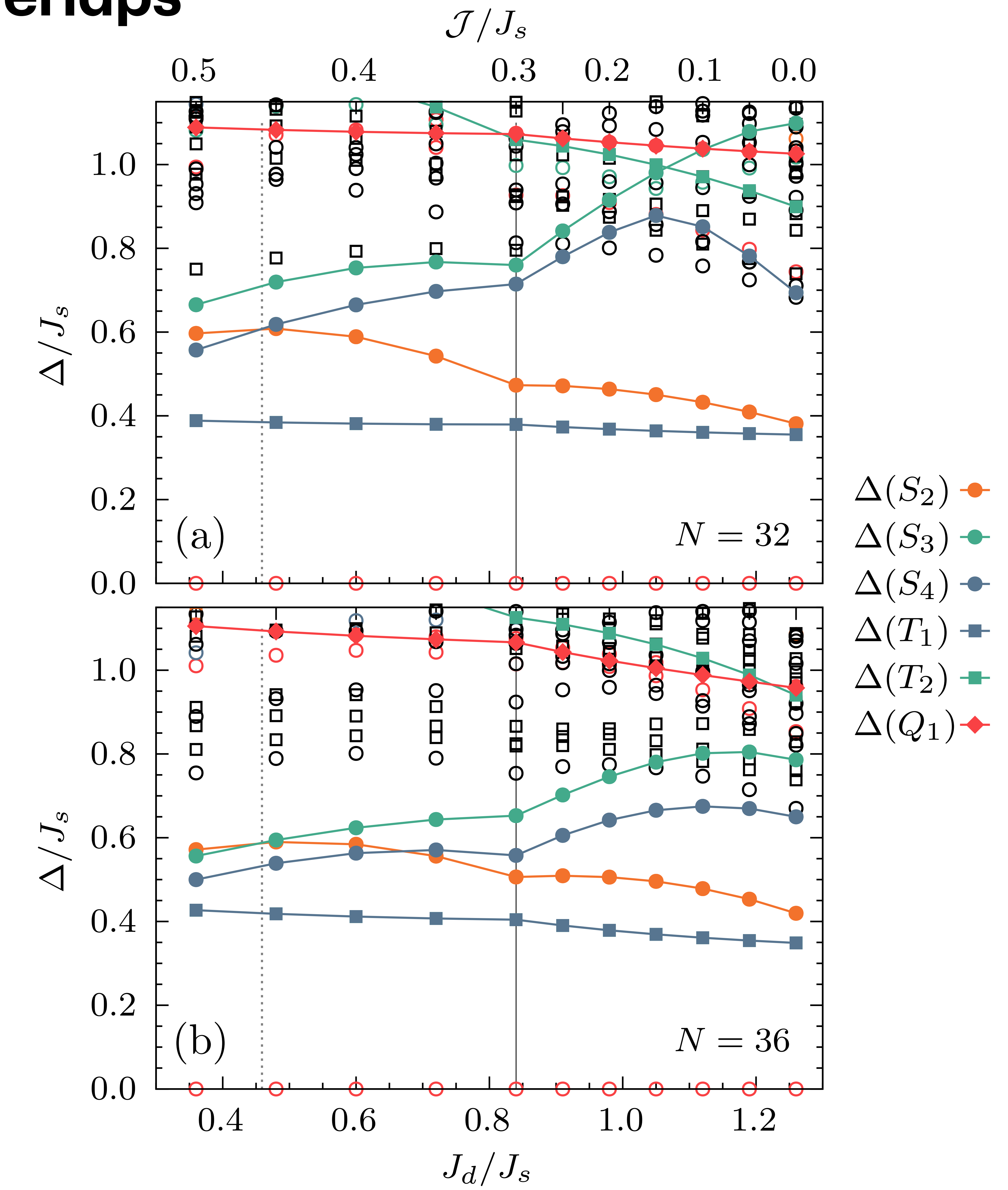




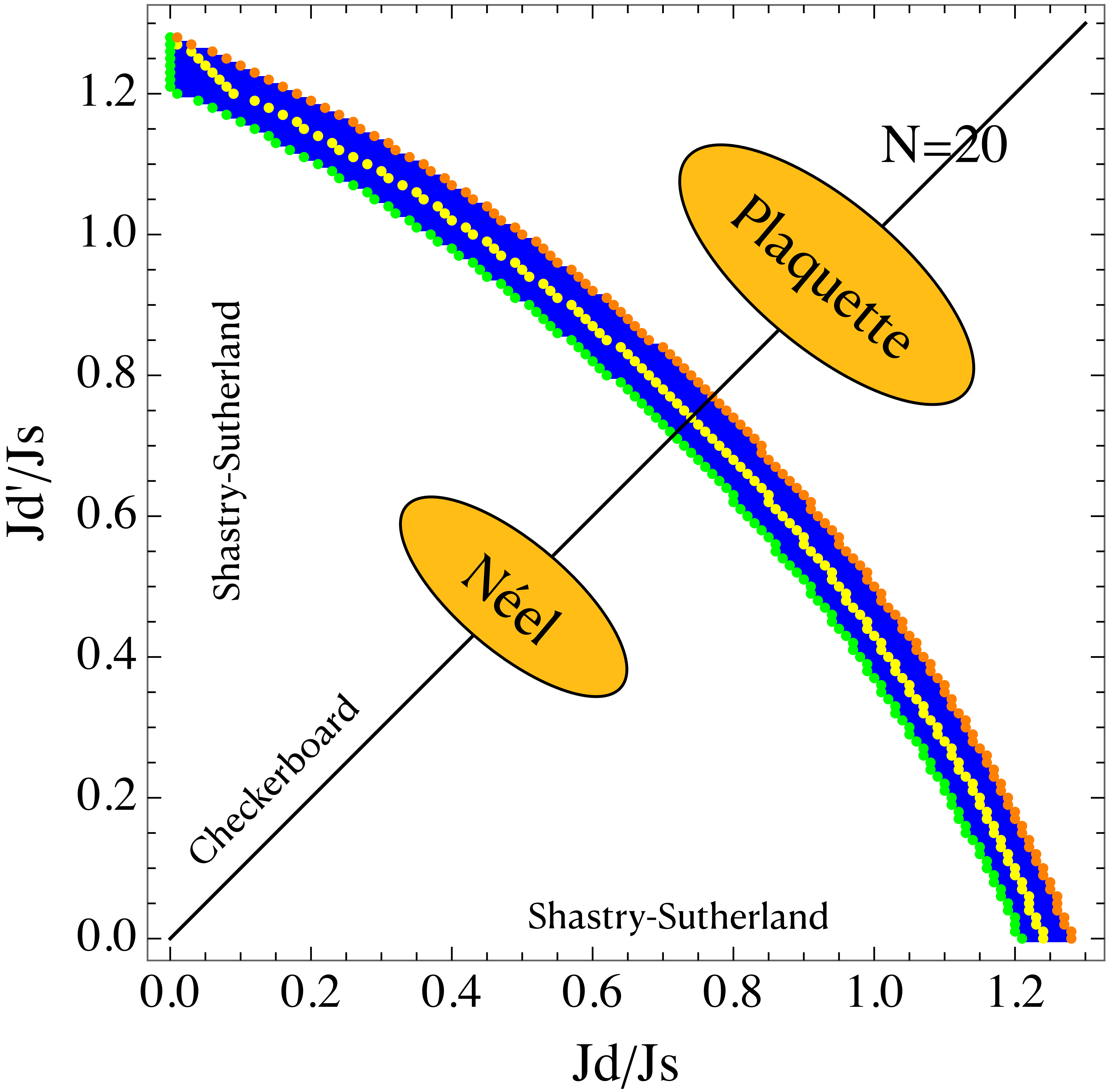
Do the J_1-J_2 square and SS share a QSL?

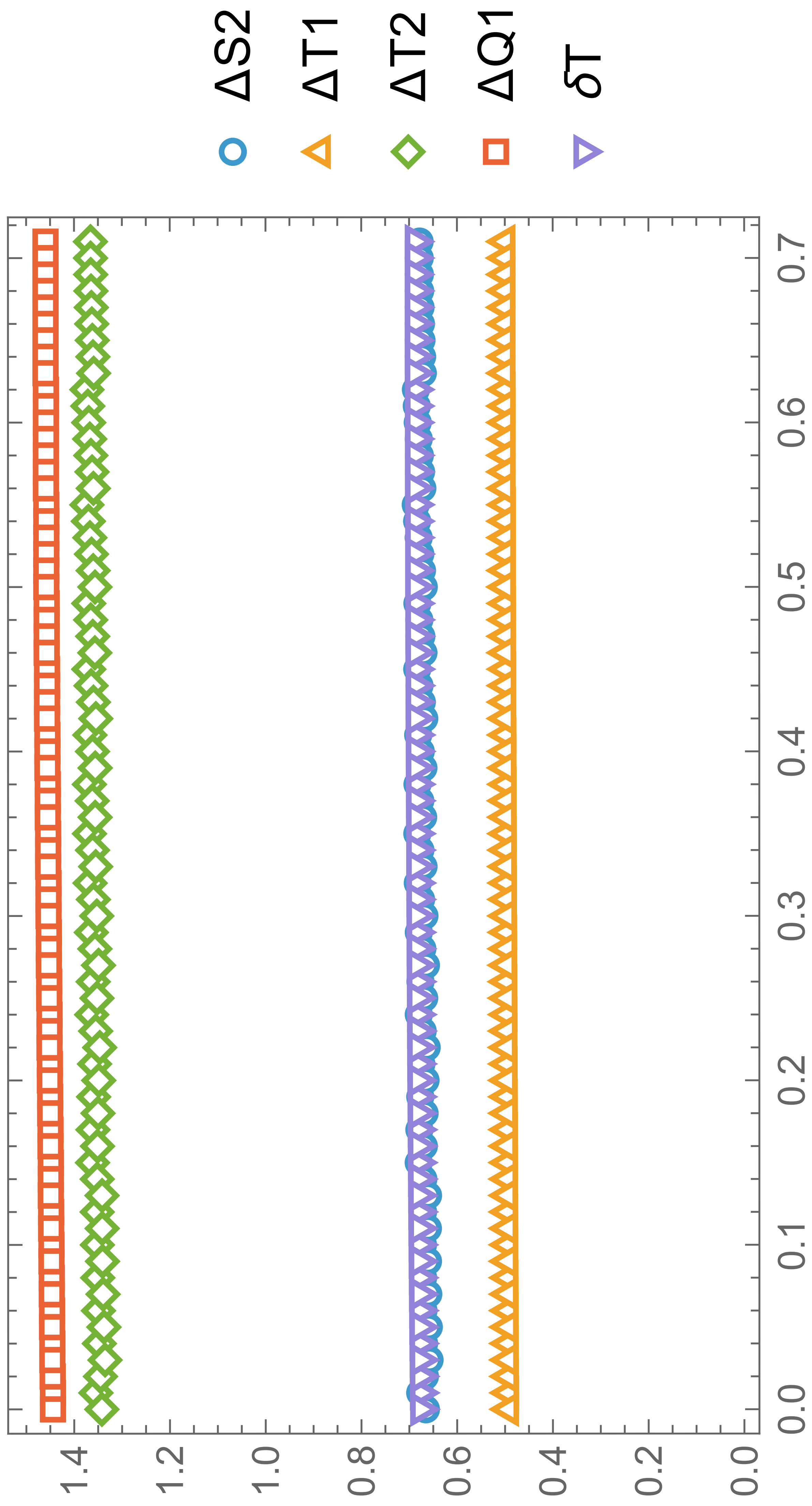
Level crossing analysis, overlaps

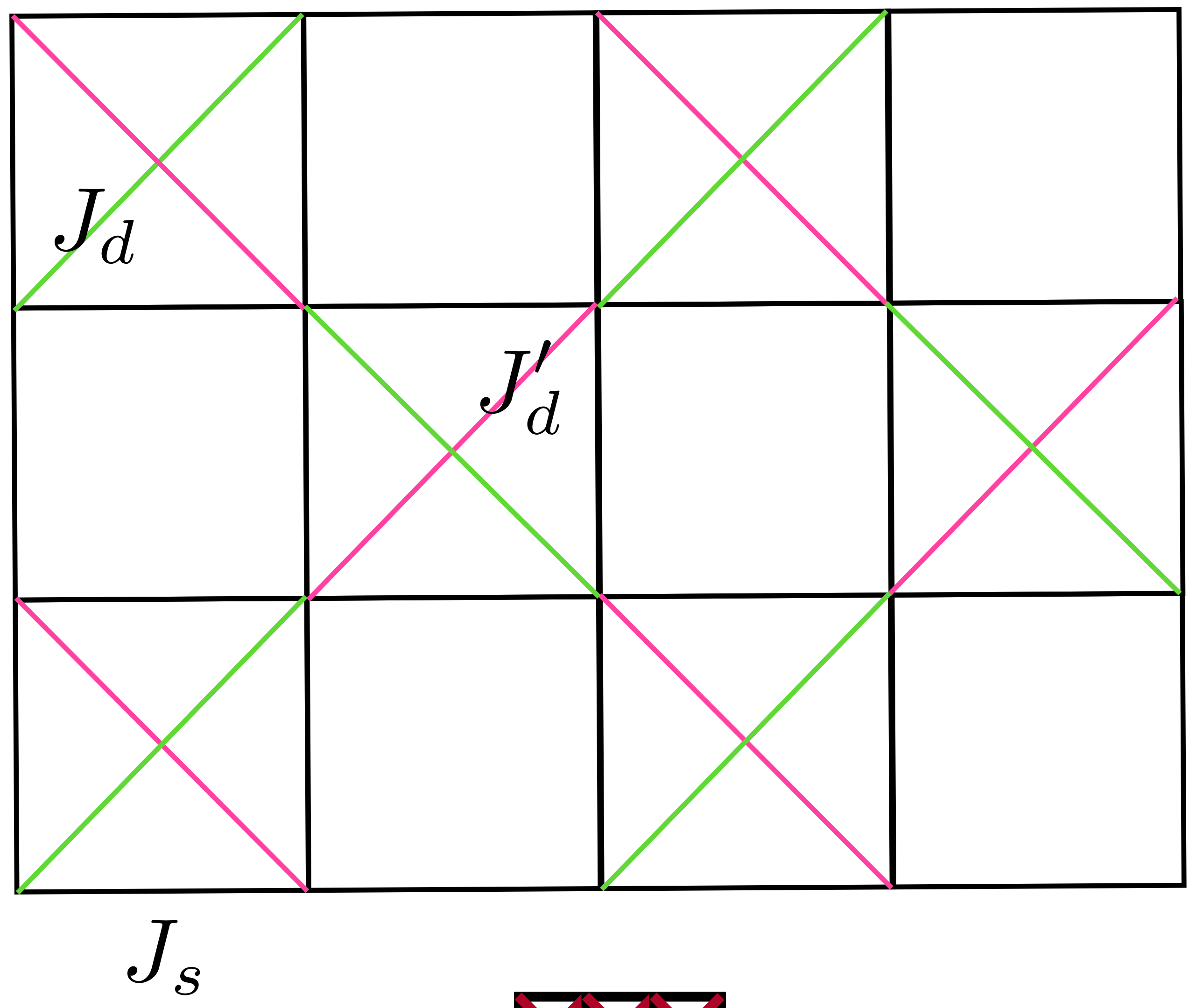


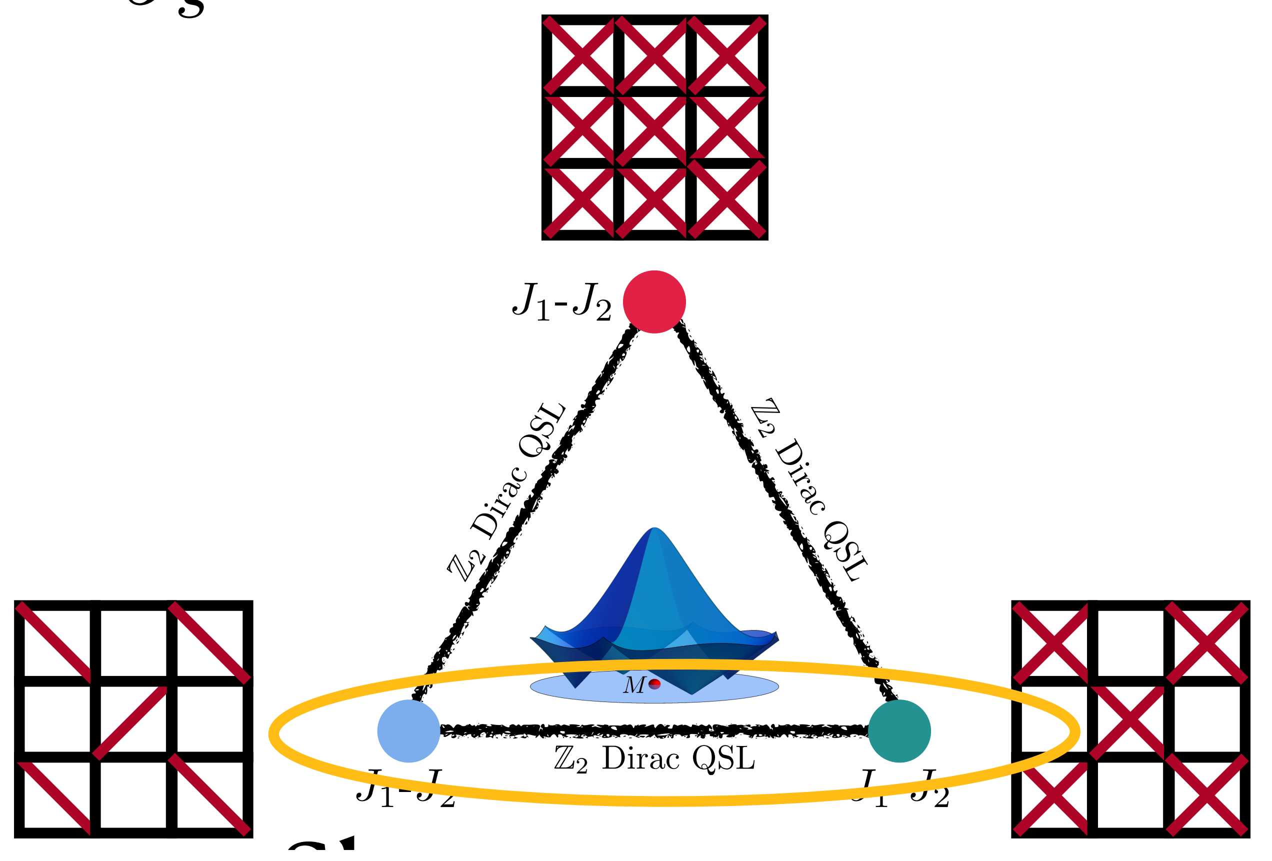


Do the SS and checkerboard share a QSL?

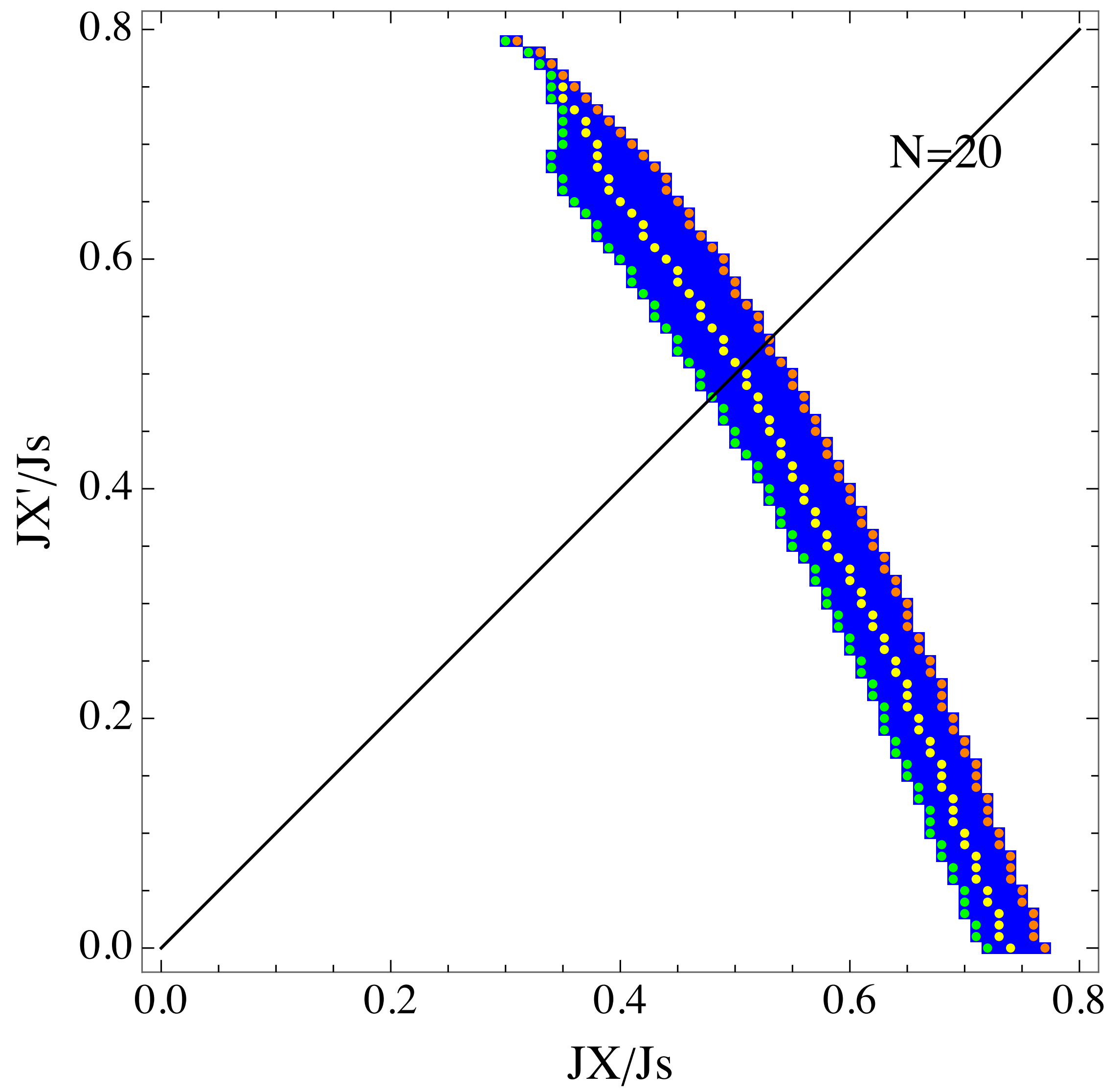


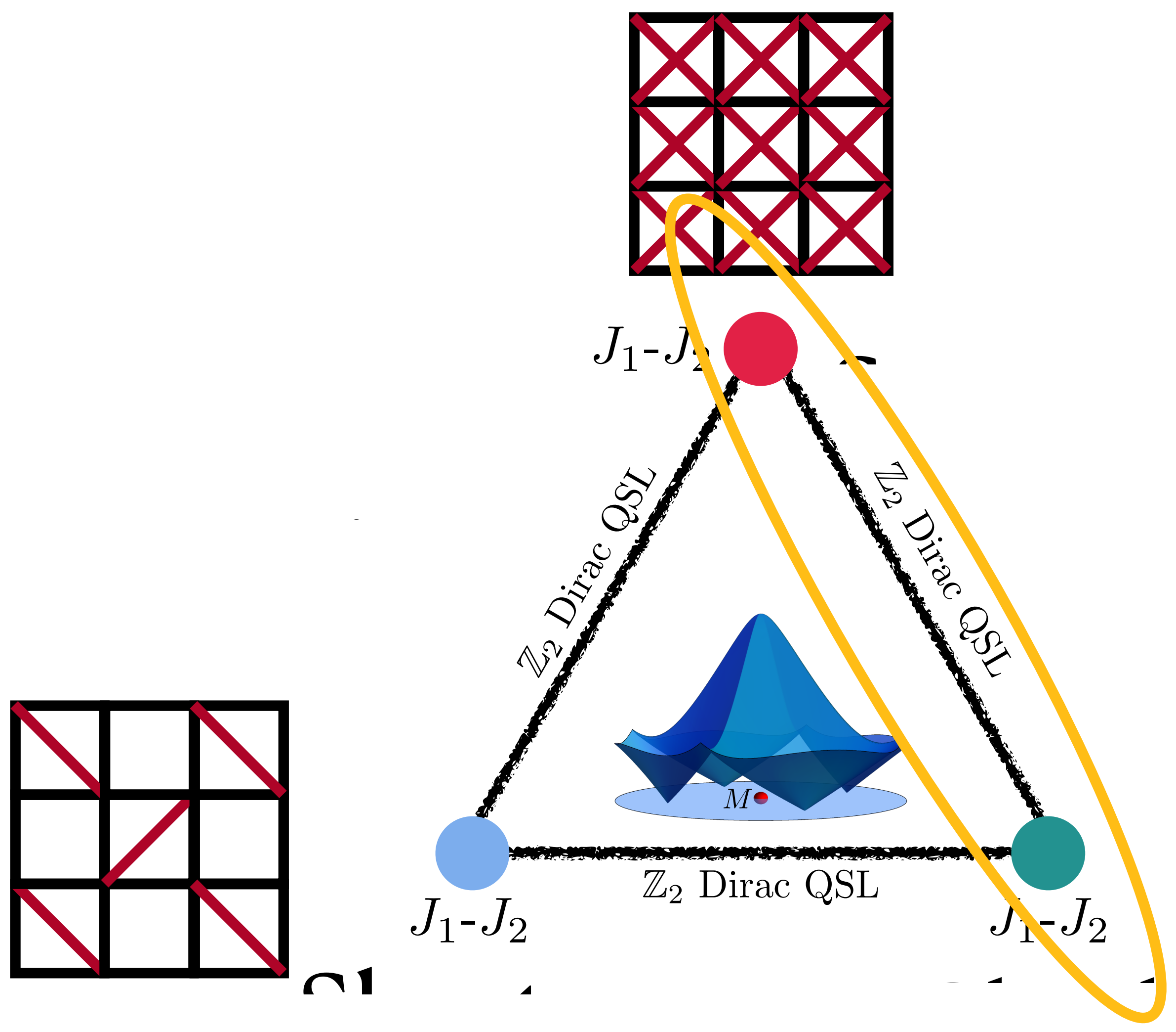


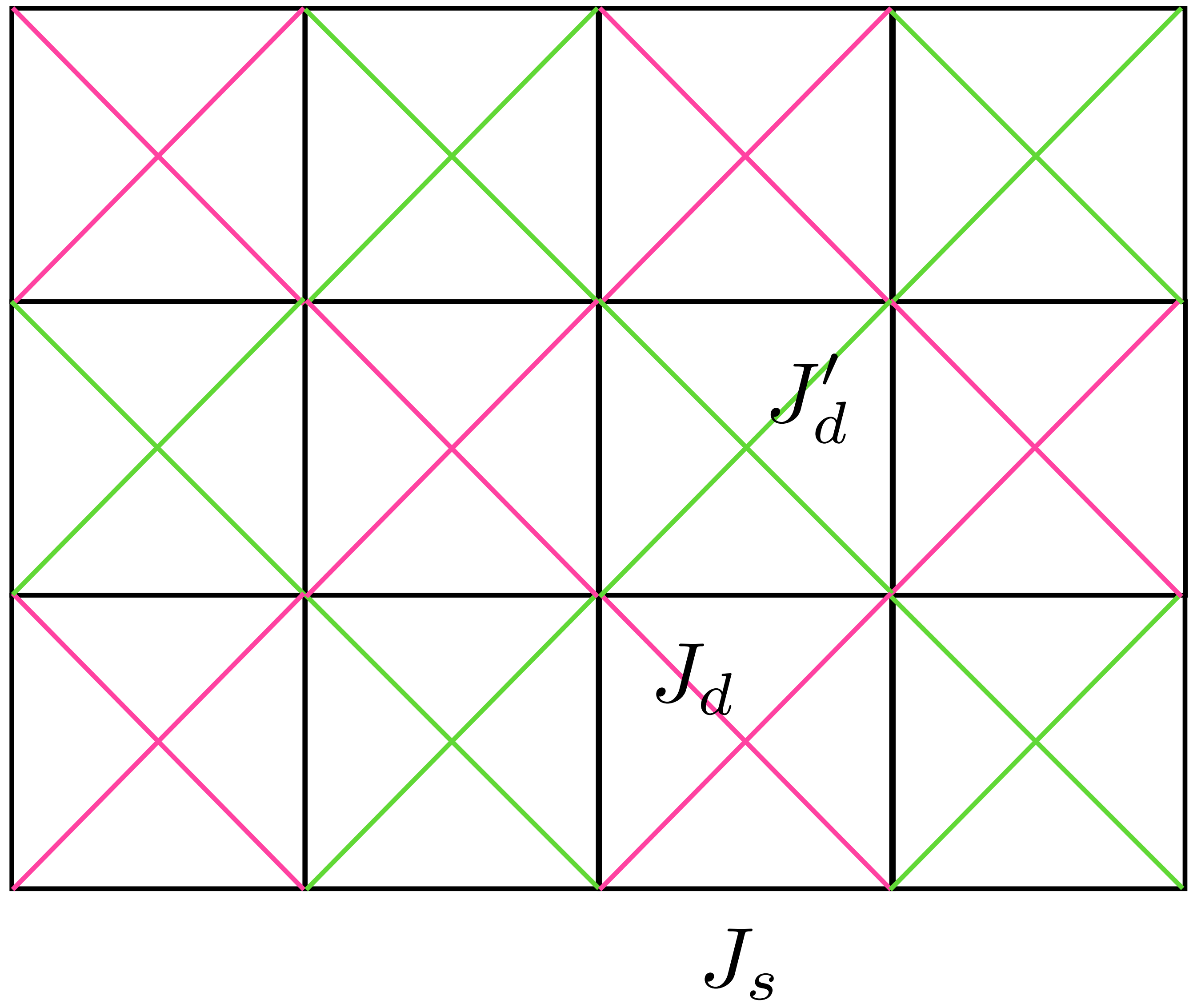


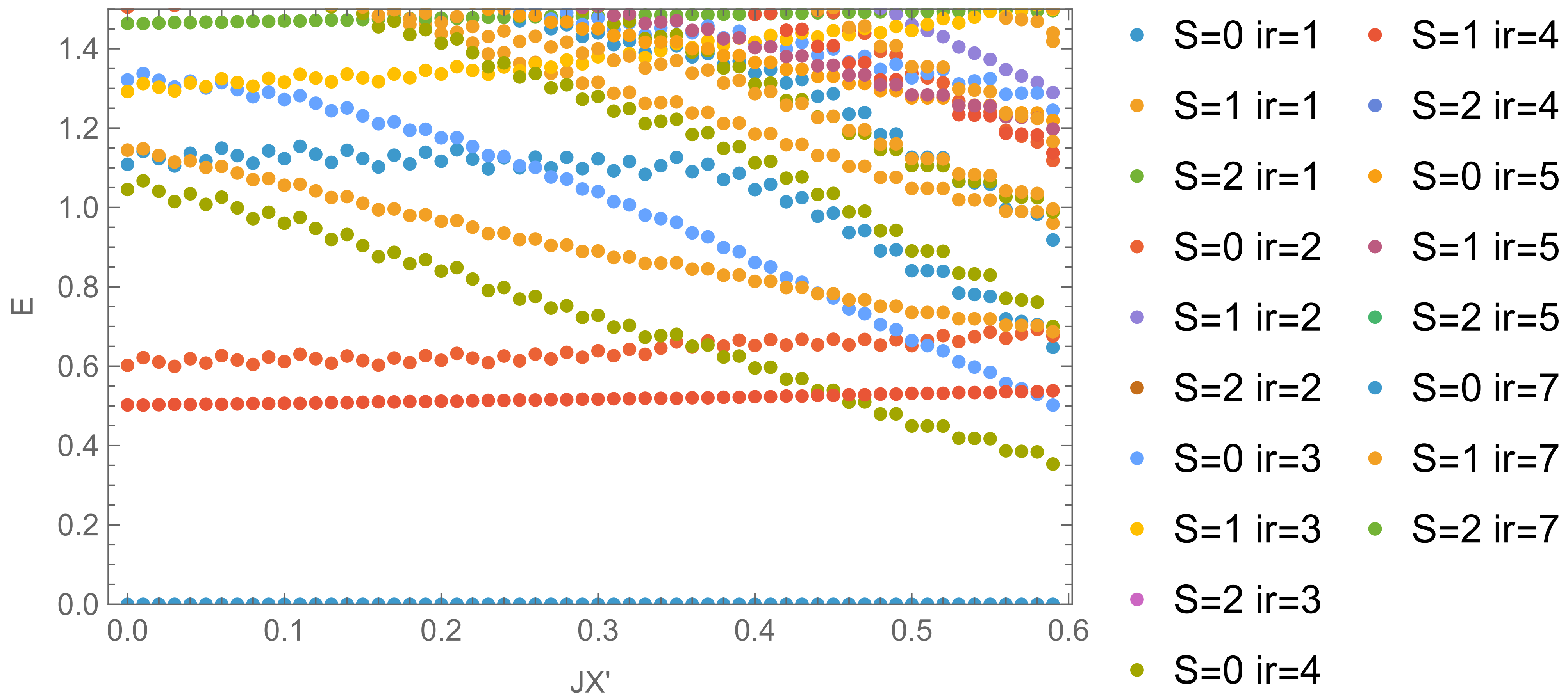


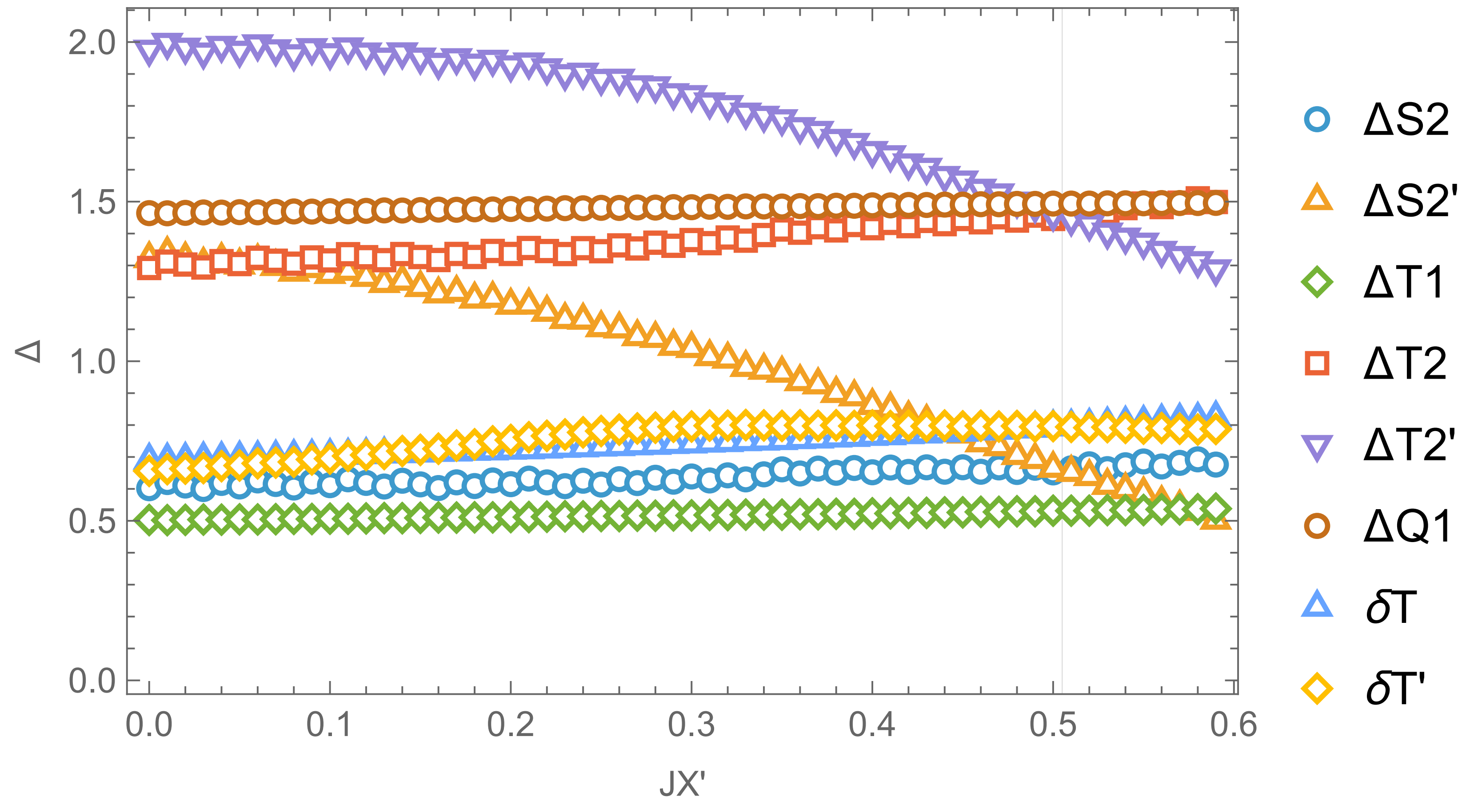
Do the checkerboard and J_1-J_2 share a QSL?











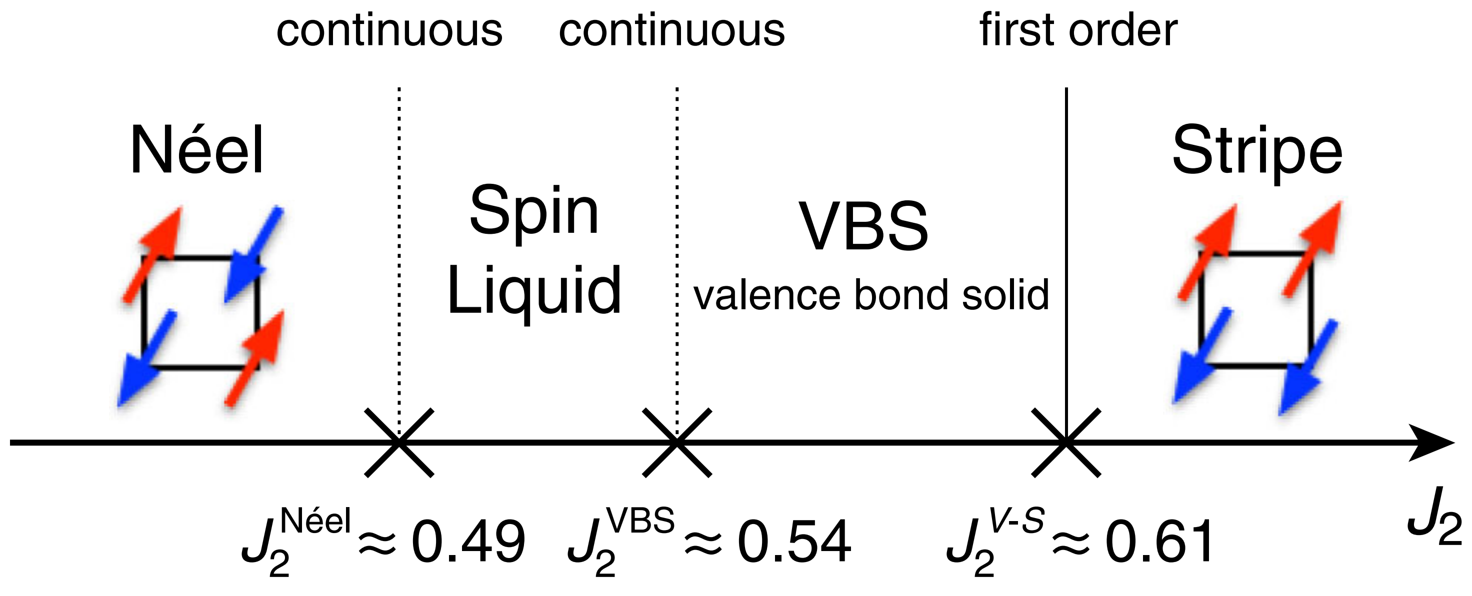
Connecting with the J_1-J_2 square lattice QSL

The gapless Z_2 Dirac spin liquid (Z2Azz13)

PHYSICAL REVIEW B **88**, 060402(R) (2013)

Direct evidence for a gapless Z_2 spin liquid by frustrating Néel antiferromagnetism

$$0.49 \lesssim J_2/J_1 \lesssim 0.54$$

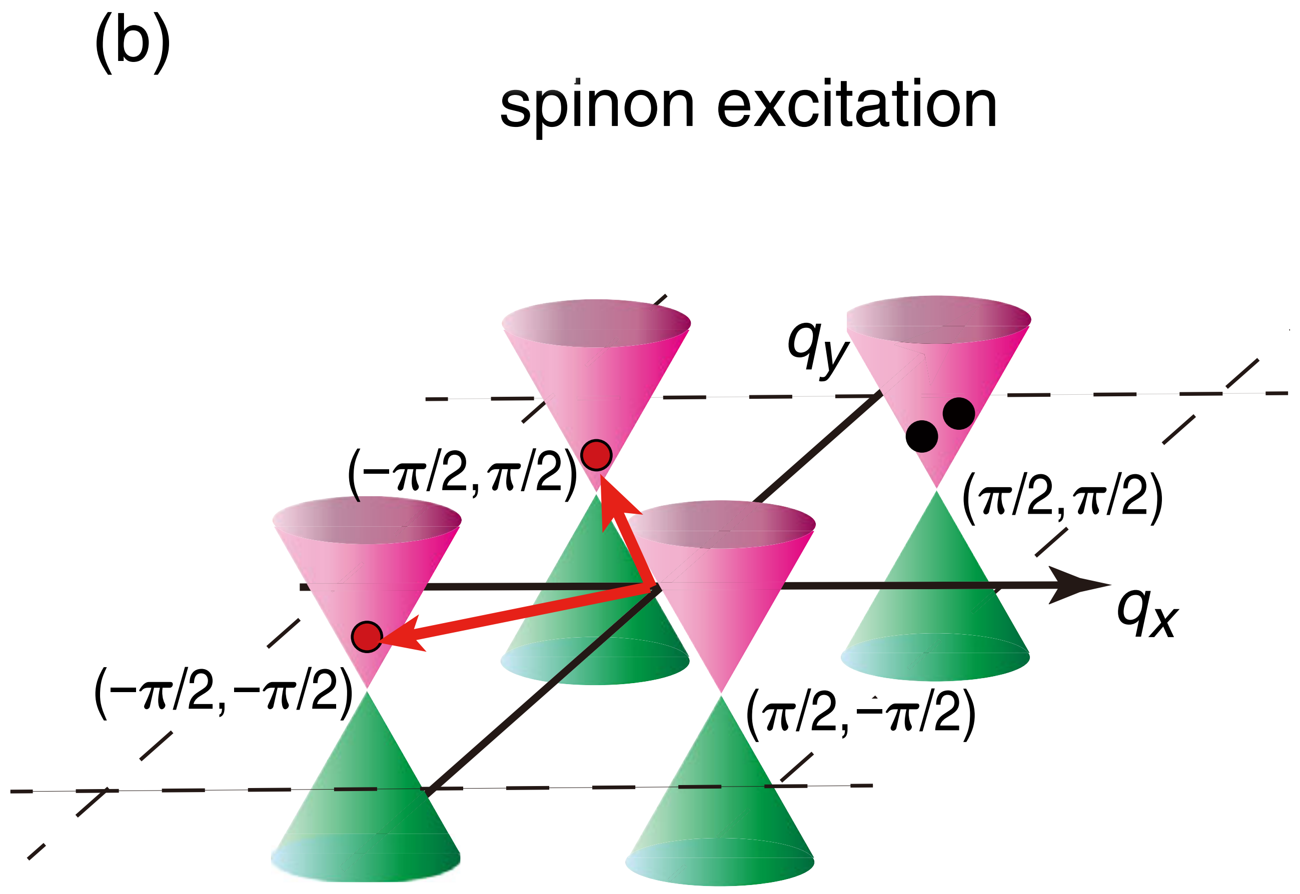


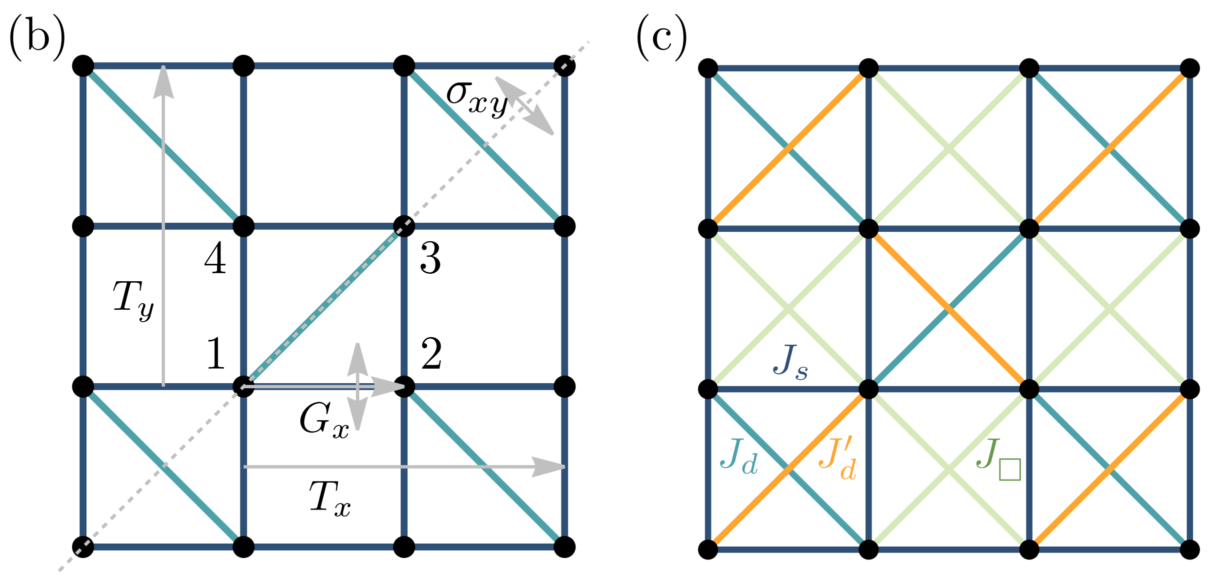
PHYSICAL REVIEW X **11**, 031034 (2021)

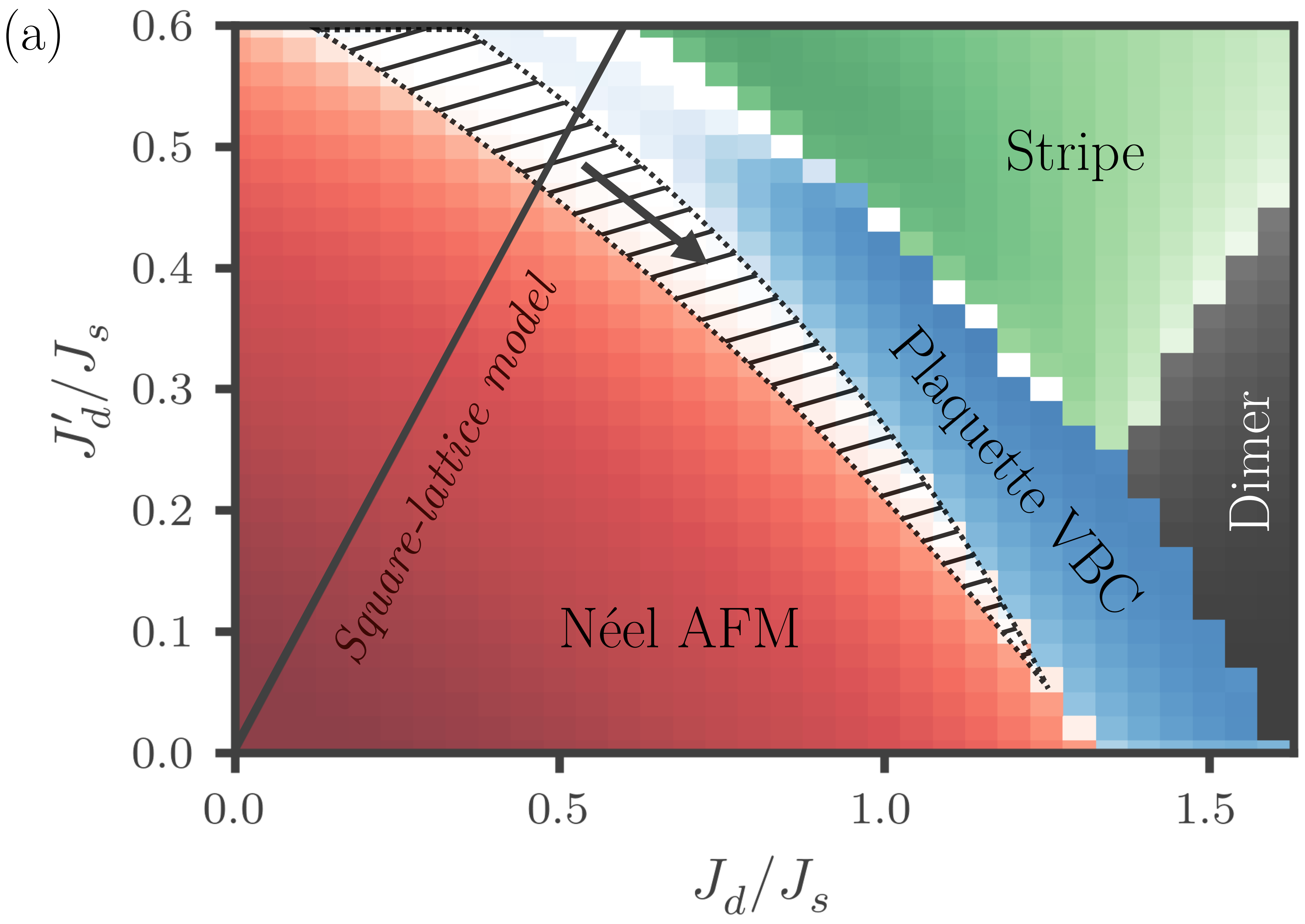
Dirac-Type Nodal Spin Liquid Revealed by Refined Quantum Many-Body Solver Using Neural-Network Wave Function, Correlation Ratio, and Level Spectroscopy

Yusuke Nomura[1,*] and Masatoshi Imada[2,3]

DMRG







Evidence for a QSL in the Shastry-Sutherland

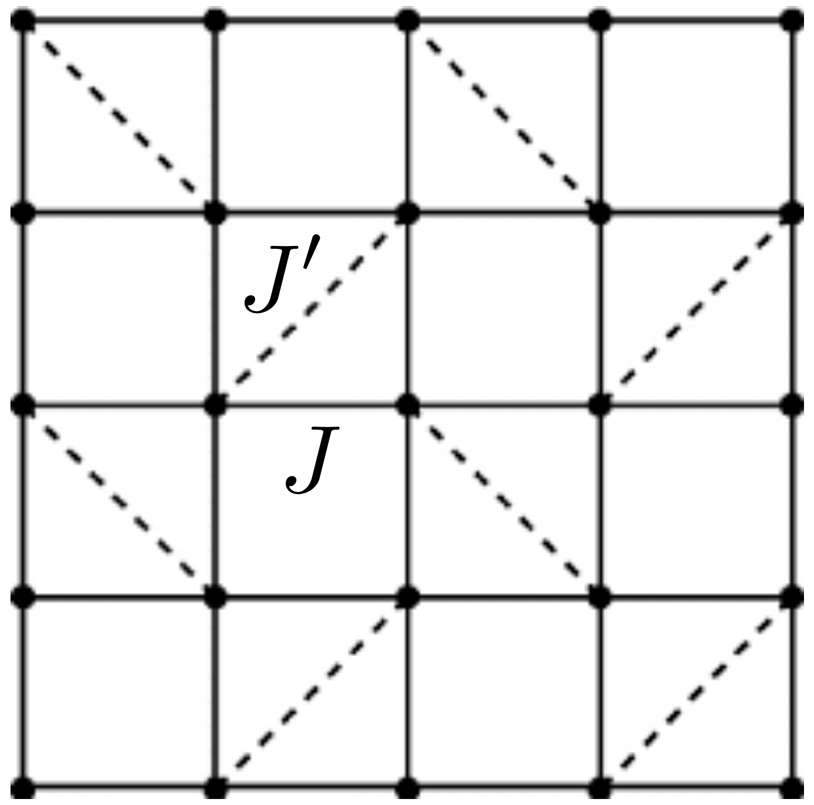
EXACT GROUND STATE OF A QUANTUM MECHANICAL ANTIFERROMAGNET

B. Sriram Shastry and Bill Sutherland

Department of Physics, University of Utah, Salt Lake City, UT 84112

We present some exact results for the ground state of a quantum mechanical antiferromagnetic model in the two dimensions with next-nearest neighbor interactions.

40+ years later



Chin. Phys. Lett. **39**, 077502 (2022) Express Letter

Quantum Spin Liquid Phase in the Shastry–Sutherland Model Detected by an Improved Level Spectroscopic Method

Ling Wang[1*], Yalei Zhang[2], and Anders W. Sandvik[3,4*]

PHYSICAL REVIEW B **105**, L060409 (2022)

Letter Editors' Suggestion

Quantum criticality and spin liquid phase in the Shastry-Sutherland model

Jianwei Yang,[1] Anders W. Sandvik,[2,3,*] and Ling Wang[4,†]

PHYSICAL REVIEW B **105**, L041115 (2022)

Letter

Rise and fall of plaquette order in the Shastry-Sutherland magnet revealed by pseudofermion functional renormalization group

Ahmet Keleş[1,*] and Erhai Zhao[2]

Quantum spin liquid phase in the Shastry-Sutherland model revealed by high-precision infinite projected entangled-pair states

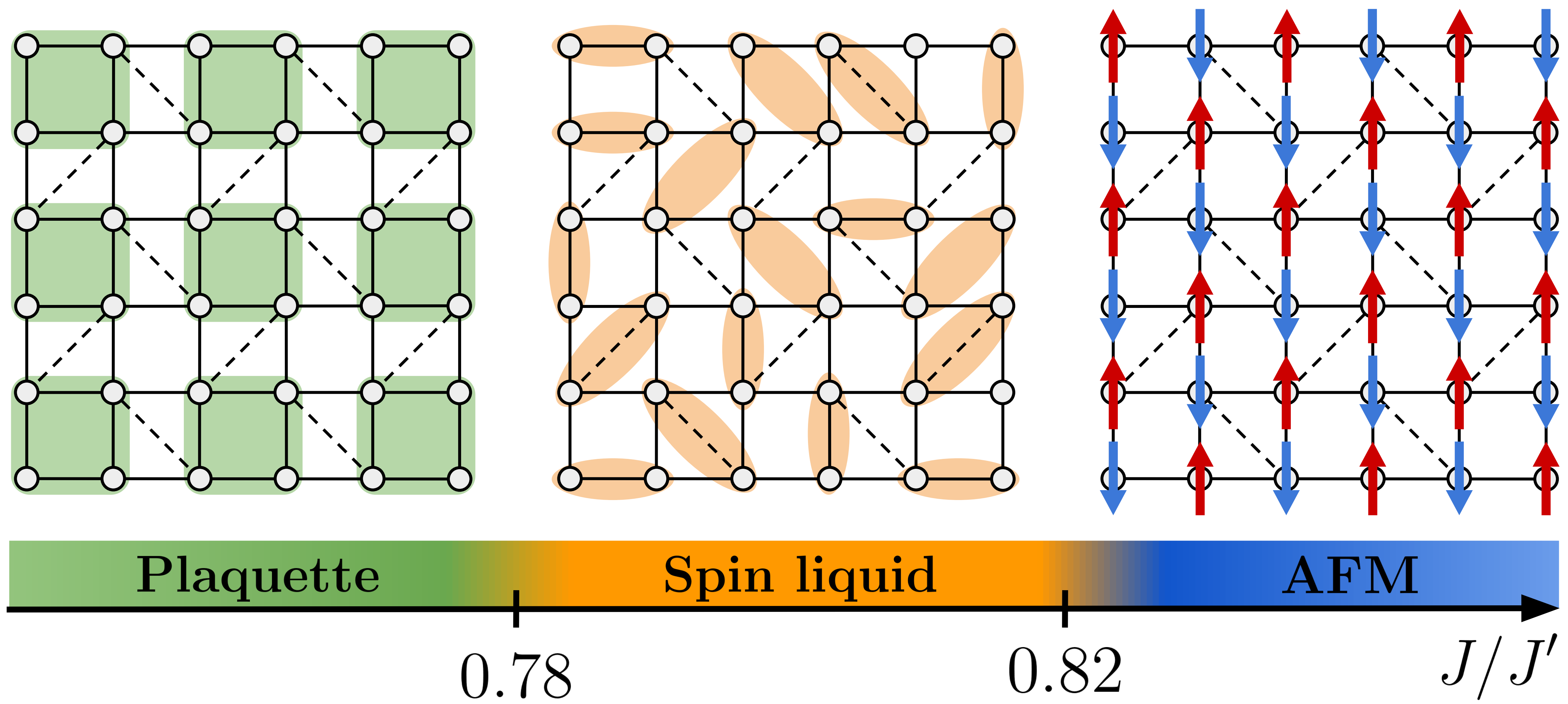
Philippe Corboz,[1] Yining Zhang,[1] Boris Ponsioen,[1] and Frédéric Mila[2]

PHYSICAL REVIEW B **111**, 134411 (2025)

Editors' Suggestion

Transformer wave function for two dimensional frustrated magnets: Emergence of a spin-liquid phase in the Shastry-Sutherland model

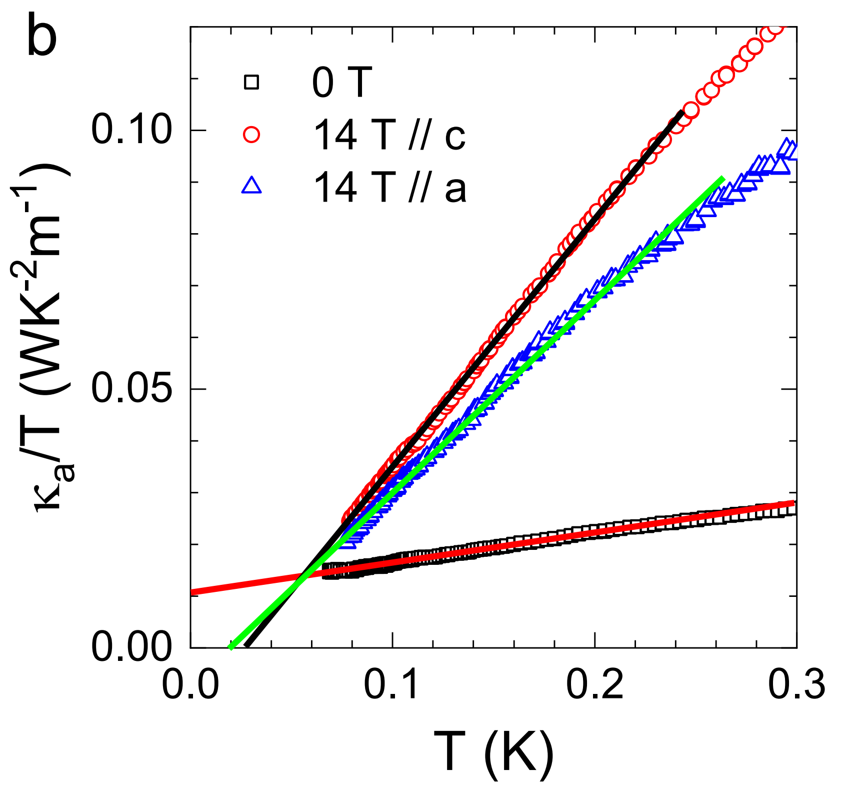
Luciano Loris Viteritti,[1,*,†] Riccardo Rende,[2,*,‡] Alberto Parola,[3] Sebastian Goldt,[2] and Federico Becca[1]

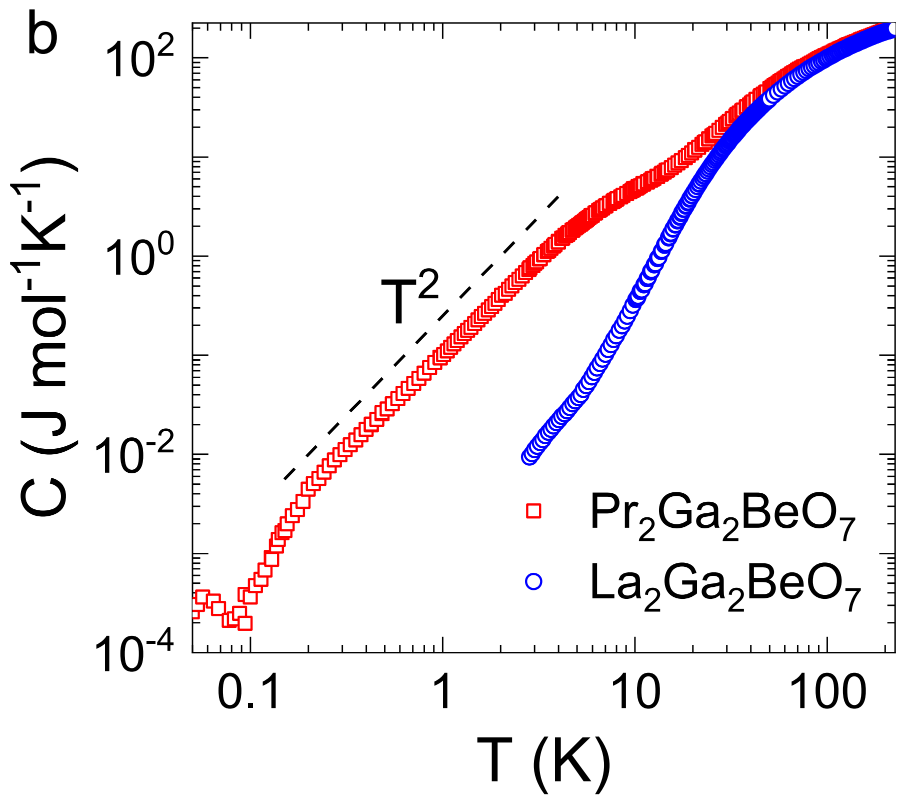


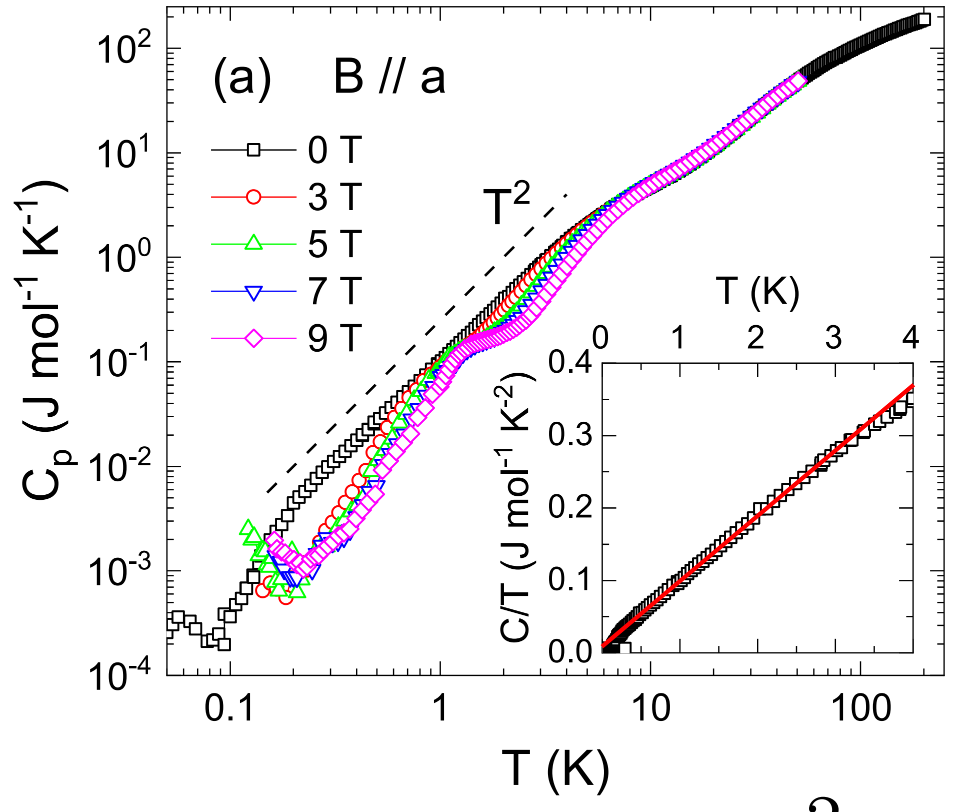
Spinons in a new Shastry-Sutherland lattice magnet $Pr_2Ga_2BeO_7$

N. Li[1,14], A. Brassington[2,14], M. F. Shu[3,14], Y. Y. Wang[1], H. Liang[1], Q. J. Li[4], X. Zhao[5], P. J. Baker[6], H. Kikuchi[7], T. Masuda[7,8], G. Duan[9], C. Liu[10], H. Wang[11], W. Xie[11], R. Zhong[12], J. Ma[3★], R. Yu[9,13★], H. D. Zhou[2★], and X. F. Sun[1★]

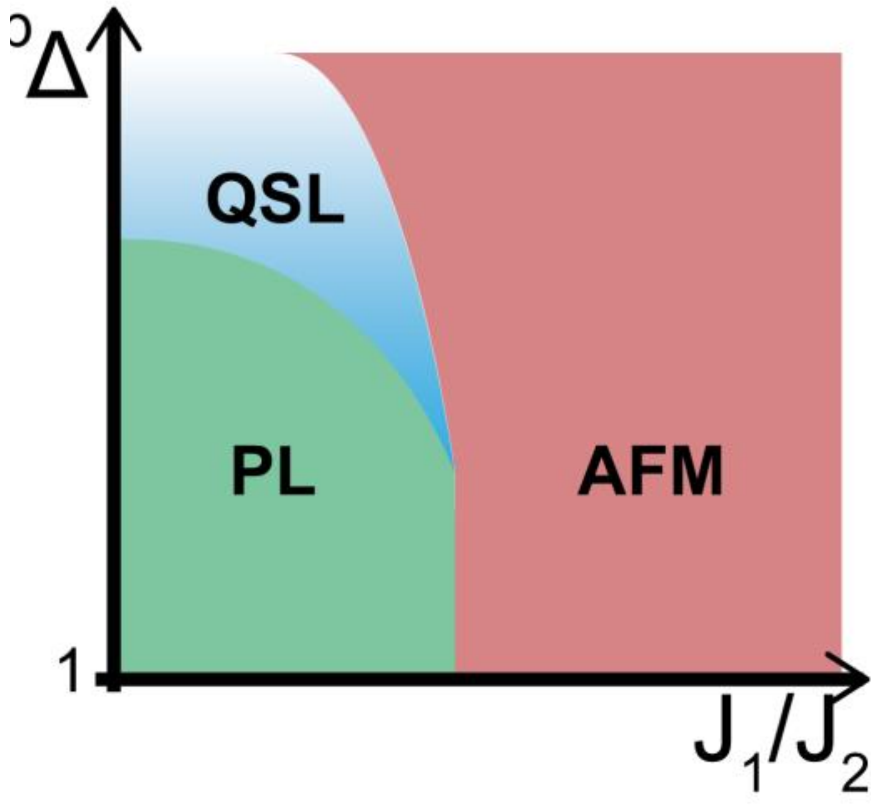
arXiv: 2405.13628 (22 May, 2024)

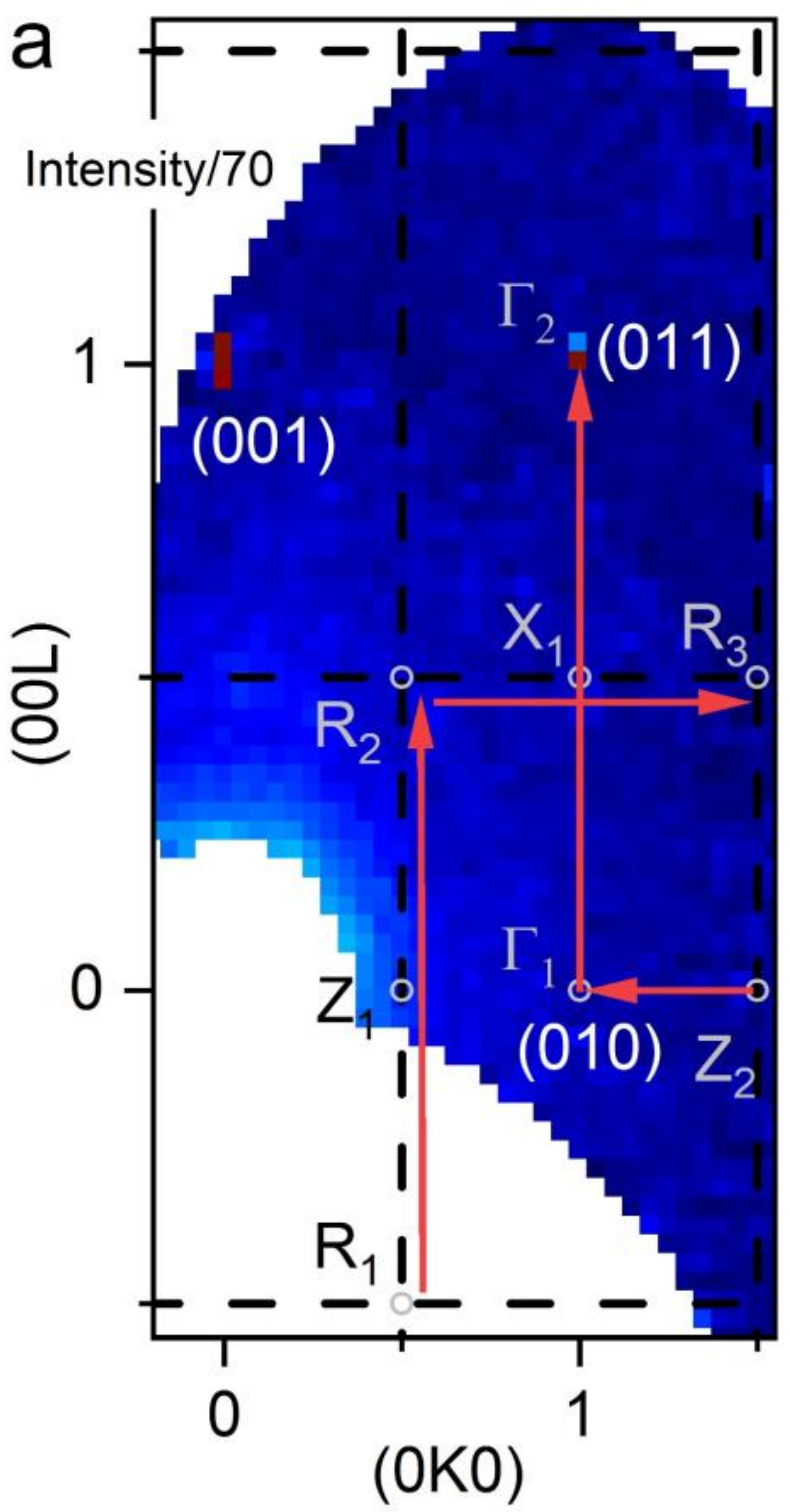


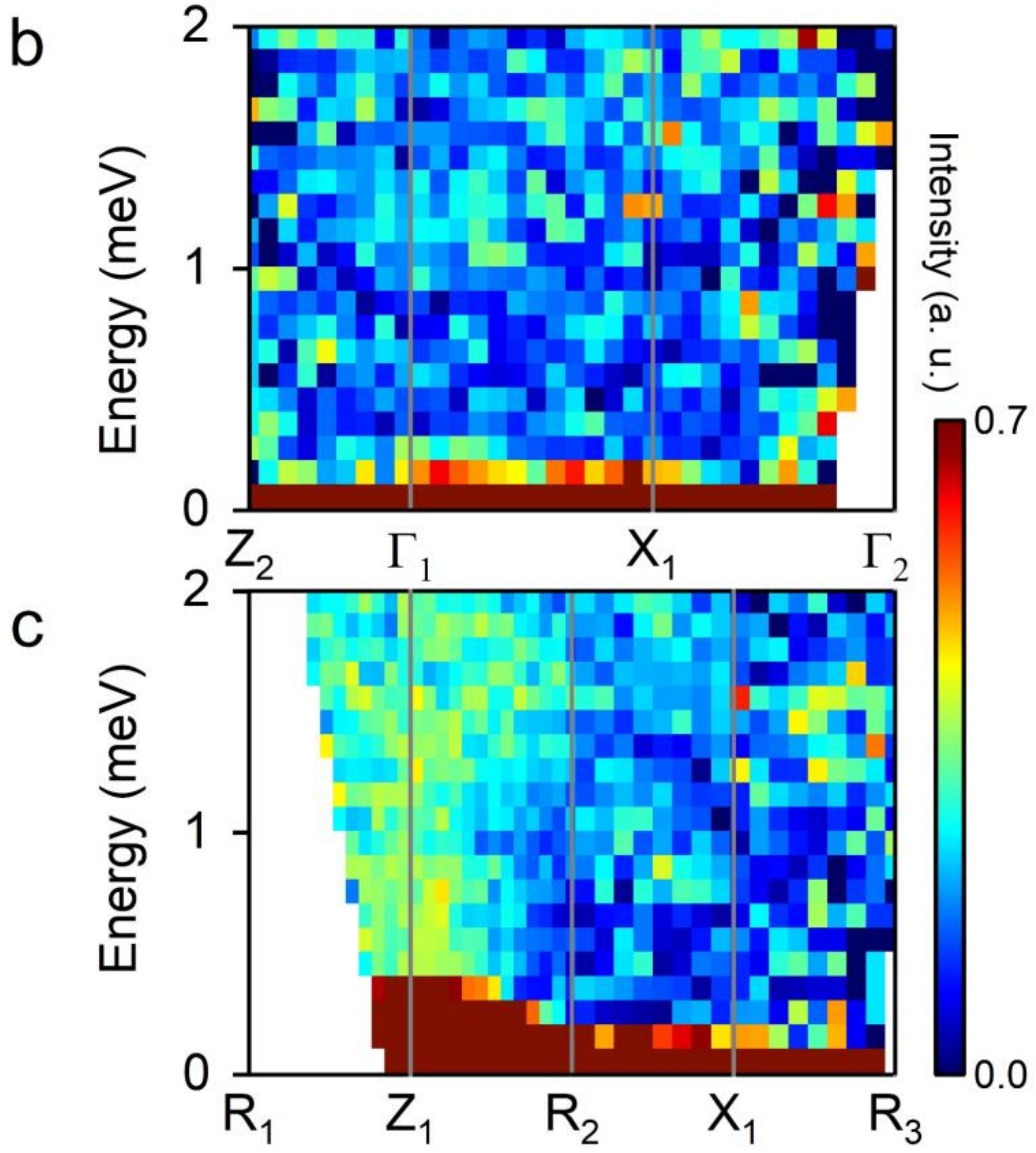




$$C \propto \gamma T + aT^2$$







Parton representation of spin operators

- Enable mean-field treatment of the spin Hamiltonian
- Acquire extra symmetries to allow distinguishing QSLs
- True low-energy excitations are fermionic spinons

$$\hat{S}_i^\gamma = \frac{1}{2}\sum_{\alpha\beta} \hat{f}_{i\alpha}^\dagger \tau_{\alpha\beta}^\gamma \hat{f}_{i\beta}$$

$$\{f_{i,\alpha}, f_{j,\beta}^\dagger\} = \delta_{ij}\delta_{\alpha\beta}$$

$$\{f_{i,\alpha}, f_{j,\beta}\} = 0$$

$$\mathbf{S}^2 = \frac{3}{4}n(2-n)$$

Unphysical

$n = 0$

$n = 2$

$$f_{i,\uparrow}^\dagger f_{i,\uparrow} + f_{i,\downarrow}^\dagger f_{i,\downarrow} = 1$$

$$\psi = (f_\uparrow, f_\downarrow^\dagger)^T \qquad \psi \to e^{i\theta\sigma_3} e^{i\phi\sigma_2} e^{i\varphi\sigma_3}\psi = g\psi \qquad g \in SU(2)$$

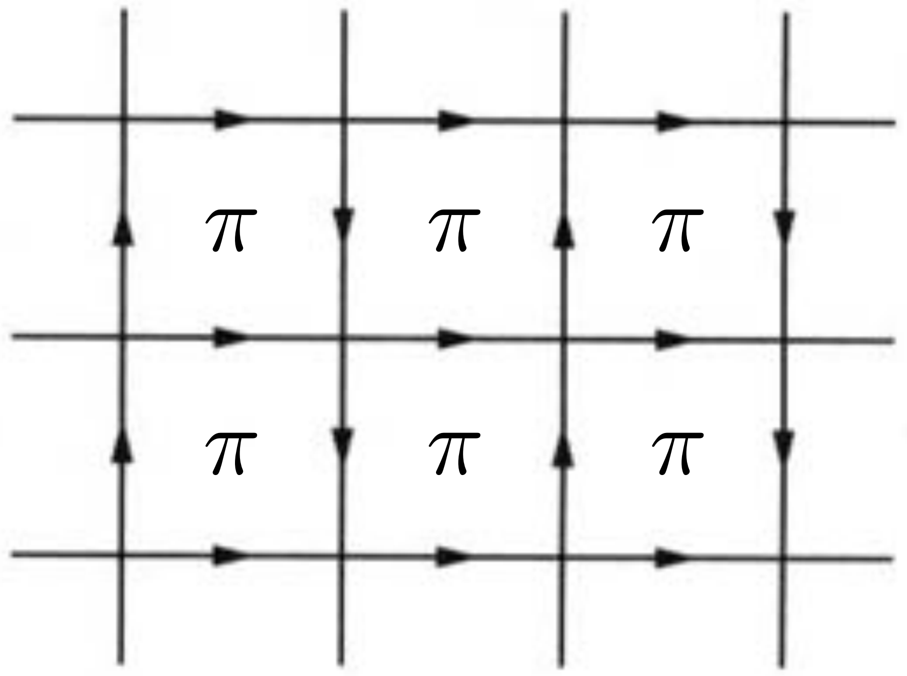
Mean-field approximation

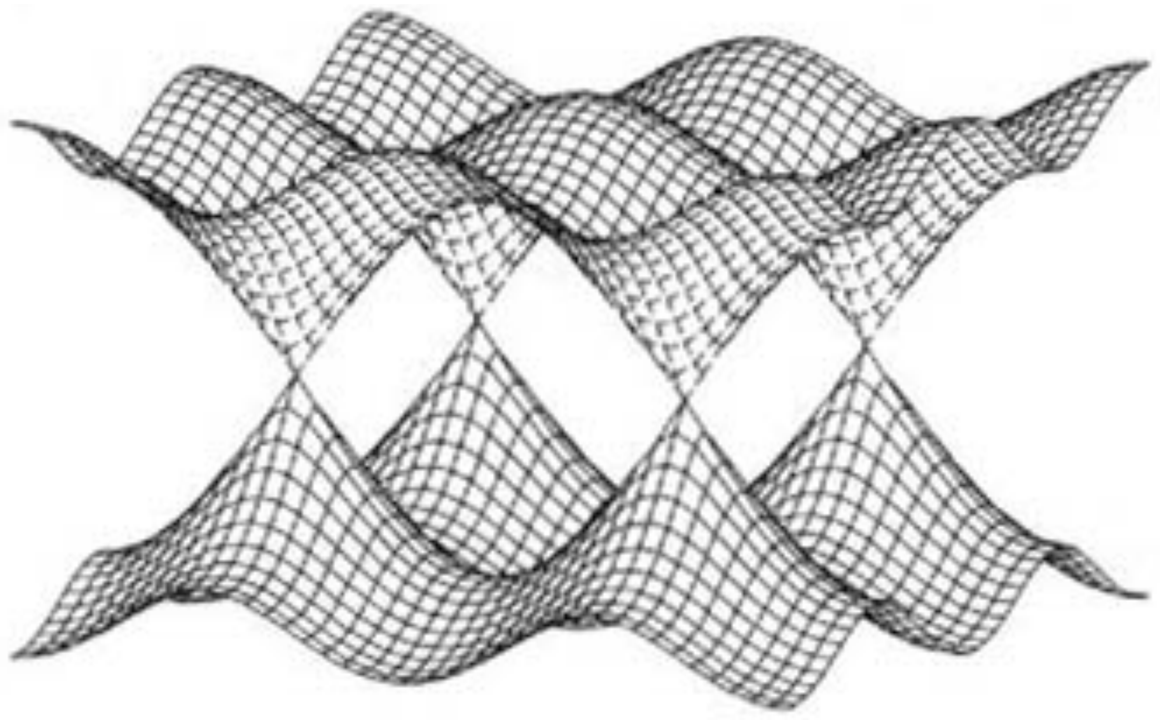
Spin-rotation invariant Hamiltonian

$$H_{\mathrm{MF}} = \sum_{ij} t_{ij}(f_{j,\uparrow}^\dagger f_{i,\uparrow} + f_{j,\downarrow}^\dagger f_{i,\downarrow}) + \Delta_{ij}(f_{j,\uparrow}^\dagger f_{i,\downarrow}^\dagger + f_{i,\uparrow}^\dagger f_{j,\downarrow}^\dagger) + \mathrm{h.c.}$$

The "parent" state

$$H_{\mathrm{MF}} = \sum_{\langle ij\rangle,\alpha} t(=\pm 1)(f_{i,\alpha}^\dagger f_{j,\alpha}) + \mathrm{h.c.}$$





- Excellent energies on kagome and triangular lattices
- Algebraic decay of spin-spin correlations
- U(1) Dirac spin liquid on non-bipartite lattices

ARTICLE

https://doi.org/10.1038/s41467-019-11727-3 OPEN

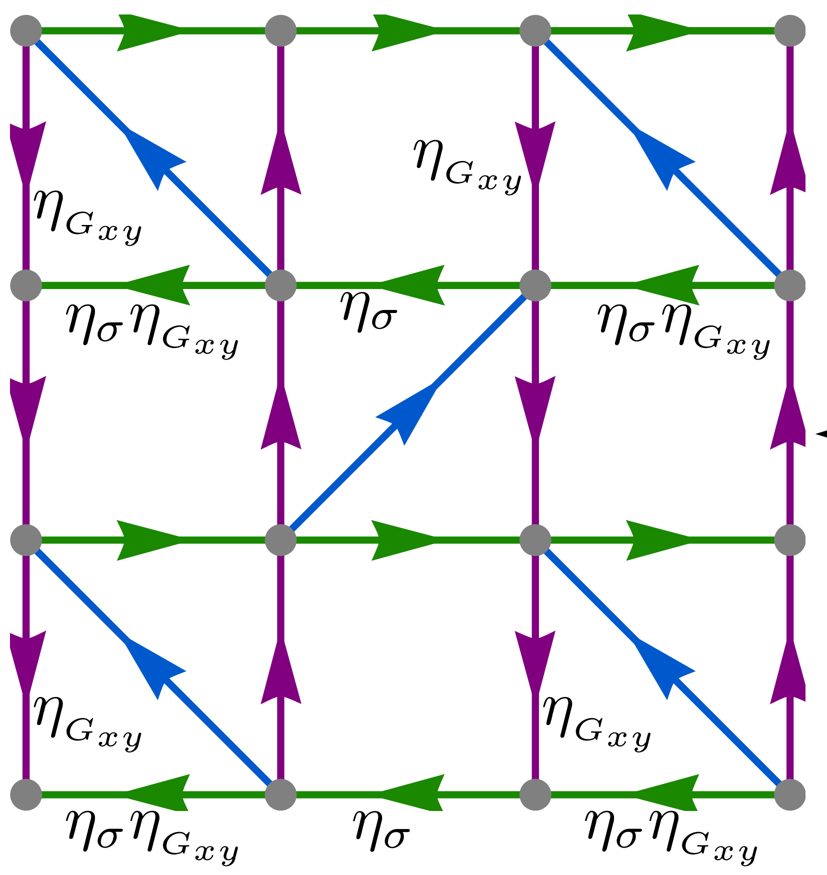
Unifying description of competing orders in two-dimensional quantum magnets

Xue-Yang Song[1], Chong Wang[1,2], Ashvin Vishwanath[1] & Yin-Chen He[1,2]

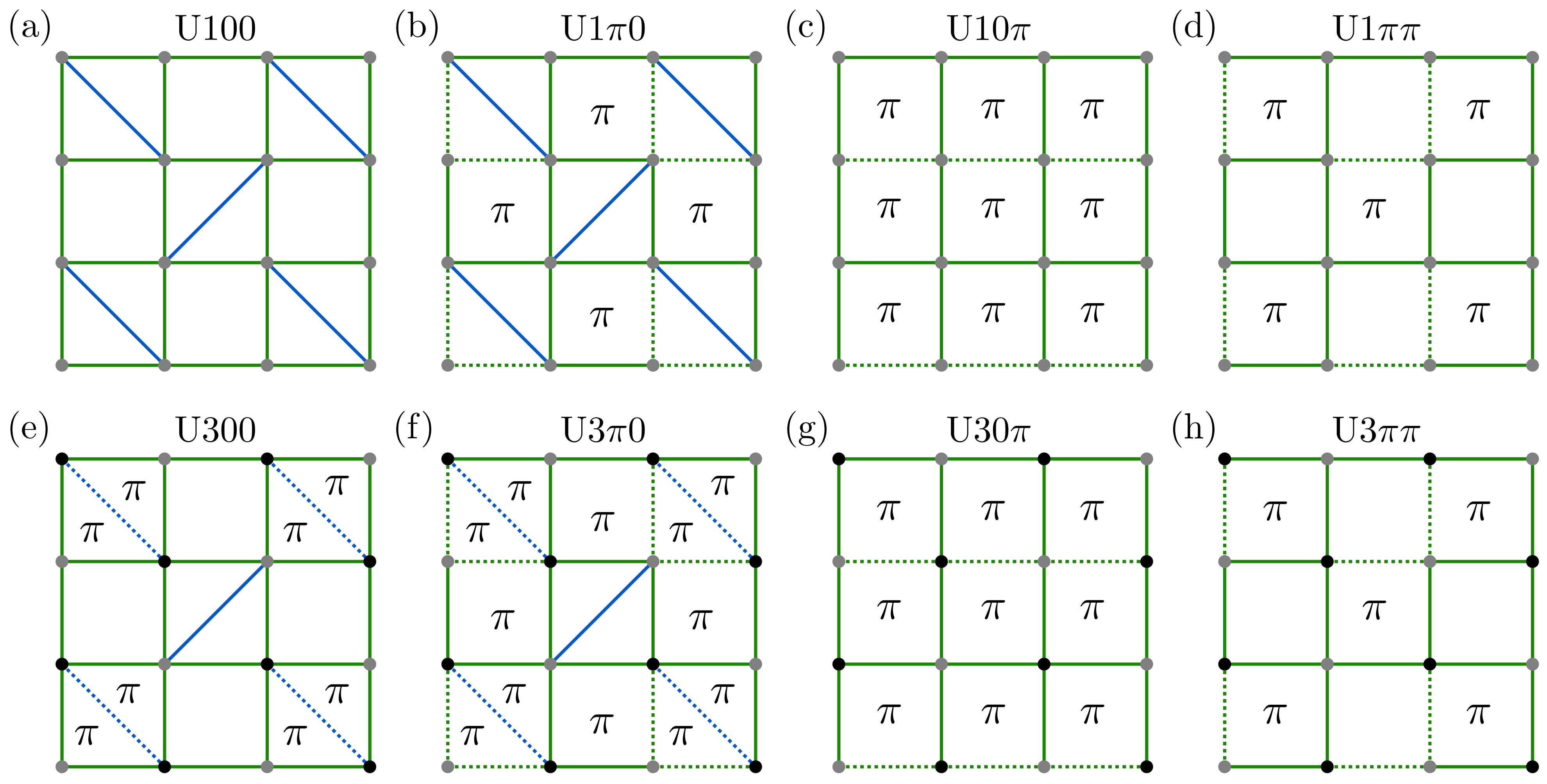
Projective Symmetry Groups

Representations of *p4g* group

64 U(1) QSLs → 12 U(1) QSLs (Short-range Ansätze)



80 Z_2 QSLs → 18 Z_2 QSLs (Short-range Ansätze)

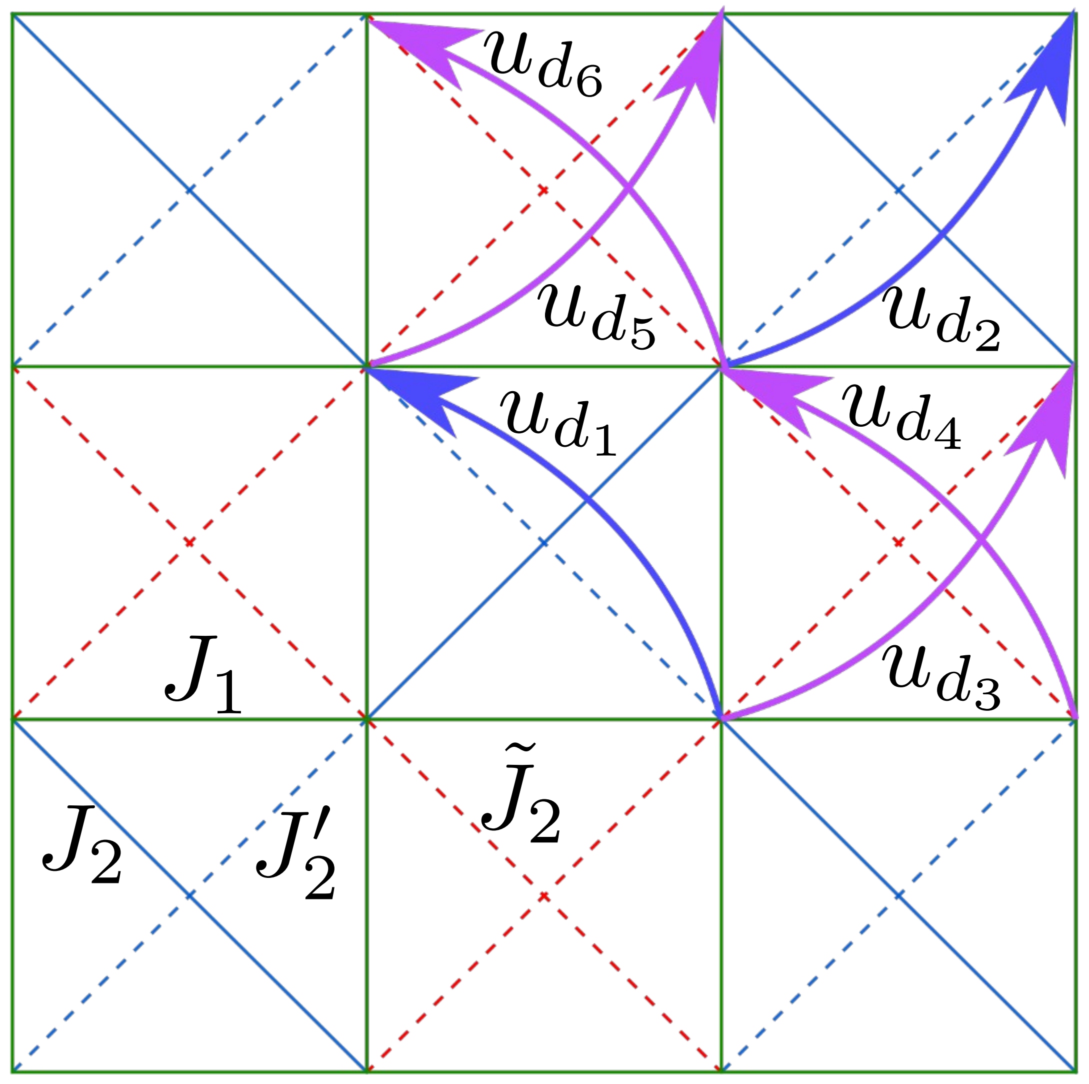


Pairing Instabilities →

Label	u_s	u_d	Onsite	Parent $U(1)$
Z2000	$\tau^{3,1}$	$\tau^{3,1}$	τ^3	U100, U300
Z2100	$\tau^{3,1}$	$\tau^{3,1}$	τ^3	U1π0, U3π0
Z2001	$\tau^{0,2}$	$\tau^{3,1}$	τ^3	U100, U300
Z2101	$\tau^{0,2}$	$\tau^{3,1}$	τ^3	U1π0, U3π0
Z2011	$\tau^{0,2}$	0	τ^3	U10π, U30π
Z2111	$\tau^{0,2}$	0	τ^3	U1$\pi\pi$, U3$\pi\pi$
Z3000	$\tau^{1,3}$	τ^3	τ^3	U100, U300
Z3100	$\tau^{1,3}$	τ^3	τ^3	U1π0, U3π0
Z3010	$\tau^{1,3}$	0	τ^3	U10π, U30π
Z3110	$\tau^{1,3}$	0	τ^3	U1$\pi\pi$, U3$\pi\pi$
Z3001	$\tau^{0,2}$	τ^3	τ^3	U100, U300
Z3101	$\tau^{0,2}$	τ^3	τ^3	U1π0, U3π0
Z3011	$\tau^{0,2}$	0	τ^3	U10π, U30π
Z3111	$\tau^{0,2}$	0	τ^3	U1$\pi\pi$, U3$\pi\pi$
Z5001	$\tau^{0,1,3}$	0	0	U500
Z5101	$\tau^{0,1,3}$	0	0	-
Z5011	$\tau^{0,1,3}$	0	0	U50π
Z5111	$\tau^{0,1,3}$	0	0	-

Interpolating between square lattice and Shastry-Sutherland PSGs

Finding the equivalent of the Z2Azz13 state



Label	u_s	u_d	Onsite	u'_d	u_{dr}
Z2000	$\tau^{3,1}$	$\tau^{3,1}$	τ^3	$\tau^{3,1}$	$\tau^{3,1}$
Z2100	$\tau^{3,1}$	$\tau^{3,1}$	τ^3	$\tau^{3,1}$	0
Z2001	$\tau^{0,2}$	$\tau^{3,1}$	τ^3	$\tau^{3,1}$	$\tau^{3,1}$
Z2101	$\tau^{0,2}$	$\tau^{3,1}$	τ^3	$\tau^{3,1}$	0
Z2011	$\tau^{0,2}$	0	τ^3	0	0
Z2111	$\tau^{0,2}$	0	τ^3	0	$\tau^{3,1}$
Z3000	$\tau^{1,3}$	τ^3	τ^3	τ^3	$\tau^{3,1}$
Z3100	$\tau^{1,3}$	τ^3	τ^3	τ^3	0
Z3010	$\tau^{1,3}$	0	τ^3	0	0
Z3110	$\tau^{1,3}$	0	τ^3	0	$\tau^{3,1}$
Z3001	$\tau^{0,2}$	τ^3	τ^3	τ^3	$\tau^{3,1}$
Z3101	$\tau^{0,2}$	τ^3	τ^3	τ^3	0
Z3011	$\tau^{0,2}$	0	τ^3	0	0
Z3111	$\tau^{0,2}$	0	τ^3	0	$\tau^{3,1}$
Z5001	$\tau^{0,1,3}$	0	0	0	0
Z5101	$\tau^{0,1,3}$	0	0	0	0
Z5011	$\tau^{0,1,3}$	0	0	0	0
Z5111	$\tau^{0,1,3}$	0	0	0	0

d-wave

$\mathbb{Z}_2$ (P43)	PSG	$(\eta_{G_x T_x}, \eta_{\sigma_x G_x})$	$\mathbb{Z}_2$ (P4)
Z3000	3a	$(+,+)$	$Z2A0013$
Z3000	3c	$(+,-)$	$Z2Azz13$
Z3010	3a	$(+,+)$	$Z2B0013$
Z2000	2b	$(+,-)$	$Z2Axx0z$
Z5011	5d	$(-,+)$	$Z2Ax2(12)n$
Z5001	5d	$(-,+)$	$Z2Bx2(12)n$

Wen 2002

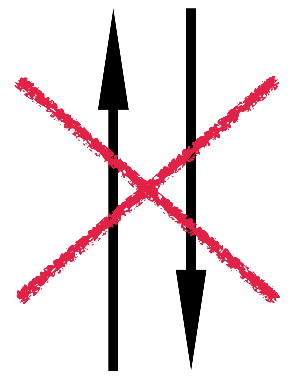
Beyond mean-field

The ground state takes the form of a BCS wave-function

$$|\Psi_{\mathrm{MF}}\rangle = \exp\left\{ \sum_{ij} c_{ij} (f^\dagger_{i,\uparrow} f^\dagger_{j,\downarrow} + f^\dagger_{j,\uparrow} f^\dagger_{i,\downarrow}) \right\} |0\rangle$$

The exact constraint $n = 1$ can be enforced exactly via Monte Carlo sampling

$$|\Psi_{\mathrm{Var\ RVB}}\rangle = P_G |\Psi_{\mathrm{MF}}\rangle \qquad P_G = \prod_i (1 - n_{i,\uparrow} n_{i,\downarrow})$$

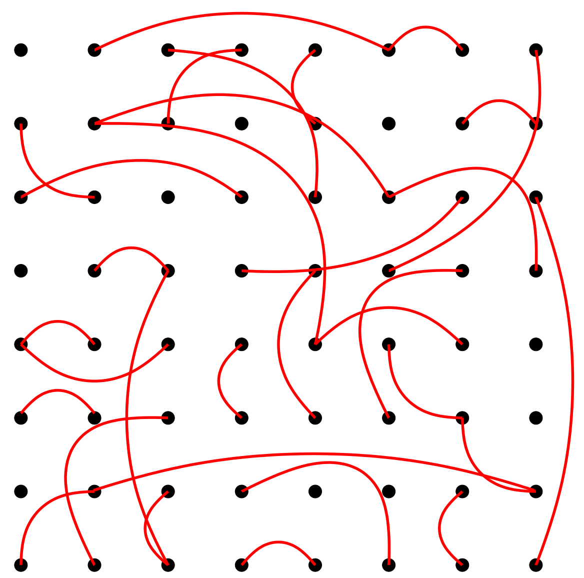


A Monte-Carlo sampling implies calculation of determinants which can be done in polynomial time

The projected wave-function

$$|\Psi_{\mathrm{MF}}\rangle = \exp\left\{\sum_{ij} c_{ij}(f^\dagger_{i,\uparrow}f^\dagger_{j,\downarrow} + f^\dagger_{j,\uparrow}f^\dagger_{i,\downarrow})\right\}|0\rangle$$

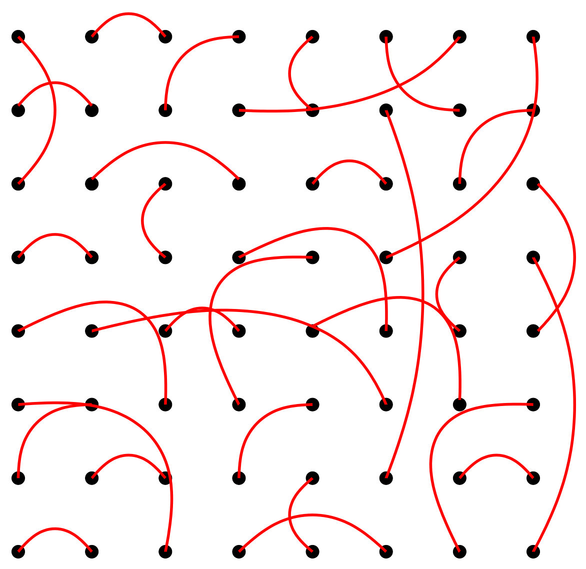
It is a linear superposition of all singlet configurations (that may overlap)



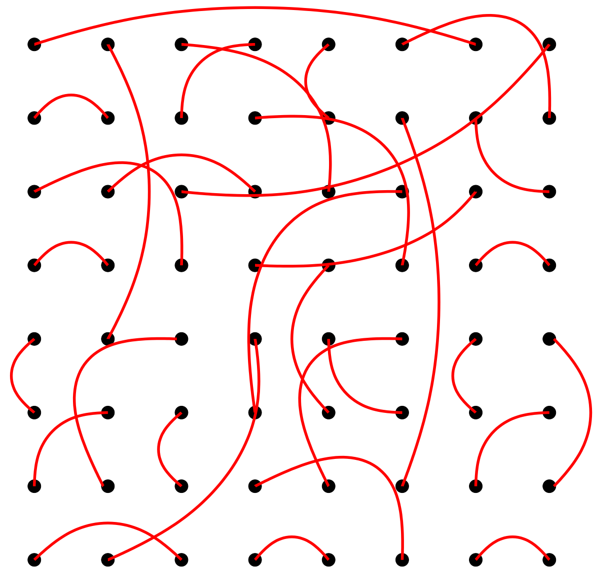
$+ \ldots$

- After projection, only non-overlapping singlets survive: the resonating valence-bond (RVB) wave function

Anderson, Science **235**, 1196 (1987)



$+$



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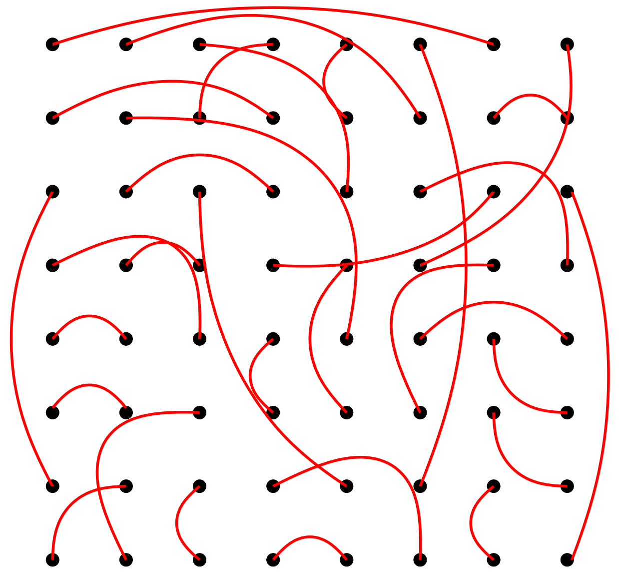
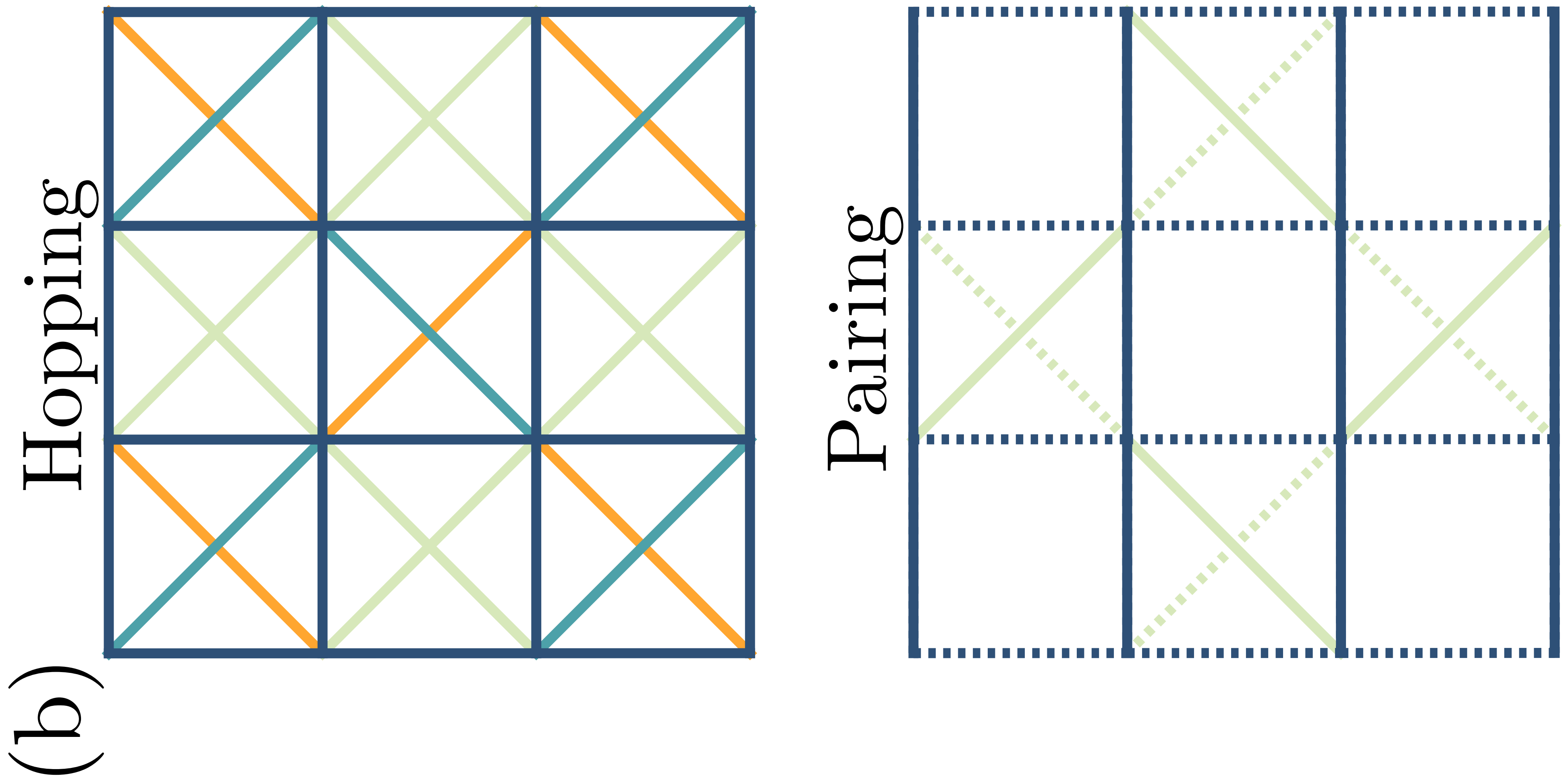
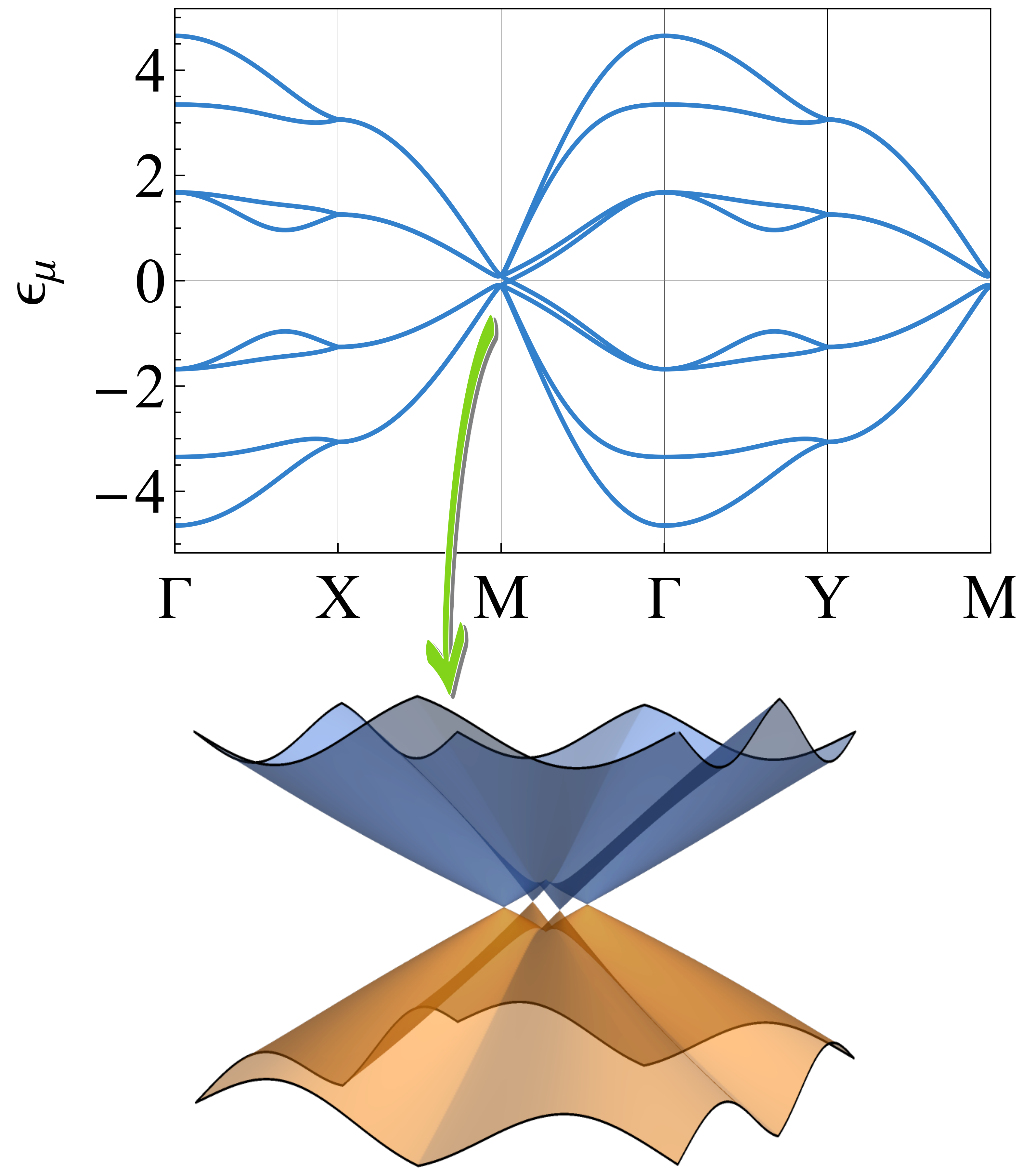


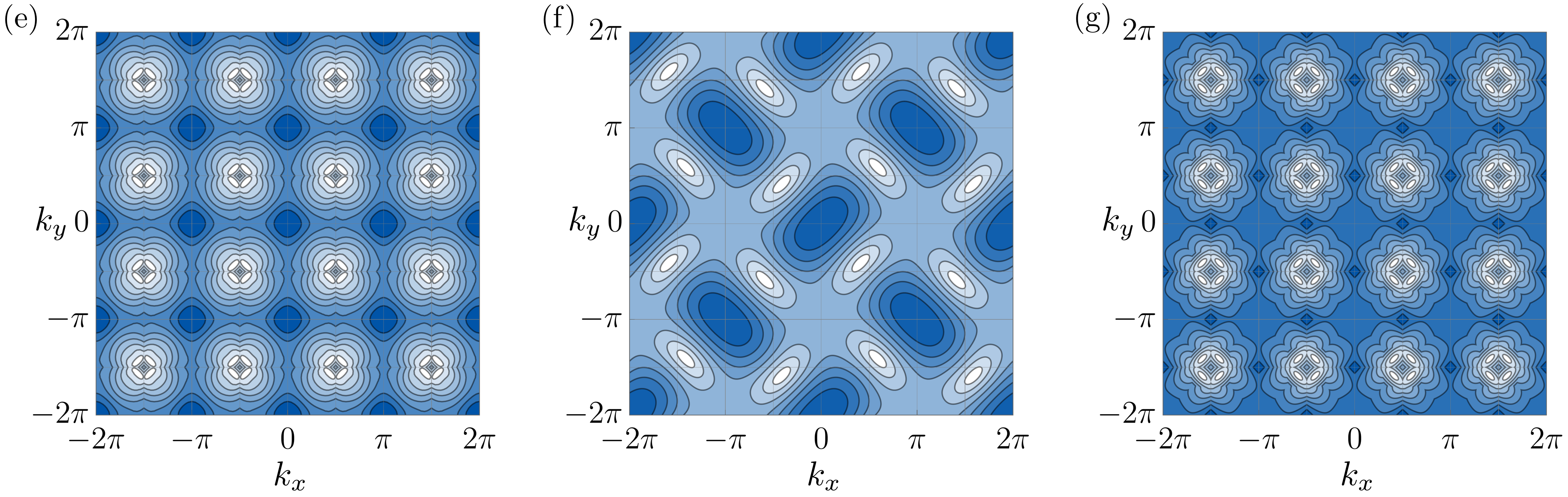
$+ \ldots$

Z_2 Dirac Spin Liquid: *d*-wave pairing

Fluxes and band structure







$(\frac{\pi}{2}, \frac{\pi}{2}) \pm (-\epsilon, \epsilon)$

$(\frac{\pi}{2}, \frac{\pi}{2}) \pm (\epsilon, \epsilon)$

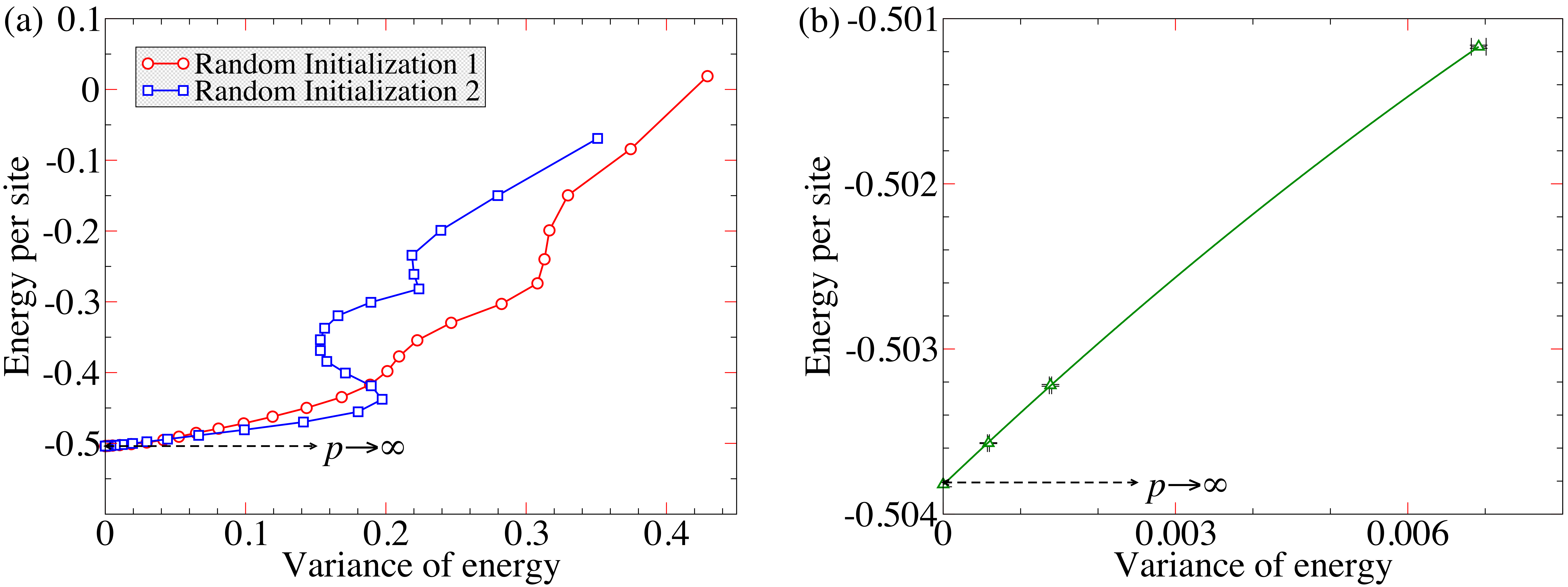
Improving the quality of liquids

$\xrightarrow{\mathcal{H}^n (n \to \infty)}$

$$|\Psi_{p-\mathrm{LS}}\rangle = \left(1 + \sum_{k=1}^{p} \alpha_k \hat{\mathcal{H}}^k\right) |\Psi_{\mathrm{VMC}}\rangle$$

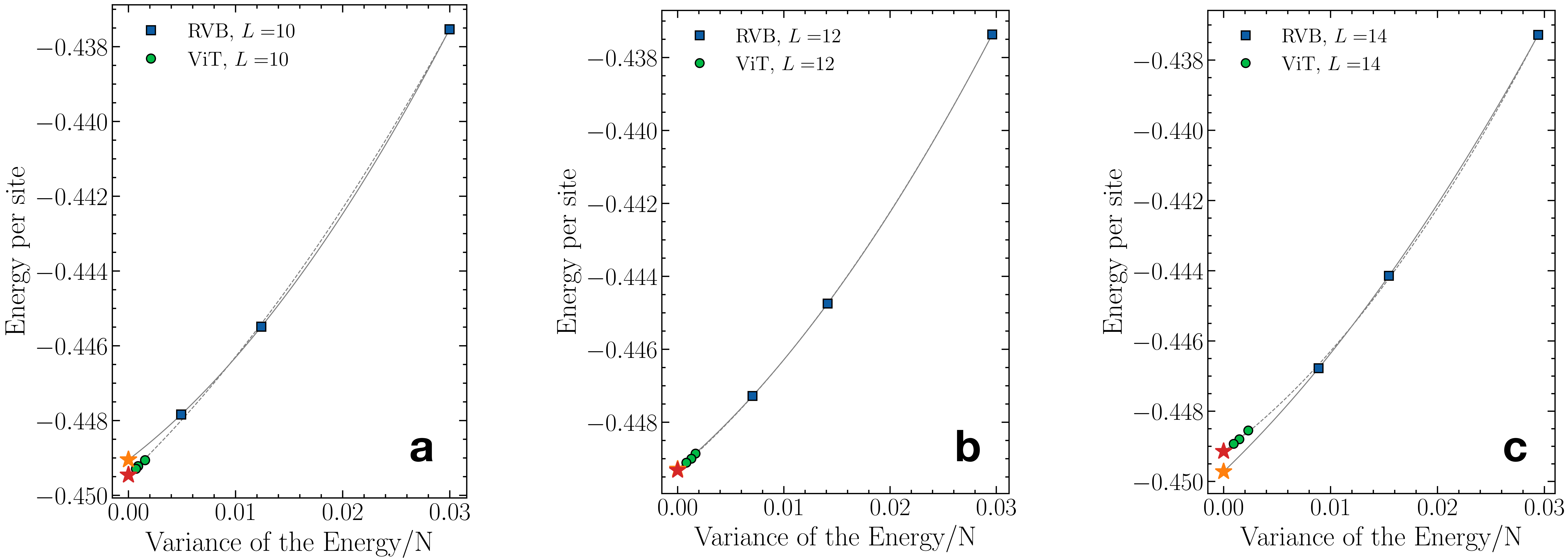
- On large cluster sizes, only a few steps can be efficiently performed ($p = 1$ and $p = 2$).
- An estimate of the exact ground-state energy may be achieved by the method of variance extrapolation: For sufficiently accurate states, we have that $E \approx E_{\mathrm{ex}} + \mathrm{constant} \times \sigma^2$, where $E = \langle \hat{\mathcal{H}} \rangle / N$ and $\sigma^2 = (\langle \hat{\mathcal{H}}^2 \rangle - \langle \hat{\mathcal{H}} \rangle^2)/N$. The exact ground-state energy E_{ex} can be extracted by fitting E vs σ^2 for $p = 0$, 1, and 2.

Criterion for a good variational state



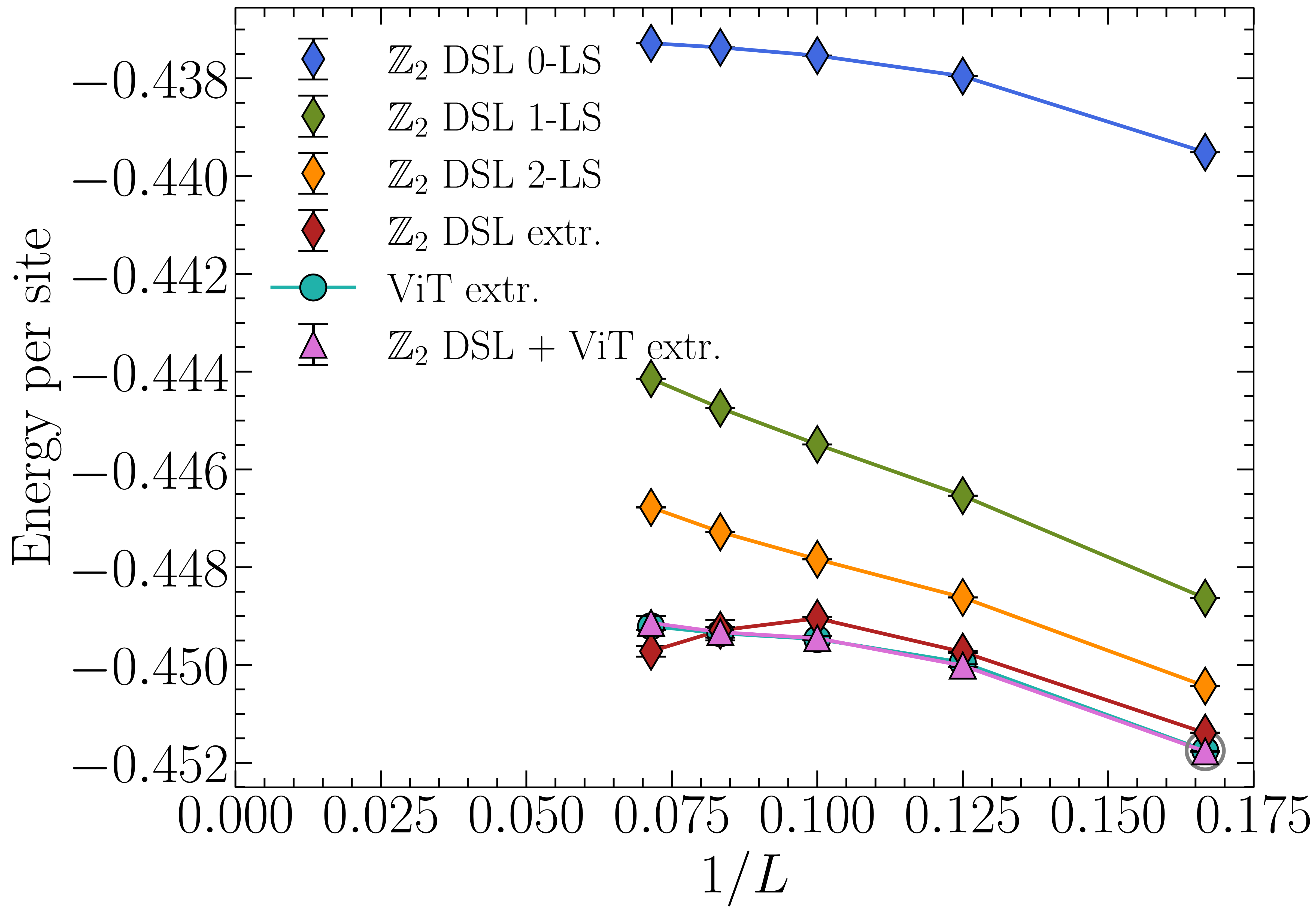
Z_2 Dirac Spin Liquid: *d*-wave pairing

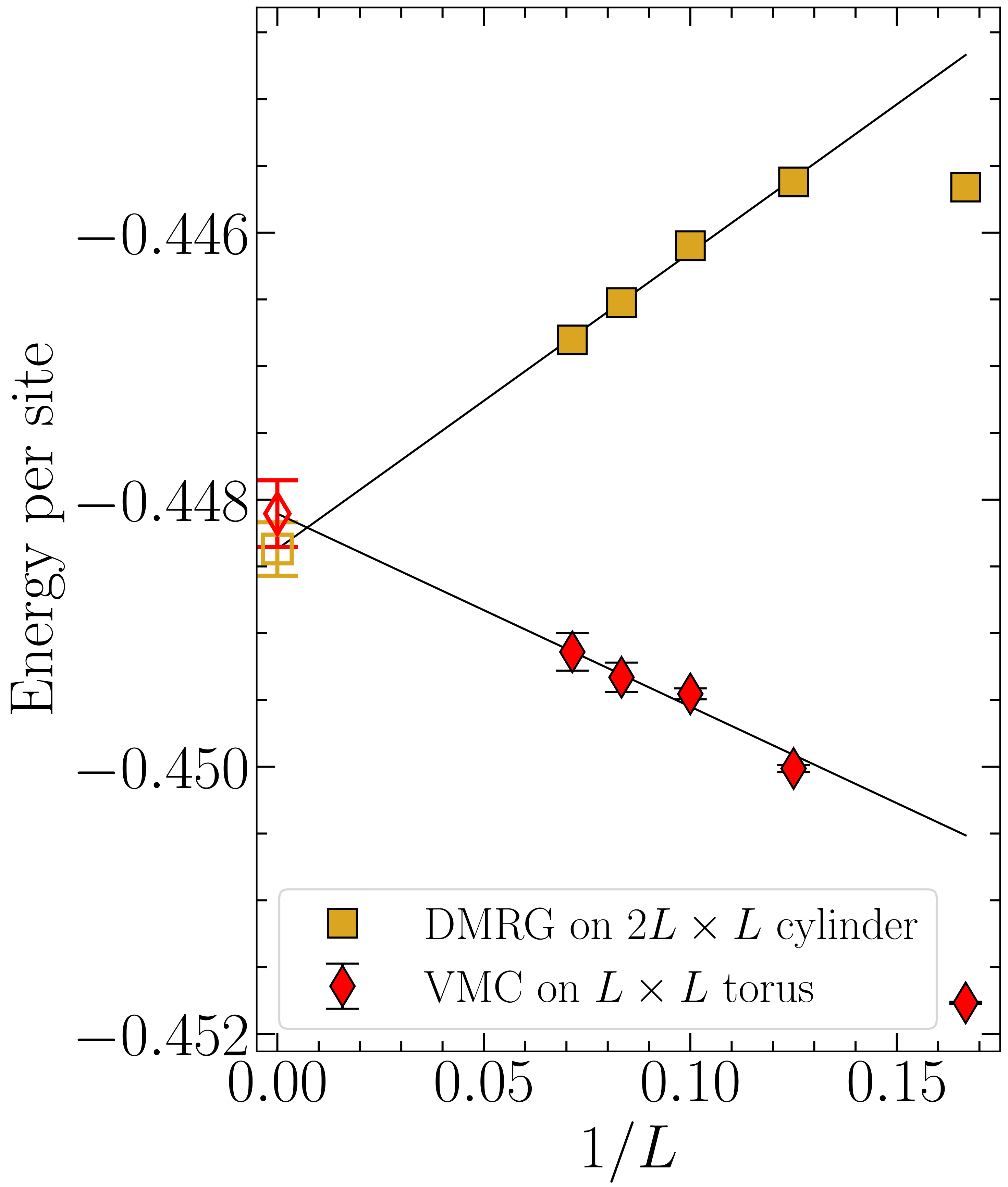
Lowest variational energy



$\mathbb{Z}_2$ Dirac Spin Liquid: *d*-wave pairing

Size-scaling



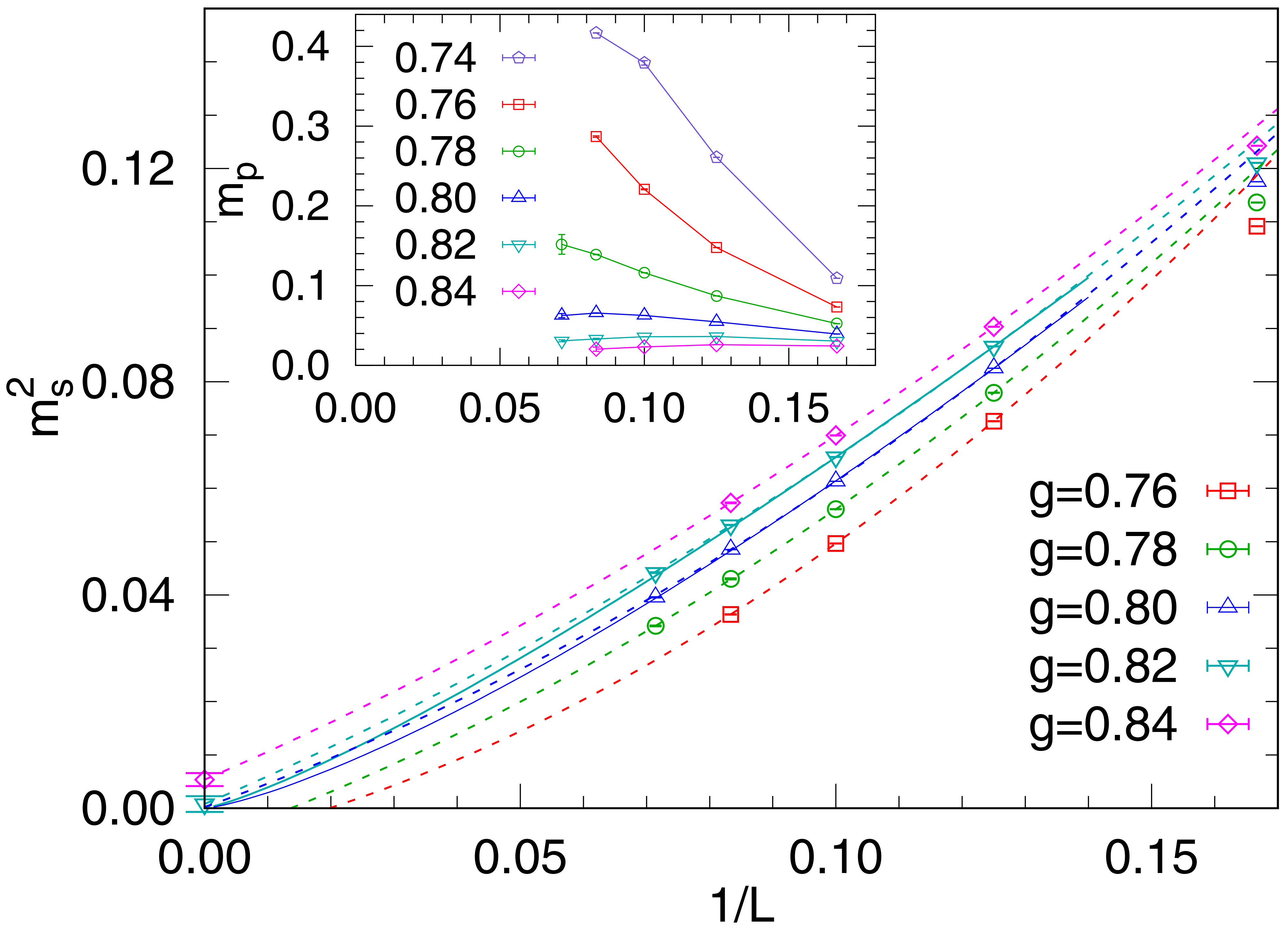


Z_2 Dirac Spin Liquid: Real-space spin-spin correlations

$$C(\mathbf{r}) \sim r^{-(z+\eta)}$$

DMRG Results

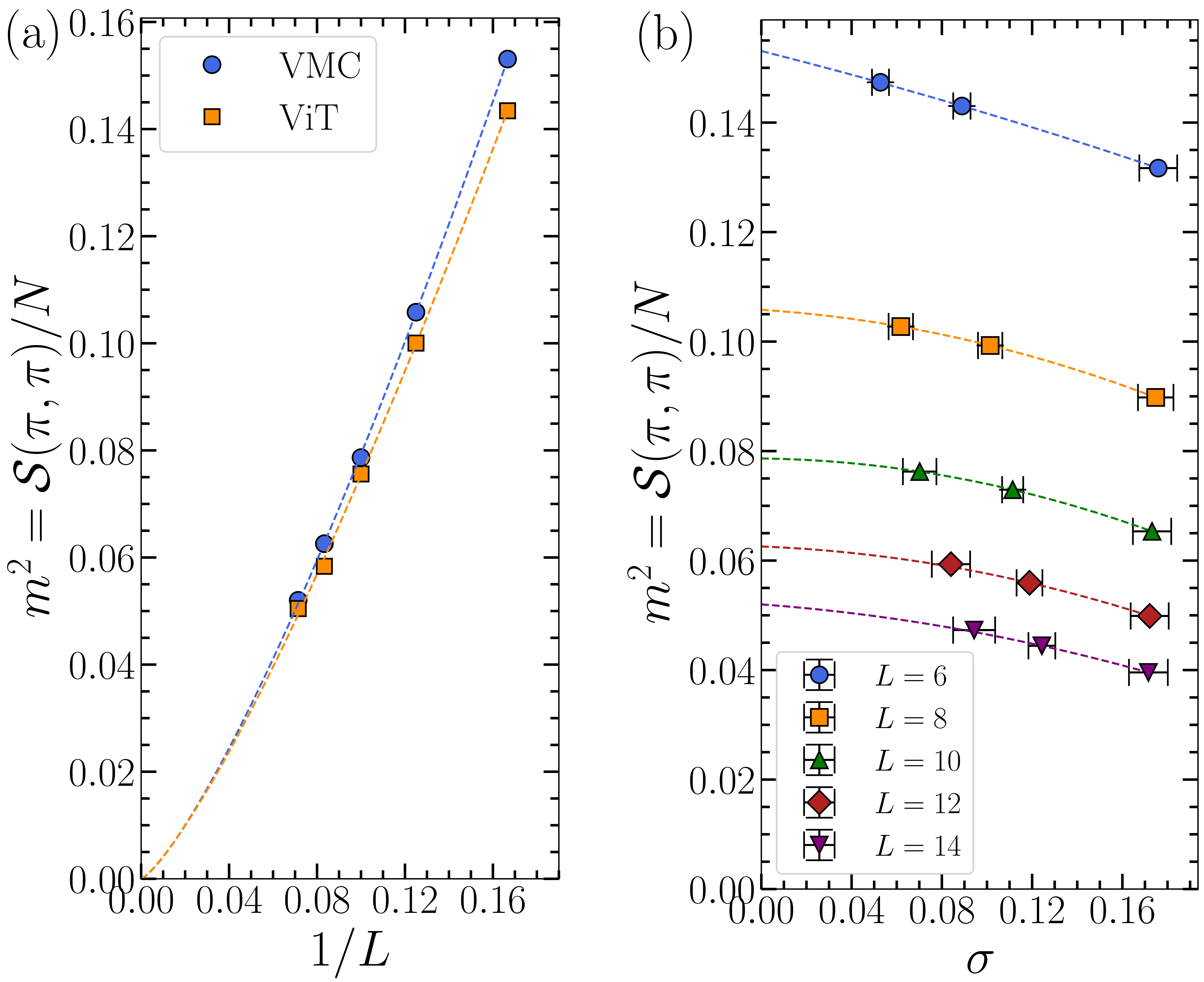
Yang, Sandvik, Wang, PRB 105, L060409 (2022)



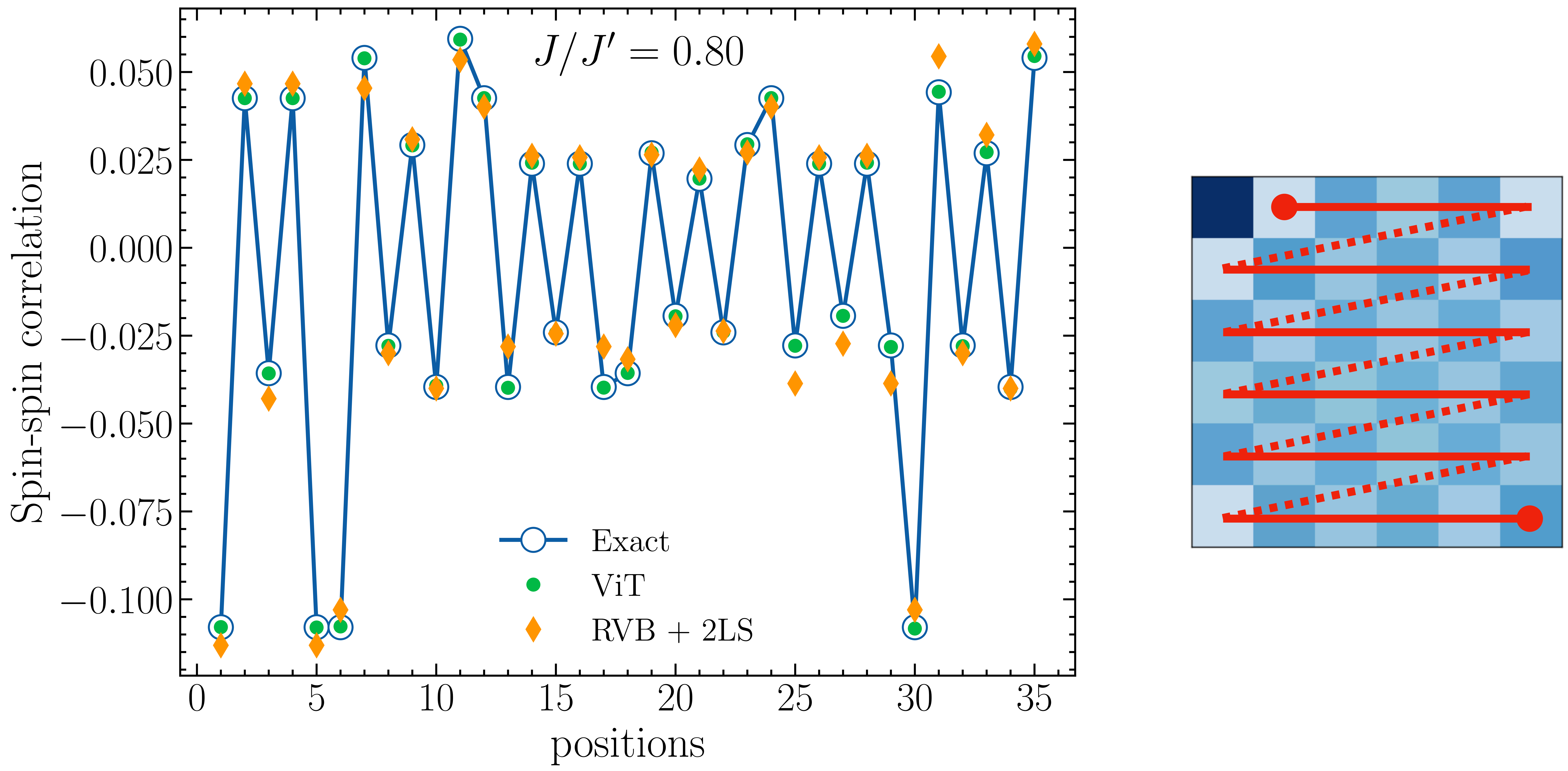
$$m^2_{AF} \propto L^{-(1+\eta)} \quad \eta \approx 0.30$$

VMC exponent

$z + \eta \sim 1.30(3)$

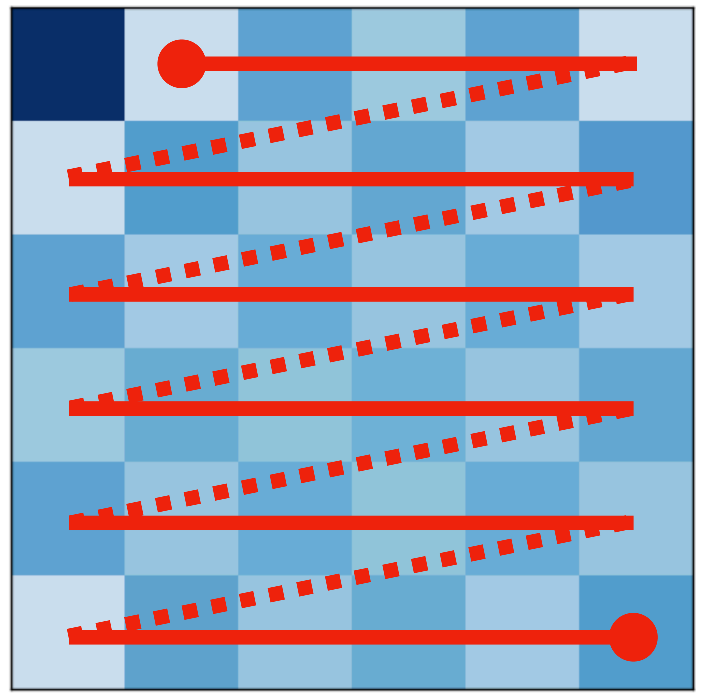


Spin-Spin Correlations on 36-site cluster



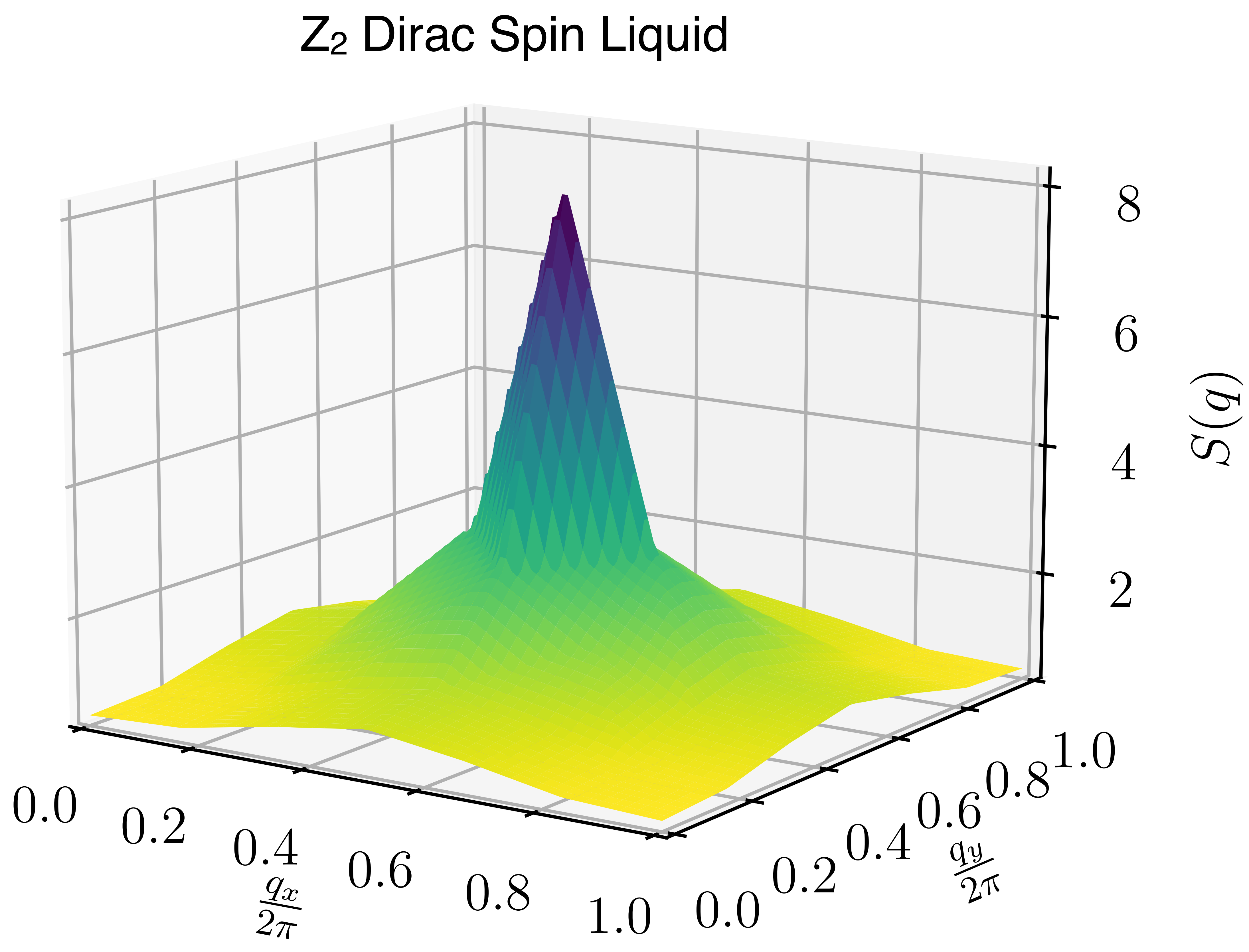
$\mathbb{Z}_2$ Dirac Spin Liquid

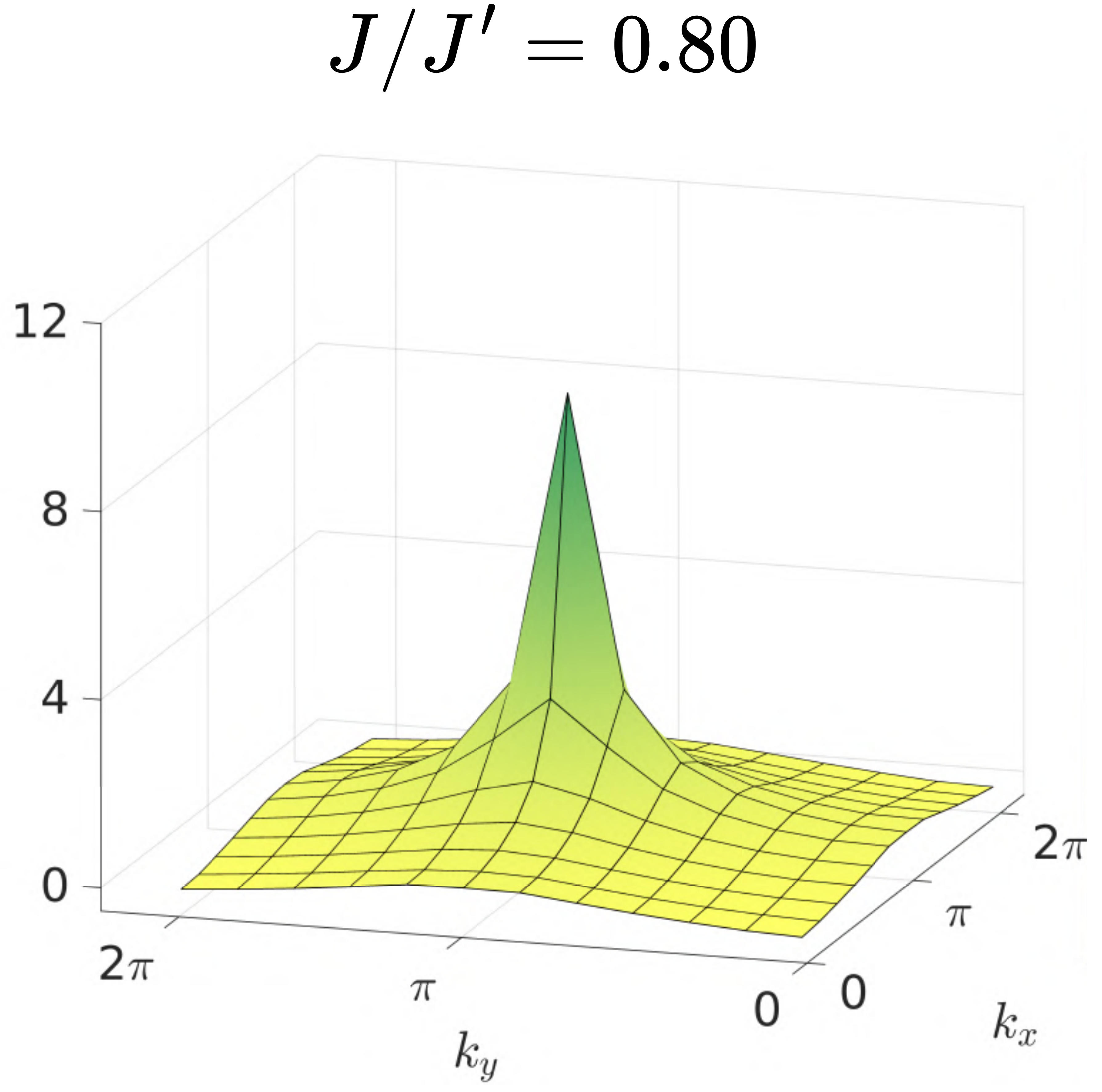
Larger system sizes: Size-consistency of the wave function

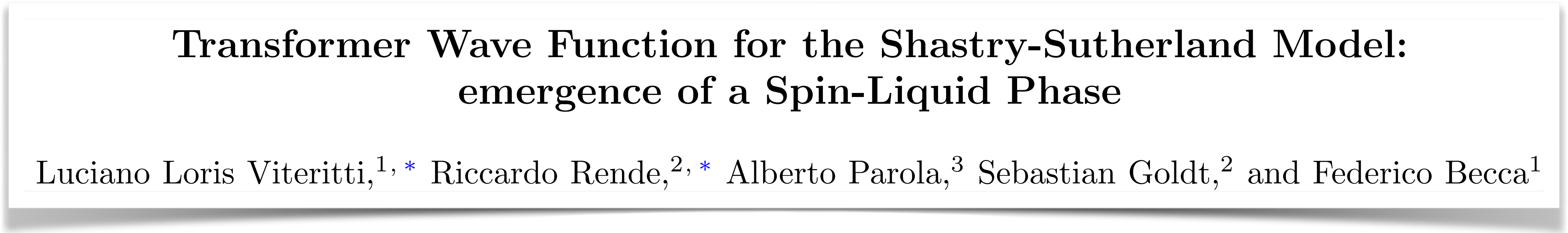


Z_2 Dirac Spin Liquid

Structure factor



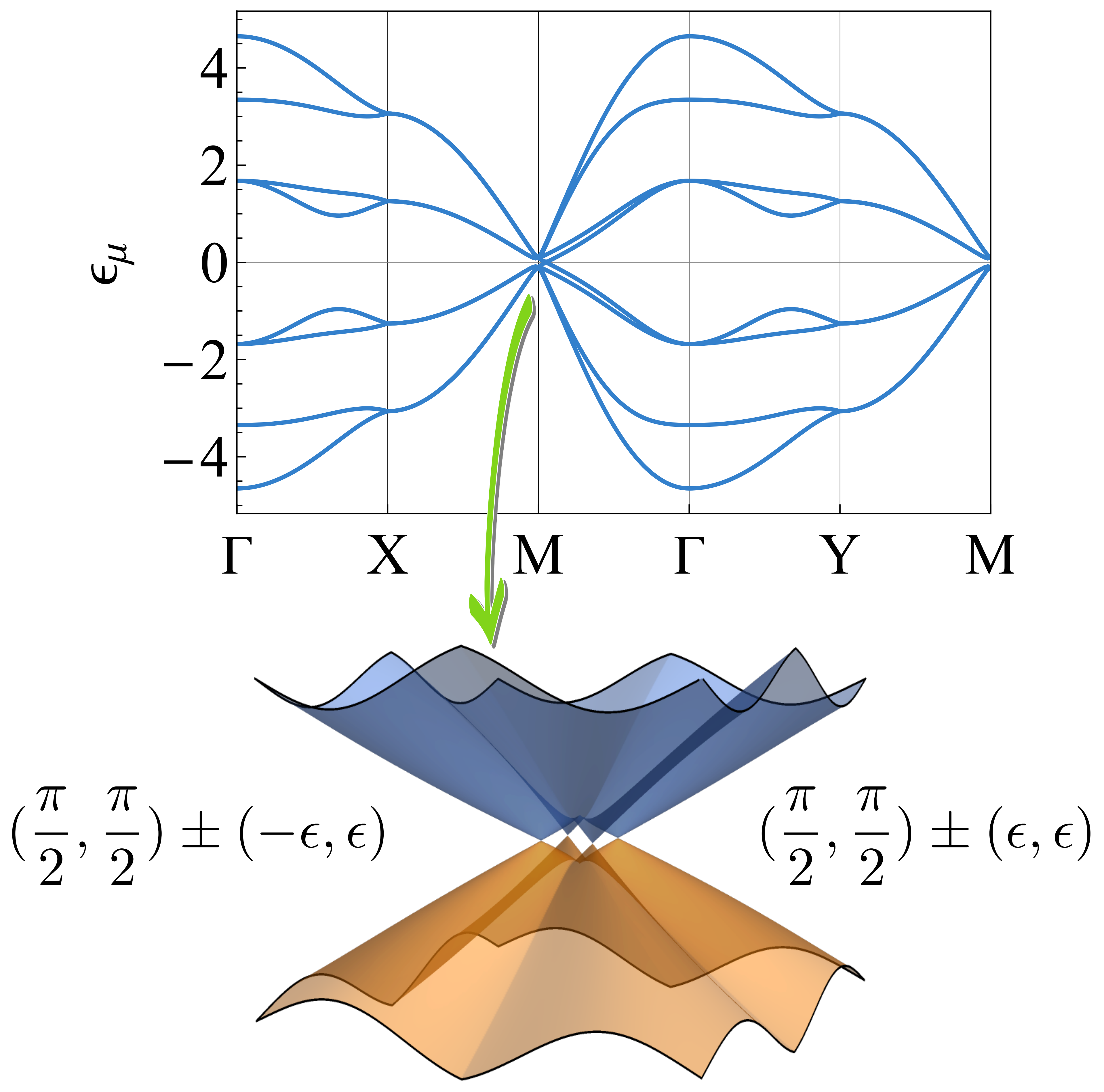




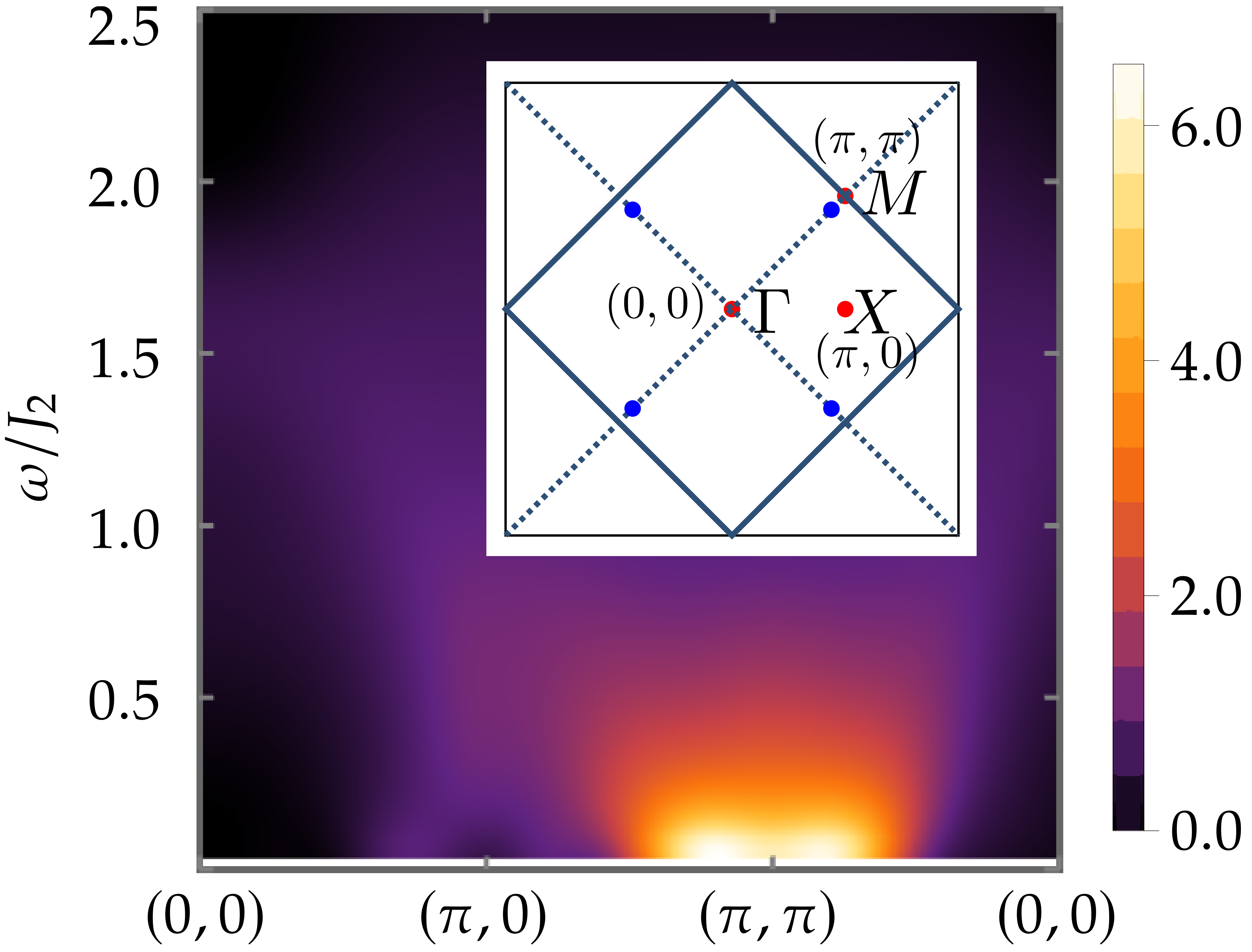
Transformer Wave Function for the Shastry-Sutherland Model: emergence of a Spin-Liquid Phase

Luciano Loris Viteritti,[1,*] Riccardo Rende,[2,*] Alberto Parola,[3] Sebastian Goldt,[2] and Federico Becca[1]

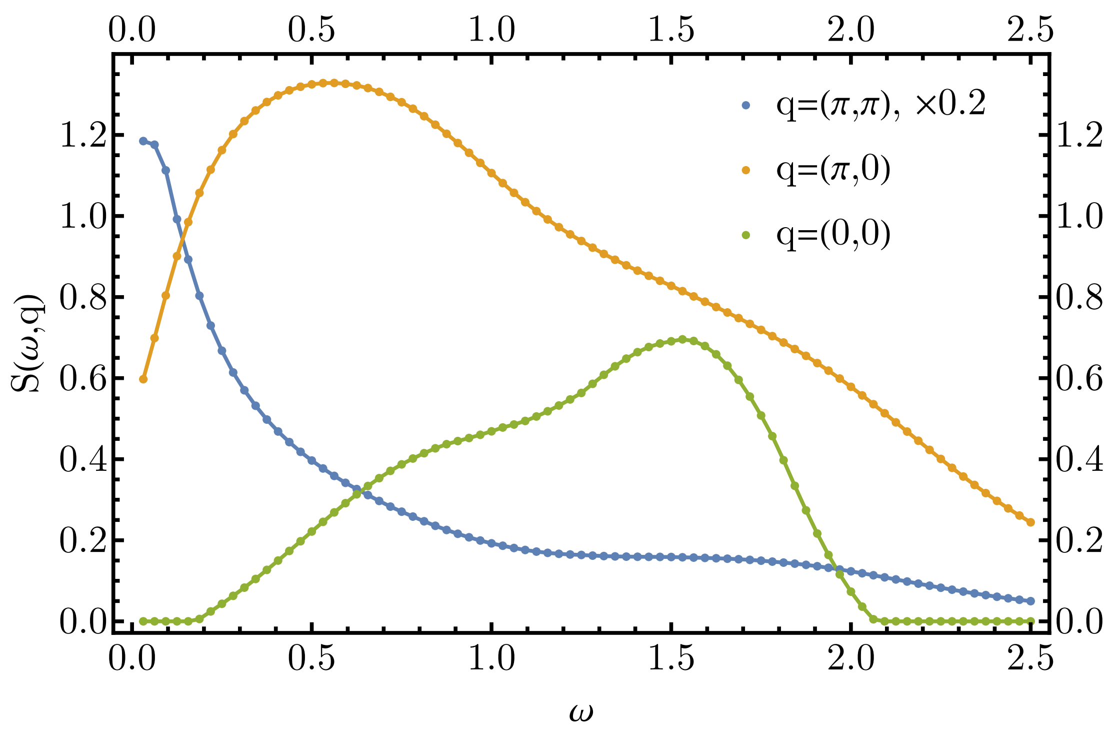
Dynamical Structure Factor: Keldysh pf-FRG

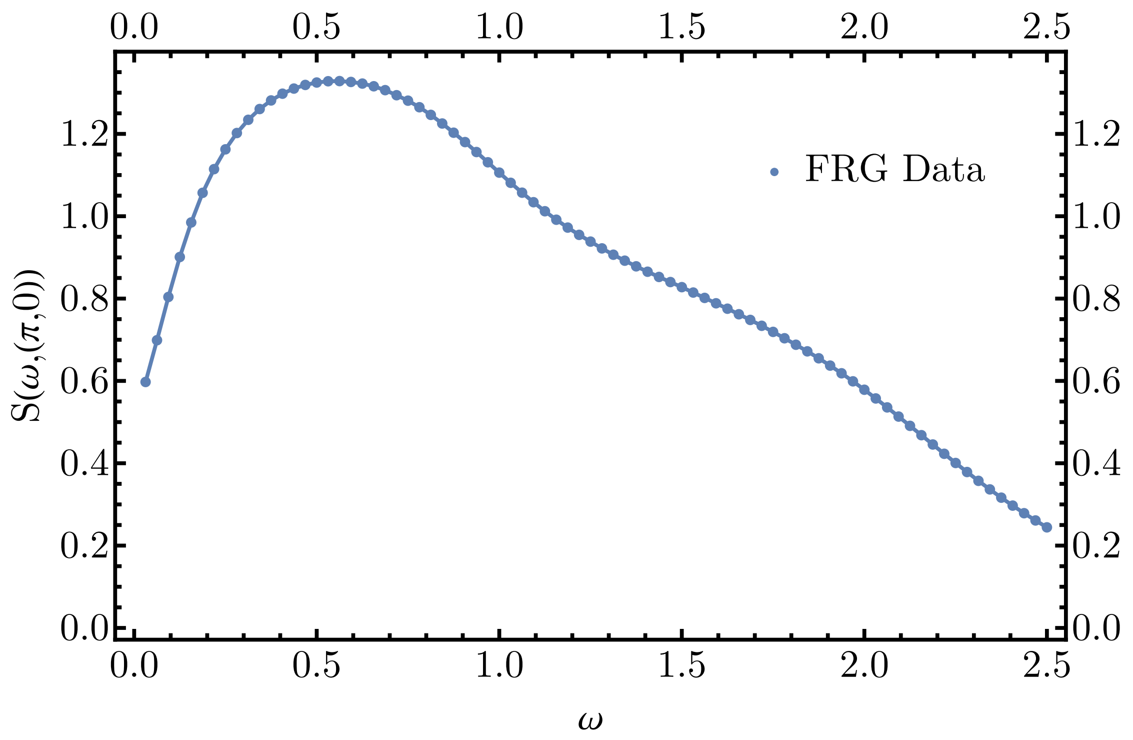


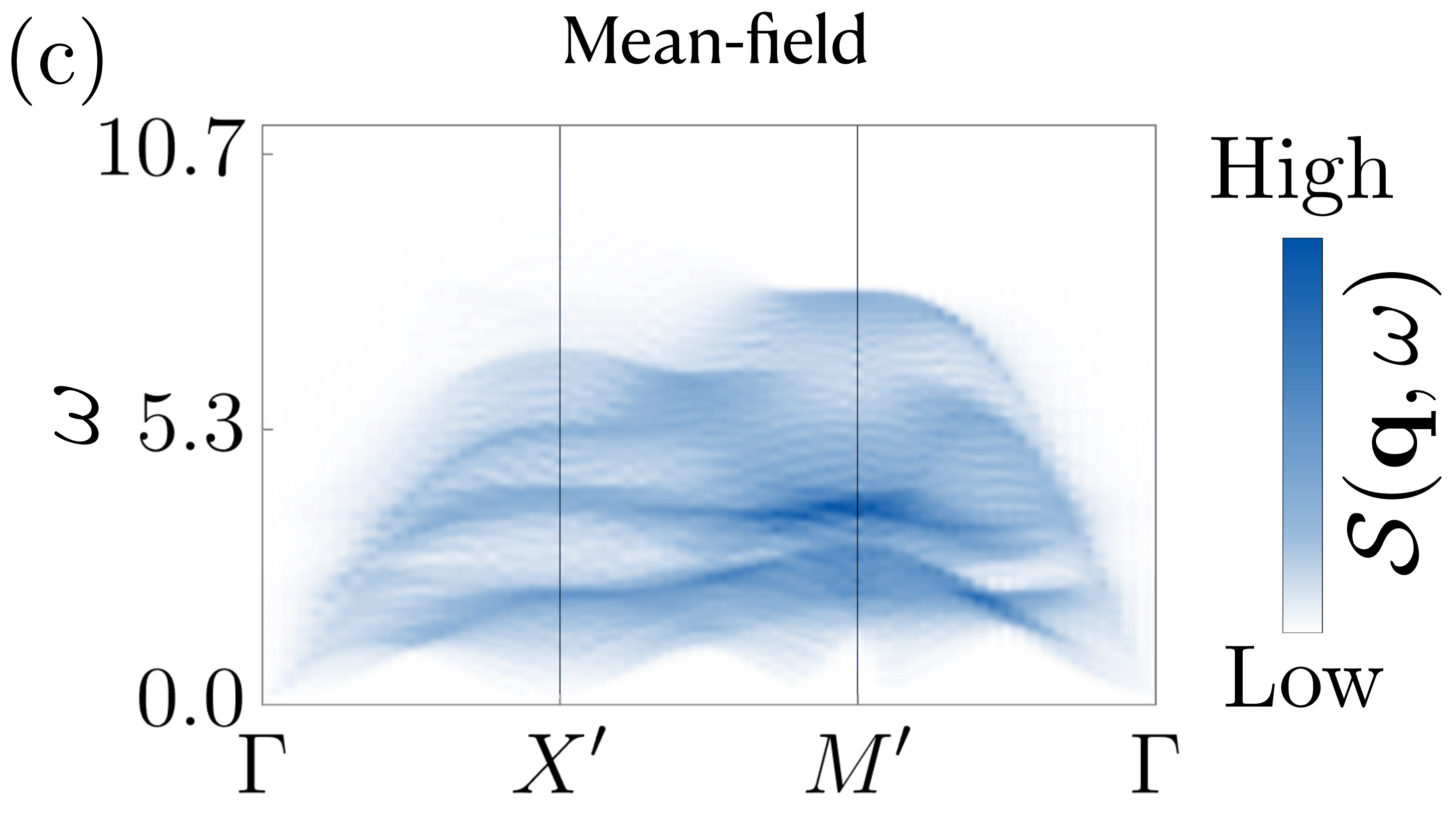
$$S^{\mathrm{Ret}}(q,\omega) = \frac{1}{\pi}(1-\mathrm{e}^{-\beta\omega})^{-1}\mathrm{Im}(\chi^{\mathrm{Ret}}(q,\omega))$$



Dynamical Structure Factor: Keldysh pf-FRG

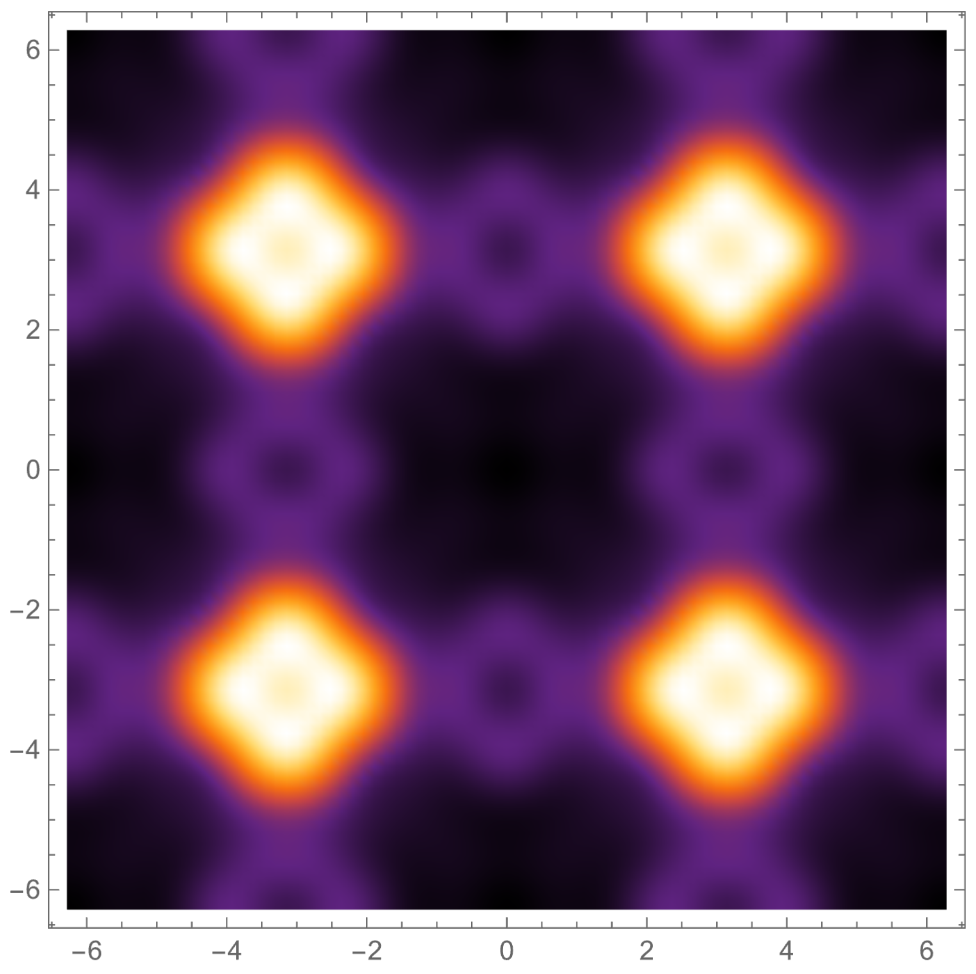




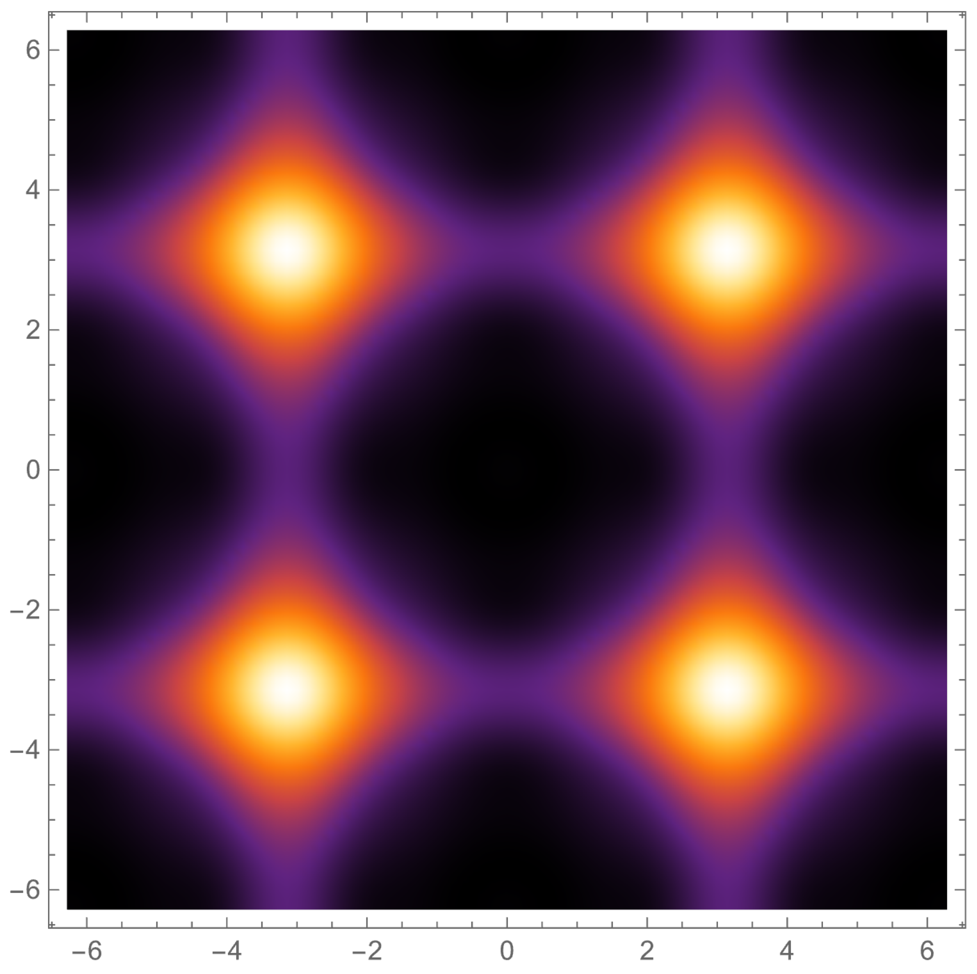


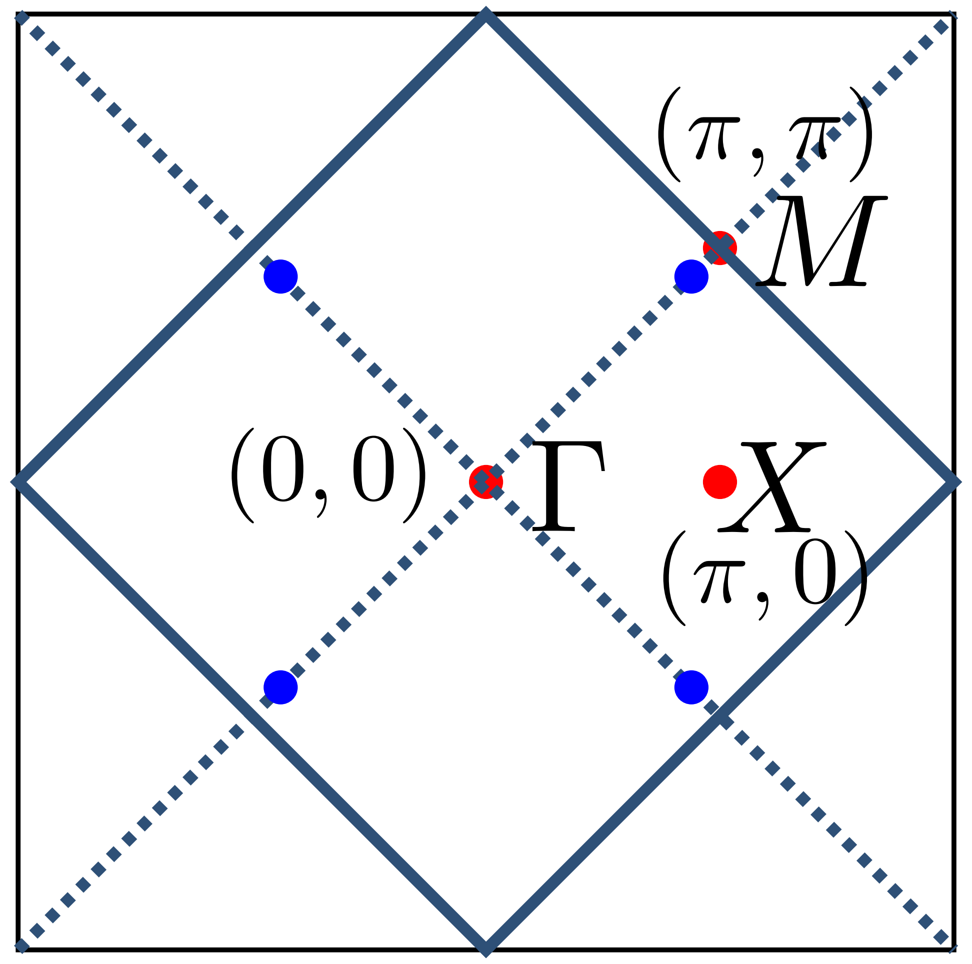
$$S^{\mathrm{Ret}}(q,\omega) = \frac{1}{\pi}(1 - \mathrm{e}^{-\beta\omega})^{-1}\mathrm{Im}(\chi^{\mathrm{Ret}}(q,\omega))$$

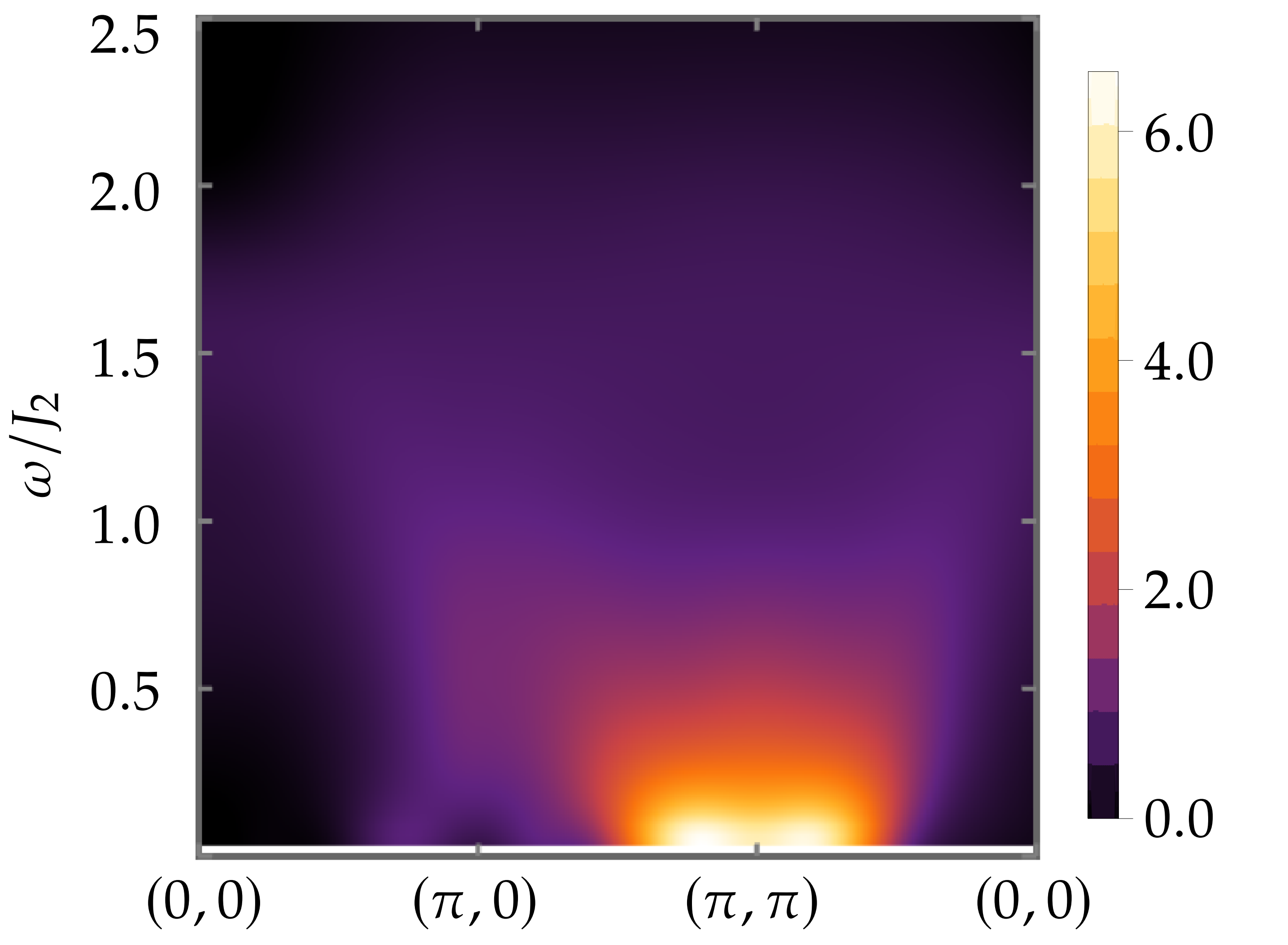
$\omega = 0$



ω integrated







Z_2 Gapless Spin Liquids

Questions of stability

$$S = \int dt d^2x [\bar{\psi}\partial_\mu \gamma^\mu \psi + (\bar{\psi} M \psi)^2]$$

$$[\psi^4] = 4 > 3$$ ✔

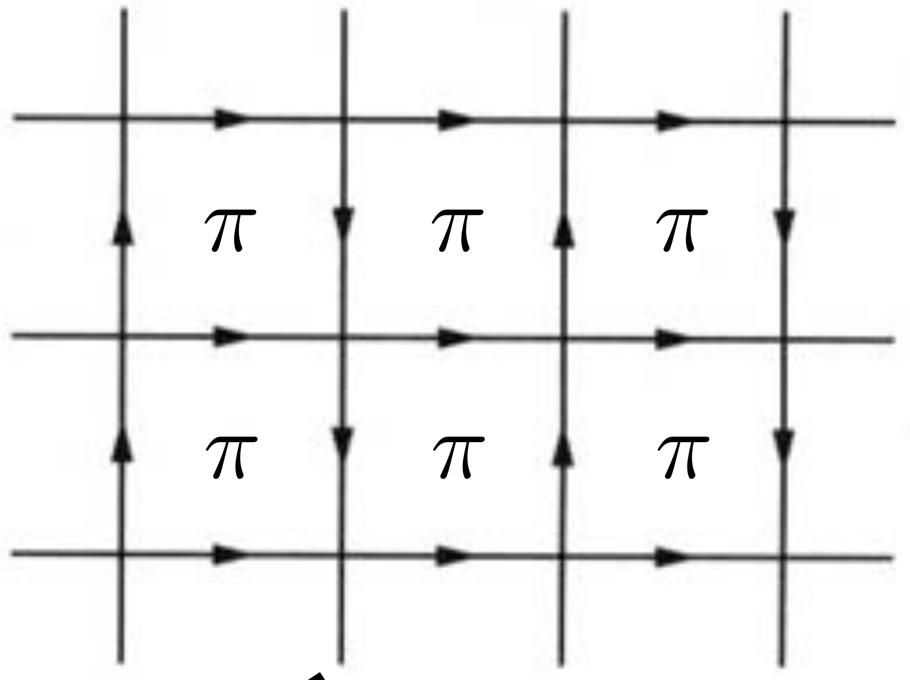
Short-range interactions between massless Dirac fermions are irrelevant in 2+1 dimensions

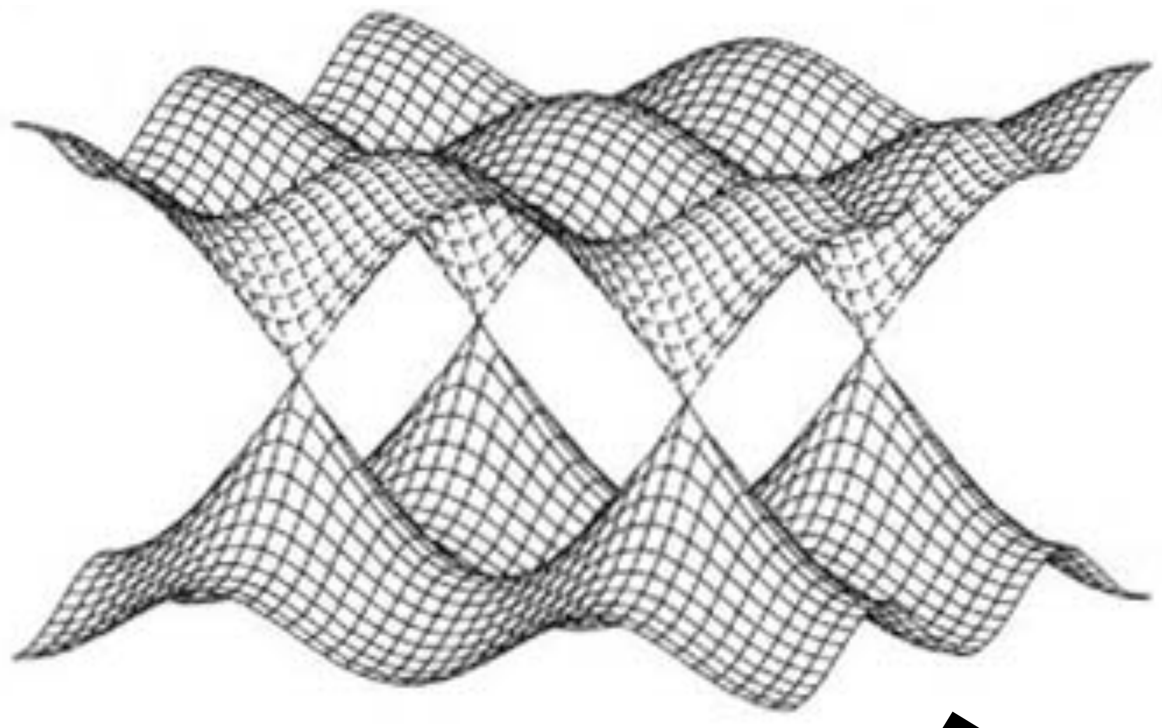
Z_2 Dirac spin liquids are stable in 2+1 dimensions!

No Topological order but quantum order

Continuum theory for Higgs transition from SU(2) to Z2

Possible Universality and community in QSLs on Shastry-Sutherland, Checkerboard and square lattices

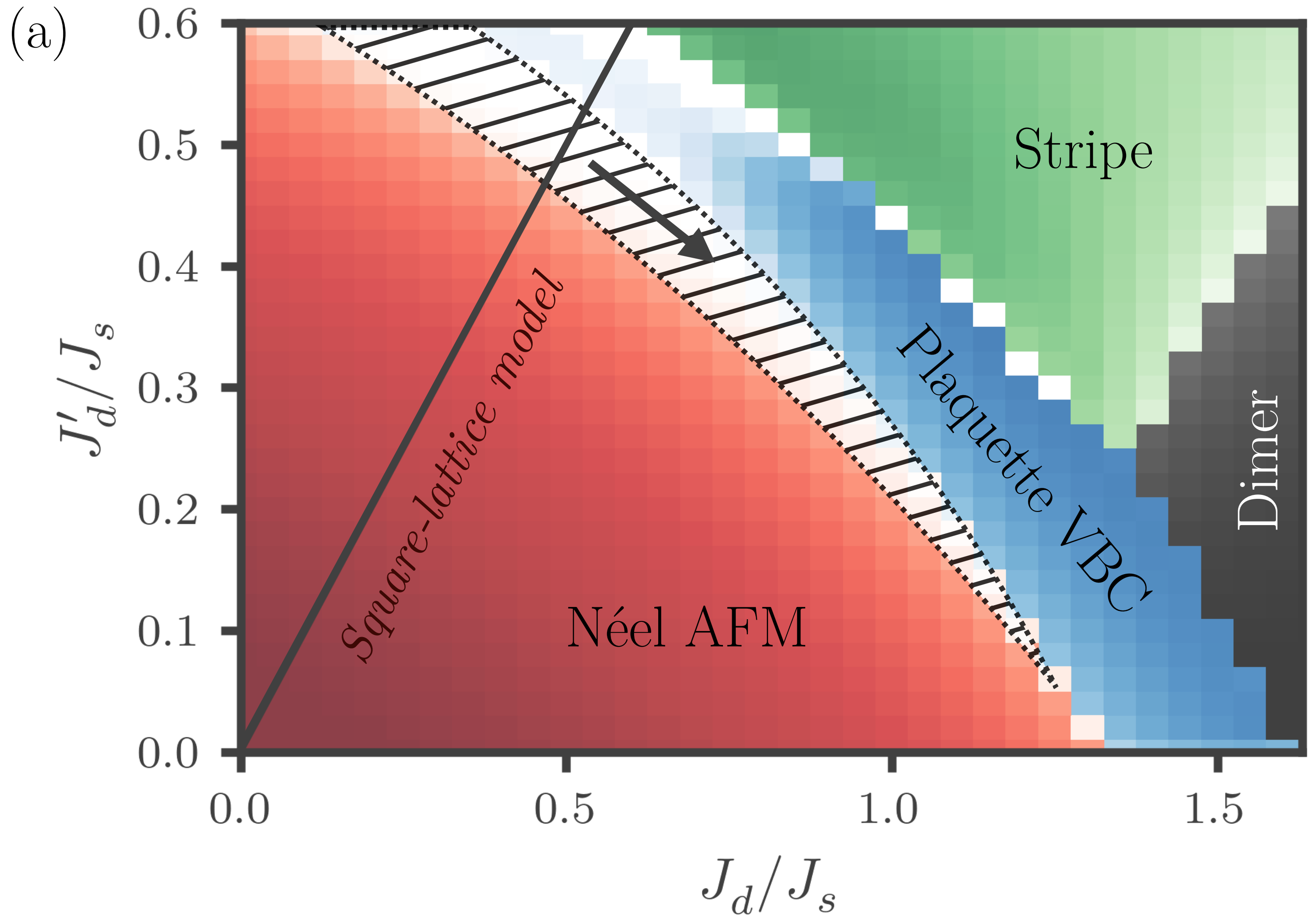




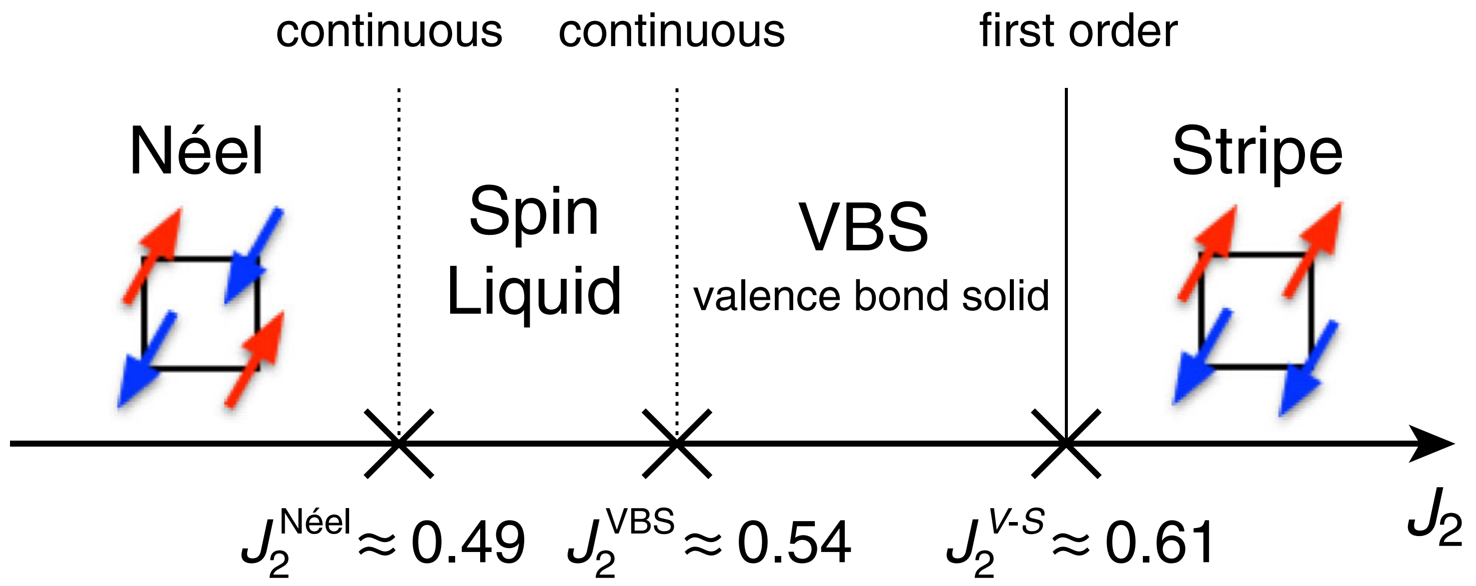
$N_f = 2$ $SU(2)$

Lorentz invariance + SO(5) symmetry

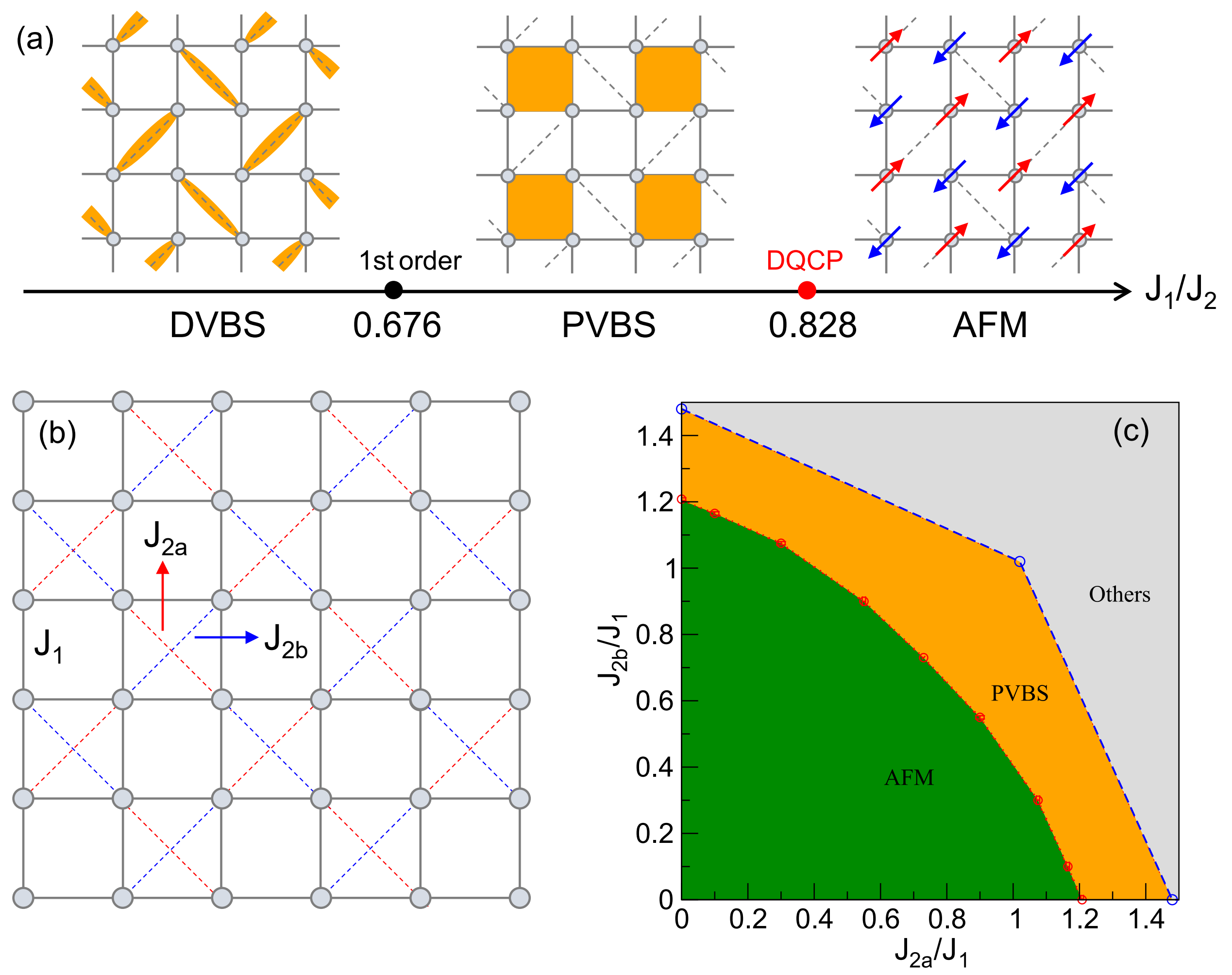
Shastry-Sutherland



Square



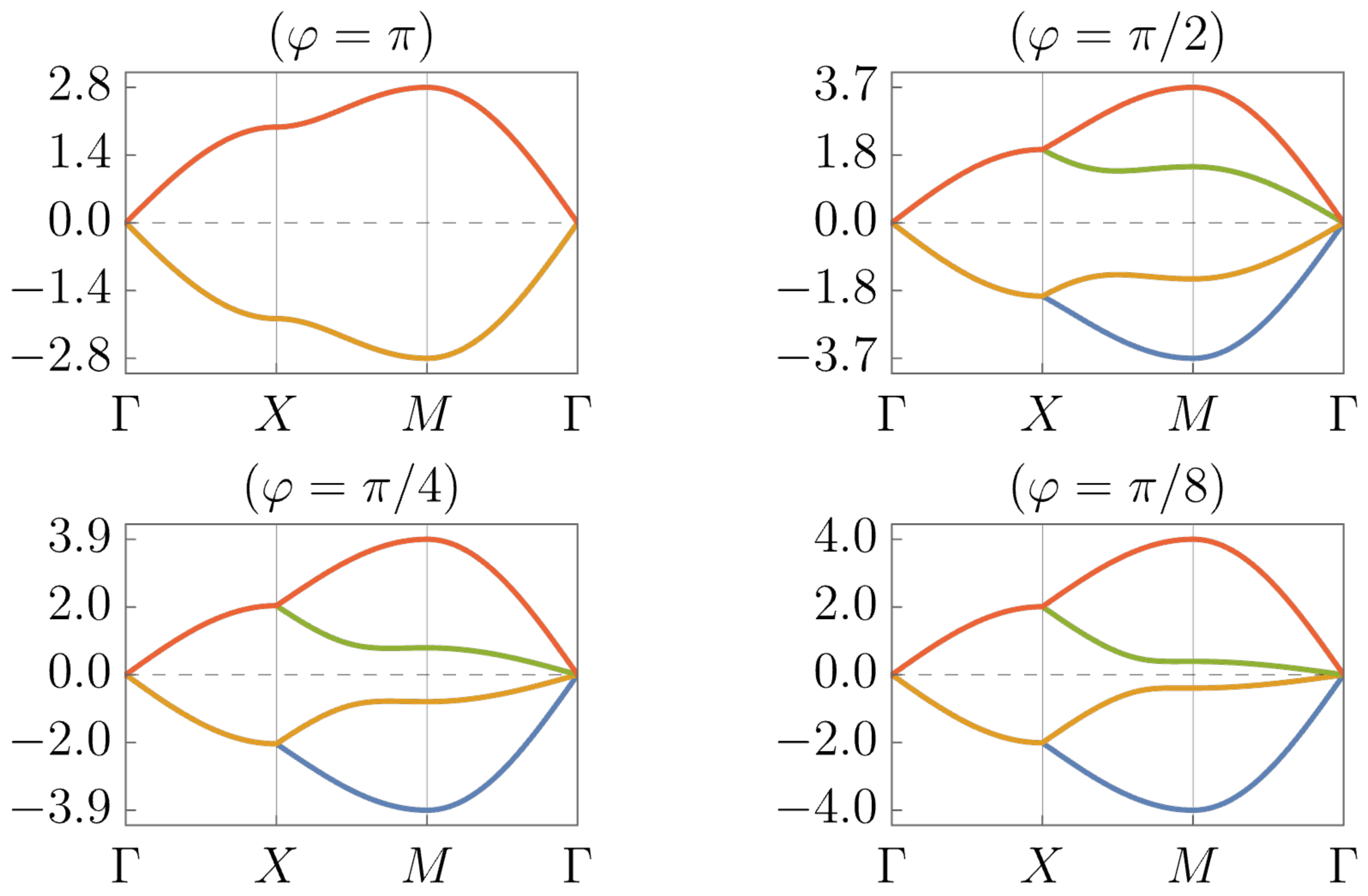
Checkerboard

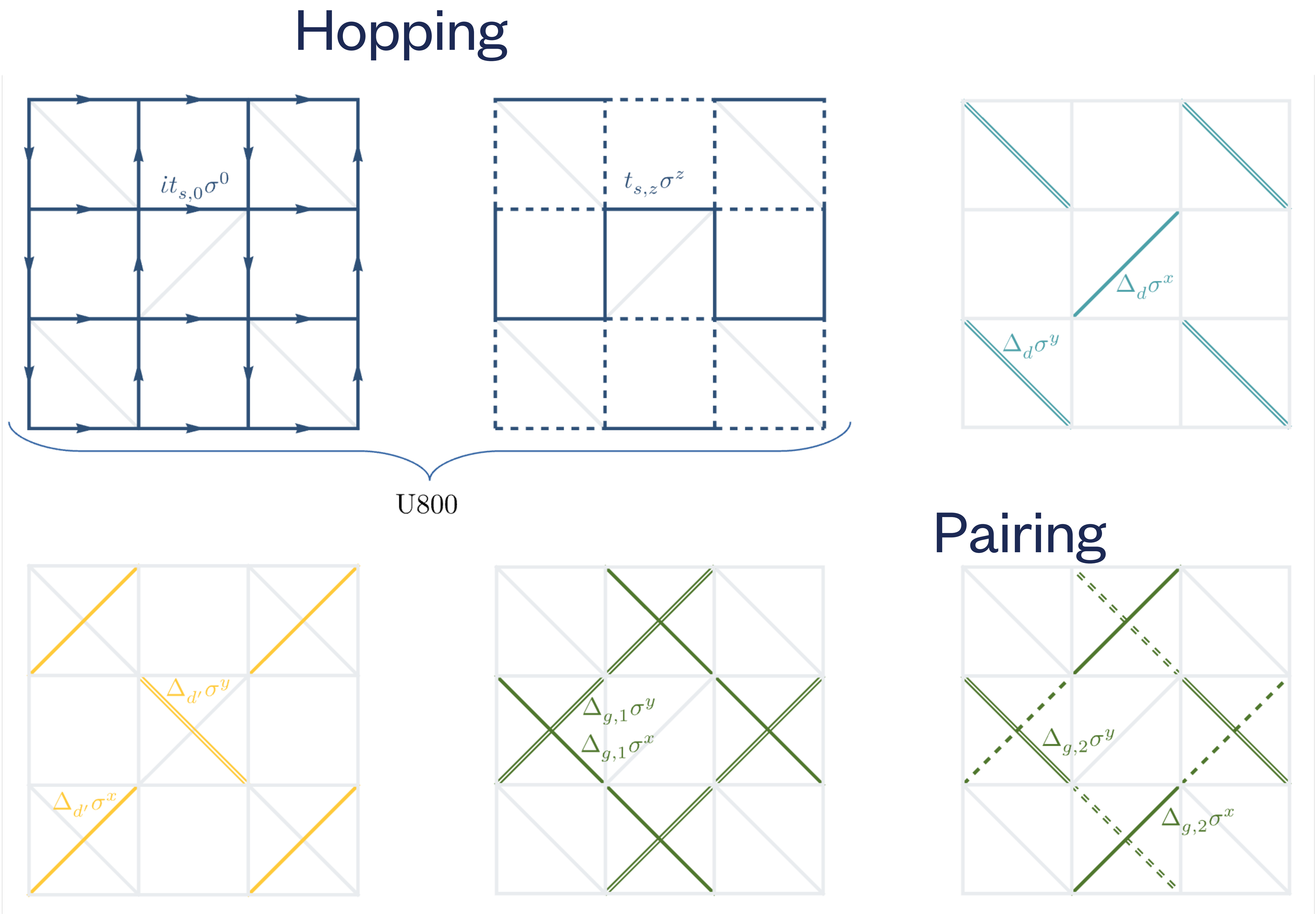


Projective Symmetry Group: Z3000

Cascade of transitions:

- $\Delta \neq 0$: Dirac $\mathbb{Z}_2$ spin liquid
- $t_{s,z} \neq 0$: U(1) staggered-flux state
- $t_{s,z} = 0$: SU(2) π-flux state





Higgs transition

$$\mathcal{F}_i = \begin{pmatrix} f_{i\uparrow} & -f_{i\downarrow} \\ f_{i\downarrow}^\dagger & f_{i\uparrow}^\dagger \end{pmatrix}$$

Expand Hamiltonian

$$H = \sum_{\langle ij \rangle} i\alpha_{ij} \operatorname{Tr}[\mathcal{F}_i^\dagger \mathcal{F}_j] + \beta_{ij}^a \operatorname{Tr}[\sigma^a \mathcal{F}_i^\dagger \mathcal{F}_j] + i\gamma_{ij} \operatorname{Tr}[\sigma^a \mathcal{F}_i^\dagger \sigma^a \mathcal{F}_j]$$

around 2 two-component low-energy degrees of freedom:

$$\mathcal{F} \sim \rho^x \chi_{v=1} + (-1)^{i_z} \chi_{v=2}$$

- Dirac Lagrangian with staggered-flux and pairing perturbations

$$\mathcal{L}_{\mathrm{MF}} = i \operatorname{Tr}[\bar{\mathcal{X}} \gamma^\mu \partial_\mu \mathcal{X}] \qquad \gamma^0 = \rho^y, \gamma^x = i\rho^z, \gamma^y = i\rho^x$$

$$\delta\mathcal{L} = -2i\delta\theta \operatorname{Tr}[\sigma^z \bar{\mathcal{X}} \mu^y (\gamma^y i\partial_x + \gamma^x i\partial_y) \mathcal{X}] + \cdots$$

- Gauge invariance necessitates similar terms with σ^a.
  => Introduce Higgs fields for perturbations:

Higgs transition

$$\delta\mathcal{L} = -2i\delta\theta\,\mathrm{Tr}[\sigma^z\bar{\mathcal{X}}\mu^y(\gamma^y i\partial_x + \gamma^x i\partial_y)\mathcal{X}] + \cdots$$

- Gauge invariance necessitates similar terms with σ^a.
  => Introduce Higgs fields for perturbations:

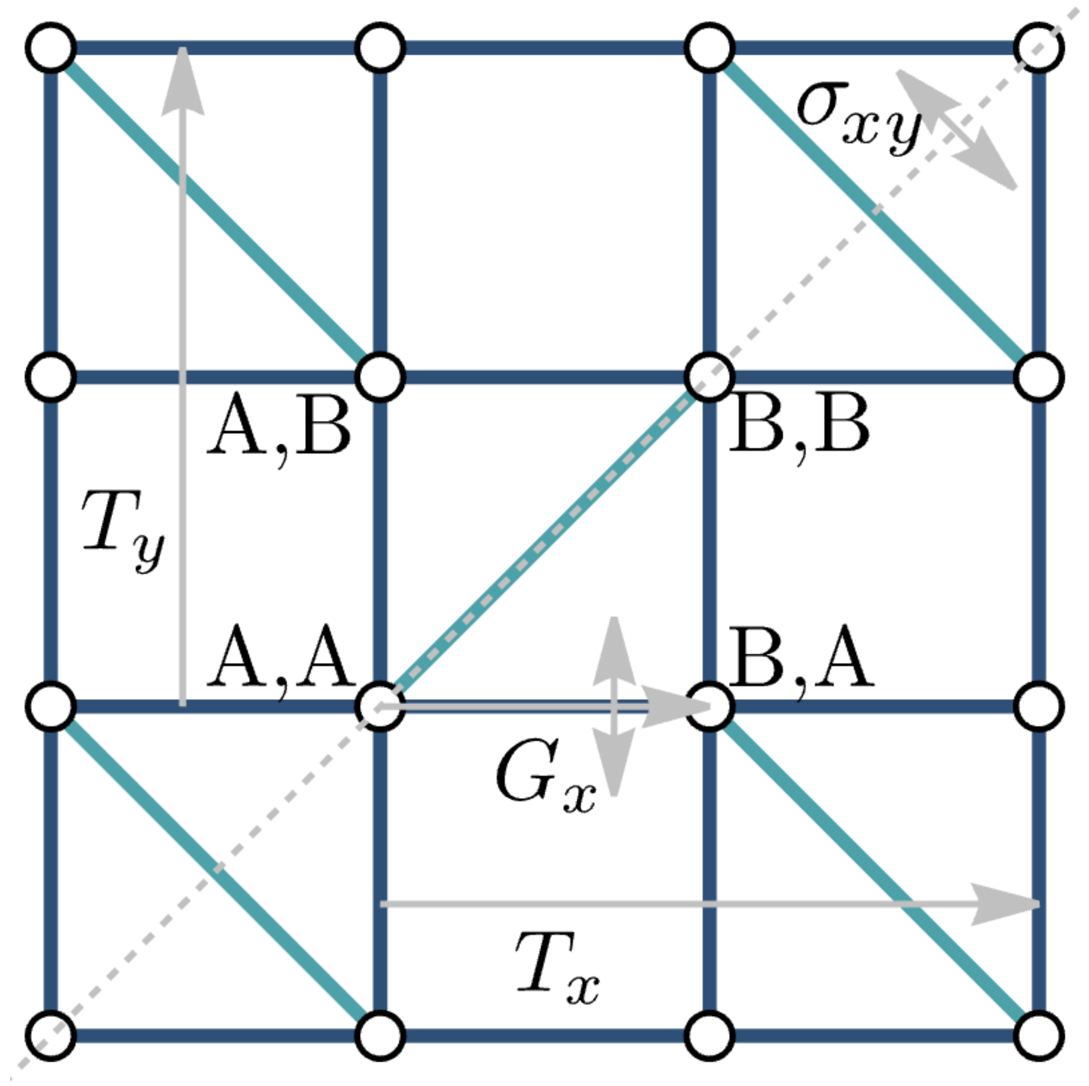
$$\begin{aligned}\mathcal{L} =& i\,\mathrm{Tr}[\bar{\mathcal{X}}\gamma^\mu\partial_\mu\mathcal{X}] + \Phi_1^a\,\mathrm{Tr}[\sigma^a\bar{\mathcal{X}}(\gamma^x\mu^z - \boxed{\delta(\gamma^0\mu^y i\partial_x - i\partial_y)})\mathcal{X}] \\ &+ \Phi_2^a\,\mathrm{Tr}[\sigma^a\bar{\mathcal{X}}(\gamma^y\mu^x + \boxed{\delta(\gamma^0\mu^y i\partial_y - i\partial_x)})\mathcal{X}] \\ &+ \Phi_3^a\,\mathrm{Tr}[\sigma^a\bar{\mathcal{X}}\mu^y(\gamma^y i\partial_x + \gamma^x i\partial_y)\mathcal{X}] + V(\Phi)\,.\end{aligned}$$

- Continuum Lagrangian identical to square lattice Z2Azz13 theory up to additional gradient terms $\propto\,\delta$ [Shackleton et al, PRB **104**, 045110, 2021].

Majorana-Higgs theory

Majorana-Higgs theory

$$\mathcal{L} = i\,\mathrm{Tr}[\bar{\mathcal{X}}\gamma^{\mu}\partial_{\mu}\mathcal{X}] + \Phi_1^a\,\mathrm{Tr}[\sigma^a\bar{\mathcal{X}}(\gamma^x\mu^z - \delta(\gamma^0\mu^y i\partial_x - i\partial_y))\mathcal{X}]$$
$$+ \Phi_2^a\,\mathrm{Tr}[\sigma^a\bar{\mathcal{X}}(\gamma^y\mu^x + \delta(\gamma^0\mu^y i\partial_y - i\partial_x))\mathcal{X}]$$
$$+ \Phi_3^a\,\mathrm{Tr}[\sigma^a\bar{\mathcal{X}}\mu^y(\gamma^y i\partial_x + \gamma^x i\partial_y)\mathcal{X}] + V(\Phi)\,.$$



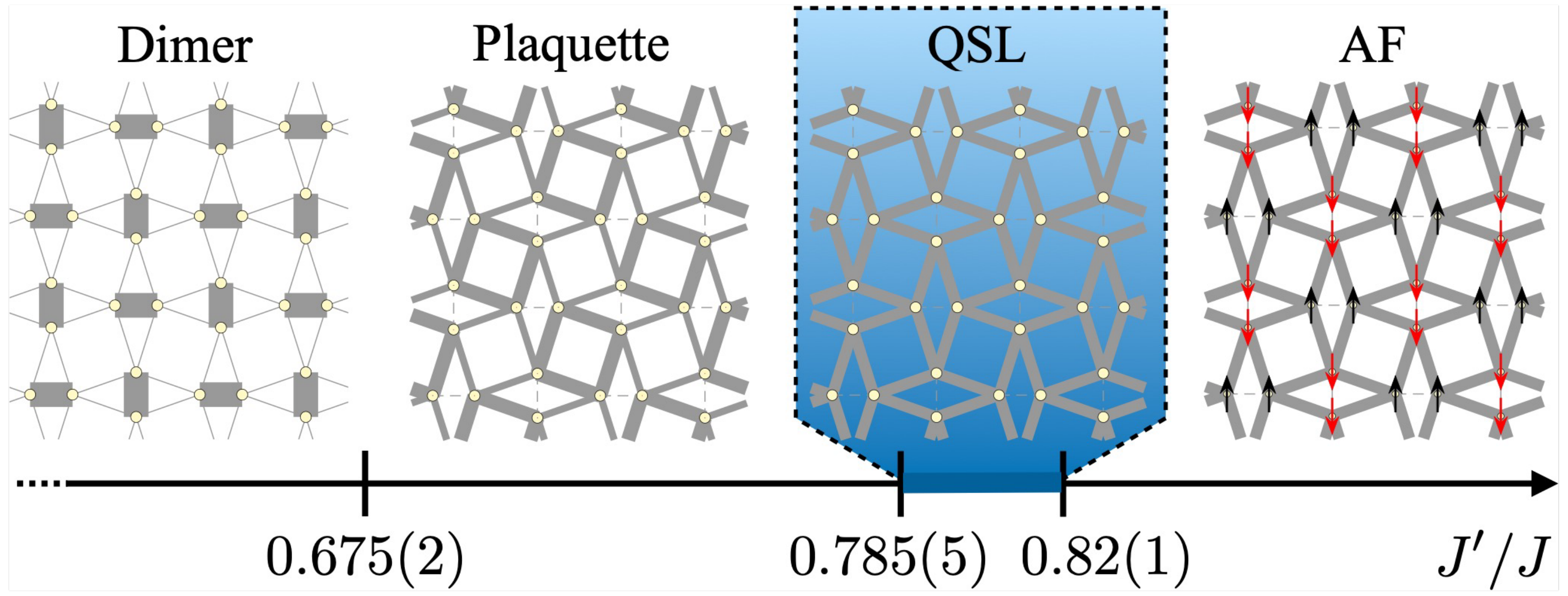
- Question: Are there any other symmetry-allowed couplings?

	T_x	T_y	P_x	P_y	$R_{\pi/2}$	G_x	$\mathcal{T}$	σ_{xy}
Φ_1^a	$-$	$+$	$-$	$-$	$-\Phi_2^a$	$+$	$-$	Φ_2^a
Φ_2^a	$+$	$-$	$-$	$-$	$-\Phi_1^a$	$-$	$-$	Φ_1^a
Φ_3^a	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$-$

=> Square lattice: No

=> Shastry-Sutherland: Yes (later: no effect on scaling dimension)

Majorana-Higgs theory

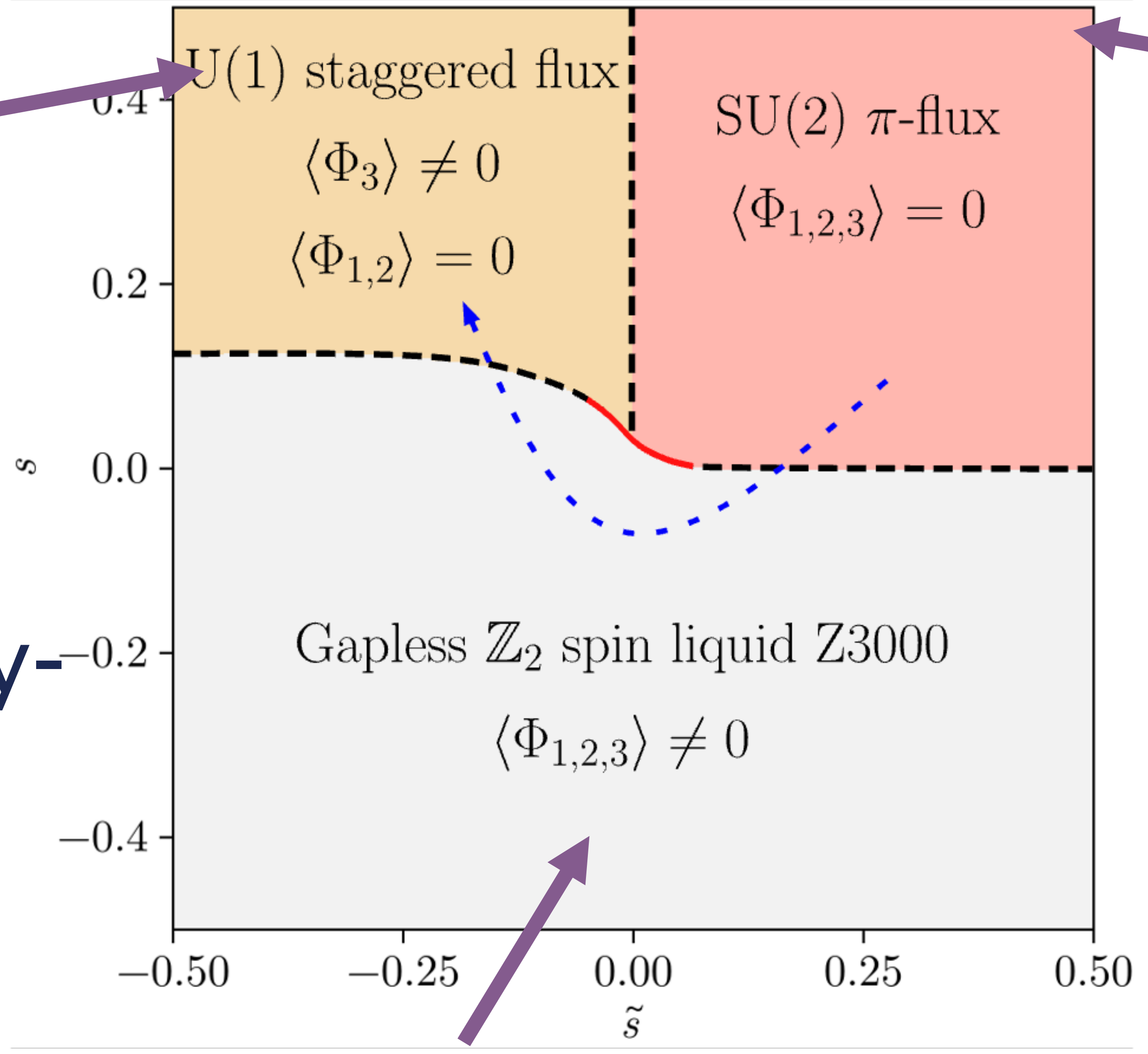


Integrating out high-energy spinon degrees of freedom

=> Higgs potential with all symmetry-allowed terms up to quartic order:

$$V(\Phi) = s(\Phi_1^a\Phi_1^a + \Phi_2^a\Phi_2^a) + \tilde{s}\Phi_3^a\Phi_3^a + w\epsilon_{abc}\Phi_1^a\Phi_2^b\Phi_3^c + u(\Phi_1^a\Phi_1^a + \Phi_2^a\Phi_2^a)^2 + \tilde{u}(\Phi_3^a\Phi_3^a)^2 + v_1(\Phi_1^a\Phi_2^a)^2$$
$$+ v_2(\Phi_1^a\Phi_1^a)(\Phi_2^b\Phi_2^b) + v_3[(\Phi_1^a\Phi_3^a)^2 + (\Phi_2^a\Phi_3^a)^2] + v_4(\Phi_1^a\Phi_1^a + \Phi_2^a\Phi_2^a)(\Phi_3^b\Phi_3^b)$$

Unstable due to monopole proliferation:
Trivial monopole on square lattice also trivial on the Shastry-Sutherland lattice
=> Néel or VBS order



Fate of the SU(2) theory unclear:
- Increasing numerical evidence for pseudocriticality and “walking” behavior
- Here: Weakly relevant Lorentz-breaking Yukawa coupling

=> Expect: Néel or VBS order

Stable in 2+1D ($\dim[\psi^4] = 4 > 3$)

=> Explanation of the phase diagram via deconfined quantum criticality

Majorana-Higgs theory

Are the Higgs potentials also structurally identical to higher order?

- Any symmetry-allowed term on the square lattice:
  Also allowed on the Shastry-Sutherland lattice due to reduced symmetry

$$G_x = P_y \circ T_x\,, \qquad \sigma_{xy} = P_x \circ R_{\pi/2}$$

- Any symmetry-allowed term on the Shastry-Sutherland lattice:
  - invariant under P_x, P_y due to $\mathcal{T}$
  - invariant under T_y due to G_x
  - invariant under $T_x, R_{\pi/2}$ due to $T_x = P_y \circ G_x$ and $R_{\pi/2} = P_x \circ \sigma_{xy}$

	T_x	T_y	P_x	P_y	$R_{\pi/2}$	G_x	$\mathcal{T}$	σ_{xy}
Φ_1^a	$-$	$+$	$-$	$-$	$-\Phi_2^a$	$+$	$-$	Φ_2^a
Φ_2^a	$+$	$-$	$-$	$-$	$-\Phi_1^a$	$-$	$-$	Φ_1^a
Φ_3^a	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$-$

Renormalization study of the critical point

Renormalization study

- Express the QFT with respect to Dirac fermions $\psi_{a,m_x,v} = i\sigma^y_{ab}[\mathcal{X}_{m_x,v}]_{1,b}$ and 2 scalar Higgs fields $\Phi_{1/2}$

- Consider the large N_f, N_b limit by letting $v = 1, \dots, N_f$ and $\alpha = 1, \dots, \frac{N_b}{2}$:

$$\mathcal{L}_\psi = i\bar{\psi}_v \gamma^\mu (\partial_\mu - iA^a_\mu \sigma^a)\psi_v ,$$

$$\mathcal{L}_\Phi = \frac{1}{2g}[(\delta_{ac}\partial_\mu - 2\epsilon_{abc}A^b_\mu)\Phi^c_{s\alpha}]^2 ,$$

$$\mathcal{L}_{\Phi\psi} = y \sum_{s,\alpha} \Phi^a_{s\alpha} \bar{\psi} X^s \sigma^a \psi , \qquad X^s = (\mu^z \gamma^x, \mu^x \gamma^y)$$

- Yukawa coupling will be treated as a weakly relevant perturbation.

- Additional terms to X^s exist on the Shastry-Sutherland lattice.

SO(5) symmetry

- $\chi_{a,v}$ has an emergent enlarged symmetry given by $SU(2)\times SU(2)$ (spin+valley rotations)

- Constraint: Reality condition

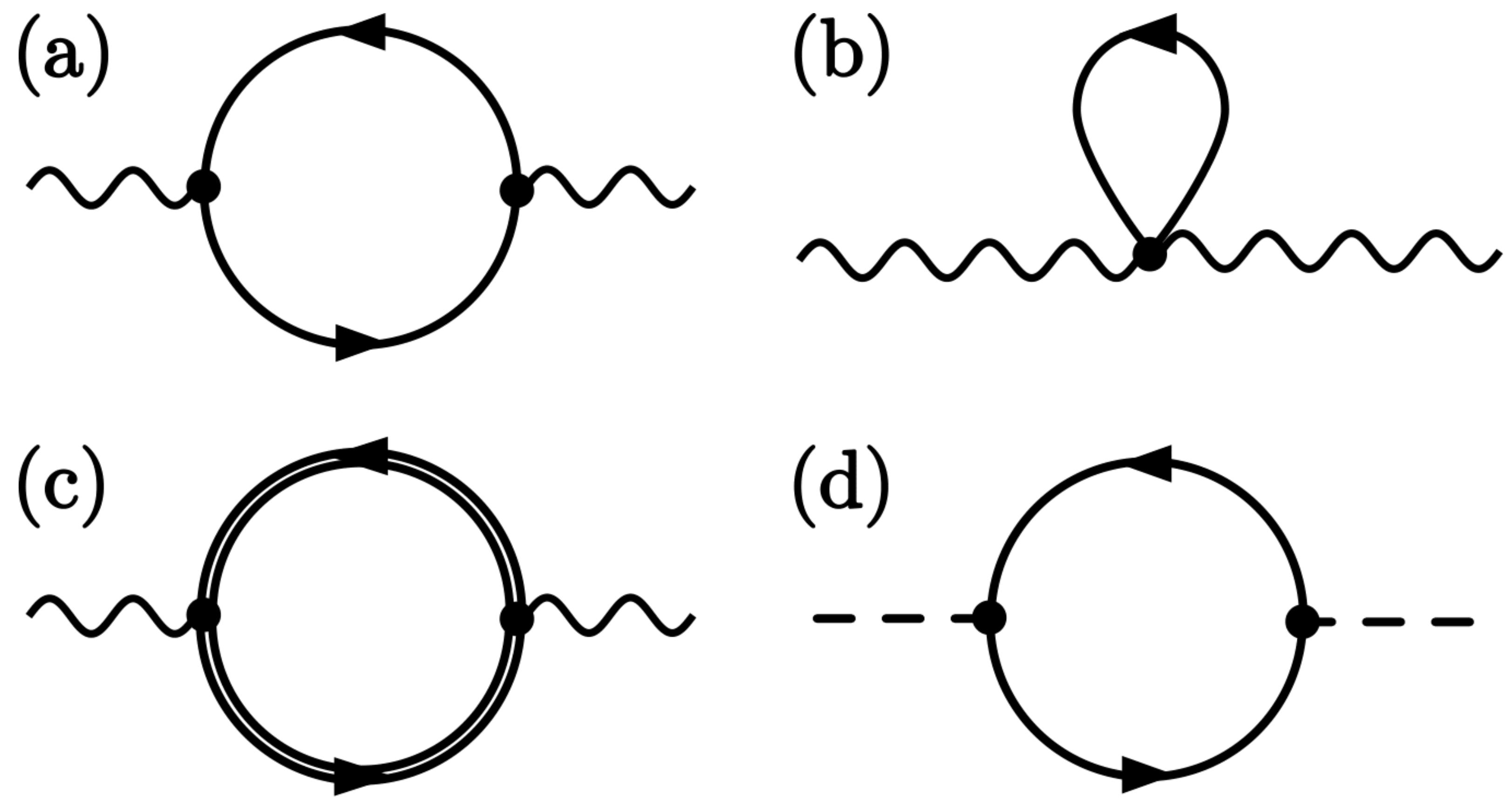
$$\mathcal{F}_i = \begin{pmatrix} f_{i\uparrow} & -f_{i\downarrow} \\ f_{i\downarrow}^\dagger & f_{i\uparrow}^\dagger \end{pmatrix} \qquad \mathcal{F}_i^\dagger = \sigma^y \mathcal{F}_i^T \sigma^y$$

=> Reduced to

$$\mathrm{Sp}(4)/\mathbb{Z}_2 \cong \mathrm{SO}(5)$$

- Néel and VBS order parameters $\bar{\psi}\Gamma^j\psi$ transform as an SO(5)-vector:

$$\Gamma^j = \{\mu^x, \mu^z, \mu^y \sigma^a\}$$



Renormalization study

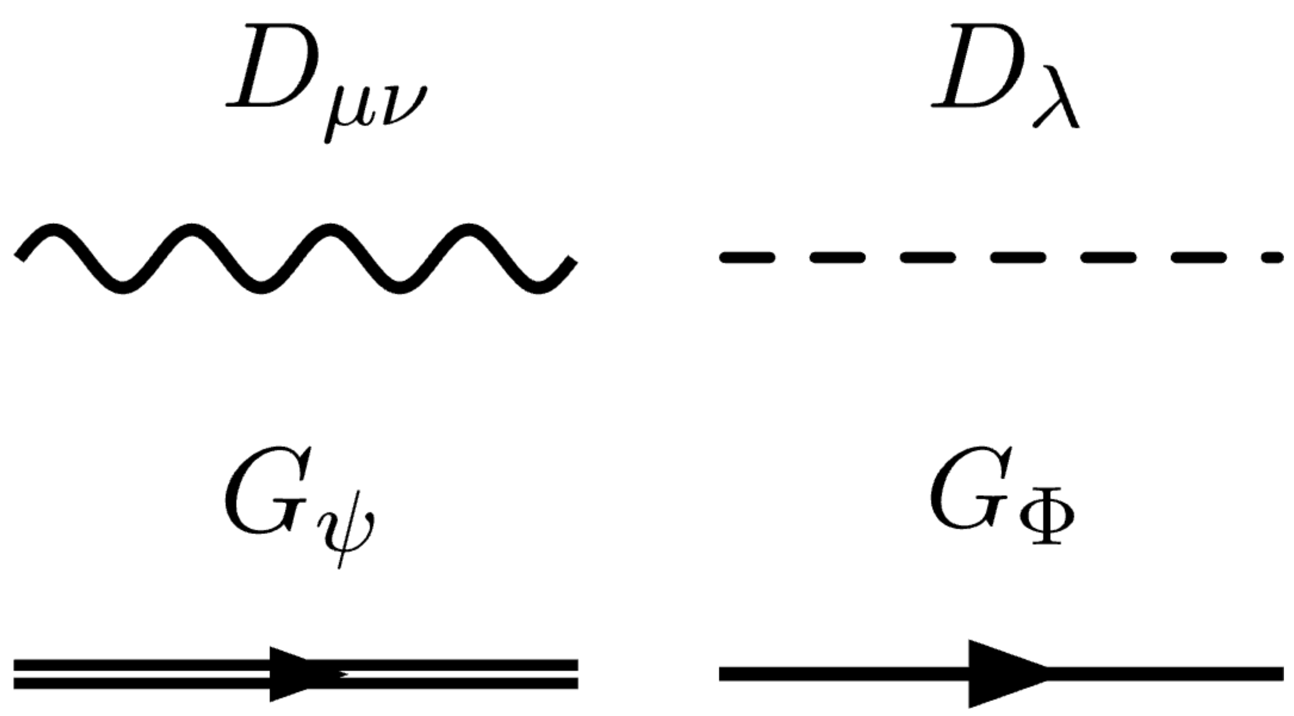
- Perform a saddle-point expansion by introducing a Lagrange multiplier field

$$\mathcal{L}_\Phi = \frac{1}{2g}\left[((\delta_{ac}\partial_\mu - 2\epsilon_{abc}A^b_\mu)\Phi^c_{s\alpha})^2 + i\lambda\left((\Phi^a_{s\alpha})^2 - \frac{3N_b}{g}\right)\right]$$

and expanding around the saddle point:

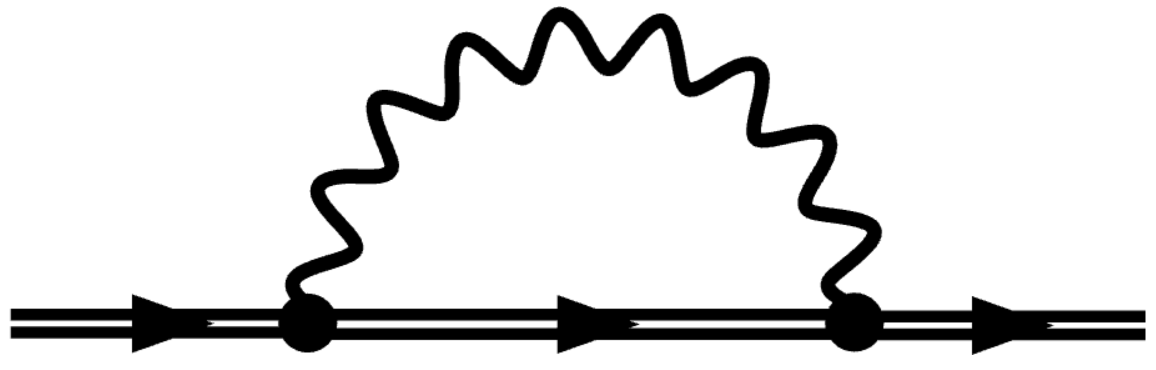
$$\mathcal{F} = \frac{1}{2}\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\left(\Pi_\lambda \lambda^2 + \Pi_A\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)A^a_\mu A^a_\nu\right)$$

- Propagators are obtained in the gauge $k_\mu A_\mu = 1 - \zeta$

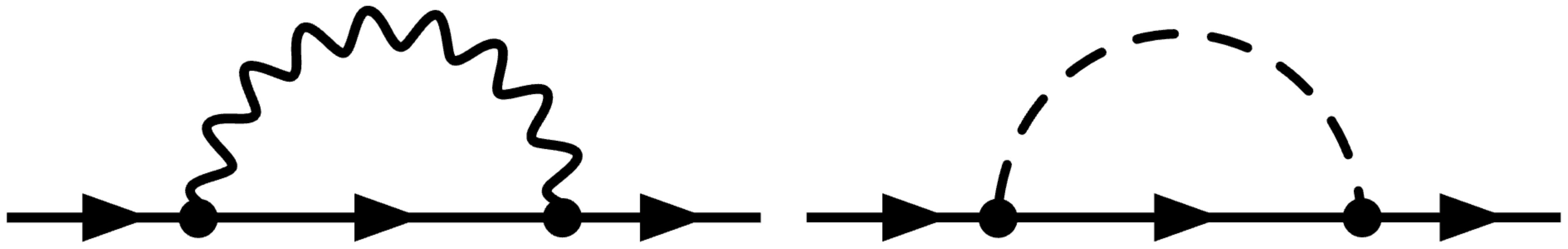


Renormalization study

- Bosons and fermions are gauge-dependent
  => Gauge-dependent fermion & boson self-energies:

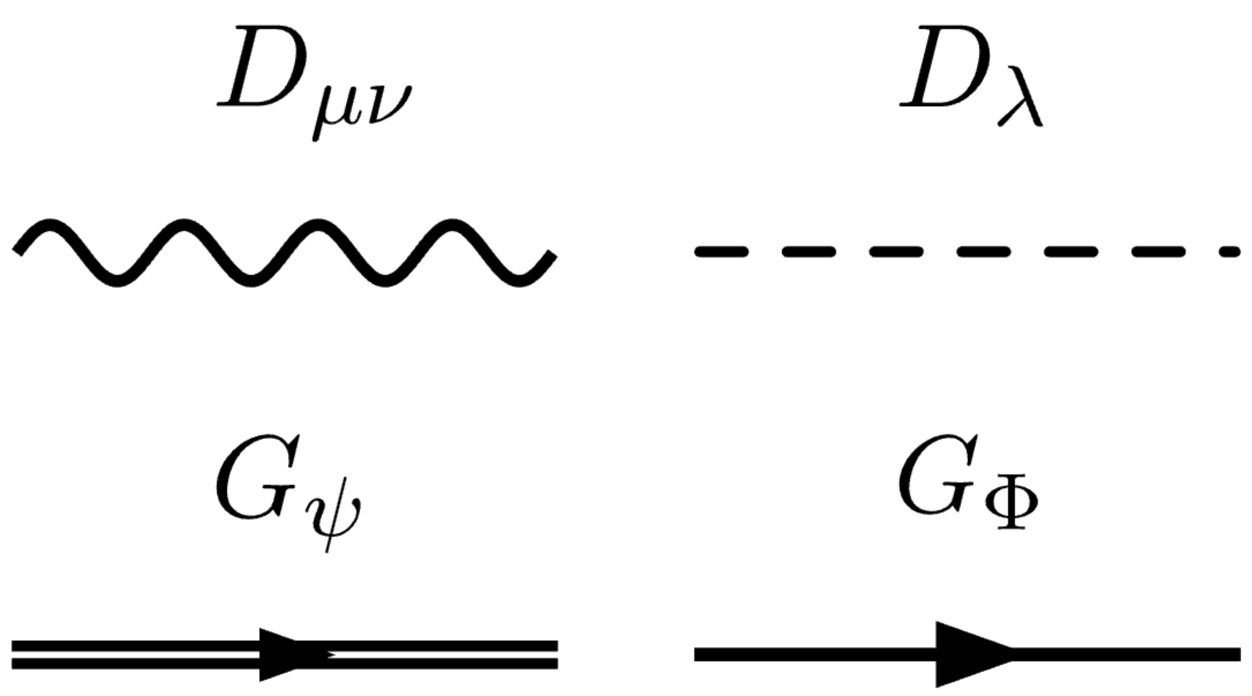


$$\eta_\psi = \frac{8(1-3\zeta)}{(N_f + 8N_b)\pi^2}$$



$$\eta_\Phi = -\frac{32}{(N_f + 8N_b)\pi^2}\left(\frac{10}{3} + 2\zeta\right) + \frac{4}{9N_b\pi^2}$$

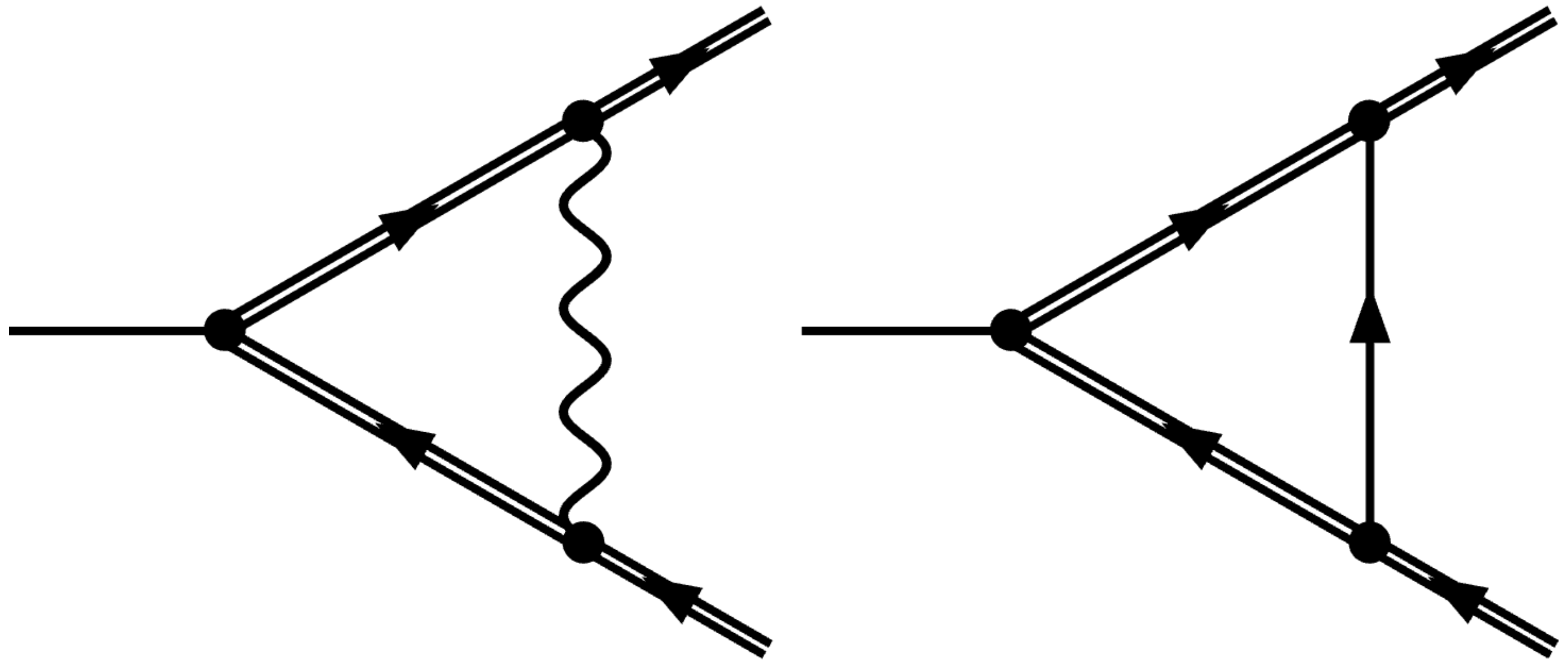
Renormalization study



Renormalization of the Néel and VBS order parameters:

$$V^i = (\bar{\psi}\mu^x\psi, \bar{\psi}\mu^z\psi) \qquad N^z = \bar{\psi}\mu^y\psi$$

$$\dim[\bar{\psi}\mu^i\psi] = 2\dim[\psi] + \eta_{\text{vrtx}} = 2 - \frac{64}{(N_f + 8N_b)\pi^2}$$



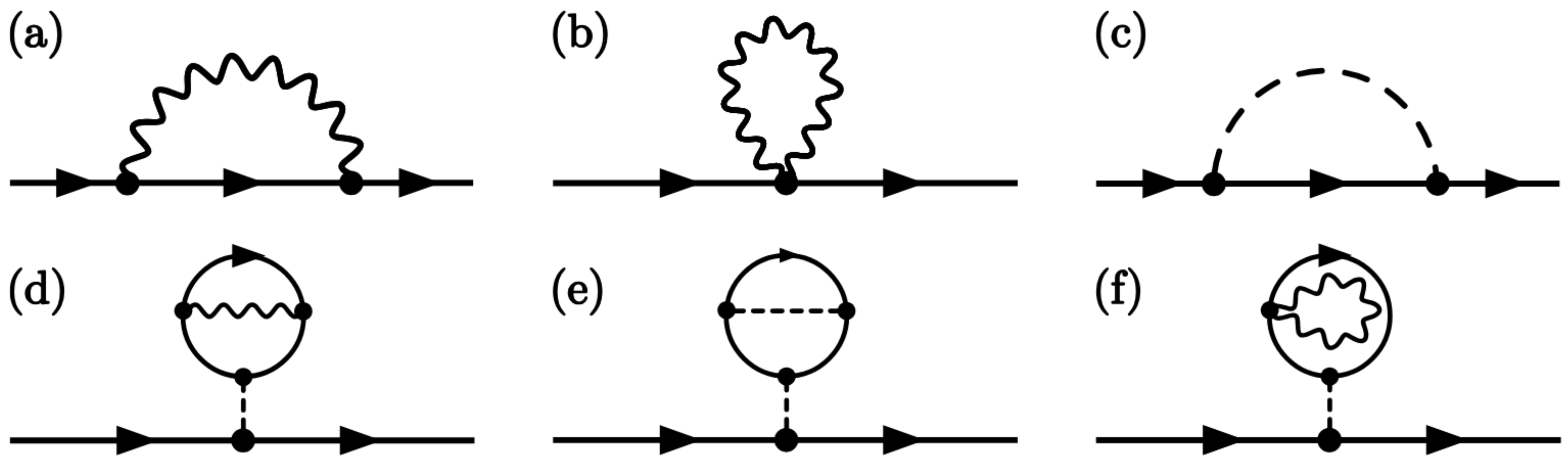
leads to strongly enhanced Néel and VBS fluctuations

$$\chi(r) \sim |r|^{-4+\frac{128}{(N_f+8N_b)\pi^2}} \approx |r|^{-3.279}$$

$\Rightarrow$ Large $\eta_{SO(5)} \approx 0.36$ hallmark of deconfined criticality

$\Rightarrow$ Consistent with $\eta \approx 0.3 - 0.35$ in J-Q models [Block et al., PRL 111(13), 137202 (2013)]

Renormalization study



- Correlation length exponent

$$\xi \propto (g - g_c)^{-\nu}$$

obtained from the mass scaling parameter

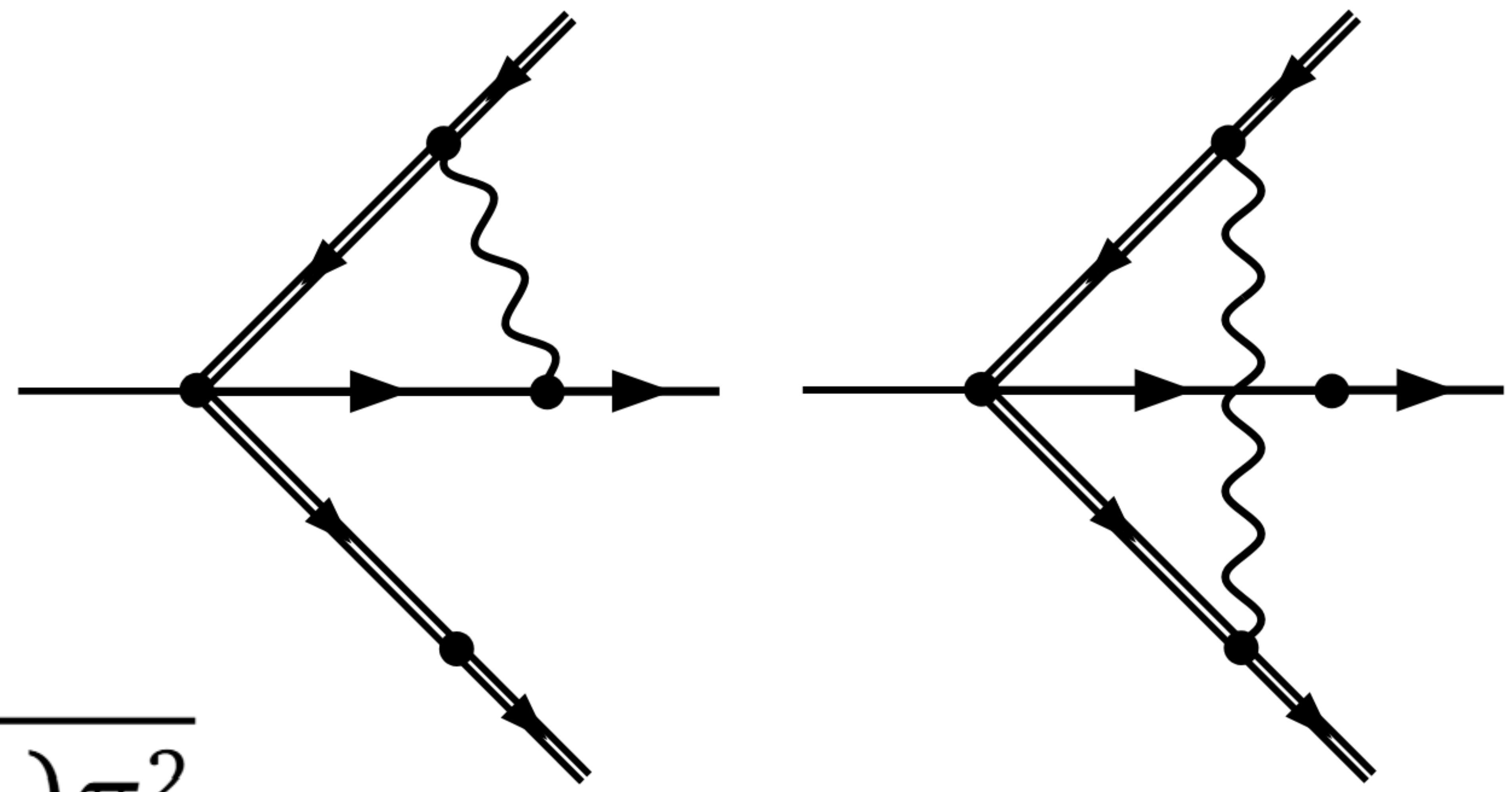
$$G_\Phi(k = 0) \propto (g - g_c)^{\gamma_\Phi}$$

via $\gamma_\Phi = \nu(2 - \eta_\Phi)$:

$$\nu \approx 1 - \underbrace{\frac{16}{9N_b\pi^2}} + \underbrace{\frac{32}{\pi^2}\frac{7N_f - 8N_b}{(N_f + 8N_b)^2} - \frac{160}{3(N_f + 8N_b)\pi^2}} \approx 0.590$$

- Both Lagrange multiplier and gauge fluctuations contribute with same sign

Renormalization study



- Vertex corrections to the Yukawa coupling:

$$\dim[y] = \frac{1}{2} - \frac{2}{9N_b\pi^2} + \frac{64}{3(N_f + 8N_b)\pi^2}$$

- Additional symmetry-allowed Yukawa couplings do not alter the scaling dimension!

$\Rightarrow$ The coupling constant is weakly relevant: $\dim[y] \approx 0.609$

$\Rightarrow$ Fluctuations of the Lagrange multiplier act to decrease the relevance, whilst gauge fluctuations increase it.

$\Rightarrow$ Ultimate fate of the SO(5) critical fixed point unknown:
Large N-expansion not reliable at estimating the relative strength of opposite trends, but only the operator content

Summary

- J_1-J_2 square lattice and Shastry-Sutherland model share a common $\mathbb{Z}_2$ Dirac spin liquid descending from the $SU(2)$ π-flux state

- They share a common QFT and Higgs potential, governing the magnetic transitions through deconfined quantum criticality: Néel → $\mathbb{Z}_2$ → VBS

- Proposed & studied a conformal field theory with SO(5) symmetry:
  - Controls the physics for BOTH square and Shastry-Sutherland lattice near the boundary between the Néel and gapless spin liquid states
  - Extends earlier studies of $N_f = 2$ QED_3 by $N_b = 2$ adjoint Higgs fields stabilizing the theory
  - Weakly relevant Yukawa coupling might destabilize it towards pseudo-critical behavior

Outlook

- Probing of critical exponents in Monte Carlo
- Study of symmetry-allowed perturbations as Dzyaloshinskii-Moriya interactions or explicit symmetry breaking present in material realizations of the Shastry-Sutherland lattice
- Finite-N sensitive methods such as Conformal Bootstrap or Functional Renormalization Group to study runaway flow
- Generalization to frustrated lattices derived from the prototypical square lattice
  => Deconfined criticality as a unifying paradigm of quantum phase transitions near a spin liquid

arXiv: 2501.00096 (2025)

Evidence for a $\mathbb{Z}_2$ Dirac spin liquid in the generalized Shastry-Sutherland model

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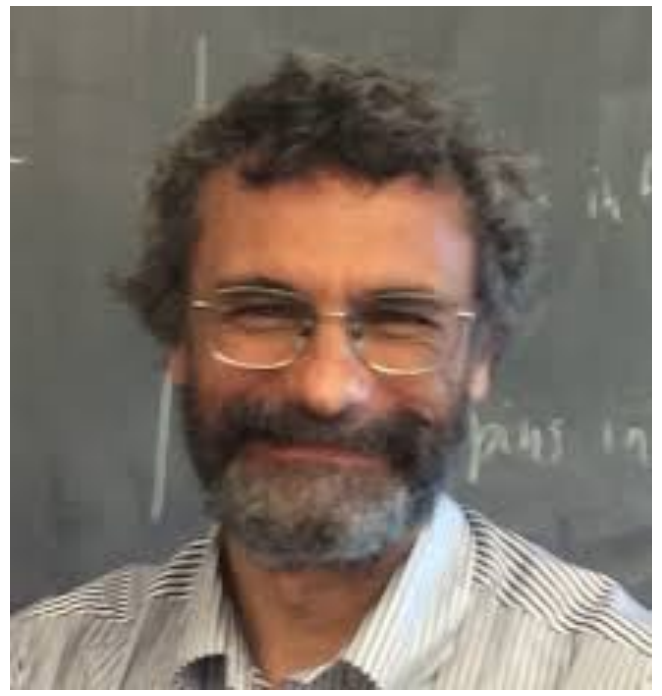

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Unifying Dirac Spin Liquids on Square and Shastry–Sutherland Lattices via Fermionic Deconfined Criticality

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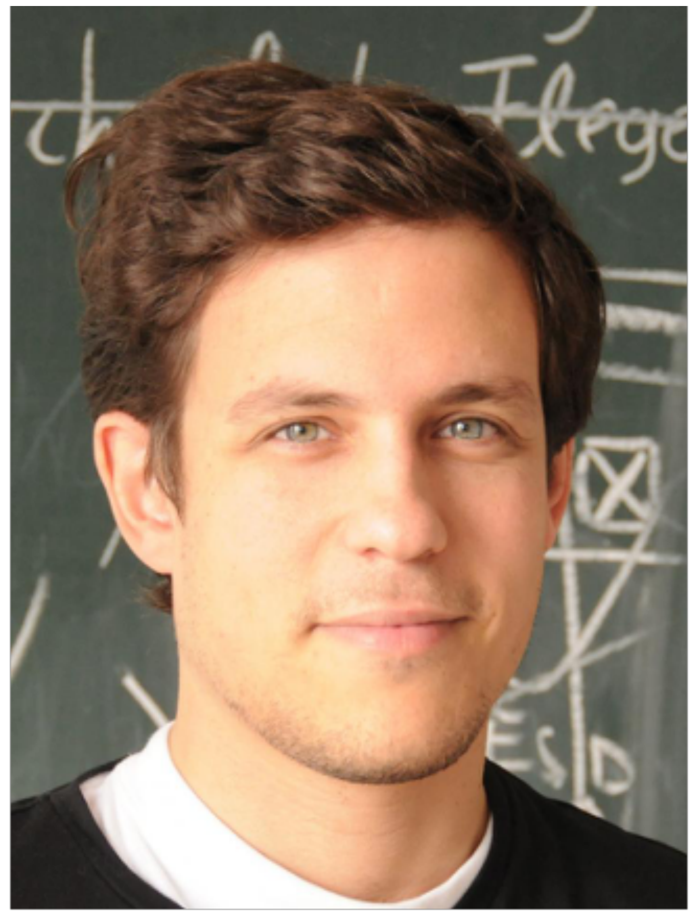

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