Hawking radiation due to a quench on the surface of a topological superconductor

Shriya Hirve
(Louisiana State University)

Co-authors:


Arthur Niwazuki
(Rice)


Andrew Allocca
(CCNY)


Matthew Foster
(Rice)


Justin Wilson
(LSU)

LSU
LOUISIANA STATE UNIVERSITY

Hawking radiation due to a quench on the surface of a topological superconductor

Shriya Hirve
(Louisiana State University)

Co-authors:


Arthur Niwazuki
(Rice)


Andrew Allocca
(CCNY)


Matthew Foster
(Rice)


Justin Wilson
(LSU)

LOUISIANA STATE UNIVERSITY

Hawking radiation due to a quench on the surface of a topological superconductor

Shriya Hirve
(Louisiana State University)

Co-authors:


Arthur Niwazuki
(Rice)


Andrew Allocca
(CCNY)


Matthew Foster
(Rice)


Justin Wilson
(LSU)

Hawking radiation due to a quench on the surface of a topological superconductor

Shriya Hirve
(Louisiana State University)

Co-authors:

Arthur Niwazuki
(Rice)

Andrew Allocca
(CCNY)

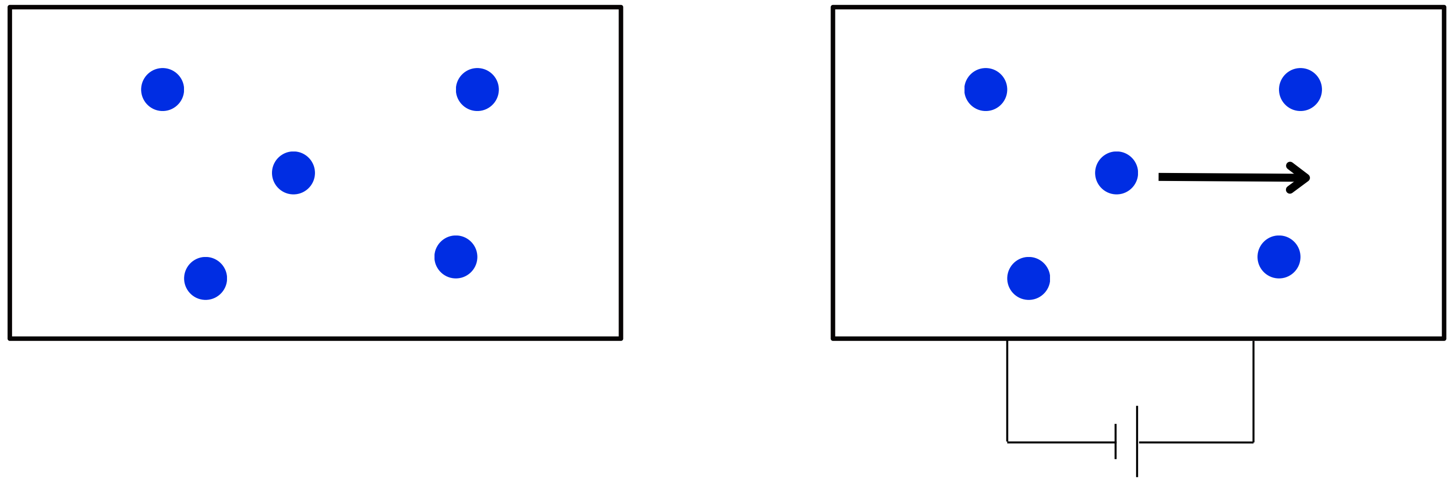
Matthew Foster
(Rice)

Justin Wilson
(LSU)

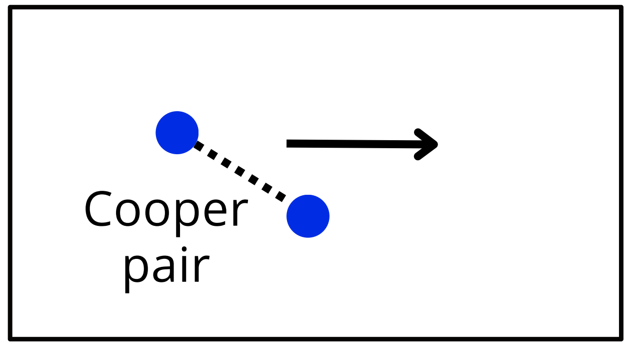
LOUISIANA STATE UNIVERSITY

Superconductor

- For regular conductors, the ground state has finite conductivity
- For superconductors, the ground state has paired up electrons ("Cooper pair")
- The Cooper pair is still charged, so it conducts and gives rise to a *supercurrent*
- The pairs don't transfer energy, so they're heat insulators too



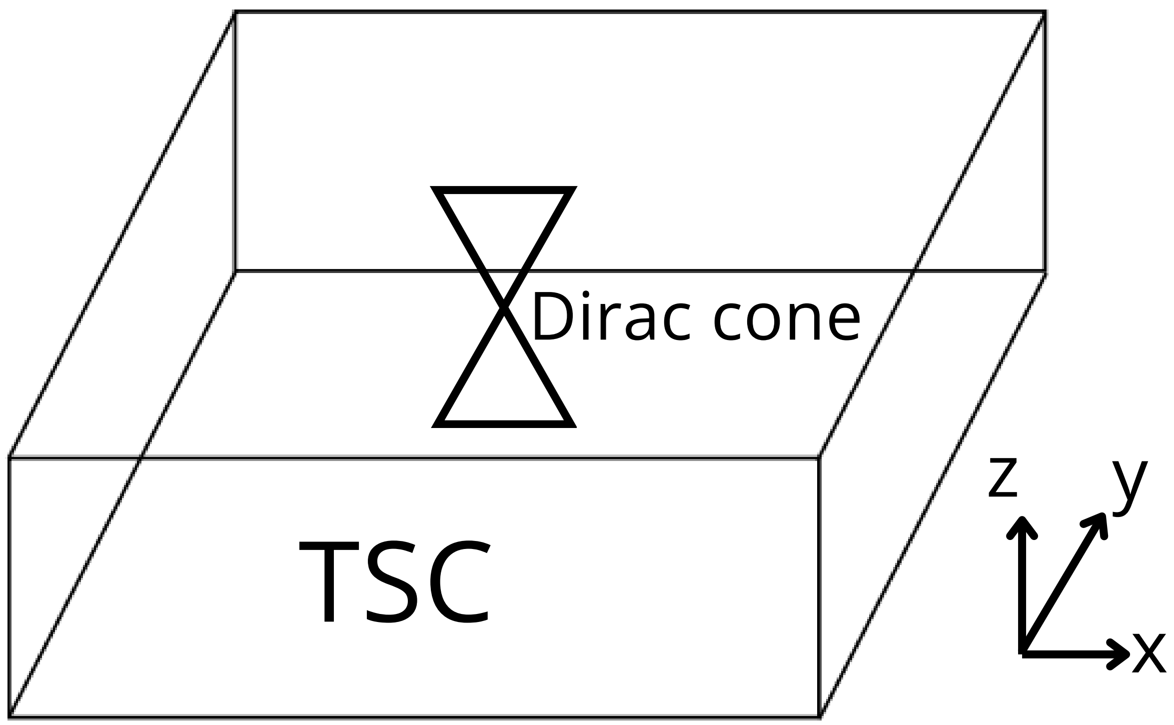
Normal conductor

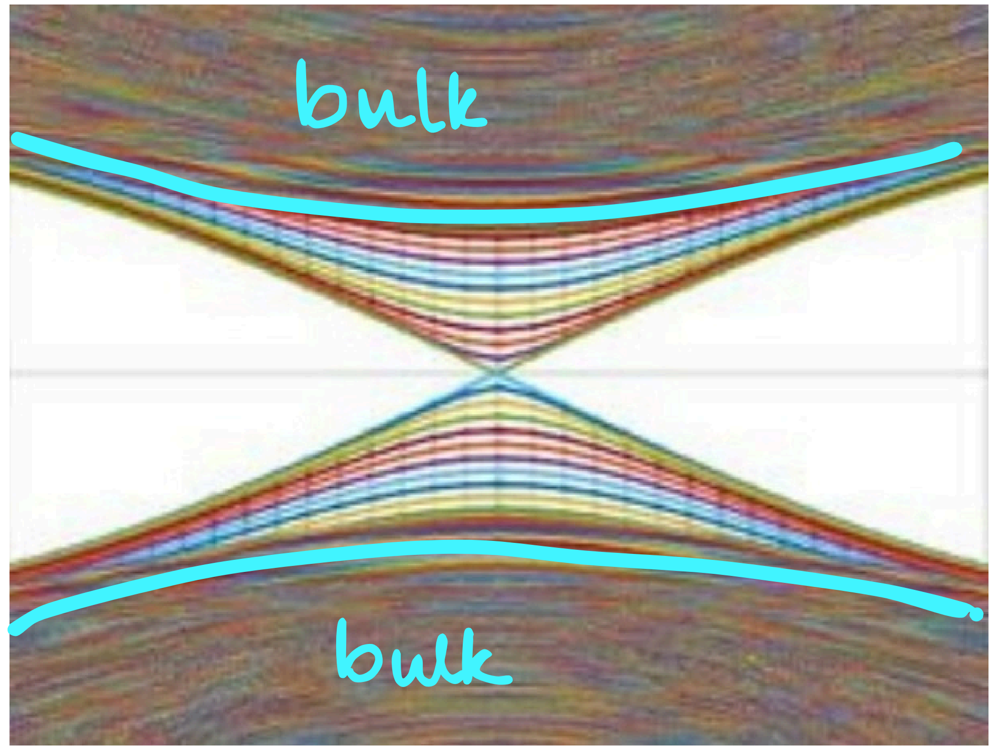


Ground state
Superconductor

Topological superconductor

- A superconductor with a non-zero topological invariant
- Allows for the existence of massless Dirac fermions on the surface
- Characteristic velocity *v* proportional to the superconducting gap *Δ*
- Makes the surface interesting from a thermal point of view: a heat current can exist



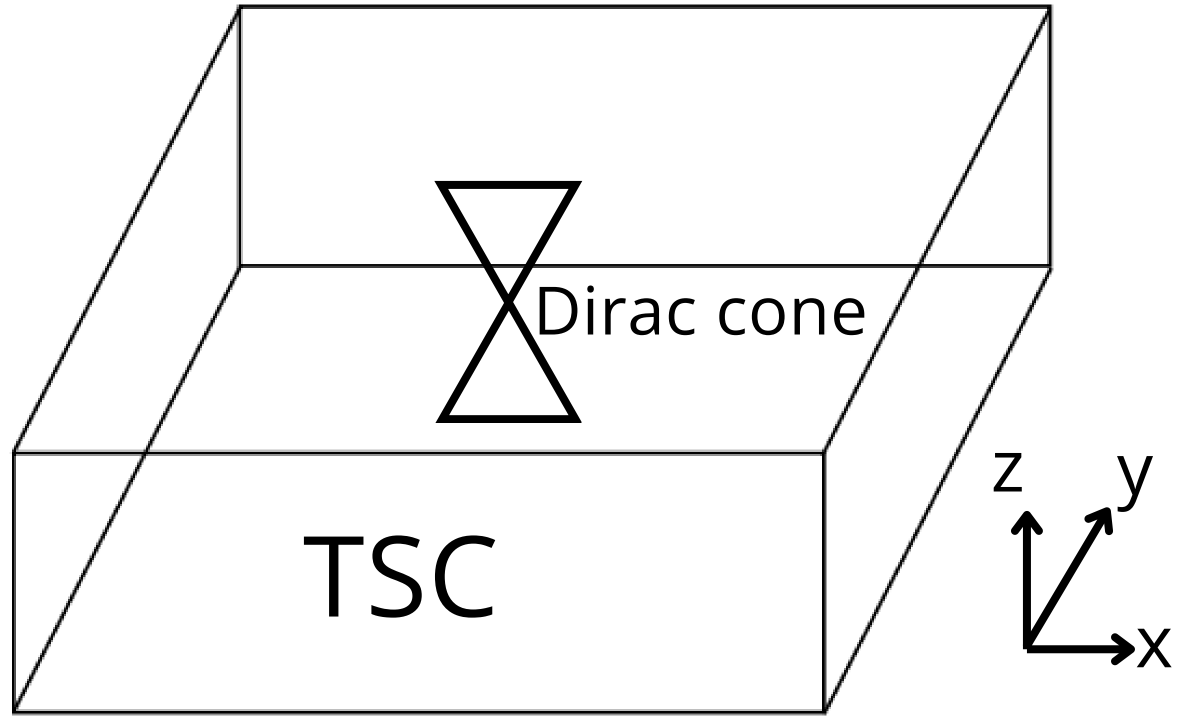


$$E(k) = v|\bar{k}|$$

Topological superconductor

Our system:

- Class DIII: has particle-hole and time-reversal symmetry
- Can have a spatially varying surface velocity profile *v(x,y)*
- Spatially varying the surface fermion velocity creates a (2+1)D curved spacetime[1]



$$\mathrm{d}s^2 = -v(x,y)^2\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2$$

1. Ghorashi et al., *Criticality across the energy spectrum from random artificial gravitational lensing in two-dimensional Dirac superconductors Phys. Rev. B 101, 214521* (2020)

How do we get the metric?

Particle-hole and time-reversal symmetry constrain the action to be of the form[1]

$$S = \int dt d^2\mathbf{r} \left[\bar{\psi}\, i\partial_t \psi + \sum_{a,b=1,2} \frac{v_{ab}(\mathbf{r})}{2} \left(\bar{\psi}\, i\hat{\sigma}^a \overset{\leftrightarrow}{\partial}_b \psi \right) \right],$$

which we compare with the covariant action for fermions

$$S = \int \sqrt{|g|} d^3x\, \bar{\psi} E^{\mu}_{A} \hat{\gamma}^{A} \left(i\partial_\mu - \frac{1}{2} \omega^{BC}_{\mu} \hat{S}^{BC} \right) \psi.$$

1. Ghorashi et al., *Criticality across the energy spectrum from random artificial gravitational lensing in two-dimensional Dirac superconductors* Phys. Rev. B 101, 214521 (2020)

How do we get the metric?

This lets us make the identification[1]

$$v_{ij}(\mathbf{r}) \equiv \frac{E_i^j(\mathbf{r})}{E_0^0(\mathbf{r})},$$

which, along with the conditions,

$$\sqrt{|g|}E_0^0 = 1,\ v_{ii}(\mathbf{r}) = v(\mathbf{r}),\ v_{12} = v_{21} = 0,$$

gives us the metric

$$\mathrm{d}s^2 = -v(x,y)^2\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2.$$

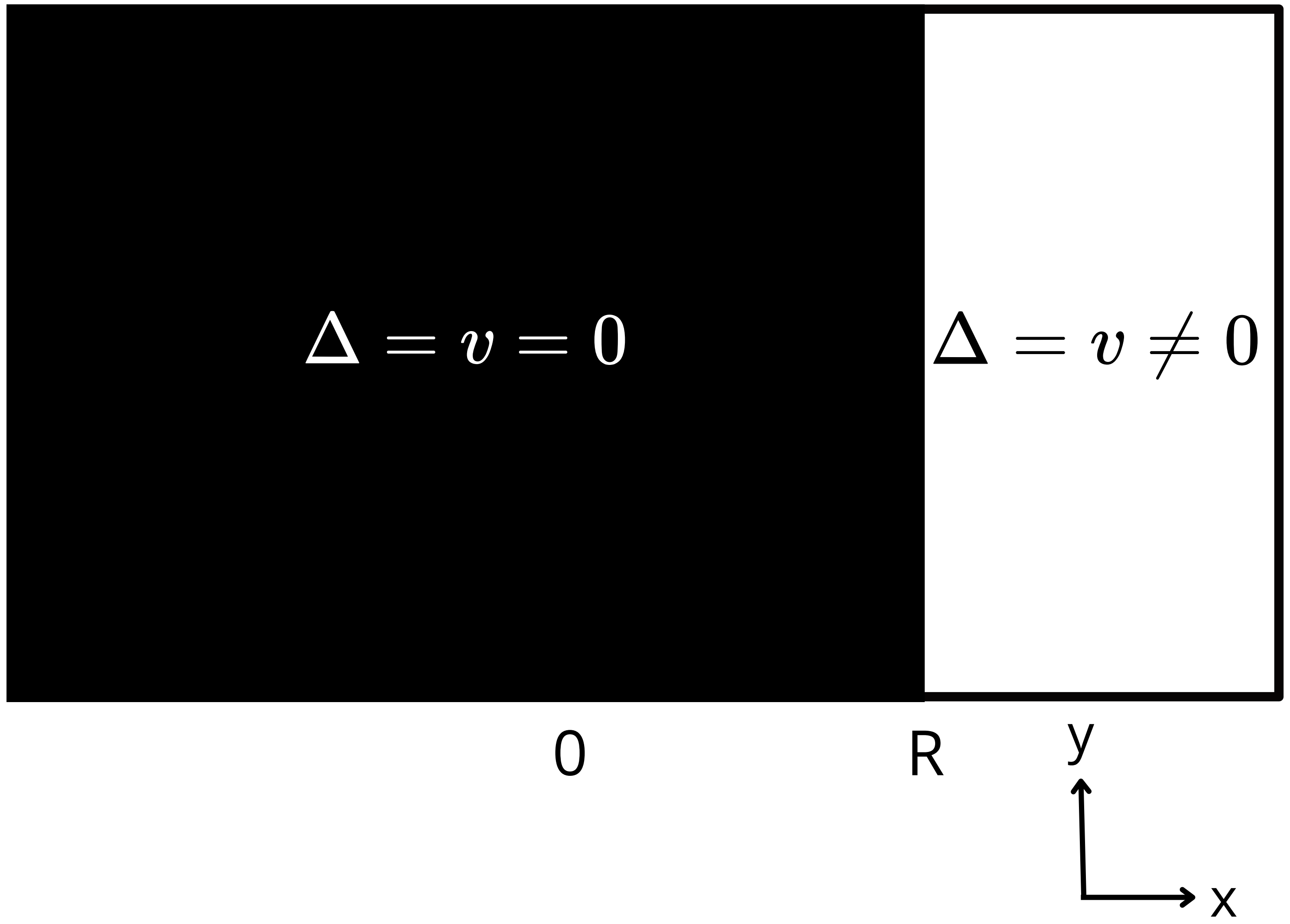
1. Ghorashi et al., *Criticality across the energy spectrum from random artificial gravitational lensing in two-dimensional Dirac superconductors* Phys. Rev. B 101, 214521 (2020)

Hawking radiation due to a quench on the surface of a topological superconductor

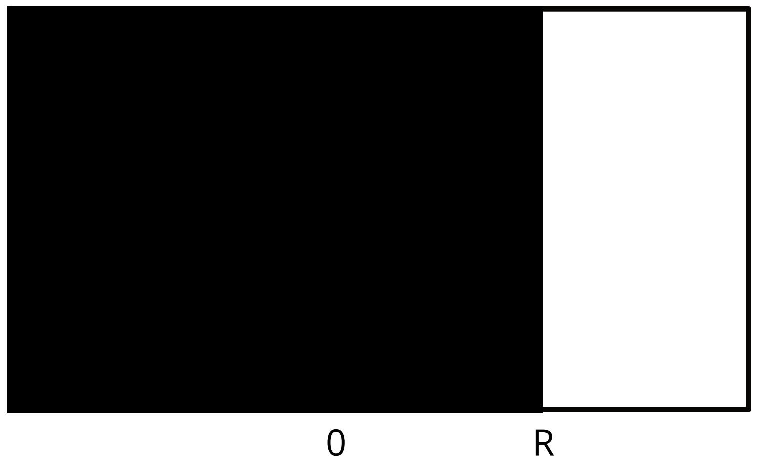
A quench on the surface

- Homogeneous velocity v_0 on the surface gives a flat spacetime
- Our quench: engineer the material such that the velocity is forced to be *zero* in a strip
- This does not necessarily kill the superconductivity in the bulk

$$\mathrm{d}s^2 = -v(x,y)^2\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2$$



Effects of the quench



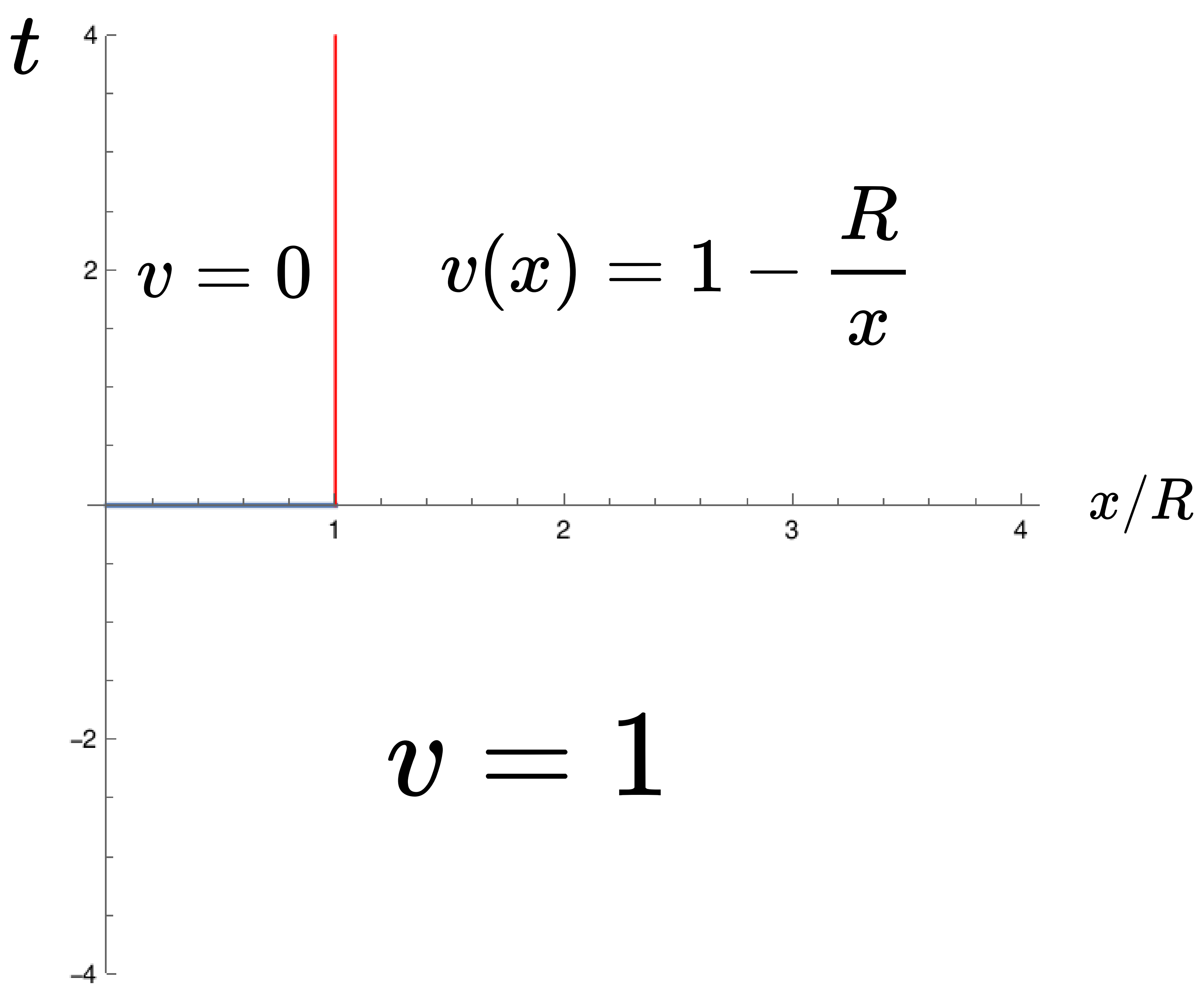
- The curvature in the quenched part blows up: an analog black hole is created
- The *event horizon* is a curvature singularity
- Choosing a velocity profile lets us get the classical particle trajectories
- Writing a quantum field theory on this system lets us calculate the thermal properties of the surface

$$\mathrm{d}s^2 = -v(x)^2\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2$$

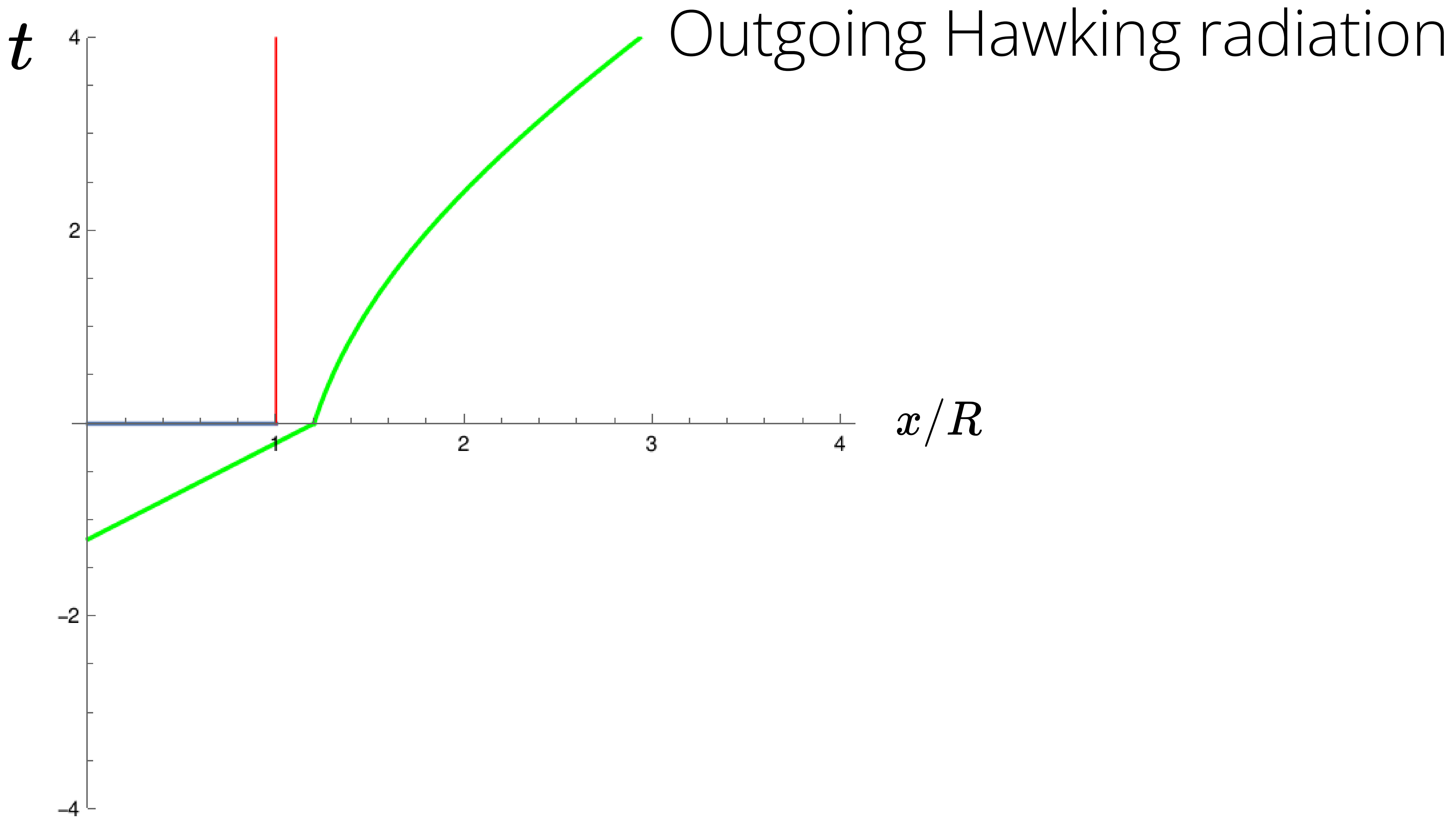
$$\text{Ricci scalar} = 2\frac{\partial_x^2 v}{v}$$

$$v(x) = 1 - \frac{R}{x},\ x > R$$
$$= 0,\ x \leq R$$

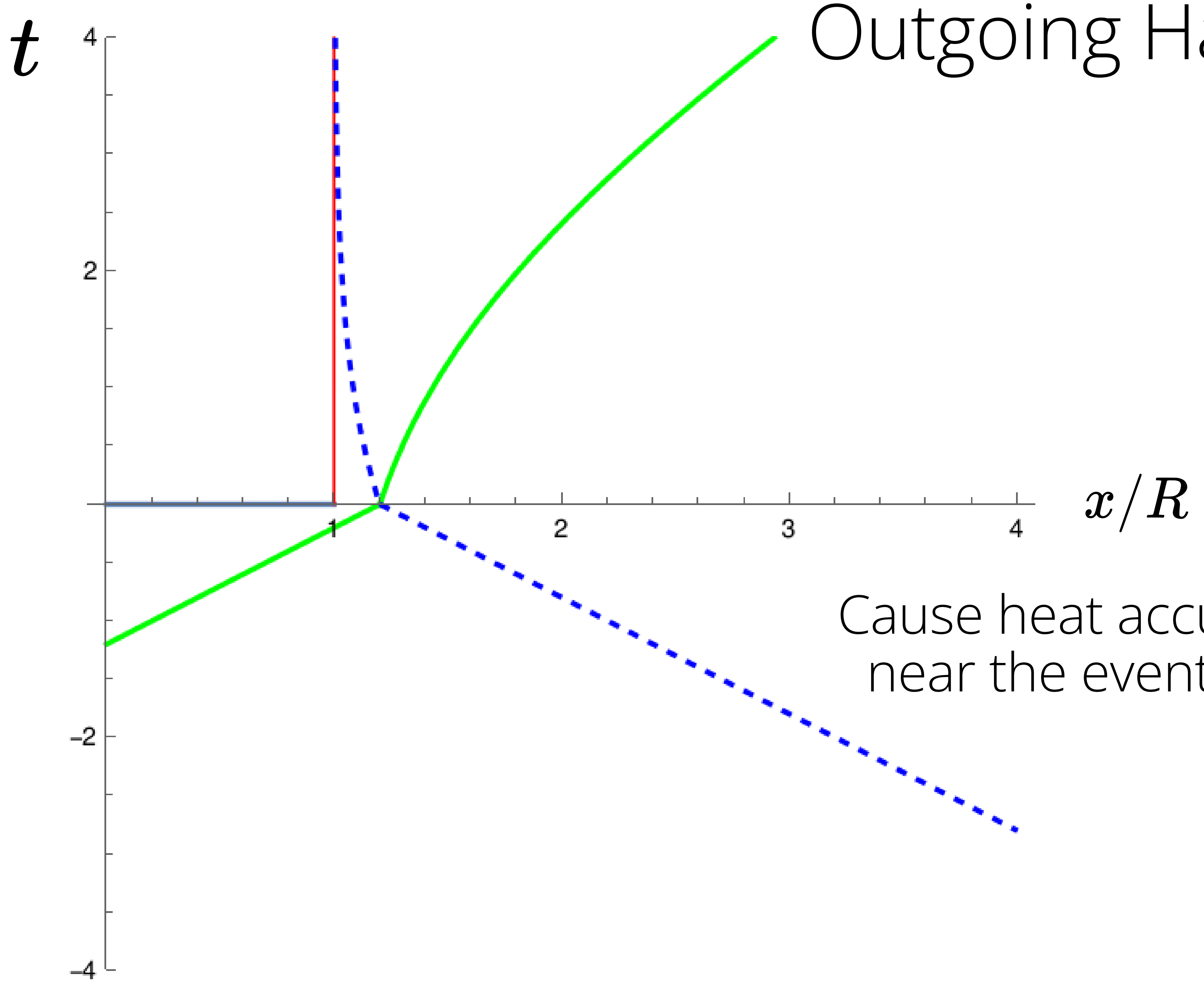
The spacetime diagram



The spacetime diagram



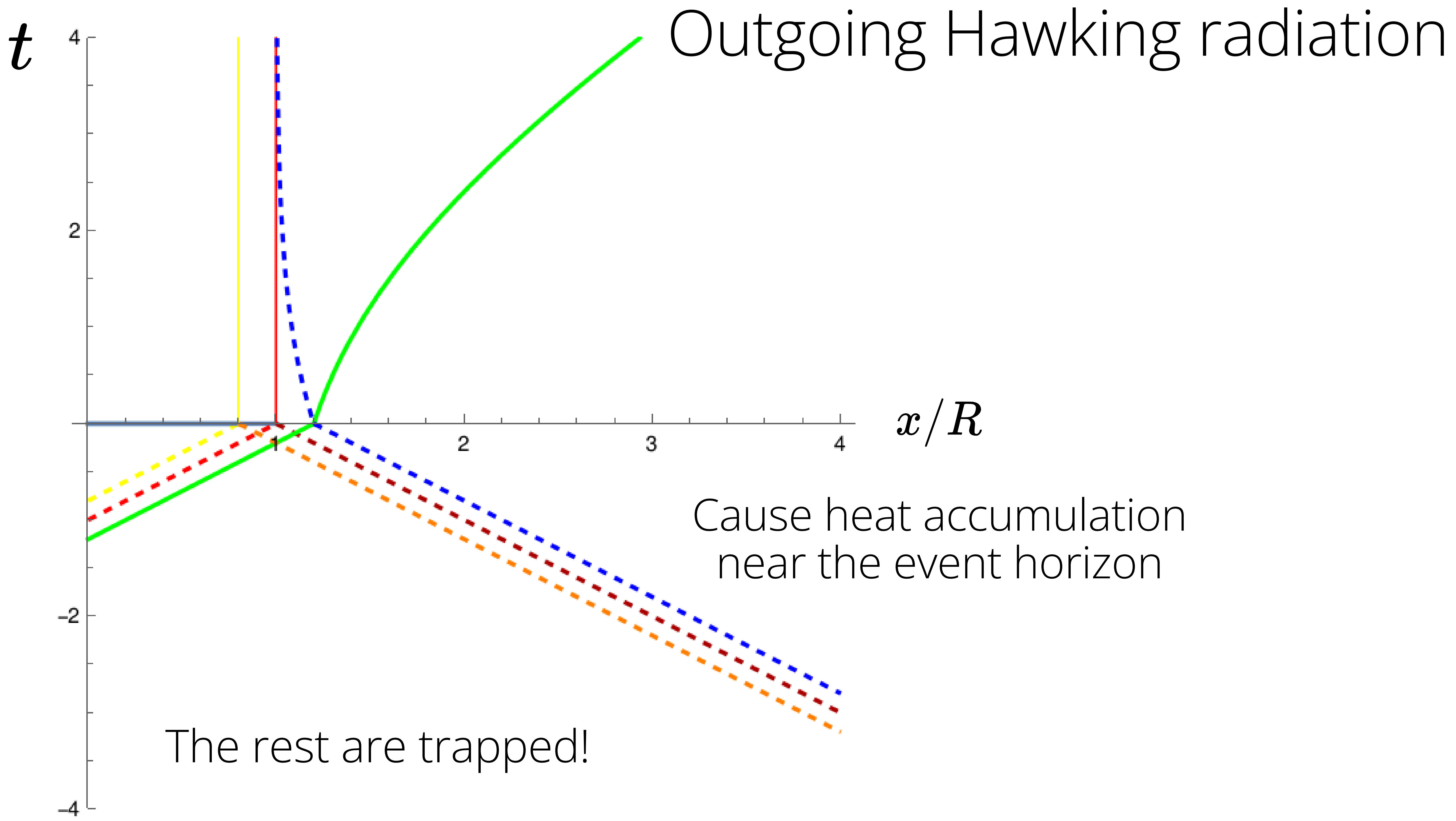
The spacetime diagram



Outgoing Hawking radiation

Cause heat accumulation
near the event horizon

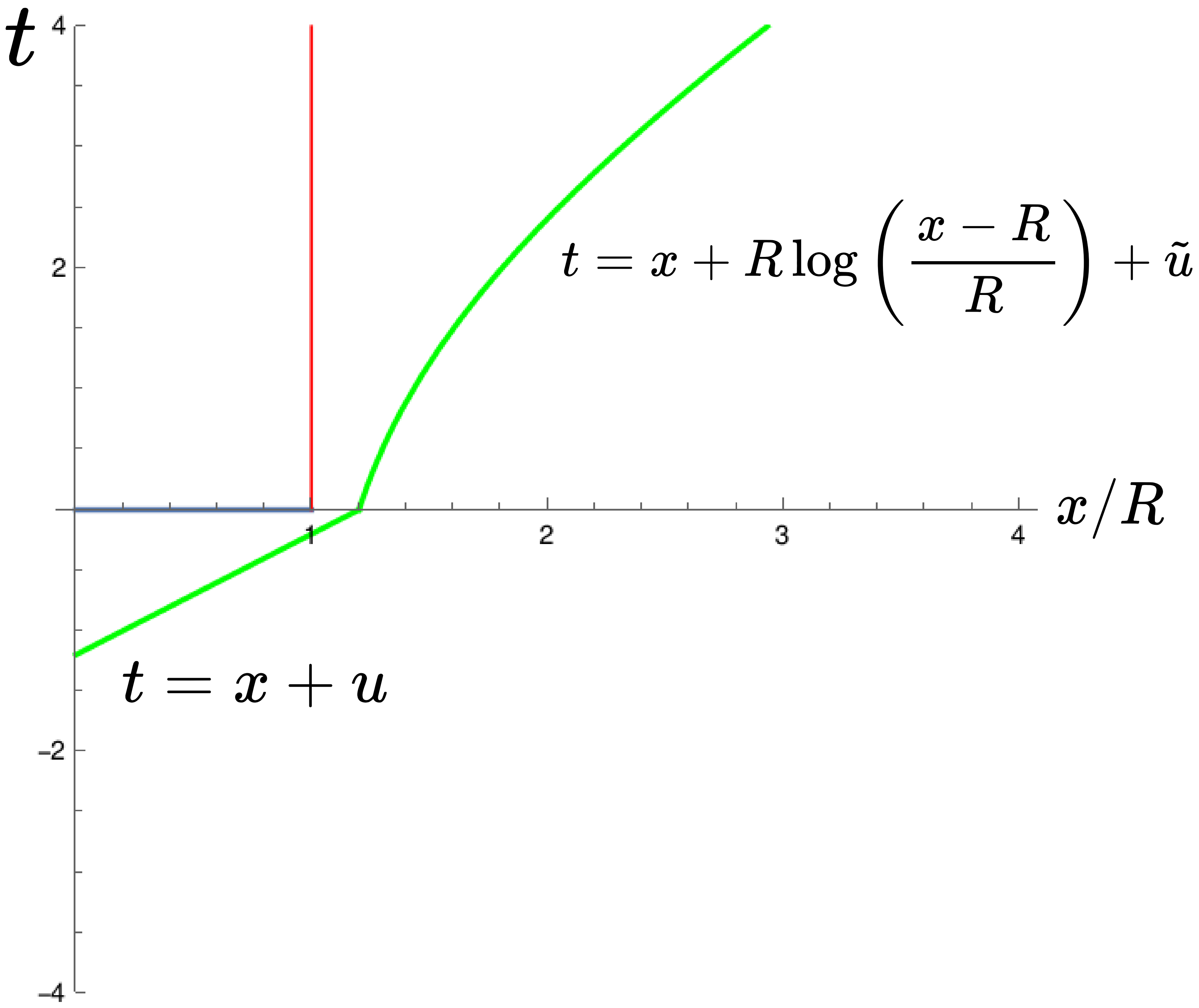
The spacetime diagram



Classical trajectories

- Can get the trajectories before and after the quench, and for all time
- Matching the two at the quench gives us the relation between the two parameters

$$\tilde{u} = u - R\log(-u - R)$$



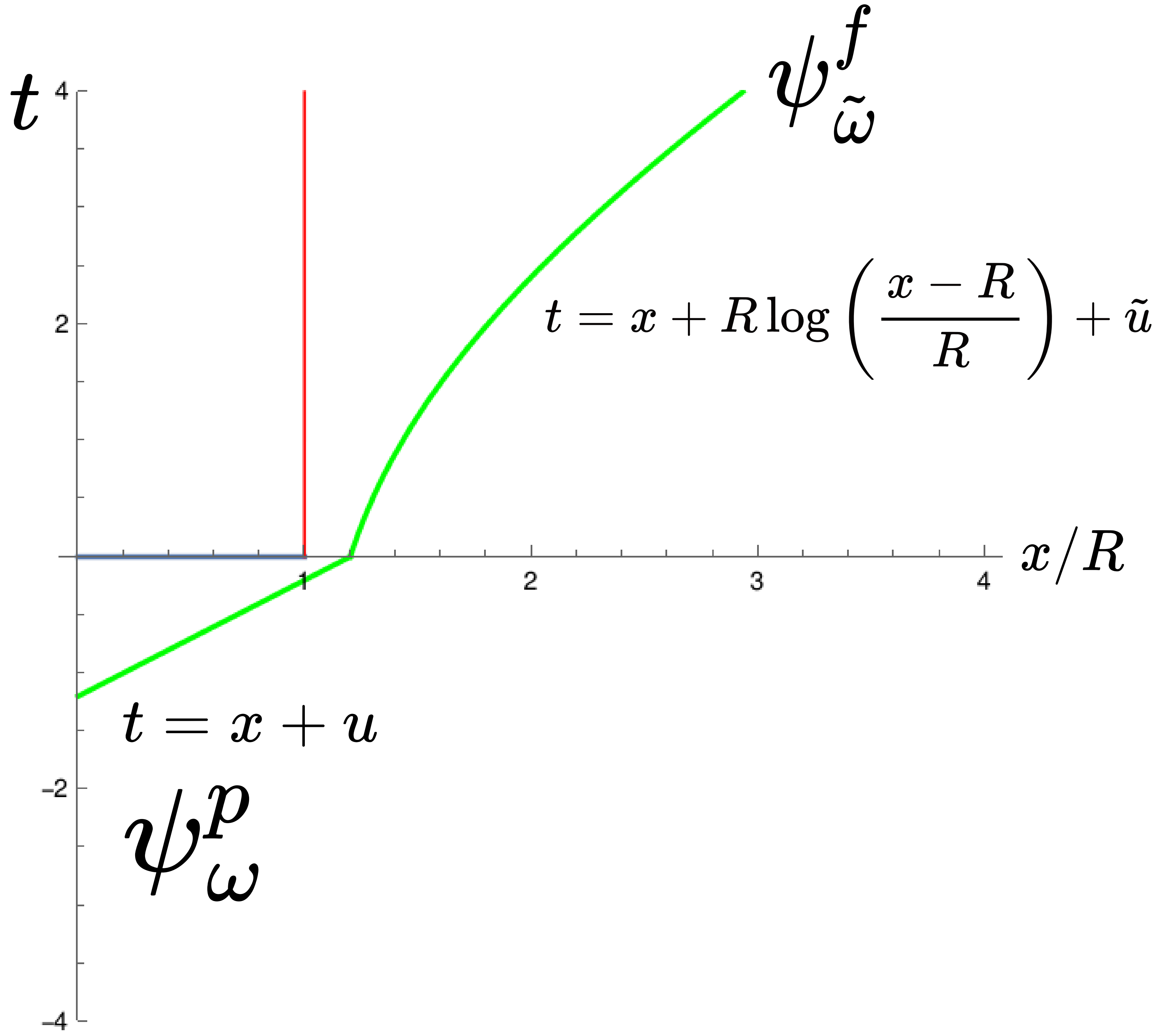
The field theory

- Can write field theories before and after the quench that are related by a Bogoliubov transformation

$$\psi^f_{\tilde{\omega}} = \int \mathrm{d}\omega [\alpha_{\tilde{\omega}\omega} \psi^p_\omega + \beta_{\tilde{\omega}\omega} \psi^{p*}_\omega]$$

- Start with the past vacuum state and get the particle number in the future

$${}_p\langle 0 | N^f_{\tilde{\omega}} | 0 \rangle_p = \int \mathrm{d}\omega |\beta_{\tilde{\omega}\omega}|^2$$



The field theory

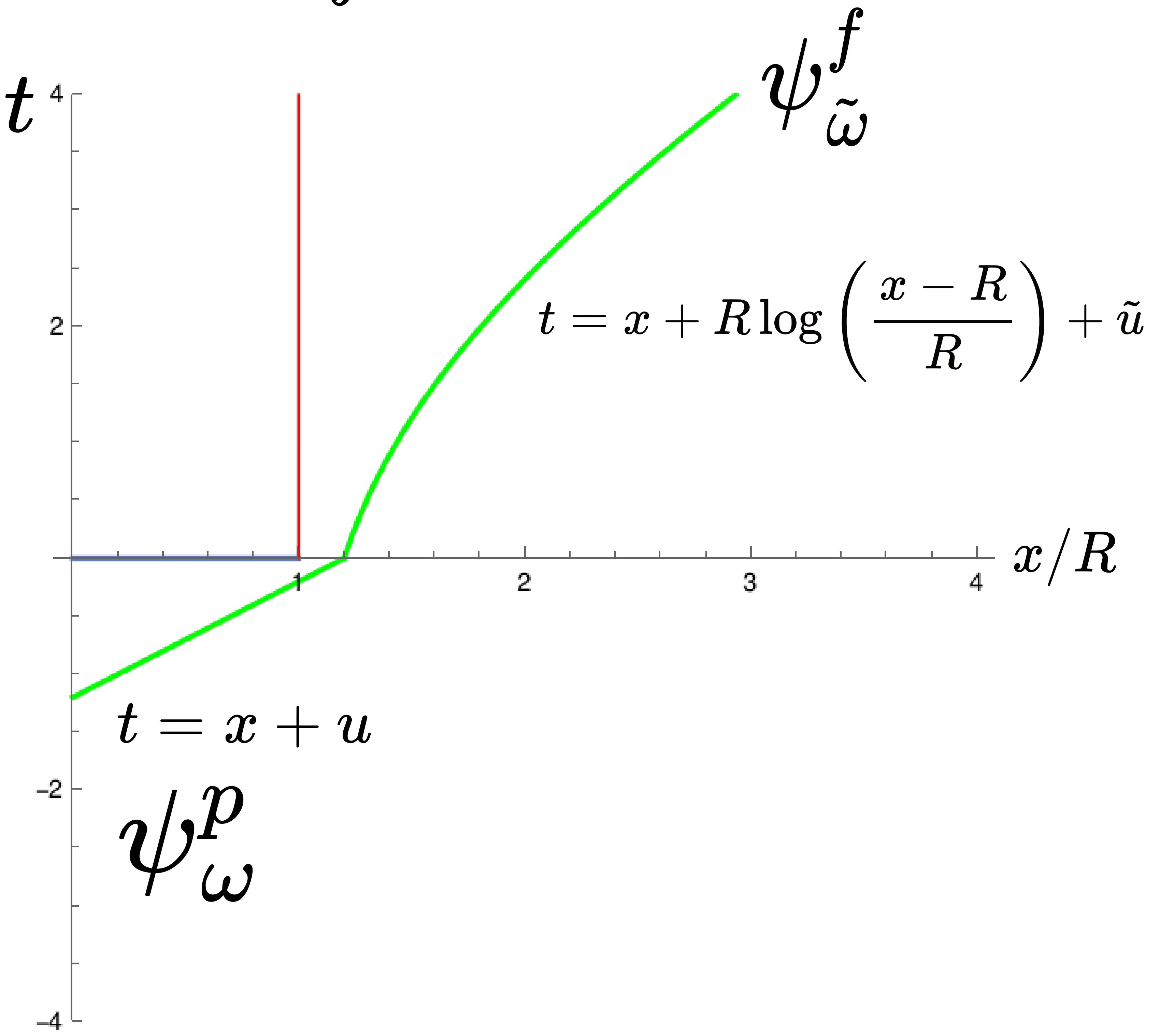
$$\psi^f_{\tilde{\omega}} = \int \mathrm{d}\omega [\alpha_{\tilde{\omega}\omega}\psi^p_\omega + \beta_{\tilde{\omega}\omega}\psi^{p*}_\omega]$$

- In the low frequency limit, we get the Fermi function

$$\int \mathrm{d}\omega |\beta_{\tilde{\omega}\omega}|^2 = \frac{1}{1 + e^{2\pi\tilde{\omega}R}}$$

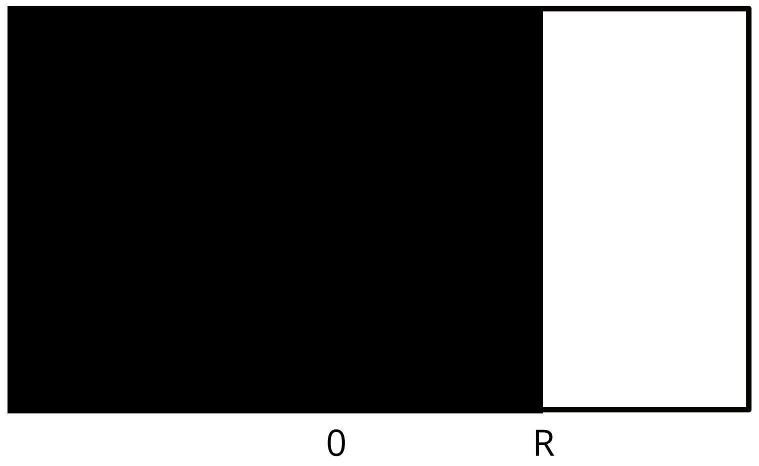
$$_p\langle 0|N^f_{\tilde{\omega}}|0\rangle_p = \int \mathrm{d}\omega |\beta_{\tilde{\omega}\omega}|^2$$

- The past vacuum state is not a vacuum state in the future, and the particle number has a thermal profile



Hawking radiation due to a quench on the surface of a topological superconductor

Hawking radiation



- Immediately after the quench, a heat wave is released
- After the wave dies down, one can observe spontaneous radiation from the black hole: Hawking radiation
- The temperature of the radiation is related to the event horizon and surface gravity κ

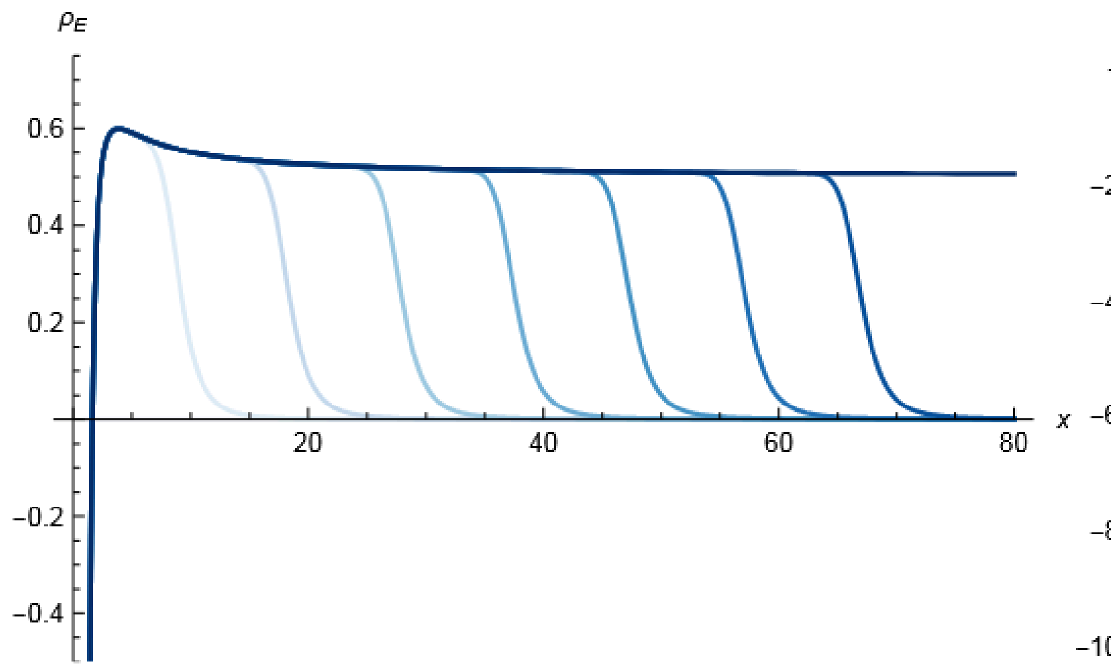
$$\mathrm{d}s^2 = -v(x)^2\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2$$

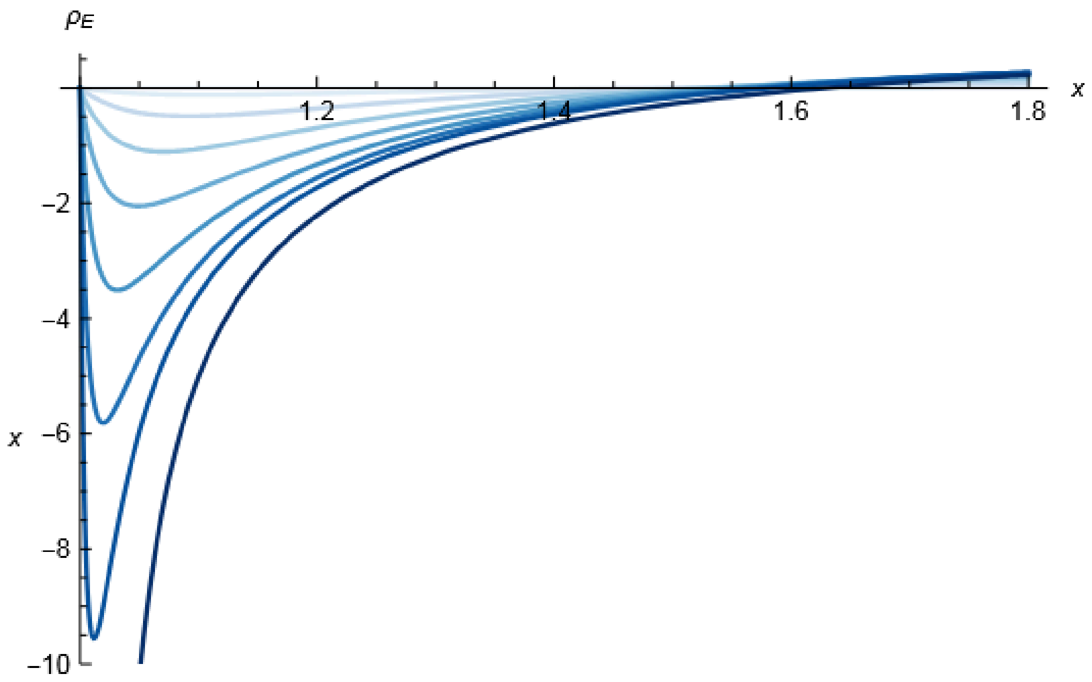
$$\left(\frac{\mathrm{d}v}{\mathrm{d}x}\right)_{x=R} = \kappa$$

$$k_B T = \frac{1}{2\pi R} = \frac{\kappa}{2\pi}$$

Outlook: Two interesting observations

- Though analytic calculations suggest Hawking radiation, simulations of a wave packet on a discretized lattice are not in agreement - currently trying to answer why (maybe a topological obstruction?)
- Negative energy builds up at the horizon while the partner lives inside - they are separated





Conclusion

- Massless Dirac fermions on the surface of topological superconductors create a (2+1)D curved spacetime on the surface
- Forcing their velocity to be zero forms a black hole on the surface, which radiates with a Hawking temperature of $\kappa/2\pi$
- An avenue to investigate the black hole information paradox - an extra dimension is readily available!

References

1. Ghorashi, S. A., Karcher, J. F., Davis, S. M., & Foster, M. S. (2020). Criticality across the energy spectrum from random artificial gravitational lensing in two-dimensional Dirac superconductors. Physical Review B, 101(21). https://doi.org/10.1103/physrevb.101.214521
2. Hawking, S. W. (1975). Particle creation by Black Holes. Communications In Mathematical Physics, 43(3), 199–220. https://doi.org/10.1007/bf02345020

Bogoliubov coefficients

α_Ωω

```
Integrate[Exp[-I (ω) x] Exp[(I Ω R - (1/2)) *Log[x]] (Sqrt[x + R]), {x, 0, Infinity}, Assumptions → {Element[R, PositiveReals]}]
```

$$\frac{2^{1-2 i R \Omega} (i \omega)^{-i R \Omega} \text{Gamma}[2 i R \Omega] \text{Gamma}[1 - i R \Omega] \text{HypergeometricU}[-\frac{1}{2}, -i R \Omega, i R \omega] \text{Sinh}[\pi R \Omega]}{\sqrt{\pi}\, \omega} \quad \text{if } \text{Im}[\omega] < 0 \,\&\&\, 2 R\, \text{Im}[\Omega] < 1$$

β_Ωω

```
Integrate[Exp[I (ω) x] Exp[(I Ω R - (1/2)) *Log[x]] (Sqrt[x + R]), {x, 0, Infinity}, Assumptions → {Element[R, PositiveReals]}]
```

$$-\frac{2^{1-2 i R \Omega} (-i \omega)^{-i R \Omega} \text{Gamma}[2 i R \Omega] \text{Gamma}[1 - i R \Omega] \text{HypergeometricU}[-\frac{1}{2}, -i R \Omega, -i R \omega] \text{Sinh}[\pi R \Omega]}{\sqrt{\pi}\, \omega} \quad \text{if } 2 R\, \text{Im}[\Omega] < 1 \,\&\&\, \text{Im}[\omega] > 0$$

Bogoliubov coefficients