# Classical Electromagnetism as a Pathway beyond Lorentzian Analogues

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# Outline

# Caveats & Conventions

- Signature: $-+++$
- $G = c = \hbar = 1$
- Space-time indices: $abcd\ldots$
- Boys–Post constitutive relations:

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{H} \end{pmatrix} = \begin{pmatrix} \epsilon & \zeta \\ \zeta^\dagger & \mu^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix} . \tag{1}$$

- PSA: EM terminology *not* gravitoelectromagnetism
- PSA: 'Analogue Space-Times' $\gg$ 'Analogue Gravity'[1]
- arXiv numbers for convenience, not to imply 'unpublished'

[1] But see, for example, Erkul & Leonhardt (2025) arXiv:2508.11300, Volovik – *The Universe in a Helium Droplet*, ...

# Electromagnetism and the Earliest Analogue

# Electromagnetism in Media for Relativists

## Reminder: Orthogonal Decomposition

For any two-form $F_{ab}$ in four dimensional space-time and any four-velocity $V^a$, there exist two vector fields $E^a$ and $B^a$, both orthogonal to $V^a$, such that

$$F_{ab} = V_a E_b - V_b E_a + \varepsilon_{abcd} V^c B^d. \tag{2}$$

- Similar statements for any possible tensor
- View our constitutive equation (1) as tensorial map of field strength $F_{ab} \xrightarrow{Z} G$, excitation tensor
- $Z$ has 21 d.o.f.[2] if compatible with an action $\mathscr{L} \propto F_{ab} Z^{abcd} F_{cd}$
- $$\epsilon_V^{ab} = -2Z^{dacb} V_d V_c, \qquad [\mu_V^{-1}]^{ab} = 2(*Z*)^{dacb} V_d V_c,$$
  $$\zeta_V^{ab} = 2(*Z)^{dacb} V_d V_c, \qquad \left[\zeta_V^{\dagger}\right]^{ab} = 2(Z*)^{dacb} V_d V_c.$$

[2] Often the axion-part $\propto \varepsilon_{abcd}$ is omitted, but see, for example, Hehl *et al.* (2009), arXiv:0903.1261.

# Link to David's Lectures

- This directly translates to the physics of moving media![3]
- An isotropic medium in its rest frame $V$ viewed by $W$:

$$
\begin{aligned}
\epsilon_W^{bd} =& -2Z^{abcd}W_aW_c,\\
=&\mu^{-1}(g^{bd}+W^bW^d)+(\epsilon-\mu^{-1})\left(g^{bd}(V\cdot W)^2\right.\\
&\left.-(W^bV^d+V^bW^d)(V\cdot W)-V^bV^d\right),\\
[\mu_W^{-1}]^{bd} =& \frac{h_W^{bd}}{\mu}+(\mu^{-1}-\epsilon)\left((V\cdot W)^2h_W^{bd}-h_W^{be}h_{ef}h_W^{fd}\right),\\
\zeta_W^{ac} =& (\epsilon-\mu^{-1})(V\cdot W)\left(\epsilon^{acef}W_eV_f\right).
\end{aligned}
$$

- Reminders: $(V\cdot W)^2=\gamma_{V,W}, \qquad 1-1/(\epsilon\mu)=1-1/n^2$
- Isotropy is lost in the moving medium!

[3] A.k.a. the Fresnel–Fizeau effect; see, *e.g.*, Post – *Formal Structure of Electromagnetics* or O'Dell - *The Electrodynamics of Magneto-Electric Media*.

# The Issue with the Gordon Analogue

- In this language, the Gordon analogue, '$g \to n$', would read as: Find $Z$ such that

$$S \stackrel{!}{=} -\frac{1}{8} \int \mathrm{d}^4 x \sqrt{-\det g_{\text{eff}}} \left( \left[g_{\text{eff}}^{-1}\right]^{ac} \left[g_{\text{eff}}^{-1}\right]^{bd} - \left[g_{\text{eff}}^{-1}\right]^{ad} \left[g_{\text{eff}}^{-1}\right]^{bc} \right) F_{ab} F_{cd}, \tag{3}$$

$$\textit{i.e.,} \quad Z^{abcd} = \frac{1}{2} \frac{\sqrt{\det g_{\text{eff}}}}{\sqrt{\det g}} \left( \left[g_{\text{eff}}^{-1}\right]^{ac} \left[g_{\text{eff}}^{-1}\right]^{bd} - \left[g_{\text{eff}}^{-1}\right]^{ad} \left[g_{\text{eff}}^{-1}\right]^{bc} \right). \tag{4}$$

- The consistency condition (for isotropic media $\epsilon = \mu$) follows from $\mathrm{d.o.f.}(g^{\text{eff}}_{ab}) \neq \mathrm{d.o.f.}(Z^{abcd})$.[4]
- Warning! $\nabla_{g_{\text{lab}}}$ is not metric-compatible with $\nabla_{g_{\text{eff}}}$ and *vice versa*

[4] See, for example, SeSc, Visser arXiv:1706.06280 for the general case.

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- Warning! $\nabla_{g_{\text{lab}}}$ is not metric-compatible with $\nabla_{g_{\text{eff}}}$ and *vice versa*
- $\Longrightarrow$ **Cartographic distortions from (not just!) non-metricity**

$$q^{\text{lab/eff}}_{bca} := \nabla^{\text{lab/eff}}_a g^{\text{eff/lab}}_{bc}, \tag{5}$$

- This has often (implicitly) been re-discovered: Cummer, Fathi, Frauendiener, Thompson (arXiv:1006.3364, arXiv:1602.08341, arXiv:1705.11108); Sawicki, Trenkler, Vikman (arXiv:2412.21169); SeSc, Visser (arXiv:1808.07987)

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# The Not-Just-Non-Metricity Distortion (arXiv:1808.07987)

**Issues of Coordinates**

- Algebraic EM analogue of Schwarzschild:

$$\epsilon^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r(r-2M)} & 0 \\ 0 & 0 & \frac{1}{r(r-2M)\sin^2\theta} \end{pmatrix},$$

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**Example: Scaled lab coordinates**

Hawking temperature of a Schwarzschild BH

- Hawking temperature can be rephrased in terms of either $M$ or $\epsilon$, $\left[\mu^{-1}\right]$, $\zeta$
- $T_{\text{H,eff}} = \left( \frac{\hbar}{4\pi\sqrt{\det g_{\text{lab}} \, \det \epsilon}} \right)_{,r_{\text{eff}}} \Bigg|_{r_{\text{eff}}=r_+}$

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- $T_{\text{H, lab}} = \frac{\hbar}{8\pi a^{9/2} M} = a^{-9/2}\, T_{\text{H, eff}}$
- $T_{\text{H,eff}}$ conformally invariant under conformal *space-time* transformations of $(M, g_{\text{eff}})$; not under *space* transformations!

# Beyond Lorentzian Geometry

# Area Metrics, Finsler Metrics, and Dispersion Relations

- For QG or EM—'area metrics'; reinterpreting $Z$ as mapping area-forms[5]

$$\text{EM:} \quad S_{\text{EM}} = -\frac{1}{8} \int_M \mathrm{d}^4x \sqrt[6]{\det G^{6\times 6}_{\text{Petrov}}} F_{ab} Z^{abcd} F_{cd},$$

$$\text{GR+:} \quad S_{\text{EH}} = \int_M \mathrm{d}^4x \sqrt[6]{\det G^{6\times 6}_{\text{Petrov}}} \text{`}R\text{'}.$$

$\implies$ 'Area metric geometry'

[5] For details: Punzi, Schuller, Witte, Wohlfarth (arXiv:hep-th/0508170, arXiv:hep-th/0612141, arXiv:0908.1016).

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- $Z$ also tied to Finsler geometry:[6]

$$\mathrm{d}s = F(x,y), \qquad x \in M, y \in T_x M, \quad \text{homogeneous of degree 1 in } y$$

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- Lastly, also tied to general/modified/...dispersion relations $\omega(\mathbf{p})$ or $P_x(q) = m^{\deg P}$, $P_x : T^*_xM \to \mathbb{R}$.[7]

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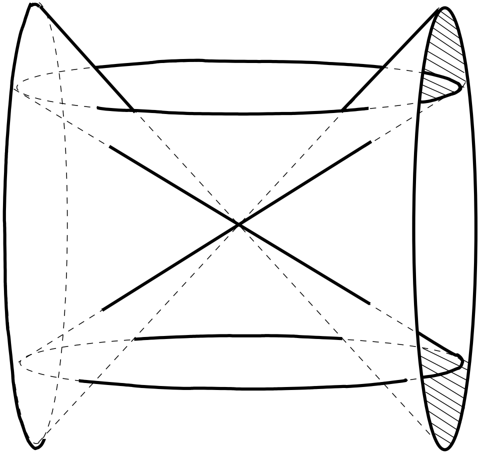
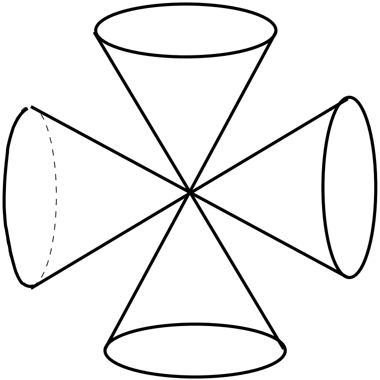
[7] See Rätzel, Rivera, Schuller (arXiv:1010.1369). Warning! Technical & physical requirements on $P$ omitted.



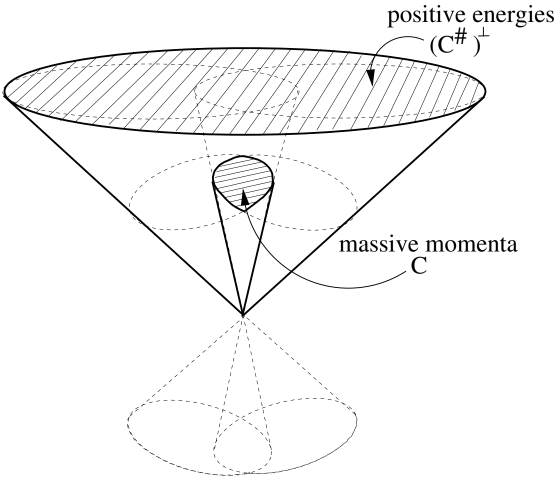
cotangent space



tangent space

Images sourced from: Rätzel, Rivera, Schuller (arXiv:1010.1369)

# What It Looks Like

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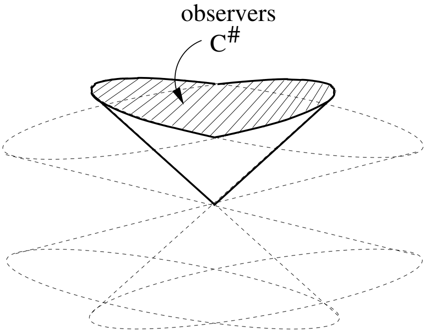


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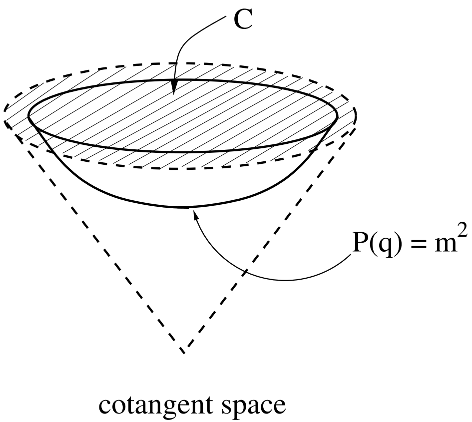


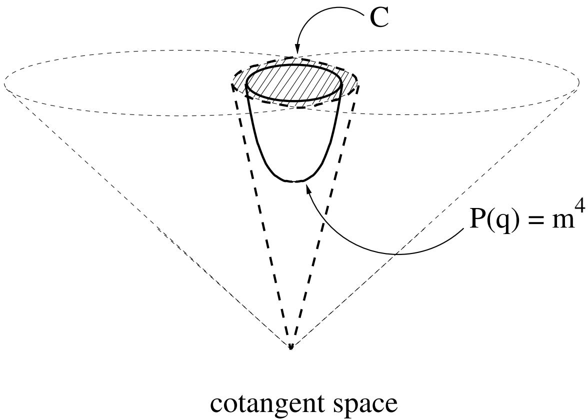


Images sourced from: Rätzel, Rivera, Schuller (arXiv:1010.1369)

# An Unruh–DeWitt Detector in a Uniaxial Crystal

# The Detector Model

- Ingredients for uniaxial crystal:
  - Flat background[8] $\eta$,
  - Crystal rest frame's worldline $U$,
  - $Z^{abcd} = \eta^{a[c}\eta^{d]b} + U^{[a}X^{b]}U^{[c}X^{d]}$

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- $\eta(\dot{x},\dot{x}) = \eta(U,U) = -1$, $\eta(M,\dot{x}) = \eta(U,X)$
- Fresnel equation[9] determines wave front momenta: $\eta^{-1}(k,k)\zeta^{-1}(k,k) = 0$ with

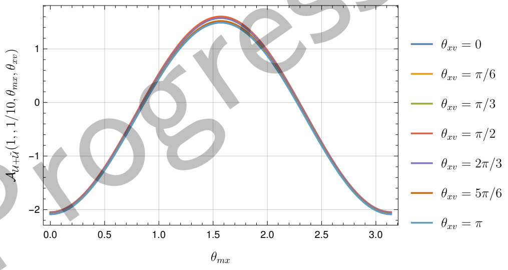
$$\zeta_{ab} = \eta_{ab} + \frac{\xi^2}{1+\xi^2}U_aU_b - \tfrac{1}{1+\xi^2}X_aX_b$$

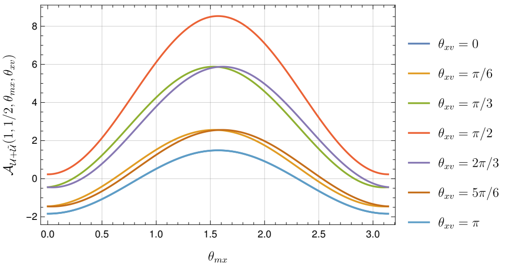
[8] Well, unless the constitutive tensor is assumed to *define* a more general (birefringent) space-time geometry.
[9] See, for example, Hehl, Obukhov – *Foundations of Classical Electrodynamics*

- Building on results of Fewster, Pfeifer, Siemssen (arXiv:1709.01760, arXiv:1602.00946)
- Currently, only (partially) done for inertial $\dot{x}$
- Calculate $\mathscr{M}^a \mathscr{M}^c \dot{x}^b \dot{x}^d \langle 0 | F_{cd}(x(\tau')) F_{ab}(x(\tau)) | 0 \rangle \big|_{\xi=0}$
- Using clever variable choices[10]

$$
\begin{aligned}
U^a &= (1, \vec{0})^a, \hat{X}^a = (0, \hat{x})^a, \\
\dot{x}^a &= \gamma (1, v)^a, \\
\eta(U, \dot{x}) &= -\gamma \\
\mathscr{M}^a &= |\mathscr{M}| (\gamma v \cos\theta_{mv}, \hat{m} + (\gamma - 1)\cos\theta_{mv} \hat{v})^a, \\
\cos\theta_{mv} &= \hat{m} \cdot \hat{v}, \\
\eta(\hat{X}, \dot{x}) &= v \gamma \cos\theta_{vx},
\end{aligned}
$$

isolate factors $A_{U+\tilde{U}}(\xi, v, \theta_{mx}, \theta_{mv})$






[10] Thank you, Finn!

# Summary

# Summary and Outlook

Summary:

- Analogues fit naturally into geometries beyond Lorentzian metrics
- One can find covariant dispersion relations
- Many application to modified theories of gravity

Outlook

- Interial UdW detector response in a uniaxial crystal
- Hopefully also accelerated UdW detector response...

# Thank you very much!

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**Microscopic Maxwell equations**

$$\nabla_{[a}F_{bc]} = 0,$$
$$\nabla_a F^{ab} = J^b$$

non-trivial constitutive relations →

**Macroscopic Maxwell equations**

$$\nabla_{[a}F_{bc]} = 0,$$
$$\nabla_a \underbrace{(Z^{abcd}F_{cd})}_{G^{ab}} = J^b,$$

$$\mathbf{D} = \epsilon\, \mathbf{E} + \zeta\, \mathbf{B},$$
$$\mathbf{H} = \zeta^\dagger \mathbf{E} + \mu^{-1}\mathbf{B}.$$

# What It Looks Like—Addendum 'Observers'

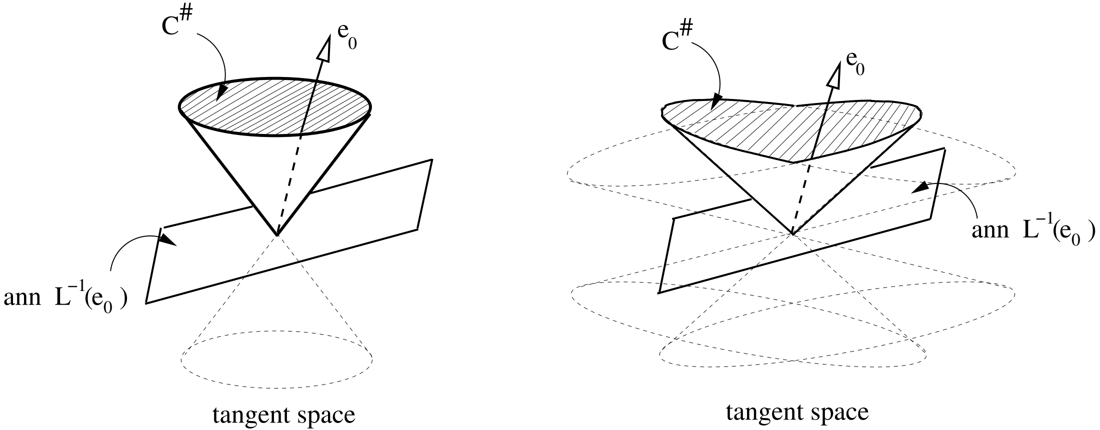


Image sourced from: Rätzel, Rivera, Schuller (arXiv:1010.1369)

# What It Looks Like—Addendum ‘Stability’

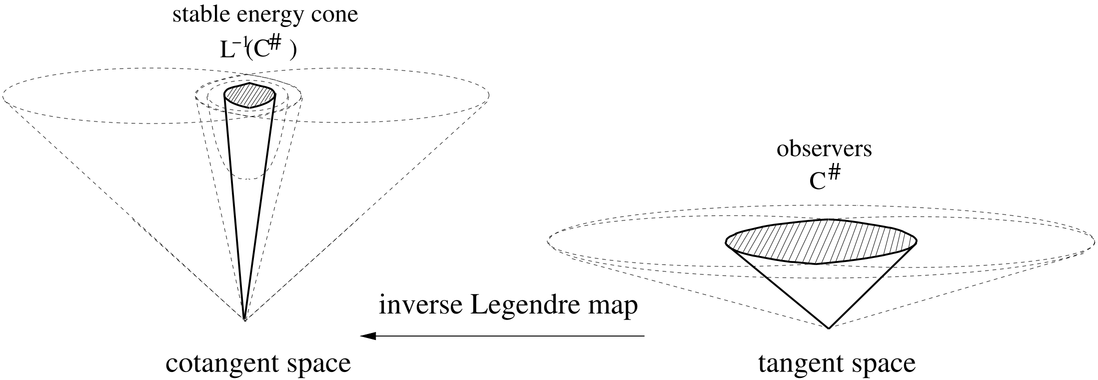


Image sourced from: Rätzel, Rivera, Schuller (arXiv:1010.1369)