Backreaction Effects in Analogue Gravity: Number-Conserving Approach

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Contents

What is Backreaction?

Quantum Field Theory in Curved Spacetime (QFTCS)

- **'Given'** classical Spacetime

$$S_{\mathrm{Grav}}(\mathfrak{g}_{\mu\nu}) \implies \mathfrak{g}_{\mu\nu}$$

$$G_{\alpha\beta}(\mathfrak{g}_{\mu\nu}) = 0$$

we choose vacuum for simplicity

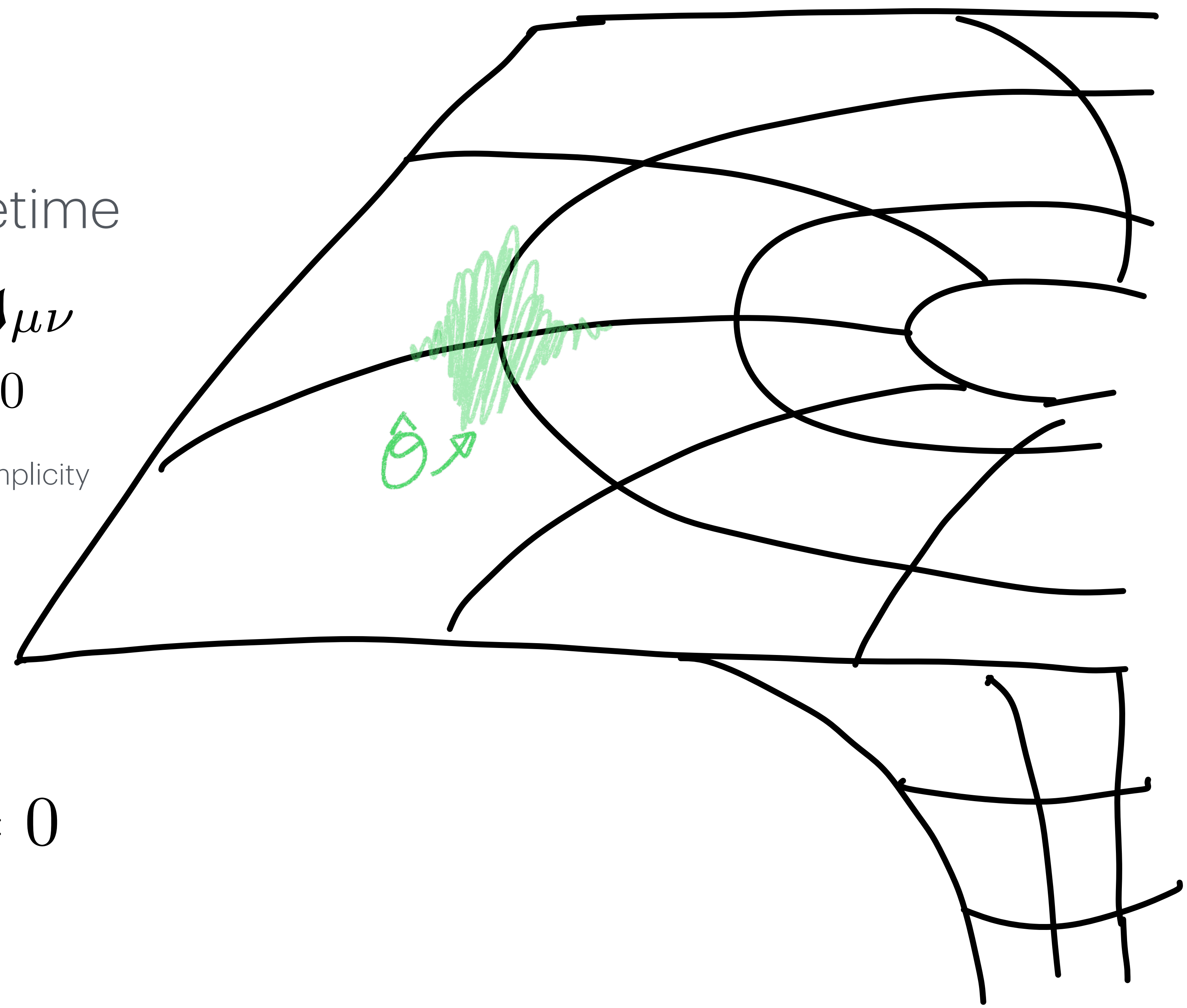
- Matter resides on given spacetime

$$S_{\mathrm{Matter}}(\mathfrak{g}_{\mu\nu}, \hat{\theta})$$

$$\implies (\Box + m^2)\hat{\theta} = 0$$

where

$$\Box = \frac{1}{\sqrt{-\mathfrak{g}}}\partial_\mu(\sqrt{-\mathfrak{g}}\mathfrak{g}^{\mu\nu}\partial_\nu)$$



Analogue Gravity

For BEC Model

- **'Given'** Spacetime

$$i\partial_t \phi_0 = \left(-\frac{1}{2}\nabla^2 + U_{\text{ext}} + g|\phi_0|^2\right)\phi_0$$

$$\implies \mathfrak{g}_{\mu\nu} = \mathfrak{g}_{\mu\nu}(g, \phi_0)$$

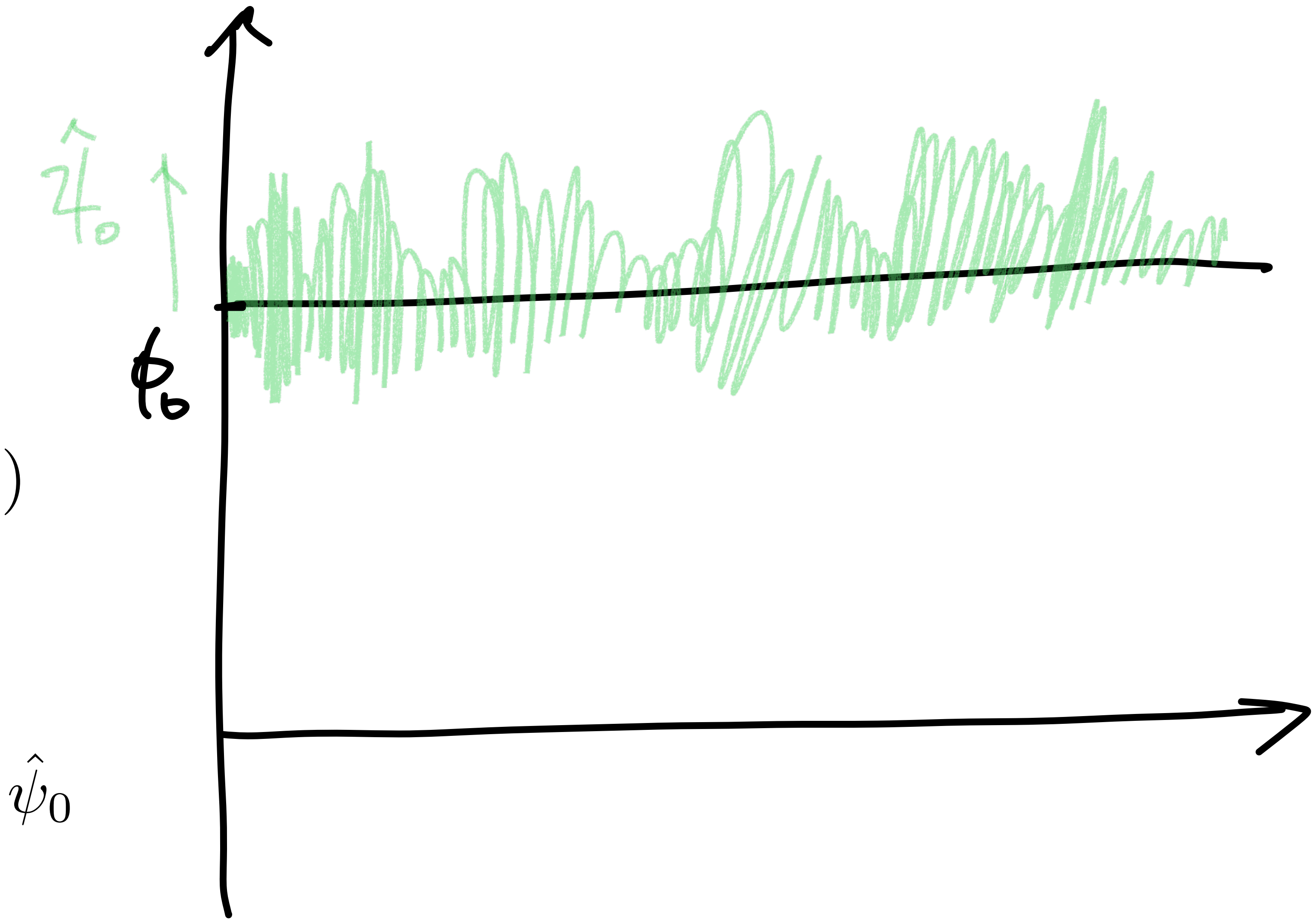
- Matter resides on given spacetime

$$i\partial_t \hat{\psi}_0 = \left(-\frac{1}{2}\nabla^2 - \frac{\nabla\phi_0}{\phi_0}\cdot\nabla\right)\hat{\psi}_0 + g|\phi_0|^2(\hat{\psi}_0 + \hat{\psi}_0^\dagger)$$

$$\implies (\Box + m^2)\hat{\theta}_0$$

where

$$m^2 = 0, \qquad \hat{\theta}_0 = \hat{\theta}_0(\hat{\psi}_0)$$



Backreaction?

- Since matter and background are interacting, i.e., (using effective action for matter)

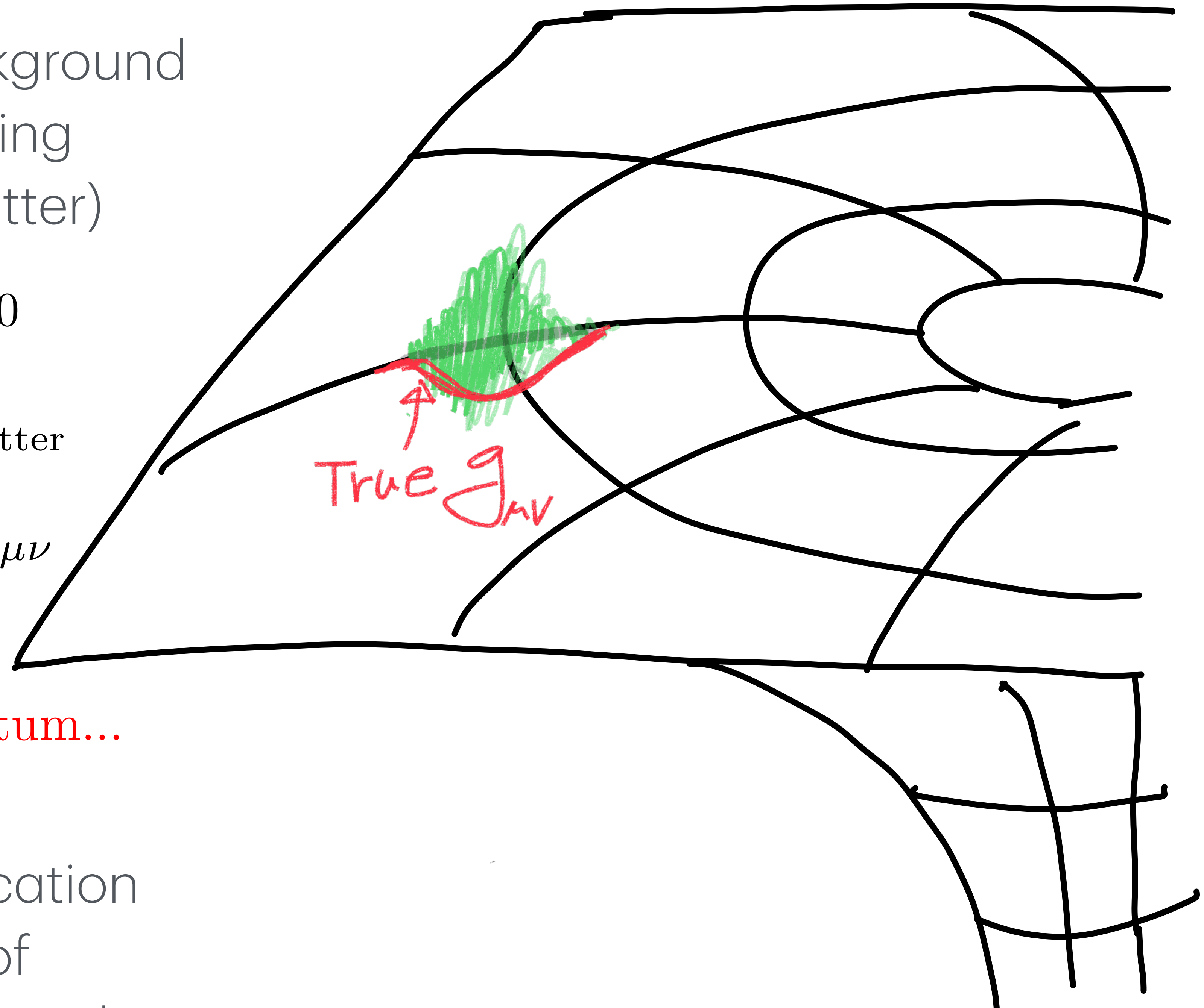
$$\frac{\delta \Gamma_{\text{Matter}}}{\delta \mathfrak{g}^{\alpha\beta}} := \langle \hat{T}_{\alpha\beta} \rangle \neq 0$$

$$S_{\text{total}} = S_{\text{Grav}} + S_{\text{Matter}}$$

$$S_{\text{Total}}\left(\mathfrak{g}_{\mu\nu}, \hat{\theta}\right) \implies \mathfrak{g}_{\mu\nu}$$

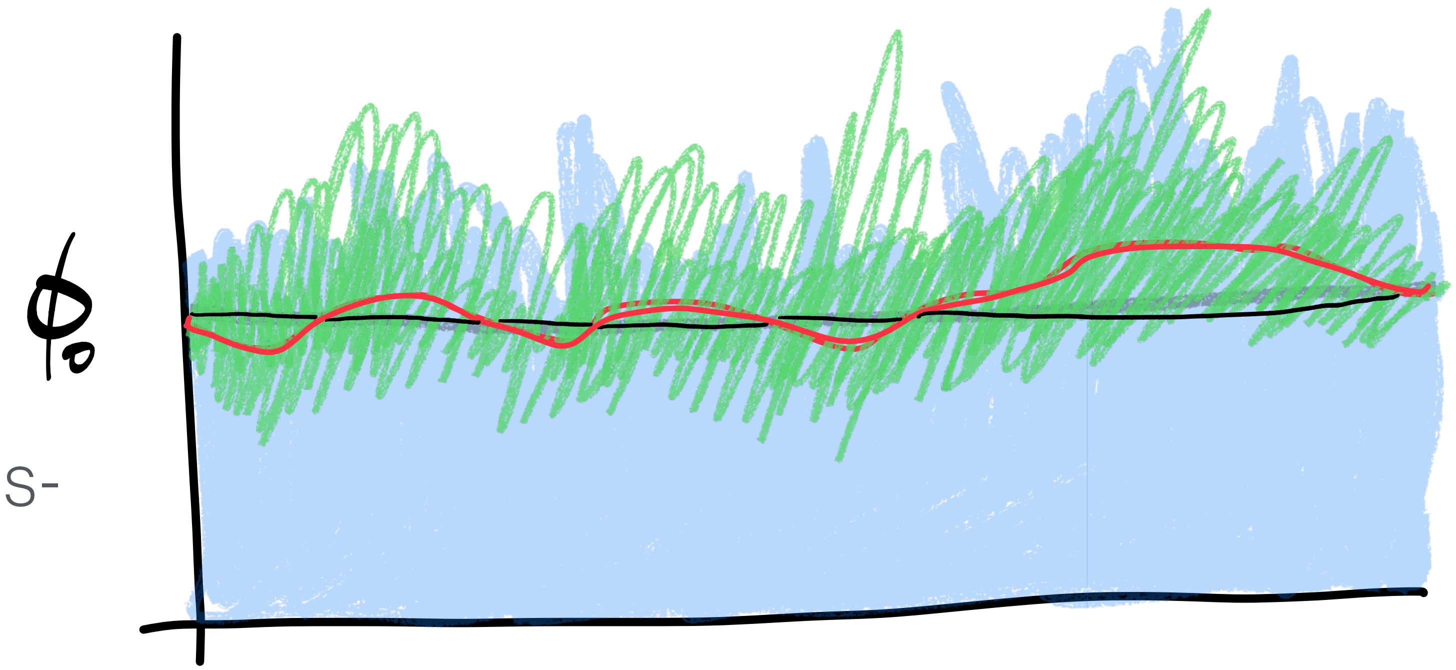
$$G_{\alpha\beta}(\mathfrak{g}_{\mu\nu}) = \langle \hat{T}_{\alpha\beta} \rangle$$

$$+\text{Quantum...}$$



- It appears as a modification term on the equation of motion for the background spacetime.

Backreaction in Analogue Gravity



$$\color{red}{\phi_c} = \phi_0 \color{red}{+\zeta}$$

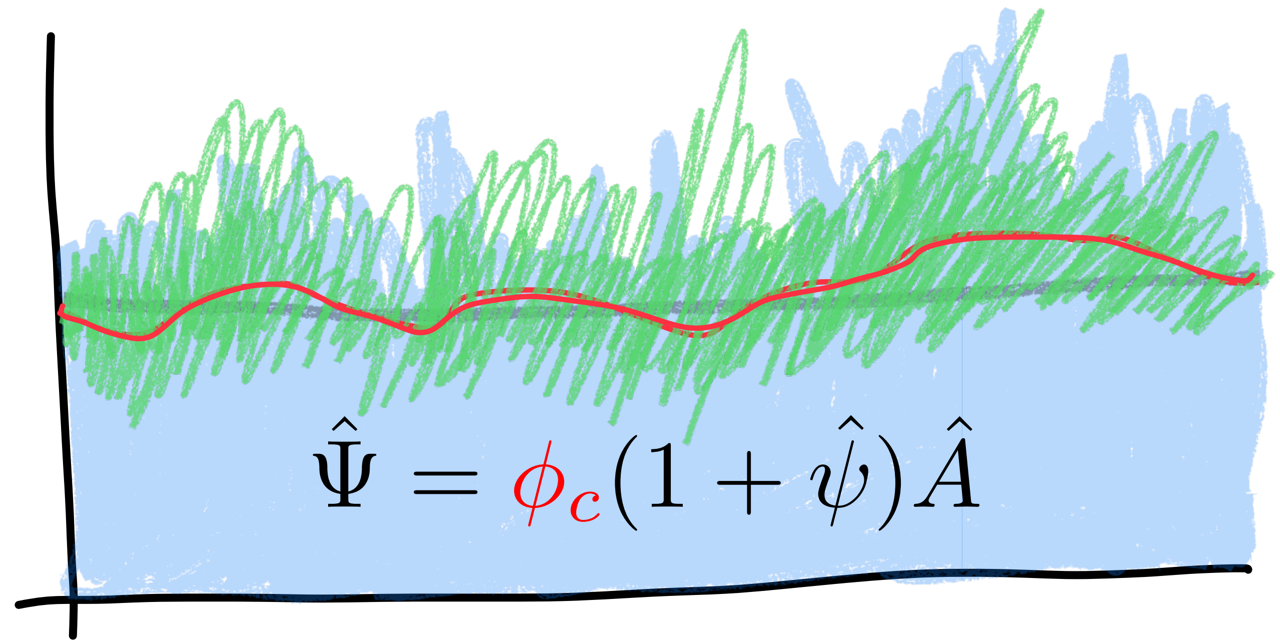
- In the linear theory, (Gross-Pitaevskii equation), quantum field does not affect the classical field dynamics.

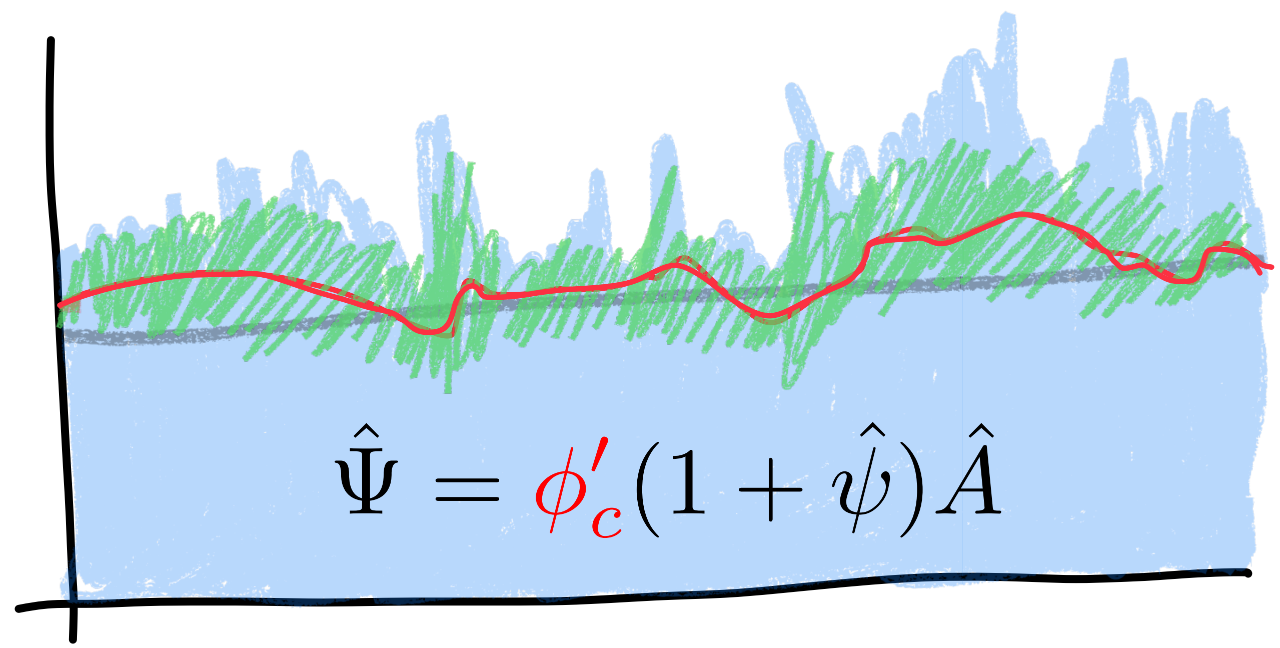
$$i\partial_t \phi_0 = \Big(-\frac{1}{2}\nabla^2 + U_{\text{ext}} + g|\phi_0|^2\Big)\phi_0$$

- One needs higher-order correction in mean field.

Backreaction
in Analogue Gravity

- No clear distinction between background and matter.
- The background (spacetime) needs to be approximated to the known solution (Mean-Field) in the limit that backreaction is neglected.
- The quantum field should be approximated to the known form (Bogoliubov de Gennes) in the limit that the backreaction is neglected.





Number-Conserving Formulation

Formalism

Note that in real experiment, the number of particles are finite

- Dynamics of Contact Interacting Bose Gas:

$$\hat{H} = \int \mathrm{d}^d x \left[\frac{1}{2} \nabla \hat{\Psi}^\dagger(\vec{x}) \cdot \nabla \hat{\Psi}(\vec{x}) + U(x) \hat{\Psi}^\dagger(x) \hat{\Psi}(x) + \frac{g}{2} \hat{\Psi}^\dagger(\vec{x}) \hat{\Psi}^\dagger(\vec{x}) \hat{\Psi}(\vec{x}) \hat{\Psi}(\vec{x}) \right]$$

$$i\partial_t \hat{\Psi} = \left(-\frac{1}{2}\nabla^2 + U_{\mathrm{ext}} + g\hat{\Psi}^\dagger \hat{\Psi} \right) \hat{\Psi}$$

- U(1)-Conserving Ansatz: $\hat{\Psi} = \phi_c (1 + \hat{\psi}) \frac{\hat{A}}{\sqrt{\hat{N}}}$ Phys. Rev. D 72, 105005 (2005).

where $\hat{N} = \hat{A}^\dagger \hat{A}$ is a total number of particle operator

$\phi_c = \mathcal{O}(\sqrt{N})$ is a U(1)-conserving order parameter

$\hat{\psi} = \mathcal{O}(N^{-1/2})$ is a U(1)-conserving quantum fluctuation.

- Dilute Gas: $g = \mathcal{O}(N^{-1})$

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Y. Castin and R. Dum, Phys. Rev. A 57, 3008 (1998).

C. W. Gardiner, Phys. Rev. A 56, 1414 (1997);

M. D. Girardeau, ibid. 58, 775 (1998).

Formalism (Classical Background)

- Amended Gross-Pitaevskii Equation:

$$i\partial_t\phi_c = \Big(-\frac{1}{2}\nabla^2 + U_{\mathrm{ext}} + g|\phi_c|^2\Big)\phi_c + g|\phi_c|^2\big(2\langle\hat{\psi}^\dagger\hat{\psi}\rangle + \langle\hat{\psi}^2\rangle\big)\phi_c$$

- Classical Fluid Variables: $\rho_c := \phi_c^*\phi_c, \qquad \vec{j}_c := \Im[\phi_c^*\nabla\phi_c], \qquad \phi_c \equiv \sqrt{\rho_c}\mathrm{e}^{i\theta_c}$

- Continuity-like Equation:

$$\partial_t\rho_c + \nabla\cdot(\vec{j}_c) = \rho_c\Delta_{\mathrm{C}}$$

- Euler-like Equation:

$$\partial_t\theta_c - \frac{1}{2\sqrt{\rho_c}}\nabla^2\sqrt{\rho_c} + \frac{1}{2}(\nabla\theta_c)^2 + U_{\mathrm{ext}} + g\rho_c + \Delta_{\mathrm{E}} = 0$$

Modification of Background Spacetime

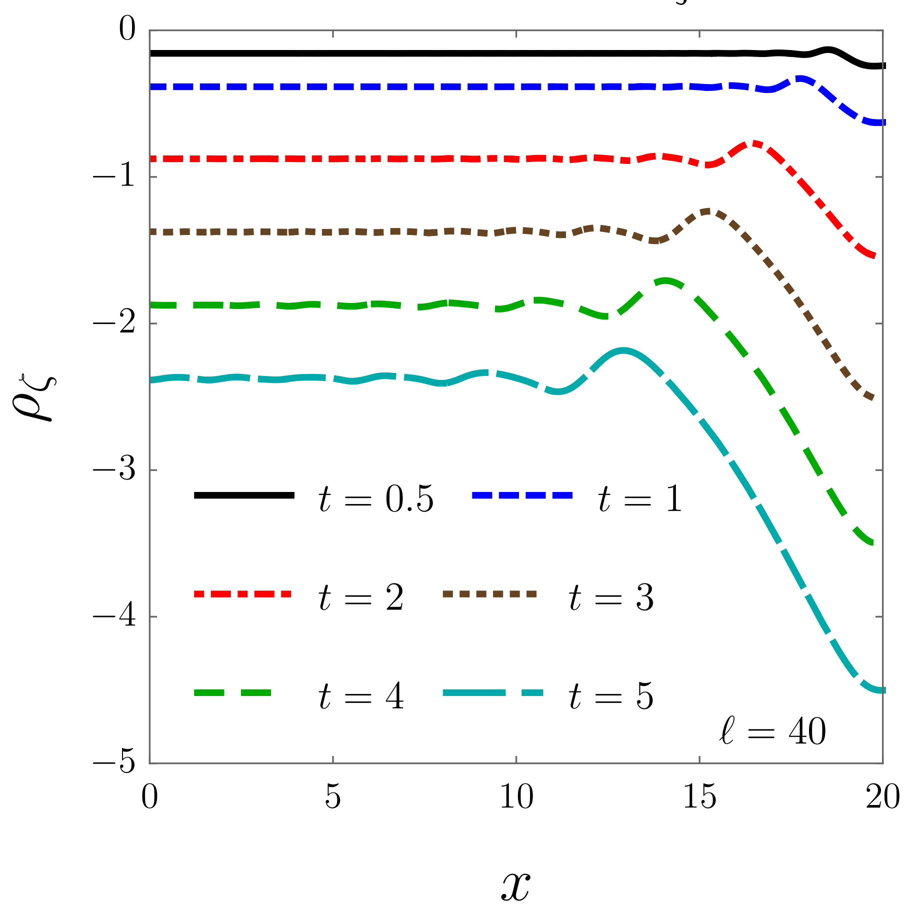
Finite-Size One-Dimensional Homogeneous Gas

system size: ℓ

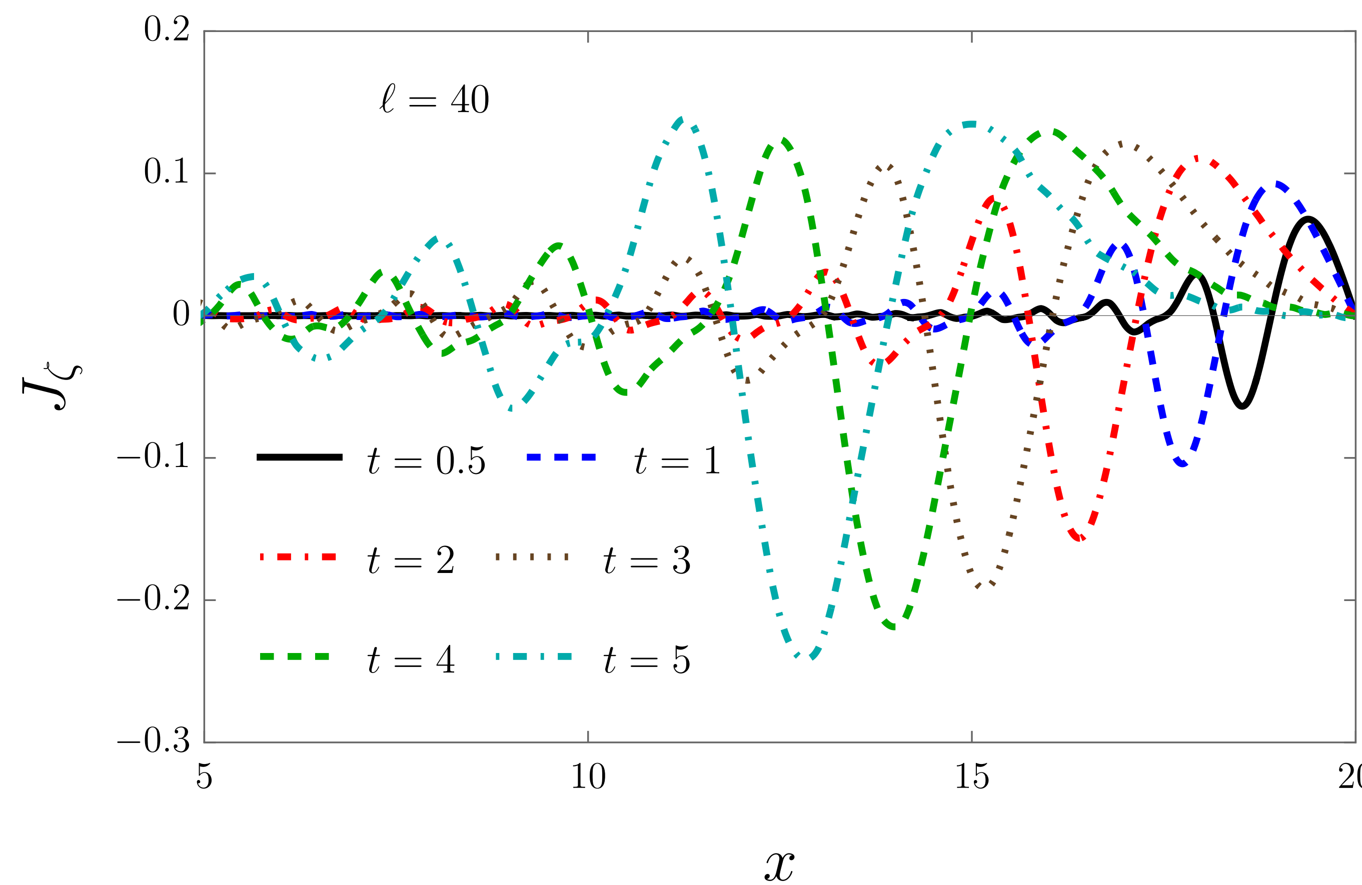
quench the interaction: $g = g_0\Theta(t)$

$\Theta(t)$ is a Heaviside step function, $g_0 = \text{constant}$

$\rho_c = \rho_0 + \rho_\zeta$



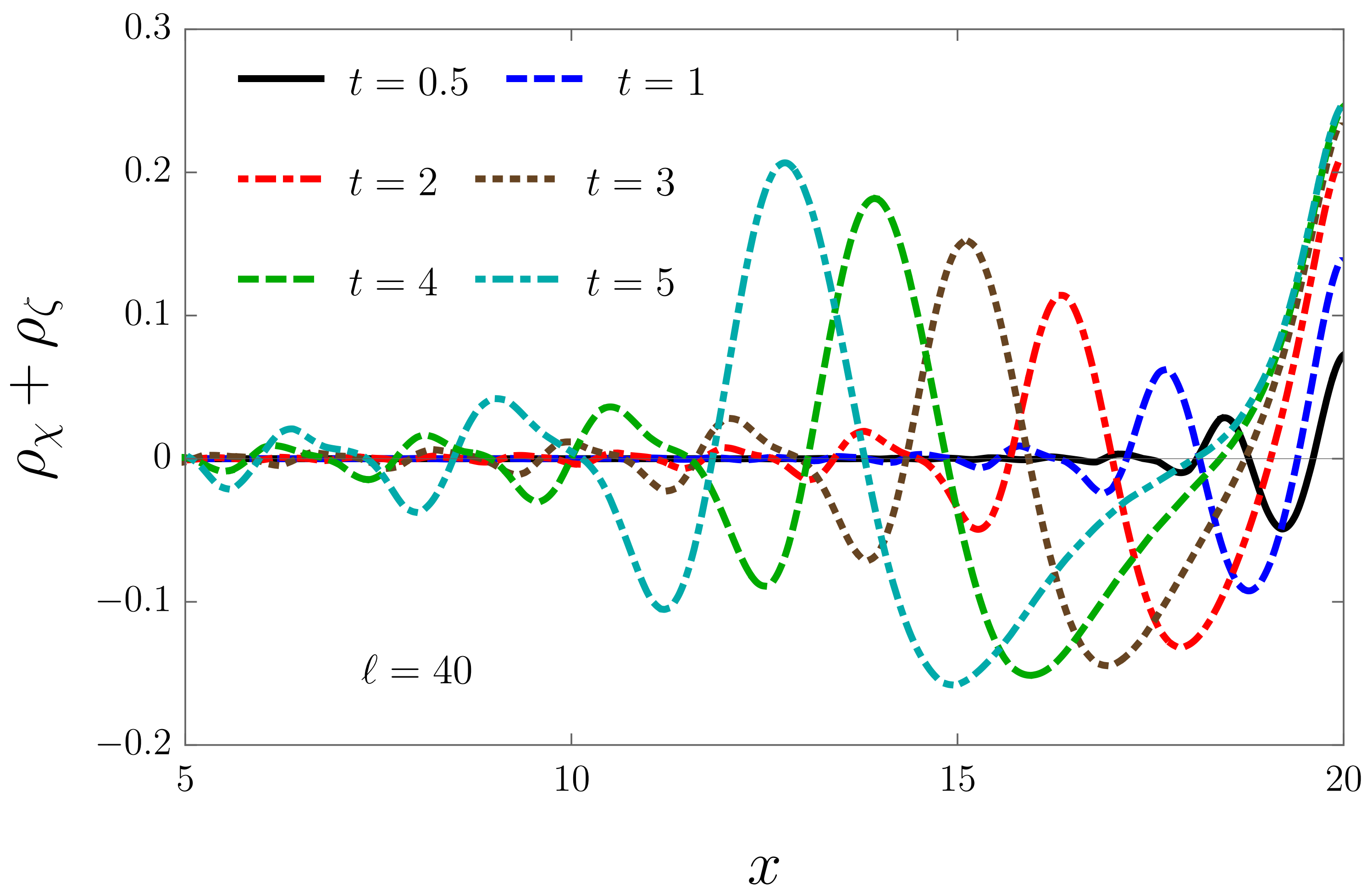
$j_c = j_0 + j_\zeta = j_\zeta$



Phys. Rev. A 106, 053319 (2022)

Measurable Effect

Modification of Density from Condensate



$$\rho_\chi = \rho_0 \langle \hat{\psi}^\dagger \hat{\psi} \rangle$$

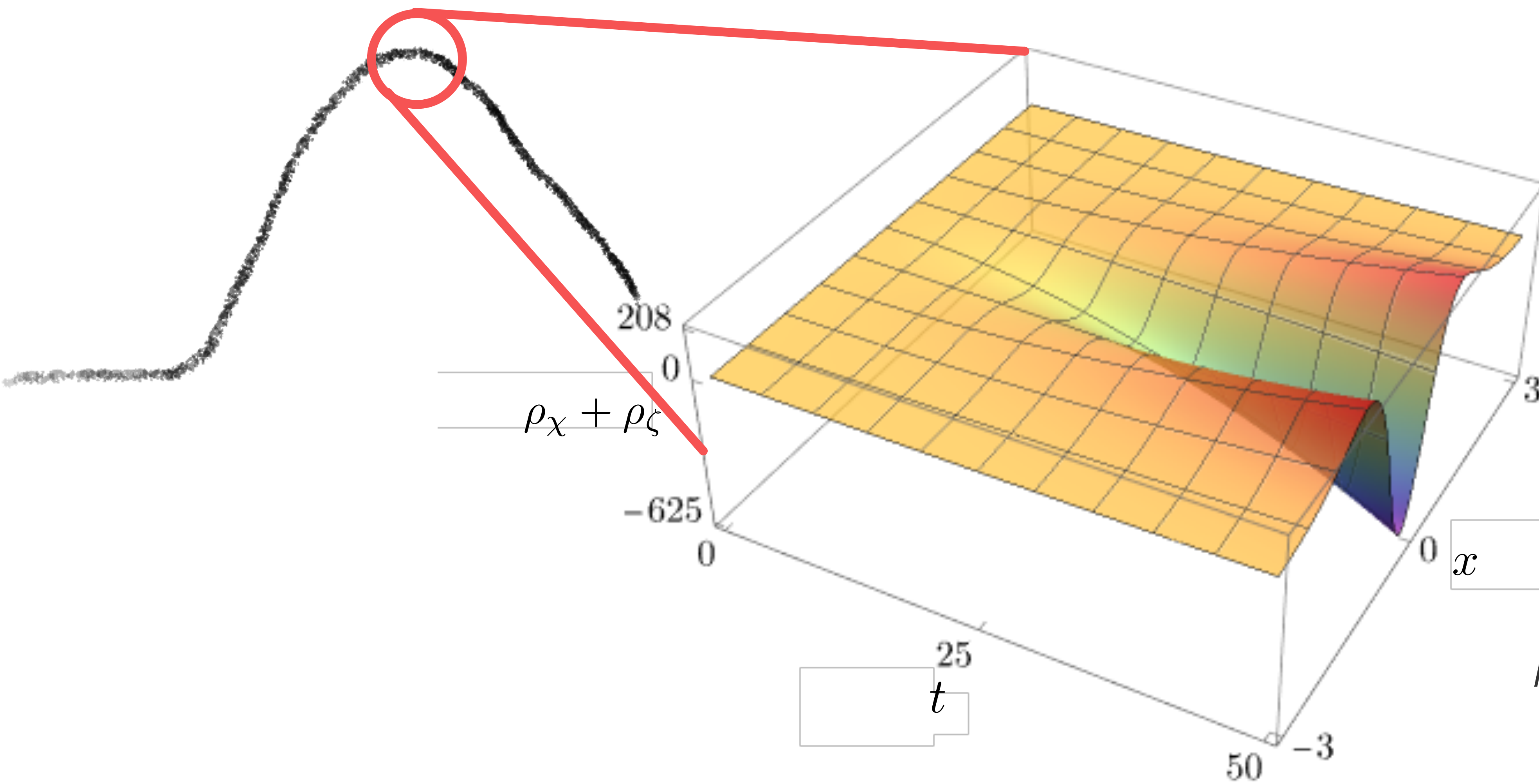
For Finite-Size Homogeneous One-Dimensional Bose Gas

Measurable Effect

For Optical Soliton

Fundamental Soliton Background: $\phi_0 = \mathrm{sech}\, x\, \mathrm{e}^{it/2}$

in soliton unit.



New J. Phys. **27** 015001 (2025)

Photon Density Distortion

Sound Wave Kinematics (Is the sound wave still in the curved spacetime?)

Formalism (Quantum Field)

- Amended Bogoliubov de Gennes Equation:

$$i\Big(\partial_t + \vec{v}_c \cdot \nabla + \frac{1}{2}\Delta_{\mathrm{C}}\Big)\hat{\psi} = -\frac{1}{2}\Big(\nabla^2 + \frac{\nabla\rho_c}{\rho_c}\cdot\nabla\Big)\hat{\psi} + g\rho_c(\hat{\psi} + \hat{\psi}^\dagger) - \Delta_{\mathrm{E}}\hat{\psi}$$

- Madelung Representation: $\hat{\Psi} = \sqrt{\rho_c}\mathrm{e}^{i\theta_c}\Big(1 + \frac{\delta\hat{\rho}}{2\rho_c} + i\delta\hat{\theta}\Big)\frac{\hat{A}}{\sqrt{\hat{N}}}$

$$\delta\hat{\rho} := \rho_c(\hat{\psi} + \hat{\psi}^\dagger), \qquad \delta\hat{\theta} := \frac{1}{2i}(\hat{\psi} - \hat{\psi}^\dagger).$$

- Equation of Motion for Phase Fluctuation:

$$-\frac{1}{\rho_c}\Big(\partial_t + \nabla\cdot\vec{v}_c - \frac{1}{2}\Delta_{\mathrm{C}}\Big)\rho_c D^{-1}\Big(\partial_t + \vec{v}_c\cdot\nabla + \frac{1}{2}\Delta_{\mathrm{C}}\Big)\delta\hat{\theta} + \Big(\frac{1}{\rho_c}\nabla\cdot(\rho_c\nabla) + 2\Delta_{\mathrm{E}}\Big)\delta\hat{\theta} = 0$$

where $D := g\rho_c - \frac{1}{4\rho_c}\nabla\cdot(\rho_c\nabla) - \frac{1}{2}\Delta_{\mathrm{E}}.$

Geometric Form

- Generalised Sound velocity (Assumption): $D = c^2$

  Hydrodynamic Approximation:

  $$c^2 = g\rho_c - \frac{1}{2}\Delta_{\mathrm{E}}$$

  Eikonal Approximation:

  $$c^2 = g\rho_c + \frac{1}{4}k^2 - \frac{1}{2}\Delta_{\mathrm{E}}$$

- Geometric form of Equation of Motion for Sound Wave:

$$(\Box + m^2)\delta\hat{\theta} = 0$$

where $m^2 = \left(\frac{c^2}{\rho_c}\right)^{\frac{1}{d-2}}\left[-\frac{1}{2c^2}\left(\partial_t + \nabla\cdot\vec{v}_c + \frac{1}{2}\Delta_{\mathrm{C}}\right)\Delta_{\mathrm{C}} + 2\Delta_{\mathrm{E}}\right],$

$$\mathfrak{g}_{\mu\nu} = \left(\frac{\rho_c}{c}\right)^{\frac{2}{d-1}}\left(\begin{array}{c|c} -(c^2 - v_c^2) & -v_c^j \\ \hline -v_c^i & \delta^{ij} \end{array}\right).$$

Correlation Function Change

Finite-Size One-Dimensional Homogeneous Gas

quench the interaction: $g = g_0\Theta(t)$

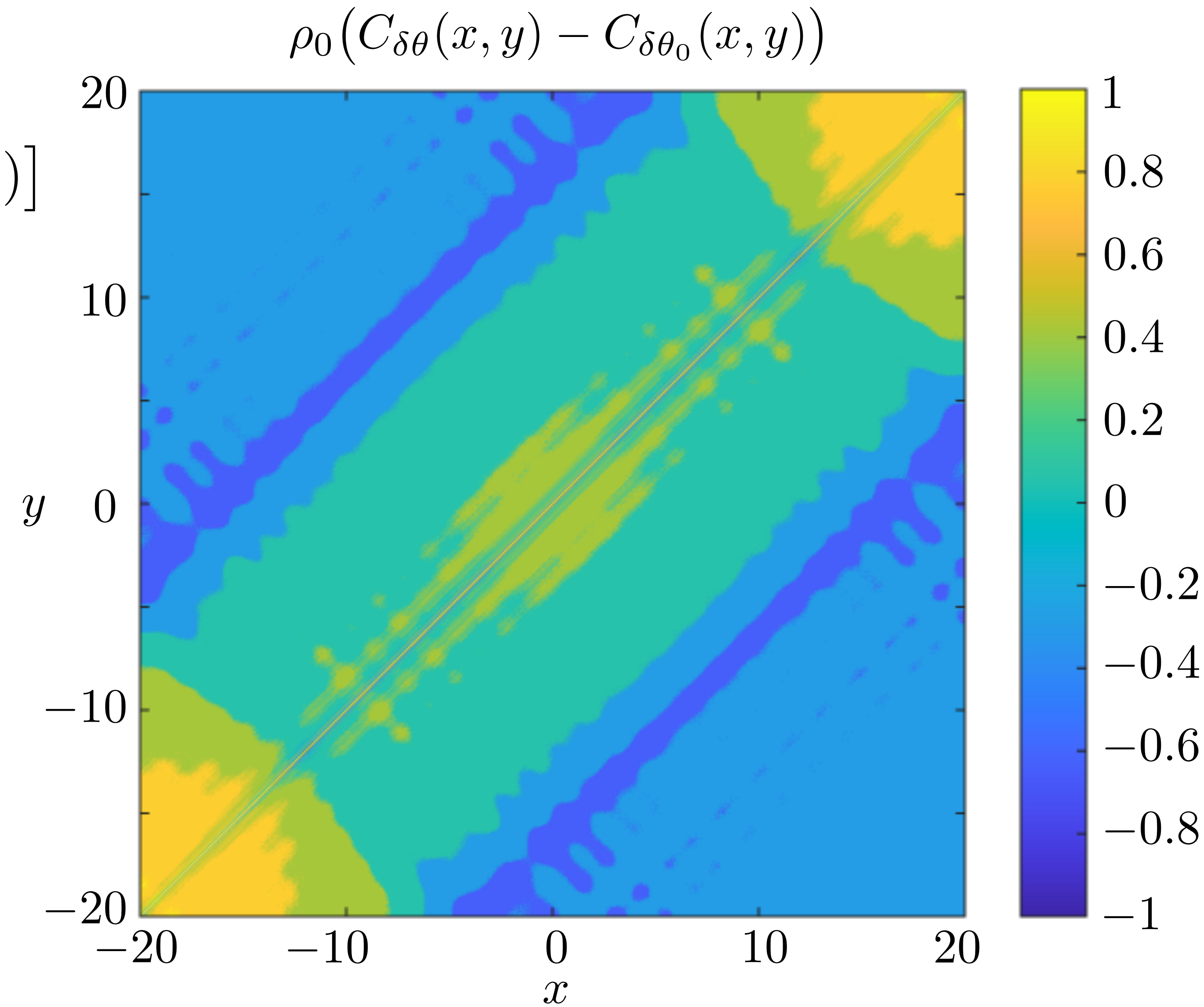
$\Theta(t)$ is a Heaviside step function, $g_0 = \text{constant}$

Mode Expansion:

$$\hat{\psi}(t,x) = \sum_{n=0}^{\infty} \left[\hat{a}_n u_n(t,x) + \hat{a}_n^{\dagger} v_n^*(t,x)\right]$$

where

$$u_n(t<0,x) = \mathrm{e}^{-i\omega_n t} u_{\omega_n}(x)$$
$$v_n(t<0,x) = 0$$



The change of correlation function $C_{\delta\theta}(x,y) = \langle \delta\hat{\theta}(t,x)\delta\hat{\theta}(t,y)\rangle$ with and without backreaction at $t = 5$.

Correlation Function Change

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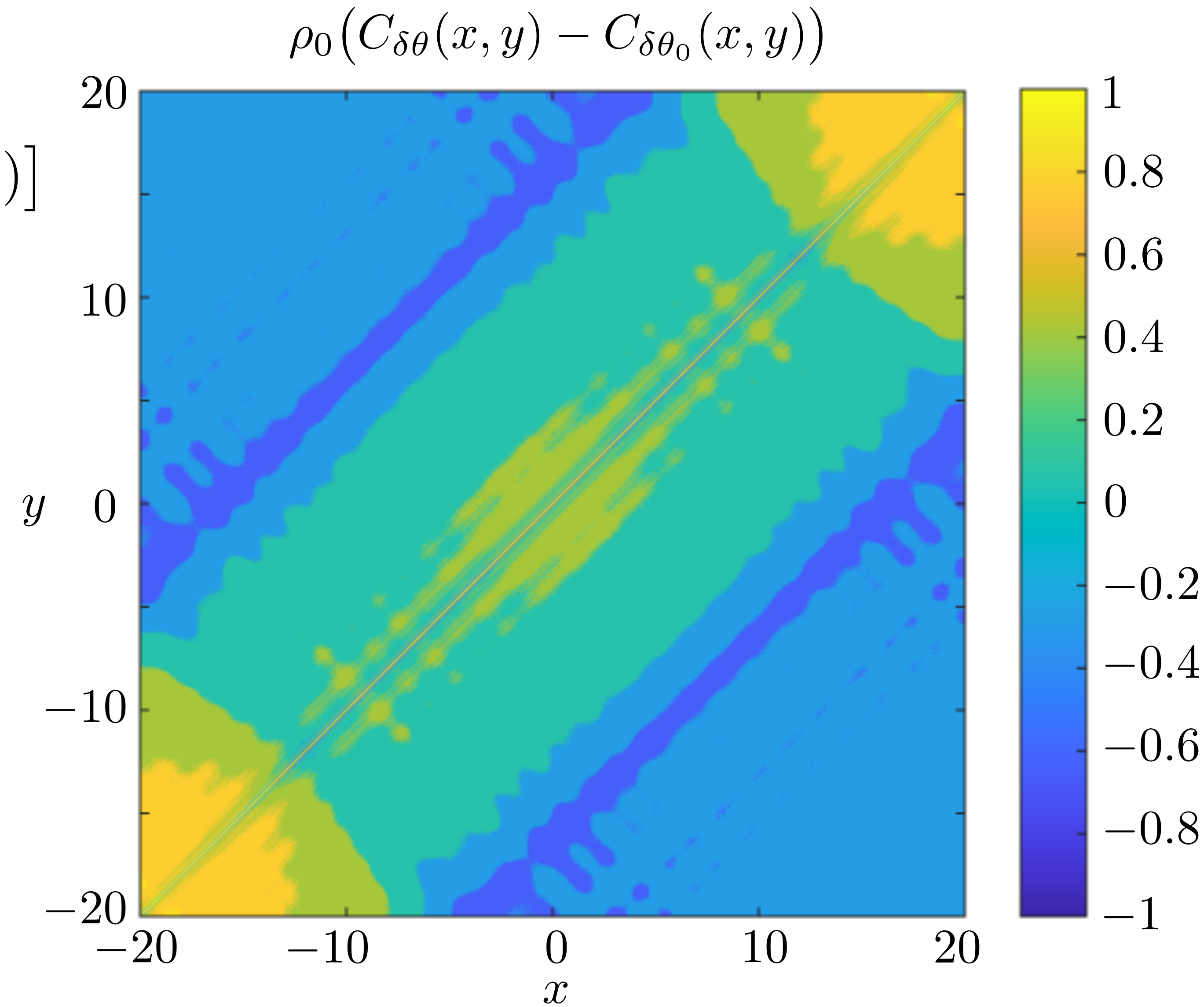

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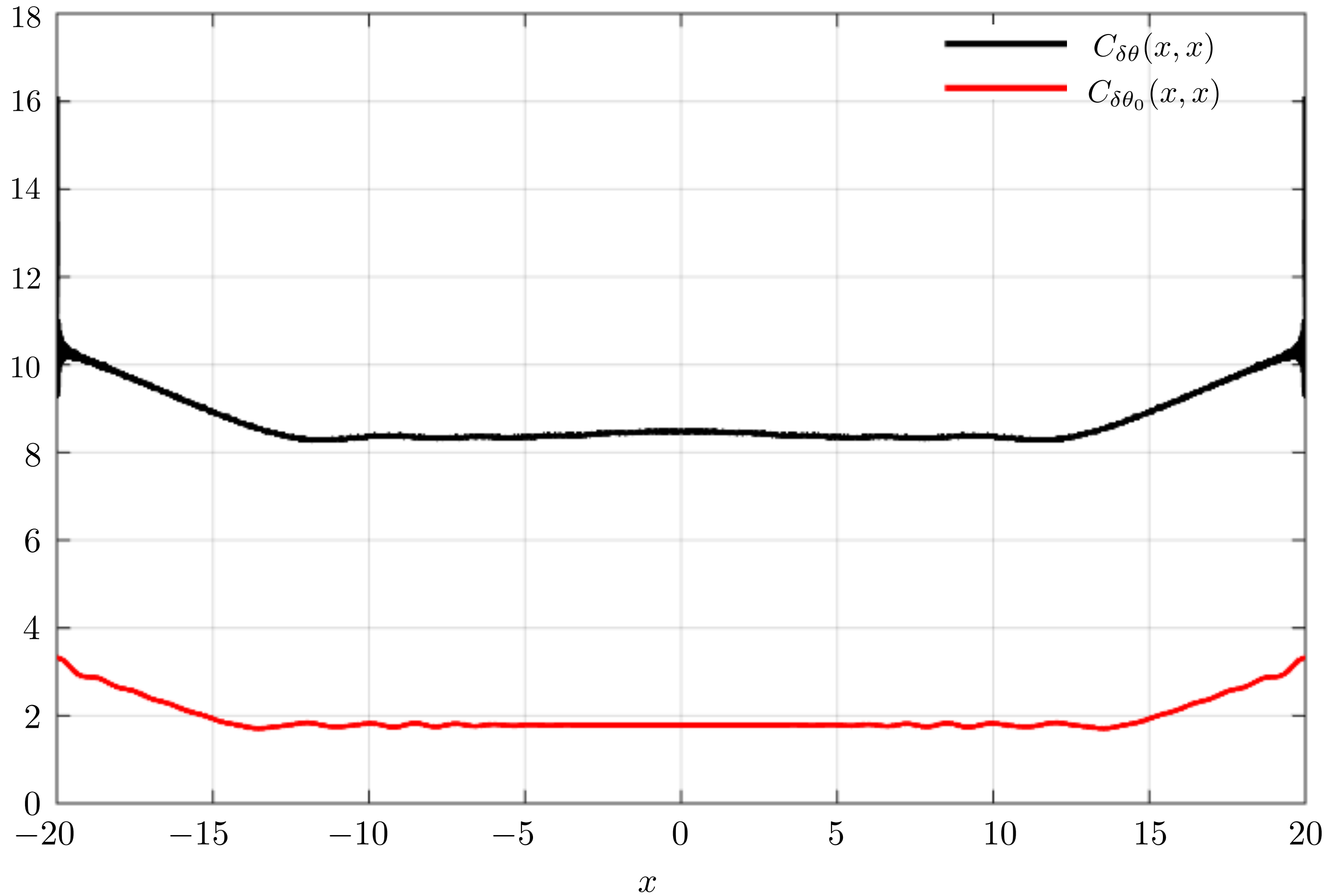
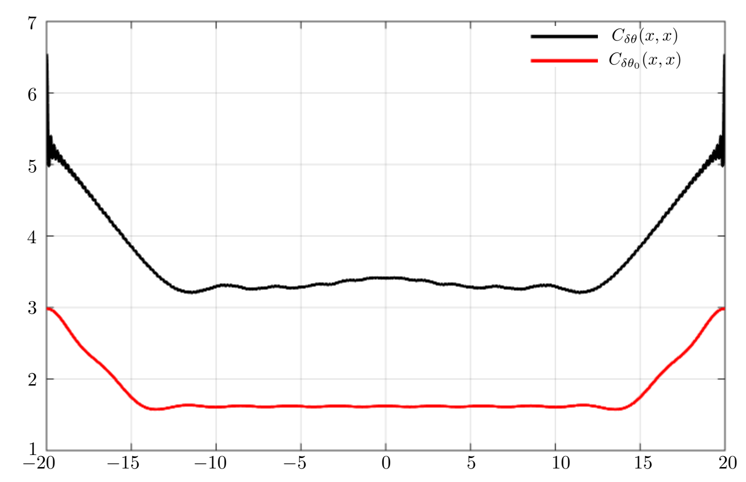
Correlation Function Change

Finite-Size One-Dimensional Homogeneous Gas

Correlation Function at the same point at $t = 5$



1000 modes

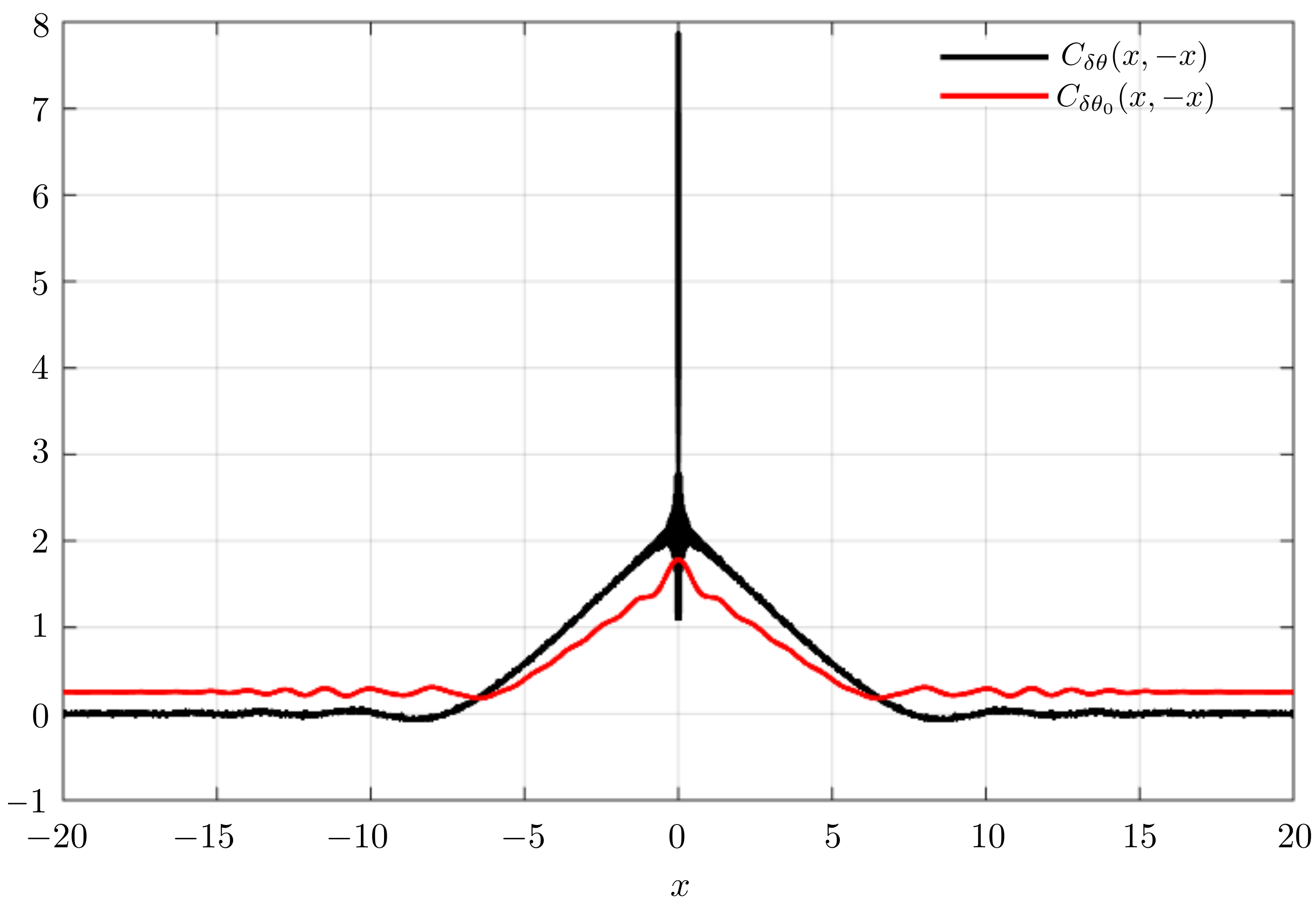


200 modes

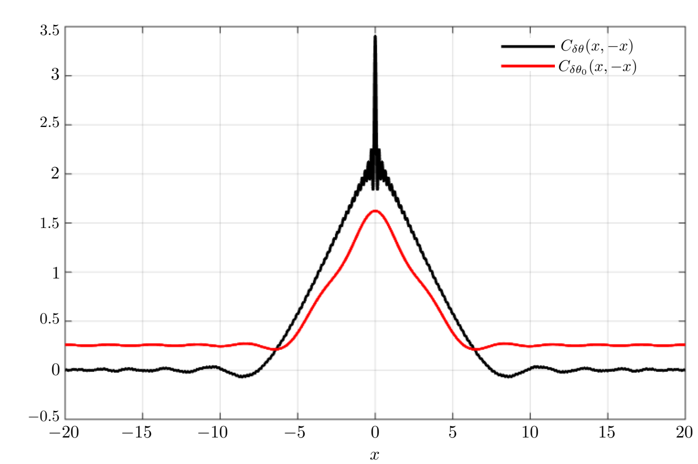
Correlation Function Change

Finite-Size One-Dimensional Homogeneous Gas

Correlation Function at the opposite point at $t = 5$



1000 modes



200 modes

Conclusion

- Backreaction is the modification of classical background dynamics due to the matter field
- In analogue gravity, choice of background and quantum field is not natural.
- Using number-conserving approach, one can show that the sound wave still resides in the classical background spacetime under the backreaction.
- As a result of backreaction, sound wave gains spacetime dependent mass.
- Modification of correlation function in backreacted spacetime can be investigated in analogue systems.
- Backreaction effect occurs in the inverse of particle number order.

Thank you

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