Motivating an analog de Sitter Schwinger effect

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In colaboration with
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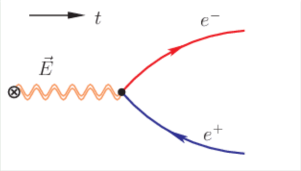
(Mostly) based on *2503.01981 and 2508.14973*

Spontaneous **particle creation**

- The Schwinger effect has for long time been known
  Fritz, Sauter (1931); W. Heisenberg, H. Euler (1936); J. Schwinger (1951)
  - Pairs of charged particles and anti-particle created by background $\vec{E}$
  - It can happen for constant $\vec{E}$, but requires time dependent vector potential $\vec{A}$
  - Strong Electric fields are required
  - Effects are exponentially suppressed by the mass



$$log\, \Gamma \propto -\frac{m^2 c^3}{ehE} \quad \rightarrow \quad E_{CR} \simeq 1.32 \times 10^{18}\ \mathrm{Vm}^{-1}$$

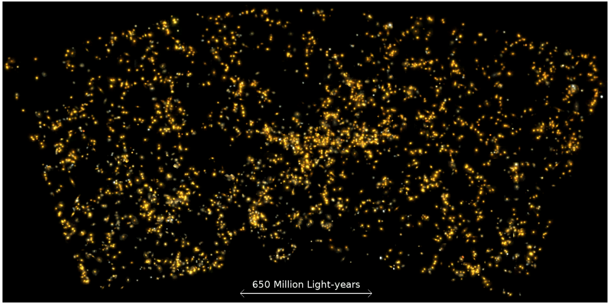
Spontaneous **particle creation**

- A similar effect can happen in Curved backgrounds

L. Parker (1966); S. W. Hawking (1975)

  - Particle production from vacuum under **time dependent gravitational field**
  - Effects are already important in cosmology
    - LSS might be seeded by accelerated expansion during inflation

Cosmological Schwinger effect, J. Martin 0704.3540



Sloan Digital Sky Survey, in Saraswati supercluster. Credit: IUCAA

Spontaneous **particle creation** by **time-varying backgrounds**

- Besides cosmic structure, **magnetic fields** observed in the Universe might also have a cosmological origin
    - Proper conditions for Schwinger **pair production** might have existed in the early universe

Exact setting to combine the two examples for particle production

Spontaneous **particle creation** by **time-varying backgrounds**

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Concrete Cosmological applications in:

- Inflationary Magnetogenesis
  - Generate the **observed magnetic fields** present in voids our Universe
- Generation of particle Dark Sectors
  - Candidates for non-thermal **dark matter**

Spontaneous **particle creation** by **time-varying backgrounds**

During inflation (χ), in practice, this could be realized with

$$S = -\int d^4x \sqrt{-g}\left[\frac{1}{2}\partial_\mu\chi\partial^\mu\chi + V(\chi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha}{4f}\chi F_{\mu\nu}\tilde{F}^{\mu\nu} + \mathcal{L}_{ch}(\phi, A_\nu)\right]$$

$$\ddot{A}_\pm + (H+\sigma)\dot{A}\pm + \left(\frac{k^2}{a^2} \mp \frac{\alpha}{f}\dot{\chi}\frac{k}{a}\right)A_\pm = 0$$

$$\dot{\rho}_\phi + 4H\rho_\phi = \sigma\langle E^2\rangle \qquad \sigma = \frac{J_\phi}{E}$$

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- No analytical solutions, difficult to test if (renormalization) results make sense
- **Forget about inflation**
  - Fix a de-Sitter background
  - Constant electric field $\vec{E}$ (along z direction)

(Scalar) QED in de-Sitter

$$S = \int d^4x\sqrt{-g}\left\{-g^{\mu\nu}\left(\partial_\mu - ieA_\mu\right)\phi^*\left(\partial_\nu + ieA_\nu\right)\phi - (m_\phi^2 + \xi R)\phi^*\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right\}$$

- Set a constant a electric field

$$A_\mu = \frac{E}{H^2\tau}\delta_\mu^z, \qquad F_{\mu\nu}F^{\mu\nu} = -2E^2$$

- Equations of motion

$$\phi_k'' + 2aH\phi_k + \omega_{\boldsymbol{k}}^2\phi_{\boldsymbol{k}} = 0$$

- Analytical solution with Whittaker functions

$$\phi_k = \frac{\mathrm{e}^{-\pi\lambda r/2}}{a\sqrt{2k}}W_{i\lambda r,\mu}(2ik\tau)$$

(Scalar) QED in de-Sitter

- A_ν e.o.m.

$$\nabla^\nu F_{\mu\nu} = J^\phi_\mu \quad \text{with} \quad J^\phi_\mu = \frac{ie}{2}\left\{\phi^\dagger\left(\partial_\mu + ieA_\mu\right)\phi - \phi\left(\partial_\mu - ieA_\mu\right)\phi^\dagger\right\} + \text{ h.c.}$$

- **Divergent expectation value**

$$\left\langle 0 \left| J^\phi_z \right| 0 \right\rangle = \frac{2e}{a^4}\int \frac{d^3k}{(2\pi)^3}\left(k_z + eA_z\right)\left|\phi_k\right|^2$$

With a cut off momentum ζ

$$\left\langle J^\phi_z \right\rangle = aH\frac{e^2E}{4\pi^2}\lim_{\zeta\to\infty}\left[\frac{2}{3}\left(\frac{\zeta}{aH}\right)^2 + \frac{1}{3}\ln\frac{2\zeta}{aH} - \frac{25}{36} + \frac{\mu^2}{3} + \frac{\lambda^2}{15} + F_\phi(\lambda,\mu)\right]$$

$$\lambda = \frac{eE}{H^2}, \qquad \mu^2 = \frac{9}{4} - \frac{m_\xi^2}{H^2} - \lambda^2 \quad \text{and} \quad m_\xi^2 = m_\phi^2 + 12\,\xi H^2$$

T. Kobayashi, N. Afshordi 2014, T. Hayashinaka, J. Yokoyama 2016, M. Banyeres, G. Domenèch, J. Garriga 2018

Revising the Renormalization

- Schwinger effect with **classical** $\vec{E}$
  - A_μ not quantized (charged particles do not accelerate in dS)
  - Only charged particles (ϕ/ψ) are quantized
    - No photon loops $\rightarrow$ ϕ/ψ propagator not corrected at loop level
    - Running of charge e $\Longleftrightarrow$ A_μ (from Ward Identity)

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        - Running of charge e $\Longleftrightarrow$ A_μ (from Ward Identity)
    - **Only one counter-term** in semi-classical Lagrangian

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{4}\delta_3(F_{\mu\nu})^2 - eA_\mu J^\mu + \dots \ ,$$

And the corrected semi-classical equations of motion will be

$$(\delta_3 + 1)\,\nabla^\nu F_{\mu\nu} = \langle J_\mu \rangle\,.$$

Revising the Renormalization

- For a constant electric field in de-Sitter $\left(A_\mu = \frac{E}{H^2\tau}\delta^z_\mu\right)$,

$$(\delta_3 + 1)\,\nabla^\nu F_{\mu\nu} = (\delta_3 + 1)\,(-2aHE\delta^z_\nu)\,.$$

- Define the **renormalized current**

$$\nabla^\nu F_{\mu\nu} = \langle J_\mu \rangle_{\rm ren}$$
$$\langle J_\mu \rangle_{\rm ren} = \langle J_\mu \rangle_{\rm reg} - (-2aHE\delta^z_\nu)\delta_{3\ \rm reg}\,.$$

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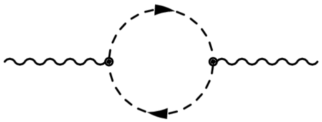
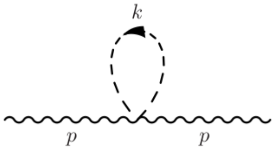
- To get ***physical*** **renormalized current**, **on-shell counter-term**!

$$\Pi(p^2 = m_A^2) = 0 \ \rightarrow\ \delta_3 = -e^2\Pi_2(m_A^2)$$

- With classical A_μ, Π_2 is fully defined (at one loop) by





Revising the Renormalization: Constant $\vec{E}$ in dS?

- In Minkowski $p^2 = m_A^2 = 0$

$$\delta_3 = -e^2\Pi_2(p^2 = 0) \;\rightarrow\; \delta_3^{PV} = -\frac{e^2}{48\pi^2}\ln\frac{\Lambda^2}{m^2}$$

- This results in $\ln m/H$ term that creates **negative conductivies** when $m \ll H$

- But does this condition actually hold for our setting?

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- But does this condition actually hold for our setting?

$$S = -\int d^4x \sqrt{-g}\,\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \;\to\; g^{\alpha\nu} g^{\beta\sigma} \nabla_\alpha F_{\nu\sigma} = 0\,.$$

Taking $A_\mu = \frac{E}{H^2\tau}\delta_\mu^z$, in e.o.m. we find

$$g^{\alpha\nu} g^{\beta\sigma} \nabla_\alpha F_{\nu\sigma} = -2a^{-4} \frac{E}{\tau^3 H^2} \delta_i^z \neq 0\,.$$

- Just a kinetic term is not compatible with constant $\vec{E}$ in de-Sitter

Revising the Renormalization: Constant $\vec{E}$ in dS?

- Introduce an effective mass in Lagrangian
  (Gauge invariance can be maintained with Stueckelberg mass)

$$S = -\int d^4x\sqrt{-g}\left(\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_A^2 A_\mu A^\mu\right).$$

  e.o.m. gives

$$-a^{-4}2\frac{E}{\tau^3 H^2}\delta_i^z - m_A^2 a^{-2}\frac{E}{\tau H^2}\delta_i^z = 0 \;\rightarrow\; m_A^2 = -2H^2.$$

- System requires an "effective" tachyonic mass
- Interpreted as effective source that ensures that $\vec{E}$ is not diluted with expansion
- **Consistency with constant electric field** background implies

$$\Pi(p^2 = m_A^2) = 0 \;\rightarrow\; \delta_3 = -e^2\Pi_2(p^2 = m_A^2 = -2H^2)$$

Finally δ_3

- Pauli-Villars to regularize both δ_3 and $\langle J_\mu \rangle$

$$\nabla^\nu F_{\mu\nu} = \langle J_\mu \rangle_{ren} = \langle J_\mu \rangle_{reg}^{PV} - (-2aHE\delta_\nu^{\ z})\delta_3^{PV}$$

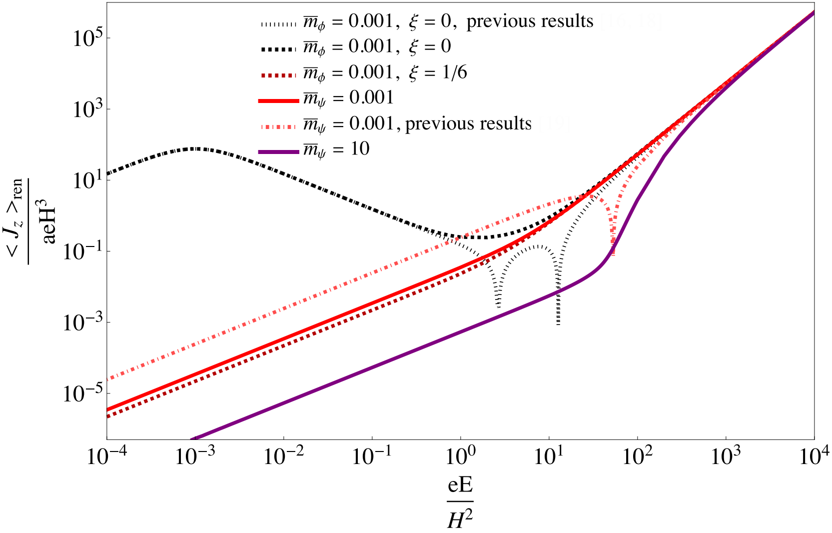
With

$$\delta_3 = \left(\frac{e}{12\pi}\right)^2 \left(3\ln\left(\frac{m^2}{\Lambda^2}\right) - 12\left(\frac{m}{H}\right)^2 + 6\left(2\left(\frac{m}{H}\right)^2 + 1\right)^{3/2} \coth^{-1}\left(\sqrt{2\left(\frac{m}{H}\right)^2 + 1}\right) - 8\right)$$

- We find the **renormalized** current to be

$$\left\langle J_z^\phi \right\rangle_{\mathrm{ren}}^{PV} = aH\frac{e^2 E}{4\pi^2}\left[\frac{1}{3}\ln\frac{m}{H} - \frac{4}{9} - \frac{2}{3}\left(\frac{m}{H}\right)^2 - \frac{2\lambda^2}{15} + F_\phi \right. \\ \left. + \frac{\left(1 + 2\left(\frac{m}{H}\right)^2\right)^{3/2}}{3}\coth^{-1}\left(\sqrt{2\left(\frac{m}{H}\right)^2 + 1}\right)\right]$$

Results


$\overline{m}_\phi = 0.001,\ \xi = 0,$ previous results
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$\overline{m}_\psi = 10$
10^5
10^3
10^1
10^{-1}
10^{-3}
10^{-5}
$\frac{< J_z >_{\text{ren}}}{\text{aeH}^3}$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
10^2
10^3
10^4
$\frac{\text{eE}}{H^2}$

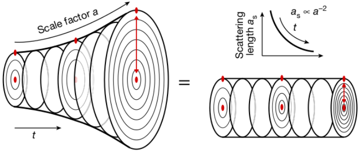
What do we need

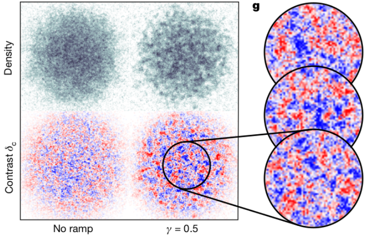
- **Analog space-time expansion**:
  - Multiple examples in both **Atomic BECs**

U. R. Fischer el al., Physical Review A 70, 063615 (2004)
Cheng-An Chen et al.,Phys. Rev. Lett. 127, 060404 (2021)
S. Eckelet al., Physical Review X 8(2), 021021 (2018)
C. Viermann et al., Nature 611(7935), 260 (2022)
M. Sparn et al., Phys.Rev.Lett. 133, 260201 (2024)
V. Gondret et al., Comptes Rendus Physique 25 S2, 1-15 (2025)
...

  - But also in **ultracold quantum fluids of light**

J. Steinhauer et al., Nature Commun. 13, 2890 (2022)





V. Gondret et al

Towards an analog model

What do we need

- Analog charged system with "**electric**" fields
  - Can be done with **synthetic gauge fields**

Couple internal dof of the atoms to external lasers

$\Downarrow$

Neutral particle acquires a geometrical phase on a close contour

$\Downarrow$

State with a modified dispersion relation

$$i\hbar\,\partial_t \Psi = \left[\frac{(-i\hbar\nabla - \mathbf{A})^2}{2m} + V_{\text{ext}} + g\,|\Psi|^2 \right] \Psi$$

J. Dalibard et al.,Reviews of Modern Physics 83(4), 1523 (2011)

Towards an analog model

Has been conceived to simulate curved space dynamics

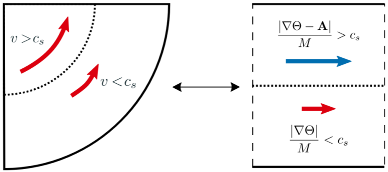
Ex: Superradiance

- In **atomic BECs**:

S. Butera et al., Class. Quantum Grav. 36 034002 (2019)
L. Giacomelli and I. Carusotto, Physical Review A 103(4), 043309 (2021)

- And for **neutral photons**:

T. Ozawa et al., Reviews of Modern Physics 91, 015006 (2019)



Analogue ergosurface
L. Giacomelli and I. Carusotto

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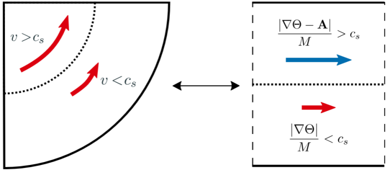

Analogue ergosurface
L. Giacomelli and I. Carusotto

But why not to **simulate** real **electric fields**?

Conclusions

- The **Schwinger effect** is one of the **most striking predictions of non perturbative QFT**
- Remains experimentally unverified since the required field strengths in lab go far beyond current technological capabilities
- Cosmology, however, offers a remarkable twist;
  - In a de Sitter background, **spacetime expansion cooperates with the electric field!**
  - Lowering the effective barrier to pair creation and enhancing production rates.

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- The **Schwinger effect** is one of the **most striking predictions of non perturbative QFT**
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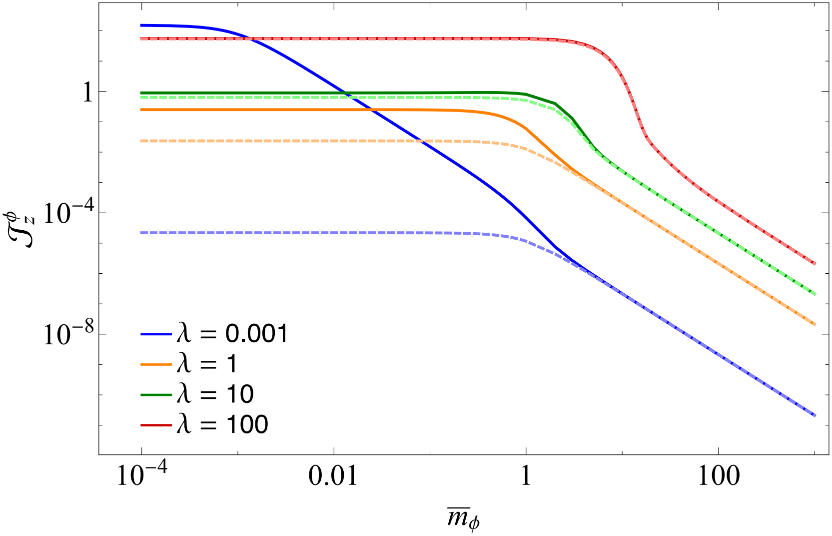
- Cosmology, however, offers a remarkable twist;
  - In a de Sitter background, **spacetime expansion cooperates with the electric field!**
  - Lowering the effective barrier to pair creation and enhancing production rates.
- Can we leverage this conspiracy to bring Schwinger physics within experimental reach?
- Not in high-energy facilities, but in **analogue quantum systems**?

Backup

Results (constant E



Charged particles do not accelerate in dS

For the electrons which are Schwinger produced in dS there are competing effects between damping due to expansion and the acceleration due to the background electric field

$$\frac{dq}{dt} + Hq = eE$$

$$\frac{dq}{d\tau} - \frac{q}{\tau} = -\frac{1}{\tau}\frac{eE}{H}$$

$$q(\tau) = q\left(\tau_0\right) e^{-N_e} + \lambda H \left(1 - e^{-N_e}\right),$$

where q is the momentum of produced pair.

- The first term is the usual damping term due to expansion.
- The second term is due to the presence of the background electric field which accelerates the electrons.
- The net effect is that during inflation very quickly reaches a terminal momentum of λH.

Revising Adiabatic Subtraction

- In a **time-dependent background** the **vacuum** of the theory is generally **evolving** making the concept of “vacuum contribution” ambiguous

- The subtraction is done **mode by mode** removing the expectation evaluated in the adiabatic approx

WKB expansion
$$q_{\boldsymbol{k}}(\tau) = \frac{1}{\sqrt{2W_{\boldsymbol{k}}(\tau)}} \exp\left\{-i\int^{\tau} d\tilde{\tau}\, W_{\boldsymbol{k}}(\tilde{\tau})\right\}$$

$$\left\langle J_z^{\phi}\right\rangle = -\frac{2e}{(2\pi)^3 a^2}\int d^3k\,(k_z + eA_z)\,\frac{1}{2W_{\boldsymbol{k}}}$$

Inserting the mode function q in the e.o.m.

$$W_{\boldsymbol{k}}^2 = \omega_{\boldsymbol{k}}^2 + \frac{3}{4}\left(\frac{W_{\boldsymbol{k}}'}{W_{\boldsymbol{k}}}\right)^2 - \frac{1}{2}\frac{W_{\boldsymbol{k}}''}{W_{\boldsymbol{k}}}$$

Expanded at the n^{th} order

$$W_{\boldsymbol{k}} = W_{\boldsymbol{k}}^{(0)} + W_{\boldsymbol{k}}^{(1)} + W_{\boldsymbol{k}}^{(2)} + \ldots$$

Running / Physical Scale Adiabatic Subtraction

- Take $\Omega_{\boldsymbol{k}}^{\bar{m}}$ with **arbitrary adiabatic expansion scale** $\bar{m}$ (opposed to automatically set $\bar{m} = m$)
  A. Ferreiro, S. Monin, J. Navarro Salas, F. Torrenti 2018, 2022, 2023

$$\Omega_{\boldsymbol{k}}^{\bar{m}2} = (k_z + eA_z)^2 + k_x^2 + k_y^2 + a^2\bar{m}^2 = \omega_{\boldsymbol{k}}^2 + a^2(\bar{m}^2 - m^2) + \frac{a''}{a}$$

And set $W_{\boldsymbol{k}}^{2\,(0)} = \Omega_{\boldsymbol{k}}^{\bar{m}2}$

Find second order $W_{\boldsymbol{k}}^2$ with e.o.m. $W_{\boldsymbol{k}}^{2\,(2)} = \Omega_{\boldsymbol{k}}^{\bar{m}2} - a^2(\bar{m}^2 - m^2) - \frac{a''}{a} + \frac{3}{4}\left(\frac{\Omega_{\boldsymbol{k}}^{\bar{m}\prime}}{\Omega_{\boldsymbol{k}}^{\bar{m}}}\right)^2 - \frac{1}{2}\frac{\Omega_{\boldsymbol{k}}^{\bar{m}\prime\prime}}{\Omega_{\boldsymbol{k}}^{\bar{m}}}$

$$\left\langle J_z^{\phi}\right\rangle^{(2)} = \lim_{\zeta\to\infty}\frac{eaH^3}{(2\pi)^2}\left[\frac{2\lambda}{3}\left(\frac{\zeta}{aH}\right)^2 - \frac{2\lambda^3}{15} - \frac{\lambda}{3}\left(\frac{m}{H}\right)^2 + \frac{\lambda}{3}\ln\left(\frac{2\zeta}{a\bar{m}}\right) + \frac{\lambda}{18}\right]$$

- And the **renormalized** current is given by

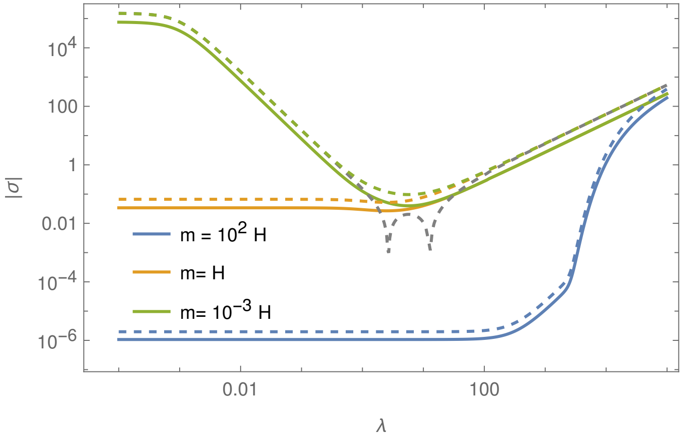
$$\left\langle J_z^{\phi}\right\rangle_{\mathrm{ren}}^{AS} = \left\langle J_z^{\phi}\right\rangle - \left\langle J_z^{\phi}\right\rangle^{(2)} = aH\frac{e^2E}{4\pi^2}\left[\frac{1}{3}\ln\frac{\bar{m}}{H} - \frac{2\lambda^2}{15} + F_{\phi}(\lambda,\mu,r)\right]$$

(Similar to Banyeres et al)

- Value of $\bar{m}$ has to be set to obtain the **appropriate adiabatic vacuum** evolution

Renormalized Conductivities Pauli-Villars vs Adiabatic Subtraction

- Successfully **removed the infrared divergences** ($\ln m/H$) that lead to negative conductivities



$$\sigma_z \equiv \frac{1}{aH}\frac{\langle J_z\rangle}{e^2 H}$$

Solid: Pauli-Villars
Dashed: Adiabatic Subtraction
Grey: Old results

(Scalar) QED in de-Sitter

- The **renormalized** current has been found, with different prescriptions, to be

$$\langle J_z \rangle_{\rm ren} = aH\frac{e^2 E}{4\pi^2}\left[\frac{1}{6}\ln\frac{m_\xi^2}{H^2} - \frac{2\lambda^2}{15} + F_\phi(\lambda,\mu)\right]$$

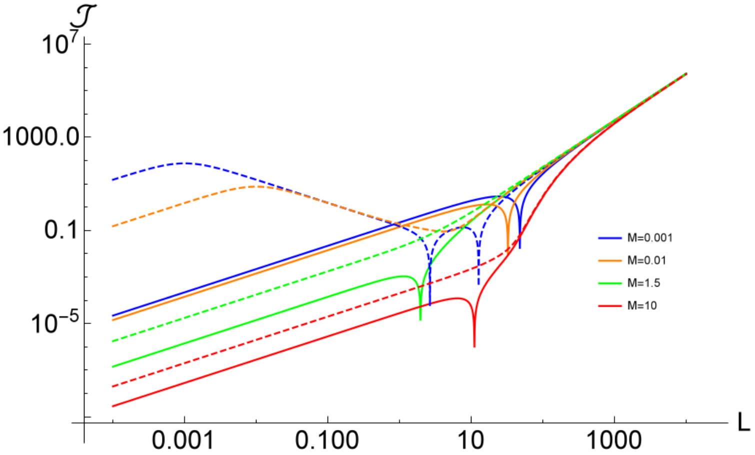
$$\lambda = \frac{eE}{H^2}, \qquad \mu^2 = \frac{9}{4} - \frac{m_\xi^2}{H^2} - \lambda^2 \quad \text{and} \quad m_\xi^2 = m_\phi^2 + 12\,\xi H^2$$

- Adiabatic Subtraction — *T. Kobayashi, N. Afshordi 2014*
- Point Splitting — *T. Hayashinaka, J. Yokoyama 2016*
- Pauli Villars — *M. Banyeres, G. Domenèch, J. Garriga 2018*

- Similar expressions for **fermions**
  - Adiabatic Subtraction — *T. Hayashinaka, T. Fujita, J. Yokoyama 2016*

- A_μ has never been considered to be dynamical (just a background)

Renormalization QED in de-Sitter



$= \lambda = \frac{eE}{H^2}$

IR divergence $\sim$ ln $\frac{m}{H}$

Dashed: Scalars $\xi = 0$

Solid: Fermions

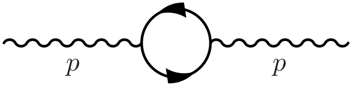
$\Downarrow$

Negative J when $\lambda < 1$

T. Hayashinaka, T. Fujita, J. Yokoyama 2016

Revising the Renormalization: Constant $\vec{E}$ in dS?

Computing δ_3 as in Minkowski with the external momentum fixed by $p^2 = m_A^2 = -2H^2$



- Corrected $\ln m/H$ factor $\rightarrow$ Currents in the massless limit become **finite**
- But for fermions and conformal scalars ($\xi = 1/6$), **when** $eE \ll H^2$ **they are negative**

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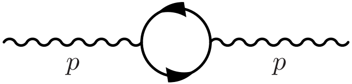

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- But for fermions and conformal scalars ($\xi = 1/6$), **when $eE \ll H^2$ they are negative**

- Minkowski propagators in the loop are not accurate
- Do not capture correctly **IR effects**

- We try a correction as exact de-Sitter does not seem doable (to us)

Scalars $m^2 \rightarrow m^2 + \xi R$ **Fermions** $m^2 \rightarrow m^2 + \frac{1}{4}R$