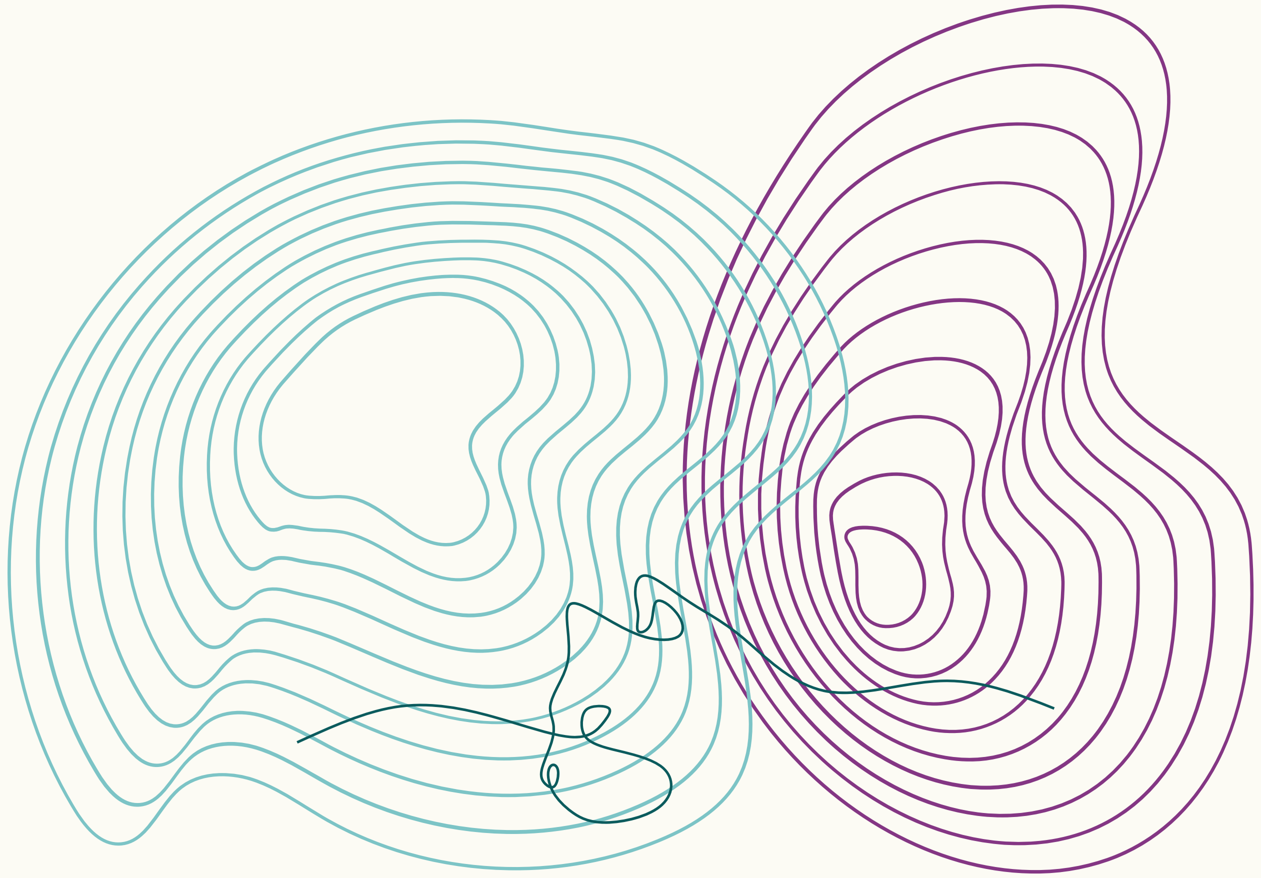
SAME SAME OR DIFFERENT:

implications of quantum trajectories for a superposition universe

Analogue Gravity in 2026
14 Jan 2026, Benasque



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[arXiv:2508.06231] - *work in collaboration with Kratika Mazde and Patrick Peter*

Institut d'astrophysique de Paris

LEVERHULME TRUST

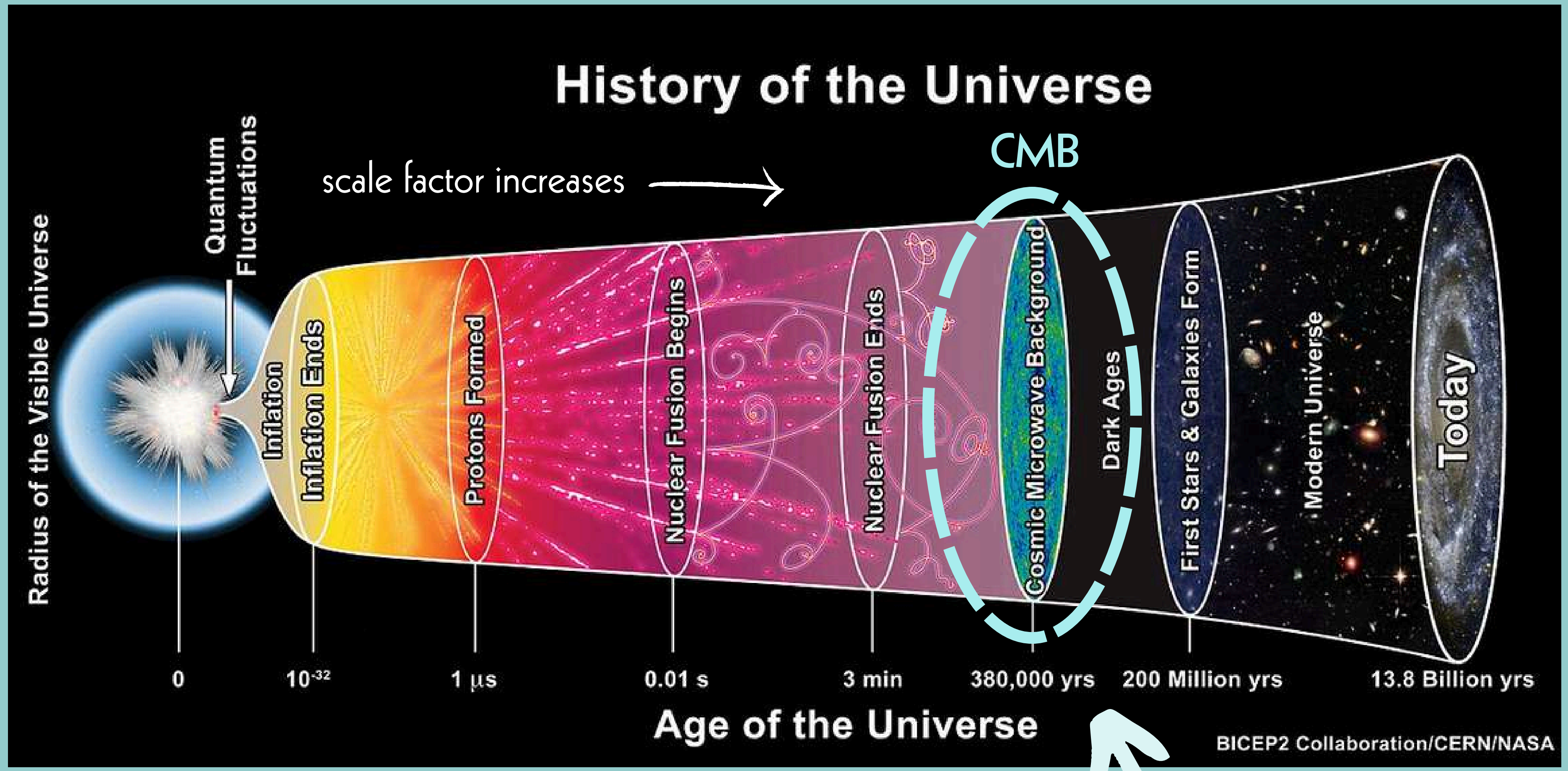
IN THE GRAND SCHEME…

- Homogeneous isotropic spacetime described by Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$\mathrm{d}s^2 = -\underbrace{N^2(t)}_{\text{time choice}}\mathrm{d}t^2 + \underbrace{a^2(t)}_{\text{scale factor}}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

- We are interested in:
  - Quantum cosmological model that treats the background and the perturbations quantum mechanically
  - Resolution of the big bang singularity

→ Effects on the primordial power spectrum



big bang singularity

radiation domination

matter domination

cosmological constant domination

quantum effects in the very early universe → primordial power spectrum

INTRO TO QUANTUM COSMOLOGY

- **Minisuperspace models**: Wheeler-DeWitt quantisation on the reduced phase space of general relativity to obtain a quantum description of the universe $(\hat{\mathcal{H}}_{\mathrm{FLRW}} + \hat{\mathcal{H}}_{\mathrm{matter}})\Psi(a,\phi) = 0$ [DeWitt ('67)]

approximation to a full theory of quantum gravity

- **Problem of time** [Isham, ('93)]
  - GR is a fully constrained system → no external time parameter
  - Evolution happens w.r.t. an internal degree of freedom serving as a clock (here: perfect fluid)

[Małkiewicz, Peter, ('19)]
[Gielen, Menéndez-Pidal, ('20, `21)]
[de Cabo Martin, Małkiewicz, Peter, ('22)]
[Bergeron, Dapor, Gazeau, Małkiewicz, ('14)]
....

- **Ambiguities**
  - Quantisation: choice of clock degree of freedom, canonical variables, quantisation scheme
  - Extraction of an effective evolution of the scale factor
  - (Semiclassical) state of the universe

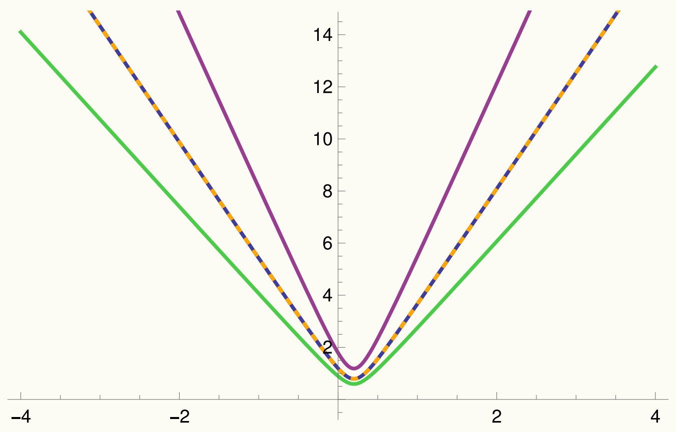
different phenomenology (e.g. singularity resolution)

IN A NUTSHELL

Minisuperspace quantisation of FLRW spacetime

bounce

Quantum **trajectories** to obtain quantum corrected evolution of the scale factor



Universe in a **superposition** $\Psi = \mathcal{N}(\psi_0 + \rho e^{i\delta}\psi_1)$

Influence on **perturbations**?

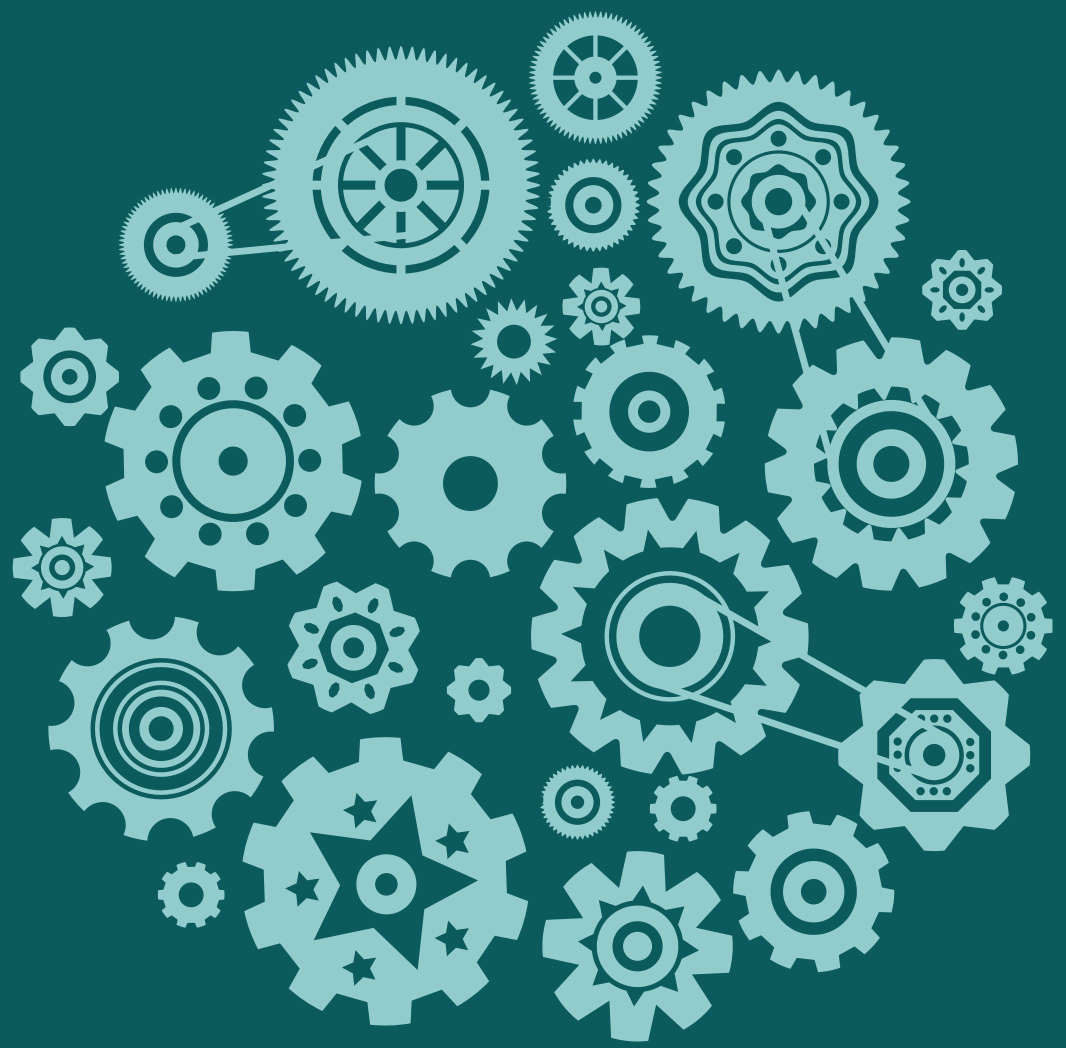
Copenhagen QM: Interaction between multiple background states and their perturbations leads to non Gaussianities in perturbations

[Bergeron, Małkiewicz, Peter, ('24)]
[Bergeron, Małkiewicz, Peter, ('25)]

Trajectories give different background evolution that alters the dynamics of perturbations

SETUP

Quantising the universe



CLASSICAL BACKGROUND

- Geometry: FLRW Hamiltonian

lapse

$$\mathcal{H}_{\mathrm{ADM}} \to \mathcal{H}_{\mathrm{FLRW}} = -\frac{\overset{\kappa = 8\pi G}{\kappa} N}{12\mathcal{V}_0 a} p_a^2$$

momentum conjugate to scale factor $\{a, p_a\} = 1$

spatial section volume

scale factor

- Matter: perfect fluid $\mathcal{H}_{\mathrm{fluid}} = \gamma(w)\frac{N}{a^{3w}}\, p_\phi^{1+w}$ [Schutz, ('70) ('71)]
  - Perfect fluid as matter clock fixes the lapse $N = -a^{3w}$ equation of state parameter

- Canonical transformations to convenient variables

$(a, p_a) \to (q, p)$ with $p \propto a^{\frac{3}{2}(1-w)} H$ $q \propto a^{\frac{3}{2}(1-w)}$ and $p_\phi \to p_\tau = -\gamma p_\phi^{1+w}$

scale factor expansion rate of the universe Hubble rate

matter clock

$\boxed{\text{Total Hamiltonian:} \quad \mathcal{H} = \mathcal{H}_{\mathrm{FLRW}} + \mathcal{H}_{\mathrm{fluid}} \propto p^2 + p_\tau}$

QUANTISATION OF THE BACKROUND

- Geometry: FLRW Hamiltonian $\mathcal{H}_{\mathrm{FLRW}} = -\frac{\kappa N}{12\mathcal{V}_0 a} p_a^2$ $\{a, p_a\} = 1$ and $\mathcal{H}_{\mathrm{fluid}} = \gamma(w)\frac{N}{a^{3w}}\, p_\phi^{1+w}$
- Matter: perfect fluid
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Total Hamiltonian: $\mathcal{H} = \mathcal{H}_{\mathrm{FLRW}} + \mathcal{H}_{\mathrm{fluid}} \propto p^2 + p_\tau$

[Małkiewicz, Peter, ('19)] [Gielen, Menéndez-Pidal, ('20, `21)] [de Cabo Martin, Małkiewicz, Peter, ('22)] [Bergeron, Dapor, Gazeau, Małkiewicz, ('14)] ....

- Quantisation map based on the affine group introduces repulsive potential that leads to a bounce

$$\mathcal{H} \longrightarrow \hat{\mathcal{H}} \propto \hat{p}^2 + \frac{K}{\hat{q}^2} + \hat{p}_\tau$$

constant $\geq \frac{3}{4}$

time evolution

introduces a bounce

with

$$\hat{p}\,\psi = -\mathrm{i}\partial_x\psi \qquad \hat{q}\,\psi = x\,\psi \qquad \hat{p}_\tau\psi = -\mathrm{i}\partial_\tau\psi$$

- Schrödinger equation recovered in perfect fluid clock variable

$$\hat{\mathcal{H}}\psi = 0 \longrightarrow \hat{\mathcal{H}}_{\mathrm{FLRW}}\psi - \mathrm{i}\partial_\tau\psi = 0$$

WAVE FUNCTION FOR THE UNIVERSE

- Semiclassical state $|\psi\rangle = e^{-\mathrm{i}\phi(\tau)}|q(\tau),p(\tau)\rangle$ that satisfies the Schrödinger equation $\hat{\mathcal{H}}_{\mathrm{FLRW}}\psi - \mathrm{i}\partial_\tau\psi = 0$ and follows dynamics generated by the semiclassical Hamiltonian

$$\mathcal{H}_{\mathrm{sem}} = p^2 + \frac{\xi_\nu}{q^2}$$

constant determined by value of K

semiclassical trajectories: $q(\tau) = q_B\sqrt{1+\omega^2(\tau-\tau_B)^2}$

initial condition

$$p(\tau) = \frac{1}{2}\dot{q}(\tau) = \frac{q_B\,\omega^2(\tau-\tau_B)}{2\sqrt{1+\omega^2(\tau-\tau_B)^2}}$$

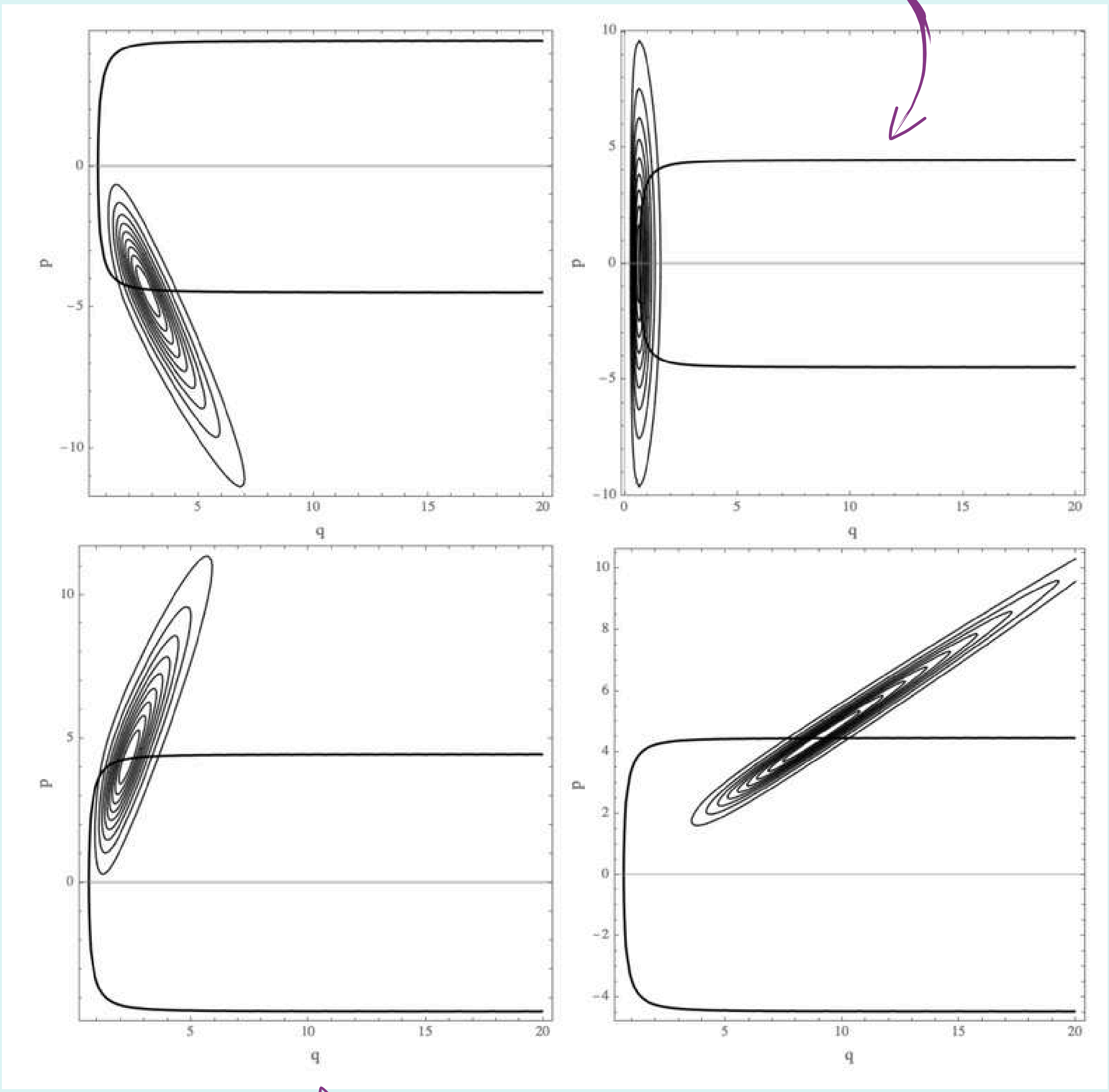
bounce time

[Bergeron, Małkiewicz, Peter ('24)]

Evolution of the wavefunction:

semiclassical trajectory

$\propto \dfrac{\mathrm{d}a}{\mathrm{d}\tau}$

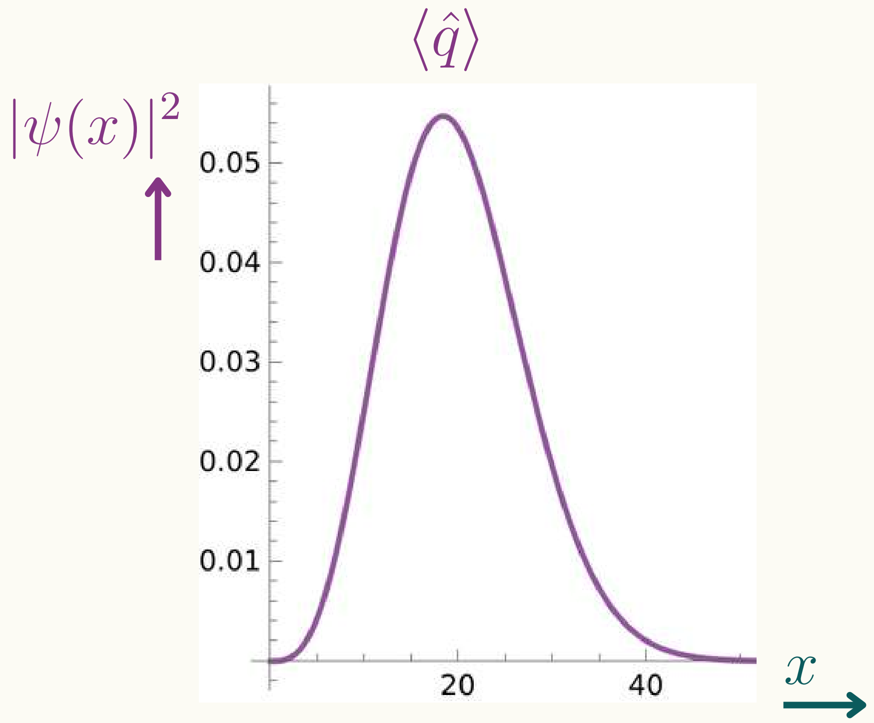


$\propto a$

[Image: Bergeron, Małkiewicz, Peter ('24)]

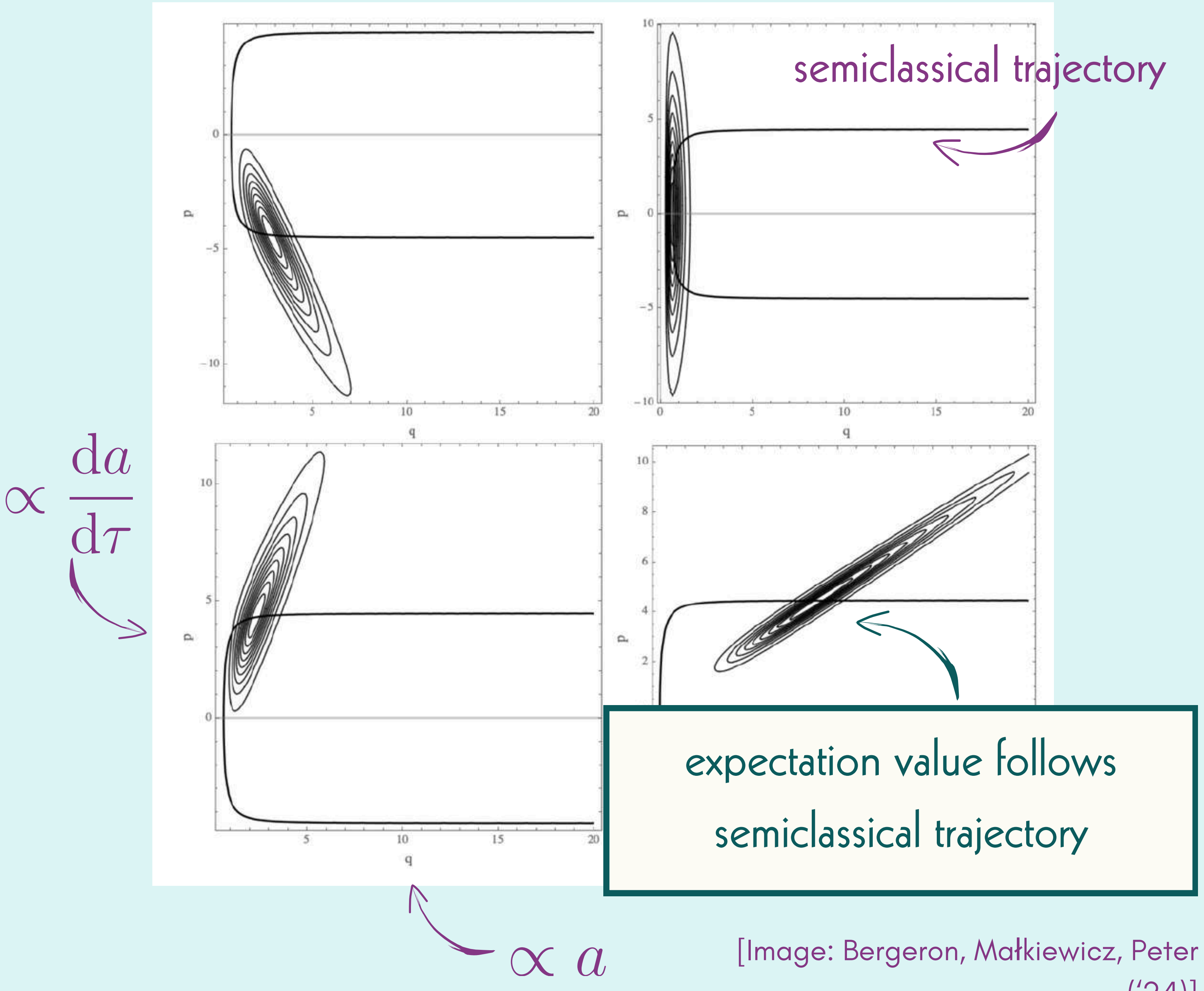
QUANTUM CORRECTED EVOLUTION

- Operator related to the scale factor $q \propto a^{\frac{3}{2}(1-w)}$ $\quad q \to \hat{q}$
- In order to connect to GR: need classical scale factor on spacetime
- *Copenhagen viewpoint*: operator expectation value in a highly peaked state semiclassical state = the most likely state for the system $\langle\psi|\hat{q}|\psi\rangle \propto a^{\frac{3}{2}(1-w)}$



Contrary to lab experiments one cannot repeat the experiment many times: no statistical distribution

Here: consider quantum trajectories instead



expectation value follows semiclassical trajectory

[Image: Bergeron, Małkiewicz, Peter ('24)]

THE TRAJECTORY APPROACH

[de Broglie ('27)] [Bohm ('52)] [Holland ('93)]

- Physical system = wave + point particle moving under guidance of the wave

$$\psi(x,t) = R(x,t)e^{\mathrm{i}S(x,t)} \longrightarrow x(t)$$ trajectory

- Wave function obeys the Schrödinger equation $\mathrm{i}\partial_t\psi = H\psi$
- Particle motion is obtained calculated from the phase (initial condition dependent) $$\dot{x} = \frac{1}{m}\nabla S(x,t)$$
  - Different initial conditions $x(t_0)$ give an ensemble of particles associated to the same wave
- Probability of the particle to lie in an interval $x + \mathrm{d}x$ is determined by the wave function amplitude $R^2(x,t)\mathrm{d}x$

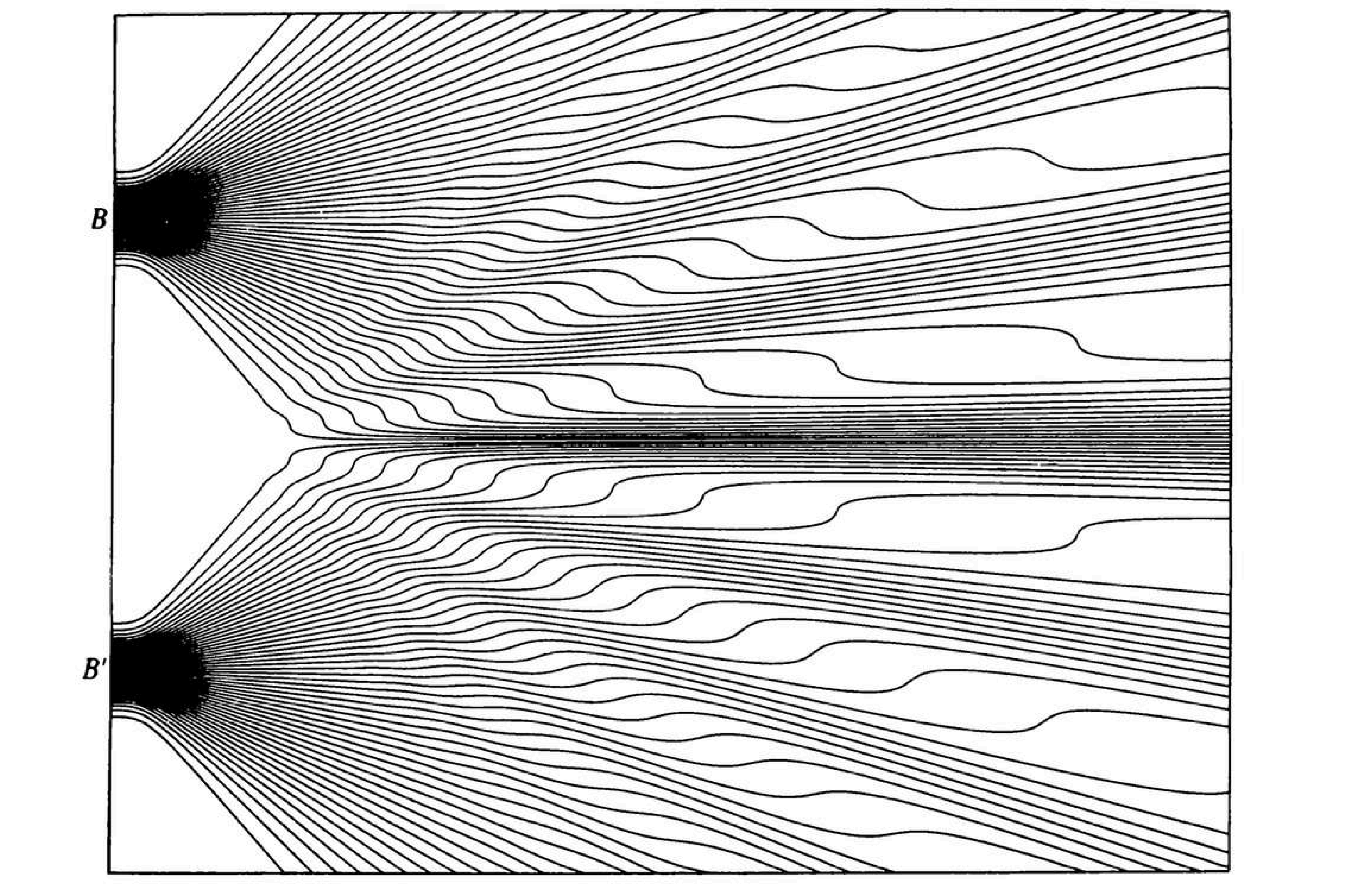
In a nutshell: The quantum system follows a trajectory, but our knowledge of system properties at any given moment is limited

- Same probabilistic predictions in trajectory and Copenhagen approaches
- Applications in e.g. theoretical chemistry

[Sanz ('18)]
[Gindensperger ('00)]

EXAMPLE: DOUBLE SLIT EXPERIMENT

[Image: Philippidis et al., ('82)]



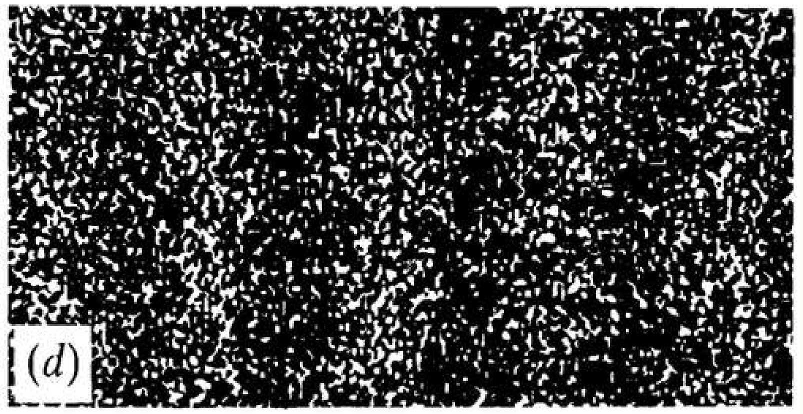
- Two wave packets emerge from the slits: Gaussian in y-direction, plane waves in x-direction

$$\psi_B(x,y,t_0) \quad \text{and} \quad \psi_{B'}(x,y,t_0)$$

- Total wave function is a superposition

$$\psi = \mathcal{N}(\psi_B(x,y,t) + \psi_{B'}(x,y,t))$$

- Obtain interference pattern from $R^2 = |\psi|^2$
- Numerically calculate trajectories
  - Can reconstruct the path of an electron that hit the screen (up to measurement uncertainty)



[Image: Tonomura et al., ('89)]

BOUNCING TRAJECTORIES

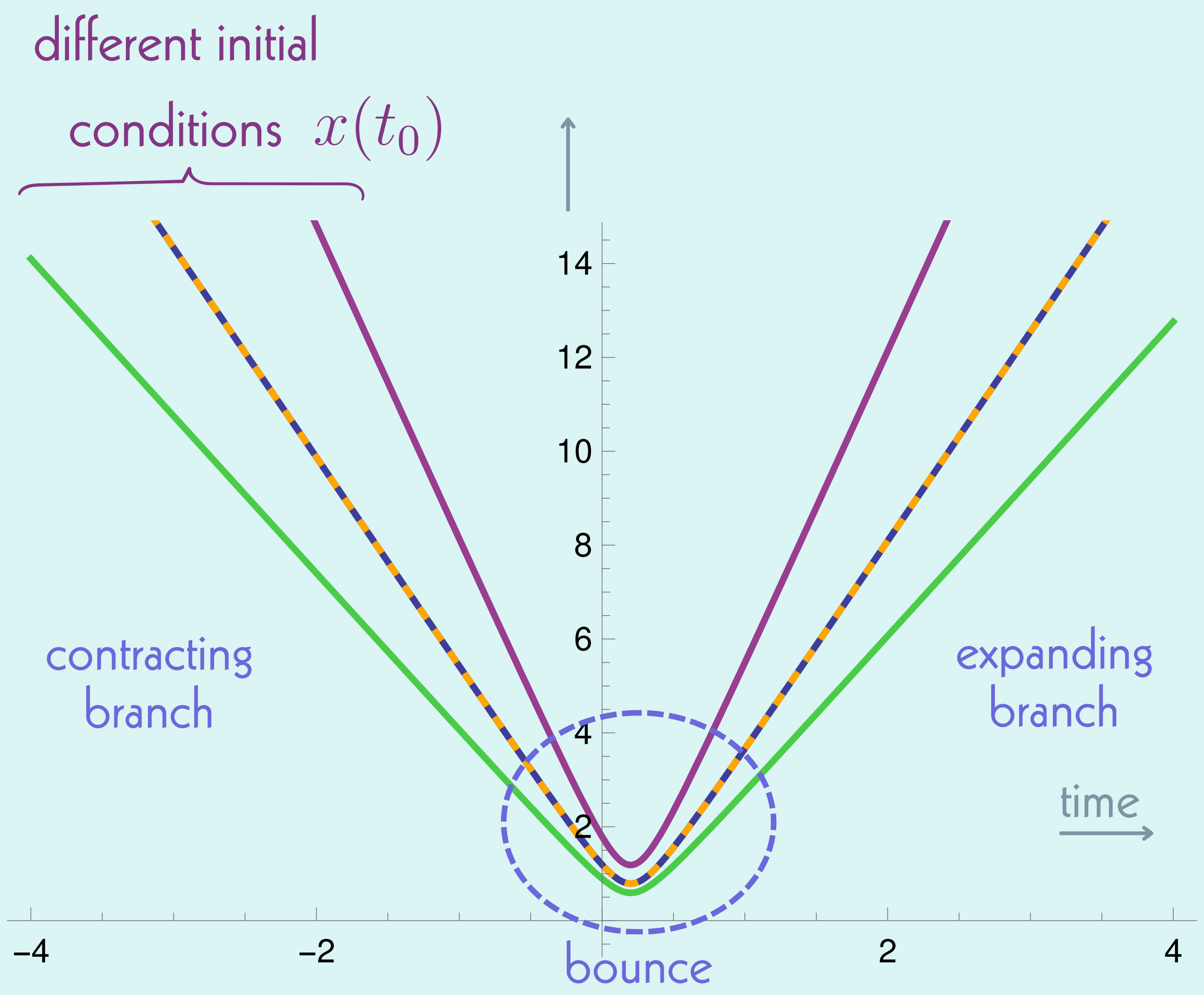
- Continuous ensemble of trajectories can be obtained from the wave function:

trajectory gives the scale factor: $$x(\tau) \propto a^{\frac{3}{2}(1-w)}$$

evolution governed by the wave function: $$\frac{\mathrm{d}x}{\mathrm{d}\tau} = -\mathrm{i}\partial_x \ln \frac{\psi}{\psi^\star}$$ with

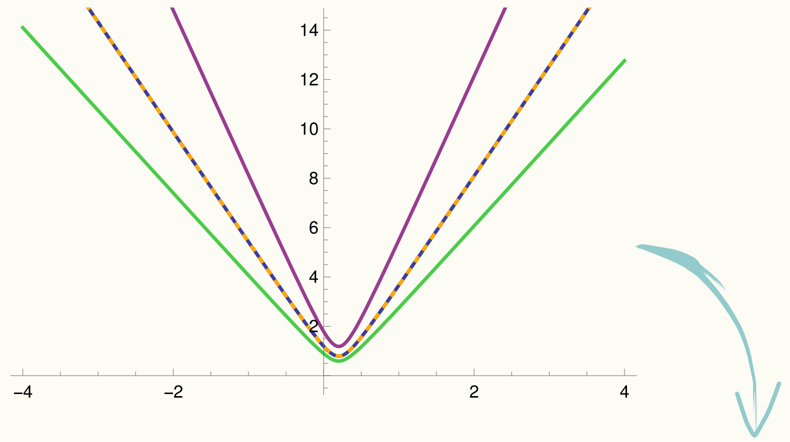
$$|\psi\rangle = e^{-\mathrm{i}\phi(\tau)}|q(\tau), p(\tau)\rangle$$
$$\hat{\mathcal{H}}_{\mathrm{FLRW}}\psi - \mathrm{i}\partial_\tau \psi = 0$$

  - Assign a concrete value to the effective scale factor at all times
  - Classical dynamics away from bounce

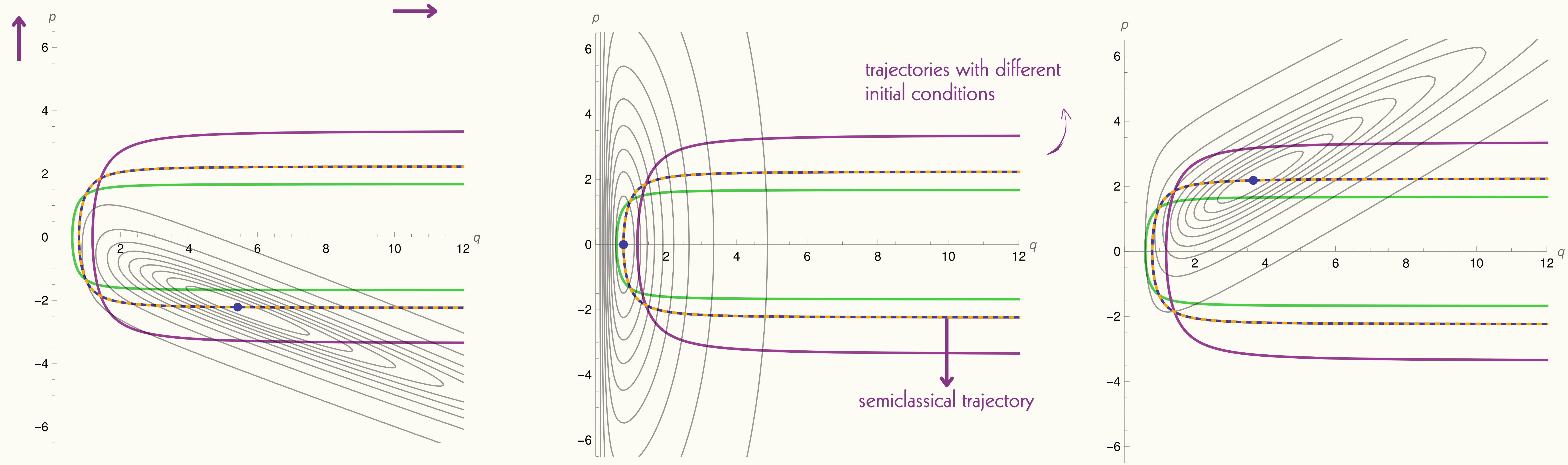


BOUNCING SINGLE STATE TRAJECTORIES

- State and trajectories follow dynamics generated by the semiclassical Hamiltonian $\mathcal{H}_{\text{sem}} = p^2 + \dfrac{K}{q^2}$



$p \propto a^{\frac{3}{2}(1-w)} H$ $\qquad$ $q \propto a^{\frac{3}{2}(1-w)}$



SUPERPOSITION UNIVERSE

Bouncing biverse

UNIVERSE IN A SUPERPOSITION

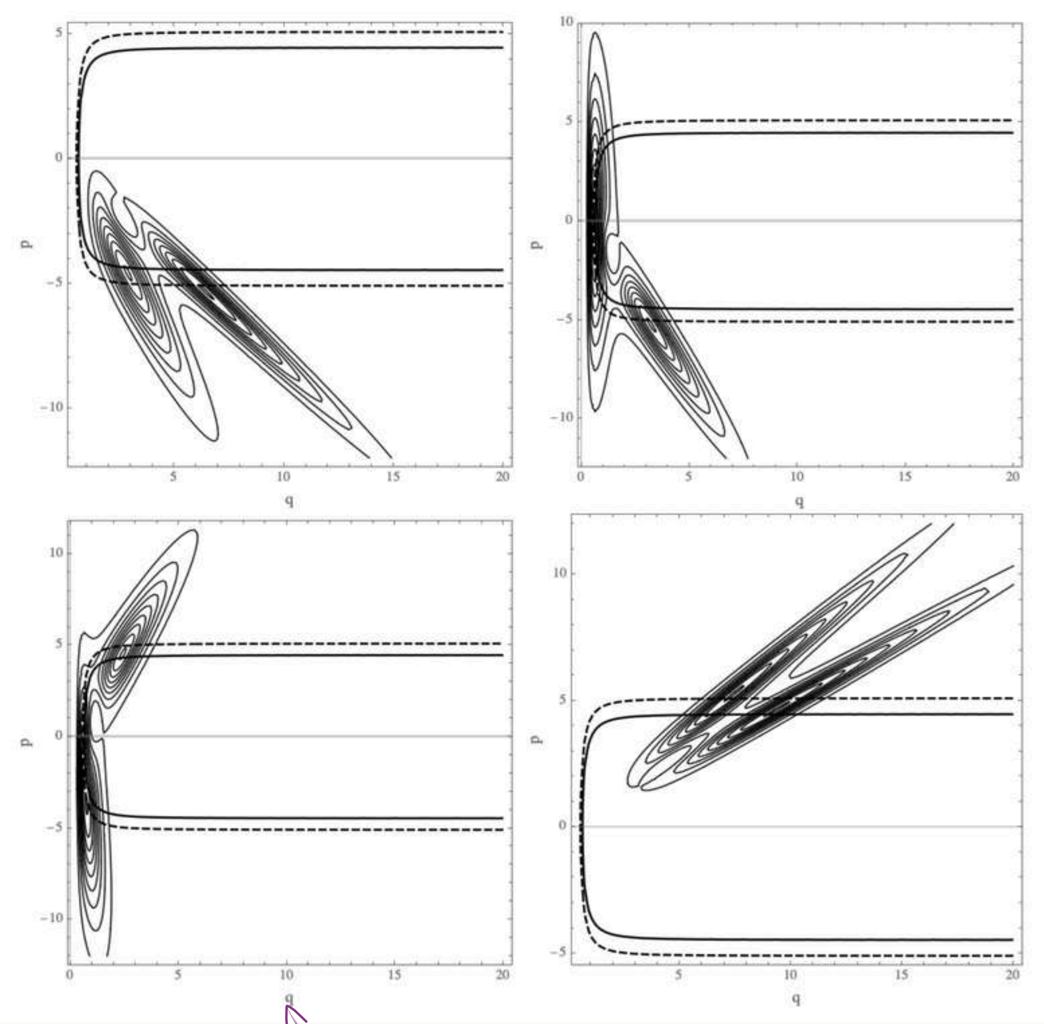
- Relax the semiclassical state assumption and consider state in superposition
  - Consider biverse example $\Psi = \mathcal{N}(\psi_0 + \rho e^{i\delta}\psi_1)$

additional phase

normalisation factor

relative contribution of second wave function

- Each of the state follows its corresponding semiclassical trajectory



$\propto \dfrac{\mathrm{d}a}{\mathrm{d}\tau}$

$\propto a$

[Image: Bergeron, Małkiewicz, Peter, ('24)]

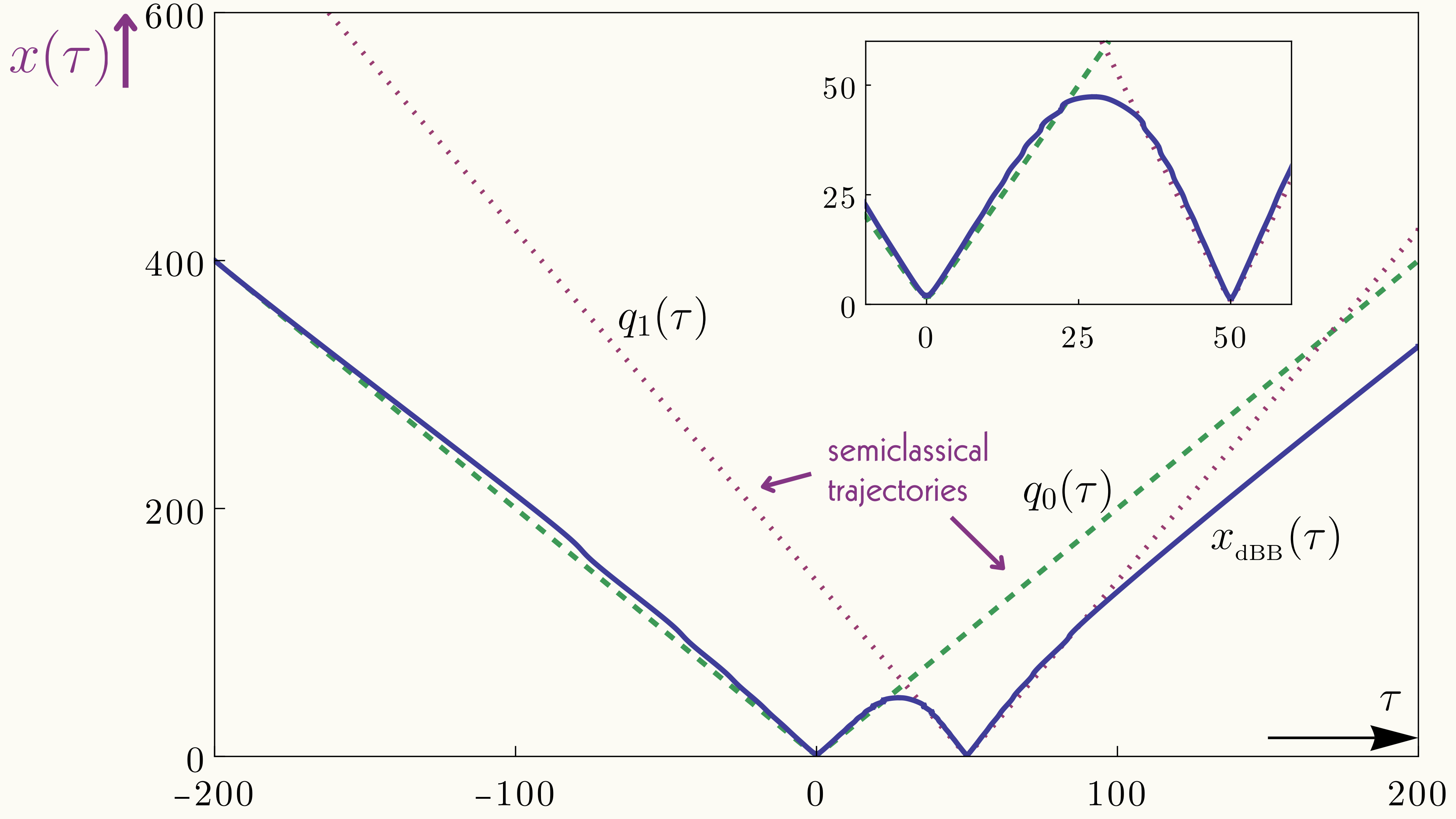
- *Copenhagen viewpoint*: assume that we live in one of these universes, the other state is "virtual"

→ no change to the perceived background dynamics for a superposition state

BIVERSE TRAJECTORY

- Biverse: $\Psi = \mathcal{N}(\psi_0 + \rho e^{i\delta}\psi_1)$
- Recall that trajectories are calculated from: $\frac{\mathrm{d}x}{\mathrm{d}\tau} = -\mathrm{i}\partial_x \ln \frac{\Psi}{\Psi^*}$ with $x \propto a^{\frac{3}{2}(1-w)}$

$r = 2, \quad \Delta\tau = 50, \quad \rho = 1 \quad \& \quad \delta = 0$

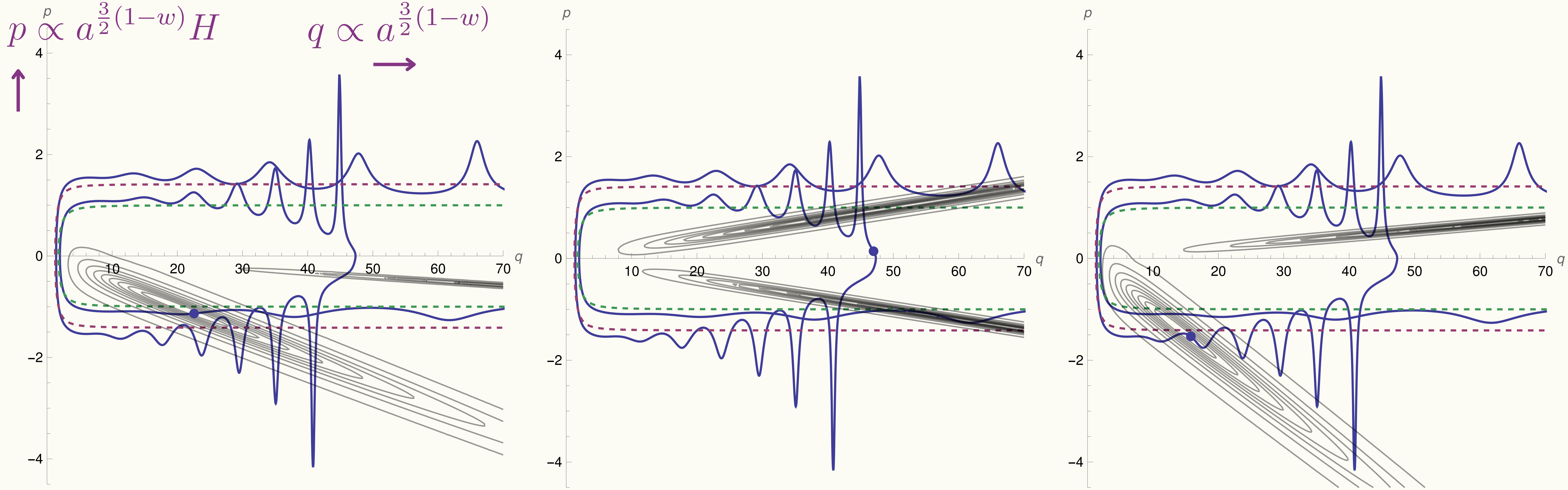


- Start on semiclassical trajectory
- Parameters:
  - r ○ ratio of late time momenta of semiclassical solutions
  - $\Delta\tau$ ○ difference in bounce times
  - $\rho,\ \delta$ ○ contribution of second wave function

BIVERSE TRAJECTORY

- Phase space portraits and wave functions for the biverse $\Psi = \mathcal{N}(\psi_0 + \rho e^{i\delta}\psi_1)$

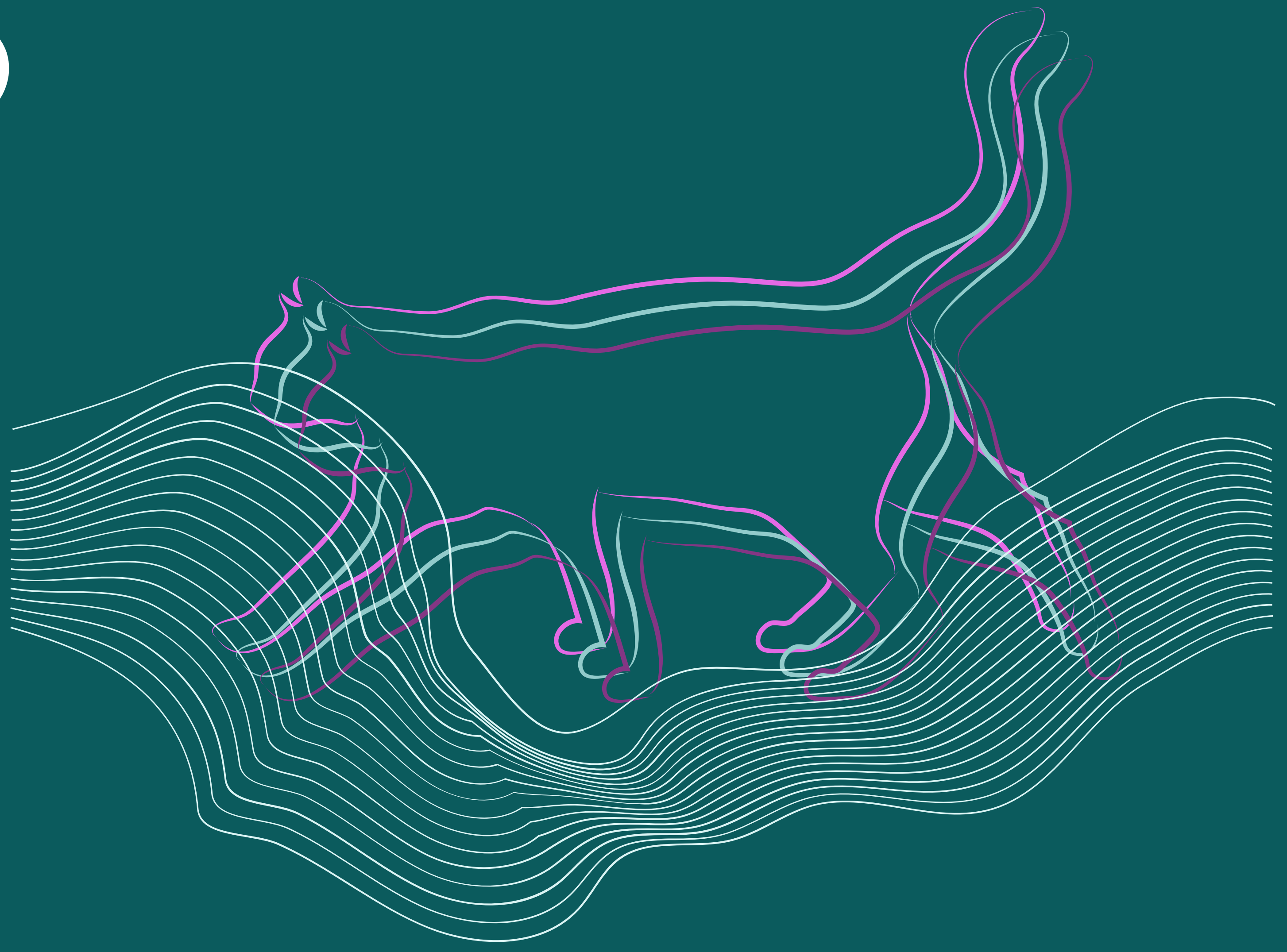
$p \propto a^{\frac{3}{2}(1-w)}H$ $\quad$ $q \propto a^{\frac{3}{2}(1-w)}$



- Trajectories highly dependent on initial conditions, but generally exhibit features that differ from single state trajectories
- Late time behaviour dominated by a single wave function → same evolution as in single state case

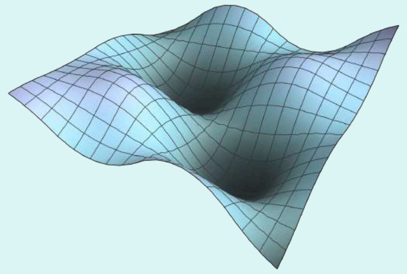
PERTURBATIONS

and the impact of biverse trajectories



FIRST APPROACH TO PERTURBATIONS

- Consider tensor perturbations in a flat FLRW spacetime with a radiation fluid (set $w = 1/3$)



$$\mathrm{d}s^2 = a^2(\eta)\left(-\mathrm{d}\eta^2 + [\delta_{ij} + h_{ij}(\eta, \vec{x})]\,\mathrm{d}x^i \mathrm{d}x^j\right)$$

[Peter, Pinho, Pinto-Neto, ('06)] [Peter, Pinho, Pinto-Neto, ('05)]

- Second order linear perturbative Hamiltonian for tensor modes: sum of the two polarisations and Fourier modes

$$\mathcal{H}_{\mathrm{FLRW}} \to \mathcal{H}_{\mathrm{FLRW}} + \mathcal{H}^{(2)}$$

$\lambda = +, \times$

$$\mathcal{H}^{(2)} = \sum_{\vec{k}} \left(\mathcal{H}^{(2)}_{\vec{k},+} + \mathcal{H}^{(2)}_{\vec{k},\times}\right) \quad \text{with} \quad \mathcal{H}^{(2)}_{\vec{k},\lambda} = \pi^{(\lambda)}_{\vec{k}} \pi^{(\lambda)}_{-\vec{k}} + \left(k^2 - \frac{a''}{a}\right) \mu^{(\lambda)}_{\vec{k}} \mu^{(\lambda)}_{-\vec{k}}$$

conjugate momentum of $\mu^{(\lambda)}_{\vec{k}}$

$h_{ij} \propto \mu_{ij}/a$

- Wave function Ansatz: $\Psi(x, \{\mu_k\}) = \psi_{\mathrm{bg}}(x)\psi_{\mathrm{pert}}(x, \{\mu_k\})$

Background:	Perturbations:
$\mathcal{H}_{\mathrm{FLRW}}\psi_{\mathrm{bg}}(x) = \mathrm{i}\partial_\tau \psi_{\mathrm{bg}}(x)$	$\mathcal{H}^{(2)}\psi_{\mathrm{pert}}(x(\tau), \{\mu_k\}) = \mathrm{i}\partial_\tau \psi_{\mathrm{pert}}(x(\tau), \{\mu_k\})$ background dependent trajectory extracted from the background

PERTURBATIVE DYNAMICS: EXAMPLE

- Dynamics of perturbation modes $$\mu_k'' + \left(k^2 - \frac{a''}{a}\right)\mu_k = 0$$

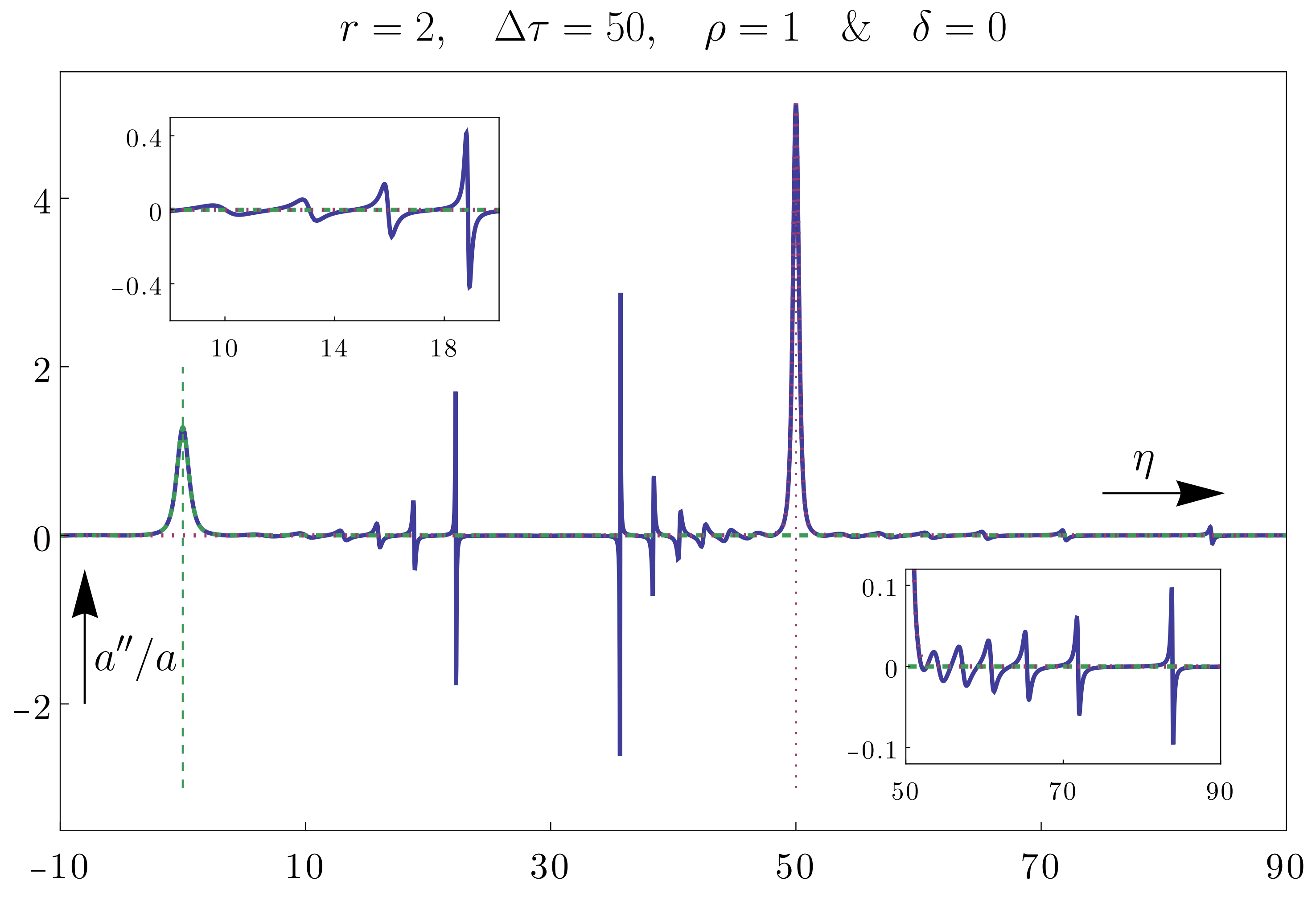
$h_{ij} \propto \mu_{ij}/a$ $\quad V_{\rm eff}$ modified by trajectories

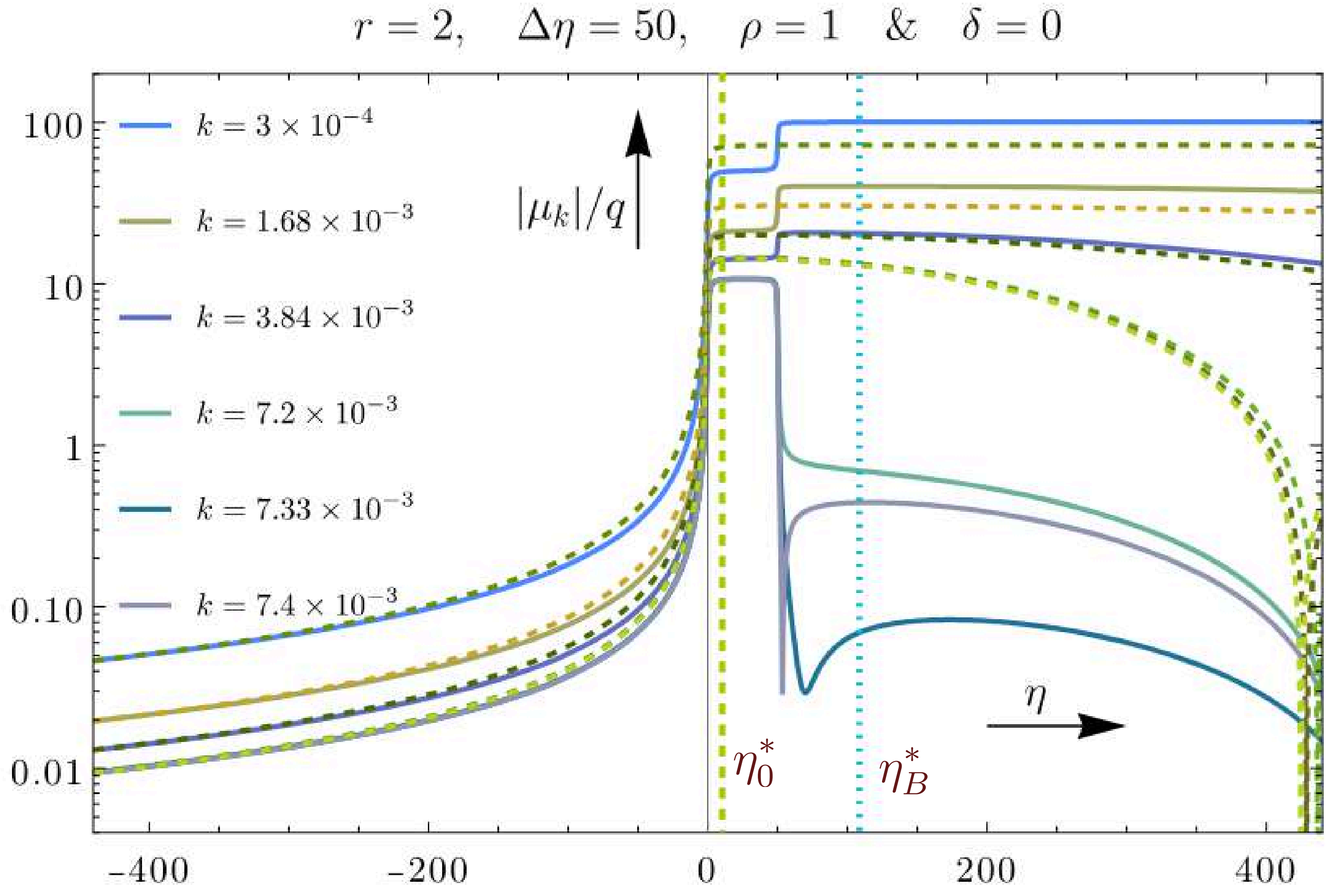
With

$$\hat{\mu}_{\vec{k}}(\eta) = \mu_k(\eta)\hat{a}_{\vec{k}} + \mu_k^*(\eta)\hat{a}^\dagger_{-\vec{k}}$$

creation and annihilation operators

- Range of Fourier modes: $k \in (8\times 10^{-4}, 3\times 10^{-3})$
- Initial conditions: "Bunch-Davies" vacuum $\mu_k(\eta) = \frac{1}{\sqrt{2k}} e^{-ik(\eta-\eta_i)}$





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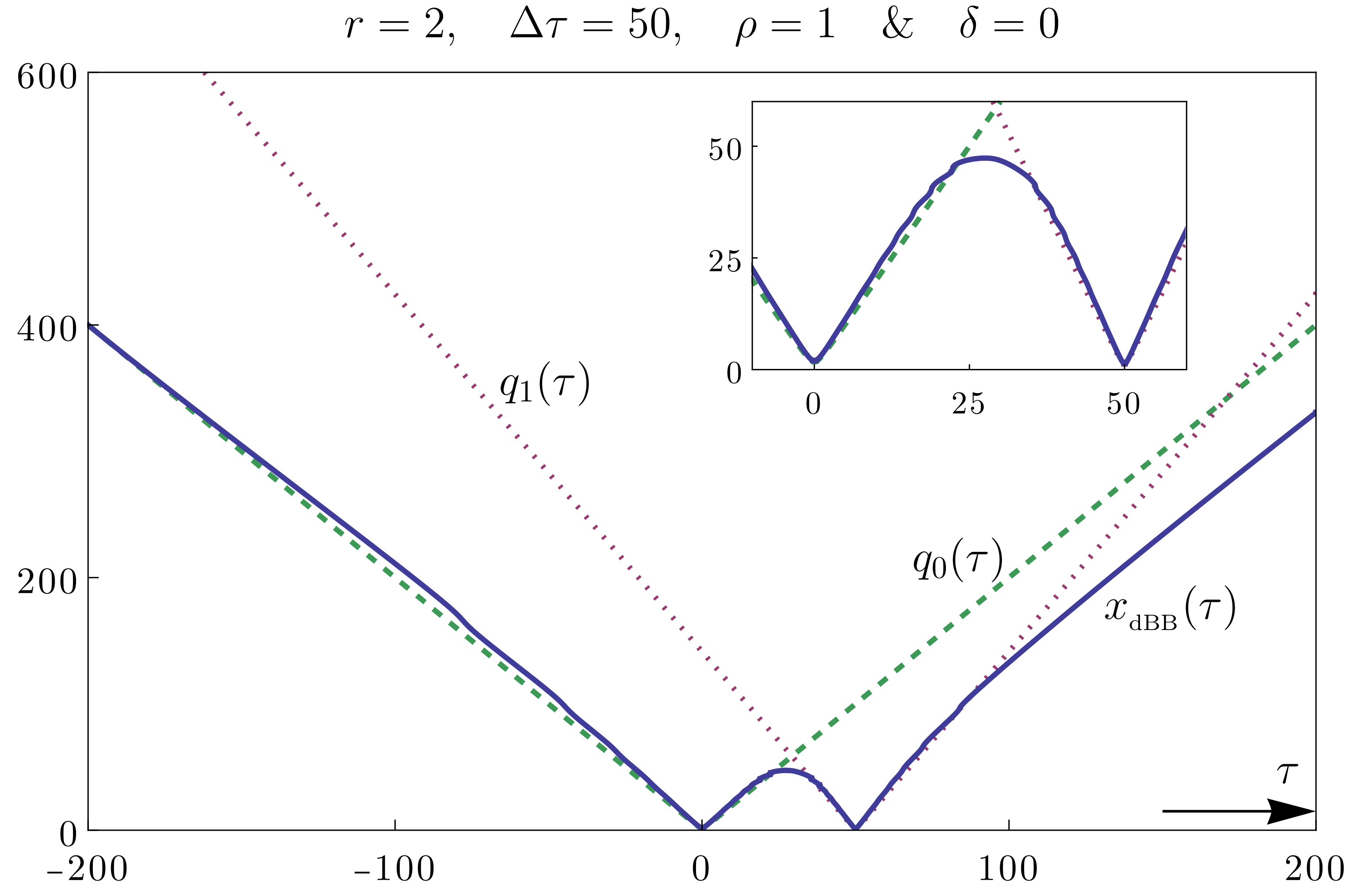
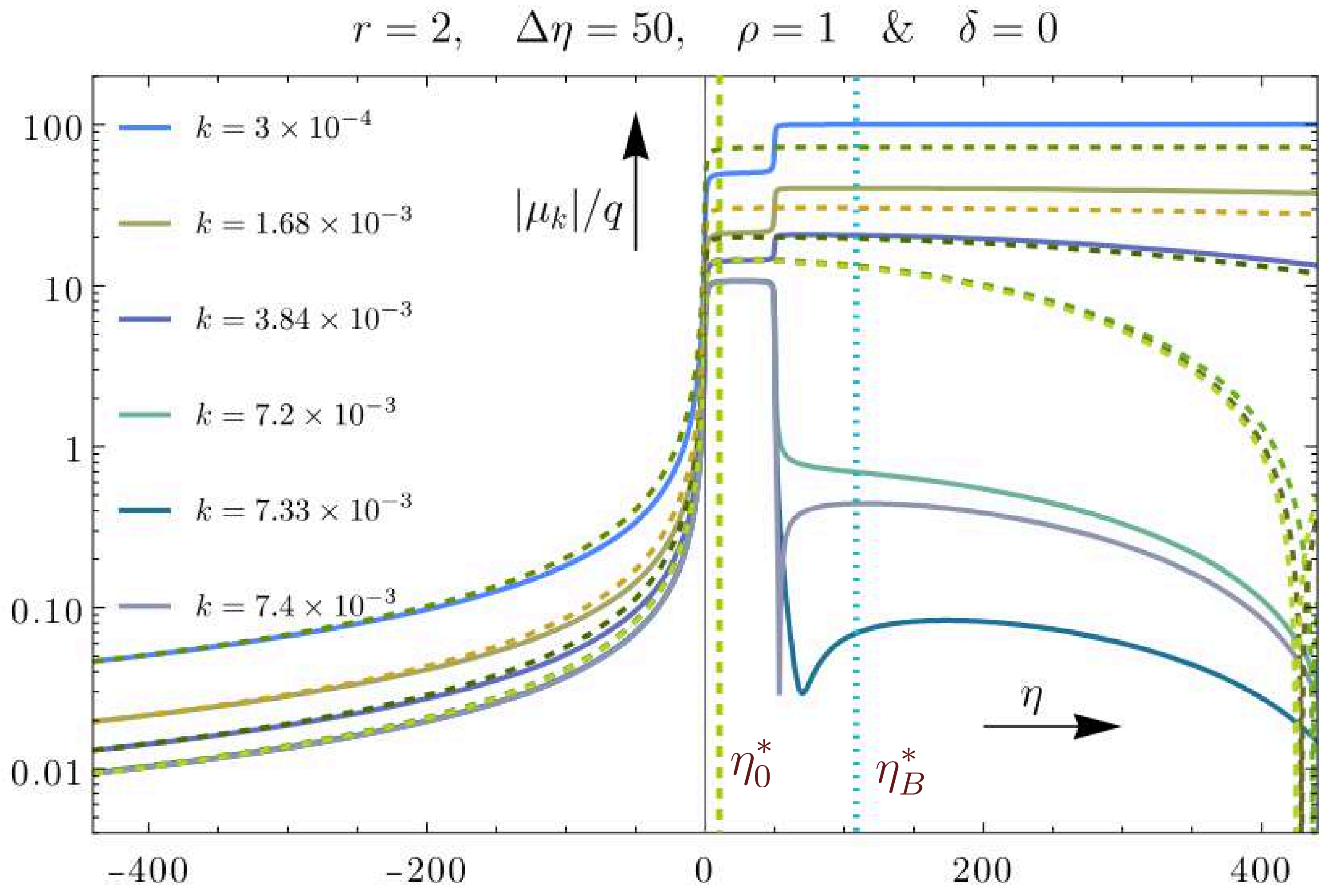
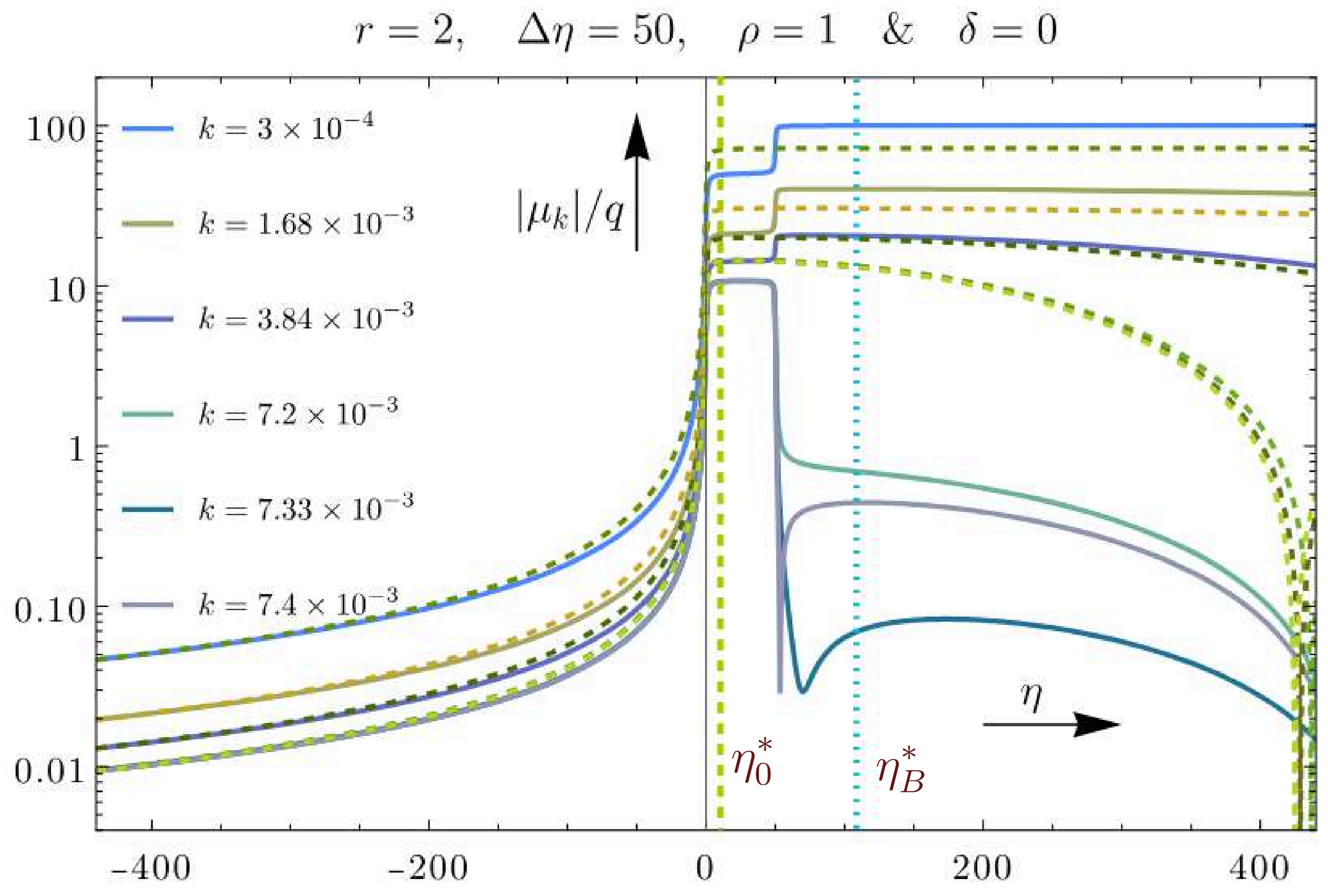
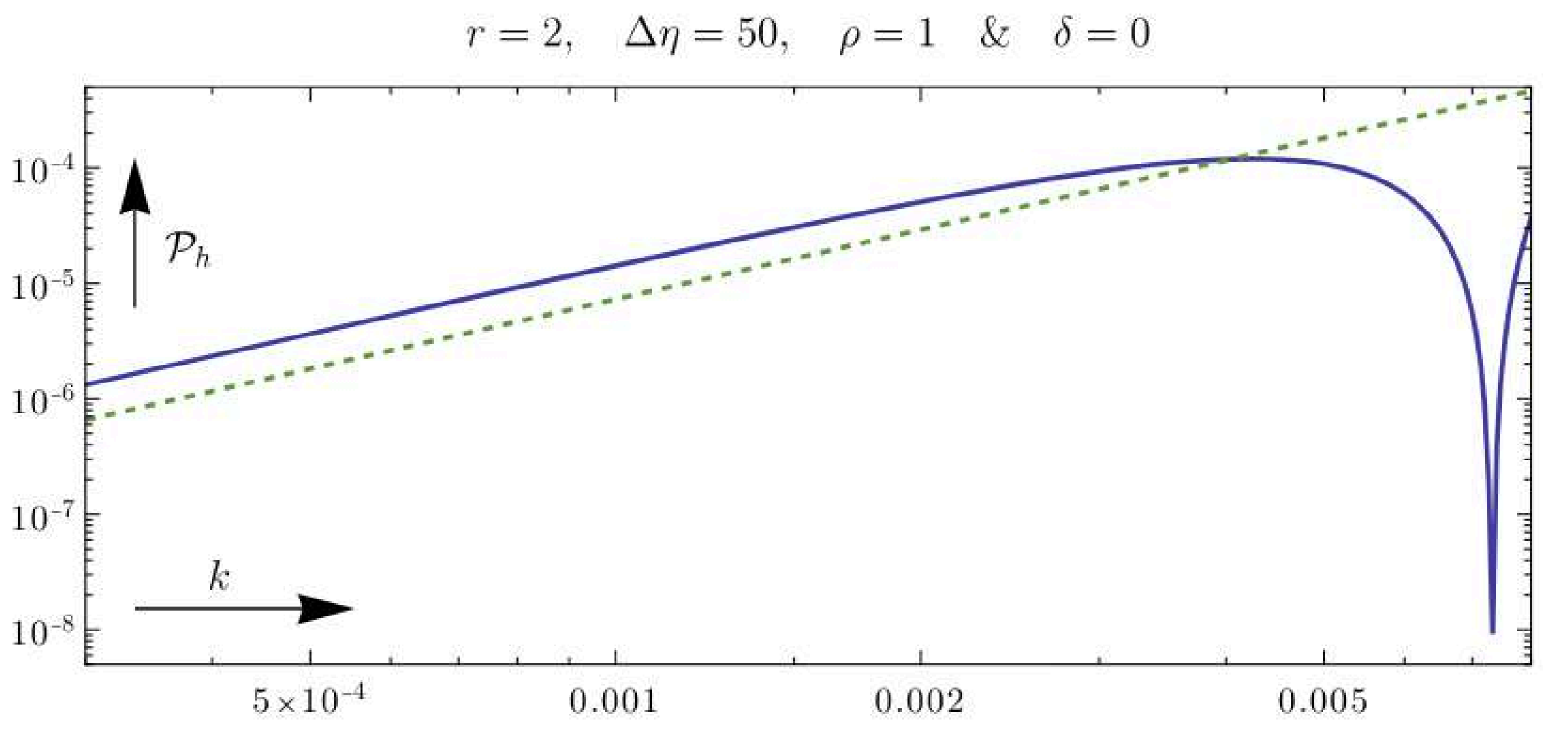




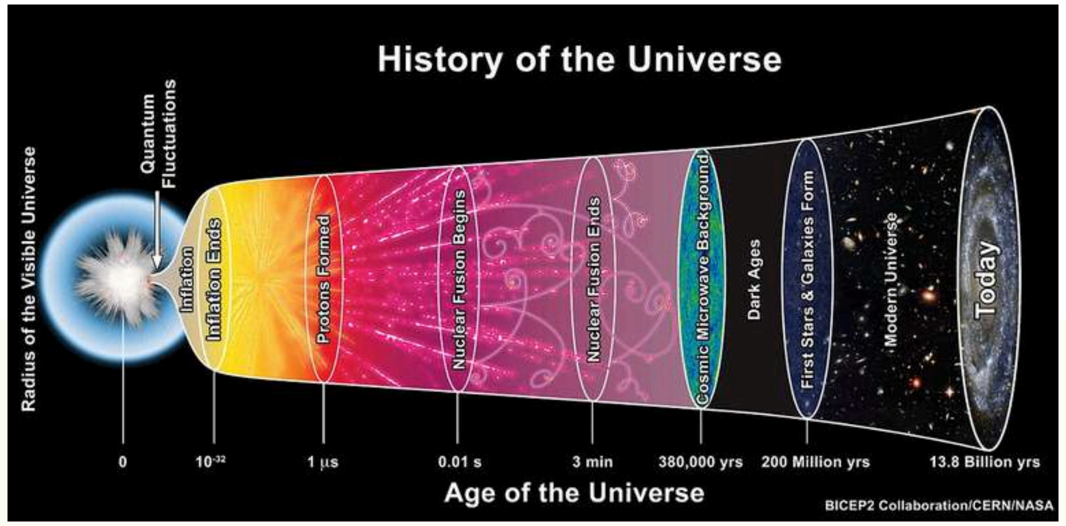
PERTURBATIVE DYNAMICS: EXAMPLE

- Power spectrum $\mathcal{P}_h \propto k^3 \left| \frac{\mu_k(\eta)}{a(\eta)} \right|^2_{\eta=\eta^*}$ taken at time when $V_{\text{eff}}(\eta^*) = k^2_{\text{max}}$
  - Lowering (reddening) of spectral index





initial conditions



CONCLUSION

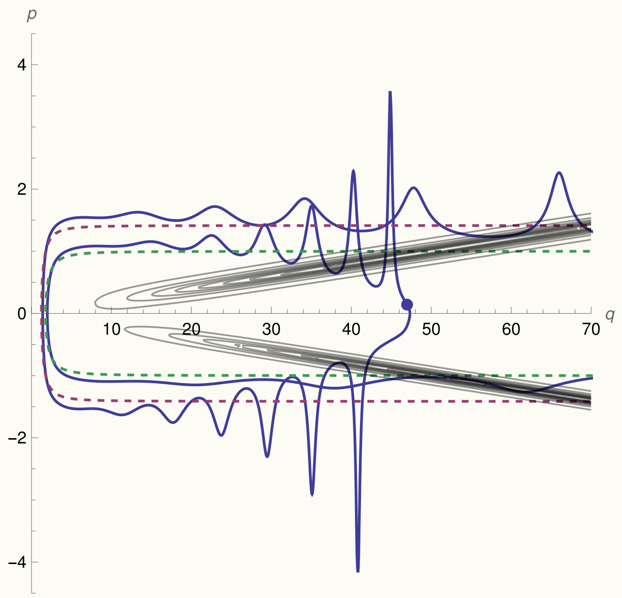
... and next steps

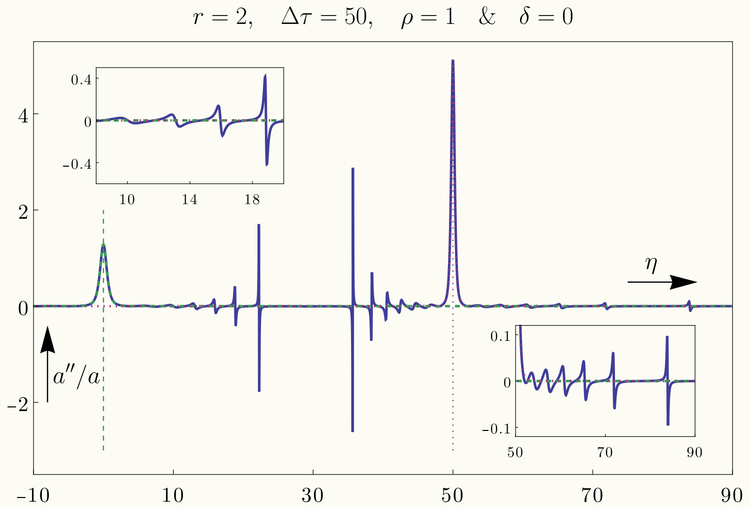
CONCLUSION

- Affinely quantise an FLRW spacetime to obtain bouncing trajectories
  - trajectories assign unambiguous value to the scale factor at all times

- Trajectories for a universe in a superposition introduce distinct features in the scale factor evolution

- These features affect the evolution of perturbations and thereby the tensor power spectrum





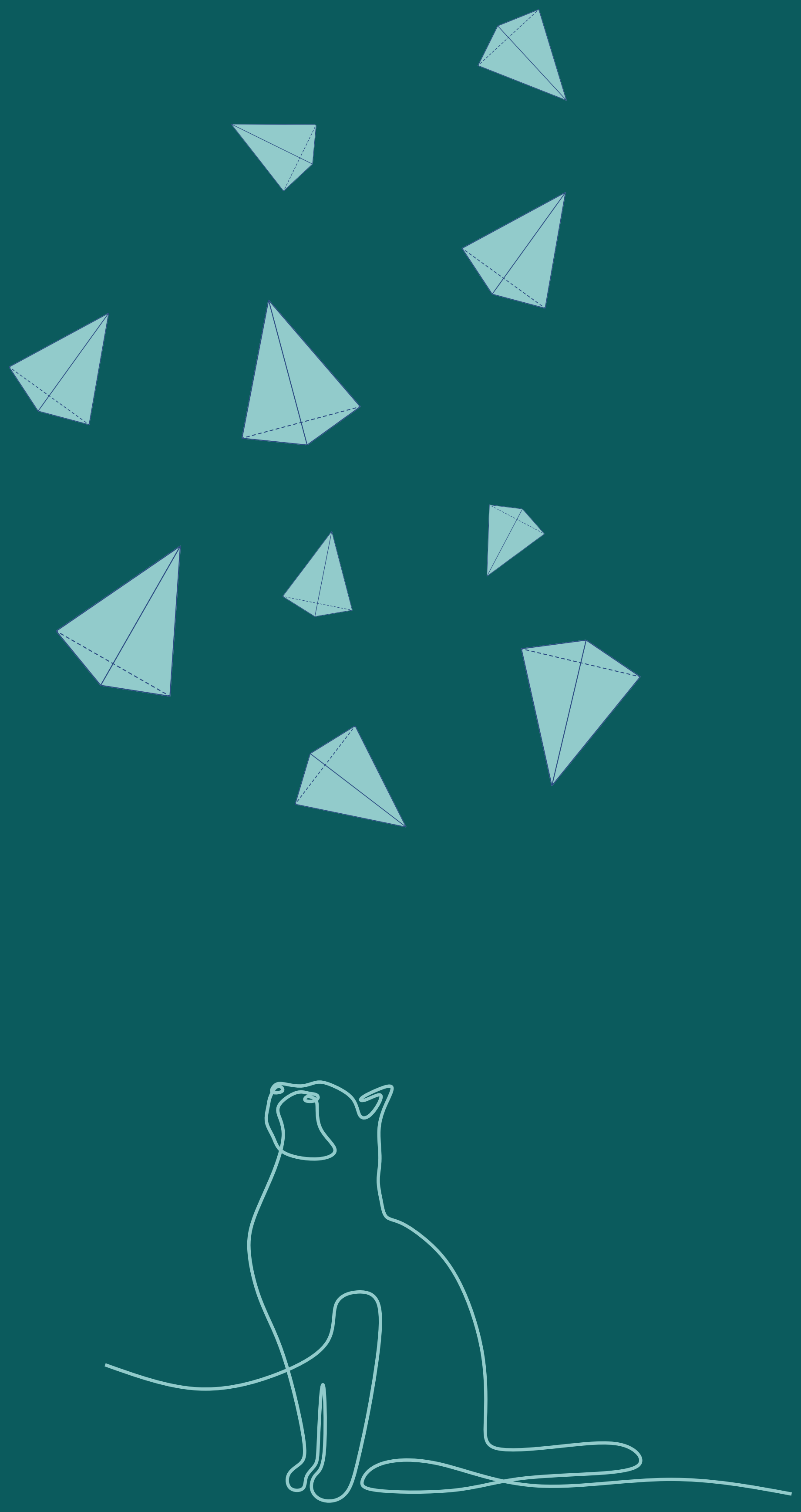
→ ***Next***: detailed study of perturbations, scalar perturbation, and perturbative trajectories

Can we find an analogue system with similar properties ?

THANK YOU!

Questions...?

BACKUP

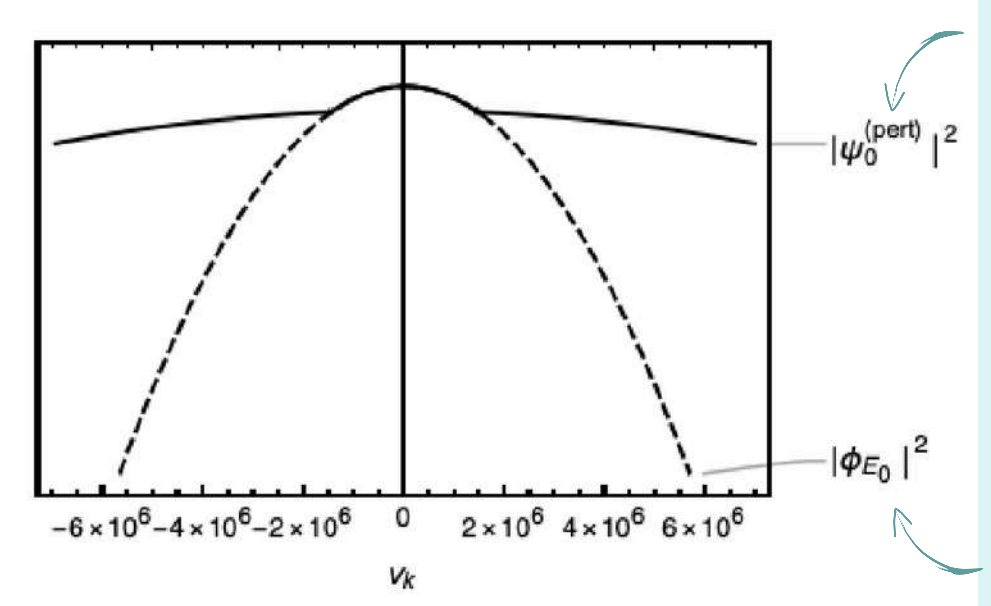


PERTURBATIONS OVER A COPENHAGEN BIVERSE

- State in which each universe has its own perturbations $\Psi = \psi_{\mathrm{bg},0}\psi_{\mathrm{pert},0} + \psi_{\mathrm{bg},1}\psi_{\mathrm{pert},1}$

Background:	Perturbations over each backgound:
$\mathcal{H}_{\mathrm{FLRW}}\psi_{\mathrm{bg}}(x) = \mathrm{i}\partial_\tau\psi_{\mathrm{bg}}(x)$	$\mathrm{i}\partial_\tau\psi_{\mathrm{pert},n} = \sum_{lm}\langle\psi_{\mathrm{bg},n}\vert\psi_{\mathrm{bg},l}\rangle^{-1}\underbrace{\langle\psi_{\mathrm{bg},l}\vert\mathcal{H}^{(2)}\vert\psi_{\mathrm{bg},m}\rangle}_{\text{effect of "virtual" states}}\psi_{\mathrm{pert},m}$

- Influence of superposition on perturbations from $\langle\psi_{\mathrm{bg},l}|\mathcal{H}^{(2)}|\psi_{\mathrm{bg},m}\rangle$
- Imprint: non-Gaussian profile in the perturbation modes



total perturbations

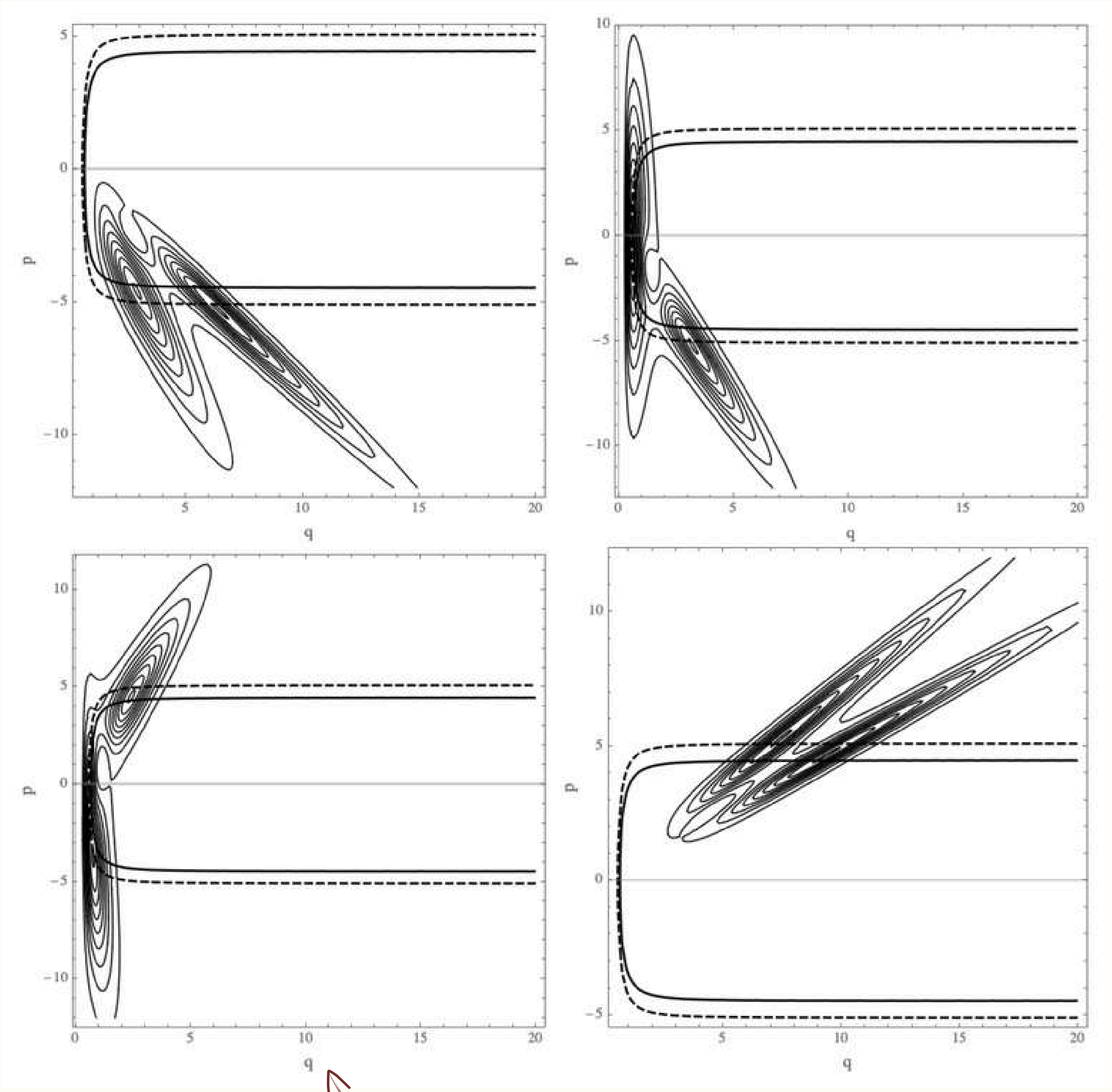
single state $\Psi = \Psi_0$

perturbation

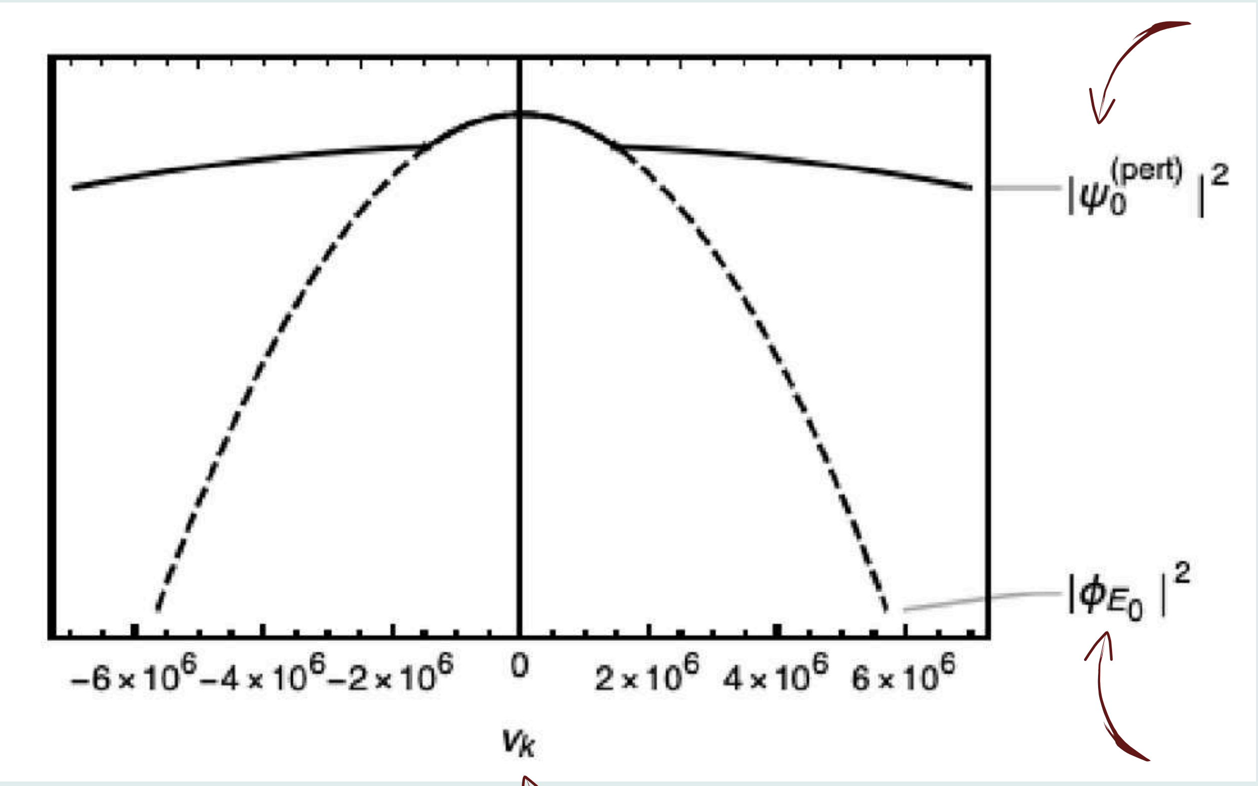
COMPARISON

[Bergeron, Małkiewicz, Peter, ('24), arXiv: 2405.09307]

- Biverse $\Psi = \mathcal{N}(\psi_0 + \alpha\,\psi_1)$
- Our universe: projection on a single wave function

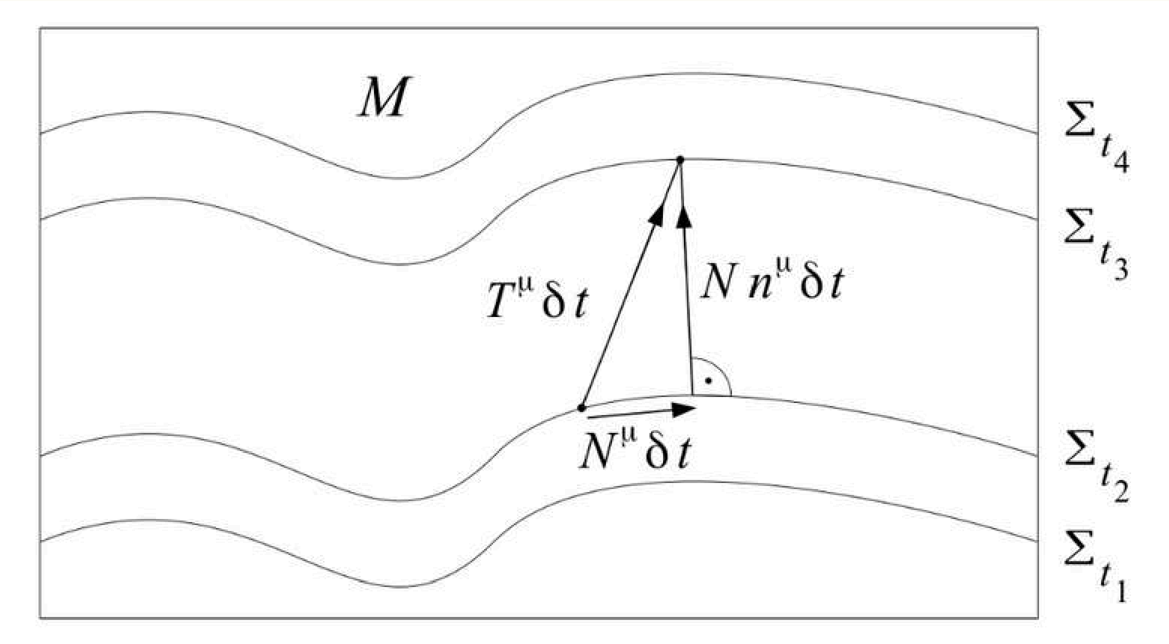


- Entanglement of perturbations between states



HAMILTONIAN FOR COSMOLOGY

- Hamiltonian formulation of general relativity:
  - Foliation of spacetime: spatial hypersurfaces with spatial metric and time direction
  - Time evolution: shift and lapse give translations parallel and orthogonal to hypersurfaces



$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^a N_a & N_a \\ N_a & q_{ab} \end{pmatrix}$$

lapse

shift

spatial metric

- Hamiltonian is the sum of two constraints: "GR is a fully constrained system"
  - Lapse and shift give gauge choice: "Problem of time"

$$\mathcal{H}_{\mathrm{GR}} = N(\underbrace{\underbrace{\mathcal{H}_{\mathrm{ADM,\,S}}}_{\text{geometry}} + \mathcal{H}_{\mathrm{matter,\,S}}}_{\approx 0}) + N_a(\underbrace{\underbrace{\mathcal{H}^a_{\mathrm{ADM}}}_{\text{geometry}} + \mathcal{H}^a_{\mathrm{matter}}}_{\approx 0})$$

- Cosmology:
  - FLRW spacetime

$$g_{\mu\nu} = \begin{pmatrix} -N(t)^2 & & & \\ & a(t)^2 & & \\ & & a(t)^2 & \\ & & & a(t)^2 \end{pmatrix}$$

$$\mathcal{H}_{\mathrm{ADM}} \to \mathcal{H}_{\mathrm{FLRW}} = -\frac{\kappa N}{12\mathcal{V}_0 a} p_a^2$$

lapse

conjugate momentum

spatial section volume

scale factor

$$\{a, p_a\} = 1$$

+ *perturbations*

MOTIVATION: QUANTUM GRAVITY

- Need for quantum gravity: prediction of singularities (big bang); quantum nature of matter...
- Quantum gravity is a hard problem!
  - "Straightforward" approach: Wheeler-DeWitt quantisation of the ADM Hamiltonian [DeWitt ('67)]

$$\mathcal{H}_{\rm GR} = N(\underbrace{\mathcal{H}_{\rm ADM,\,S}}_{\text{geometry}} + \mathcal{H}_{\rm matter,\,S})_{\approx 0} + N_a(\underbrace{\mathcal{H}^a_{\rm ADM}}_{\text{geometry}} + \mathcal{H}^a_{\rm matter})_{\approx 0}$$

    - Wave functional $\Psi[\underbrace{q_{ab}(x)}_{\text{spatial metric}}, \underbrace{\Phi(x)}_{\text{matter fields}}]$ with constraints $\hat{\mathcal{H}}^a\Psi = 0$ and $\hat{\mathcal{H}}_{\rm S}\Psi = 0$

Issues: Definition of Hilbert space – inner product

- *Here:* minisuperspace model: Wheeler-DeWitt quantisation of the FLRW Hamiltonian

$$(\hat{\mathcal{H}}_{\rm FLRW} + \hat{\mathcal{H}}_{\rm matter})\Psi(a, \phi) = 0$$

  - Finite number of degrees of freedom instead of infinite
    - Mathematically consistent: *we can make calculations*!

understood as low energy limit of a full quantum gravity theory

THE SYSTEM

- Quantise phase space of FLRW spacetime $\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$

lapse

$\kappa = 8\pi G$

$$\mathcal{H}_{\text{ADM}} \to \mathcal{H}_{\text{FLRW}} = -\frac{\kappa N}{12\mathcal{V}_0 a} p_a^2$$

momentum conjugate to scale factor

scale factor

spatial section volume

- Perfect fluid as matter clock fixes the lapse $N = -a^{3w}$

equation of state parameter

[Schutz, ('70) ('71)]

$$\mathcal{H}_{\text{fluid}} = \gamma(w)\frac{N}{a^{3w}}\, p_\phi^{1+w}$$

- Canonical transformation to convenient variables
  - $(a, p_a) \to (q, p)$ with $p \propto a^{\frac{3}{2}(1-w)} H$, $q \propto a^{\frac{3}{2}(1-w)}$

    Hubble rate
  - $p_\phi \to p_\tau = -\gamma p_\phi^{1+w}$

- Total Hamiltonian after deparametrisation: $\mathcal{H} = \mathcal{H}_{\text{FLRW}} + \mathcal{H}_{\text{fluid}} \propto p^2 + p_\tau$

matter clock

QUANTISATION $\mathcal{H} \to \hat{\mathcal{H}}$

- Quantisation based on the Weyl-Heisenberg group $x \in \mathbb{R}$ $p \in \mathbb{R}$
  - $U(q,p)\psi(x) = e^{ip(x-q/2)}\psi(x-q)$

- Here: $x \geq 0$, $p \in \mathbb{R}$ $\longrightarrow$ use affine group $U(q,p)\psi(x) = \frac{e^{ipx}}{\sqrt{q}}\psi\left(\frac{x}{q}\right)$

- Quantisation map $\hat{A}_f = \mathcal{N}\int_{\mathbb{R}\times\mathbb{R}^+} \mathrm{d}p\mathrm{d}q\, |q,p\rangle\, f(p,q)\, \langle q,p|$ where $|q,p\rangle = U(q,p)|\psi_0\rangle$

  fiducial state

  phase space function

  coherent state

- Quantisation of FLRW Hamiltonian $\mathcal{H} \propto p^2 + p_\tau$ leads to a repulsive potential

$$\boxed{\hat{A}_{p^2}\,\psi = -\partial_x^2\,\psi + \frac{K}{x^2}\psi} \longrightarrow \boxed{\hat{\mathcal{H}} \propto \hat{p}^2 + \frac{K}{\hat{q}^2} + \hat{p}_\tau}$$

time evolution

introduces a bounce

with

$$\boxed{\begin{aligned} \hat{p}\,\psi &= -\mathrm{i}\partial_x\psi \\ \hat{q}\,\psi &= x\,\psi \\ \hat{p}_\tau\psi &= -\mathrm{i}\partial_\tau\psi \end{aligned}}$$

[Klauder, ('99)] [Bergeron, Dapor, Gazeau, Małkiewicz, ('14)]

THE CHOICE OF STATE

- Use semiclassical state $|\psi\rangle = e^{-\mathrm{i}\phi(\tau)}|q(\tau), p(\tau)\rangle$
  - Satisfies the Schrödinger equation $\hat{\mathcal{H}}_{\mathrm{FLRW}}\psi - \mathrm{i}\partial_\tau\psi = 0$
  - Follows dynamics generated by the semiclassical Hamiltonian $\mathcal{H}_{\mathrm{sem}} = p^2 + \dfrac{K}{q^2}$

$$q(\tau) = q_B\sqrt{1+\omega^2(\tau-\tau_B)^2} \qquad p(\tau) = \frac{1}{2}\dot{q}(\tau) = \frac{q_B\,\omega^2(\tau-\tau_B)}{2\sqrt{1+\omega^2(\tau-\tau_B)^2}}$$

  - Specific choice of **fiducial state** $\langle x|q(\tau), p(\tau)\rangle = \dfrac{1}{\sqrt{q(\tau)}}\exp\left(\mathrm{i}\dfrac{p(\tau)}{2q(\tau)}x^2\right)\Phi_n\left(\dfrac{x}{q(\tau)}\right)$

[Bergeron, Gazeau, Małkiewicz, Peter ('23)]

Note: use different representation

$$V(q,p)\psi(x) = \frac{1}{\sqrt{q}}\exp\left(\mathrm{i}\frac{p}{2q}x^2\right)\psi\left(\frac{x}{q}\right)$$

- Wave function of the universe: $\psi_a(x) = \langle x|\psi_a\rangle$ with $x \propto a^{\frac{3}{2}(1-w)}$

$n \in \mathbb{N}$

$\nu^2 \geq 1$

$$\psi_a(x) = \sqrt{\frac{2\;n_a!}{\Gamma(\nu+n_a+1)}}\left(\frac{\xi_{\nu,n_a}-\mathrm{i}q_a p_a}{\xi_{\nu,n_a}+\mathrm{i}q_a p_a}\right)^{\frac{1}{2}(2n_a+\nu+1)}\xi_{\nu,n_a}^{\frac{\nu+1}{2}}\frac{x^{\nu+1/2}}{q_a^{\nu+1}}L_{n_a}^{\nu}\left(\xi_{\nu,n_a}\frac{x^2}{q_a^2}\right)\exp\left(-\frac{1}{2}(\xi_{\nu,n_a}-\mathrm{i}\,q_a p_a)\frac{x^2}{q_a^2}\right)$$

Phase: $e^{-\mathrm{i}\phi_a(\tau)} = \left(\dfrac{\xi_{\nu,n_a}-\mathrm{i}q_a p_a}{\xi_{\nu,n_a}+\mathrm{i}q_a p_a}\right)^{\frac{1}{2}(2n_a+\nu+1)}$

$$\xi_{\nu,n} = \left(\frac{n!}{\Gamma(n+\nu+1)}\int_0^\infty y^{\nu+\frac{1}{2}}\left(L_n^{(\nu)}(y)\right)^2 e^{-y}\mathrm{d}y\right)^2$$

QUANTUM CORRECTED SCALE FACTOR

- Quantum corrected scale factor has the same dynamics as a semiclassical trajectory → recover GR dynamics at late times

  $$x(\tau) \to x_0\,\omega\,\tau \Rightarrow a \propto t^{2/(3(1+w))} \quad \text{with} \quad x \propto a^{\frac{3}{2}(1-w)} \quad \text{and} \quad N\mathrm{d}\tau = \mathrm{d}t$$

  - Initial condition related to the expansion rate of the universe $\dot{x}(\tau) \to x_0\,\omega$

- Quantum potential $Q(x,\tau) = \frac{2\xi_\nu(\nu+1)}{q^2} - \frac{\xi_\nu^2 x^2}{q^4} - \frac{\nu^2 - \frac{1}{4}}{x^2}$ gives $\ddot{x} = -2\frac{\partial}{\partial x}\left(\frac{\nu^2-\frac{1}{4}}{x^2} + Q(x,\tau)\right) = \frac{4\xi_\nu^2}{q(\tau)^4}x$

- Hubble rate from trajectories
  - FLRW with perfect fluid for $\partial_x S = \text{const.}$

  $$H^2 = \left(\frac{\dot{a}}{Na}\right)^2 = \frac{4}{9(1-w)^2}\frac{\dot{x}^2}{N^2x^2} \propto \frac{(\partial_x S)^2}{a^{3(1+w)}} \propto \frac{(\partial_a S)^2}{a^4}$$

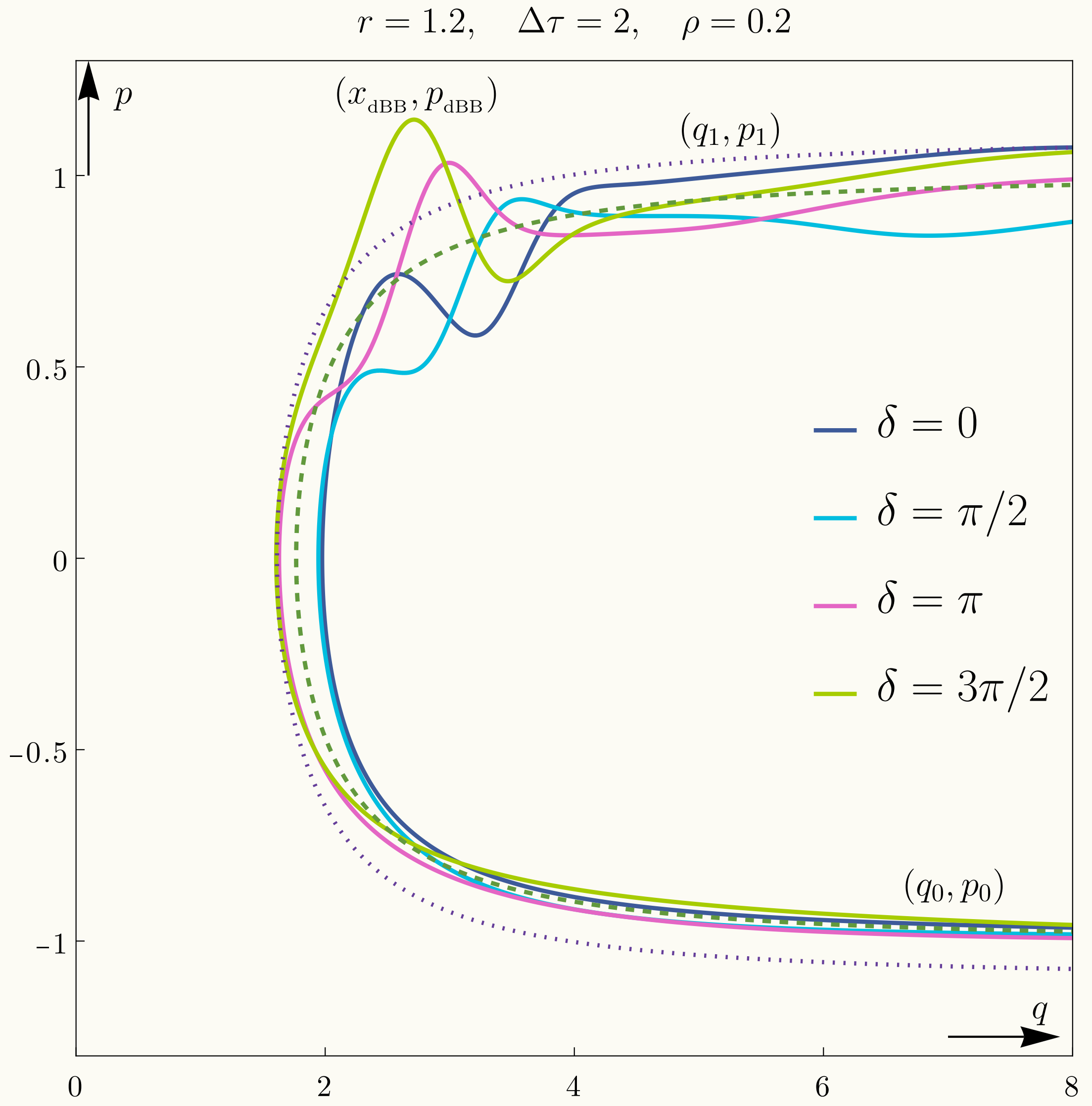
- Acceleration
  - FLRW for $\ddot{x} \to 0$

  $$\dot{H} = \frac{2}{3(1-w)}\frac{\ddot{x}}{Nx} - \frac{3}{2}(1+w)NH^2$$

UNIVERSE IN A SUPERPOSITION

- Phase space portraits and wave functions for the biverse $\Psi = \mathcal{N}(\psi_0 + \rho e^{i\delta}\psi_1)$

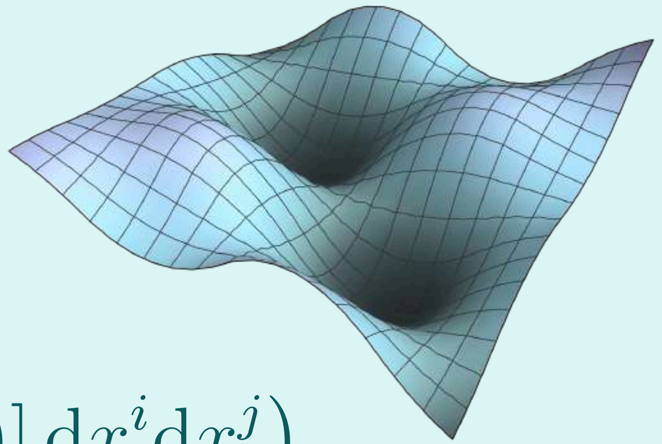
$p \propto a^{\frac{3}{2}(1-w)} H$



$q \propto a^{\frac{3}{2}(1-w)}$

- Trajectories highly dependent on initial conditions, but generally exhibit features that differ from single state trajectories

INCLUDING PERTURBATIONS



- Tensor perturbations
  - Pure gravity and do not require a matter source

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j \quad \xrightarrow[\text{matter = radiation}]{} \quad \mathrm{d}s^2 = a^2(\eta)\left(-\mathrm{d}\eta^2 + \left[\delta_{ij} + \underbrace{h_{ij}}_{\text{tensor perturbations}}(\eta,\vec{x})\right]\mathrm{d}x^i\mathrm{d}x^j\right)$$

$w = 1/3$

- Dynamics of tensor modes included in the Hamiltonian $\mathcal{H}_{\mathrm{FLRW}} \to \mathcal{H}_{\mathrm{FLRW}} + \mathcal{H}^{(2)}$
- Describe perturbations in Fourier space, encoded by $\mu_{\vec{k}}$ and the conjugate momentum $\pi_{\vec{k}}$

[Peter, Pinho, Pinto-Neto, ('05)]
[Peter, Pinho, Pinto-Neto, ('06)]
[Peter, MIcheli, ('23)]

with

$$h_{ij}(\vec{x},\eta) \propto \sum_{\vec{k},\lambda} \varepsilon_{ij}^{(\lambda)} \frac{\mu^{(\lambda)}(\vec{k},\eta)}{a(\eta)} e^{\mathrm{i}\vec{k}\cdot\vec{x}}$$

- Hamiltonian for perturbations

$\lambda = +, \times$

$$\mathcal{H}^{(2)} = \sum_{\vec{k}} \left(\mathcal{H}^{(2)}_{\vec{k},+} + \mathcal{H}^{(2)}_{\vec{k},\times}\right) \quad \text{with} \quad \mathcal{H}^{(2)}_{\vec{k},\lambda} = \pi^{(\lambda)}_{\vec{k}}\pi^{(\lambda)}_{-\vec{k}} + \left(k^2 - \frac{a''}{a}\right)\mu^{(\lambda)}_{\vec{k}}\mu^{(\lambda)}_{-\vec{k}}$$

$h_{ij} \propto \mu_{ij}/a$

conjugate momentum of $\mu^{(\lambda)}_{\vec{k}}$

PERTURBED SYSTEM

- Total Schrödinger equation $$\left(\mathcal{H}_{\text{FLRW}} + \mathcal{H}^{(2)}\right) \Psi(x, \{\mu_k\}) = \mathrm{i}\partial_\tau \Psi(x, \{\mu_k\})$$
- Born-Oppenheimer approximation $$\Psi(x, \{\mu_k\}) = \psi_{\text{bg}}(x)\psi_{\text{pert}}(x, \{\mu_k\})$$

Background:

$$\mathcal{H}_{\text{FLRW}}\psi_{\text{bg}}(x) = \mathrm{i}\partial_\tau \psi_{\text{bg}}(x)$$

Perturbations:

$$\mathcal{H}^{(2)}\psi_{\text{pert}}(x(\tau), \{\mu_k\}) = \mathrm{i}\partial_\tau \psi_{\text{pert}}(x(\tau), \{\mu_k\})$$

background dependent

trajectory extracted from the background

$$\mathcal{H}^{(2)}_{\vec{k},\lambda} = \pi^{(\lambda)}_{\vec{k}}\pi^{(\lambda)}_{-\vec{k}} + \left(k^2 - \frac{a''}{a}\right)\mu^{(\lambda)}_{\vec{k}}\mu^{(\lambda)}_{-\vec{k}}$$

What happens if the background is in a superposition?

$\rightarrow$ different treatments for trajectories and Copenhagen approach

FIRST APPROACH TO PERTURBATIONS

- Consider tensor perturbations in a flat FLRW spacetime with a radiation fluid (set $w = 1/3$)

$$\mathrm{d}s^2 = a^2(\eta)\left(-\mathrm{d}\eta^2 + \left[\delta_{ij} + h_{ij}(\eta, \vec{x})\right]\mathrm{d}x^i\mathrm{d}x^j\right)$$

[Peter, Pinho, Pinto-Neto, ('05)]

[Peter, Pinho, Pinto-Neto, ('06)]

- Second order linear perturbative Hamiltonian for tensor modes: sum of the two polarisations and Fourier modes

$$\mathcal{H}_{\mathrm{FLRW}} \to \mathcal{H}_{\mathrm{FLRW}} + \mathcal{H}^{(2)}$$

$$\mathcal{H}^{(2)} = \sum_{\vec{k}}\left(\mathcal{H}^{(2)}_{\vec{k},+} + \mathcal{H}^{(2)}_{\vec{k},\times}\right) \quad \text{with} \quad \mathcal{H}^{(2)}_{\vec{k},\lambda} = \pi^{(\lambda)}_{\vec{k}}\pi^{(\lambda)}_{-\vec{k}} + \left(k^2 - \frac{a''}{a}\right)\mu^{(\lambda)}_{\vec{k}}\mu^{(\lambda)}_{-\vec{k}}$$

$\lambda = +, \times$

conjugate momentum of $\mu^{(\lambda)}_{\vec{k}}$

$h_{ij} \propto \mu_{ij}/a$

- Use scale factor as given by quantum trajectories in perturbed Hamiltonian $\Psi_{\mathrm{pert}} = \Psi_{\mathrm{pert}}(a(\eta), \mu_k)$

- Canonical quantisation of the tensor perturbations leads to mode equation $\mu_k'' + \left(k^2 - \frac{a''}{a}\right)\mu_k = 0$

modified by trajectories

  - With perturbation operators decomposed as $\hat{\mu}_{\vec{k}}(\eta) = \mu_k(\eta)\hat{a}_{\vec{k}} + \mu_k^*(\eta)\hat{a}^\dagger_{-\vec{k}}$

creation and annihilation operators

EXAMPLE: DOUBLE SLIT EXPERIMENT

- Weak measurements – reconstruct trajectories

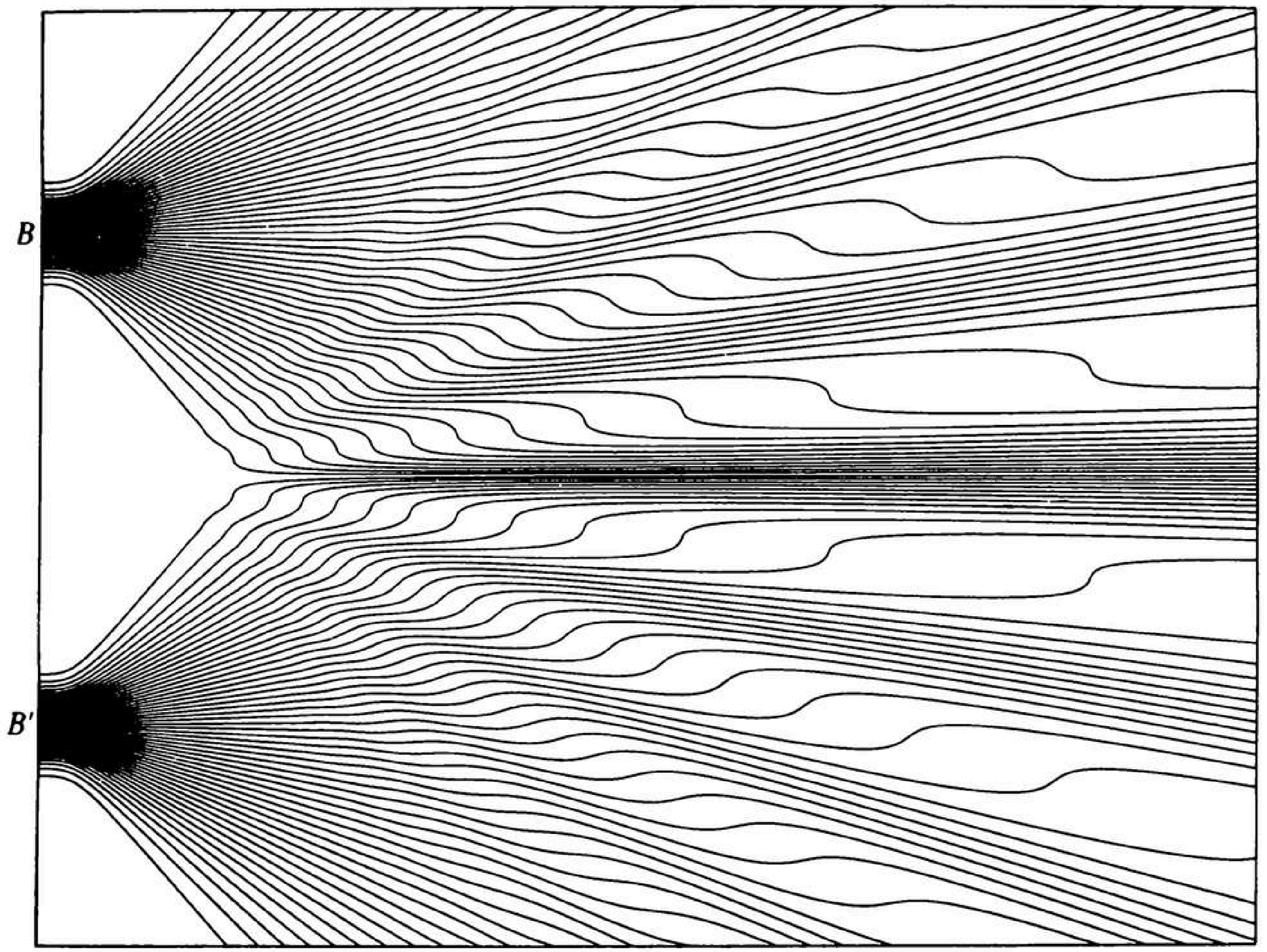


Fig. 5.7 Trajectories for two Gaussian slits with a Gaussian distribution of initial positions at each slit. The probability density is proportional to the number of lines per unit length in the *y*-direction (from Philippidis *et al.* (1982)).