Probing Penrose-type singularities inside sonic black holes

Satadal Datta and Uwe R. Fischer 2025 Class. Quantum Grav. 42 245009

Satadal Datta

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Outline

- Penrose black hole singularity in Einstein gravity,

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- Penrose singularity in the light of analogue gravity.

Einstein Gravity

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \qquad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

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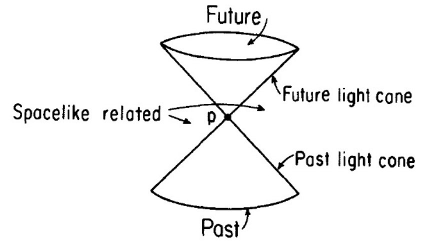
- Causality



[Source: Robert M. Wald, “General Relativity” textbook]

Spacetime singularities in Einstein Gravity

- Schwarzschild black hole,

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right).$$

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- Coordinate singularity, $r = \frac{2GM}{c^2}$.
- True singularity at $r = 0$.
  Kretschmann scalar, $K = R_{\mu\nu\lambda\kappa} R^{\mu\nu\lambda\kappa} = \frac{2G^2M^2}{c^2 r^6}$.

Gravitational Collapse

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⇒ spherically symmetric gravitational collapse of neutron stardust cloud $M > 0.7 M_\odot$.

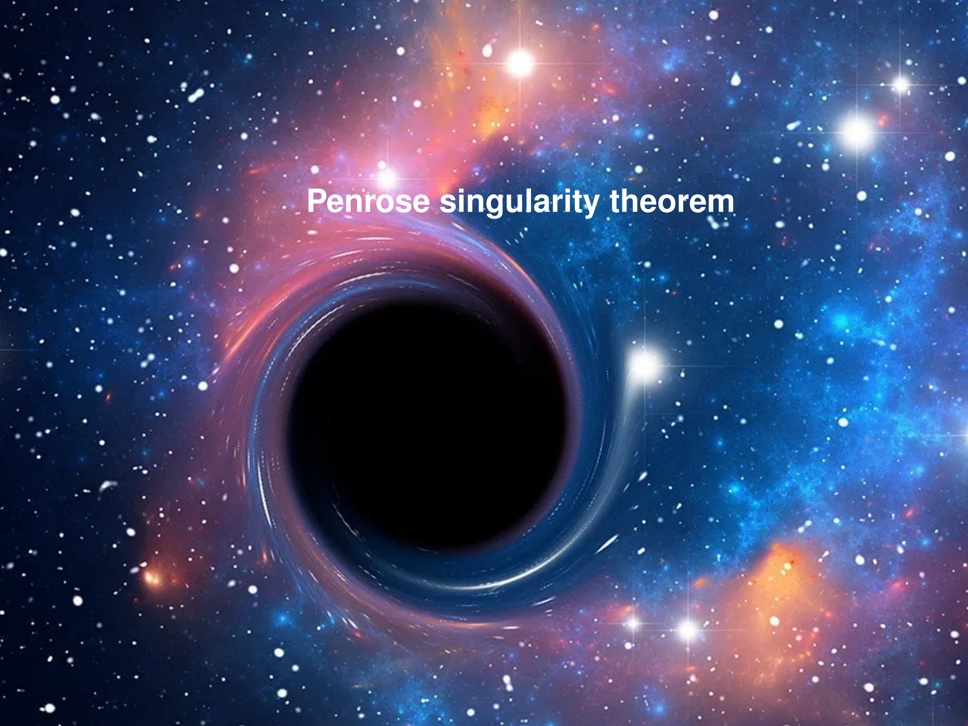
SEPTEMBER 1, 1939 PHYSICAL REVIEW VOLUME 56

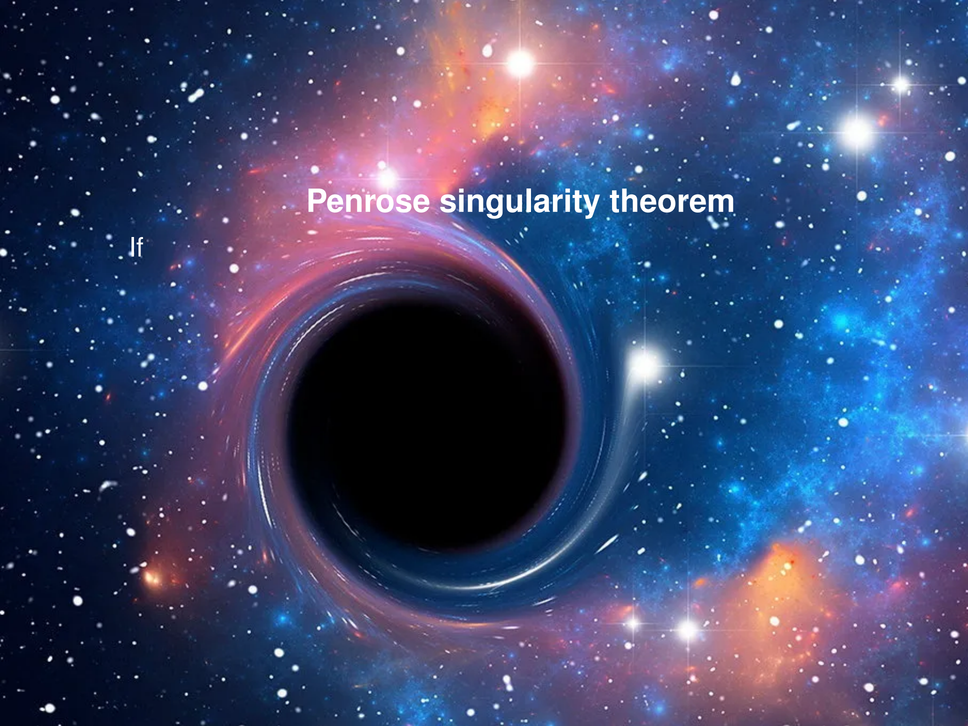
On Continued Gravitational Contraction

J. R. Oppenheimer and H. Snyder
University of California, Berkeley, California
(Received July 10, 1939)

When all thermonuclear sources of energy are exhausted a sufficiently heavy star will collapse. Unless fission due to rotation, the radiation of mass, or the blowing off of mass by radiation, reduce the star's mass to the order of that of the sun, this contraction will continue indefinitely. In the present paper we study the solutions of the gravitational field equations which describe this process. In I, general and qualitative arguments are given on the behavior of the metrical tensor as the contraction progresses: the radius of the star approaches asymptotically its gravitational radius; light from the surface of the star is progressively reddened, and can escape over a progressively narrower range of angles. In II, an analytic solution of the field equations confirming these general arguments is obtained for the case that the pressure within the star can be neglected. The total time of collapse for an observer comoving with the stellar matter is finite, and for this idealized case and typical stellar masses, of the order of a day; an external observer sees the star asymptotically shrinking to its gravitational radius.

Penrose singularity theorem





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Null geodesics orthogonal to the trapped surface truncates at a caustic in future within finite affine parameter length.

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Trapped surface, closed spacelike codimension 2 submanifold, null rays orthogonal to it, initially converge.

Null energy condition, $T_{\mu\nu}l^\mu l^\nu \geq 0$, $l_\mu l^\mu = 0 \Rightarrow$ in Einstein gravity, $G_{\mu\nu}l^\mu l^\nu \geq 0 \Rightarrow R_{\mu\nu}l^\mu l^\nu \geq 0$.
For, $T_{\mu\nu} = pg_{\mu\nu} + (p+\rho)v_\mu v_\nu$, null energy condition implies $(p+\rho) \geq 0$.

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In affine parametrization ($x^\mu = x^\mu(\lambda)$), geodesic equation takes the form,

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In an arbitrary parametrization ($x^\mu = x^\mu(t)$)

$$\frac{d^2x^\mu}{dt^2} + \Gamma^\mu_{\nu\sigma}\frac{dx^\nu}{dt}\frac{dx^\sigma}{dt} = f(t)\frac{dx^\mu}{dt}.$$

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In affine parametrization $(x^\mu = x^\mu(\lambda))$ length of a tangent vector $\frac{dx^\mu}{d\lambda}$ is preserved; $\frac{d}{d\lambda}\left(g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}\right)^{\frac{1}{2}} = 0.$

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Ex. 1:- In Euclidean geometry ($ds^2 = dx^2 + dy^2$), parametrization of a starigt line $y = x$; $x = \lambda$, $y = \lambda$ is affine but $x = t^3$, $y = t^3$, ($t \in \mathbb{R}$) is not affine.

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Ex. 2:- Proper time of a timelike geodesic.

Caustic, where null expansion Θ diverges. $\Theta = l^{\mu}{}_{;\mu}$, for affine parametrized l^{μ} in a null geodesic congruence ($l^{\mu} = l^{\mu}(x)$).

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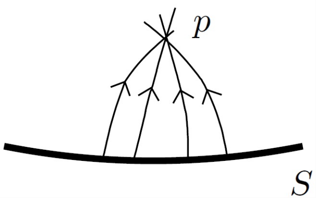
Lemma:- If $R_{\mu\nu}l^{\mu}l^{\nu} \geq 0$ for a null geodesic congruence which are initially converging and orthogonal to a spacelike codimension 2 submanifold S, then Θ diverge within finite affine parameter interval from S. Proof:- (Using Raychaudhuri equation for null geodesic congruence).



E. Witten, "Light rays, singularities, and all that", Rev. Mod. Phys. 92, 045004 (2020).

Null Completeness

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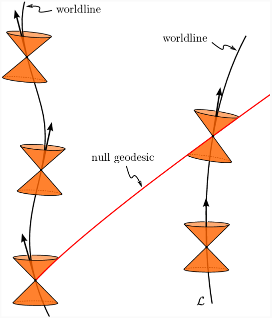

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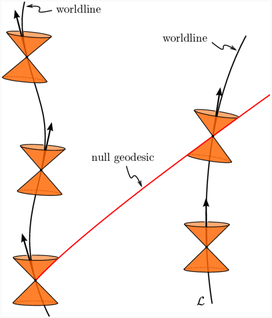

“Keep in mind how fast things pass by and are gone—those that are now, and those to come. Existence flows past us like a river: the ‘what’ is in constant flux, the ‘why’ has a thousand variations. Nothing is stable, not even what’s right here. The infinity of past and future gapes before us-a chasm whose depths we cannot see.”-Marcus, Aurelius, Meditation

Penrose Singularity

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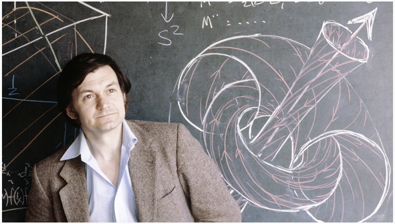

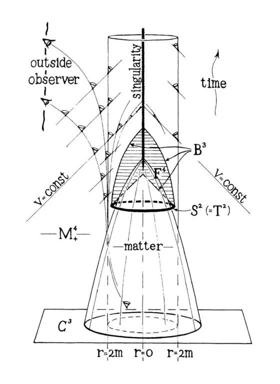

R. Penrose, “Gravitational Collapse and Space-Time Singularities”, Phys. Rev. Lett. 14, 57 (1965).

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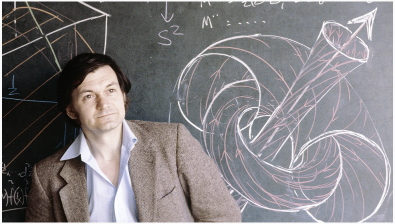

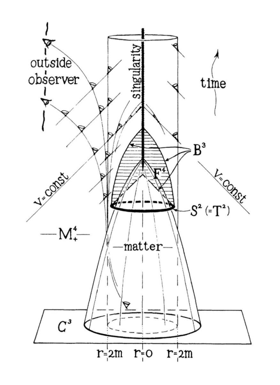

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Nobel Prize in Physics 2020.

Need for New Physics!

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Karen Crowther, Sebastian De Haro, "Four Attitudes Towards Singularities in the Search for a Theory of Quantum Gravity", arXiv:2112.08531 [gr-qc].

Analogue Gravity

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Linear perturbation in a barotropic, inviscid, irrotational fluid behaves like a massless Klein-Gordon scalar field in a curved spacetime.

Penrose singularity in analogue black holes?

Cosmic Censorship Conjecture by R. Penrose prohibits access to black hole singularity. System governing an analogue black hole is very much known to all relevant scales, we can probe into the interior of an analogue black hole.

- 2D axially symmetric BEC flow (Thomas-Fermi approximation),

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$$\frac{d}{dr}(r\rho_0 v_0^r) = 0,$$

$$r\rho_0 v_0^r = \text{constant} = C_1$$

$$v_0^r \frac{dv_0^r}{dr} - \frac{(v_0^\phi)^2}{r} = -\frac{1}{\rho_0}\frac{dp_0}{dr} - \frac{dV_{ext}}{dr}.$$

$$r v_0^\phi = \text{constant} = l.$$

$$p_0 = \frac{1}{2} g_{2D} \rho_0^2.$$

In our unit, g_{2D} is unity. Therefore, $c_{s0}^2 = \frac{dp_0}{d\rho_0} = \rho_0$.

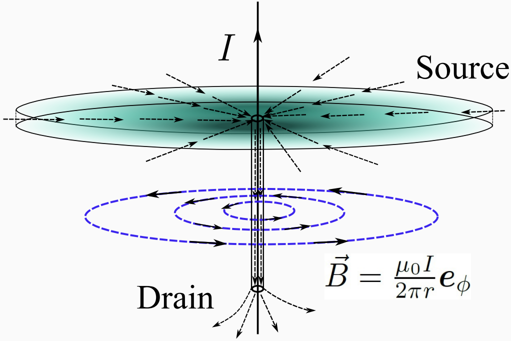
$$\frac{1}{2}(v_0^r)^2 + \frac{l^2}{2r^2} + \frac{c_{s0}^2}{(\gamma - 1)} + V_{ext}(r) = \text{constant} = C_2.$$

Acoustic metric,

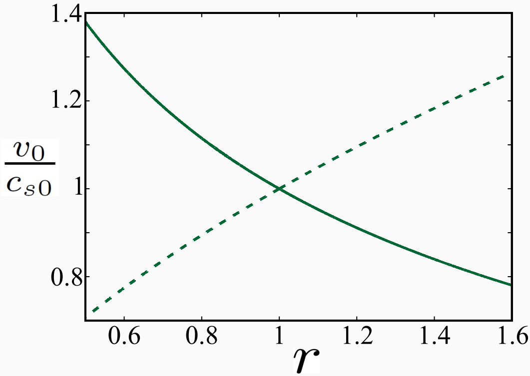
$$g_{\mu\nu} = \left(\frac{\rho_0}{c_{s0}}\right)^2 \begin{bmatrix} -\left(c_{s0}^2 - (v_0^r)^2 - \frac{l^2}{r^2}\right) & -v_0^r & -l \\ -v_0^r & 1 & 0 \\ -l & 0 & r^2 \end{bmatrix}.$$

$$\Omega^2 = \left(\frac{\rho_0}{c_{s0}}\right)^2 = c_{s0}^2.$$

- Design of an experimental Set up:



$$V_{ext}(r) = -\vec{\mu}\cdot\vec{B} = -\frac{\mu\mu_0 I}{2\pi r}.$$



Transonic purely radial flow ($l = 0$).

Penrose type singularity?

- Affine length, $d\lambda = \Omega^2 c_{s0}^2 dr = c_{s0}^3 dr$ for radially ingoing $(\frac{dr}{dt} = -c_{s0} - v_0)$/outgoing modes $(\frac{dr}{dt} = c_{s0} - v_0)$.

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  In our unit, $r_H = 1$, $c_{s0}(r = r_H) = v_0(r = r_H) = 1$, where $v_0 = |v_0^r|$.
  From continuity, and equation of state of BEC,

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  $\Rightarrow$ for $v_0(r) > c_{s0}(r)$, $c_{s0}^3 < \frac{1}{r}$.

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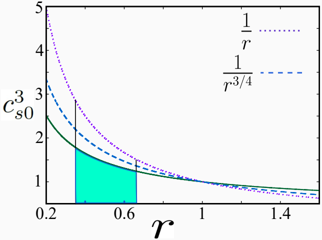

Bounded affine parameter interval.

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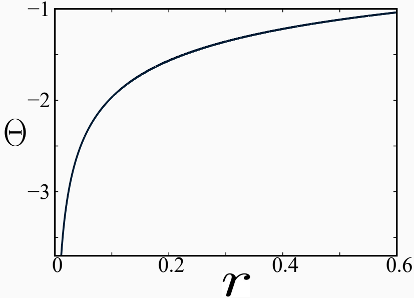

$r = 0$ is the caustic, and the location of Penrose singularity.

Penrose singularity in the light of our analogue black hole model

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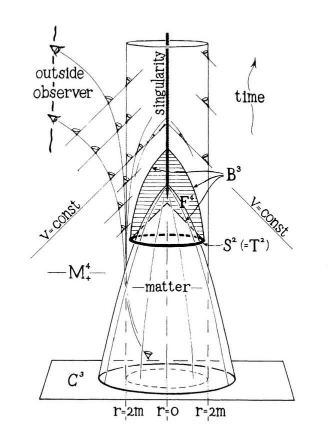

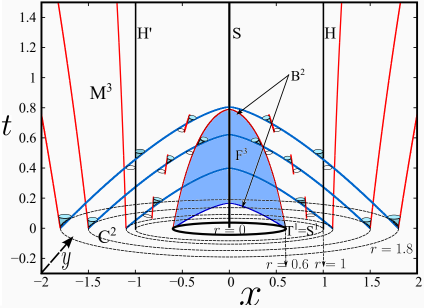

Discussion

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- Curvature singularity and Penrose type singularity, Fluid dynamical singularities correspond to singularities in analogue spacetime of some sort? Uwe R. Fischer and Satadal Datta, “Dispersive censor of acoustic spacetimes with a shock-wave singularity”, PhysRevD.107.084023 (2023).

Thank you