Collective excitations of quantum gravity condensates

Andrea Calcinari

in collaboration with Adrià Delhom and Daniele Oriti

Universidad Complutense Madrid

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Outline

BEC physics

- GPE Hamiltonian describes bosonic atoms with a contact interaction

$$\hat{H} = \int_{\Sigma} \mathrm{d}V \left[\frac{\hbar^2}{2m} \nabla \hat{\Psi}^\dagger(x) \nabla \hat{\Psi}(x) + + \frac{g}{2} \hat{\Psi}^\dagger(x) \hat{\Psi}(x) \hat{\Psi}^\dagger(x) \hat{\Psi}(x) \right]$$

- At $T \to 0$ most atoms condense in the ground state $\Psi_0 = \sqrt{n_0} \mathrm{e}^{-\mathrm{i}\mu t/\hbar}$
- Relevant physics = **macroscopic** occupation of a single state
- The atomic field operator can be expanded as $\hat{\Psi} = \frac{1}{2\pi\sqrt{V}} \int \mathrm{d}\mathbf{k} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} \hat{a}_{\mathbf{k}}$
- **Bogolyubov approximation:** Treat ground state as classical field

$$\hat{a}_0 \to \langle \hat{a}_0 \rangle = \sqrt{N} \qquad \text{and} \qquad \hat{a}_0^\dagger \to \langle \hat{a}_0^\dagger \rangle = \sqrt{N}$$

Bogolyubov excitations

- Low energy excitations of the condensate as leading order terms in $\hat{a}_{\mathbf{k}\neq 0}$ and $\hat{a}^\dagger_{\mathbf{k}\neq 0}$

$$\hat{H}_2 = \int \mathrm{d}\mathbf{k} \left[\frac{\hbar^2 \mathrm{k}^2}{2m} \hat{a}^\dagger_{\mathbf{k}} \hat{a}_{\mathbf{k}} + \frac{g}{2V} \left(2\hat{a}^\dagger_{\mathbf{k}} \hat{a}_{\mathbf{k}} + \hat{a}^\dagger_{\mathbf{k}} \hat{a}^\dagger_{-\mathbf{k}} + \hat{a}_{\mathbf{k}} \hat{a}_{-\mathbf{k}} \right) \right]$$

- Diagonalisation is achieved through a Bogolyubov transformation

$$\hat{a}_{\mathbf{k}} = u_{\mathrm{k}} \hat{b}_{\mathbf{k}} + v^*_{\mathrm{k}} \hat{b}^\dagger_{-\mathbf{k}} \qquad \text{and} \qquad \hat{a}^\dagger_{\mathbf{k}} = u^*_{k} \hat{b}^\dagger_{\mathbf{k}} + v_{\mathrm{k}} \hat{b}_{-\mathbf{k}}$$

  where $|u_{\mathrm{k}}|^2 - |v_{\mathrm{k}}|^2 = 1$ is required to ensure $[\hat{b}_{\mathbf{k}}, \hat{b}^\dagger_{\mathbf{q}}] = \delta(\mathbf{k} - \mathbf{q})$

- $v_{\mathbf{k}} \neq 0$ implies that the vacuum of $\hat{b}_{\mathbf{k}}$ modes is not the atomic vacuum
- $\hat{b}_{\mathbf{k}}$ annihilate *collective* modes of the system, rather than individual atoms

Can we do similar things in Quantum Gravity?

- Quantum gravity framework with "**atoms**" and perhaps "**condensates**"?

Group Field Theory (GFT)

What is Group Field Theory? [Reisenberger, Rovelli, De Pietri, Krasnov, Oriti, ...]

- GFT is a nonperturbative and background-independent approach to quantum gravity

    $\Rightarrow$ field theory *of* spacetime, so it cannot be defined *on* spacetime

- In a nutshell: scalar "group field" φ defined on an abstract configuration space (a Lie group rather than spacetime, eg $\varphi : U(1)^4 \times \mathbb{R} \to \mathbb{R}$)

- There is **no spacetime** structure in the fundamental theory

    $\Rightarrow$ geometry is *emergent* (some sort of condensate of GFT)

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NB: In analogues the condensate is the "stage", here it will be dynamical!

- Start from action (field written in terms of "Fourier" modes $\mathbf{n}$ and a real number χ)

$$S[\varphi] = \frac{1}{2}\int \mathrm{d}\chi \sum_{\mathbf{n}} \varphi_{-\mathbf{n}}(\chi)\left(K_{\mathbf{n}}^{(0)} + K_{\mathbf{n}}^{(2)}\partial_\chi^2\right)\varphi_{\mathbf{n}}(\chi) + \cancel{V[\varphi]}$$

- *Canonical quantisation* of GFT enables the use of second quantisation methods
  $\Rightarrow$ **"quanta of geometry" of an underlying QFT of gravity**

$$\hat{a}_{\mathbf{n}}|0\rangle = 0\,,$$

$\Big|$  $\Big\rangle = |1\rangle = \hat{a}^\dagger_{\mathbf{n}}|0\rangle$

with $[\hat{a}_{\mathbf{n}}\,,\,\hat{a}^\dagger_{\mathbf{m}}] = \delta_{\mathbf{n},\mathbf{m}}$ (build GFT Fock space)

- NB: the quanta carry fundamental units of "quantum" volume

- Treat χ as **relational time**[1], construct number and volume operators:

$$\hat{V}_{\mathbf{n}}(\chi) = v_{\mathbf{n}}\, \hat{N}_{\mathbf{n}}(\chi) = v_{\mathbf{n}}\, \hat{a}^{\dagger}_{\mathbf{n}}(\chi)\hat{a}_{\mathbf{n}}(\chi)$$

- Evolve operators in Heisenberg picture using $\hat{H}$ as $\mathrm{d}\hat{O}/\mathrm{d}\chi = [\hat{O}, \hat{H}]$ and study dynamics
- In the ladder operator basis the Hamiltonian reads (depending on the modes: HO, SQ)

$$\hat{H} = \sum_{\mathbf{n}\in\mathfrak{N}^{\mathrm{HO}}} \omega_{\mathbf{n}} \left(\hat{a}^{\dagger}_{\mathbf{n}}\hat{a}_{\mathbf{n}} + \frac{1}{2}\right) + \sum_{\mathbf{n}\in\mathfrak{N}^{\mathrm{SQ}}} \frac{\omega_{\mathbf{n}}}{2} \left(\hat{a}^{\dagger}_{\mathbf{n}}\hat{a}^{\dagger}_{-\mathbf{n}} + \hat{a}_{\mathbf{n}}\hat{a}_{-\mathbf{n}}\right)\,,$$

with $\omega_{\mathbf{n}} = -\,\mathrm{sgn}\left(K^{(0)}_{\mathbf{n}}\right)\sqrt{|K^{(0)}_{\mathbf{n}}/K^{(2)}_{\mathbf{n}}|}$

- Let us study the total volume $V(\chi)$ wrt to scalar field $\chi \Rightarrow$ **GFT cosmology**

[1] Can be done with "quantum clocks" using Page–Wootters formalism [AC, S. Gielen]

- Focus on the *free* theory: dynamics are dominated by the mode for which squeezing rate $|\omega_{\mathbf{n_0}}|$ is maximised $\Rightarrow$ GFT naturally selects a mode for a macroscopic theory!

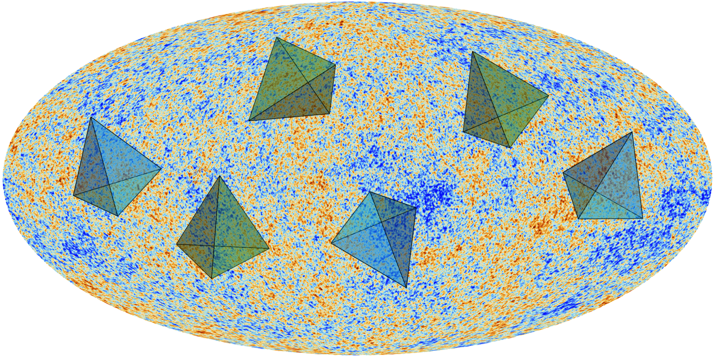
- Take EV[2] of that volume mode $V(\chi) \equiv \langle \hat{V}_{\mathbf{n_0}}(\chi) \rangle$ and cast evolution of the universe as:

$$\left(\frac{1}{V(\chi)} \frac{\mathrm{d}V(\chi)}{\mathrm{d}\chi} \right)^2 = 4\omega_{\mathbf{n_0}}^2 \left(1 + \frac{C_0}{V(\chi)} - \frac{E_0}{V^2(\chi)} \right) \xrightarrow{\text{late time}} 12\pi G_{\mathrm{N}} \quad \text{with} \quad \omega_{\mathbf{n_0}}^2 = 3\pi G_{\mathrm{N}}$$

  - matching classical cosmology $\rightarrow$ *emergence of Newton's constant* from GFT
  - exponential volume expansion after a "big bounce" instead of the big bang singularity

[2] EV with semiclassical states: good properties shown for rich family of Gaussian states [AC, S. Gielen]

Cosmology as "condensation"



Macroscopic cosmological spacetime emerges as a collection of microscopic dof

Dynamical condensation (built-in quantum gravity mechanism)

- **Recap:** There is a mode $\mathbf{n_0} \neq \mathbf{0}$ that will necessarily dominate dynamically

- We interpret $\mathbf{n_0}$ as the **condensate**: this becomes the most populated (by far), and gives the GFT cosmology dynamics shown earlier (**macroscopic** occupation)

- **Number of quanta in the condensate grows** in time $\Rightarrow$ expanding universe

Condensation does not happen by manually lowering the temperature in the lab – the mode $\mathbf{n_0}$ gets populated as a dynamical process occurring naturally in GFT! (squeezing)

- Interpreting this as condensation, then natural questions arise:
  - Can we see some of the typical BEC features? Can we study collective excitations, corrections from "out of condensate" modes, etc?
  - Importantly: what are the resulting cosmological implications?

- Add **local** (weak) interaction in the GFT model ($\sim \lambda\varphi^4$)
- **BEC techniques** (split '*cond*' and '*out-of-cond*' modes, large N_c, leading order)
  - replace $\hat{a}_{\mathbf{n_0}} \to \sqrt{N_c}$ while keeping $\hat{a}_{\mathbf{n}}$ describing fluctuations

$\Rightarrow$ Hamiltonian splits as "condensate + perturbations": $\hat{H} = \hat{H}_0 + \hat{H}_2$

- **Diagonalise** $\hat{H}_2$

$$\hat{H}_2 = \sum\!\!\!\!\!\!\int_{\boldsymbol{\nu}\in\mathfrak{S}^{\mathrm{HO}}} \omega^\lambda_{\boldsymbol{\nu}} \left(\hat{b}^\dagger_{\boldsymbol{\nu}}\hat{b}_{\boldsymbol{\nu}} + \frac{1}{2}\right) + \frac{1}{2}\sum\!\!\!\!\!\!\int_{\boldsymbol{\nu}\in\mathfrak{S}^{\mathrm{SQ}}} \omega^\lambda_{\boldsymbol{\nu}} \left(\hat{b}^\dagger_{\boldsymbol{\nu}}\hat{b}^\dagger_{-\boldsymbol{\nu}} + \hat{b}_{\boldsymbol{\nu}}\hat{b}_{-\boldsymbol{\nu}}\right),$$

with **Bogolyubov transformation** ($\boldsymbol{\nu}$ just means 'out of condensate mode')

$$\hat{b}_{\boldsymbol{\nu}} = u_{\boldsymbol{\nu}}\,\hat{a}_{\boldsymbol{\nu}} + v_{\boldsymbol{\nu}}\,\hat{a}^\dagger_{-\boldsymbol{\nu}}\,, \qquad \hat{b}^\dagger_{\boldsymbol{\nu}} = v_{\boldsymbol{\nu}}\,\hat{a}_{-\boldsymbol{\nu}} + u_{\boldsymbol{\nu}}\,\hat{a}^\dagger_{\boldsymbol{\nu}}\,, \qquad (u^2_{\boldsymbol{\nu}} - v^2_{\boldsymbol{\nu}} = 1)$$

Time dependence and stability

[AC, A. Delhom, D. Oriti, To Appear]

- We will focus on the first Hamiltonian ("stable", HO-like)

- The Hamiltonian $\hat{H}_2$ shows a new (time-dependent) frequency

$$\omega_{\boldsymbol{\nu}}^{\lambda} = -\operatorname{sgn}\left(K_{\boldsymbol{\nu}}^{(0)} + \lambda N_{\mathrm{c}} f_{\boldsymbol{\nu}}\right)\sqrt{\frac{\left|K_{\boldsymbol{\nu}}^{(0)} + \lambda N_{\mathrm{c}} f_{\boldsymbol{\nu}}\right|}{\left|K_{\boldsymbol{\nu}}^{(2)}\right|}}$$

where $f_{\boldsymbol{\nu}}$ is like a "Bloch factor" (function of the mode)

- λ multiplies $N_{\mathrm{c}}(\chi)$ – interaction not felt during condensate formation but important in Bogolyubov regime (when number of quanta in condensate is large)

Why not squeezing normal modes?

Consistency check: out-of-condensate vs condensate quanta

$$R_{\nu}(\chi) := \frac{\langle \hat{N}_{\nu}(\chi) \rangle}{N_{\mathrm{c}}(\chi)}$$

gives a criterion to check in which **region of the GFT parameter space** the Bogolyubov approach is well-defined. **In a nutshell** we find:

HO normal modes are safe as their growth is much slower than that of the condensate: approximation gets better over time

SQ normal modes inevitably take over: R_{SQ} increases and breaks Bogolyubov approx. (different phases? pre-condensate, double condensate - 'beyond Bogolyubov')

[**Question for audience:** are there techniques to study unstable collective modes?]

Cosmological consequences: corrections to Friedmann equation

- The total number of quanta is $\hat{N}(\chi) = N_c(\chi) + \sum\!\!\!\!\!\!\int_{\nu} \hat{N}_{\nu}(\chi)$

Normal modes are independent $\rightarrow \hat{N}(\chi) = N_c(\chi) + \hat{N}_{\nu}^{HO}(\chi)$ for simplicity

- From this, one calculates again the dynamics of the volume of the universe

$$\left(\frac{V'(\chi)}{V(\chi)}\right)^2 = A_\lambda + \frac{B_\lambda}{V^{1/2}(\chi)} + \frac{C_\lambda}{V(\chi)} + \frac{D_\lambda}{V^{3/2}(\chi)} + \frac{E_\lambda}{V^2(\chi)} + \frac{F_\lambda}{V^{5/2}(\chi)} + \cdots$$

  - when $\lambda = 0$ red coefficient vanish; $A_\lambda, C_\lambda, E_\lambda$ reduce to "old" values (condensate)
  - These are genuinely **new effects** coming from **"beyond condensate"** contributions

Cosmological consequences: corrections to Friedmann equation

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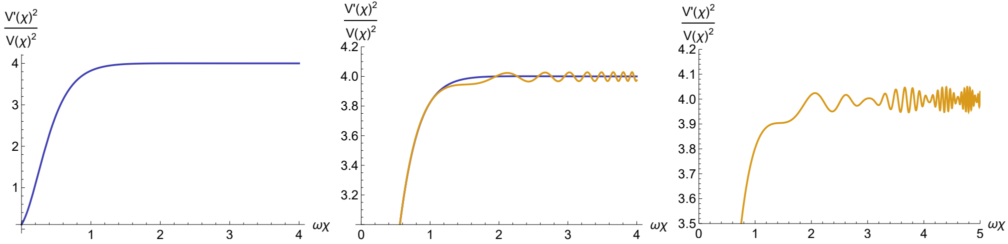
- From this, one calculates again the dynamics of the volume of the universe

$$\left(\frac{V'(\chi)}{V(\chi)}\right)^2 = \left(\frac{V'_{\mathrm{c}}(\chi)}{V_{\mathrm{c}}(\chi)}\right)^2 + \mathcal{B}^{(1)}_{\boldsymbol{\nu}}\sqrt{|\lambda|} + \mathcal{B}^{(2)}_{\boldsymbol{\nu}}\,|\lambda| + \mathcal{O}(|\lambda|^{3/2})$$

  - where $\mathcal{B}^{(i)}_{\boldsymbol{\nu}}$ govern oscillations and depend on the mode, initial conditions, etc
  - These are genuinely **new effects** coming from **"beyond condensate"** contributions

What are these corrections?

[AC, A. Delhom, D. Oriti, To Appear]



- **Showing:** condensate only, one 'out of condensate' mode, multiple modes
- **Interpretation:** fluctuations understood as small oscillations in the overall expansion
  - The perturbations here only depend on χ so we are studying the **fluctuations of the whole universe as functions of time**
  - (by adding *relational spatial coordinates* (rods) we could include inhomogeneities)

Conclusion and outlook

- **Quantum gravity** formalism that resembles condensed matter physics

$\Rightarrow$ **GFT cosmology** is seen as condensate (macro) of quantum gravity (micro) dofs

- **Upshot of our work:** Genuine "beyond condensate" effects (quantum corrections) on cosmological evolution in quantum gravity (GFT) using BEC-like techniques

$\Rightarrow$ the paradigm admits collective excitations of quantum gravity condensates

- Work in progress: include **rods**, ie (relational) spatial coordinates $\Rightarrow$ inhomogeneities
  - compare with Bogolyubov dispersion relation (phonons)
  - compare to cosmology (inhomogeneous perturbations)

- Can we connect with **experiments**? (Growing condensates in the lab? Polaritons?)

$\Rightarrow$ Can we establish a new field called "analogue quantum gravity"?

Extra material

Normal modes dynamics

• Normal modes describing quasi-particles evolve with a HO- or a SQ-like Hamiltonian, but $\Phi^2(\chi)$ can dynamically change the type of dynamics for some modes $\rightarrow \Phi^2_T := -K^{(0)}_\nu/(\lambda f_\nu)$

Case	Sign pattern	Type of dynamics
I	$\text{sgn}(K^{(0)}_\nu) = \text{sgn}(K^{(2)}_\nu) = \text{sgn}(\lambda)$	HO for all Φ^2
II	$\text{sgn}(K^{(0)}_\nu) = \text{sgn}(K^{(2)}_\nu) = -\text{sgn}(\lambda)$	HO for $\Phi^2 < \Phi^2_T$ SQ for $\Phi^2 > \Phi^2_T$
III	$\text{sgn}(K^{(0)}_\nu) = -\text{sgn}(K^{(2)}_\nu) = \text{sgn}(\lambda)$	SQ for all Φ^2
IV	$-\text{sgn}(K^{(0)}_\nu) = \text{sgn}(K^{(2)}_\nu) = \text{sgn}(\lambda)$	SQ for $\Phi^2 < \Phi^2_T$ HO for $\Phi^2 > \Phi^2_T$

Bogolyubov regime: $N_c \sim \Phi^2 \gg 1$

$\downarrow$

we focus on the regimes past the threshold Φ^2_T for cases II and IV

• **'Stability'**: does the condensate *remain the condensate* when including quasi-particles?

Not obvious: frequency, Hamiltonian, $\hat{b}$-operators and Bogolyubov coefficients – all evolve in time

Validity of Bogolyubov approximation in GFT

- To be precise, one finds the out-of-the-condensate number operator (from Bogolyubov tranf.)

$$\hat{N}_\nu(\chi) = \hat{a}^\dagger_\nu \hat{a}_\nu = (u_\nu^2 + v_\nu^2)\hat{b}^\dagger_\nu \hat{b}_\nu + v_\nu^2 - u_\nu v_\nu \left(\hat{b}_\nu^2 + \hat{b}_\nu^{\dagger\,2}\right)$$

($\neq \hat{N}^{\mathrm{bog}}_\nu := \hat{b}^\dagger_\nu \hat{b}_\nu$ in general) which can then be summed over modes $\hat{N}_{\mathrm{out}} := \sum\!\!\!\!\!\!\int_\nu \hat{N}_\nu$.

- Then, one checks *dynamically* whether the ratio

$$R_\nu(\chi) := \frac{\langle \hat{N}_\nu(\chi)\rangle}{N_c(\chi)}$$

increases or decreases – this gives a criterion to check in which **region of the GFT parameter space** the Bogolyubov approach is well-defined. **In a nutshell** we find:

HO normal modes are safe as their growth is much slower than that of the condensate: R_{HO} decreases in χ and approximation gets better over time

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Expansion for small λ

- One can also expand the dynamics as (only showing leading contribution)

$$\left(\frac{V'(\chi)}{V(\chi)}\right)^2 = \left(\frac{V'_{\text{cond}}}{V_{\text{cond}}}\right)^2 - \left[k_{\boldsymbol{\nu}}\frac{(V'_{\text{cond}})^2}{V_{\text{cond}}^{5/2}}\left(N_{\boldsymbol{\nu}}^{\text{bog}} + \tfrac{1}{2} - \mathcal{B}_{\boldsymbol{\nu}}\right)\right]\sqrt{|\lambda|} + \left[\cdots\right]|\lambda| + \mathcal{O}(|\lambda|^{3/2})$$

where

- V_{cond} gives "old" Friedmann dynamics
- $k_{\boldsymbol{\nu}}$ is a mode-dependent constant
- initial conditions, eg: $N_{\boldsymbol{\nu}}^{\text{bog}} = \langle \hat{b}_{\boldsymbol{\nu}}^{\dagger}(0)\hat{b}_{\boldsymbol{\nu}}(0)\rangle$
- $\mathcal{B}_{\boldsymbol{\nu}} \sim \exp\left(\pm 2\mathrm{i}\int^{\chi}\omega_{\boldsymbol{\nu}}^{\lambda}\right)$ governs oscillations

Classical **relational** cosmology

- In GR, consider flat FLRW Universe with metric: $\mathrm{d}s^2 = -N(t)^2\mathrm{d}t^2 + V(t)^{2/3}\mathrm{d}\mathbf{x}^2$
- We want to describe the volume $V = a^3$ as function of χ, **not t**
- Start from the usual FE

$$H^2 = \left(\frac{\dot{a}}{aN}\right)^2 = \left(\frac{\dot{V}}{3VN}\right)^2 = \frac{8\pi G}{3}\rho_\chi$$

substitute energy density for scalar and the lapse corresponding to choosing χ as clock

$$\rho_\chi = \frac{p_\chi^2}{2V^2}\,, \qquad \dot{\chi} = 1 \Rightarrow N = \frac{V}{p_\chi}$$

- Obtain **relational Friedmann equation**

$$\left(\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}\chi}\right)^2 = 12\pi G$$