Toward probing QFT vacuum entanglement

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in collaboration with

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IPARCOS

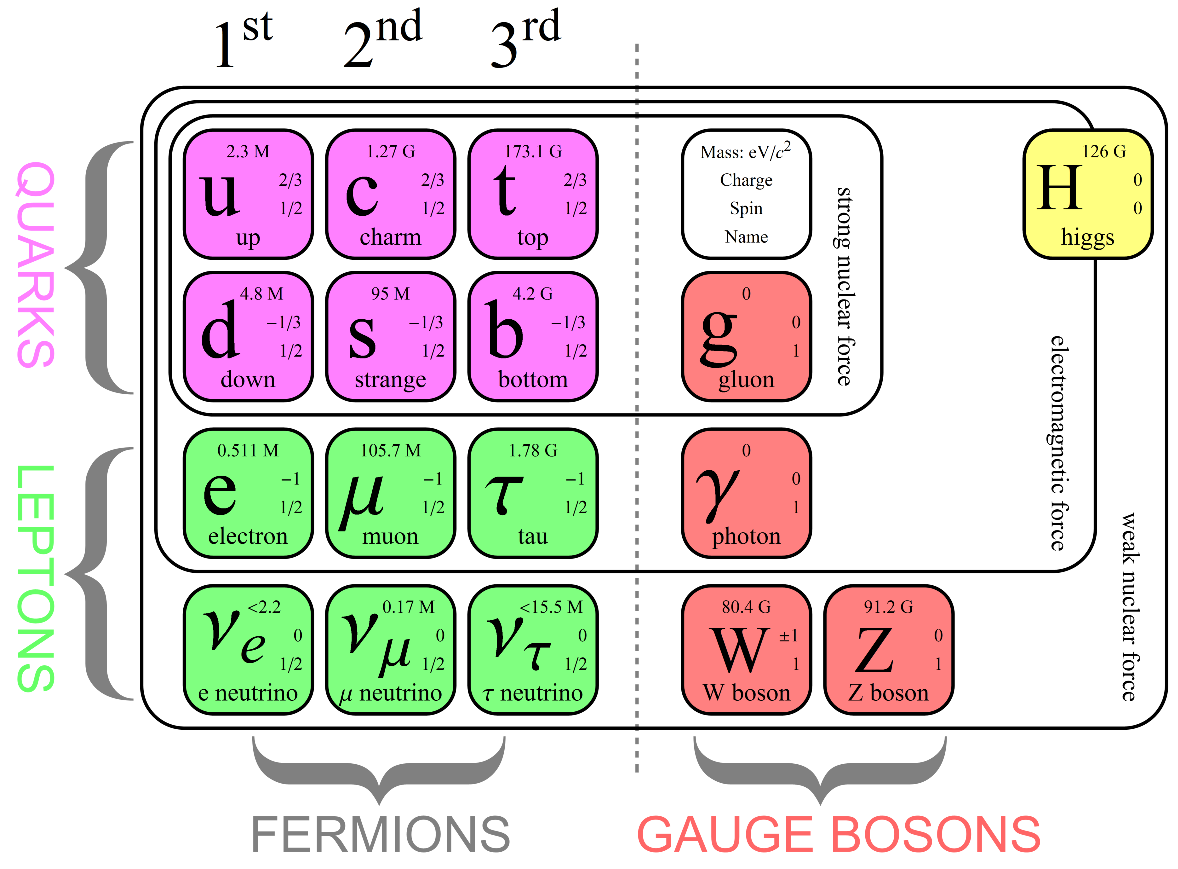
Outline

- Introduction
- A pedagogical introduction to vacuum entanglement
  - Coupled harmonic oscilators
  - Field theory
  - Quantum field theory (in the continuum)
- A QFT in the lab
  - Collective excitations of quantum fluids
  - Relativistic regime at long wavelengths
- Results for current BEC and Polariton experiments
- Outlook

Introduction

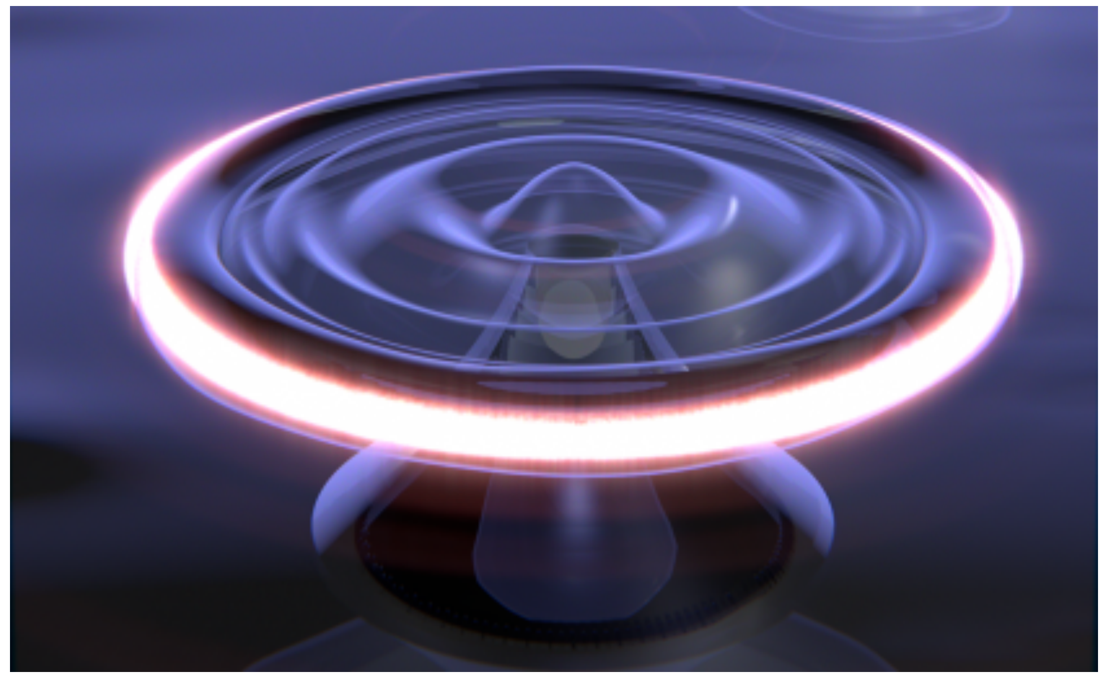
Applications of the QFT framework

Particle physics (weak field)



Standard Quantum Field Theory (QFT)

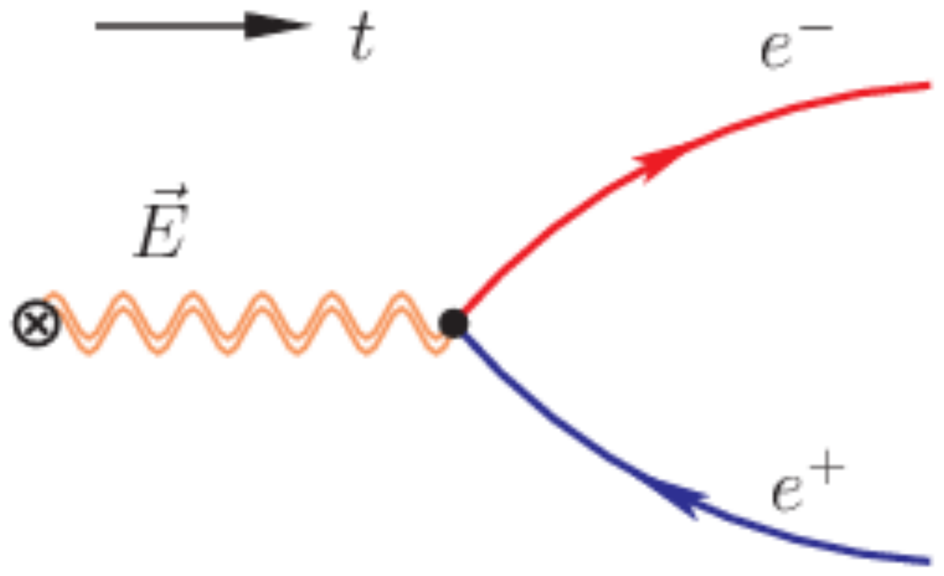
Condensed Matter



Credit: C. Baker and W. Bowen

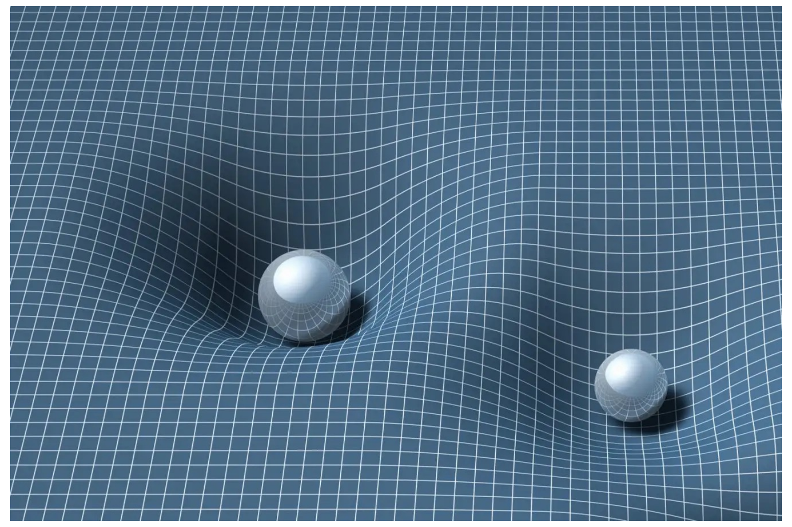
Collective excitations: QFT in presence of external fields

Particle physics (strong field)



QFT in presence of external fields

Quantum matter interacting with gravity

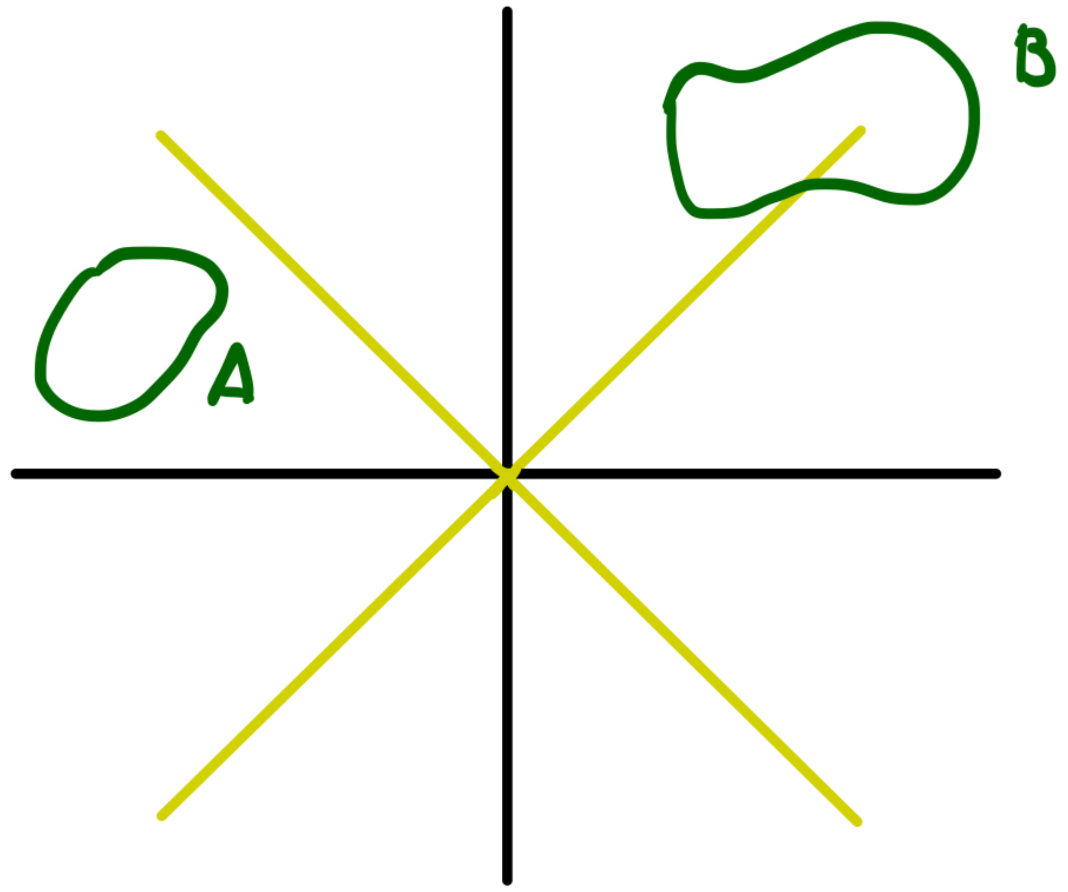


QFT in curved spacetimes

Richness of the QFT vacuum

Reeh-Schlieder: **any two "regions"** of spacetime **are entangled**

Reeh, Schlieder '61



$$\hat{\rho}_{AB} \neq \sum_i p_i \hat{\rho}_A^i \otimes \hat{\rho}_B^i$$

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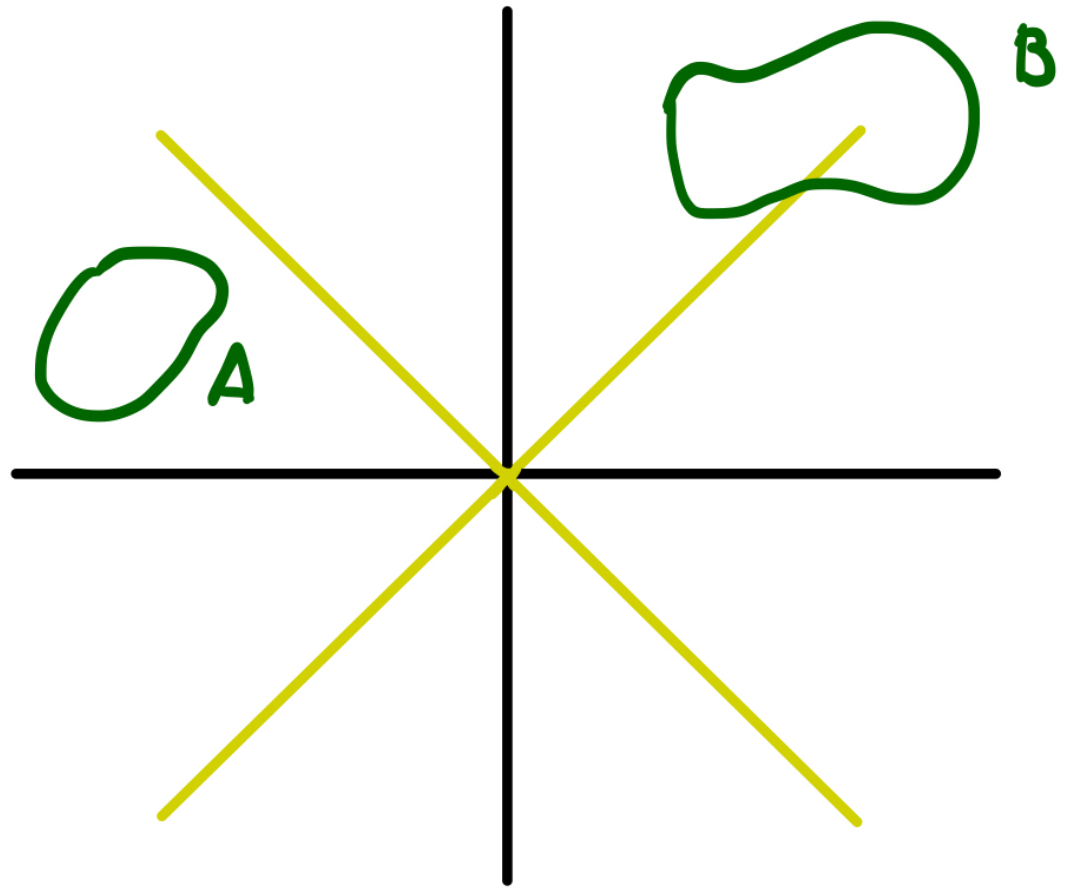
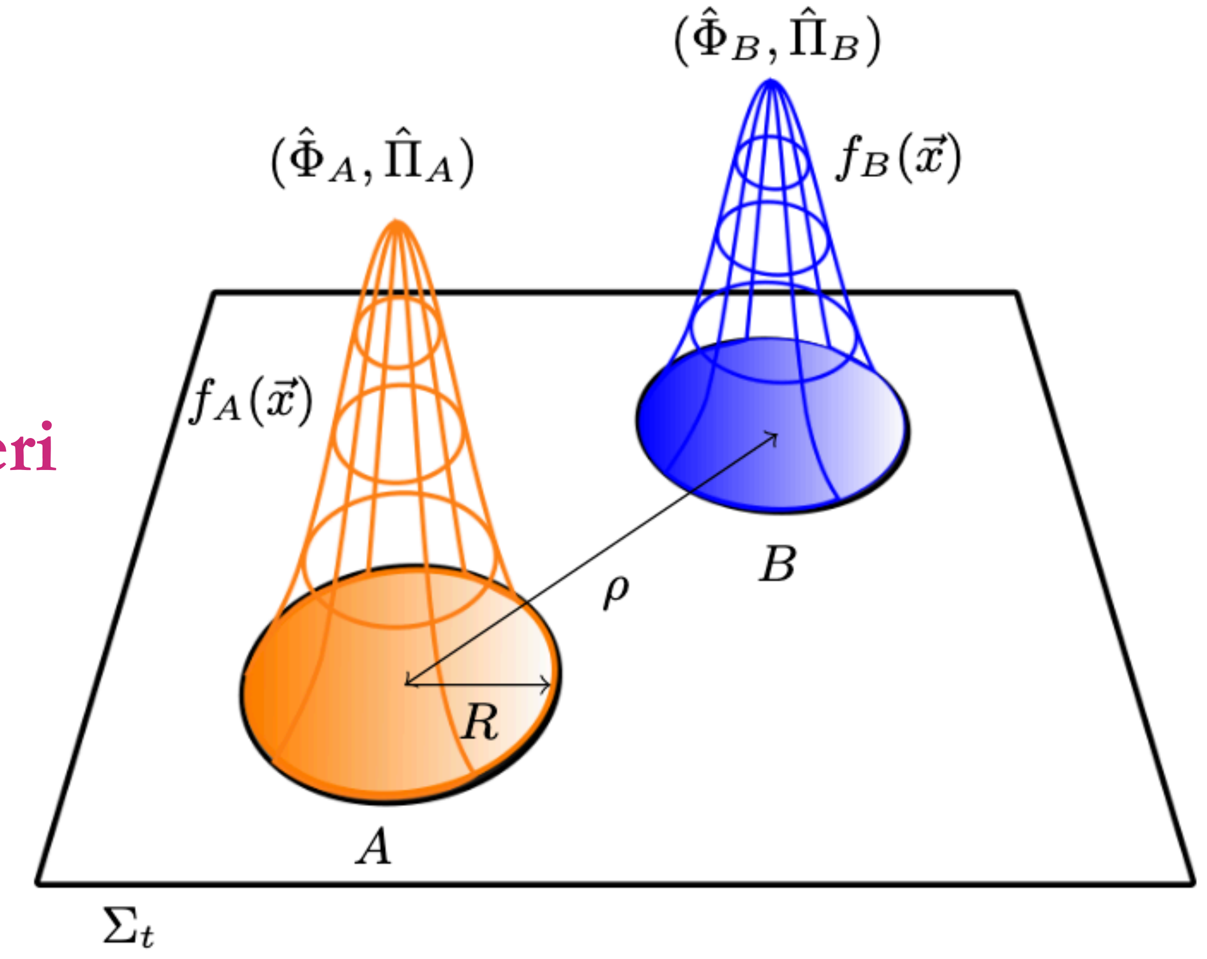


$$\hat{\rho}_{AB} \neq \sum_i p_i \hat{\rho}_A^i \otimes \hat{\rho}_B^i$$

Figure credit to
Patricia Ribes-Metidieri
PRD 108 085005



However:

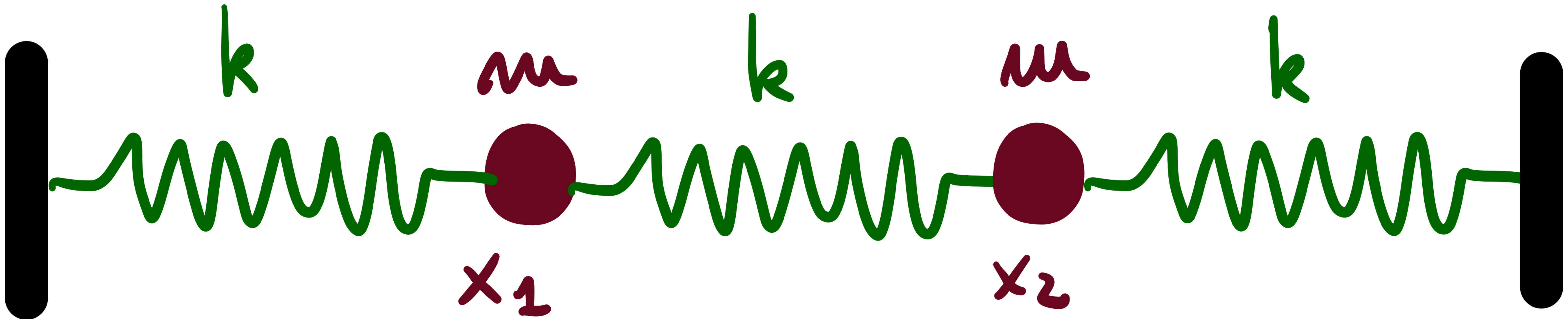
Entanglement between two local DOFs is sparse, harder to find the higher the dimension.

Agullo et. al. 23'

A pedagogical introduction to vacuum entanglement

Entanglement in vacuum states

Consider two coupled harmonic oscillators: $\mathscr{H} = \mathscr{H}_1 \otimes \mathscr{H}_2$



$$\hat{H} = \frac{\hat{p}_1^2 + \hat{p}_2^2}{2m} + \frac{k}{2}\left(\hat{x}_1^2 + \hat{x}_2^2 + (\hat{x}_2 - \hat{x}1)^2\right)$$

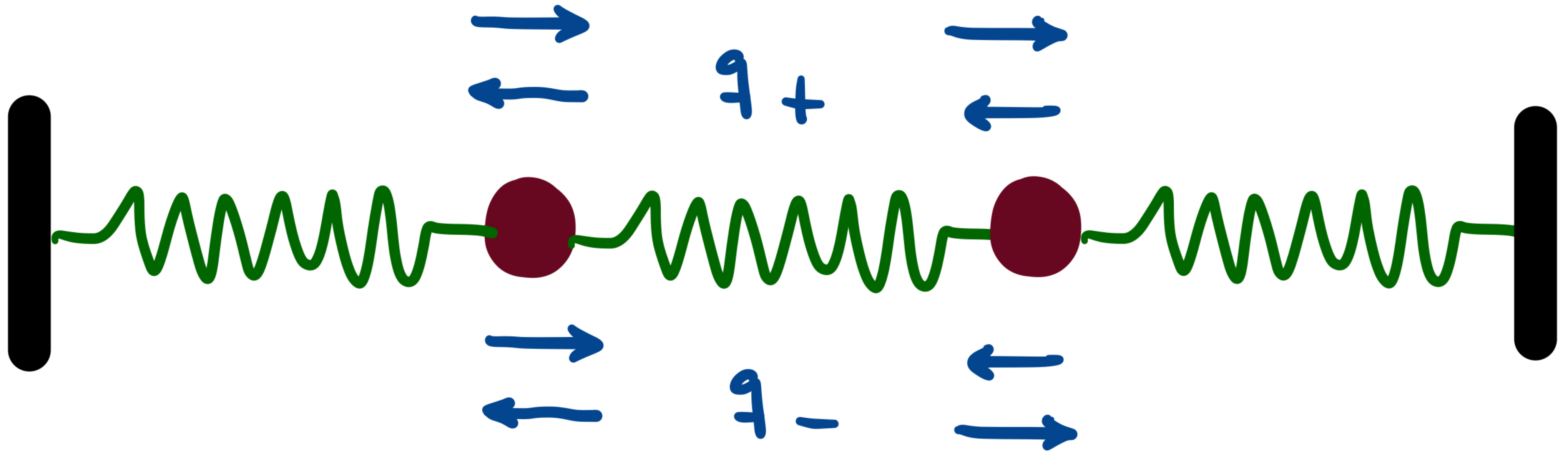
Is there entanglement in the vacuum state of this system?

Entanglement in vacuum states

To compute the vacuum state, we first find the normal modes

$$\hat{q}_+ = \frac{\hat{x}_1 + \hat{x}_2}{\sqrt{2}} \qquad \hat{q}_- = \frac{\hat{x}_1 - \hat{x}_2}{\sqrt{2}}$$

$$\hat{p}_+ = \frac{\hat{p}_1 + \hat{p}_2}{\sqrt{2}} \qquad \hat{q}_- = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{2}}$$



The Hamiltonian is now written as two decoupled oscillators with frequencies $\omega_\pm$

$$\hat{H} = \frac{\hat{p}_+^2}{2m} + \frac{m\omega_+^2}{2}\hat{q}_+^2 + \frac{\hat{p}_-^2}{2m} + \frac{m\omega_-^2}{2}\hat{q}_-^2 \qquad \text{with} \qquad \omega_+ = \sqrt{\frac{k}{m}} \qquad \omega_- = \sqrt{\frac{3k}{m}}$$

Vacuum is separable: $|0\rangle = |0\rangle_+ \otimes |0\rangle_- \longrightarrow$ no entanglement?

Entanglement in vacuum states

Normal modes are not entangled, But:

Is there entanglement among each oscillator in the vacuum state of this system?

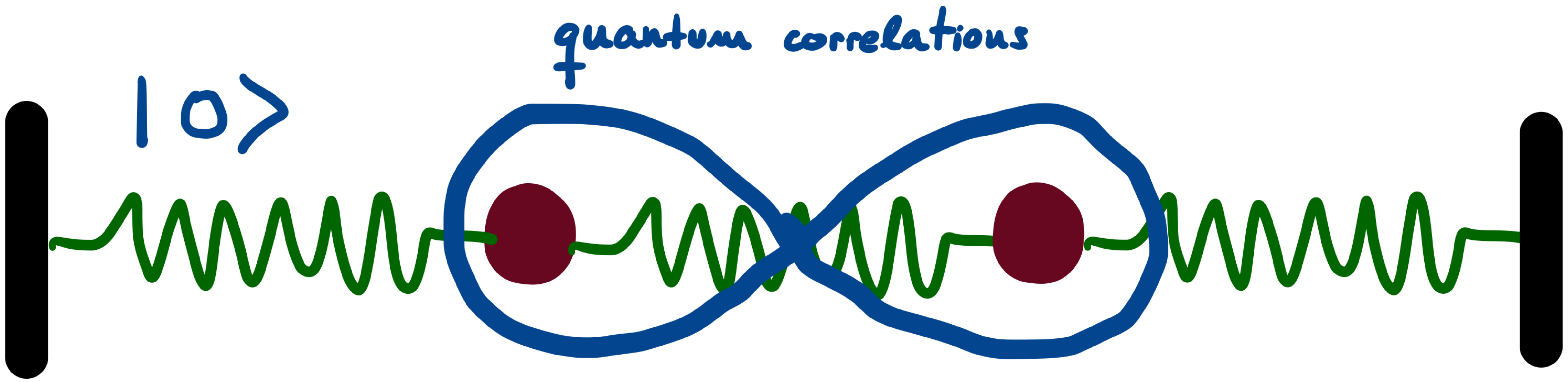
Entanglement in vacuum states

Normal modes are not entangled, But:

Is there entanglement among each oscillator in the vacuum state of this system?

$$|0\rangle = |0\rangle_+ \otimes |0\rangle_- \neq |\psi\rangle_1 \otimes |\phi\rangle_2$$

vacuum not separable in terms of each oscillator → The oscillators are entangled!



Entanglement in vacuum states

1D Lattice Field Theory

$$\hat{H} = \sum_{i=-\infty}^{\infty} \frac{1}{2} \left(\hat{\pi}_i^2 + \frac{\left(\hat{\phi}_{i+1} - \hat{\phi}_i\right)^2}{\epsilon} + m\hat{\phi}_i^{\,2} \right)$$

Normal modes are discrete Fourier modes: they are global modes of vibration.

Normal modes are not entangled.

Local modes all have mixed reduced states: they are entangled with the rest.

Entanglement in vacuum states

1D Lattice Field Theory

More general local single-mode subsystems:

$$\hat{q} = \sum_i \hat{f}_i \hat{\phi}_i \qquad \hat{p} = \sum_i g_i \hat{\pi}_i$$

If $\sum_i f_i g_i = 1$ then $(\hat{q}, \hat{p})$ define a single mode subsystem $\longleftarrow$ $[\hat{q}, \hat{p}] = i \sum_i f_i g_i$

for instance $$\hat{q} = \frac{1}{\sqrt{2}}\hat{\phi}_1 + \frac{1}{\sqrt{2}}\hat{\phi}_3 + \frac{1}{\sqrt{2}}\hat{\phi}_7 \qquad \hat{p} = \frac{1}{\sqrt{2}}\hat{\phi}_1 + \frac{1}{\sqrt{2}}\hat{\phi}_4 + \frac{1}{\sqrt{2}}\hat{\phi}_7$$

$\{f_i\}$ and $\{g_i\}$ can be seen as weight (or "smearing") functions.

QFT and entanglement in local modes

(d+1)-dimensional Field Theory

$$\hat{H} = \int_\Sigma \mathrm{dVol}_d \frac{1}{2}\left(\hat{\pi}^2 + |\nabla\hat{\phi}|^2 + m\hat{\phi}^2\right)$$

Minkowski vacuum $|0\rangle$: $\quad \hat{\phi} = \int \mathrm{d}^d\mathbf{k}\,\left(e^{-i\omega_k t + i\mathbf{k}\cdot\mathbf{x}}\,\hat{a}_\mathbf{k} + e^{i\omega_k t - i\mathbf{k}\cdot\mathbf{x}}\,\hat{a}^\dagger_\mathbf{k}\right) \quad ; \quad \hat{a}_\mathbf{k}|0\rangle = 0$

$$\hat{H} = \int \mathrm{d}^d\mathbf{k}\,\left(\hat{a}^\dagger_\mathbf{k}\hat{a}_\mathbf{k} + \frac{1}{2}\right)$$

Normal modes are global (plane waves) and not entangled

QFT and entanglement in local modes

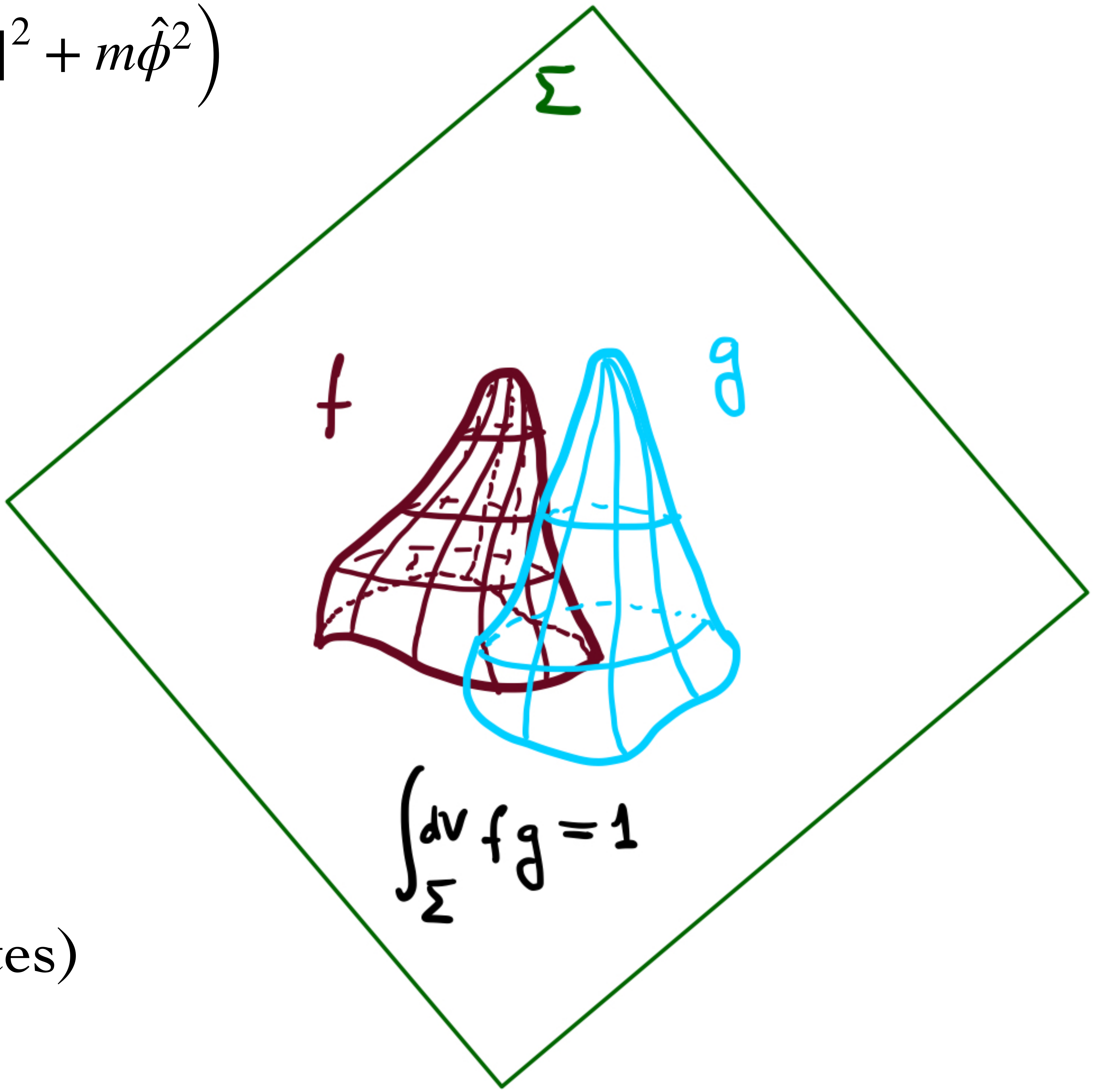
(d+1)-dimensional Field Theory

$$\hat{H} = \int_\Sigma \mathrm{dVol}_d \, \frac{1}{2}\left(\hat{\pi}^2 + |\nabla \hat{\phi}|^2 + m\hat{\phi}^2\right)$$

Local modes defined as smeared field operators

$$\hat{\phi}_f = \int_\Sigma \mathrm{dVol}_d \, f(\mathbf{x})\hat{\phi}(\mathbf{x}) \qquad \hat{\pi}_g = \int_\Sigma \mathrm{dVol}_d \, g(\mathbf{x})\hat{\pi}(\mathbf{x})$$

$$\left[\hat{\phi}_f, \hat{\pi}_g\right] = i\int_\sigma \mathrm{dVol}_d \, f(\mathbf{x})g(\mathbf{x})$$



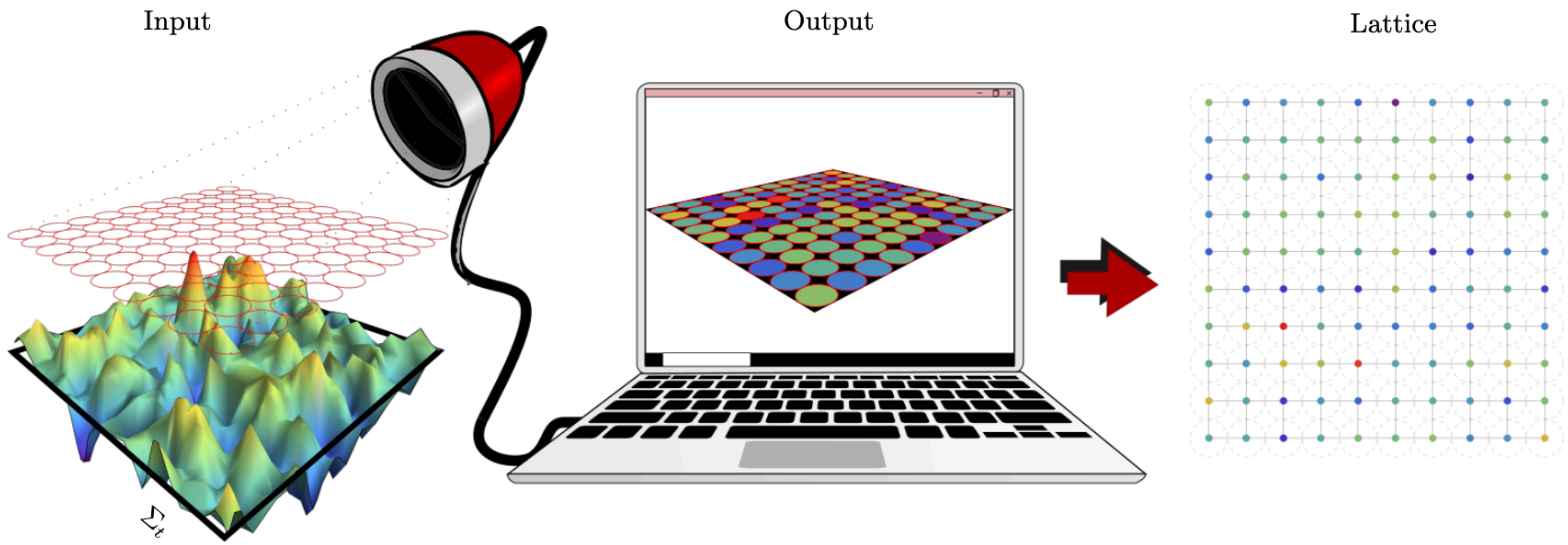
Local modes are all entangled (mixed reduced states)

QFT and entanglement in local modes

Why local DOFs?

Experiments only access a finite region and with finite resolution.

Figure credit to
Patricia Ribes-Metidieri
PRD 108 085005



Finitely many compactly supported DOFs.

Testable predictions

The entanglement structure of the QFT vacuum is rich and intriguing

Some recent results and predictions: Agullo, Beck, Bonga, Elizaga-Navascués, Klco, Kranas, Martín-Martínez, Nadal-Gisbert, Perche, Polo-Gómez, Ribes-Metidieri, Savage, Torres, Yamaguchi…

- Single-mode becomes sparser increasing spatial dimension.
- Multimode entanglement is rather generic.
- Varies with distance between subsystems, spacetime curvature, …
- Definition of partner modes

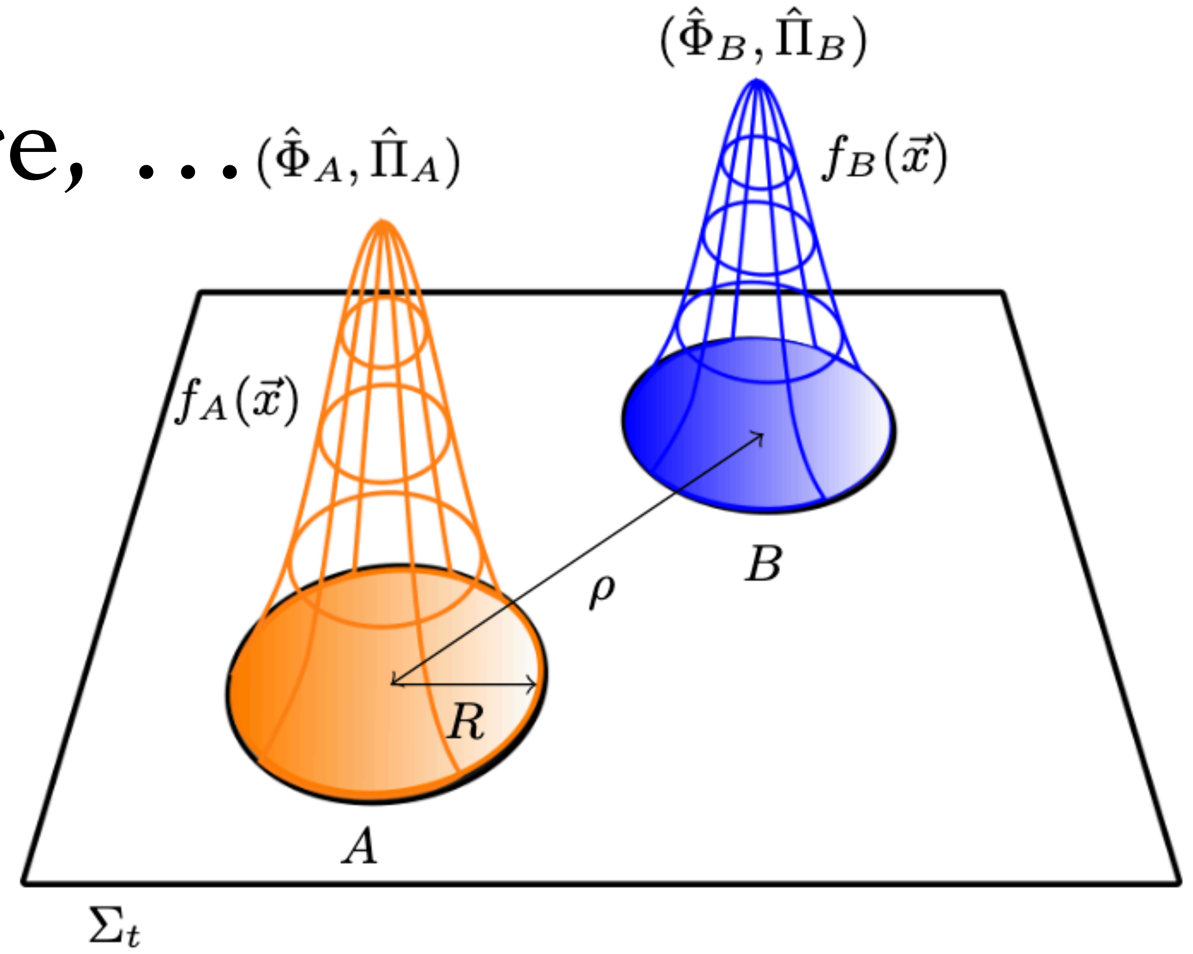


Figure credit to P. Ribes-Metidieri, in PRD 108 085005

Testable predictions

Can we **test** these aspects of the quantum vacuum?

Testable predictions

Can we **test** these aspects of the quantum vacuum?

“Yo puedo asegurarles que haré todo lo que pueda y un poco más de lo que pueda si es que eso es posible, y haré todo lo posible e incluso lo imposible si también lo imposible es posible”

M. Rajoy (2018)

Literal translation:

“I can assure you that I will do what is possible and a little more than what is possible if that is possible, and I will do all that is possible and even the impossible if also the impossible is possible”

M. Rajoy (2018)

A QFT in the lab

A QFT in the lab

Analogue gravity: simulators for field theory in curved spacetimes

Pressure —sound— waves in inviscid, barotropic, irrotational fluids
satisfy a curved **Klein Gordon equation**

Unruh (1980)
Visser (1993)

$$\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}\,g^{\mu\nu}\partial_\nu\Phi(t,\boldsymbol{x}) = 0 \qquad \text{with the metric} \qquad g \propto -(c_s^2 - v^2)dt^2 - 2\boldsymbol{v}\cdot d\boldsymbol{x}dt + d\boldsymbol{x}\cdot d\boldsymbol{x}$$

$\boldsymbol{v}$ = fluid velocity $\quad c_s$ = speed of sound

Includes expanding universes, black holes, ergoregions among others!

Barceló, Liberati, Visser (2005)

From microscopic physics to collective excitations

Low temperature: Bose-Einstein Condensates, quantum fluids of light,…

described by a (constrained) complex field $\hat{\Psi}$ that creates/destroys microscopic DOFs

Mean field description $\langle \hat{\Psi} \rangle = \Psi_0 = \sqrt{n_0} e^{i\theta_0}$

(θ_0, n_0) are a canonically conjugated pair: $\pi_{\theta_0} = \dfrac{\partial \mathscr{L}}{\partial \dot{\theta}_0} = -\hbar n_0$

Mean field dynamics = continuity and Euler equations

$$\partial_t n_0 + \nabla \cdot (n_0 \mathbf{v}) = 0$$

$$(\partial_t + \mathbf{v} \cdot \nabla)\mathbf{v} + \nabla P = 0$$

where $\mathbf{v} = \dfrac{\nabla \theta_0}{m}$

Acoustic metric theorem (almost) applies!

Relativistic limit in the hydrodynamic regime

Linear fluctuations of density and phase $n = n_0 + \eta$ and $\theta = \theta_0 + \varphi$

(φ, η) are a canonically conjugated pair: $\pi_\varphi = \dfrac{\partial \mathscr{L}}{\partial \dot{\varphi}} = -\eta$

Dynamics = generalized KG equation $\left(1 - \dfrac{\xi^2}{2} \nabla^2\right)^{-1} \left(\partial_t^2 - c_s^2 \nabla^2\right) \varphi = 0$

Homogeneity assumed, but holds more generally!

Long wavelength limit $\xi^2 \nabla^2 \varphi \ll \varphi$ = relativistic limit

Causal speed of propagation is $c_s = \sqrt{\dfrac{g n_0}{m}}$

Healing length: $\xi = \dfrac{1}{\sqrt{m g n_0}}$

Relativistic limit in the hydrodynamic regime

Quantum fluctuations over mean field $\hat{\Psi} = \Psi_0 \hat{\mathbb{I}} + \delta\hat{\Psi}$ well described by quantizing (φ, η)

Equal time CCR $[\hat{\eta}(t, \mathbf{x}), \hat{\varphi}(t, \mathbf{y})] = i\delta(\mathbf{x} - \mathbf{y})$

Localized modes: $\hat{\varphi}_f = \int_\Sigma \mathrm{dVol}_d\, f(x)\hat{\varphi}(\mathbf{x})$, $\hat{\eta}_f = \int_\Sigma \mathrm{dVol}_d\, f(x)\hat{\eta}(\mathbf{x})$

where $[\hat{\eta}_f, \hat{\varphi}_f] = i$ requires $\int_\Sigma \mathrm{dVol}_d\, f^2(\mathbf{x}) = 1$

Careful: make sure we stay relativistic by considering big enough support for f

Negligible Fourier components for $\mathrm{k} \gtrsim \xi^{-1}$

Results for polariton and BEC experiments

Thermal or vacuum fluctuations?

Detecting entanglement requires thermal correlations < vacuum correlations

Vacuum correlations

A quick estimate: $\sigma_\omega = 1 + 2n_\omega$ with $n_\omega = \dfrac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1}$

Thermal correlations

$$n_\omega < \frac{1}{2} \quad \longleftrightarrow \quad \omega > \frac{k_B T}{\hbar} \ln 3 \quad \longleftrightarrow \quad \mathrm{k} > \frac{k_B T}{\hbar c_s} \ln 3$$

Parameters to play with to optimize:

- Tempreature
- Speed of sound
- Healing length
- System size

(Healing length, System size) $\longrightarrow$ Mode size

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Throwing in some realistic numbers…

BECs: $c_s \sim 2\mathrm{mm\,s^{-1}} \quad \xi \sim 0.6\mu\mathrm{m}$

Size $\sim 50\mu\mathrm{m} \rightarrow \mathrm{k_{min}} \sim 0.25\mu\mathrm{m}^{-1}$

$T \lesssim 3.5\mathrm{nK}$

Polaritons: $c_s \sim 1\mu\mathrm{m\,ps^{-1}} \quad \xi \sim 1\mu\mathrm{m}$

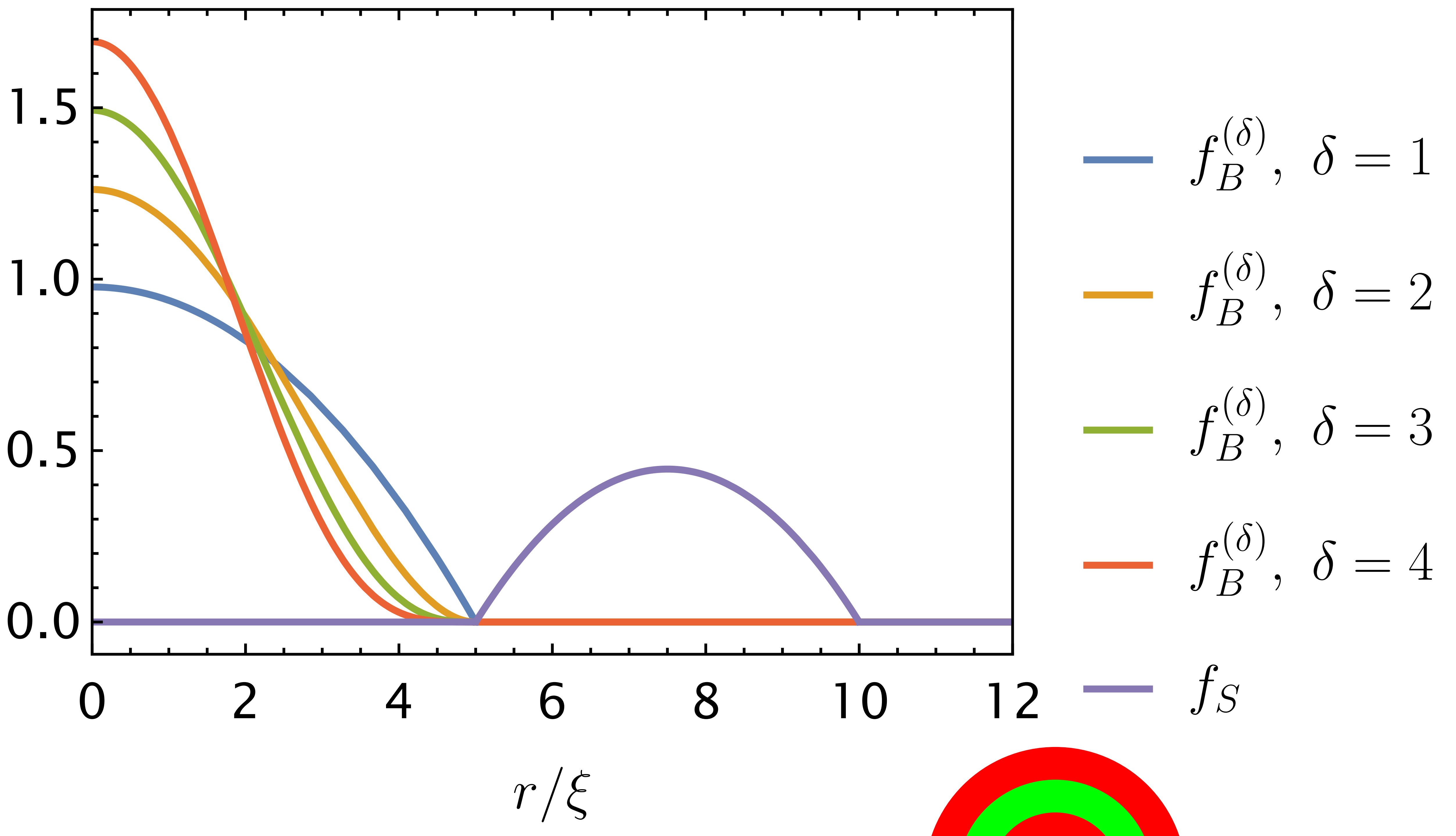
Size $\sim 100\mu\mathrm{m} \rightarrow \mathrm{k_{min}} \sim 0.15\mu\mathrm{m}^{-1}$

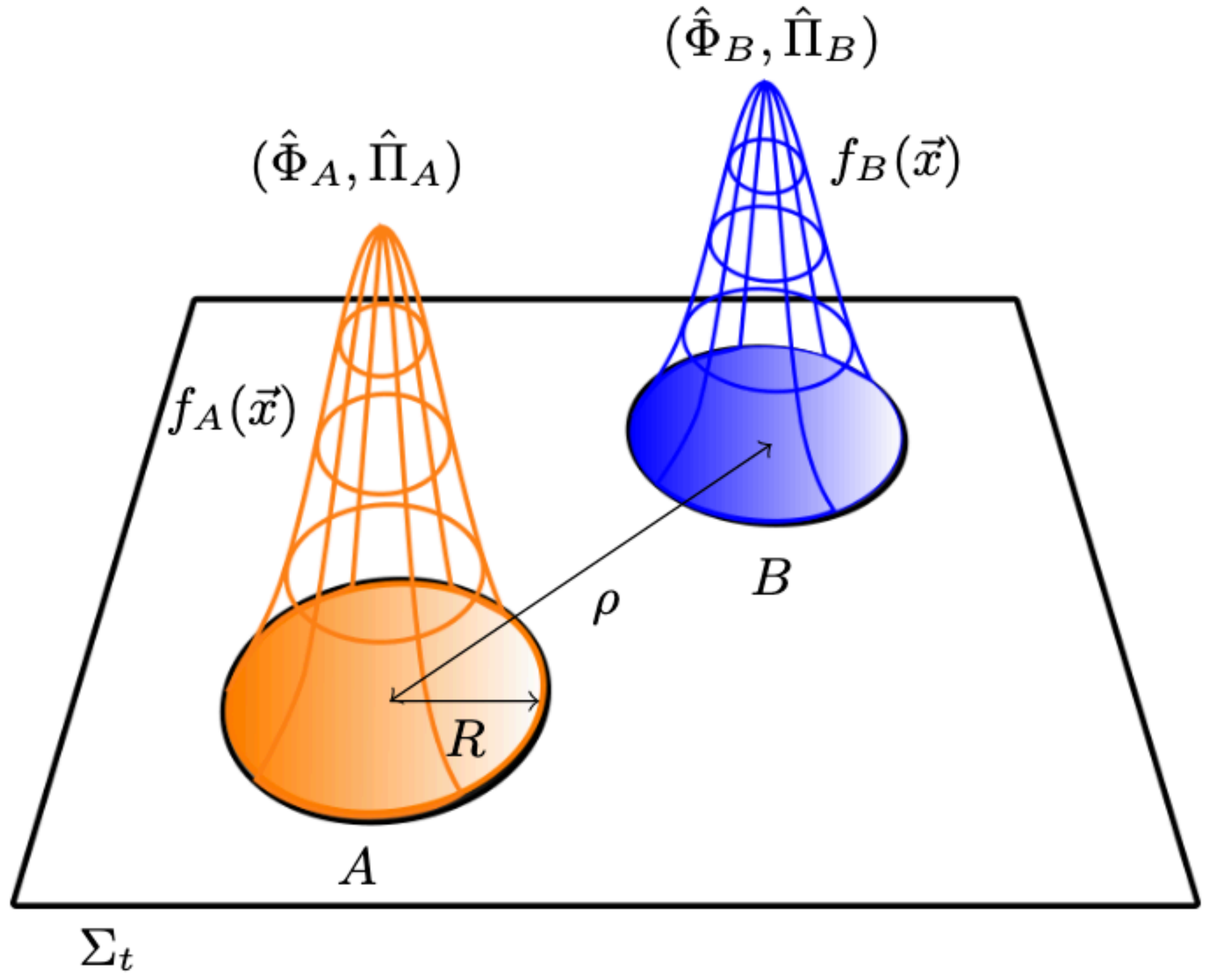
$T \lesssim 1\mathrm{K}$

What degrees of freedom?

Take two modes defined by smearing functions f_A and f_B

Smearing functions





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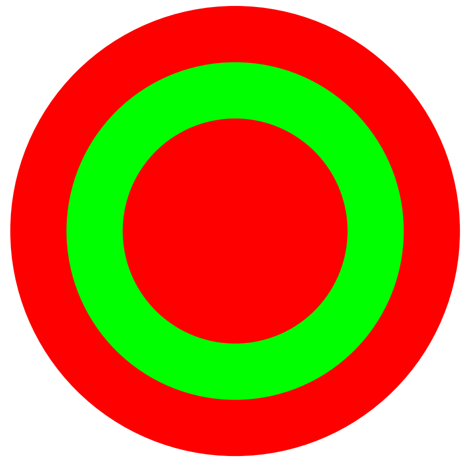


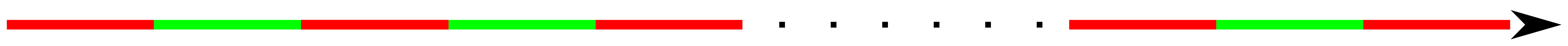
Preliminary results for BECs

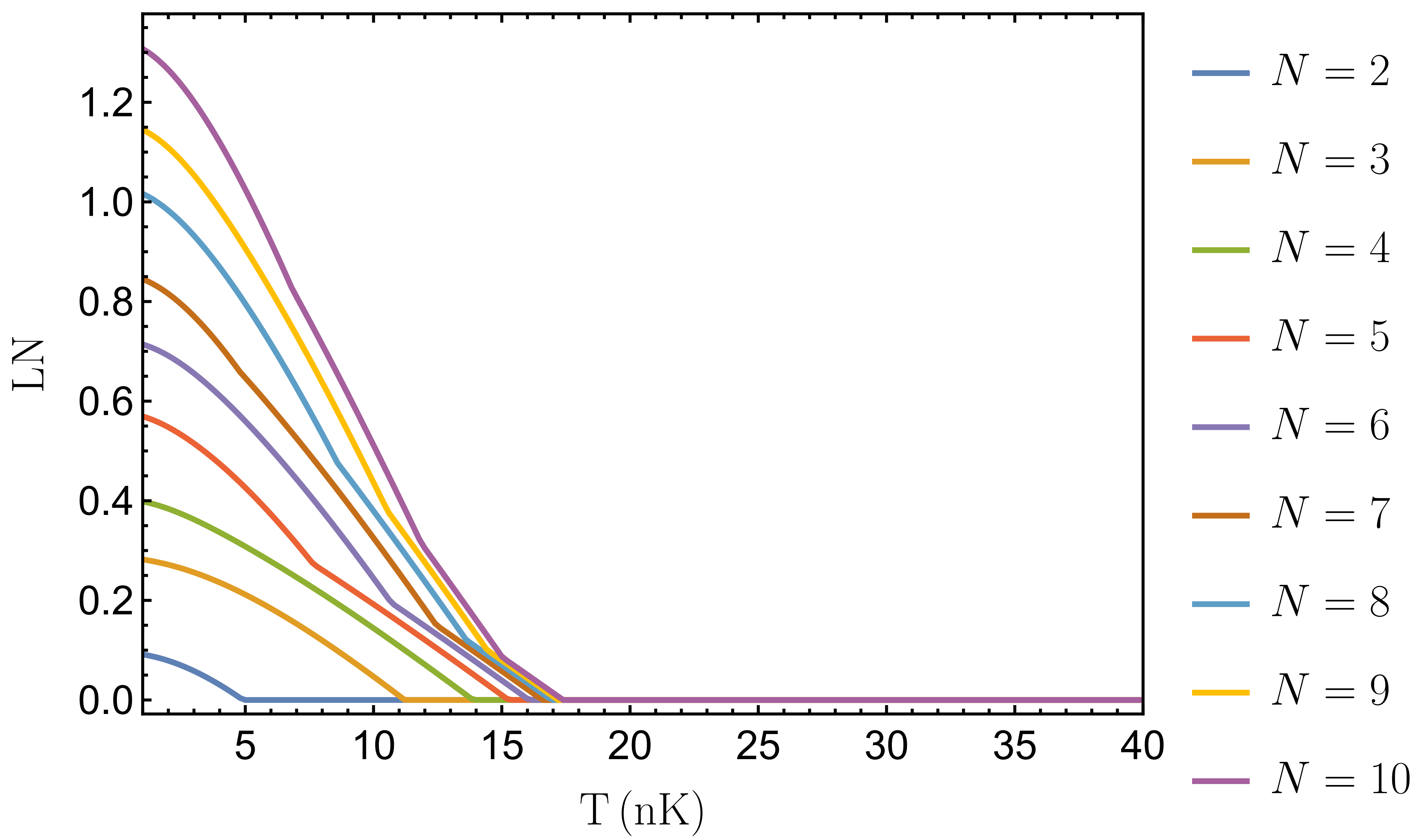
Predictions for current BEC experiments

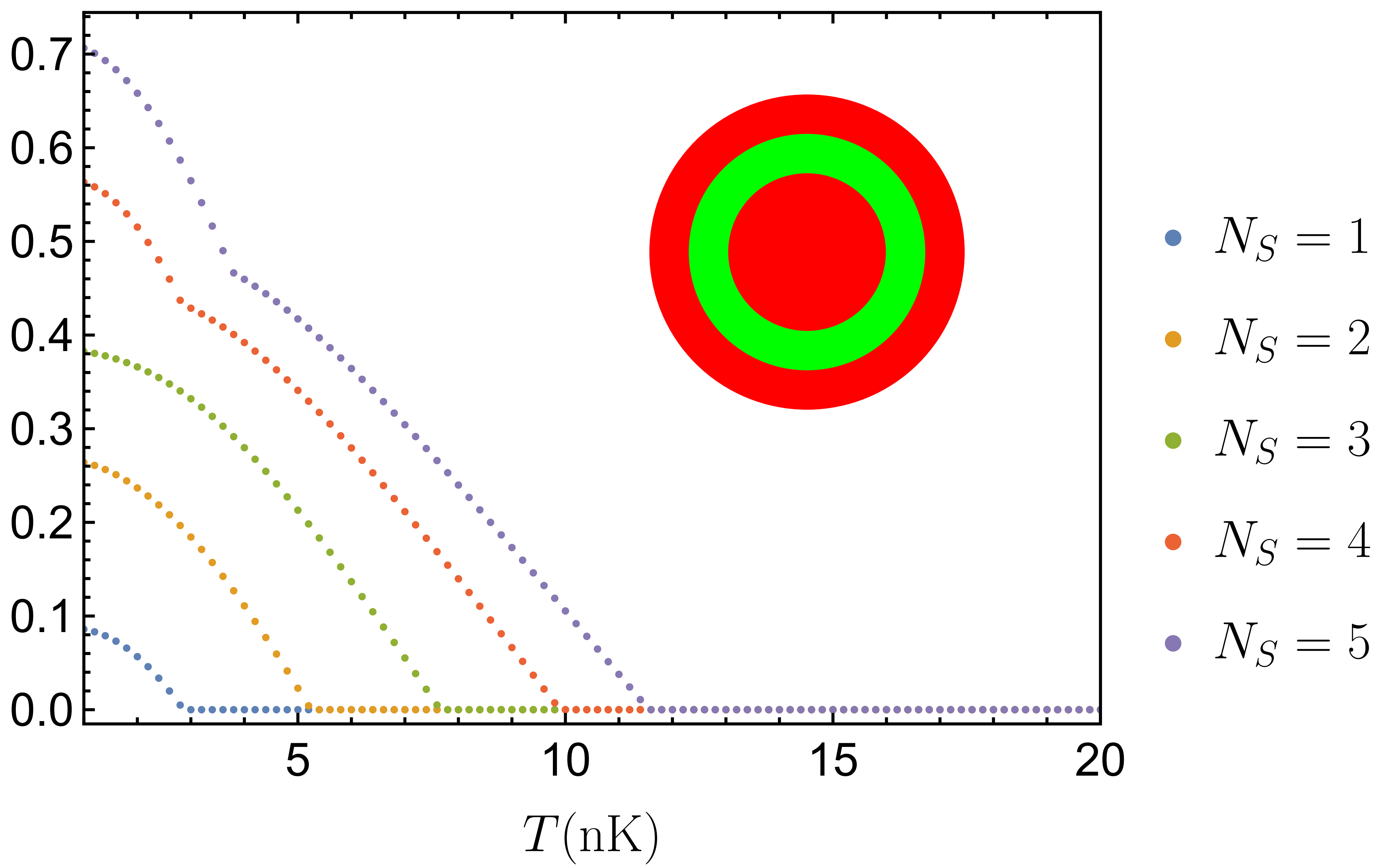
Mode size $\sim 10\xi$

1+1 dimensions

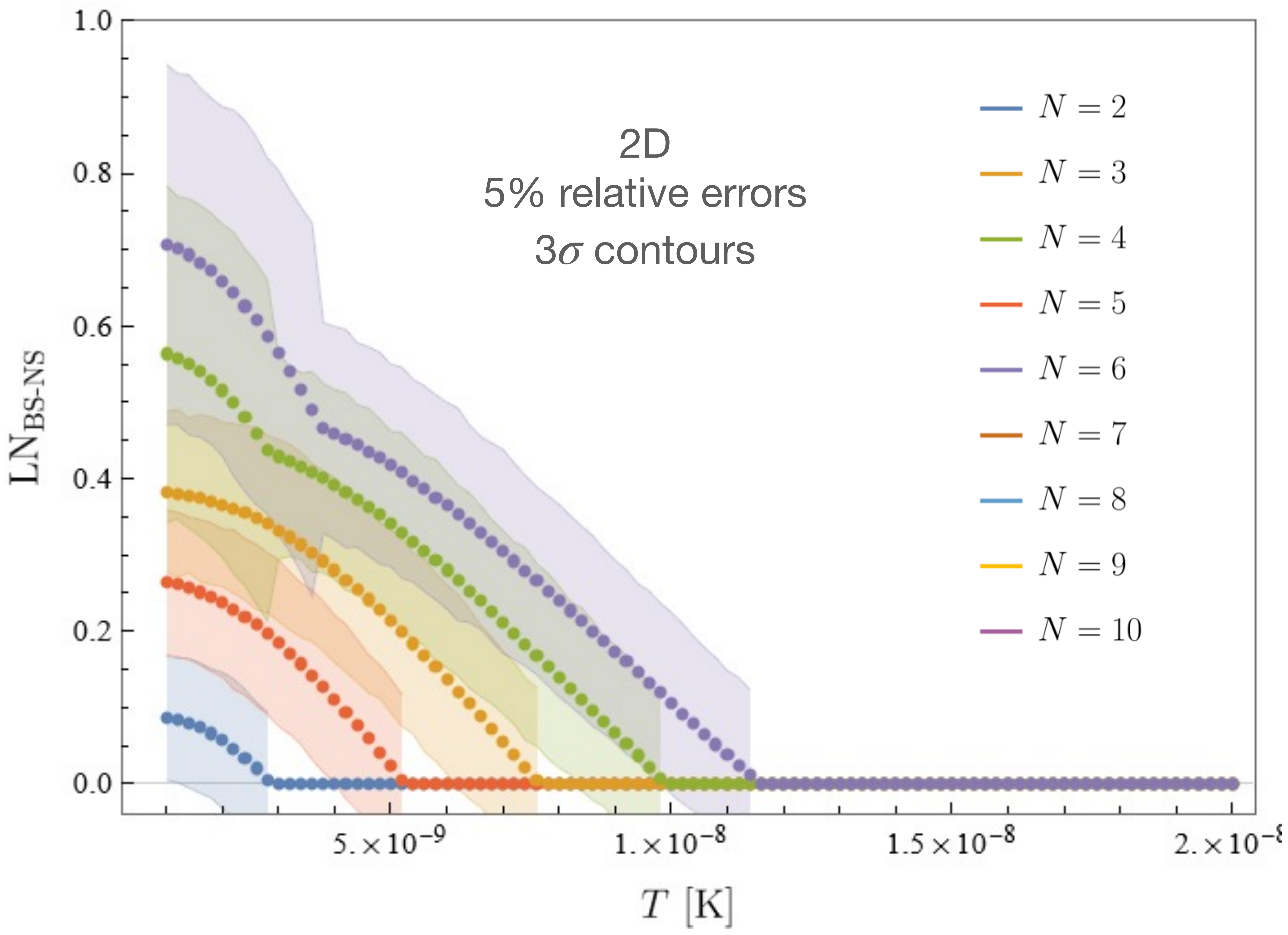
2+1 dimensions

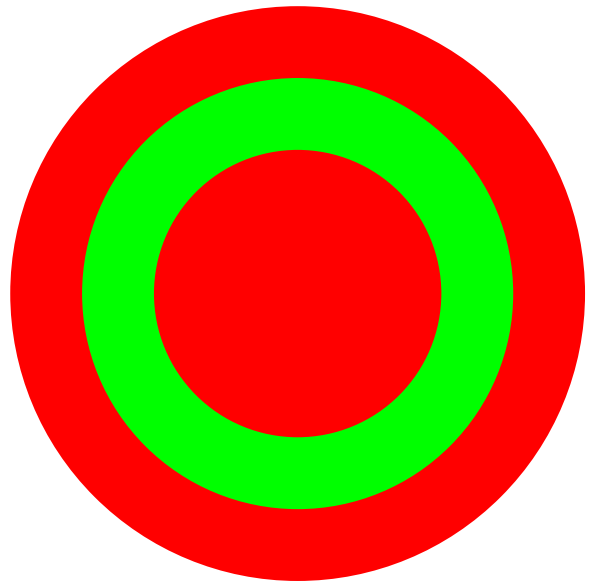






Preliminary results for BECs

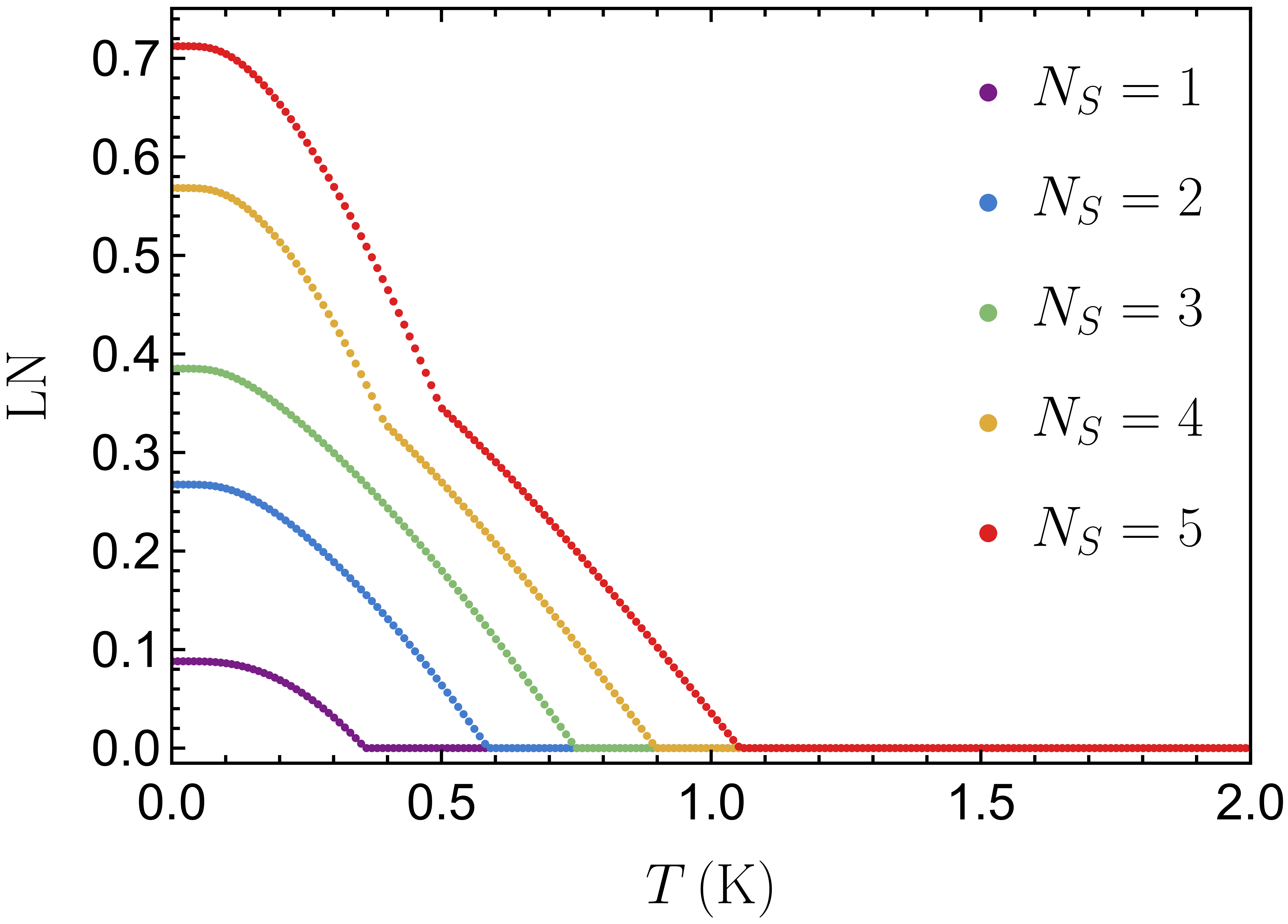


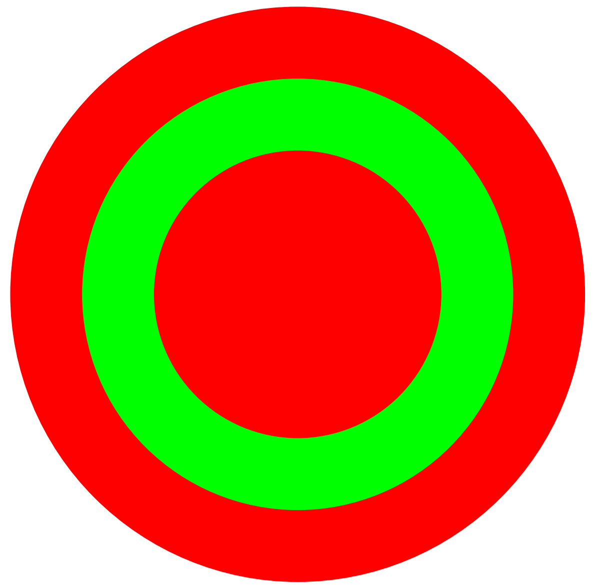


Preliminary results for Polaritons

Predictions for current Polariton experiments

Mode size $\sim 20\xi$





Measurement schemes

Measurement scheme for Polaritons is straightforward:

- Homodyne detection of photons = state tomography of acoustic field

Measurement scheme for BECs is trickier:

- Measure density contrast $\eta = n - n_0$ and its time evolution.
- Assuming dynamics, allows to find out phase fluctuations φ
- Compute one- and two-point correlation functions.
- Caveat: method needs some time evolution to work.

The team

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Outlook

- **The structure of the QFT vacuum is surprisingly rich.**
- **Vacuum entanglement among local modes can be empirically accessed.**
- **Polariton fluids at $T \sim$ 1K are promising platforms thanks to quantum optics tools.**
- **BEC at $T \sim 1$nK are also promising, but measurement techniques need improvement**
- **A new window for RQI in the lab?**

THANKS FOR ABIDING!