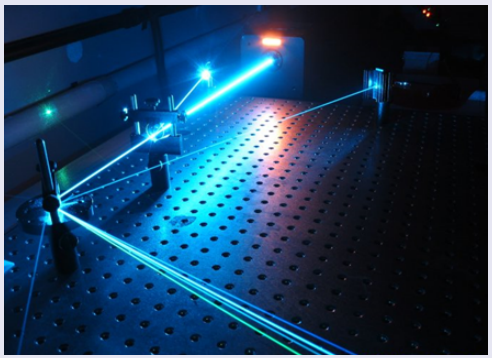
Hawking Time Crystals

Analogue Gravity in 2026, Benasque, España

JRMdN, F. Sols, arXiv 2507.10862 (2025)

Juan Ramón Muñoz de Nova, Fernando Sols

12/01/2026



Analogues: Intuitive picture

- Light cannot escape from black hole=Sound cannot travel upstream in supersonic flow
- Subsonic/supersonic flows = Exterior/interior black hole
- Subsonic-supersonic interface = Event horizon

BEC analogues: Andreev-Hawking effect

- Bogoliubov approximation for atomic condensates:

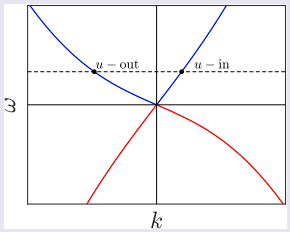
- Homogeneous condensate flow:

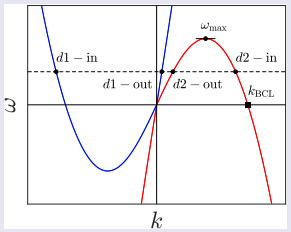
$$\Psi_0(x) = \sqrt{n_0} e^{iqx}$$

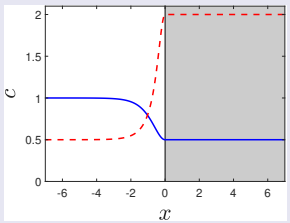
- Bogoliubov dispersion relation:

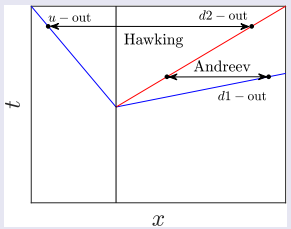
$$(\omega - vk)^2 = c^2k^2 + \frac{\hbar^2 k^4}{4m^2} = c^2k^2\left(1 + \frac{(k\xi)^2}{4}\right)$$

$$c = \sqrt{gn_0/m},\ v = \frac{\hbar q}{m},\ \xi = \frac{\hbar}{mc}$$









- Black-hole GP solution: Subsonic/Supersonic Interface
- $\nexists$ *global* vacuum $\rightarrow$ Spontaneous, correlated quasiparticle emission $\rightarrow$ Andreev-Hawking effect $\rightarrow$ No need for metrics!

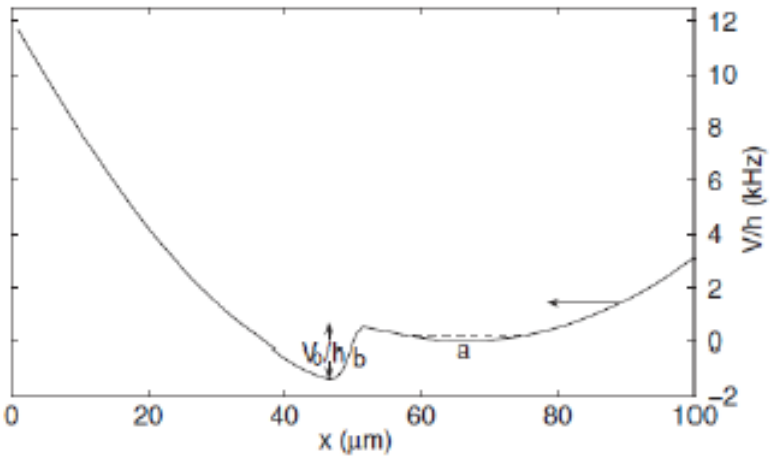
I. Zapata, F. Sols, PRL 102, 180405 (2009); *A. Recati et al., PRA 80, 043603 (2009)*; **JRMdN** *et al., Comptes Rendus Physique 25, S2 (2024)*

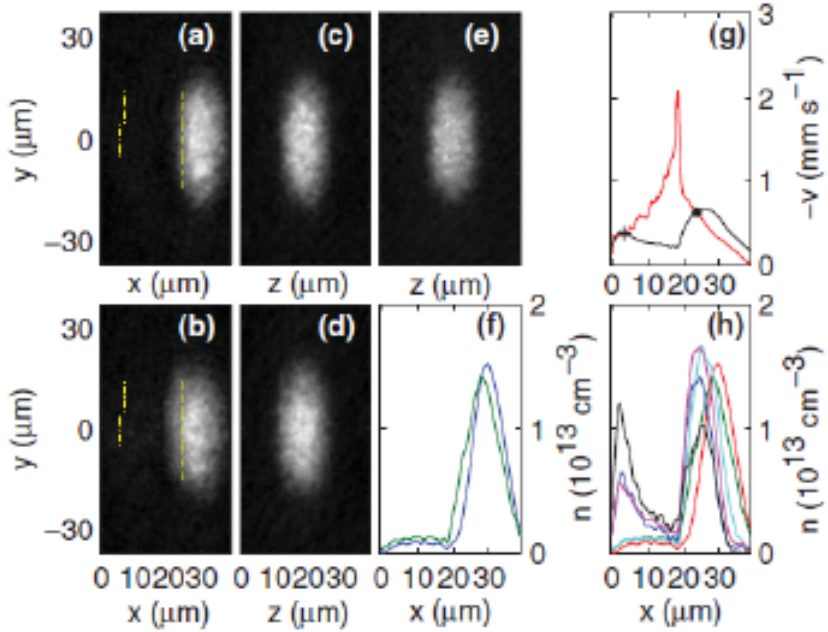
BEC (Experiment): 2010 Black Hole

PRL 105, 240401 (2010) PHYSICAL REVIEW LETTERS week ending 10 DECEMBER 2010

Realization of a Sonic Black Hole Analog in a Bose-Einstein Condensate

Oren Lahav, Amir Itah, Alex Blumkin, Carmit Gordon, Shahar Rinott, Alona Zayats, and Jeff Steinhauer
Technion—Israel Institute of Technology, Haifa, Israel





BEC (Experiment): 2016 Observation

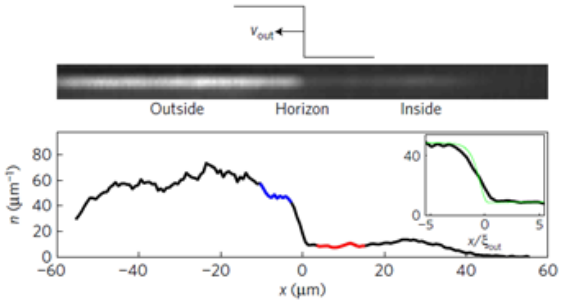
nature physics

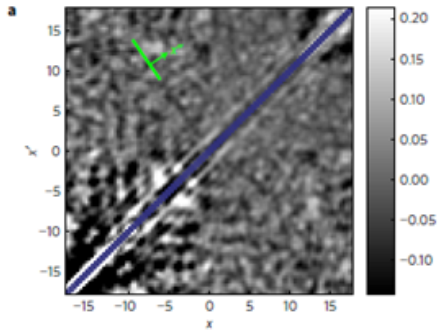
ARTICLES

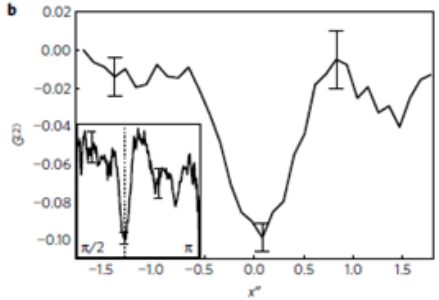
PUBLISHED ONLINE: 15 AUGUST 2016 | DOI: 10.1038/NPHYS3863

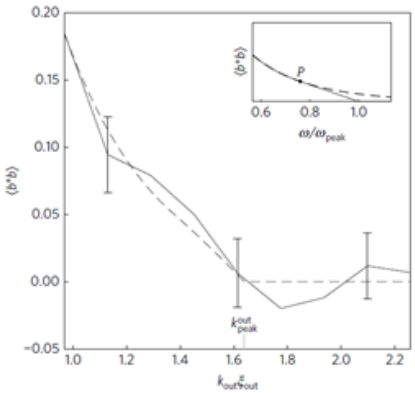
Observation of quantum Hawking radiation and its entanglement in an analogue black hole

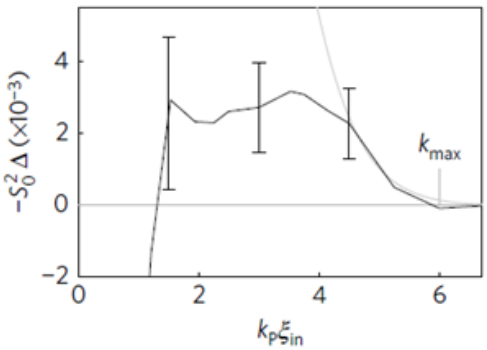
Jeff Steinhauer





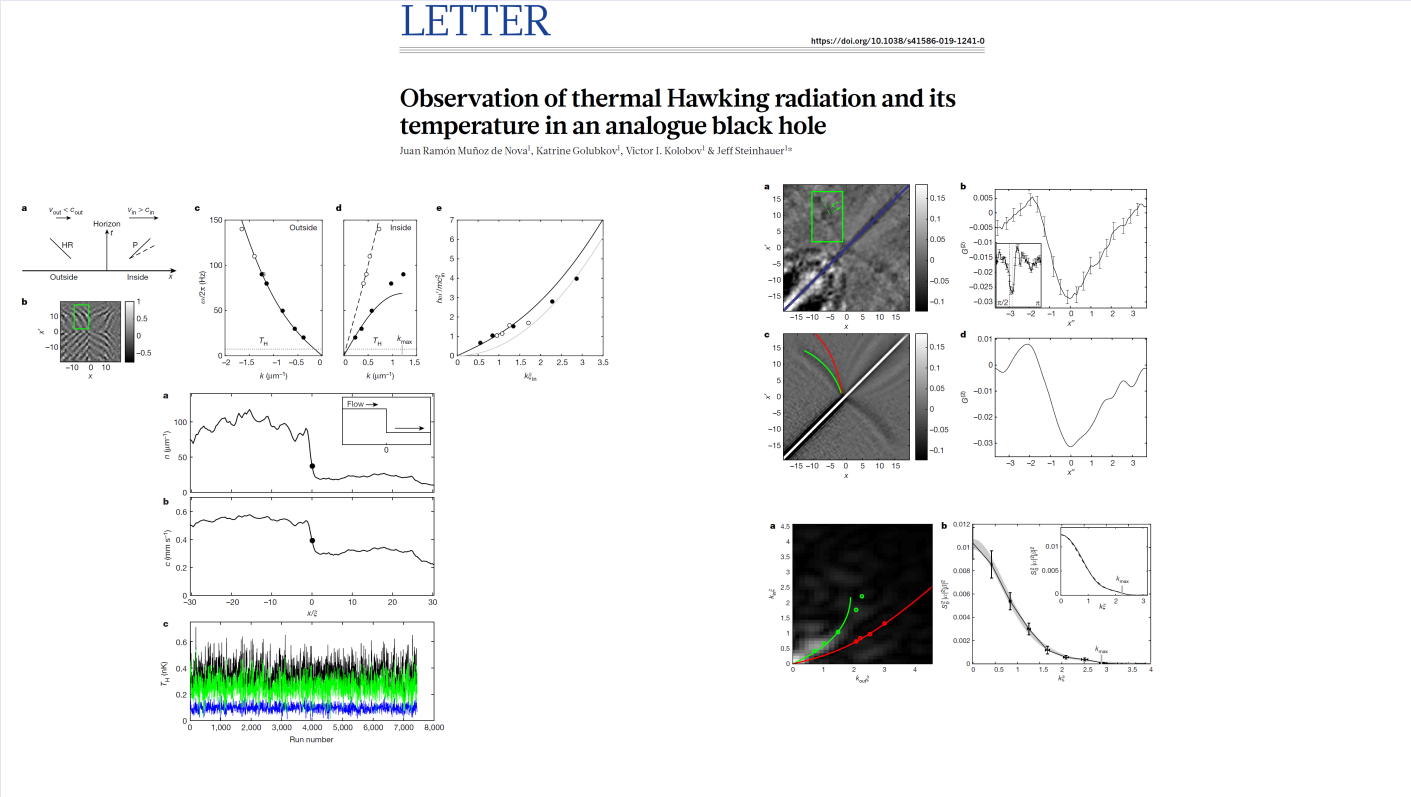






BEC (Experiment): 2019 Observation+Hawking Temperature

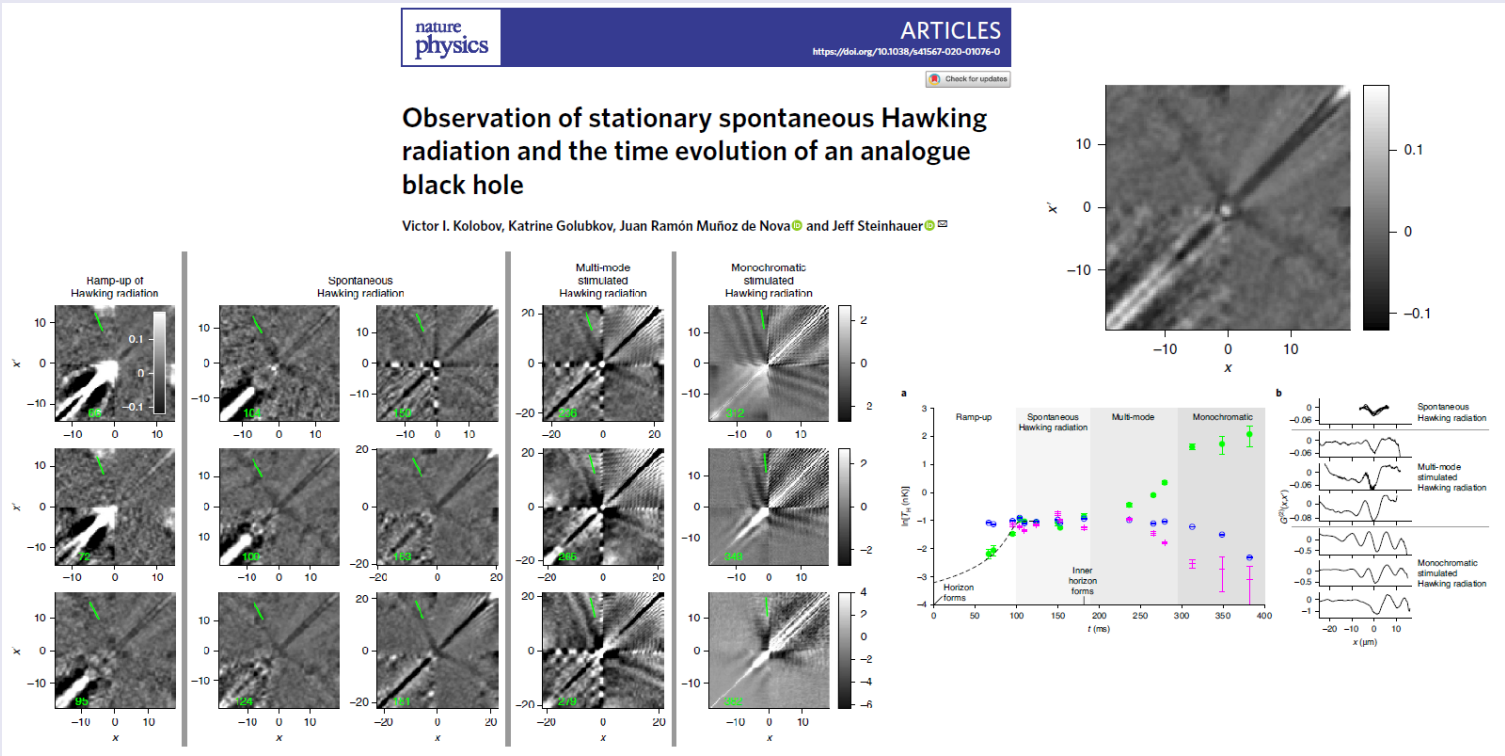
- 2016 experimental data did not quantitatively fit theory $\rightarrow$ *Thin-moustache mistery!*



JRMdN, *K. Golubkov, V. I. Kolobov, J. Steinhauer, Nature 569, 688 (2019)*

BEC (Experiment): 2021 Observation Stationarity + Life of Black Hole

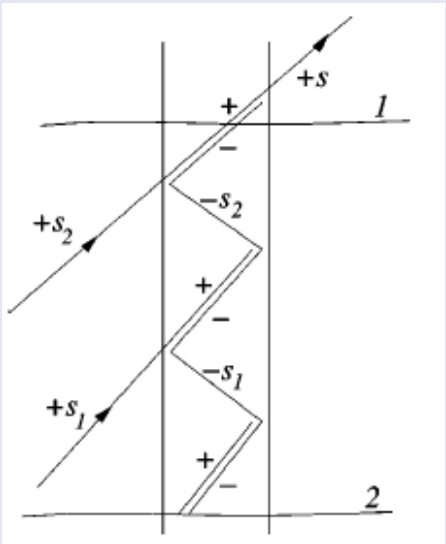
- Experiment extended (97000 repetitions over 124 days!) to observe life of analogue black hole and birth of inner horizon.

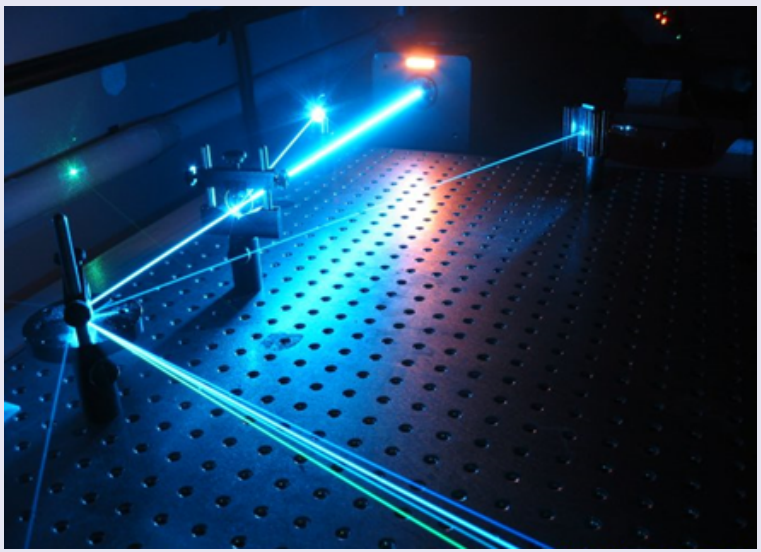


V. I. Kolobov, K. Golubkov, **JRMdN**, *J. Steinhauer, Nature Physics 17, 362 (2021)*

Black-hole laser

- Pair of horizons: Hawking radiation bounces back and forth $\rightarrow$ Self-stimulation $\rightarrow$ Dynamical instability (Complex BdG modes)
- Hawking radiation must be able to travel back from inner horizon $\rightarrow$ Superluminal dispersion relation $\omega^2 = c^2 k^2 \left(1 + \frac{k^2}{\Lambda^2}\right)$
- BEC quasiparticles are superluminal! $\rightarrow$ Planck scale $\Lambda \leftrightarrow$ Healing length ξ





S. Corley and T. Jacobson PRD 59, 124011 (1999).

Degenerate BHL: Degenerate parametric amplifier

- Lasing cavity with single degenerate mode $\Omega = i\Gamma = -\Omega^*$:

$$M_0 z_I = \Omega z_I, \;\; M_0 z_S = \Omega^* z_S, \;\; (z_I|z_I) = (z_S|z_S) = 0.$$

- Convention: $(z_I|z_S) = i$, $\bar{z}_{I,S} = z_{I,S} \rightarrow$ Properly normalized mode:

$$Z = \frac{z_I - i z_S}{\sqrt{2}} \Longrightarrow \hat{a} = (Z|\hat{\Phi}) = \frac{\hat{X} + i\hat{P}}{\sqrt{2}}$$

- BHL dynamics: *Degenerate* parametric amplifier

$$\hat{K} = \hat{K} - \mu \hat{N} \simeq i\Gamma \frac{(\hat{a}^\dagger)^2 - \hat{a}^2}{2} = \Gamma \frac{\hat{X}\hat{P} + \hat{P}\hat{X}}{2}$$

- Lasing amplitude=Position operator $\hat{X}$ in DPA:

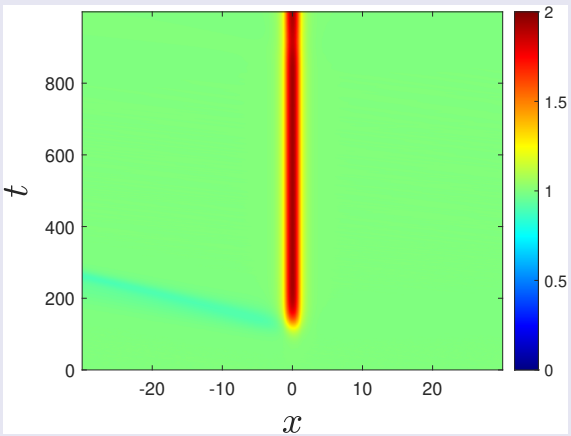
$$\hat{\Phi}(x,t) \simeq \hat{X} z_I(x) e^{\Gamma t} + \hat{P} z_S(x) e^{-\Gamma t}$$

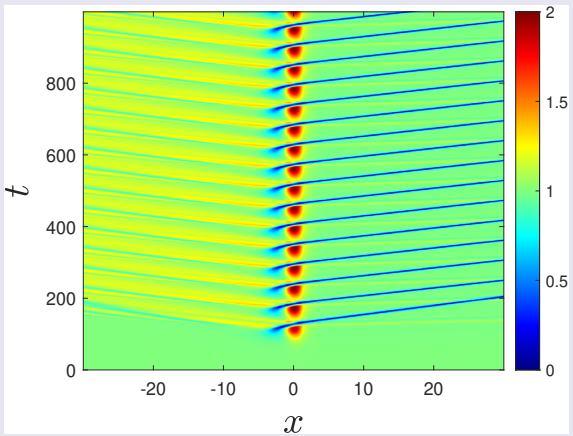
- Time evolution $U(t) = e^{-i\hat{K}t}$:

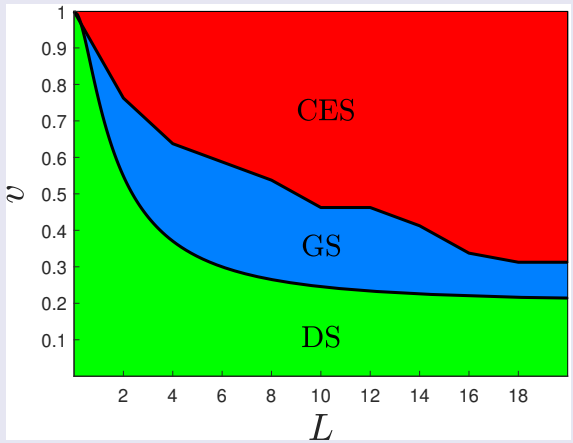
$$U(t)\,|X\rangle = e^{\frac{\Gamma t}{2}}\,|X e^{\Gamma t}\rangle\,, \;\; \hat{X}(t) = U^\dagger(t)\hat{X}U(t) = e^{\Gamma t}\hat{X}$$

BHL Phase Diagram

- Linear instability exponentially amplified until... What is the final fate of a BHL?
- Universal mean-field dynamical phase diagram: Ground state vs. periodic continuous emission of solitons (CES) state.
- GS/CES transition above critical velocity $v > v_c \rightarrow$ Nonlinear Landau criterion for superfluidity!







JRMdN, *S. Finazzi, I. Carusotto, PRA 94, 043616 (2016)*
JRMdN, *P. F. Palacios, I. Carusotto, F. Sols, NJP 23, 023040 (2021)*

Spontaneous Floquet states

- CES state: simplest realization of spontaneous Floquet state
- Spontaneous Floquet state: Floquet state from time-independent Hamiltonian due to interactions
- Condensates: Self-consistent Floquet solutions to GP equation $\Psi(x,t) = e^{-i\mu t}\Psi_0(x,t),\ \Psi_0(x,t+T) = \Psi_0(x,t)$:

$$[i\hbar\partial_t - \mu]\Psi_0(x,t) = H_{GP}(t)\Psi_0(x,t)$$

$$H_{GP}(t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}) + g|\Psi_0(\mathbf{x},t)|^2\right] = H_{GP}(t+T)$$

- Feasible in a number of variational many-body ansätze: MCTDH method, Hartree-Fock, Gutzwiller ansatz...

JRMdN, *F. Sols, PRA 105, 043302 (2022)*

Spontaneous Floquet states

- Intriguing features:
  - Temporal Floquet-Nambu-Goldstone mode with zero quasi-frequency in the Bogoliubov-Floquet spectrum
  - Energy conservation $\rightarrow$ Conserved Floquet charge $F \rightarrow$ Spontaneous Floquet states are "isofloquetic" ($\sim$ "isochoric")

$$dE = \omega dF + \sum_i \lambda^i dQ_i$$

  - Driven Floquet states are "isoperiodic" ($\sim$"isobaric") $\rightarrow$ Floquet enthalphy:

$$I = E - \omega F \rightarrow dI = -Fd\omega + \sum_i \lambda^i dQ_i$$

  - Quantum amplitude of temporal FNG$\rightarrow$ Tangible realization of Time Operator in Quantum Mechanics!

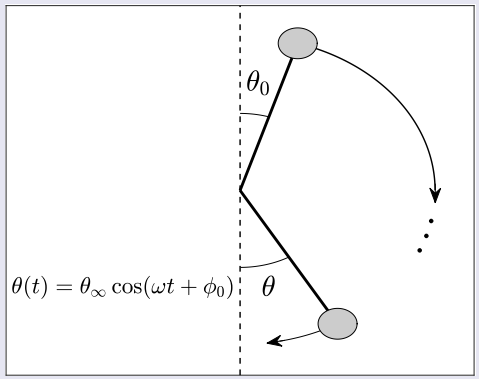
JRMdN, *F. Sols, Quantum 9, 1850 (2025)*

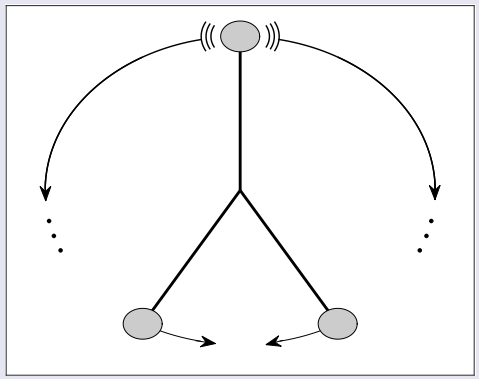
Quantum-Classical BHL

- So far, only mean-field spontaneous Floquet state $\rightarrow$ Fixed oscillation phase $\phi_0 \rightarrow$ Not truly spontaneous symmetry breaking!

$$\Psi_0(x,t) = \Psi_0(x, \phi_0 + \omega t), \ \Psi_0(x, \phi + 2\pi) = \Psi_0(x, \phi)$$

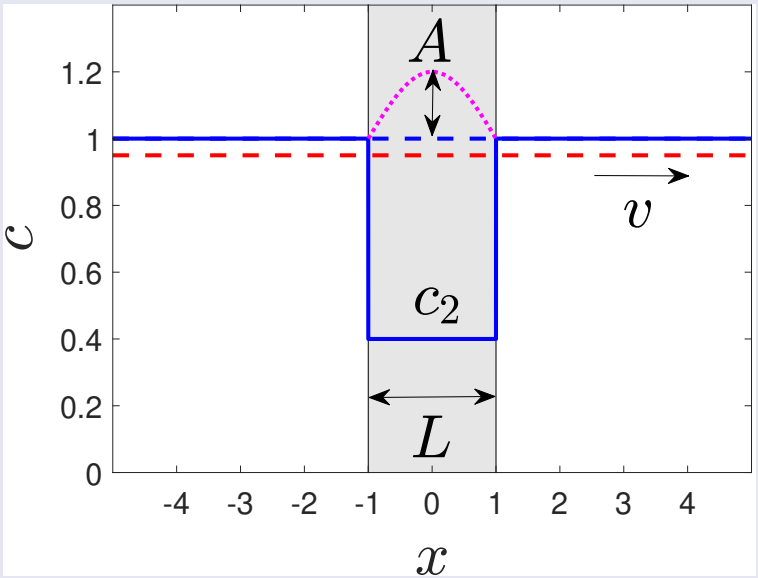
- Why not leveraging the quantum nature of Hawking radiation?
- BHL=DPA=Unstable pendulum:
  - Large initial amplitude $\longleftrightarrow$ Classical (coherent) trajectory$\rightarrow$ Late well-defined trajectory
  - Unstable position $\longleftrightarrow$ Amplification of vacuum fluctuations through squeezing $\rightarrow$ Superposition of eventual oscillations

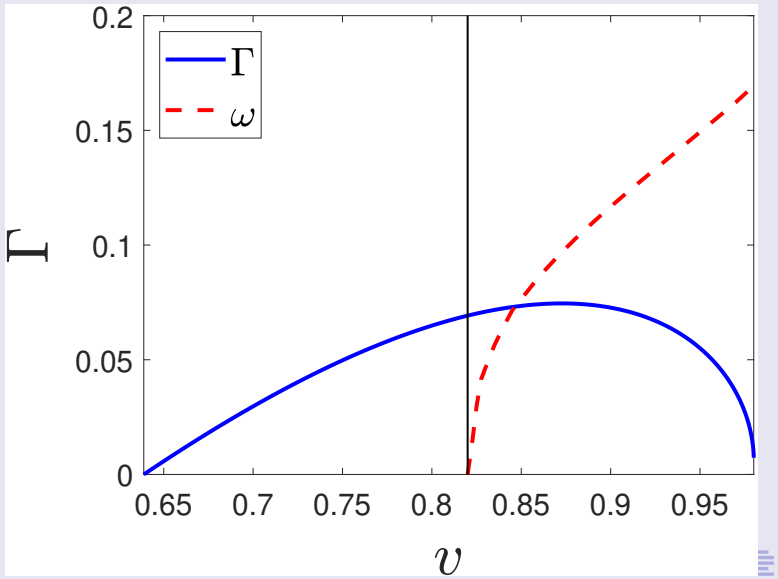




Flat-profile BHL

- $t < 0$: Homogeneous subsonic condensate $\Psi_0(x) = e^{ivx}$
- $t \geq 0$: Inhomogeneous quench $g(x) + V(x) = 1 \rightarrow$ Finite size supersonic region $\rightarrow$ BHL
  - Condensate does NOT experience dynamics $\rightarrow \Psi_0$ remains stationary
  - Quantum fluctuations DO experience dynamics $\rightarrow$ Lasing amplification
- Short cavity containing only one degenerate lasing mode ($\Omega = i\Gamma$)
- Initial quantum state: $T = 0$ quasiparticle vacuum
- Small bump of amplitude A is added within the lasing region $\rightarrow$ Coherent amplitude $\rightarrow$ Small pendulum kick





Numerical results: Density & ETCF

- Truncated Wigner computation ($v = 0.95,\ c_2 = 0.4,\ L = 2$)
  - Ensemble-averaged density: $n(x,t) \equiv \langle \hat{n}(x,t) \rangle = \langle \hat{\Psi}^\dagger(x,t)\hat{\Psi}(x,t) \rangle$
  - Out-of-time density-density correlation function (OTCF):

$$g^{(2)}(x,x',t,t') \equiv \langle \hat{n}(x',t')\hat{n}(x,t) \rangle - n(x',t')n(x,t).$$

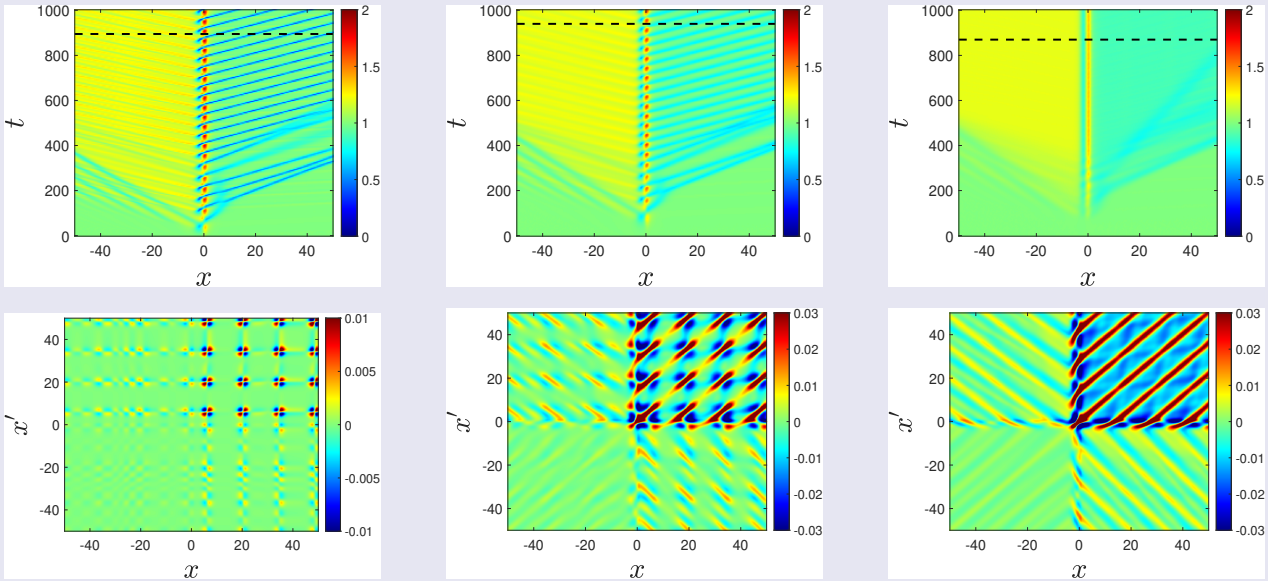


Figure: *Classical-to-Quantum. Left to right:* $A = 0.05, 0.005, 0$. Upper row: $n(x,t)$. Lower row: ECTF $g^{(2)}(x,x',t) \equiv g^{(2)}(x,x',t,t)$.

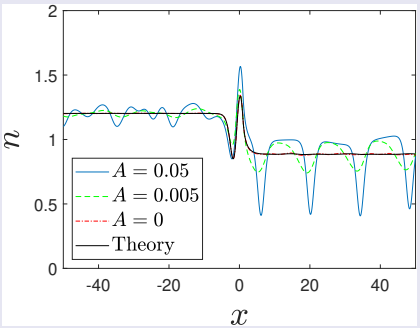
Numerical results: Density & ETCF

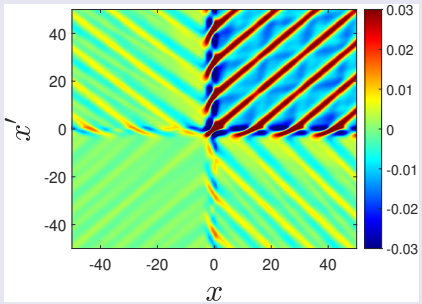
- Theoretical HTC prediction: Averages over different time-shifts $t_0 \in [0, T) \rightarrow$ Time-independent expectation values
  - Ensemble-averaged density: $n(x,t) = n_{\mathrm{HTC}}(x)$

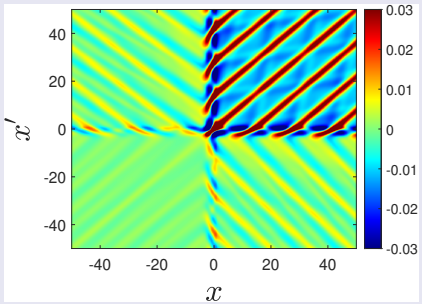
$$n_{\mathrm{HTC}}(x) = \frac{1}{T}\int_0^T \mathrm{d}t_0\, n_0(x, t_0),$$

  - ETCF: $g^{(2)}(x, x', t) = g^{(2)}_{\mathrm{HTC}}(x, x', \tau = 0)$

$$g^{(2)}_{\mathrm{HTC}}(x, x', \tau) = \frac{1}{T}\int_0^T \mathrm{d}t_0\, n_0(x, t_0+\tau) n_0(x', t_0) - n_{\mathrm{HTC}}(x) n_{\mathrm{HTC}}(x')$$







Numerical results: OTCF

- Periodicity manifested in off-diagonal OTCF:

$$g^{(2)}(x, x', \tau; t) = g^{(2)}(x, x', t, t + \tau)$$

- HTC: $g^{(2)}(x, x', \tau; t) = g^{(2)}_{\mathrm{HTC}}(x, x', \tau) = g^{(2)}_{\mathrm{HTC}}(x, x', \tau + T)$
- Figure of merit: Normalized peak of Downstream-Upstream $(x > 0, x' < 0)$ Fourier transform $\mathcal{G}^{(2)}(t, t')$
- $\mathcal{G}^{(2)}(t, t')$ becomes function only of $\tau = t' - t$

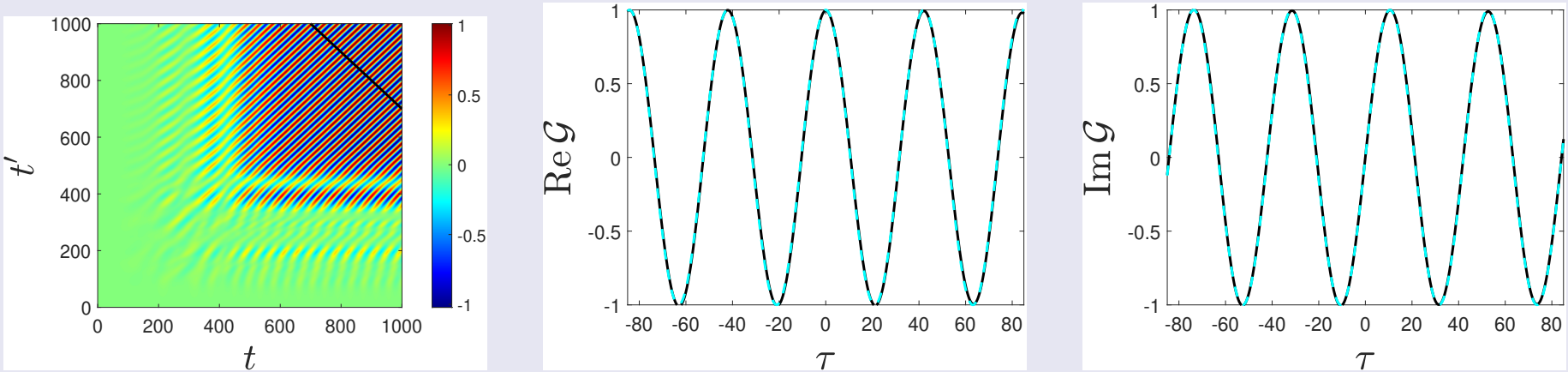
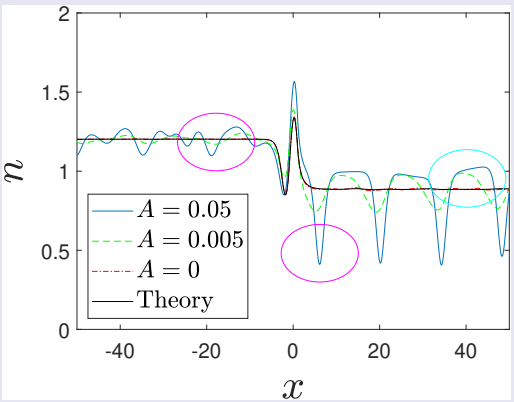
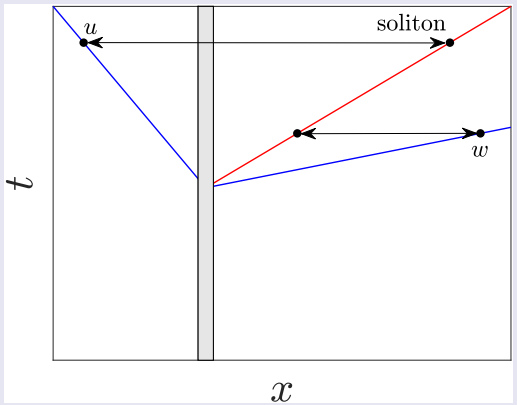


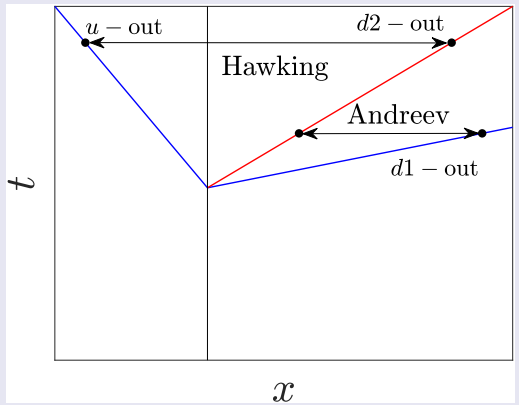
Figure: Left: $\mathrm{Re}\,\mathcal{G}(t, t')$. Center: $\mathrm{Re}\,\mathcal{G}(t, t')$ along black line. Right: $\mathrm{Im}\,\mathcal{G}(t, t')$ along black line. Dashed cyan: Theory $\mathcal{G}_{\mathrm{HTC}}(\tau) = e^{i\omega\tau}$.

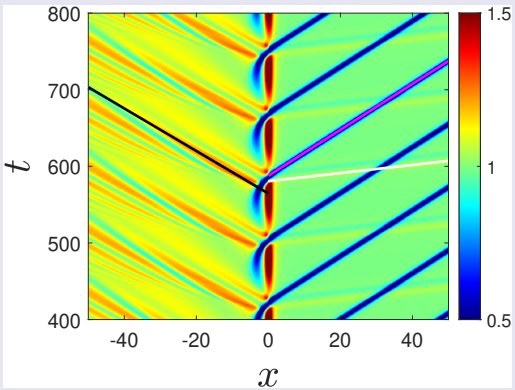
Nonlinear Andreev-Hawking effect

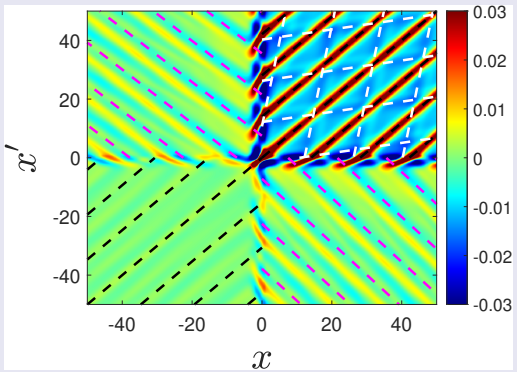
- Spatial correlation patterns arise from correlated quantum emission:
  - Downstream soliton (d2)
  - Upstream wave (u)
  - Downstream wave (d1)
- Nonlinear periodic analogue of Andreev-Hawking effect
- Time-periodicity+Traveling features=Space-periodicity

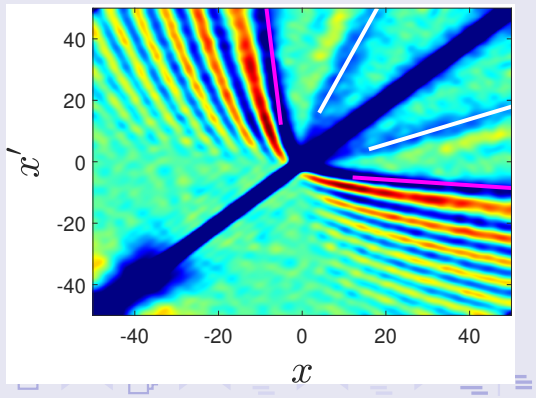






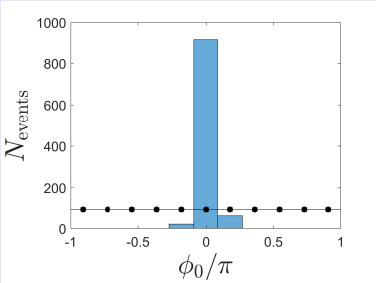


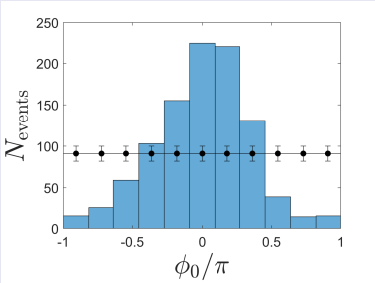


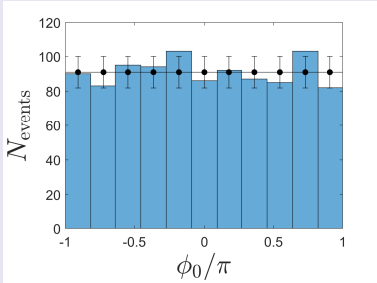


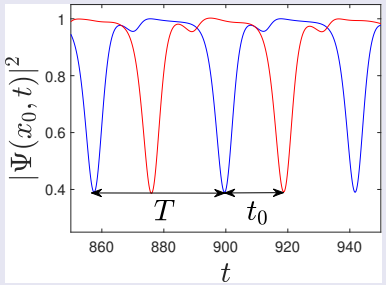
Histogram

- Late-time histogram of CES oscillation phase-shift $\phi_0 = -\omega t_0$:



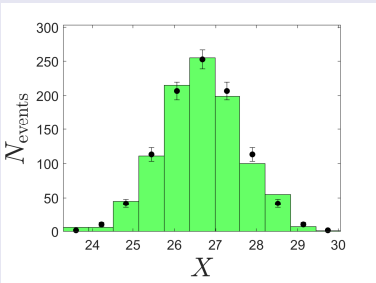


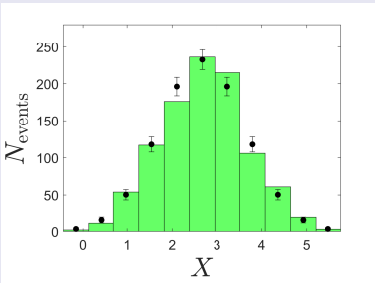


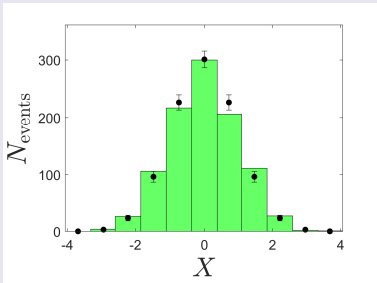


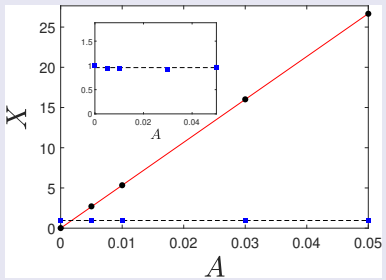
- Simple model: Final CES state determined by initial lasing distribution

$$W_0(X) = \frac{1}{\sqrt{2\pi\Delta X_Q^2}} e^{-\frac{(X-X_C)^2}{2\Delta X_Q^2}}$$









  - $X_C \sim A$=Mean=Classical component $\rightarrow$ Mean-field trajectory
  - ΔX_Q=Width=Quantum fluctuations
- Crystallization $\rightarrow$ Random oscillation phase-shifts $\rightarrow \Delta X_Q \gtrsim X_C$

DPA time operator

- Time evolution $U(t) = e^{-i\hat{K}t}$:

$$U(t)\,|X\rangle = e^{\frac{\Gamma t}{2}}\,|Xe^{\Gamma t}\rangle\,,\ \hat{X}(t) = U^{\dagger}(t)\hat{X}\,U(t) = e^{\Gamma t}\hat{X}$$

- Hamiltonian not bounded $\rightarrow$ DPA time operator:

$$\hat{t} \equiv -\frac{1}{\Gamma}\ln\left|\frac{\hat{X}}{X_R}\right| \Longrightarrow [\hat{t},\hat{K}] = -i$$

- Continuous degenerate spectrum with two branches:

$$\begin{aligned}
|t\rangle_{\pm} &\equiv \sqrt{\left|\frac{dX_{\pm}}{dt}\right|}\,|X_{\pm}(t)\rangle\,,\ X_{\pm}(t) = \pm|X_R|e^{-\Gamma t} \\
\sum_{a=\pm}\int_{-\infty}^{\infty}\mathrm{d}t\ |t\rangle_a\,{}_a\langle t| &= \int_0^{\infty}\mathrm{d}X\ |X\rangle\langle X| + \int_{-\infty}^{0}\mathrm{d}X\ |X\rangle\langle X| = 1 \\
{}_{\pm}\langle t|t'\rangle_{\pm} &= \delta(t-t'),\ {}_{\pm}\langle t|t'\rangle_{\mp} = 0
\end{aligned}$$

- Physics: Time-shift with respect to certain reference amplitudes $\pm|X_R|$. Different X_R, different time origin.

DPA time operator

- Whole spectrum generated through time translations:

$$|t\rangle_{\pm} = U(-t)\,|0\rangle_{\pm}$$

- Dynamics only needs to be computed for the two time origins $|t=0\rangle_{\pm}$ (i.e., reference lasing amplitudes $|X=\pm X_R\rangle$).
- Any state is a quantum superposition of different time realizations:

$$|\Psi\rangle = \sum_{a=\pm}\int_{-\infty}^{\infty} \mathrm{d}t\ \Psi_a(t)\,|t\rangle_a = \sum_{a=\pm}\int_{-\infty}^{\infty} \mathrm{d}t\ \Psi_a(t)U(-t)\,|0\rangle_a$$

- Time evolution simply shifts temporal wavefunction:

$$U(t)\,|\Psi\rangle = \sum_{a=\pm}\int_{-\infty}^{\infty} \mathrm{d}t'\ \Psi_a(t')\,|t'-t\rangle_a = \sum_{a=\pm}\int_{-\infty}^{\infty} \mathrm{d}t'\ \Psi_a(t'+t)\,|t'\rangle_a$$

- $\hat{K}$ acts as $i\partial_t$ in temporal representation:

$$\hat{K}\,|\Psi\rangle = \sum_{a=\pm}\int_{-\infty}^{\infty} \mathrm{d}t\ i\partial_t\Psi_a(t)]\,|t\rangle_a\,.$$

Time operators

- Physics: Each $X = X_C e^{-\Gamma t_0}$ leads to different trajectory time-shifted t_0 with respect to mean-field

$$t_0 = -\frac{1}{\Gamma} \ln \frac{X}{X_C} \rightarrow \phi_0(X) = -\omega t_0$$

- Late ϕ_0 distribution determined by initial lasing distribution:

$$W(\phi_0) = \left| \frac{dX}{d\phi_0} \right| W_0(X)$$

- But t_0 is also the time amplitude of DPA time operator with $X_R = X_C$ $\rightarrow$ Connection between initial (DPA) and final (CES) time operators!

***Disclaimer: Simplified (albeit illustrative) picture.*

HTC Conclusions

- Numerical observation of Hawking Time Crystal:
  - Time-independent density and ETCF
  - Periodic OTCF
  - Uniform oscillation phase-shift distribution
- Textbook example of time crystal! *K. Sacha, Time crystals (2020)*
- Nonlinear version of Andreev-Hawking effect.
- Unique characterization in terms of time operators $\rightarrow$ Novel time operator in a fundamental paradigm in quantum optics such as DPA!
- Analogue concepts can inspire applications in tabletop experiments:
  - Low-pass filters **JRMdN**, *F. Sols, I. Zapata, AdP 529, 1600385 (2017)*
  - Quantum amplifiers **JRMdN**, *F. Sols, PRR 5, 043282 (2023)*
  - Quantum Information in High-Energy Physics *Y. Afik*, **JRMdN**, *EPJP 136, 907 (2021); Aad . . .* **JRMdN** *. . . Zwalinski (ATLAS), Nature 633, 542 (2024)*
- Time operator+Gravitational analogues=Fundamental platform to research on the quantum nature of time

The End

THANK YOU!

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Temporal Floquet-Goldstone mode

- BdG expansion for Floquet states:

$$\begin{aligned}\hat{\Phi}(\mathbf{x},t) &= [\hat{X}^{\alpha}+\partial_A\lambda_B\hat{P}^B t]z_{\alpha}(\mathbf{x},t)+\hat{P}^A z_A(\mathbf{x},t)\\ &+ \sum_n \hat{\gamma}_n z_n(\mathbf{x},t)e^{-i\varepsilon_n t}+\hat{\gamma}_n^{\dagger}\bar{z}_n(\mathbf{x},t)e^{i\varepsilon_n t}\end{aligned}$$

- Spontaneous Floquet states: Genuine temporal Floquet-Goldstone mode z_{ϕ} arising from phase translations.
- Actual temporal Goldstone mode $z_t=\omega z_{\phi}$ whose conjugate are energy fluctuations:

$$\hat{X}^t=\hat{X}^{\phi}/\omega,\ \delta\hat{K}=\omega\hat{P}^F \Longrightarrow [\hat{X}^t,\delta\hat{K}]=i$$

- $\hat{t}_0\equiv-\hat{X}^t$: Time operator describing quantum fluctuations of mean-field time origin $t_0=-\phi_0/\omega$

Time operator

- Why there is no time operator $[\hat{T}, \hat{H}] = -i$? Pauli no-go theorem:

$$e^{-i\hat{T}E_0}\hat{H}e^{i\hat{T}E_0} = \hat{H} - E_0$$

- HORROR: Energy is bounded!
- Same for particle number/phase $[\hat{N}, \hat{\theta}] = i$:

$$e^{-i\hat{\theta}N_0}\hat{N}e^{i\hat{\theta}N_0} = \hat{N} - N_0$$

- HORROR: Particle number is non-negative and discrete!
- However, one can define phase fluctuations for large $N \rightarrow$ One can define time fluctuations for large E
- Spontaneous Floquet states must be highly-excited states! $\rightarrow$ *Tangible* realization of time operator