Many body physics with Rydberg arrays – Lecture 3

Lecture 1: Many-body problem and quantum simulation
Arrays of atoms & "Rydbergology"
Interactions between atoms

Lecture 2: Rydberg Interactions and spin models
Engineering many-body Hamiltonians

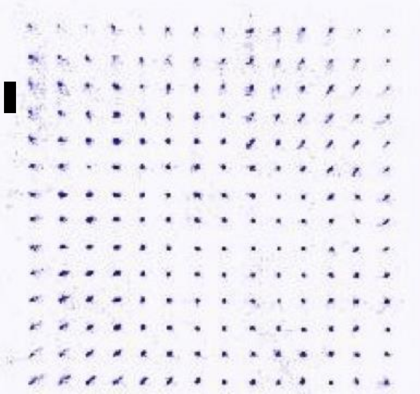
Mainly 2 atoms

Lecture 3: Examples of quantum simulations in and out-of-equilibrium: quantum magnetism

Many atoms…

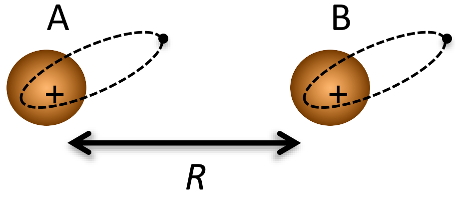
Combining arrays of atoms and Rydberg interactions

5 µm



+

Rydberg interactions



Van der Waals

$$\frac{C_6}{R^6}$$

resonant

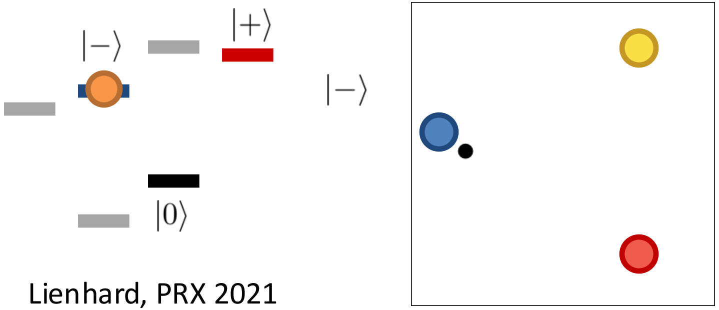
$$\frac{C_3}{R^3}$$

Quantum Ising
s = 1/2

Hardcore
boson

Bosons/ Fermions
Softcore
potential

Spin-orbit coupling



Lienhard, PRX 2021

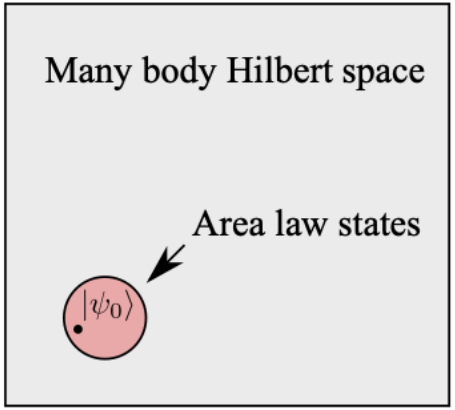
XY, s = 1/2
$\frac{1}{R^3}, \frac{1}{R^6}$

XYZ
Heisenberg
s = 1/2
Floquet

t- J model

How to study a many-body system?

Ground-state



g.s. of *gapped* many-body system
= weakly entangled (area laws)

"Easy" to simulate: Monte Carlo, MPS
(when no frustration, away from QPT)

Out-of-equilibrium
Dynamics following "quench"

Involve ~ *all states* of Hilbert space

Very hard to simulate:
Massive generation of entanglement

(Volume law)

Hauschild & Pollman, SciPost 2016

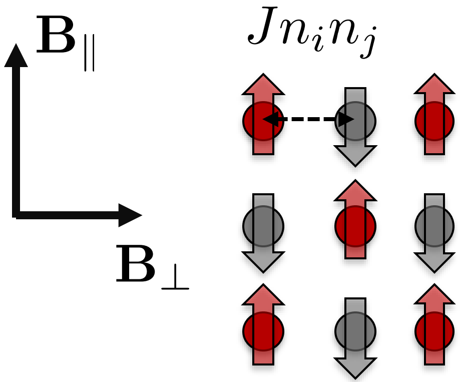
Outline – Lecture 3

1. Studying the ground state of quantum magnets
    - Ising model in 2D
    - Dipolar XY model in 2D

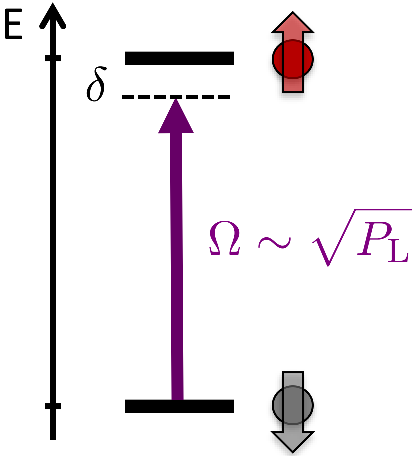
2. Out-of-equilibrium dynamics: Quench dynamics in Ising model. Thermalization or not…?

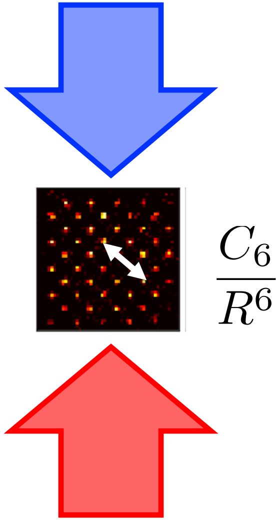
3. Outlook: what we did not discuss… & beyond

From van der Waals interaction to spin models…



$$\sim \mu B_{\perp} \hat{\sigma}_x + \mu B_{\parallel} \hat{\sigma}_z$$





Transverse Field Ising model:

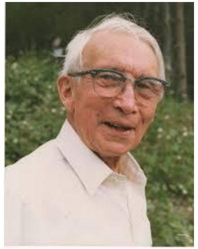
$$H = \frac{\hbar\Omega}{2} \sum_i \hat{\sigma}_x^i + \hbar\delta \sum_i \hat{\sigma}_z^i + \sum_{i<j} \frac{C_6}{R_{ij}^6} \hat{n}_i \hat{n}_j$$

Laser: $B_{\perp}$ $B_{\parallel}$ spin-spin interactions

Controlled parameters:

From negligible to dominant interactions

Quantum Ising model: an old problem…with open questions



Ernst Ising (1900 - 98)

Classical Ising model $$H_C = \sum_{i\neq j} V_{ij} n_i n_j - \delta \sum_i n_i \ , \quad n_{i,j} = 0, 1$$

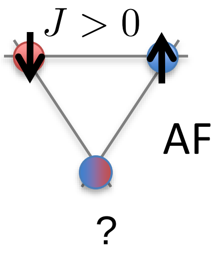
1D solved (Ising, 1924); 2D solved (Onsager, 1944)

Quantum Ising model $$H_Q = \sum_{i\neq j} V_{ij} \hat{n}_i \hat{n}_j - \delta \sum_i \hat{n}_i + \Omega \sum_i \sigma_x^i \qquad [\hat{n}_i, \sigma_x^i] \neq 0$$

Solved in 1D (Lieb, 1961)

Monte-Carlo in 2D: critical exponent, critical point
but never measured…

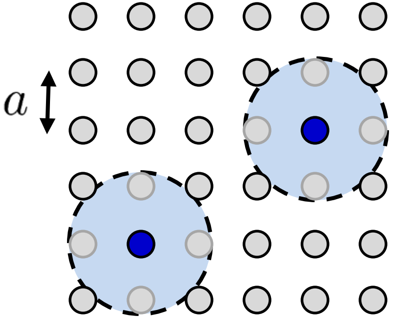
Triangular, Kagomé geometry: geometrical frustration
critical exponents (MC ~2000), spin liquids…??



Dynamics: growth of entanglement (>2D)??
disordered V_{ij} (even in 1D)…??

2D Ising anti-ferromagnet on a square

Nearest neighb. interaction

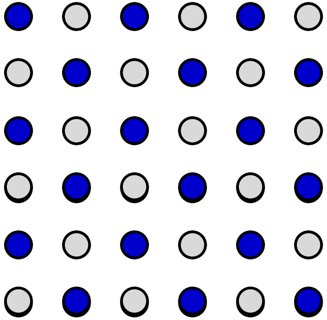


$$\frac{C_6}{a^6} \sim \Omega$$

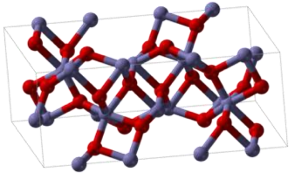
○ $= |g\rangle = |\downarrow\rangle$

● $= |r\rangle = |\uparrow\rangle$

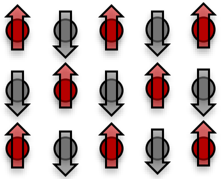
Anti-ferromagnetic ground state



Ex of antiferromagnets:
MnO, FeO, CoO, NiO, $FeCl_2$…

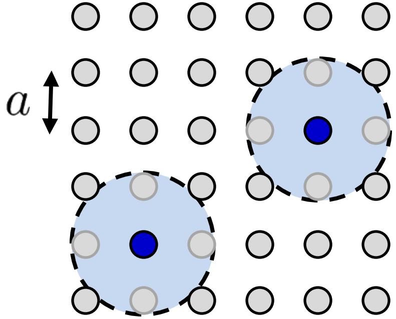


AFM (Néel) ordering (Z_2 phase)



2D Ising anti-ferromagnet on a square

Nearest neighb. interaction

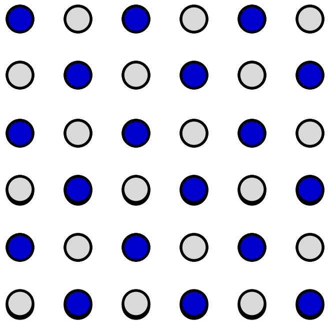


$\frac{C_6}{a^6} \sim \Omega$

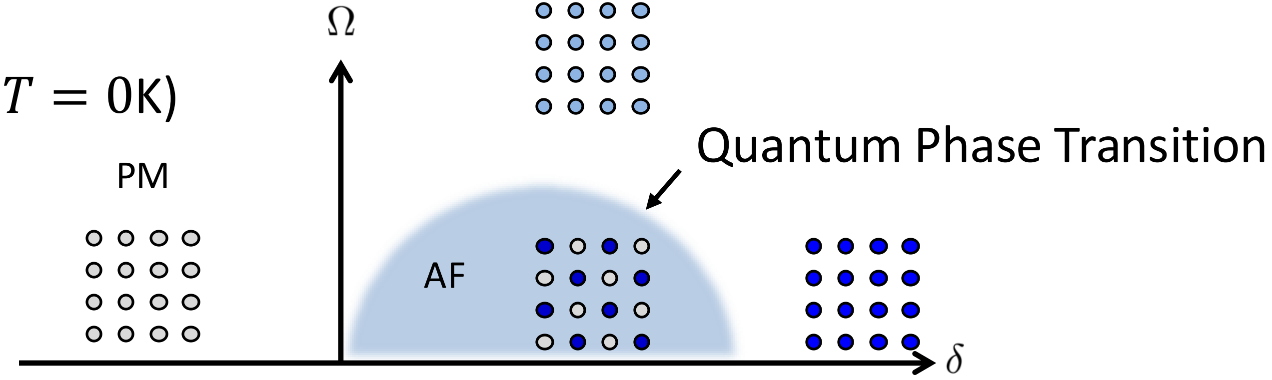
$\bigcirc = |g\rangle = |\downarrow\rangle$

$\bullet = |r\rangle = |\uparrow\rangle$

Anti-ferromagnetic ground state



2D phase diagram $(T = 0\text{K})$
(1970)

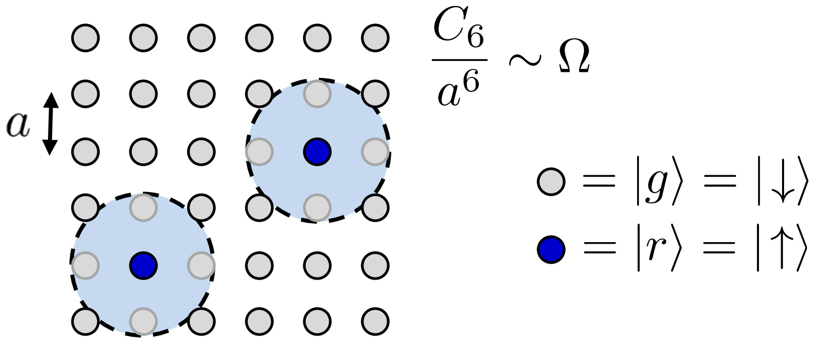


Known by Quantum Monte-Carlo

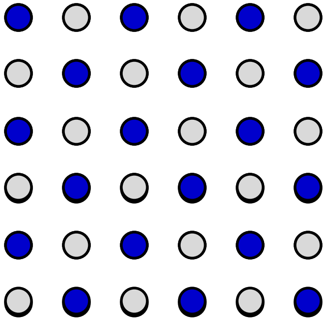
Never implemented and measured in 2D… (approximation in material)

2D Ising anti-ferromagnet on a square

Nearest neighb. interaction



Anti-ferromagnetic ground state



2D phase diagram ($T = 0$K)
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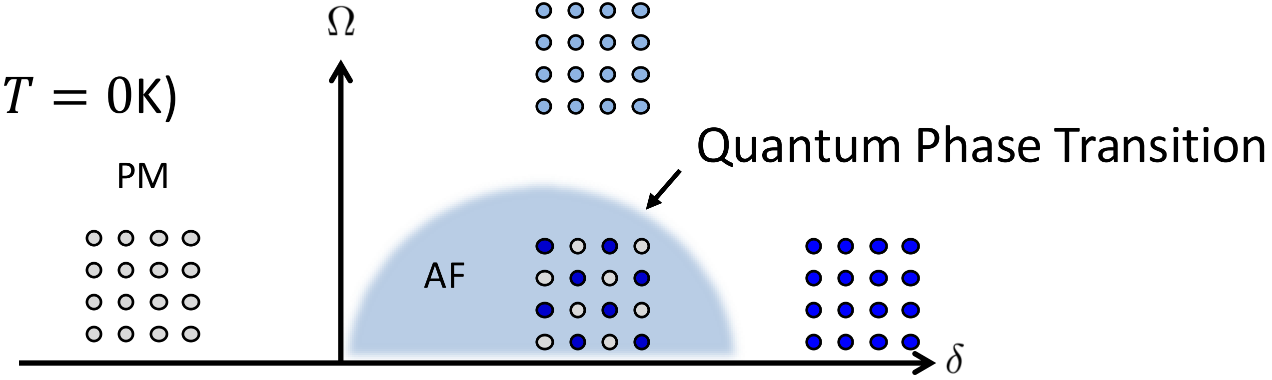
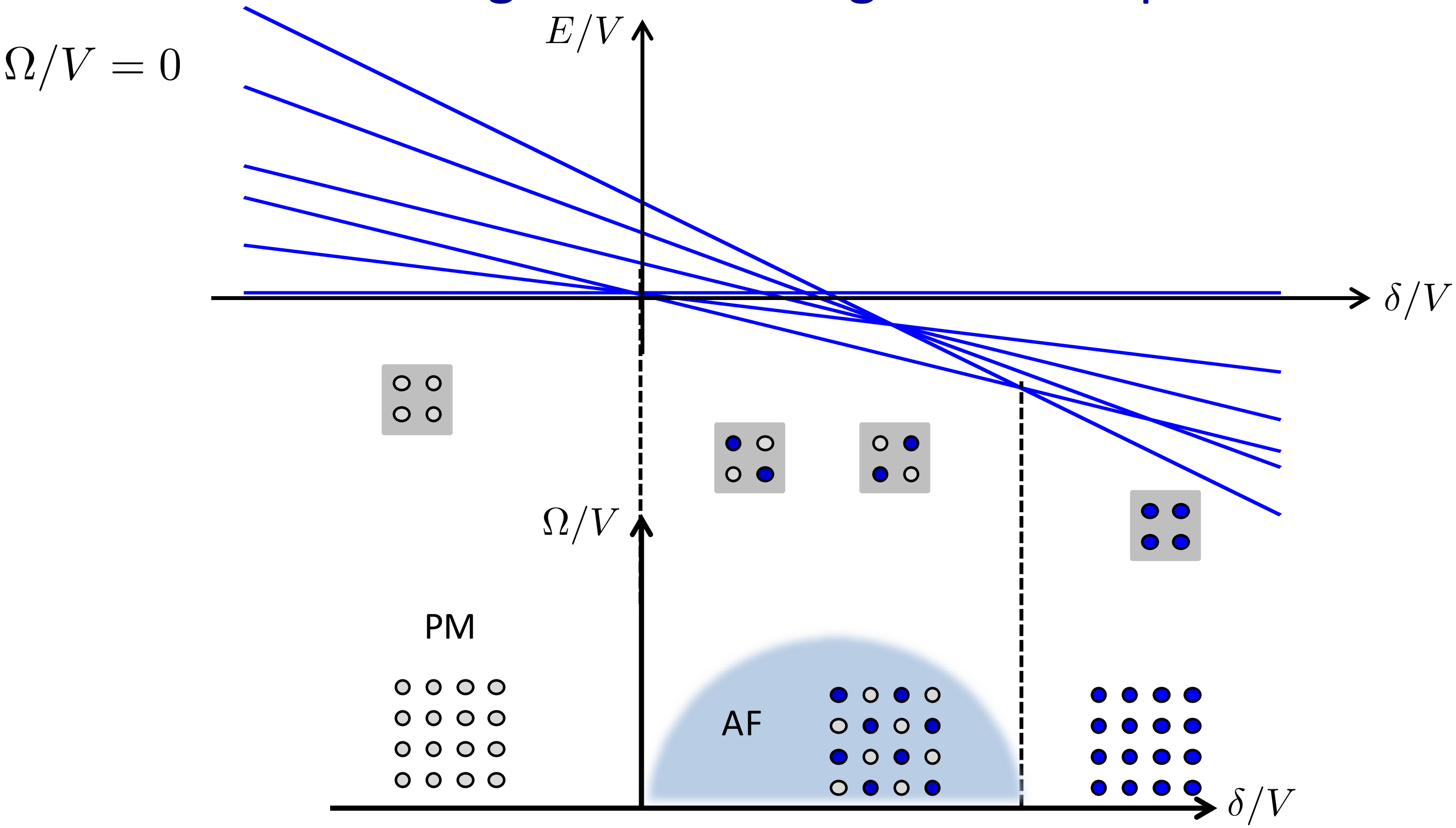


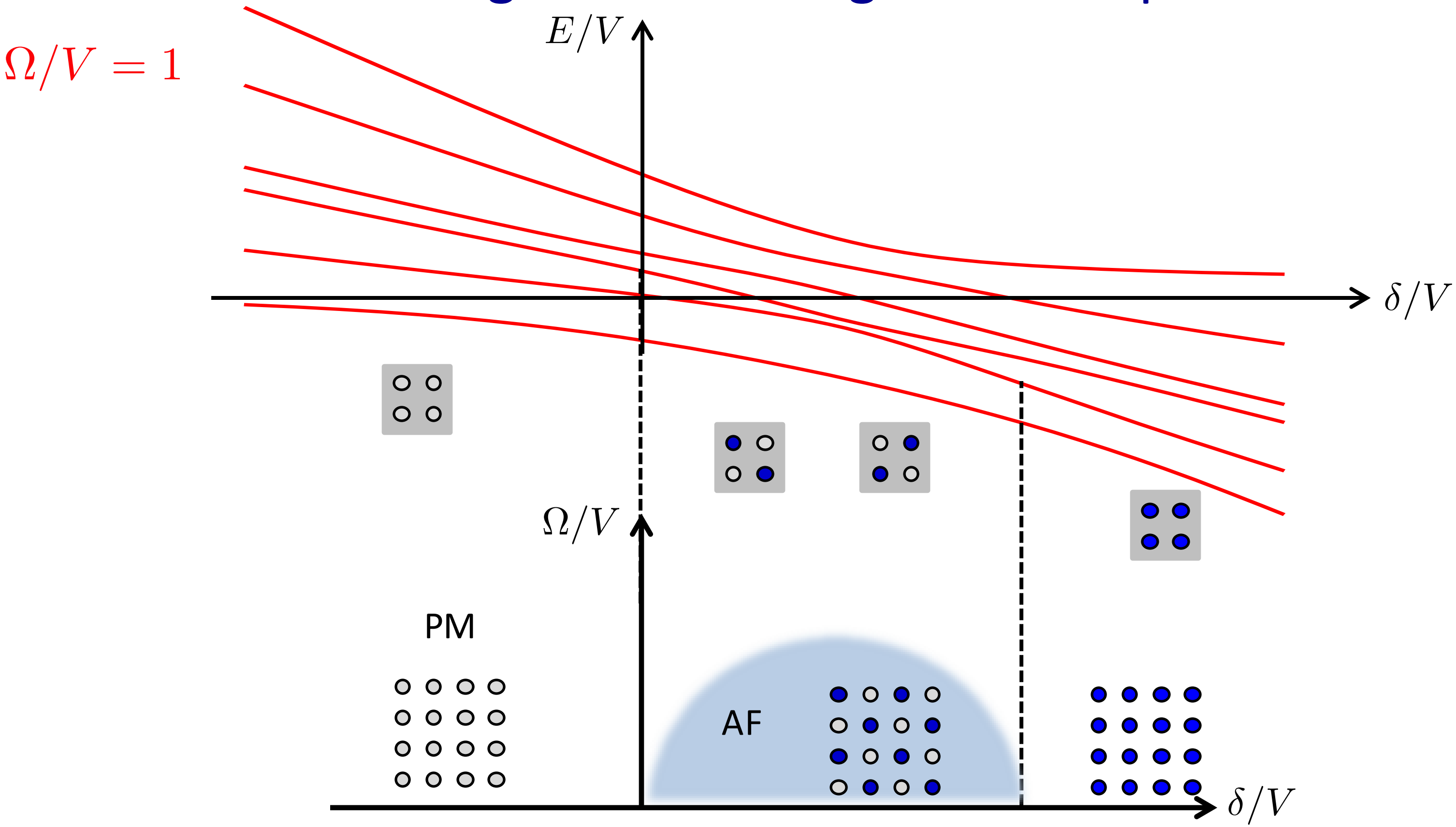
$$H = \frac{\hbar\Omega}{2}\sum_i \sigma_x^i - \hbar\delta\sum_i \hat{n}_i + \frac{V}{2}\sum_i \hat{n}_i\hat{n}_{i+1}$$

2D Ising anti-ferromagnet on a square



$$H = \quad -\hbar\delta\sum_i \hat{n}_i + \frac{V}{2}\sum_i \hat{n}_i\hat{n}_{i+1}$$

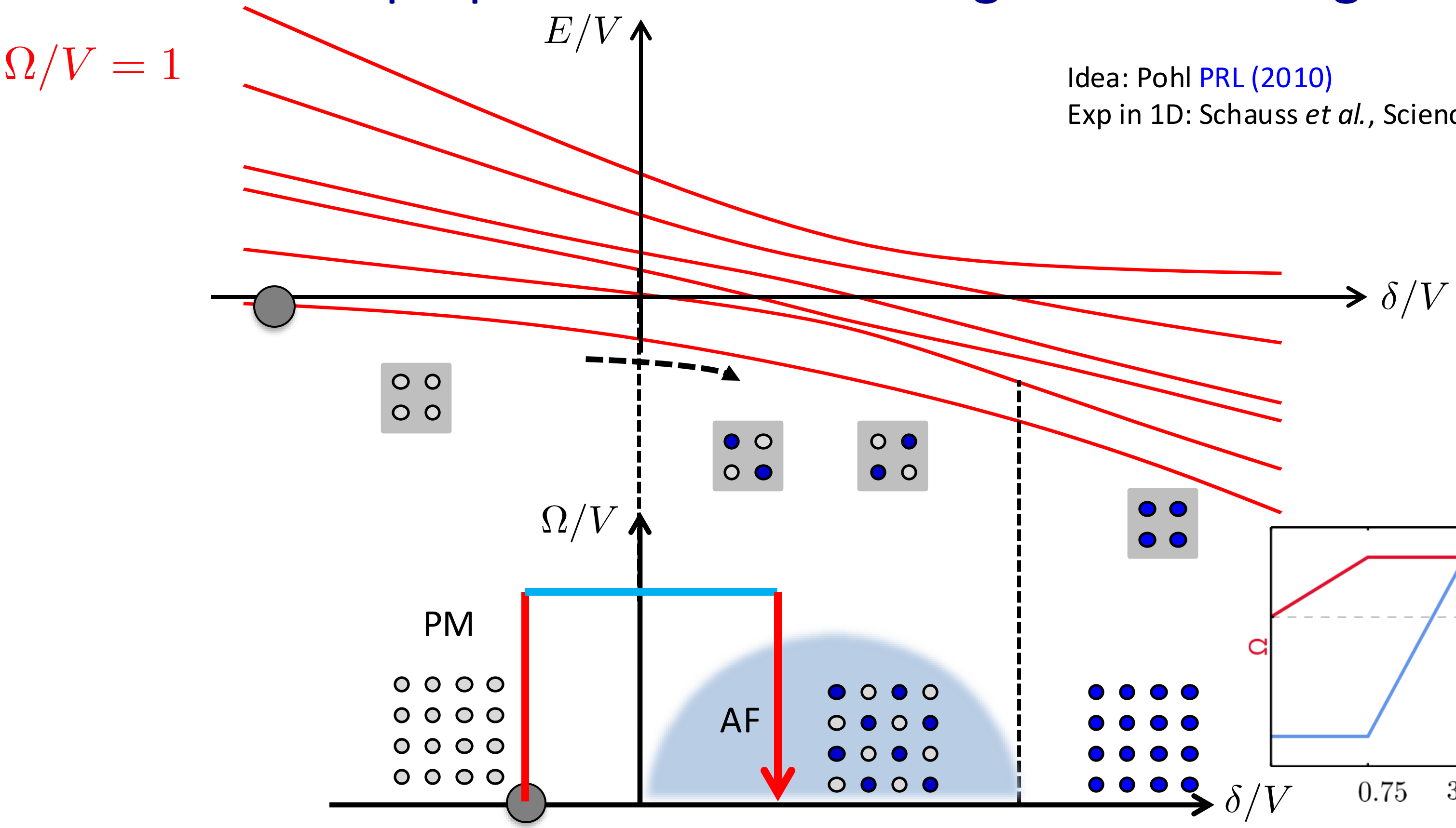
2D Ising anti-ferromagnet on a square

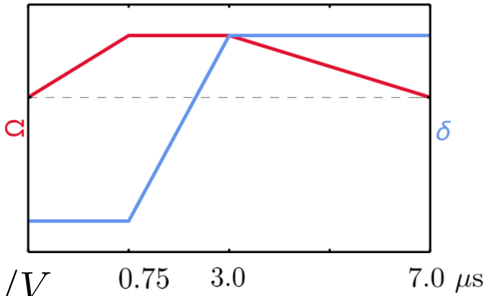


$$H = \frac{\hbar\Omega}{2}\sum_i \sigma_x^i - \hbar\delta\sum_i \hat{n}_i + \frac{V}{2}\sum_i \hat{n}_i\hat{n}_{i+1}$$

Adiabatic preparation of a 2D Ising anti-ferromagnet

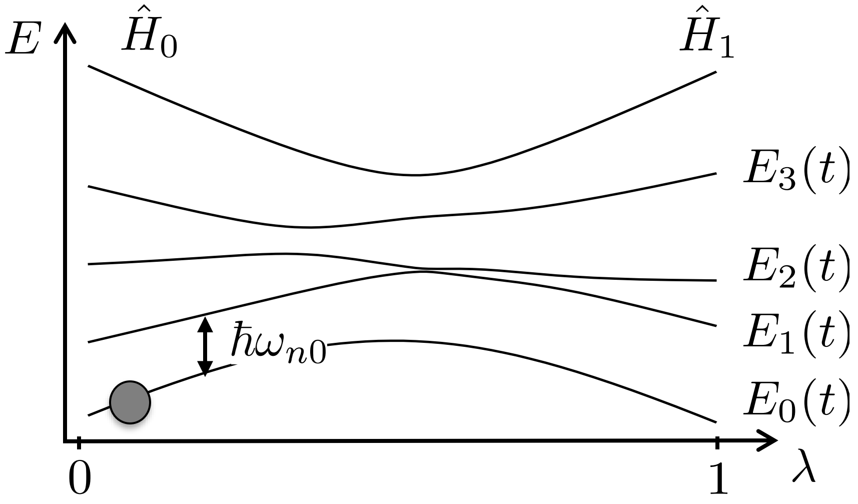
Idea: Pohl PRL (2010)
Exp in 1D: Schauss *et al.*, Science (2015)





$$H = \sum_i \left(\frac{\hbar\Omega(t)}{2}\sigma_x^i - \hbar\delta(t)\hat{n}_i \right) + \sum_{i<j} \frac{C_6}{R_{ij}^6}\hat{n}_i\hat{n}_j$$

Adiabatic preparation of a ground state: quantum annealing



Sakurai, Quantum Mechanics & Wikipedia

$$\hat{H}(t) = (1-\lambda(t))\hat{H}_0 + \lambda(t)\hat{H}_1$$

Instantaneous eigenstates:

$$\hat{H}(t)|\phi_n(t)\rangle = E_n(t)|\phi_n(t)\rangle$$

Solve: $i\hbar \frac{d}{dt}|\psi(t)\rangle = \hat{H}(t)|\psi(t)\rangle$ with $|\psi(t)\rangle = \sum_n a_n(t)|\phi_n(t)\rangle, \; a_n(0) = \delta_{n,0}$

$$\Rightarrow |a_n(t)| \sim \frac{|\langle\phi_n|\frac{d\hat{H}}{dt}|\phi_0\rangle|^2}{\hbar\omega_{n0}^2}$$

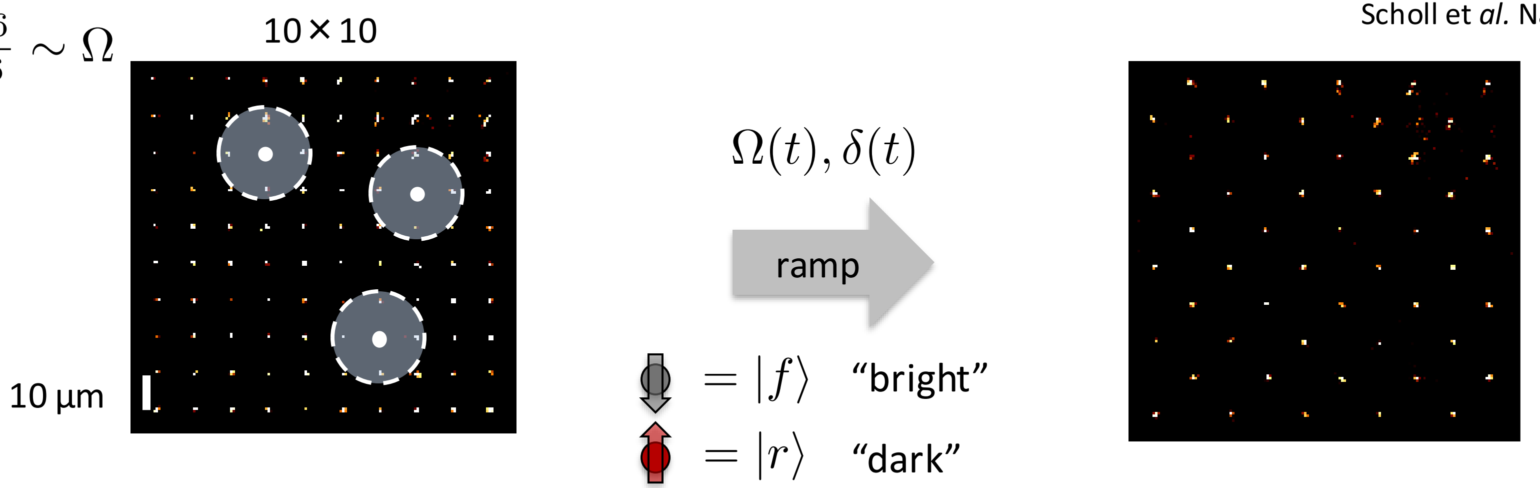
Adiabatic following: $|\langle\phi_n|\frac{d\hat{H}}{dt}|\phi_0\rangle|^2 \ll \hbar\omega_{n0}^2$

Rate of change of *H* slow with respect to the energy gaps…

Adiabatic preparation of an antiferromagnet on a square array

Scholl et *al.* Nature (2021)

$\frac{C_6}{a^6} \sim \Omega$



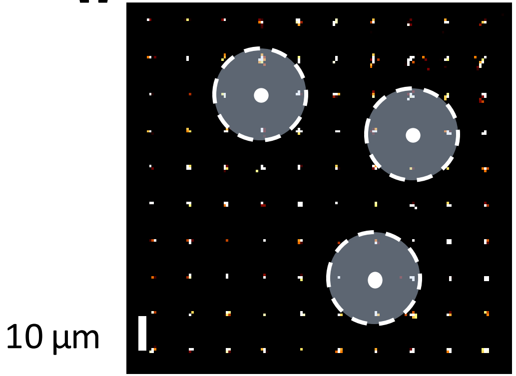
1D: Pohl PRL 2010; Bloch Science 2015; Lukin Nature 2017, 2019;
2D: Lienhard PRX 2018, Bakr PRX 2018; Lukin Nature 2021

Adiabatic preparation of an antiferromagnet on a square array

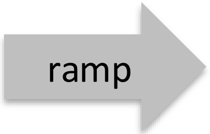
Scholl et *al.* Nature (2021)

$$\frac{C_6}{a^6} \sim \Omega$$

10×10

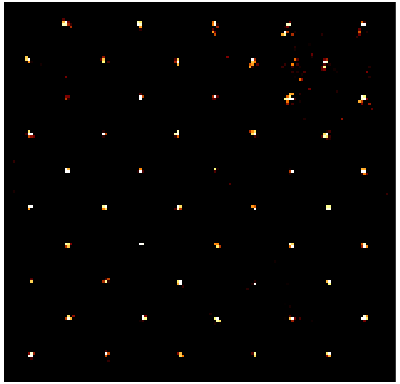


$\Omega(t), \delta(t)$



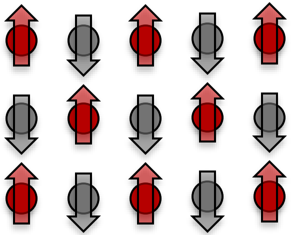
$= |f\rangle$ "bright"

$= |r\rangle$ "dark"



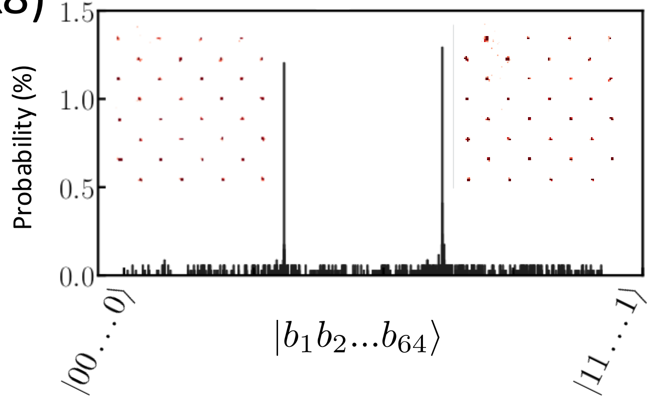
Perfect AF (Néel) ordering!

(proba < 1%)



(8x8)

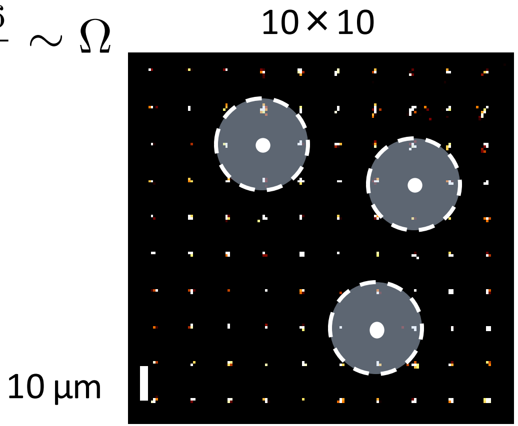
2^{64} states!!!



Adiabatic preparation of an antiferromagnet on a square array

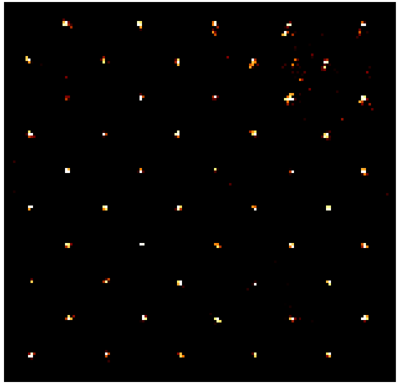
$$\frac{C_6}{a^6} \sim \Omega$$

Scholl et al. Nature (2021)



$\Omega(t), \delta(t)$

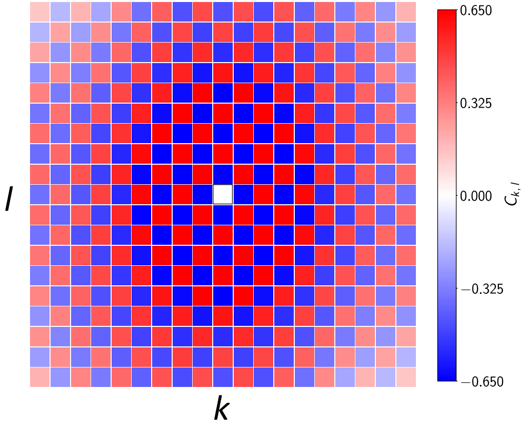
ramp

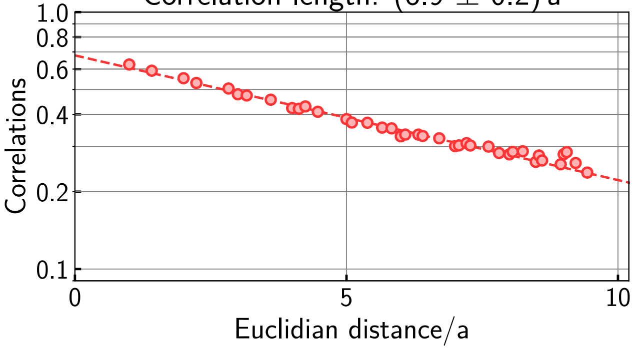


$= |f\rangle$ "bright"

$= |r\rangle$ "dark"

$$C_{kl} \sim \langle n_k n_l \rangle - \langle n_k \rangle \langle n_l \rangle$$



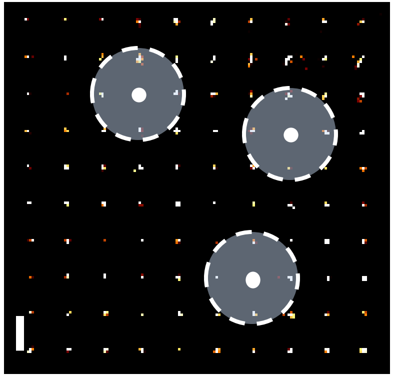


Also: Lukin Nature 2021

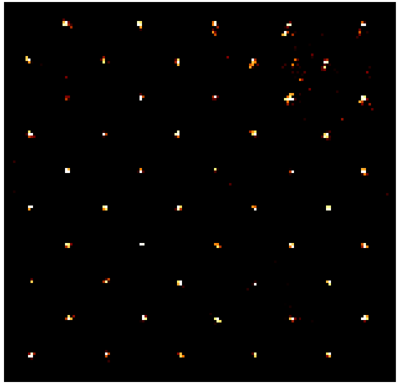
Classical simulation of the preparation

$\frac{C_6}{a^6} \sim \Omega$

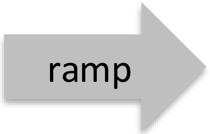
10 × 10

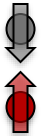


Scholl et *al.* Nature (2021)



$\Omega(t), \delta(t)$





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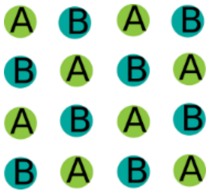
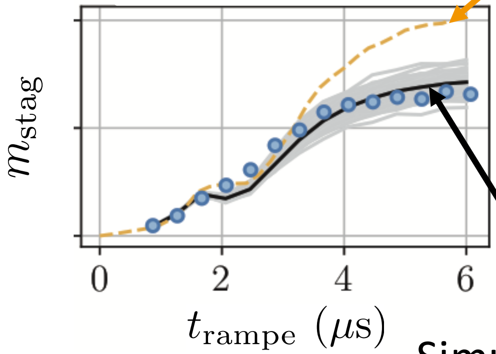


Dynamics of magnetization

$$m_{\text{stag}} = \langle |n_A - n_B| \rangle$$



Perfect

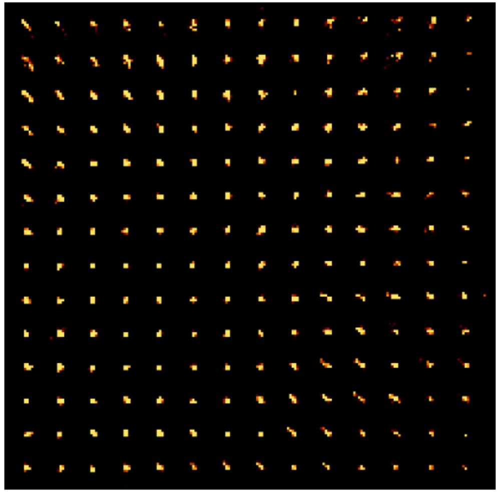
Simulation with imperfections

State of-the-art simulation (2021):
MPS limited to 10 x 10
(14 days on super supercomputer!!)

But we can push the atom number… by a lot…

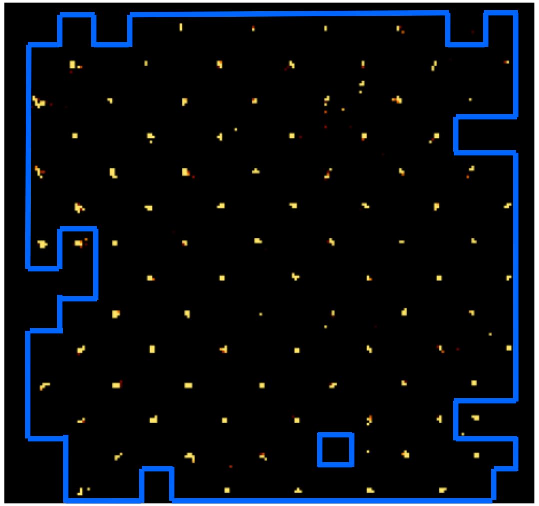
Scholl et *al.* Nature (2021)

14x14



$$\Omega(t), \delta(t)$$

ramp

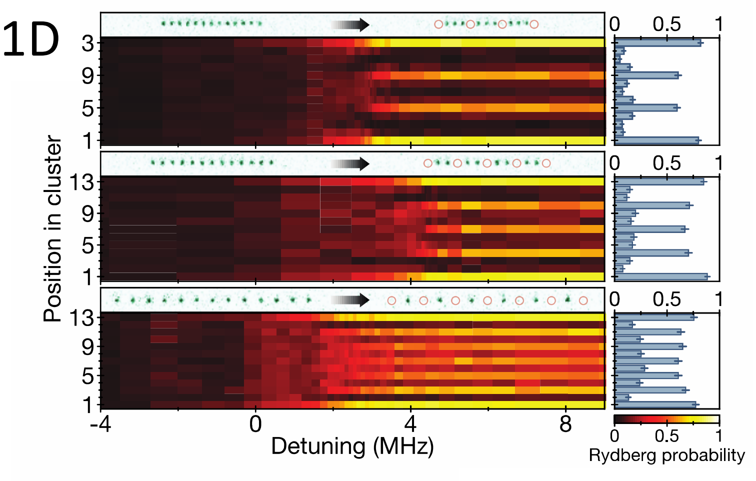


Antiferromagnetic cluster:
182 atomes

Since 2022: more elaborate numerical methods …!!

Ising model in other geometries

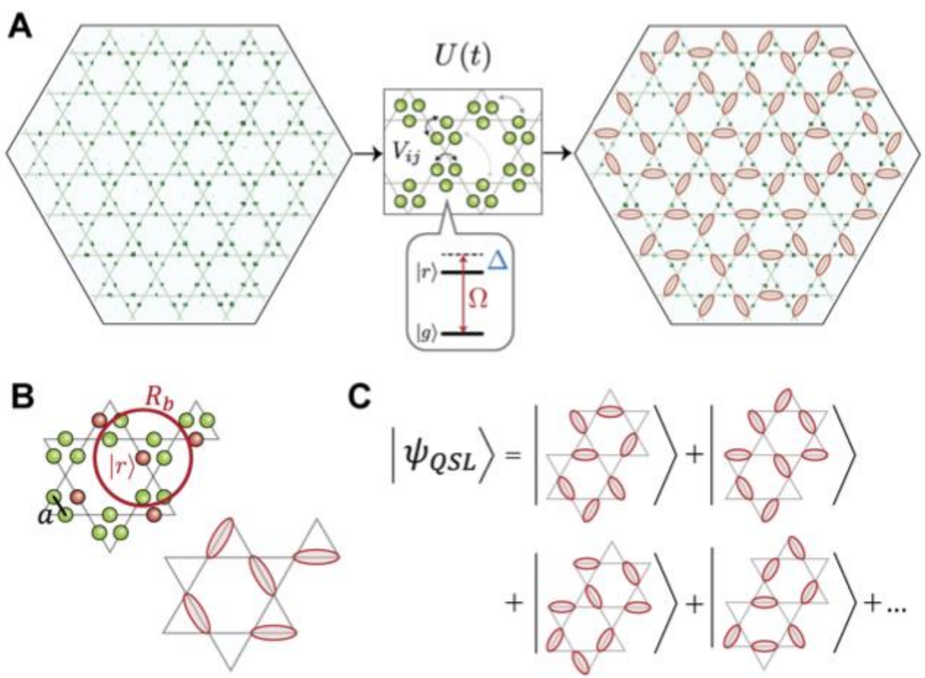
1D



Bernien, Nature 2017

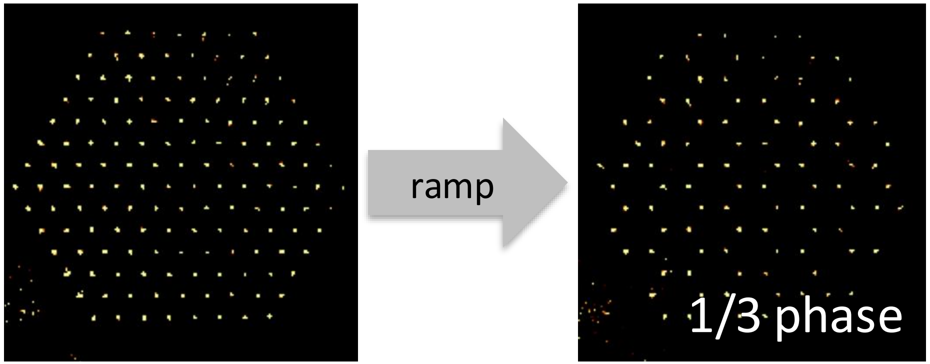
Ruby lattice: spin liquid?

Lukin, Science 2021



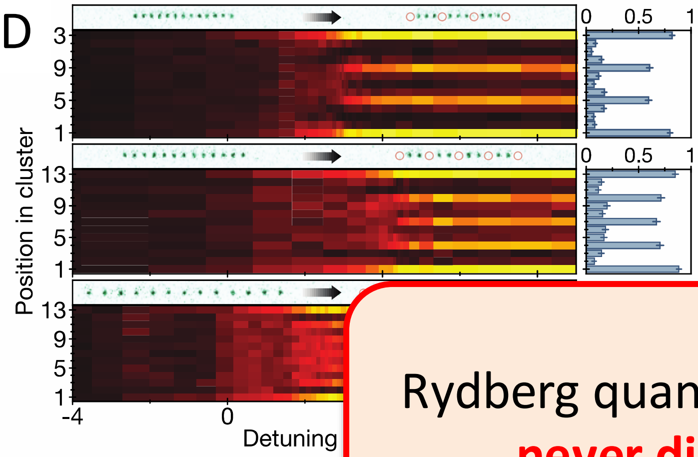
Triangle (frustration)

Scholl et *al.* Nature (2021)



Ising model in other geometries

1D



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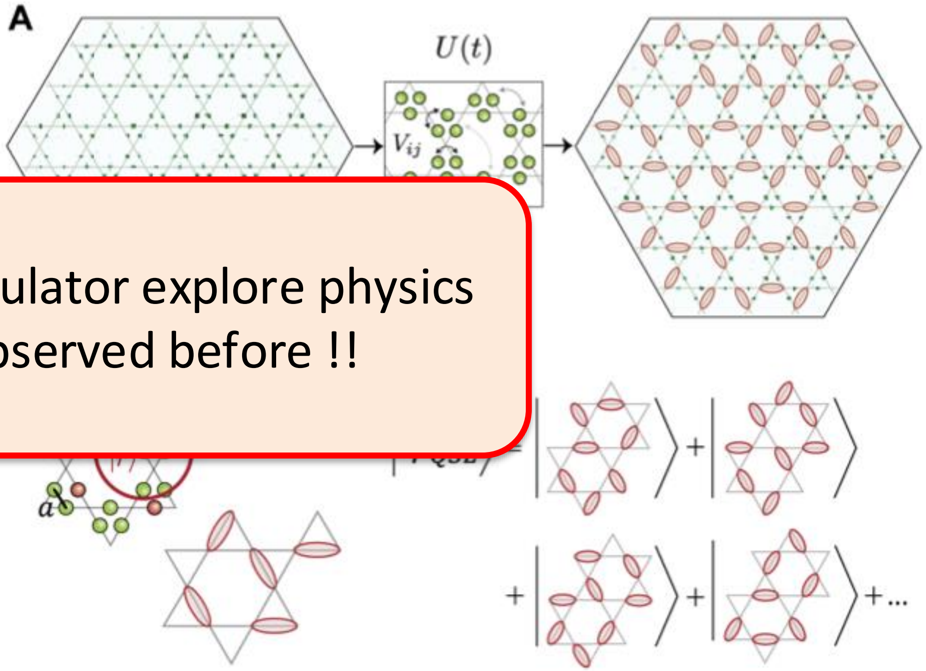
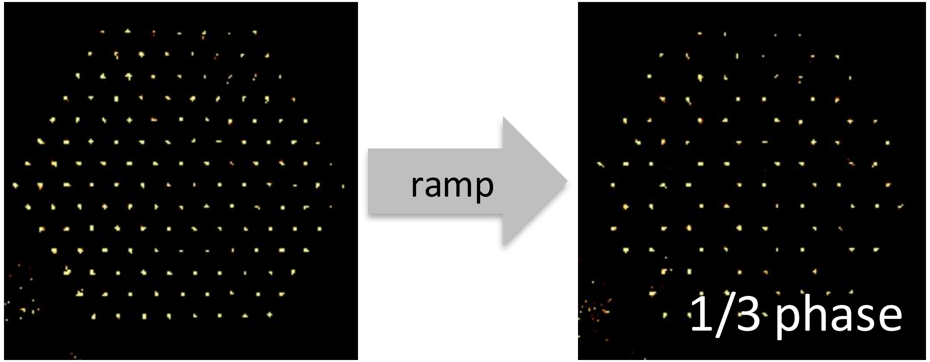
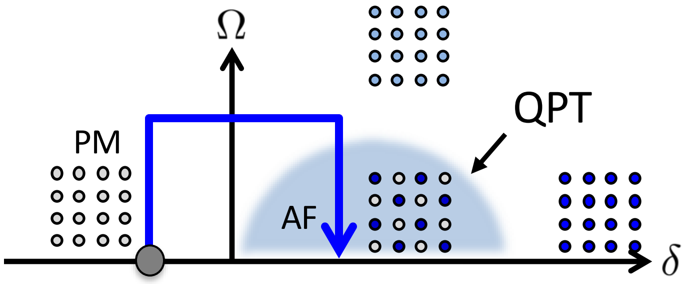


Rydberg quantum simulator explore physics **never directly** observed before !!

Triangle (frustration)



Use failure of adiabaticity to study quantum phase transition



Adiabaticity criteria:

$$H(t) = (1-\lambda(t))H_0 + \lambda(t)H_{\mathrm{MB}}$$

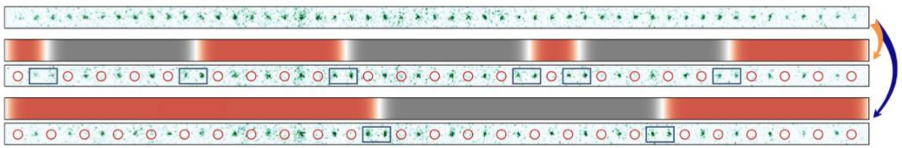
$$|\langle\psi_1(t)|\frac{dH}{dt}|\psi_0(t)\rangle| \ll \frac{\Delta E(t)^2}{\hbar}$$

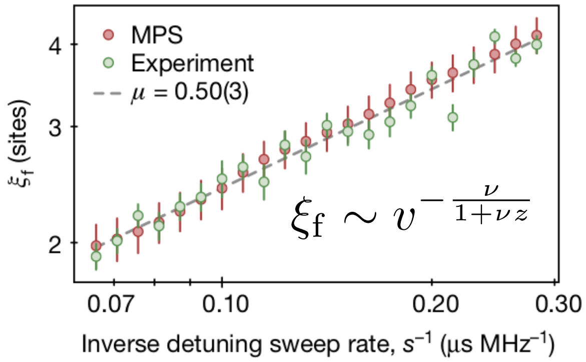
But…gaps close at the QPT!!

Sweeping too fast $\Rightarrow$ create defects

1D: Keesling, Nature (2019), 2D: arXiv.2012.12281

$R_b \sim a$ 51 atoms





Kibble-Zurek mechanism:

statistics of defects $\Rightarrow$ critical exponent

ν_{1D} = 0.50(3) (ν_{MF} = 1/3)

$\nu_{2D,square}$ = 0.62(4) (ν_{MF} = 1/2)

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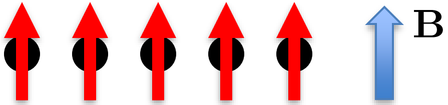
1. Studying the ground state of quantum magnets
    - Ising model in 2D
    - Dipolar XY model in 2D

2. Out-of-equilibrium dynamics: Quench dynamics in Ising model. Thermalization or not…?

3. Outlook: what we did not discuss… & beyond

Quench in Ising Hamiltonian with Rydberg simulator

Labuhn *et al.*, Nature 2016



$\Rightarrow ??$

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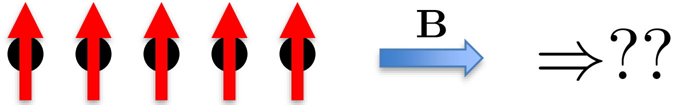
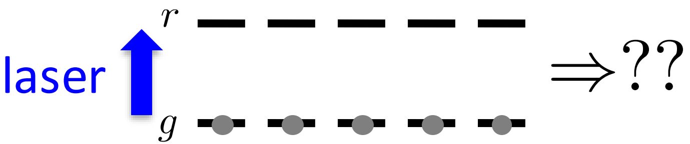
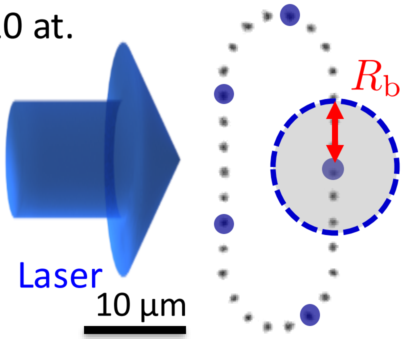




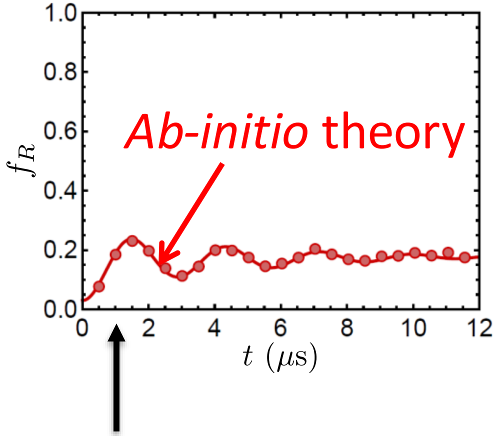
1D with periodic boundaries

20 at.



"Magnetization"

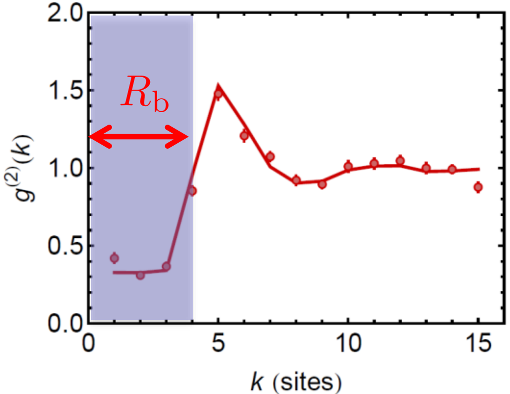
$$f_r = \frac{\langle N_r \rangle}{N}$$



Spin-spin correlation

$$\sim \langle n_j n_{j+k} \rangle$$

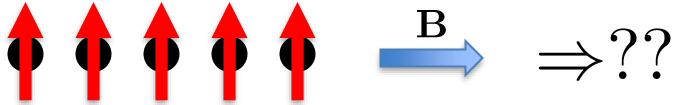
Schauss, Nature 2012
Lesanovsky, PRA 2012
Petrosyan, PRA 2013

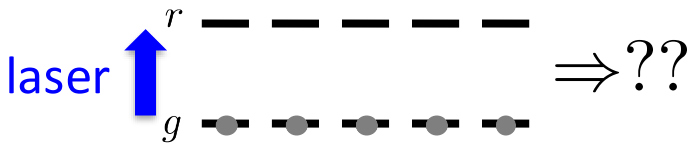


1 Rydberg atom
= hard sphere R_b

Quench in Ising Hamiltonian with Rydberg simulator

Labuhn *et al.*, Nature 2016





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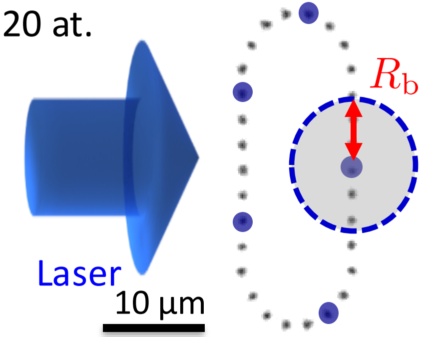

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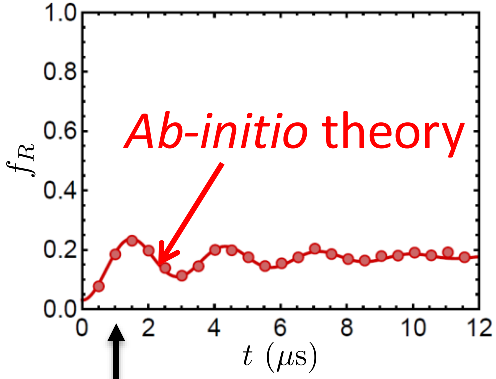

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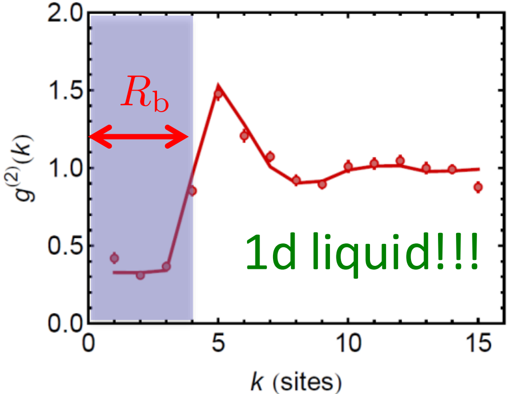

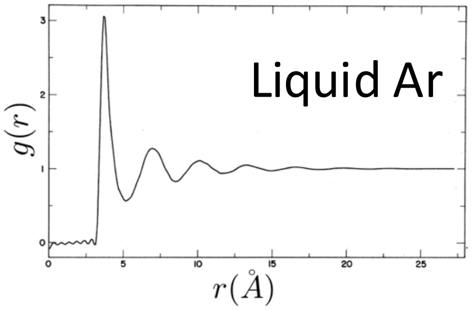

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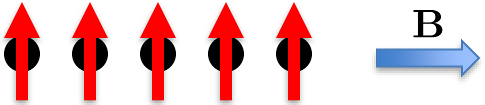

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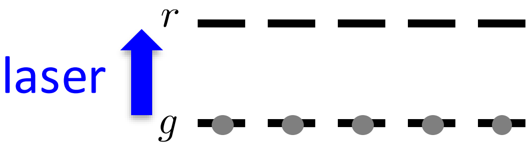

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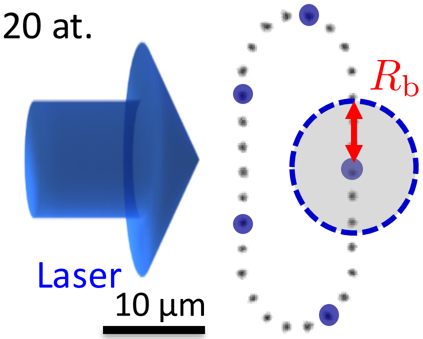

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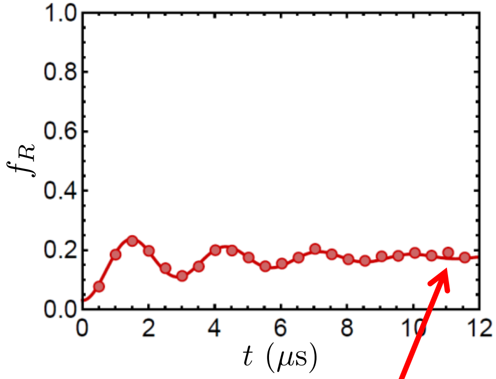

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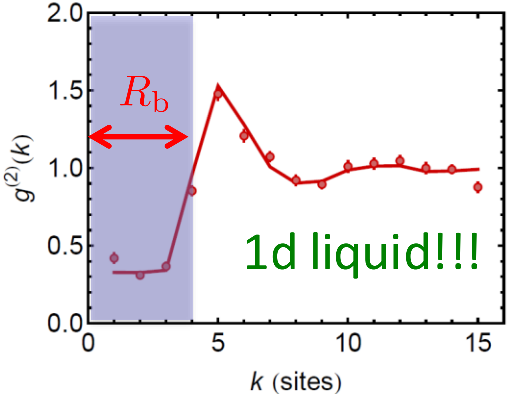
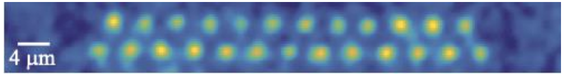


Thermalization??

Kim,… Ahn, PRL 2018



Schauss, Nature 2012
Lesanovsky, PRA 2012
Petrosyan, PRA 2013

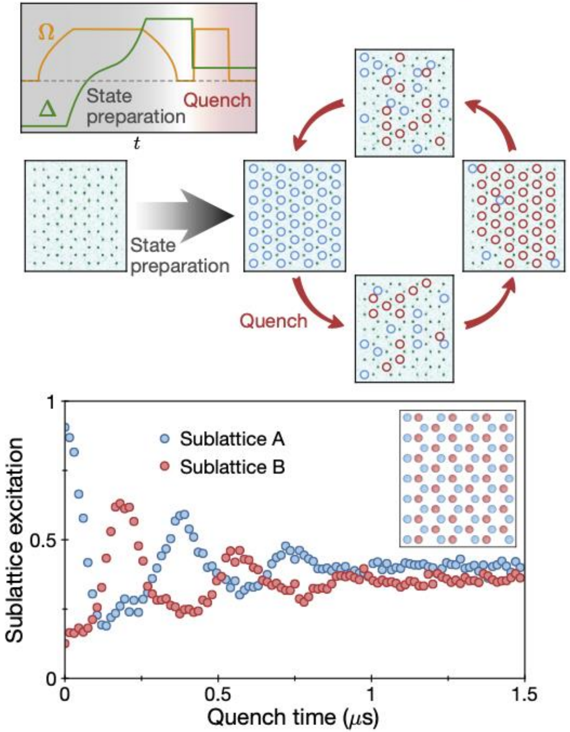
Thermalization of closed Many-Body systems

Question: do closed systems always reach equilibrium?

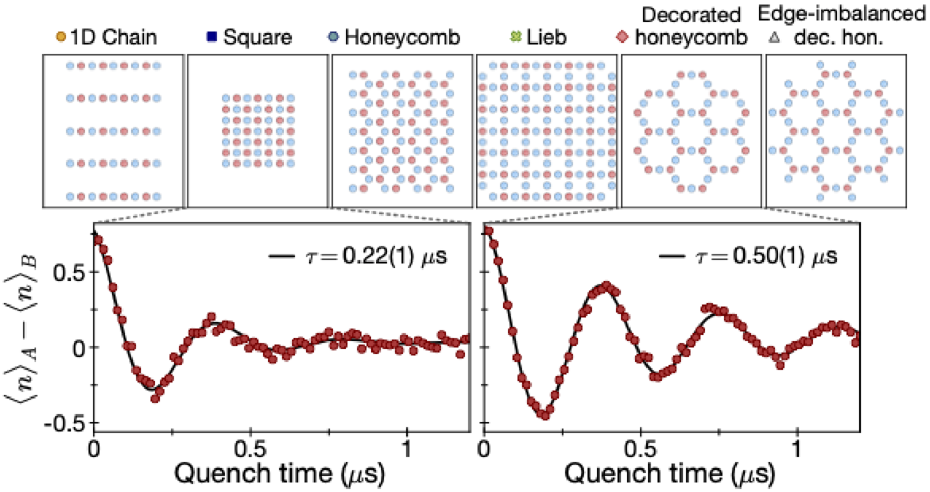
Answer: it depends… ETH, many-body localization and Quantum Scars

Quantum scarrs in 2D (**1D:** Lukin Nature 2019)

Bluvstein…Lukin, Science 2021



Scarrs depends on geometry



Blockade constraint breaks ergodicity

Outline – Lecture 3

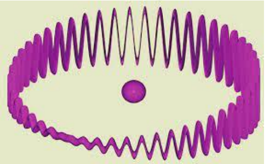
1. Studying the ground state of quantum magnets
    - Ising model in 2D
    - Dipolar XY model in 2D

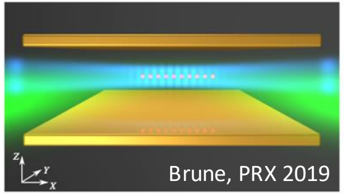
2. Out-of-equilibrium dynamics: Quench dynamics in Ising model. Thermalization or not…?

3. Outlook: what we did not discuss… & beyond

Outlook: what we did not discuss…

New developments: circular Rydberg states ⇒ lifetimes > 50 s…





Brune (Paris): arXiv:2407.04109, Nat. Phys. 2022
Covey (Urbana Champaign)
Thompson (Princeton)
Meinert & Pfau (Stuttgart)…

overhead:
Rydberg trapping

High precision quantum simulation: validation of the simulation

Article

Benchmarking highly entangled states on a 60-atom analogue quantum simulator

https://doi.org/10.1038/s41586-024-07173-x
Received: 18 August 2023

Adam L. Shaw[1,5]✉, Zhuo Chen[2,3,5], Joonhee Choi[1,4,5], Daniel K. Mark[2,5], Pascal Scholl[1], Ran Finkelstein[1], Andreas Elben[1], Soonwon Choi[2]✉ & Manuel Endres[1]✉

First attempt of **digital quantum simulation** (and hybrid analog-digital)

Variational simulation of the Lipkin-Meshkov-Glick model on a neutral atom quantum computer

R. Chinnarasu,[1] C. Poole,[1] L. Phuttitarn,[1] A. Noori,[1,2] T. M. Graham,[1] S. N. Coppersmith,[3,1] A. B. Balantekin,[1] and M. Saffman[1,4]

arXiv:2501.06097

Probing topological matter and fermion dynamics on a neutral-atom quantum computer

Simon J. Evered[1,*], Marcin Kalinowski[1,*], Alexandra A. Geim[1], Tom Manovitz[1], Dolev Bluvstein[1], Sophie H. Li[1], Nishad Maskara[1], Hengyun Zhou[1,2], Sepehr Ebadi[1,3], Muqing Xu[1], Joseph Campo[2], Madelyn Cain[1], Stefan Ostermann[1], Susanne F. Yelin[1], Subir Sachdev[1], Markus Greiner[1], Vladan Vuletić[4], and Mikhail D. Lukin[1,†]

arXiv:2501.18554

Digital quantum simulation: resource estimates…

Quantum Science and Techno. **7**, 045025 (2022)

Number of *perfect* gates to reproduce current *imperfect* analog simulation

Gate	Gate Count	Depth
CNOT	1.7×10^5	8.4×10^3
$R_Z(\theta)$	6.8×10^4	6.7×10^2

TABLE I. Gate count and depth estimates for digital quantum simulation of the Hubbard model with $J\tau = 2.7$, $M = 100$ and $tJ = 10$.

M sites

Gate	Gate Count	Depth
CNOT	1.6×10^3	5.5×10^2
$R_Z(\theta)$	2.1×10^4	3.5×10^2

TABLE III. Gate count and depth estimates for digital quantum simulation of the nearest neighbour Ising model with $J\tau = 2.6$, $M = 100$, $tJ = 10$.

Gate	Gate Count	Depth
CNOT	6.9×10^5	1.4×10^4
$R_Z(\theta)$	3.5×10^5	7.0×10^3

TABLE II. Gate count and depth estimates for digital quantum simulation of the long-range Ising model with $J\tau = 2.6$, $M = 100$ and $tJ = 10$.

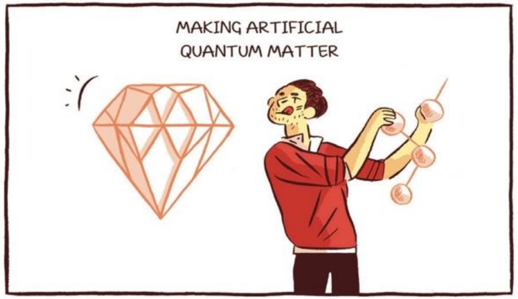
Numbers explode when analog errors $\rightarrow 0$

Many-body physics with synthetic systems or Quantum Simulation?

Experiments are imperfect… ⇒ **Not a pristine quantum simulation** of a model…

Study the noisy many-body system for itself…

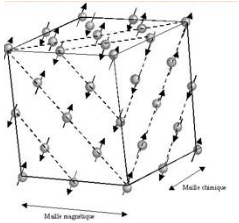
Héloïse Chochois, Labex PALM



Use "toy many-body systems" to

- Develop intuition ("simple to complex", noise…)
- Trigger new theoretical methods
- Generate "interesting" quantum states (squeezed…)

Understand better "real" systems?

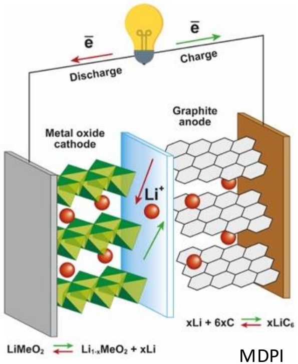


Strongly correlated matter

Develop applications?

Fraunhofer

Material



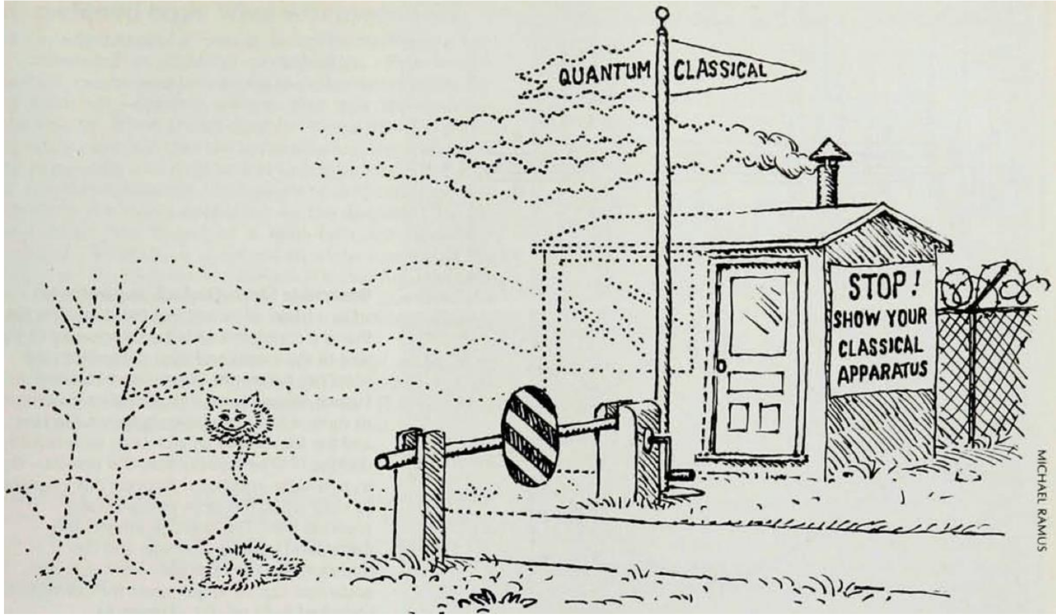
MDPI

Chemistry

Catalysis

…

How large can a quantum system be?



Zurek, Physics Today 1991