Townes soliton beyond mean field

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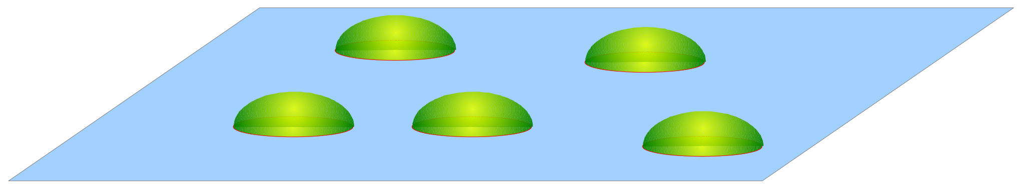
2D attractive bosons

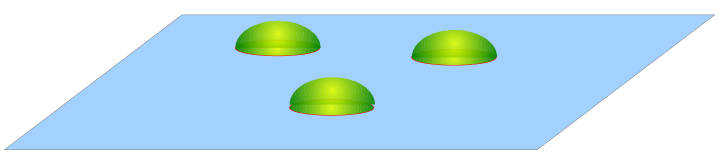
Bosons, dimension=2, zero-range attraction (*a* – unit of length)

single parameter: number of bosons *N*



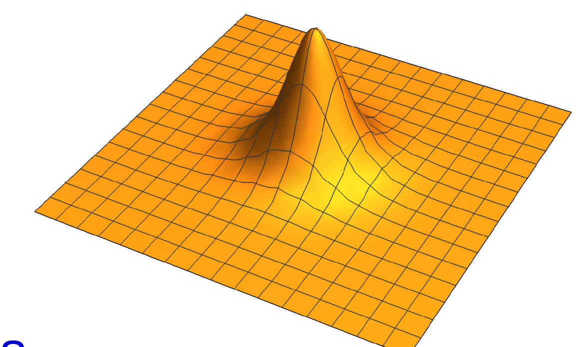
N=3,4,...

Few-body methods,
Efimov ?...



N >> 1

Mean field (classical) soliton,
scaling invariance, optical
and ultracold expts.,...



? ?

N=10-20

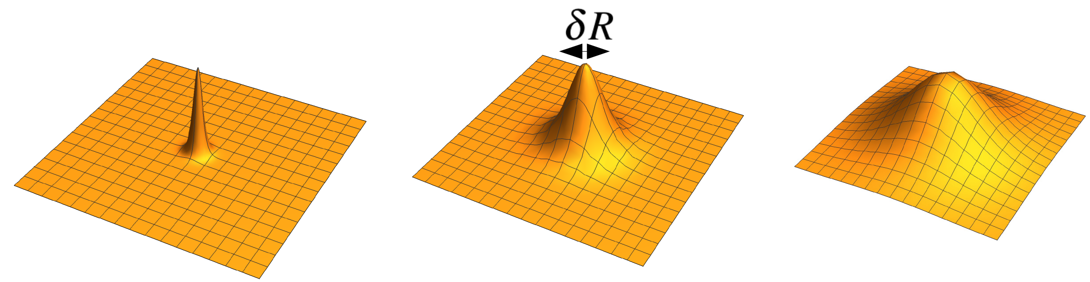
1/*N* expansion, inhomogeneous
Bogoliubov theory, excitations, etc.
possibilities for ultracold expts

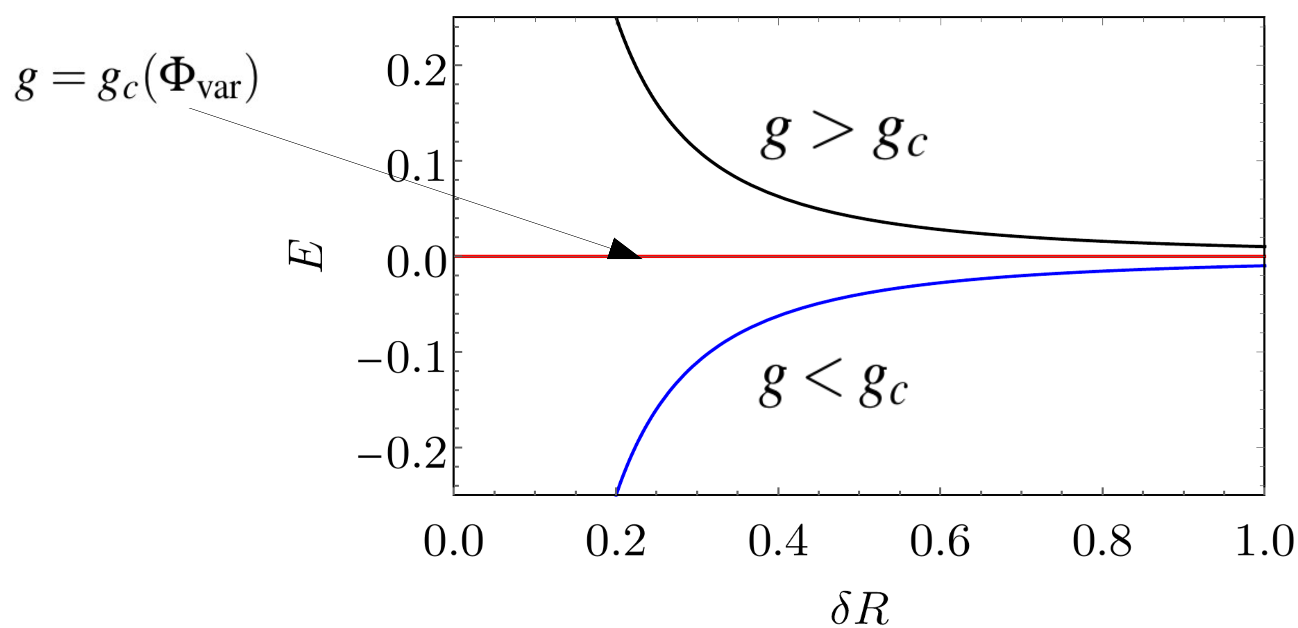
MF scaling invariance

Choose arbitrary profile $\Psi(\boldsymbol{\rho}) = \Phi_{\mathrm{var}}(\boldsymbol{\rho})$ with $N = \int |\Phi_{\mathrm{var}}(\boldsymbol{\rho})|^2 d^2\rho$

$E_{\mathrm{MF}}(\Psi,\Psi^*) = (1/2)\int d^2\rho[|\nabla_{\rho}\Psi(\boldsymbol{\rho},t)|^2 + g|\Psi(\boldsymbol{\rho},t)|^4]$ ⟹ $E_0 = E_{\mathrm{MF}}[\Phi_{\mathrm{var}}(\boldsymbol{\rho})]$

Norm-conserving rescaling $\Phi_{\mathrm{var}}(\boldsymbol{\rho}) \to \Phi_{\mathrm{var}}(\boldsymbol{\rho}/\delta R)/\delta R$ ⟹ $E_0 \to E_0/\delta R^2$





Note that we variationally force the shape to be conserved !

MF scaling invariance

Gross-Pitaevskii equation : $i\partial_t \Psi(\rho,t) = -(1/2)\nabla_\rho^2 \Psi(\rho,t) + g|\Psi(\rho,t)|^2 \Psi(\rho,t)$

Initial condition : $\Psi(\rho, t=0) = \Phi_{\mathrm{var}}(\rho)$

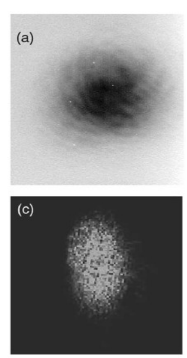
$$\Downarrow$$

$$\Psi(\rho,t)$$

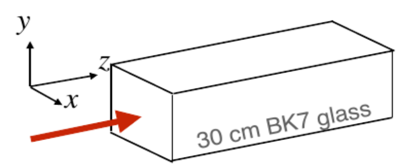
A few facts about this dynamics : $N \dfrac{d^2}{dt^2}\langle \rho^2 \rangle = 4E$ [Vlasov, Petrishchev,Talanov'71, Pitaevskii'96]

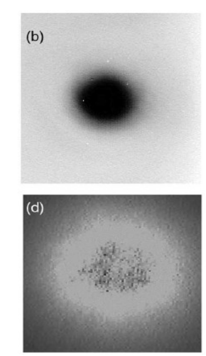
$$\frac{1}{N}\int |\Psi(\rho,t)|^2 \rho^2 d^2\rho \qquad E_{\mathrm{MF}}[\Psi(\rho,0),\Psi^*(\rho,0)]$$

Optical wave collapse
[Moll, Gaeta, and Fibich'2003]



Their z is our t



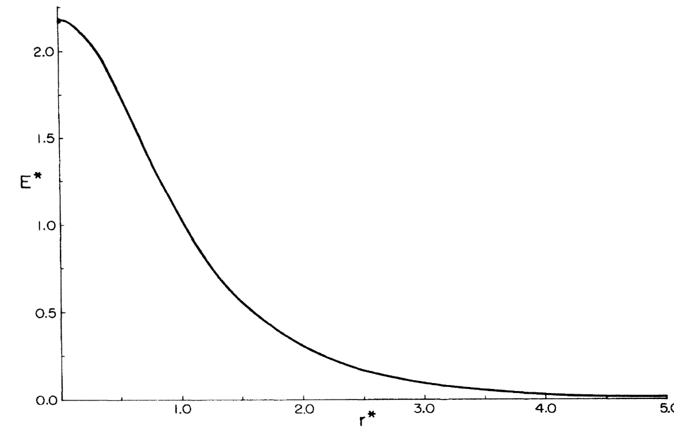


Input : randomly distorted or elliptic ⇒ Output : clean isotropic ``Townsian''

Townes soliton [Chiao, Garmire, Townes'1964]

Townes profile [Chiao, Garmire, Townes'1964]

$$\frac{d^2E^*(r^*)}{dr^{*2}}+\frac{1}{r^*}\frac{dE^*(r^*)}{dr^*}-E^*(r^*)+E^{*3}(r^*)=0$$



$$-f''(r)-f'(r)/r-f(r)^3=-f(r)$$

$$C:=\int_0^\infty drrf^2(r)=\int_0^\infty drr[f'(r)]^2=\frac{1}{2}\int_0^\infty drrf^4(r)=1.862$$

$$M_2:=\int_0^\infty drr^3f^2(r)=2.211$$

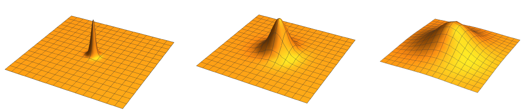
The stationary state (R is arbitrary but time-independent)

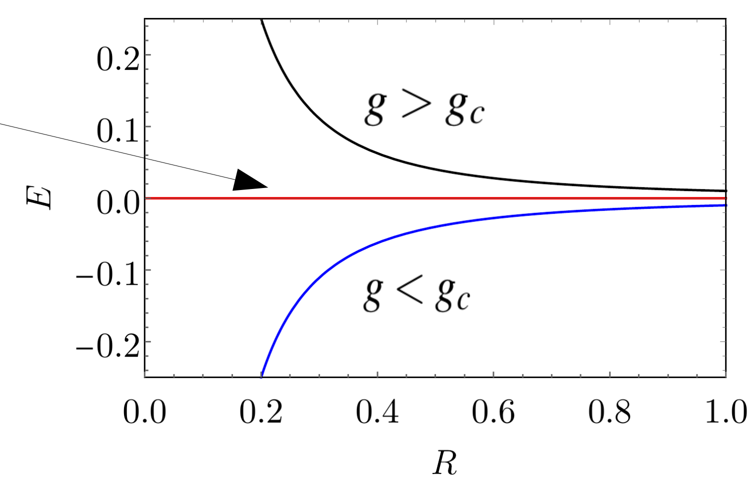
$$\Psi_R(\rho,t)=e^{it/(2R^2)}\Psi_R(\rho)=e^{it/(2R^2)}\sqrt{N/(2\pi C)}f(\rho/R)/R$$

solves GPE

$$i\partial_t\Psi(\rho,t)=-(1/2)\nabla_\rho^2\Psi(\rho,t)+g|\Psi(\rho,t)|^2\Psi(\rho,t)$$

Under the condition $g=g_c=-\pi C/N=-5.85/N$





Dynamics of Townes soliton

Consider the anzats $\Psi(\rho,t)=\sqrt{N/(2\pi C)}e^{i\theta(\rho,t)}f[\rho/R(t)]/R(t)$

Determine $\theta(\rho,t)$ and $R(t)$ by minimizing the action $S=\int dt d^2\rho \mathscr{L}(\Psi,\Psi^*)$ with Lagrangian density

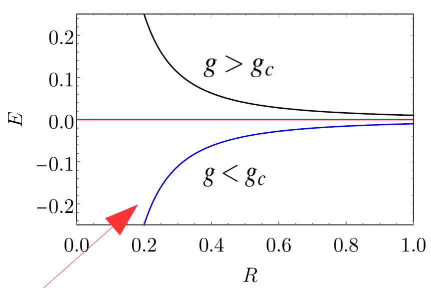
$$\mathscr{L}(\Psi,\Psi^*)=\mathrm{Re}[i\Psi^*(\rho,t)\partial_t\Psi(\rho,t)]-|\nabla_\rho\Psi(\rho,t)|^2/2-g|\Psi(\rho,t)|^4/2$$

Minimization wrt $\theta(\rho,t)$

Continuity equation : $\partial_t|\Psi|^2+\nabla_\rho(|\Psi|^2\nabla_\rho\theta)=0$

$$\theta(\rho,t)=[\dot{R}(t)/R(t)]\rho^2/2-\mu' t$$

$$S=\mathrm{const}+\int dt\,\{NM_2\dot{R}^2(t)/(2C)-(g-g_c)N^2/[2\pi CR^2(t)]\}$$



Classical motion of a particle of mass $m_{\mathrm{eff}}=NM_2/C$ in external potential $(g-g_c)N^2/(2\pi CR^2)$

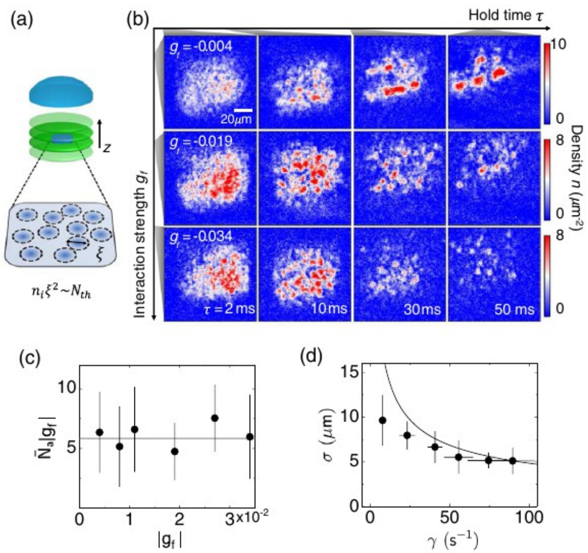
with total energy $E=m_{\mathrm{eff}}\dot{R}^2/2+(g-g_c)N^2/(2\pi CR^2)=\mathrm{const}$ ...check that $d^2R^2/dt^2=4E/m_{\mathrm{eff}}$

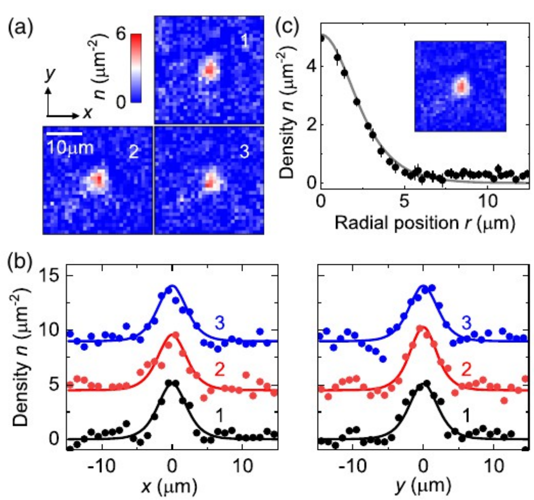
$$R(t)=\sqrt{(g-g_c)N/(\pi A)+A(t-t_0)^2/M_2}$$

where A and t_0 are determined by $R(0)$ and $\dot{R}(0)$

Purdue Cs exp

[Chen&Hung'20]





prepare repulsive
2D Cs condensate

$$g = \sqrt{8\pi} a_{3D}/l_\perp > 0$$

quench to

$$g < 0$$

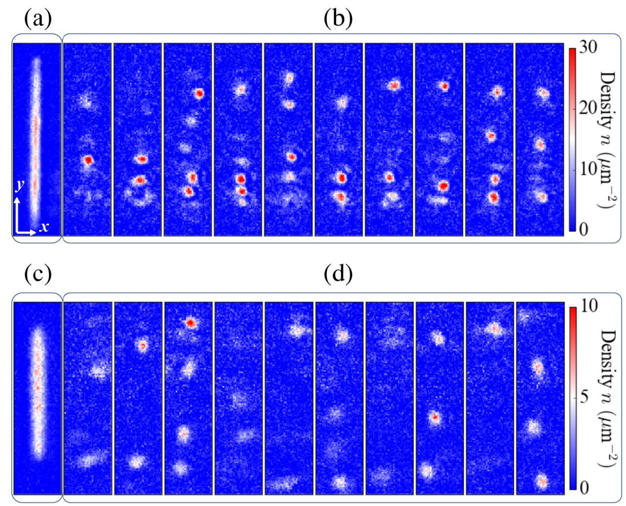
Depending on g and init density → Modulational instability gives solitons with more or less expected R and N

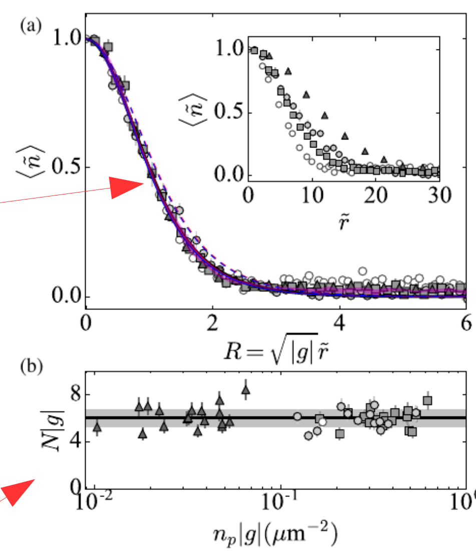
« self-cleaned »
|Townes profile|^2

$$|\Psi_R(\rho)|^2 = \frac{N}{2\pi C R^2} f^2(\rho/R)$$

$$g = g_c = -\pi C/N = -5.85/N$$

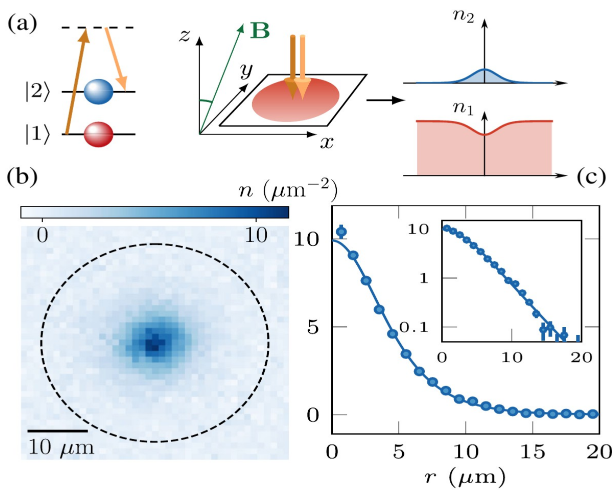
[Chen&Hung'21]





Collège de France Rb exp

[Bakkali-Hassani et al.'21, see also Bakkali-Hassani & Dalibard, Varenna'2022]



3D scattering lengths

- a_{11}=100.9 a_0
- a_{12}=100.4 a_0
- a_{22}= 94.9 a_0

2D coupling strengths

- g_{11}=0.160
- g_{12}=0.159
- g_{22}=0.151

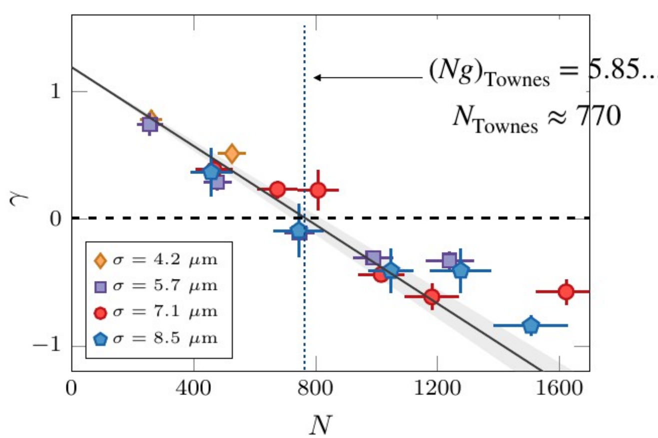
$$g_{\rm eff} = g_{22} - \frac{g_{12}^2}{g_{11}} \approx -0.0076$$

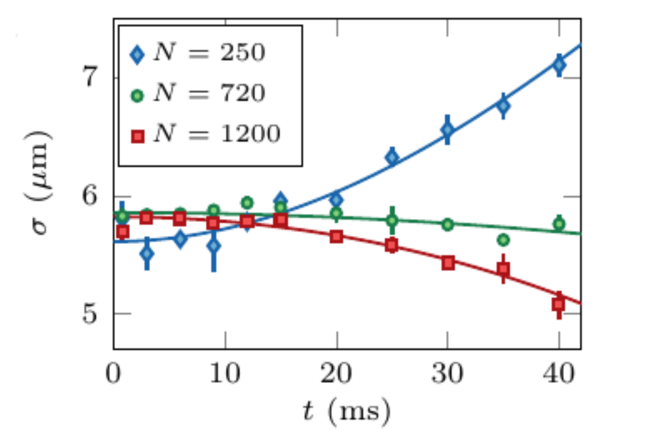
$$N_{\rm Townes}\,|g| = 5.85$$

$$N_{\rm Townes} \approx 770$$

Expansion factor

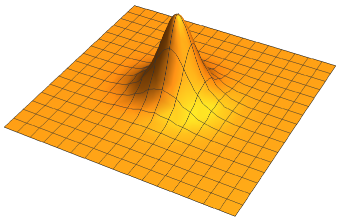
$$\propto \frac{\rm d}{{\rm d}t}\langle r^2\rangle$$





Summary (up to this point)

Townes soliton :

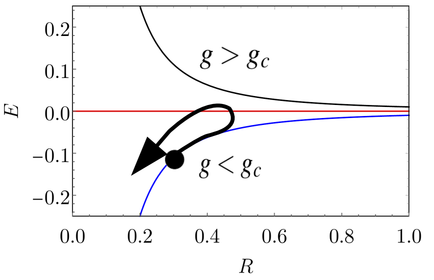


- universal profile
- Self-similar dynamics (if proper phase factor) parametrized by collective var. $R(t)$
- No periodic dynamics, no collective excitations (self-evaporation or self-cleaning)
- Vanishing breating mode frequency
- If stationary, $E = Kin + Int = 0$ ⇨ automatic cancellation of the MF energy

⇩

- higher-order terms become important ! Beyond-mean-field terms may be too weak in current expts. Need lower N ...
- $1/R^2$ scaling of the energy... relation to Efimov ?

$$g = g_c = -5.85/N$$



Few-body results

Few-body studies

3 bosons Bruch&Tjon'1979: no Thomas collapse, no Efimov effect, two trimer states

Good for lifetime !

$$B_3 = 16.522688(1)\, B_2 \qquad B_3^{ex} = 1.2704091(1)\, B_2$$

[Bruch&Tjon'79; Adhikari et al.'88; NielsenFedorovJensen'99; Hammer&Son'04; Kartavtsev&Malykh'06...]

4 bosons : $B_4 = 197.3(1)\, B_2 \qquad B_4^{ex} = 25.5(1)\, B_2$

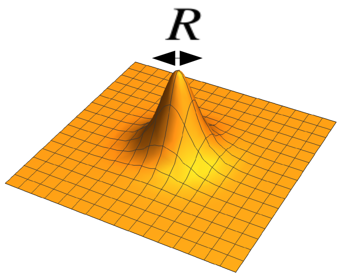
[Platter,Hammer&Meissner'04; Brodsky et al'06]

$N \to \infty$ Hammer&Son'04;

$$B_N / B_{N-1} \underset{N\to\infty}{\to} e^{4/C} = 8.567\ldots \quad \Leftrightarrow \quad B_N \propto B_2 e^{4N/C}$$

$$R_N / R_{N-1} \underset{N\to\infty}{\to} e^{-2/C} = 0.3417\ldots \quad \Leftrightarrow \quad R_N \propto a_{2D} e^{-2N/C}$$

Hammer&Son'2004: $g \to g(R) = \dfrac{4\pi}{\ln(|B_2|R^2 \times \mathrm{const})}$



Renormalized coupling constant

$$\Psi_R(\rho) = \sqrt{N/(2\pi C)}\, f(\rho/R)/R$$

$$E_{\mathrm{MF}}(R) = (1/2)\int d^2\rho\, [|\nabla_\rho \Psi_R(\rho)|^2 + g(R)|\Psi_R(\rho)|^4]$$

$E_{\mathrm{MF}}(R) = [g(R) - g_c]N^2/(2\pi C R^2)$ has minimim at R_{min} : $g(R_{\mathrm{min}}) = g_c + O(N^{-2})$

$$g_c = -\pi C/N = -5.85/N$$

$$R_{min} = R_N \propto |B_2|^{-1/2} e^{-2\mathrm{N}/C}$$

$$B_N \propto B_2 e^{4\mathrm{N}/C}$$

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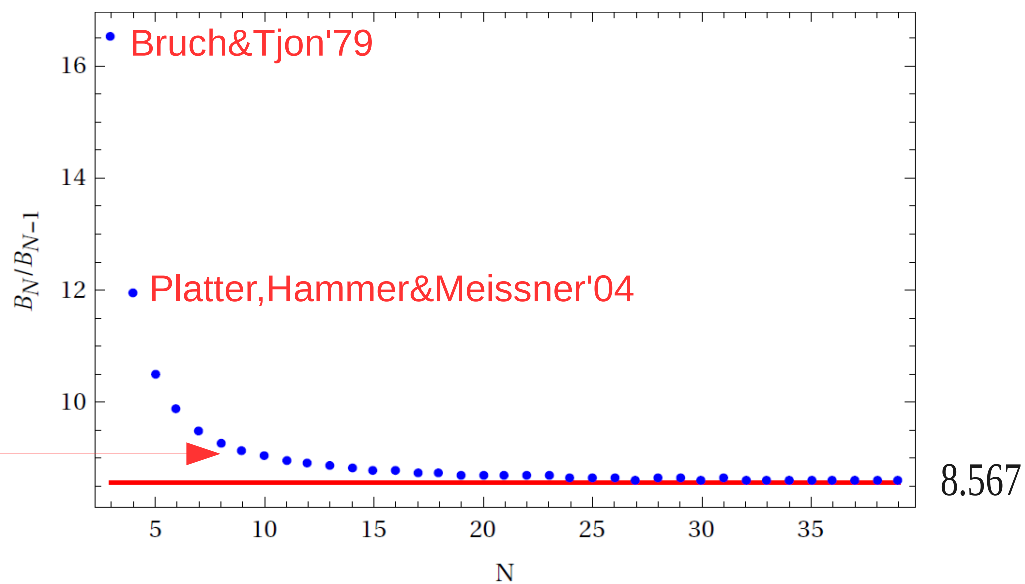
$N \leq 7$ finite-range calculations [Blume'05] (inconclusive)

$N \leq 10$ lattice EFT [Lee'06] $B_N / B_{N-1} \to 8.3(6)$

$N \leq 26$ STM-DMC approach [Bazak&DSP'18]



For N>4 no information about excited states:(

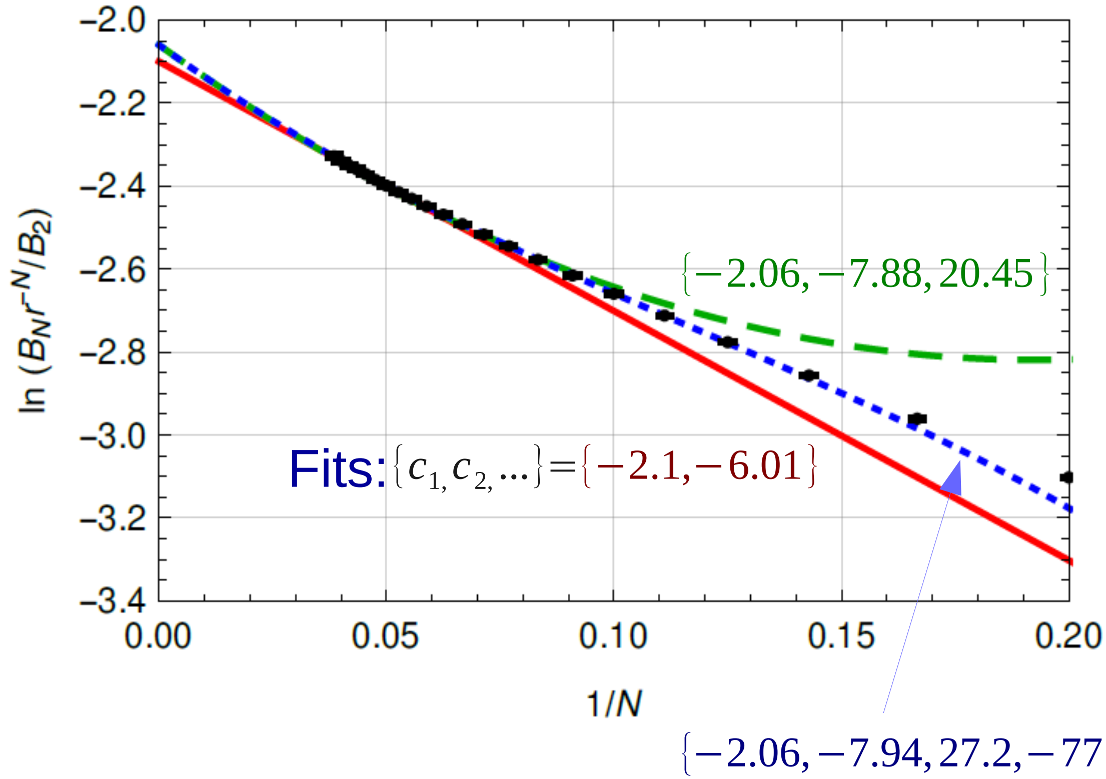
STM-DMC [Bazak&DSP'18]

N	B_N/B_2	N	B_N/B_2
3	$1.65225(2)\times10^{1}$	15	$8.135(2)\times10^{12}$
4	$1.9720(1)\times10^{2}$	16	$7.129(4)\times10^{13}$
5	$2.0745(1)\times10^{3}$	17	$6.232(2)\times10^{14}$
6	$2.0471(1)\times10^{4}$	18	$5.438(3)\times10^{15}$
7	$1.9462(1)\times10^{5}$	19	$4.734(2)\times10^{16}$
8	$1.8070(1)\times10^{6}$	20	$4.119(2)\times10^{17}$
9	$1.6508(4)\times10^{7}$	21	$3.577(2)\times10^{18}$
10	$1.4905(2)\times10^{8}$	22	$3.108(4)\times10^{19}$
11	$1.3345(2)\times10^{9}$	23	$2.694(5)\times10^{20}$
12	$1.1873(4)\times10^{10}$	24	$2.332(4)\times10^{21}$
13	$1.0508(3)\times10^{11}$	25	$2.018(4)\times10^{22}$
14	$9.2596(9)\times10^{11}$	26	$1.748(4)\times10^{23}$

Conjecture...wrong !

$$B_N = B_2 e^{\ln(8.567) N + c_1 + c_2/N + \ldots}$$

$$\ln\left(B_N 8.567^{-N}/B_2\right) = c_1 + c_2/N + \ldots$$

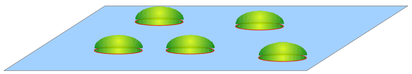


Updated conjecture [Petrov'2024] : $B_N = B_2 e^{4N/C + c_1 - 2\sqrt{2}/\sqrt{M_2 N} + \ldots}$

MF LO BMF « free bonus »

Bogoliubov analysis

Renormalization



$$\hat{H} = \frac{1}{2}\int d^2\rho \left(-\hat{\Psi}^\dagger_\rho \nabla^2_\rho \hat{\Psi}_\rho + g\hat{\Psi}^\dagger_\rho \hat{\Psi}^\dagger_\rho \hat{\Psi}_\rho \hat{\Psi}_\rho\right)$$ - needs regularization

square lattice

$$\hat{H} = \sum_{k_x,k_y\in[-\pi/h,\pi/h]} \varepsilon^{(0)}_{\mathbf{k}} \hat{a}^\dagger_{\mathbf{k}} \hat{a}_{\mathbf{k}} + \frac{g}{2}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}} \hat{a}^\dagger_{\mathbf{k}_1+\mathbf{q}} \hat{a}^\dagger_{\mathbf{k}_2-\mathbf{q}} \hat{a}_{\mathbf{k}_1} \hat{a}_{\mathbf{k}_2}$$

$$\varepsilon^{(0)}_{\mathbf{k}} = [2-\cos(k_x h)-\cos(k_y h)]/h^2 \approx k^2/2$$

cutoff

$$\hat{H} = \sum_{\mathbf{k}} \frac{k^2}{2} \hat{a}^\dagger_{\mathbf{k}} \hat{a}_{\mathbf{k}} + \frac{1}{2}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}} \hat{a}^\dagger_{\mathbf{k}_1+\mathbf{q}} \hat{a}^\dagger_{\mathbf{k}_2-\mathbf{q}} U(\mathbf{q}) \hat{a}_{\mathbf{k}_1} \hat{a}_{\mathbf{k}_2}$$

$$U(\mathbf{q}) = \begin{cases} g, \text{ for } |\mathbf{q}| < \kappa \\ 0, \text{ for } |\mathbf{q}| > \kappa \end{cases}$$

Both models should correspond to $\psi(\rho) \underset{\rho\to 0}{\to} C \ln\left(\frac{\rho}{a_{2D}}\right)$ or, equivalently, reproduce $B_2 = -4e^{-2\gamma}/a^2_{2D}$

$$\frac{1}{g} \approx \frac{\ln(|B_2|h^2/32)}{4\pi}$$

$$\frac{1}{g} \approx \frac{\ln(|B_2|/\kappa^2)}{4\pi}$$

Both valid when $|g| \ll 1$ i.e., for large (repulsion) or small (attraction) $|B_2|/\kappa^2$

Trade a single (universal) interaction parameter B_2<0 for a pair (g,κ), but can now do perturbation theory in $|g|$<<1!

Bogoliubov for homogeneous gas

$$\hat{H}=\sum_{\mathbf{k}}\frac{k^2}{2}\hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{k}}+\frac{1}{2}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}}\hat{a}^\dagger_{\mathbf{k}_1+\mathbf{q}}\hat{a}^\dagger_{\mathbf{k}_2-\mathbf{q}}U(\mathbf{q})\hat{a}_{\mathbf{k}_1}\hat{a}_{\mathbf{k}_2}$$

$\hat{a}_0,\hat{a}_0^\dagger\to a_0$ - assume real

$$\hat{H}=H_0+\hat{H}_2+\hat{H}_3+\hat{H}_4$$

$$H_0=\frac{1}{2}ga_0^4 \qquad a_0^2=n-\sum_{\mathbf{k}}{}'\hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{k}}$$

$$\hat{H}_2=\sum_{\mathbf{k}}{}'\left[\frac{k^2}{2}+ga_0^2\right]\hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{k}}+\frac{1}{2}\sum_{\mathbf{k}}{}'U(\mathbf{k})a_0^2(\hat{a}^\dagger_{\boldsymbol{k}}\hat{a}^\dagger_{-\boldsymbol{k}}+\hat{a}_{\boldsymbol{k}}\hat{a}_{-\boldsymbol{k}}+2\hat{a}^\dagger_{\boldsymbol{k}}\hat{a}_{\boldsymbol{k}})$$

$$\hat{H}_3=\sum_{\boldsymbol{k}_1,\boldsymbol{k}_2}{}'U(\mathbf{k}_1)a_0\hat{a}^\dagger_{\boldsymbol{k}_1+\boldsymbol{k}_2}(\hat{a}_{\boldsymbol{k}_1}+\hat{a}^\dagger_{-\boldsymbol{k}_1})\hat{a}_{\boldsymbol{k}_2}$$

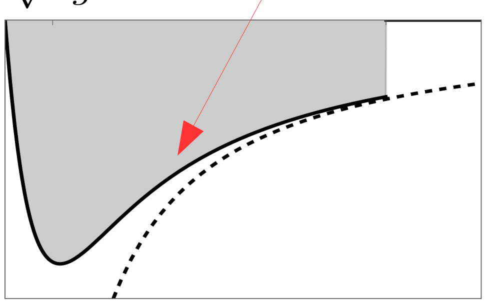
$$\hat{H}_4=\frac{1}{2}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}}{}'\hat{a}^\dagger_{\mathbf{k}_1+\mathbf{q}}\hat{a}^\dagger_{\mathbf{k}_2-\mathbf{q}}U(\mathbf{q})\hat{a}_{\mathbf{k}_1}\hat{a}_{\mathbf{k}_2}$$

Diagonalization

$$\frac{E}{\text{Surface}}=\frac{gn^2}{2}+\frac{1}{2}\sum_k^{\kappa}[\sqrt{k^4/4+gnk^2}-k^2/2-gn]\approx\frac{gn^2}{2}+\frac{g^2n^2}{8\pi}\ln\frac{gn\sqrt{e}}{\kappa^2}$$

Higher orders [Mora&Castin'2009]

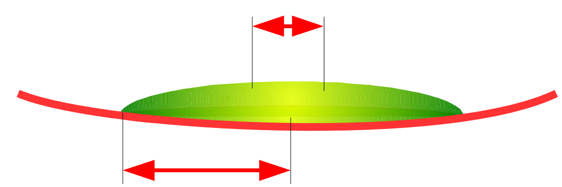
$$+\sim n^2g^3$$



$$\propto\int\frac{dk}{k}$$

BMF correction is local !

Healing length $\xi=1/\sqrt{gn}$



$R_{\rm TF}=\sqrt{2gn}/\omega$

Local-density approx when $\omega\ll gn$

Independence of the cutoff

$$\frac{E}{\text{Surface}}=\frac{gn^2}{2}+\frac{1}{2}\sum_k^{\kappa}[\sqrt{k^4/4+gnk^2}-k^2/2-gn]\approx\frac{gn^2}{2}+\frac{g^2n^2}{8\pi}\ln\frac{gn\sqrt{e}}{\kappa^2}$$

$$g\approx 4\pi/\ln(|B_2|/\kappa^2) \Rightarrow dg/d\kappa=-g^2/(2\pi\kappa) \Rightarrow dE/d\kappa\propto g^3$$

Beyond Bogoliubov

Cutoff independence on the Bogoliubov level
(g just has to remain small)

$$\kappa^2=gn\sqrt{e}$$

Implicit eq. : $g=\frac{4\pi}{\ln\frac{|B_2|}{gn\sqrt{e}}}$ $\Rightarrow$ $g(n,B_2)$

$$\frac{E}{\text{Surface}}=\frac{g(n,B_2)n^2}{2}$$

cf. [Schick'1971]:

$$g=\frac{4\pi}{\ln\frac{1}{a_{2D}^2 n}}$$

Leading BMF result [Popov'1972]
LHY (or Bogoliubov) accuracy

Jargon : « density-dependent interaction» $\Rightarrow$ LDA +GPE ...

Inhomogeneous Bogoliubov [Petrov'2024]

$$\hat{H} = \frac{1}{2}\int d^2\rho(-\hat{\Psi}^\dagger_\rho \nabla^2_\rho \hat{\Psi}_\rho + g\hat{\Psi}^\dagger_\rho\hat{\Psi}^\dagger_\rho\hat{\Psi}_\rho\hat{\Psi}_\rho)$$

Cannot do LDA !!!

$$1/\sqrt{|g|n} \sim R$$

$$\Downarrow \quad \hat{\Psi}_\rho = \Psi_R(\boldsymbol{\rho}) + \delta\hat{\Psi}_\rho$$

$$E_{\rm MF} = (g-g_c)N^2/(2\pi C R^2) \quad + \quad \hat{H}_2 = \frac{1}{2}\int d^2\rho \begin{pmatrix}\delta\hat{\Psi}^\dagger_\rho & \delta\hat{\Psi}_\rho\end{pmatrix}\begin{pmatrix}\hat{A} & \hat{B}\\ \hat{B} & \hat{A}\end{pmatrix}\begin{pmatrix}\delta\hat{\Psi}_\rho\\ \delta\hat{\Psi}^\dagger_\rho\end{pmatrix} - \mathrm{Tr}(\hat{A})/2$$

$$\hat{A} = -\nabla^2_\rho/2 - \mu + 2g_c\Psi^2_R(\boldsymbol{\rho}),\ \hat{B} = g_c\Psi^2_R(\boldsymbol{\rho})$$

Almost complete cancellation Kin + Int

$$|g-g_c| \sim 1/N^2 \ll |g| \approx \pi C/N$$

Blaizot & Ripka
"Quantum Theory of Finite Systems"

Inhomogeneous Bogoliubov [Petrov'2024]

$$\hat{H} = \frac{1}{2}\int d^2\rho(-\hat{\Psi}^\dagger_\rho \nabla^2_\rho \hat{\Psi}_\rho + g\hat{\Psi}^\dagger_\rho \hat{\Psi}^\dagger_\rho \hat{\Psi}_\rho \hat{\Psi}_\rho)$$

Cannot do LDA !!!

$$1/\sqrt{|g|n} \sim R$$

$$\hat{\Psi}_\rho = \Psi_R(\rho) + \delta\hat{\Psi}_\rho$$

$$E_{\mathrm{MF}} = (g-g_c)N^2/(2\pi C R^2) \quad + \quad \hat{H}_2 = \frac{1}{2}\int d^2\rho \begin{pmatrix}\delta\hat{\Psi}^\dagger_\rho & \delta\hat{\Psi}_\rho\end{pmatrix} \begin{pmatrix}\hat{A} & \hat{B} \\ \hat{B} & \hat{A}\end{pmatrix} \begin{pmatrix}\delta\hat{\Psi}_\rho \\ \delta\hat{\Psi}^\dagger_\rho\end{pmatrix} - \mathrm{Tr}(\hat{A})/2$$

$$\hat{A} = -\nabla^2_\rho/2 - \mu + 2g_c\Psi^2_R(\rho),\ \hat{B} = g_c\Psi^2_R(\rho)$$

Almost complete cancellation Kin + Int

$$|g-g_c| \sim 1/N^2 \ll |g| \approx \pi C/N$$

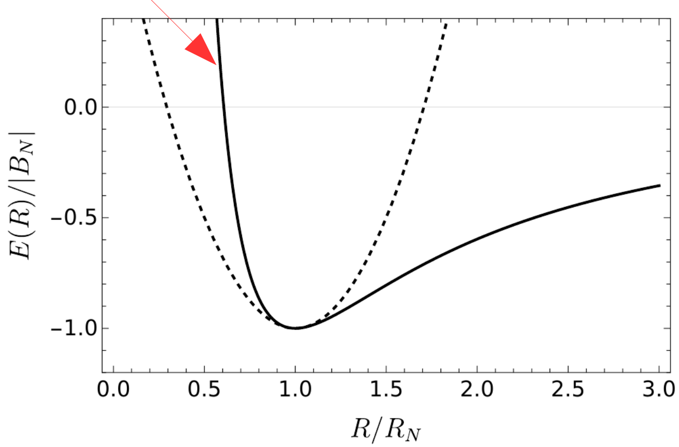
Blaizot & Ripka
"Quantum Theory of Finite Systems"

$$E(R) = \frac{(g-g_c)N^2}{2\pi C R^2} - \frac{C}{4R^2}\ln(\xi R/h) = -2|B_N|\frac{R_N^2}{R^2}\ln\frac{R\sqrt{e}}{R_N}$$

$$R_N = \sqrt{\frac{C}{8|B_2|}}e^{-2N/C-c_1/2}$$

$$B_N = B_2 e^{4N/C+c_1}$$

$$c_1 = -1.91(1)$$

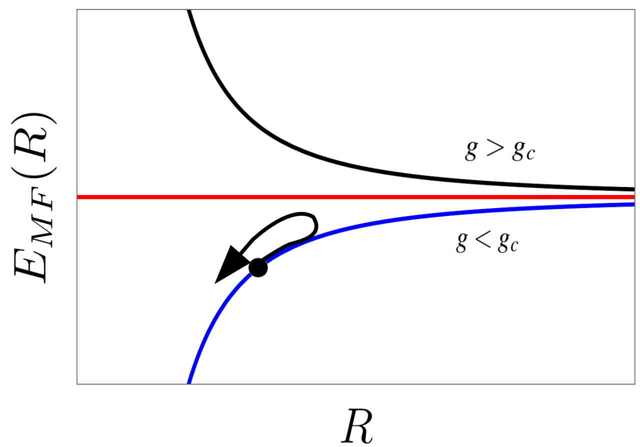


Breathing dynamics [Petrov'2024]

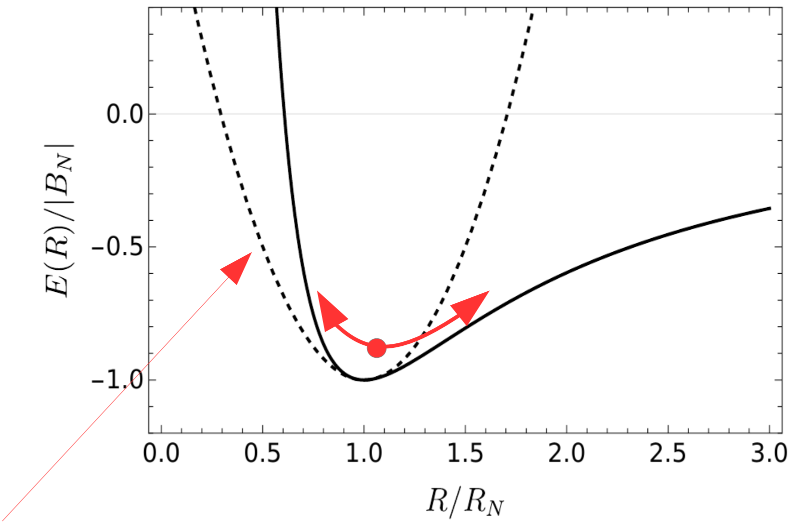
$$E(R) = -2|B_N|\frac{R_N^2}{R^2}\ln\frac{R\sqrt{e}}{R_N}$$

Classical dynamics of collective coordinate *R*

$$S = \int dt \left[\frac{NM_2}{C}\frac{\dot{R}^2(t)}{2} - E[R(t)]\right]$$



+ BMF corr.



Harmonic approx.

low-amplitude breathing mode frequency :

$$\Omega = \frac{4\sqrt{2}}{\sqrt{NM_2}}|B_N|$$

$$\tau = 2\pi/\Omega = 7.1 m R_N^2\sqrt{N}/\hbar \quad \Longrightarrow \quad \tau \approx 100\text{ms}$$

Cs mass

$N = 16$

$R_N = 1.3\mu\text{m}$

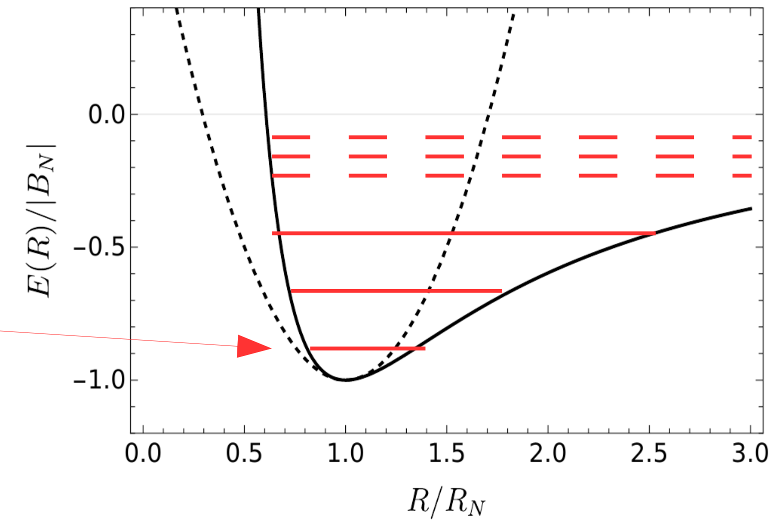
Conjectures. Attention: illegal stuff!

quantize $S = \int dt \left[\frac{NM_2}{C} \frac{\dot{R}^2(t)}{2} - E[R(t)] \right]$

$$\Omega = \frac{4\sqrt{2}}{\sqrt{NM_2}} |B_N|$$

Discrete « breathing » spectrum

With level spacing/well depth ~ $N^{-1/2}$



$B_3^{ex} = 1.2704091(1) B_2$ $\quad B_4^{ex} = 25.5(1) B_2$

$B_3 = 16.522688(1) B_2$ $\quad B_4 = 197.3(1) B_2$

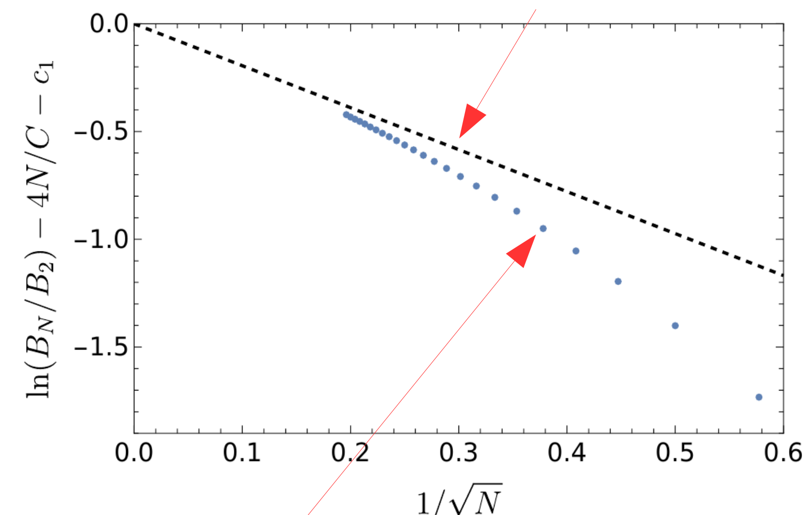
Conjecture #1 : with increasing N there will be second, third,... excited states

Conjecture #2 : include zero-point energy $\Omega/2$

$$B_N \to B_N + \Omega/2 \approx B_2 e^{4N/C + c_1 - 2\sqrt{2}/\sqrt{M_2 N}}$$

Very special term due to quantum anomaly. Otherwise. expect integer powers of $g \sim 1/N$ (cf. repulsive case [Mora&Castin'2009])

$-2\sqrt{2}/\sqrt{NM_2}$



[Bazak&DSP'18]

Summary/Outlook

Scaling invariance (1/R^2 potential)

Few-body phys. : model nuclei (example Dy+K+K ~ alpha+n+n)

Wish to control three-body and elasticity parameters

Include dipole-dipole interactions (think of Dy-Li...)

Universal fermionic clusters : 5+1 in 3D ? Method ?

2D self-bound clusters : automatic cancellation of MF : many things to think about...

Quantum Townes : to calculate : excited states for moderate N, precise E for N~100

Quantization of the breathing mode : nonequidistant spectrum : dynamics ?

Topological excitations of Q Townes solitons (excited Townes profiles)

Trapped case ? 2D $\rightarrow$ quasi-2D ?

Thank you !