Introduction to Many-body Theory III

Part III: Linear response and examples

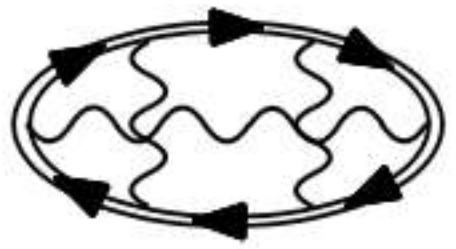
- Conserving approximations and TDDFT
- The 2-particle Green's function and optical spectra
- Linear response
- Examples: Time-dependent screening in an electron gas

The Phi-functional

The self-energy can be written as the functional derivative of a so-called Phi-functional.

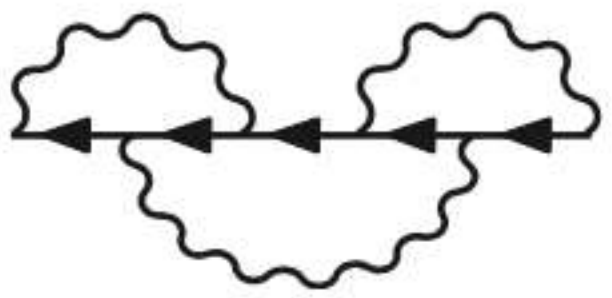
$$\Sigma(1,2) = \frac{\delta \Phi}{\delta G(2,1)}$$

$$\Phi_D[G] = \frac{1}{2}$$



Σ_1 Σ_2 Σ_3

$$\Sigma_D[G](1,2) = \frac{\delta \Phi_D}{\delta G(2,1)} =$$



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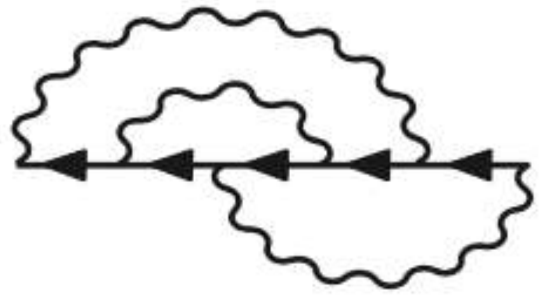

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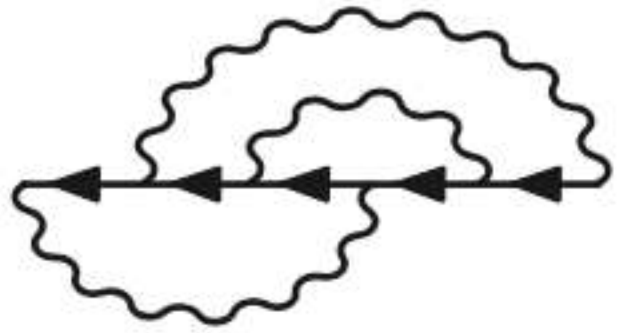


The Phi-functional can be defined as

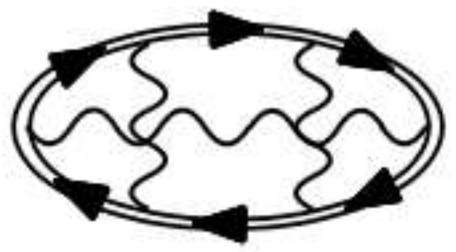
n-th order self-energy

$$\Phi[G] = \sum_{n=1}^{\infty} \frac{1}{2n} \int d1d2\, \Sigma^{(n)}[G](1,2)G(2,1)$$

Proof:
Textbook Stefanucci,RvL
"Nonequilibrium many-body theory for quantum systems"

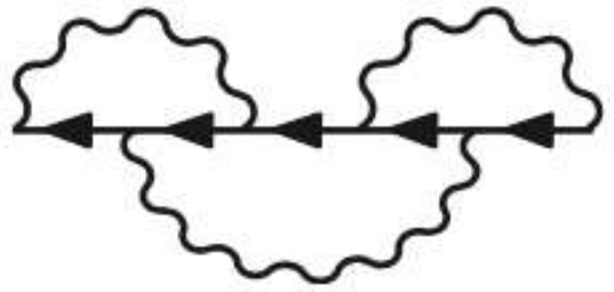
In our example

$$\frac{1}{2 \times 3}\left(\int \Sigma_1 G + \int \Sigma_2 G + \int \Sigma_3 G\right) = \frac{1}{2}$$



Approximate self-energies need not be Phi-derivable, for example

$$\Sigma(1,2) =$$



is not a Phi-derivable self-energy

Theorem (Baym): If a self-energy is Phi-derivable and we solve the Dyson equation self-consistently with this self-energy then the conserving laws of energy, momentum and particle number are satisfied

The theorem is a consequence of the invariance of the Phi-functional under space and time translations as well as gauge transformations

It is a many-body version of the Noether theorem

For example: self-consistent GW is a Phi-derivable approximation

Conserving approximations in TDDFT

Ulf von Barth et al.
"Conserving approximations in TDDFT", Phys.Rev.B72, 235109 (2005)

We define the Hartree-exchange-correlation action functional by

$$A_{\mathrm{Hxc}}[n] = -i\Phi[G_s[n]]$$

Theorem 1: The Hxc potential

$$v_{\mathrm{Hxc}}[n](1) = \frac{\delta A_{\mathrm{Hxc}}}{\delta n(1)}$$

satisfies the linearised Sham-Schlüter equation with a Phi-derivable self-energy

Proof:

$$v_{\mathrm{Hxc}}[n](1) = \frac{\delta A_{\mathrm{Hxc}}}{\delta n(1)} = -i\int d2d3d4 \frac{\delta \Phi}{\delta G_s(3,2)} \frac{\delta G_s(3,2)}{\delta v_s(4)} \frac{\delta v_s(4)}{\delta n(1)}$$

inverse density response function

$$= -i\int d2d3d4\, \Sigma[G_s](2,3) G_s(3,4) G_s(4,2) \chi_s^{-1}(4,1)$$

$$\Rightarrow \int d1 \chi_s(4,1) v_{\mathrm{Hxc}}(1) = -i\int d2d3d4\, G_s(4,2)\Sigma[G_s](2,3)G_s(3,4)$$

which is precisely the LSS equation

Theorem 2: The Hxc potential from the last equation satisfies the zero-force theorem

$$0 = \int d\mathbf{x}\, n(\mathbf{x},t) \nabla v_{\text{Hxc}}(\mathbf{x},t)$$

Proof: We use the relation $-i\delta\Phi = \int d1\, v_{\text{Hxc}}(1)\delta n(1)$

and use that the Phi-functional is invariant under the coordinate change $\mathbf{r} \to \mathbf{r} + \mathbf{R}(t)$

To first order in R(t) we have $\delta n(\mathbf{r},t) = n(\mathbf{r}+\mathbf{R}(t),t) - n(\mathbf{r},t) = \mathbf{R}(t) \cdot \nabla n(\mathbf{r},t)$

and therefore

$$0 = -i\delta\Phi = \int d1\, v_{\text{Hxc}}(1)\delta n(1) = \int dt_1 d\mathbf{r}_1\, v_{\text{Hxc}}(\mathbf{r}_1,t_1)\mathbf{R}(t_1) \cdot \nabla n(\mathbf{r}_1,t_1)$$

but since this is valid for arbitrary R(t) this implies

$$0 = \int d\mathbf{r}\, v_{\text{xc}}(\mathbf{r},t) \nabla n(\mathbf{r},t) \quad \Rightarrow \quad 0 = \int d\mathbf{x}\, n(\mathbf{x},t) \nabla v_{\text{Hxc}}(\mathbf{x},t)$$

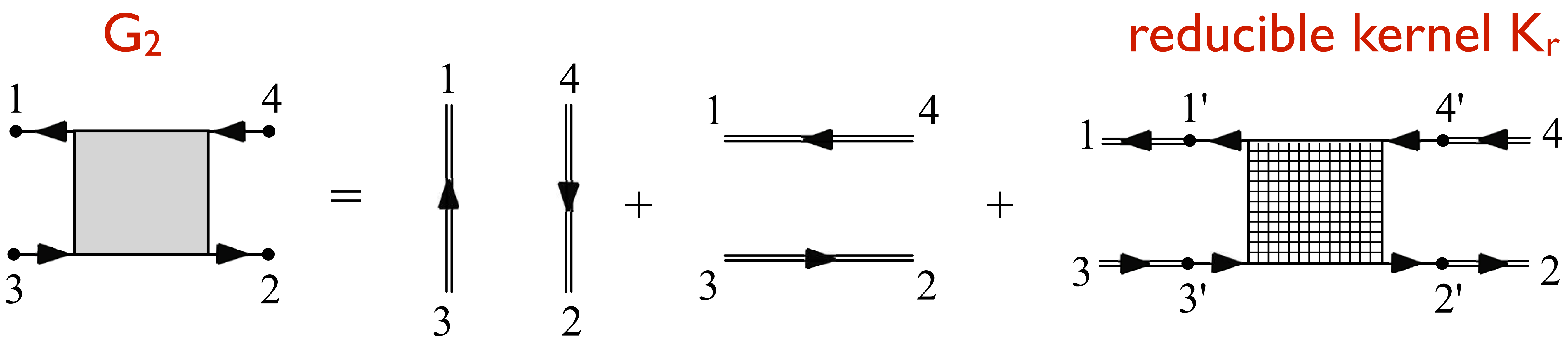
The 2-particle Green's function

We can expand the two-particle Green's function using Wick's theorem

$$G_2(a,b;c,d)$$

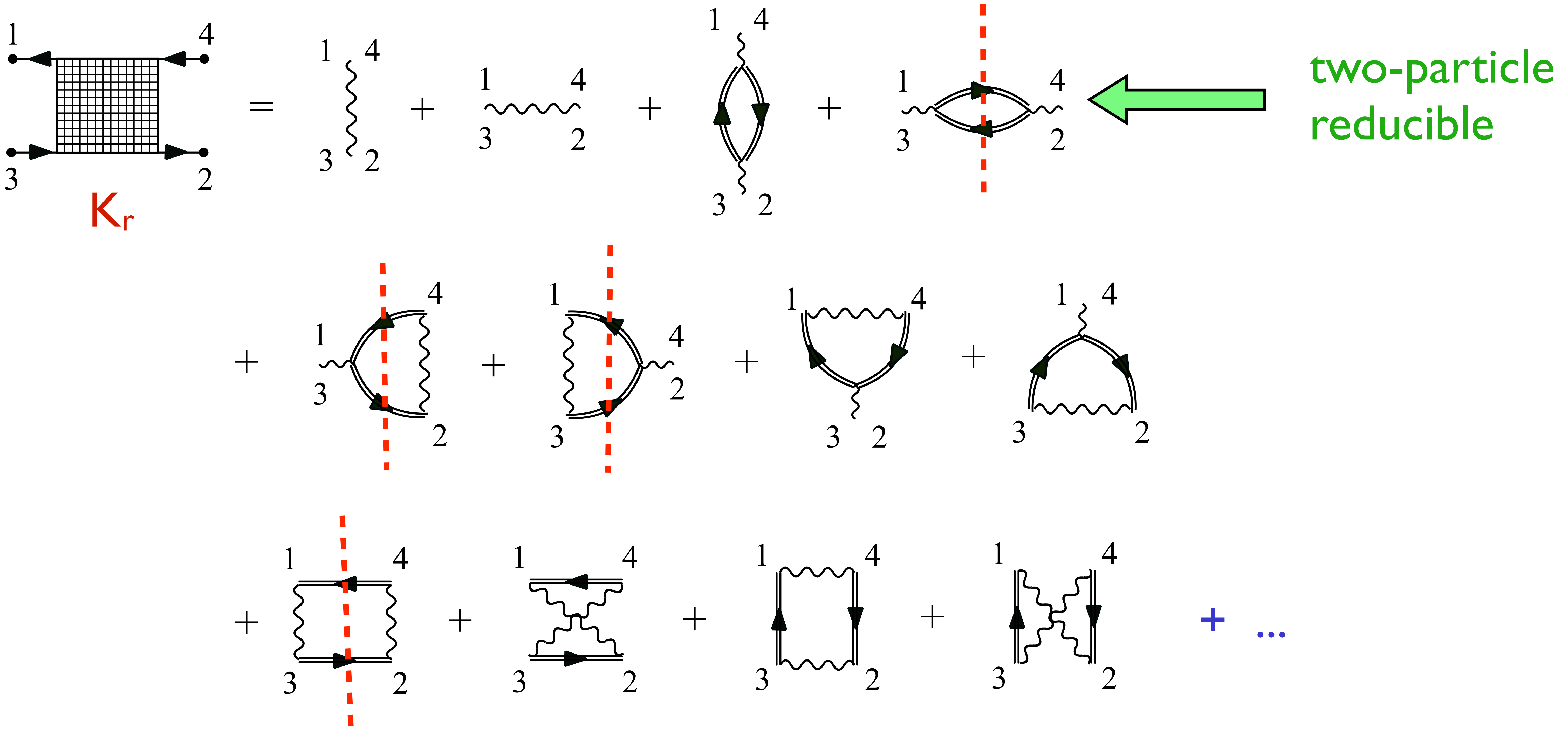
$$= \frac{\sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{\mathrm{i}}{2}\right)^k \int v(1;1')\dots v(k;k') \begin{vmatrix} G_0(a;c) & G_0(a;d) & \dots & G_0(a;k'^+) \\ G_0(b;c) & G_0(b;d) & \dots & G_0(b;k'^+) \\ \vdots & \vdots & \ddots & \vdots \\ G_0(k';c) & G_0(k';d) & \dots & G_0(k';k'^+) \end{vmatrix}_\pm}{\sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{\mathrm{i}}{2}\right)^k \int v(1;1')\dots v(k;k') \begin{vmatrix} G_0(1;1^+) & G_0(1;1'^+) & \dots & G_0(1;k'^+) \\ G_0(1';1^+) & G_0(1';1'^+) & \dots & G_0(1';k'^+) \\ \vdots & \vdots & \ddots & \vdots \\ G_0(k';1^+) & G_0(k';1'^+) & \dots & G_0(k';k'^+) \end{vmatrix}_\pm}$$

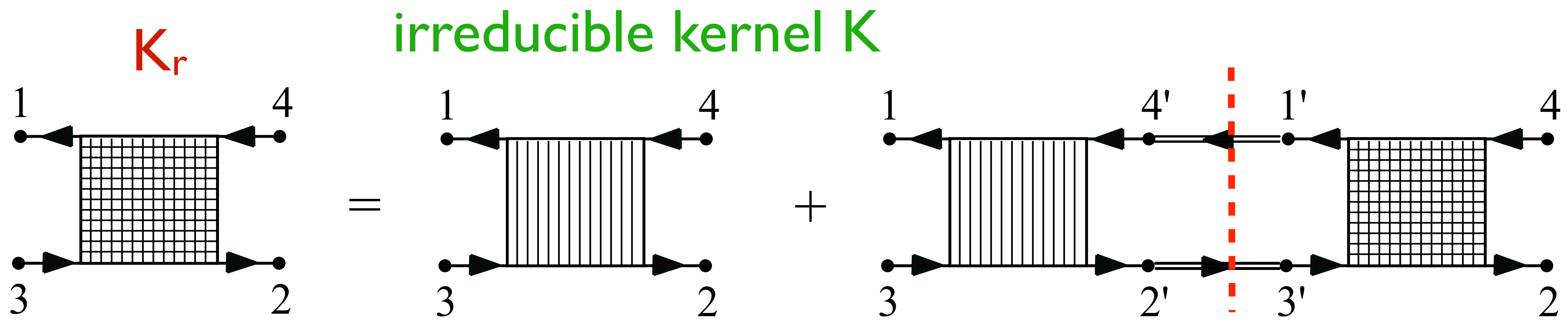
Again only connected diagrams contribute. In the same way as before non-connected diagrams cancel and we can expand in G-skeletons by removing self-energy insertions



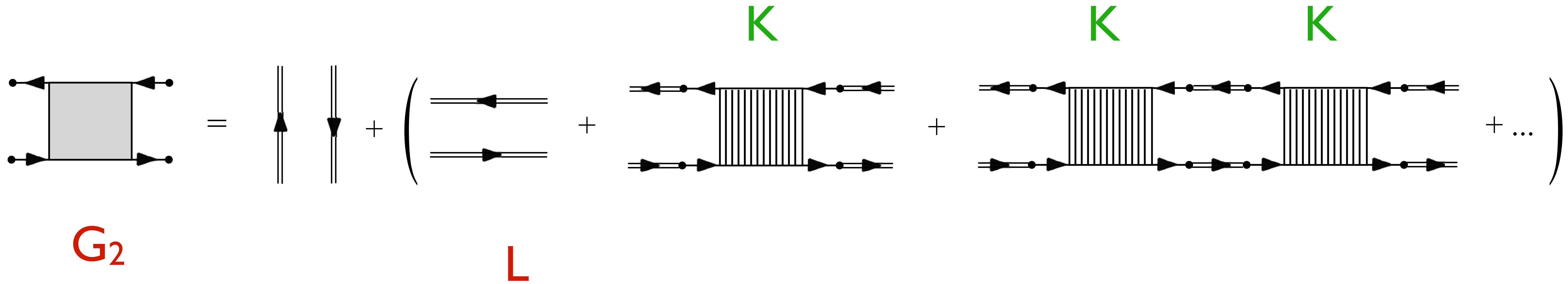
$$G_2(1,2;3,4) = G(1;3)G(2;4) \pm G(1;4)G(2;3)$$
$$+ \int G(1;1')G(3';3)K_r(1',2';3',4')G(4';4)G(2;2')$$

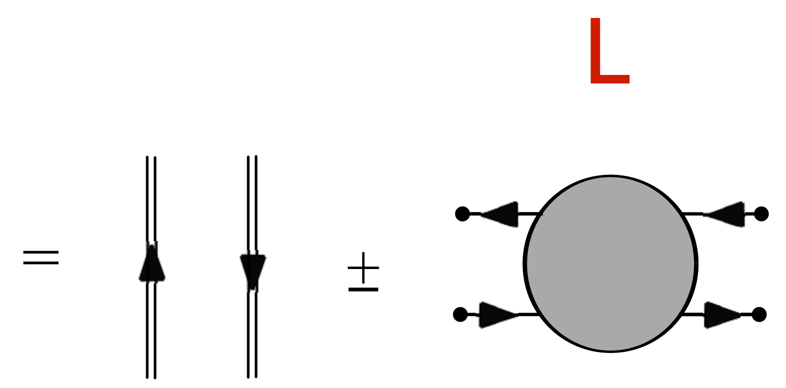
noninteracting form



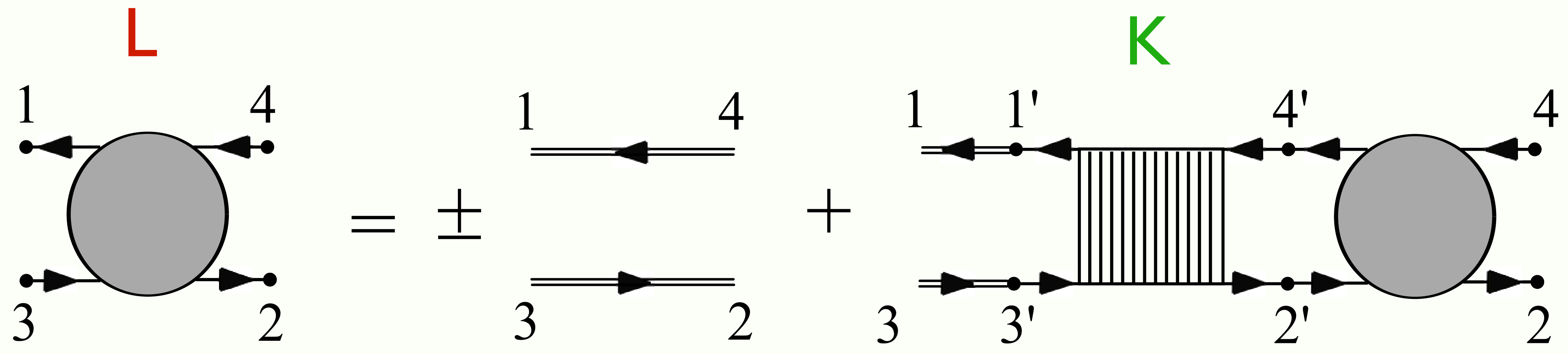


$$K_r(1,2;3,4) = K(1,2;3,4) \pm \int K(1,2';3,4')G(4';1')G(3';2')K_r(1',2;3',4)$$





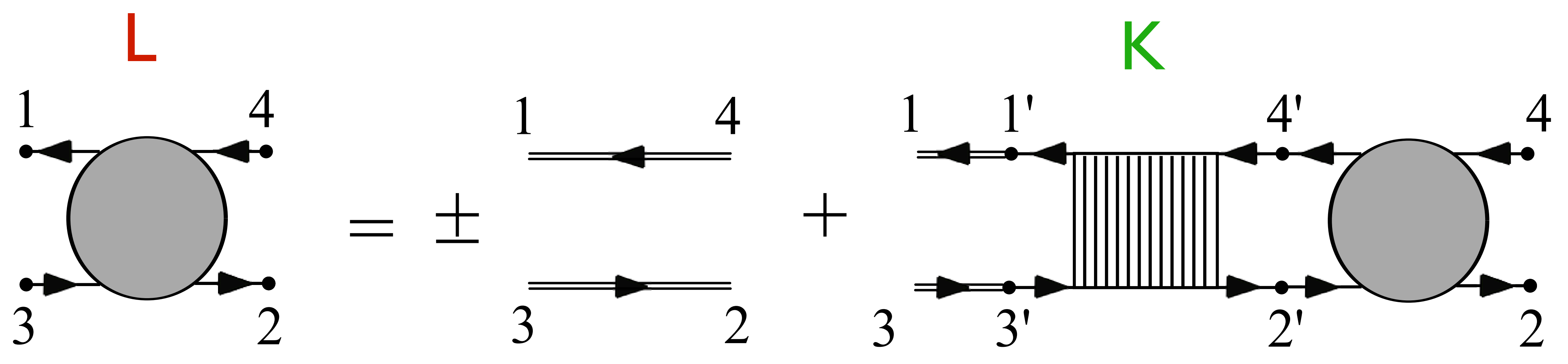
$$L(1,2;3,4) \equiv \pm\left[G_2(1,2;3,4) - G(1;3)G(2;4)\right]$$



Bethe-Salpeter equation

$$L(1,2;3,4) = G(1;4)G(2;3) \pm \int G(1;1')G(3';3)K(1',2';3',4')L(4',2;2',4)$$

To find the 2-particle Green's function we have to solve the Bethe-Salpeter equation



Bethe-Salpeter equation

$$L(1,2;3,4) = G(1;4)G(2;3) \pm \int G(1;1')G(3';3)K(1',2';3',4')L(4',2;2',4)$$

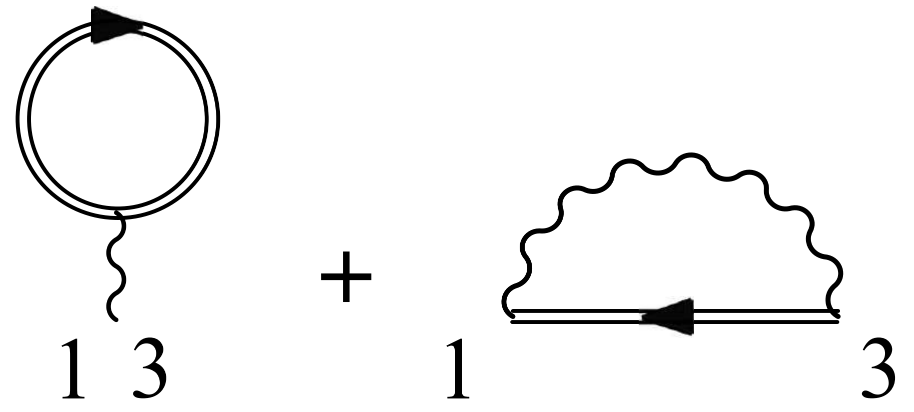
If we expand the self-energy in G-skeletonic diagrams then the following important relation is valid

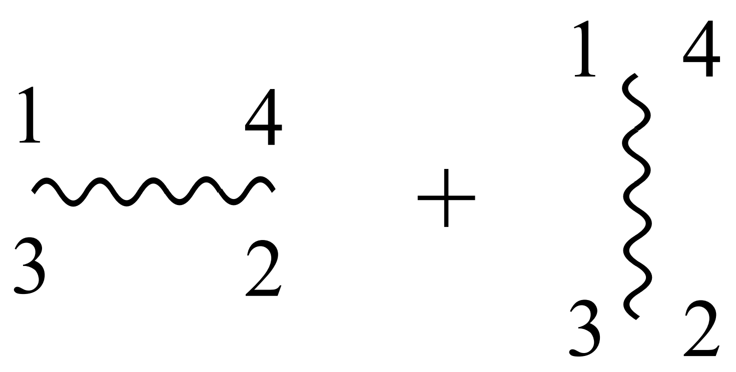
$$K(1,2;3,4) = \pm \frac{\delta\Sigma(1;3)}{\delta G(4;2)}$$

One can prove this diagrammatically

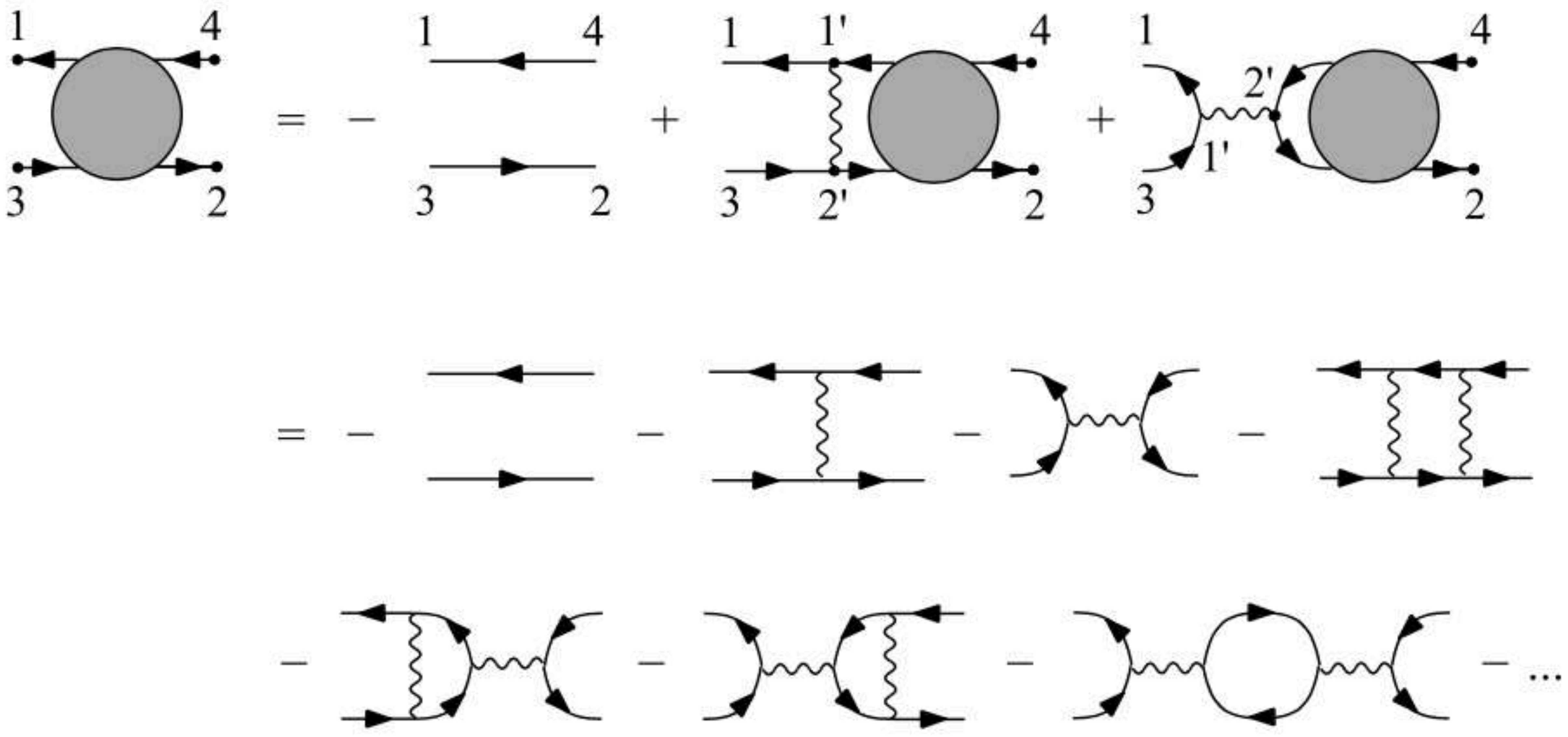
Let us give an example

K

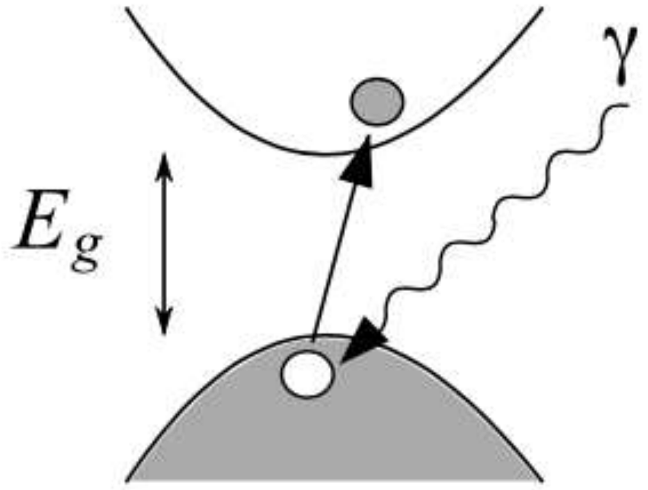
$\Sigma_{\mathrm{HF}}(1;3) =$ 

$\pm \frac{\delta \Sigma_{\mathrm{HF}}(1;3)}{\delta G(4;2)} =$ 

The Bethe-Salpeter equation is then given by



This equation is relevant for describing excitons in semiconductors



Linear response functions

$$\langle \hat{n}(\mathbf{x},t)\rangle = \frac{\mathrm{Tr}\ \mathcal{T}\left\{e^{-i\int_\gamma d\bar{z}\,\hat{H}(\bar{z})}\hat{n}(\mathbf{x},t)\right\}}{\mathrm{Tr}\ \mathcal{T}\left\{e^{-i\int_\gamma d\bar{z}\,\hat{H}(\bar{z})}\right\}}$$

If we make the variation $\hat{H}(z) \to \hat{H}(z) + \delta\hat{V}(z)$ $\quad \delta\hat{V}(z) = \int d\mathbf{x}\,\hat{n}(\mathbf{x})\,\delta v(\mathbf{x}z)$

then

$$\delta\langle \hat{n}(\mathbf{x},t)\rangle = -i\int_\gamma dz_1 \frac{\mathrm{Tr}\ \mathcal{T}\left\{e^{-i\int_\gamma d\bar{z}\,\hat{H}(\bar{z})}\hat{n}(\mathbf{x},t)\delta\hat{V}(z_1)\right\}}{\mathrm{Tr}\ \mathcal{T}\left\{e^{-i\int_\gamma d\bar{z}\,\hat{H}(\bar{z})}\right\}} + i\langle \hat{n}(\mathbf{x},t)\rangle \int_\gamma dz_1 \frac{\mathrm{Tr}\ \mathcal{T}\left\{e^{-i\int_\gamma d\bar{z}\,\hat{H}(\bar{z})}\delta\hat{V}(z_1)\right\}}{\mathrm{Tr}\ \mathcal{T}\left\{e^{-i\int_\gamma d\bar{z}\,\hat{H}(\bar{z})}\right\}}$$

which can be rewritten as

$$\delta n(1) = \int d2\,\chi(1,2)\,\delta v(2)$$

$$\chi(1,2) = -i\left[\,\langle \mathcal{T}\left\{\hat{n}_H(1)\hat{n}_H(2)\right\}\rangle - n(1)n(2)\,\right]$$

There is a close relation between the density response function and the Bethe-Salpeter equation. We have

$$L(1,2;1',2') = -\left[G_2(1,2;1',2') - G(1,1')G(2,2')\right]$$
$$= \langle\mathcal{T}\left\{\hat{\psi}_H(1)\hat{\psi}_H(2)\hat{\psi}^\dagger_H(2')\hat{\psi}^\dagger_H(1')\right\}\rangle - \langle\mathcal{T}\left\{\hat{\psi}_H(1)\hat{\psi}^\dagger_H(1')\right\}\rangle\langle\mathcal{T}\left\{\hat{\psi}_H(2)\hat{\psi}^\dagger_H(2')\right\}\rangle$$

and therefore

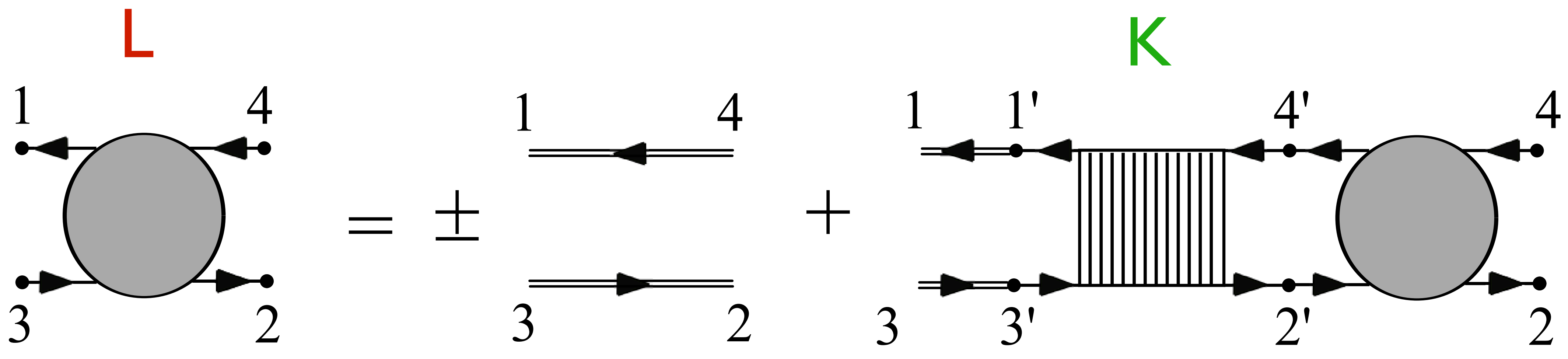
$$\chi(1,2) = -i\,[\,\langle\mathcal{T}\,\{\hat{n}_H(1)\hat{n}_H(2)\}\rangle - n(1)n(2)\,] = -i\,L(1,2;1^+,2^+)$$

In combination with the Bethe-Salpeter equation we can then further derive that

$$\chi(1,2) = P(1,2) + \int d3d4\,P(1,3)\,w(3,4)\,\chi(4,2)$$

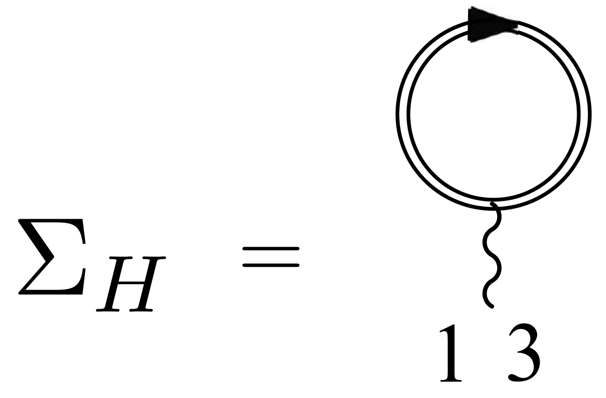
A diagrammatic expansion of the polarisability therefore directly gives an approximation for the density response function

Random Phase Approximation and plasmons

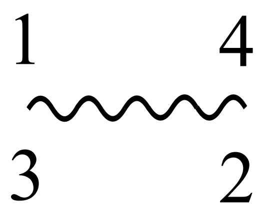


Bethe-Salpeter equation

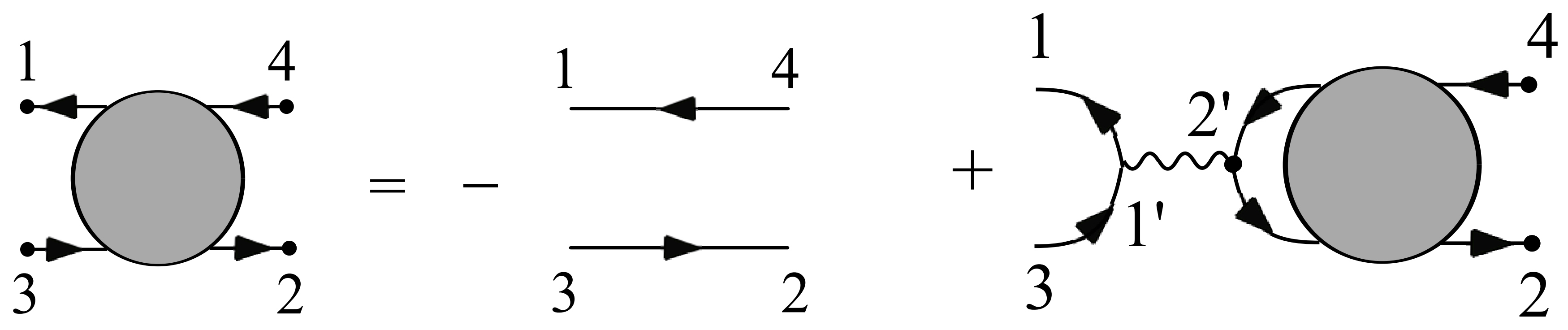
If we calculate the Bethe-Salpeter from the Hartree self-energy



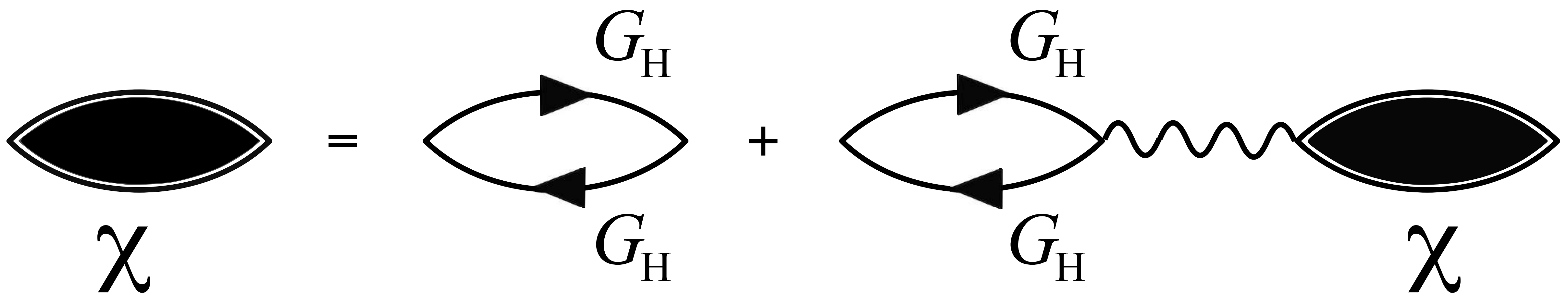
$$K_H(1,2;3,4) = -\frac{\delta\Sigma_H(1,3)}{\delta G(4,2)} =$$



then the Bethe-Salpeter equation becomes



From $\chi(1,2) = -i\,L(1,2;1^+,2^+)$ it then follows



if we take the retarded component of this expression and Fourier transform then we find

$$\chi^{\rm R}(\mathbf{x}_1,\mathbf{x}_2;\omega) = \chi_0^{\rm R}(\mathbf{x}_1,\mathbf{x}_2;\omega) + \int d\mathbf{x}_3 d\mathbf{x}_4\, \chi_0^{\rm R}(\mathbf{x}_1,\mathbf{x}_3;\omega) v(\mathbf{x}_3,\mathbf{x}_4)\chi^{\rm R}(\mathbf{x}_4,\mathbf{x}_2;\omega)$$

This approximation for the density response function is also known as the Random Phase Approximation (RPA).

A better name is the Time-Dependent Hartree Approximation (it amounts to TDDFT with zero xc-kernel)

Let us now take the case of the homogeneous electron gas. Since the system is translational invariant we can write

$$\sum_{\sigma\sigma'} \chi^{\mathrm{R}}(\mathbf{x},\mathbf{x}';\omega) = \int \frac{d\mathbf{p}}{(2\pi)^3} e^{i\mathbf{p}\cdot(\mathbf{r}-\mathbf{r}')} \chi^{\mathrm{R}}(\mathbf{p},\omega)$$

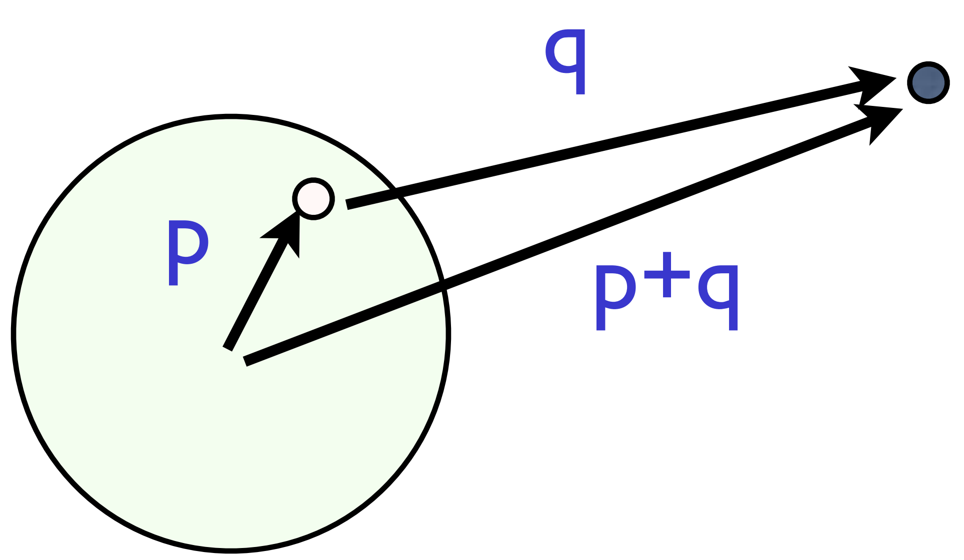
$$\chi^{\mathrm{R}}(\mathbf{q},\omega) = \frac{\chi_0^{\mathrm{R}}(\mathbf{q},\omega)}{1-\tilde{v}_{\mathbf{q}}\chi_0^{\mathrm{R}}(\mathbf{q},\omega)}, \qquad \tilde{v}_{\mathbf{q}} = \frac{4\pi}{q^2}$$

Fourier transform Coulomb potential

The RPA response function has poles at the poles of $\chi_0(\mathbf{q},\omega)$ and when

$$1 - \tilde{v}_{\mathbf{q}}\,\chi_0(\mathbf{q},\omega) = 0$$

The extra pole corresponding to this condition is known as the plasmon and corresponds to a collective mode of the electron gas



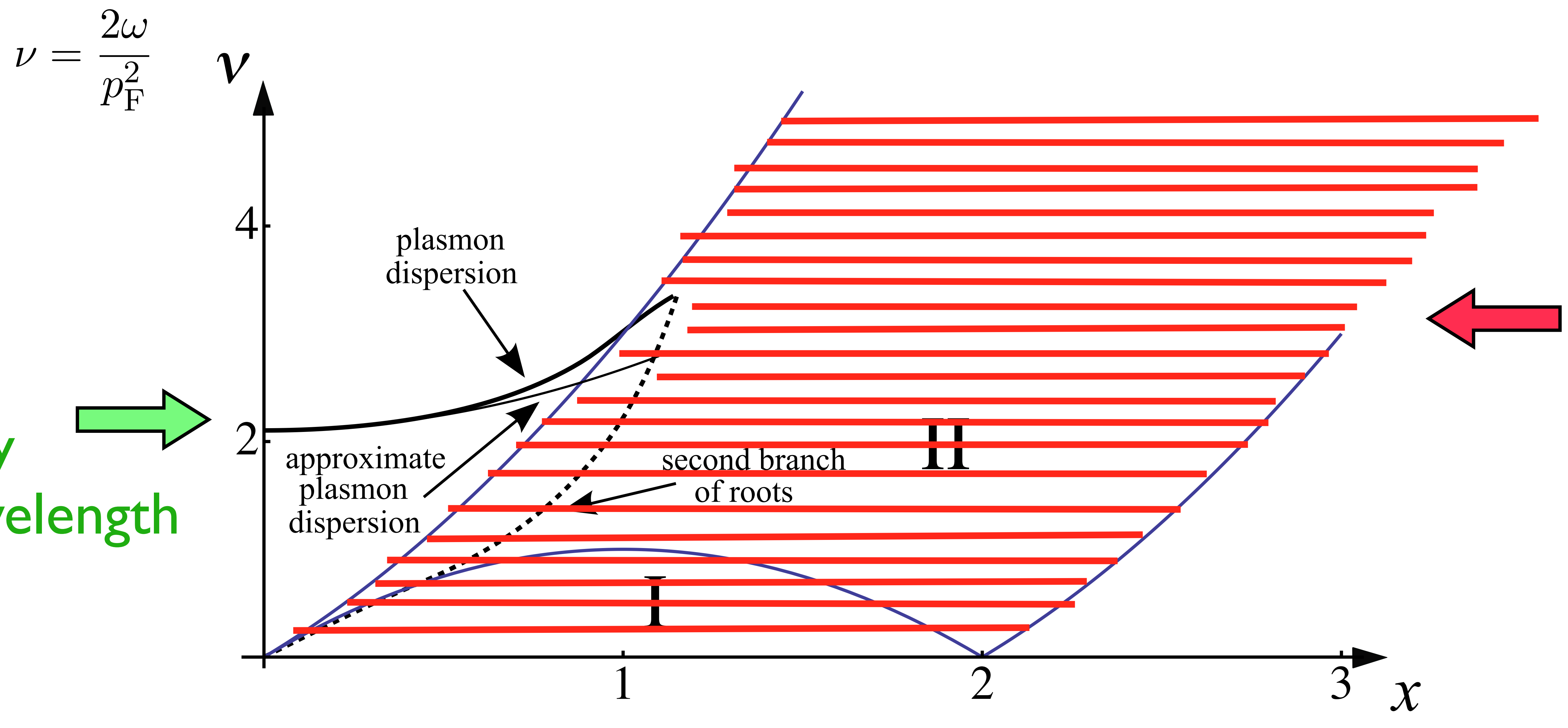
Fermi sphere
with radius p_F

$$\epsilon = \frac{(\mathbf{p}+\mathbf{q})^2}{2} - \frac{\mathbf{p}^2}{2} = \frac{\mathbf{q}^2}{2} + |\mathbf{p}||\mathbf{q}|\cos\theta$$

$$\frac{q^2}{2} - q\,p_{\mathrm{F}} \leq \epsilon \leq \frac{q^2}{2} + q\,p_{\mathrm{F}} \qquad q = |\mathbf{q}|$$

The particle-hole excitations lie between two parabolas in the q-ω plane

ω_p
plasma
frequency
Long wavelength
plasmon



particle-hole
continuum

$$x = \frac{q}{p_{\mathrm{F}}}$$

Sudden creation of a positive charge (such as in the creation of a core-hole)

$$\delta V(\mathbf{x},t) = \theta(t)\frac{Q}{r} = \int \frac{d\mathbf{q}}{(2\pi)^3}\int \frac{d\omega}{2\pi} e^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}-\mathrm{i}\omega t}\,\delta V(\mathbf{q},\omega)$$

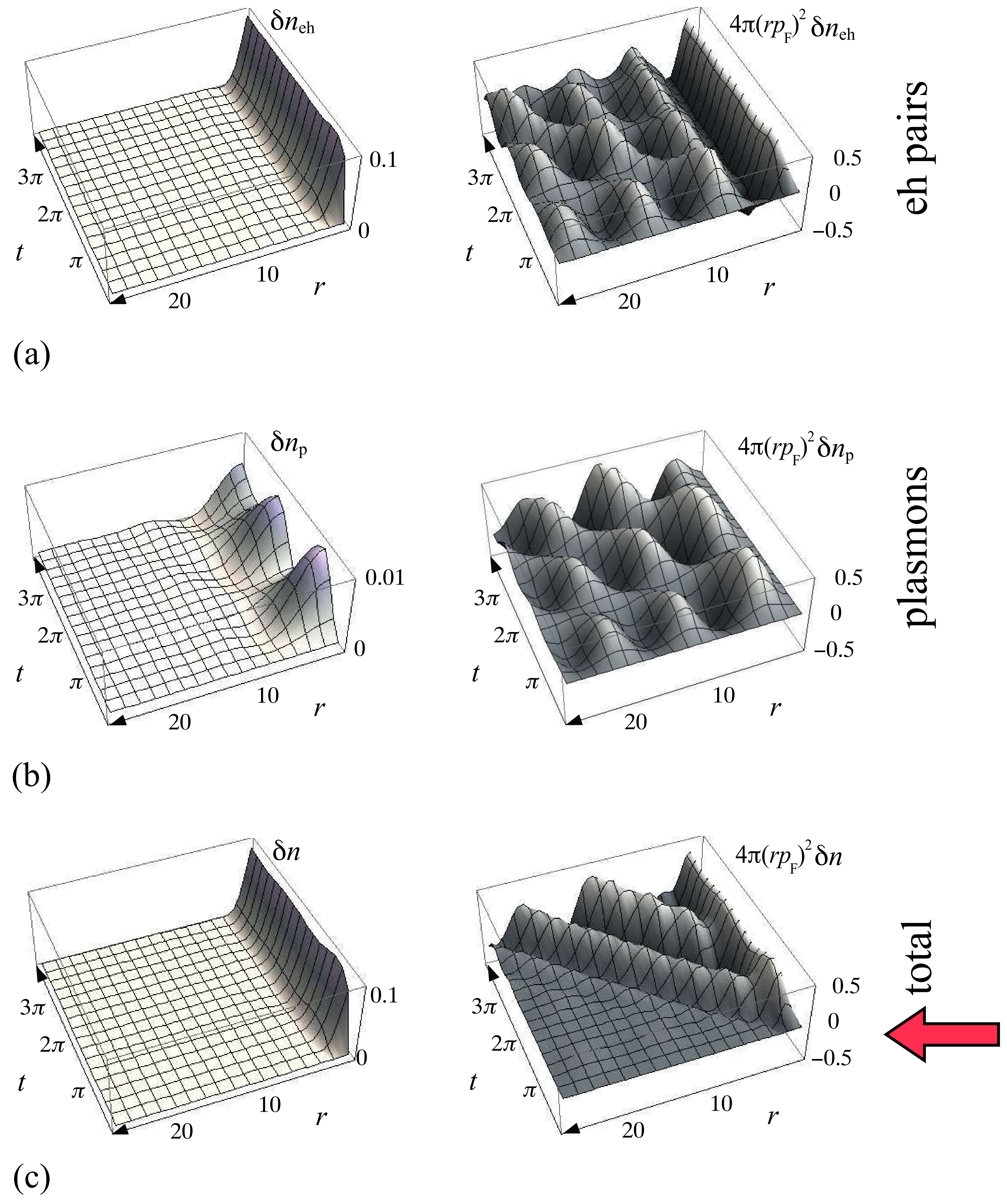
$$\delta V(\mathbf{q},\omega) = \frac{4\pi Q}{q^2}\frac{\mathrm{i}}{\omega+\mathrm{i}\eta} = \tilde{v}_{\mathbf{q}}\,Q\frac{\mathrm{i}}{\omega+\mathrm{i}\eta}.$$

We can calculate the induced density change from the RPA response function. A few manipulations lead to

$$\delta n(\mathbf{r},t) = -\frac{16\pi Q}{(2\pi)^4}\frac{1}{r}\int_0^\infty dq\,q\,\sin qr\int_0^\infty \mathrm{Im}\chi^{\mathrm{RPA}}(q,\omega)\,\tilde{v}_{\mathbf{q}}\frac{1-\cos\omega t}{\omega}$$

⇓

The integral can be split into a contribution from particle-hole excitations and a contribution from the plasmon



The positive charge is screened at a time-scale of the inverse plasmon frequency

Figure 15.7: This figure shows the 3D plot of the transient density in an electron gas with $r_s = 3$ induced by the sudden creation of a point-like positive charge $Q = 1$ in the origin at $t = 0$. The contribution due to the excitation of electron-hole pairs (a) and plasmons (b) is, for clarity, multiplied by $4\pi(rp_F)^2$ in the plots to the right. Panel (c) is simply the sum of the two contributions. Units: r is in units of $1/p_F$, t is in units of $1/\omega_p$ and all densities are in units of p_F^3.

In the long time limit we have

$$\delta n_s(\mathbf{r}) \equiv \lim_{t\to\infty} \delta n(\mathbf{r},t) = -\frac{Q}{2\pi^2}\frac{1}{r}\int_0^\infty dq\, q \sin(qr)\tilde{v}_{\mathbf{q}}\, \chi^{\mathrm{R}}(\mathbf{q},0)$$

has spatial oscillations known as Friedel oscillations

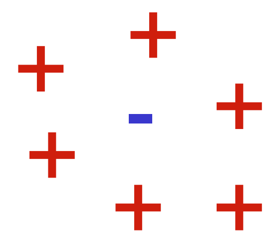
Suppose now that $Q = q = -1$ is the same a the electron charge. The total density change due to this test charge is

$$q\,\delta n_{\mathrm{tot}}(\mathbf{r}) = q[\delta(\mathbf{r}) + \delta n_s(\mathbf{r})]$$

The interaction energy between this charge and a generic electron is

$$e_{\mathrm{int}}(\mathbf{r}) = \int d\mathbf{r}' v(\mathbf{r},\mathbf{r}')\delta n_{\mathrm{tot}}(\mathbf{r}')$$

$$\begin{aligned} e_{\mathrm{int}}(\mathbf{r}) &= \int d\mathbf{r}' v(\mathbf{r},\mathbf{r}') \left[\delta(\mathbf{r}) + \int \frac{d\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}'}\, \tilde{v}_{\mathbf{q}}\, \chi^{\mathrm{R}}(\mathbf{q},0)\right] \\ &= \int \frac{d\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \left[\tilde{v}_{\mathbf{q}} + \tilde{v}_{\mathbf{q}}^2 \chi^{\mathrm{R}}(\mathbf{q},0)\right] \\ &= \int \frac{d\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}}\, W^{\mathrm{R}}(\mathbf{q},0) \xrightarrow[r\to\infty]{} \frac{e^{-r/\lambda_{\mathrm{TF}}}}{r} \end{aligned}$$



In the static limit W describes the interaction between a test charge an an electron

Linear response: Take home message

- We can derive a diagrammatic expansion for the linear response function from the diagrammatic rules for the 2-particle Green's function

- The linear response function gives direct information on neutral excitation spectra such as measured in optical absorption experiments

- The random phase approximation to the linear response function describes the phenomena of plasmon excitation in metallic systems

- The screening of a an added charge in the electron gas happens at a time-scale of the inverse plasmon frequency

Spectral properties of an electron gas : GW

We have seen that the spectral function describes the energy distribution of excitations upon addition or removal of an electron. We therefore expect to see both plasmon and particle-hole excitations when we do a photo-emission experiment on an electron gas (or electron gas like metals such a sodium)

Dyson equation

$$G^R(\mathbf{q},\omega) = g^R(\mathbf{q},\omega) + g^R(\mathbf{q},\omega)\Sigma^R(\mathbf{q},\omega)G^R(\mathbf{q},\omega)$$

$$g^R(\mathbf{q},\omega) = \frac{1}{\omega - \epsilon_{\mathbf{q}} + i\eta} \qquad \epsilon_{\mathbf{q}} = \frac{|\mathbf{q}|^2}{2}$$

$$\Rightarrow \quad G^R(\mathbf{q},\omega) = \frac{g^R(\mathbf{q},\omega)}{1 - g^R(\mathbf{q},\omega)\Sigma^R(\mathbf{q},\omega)} = \frac{1}{\omega - \epsilon_{\mathbf{q}} - \Sigma^R(\mathbf{q},\omega)}$$

We calculate the self-energy in the GW approximation using noninteracting Green's function we find

$$\Sigma^{\lessgtr}(p,\omega) = \frac{\mathrm{i}}{(2\pi)^3\, p} \int d\omega' \int_0^\infty dk\, k\, G^{\lessgtr}(k,\omega') \int_{|k-p|}^{k+p} dq\, q\, W^{\gtrless}(q,\omega'-\omega)$$

The greater and lesser self-energies describe scattering rates for added or removed particles with energy ω and momentum p

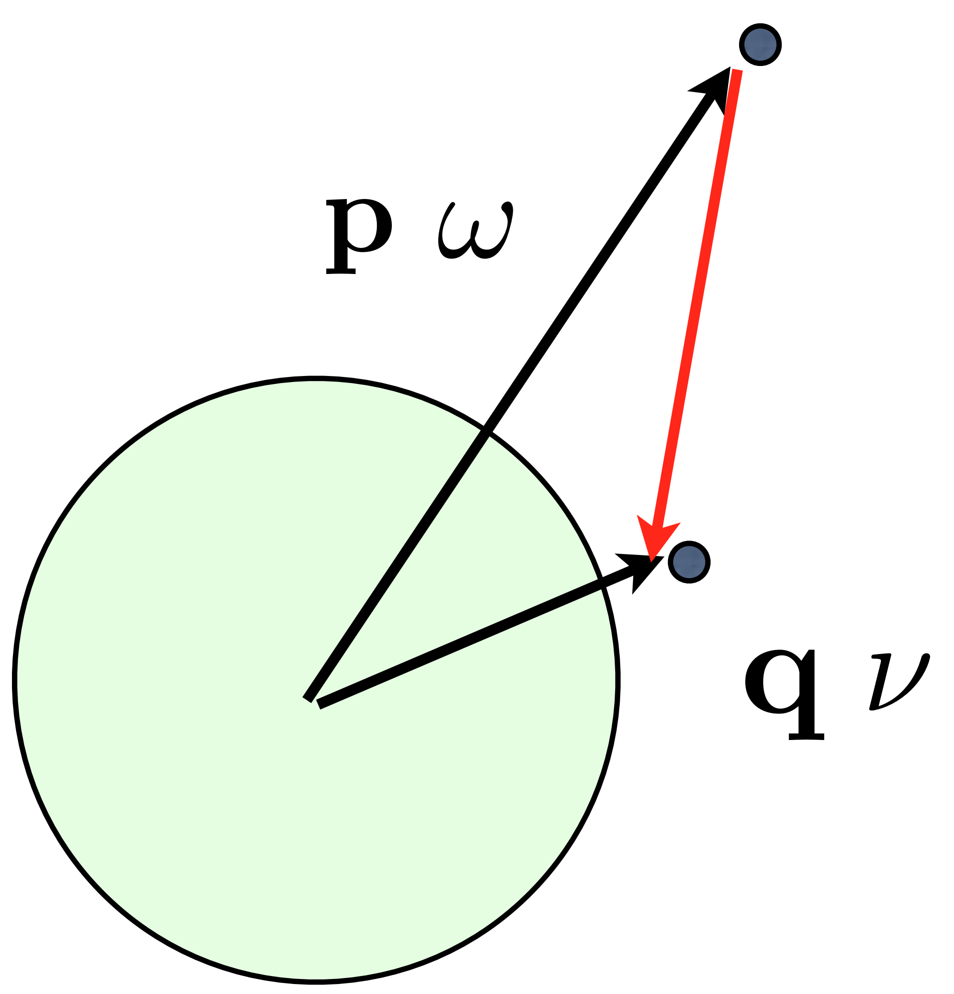
The self-energy vanishes when $\omega \to \mu$ due to the fact an added particle can maximally lose energy $\omega - \mu$ as states below the Fermi energy are occupied

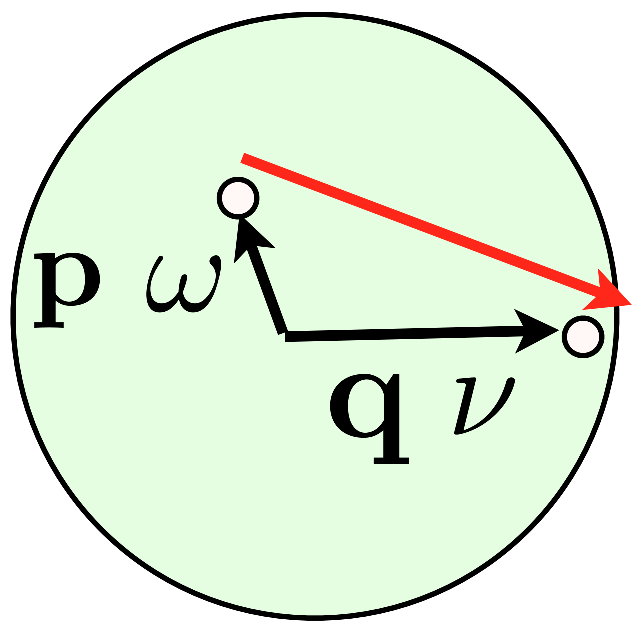
$$i(\Sigma^{>}(\mathbf{q},\omega) - \Sigma^{<}(\mathbf{q},\omega)) = -2\,\mathrm{Im}\,\Sigma^{R}(\mathbf{q},\omega) = \Gamma(\mathbf{q},\omega)$$

$$\lim_{\omega\to\mu} \mathrm{Im}\,\Sigma^{R}(\mathbf{q},\omega) = 0$$

$$\Sigma^{R}(\mathbf{q},\omega) = \Lambda(\mathbf{q},\omega) - \frac{i}{2}\Gamma(\mathbf{q},\omega)$$

Scattering processes





Loss of energy by a particle.
Scattering rate given by $i\,\Sigma^{>}(\mathbf{p},\omega)$

Only relevant when $p \geq p_{\mathrm{F}}$

A plasmon can be excited
only when $\omega \geq \mu + \omega_p$

Absorption of energy by a hole.
Scattering rate given by $-\,i\,\Sigma^{<}(\mathbf{p},\omega)$

Only relevant when $p \leq p_{\mathrm{F}}$

A plasmon can be absorbed
only when $\omega \leq \mu - \omega_p$

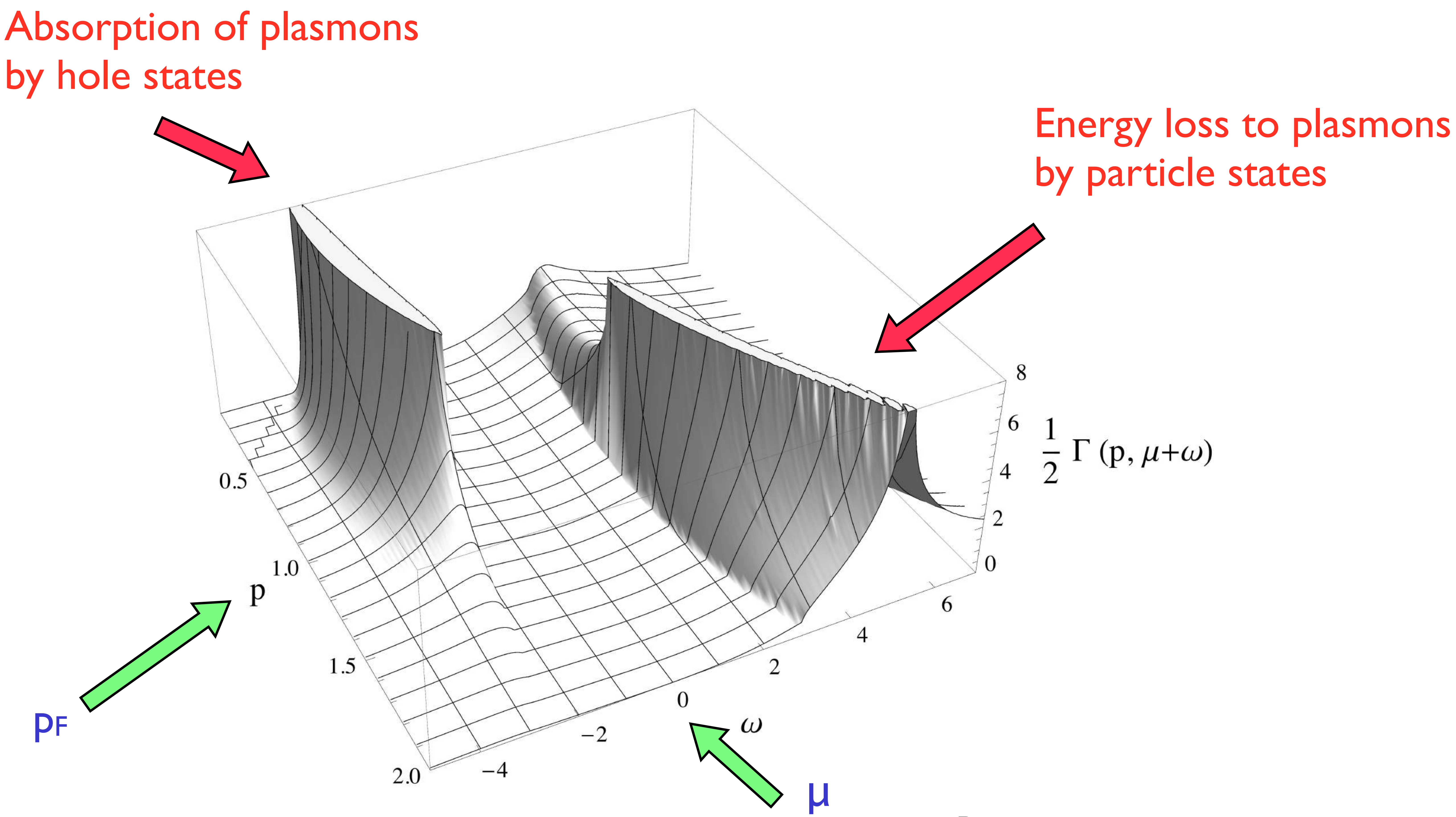


Figure 15.9: The imaginary part of the retarded self-energy $-\mathrm{Im}[\Sigma^{\mathrm{R}}(p, \omega + \mu)] = \Gamma(p, \omega + \mu)/2$ for an electron gas at $r_s = 4$ within the G_0W_0 approximation as a function of the momentum and energy. The momentum p is measured in units of p_{F} and the energy ω and the self-energy in units of $\epsilon_{p_{\mathrm{F}}} = p_{\mathrm{F}}^2/2$.

For the spectral function this implies the following

$$A(\mathbf{q},\omega) = -2\,\mathrm{Im}\,G^R(\mathbf{q},\omega) = \frac{\Gamma(\mathbf{q},\omega)}{(\omega - \epsilon_{\mathbf{q}} - \Lambda(\mathbf{q},\omega))^2 + \left(\frac{\Gamma(\mathbf{q},\omega)}{2}\right)^2}$$

If $\Gamma(\mathbf{q},\omega)$ is small then the spectral function can only become large ($\sim 1/\Gamma$) when

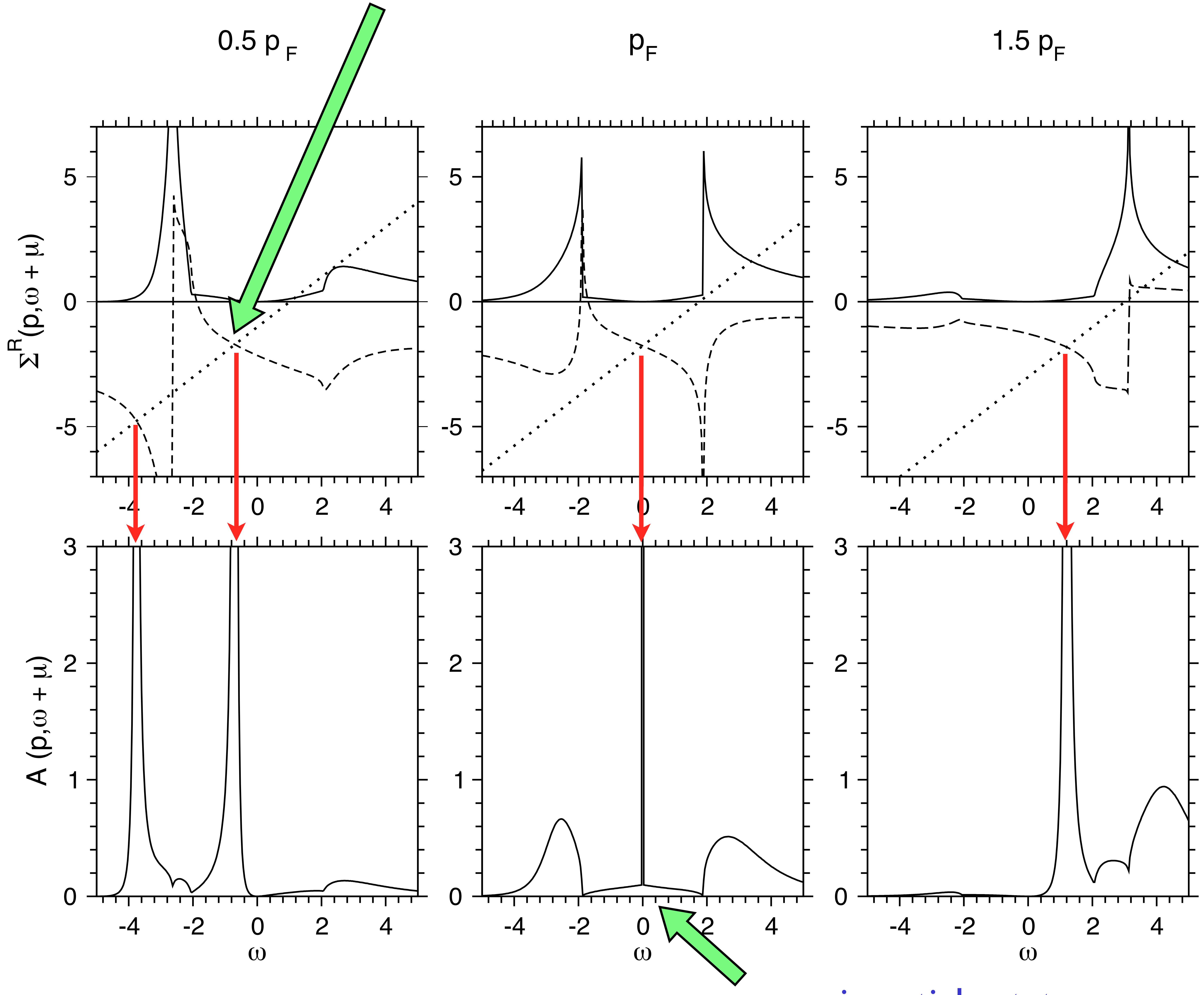
$$\omega - \epsilon_{\mathbf{q}} - \Lambda(\mathbf{q},\omega) = 0$$

The Luttinger-Ward theorem tells that this happens when $q = p_{\mathrm{F}}\ ,\ \omega = \mu$

$$\mu - \epsilon_{p_{\mathrm{F}}} - \Lambda(p_{\mathrm{F}},\mu) = 0$$

(not explained in these lectures, requires a derivation of the Luttinger-Ward functional, see G.Stefanucci, RvL, Nonequilibrium Many-Body Theory of Quantum Systems)

$$\omega - \epsilon_{\mathbf{p}} - \Lambda(\mathbf{p}, \omega) = 0$$



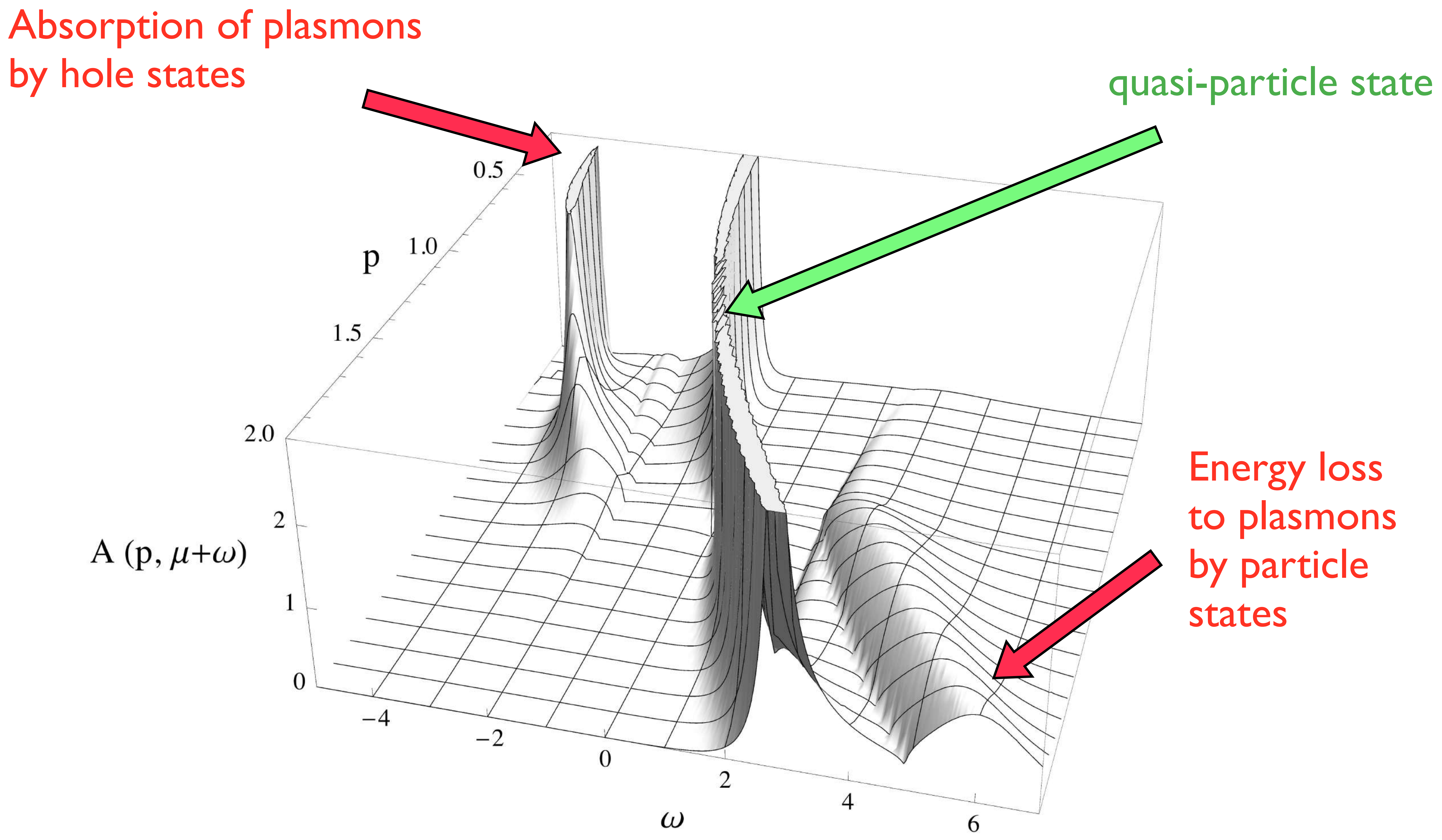
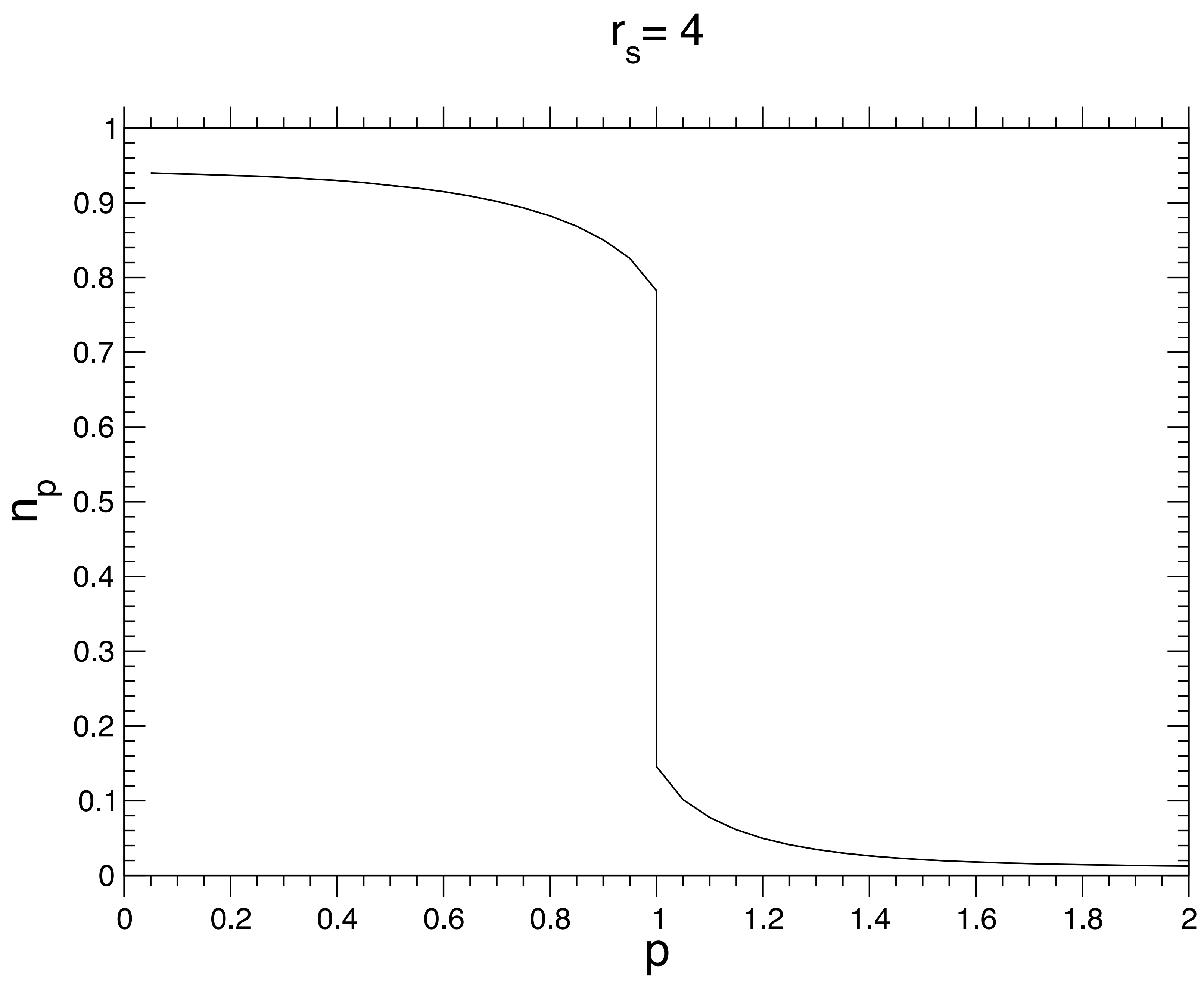


Figure 15.12: The spectral function $A(p, \mu+\omega)$ as a function of the momentum and energy for an electron gas at $r_s = 4$ within the G_0W_0 approximation. The momentum p is measured in units of p_F and the energy ω and the spectral function in units of $\epsilon_{p_\mathrm{F}} = p_\mathrm{F}^2/2$.

The momentum distribution in the electron gas is given by

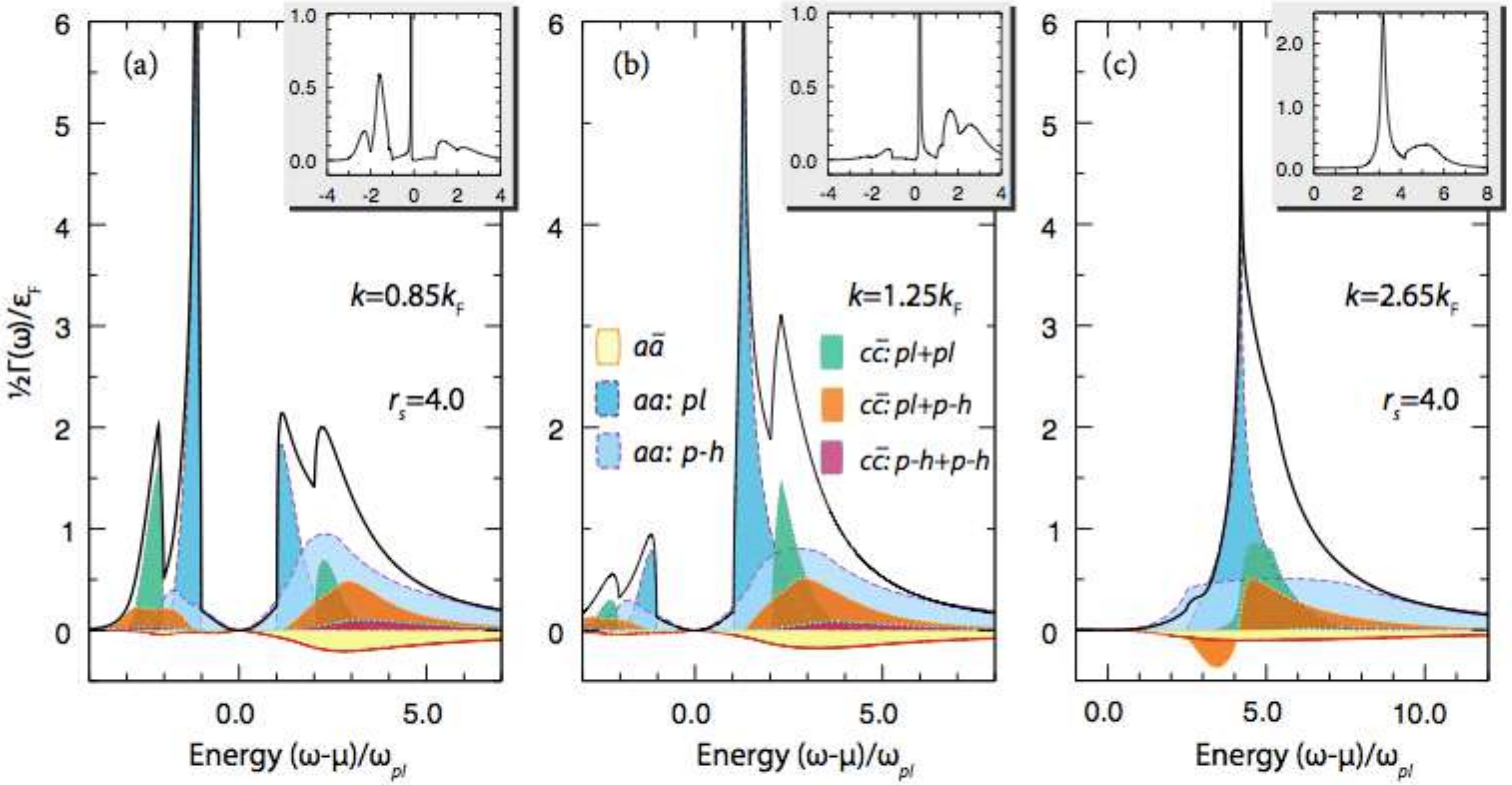
$$n_p = \int_{-\infty}^{\mu} \frac{d\omega}{2\pi}\, A(p,\omega)$$

Due to the appearance of a delta peak in the spectral function at the Fermi momentum p_F the momentum distribution jumps discontinuously at the Fermi momentum. The jump is the strength of the quasi-particle peak.

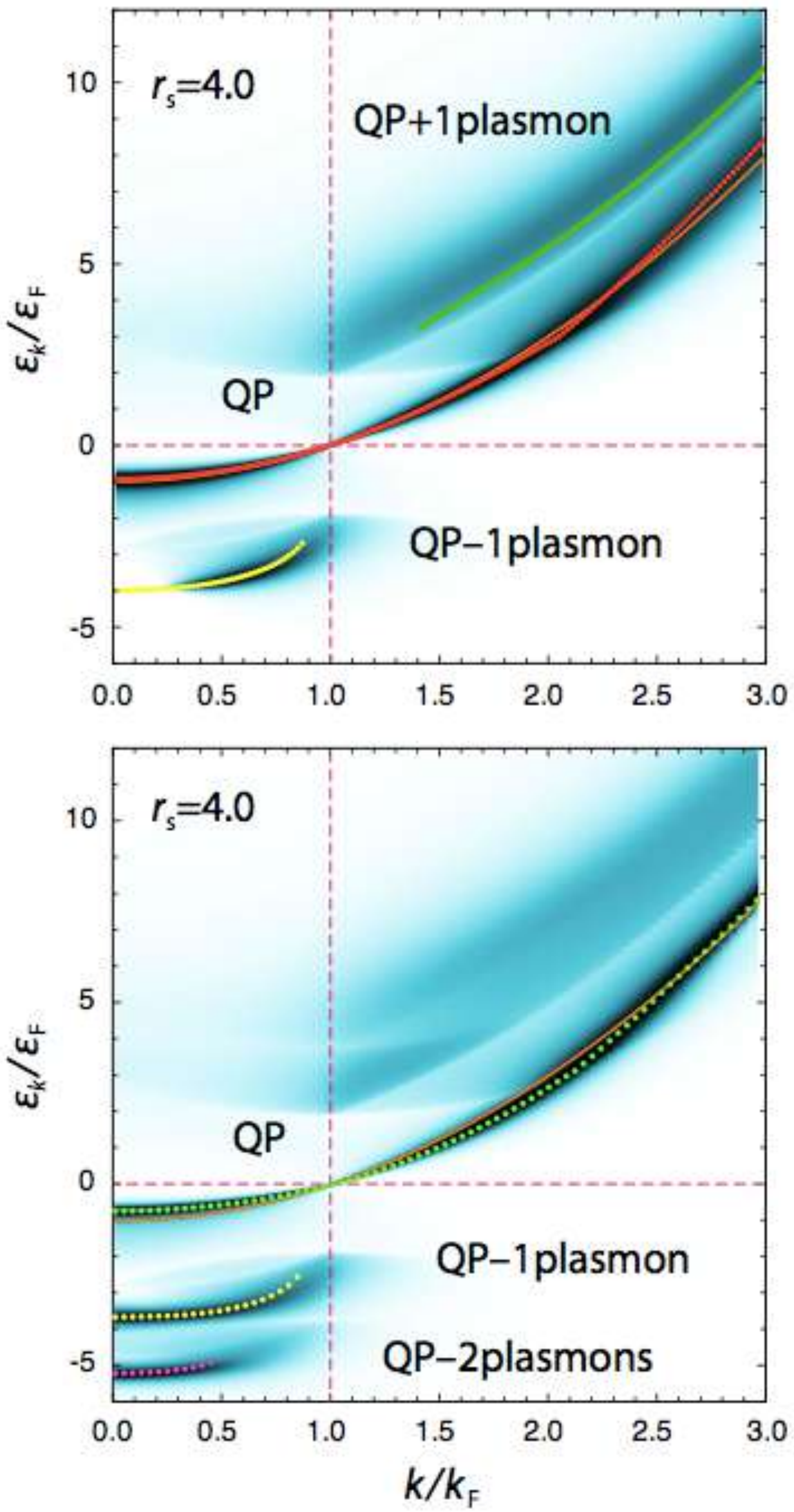


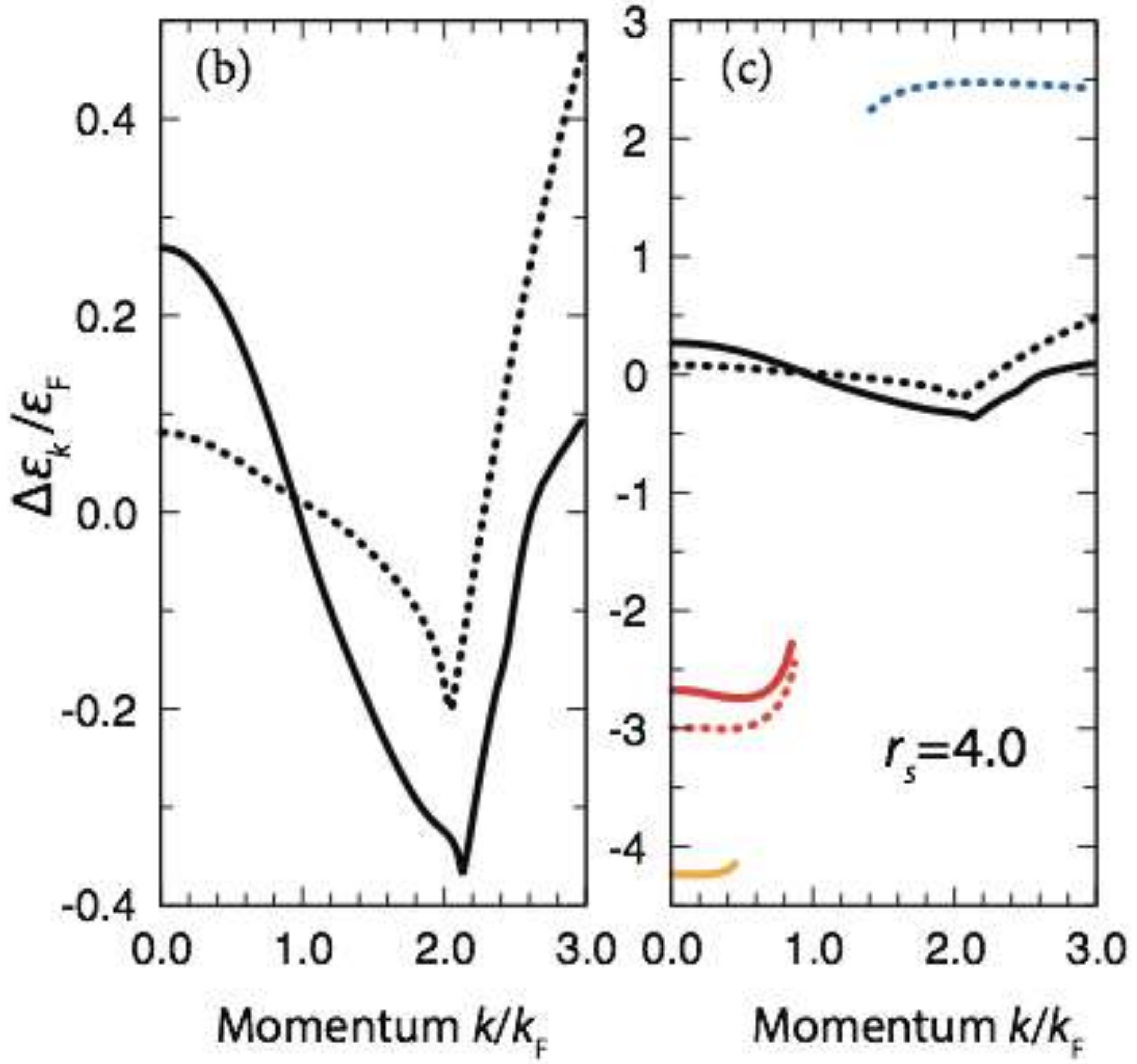
Beyond GW

Y.Pavlyukh, A.-M. Uimonen, G.Stefanucci, RvL, PRL 2016



Due to negative corrections around the chemical potential in the rate function, vertex corrections sharpen the quasi-particle peak as compared to G_0W_0





Vertex corrections:

- Reduce the band width by 27 percent (sc GW
  increases by 20 percent
- Wash out the plasmon above the chemical potential
- Reduce the first plasmon energy

Spectral properties of the electron gas: Take home message

- By addition or removal of an electron we create particle-hole and plasmon excitations

- The self-energy at the Fermi-surface vanishes due to phase-space restrictions. This has various consequences:

  1) The momentum distribution of the electron gas jumps discontinuously at the Fermi momentum

  2) Quasi-particles at the Fermi surface have an infinite life-time.

- The GW approximation gives extra plasmon structure in the spectral function due to plasmons

- Multiple-plasmons excitations (satellites) are beyond GW and require vertex corrections.

That’s all folks!