Introduction to Octopus: a real-space (TD)DFT code

Martin Lüders, Alex Buccheri, Heiko Appel, David Strubbe, Nicolas Tancogne-Dejean

Benasque 2025

Schedule

- Day 1 (08 Apr):
 - Octopus basics tutorial series
- Day 2 (09 Apr):
 - Optical absorption tutorial series
- Day 3 (10 Apr):
 - Solids tutorial series
- Day 4 (11 Apr):
 - Maxwell tutorials
- Day 5 (13 Apr):
 - BerkeleyGW tutorials

The Octopus code

Purpose: simulate the dynamics of electrons and nuclei under the influence of external time-dependent fields in the framework of Time-Dependent Density Functional Theory (TDDFT)



- DFT with many functionals (from Libxc), Hartree-Fock, Hartree, RDMFT
- $\bullet\,$ Fortran 2008, C, C++, OpenCL/CUDA and some Python and Perl.
- extensive use of mathematical libraries: BLAS/LAPACK, FFTW, GSL, etc.
- Interfaces to external libraries: libxc, libvdwxc, wannier90, berkeleygw, etc.
- Free open-source software (GNU Public License).
- Current version is 16.0.
- Framework to implement and test new ideas

• Ground-state DFT calculations



- Ground-state DFT calculations
- Excited states calculations (real-time propagation, linear response)



- Ground-state DFT calculations
- Excited states calculations (real-time propagation, linear response)
 - optical (e.g. ARPES)



- Ground-state DFT calculations
- Excited states calculations (real-time propagation, linear response)
 - optical (e.g. ARPES)
 - magnetic (magnons)



- Ground-state DFT calculations
- Excited states calculations (real-time propagation, linear response)
 - optical (e.g. ARPES)
 - magnetic (magnons)
 - vibrations



- Ground-state DFT calculations
- Excited states calculations (real-time propagation, linear response)
 - optical (e.g. ARPES)
 - magnetic (magnons)
 - vibrations
- electric and thermal conductivities



- Ground-state DFT calculations
- Excited states calculations (real-time propagation, linear response)
 - optical (e.g. ARPES)
 - magnetic (magnons)
 - vibrations
- electric and thermal conductivities
- Theory levels include:



- Ground-state DFT calculations
- Excited states calculations (real-time propagation, linear response)
 - optical (e.g. ARPES)
 - magnetic (magnons)
 - vibrations
- electric and thermal conductivities
- Theory levels include:
 - Kohn-Sham with LDA, GGA



- Ground-state DFT calculations
- Excited states calculations (real-time propagation, linear response)
 - optical (e.g. ARPES)
 - magnetic (magnons)
 - vibrations
- electric and thermal conductivities
- Theory levels include:
 - Kohn-Sham with LDA, GGA
 - generalized Kohn-Sham: MGGA, hybrid functionals



- Ground-state DFT calculations
- Excited states calculations (real-time propagation, linear response)
 - optical (e.g. ARPES)
 - magnetic (magnons)
 - vibrations
- electric and thermal conductivities
- Theory levels include:
 - Kohn-Sham with LDA, GGA
 - generalized Kohn-Sham: MGGA, hybrid functionals
 - Optimized effective potentials (OEP)



- Ground-state DFT calculations
- Excited states calculations (real-time propagation, linear response)
 - optical (e.g. ARPES)
 - magnetic (magnons)
 - vibrations
- electric and thermal conductivities
- Theory levels include:
 - Kohn-Sham with LDA, GGA
 - generalized Kohn-Sham: MGGA, hybrid functionals
 - Optimized effective potentials (OEP)
 - reduced density matrix functional theory (RDMFT)



- Ground-state DFT calculations
- Excited states calculations (real-time propagation, linear response)
 - optical (e.g. ARPES)
 - magnetic (magnons)
 - vibrations
- electric and thermal conductivities
- Theory levels include:
 - Kohn-Sham with LDA, GGA
 - generalized Kohn-Sham: MGGA, hybrid functionals
 - Optimized effective potentials (OEP)
 - reduced density matrix functional theory (RDMFT)
 - van der Waals interactions



Big changes going on:

• Multi-system mode



- Multi-system mode
 - classical particles (nuclei)



- Multi-system mode
 - classical particles (nuclei)
 - electrons



- Multi-system mode
 - classical particles (nuclei)
 - electrons
 - Maxwell fields



- Multi-system mode
 - classical particles (nuclei)
 - electrons
 - Maxwell fields
- Maxwell calculations



- Multi-system mode
 - classical particles (nuclei)
 - electrons
 - Maxwell fields
- Maxwell calculations
 - propagation of \vec{E} and \vec{B} fields, and propagation in linear media.



- Multi-system mode
 - classical particles (nuclei)
 - electrons
 - Maxwell fields
- Maxwell calculations
 - propagation of \vec{E} and \vec{B} fields, and propagation in linear media.
 - see upcoming tutorial



- Multi-system mode
 - classical particles (nuclei)
 - electrons
 - Maxwell fields
- Maxwell calculations
 - propagation of \vec{E} and \vec{B} fields, and propagation in linear media.
 - see upcoming tutorial
- Move to object oriented design



The Octopus code

• https://octopus-code.org

• https://gitlab.com/octopus-code



M.A.L. Marques, A. Castro, G. F. Bertsch, and A. Rubio, "octopus: a first-principles tool for excited electron-ion dynamics", Comput. Phys. Commun. 151, 60-78 (2003).

A. Castro, H. Appel, M.J.T. Oliveira, C.A. Rozzi, X. Andrade, F. Lorenzen, M.A.L. Marques, E.K.U. Gross, and A. Rubio, "octopus: a tool for the application of time-dependent density functional theory", *Phys. Stat. Sol. B* 243, 2465-2488 (2006).

X. Andrade, J. Alberdi-Rodriguez, D.A. Strubbe, M.J.T. Oliveira, F. Nogueira, A. Castro, J. Muguerza, A. Arruabarrena, S.G. Louie, A. Aspuru-Guzik, A. Rubio, and M.A.L. Marques, "Time-dependent density-functional theory in massively parallel computer architectures: the octopus project", *J. Phys.: Cond. Matt.* 24, 233202 (2012).

X. Andrade, D.A. Strubbe, U. De Giovannini, A.H. Larsen, M.J.T. Oliveira, J. Alberdi-Rodriguez, A. Varas, I. Theophilou, N. Helbig, M.J. Verstraete, L. Stella, F. Nogueira, A. Aspuru-Guzik, A. Castro, M.A.L. Marques, and A. Rubio, "Real-space grids and the Octopus code as tools for the development of new simulation approaches for electronic systems", *Phys. Chem. Chem. Phys.* **17**, 31371 (2015).

N. Tancogne-Dejean, M. J. T. Oliveira, X. Andrade, H. Appel, C. H. Borca, G. Le Breton, F. Buchholz, A. Castro, S. Corni, A. A. Correa, U. De Giovannini, A. Delgado, F. G. Eich, J. Flick, G. Gil, A. Gomez, N. Helbig, H. Hübener, R. Jestädt, J. Jornet-Somoza, A. H. Larsen, I. V. Lebedeva, M. Lüders, M. A. L. Marques, S. T. Ohlmann, S. Pipolo, M. Rampp, C. A. Rozzi, D. A. Strubbe, S. A. Sato, C. Schäfer, I. Theophilou, A. Welden, and A. Rubio, "Octopus, a computational framework for exploring light-driven phenomena and quantum dynamics in extended and finite systems", *The Journal of Chemical Physics* 152, 124119 (2020)

• Real-space grid representation



- Real-space grid representation
- Finite differences for the calculation of derivatives



- Real-space grid representation
- Finite differences for the calculation of derivatives
- Pseudopotential approximation



- Real-space grid representation
- Finite differences for the calculation of derivatives
- Pseudopotential approximation
- Finite systems



- Real-space grid representation
- Finite differences for the calculation of derivatives
- Pseudopotential approximation
- Finite systems
- Periodic systems



- Real-space grid representation
- Finite differences for the calculation of derivatives
- Pseudopotential approximation
- Finite systems
- Periodic systems
- Arbitrary number of dimensions



- Real-space grid representation
- Finite differences for the calculation of derivatives
- Pseudopotential approximation
- Finite systems
- Periodic systems
- Arbitrary number of dimensions
- Three methods to obtain excited states properties within TDDFT:



- Real-space grid representation
- Finite differences for the calculation of derivatives
- Pseudopotential approximation
- Finite systems
- Periodic systems
- Arbitrary number of dimensions
- Three methods to obtain excited states properties within TDDFT:
 - Real-time TDDFT



- Real-space grid representation
- Finite differences for the calculation of derivatives
- Pseudopotential approximation
- Finite systems
- Periodic systems
- Arbitrary number of dimensions
- Three methods to obtain excited states properties within TDDFT:
 - Real-time TDDFT
 - Casida Linear response



- Real-space grid representation
- Finite differences for the calculation of derivatives
- Pseudopotential approximation
- Finite systems
- Periodic systems
- Arbitrary number of dimensions
- Three methods to obtain excited states properties within TDDFT:
 - Real-time TDDFT
 - Casida Linear response
 - Sternheimer linear response



7 / 18

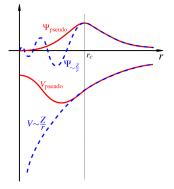
- Real-space grid representation
- Finite differences for the calculation of derivatives
- Pseudopotential approximation
- Finite systems
- Periodic systems
- Arbitrary number of dimensions
- Three methods to obtain excited states properties within TDDFT:
 - Real-time TDDFT
 - Casida Linear response
 - Sternheimer linear response
- Quantum optimal control theory



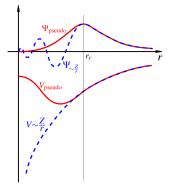
- Real-space grid representation
- Finite differences for the calculation of derivatives
- Pseudopotential approximation
- Finite systems
- Periodic systems
- Arbitrary number of dimensions
- Three methods to obtain excited states properties within TDDFT:
 - Real-time TDDFT
 - Casida Linear response
 - Sternheimer linear response
- Quantum optimal control theory
- Many other features



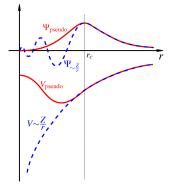
• The atomic potential is very strong and "hard" (small spacing or high plane-wave cutoff required).



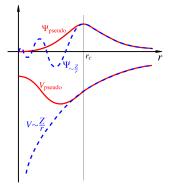
- The atomic potential is very strong and "hard" (small spacing or high plane-wave cutoff required).
- Core electrons are strongly bound and do not play a significant role in the chemical binding of atoms.



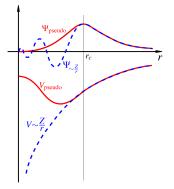
- The atomic potential is very strong and "hard" (small spacing or high plane-wave cutoff required).
- Core electrons are strongly bound and do not play a significant role in the chemical binding of atoms.
- Replace the potential and core electrons by a pseudo-potential.



- The atomic potential is very strong and "hard" (small spacing or high plane-wave cutoff required).
- Core electrons are strongly bound and do not play a significant role in the chemical binding of atoms.
- Replace the potential and core electrons by a pseudo-potential.



- The atomic potential is very strong and "hard" (small spacing or high plane-wave cutoff required).
- Core electrons are strongly bound and do not play a significant role in the chemical binding of atoms.
- Replace the potential and core electrons by a pseudo-potential.



Norm-conserving pseudo-potentials in Kleinman-Bylander form

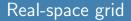
$$V = V_{\rm loc} + \sum_{lm} |lm\rangle (V_l - V_{\rm loc}) \langle lm|$$

• all electron potentials (delta or Gaussian nucleus)

- all electron potentials (delta or Gaussian nucleus)
- user defined potentials

- all electron potentials (delta or Gaussian nucleus)
- user defined potentials
- jellium spheres

- all electron potentials (delta or Gaussian nucleus)
- user defined potentials
- jellium spheres
- and others...



• We have a partial differential equation with infinite degrees of freedom

- We have a partial differential equation with infinite degrees of freedom
- We want a finite number of degrees of freedom

- We have a partial differential equation with infinite degrees of freedom
- We want a finite number of degrees of freedom
- Discretization: functions are represented by values on a set of points

- We have a partial differential equation with infinite degrees of freedom
- We want a finite number of degrees of freedom
- Discretization: functions are represented by values on a set of points
- Point distribution:

- We have a partial differential equation with infinite degrees of freedom
- We want a finite number of degrees of freedom
- Discretization: functions are represented by values on a set of points
- Point distribution:
 - Uniformly spaced grid

- We have a partial differential equation with infinite degrees of freedom
- We want a finite number of degrees of freedom
- Discretization: functions are represented by values on a set of points
- Point distribution:
 - Uniformly spaced grid
 - Distance between points is constant: Spacing

- We have a partial differential equation with infinite degrees of freedom
- We want a finite number of degrees of freedom
- Discretization: functions are represented by values on a set of points
- Point distribution:
 - Uniformly spaced grid
 - Distance between points is constant: Spacing
 - (Non-uniform grids also possible)

- We have a partial differential equation with infinite degrees of freedom
- We want a finite number of degrees of freedom
- Discretization: functions are represented by values on a set of points
- Point distribution:
 - Uniformly spaced grid
 - Distance between points is constant: Spacing
 - (Non-uniform grids also possible)
- Finite region of the space: Box

• Natural boundary conditions for different problems: zero, one, two, or three periodic dimensions for molecules, wires, sheets, and solids

- Natural boundary conditions for different problems: zero, one, two, or three periodic dimensions for molecules, wires, sheets, and solids
- Can systematically improve discretization quality:

- Natural boundary conditions for different problems: zero, one, two, or three periodic dimensions for molecules, wires, sheets, and solids
- Can systematically improve discretization quality:
 - Decrease the spacing

- Natural boundary conditions for different problems: zero, one, two, or three periodic dimensions for molecules, wires, sheets, and solids
- Can systematically improve discretization quality:
 - Decrease the spacing
 - Increase the box size (in finite directions)

- Natural boundary conditions for different problems: zero, one, two, or three periodic dimensions for molecules, wires, sheets, and solids
- Can systematically improve discretization quality:
 - Decrease the spacing
 - Increase the box size (in finite directions)
- Orthogonal "basis set"

- Natural boundary conditions for different problems: zero, one, two, or three periodic dimensions for molecules, wires, sheets, and solids
- Can systematically improve discretization quality:
 - Decrease the spacing
 - Increase the box size (in finite directions)
- Orthogonal "basis set"
- Unbiased, independent of atomic positions (no Pulay forces)

- Natural boundary conditions for different problems: zero, one, two, or three periodic dimensions for molecules, wires, sheets, and solids
- Can systematically improve discretization quality:
 - Decrease the spacing
 - Increase the box size (in finite directions)
- Orthogonal "basis set"
- Unbiased, independent of atomic positions (no Pulay forces)
- Problems:

- Natural boundary conditions for different problems: zero, one, two, or three periodic dimensions for molecules, wires, sheets, and solids
- Can systematically improve discretization quality:
 - Decrease the spacing
 - Increase the box size (in finite directions)
- Orthogonal "basis set"
- Unbiased, independent of atomic positions (no Pulay forces)
- Problems:
 - Breaking of translational invariance: egg-box effect

- Natural boundary conditions for different problems: zero, one, two, or three periodic dimensions for molecules, wires, sheets, and solids
- Can systematically improve discretization quality:
 - Decrease the spacing
 - Increase the box size (in finite directions)
- Orthogonal "basis set"
- Unbiased, independent of atomic positions (no Pulay forces)
- Problems:
 - Breaking of translational invariance: egg-box effect
 - Breaking of rotational invariance

- Natural boundary conditions for different problems: zero, one, two, or three periodic dimensions for molecules, wires, sheets, and solids
- Can systematically improve discretization quality:
 - Decrease the spacing
 - Increase the box size (in finite directions)
- Orthogonal "basis set"
- Unbiased, independent of atomic positions (no Pulay forces)
- Problems:
 - Breaking of translational invariance: egg-box effect
 - Breaking of rotational invariance
 - (Decreasing spacing helps both)

• Derivative at a point: weighted sum over neighboring points

- Derivative at a point: weighted sum over neighboring points
- General form for the Laplacian:

$$\nabla^2 f(n_x h, n_y h) = \sum_{i}^{n} \sum_{j}^{n} \frac{c_{ij}}{h} f(n_x h + ih, n_y h + jh)$$

- Derivative at a point: weighted sum over neighboring points
- General form for the Laplacian:

$$\nabla^2 f(n_x h, n_y h) = \sum_{i}^{n} \sum_{j}^{n} \frac{c_{ij}}{h} f(n_x h + ih, n_y h + jh)$$

• The coefficients c_{ij} depend on the mesh and number of points used: the stencil

- Derivative at a point: weighted sum over neighboring points
- General form for the Laplacian:

$$\nabla^2 f(n_x h, n_y h) = \sum_{i}^{n} \sum_{j}^{n} \frac{c_{ij}}{h} f(n_x h + ih, n_y h + jh)$$

- The coefficients c_{ij} depend on the mesh and number of points used: *the stencil*
- $\bullet \ \ \text{More points} \to \text{more precision}$

- Derivative at a point: weighted sum over neighboring points
- General form for the Laplacian:

$$\nabla^2 f(n_x h, n_y h) = \sum_{i}^{n} \sum_{j}^{n} \frac{c_{ij}}{h} f(n_x h + ih, n_y h + jh)$$

- The coefficients c_{ij} depend on the mesh and number of points used: *the stencil*
- More points \rightarrow more precision

- Derivative at a point: weighted sum over neighboring points
- General form for the Laplacian:

$$\nabla^2 f(n_x h, n_y h) = \sum_{i}^{n} \sum_{j}^{n} \frac{c_{ij}}{h} f(n_x h + ih, n_y h + jh)$$

- The coefficients c_{ij} depend on the mesh and number of points used: *the stencil*
- $\bullet \ \ \mathsf{More \ points} \to \mathsf{more \ precision}$

The Hamiltonian becomes a finite-size matrix

• Find the eigenvectors and eigenvalues of a matrix

- Find the eigenvectors and eigenvalues of a matrix
- Very large matrix with lots of zero components (Sparse)

- Find the eigenvectors and eigenvalues of a matrix
- Very large matrix with lots of zero components (Sparse)
- Use iterative solvers where only the application of the matrix is required (various options available in the code)

• Force functions to go to zero on the border of the box

- Force functions to go to zero on the border of the box
- The box has to be large enough to contain the functions

- Force functions to go to zero on the border of the box
- The box has to be large enough to contain the functions
- Other BCs are possible:

- Force functions to go to zero on the border of the box
- The box has to be large enough to contain the functions
- Other BCs are possible:
 - periodic

- Force functions to go to zero on the border of the box
- The box has to be large enough to contain the functions

• Other BCs are possible:

- periodic
- zero derivative

- Force functions to go to zero on the border of the box
- The box has to be large enough to contain the functions

• Other BCs are possible:

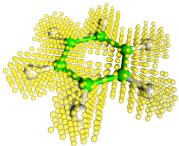
- periodic
- zero derivative
- absorbing

- Force functions to go to zero on the border of the box
- The box has to be large enough to contain the functions

• Other BCs are possible:

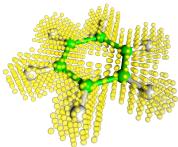
- periodic
- zero derivative
- absorbing
- etc

- Optimize the shape of the box to minimize the number of points needed
- Available box shapes:



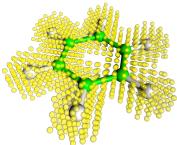
Benzene molecule in minimal box

- Optimize the shape of the box to minimize the number of points needed
- Available box shapes:
 - Sphere



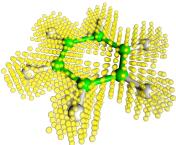
Benzene molecule in minimal box

- Optimize the shape of the box to minimize the number of points needed
- Available box shapes:
 - Sphere
 - Cylinder



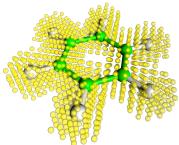
Benzene molecule in minimal box

- Optimize the shape of the box to minimize the number of points needed
- Available box shapes:
 - Sphere
 - Cylinder
 - Parallelepiped



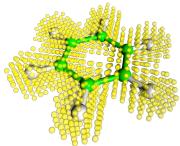
Benzene molecule in minimal box

- Optimize the shape of the box to minimize the number of points needed
- Available box shapes:
 - Sphere
 - Cylinder
 - Parallelepiped
 - Minimum box: union of spheres around each atom



Benzene molecule in minimal box

- Optimize the shape of the box to minimize the number of points needed
- Available box shapes:
 - Sphere
 - Cylinder
 - Parallelepiped
 - Minimum box: union of spheres around each atom
 - Arbitrary (e.g. 2D image!)



Benzene molecule in minimal box

$$\varphi_i(\boldsymbol{r}, t + \Delta t) = \hat{T} \exp\left\{-\mathrm{i} \int_t^{t + \Delta t} \mathrm{d}t \, \hat{H} \varphi_i(\boldsymbol{r}, t)\right\}$$

• Several numerical methods available for doing the time-propagation

$$\varphi_i(\boldsymbol{r}, t + \Delta t) = \hat{T} \exp\left\{-\mathrm{i} \int_t^{t + \Delta t} \mathrm{d}t \, \hat{H} \varphi_i(\boldsymbol{r}, t)\right\}$$

Several numerical methods available for doing the time-propagation
Exponential must also be calculated numerically

$$\varphi_i(\boldsymbol{r}, t + \Delta t) = \hat{T} \exp\left\{-\mathrm{i} \int_t^{t + \Delta t} \mathrm{d}t \, \hat{H} \varphi_i(\boldsymbol{r}, t)\right\}$$

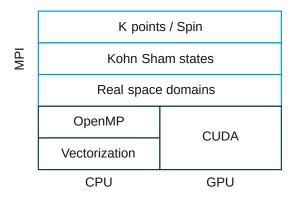
- Several numerical methods available for doing the time-propagation
- Exponential must also be calculated numerically
- Many properties can be obtained

$$\varphi_i(\boldsymbol{r}, t + \Delta t) = \hat{T} \exp\left\{-\mathrm{i} \int_t^{t + \Delta t} \mathrm{d}t \, \hat{H} \varphi_i(\boldsymbol{r}, t)\right\}$$

- Several numerical methods available for doing the time-propagation
- Exponential must also be calculated numerically
- Many properties can be obtained
- Response to time-dependent fields: lasers

$$\varphi_i(\boldsymbol{r}, t + \Delta t) = \hat{T} \exp\left\{-\mathrm{i} \int_t^{t+\Delta t} \mathrm{d}t \, \hat{H} \varphi_i(\boldsymbol{r}, t)\right\}$$

- Several numerical methods available for doing the time-propagation
- Exponential must also be calculated numerically
- Many properties can be obtained
- Response to time-dependent fields: lasers
- See e.g. tutorials on optical absorption



You can find the tutorials under this link: https://www.octopus-code.org/documentation/16/tutorial/ You can find the tutorials under this link: https://www.octopus-code.org/documentation/16/tutorial/

Have fun!

