Emerging topological phases of interacting driven-dissipative systems

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INSTITUTO DE FÍSICA FUNDAMENTAL

CSIC
Consejo Superior de Investigaciones Científicas

Outline

Part I
- Introduction: topology in quantum dissipative systems
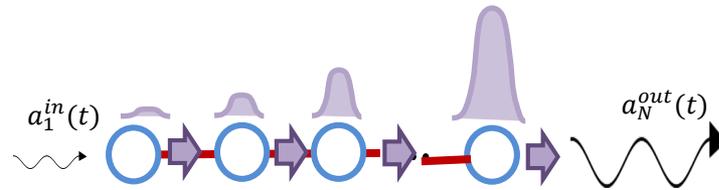- Topological amplification
- Superconducting circuit implementation

Part II
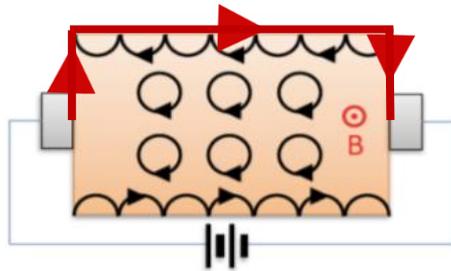- Emergent topology in the chiral driven-dissipative Bose-Hubbard model

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Part I:
Topological amplification

Topology in quantum dissipative systems
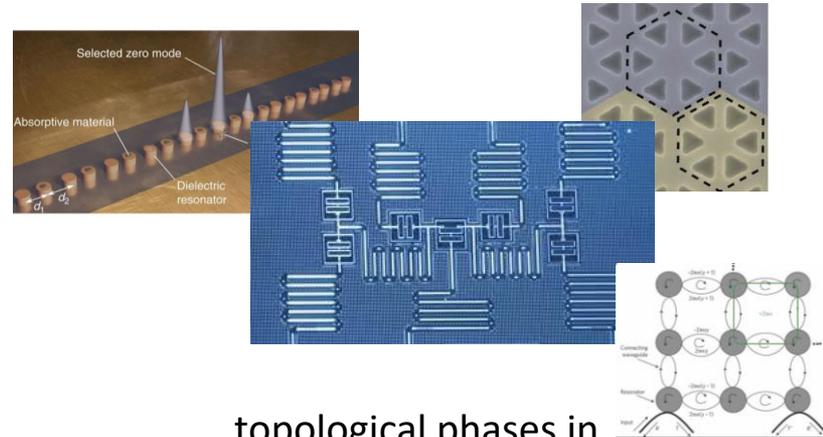
topological phases in
closed electronic systems

conductivity

conductivity $\sigma_{xy} = C \dfrac{e^2}{h}$

Chern number
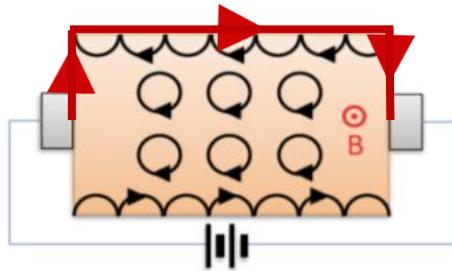
topological phases in
open photonic systems

- non-Hermitian modes (although not easy to address or measure)

- Motivate topological invariants with **quantum optical response functions**?

Topology in quantum dissipative systems

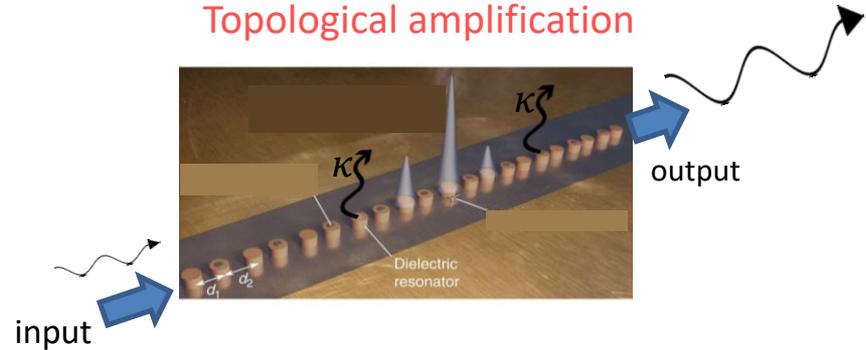topological phases in
closed electronic systems

conductivity

conductivity $\sigma_{xy} = C \dfrac{e^2}{h}$

Chern number

Topological amplification

input

output

Transmission & amplification of light

topological properties of **open photonic** system

DP, S. Fernández-Lorenzo PRL **122**, 143901 (2019)

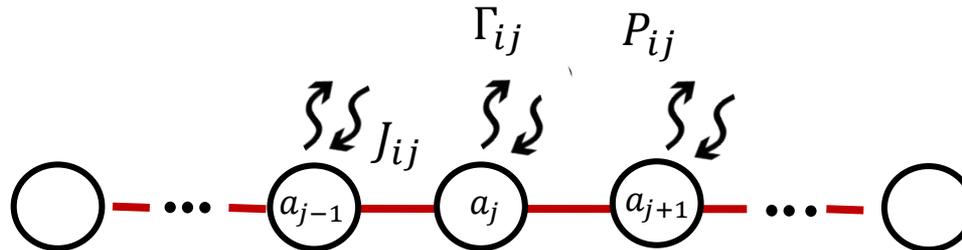C.C. Wanjura, M. Brunelli, A. Nunnenkamp, Nat. Comm. 11, 3149 (2020)

Topological input-output theory

- Consider quadratic bosonic Liouvillians of the form

$$\mathcal{L}(\rho) = -i\left[\sum_{ij} J_{ij} a_i^+ a_j, \rho\right] + \sum_{ij} \frac{\Gamma_{ij}}{2}(a_i \rho a_j^+ - a_j^+ a_i \rho - \rho a_j^+ a_i)$$

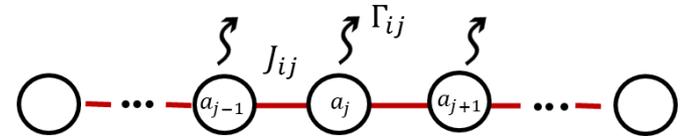$$+ \sum_{ij} \frac{P_{ij}}{2}(a_i^+ \rho a_j - a_j \ a_i^+ \rho - \rho a_j \ a_i^+)$$

boson hopping terms
(unitary)

Dissipative terms: diagonal or
collective decay/pump

Topological input-output theory

- Input-Output formalism: linear equations of motion

$$\dot{a}_j(t) = -i \sum_l H_{jl}\, a_l + \xi_j^{in}(t)$$

Langevin forces
(stochastic term)

Non-Hermitian coupling
matrix (dynamical matrix)

$$H_{jl} = G_{ij} + i\left(-\frac{\Gamma_{ij}}{2} + \frac{P_{ij}}{2}\right)$$

- Solution in frequency-space: Green's function

$$a_j(\omega) = i \sum_l G_{jl}(\omega)\, \xi_l^{in}(\omega)$$

$$G(\omega) = \frac{1}{\omega\mathbf{1} - H}$$

Green's function → input-output relations

Topological?

Analysing the non-Hermitian dynamical matrix, what eigensystem?

- Let us focus on the Green's function at $\omega = 0$

- The Green's function is the inverse of the dynamical matrix

$$G(0) = -\frac{1}{H}$$

- Analysis in terms of the eigenstates of H is problematic:

$$H = V^{-1} D V \quad \Rightarrow \quad \begin{array}{l} H \text{ is non-Hermitian} \\ \text{eigenstates} \neq \text{orthonormal basis} \end{array}$$

- Instead, use the singular value decomposition

$$H = U S V^{\dagger} \quad \Rightarrow \quad \begin{array}{l} U, V \text{ orthonormal basis (singular vectors)} \\ S \text{ (singular values)} \end{array}$$

$$G(0) \propto V S^{-1} U^{\dagger}$$

Analysing the non-Hermitian dynamical matrix, what eigensystem?

- The singular vectors/values have a neat interpretation for input-output theory:

$$G(0) \propto V \, S^{-1} U^\dagger$$

$$a_j(0) \propto \sum_l G_{jl}(0) \, \xi_l^{in}(0)$$

$$a_j \propto \sum_{l.n} V_{jn} \frac{1}{s_n} U_{ln}^* \, \xi_l^{in}$$

OUTPUT ⬅

singular vector V_{jn} = n'th output channel

INPUT ⬅

singular vector U_{jn} = n'th input channel

s_n^{-1} = inverse singular values are the strength of the channel

Topological properties from the SVD

- The SVD is deeply connecting to topological properties (DP, S. Fernández-Lorenzo PRL **122**, 143901 (2019))

- Singular vectors of H = eigenvectors of an effective Hamiltonian $\mathcal{H}$:

$$H = USV^\dagger \qquad \Longleftrightarrow \qquad \mathcal{H} = \begin{pmatrix} 0 & H \\ H^\dagger & 0 \end{pmatrix}$$

$$\mathcal{H} \begin{pmatrix} U \\ V \end{pmatrix} = \begin{pmatrix} 0 & H \\ H^\dagger & 0 \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix} = \begin{pmatrix} USV^\dagger V \\ VSU^\dagger U \end{pmatrix} = \begin{pmatrix} U\,S \\ V\,S \end{pmatrix}$$

Topological properties from the SVD

- The SVD is deeply connecting to topological properties (DP, S. Fernández-Lorenzo PRL **122**, 143901 (2019))

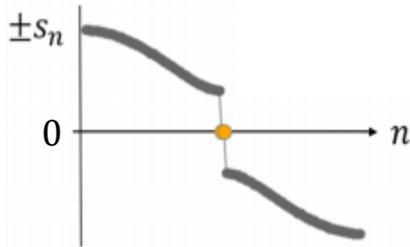- Singular vectors of H = eigenvectors of an effective Hamiltonian $\mathcal{H}$:

$$H = USV^\dagger \qquad \Longleftrightarrow \qquad \mathcal{H} = \begin{pmatrix} 0 & H \\ H^\dagger & 0 \end{pmatrix}$$

- Chiral symmetry of $\mathcal{H}$ $\qquad \Longrightarrow \qquad$ Winding number: $\nu = \int \dfrac{dk}{2\pi i} \partial_k \log(H(\mathrm{k}))$
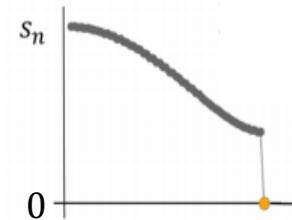
If $\nu \neq 0$ $\qquad \mathcal{H} = \begin{pmatrix} 0 & H \\ H^\dagger & 0 \end{pmatrix}$ $\qquad\qquad H = USV^\dagger$

Zero eigenvalue pairs $\pm s_0$

Zero singular values s_0

Edge singular vectors $\boldsymbol{v}_0 , \boldsymbol{u}_0 \rightarrow$ *Left /Right Edge states*

Topological properties from the SVD

- Zero-singular values dominate the Green's function and lead to exponential amplification of incoming signals.

$$G_{jk}(\omega) = \left(\frac{1}{\omega\mathbf{1} - H}\right)_{jk} = \sum_n (v_n)_j \frac{1}{s_n} (u_n^*)_k$$

- In the general ($\omega \neq 0$) case, we get a frequency dependent winding number

$$\nu(\omega) = \int \frac{dk}{2\pi i} \partial_k \log(\omega\mathbf{1} - H(\mathrm{k}))$$

DP, S. Fernández-Lorenzo PRL **122**, 143901 (2019)

T. Ramos, J.J. García-Ripoll, DP, Phys. Rev. A **103**, 033513 (2021)

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$$G_{jk}(\omega) = \left(\frac{1}{\omega\mathbf{1}-H}\right)_{jk} = \sum_n (v_n)_j \frac{1}{s_n}(u_n^*)_k$$

If $\nu(\omega) \neq 0 \rightarrow s_0 \propto e^{-N/\xi}$

$$G_{jk}(\omega) = \left(\frac{1}{\omega\mathbf{1}-H}\right)_{jk} \approx (v_0)_j (u_0^*)_k e^{+N/\xi}$$

Inverse zero-singular value dominates

- Localized left/rigth edge-singular vectors:

$$a_j(\omega) \propto (v_0)_j \sum_l (u_0^*)_l e^{+N/\xi} \xi_l^{in}(\omega)$$
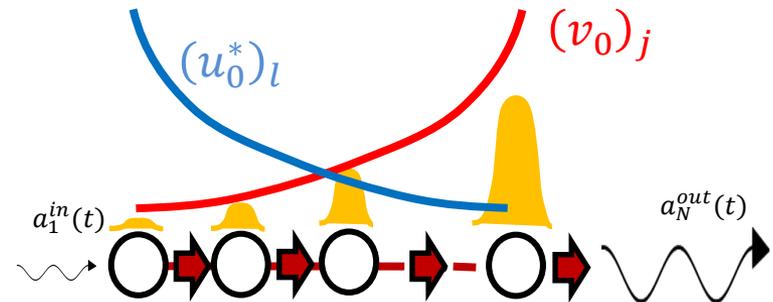
$(u_0^*)_l$ $(v_0)_j$

$a_1^{in}(t)$ $a_N^{out}(t)$

DP, S. Fernández-Lorenzo PRL **122**, 143901 (2019)

T. Ramos, J.J. García-Ripoll, DP, Phys. Rev. A **103**, 033513 (2021)

Topological properties from the SVD

- To summarize:

    o The singular value decomposition allow us to classify topological phases of quadratic bosonic Liouvillians

    o A mapping from the SVD to an effective "doubled Hamiltonian" allows us to use Hermitian topological insulator

    o **Topological zero singular values = Directional amplification**

    o Topological singular edge-states → input/output signals localized at the edges.

    o Topology does not require an internal symmetry → **intrinsic chiral symmetry** of the SVD

DP, S. Fernández-Lorenzo PRL **122**, 143901 (2019)

T. Ramos, J.J. García-Ripoll, DP, Phys. Rev. A **103**, 033513 (2021)

Example: Hatano-Nelson chain

- Quantum Hatano-Nelson chain: dissipative couplings + complex photon tunneling terms
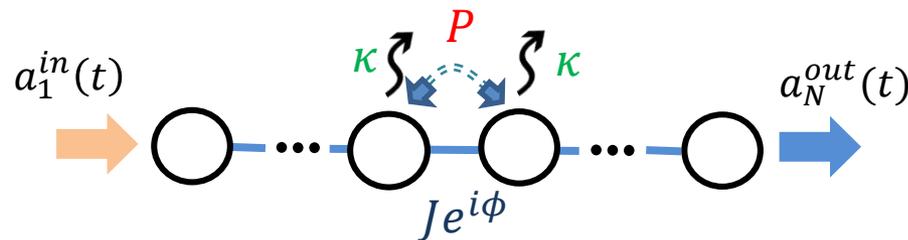
$$P_{j,j+1} = P$$

$$P_{j,j} = 2\,P$$

Collective photon gain

κ local decay

$Je^{i\phi}$ Photon tunneling

$$\mathcal{L}(\rho) = \frac{P}{2} \sum_j 2\left(a_j^\dagger - a_{j+1}^\dagger\right)\rho\left(a_j - a_{j+1}\right) - \cdots$$

$$+ \frac{\kappa}{2} \sum_j \left(2\,a_j\rho a_j^\dagger - a_j^\dagger a_j\rho - \rho a_j^\dagger a_j\right)$$

$$-\,i\,[H,\rho])$$

$$H = \sum_j \left(Je^{i\phi}\,a_j^\dagger a_{j+1} + \mathrm{H.c.}\right)$$

Example: Hatano-Nelson chain

- Quantum Hatano-Nelson chain: dissipative couplings + complex photon tunneling terms
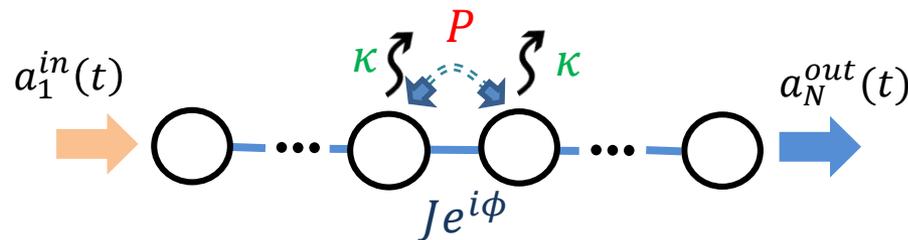
$$P_{j,j+1} = P$$
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Collective photon gain

κ local decay

$J e^{i\phi}$ Photon tunneling

$$\frac{d\langle a_j \rangle}{dt} = J_{\text{left}}\langle a_{j-1} \rangle + J_{\text{right}}\langle a_{j+1} \rangle$$
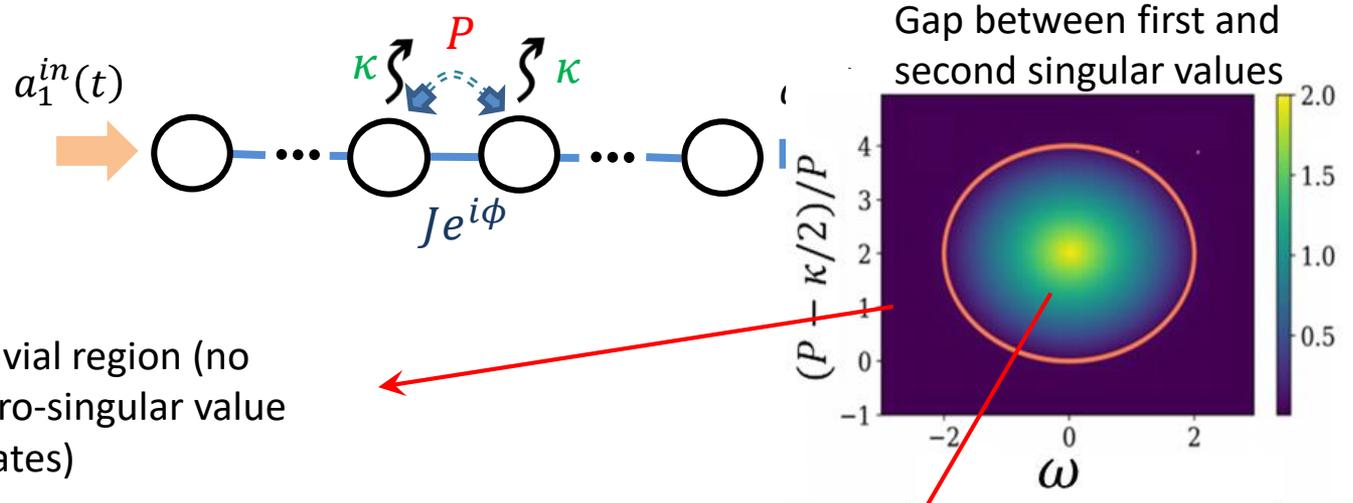
$$J_{\text{left}} = -i\,J e^{i\phi} + P$$
$$J_{\text{right}} = -i\,J e^{-i\phi} + P$$
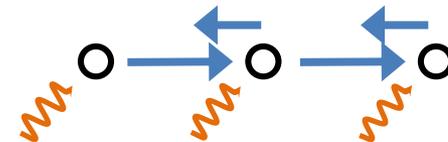
Example: Hatano-Nelson chain

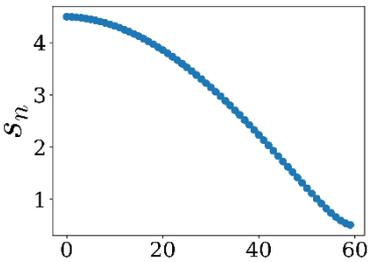T. Ramos, J.J. García-Ripoll, DP, Phys. Rev. A **103**, 033513 (2021)

- Quantum Hatano-Nelson chain: dissipative couplings + complex photon tunneling terms

Gap between first and second singular values

Trivial region (no zero-singular value states)

Topological region: singular edge-vectors and topological amplification

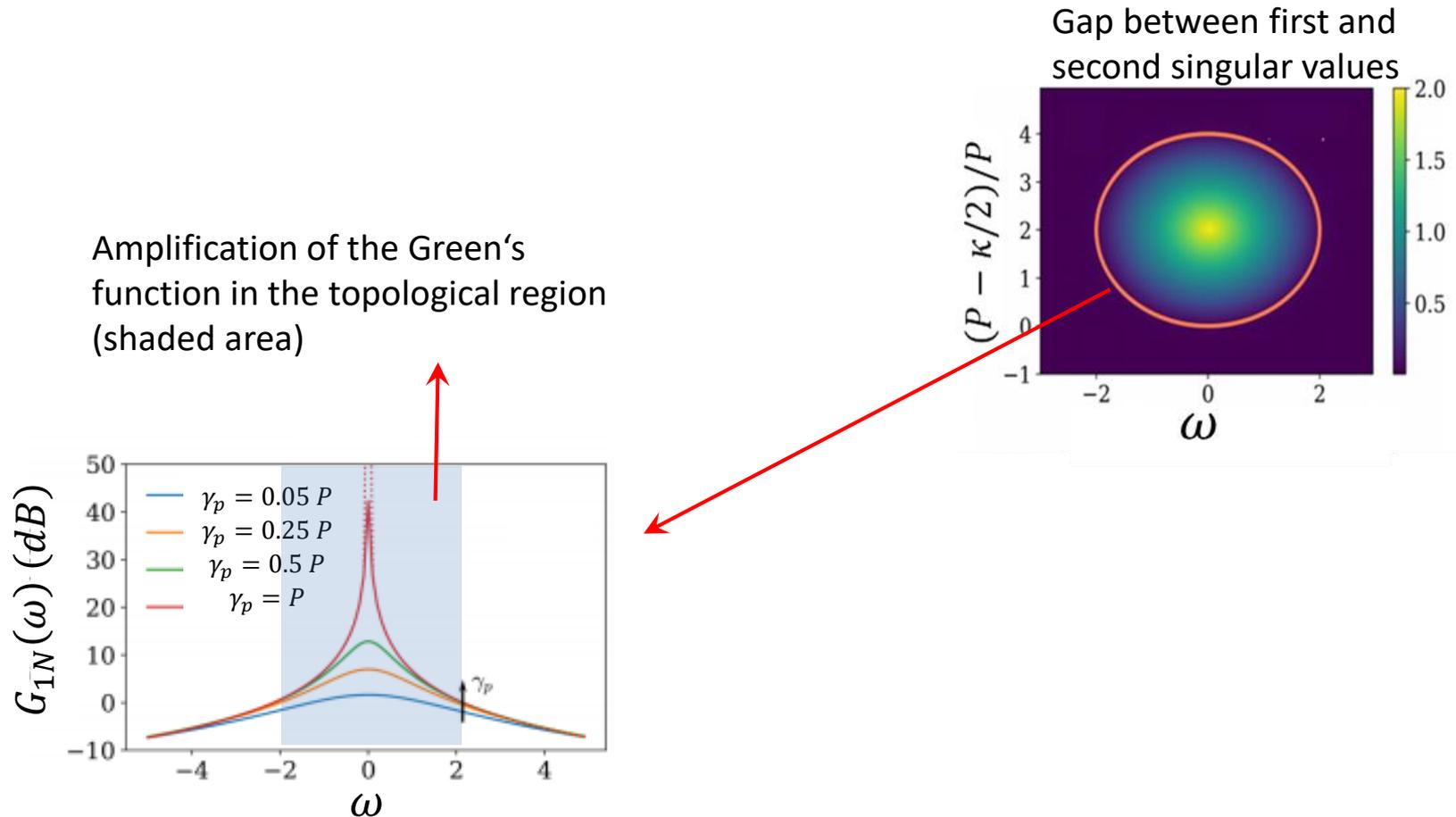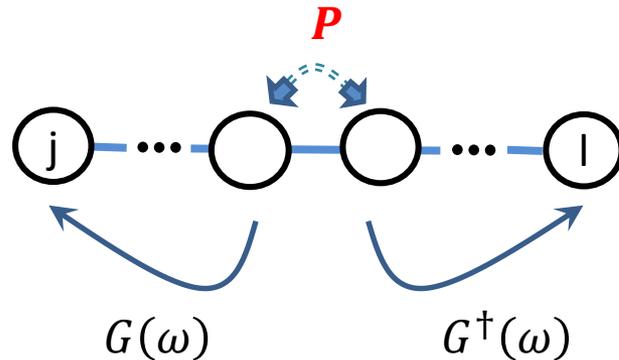$$G_{jl}(\omega) \propto e^{N/\xi(\omega)} \, (v_0)_j (u_0)_l$$
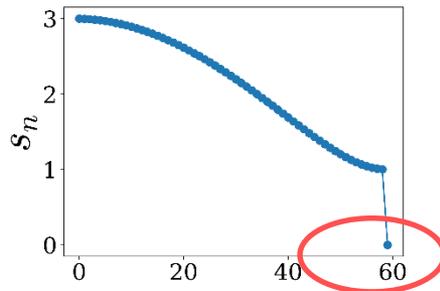
Example: Hatano-Nelson chain

T. Ramos, J.J. García-Ripoll, DP, Phys. Rev. A **103**, 033513 (2021)

- Quantum Hatano-Nelson chain: dissipative couplings + complex photon tunneling terms

Gap between first and second singular values

Amplification of the Green's function in the topological region (shaded area)

Topological amplification of quantum fluctuations

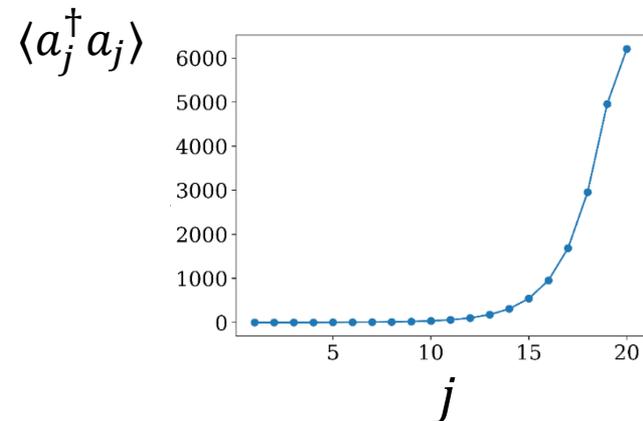- The Green's function also determines steady-state quantum fluctuations

$$M_{jl}(\omega) = \langle a_j^\dagger(\omega) a_l(\omega) \rangle = \left(G^*(\omega)\, \boldsymbol{P}\, G^T(\omega) \right)_{jl}$$

Propagation of excitations generated by the gain term (matrix P)

- In the topological phase, boson fluctuations also get amplified

- Steady-state (incoherent) bosons in the quantum Hatano-Nelson model replicate the singular edge- vector structure

$$\boldsymbol{P}$$

j ... ○ ○ ... l

$$G(\omega) \qquad G^\dagger(\omega)$$

$\langle a_j^\dagger a_j \rangle$

Bridging the gap between topological non-Hermitian physics and open quantum system,
Á. Gómez-León, T. Ramos, A. González-Tudela, DP, Phys. Rev. A 106, L011501 (2022)

How to implement a "topological amplifier"?

- Ingredients for topological amplification in the Hatano-Nelson model

  o **Breaking time-reversal invariance (complex photon tunneling rates)**
  o **Incoherent pumping**
  o **Collective dissipation (reservoir engineering)**

- Not impossible, e.g. with superconducting circuits, but challenging!!

How to implement a "topological amplifier"?

- Simpler setup for topological amplification? → coupled parametric amplifiers
  (motivated by Josephson TWPA's)
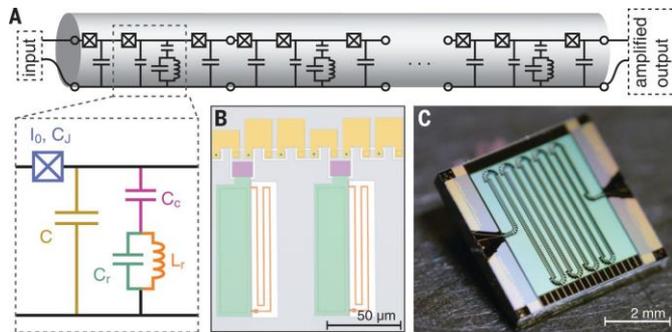
Martina Esposito, Arpit Ranadive, Luca Planat, and Nicolas Roch,
*Perspective on traveling wave microwave parametric amplifiers
featured*,   Appl. Phys. Lett. **119**, 120501 (2021)

- Paremetric terms in these systems of the form $a_j^2, a_j a_{j+1}, \ldots$ generate photons
  (no need for incoherent pump)

- We will see that no synthetic gauge fields are also not necessary.

Topological amplification in parametric chains

- "Bose-Kitaev dissipative chain: coherent photon tunnelling (J_a) and parametric interactions (local terms, g_s, and nearest-neighbor, g_c)
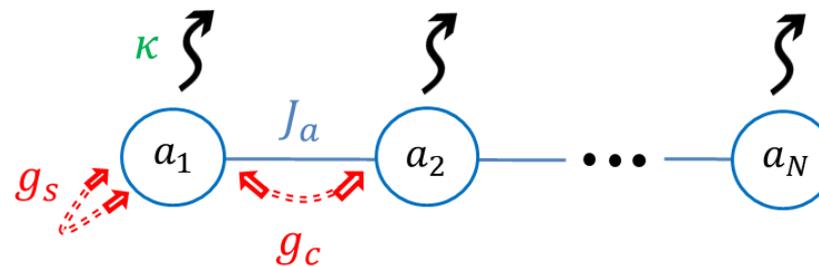
$$\dot{\rho} = -i\,[H,\rho] + \mathcal{L}_\mathrm{d}(\rho)$$

parametric couplings

photon tunneling

$$H = \sum_j \left(J_a\, a_j^+ a_{j+1} e^{-i\theta} + H.c.\right) + g_s \sum_j \left(a_j^2 + a^{+\,2}_j\right) + g_c \sum_j \left(a_j a_{j+1} + a_j^+ a_{j+1}^+\right)$$

$$\mathcal{L}_\mathrm{d}(\rho) = \sum_{j,k} \kappa\,\left(a_j \rho a_j^+ - \frac{1}{2} a_j^+ a_j \rho - \frac{1}{2}\rho a_j^+ a_j\right)$$

collective decay

Topological amplification in parametric chains

A. Gómez-León, T Ramos, A González-Tudela, DP, Quantum **7**, 1016 (2023)
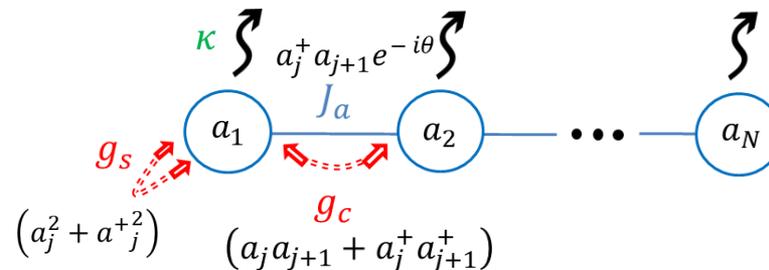
- Bogoliubov topological input-output theory:

$$\dot{a}_j = \sum_l (\Gamma_{jl} - i J_{jl} - i \epsilon_j \delta_{jl}) a_l - i \sum_l K_{jl} a_l^+ + \xi_j^{in}$$

$$\begin{pmatrix} a_j(\omega) \\ a_j^+(-\omega) \end{pmatrix} = i \sum_l H_{jl}(\omega) \begin{pmatrix} \xi_l(\omega) \\ \xi_l^+(-\omega) \end{pmatrix}$$

$$W_1(\omega) = \int \frac{dk}{2\pi i} \text{Tr}\{\partial_k(\omega - H(k))\}$$

Topological amplification region
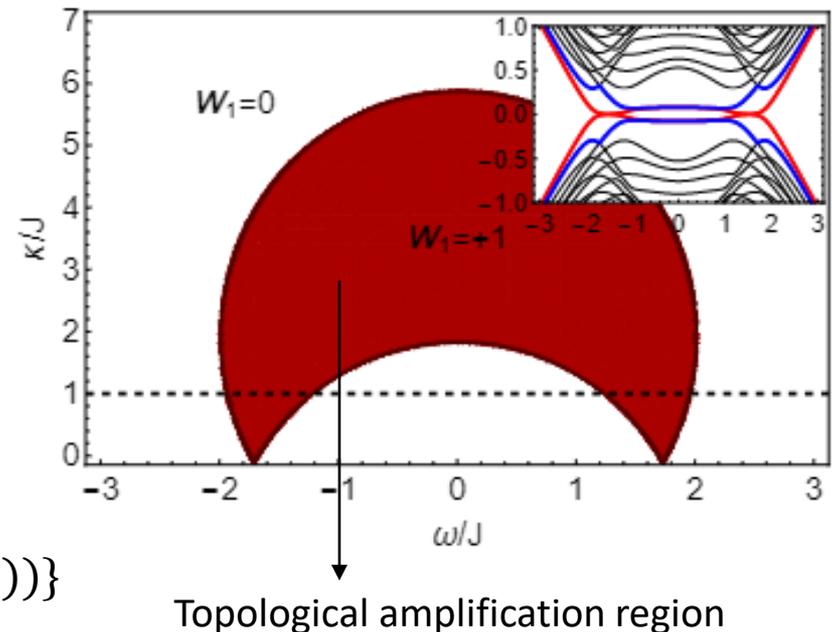
Long-range topological order

M. Clavero, T Ramos, DP, in preparation

- One of the signatures of topological amplification is the emergence of long-range order in frequency-resolved two-point correlations

$$\langle a_j^+(\omega)a_l(\omega)\rangle = \left(G^*(\omega)\, \boldsymbol{P}\, G^T(\omega)\right)_{jl} \propto (v_0)^*_{\ j}(u_0)_l$$

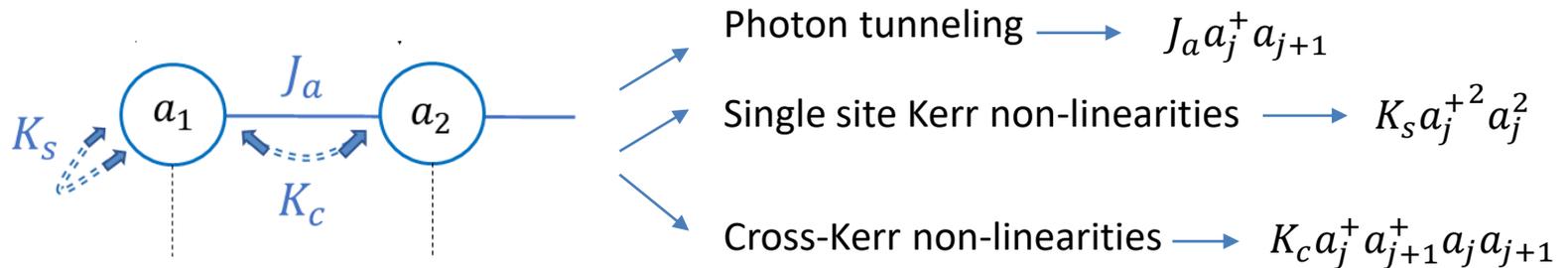- In the topological amplification regime, the Green's function is dominated by a single singular value, which leads to a "topological synchronization effect"

$$\text{If } v(\omega) \neq 0: \ \bar{N}_{ij}(\omega) = \frac{\langle a_j^+(\omega)a_l(\omega)\rangle}{\sqrt{n_i(\omega)n_j(\omega)}} \rightarrow 1$$
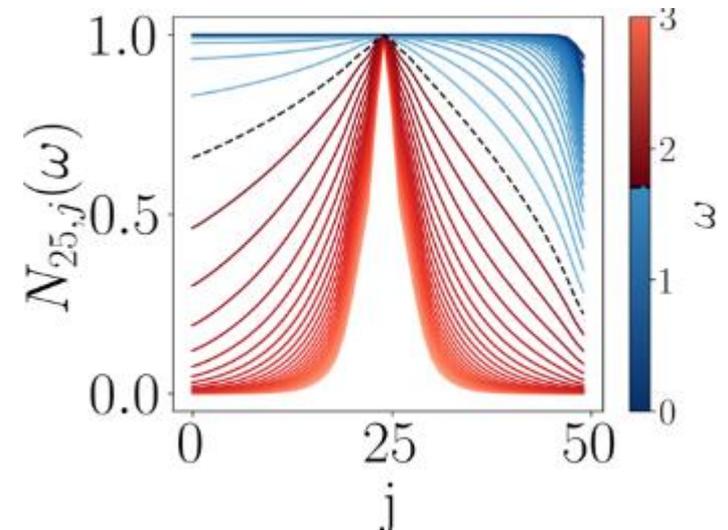
Implementation in circuit QED: Kerr non-linearities + coherent drive with a site-dependent phase

- No need of Floquet or reservoir engineering in a circuit QED setup: everything comes from driven Kerr non-linearities

Photon tunneling $\longrightarrow J_a a_j^+ a_{j+1}$

Single site Kerr non-linearities $\longrightarrow K_s a_j^{+2} a_j^2$

Cross-Kerr non-linearities $\longrightarrow K_c a_j^+ a_{j+1}^+ a_j a_{j+1}$

- Coherent driving by a travelling field in a nearby cavity array
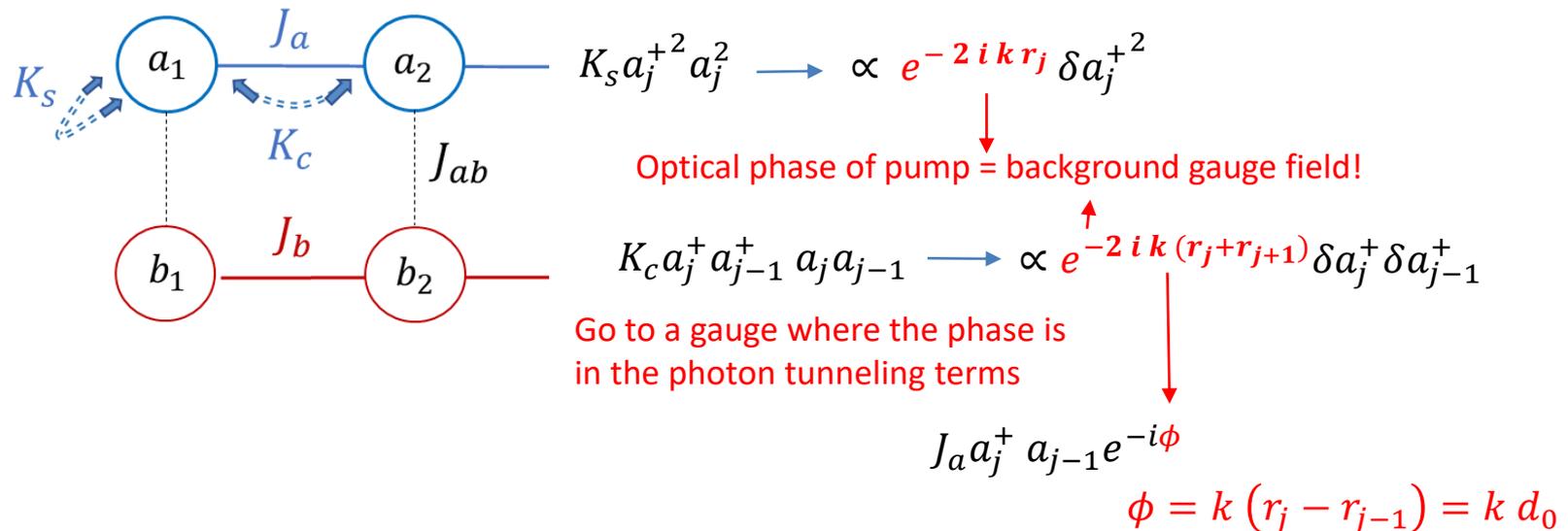
$K_s a_j^{+2} a_j^2 \longrightarrow \propto e^{-2\,i\,k\,r_j}\,\delta a_j^{+2}$

Optical phase of pump = background gauge field!

$K_c a_j^+ a_{j-1}^+ \, a_j a_{j-1} \longrightarrow \propto e^{-2\,i\,k\,(r_j+r_{j+1})}\,\delta a_j^+ \delta a_{j-1}^+$

Go to a gauge where the phase is in the photon tunneling terms

$J_a a_j^+ \, a_{j-1} e^{-i\phi}$

$\phi = k\,(r_j - r_{j-1}) = k\,d_0$

Implementation in circuit QED: Kerr non-linearities + coherent drive with a site-dependent phase

T. Ramos, A Gómez-León, JJ García-Ripoll, A González-Tudela, DP, arXiv:2207.13728
[European patent (approved): EP24382293.9]

Exponential amplification in forward gain
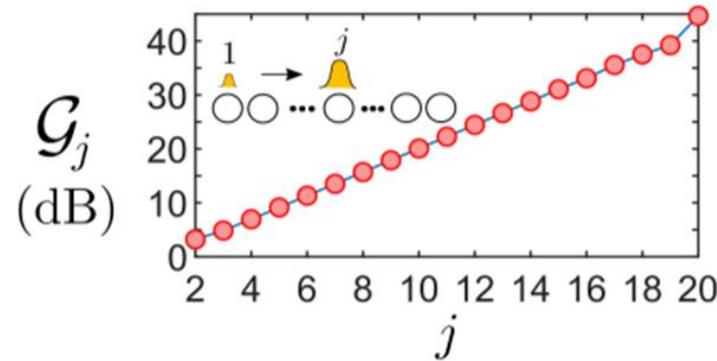
Exponential suppression in reverse gain
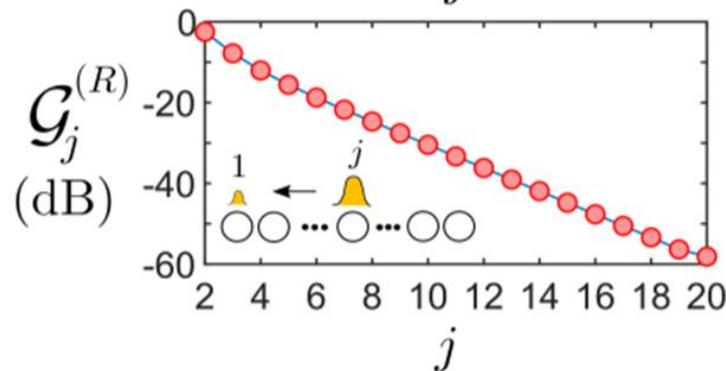

Tomás Ramos (IFF-CSIC)

Implementation in circuit QED: Kerr non-linearities + coherent drive with a site-dependent phase

T. Ramos, A Gómez-León, JJ García-Ripoll, A González-Tudela, DP, arXiv:2207.13728
[European patent (approved): EP24382293.9]

- ✓ Directional amplifier

- ✓ Broadband (bandwidth depends on inter JJ couplings)

- ✓ Quantum-limited (for long enough chains)

Example: N = 10 sites (bandwidth of the order of GHz)
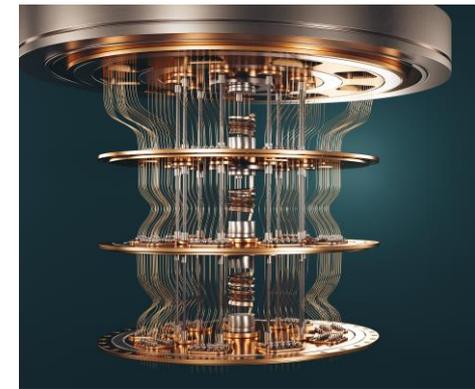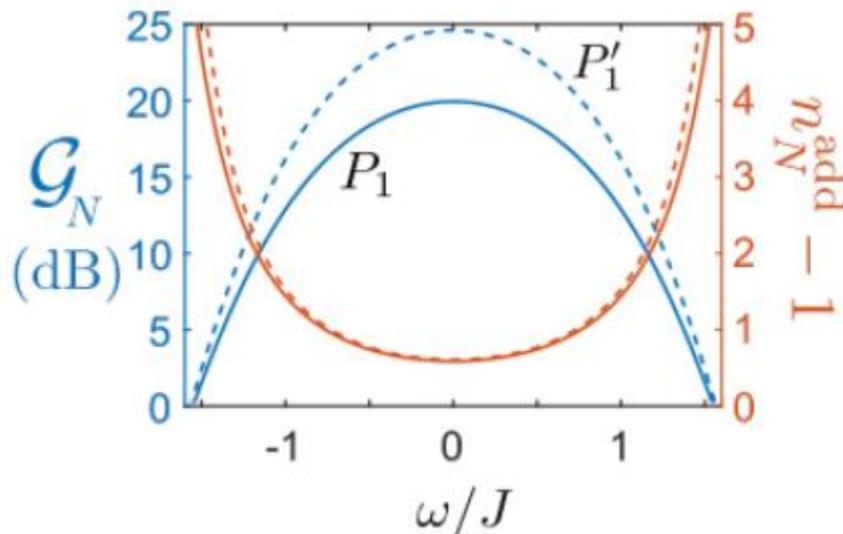
Applications: on-chip amplifier for read-out of quantum states

On-going collaboration with Dian Tan's group at Shanghai Instute of Technology

Part II:
Chiral driven-dissipative Bose-Hubbard chains

Chiral driven-dissipative Bose-Hubbard model

- So far, we have defined topological amplification phases are defined in quadratic (Gaussian) Liouvillians.

- Driven-dissipative Bose-Hubbard chains may exhibit those phases in certain regions of the non-equilibrium steady-state phase diagram

- Example: chiral driven-dissipative Bose-Hubbard chain:

$$\kappa \qquad J \qquad U\left(a_j^+ a_j\right)^2$$

$$\epsilon_j e^{i \phi j}$$

Chiral driven-dissipative Bose-Hubbard model

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- Example: chiral driven-dissipative Bose-Hubbard chain:

$$H = \sum_j \Delta_j a_j^+ a_j + J \sum_j a_j^+ a_{j+1} + U \sum_j (a_j^+ a_j)^2 + \sum_j \epsilon_j (a_j^+ e^{i\phi j} + a_j e^{-i\phi j})$$

$$\mathcal{L}(\rho) = -i[H, \rho] + \kappa \sum_j (2\, a_j \rho a_j^+ - a_j^+ a_j \rho - \rho a_j^+ a_j)$$

- We expect similar physics to parametric chains:

$$U \sum_j (a_j^+ a_j)^2 \rightarrow U \langle a_j^+ \rangle^2 a_j^2 + U \langle a_j \rangle^2 (a_j^+)^2$$

Chiral driven-dissipative Bose-Hubbard model

- Topological amplification phases are defined in quadratic (Gaussian) Liouvillians.

- Driven-dissipative Bose-Hubbard chains may exhibit those phases in certain regions of the non-equilibrium steady-state phase diagram

- Example: chiral driven-dissipative Bose-Hubbard chain:

$$H = \sum_j \Delta_j a_j^+ a_j + J \sum_j a_j^+ a_{j+1} + U \sum_j (a_j^+ a_j)^2 + \sum_j \epsilon_j \left(a_j^+ e^{i\phi j} + a_j e^{-i\phi j} \right)$$

$$\mathcal{L}(\rho) = -i[H, \rho] + \kappa \sum_j (2 a_j \rho a_j^+ - a_j^+ a_j \rho - \rho a_j^+ a_j)$$

- In the limit $|\epsilon_j| \gg U \rightarrow |\langle a_j \rangle| \gg 1 \rightarrow$ Variational Gaussian ansatz

$$\langle a_j a_k a_l \rangle \approx \langle a_j a_k \rangle \langle a_l \rangle + \langle a_j \rangle \langle a_k a_l \rangle + \cdots \quad \text{(Wick's theorem)}$$

Tao Shi, E. Demler, I. Cirac, Annals of Physics **390**, 245 (2018)
R. Menu, T. Roscilde, SciPost Phys. **14**, 151 (2023)

Variational Gaussian ansatz predictions ($\phi = 0$)

$$H = \Delta \sum_j a_j^+ a_j + J \sum_j a_j^+ a_{j+1} + U \sum_j (a_j^+ a_j)^2 + \sum_j \epsilon_j (a_j^+ + a_j)$$

- Let us review what happens in the "non-chiral" version of this system. In the steady-state, our Gaussian ansatz predicts a first-order phase transition
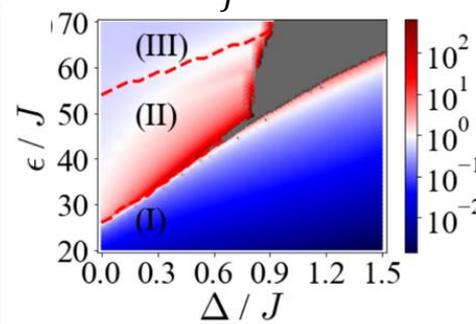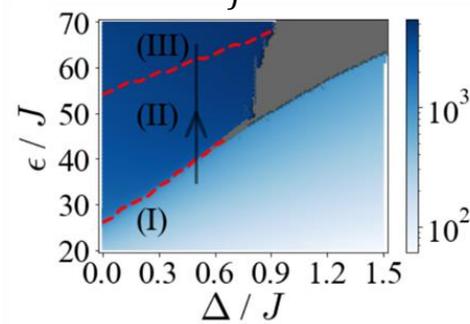
Large boson number phase ("bright")

$$\frac{1}{N} \sum_j |\langle a_j \rangle|^2$$

$U = 10^{-4} J$

$\times 10^3$
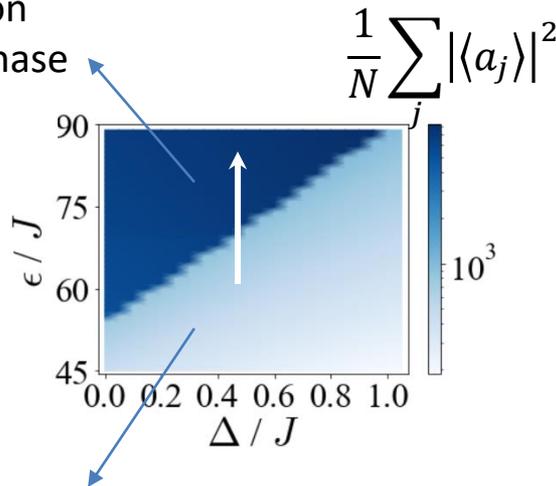
$$\frac{1}{N} \sum_j |\langle a_j \rangle|^2$$

Small boson number phase ("dark")
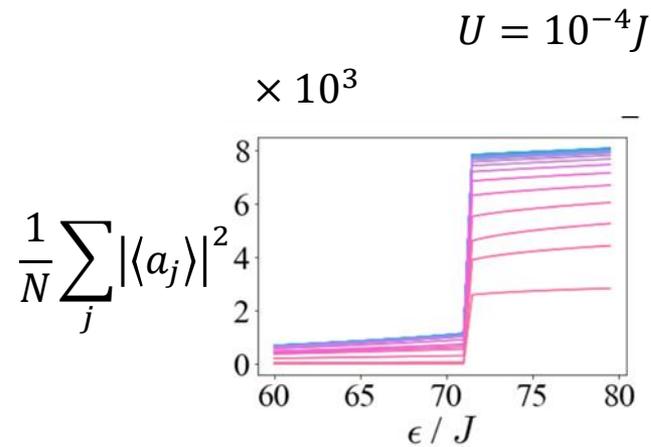
Variational Gaussian ansatz predictions ($\phi \neq 0$)

$$H = \Delta \sum_j a_j^+ a_j + J \sum_j a_j^+ a_{j+1} + U \sum_j (a_j^+ a_j)^2 + \sum_j \epsilon_j \, (a_j^+ e^{i\,\phi\,j} + a_j e^{-i\phi j})$$
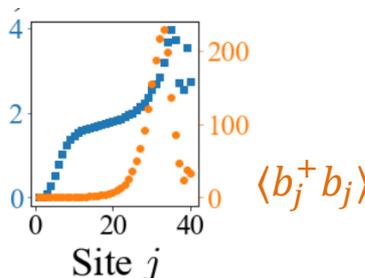
$$\frac{1}{N} \sum_j |\langle a_j \rangle|^2 \qquad \frac{1}{N} \sum_j \langle b_j^+ b_j \rangle \qquad b_j = a_j - \langle a_j \rangle$$

$$U = 10^{-4} J$$
$$\phi = \pi/3$$
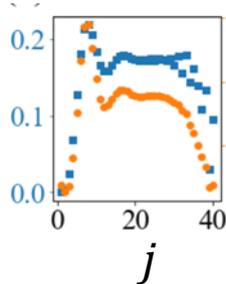
Phase I (small $\langle a_j \rangle$)

Phase III (large $\langle a_j \rangle$)

$|\langle a_j \rangle|^2 / 10^3$
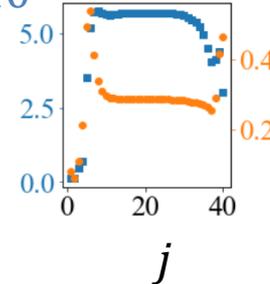
$\langle b_j^+ b_j \rangle$

$|\langle a_j \rangle|^2 / 10^3$

$\langle b_j^+ b_j \rangle$

Laszlo Rassaert, Tomás Ramos, Tommaso Roscilde, DP, arXiv:2411.08965

Variational Gaussian ansatz predictions ($\phi \neq 0$)

$$H = \Delta \sum_j a_j^+ a_j + J \sum_j a_j^+ a_{j+1} + U \sum_j (a_j^+ a_j)^2 + \sum_j \epsilon_j \, (a_j^+ e^{i\phi j} + a_j e^{-i\phi j})$$

$$\frac{1}{N} \sum_i |\langle a_j \rangle|^2 \qquad \frac{1}{N} \sum_i \langle b_j^+ b_j \rangle \quad b_j = a_j - \langle a_j \rangle$$

$$U = 10^{-4} J$$
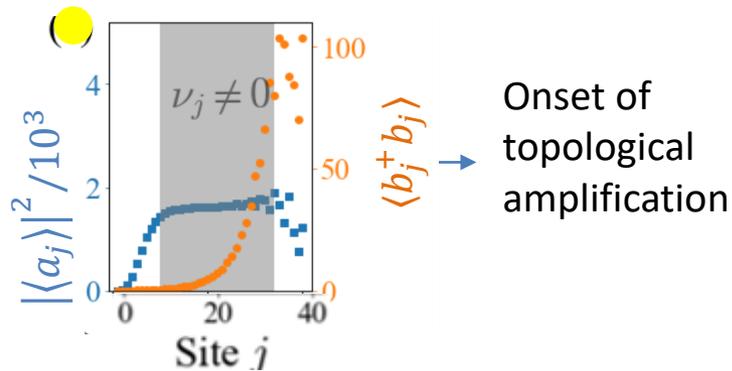$$\phi = \pi/3$$

$|\langle a_j \rangle|^2 / 10^3$ $\langle b_j^+ b_j \rangle$
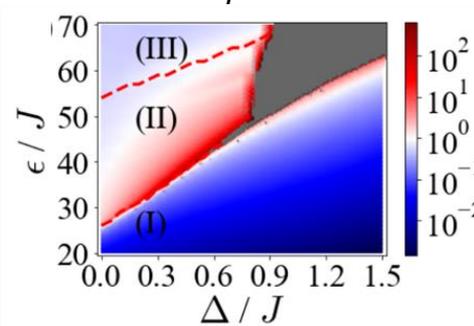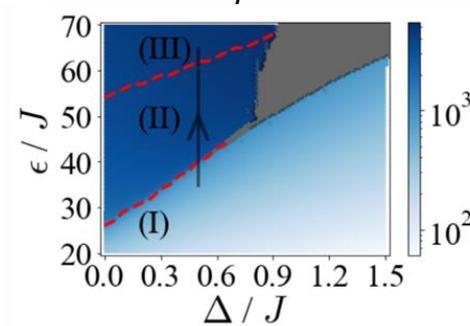
Phase II
- Phase coexistence
- Large fluctuations

Why? Topology!

Variational Gaussian ansatz predictions ($\phi \neq 0$)

Onset of topological amplification

$$\frac{1}{N}\sum_j \langle b_j^+ b_j \rangle$$

Regions can be described by a local winding number:

$$\nu_j = \int \frac{dk}{2\pi i} \text{Tr}\{\partial_k(H_j(k))\}$$

Dynamical (Bogoliubov) matrix at site j

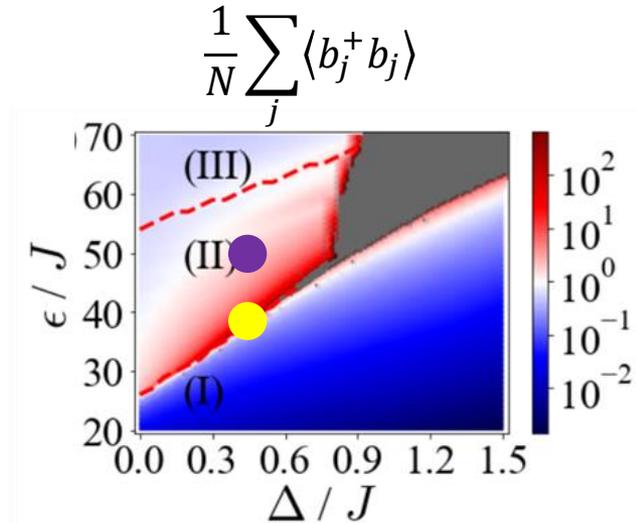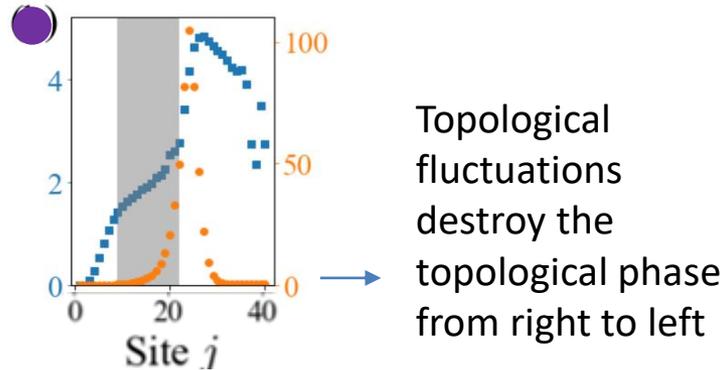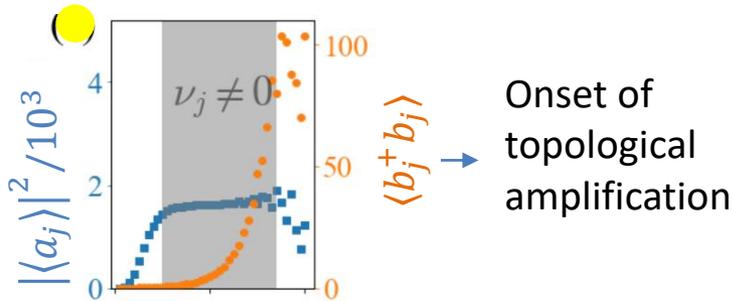$\nu_j = 1 \rightarrow$ top. amplification

$\nu_j = 0 \rightarrow$ trivial phase
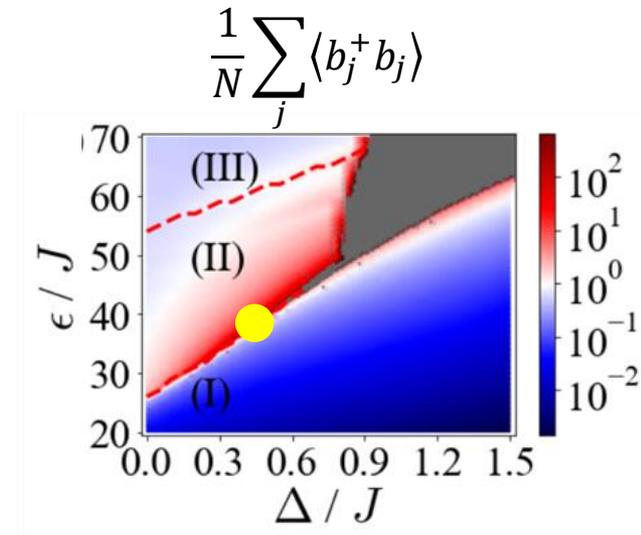
Variational Gaussian ansatz predictions ($\phi \neq 0$)

Increase coherent drive in phase (II)
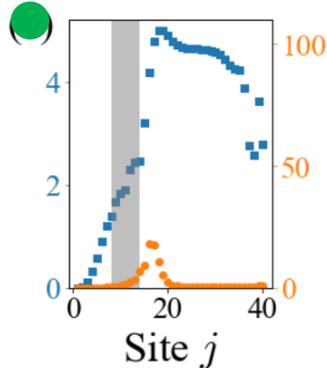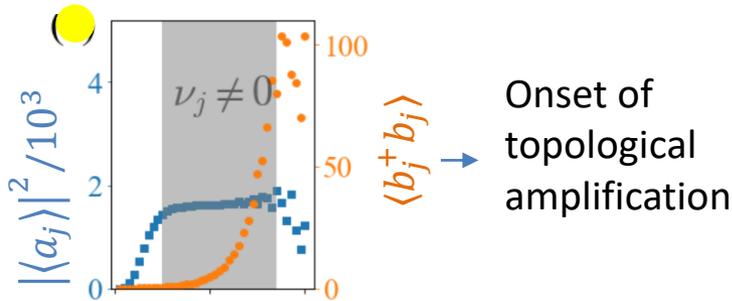

Onset of topological amplification

Topological fluctuations destroy the topological phase from right to left

$$\frac{1}{N}\sum_j \langle b_j^+ b_j \rangle$$

Variational Gaussian ansatz predictions ($\phi \neq 0$)

Increase coherent drive in phase (II)

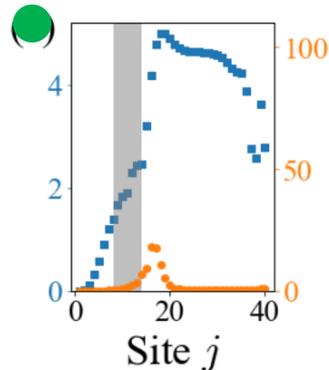Onset of topological amplification

Topological fluctuations destroy the topological phase from right to left

$$\frac{1}{N}\sum_j \langle b_j^+ b_j \rangle$$

Variational Gaussian ansatz predictions ($\phi \neq 0$)

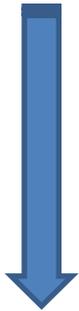$$\nu_j = \int \frac{dk}{2\pi i} \text{Tr}\{\partial_k(H_j(k))\}$$

Local winding number

Increase coherent drive in phase (II)

Onset of topological amplification

Fluctuations destroy the topological phase from right to left
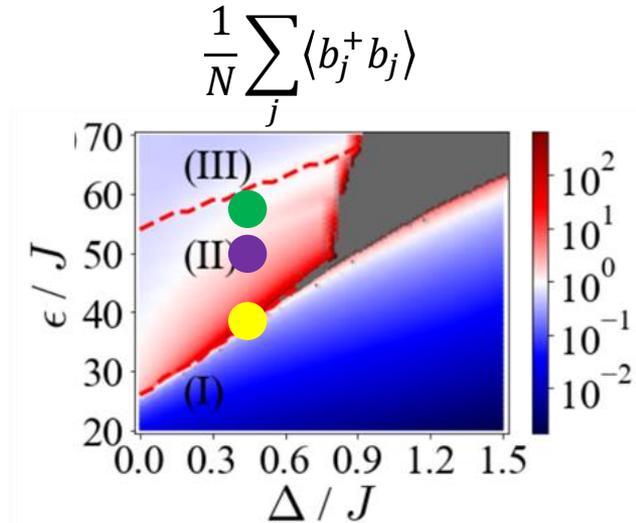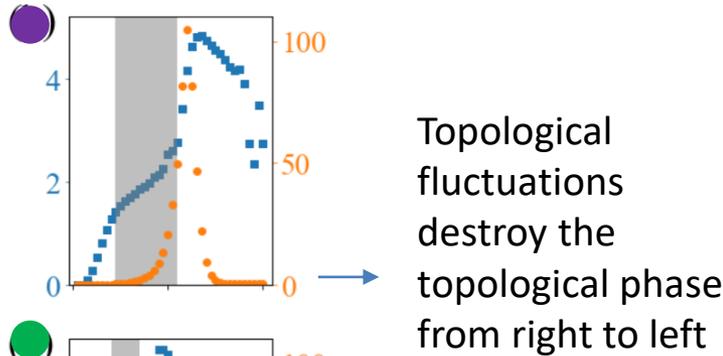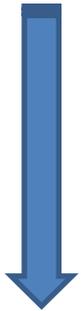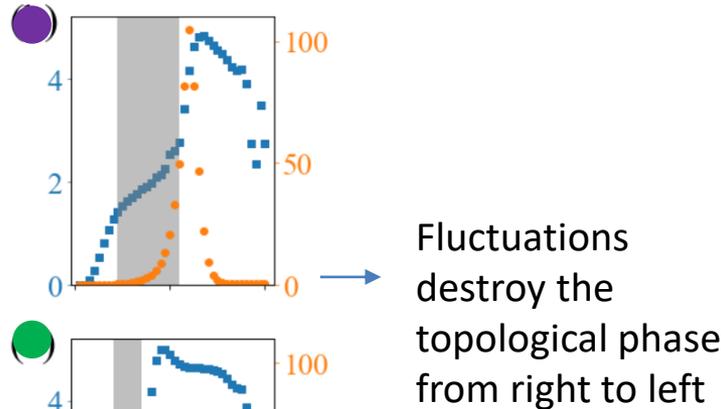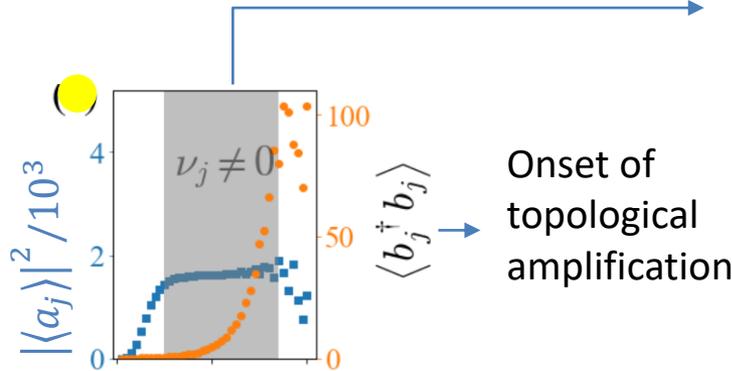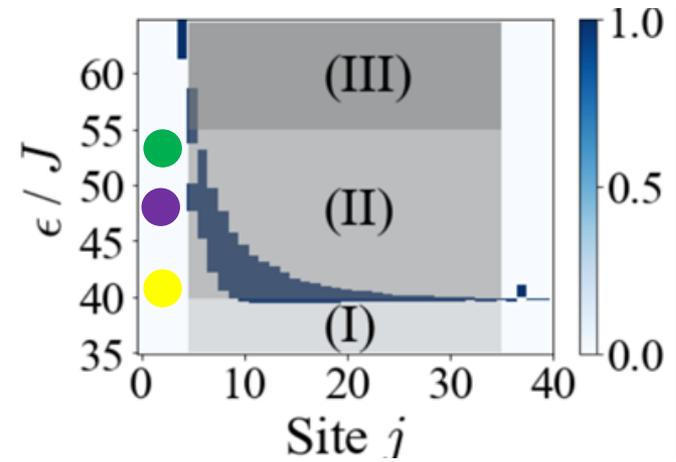
Phase II: **Topological fluctuations drive the transition between the small to large field phases**

Laszlo Rassaert, Tomás Ramos, Tommaso Roscilde, DP, arXiv:2411.08965

Conclusions

- Topological description of driven-dissipative systems through the singular value decomposition. Non-trivial topology = directional amplification

- Applications in the design of directional amplifiers (superconducting qubit readout)

- Theory can also be applied to non-linear (driven-dissipative Bose-Hubbard) systems. Non-equilibrium topological phase transitions.

Outlook

- Proper theoretical characterization, Keldysh path integral?

- Quantum many-body effects (beyond the Gaussian or semiclassical limit, e.g. in non-reciprocal quantum spin chains). Theoretical description with tensor networks (collaboration with Luca Tagliacozzo)

- Topological amplification in Floquet systems (work in progress)

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Jing Chao (SUSTech, China)
(Experimental
collaboration)

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Tommaso Roscilde (ENS Lyon)