

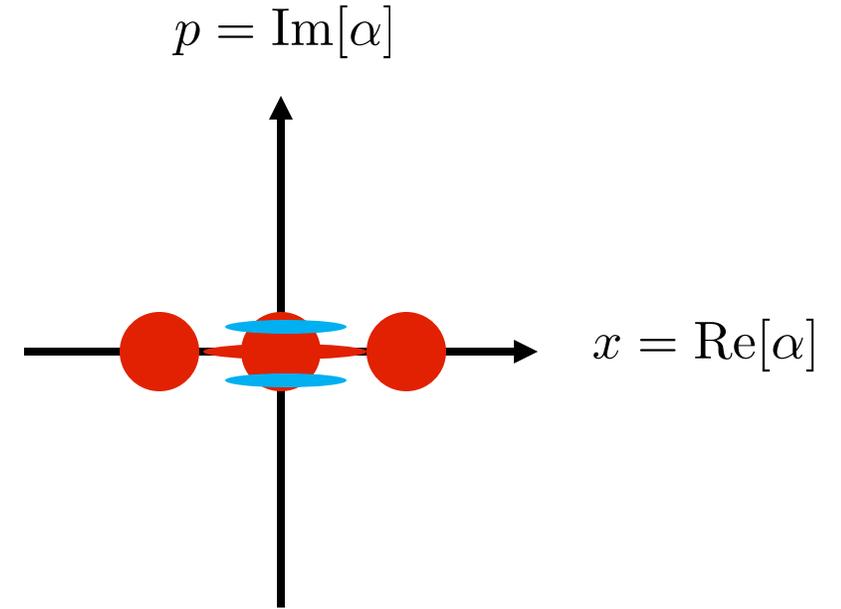
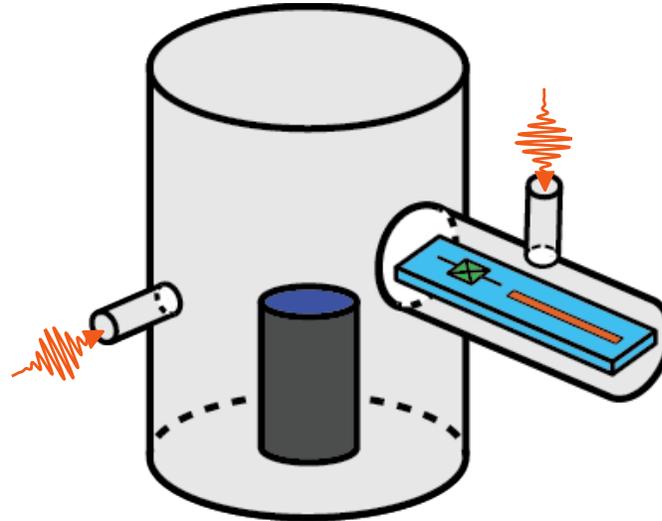
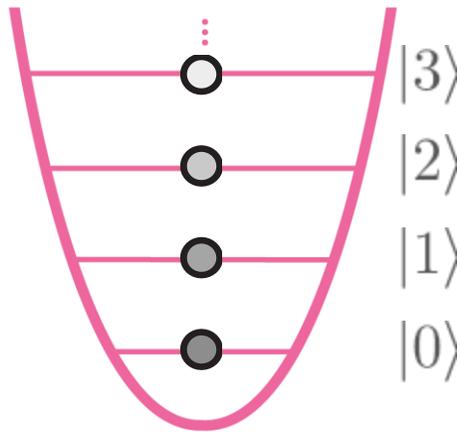
Bosonic cQED:

A rich playground for light-matter interaction

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Recap of yesterday's discussion

LC oscillators



- Superconducting cavities are good, controllable oscillators in stationary modes of superconducting cavities, coupled to non-linear ancillary modes
- Bosonic cQED provides a versatile playground for creating and manipulating quantum states of light



Plans for the lectures – day 2

1. Brief introduction to cQED
2. What goes into a minimal bosonic cQED hardware module
3. Bosonic cQED as a playground for good old quantum optics concepts
 - States
 - Gates
 - **Measurements**
4. **Leverage bosonic cQED devices for quantum information processing**
 - **Continuous-variable logical qubits encoded in bosonic modes**
 - **Non-Gaussian bosonic resources for simulation and metrology**
5. **Looking ahead: challenges and exciting developments**



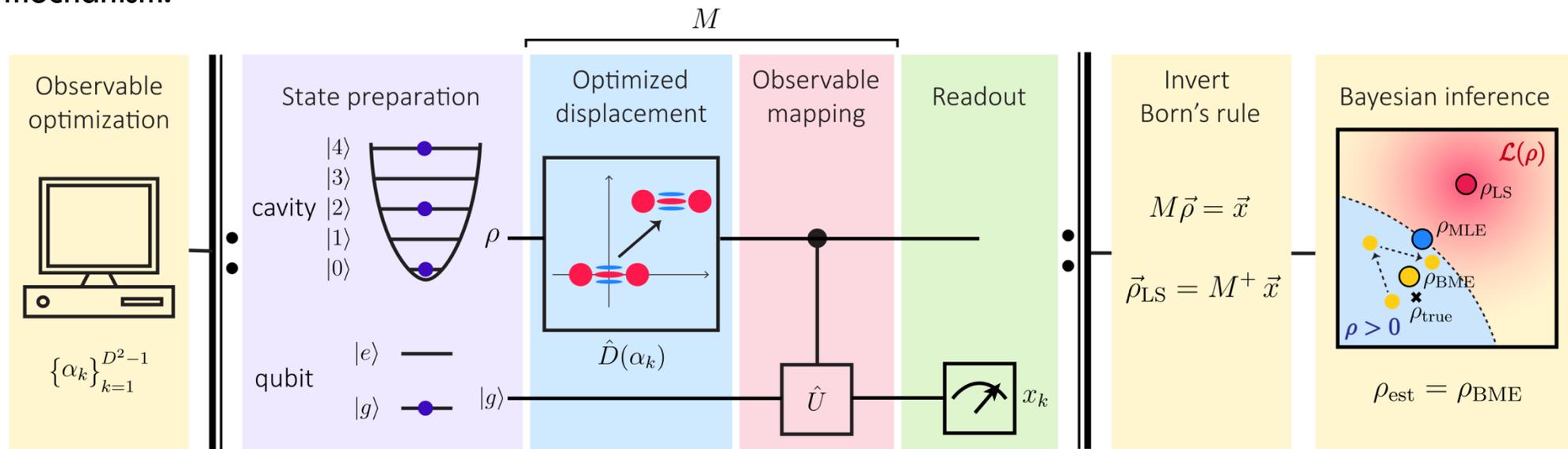
How to measure bosonic states – tailored observable mapping

Access to a few specific observables that enable full tomography

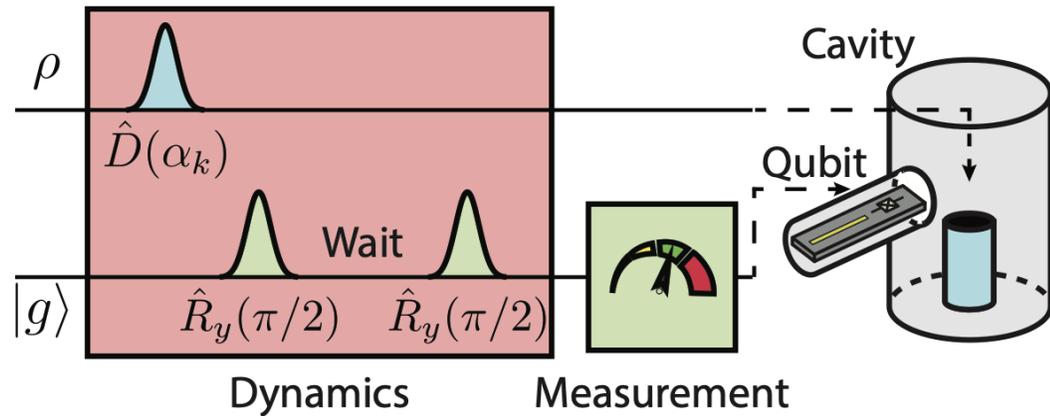
Common choices:

- Parity operator
- Displacement operator
- Photon number operator

Key mechanism:



Example: Wigner tomography

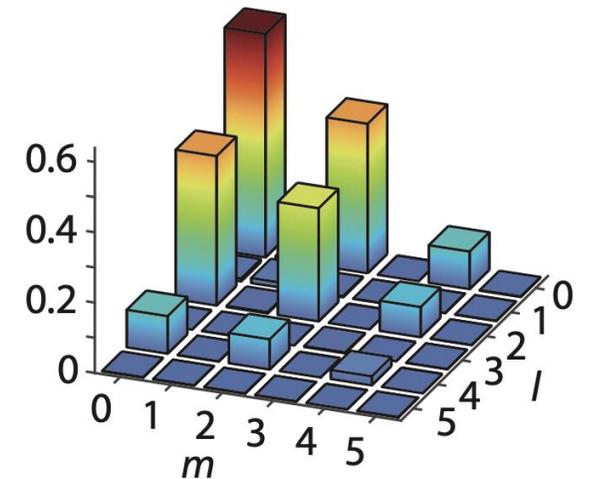
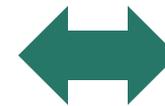
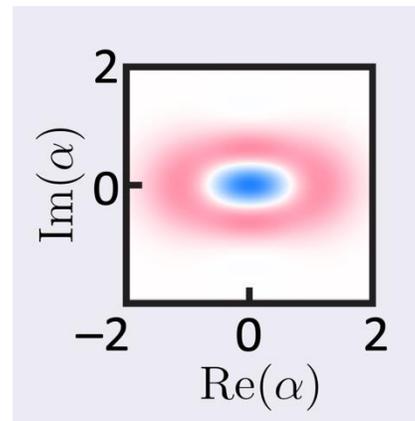
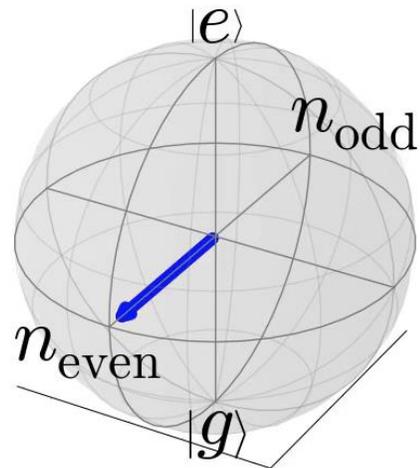


- Simple analytical sequence that only uses dispersive coupling
- Measurement outcome relates to density matrix via a linear equation

$$\vec{X}_n = M\vec{Y}_n + \vec{V}$$

- Invert the mapping matrix to get DM

$$\vec{\rho}_{\text{est}} = M^{-1}\vec{X}$$



<https://www.nature.com/articles/s41467-024-48573-x>

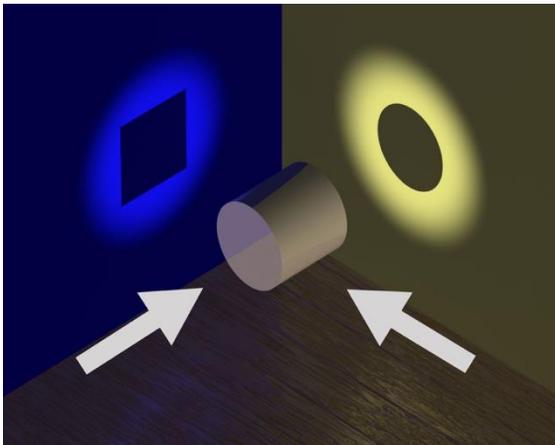
<https://journals.aps.org/prxquantum/abstract/10.1103/PRXQuantum.6.010303>

What if we want to probe other observables?

Challenges with full tomography:

- Large Hilbert space, need to sample many points, minimum D^2-1 (D =truncation dimension)
- Time consuming, prone to drifts in system parameters

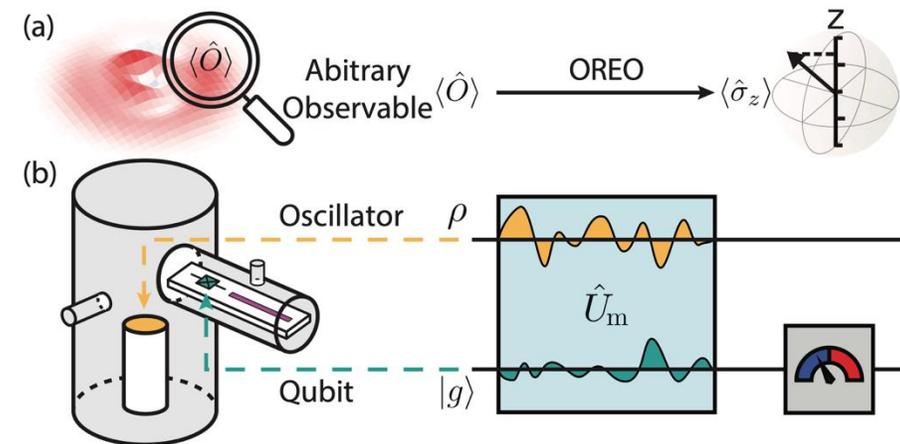
Often do not need the full DM, but rather specific observable values of the bosonic state!



Shadow tomography

<https://www.nature.com/articles/s41567-020-0932-7>

<https://ieeexplore.ieee.org/document/10415254>



Numerically optimised mapping to arbitrary unitaries

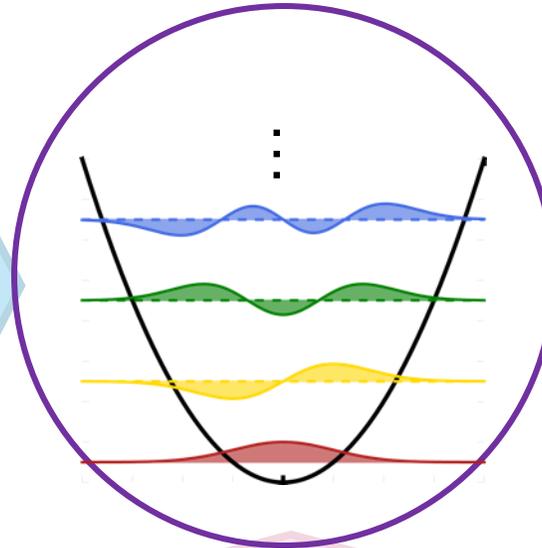
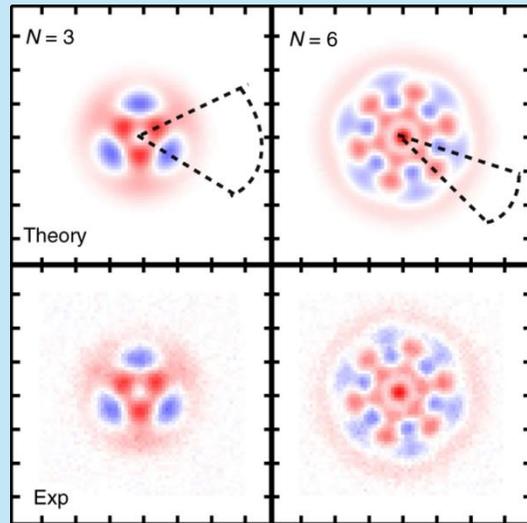
<https://arxiv.org/abs/2503.10436>

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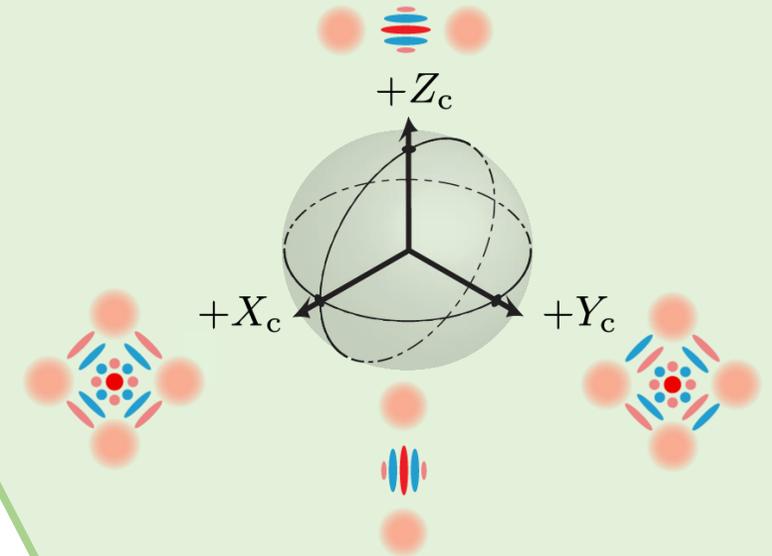


Bosonic cQED devices are versatile quantum testbeds

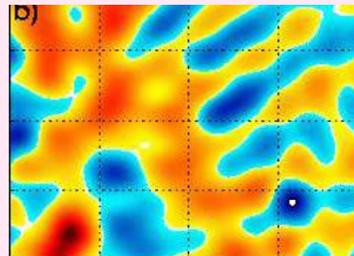
metrology & simulation



quantum computing



quantum foundations

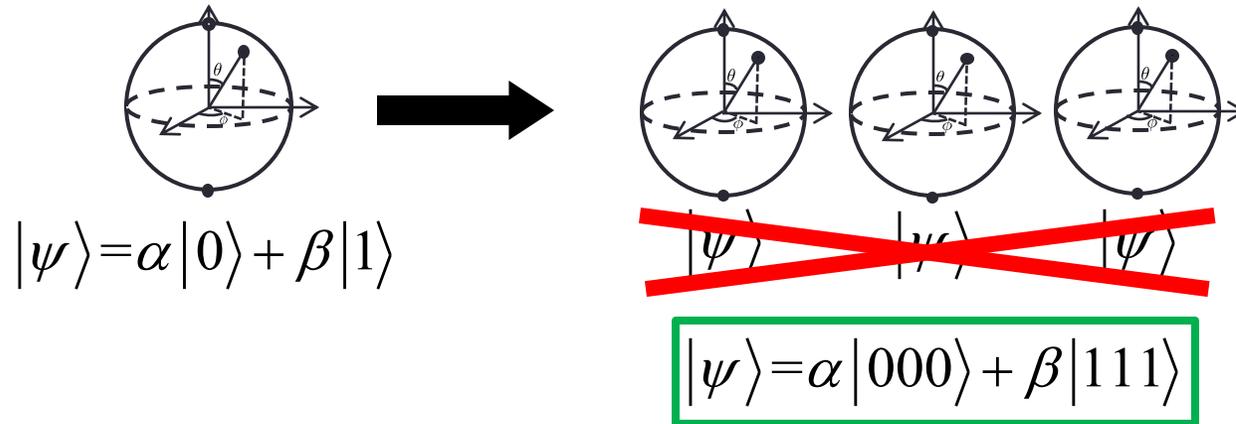


Knott et al. PRA (2016)
Mirrahimi, NJP (2014)
Ofek, Nature (2016)
Wang, PRX (2020)
Zurek, Nature (2020)
etc. etc.



Recap on QEC:

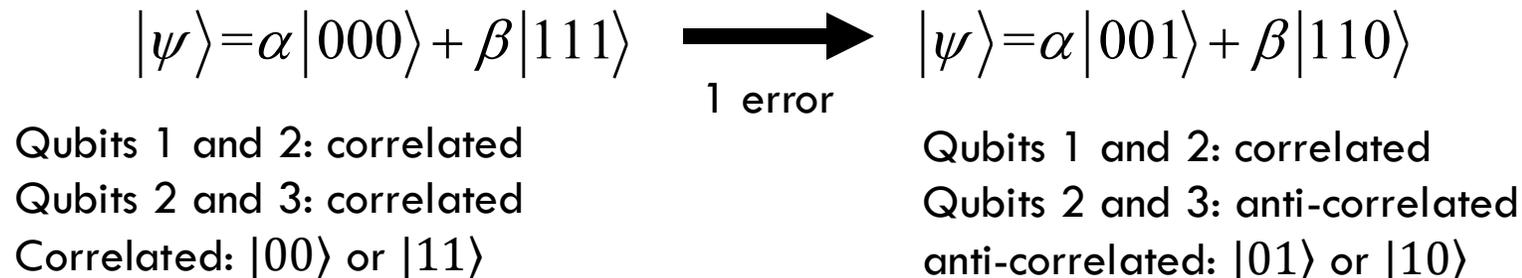
Encoding quantum information:



Cannot CLONE quantum information!

Distribute quantum information across multiple qubits, leverage their entanglement

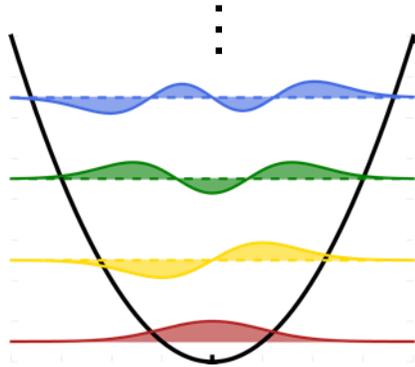
Detecting an error:



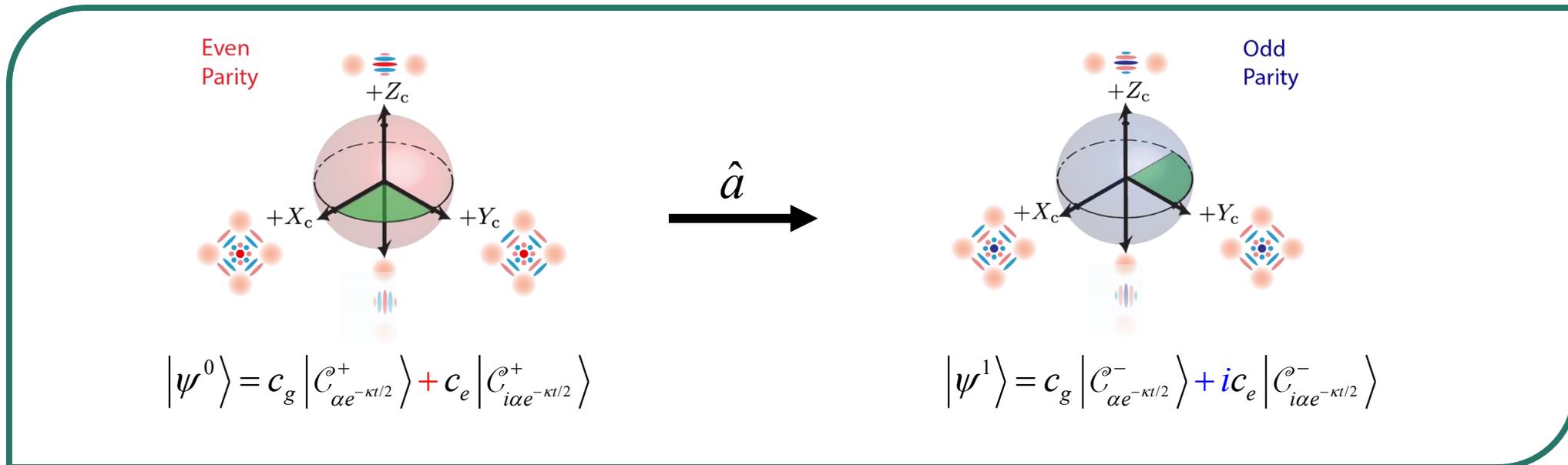
Detecting quantum errors: use symmetry in encoding to extract error syndrome



Overview of different bosonic codes

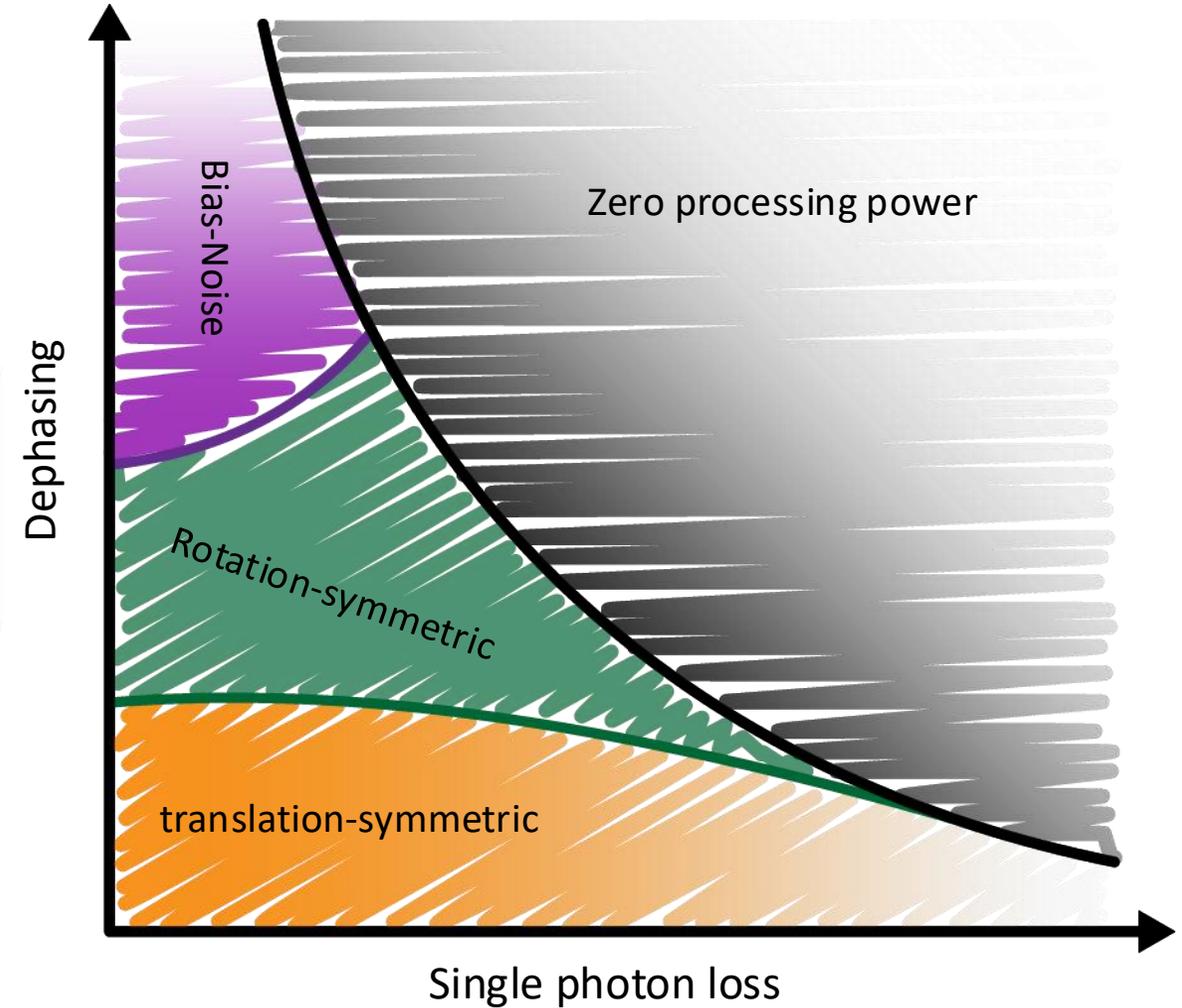
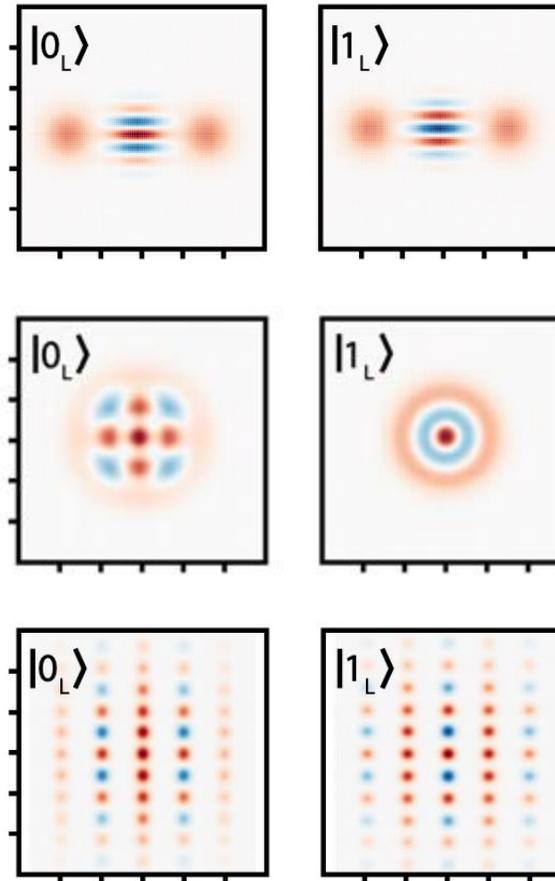


- Many levels in a single hardware, reduces control overhead
- Naturally biased in loss channels, reduces the types of errors
 - Dominated by single photon loss
 - Almost no dephasing
- Convenient error detection by via parity/displacement, reduces measurement overhead.



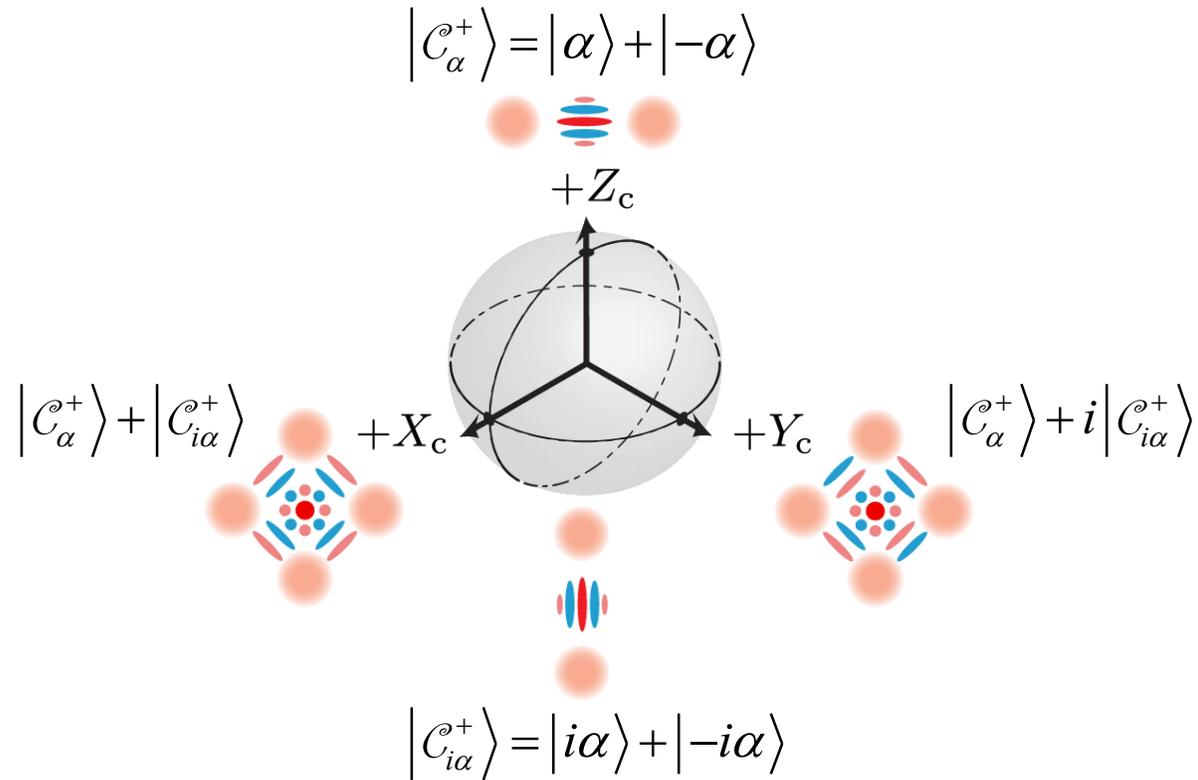
Errors and error correction codes in bosonic cQED

- Leghtas, Science (2013)
- Lescanne, Nat Phys (2019)
- Grimm, Nature (2021)
- ...
- Mirrahimi, NJP (2014)
- Ofek, Nature (2016)
- Ni, Nat Phys (2023)
- ...
- GKP (1999)
- Campagne-Ibarc, Nature (2021)
- Sivak, Nature (2023)
- ...

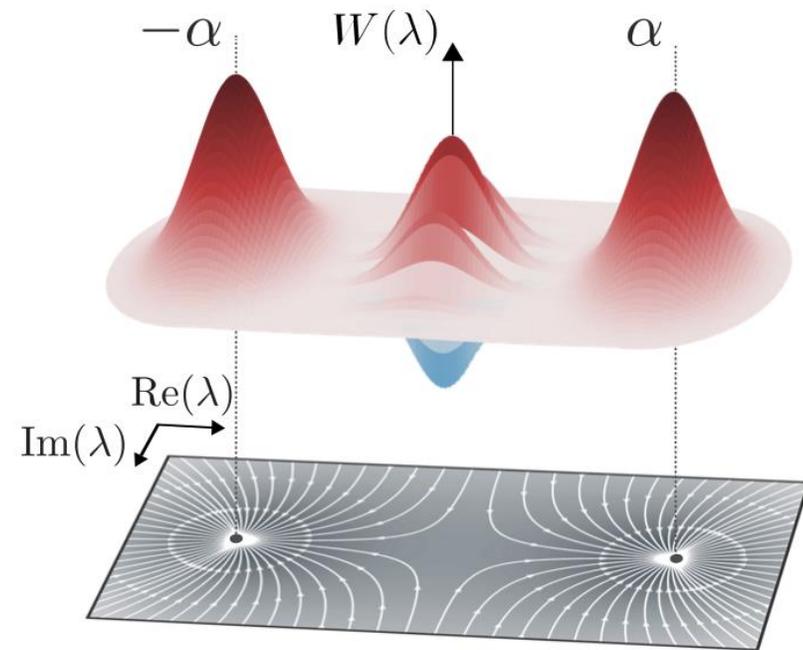


Encoding logical qubits in cat states

hardware-efficient quantum error correction
beyond break-even though parity tracking,
Ofek, Nature (2016)...



Dissipative cat qubits with exponentially-suppressed bit-flip rate as biased noise qubits
Reglade, Nature (2024)...

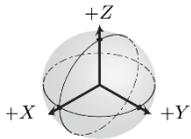


Example: 4-legged cat code full implementation

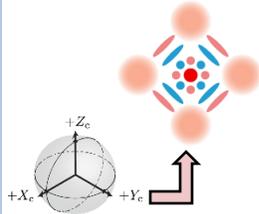
Ofek, Petrenko, et al, Nature (2016) - Yale

Optimal Control Pulse: Cavity and Qubit

Initialize



Encode

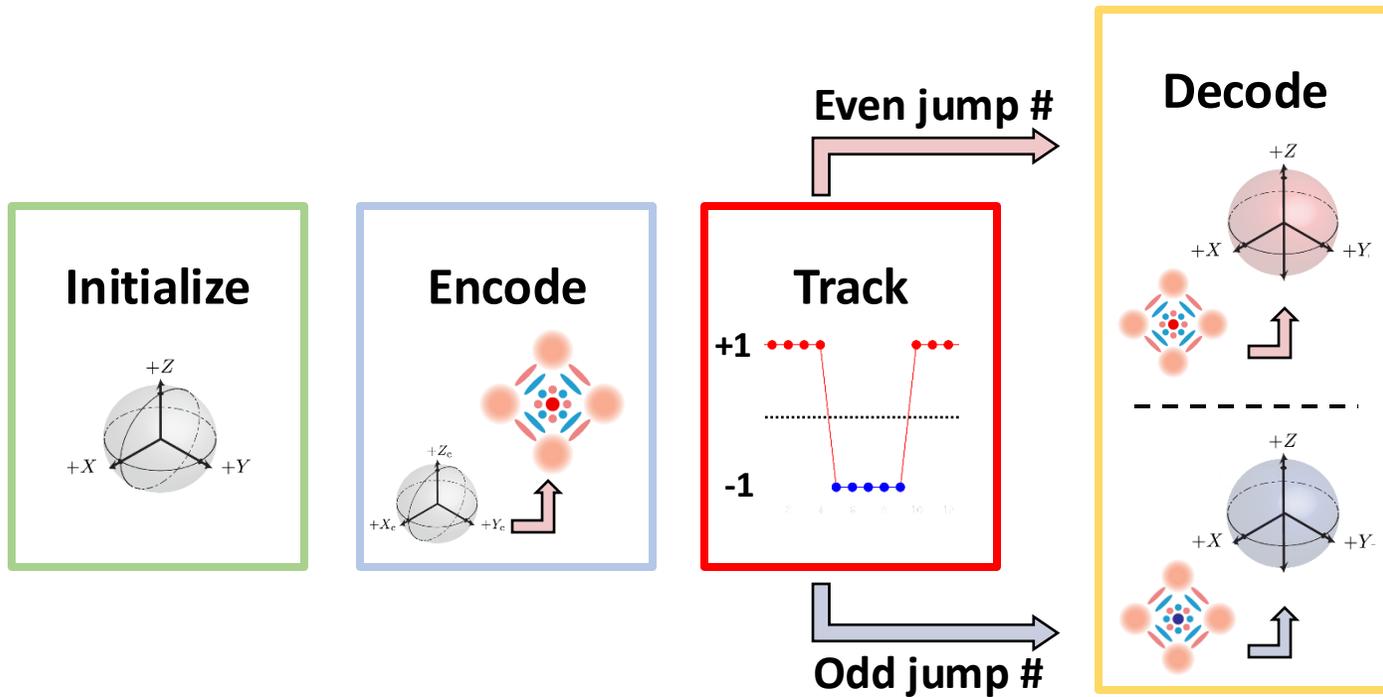


$$|\psi_L\rangle = (c_g |C_\alpha^+\rangle + c_e |C_{i\alpha}^+\rangle) |g\rangle$$



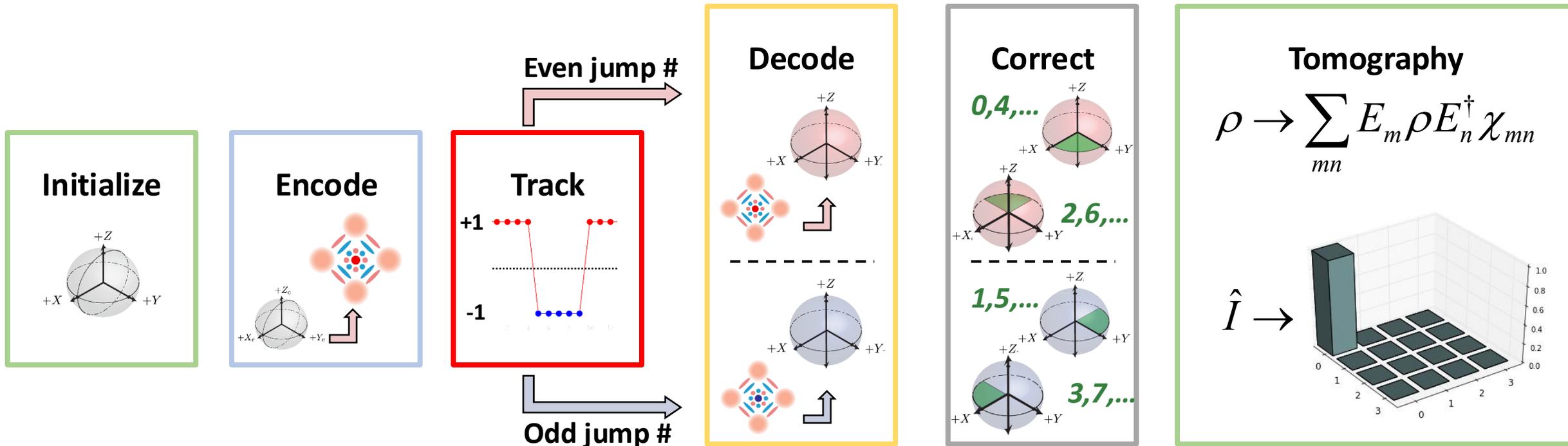
Example: 4-legged cat code full implementation

Ofek, Petrenko, et al, Nature (2016) - Yale



Example: 4-legged cat code full implementation

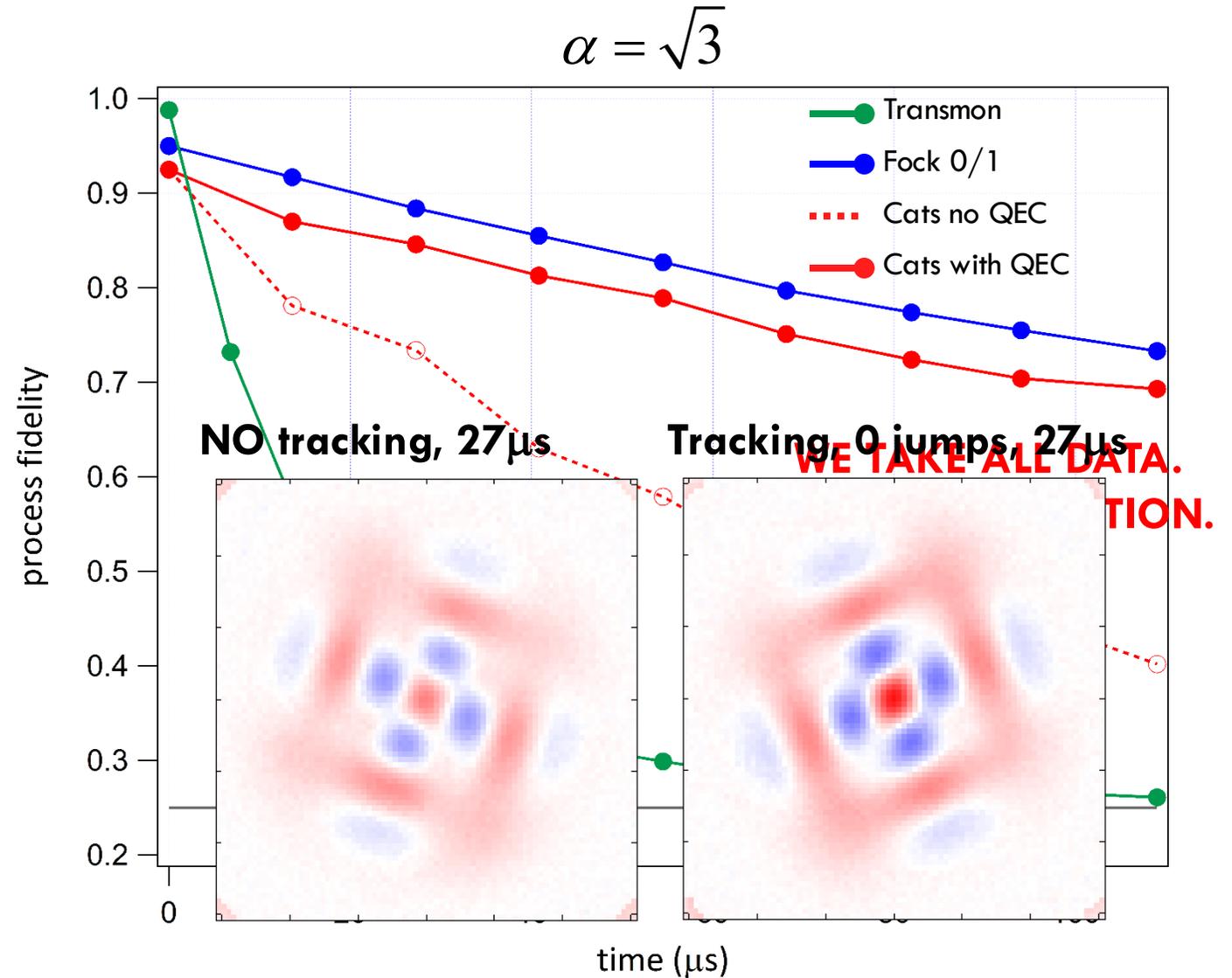
Ofek, Petrenko, et al, Nature (2016) - Yale



Some sources of imperfections

Sources of Loss

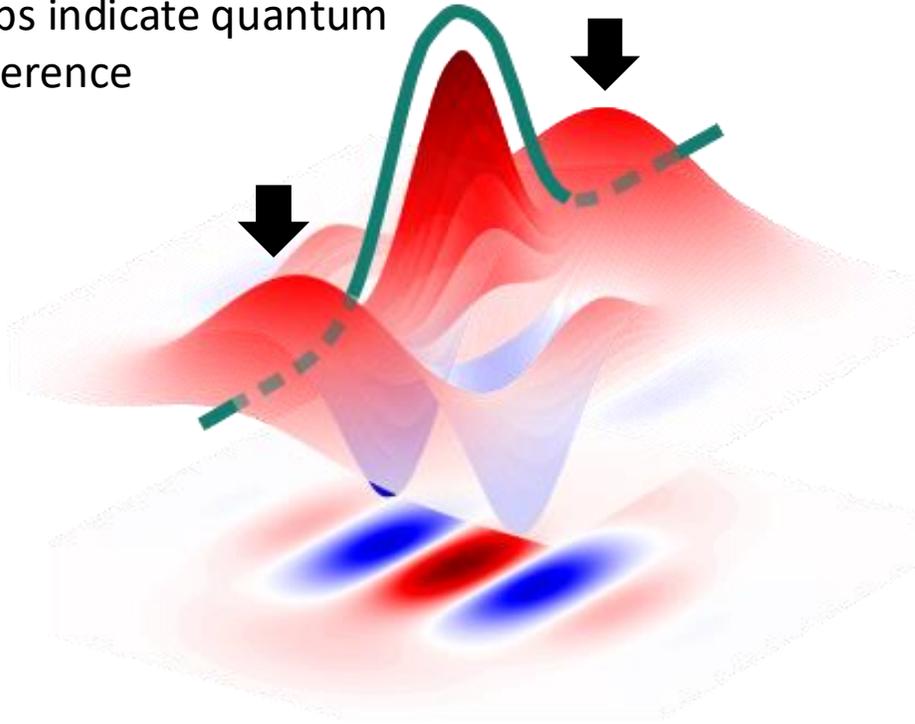
- Post-readout decay from e to g
- Uncertainty in jump time
- Presence of Kerr
- Missing two jumps
- Parity/readout infidelity
- Measurement-induced dephasing
- Transmon jumps
- ...



Intuition about how photon loss affects quantum interference

$$|\Psi\rangle_{\text{cat}} = \mathcal{N}(|\alpha\rangle + |-\alpha\rangle)$$

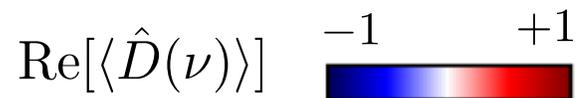
Blobs indicate quantum coherence



Dominant loss channel in cavity mode = photon loss!

Acts as a Gaussian filter!

- Centred at the origin, width $\sim 1/\kappa t$
- Cuts off features further away in phase space
- Coherence degrades faster than population

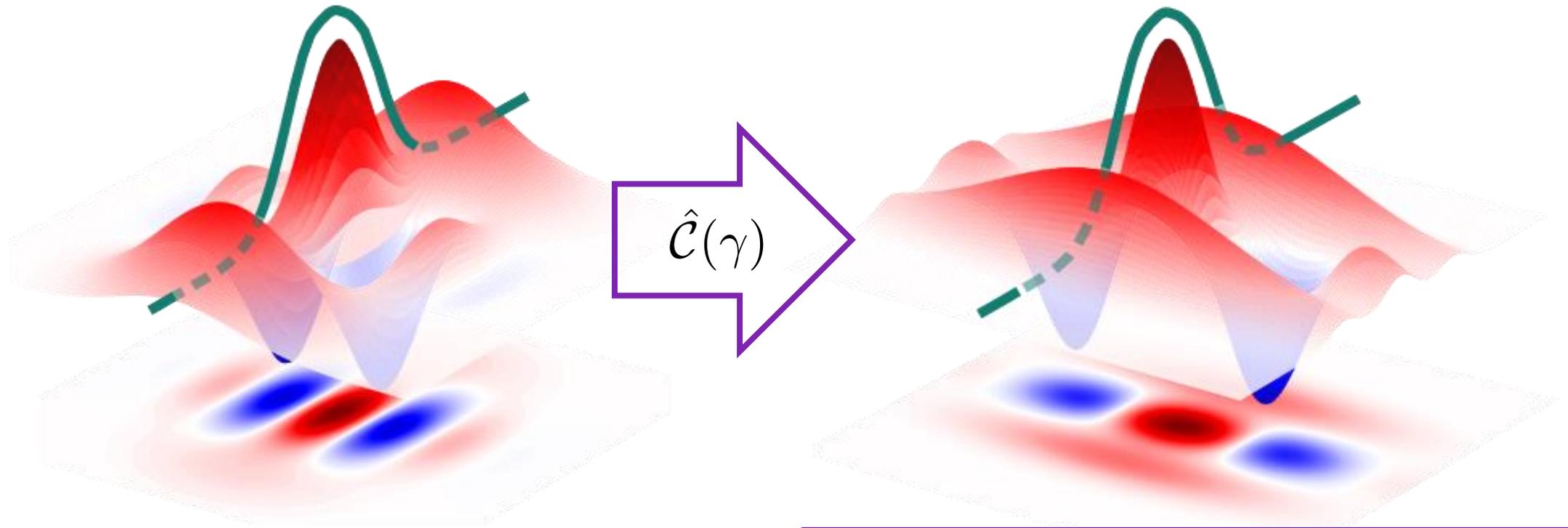


characteristic function picture

Serafini et al. J. Opt. B 6, S591 (2004), etc



Noise-resilience through phase-space engineering

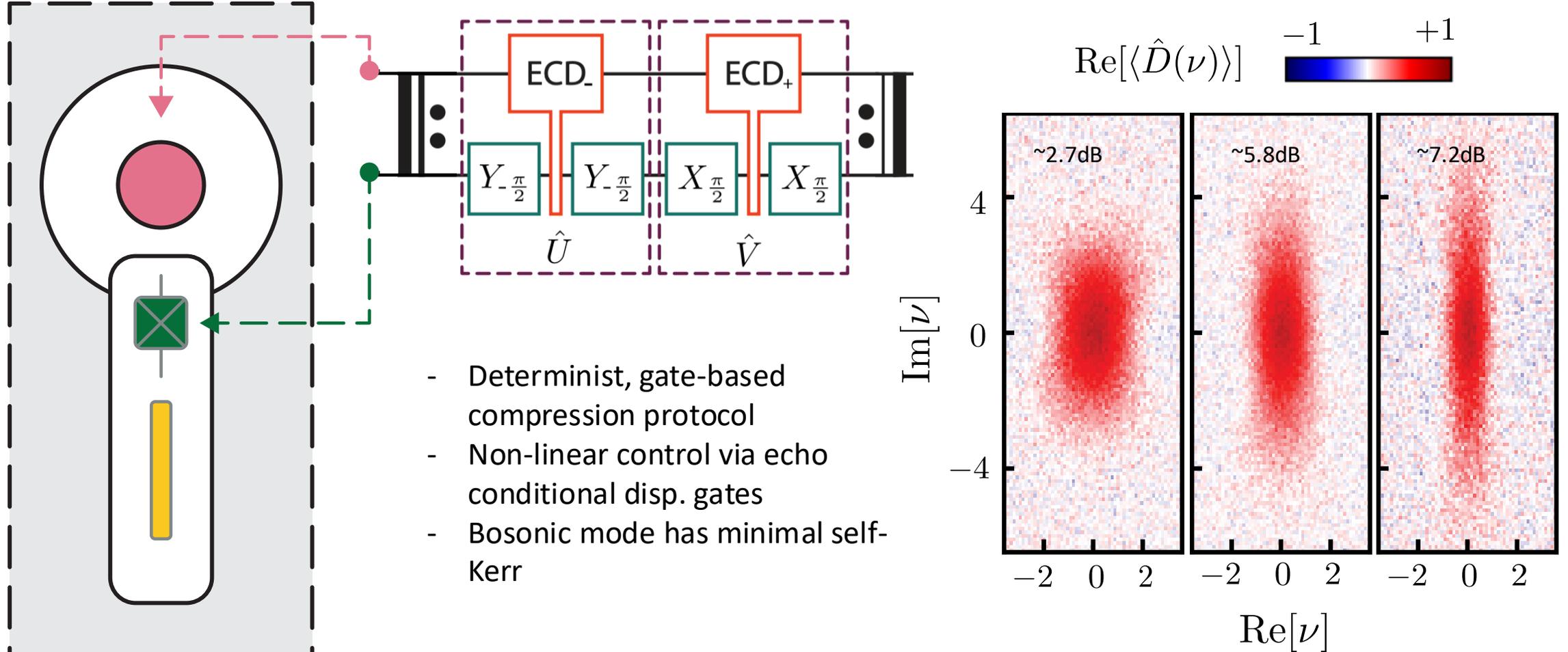


$\text{Re}[\langle \hat{D}(\nu) \rangle]$ -1 +1

compress the features to fit under the filter
compactness preserves quantum interference



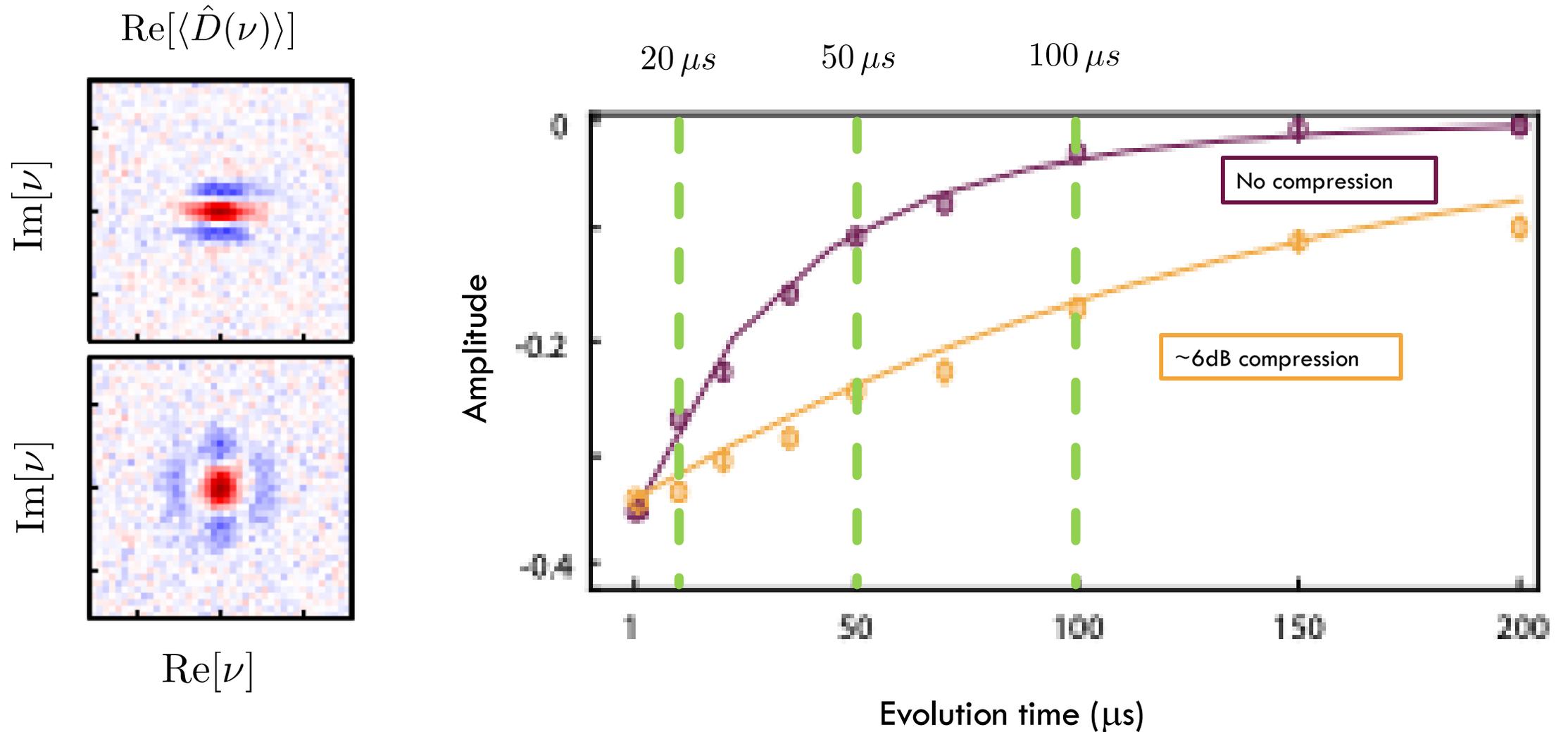
Phase-space compression via quantum interference



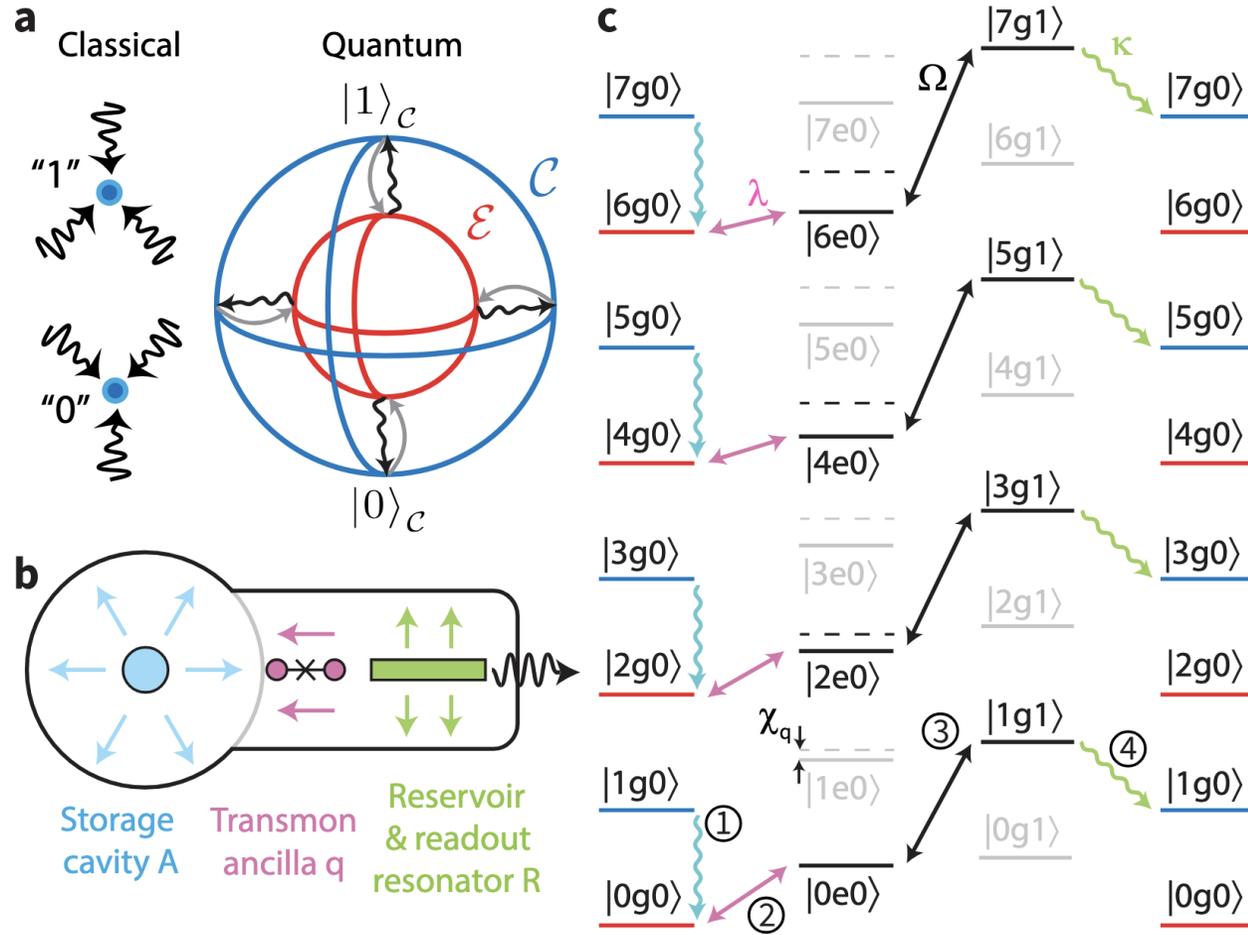
Hastrup et al. PRL (2019)
Campagne-Ibarcq, Nature (2020)



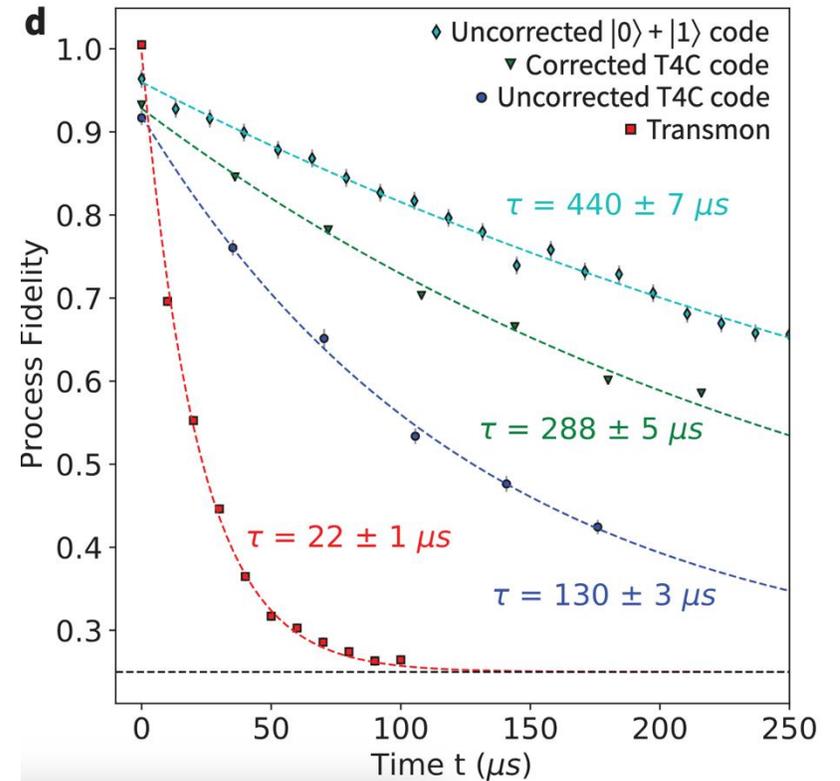
Noise-resilience through phase-space engineering



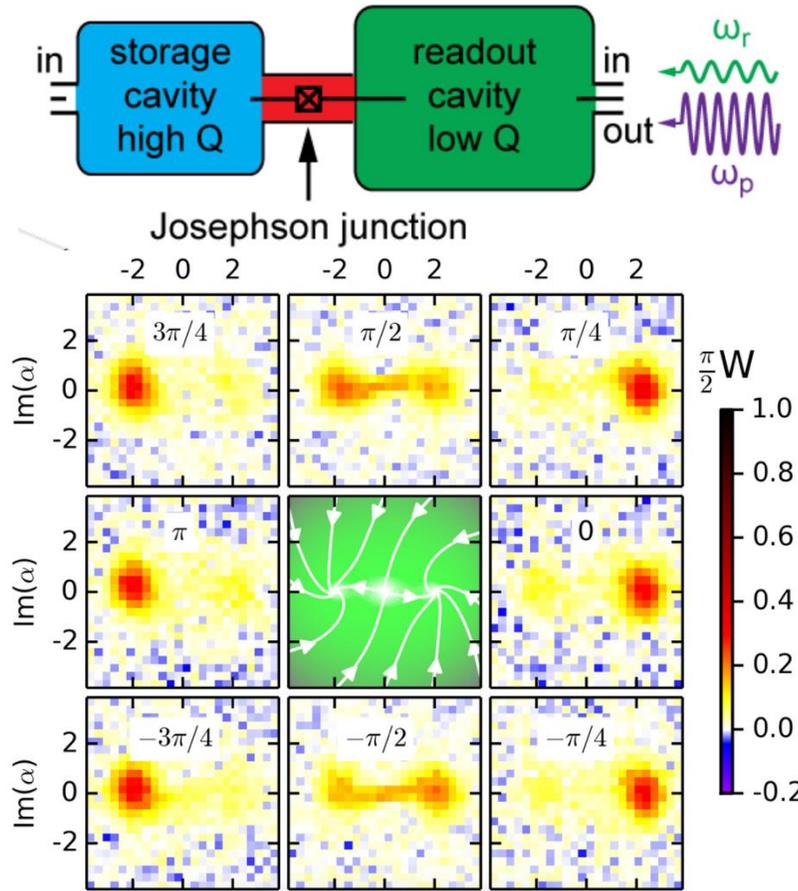
Autonomous correction by putting back the photon



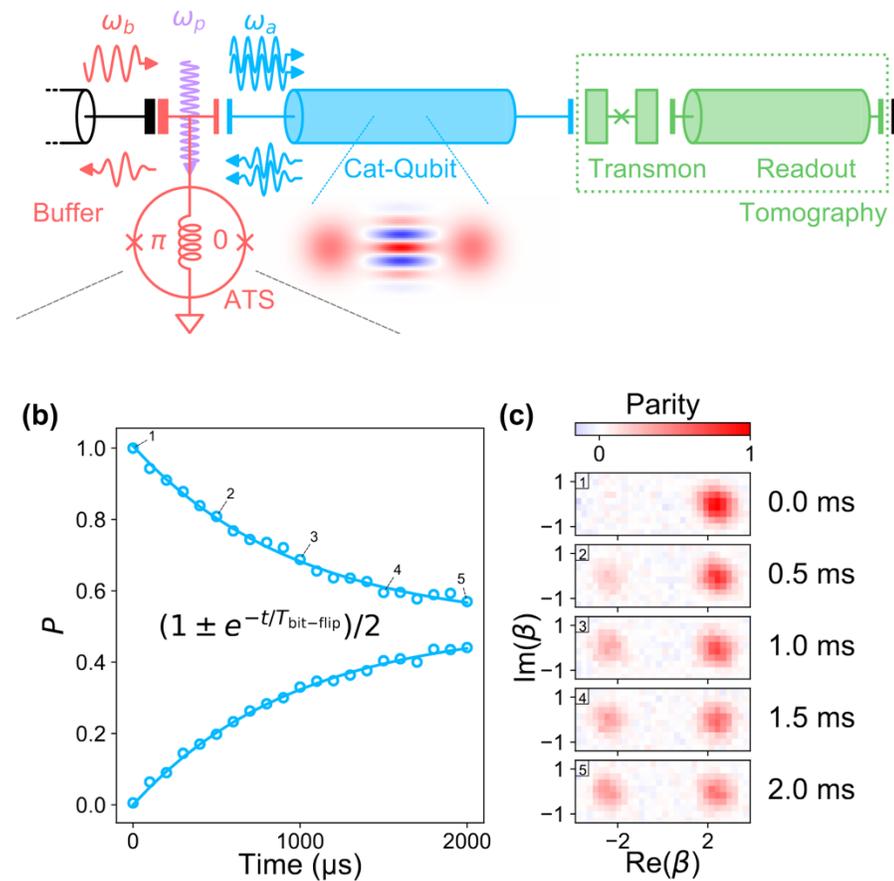
Engineer a continuous drive to autonomously restore the lost photon



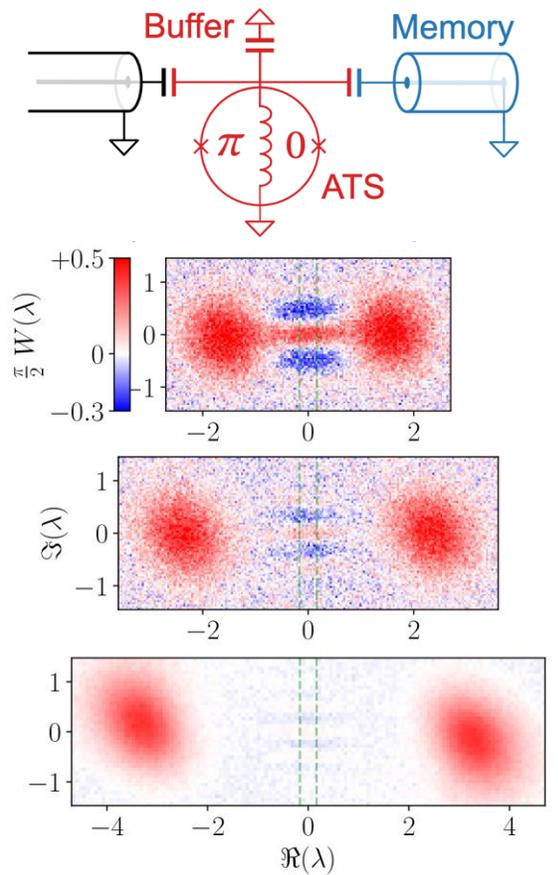
Engineering strong noise bias – a bit of history



Bit flip $\sim \mu\text{s}$,
Leghtas, Science (2015)



Bit flip $\sim \text{ms}$,
Lescanne, Nat Phys (2020)



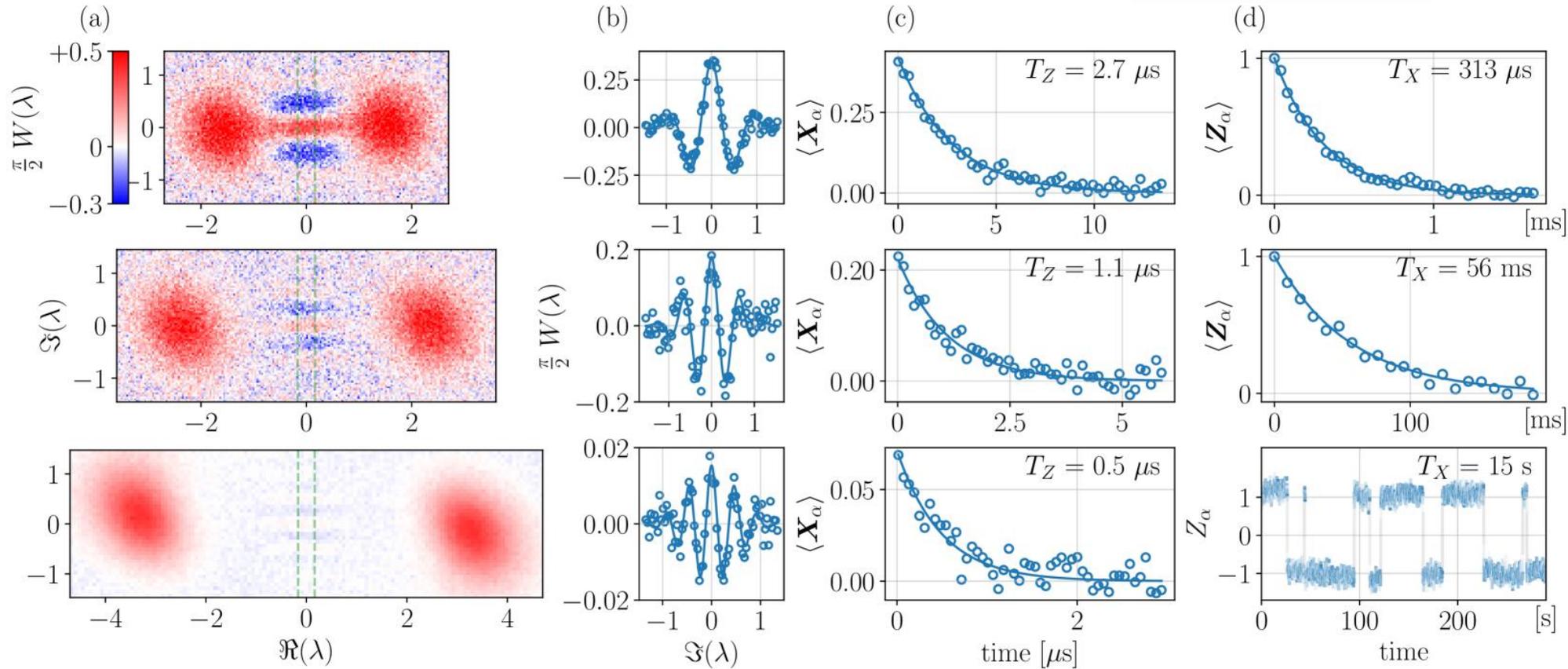
Bit flip $\sim \text{seconds}$,
Reglade, Nature (2024)

Engineering strong noise bias



ALICE & BOB

Effective suppression of 1 type of error and then concatenate



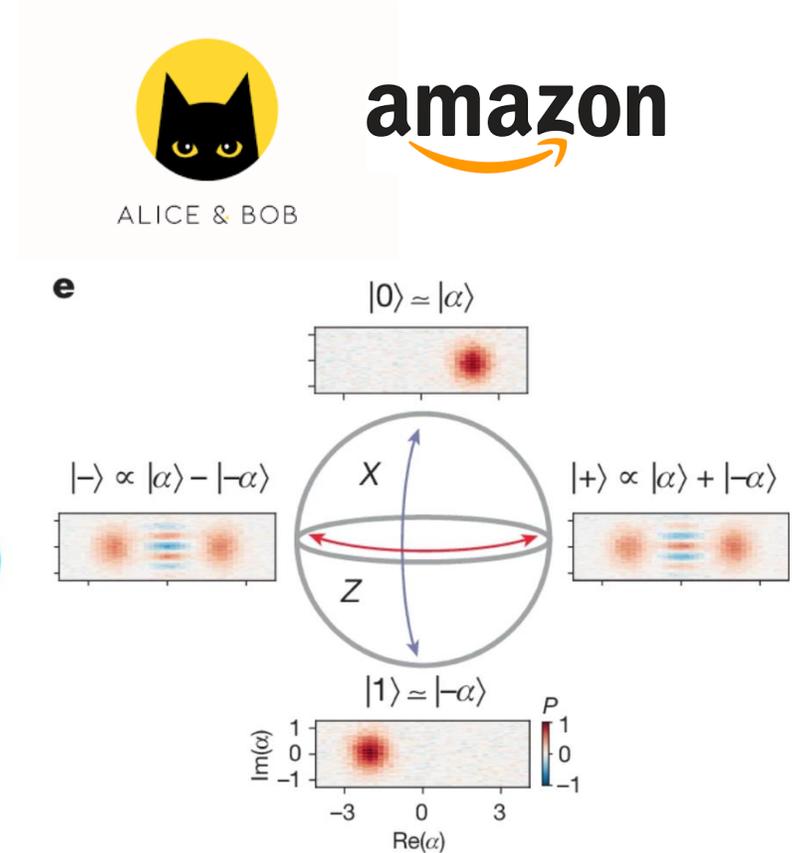
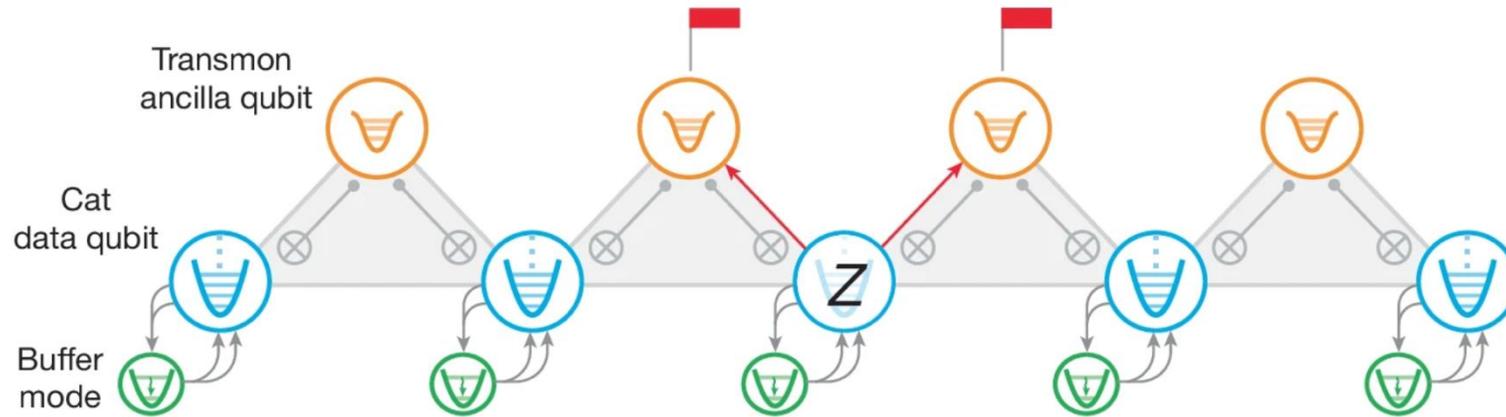
<https://www.nature.com/articles/s41586-025-08642-7>

<https://www.nature.com/articles/s41586-024-07294-3>



Engineering strong noise bias

Effective suppression of 1 type of error and then concatenate



Open questions:

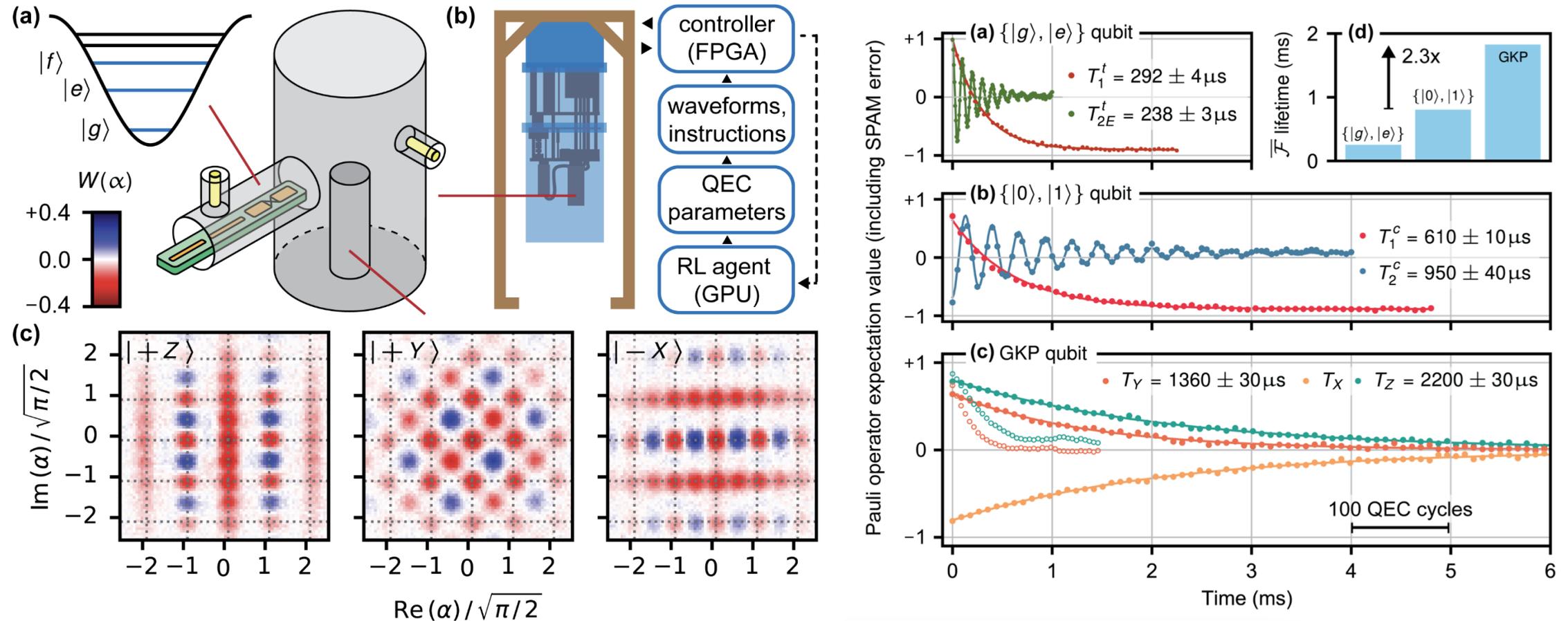
- Two-qubit gates that preserve noise bias?
- What kind of ancillary modes to use?

<https://www.nature.com/articles/s41586-025-08642-7>

<https://www.nature.com/articles/s41586-024-07294-3>



Stabilising code words through measurement

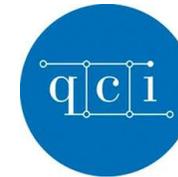
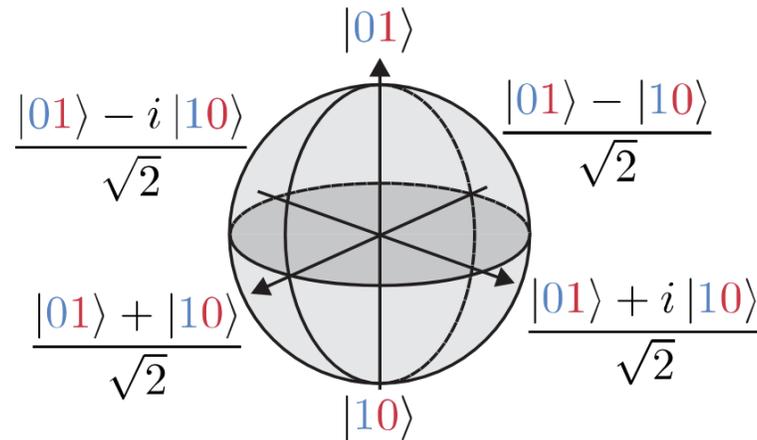
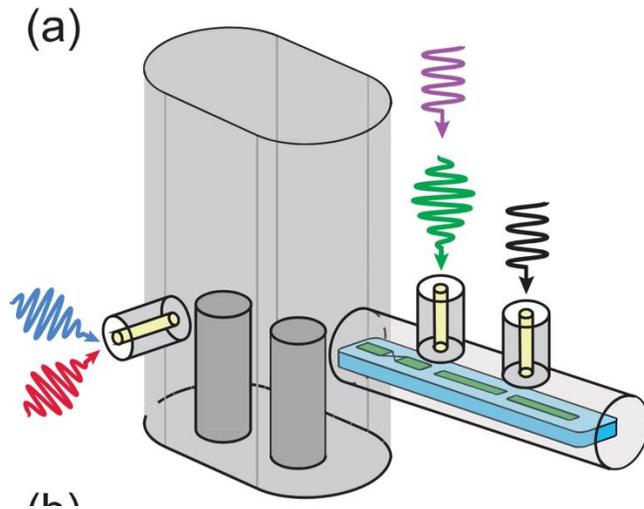


It's hard to realise the code words, but all logical errors show up as displacements**



Other strategies for bosonic error correction

Multimode codes: Pair-cat, Dual-rail ...



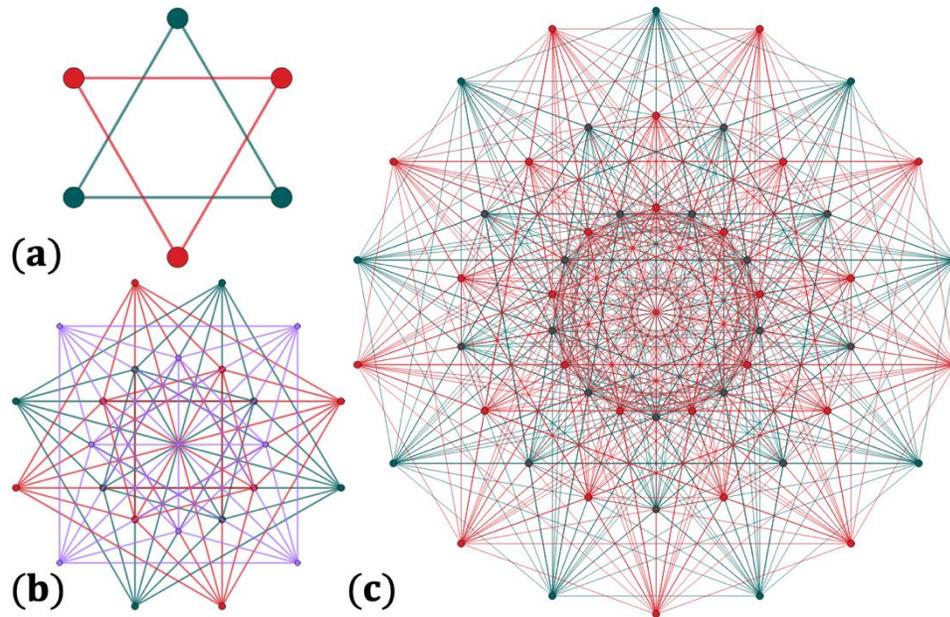
Example: Dual-rail code

- first-order hardware errors in the cavities and the transmon can be detected and converted to erasure errors (partial trace out one mode) in all operations
- Easy to prepare the state, single qubit gates are beamsplitters
- Measure joint parity to detect errors



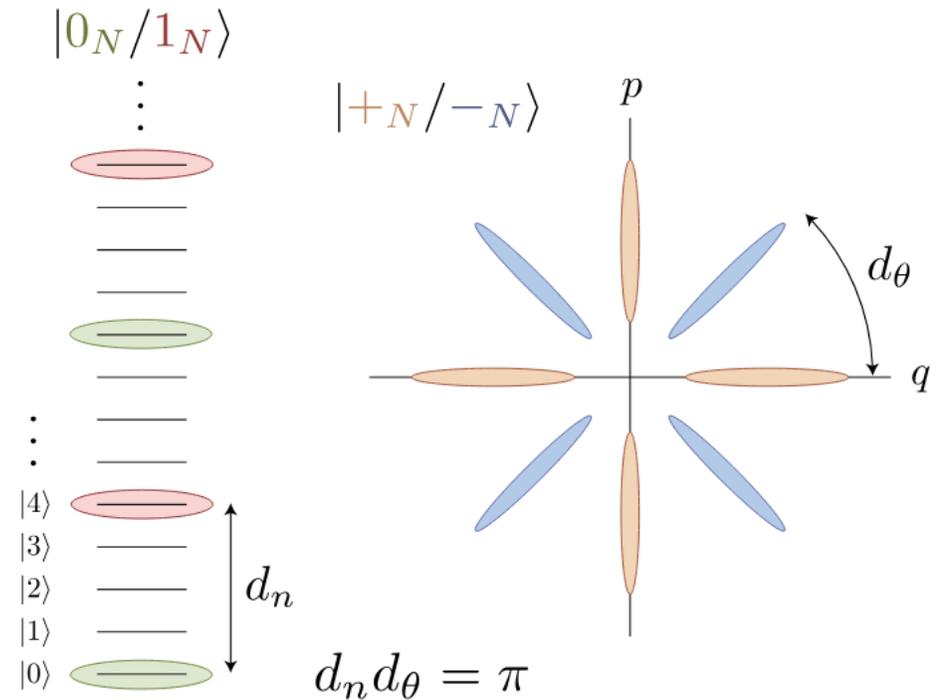
Many other theory ideas on bosonic codes

Spherical code



<https://www.nature.com/articles/s41567-024-02496-y>

Number phase code



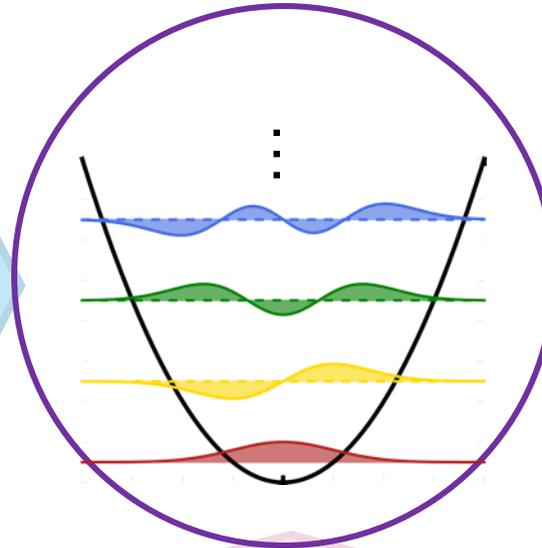
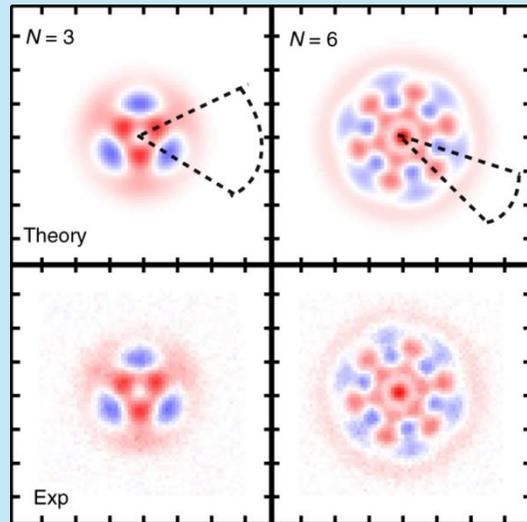
<https://journals.aps.org/prx/abstract/10.1103/PhysRevX.10.011058>

<https://errorcorrectionzoo.org/c/oscillators>

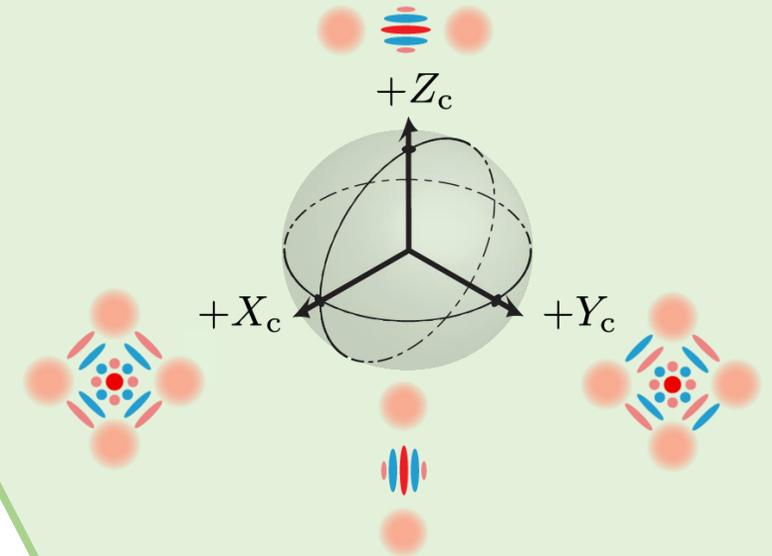


Bosonic cQED devices are versatile quantum testbeds

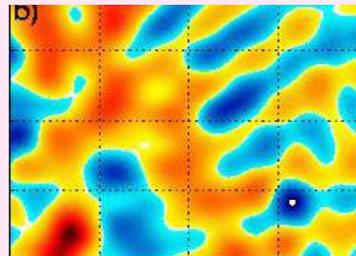
metrology & simulation



quantum computing



quantum foundations



Knott et al. PRA (2016)
Mirrahimi, NJP (2014)
Ofek, Nature (2016)
Wang, PRX (2020)
Zurek, Nature (2020)
etc. etc.



Quick recap on quantum metrology concepts

Why? -- Superior scaling for measurement precision $\Delta\beta$

Classical: $\Delta\beta \propto 1/\sqrt{n}$ The standard quantum limit (SQL)

Quantum: $\Delta\beta \propto 1/n$ The Heisenberg limit (HL), a \sqrt{n} enhancement

Main metric: Fischer Information quantifies the precision $\Delta\beta$ when we measure an observable \hat{B}

$$\text{FI}(\beta) = \sum_b \frac{1}{p(\beta|b)} \left(\frac{\partial p(\beta|b)}{\partial \beta} \right)^2 \quad \{p(\beta|b)\} \text{ probabilities from measuring } \hat{B}$$
$$\Delta\beta \geq \frac{1}{\sqrt{\text{FI}(\beta)}} \quad \text{Cramér-Rao bound, Phys. Rev. Lett. 72, 3439 (1994)}$$

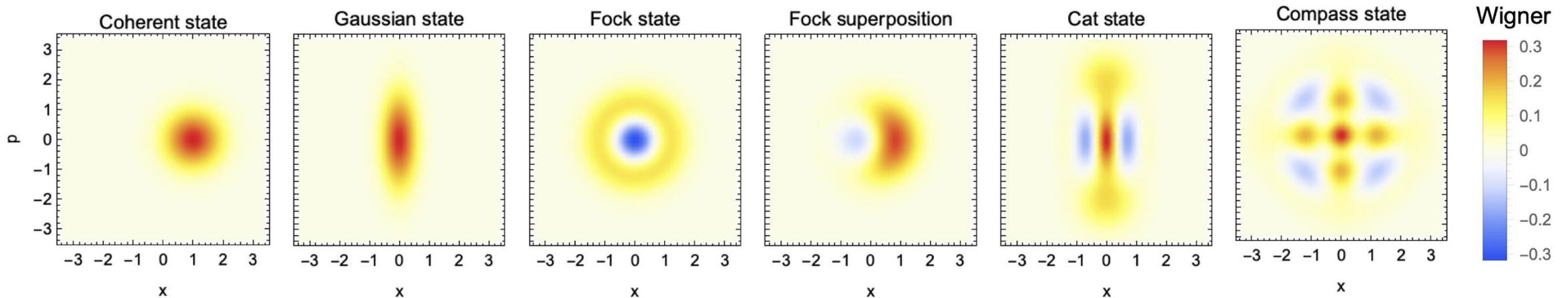
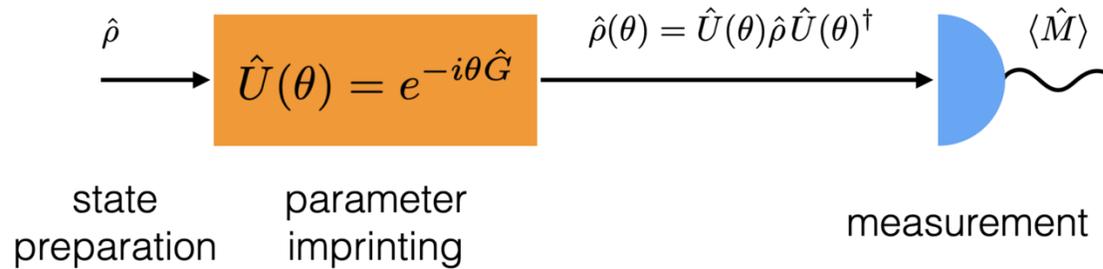
Quantum FI is the upper bound for FI

$$\max_{\beta} \text{FI} = \text{QFI} = 4 \left(\langle \psi | \hat{B}^2 | \psi \rangle - \langle \psi | \hat{B} | \psi \rangle^2 \right)$$

Strategy: choose the best states and measurements to achieve the highest FI and saturate HL



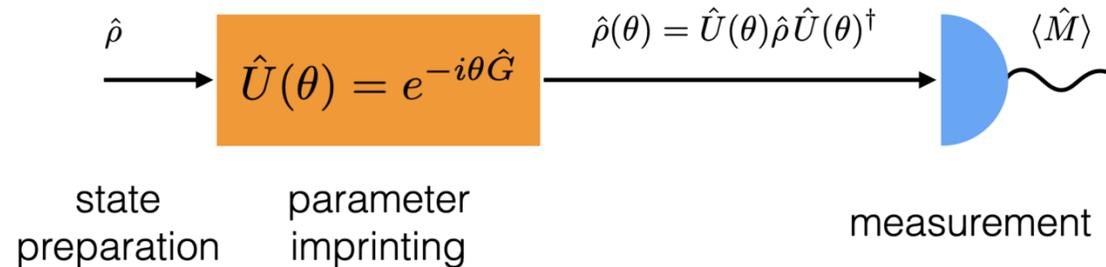
Quantum metrology using non-Gaussian bosonic states



A nice overview: <https://arxiv.org/abs/2411.04122>



The pros and cons of bosonic cQED for metrology

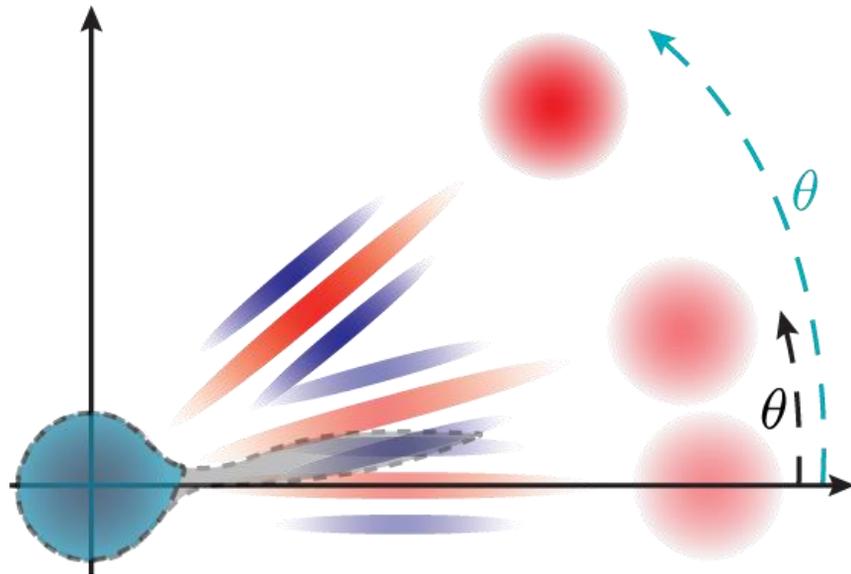


- High-Q cavities store CV states coherently
- Versatile nonlinear ancillary elements allow access to tailored resource states
- **Can only use artificial processes to imprint parameters!**
- **Not really “sensing” anything in practice**
- High quality measurement (SNR) via transmon readout
- Access to non-linear observables (e.g. parity)
- **Great testbed for metrology protocols**
- **Not so great as a practical sensor (yet)**

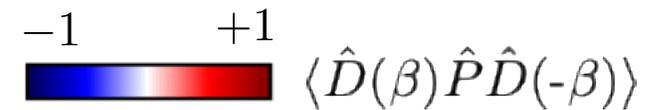
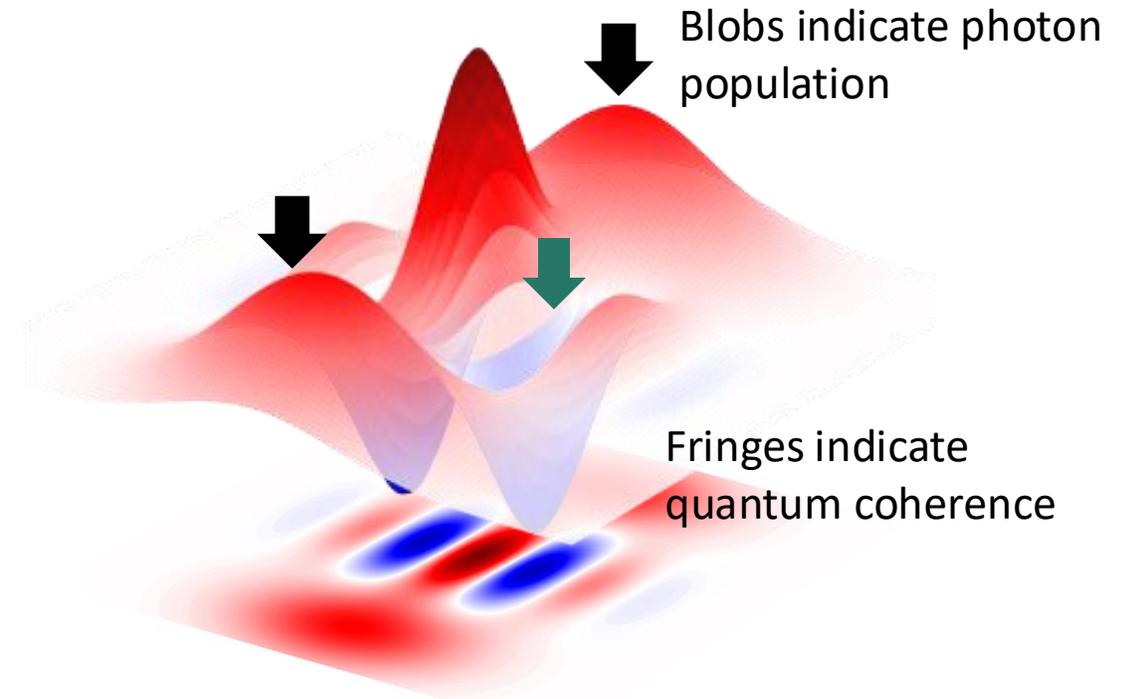


Quantum parameter estimation with bosonic cats

$$|\Psi\rangle_{\text{cat}} = \mathcal{N}(|\alpha\rangle + |-\alpha\rangle)$$



$$|\Psi\rangle_{\text{SCS}} = \mathcal{N}(|0\rangle + |\alpha\rangle)$$

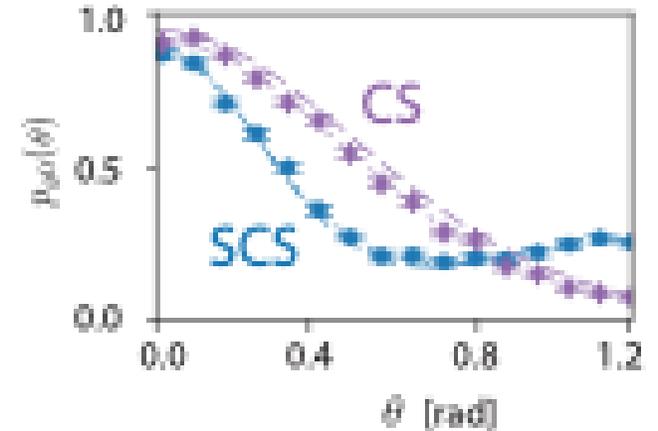
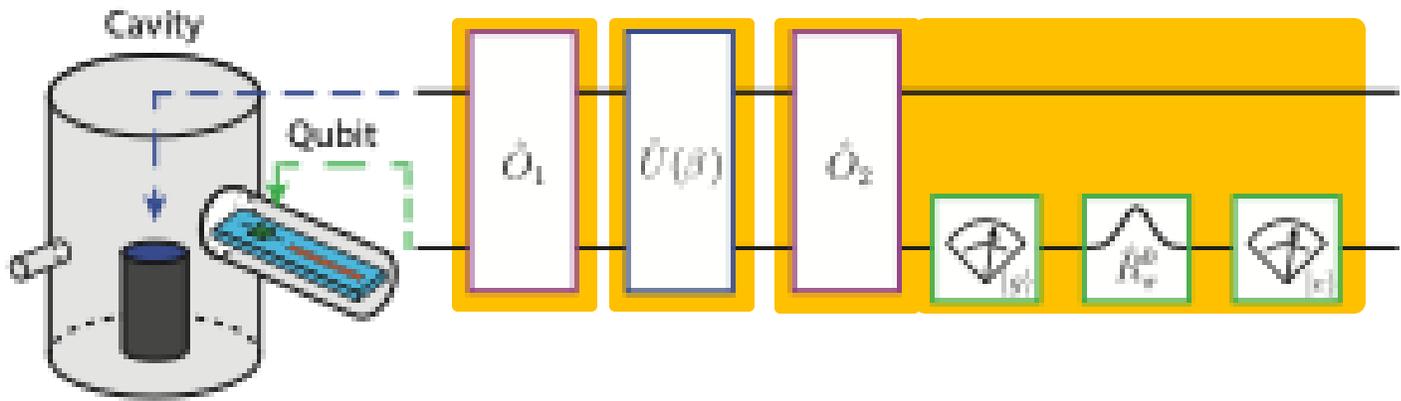


Wigner function picture



Simple and versatile metrology scheme

Use the strong dispersive regime to leverage conditional phase operations ($e^{i\chi t \hat{n}\sigma_z}$)



1. Two state transfer processes:

$$|g\rangle|\psi\rangle = \hat{O}_1|g\rangle|0\rangle \quad |g\rangle|0\rangle = \hat{O}_2|g\rangle|\psi\rangle$$

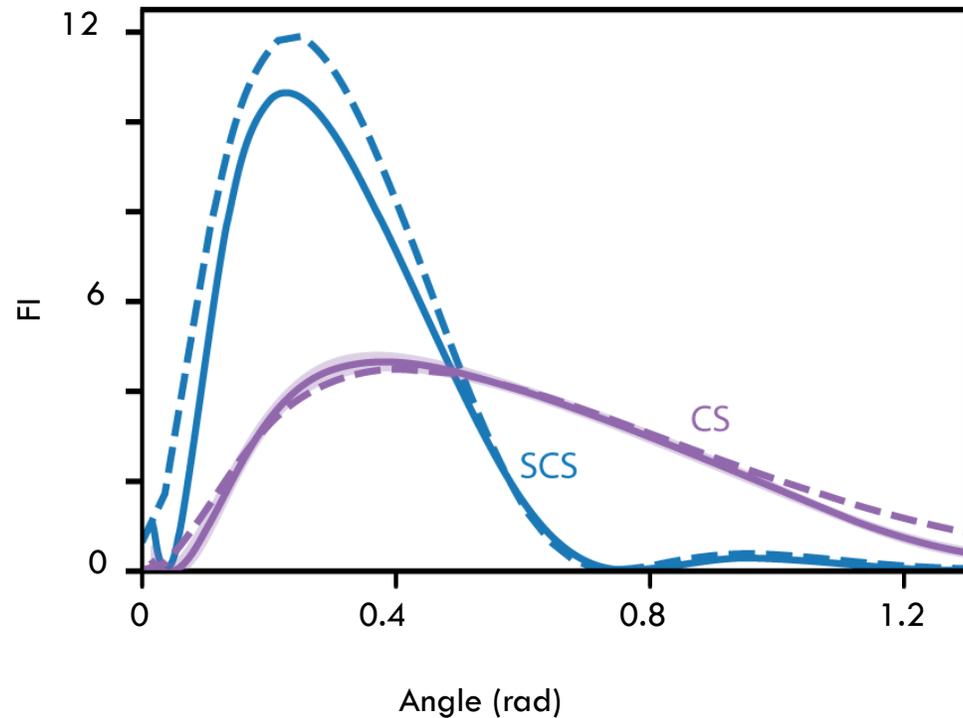
2. Impart phase via qubit or detuning

3. Directly extract Fischer information from projective measurement of $P_{g0}(\beta)$

$$F(\beta) = \frac{1}{p(\beta)(1-p(\beta))} \left(\frac{\partial p(\beta)}{\partial \beta} \right)^2$$

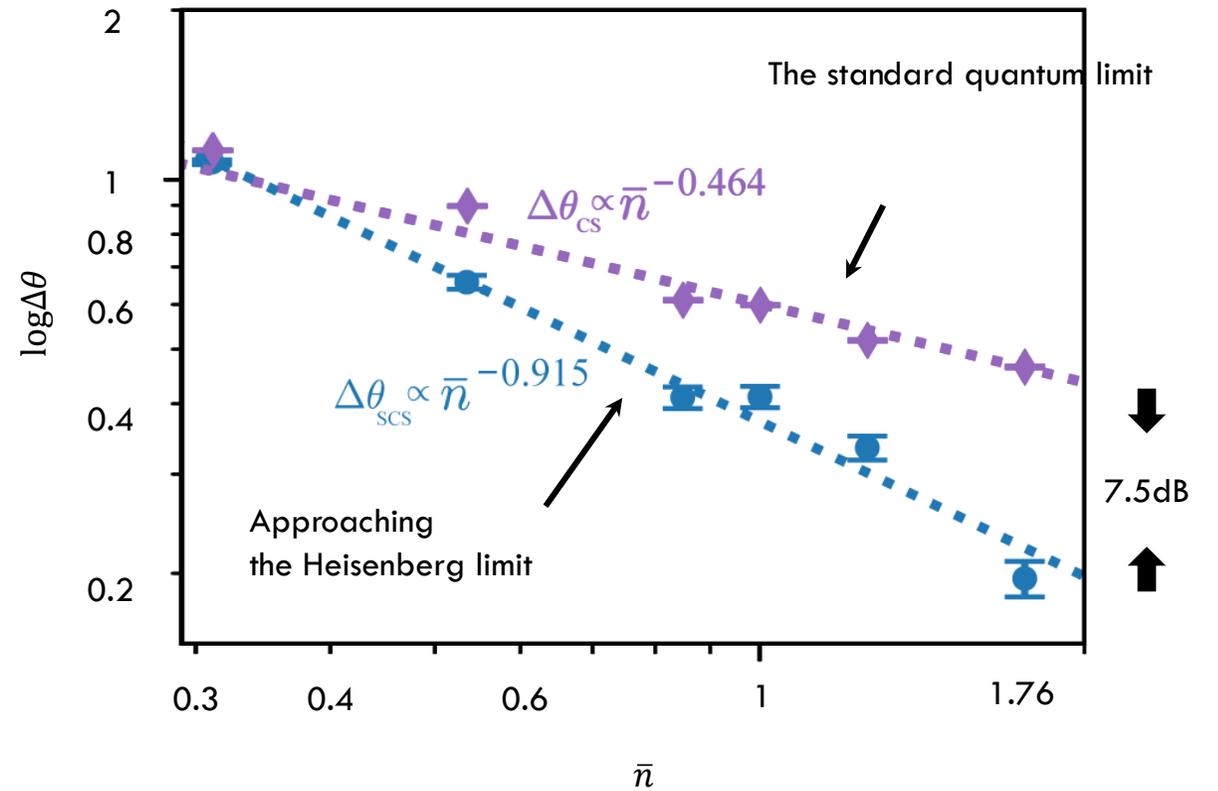


Quantum-enhanced precision with small cats

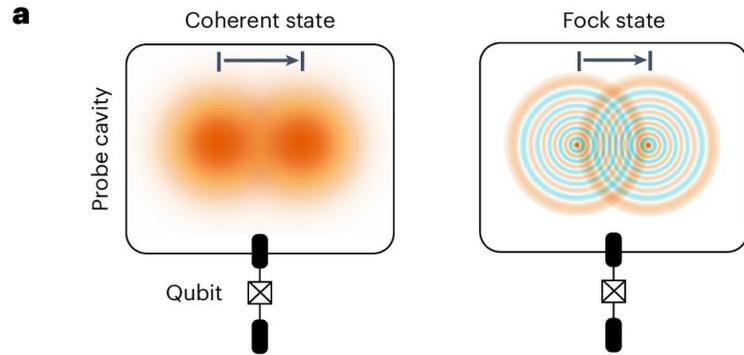


Higher Fisher information obtained with SCS compared to using semi-classical states

Significant quantum enhancement in precision achieved with low energy states

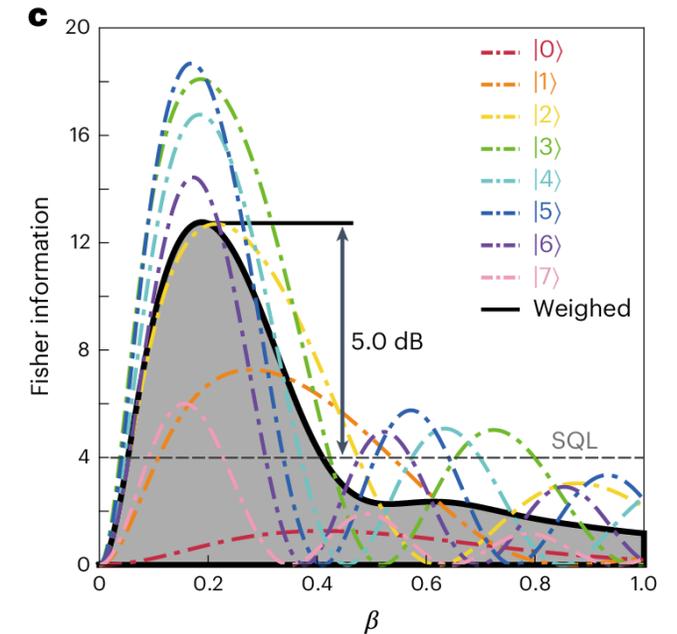
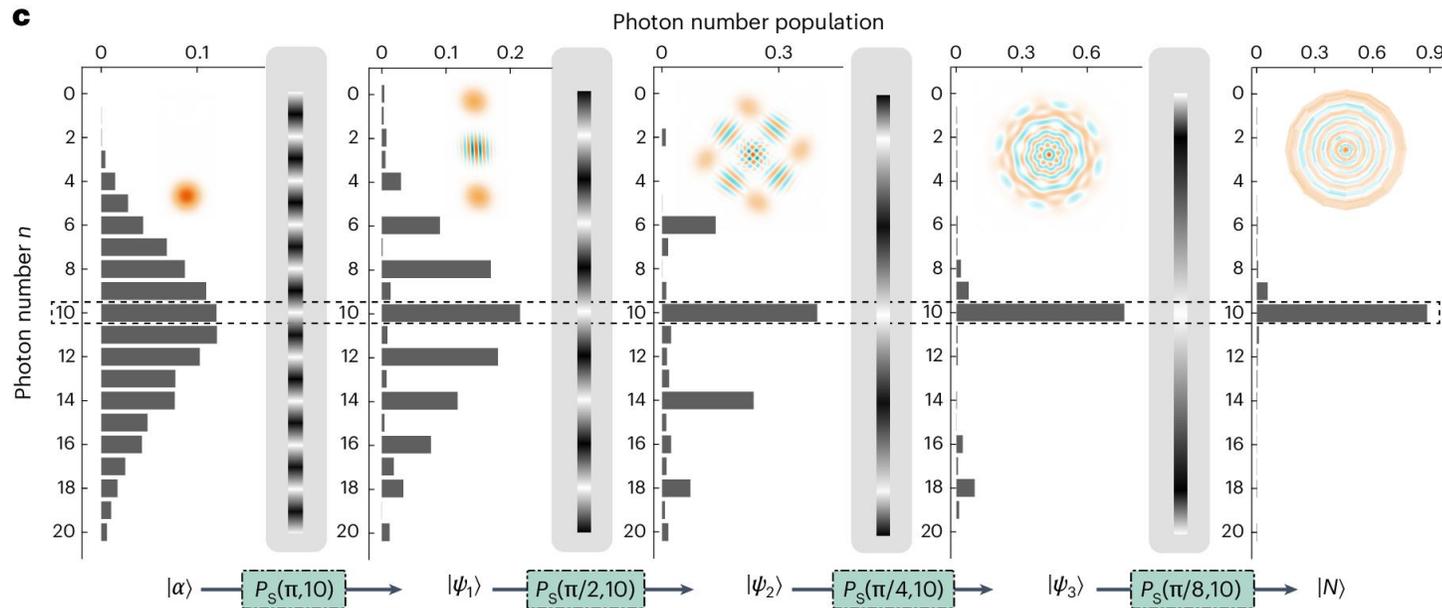


Quantum parameter estimation with large Fock states

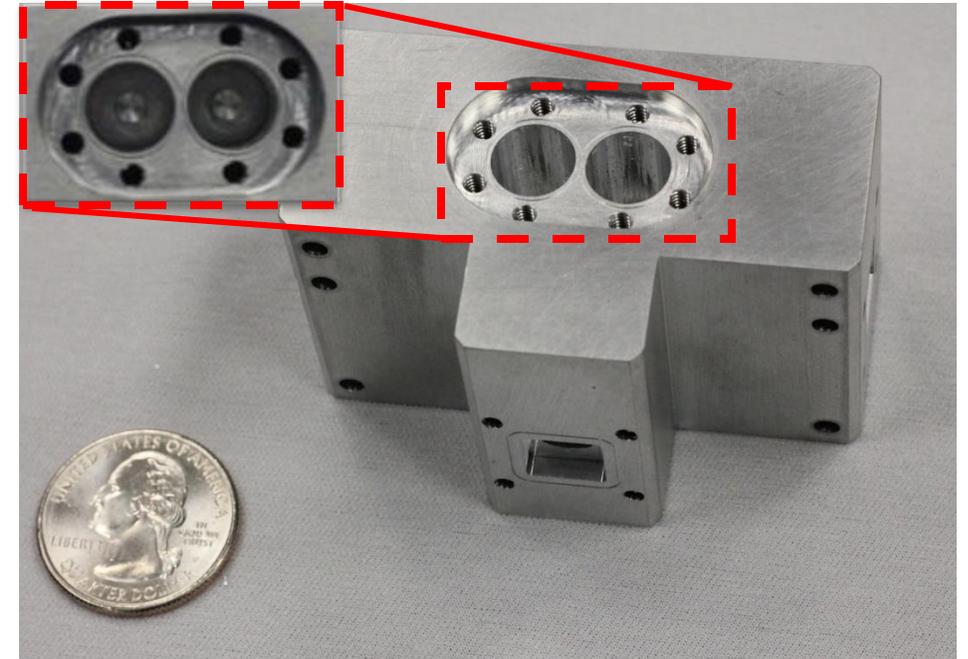
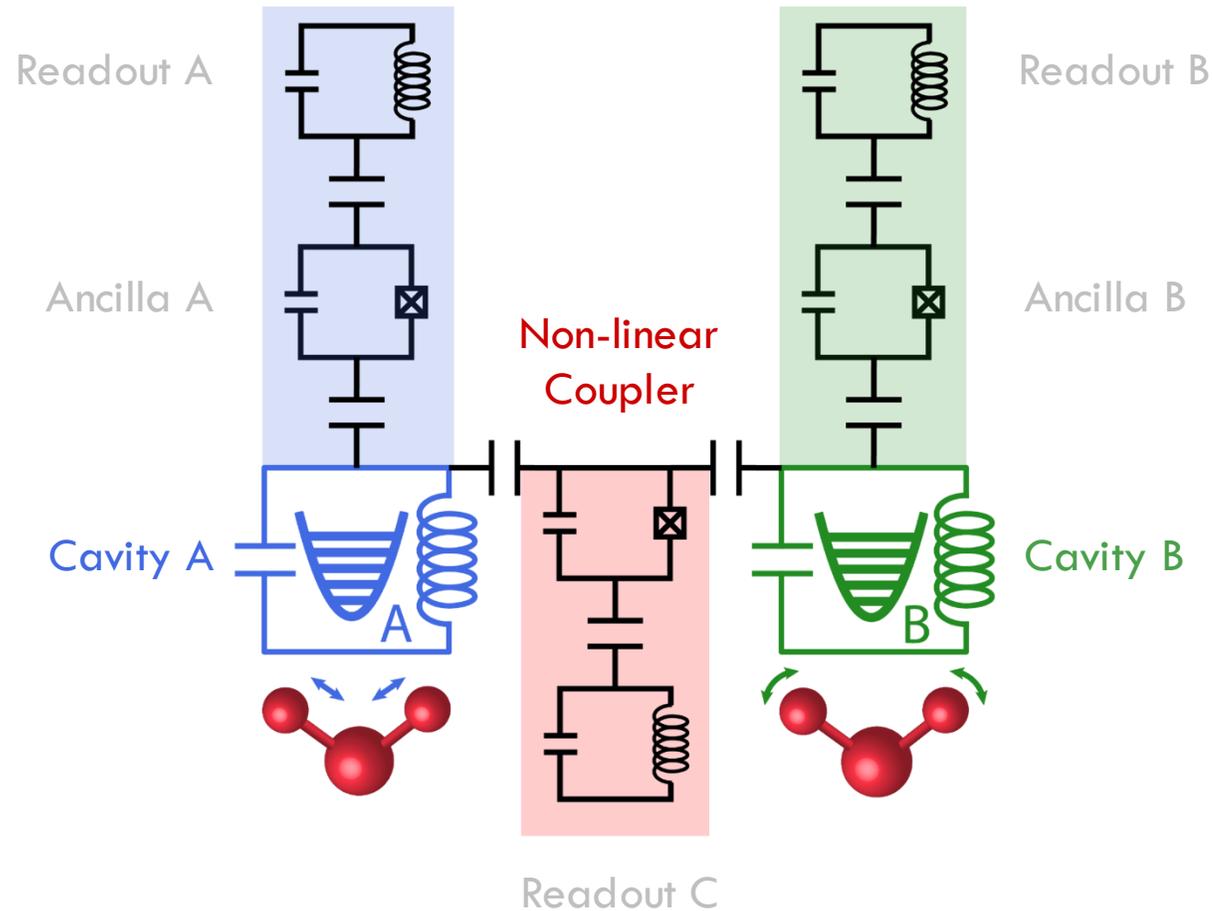


Use the “rings” to gain sensitivity
 The larger the Fock state, the higher the FI

Open question:
 - is there a fair comparison when the state is hard to prepare?



Analogue quantum simulation in bosonic cQED



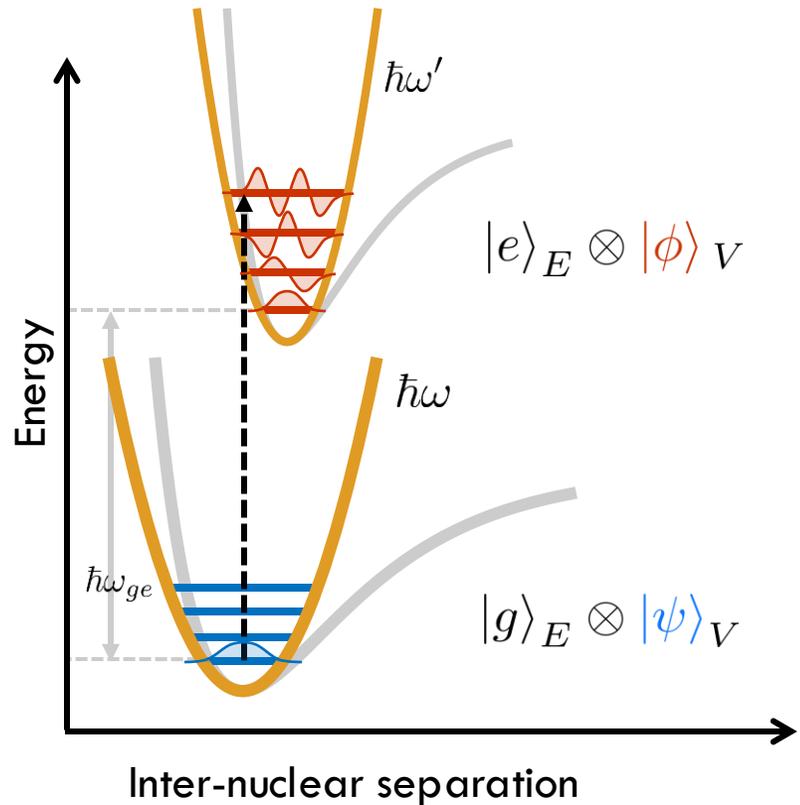
- Bosonic cQED provides high-Q oscillators with
 - Arbitrary state preparation
 - Gaussian operations
 - Photon-number resolving measurements



Direct extraction of Frank Condon factors

Frank-Condon factors $|\langle \phi_{n'} | \psi_n \rangle|^2$

Idea: relate two sets of harmonic oscillators with a basis change



$$\mathbf{U}_{\text{Dok}} = \mathbf{D}(\alpha) \mathbf{S}^\dagger(\zeta') \mathbf{R}(U) \mathbf{S}(\zeta)$$

□ Gaussian operations

Strategy: prepare pre-transition state, implement basis change, measure

$$\text{FCFs} = \left| \langle n^r | \mathbf{U}_{\text{Dok}} | \psi_0 \rangle \right|^2$$

□ Non-Gaussian state preparation

□ photon number resolving detection

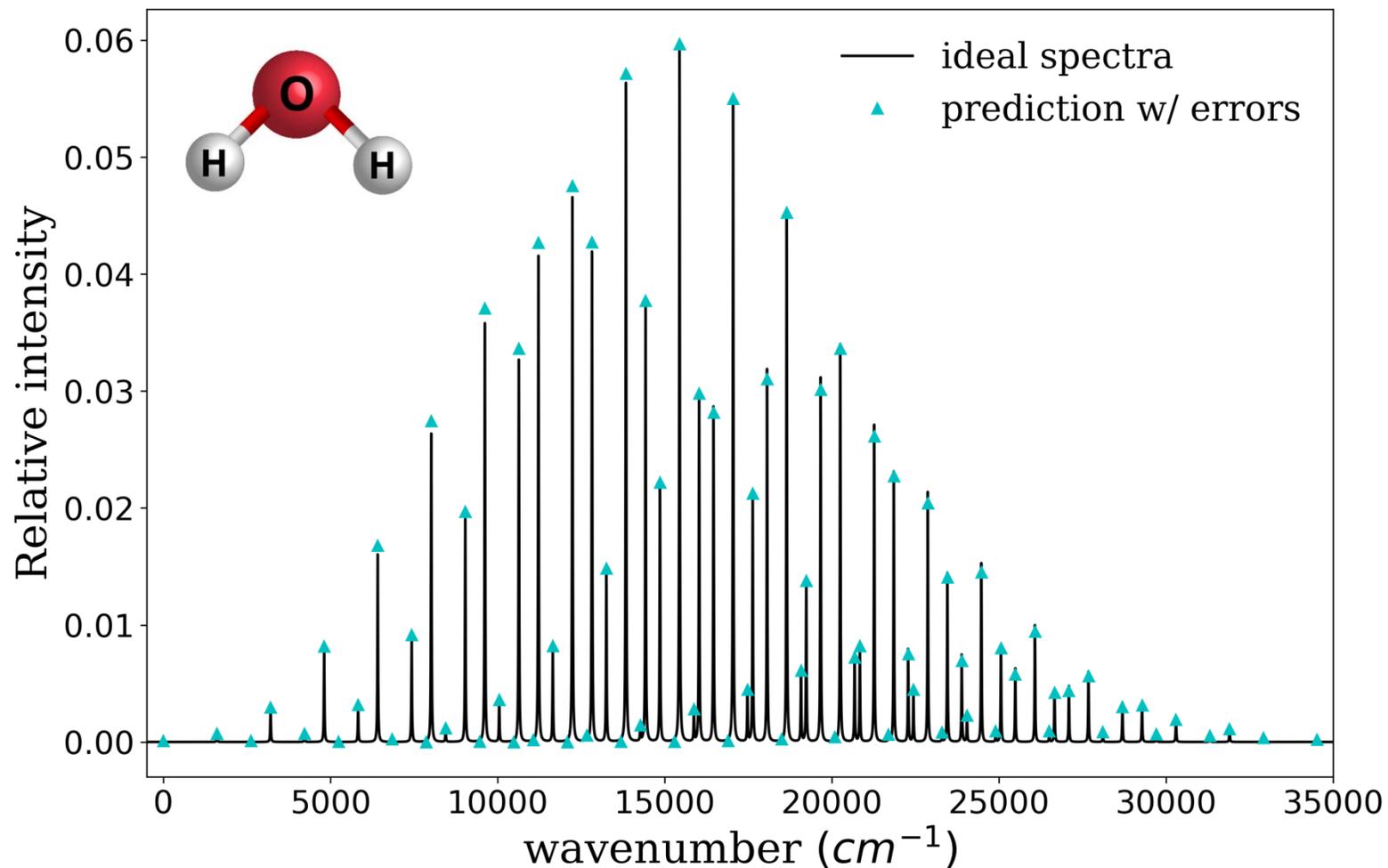
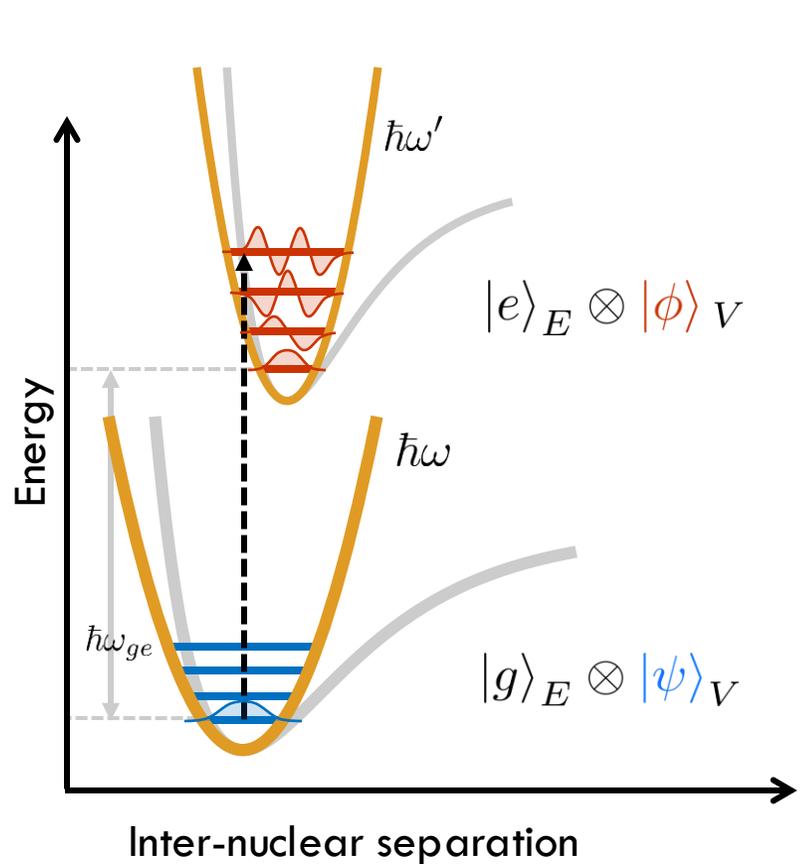
Doktorov et al. *J. Mol. Spec.* **64** 302-326 (1977)

Huh et al. *Nature Photonics* **9** 615-620 (2015)

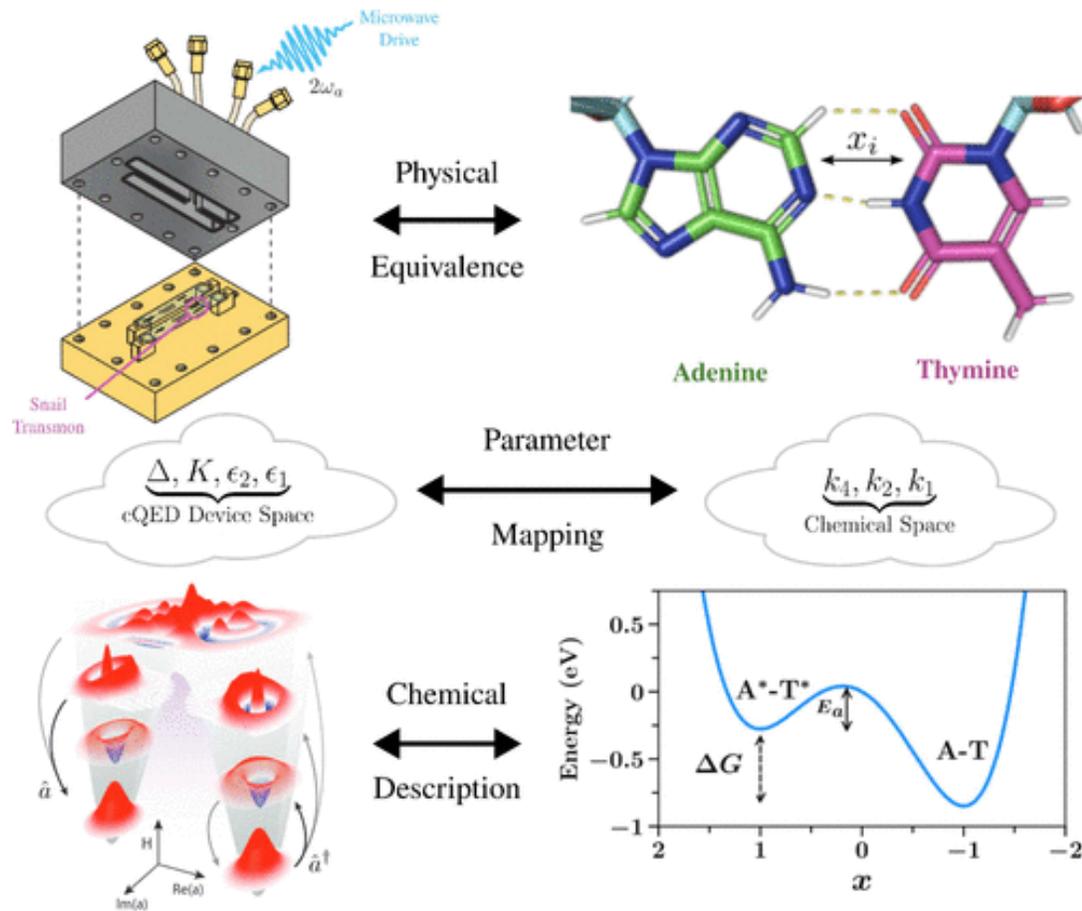


Direct extraction of Frank Condon factors

Frank-Condon factors $|\langle \phi_{n'} | \psi_n \rangle|^2$ $\text{H}_2\text{O} \rightarrow \text{H}_2\text{O}^+(\tilde{\text{B}}^2\text{B}_2) + \text{e}^-$



Analogue quantum simulation in bosonic cQED



Example:

- We can map between the Hamiltonian parameter space of the Kerr-Cat Hamiltonian and a generic chemical double-well.
- The bosonic states under the Hamiltonian is analogous to the states associated with a chemical double-well potential.

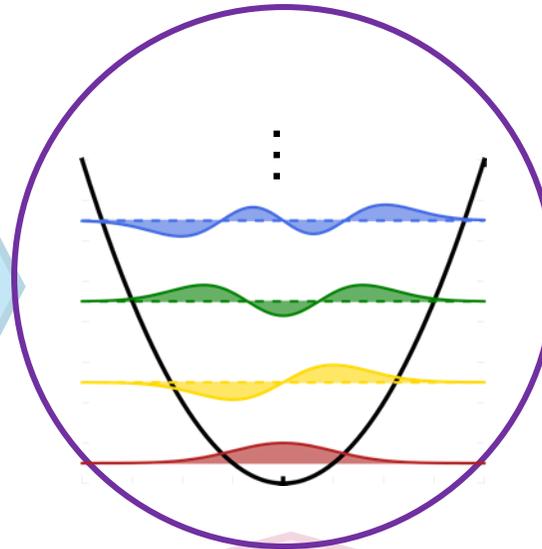
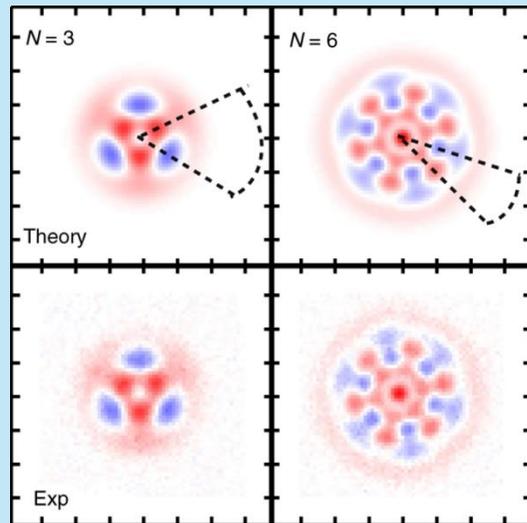
Simulating Chemistry on Bosonic Quantum Devices

<https://pubs.acs.org/doi/full/10.1021/acs.jctc.4c00544>

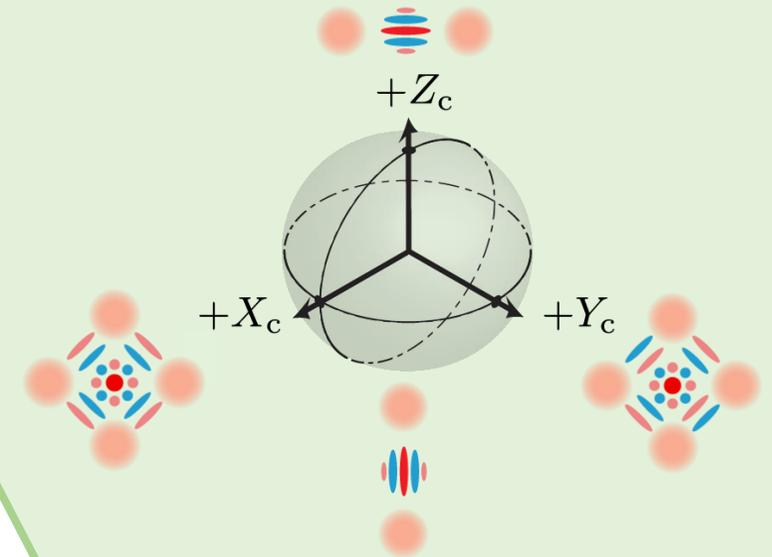


Bosonic cQED devices are versatile quantum testbeds

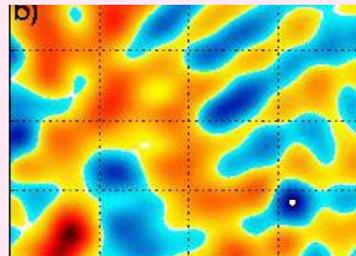
metrology & simulation



quantum computing



quantum foundations



Knott et al. PRA (2016)
Mirrahimi, NJP (2014)
Ofek, Nature (2016)
Wang, PRX (2020)
Zurek, Nature (2020)
etc. etc.



Probe interesting physics with bosonic cQED

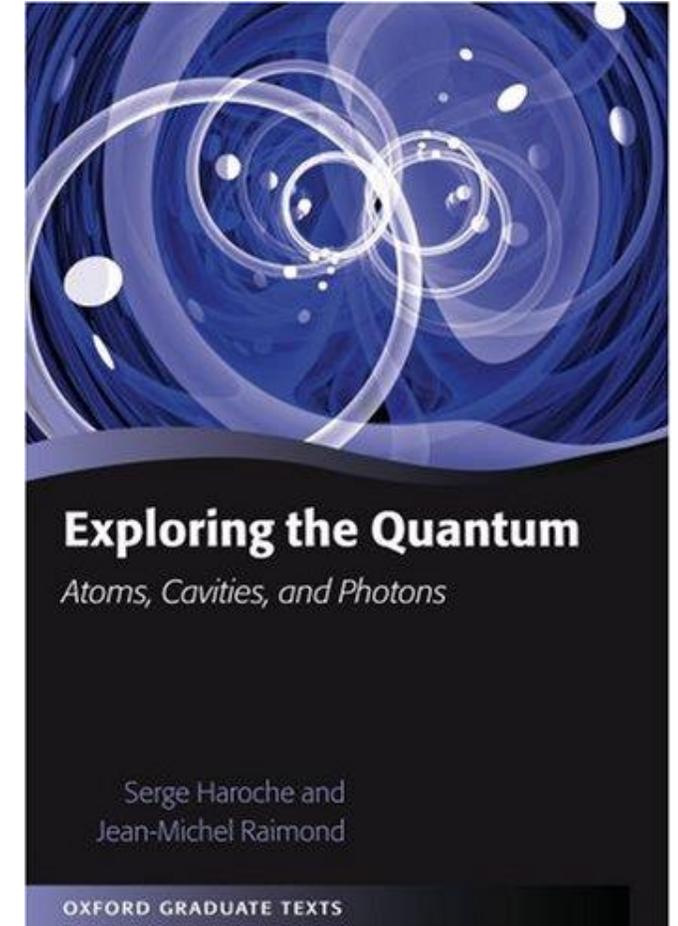
Multimode mesoscopic (macroscopic) entanglement

7.6 Non-local cats

The control of a mesoscopic field by a single atom can be extended to even stranger situations. The fields of spatially separated cavities can be entangled together via their interaction with the atom, leading to situations which would add the flavour of non-locality to the strangeness of cat states. These experiments are, at the time of the writing of this book, in planning stage in our laboratory (Milman *et al.* 2005). We give in this section a brief description of what we hope to achieve.

7.6.1 Two weirdnesses combined: toward mesoscopic non-locality

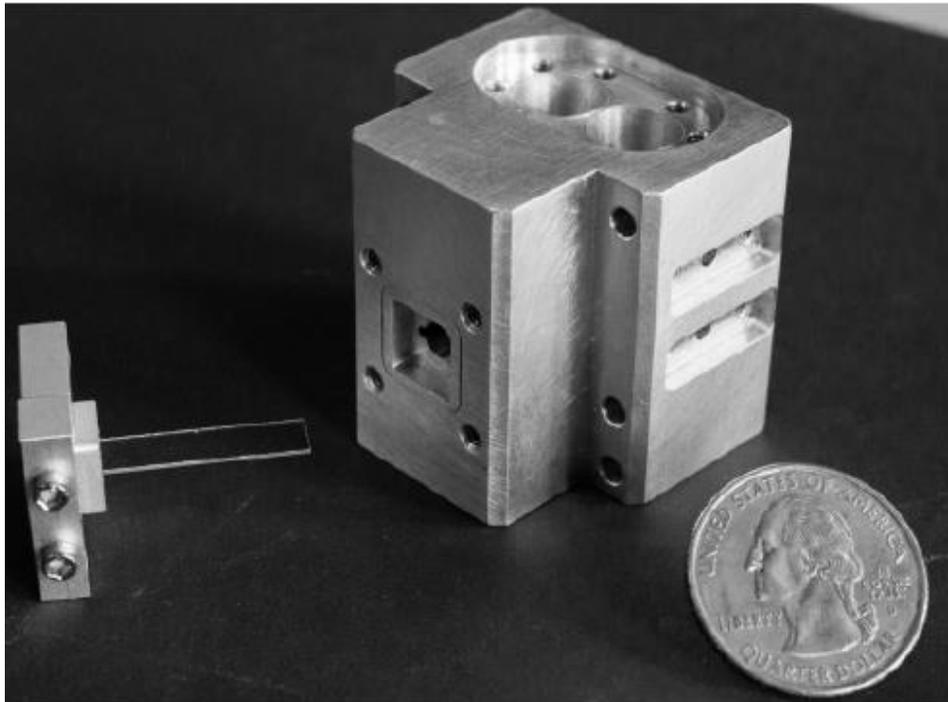
The principle of the experiment consists in combining together two (or more) set-ups of the kind described above, in order to knit entanglement between mesoscopic fields stored in distinct boxes. Restricting ourselves to the case of two cavities, we have in mind the set-up sketched in Fig. 7.36. An atom crosses successively two identical



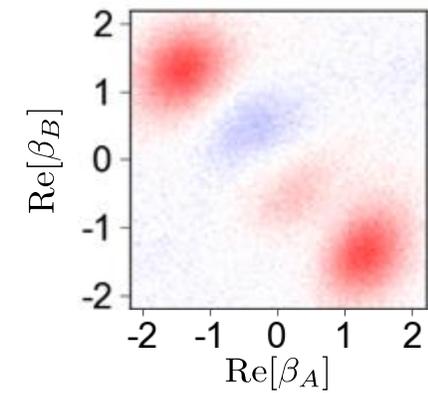
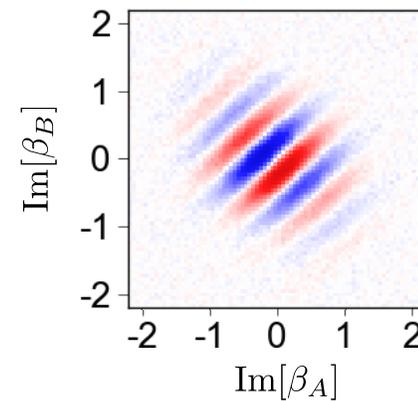
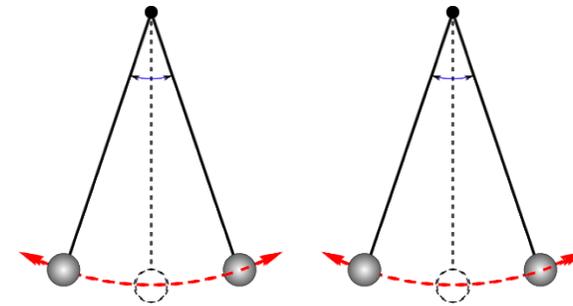
Probe interesting physics with bosonic cQED

Multimode mesoscopic (macroscopic) entanglement

- quantum-classical boundary?
- New classes of entanglement tests and witnesses?



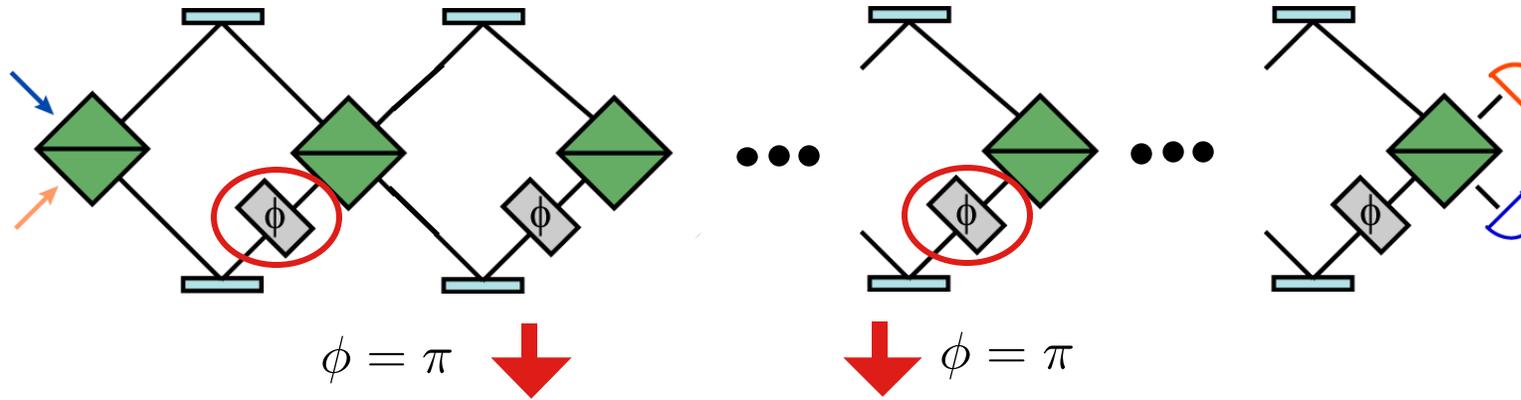
$$|\alpha\rangle_A |-\alpha\rangle_B + i|-\alpha\rangle_A |\alpha\rangle_B$$



Science 352 (6289), 1087-1091 (2016)
Nature 566 (7745), 509-512 (2019)

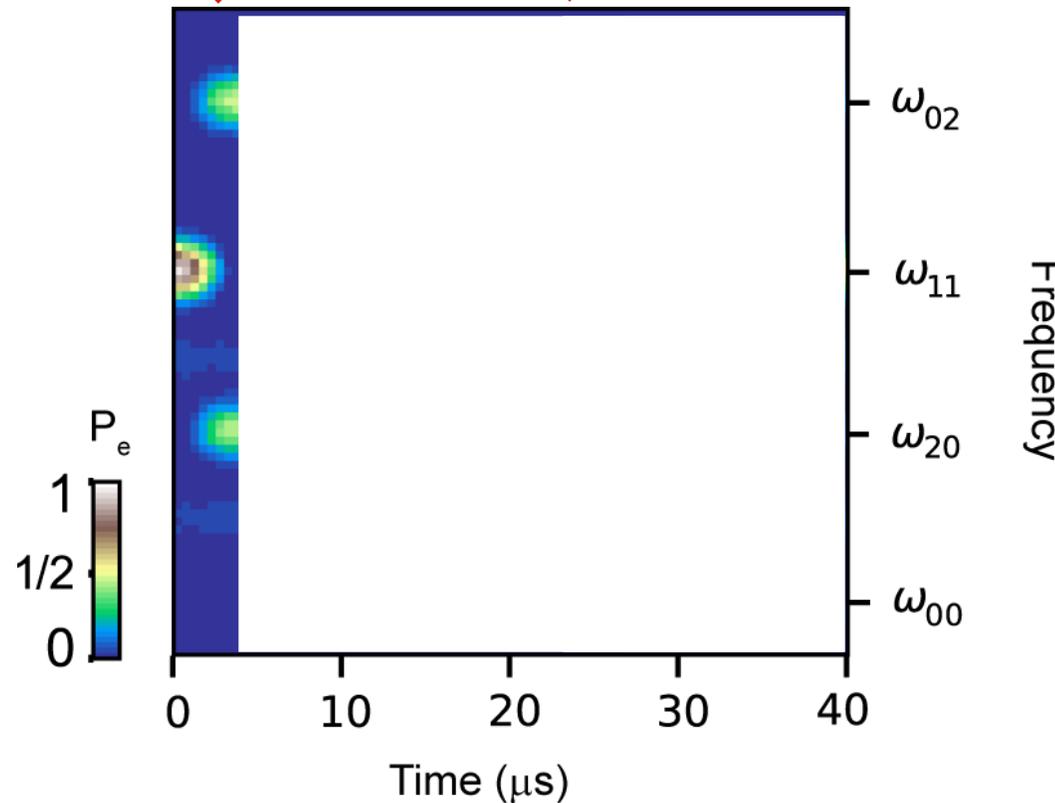


Probe interesting physics with bosonic cQED



Programmable quantum interferometers with strong nonlinearity

- HOM on steroids
- Boson sampling?
- Swap tests?



The Hong-Ou-Mandel effect is really odd

<https://arxiv.org/abs/2410.11800v1>

Y. Gao, PRX (2018);
Chapman, PRXQ (2023)



What must be further improved?

Non-linearity is a double-edged sword!

- Good for fast control, engineering new dynamics, etc
- But the harmonic oscillator is not truly “harmonic”!
 - Inherited nonlinearity distorts bosonic resource states
 - Always on coupling causes leakage out of code space
- Exploration of new ancillary elements or control techniques!

Large Hilbert space of the bosonic mode is also a double-edged sword!

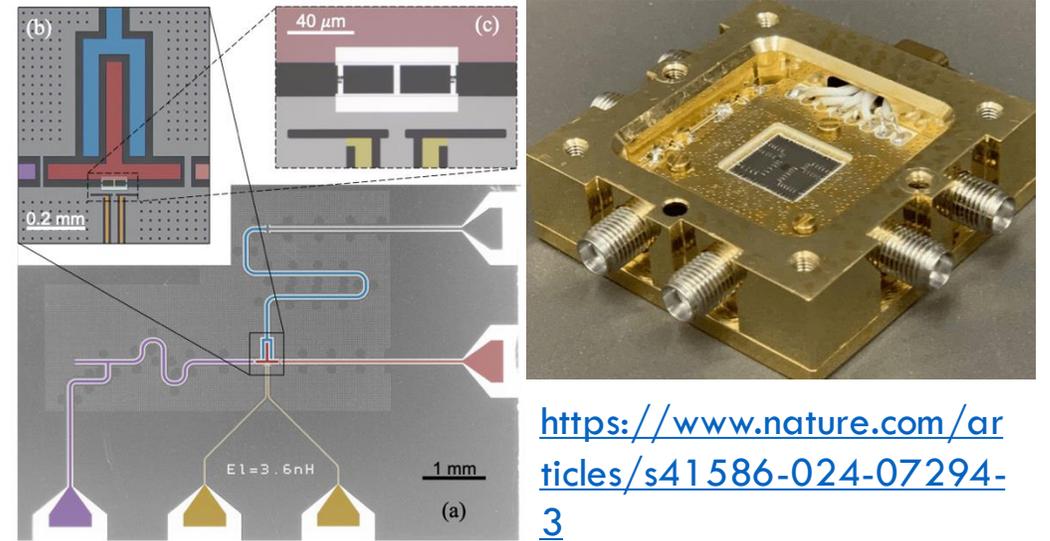
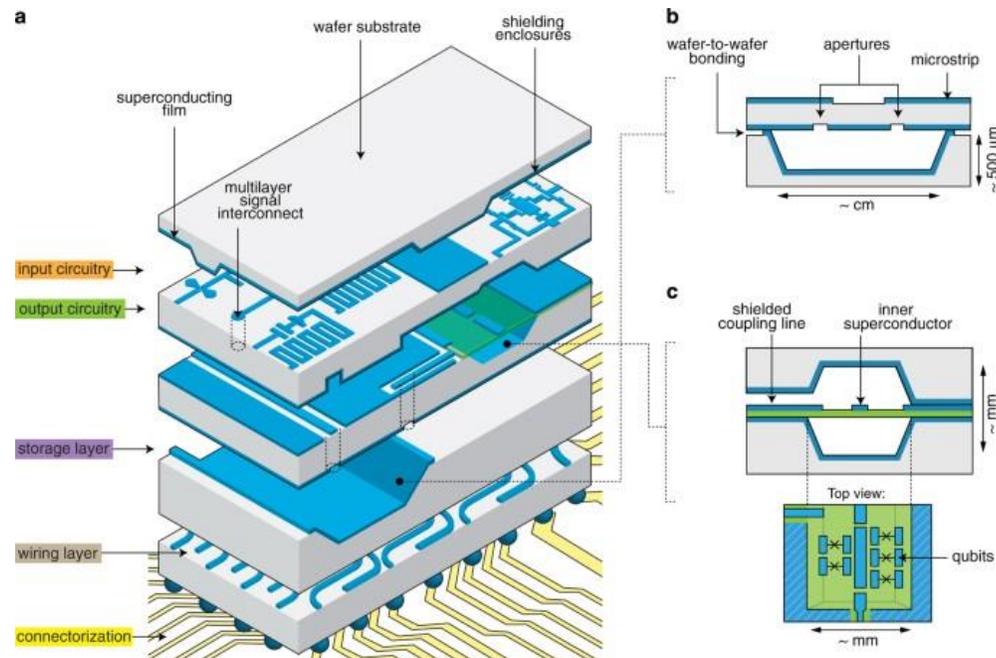
- Great for creating CV resource states and offers rich dynamics
- Simulating its dynamics are hard even with just a few modes!
- new basis for simulation/modeling? <https://doi.org/10.1364/OPTICAQ.504311>



What must be further improved?

Hardware optimisation needs to be continued

- 3D cavities have a great coherence
- They are bulky!
- Limited control flexibility and connectivity



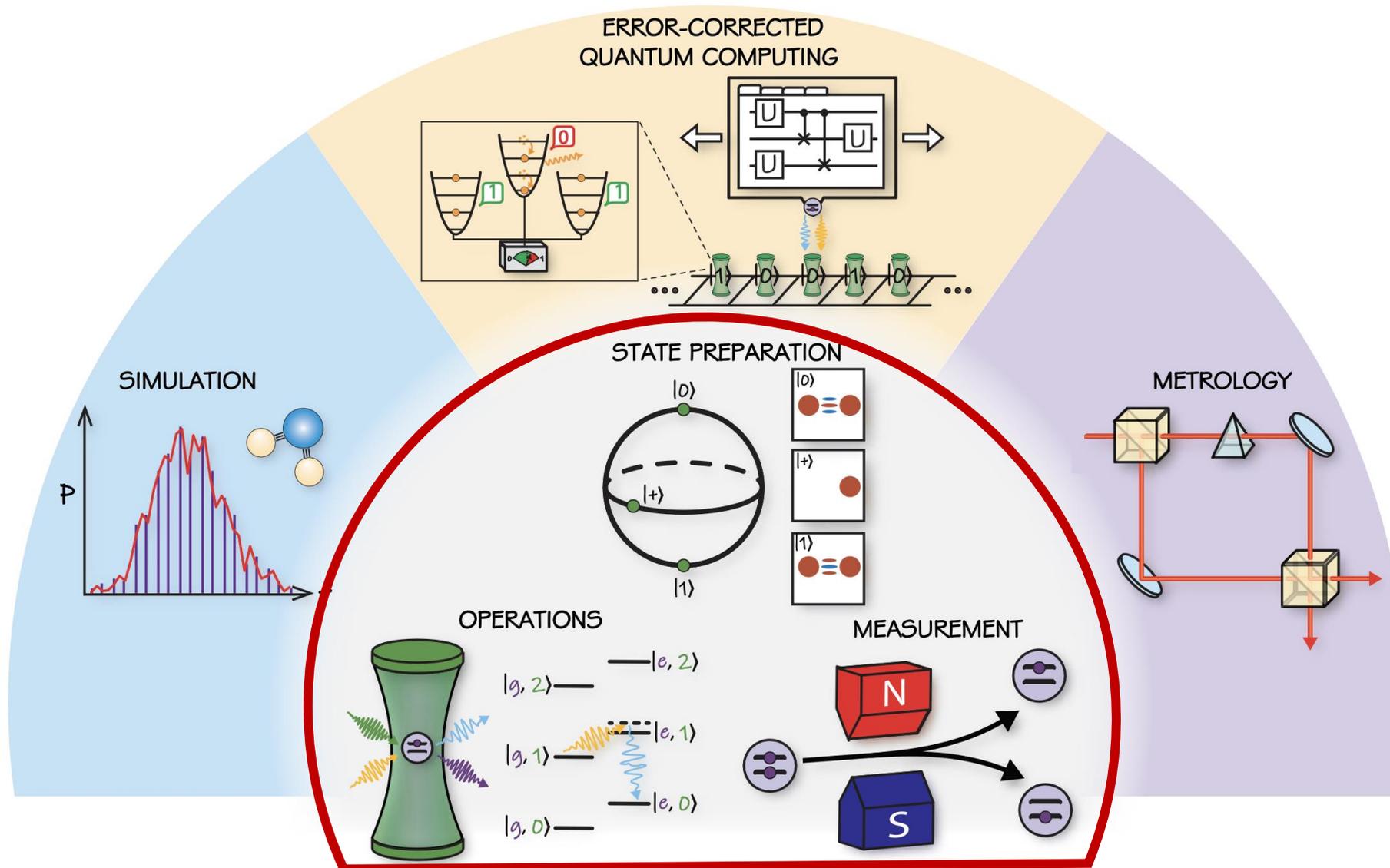
<https://www.nature.com/articles/s41586-024-07294-3>

investigations of alternative architectures:

- Balance coherence with controllability and form factor
- Need materials studies, RF engineering, and lots of creativity!

Brecht, APL (2015), Lei, APL (2020), Ganjam, Nature Comm (2024), etc

A versatile quantum playground even at small scale



Many exciting explorations ahead!

RESEARCH ARTICLE | NOVEMBER 18 2024

Bosonic Mpemba effect with non-classical states of light

Stefano Longhi 



+ Author & Article Information

APL Quantum 1, 046110 (2024)

<https://doi.org/10.1063/5.0234457>

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