

QUANTUM ANNEALING

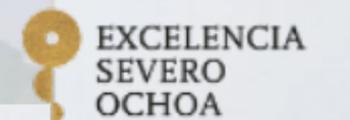
Pol Forn-Díaz

Institut de Física d'Altes Energies (IFAE), Barcelona
Quantum Computing Technology group: <https://qct.ifae.es/>

SPRING SCHOOL ON SUPERCONDUCTING QUBIT TECHNOLOGY
Benasque Center for Science, May 29th 2025



Institut de Física
d'Altes Energies



QCT group

<https://qct.ifaes.es>

Team members



P. Forn-Díaz, PI



Manel Martínez, staff



Elia Bertoldo, Postdoc



Ariadna Gómez del Pulgar, PhD



Guillermo del Riego, PhD



Luca Cozzolino, PhD



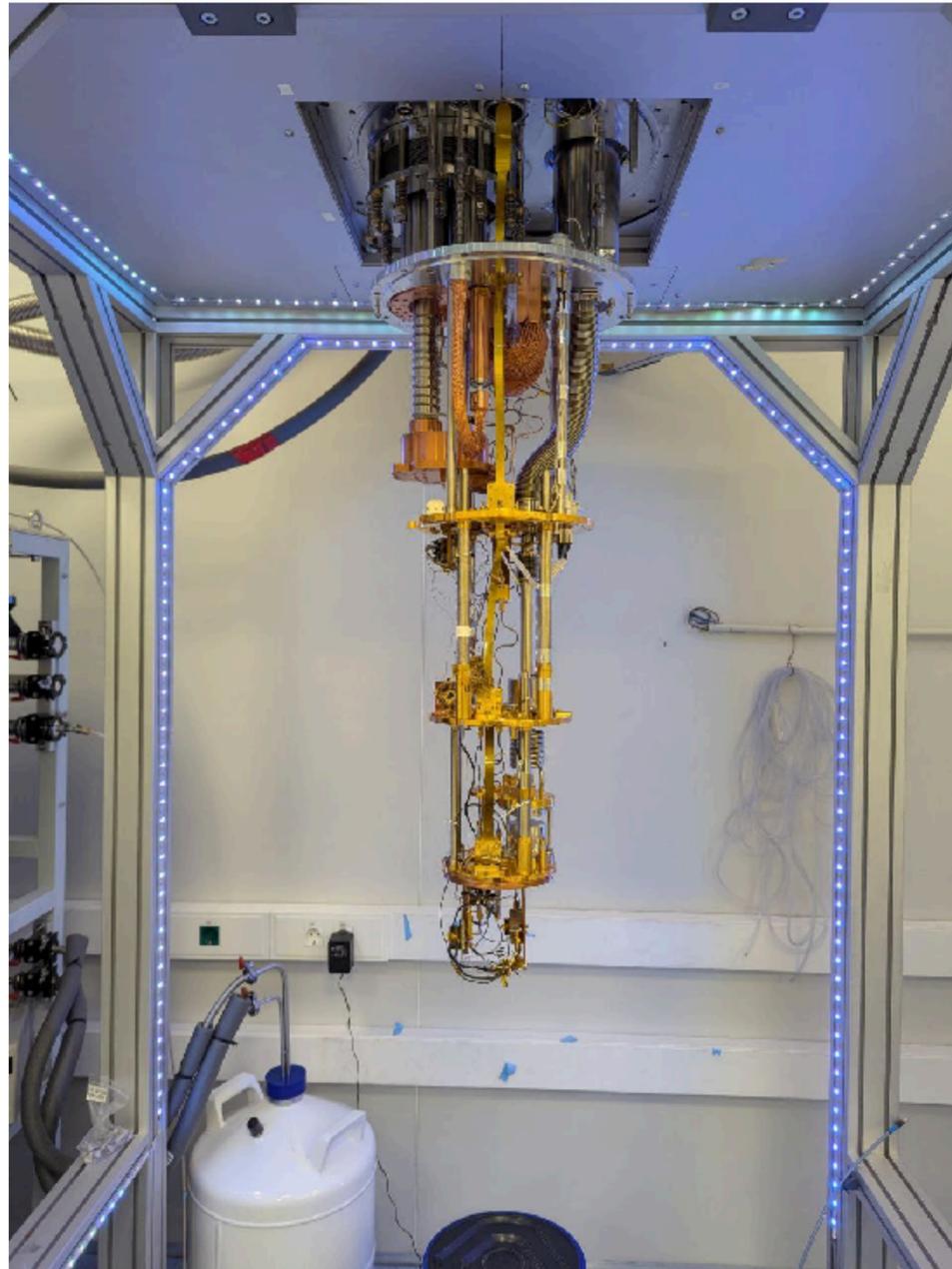
Alba Torras, PhD

Alumni

- Oriol de Miguel Ribas (TFG) 2023-2024
- David López Núñez (PhD) 2018-2024
- Miquel Àngel Pérez Puigdomènech (TFG) 2023-2024
- Pol Rayo Sánchez (TFG) 2022-2023
- Fabian Zwiehoff (PhD) 2019-2023
- Boris Nedyalkov (PhD) 2020-2023
- Raquel Garcia Bellés (TFG) 2020-2021
- Queralt Portell (TFG) 2021-2022
- Adrià Riera (TFG) 2020-2021
- Raquel Garcia (TFG) 2020-2021
- Olga Marco (TFG) 2019-2020
- Ivan Alsina (TFG) 2019-2020
- Alberto Lajara (TFM) 2019-2020
- Barkay Guttel (TFM) 2019-2020
- Santi Vallés (TFG) 2019-2020
- Rafael Luque (TFM) 2019-2020
- Adrià Grabulosa (TFG) 2018-2019

QCT group

Laboratory space: Dilution refrigerators



QCT group

IFAE cleanroom space



QILIMANJARO
QUANTUM · TECH

<https://www.qilimanjaro.tech/>

IFAE
Institut de Física
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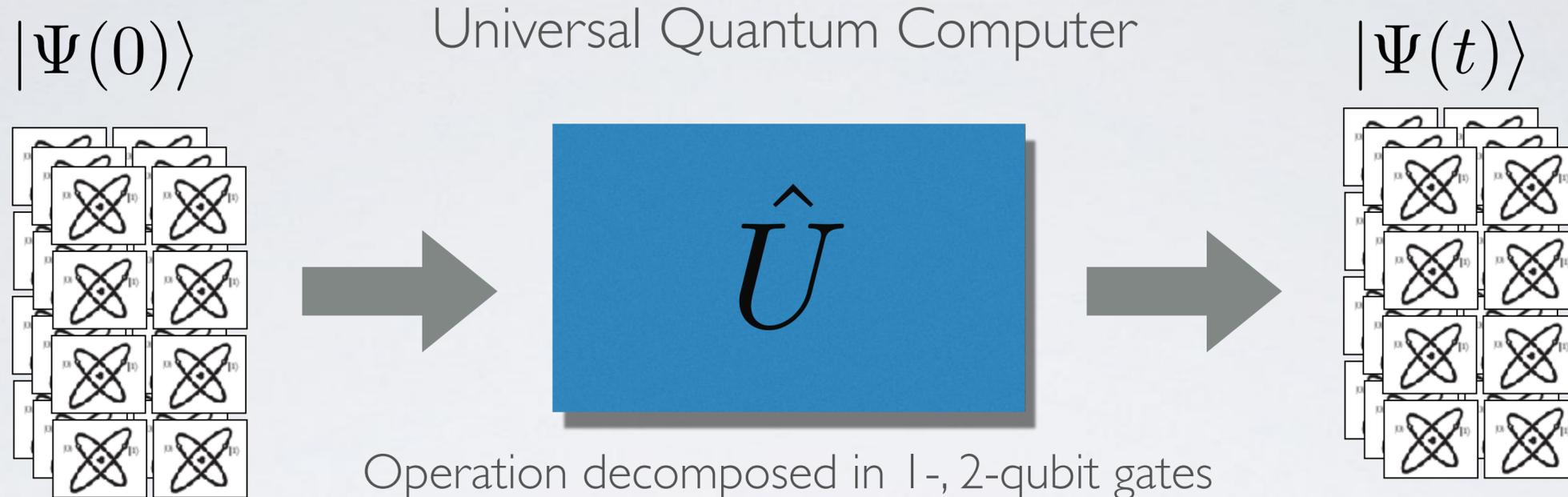
BIST Barcelona Institute of
Science and Technology

OUTLOOK

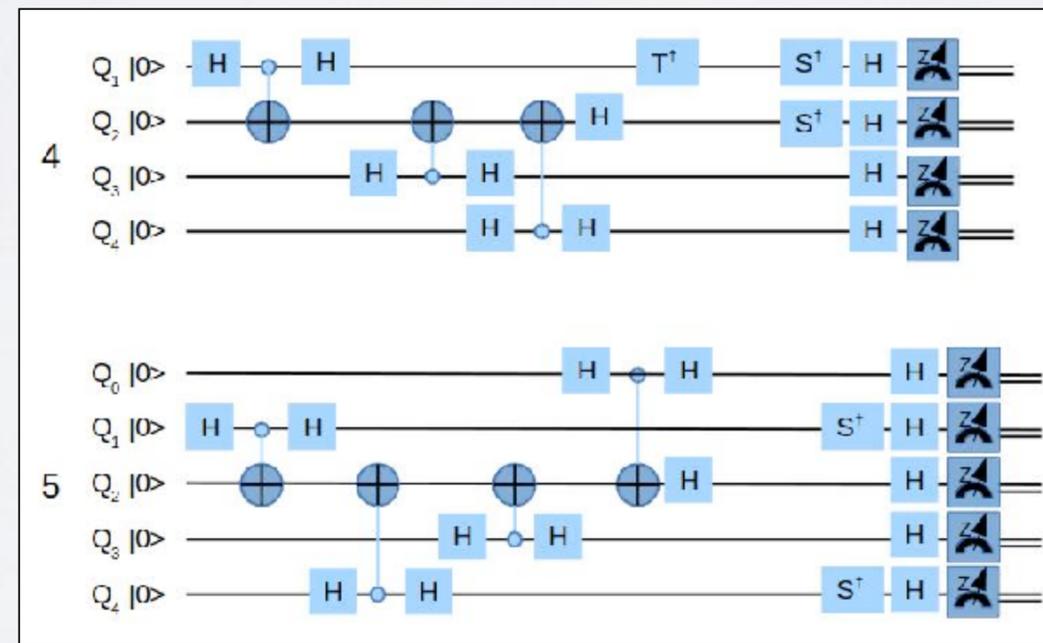
- Adiabatic Quantum Computation and Quantum Annealing
- Superconducting circuits with Josephson junctions

break
- Commercial (incoherent) Quantum Annealers
- Coherent Quantum Annealing technology

DIGITAL VS ANALOG QC



PROS:
Applicable to any quantum algorithm



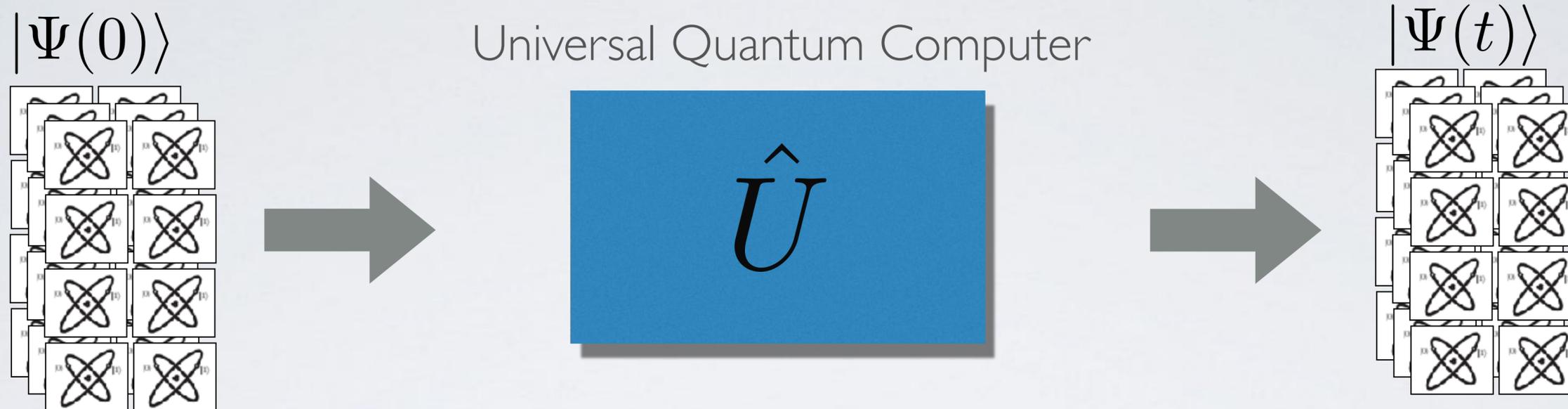
Quantum gates circuit

Processing 2^N states simultaneously

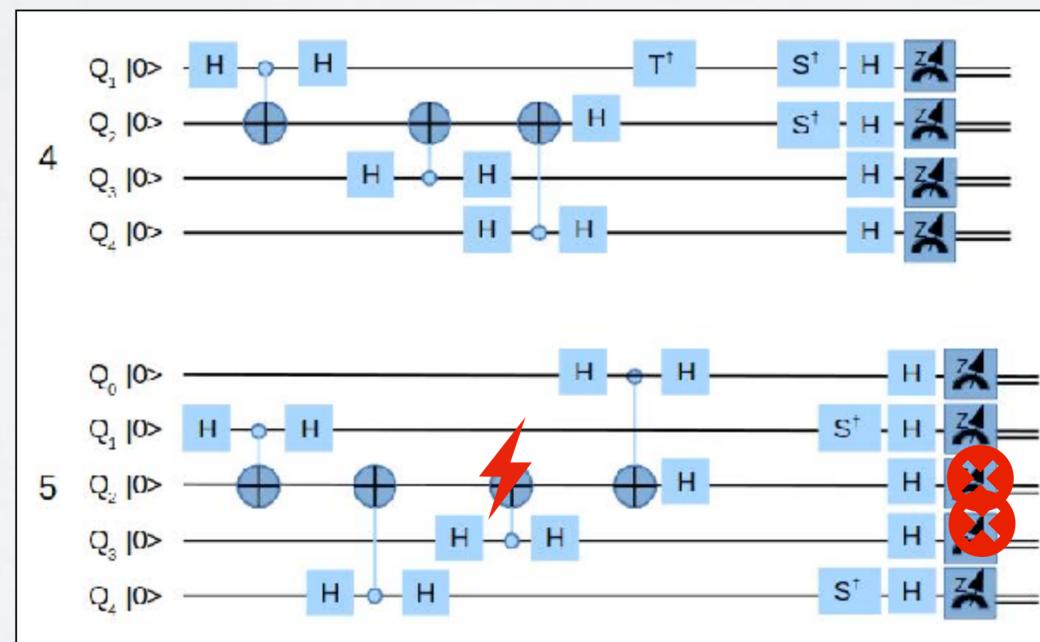
CONS:
Requires quantum error correction: $\sim 10^6$
Long-term

Examples 4/5, qubit sequences from D. Alsina, J.I. Latorre. Physical Review A 94 (1), 012314 (2016)

DIGITAL VS ANALOG QC



Operation decomposed in 1-, 2-qubit gates



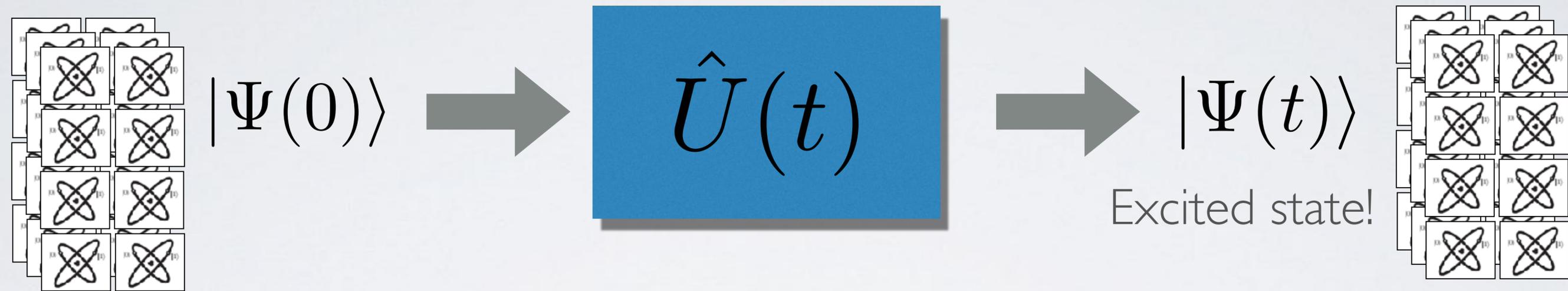
If error occurs, algorithm fails

Unless Quantum Error Correction is implemented, approach is unreliable

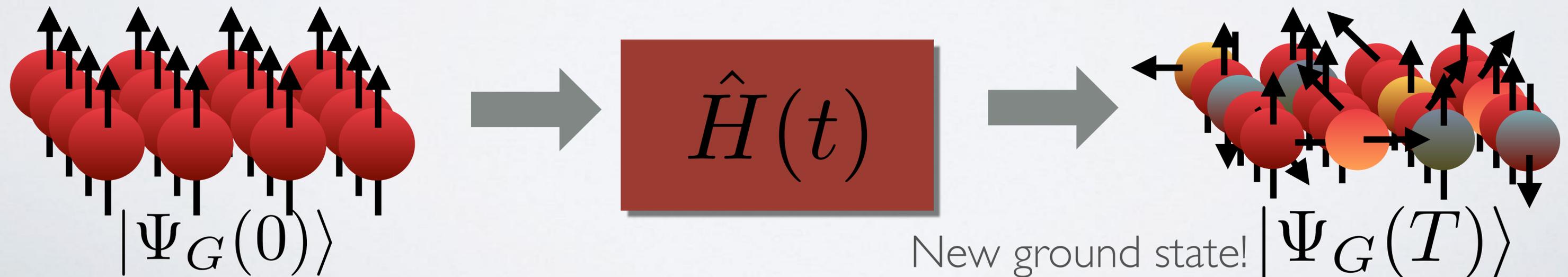
DIGITAL VS ANALOG QC

Digital quantum computers are most well-known. Alternative approach is by using analog quantum processors, also known as quantum annealers.

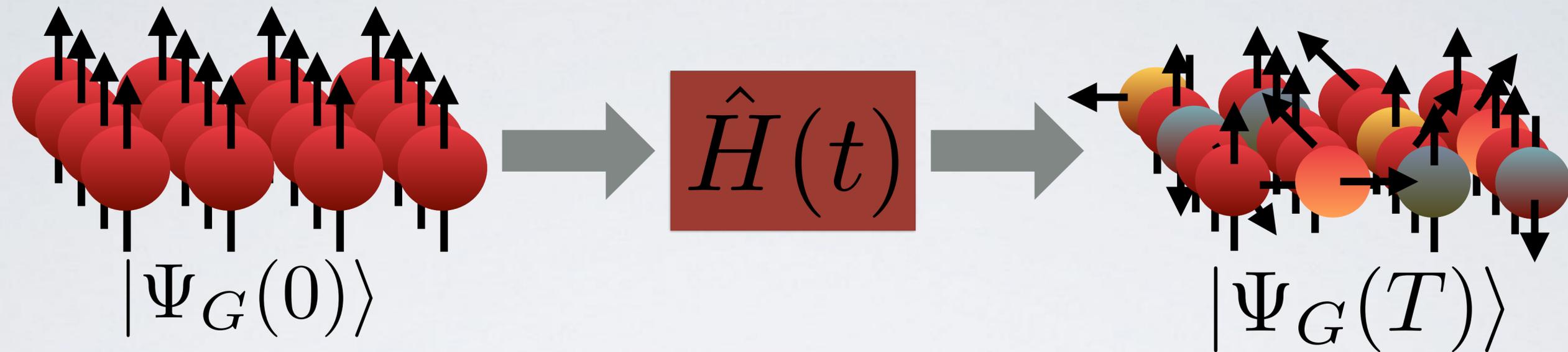
Digital: Evolution of quantum state by change of state:



Analogue: Evolution of quantum state by change of H. Also known as AQC.



DIGITAL VS ANALOG QC



Adiabatic Quantum Computation is Equivalent to Standard Quantum Computation
Dorit Aharonov, Wim van Dam, Julia Kempe, Zeph Landau, Seth Lloyd, Oded Regev
SIAM Journal of Computing, Vol. 37, Issue 1, p. 166-194 (2007)
arXiv:quant-ph/0405098

We know how to realize discrete sets of gate sets in digital QC.
How do we implement analog quantum computing protocols in practice?

ADIABATIC QUANTUM COMPUTATION

ADIABATIC QUANTUM COMPUTATION (AQC)

The adiabatic theorem

Practical example: 2-SAT problem

Blackboard

QUBO formulation

Classical Constraint	Equivalent Penalty
$x + y \leq 1$	$P(xy)$
$x + y \geq 1$	$P(1 - x - y + xy)$
$x + y = 1$	$P(1 - x - y + 2xy)$
$x \leq y$	$P(x - xy)$
$x_1 + x_2 + x_3 \leq 1$	$P(x_1x_2 + x_1x_3 + x_2x_3)$
$x = y$	$P(x + y - 2xy)$

Table of a few Known constraint/penalty pairs

ADIABATIC QUANTUM COMPUTATION (AQC)

Quantum solution

Quantum solution: build Hamiltonian whose ground state energy is the minimum of f :

$$H_P(|x_i\rangle) = f(x_{i_C}, x_{j_C})(|x_i\rangle)$$

equivalence only possible in ground state $|x_i\rangle$

Finds **equivalence** between mathematical formulation of real problem with physics from a physical system.

ADIABATIC QUANTUM COMPUTATION (AQC)

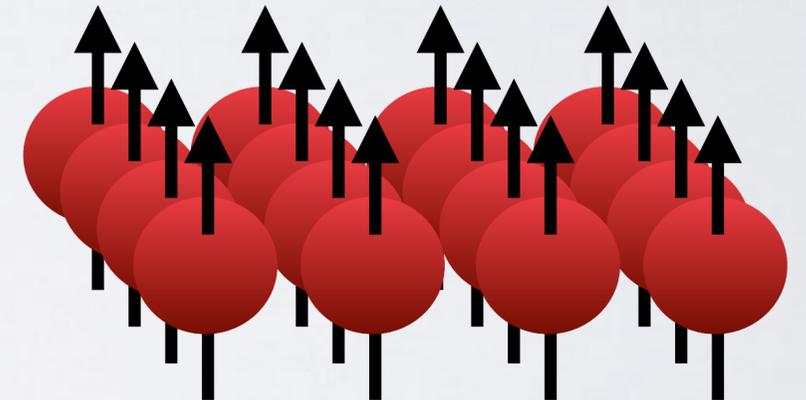
The problem Hamiltonian

Start with cost function $f(x_i) = \sum_{i,j} C_{ij}x_ix_j + \sum_i B_ix_i$

Replace bits x_i by spin 1/2 qubits $|z_i\rangle$ eigenstates of σ_i^z

$$\frac{1}{2}(1 - \sigma_i^z)|z_i\rangle = z_i|z_i\rangle \quad \sigma_i^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$H_P(|z_i\rangle) = \sum_{ij} J_{ij}\sigma_{z_i}\sigma_{z_j} + \sum_i h_i\sigma_{z_i}$$



Corresponds to the Ising model of interacting spins with on-site magnetic field

One-to-one correspondence with cost function f

We need to emulate Ising physics!

ADIABATIC QUANTUM COMPUTATION (AQC)

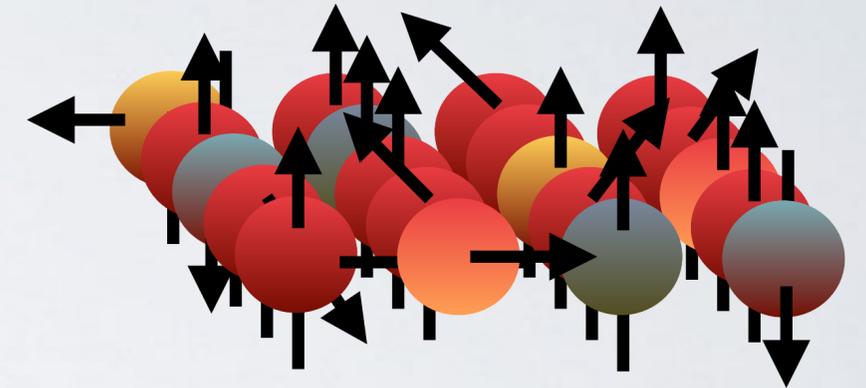
The problem Hamiltonian

Clause C now associated with operator $H_{P,C}$

$$H_{P,C}(|z_1\rangle|z_2\rangle \dots |z_N\rangle) = f(z_{i_C}, z_{j_C})|z_1\rangle|z_2\rangle \dots |z_N\rangle$$

Hamiltonian associated with all clauses, $H_P = \sum_C H_{P,C}$

$H_P|\psi\rangle = 0$ if and only if $|\psi\rangle$ is a superposition of states of the form $|z_1\rangle|z_2\rangle \dots |z_N\rangle$ where z_1, z_2, \dots, z_N satisfy all clauses.



Minimum of Hamiltonian eigenvalue corresponds to optimal configuration to minimize cost function

ADIABATIC QUANTUM COMPUTATION (AQC)

The initial/driving Hamiltonian

Ground state is **easy to find**.

$$H_{D_i} = \frac{1}{2}(1 - \sigma_i^x) \quad H_D = \sum_i H_{D_i}$$

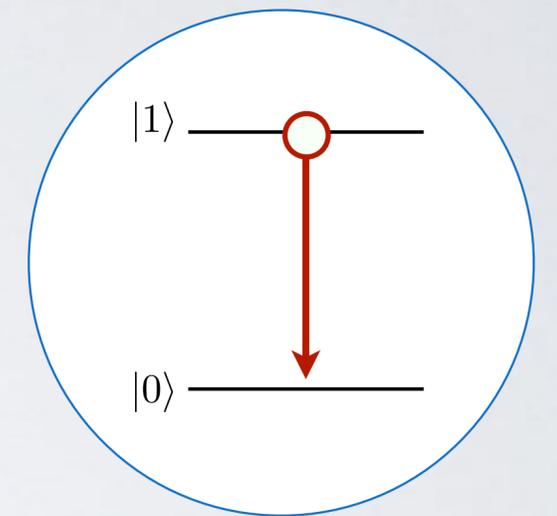
No interaction between qubits:

preparing ground state is equivalent to individual qubit ground state

Ground state is $|\Psi_{\text{GS}}(0)\rangle = |x_1 = 0\rangle |x_2 = 0\rangle \dots |x_N = 0\rangle$

$$|x_1 = 0\rangle |x_2 = 0\rangle \dots |x_N = 0\rangle = \frac{1}{2^{n/2}} \sum_{z_1} \sum_{z_2} \dots \sum_{z_n} |z_1\rangle |z_2\rangle \dots |z_N\rangle$$

In x-basis, initial state becomes a superposition of states in z-basis, known as the **computational basis. Samples all classical possibilities at once.**



Preparation easy by decay, measurement, etc.

ADIABATIC QUANTUM COMPUTATION (AQC)

Adiabatic evolution

$$H(t) = \sum_i \frac{\Delta_i(t)}{2} (1 - \sigma_{x_i}) + \sum_{ij} J_{ij}(t) \sigma_{z_i} \sigma_{z_j} + \sum_i h_i(t) \sigma_{z_i}$$

Simplest approach: **linear interpolation:**

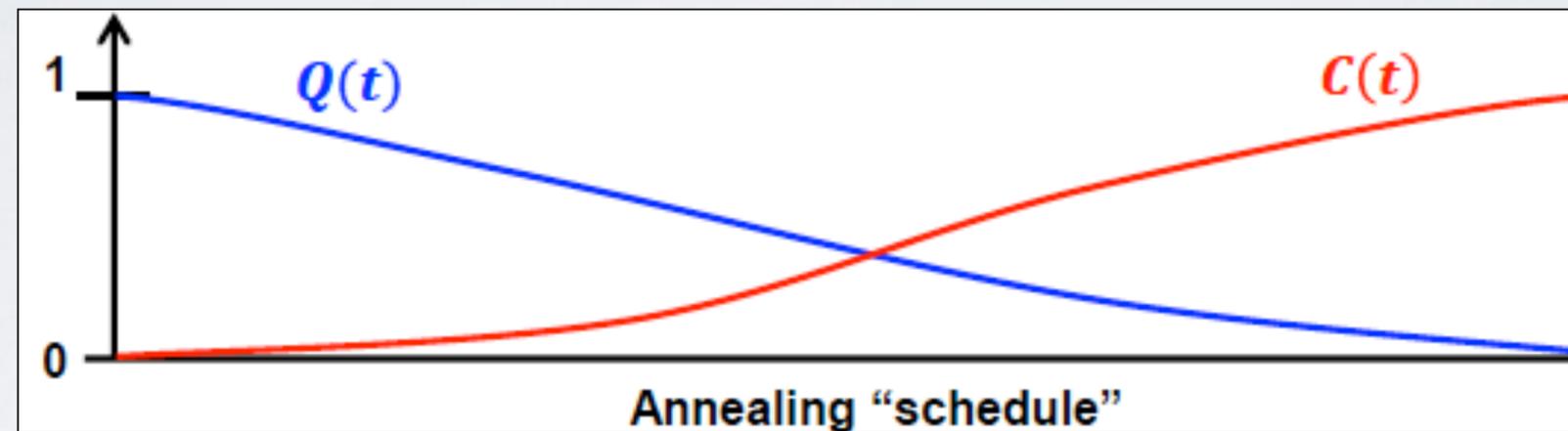
For a single clause: $H_C(t/T) = (1 - t/T)H_{D,C} + (t/T)H_{P,C}$

Total Hamiltonian = sum of clause Hamiltonians $H(t/T) = \sum_C H_C(t/T)$

ADIABATIC QUANTUM COMPUTATION (AQC)

Adiabatic evolution

In general, nonlinear time evolution



$$\mathcal{H}(t) = Q(t)\mathcal{H}_D + C(t)\mathcal{H}_P$$

Trivial Problem

Grover's algorithm in AQC:

Quantum Search by local adiabatic evolution
J. Roland, N. J. Cerf, Phys. Rev. A 65, 042308 (2002)

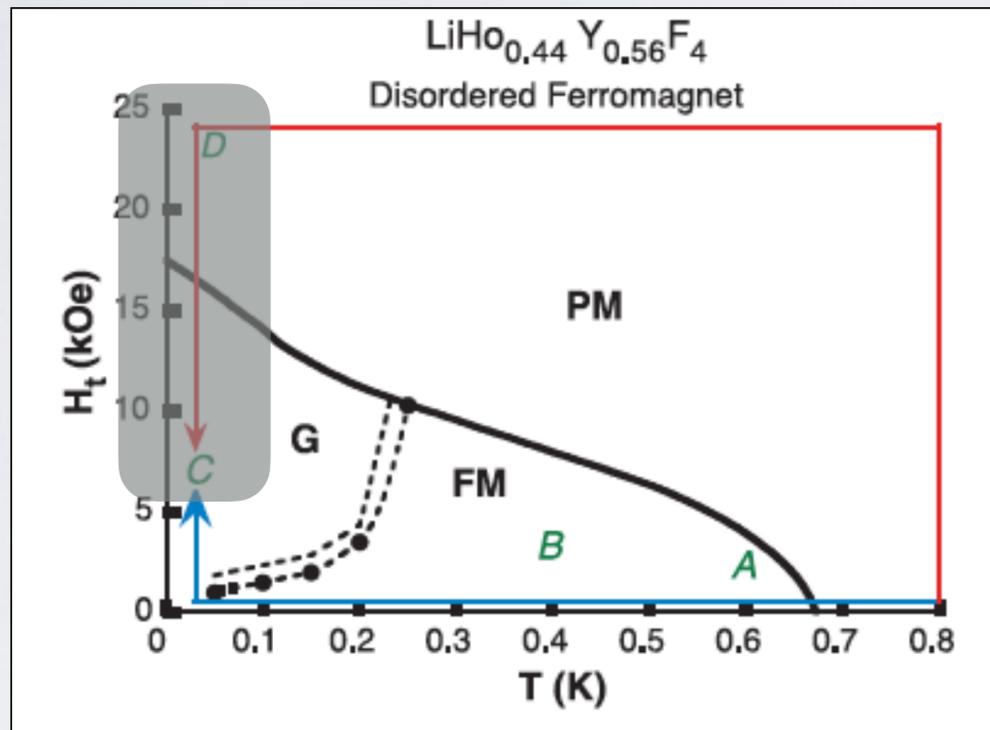
So far, only considered unitary evolution of closed system. In the real world: finite temperature, finite times.

Need open-system framework: Quantum Annealing

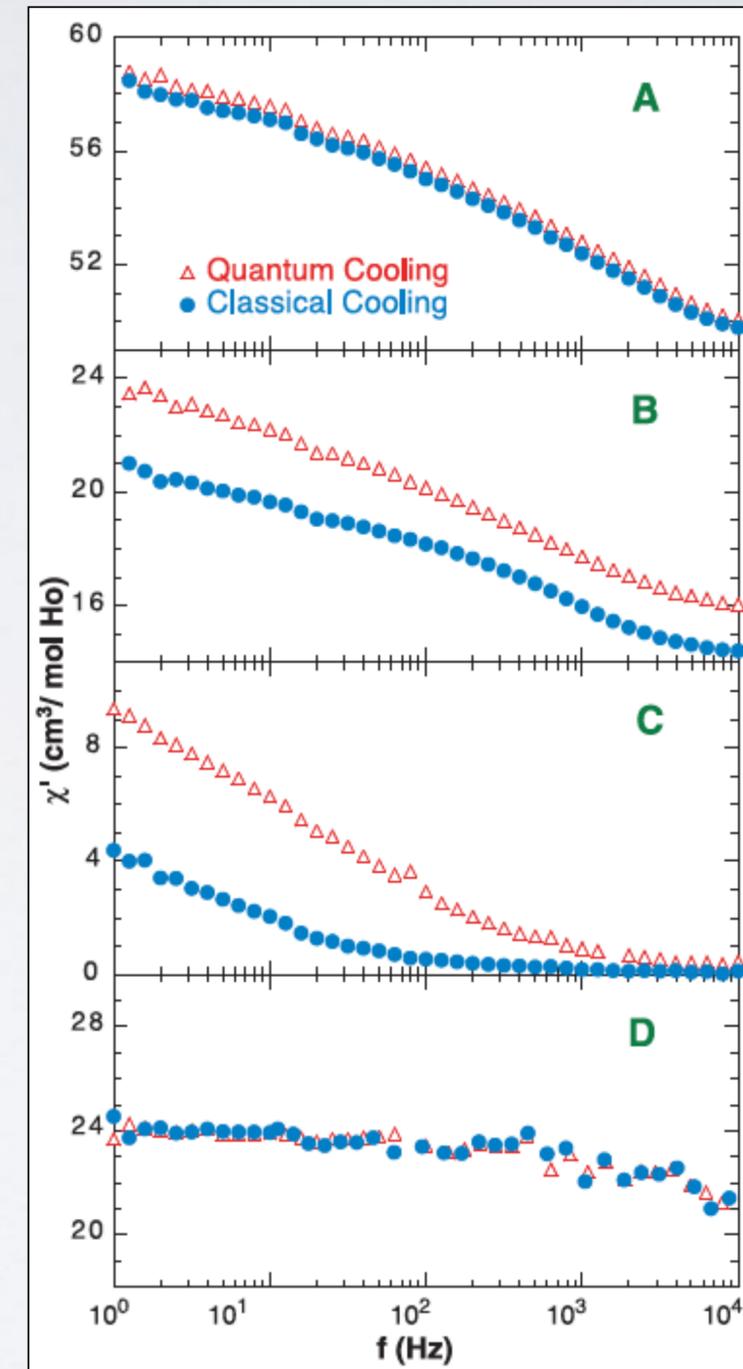
QUANTUM ANNEALING

WHERE IT ALL STARTED: MAGNETISM

Cooling a disordered ferromagnet in two paths: open system



Magnetic susceptibility shows signatures of cooling system to a different state



Guided phase transition between two ground states!

Motivated the idea to reach arbitrary ground states...

Brooke *et al.*, Nature **284**, 779 (1999)

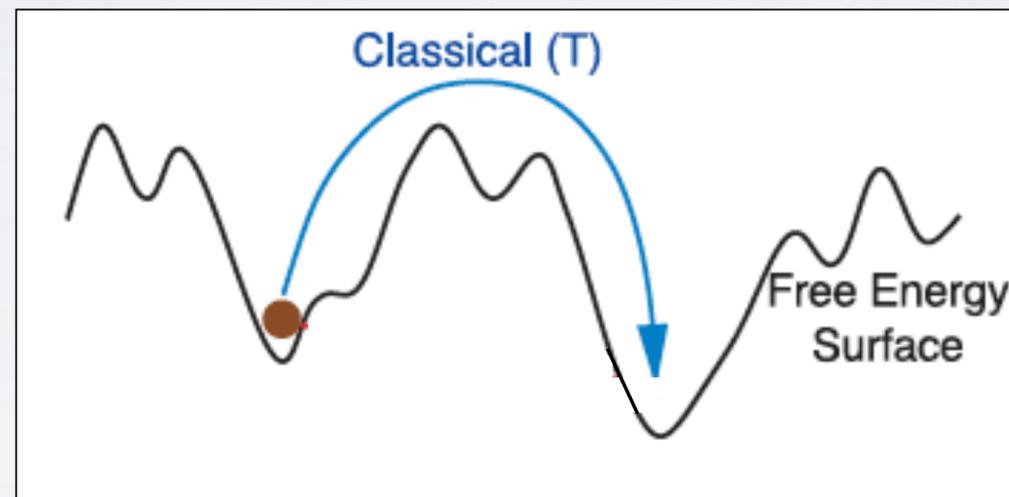
QUANTUM ANNEALING

Simulated annealing

Kirkpatrick, Gelatt, Vecchi, Science **220**, 671 (1983).

Connection between **statistical mechanics** and combinatorial **optimization**

Cost function: $f(x_i)$



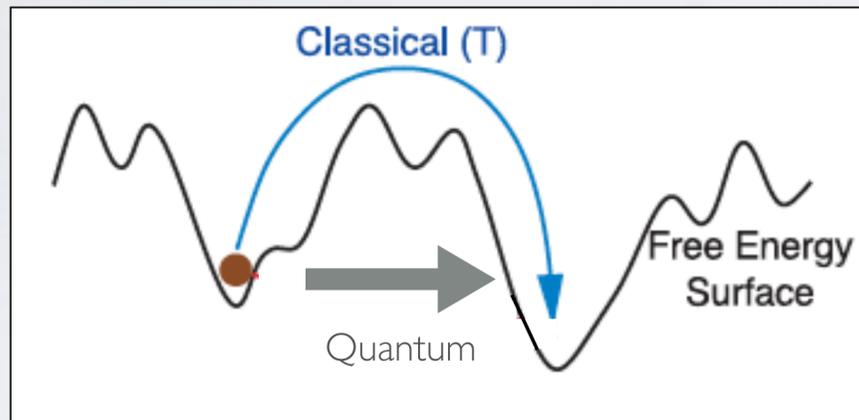
Thermal fluctuations help escape from local minima under annealing schedule by **decreasing temperature**

QUANTUM ANNEALING

Quantum annealing

Kadowaki, Nishimori, Phys. Rev. E **58**, 5355 (1998).

Ising model in a transverse magnet



$$\mathcal{H}_{IT} = - \sum_{i,j}^N J_{ij} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^z - \Gamma(t) \sum_i \sigma_i^x$$

Example of 2-SAT problem:

$$\mathcal{H}_{IT}(|x_1\rangle|x_2\rangle \dots |x_N\rangle) = f(x_{i_C}, x_{j_C})(|x_1\rangle|x_2\rangle \dots |x_N\rangle)$$

Many classical optimization problems can be cast into the classical Ising model

$$\Gamma \neq 0 \Rightarrow [\mathcal{H}_{IT}, \sigma_i^z] \neq 0$$

σ_i^z is not a conserved quantity, it **fluctuates**

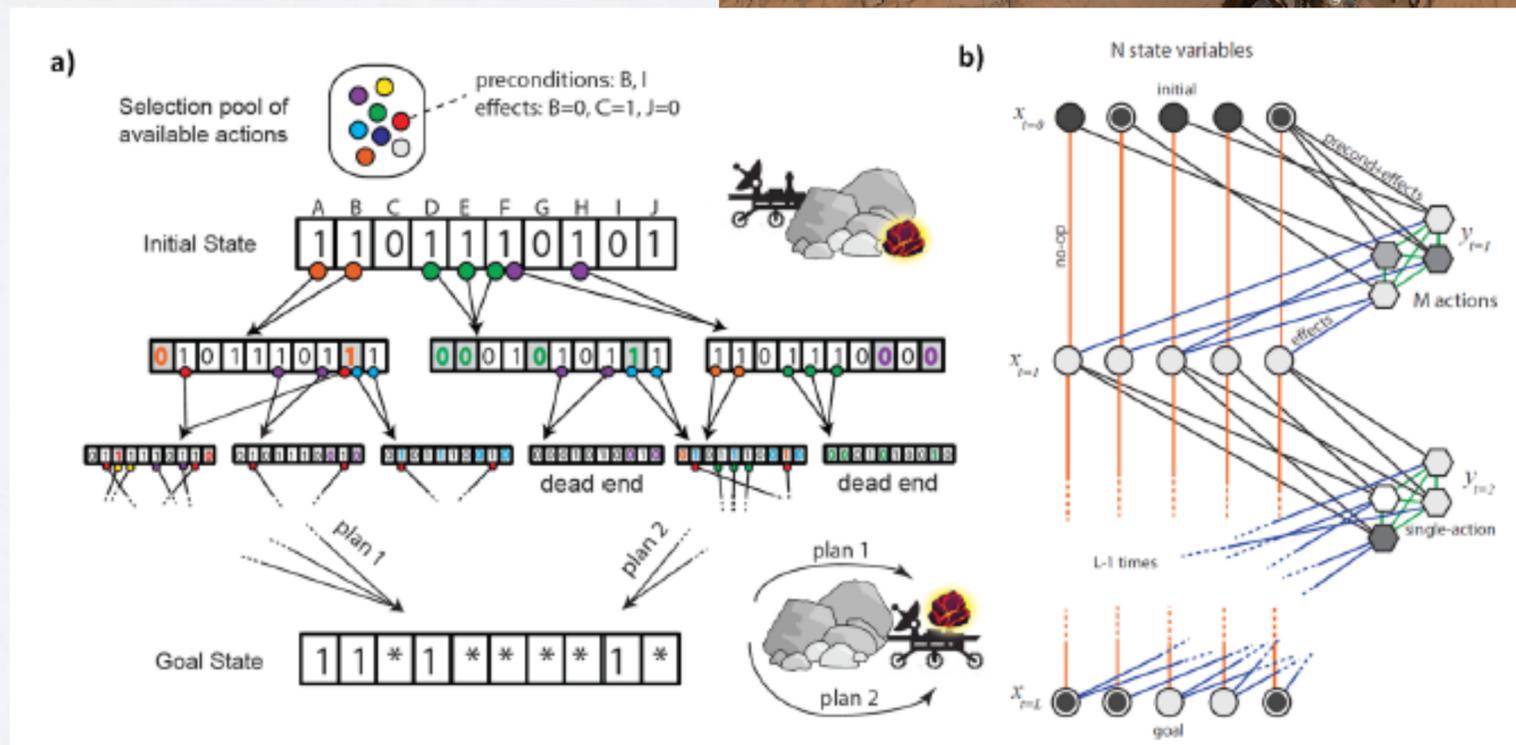
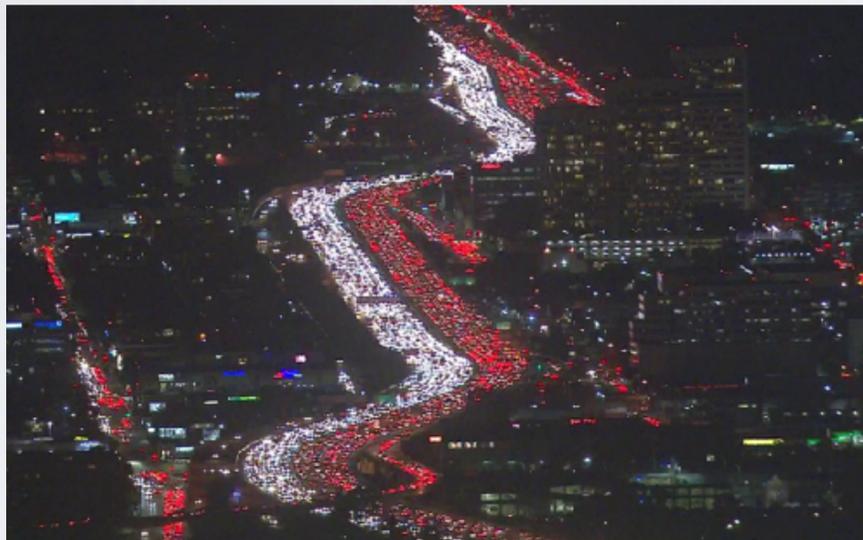
Γ controls effective precession rate of spins!

We need spin 1/2-like particles to implement algorithm!

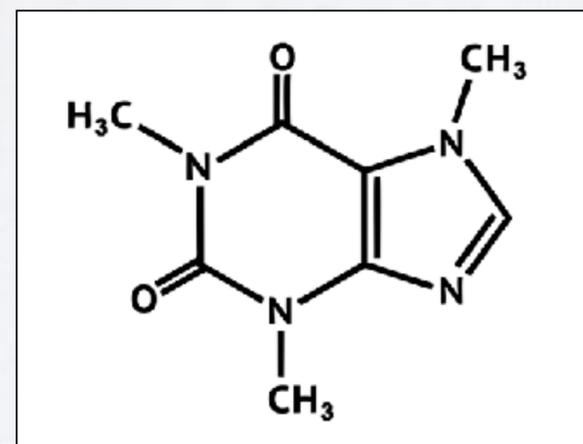
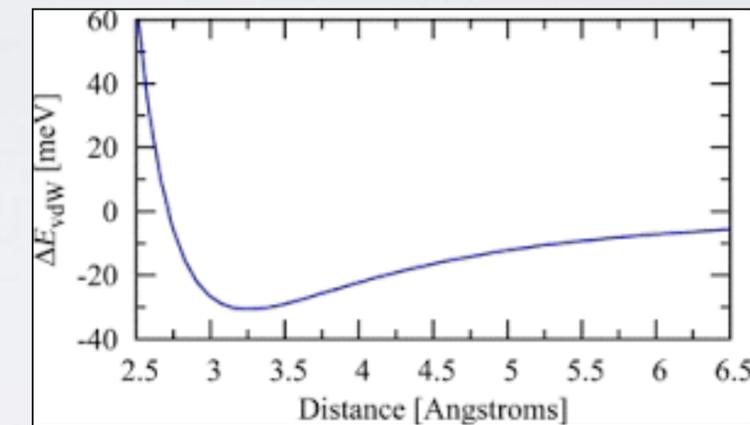
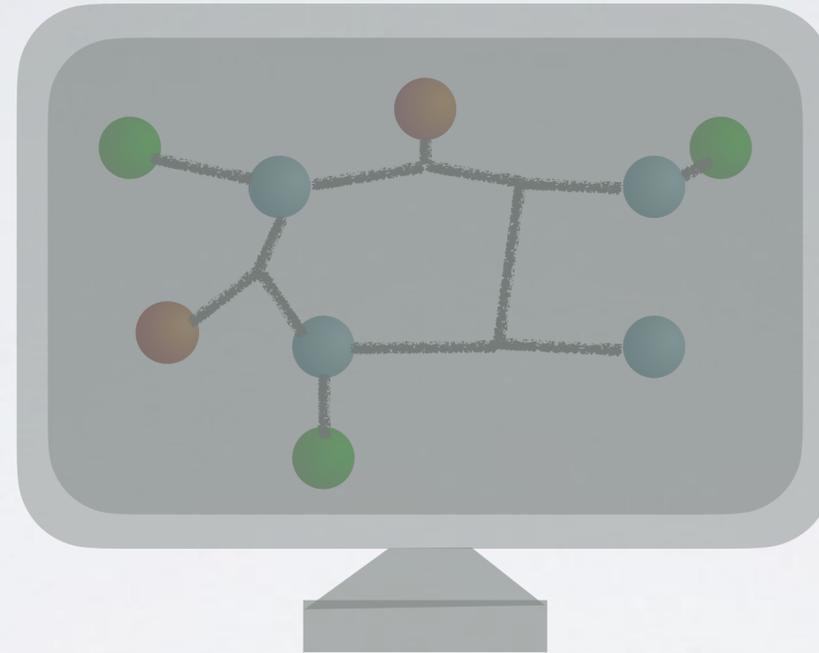
QUANTUM ANNEALING

Why do we care about small Quantum Annealers?

- Applications where small-sized quantum processors can outperform classical computers
 - I. Optimization: traffic, navigation, scheduling, machine learning, etc.



QUANTUM ANNEALING

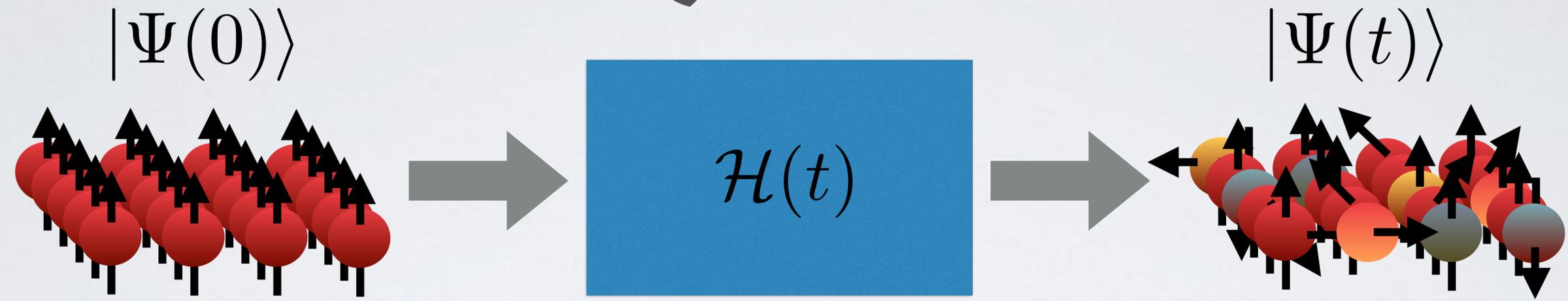


Using intrinsic properties of quantum systems can we map **and compute** complex problems?

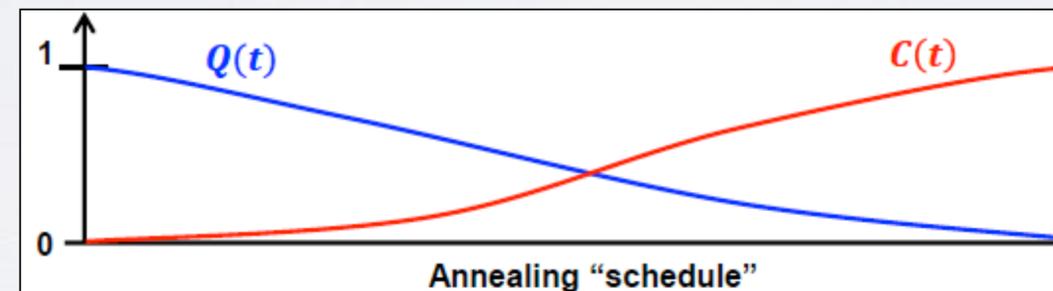
R. P. Feynman, *Simulating physics with computers*, *Int. J. Theor. Phys.* **21**, 467 (1982)

QUANTUM ANNEALING

Adiabatic Quantum Simulator



Initial state may be a superposition of each qubit's eigenstates



Final state contains the Hamiltonian of molecule/reaction in question. May **not** be ground state.

$$\mathcal{H}(t) = Q(t)\mathcal{H}_D + C(t)\mathcal{H}_P$$

Trivial

Problem

Requires system to hold quantum coherence throughout evolution. Must contain complex qubit-qubit interactions (beyond Ising): **Coherent Quantum Annealer.**

QUANTUM ANNEALING

All very nice, but....

- The gap problem: $\Delta \sim e^{-kn}$
- Gap size + gap position (NP hard)
- Thermal noise / decoherence
- Quantum Monte Carlo
- ...

Some solutions exist, no general prescription:

- Reverse annealing: partial knowledge of solution
- Non-homogeneous annealing: qubit-specific schedule

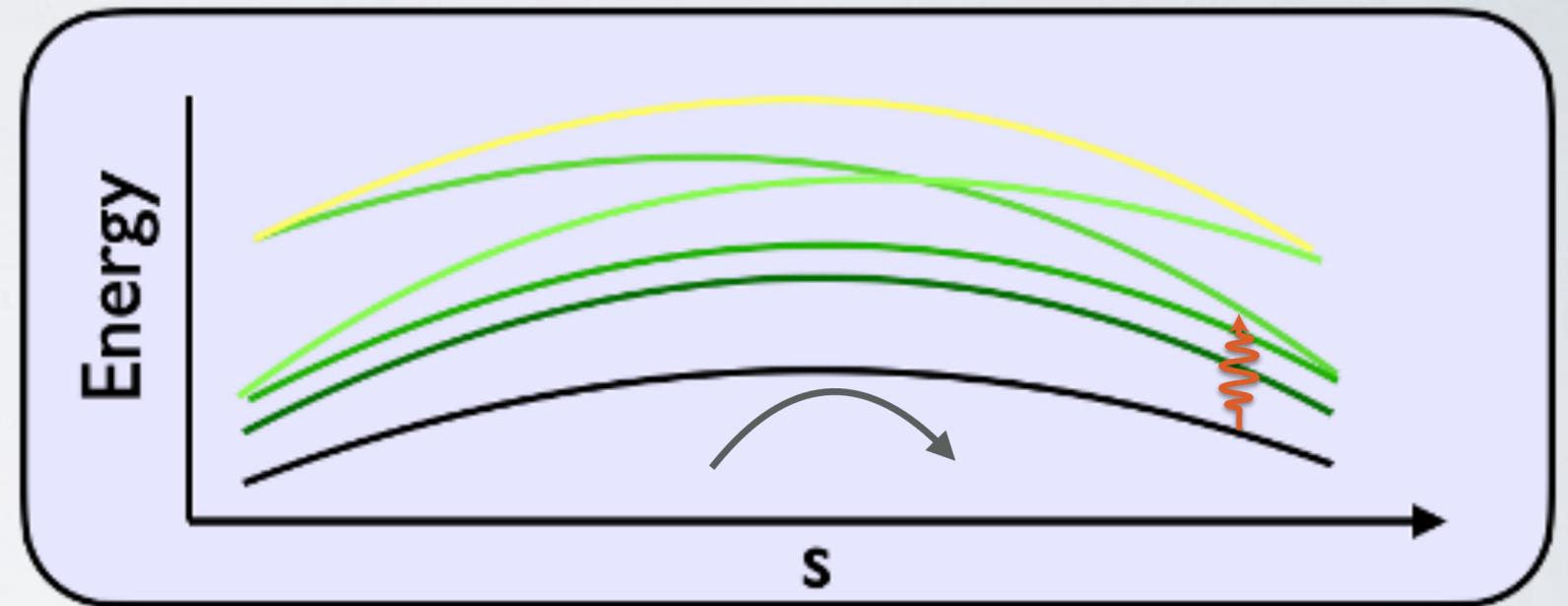
- Non-stoquastic terms $\mathcal{H}(t) = (1 - t/T)\mathcal{H}_D + (t/T)\mathcal{H}_P + (t/T)(1 - t/T)\mathcal{H}_{NS}$

Much more work needed!

Perspectives of quantum annealing: Methods and implementations

P. Hauke, H. G. Katzgraber, W. Lechner, H. Nishimori, W. D. Oliver

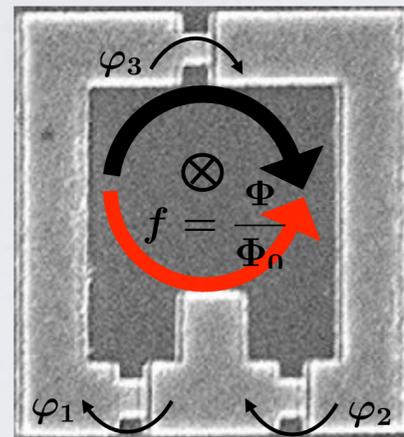
Reports on Progress in Physics 83, 054401 (2020) | arXiv:1903.06559 (2019)



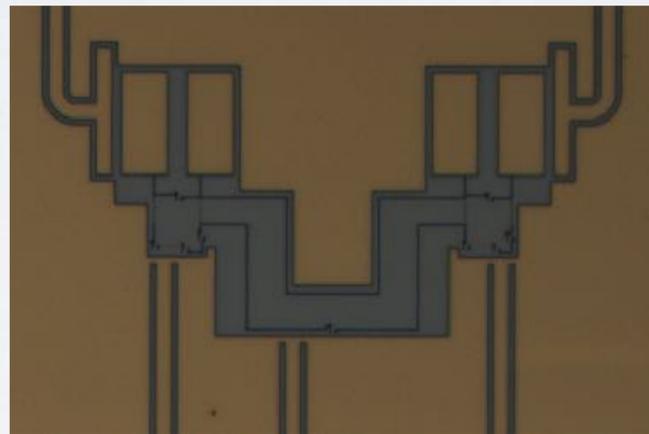
EXPERIMENTAL QA

Ingredients to build a quantum annealer:

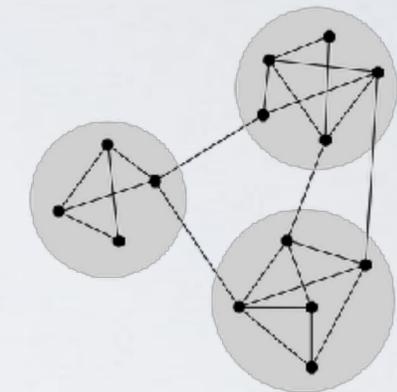
Spin-1/2-like Qubits



Qubit-qubit couplers



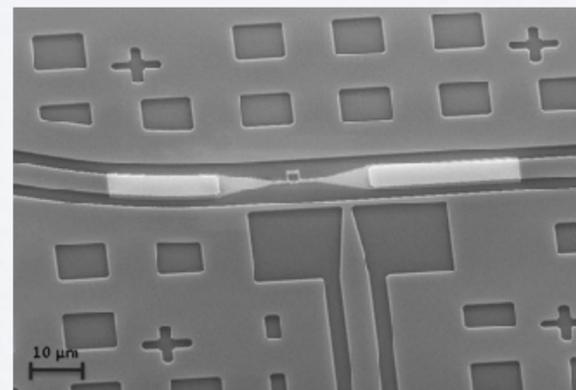
Qubit connectivity map



Readout



Individual qubit control



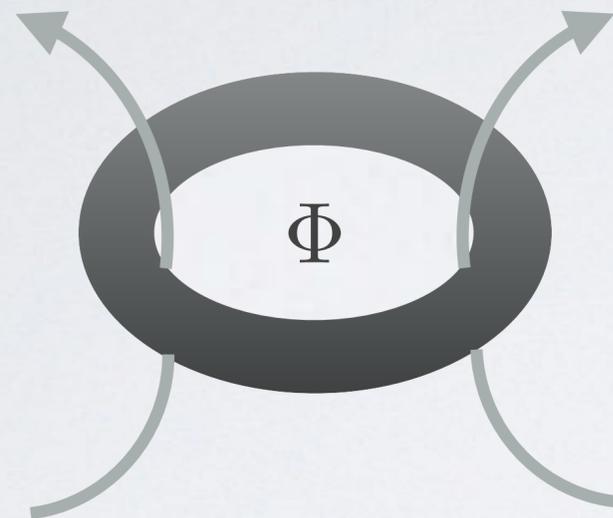
Classical control



SUPERCONDUCTING QUBITS WITH JOSEPHSON JUNCTIONS

SUPERCONDUCTING QUBITS

Superconducting ring



$$\Psi = |\Psi(n_{CP})|e^{i\varphi}$$

$$\Delta\varphi = \varphi + 2\pi N$$

fixed upon cooldown:
Number of flux quanta trapped

M. Tinkham, Introduction to Superconductivity

If ring is very thick, supercurrent only runs on the surface, and the magnetic **flux becomes quantized** to a **flux quantum**:

$$\Phi = N\Phi_0$$

$$\Phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15} \text{ Wb}$$

Earth's field: Check phone

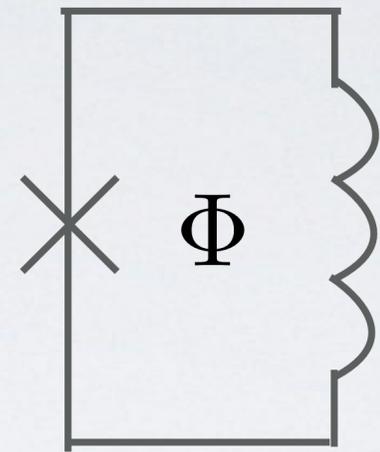
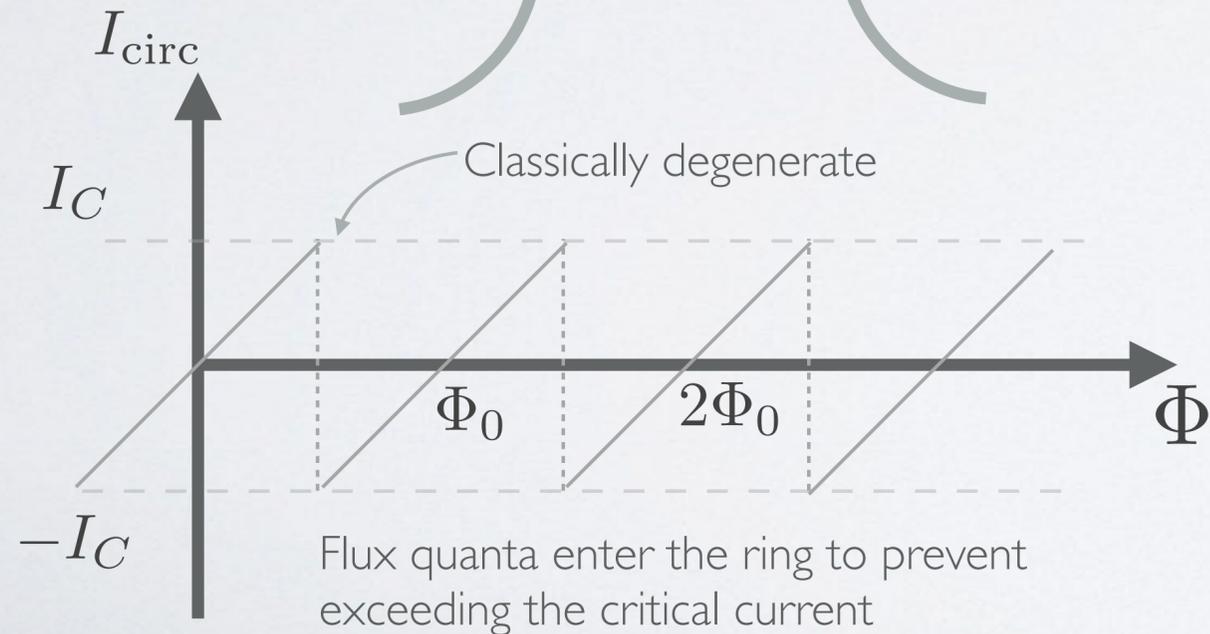
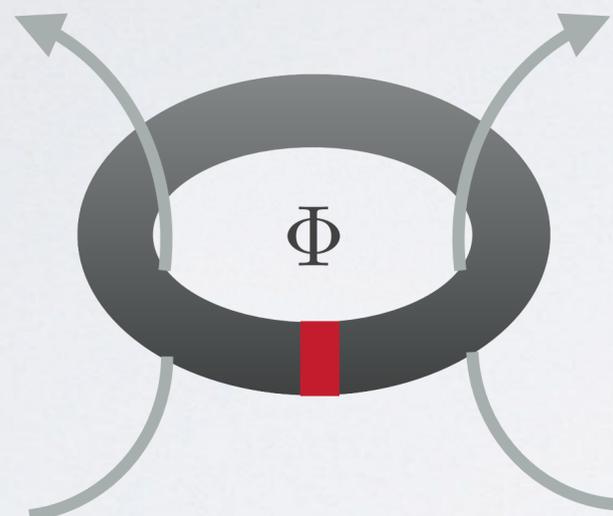
Area for 1 flux quantum?



SUPERCONDUCTING QUBITS

Hydrogen atom Flux qubit: rf-SQUID

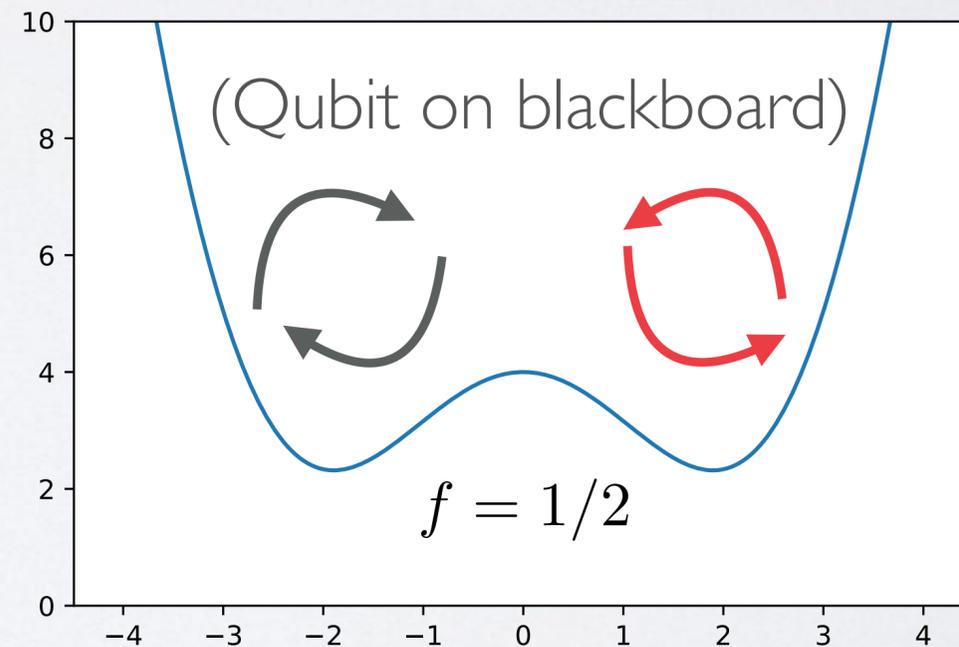
Single-junction loop



$$\beta = \frac{L_g}{L_J} = \frac{E_J}{E_L} > 1$$

Bistable, good qubit regime. Condition requires **large loop inductance**.

Qubit potential energy $U = -E_J \cos \varphi + (E_L/2)(2\pi f - \varphi)^2$



$$f = \Phi/\Phi_0$$

$$E_L = \frac{1}{L} \left(\frac{\Phi_0}{2\pi} \right)^2$$

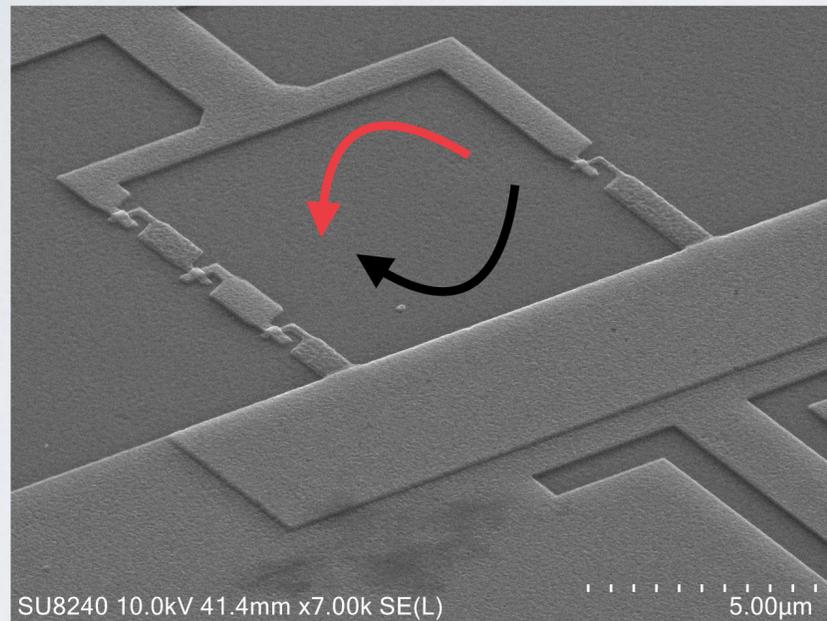
$$E_J = \frac{I_C \Phi_0}{2\pi}$$

Usually, large inductance implies large loops, too sensitive to flux noise...

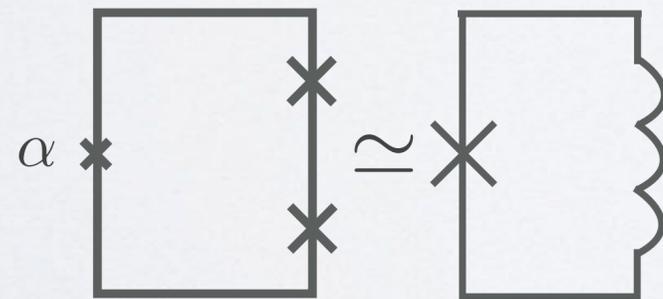
SUPERCONDUCTING QUBITS

Persistent current qubit: multiple junctions to reduce loop size

Aluminum loop with 3 Josephson junctions



Circuit diagram:

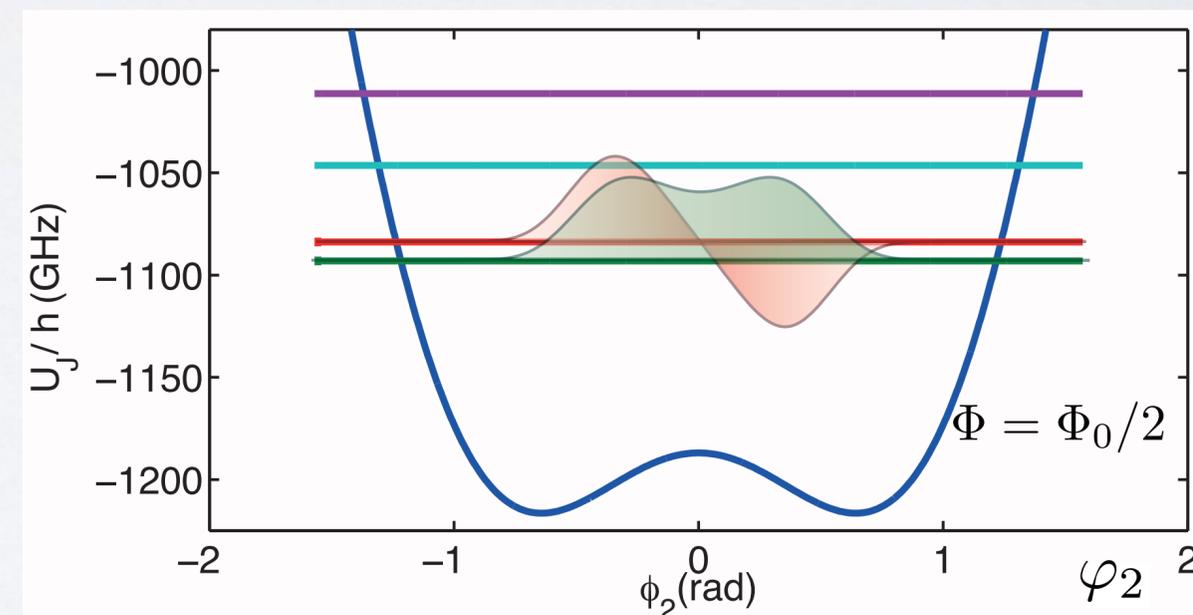


T. P. Orlando, et al. PRB 60 15398 (1999)

J. E. Mooij, et al. Science 285 1036 (1998)

Circuit Hamiltonian:

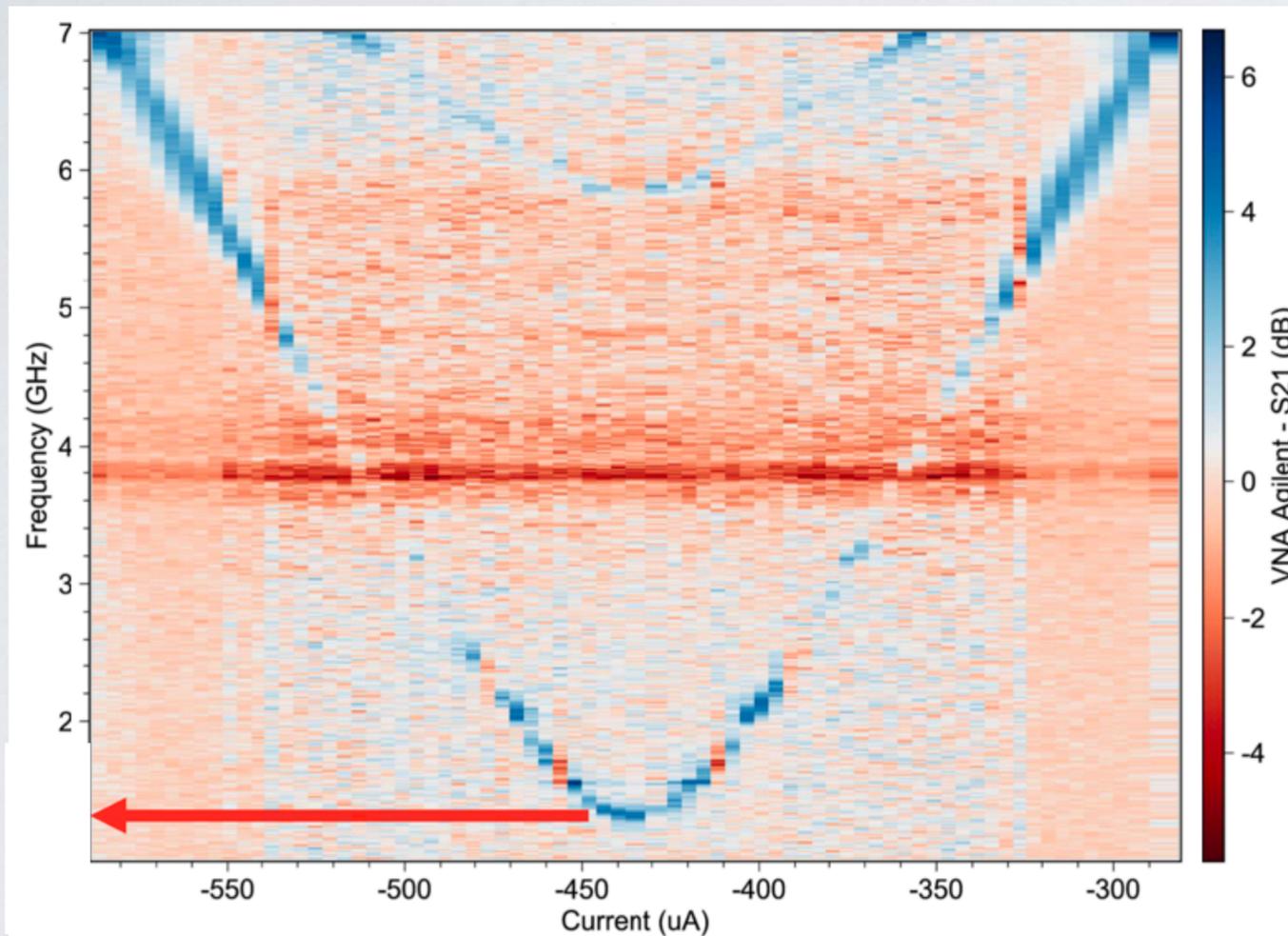
$$\mathcal{H}_{\text{PCQ}} = \frac{E_C}{1 + 2\alpha} [n_1^2 - 2\alpha n_1 n_2 + (1 + \alpha)n_2^2] + E_J [\cos \varphi_1 + \cos \varphi_2 + \alpha \cos(\varphi_1 + \varphi_2 + 2\pi\Phi/\Phi_0)]$$



The tunneling through or over the barrier couples states into superposition of phase, hence current states

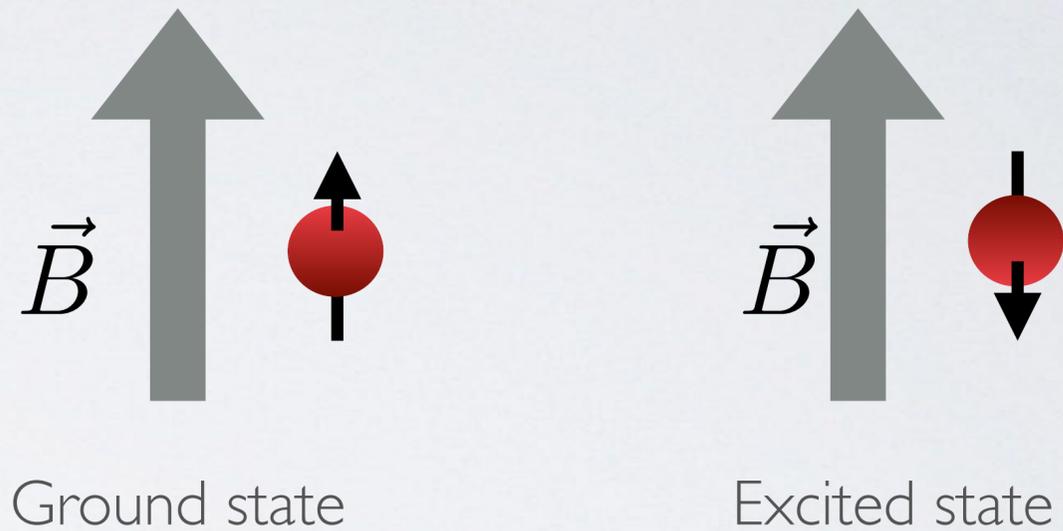
SUPERCONDUCTING QUBITS

Persistent current qubit



Ground state energy changes slope: persistent current flip
External field determines current orientation in ground state!

Qubit spectrum resembles that of a spin-1/2 particle in a magnetic field!



Zeeman energy:

$$U_Z = -\vec{B} \cdot \vec{S}$$

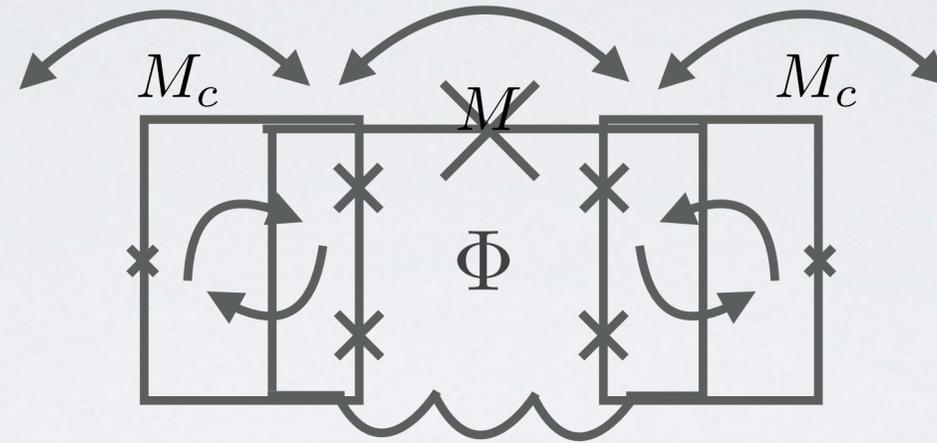
Zero-field splitting (could be from spin-orbit coupling effects)

Flux qubit is a good emulator of spin-1/2 particle!

QUBIT-QUBIT COUPLING

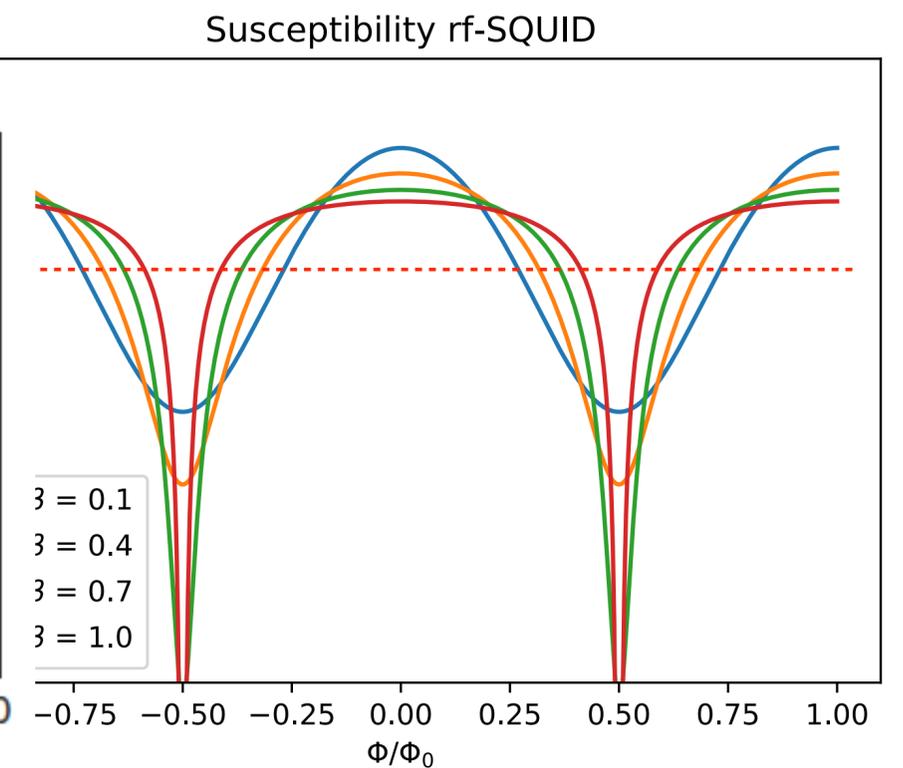
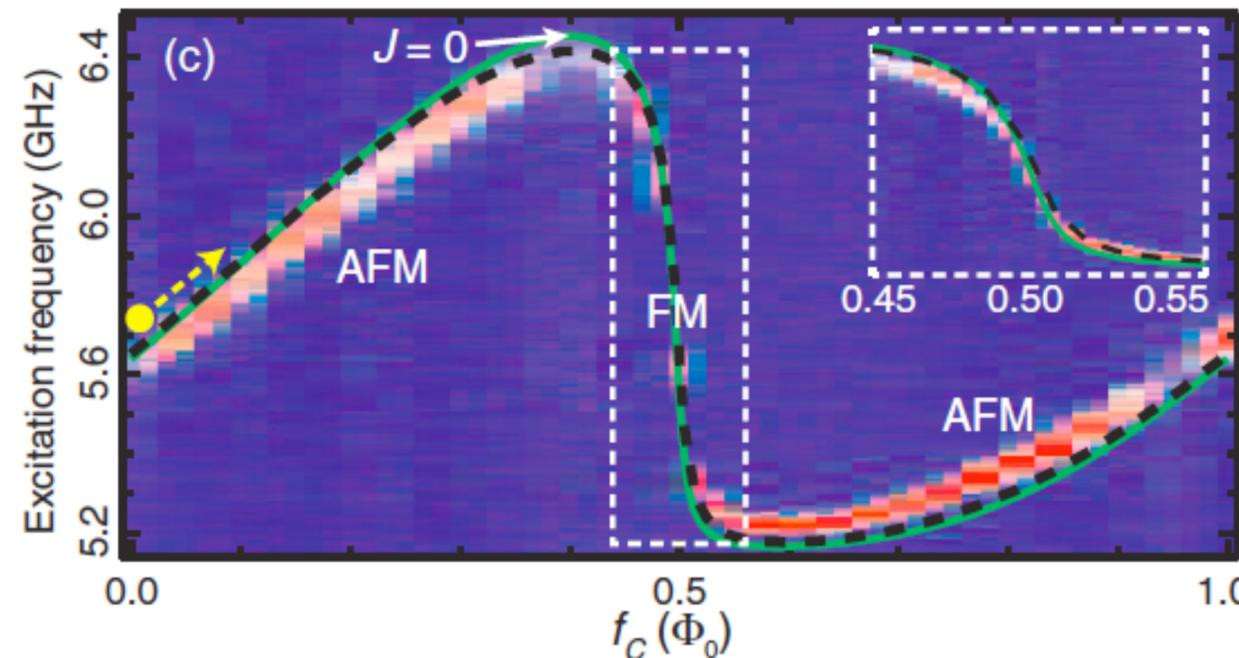
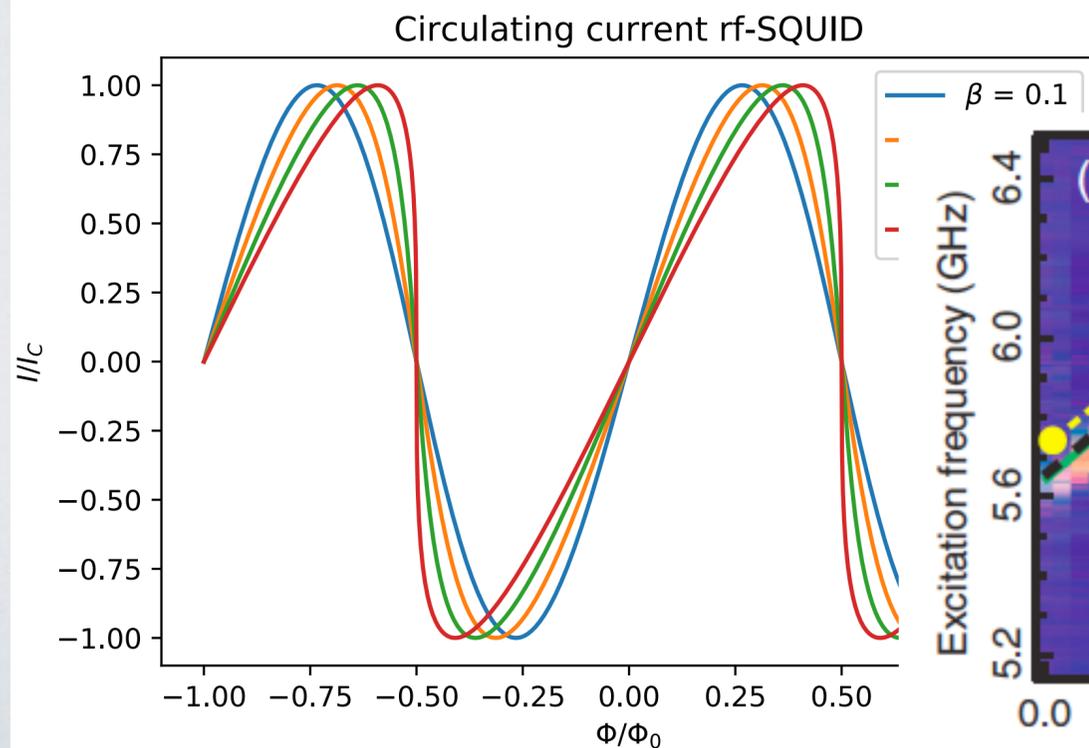
rf-SQUID inductive coupler

S. J. Weber et al., PHYSICAL REVIEW APPLIED 8, 014004



Circuit susceptibility / Effective inductance

$$\chi = L_{\text{eff}}^{-1} = \frac{1}{L_J} \frac{\beta \cos(2\pi f - \Delta\varphi_L)}{1 + \beta \cos(2\pi f - \Delta\varphi_L)} \quad 10$$

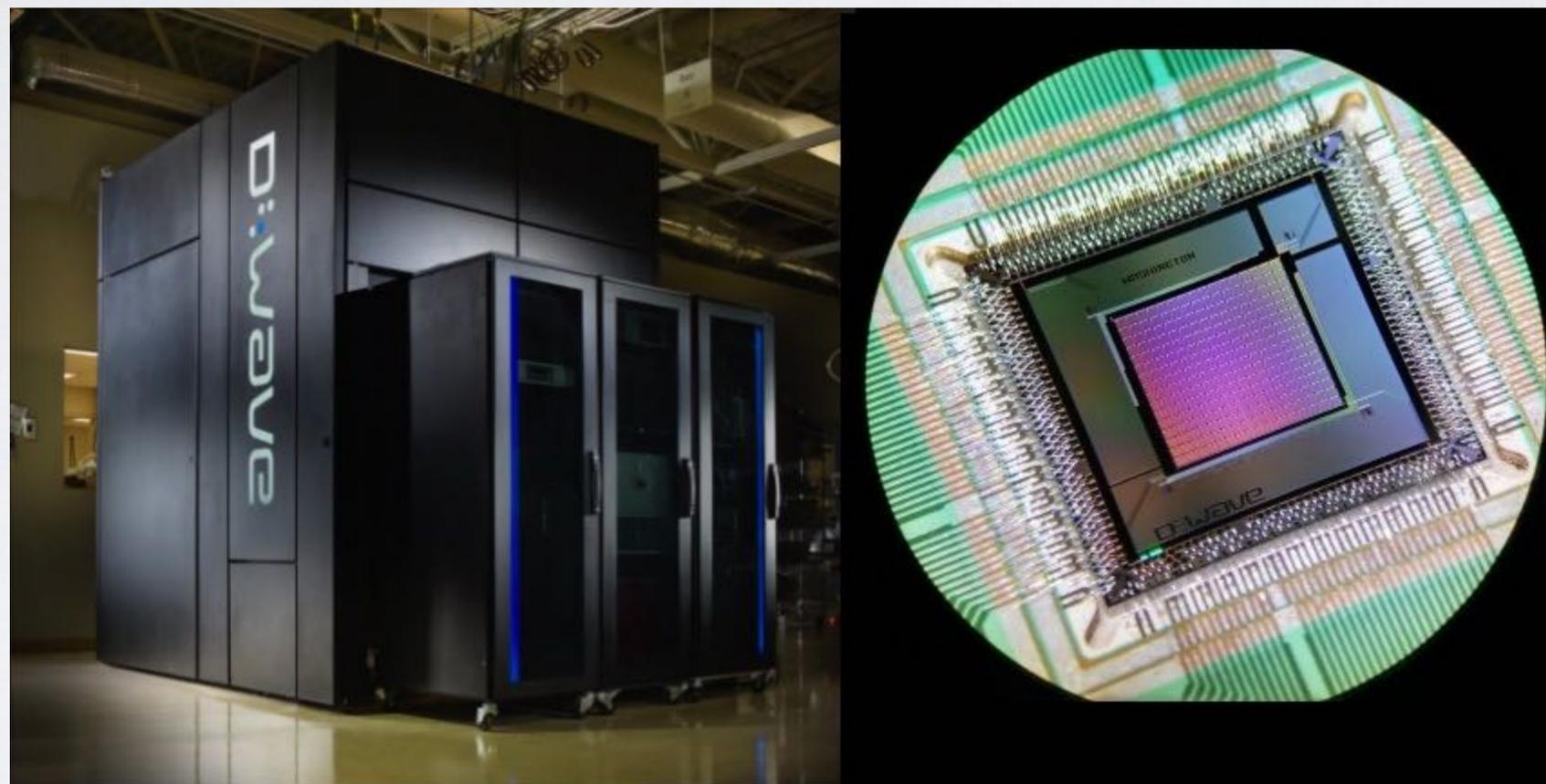


END OF PART I

COMMERCIAL (INCOHERENT) QUANTUM ANNEALERS

D-WAVE APPROACH

D-Wave, 2000Q (2017)



Quantum annealing, simulators

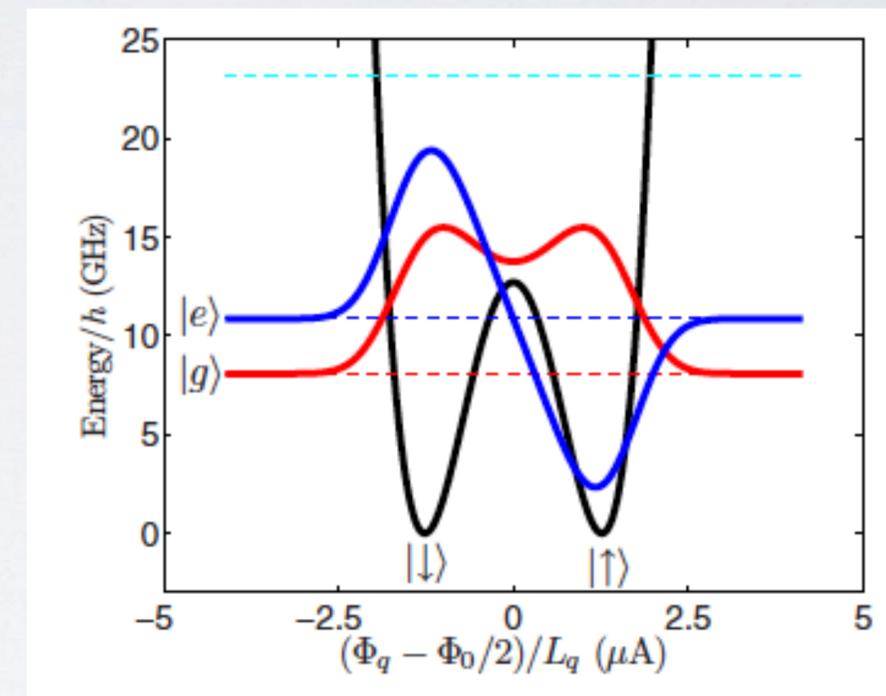
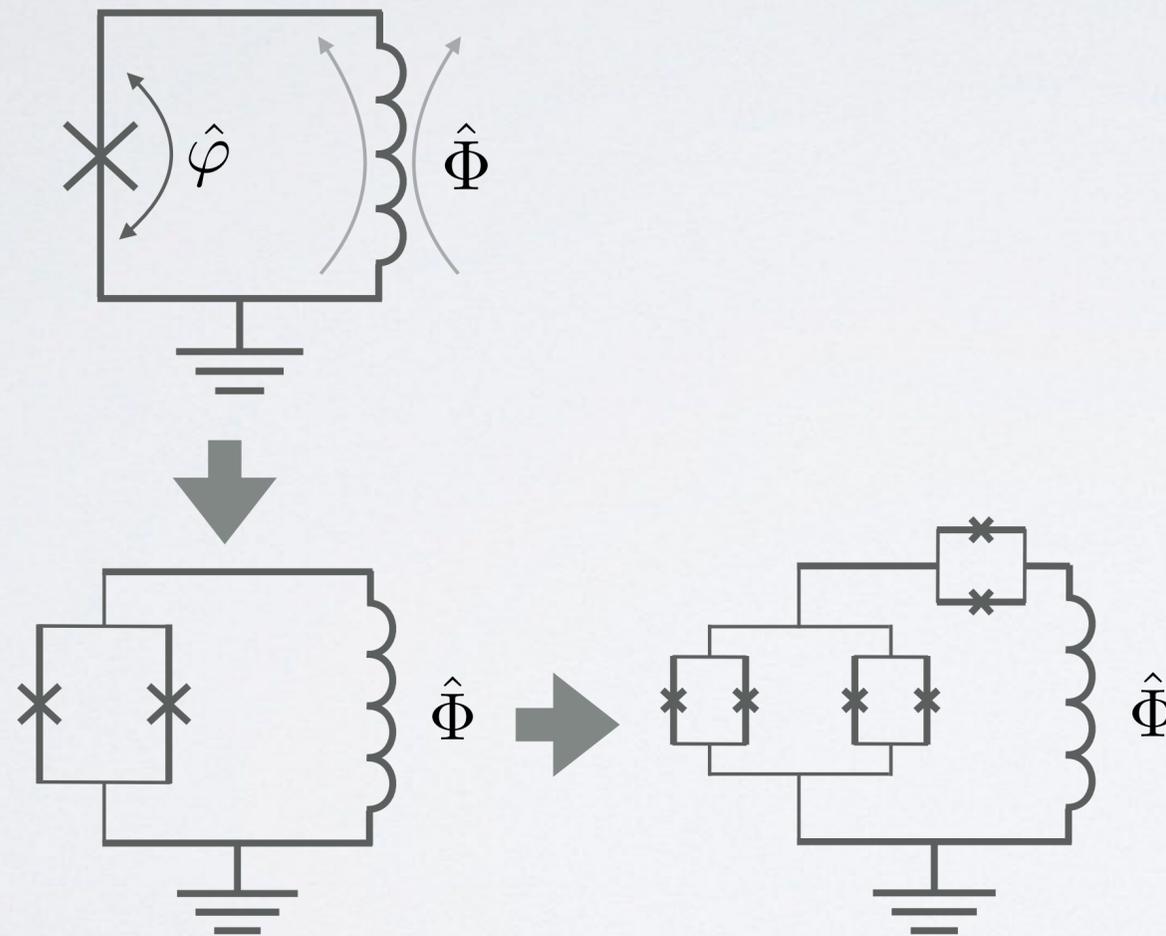
D-WAVE APPROACH

	D-Wave One	D-Wave Two	D-Wave 2X	D-Wave 2000Q ^{[58][59]}	Advantage ^{[60][61]}	Advantage 2 ^{[62][63]}
Release date	May 2011	May 2013	August 2015	January 2017	2020	2023-2024
Topology	Chimera	Chimera	Chimera	Chimera	Pegasus	Zephyr
Code-name	Rainier	Vesuvius	W1K	W2K	Pegasus P16	
Qubits	128	512	1152	2048	5640	7440
Couplers^[64]	352	1,472	3,360	6,016	40,484	
Connectivity	6	6	6	6	15	20
Josephson junctions	24,000	?	128,000	128,472 ^[61]	1,030,000	
I/O lines / Control lines	?	192	192	200 ^[65]	?	
Active area				5.5 mm × 5.5 mm	8.4 mm × 8.4 mm	
On-chip memory				22 kB	130 kB	
Operating temperature (K)	?	0.02	0.015	0.015	<0.015	
Power consumption (kW)	?	15.5	25	25	25	

Wikipedia (not anymore available)

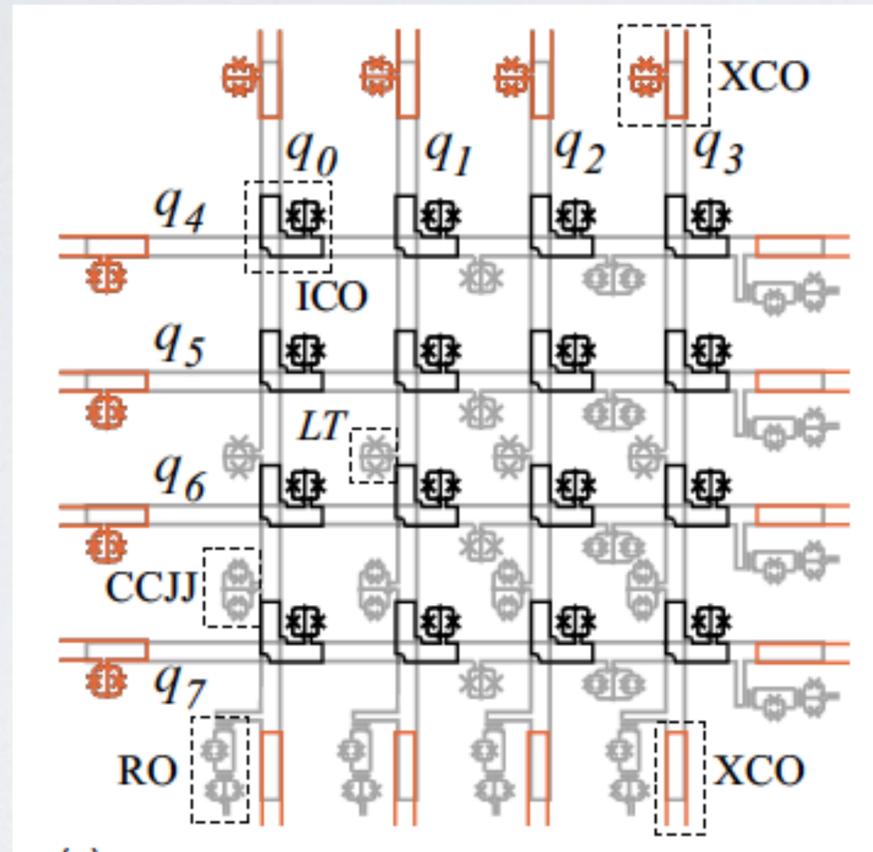
D-WAVE APPROACH

- Qubit: rf-SQUID

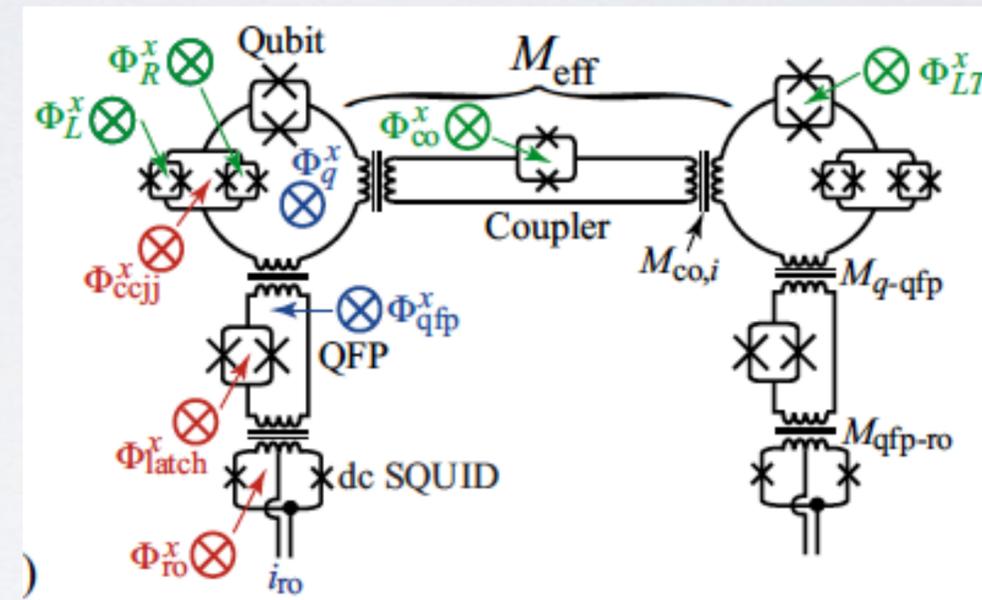


D-WAVE APPROACH

8-qubit cell



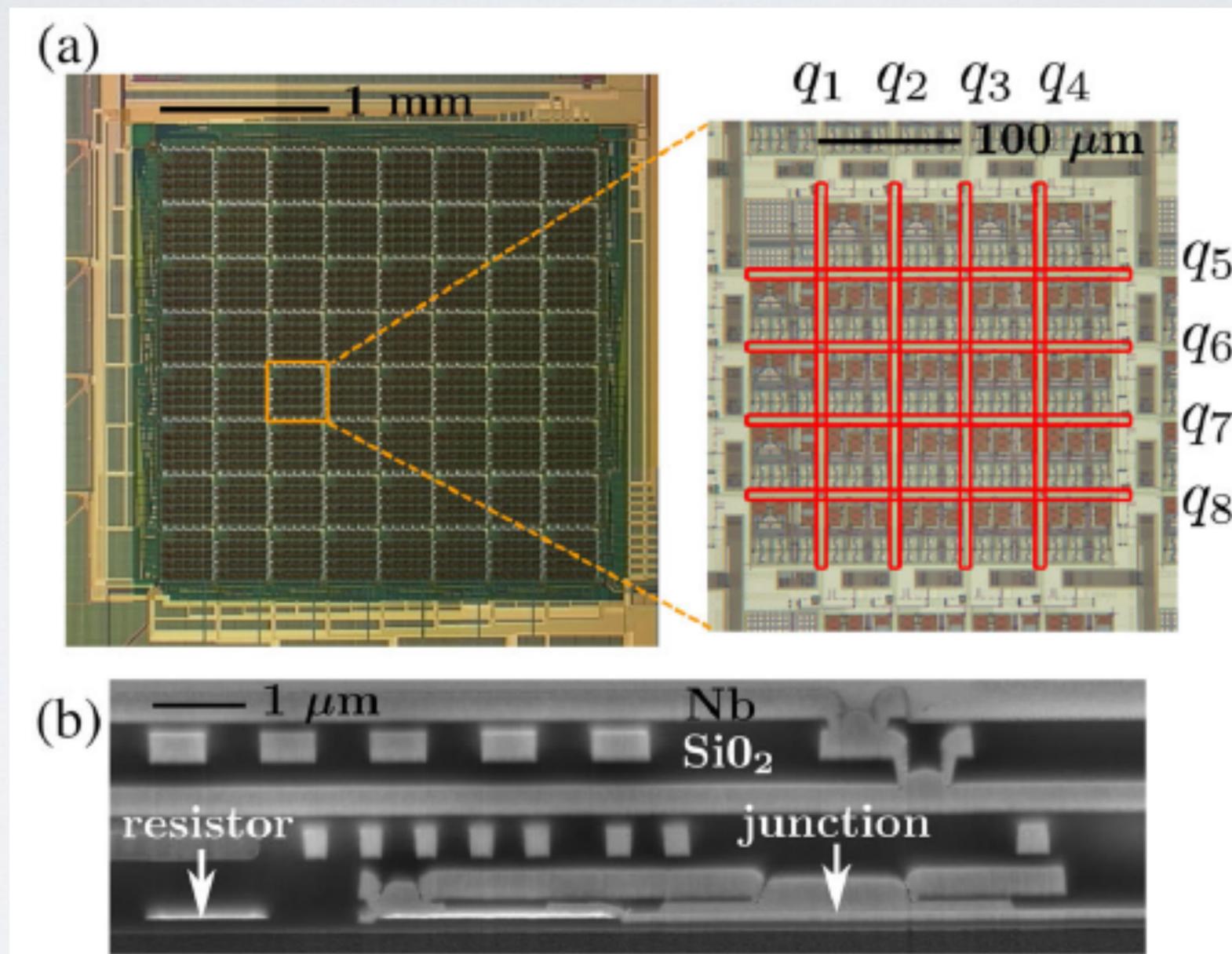
Real superconducting circuit



DC sources used to lock parameters

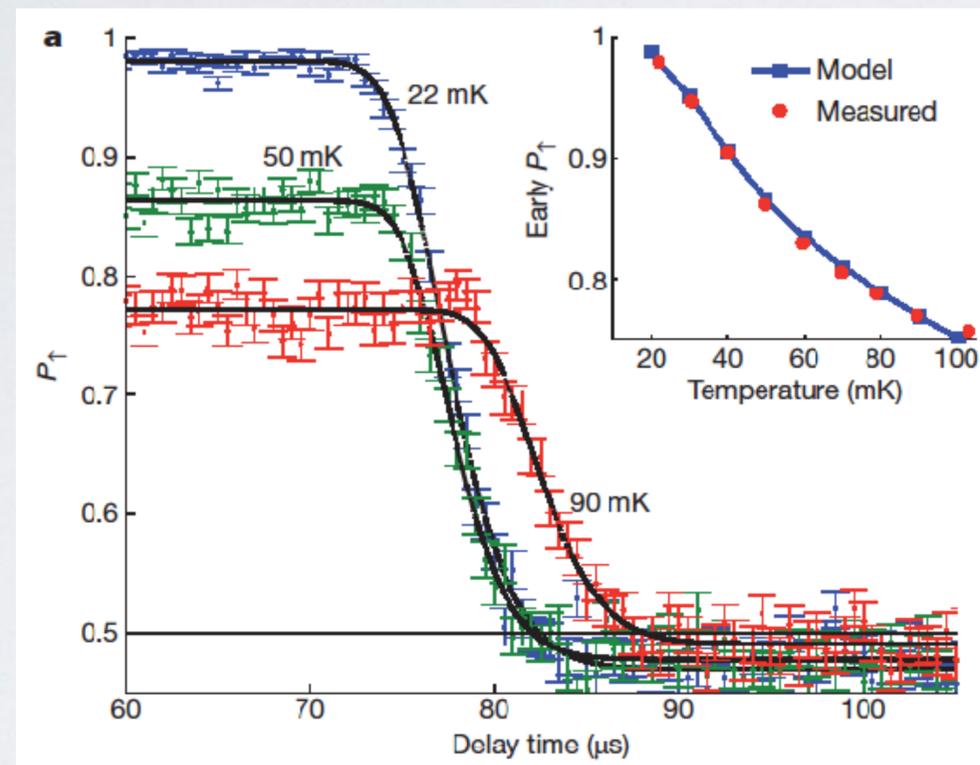
External sources to run schedule

D-WAVE APPROACH

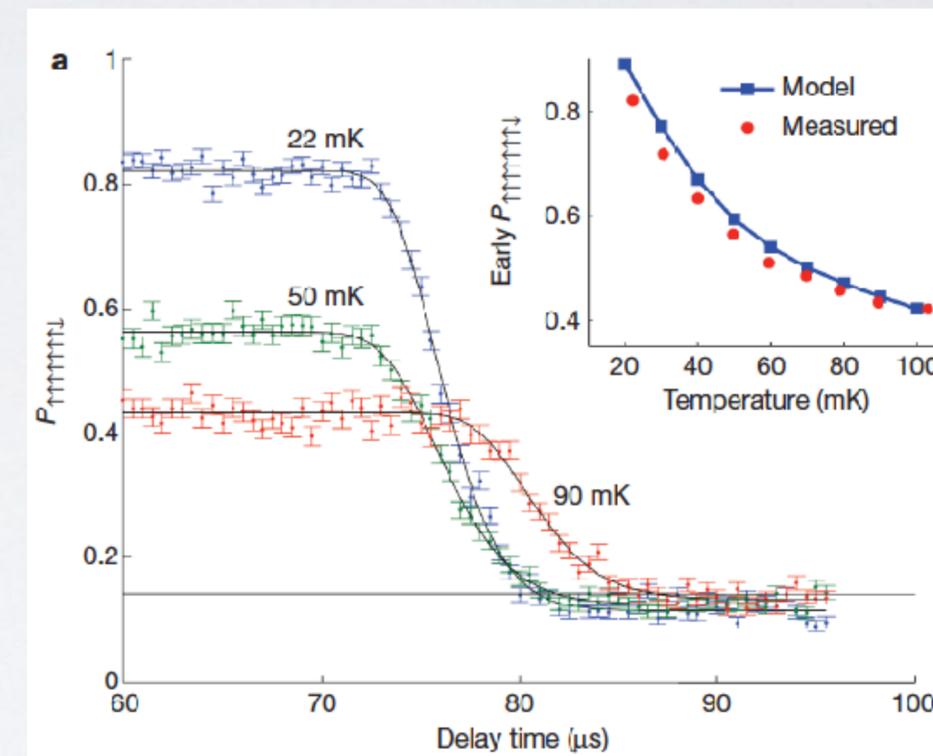


D-WAVE APPROACH

Single qubit evolution



8-qubit evolution

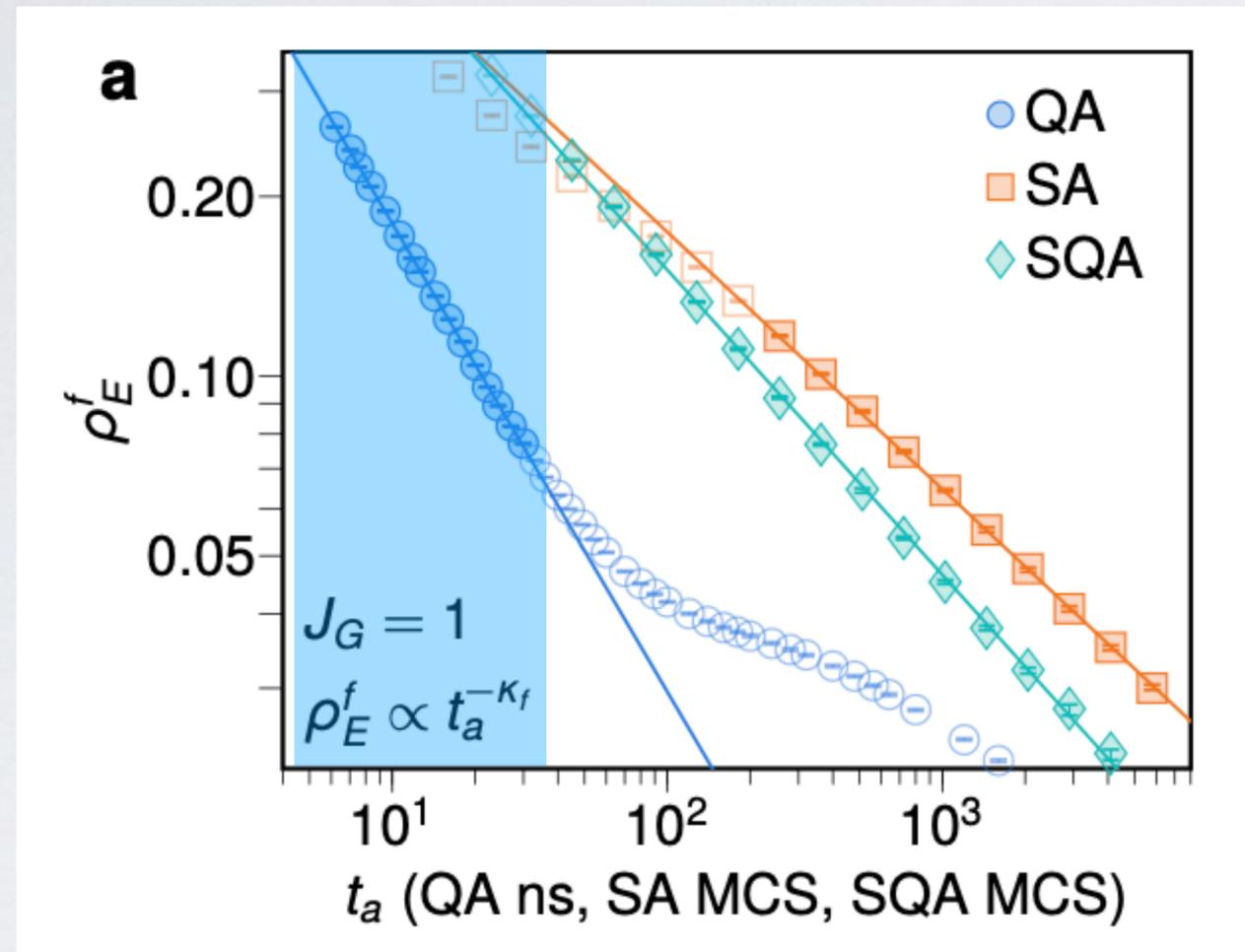


Ferromagnetic coupling, domain wall formation

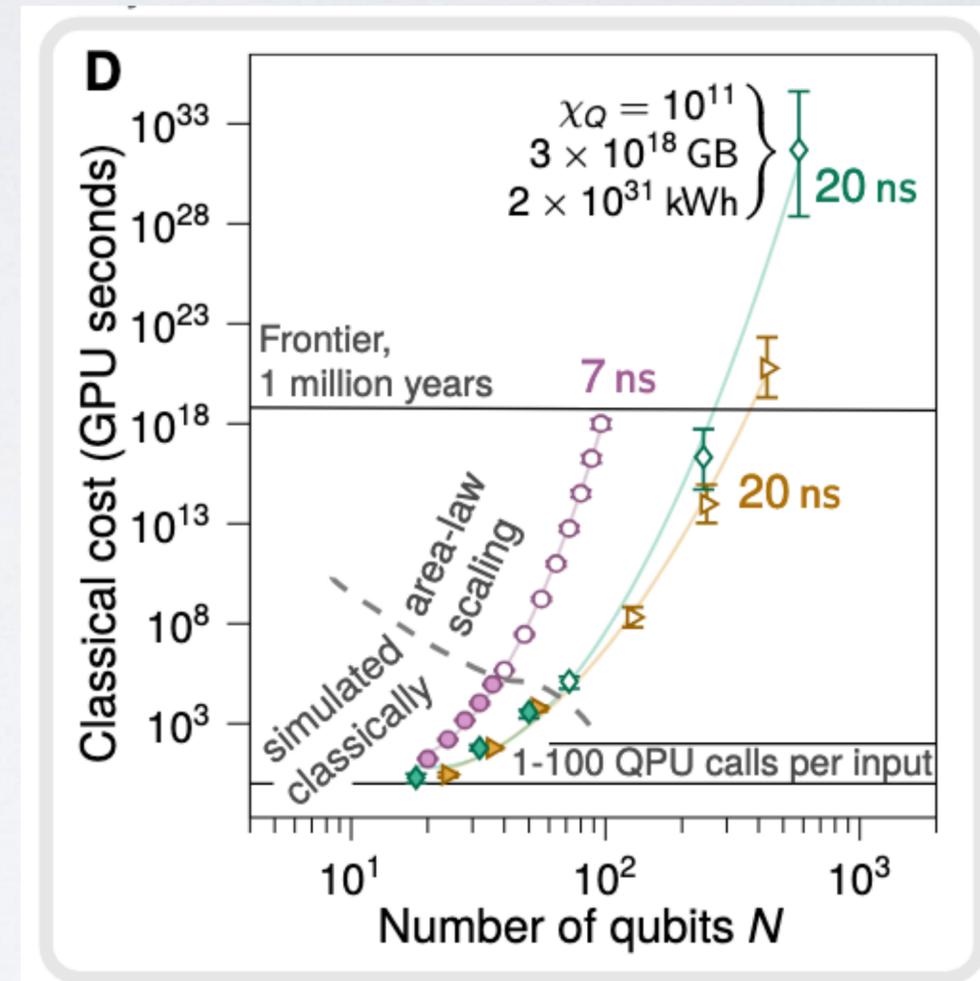
Johnson *et al.*, Nature **473**, 194 (2011)

D-WAVE APPROACH

But... there is no quantum advantage in optimization. There is in simulation!



King et al., Nature volume 617, pages 61–66 (2023)



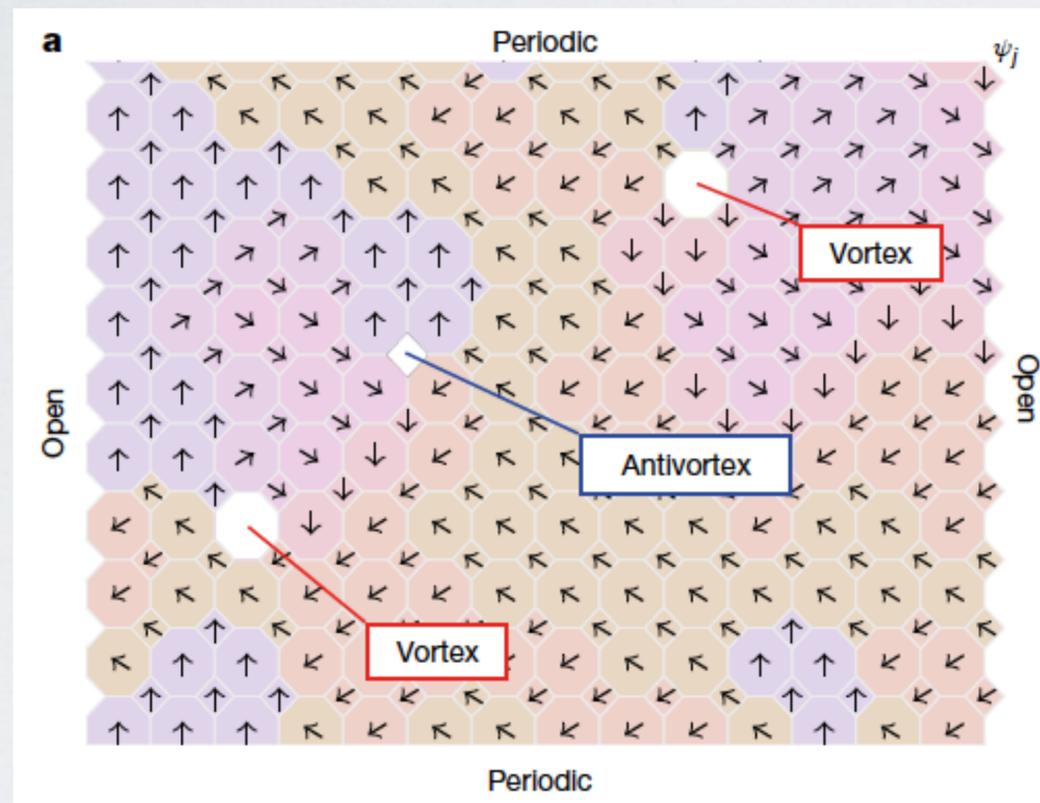
Note the (very) short coherent regime

King et al., <https://arxiv.org/abs/2403.00910> (2024)

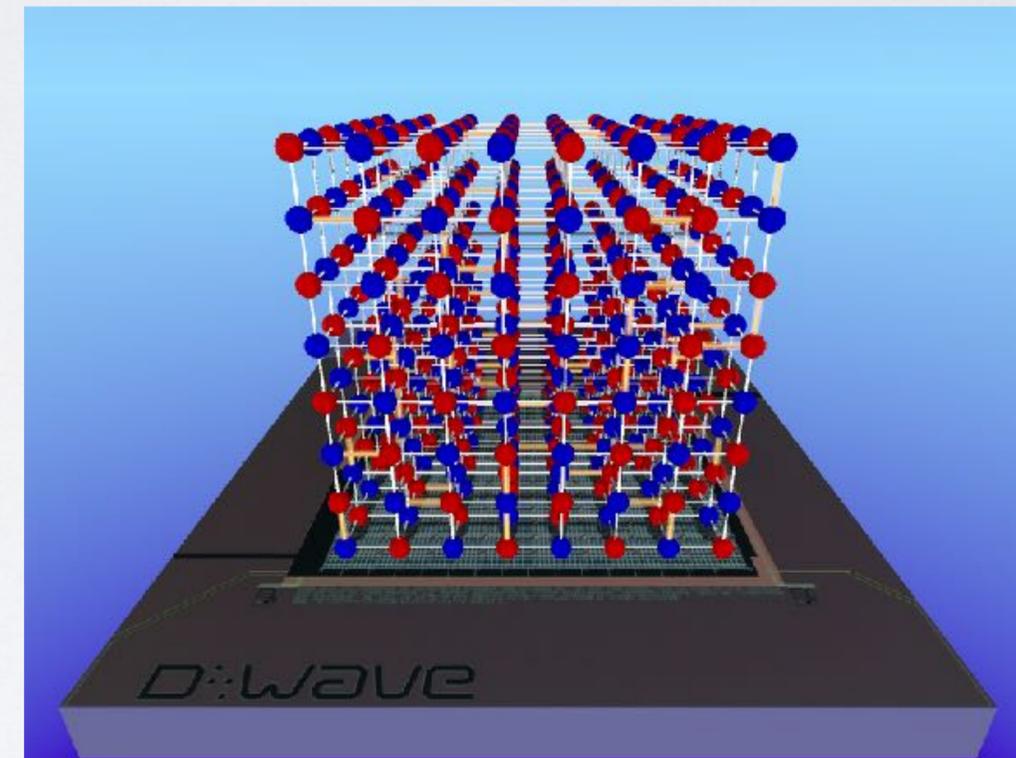
D-WAVE APPROACH

STATE OF THE ART

Quantum simulation of magnetism with D-Wave system of 2000 spins



King *et al.*, Nature **560**, 458 (2018)



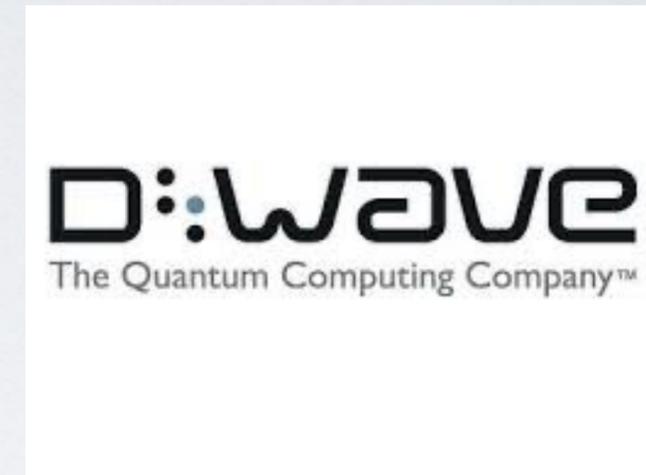
Harris *et al.*, Science **361**, 162 (2018)

Ingredients to build a quantum annealer:

- Qubits: controllable (coherent) quantum systems 
- Couplers: addressable, noiseless(?) 
- Readout: distinguish states in computational basis 
- Connectivity: ideally all-to-all 
- Addressability: engineering 
- Tunability: annealing schedule 

Existing quantum annealers are faulty:

1. Qubit coherence is **poor** (ns vs us)
2. Qubit-qubit interactions are **Ising-type**
3. Qubit connectivity is **sparse**
4. Only **2-body** interactions



As a result of this:
No quantum speedup has been found

Is there a way ahead?

COHERENT QA

Since theorists have not settled whether quantum annealing is powerful, **further experiments are needed**. Experiments with quantum annealers over the next few years are likely to be quite informative. In particular, aside from the applications of quantum annealers to classical optimization problems, **applications to quantum simulation problems** should also be explored [34].

John Preskill, “NISQ” Quantum 2, 79 (2018)

COHERENT QA

Cooking recipe:

1. Qubits with long coherence (w.r.t. relevant time scales)
2. Qubit-qubit interactions beyond **Ising-type**
3. Qubit connectivity dense
4. Beyond **2-body** interactions

Along lines of demonstration equivalence AQC - Standard QC

Aharonov *et al.*, SIAM Journal of Computing, Vol. 37, Issue 1, p. 166-194 (2007). arxiv: 0405098

Can they be realized?

COHERENT QUANTUM ANNEALING EFFORTS

- UCSB (Martinis group)



- QEO program: MIT-NG-USC (Oliver, Lidar, Wilhelm, Fergusson,...)



- AVaQus: IFAE-KIT-CNRS-CSIC-UG-QQT-HQS-DC
(Forn-Díaz, Pop, Roch, García-Ripoll,...)

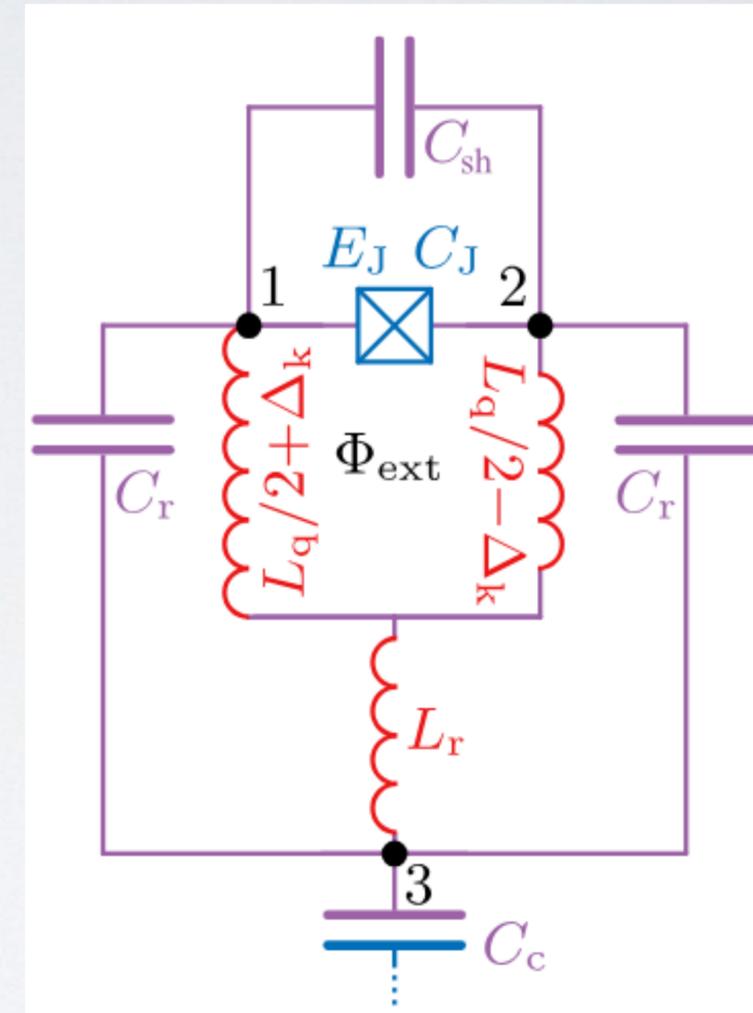
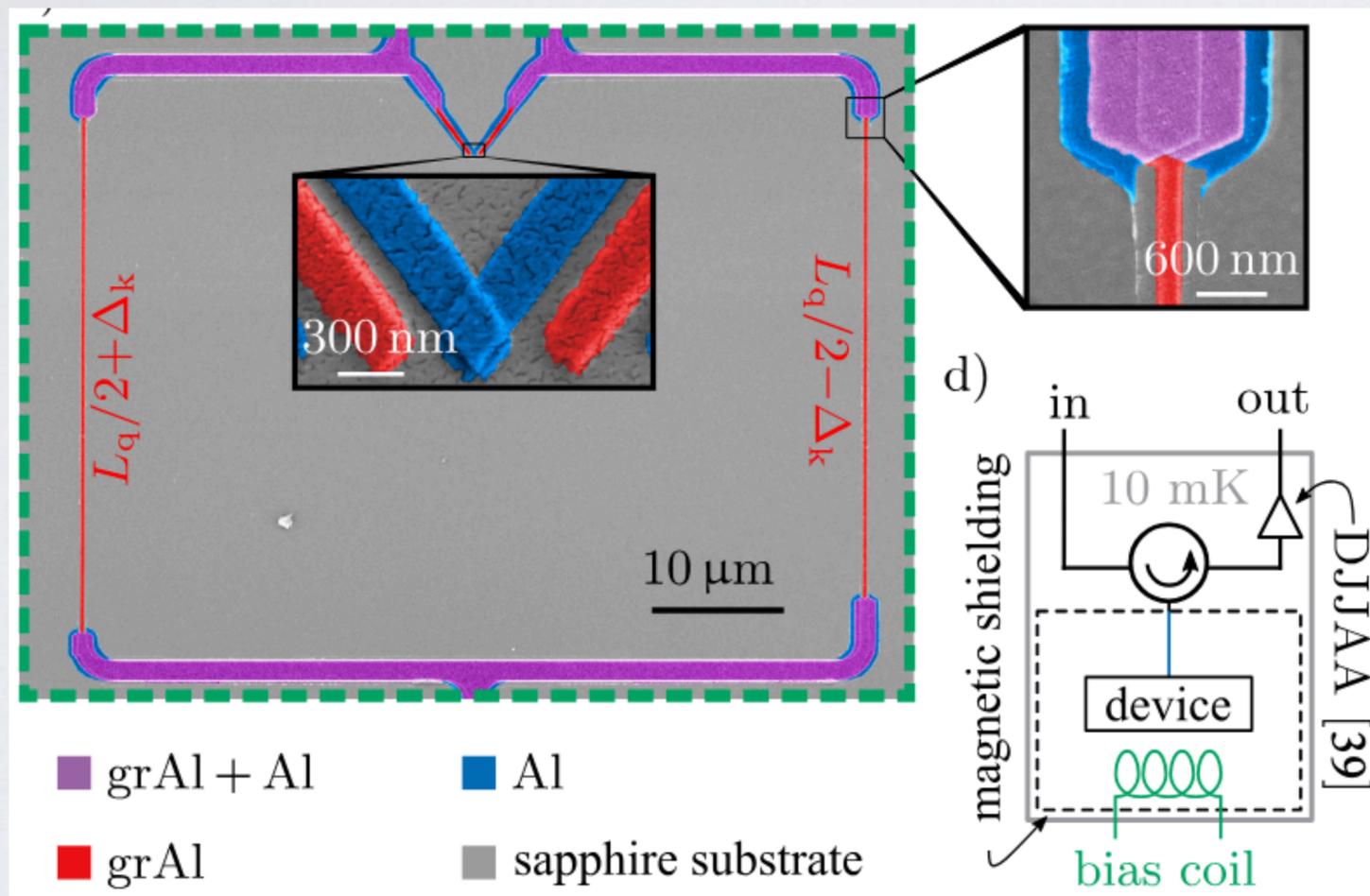


CQA EFFORTS

- Coherent, tunable qubits
- Readout of coherent qubits
- Coupling coherent qubits, stoquastic
- Non-stoquastic qubit-qubit couplings
- Scalable circuits for CQA

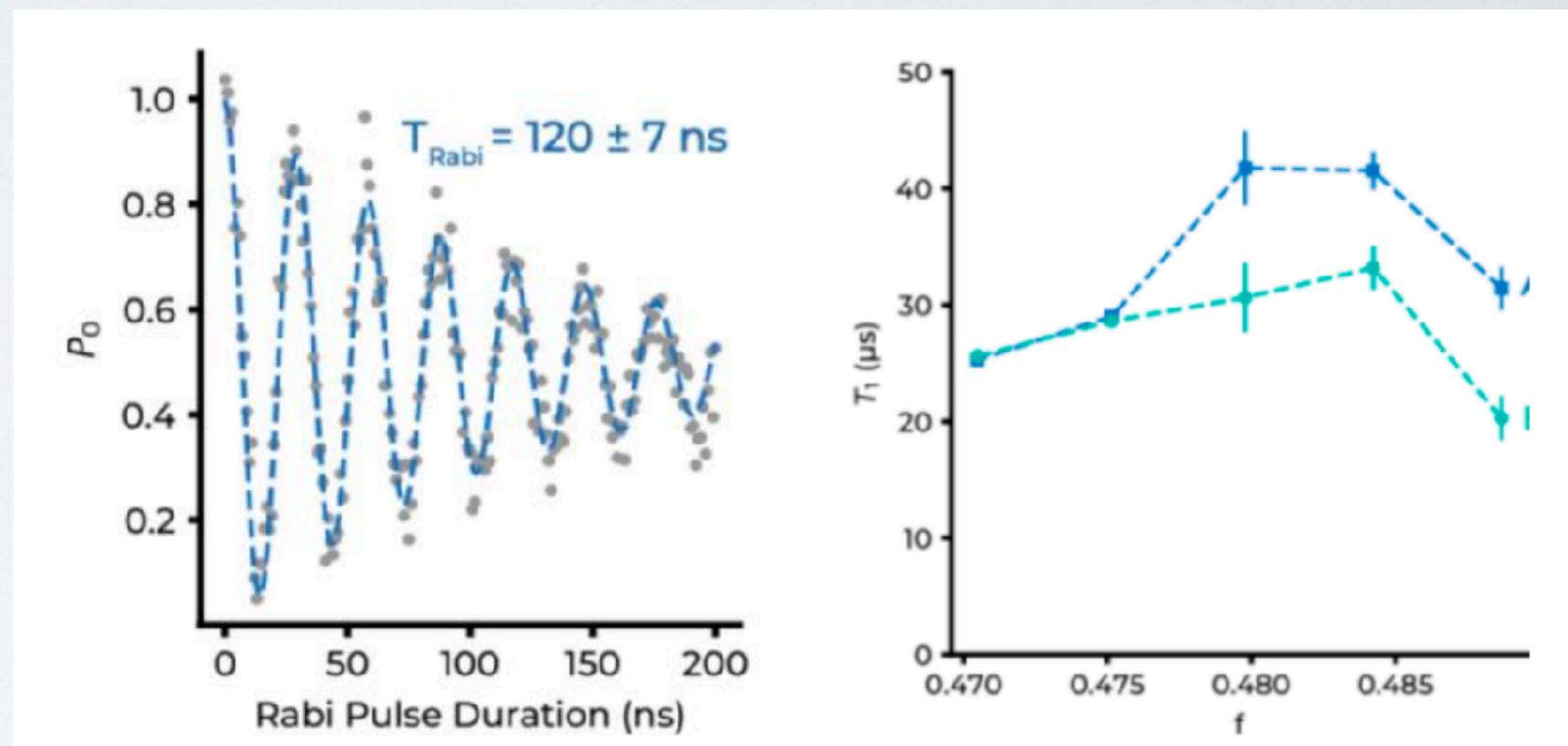
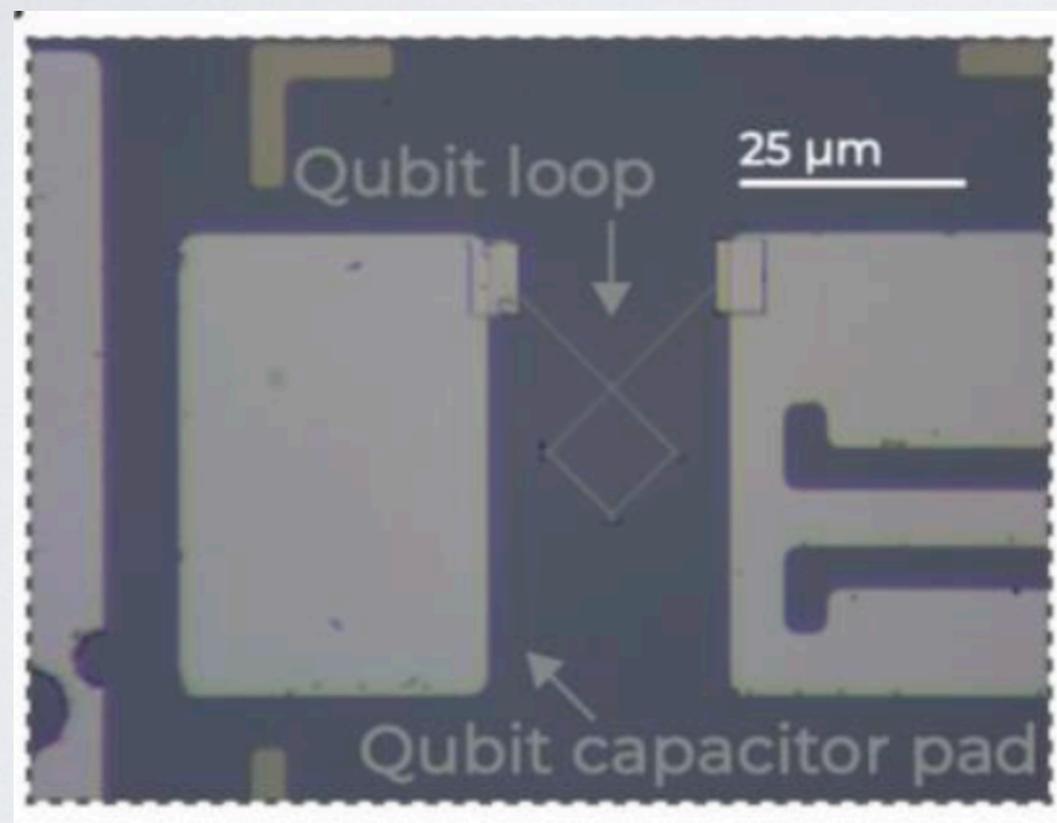
COHERENT QUBITS FOR QA

rf-SQUID-based superinductor flux qubit (KIT)



COHERENT QUBITS FOR QA

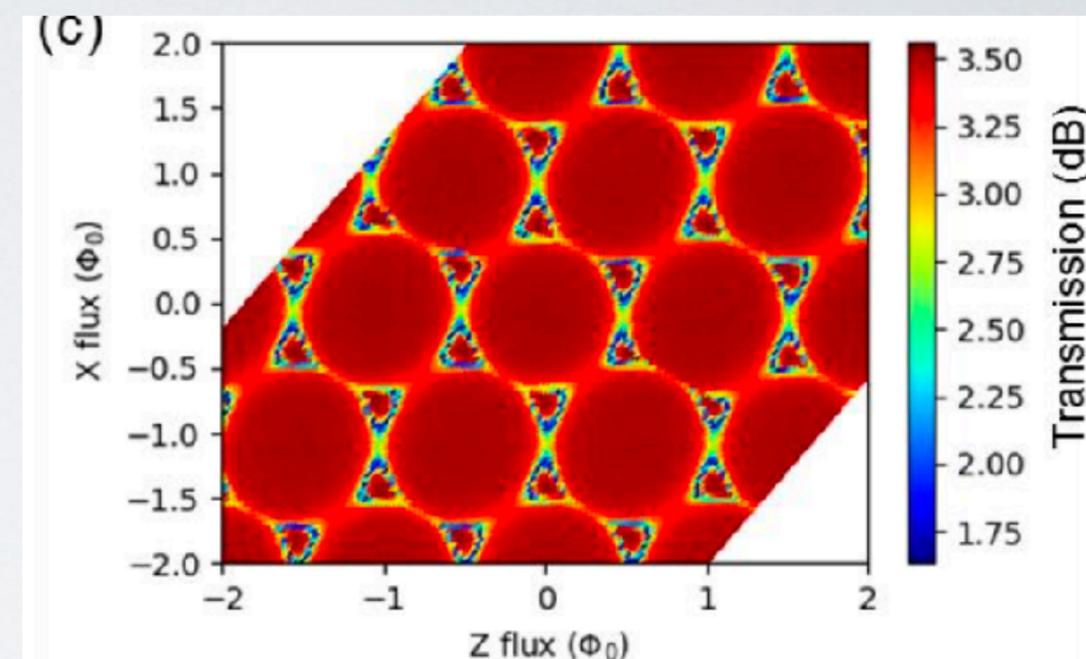
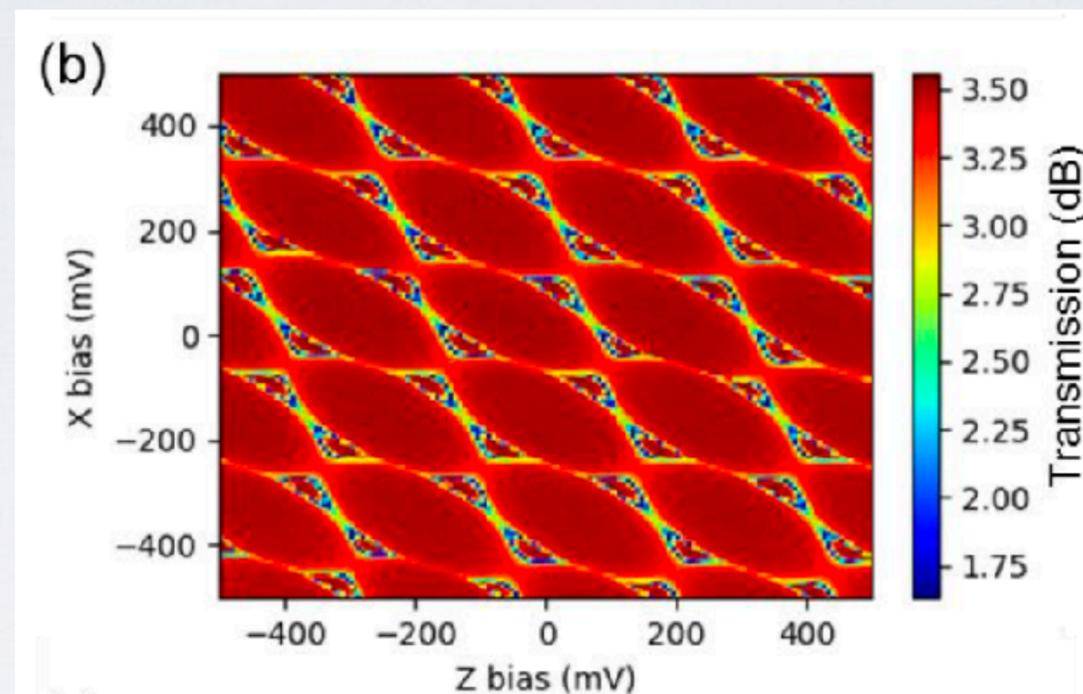
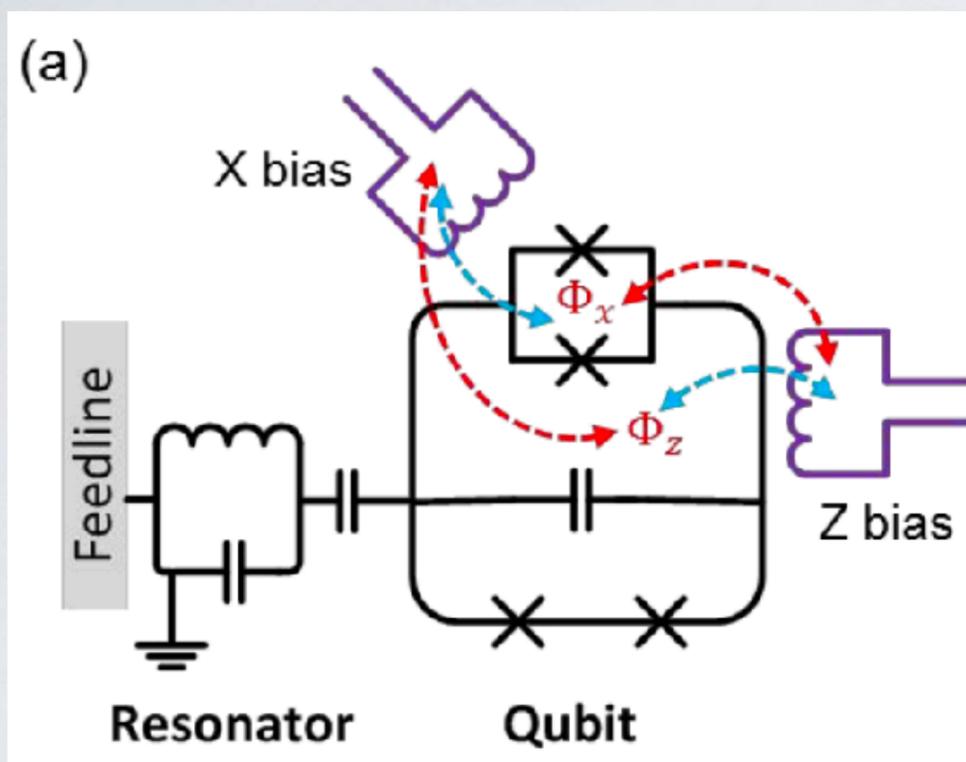
C-shunted flux qubits



D. López-Núñez, PhD thesis

AVa⁺QUS

COHERENT QUBITS FOR QA

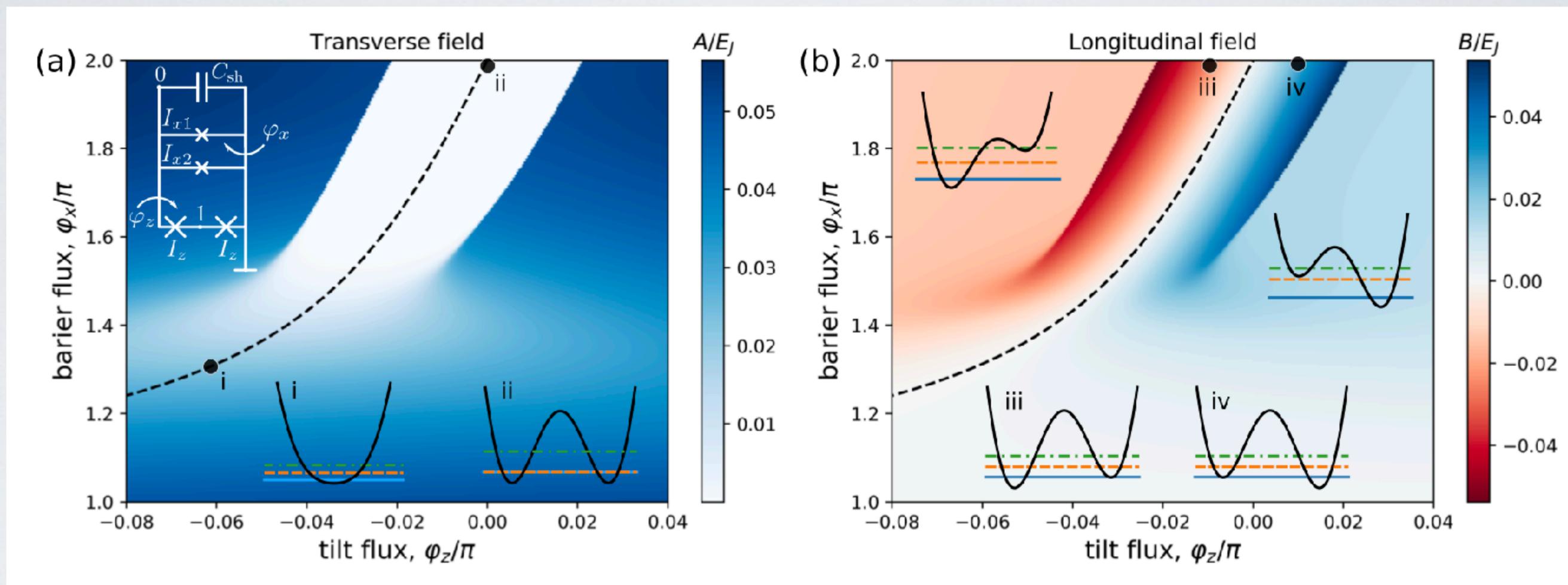


Novikov et al., 2018 IEEE International Conference on Rebooting Computing (ICRC), McLean, VA, USA, 2018, pp. 1-7

X. Dai et al., PRX Quantum 2, 040313 (2021)

COHERENT QUBITS FOR QA

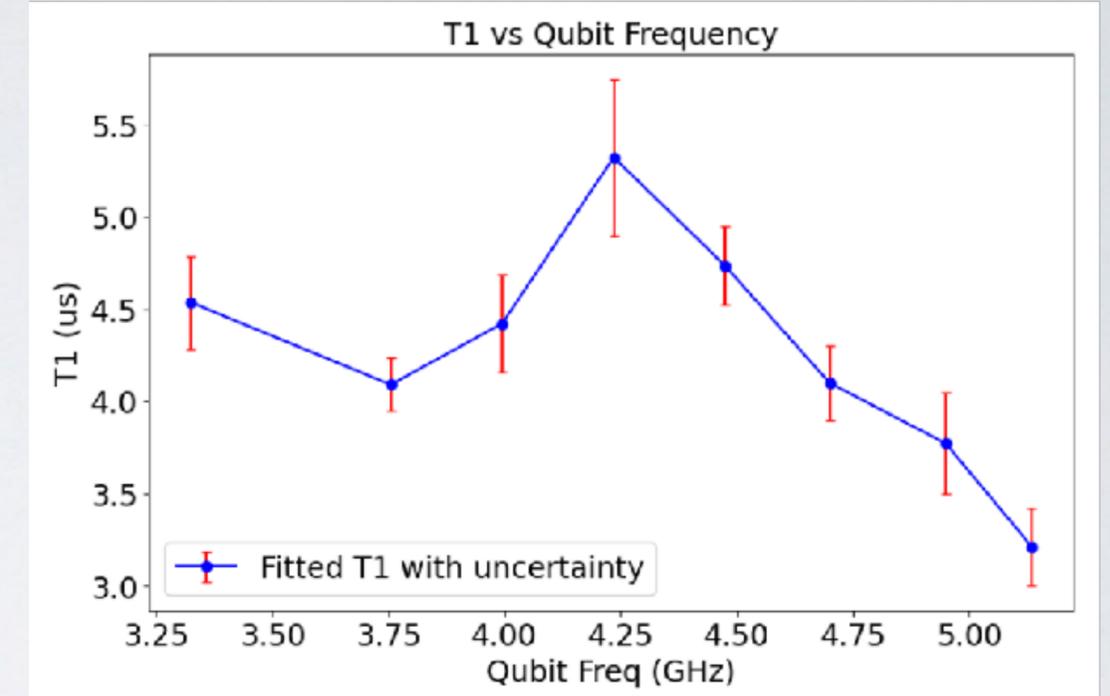
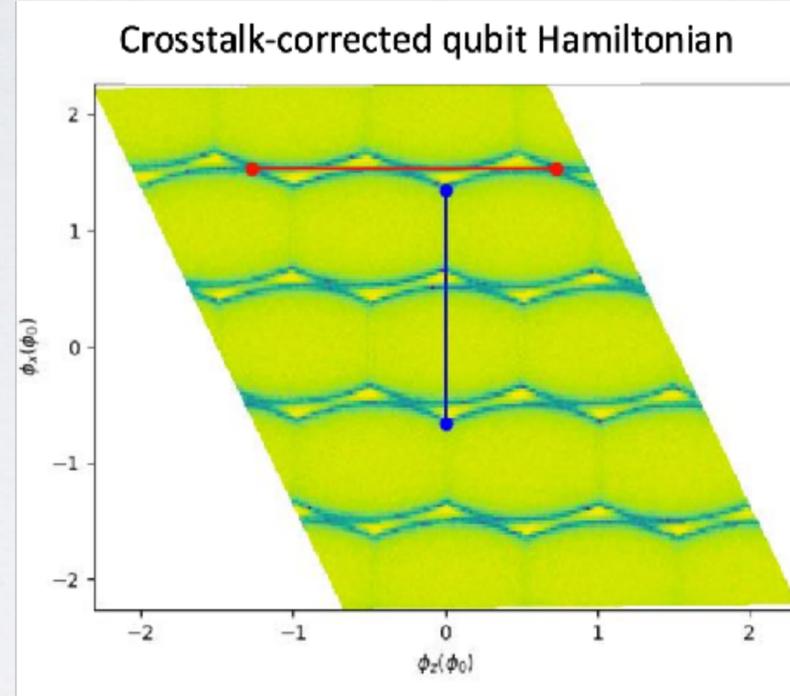
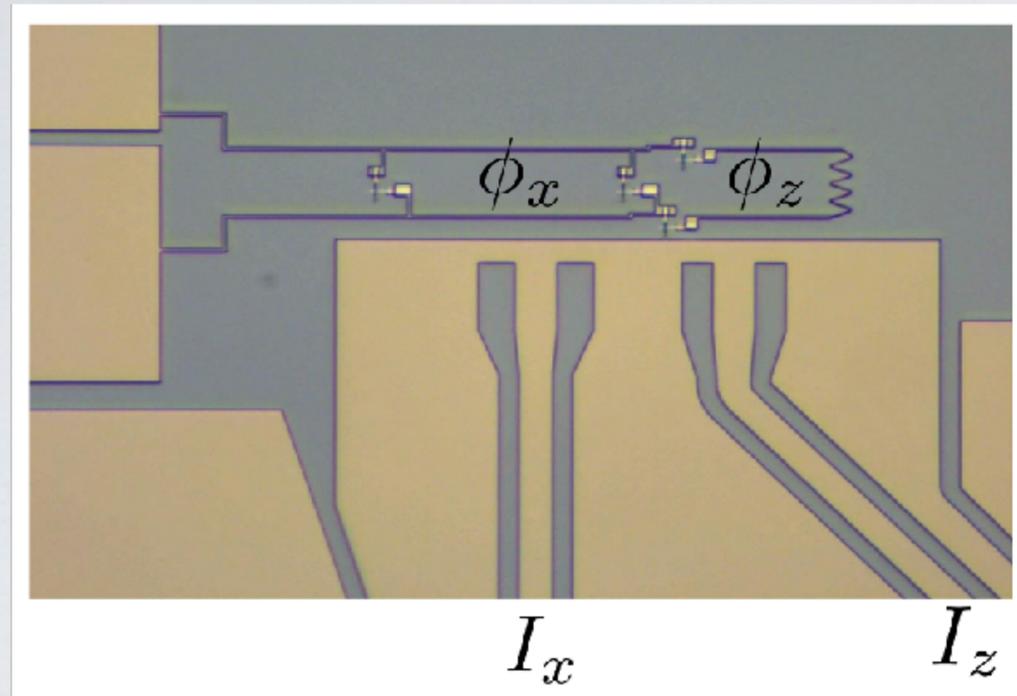
Anneal path correction in flux qubits



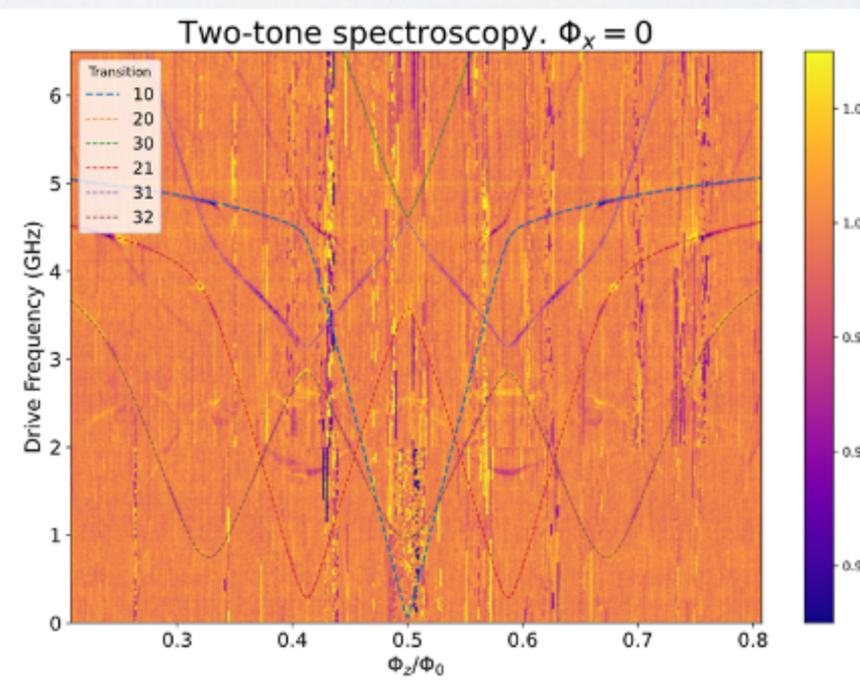
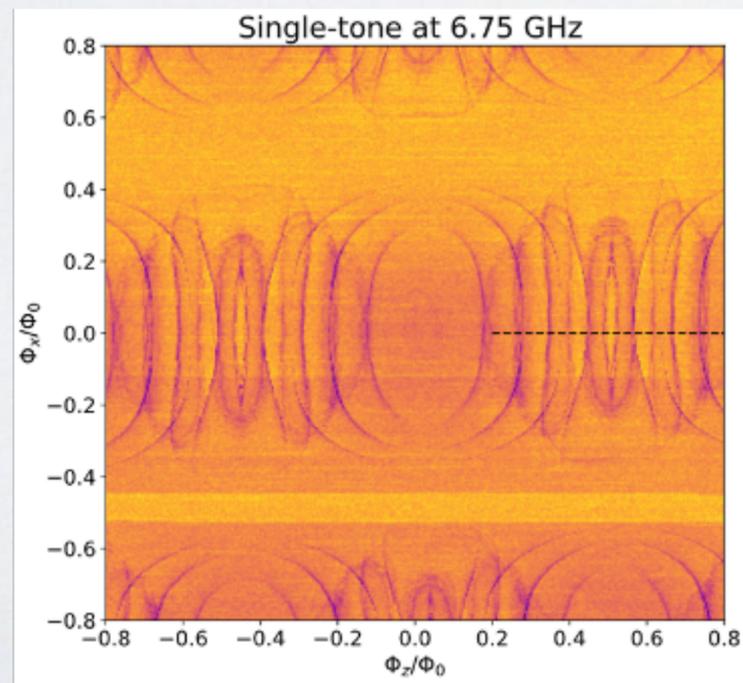
M. Khezri et al., npj Quantum Inf. 7, 36 (2021)

COHERENT QUBITS FOR QA

From C-shunt flux qubits...

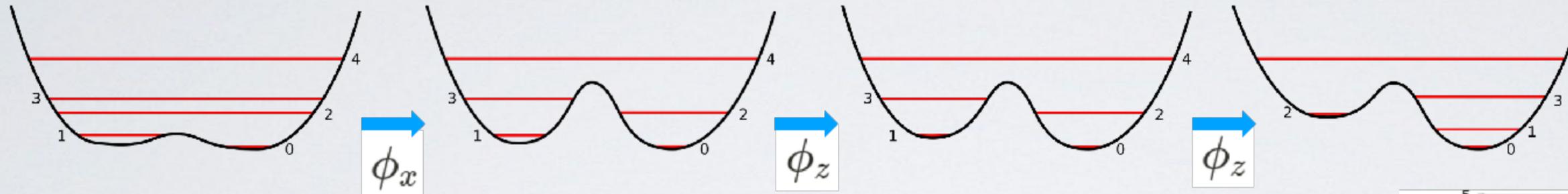


...to fluxonium qubits for QA



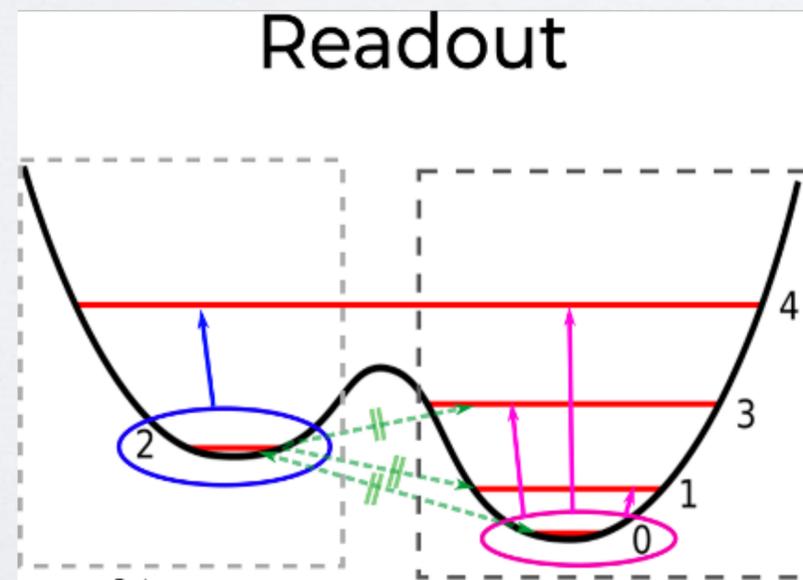
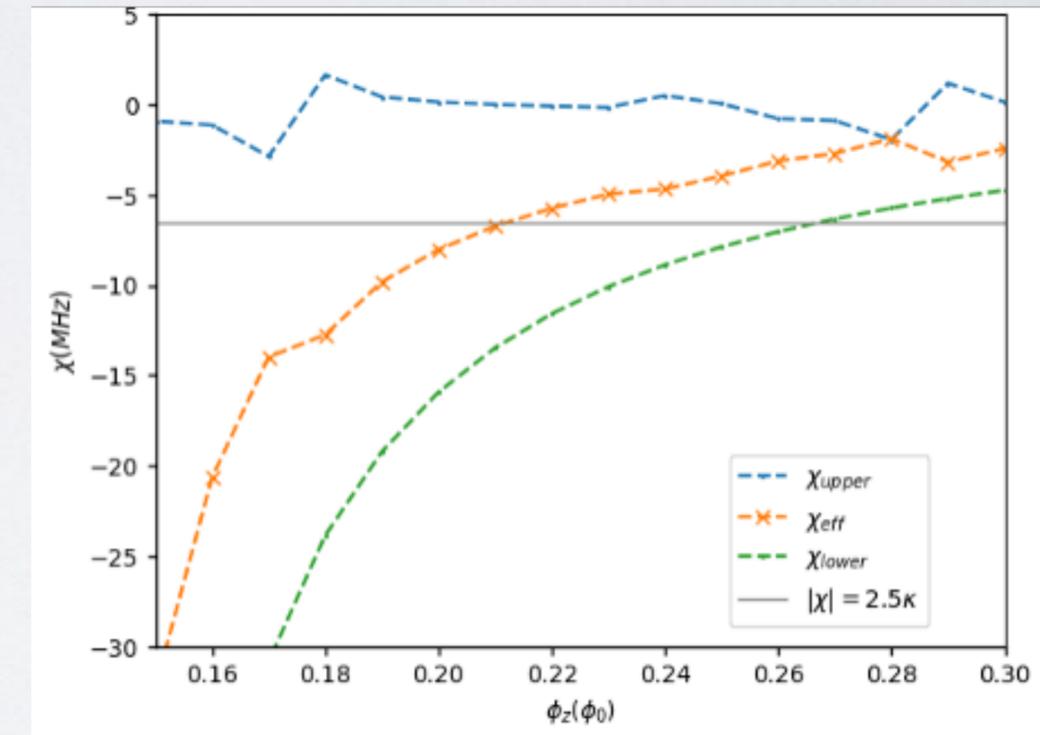
READOUT OF QUBITS FOR QA

Which-well readout



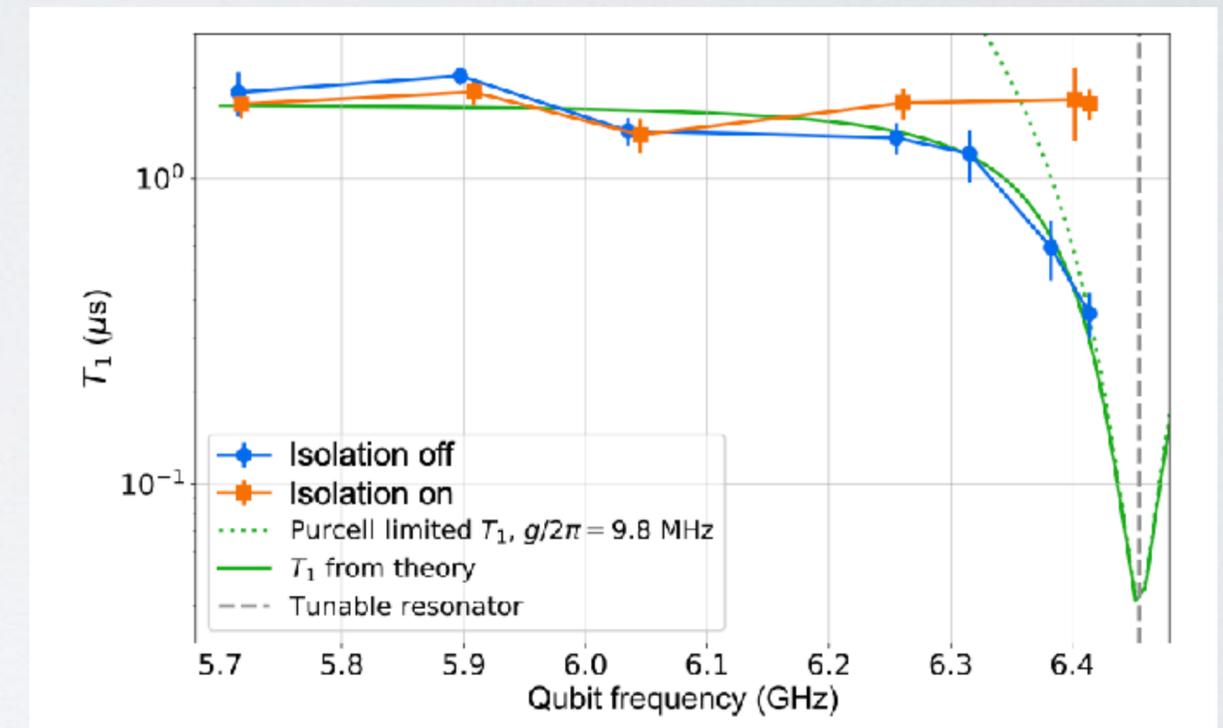
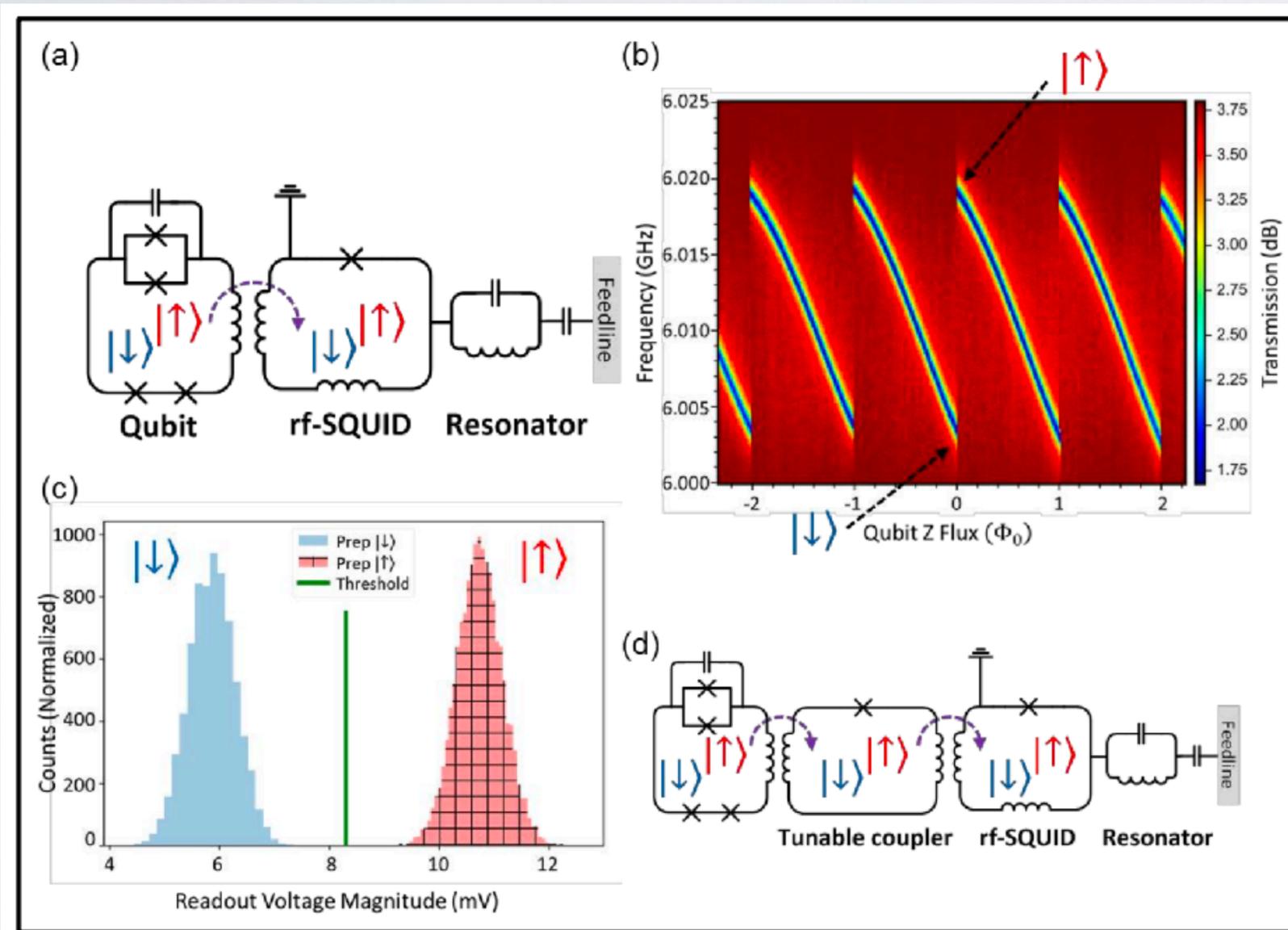
Total dispersive shift:

$$X_{PCR} = \frac{1}{2} (X_{0,R} - X_{0,L}) = \frac{1}{2} \left\{ \sum_j (\chi_{j0,R} - \chi_{0j,R}) - \sum_j (\chi_{j0,L} - \chi_{0j,L}) \right\}$$



READOUT OF QUBITS FOR QA

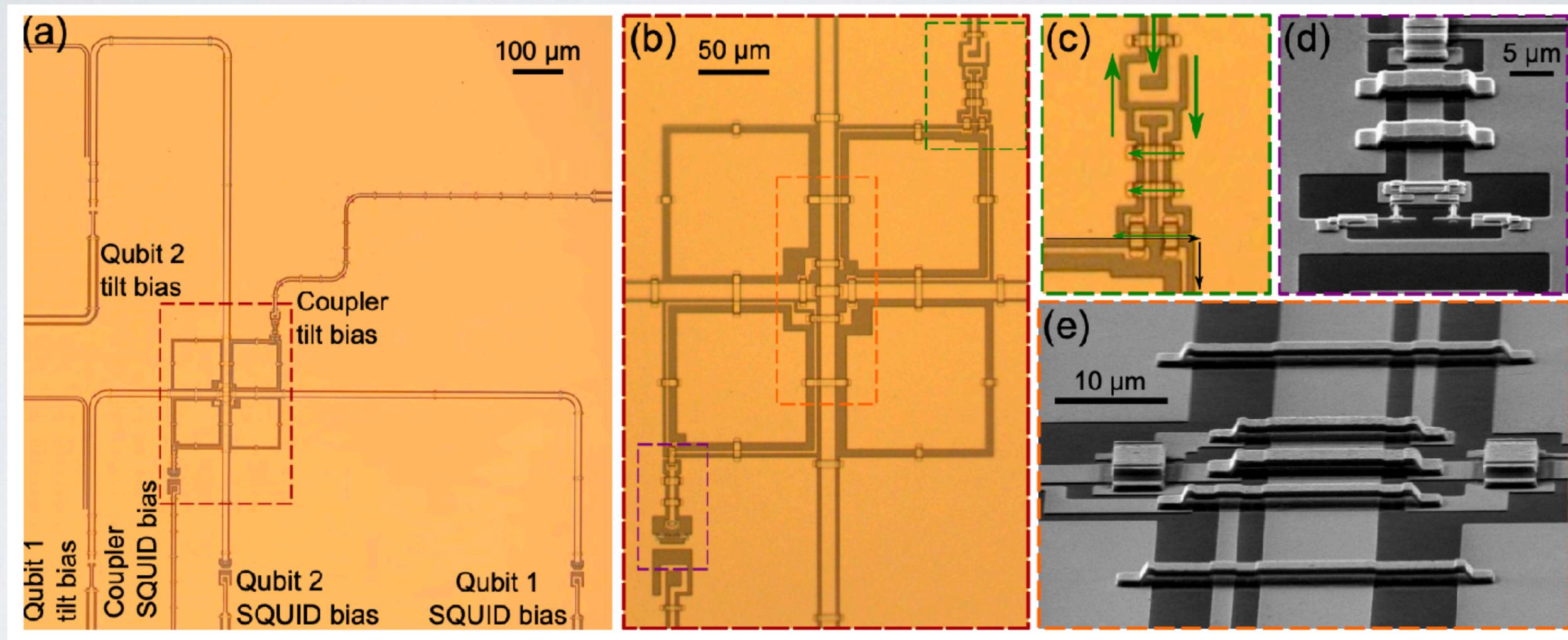
Dispersive-like readout



Novikov et al., 2018 IEEE International Conference on Rebooting Computing (ICRC), McLean, VA, USA, 2018, pp. 1-7

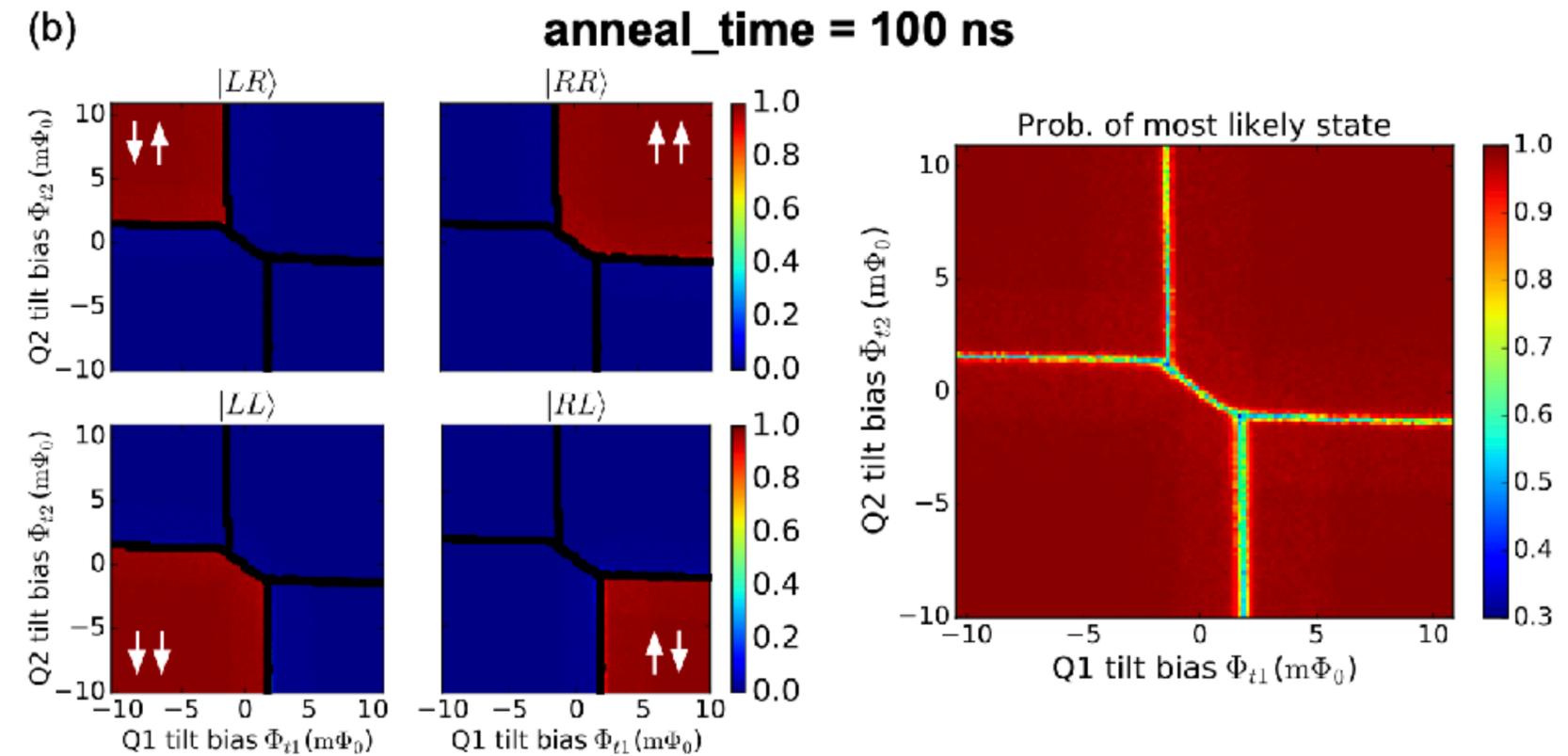
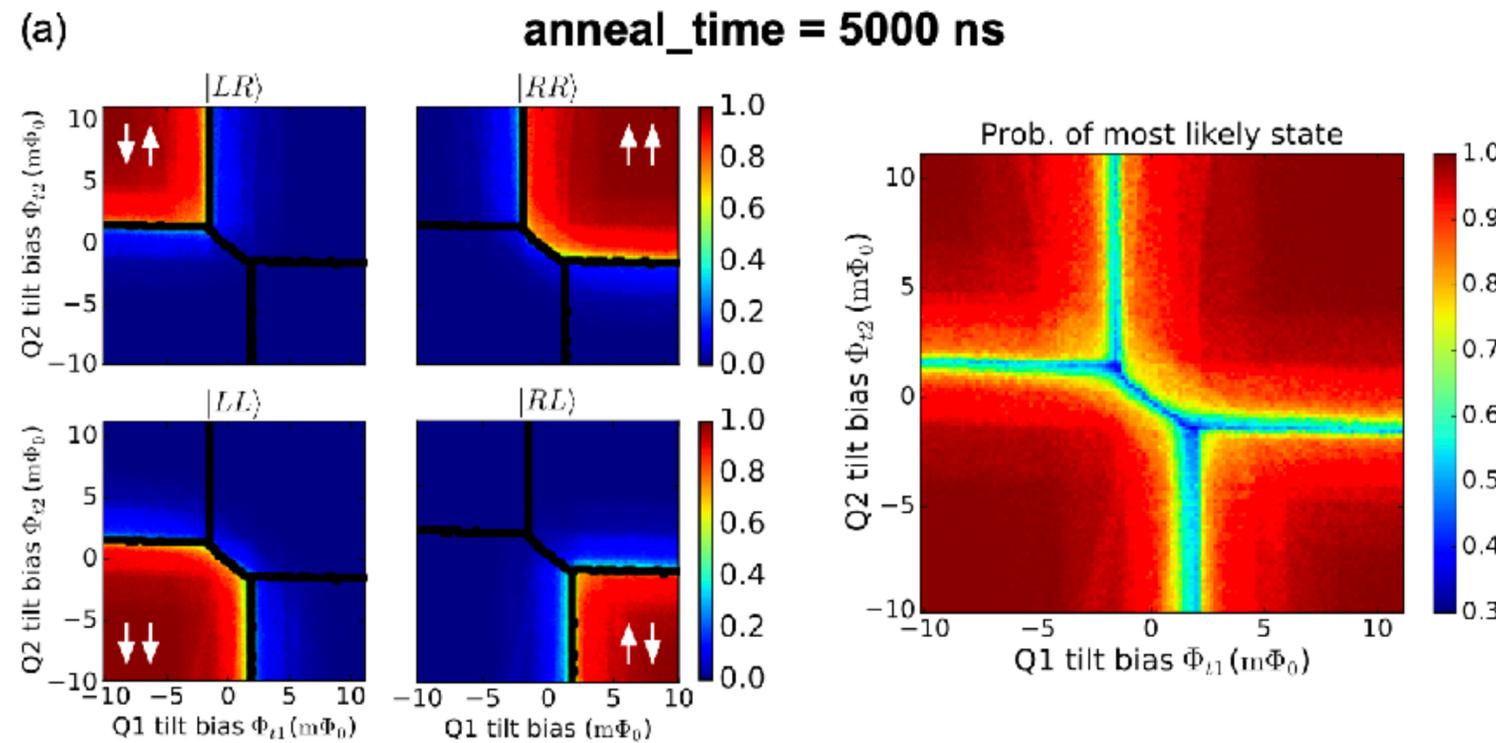
Grover et al., PRX QUANTUM 1, 020314 (2020)

COUPLING OF COHERENT QUBITS

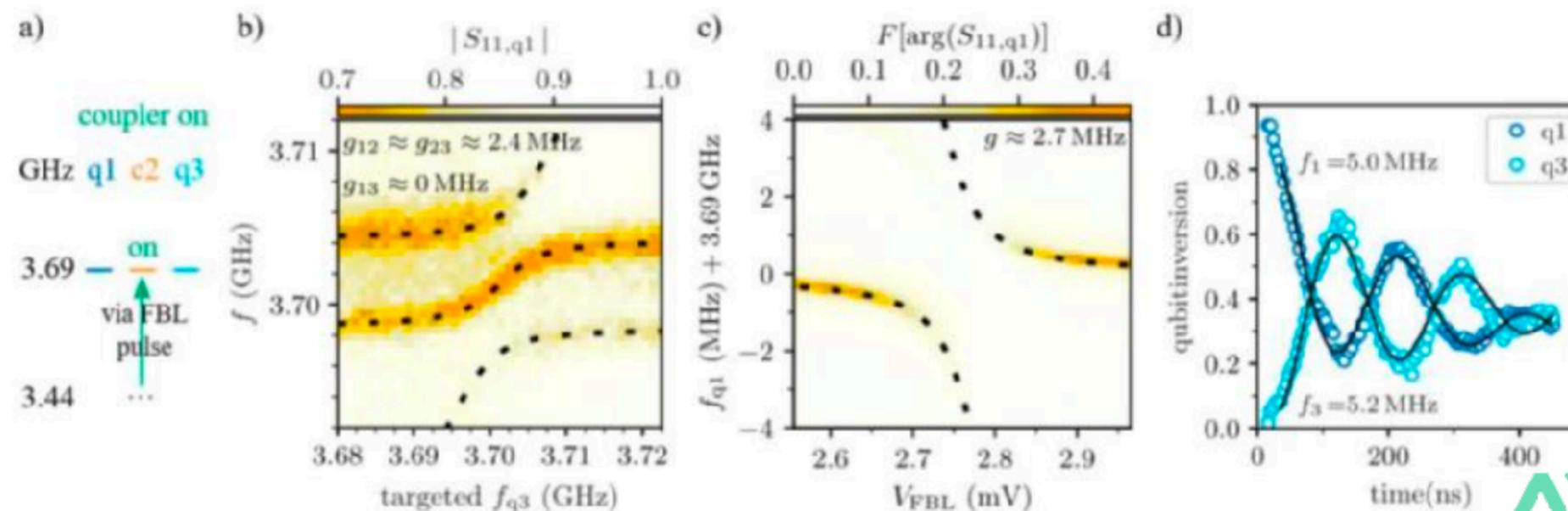
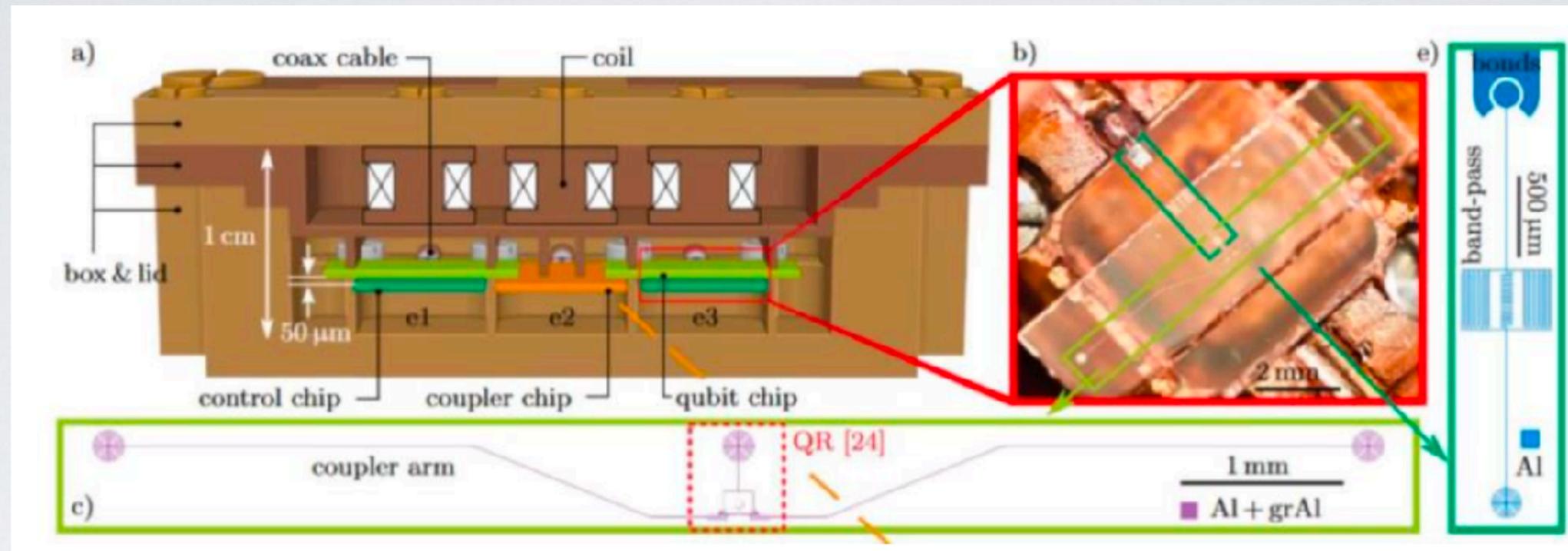


COUPLING OF COHERENT QUBITS

Two-qubit annealing: $M_{\text{eff}} = -5$ pH

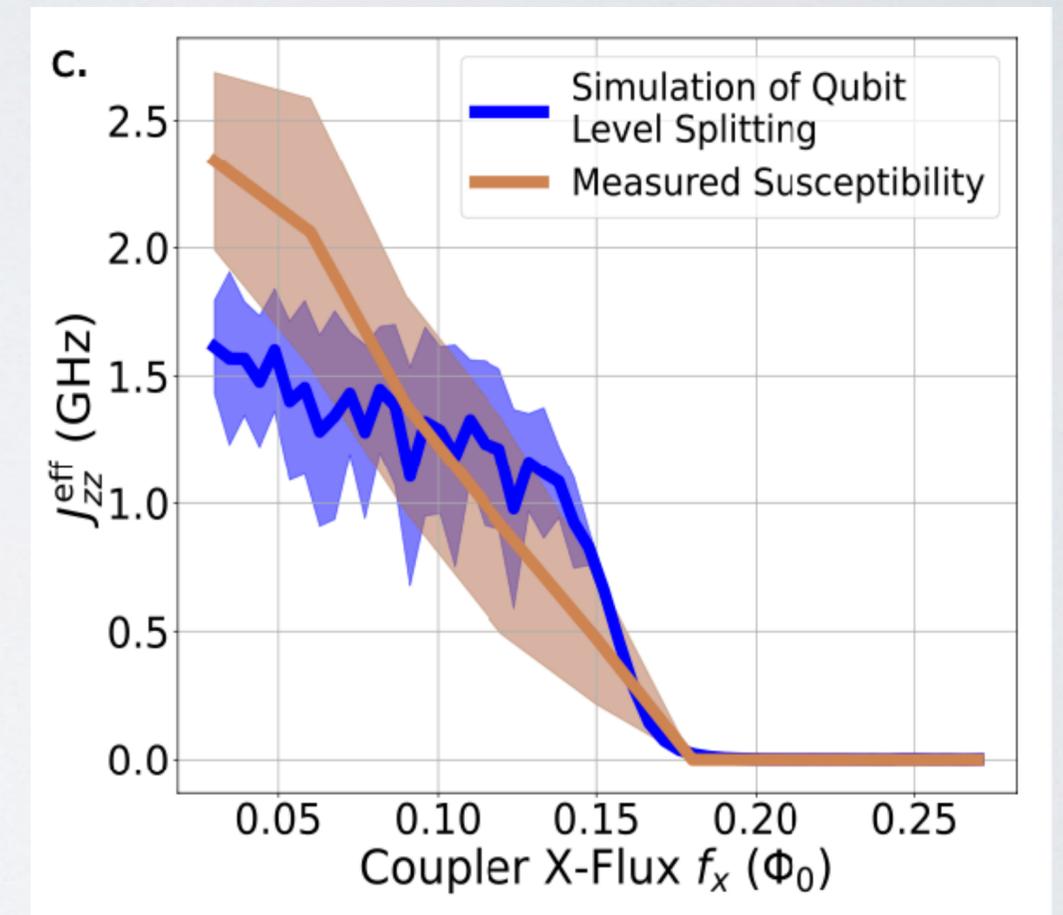
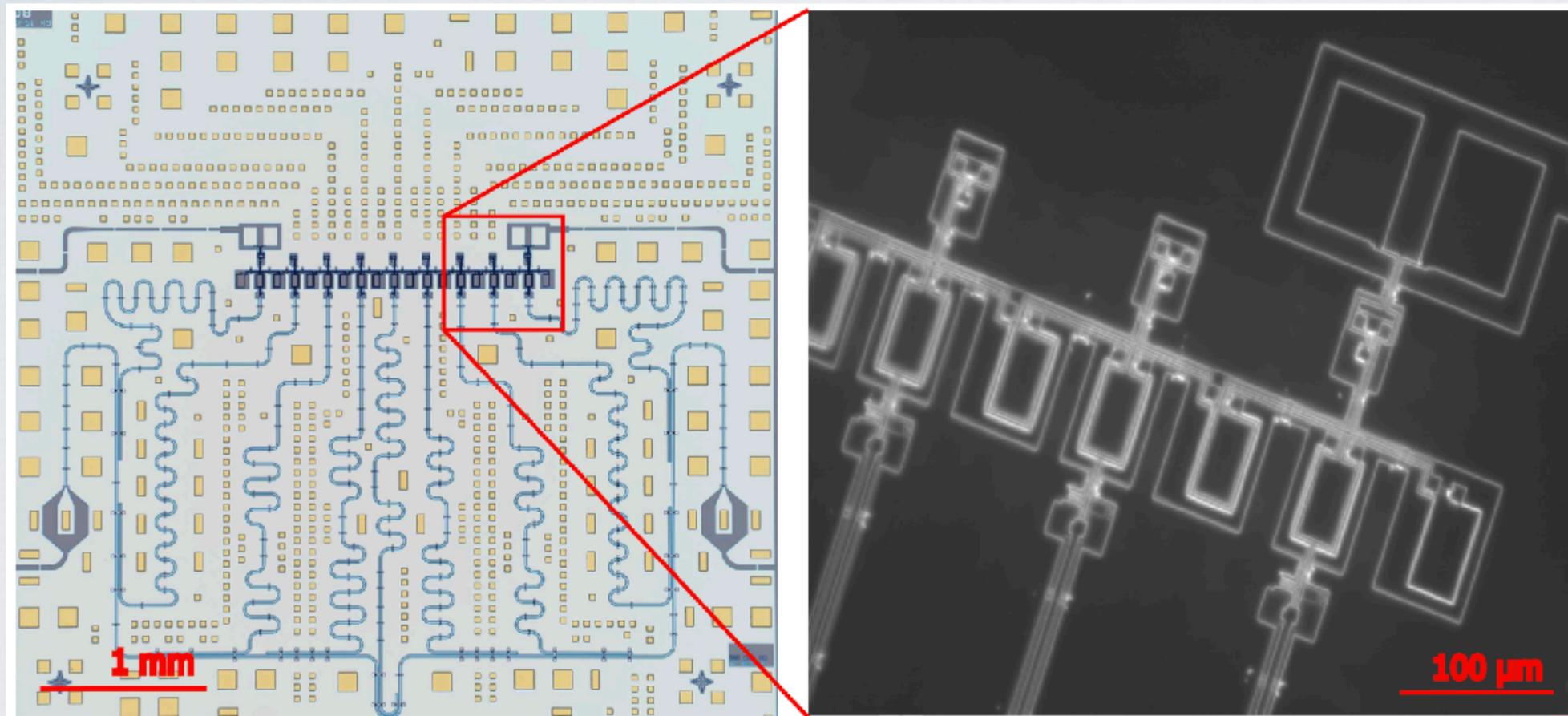


SCALABLE CIRCUITS FOR QA



SCALABLE CIRCUITS FOR QA

Long-range qubit-qubit coupling through coupler chain



CQA EFFORTS

- Coherent, tunable qubits ✓
- Readout of coherent qubits ✓
- Coupling coherent qubits, stoquastic ✓
- Non-stoquastic qubit-qubit couplings ✗
- Scalable circuits for CQA ✗✓

Lots do be done!
Interested? Contact us!

END OF PART II

ACKNOWLEDGEMENTS

AVaQus consortium
The QCT group at IFAE
The Qilimanjaro team



THANK YOU!

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www.qilimanjaro.tech

