Bosonic cQED:

A rich playground for light-matter interaction

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A bit about me & my team

2008 – 2011: Undergrad in Physics, Oxford

- I love physics and tinkering with things

2011 – 2012: Gap year research internship

- I HATE aligning mirrors

2012 – 2018: Ph.D in expt cQED, Yale

- Playing with photons, but only microwave ☺

2018 – 2019: Exploratory year

- Start-ups are cool!
- But I miss curiosity-driven research

2020 – now: assistant professor in NUS

- physics with small bosonic cQED devices

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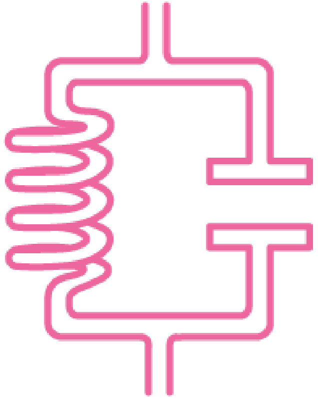
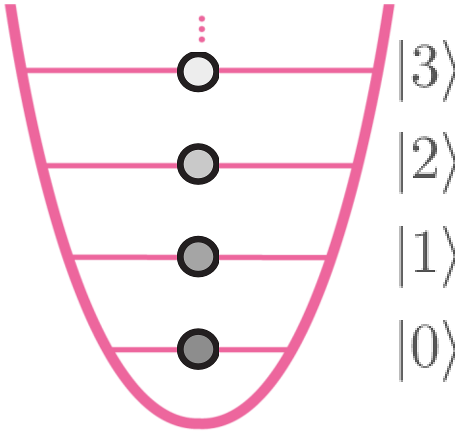
Plans for today

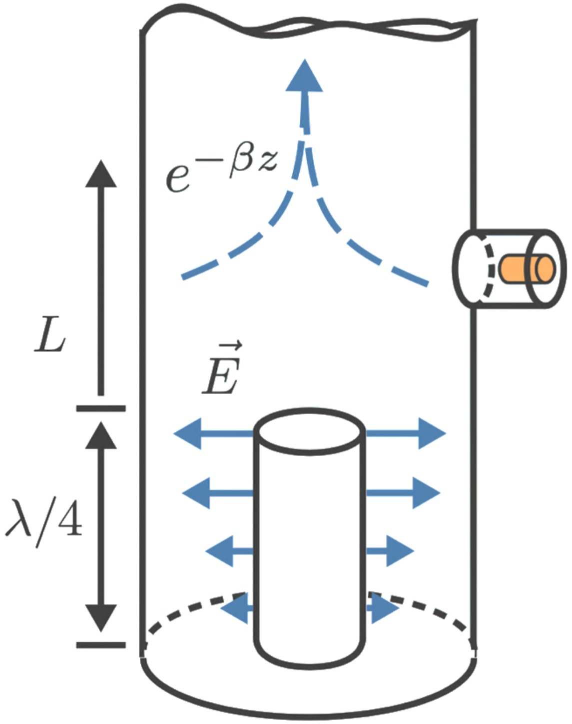
1. **Brief introduction to cQED**
2. **What goes into a minimal bosonic cQED hardware module**
3. **Bosonic cQED as a playground for good old quantum optics concepts**
    - **States**
    - **Gates**
    - Measurements
4. Leverage bosonic cQED devices for quantum information processing
    - Continuous-variable logical qubits encoded in bosonic modes
    - Non-Gaussian bosonic resources for simulation and metrology
5. Looking ahead: challenges and exciting developments

Main building blocks of cQED – the harmonic oscillator



LC oscillators
(cavity)





3D superconducting
cavities storing
microwave photons

High quality factor:

- Typically made using high-purity aluminium;
- Chemically etched after machining
- Geometry is optimised to minimise surface and seam participation

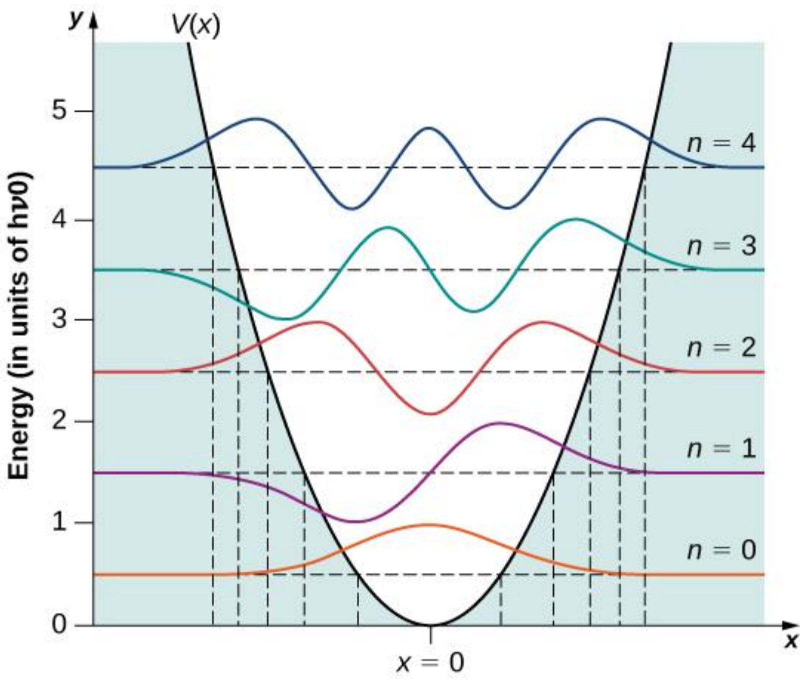
Straightforward coupling to non-linear ancillary element(s)

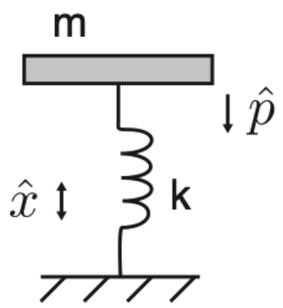
Reagor (2014, 2016)

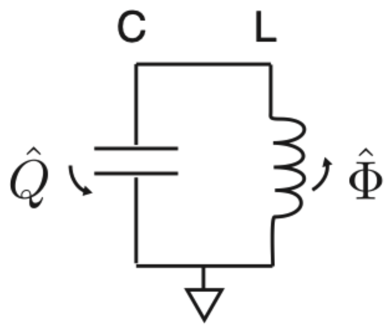
Recap about quantum harmonic oscillators

$$-\frac{1}{2}\frac{d^2\psi}{dx^2}+\frac{1}{2}x^2\psi=\frac{E}{\hbar\omega}\psi=\epsilon\psi$$

Solutions are Hermite polynomials







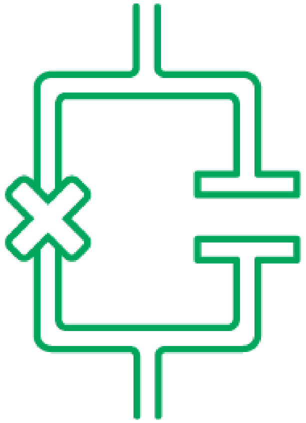
$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

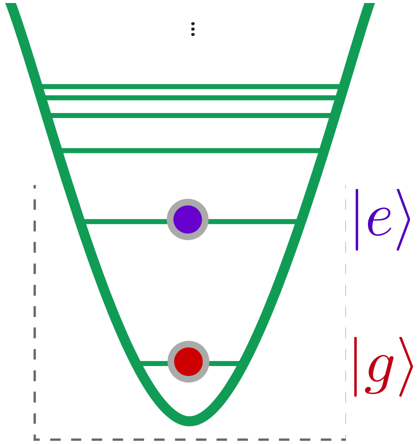
Standard quantisation procedure

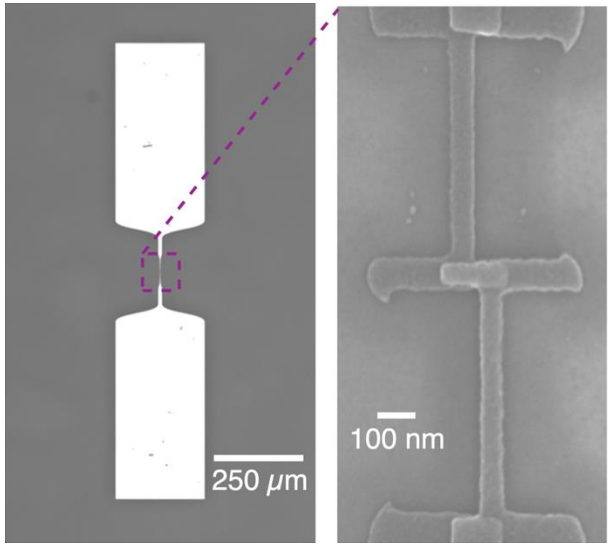
$$E=(n+\frac{1}{2})\hbar\omega$$

Universal control on oscillator provided by transmon



Non-linear coupler/controller (transmon)

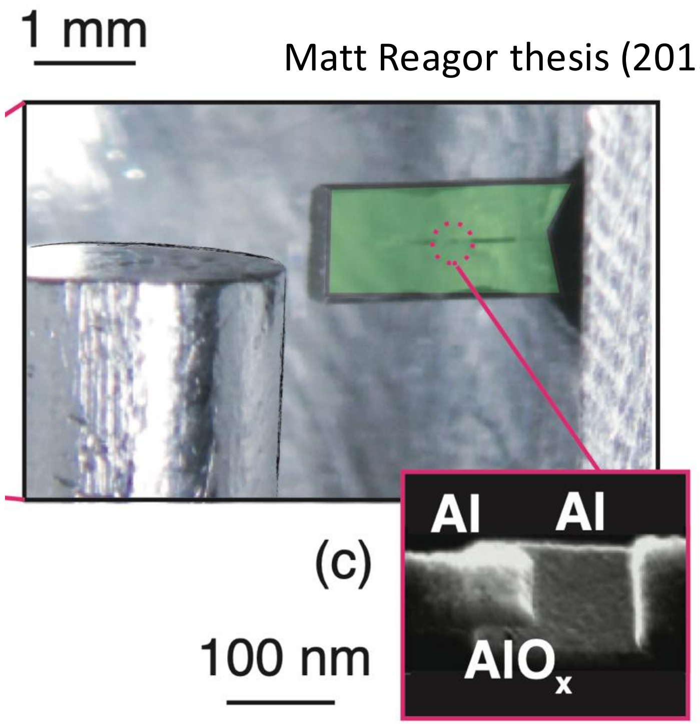




non-linear elements based on Josephson junctions fabricated on chip

In 3D devices, the choice of substrate is usually sapphire

Matt Reagor thesis (2015)



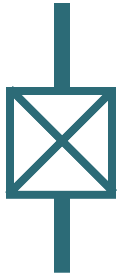
Couples to the cavity mode when their electric fields overlap

Choices of nonlinear elements based on JJs

Bosonic cQED:

- nonlinear elements are used as ancillary modes and couplers
- they don‘t need to be qubits
- just need to have the right form of nonlinearity for the specific tasks

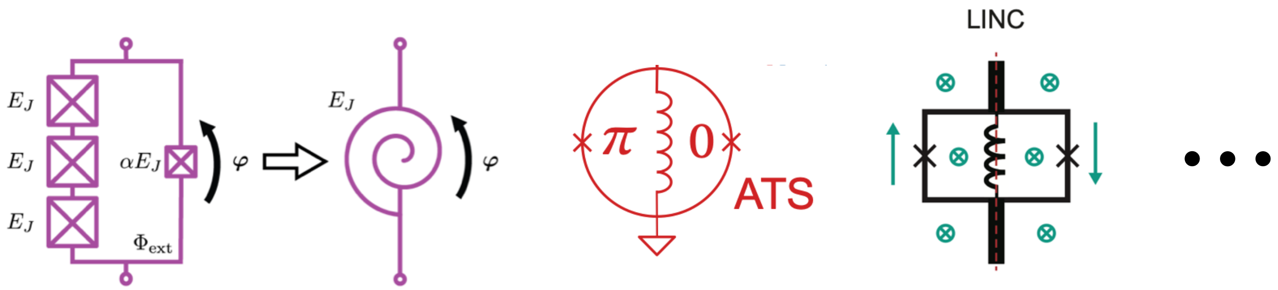
Transmon: 4 wave mixing



$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$

i.e. energy conservation

When flux is incorporated, things can get more interesting



SNAIL: Nick Frattini Ph.D thesis: Three-wave Mixing in Superconducting Circuits: Stabilizing Cats with SNAILs
ATS: Raphaël Lescanne Ph.D Thesis: Engineering multi-photon dissipation in superconducting circuits for quantum error correction (2020)
LINC: Aniket Maiti; https://arxiv.org/pdf/2501.18025

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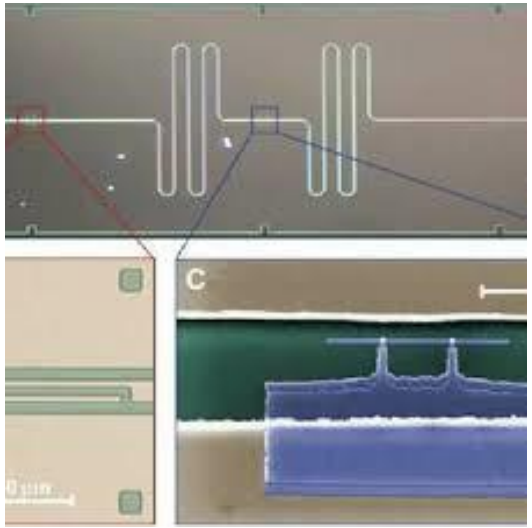
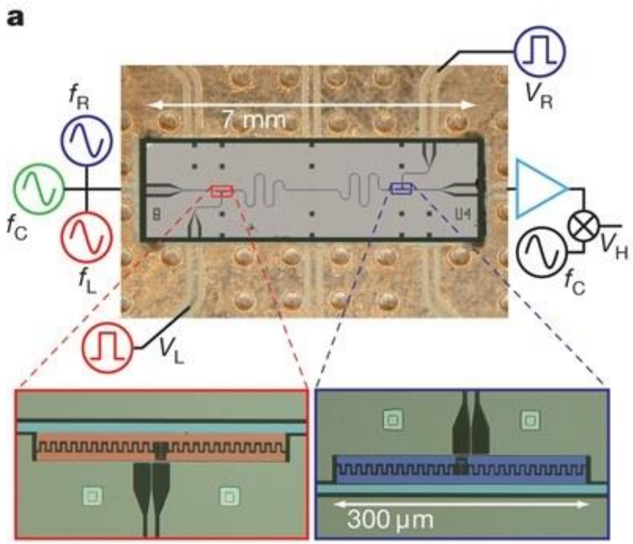
The bosonic element has always been there!



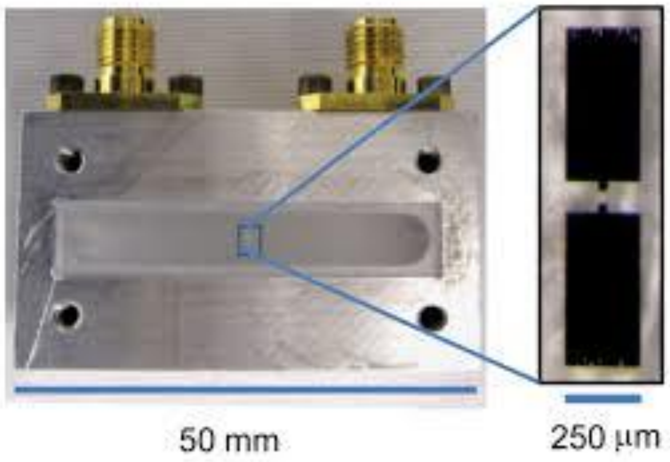
- Resonator strongly coupled to a cooper pair box qubit
- Vacuum Rabi between microwave photon and qubit

Wallraff, Nature (2004)



- Resonator coupled to two transmon qubits
- Resonator photon acts as bus to mediate two-qubit gate

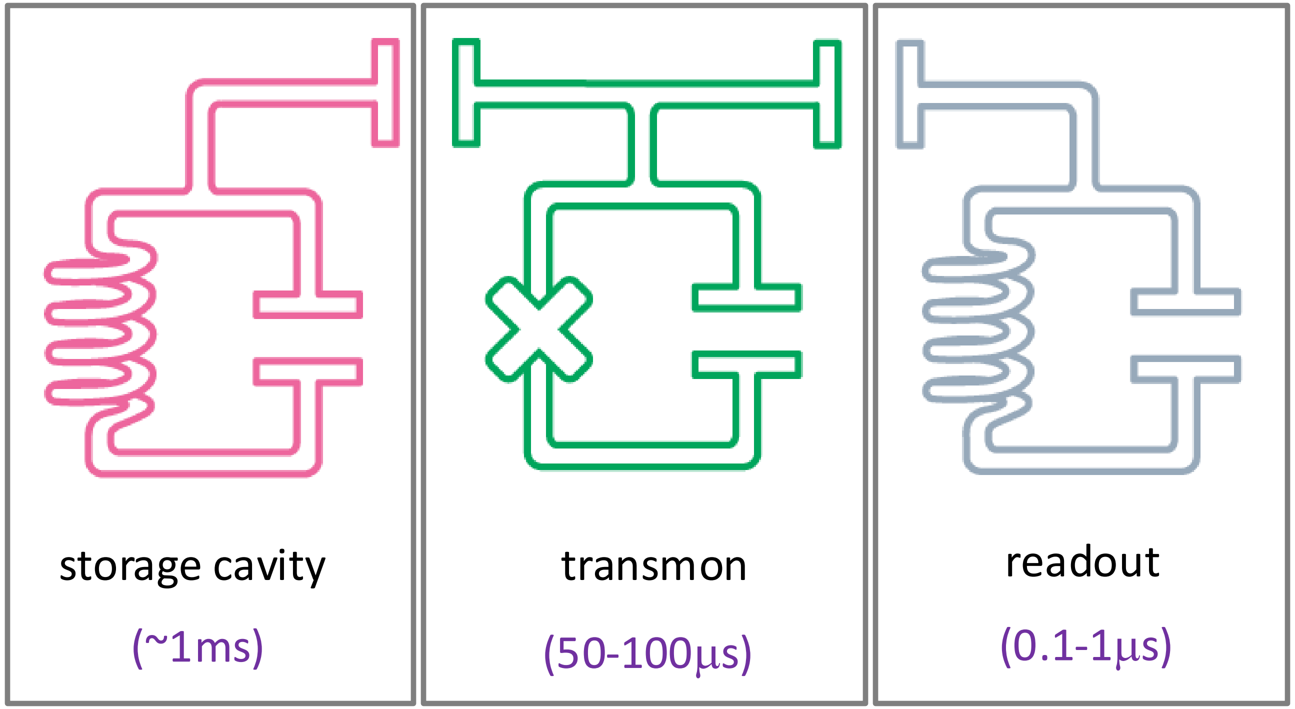
Dicarlo, Nature (2009)

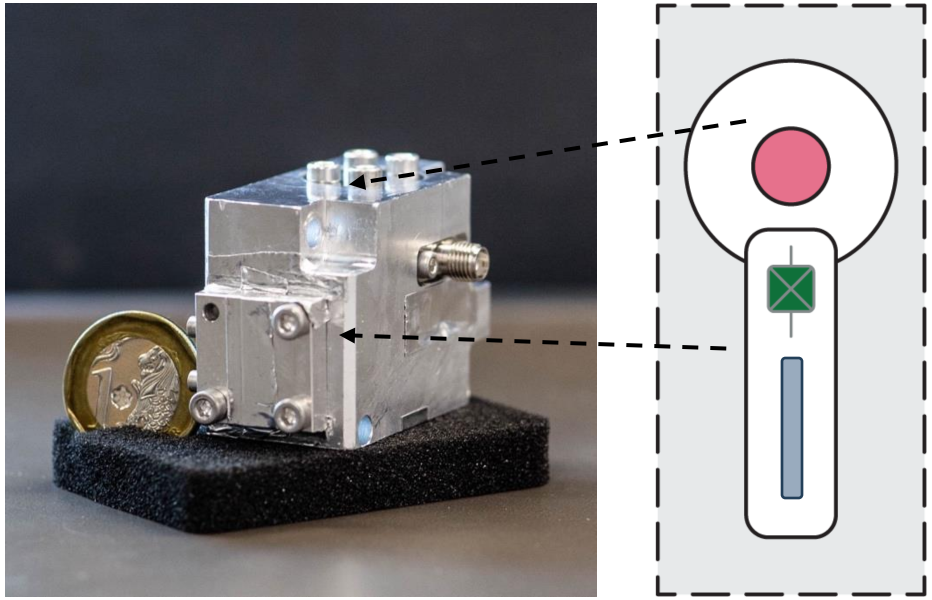


- 3D cavity used as a readout element for a transmon qubit
- Quick and handy way to test/characterise a qubit

Park, PRL (2011)

Unique playground for on-demand light-matter dynamics

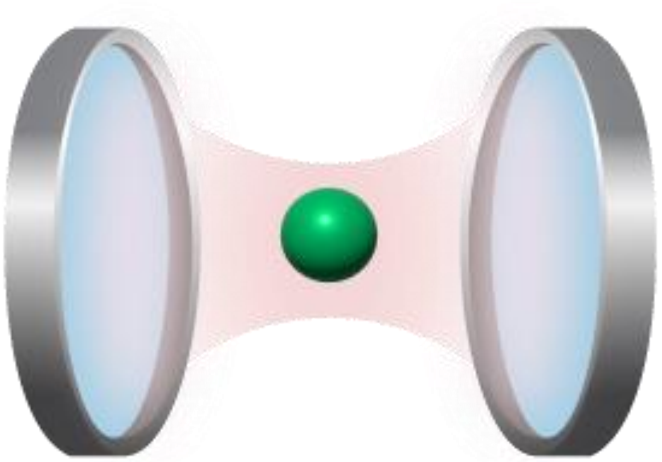




a basic module equipped with

1. harmonic, long-lived bosonic mode
2. engineered non-linear operations via transmon ancilla
3. efficient information measurement via fast dispersive readout

The Jaynes-Cummings Hamiltonian



Light matter interaction 101:

$$\hat{H}^I = -\hat{d} \cdot \hat{E}$$

Matter
– 2 level atom:

$$\hat{\sigma}_+ = |e\rangle\langle g|, \quad \hat{\sigma}_- = |g\rangle\langle e|$$

Light
– single mode EM field in a cavity

$$\hat{E} = \vec{e}\left(\frac{\hbar\omega}{\epsilon_0 V}\right)^{\frac{1}{2}} (\hat{a} + \hat{a}^\dagger)\sin(kz),$$

$$\boxed{\hat{H}^I = g\hat{d}(\hat{a} + \hat{a}^\dagger) = dg(\sigma_+ + \sigma_-)(\hat{a} + \hat{a}^\dagger)}$$

Account for time dependence/frequencies, and take RWA

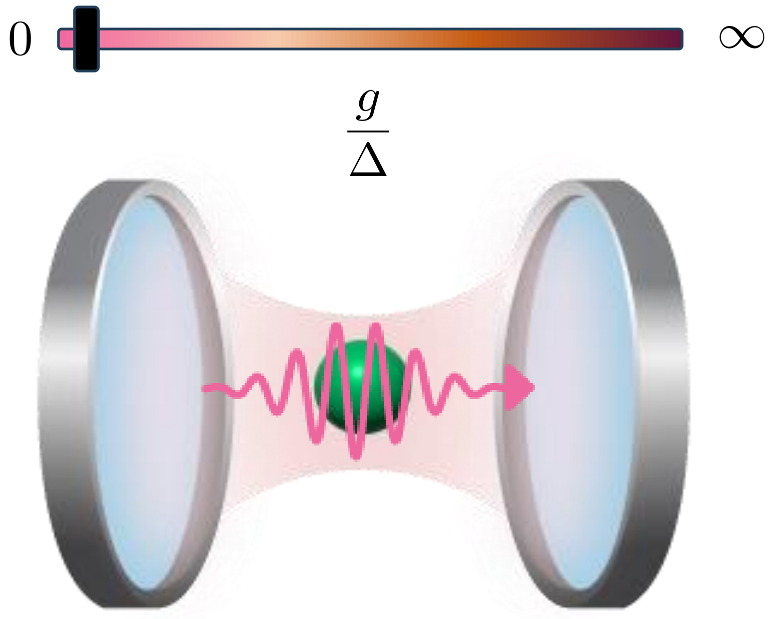
$$\hat{\sigma_+}\hat{a} \sim e^{i(\omega_0-\omega)t}; \quad \hat{\sigma_-}\hat{a}^\dagger \sim e^{-i(\omega_0-\omega)t}$$

$$\hat{\sigma_+}\hat{a}^\dagger \sim e^{i(\omega_0+\omega)t}; \quad \hat{\sigma_-}\hat{a} \sim e^{-i(\omega_0+\omega)t}.$$

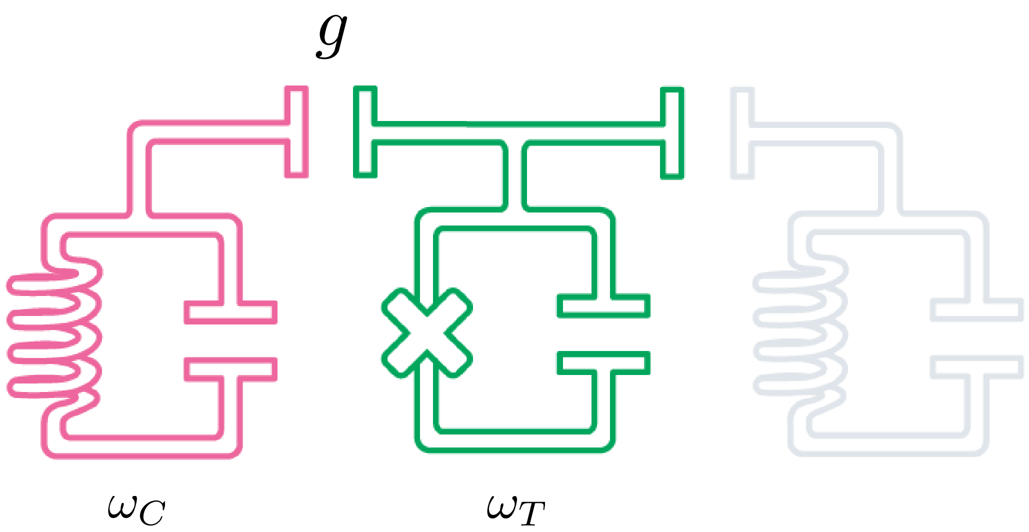
Assume $\omega_0 \approx \omega$

$$\boxed{\hat{H}^I = dg(\sigma_+\hat{a} + \sigma_-\hat{a}^\dagger)}$$

Harness on-demand light-matter interaction



No Interaction

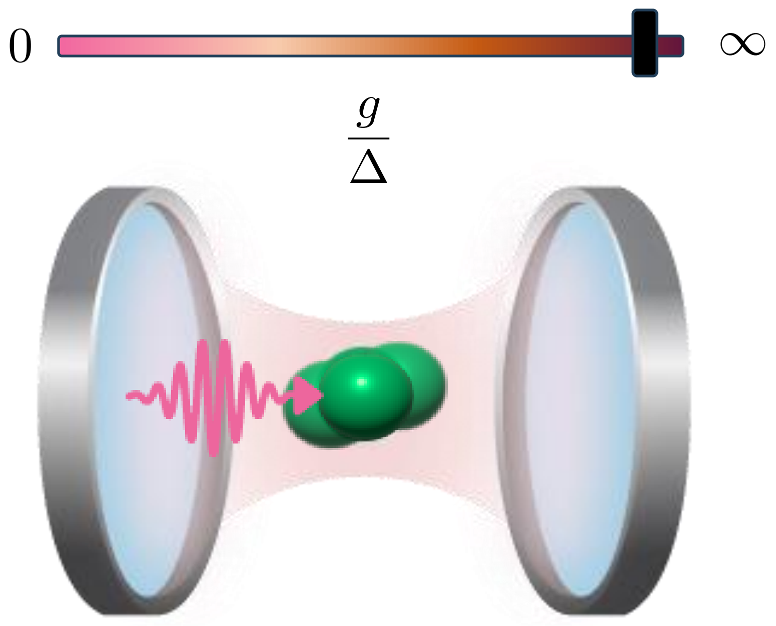


$$\Delta = |\omega_C - \omega_T|$$

Atom and light do not influence one another, useful for:

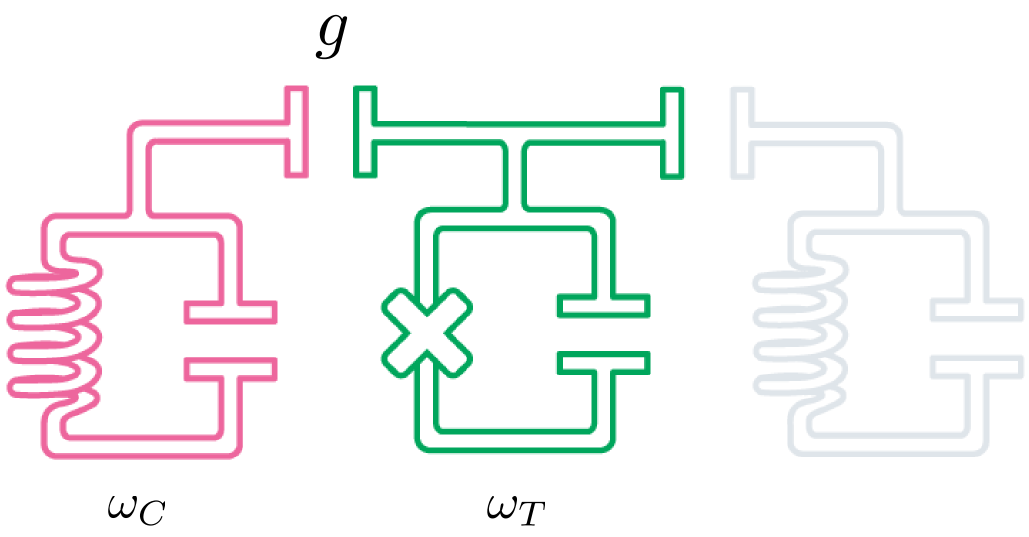
1. Probing individual manipulations & dynamics
2. Enacting processes with significant idle periods

Harness on-demand light-matter interaction



Resonant interaction:

$$H = g(a^{\dagger}\sigma_{-} + a^{-}\sigma_{+})$$

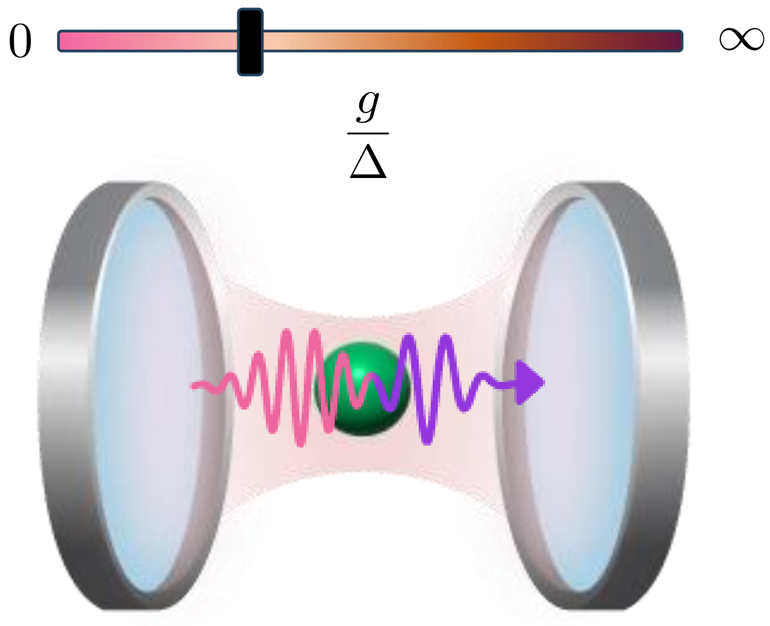


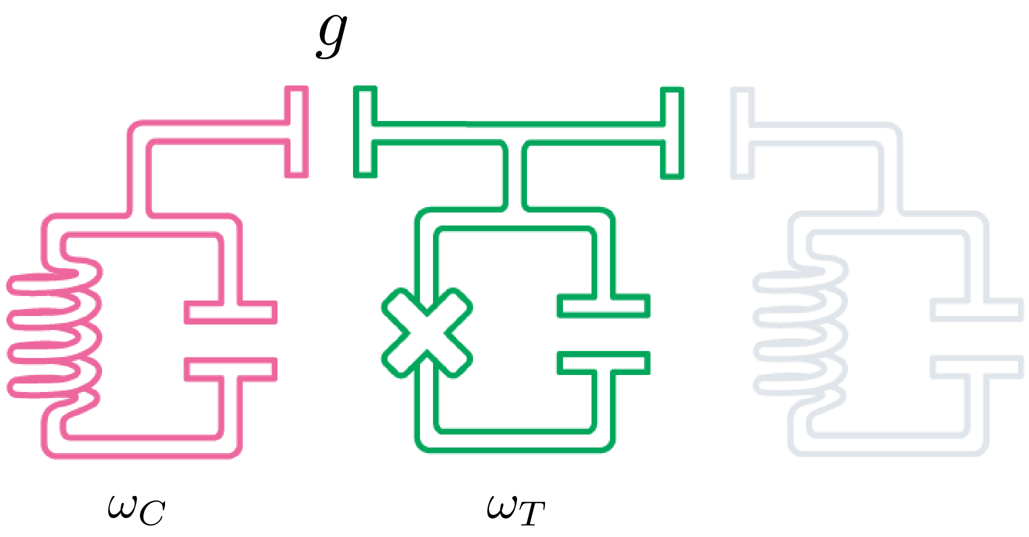
$$\Delta = |\omega_C - \omega_T|$$

Direct exchange of excitations, allowing:

1. Resonant control of light/atom
2. Direct mapping of information

Harness on-demand light-matter interaction





$$\Delta = |\omega_C - \omega_T|$$

Dispersive Interaction:

$$\boldsymbol{H} = -\frac{\chi}{2}\boldsymbol{a}^\dagger \boldsymbol{a}\boldsymbol{\sigma}_z$$

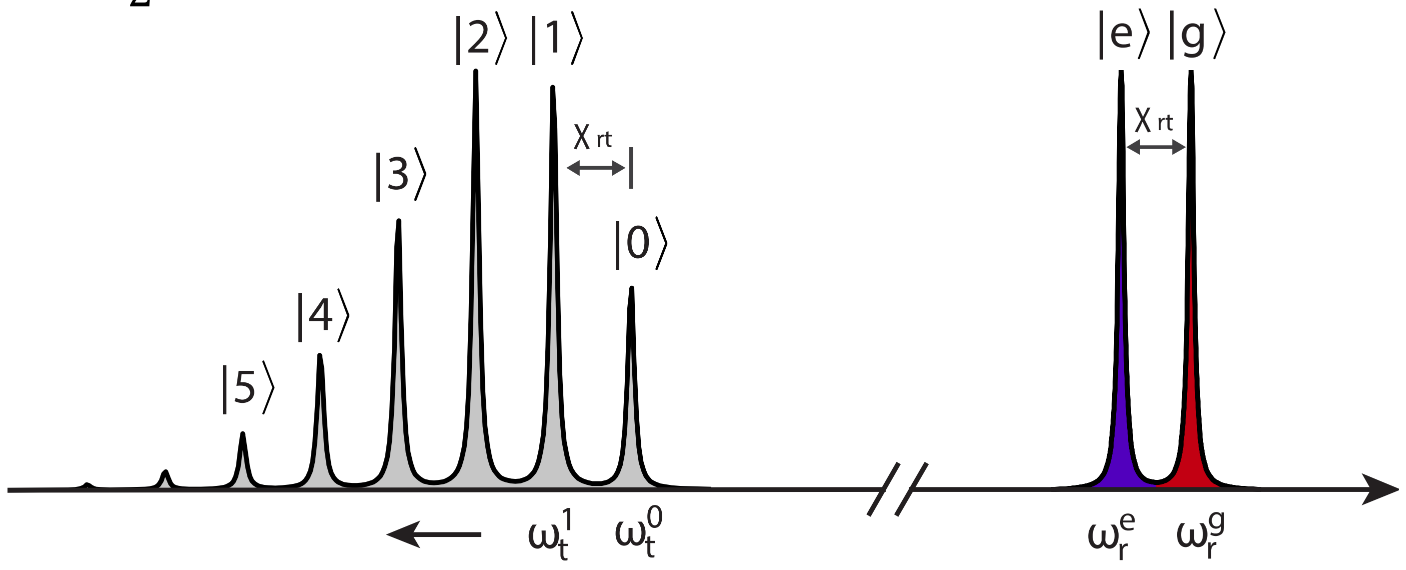
State-dependent frequency shift, enabling

1. extraction of information from atom
2. fast non-linear operations on light field

More on the dispersive coupling

1. Start with the system Hamiltonian
2. Go to interaction picture, find time evolution
3. Second order rotating wave approximation
4. Take RWA
5. Get new effective Hamiltonian under the limit $g \ll \Delta$

$$\boldsymbol{H} = -\frac{\chi}{2}\boldsymbol{a}^{\dagger}\boldsymbol{a}\boldsymbol{\sigma}_z$$



Transmon spectrum

cavity spectrum

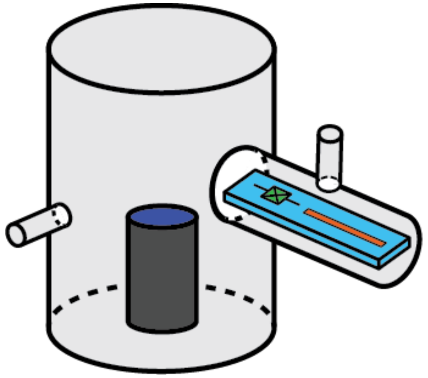
Why is bosonic cQED appealing?

Long-lived bosonic mode
Large, controlled non-linearity with tailored dynamics
Non-propagating modes makes coupling more effective

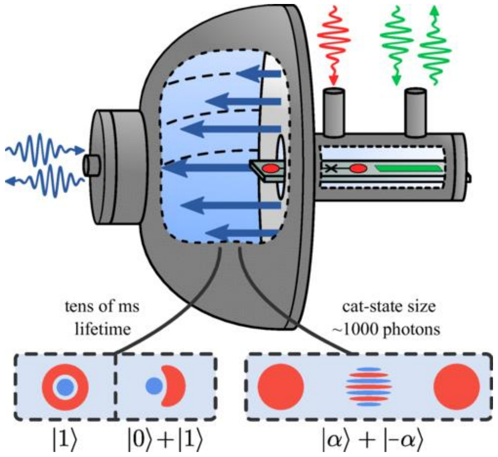
Bosonic vs Photonic?
(personal take)

Bosonic: microwave photons trapped in cavities, not propagating

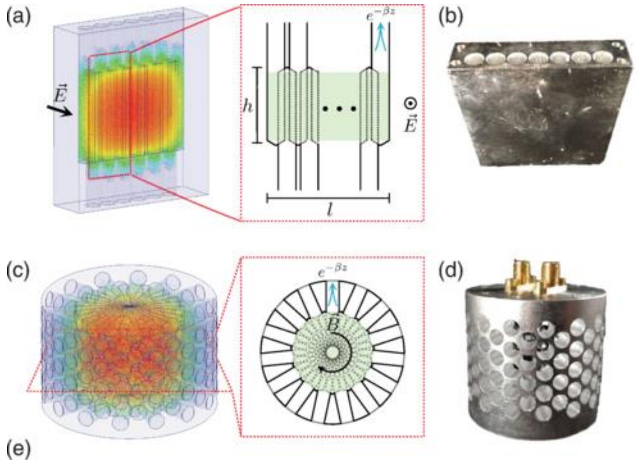
Photonic: optical/telecom photons that fly



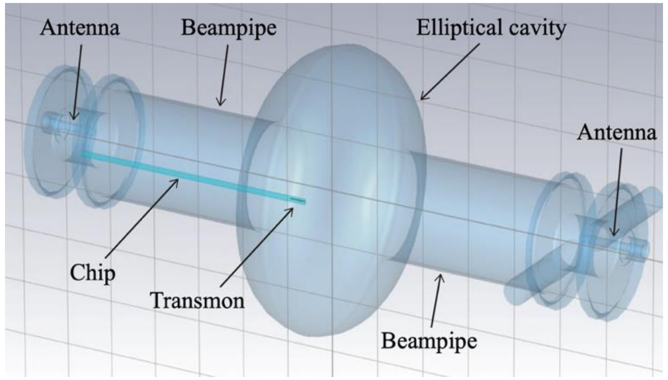
Yale, NUS, etc



Rosenblum group, Weizmann



Schuster group, Stanford



Fermilab

Plans for the lectures

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Useful states of light we can have in the bosonic mode

Coherent states (semiclassical states)

The displacement operator

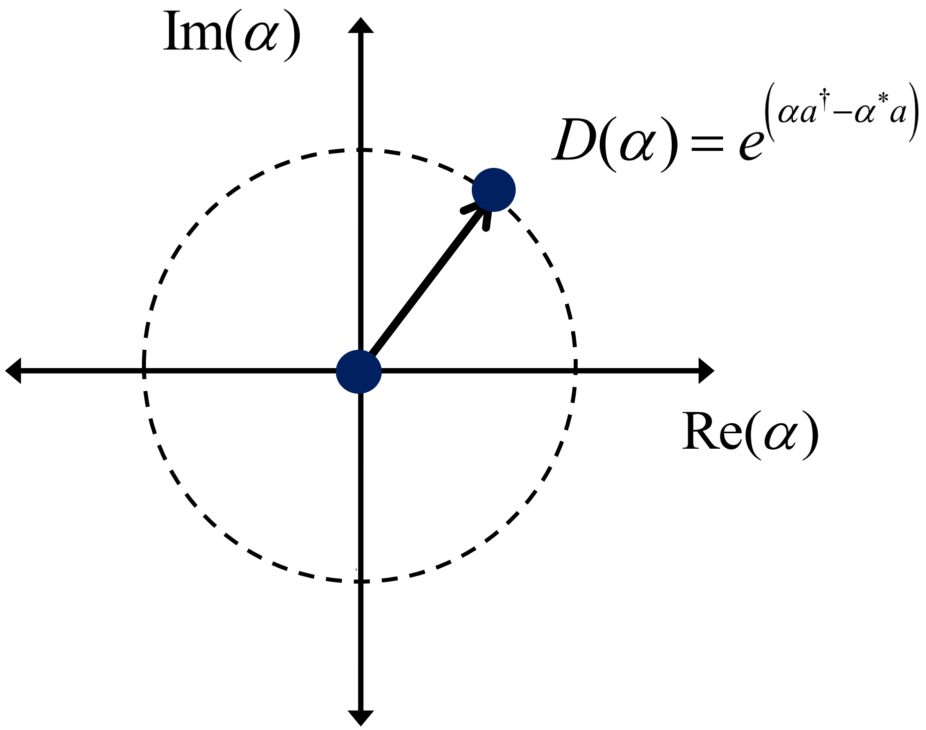
$$D(\alpha) = e^{\left(\alpha a^\dagger - \alpha^* a\right)} \longrightarrow D(\alpha)|0\rangle = |\alpha\rangle$$

Where:

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \qquad \alpha = |\alpha| e^{i\phi}$$

$$\langle I \rangle = |\alpha| \cos(\phi + \omega t) = \mathrm{Re}(\alpha)$$
$$\langle Q \rangle = |\alpha| \sin(\phi + \omega t) = \mathrm{Im}(\alpha)$$

Phase-space representation (IQ plane)



Different representations in phase space

Wigner Function $W(\alpha) = \frac{2}{\pi}\mathrm{Tr}[\hat{P}\hat{D}^\dagger(\alpha)\hat{\rho}\hat{D}(\alpha)]$

- Most native in cQED due to access to parity mapping
- Always real
- Negativity directly signifies non-Gaussianity

Gaussian convolution

2D Fourier transform

Husimi Q-Function

$$Q(\alpha) = \frac{1}{\pi}\langle\alpha|\hat{\rho}|\alpha\rangle = \mathrm{Tr}[\hat{\rho}|\alpha\rangle\langle\alpha|]$$

- Always positive, so no negativity
- Could be hard to visually identify if a state has quantum coherence
- Intuitive picture for Gaussian states

Characteristic function

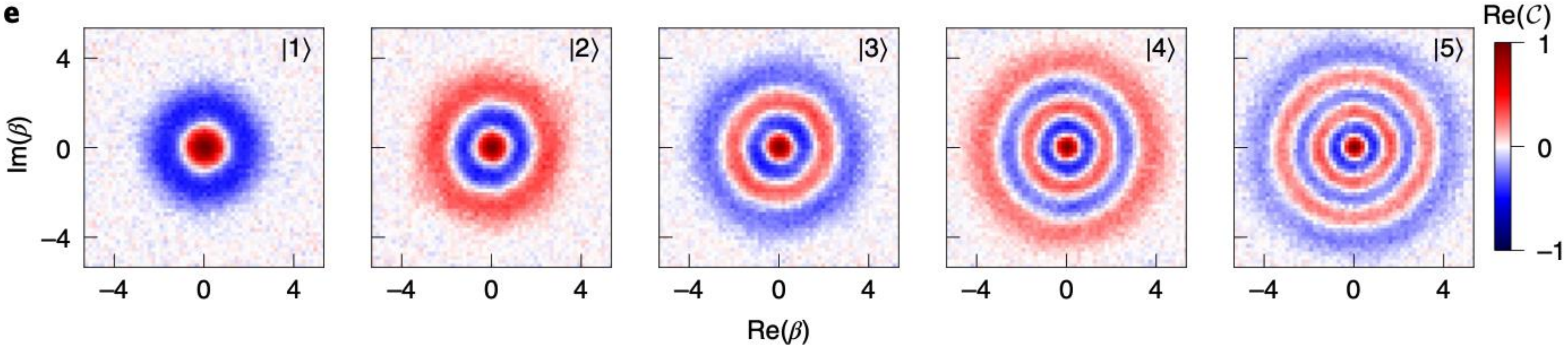
$$C(\alpha) = \mathrm{Tr}[\hat{D}(\alpha)\hat{\rho}]$$

- Need both real and imaginary components to capture the state
- Useful picture for investigating spectral properties of noise

Non–Gaussian states of light: Fock states

$$|\Psi\rangle_{\mathrm{Fock}} = |n\rangle$$

- Energy eigenstates of the oscillator
- Conceptually simple but rather hard to create in experiments
- minimum uncertainty in energy, maximum uncertainty in phase!



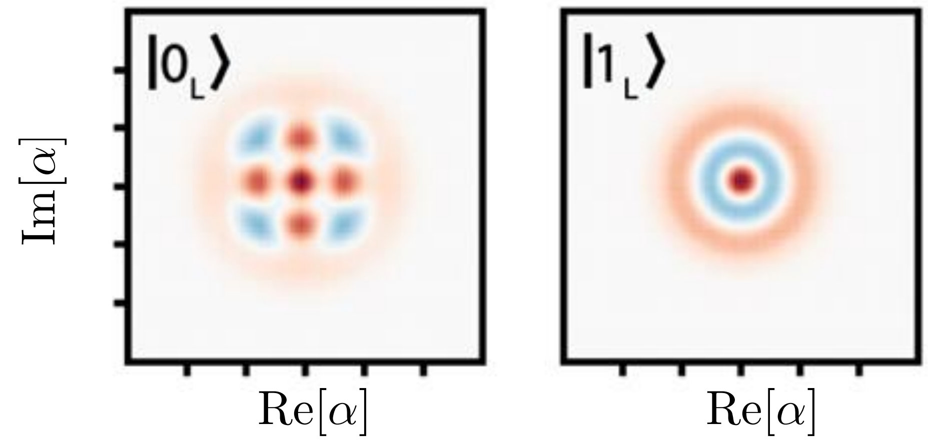
Eickbusch et al, Nature Physics (2022)

Non–Gaussian states of light: binomial states

$$|\eta, M\rangle = \sum_{n=0}^{M} \left[\binom{M}{n} \eta^n (1-\eta)^{M-n} \right]^{1/2} |n\rangle$$

- Superpositions of Fock states with binomial coefficients,
- in cQED it's often used as a code word for logical information
- Example of the smallest binomial state for QEC application

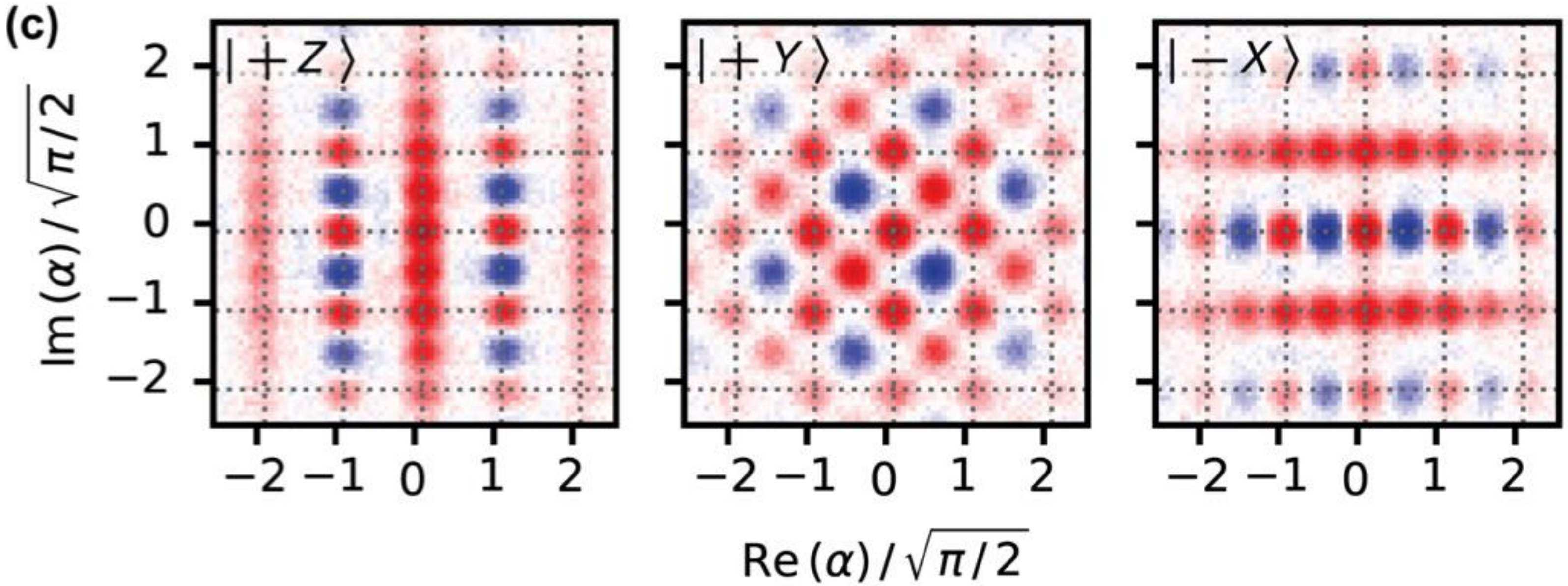
$$|W_\uparrow\rangle = \frac{|0\rangle + |4\rangle}{\sqrt{2}}, \qquad |W_\downarrow\rangle = |2\rangle.$$



First introduced in: Stoler et al Binomial states of the quantized radiation field. Journal of Optics, 32(3):345–355, 1985
Translation to QEC code: https://journals.aps.org/prx/pdf/10.1103/PhysRevX.6.031006

Non–Gaussian states of light: GKP states

- Simultaneous eigenstates of two displacement operators (many choices, as long as they are not colinear)
- Single photon loss translate into displacements
- Can be used to encode a qudit (both qubits and qudits shown experimentally)

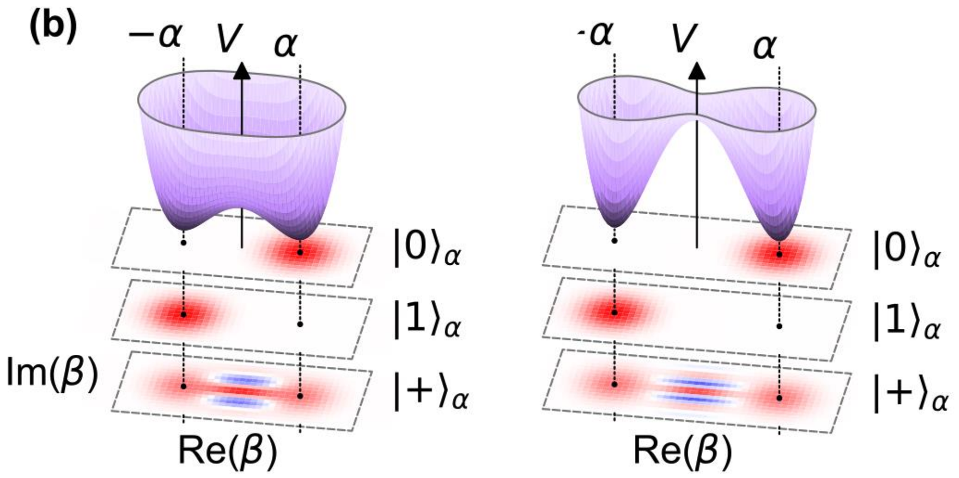


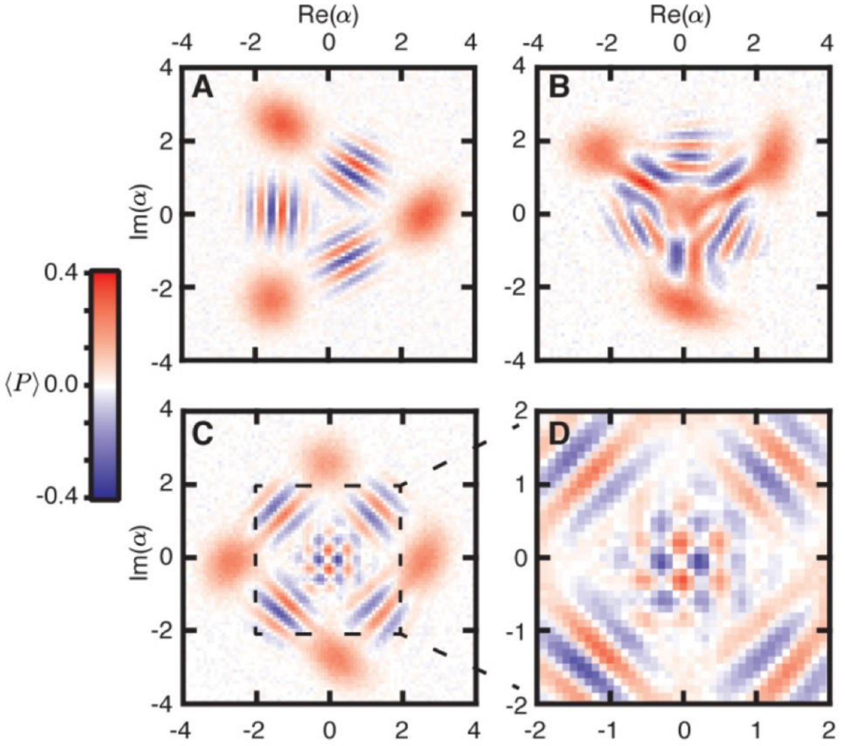
Sivek et al, Real-time quantum error correction beyond break-even, Nature 2023

Non–Gaussian states of light: cat states

$$|\Psi\rangle_{\mathrm{cat}} = \mathcal{N}(|\alpha\rangle + |\text{-}\alpha\rangle)$$

- Superpositions of coherent states
- Single photon loss leads to a parity flip
- Extends beyond just two components





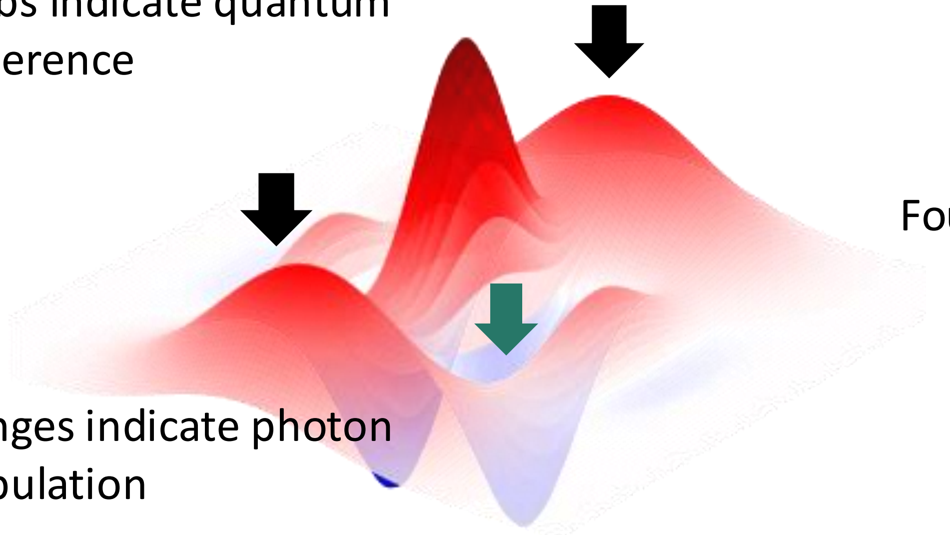
Lescanne, Nature Physics 2020; Vlastakis, Science 2013

Unique non-Gaussian features of bosonic states

$$|\Psi\rangle_{\text{cat}} = \mathcal{N}(|\alpha\rangle + |\text{-}\alpha\rangle)$$

Blobs indicate quantum coherence

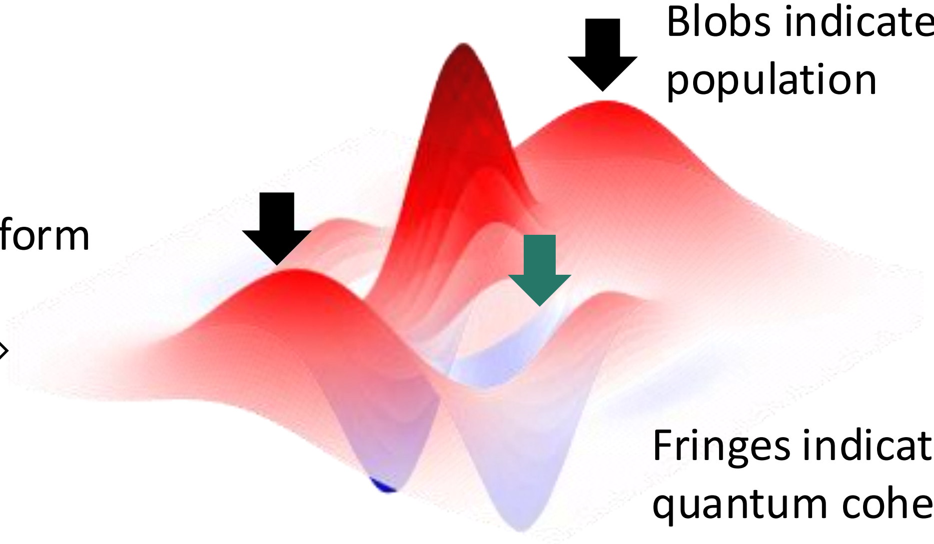
Fringes indicate photon population



Fourier transform

Blobs indicate photon population

Fringes indicate quantum coherence



We get valuable information and insights by looking (very hard) at the phase space

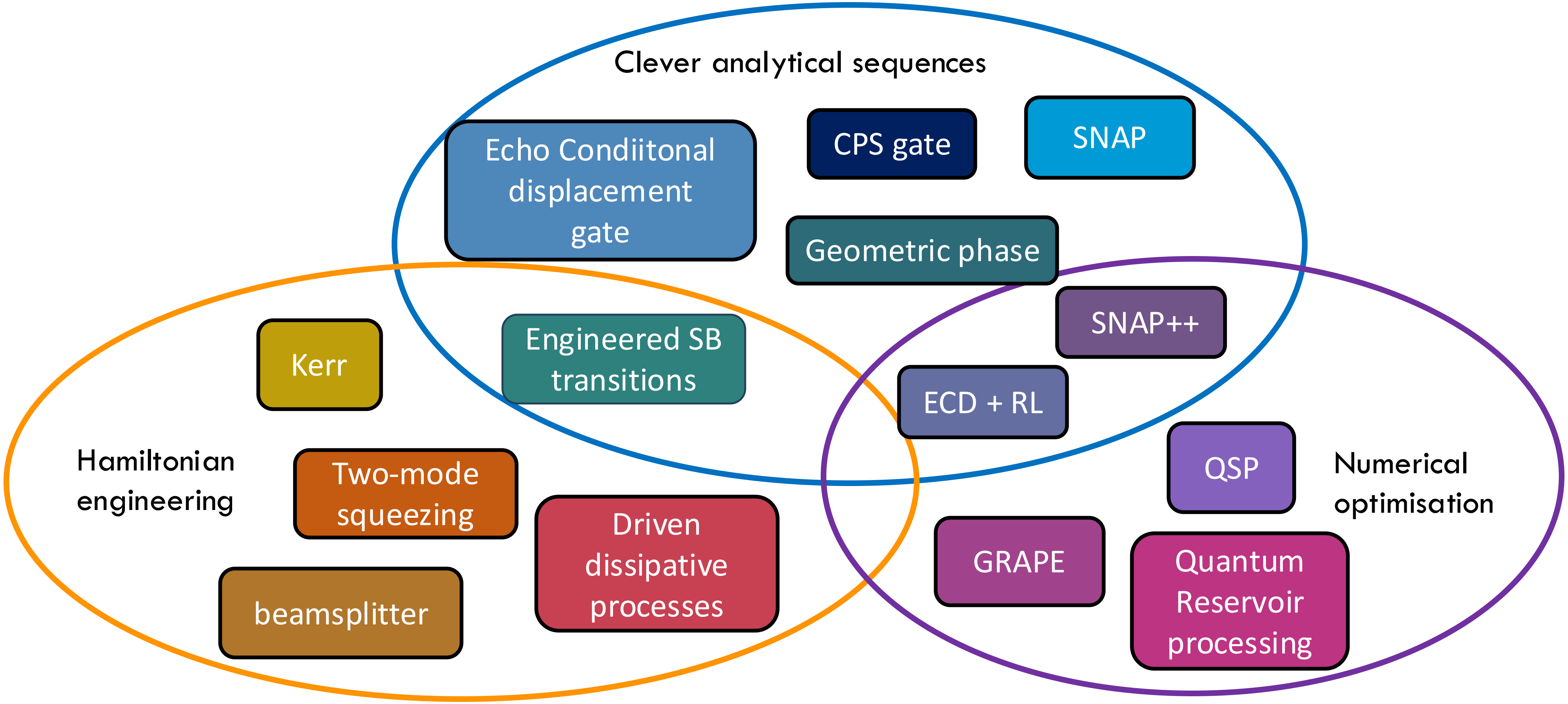
$\text{Re}[\langle\hat{D}(\nu)\rangle]$ -1 $+1$

characteristic function picture

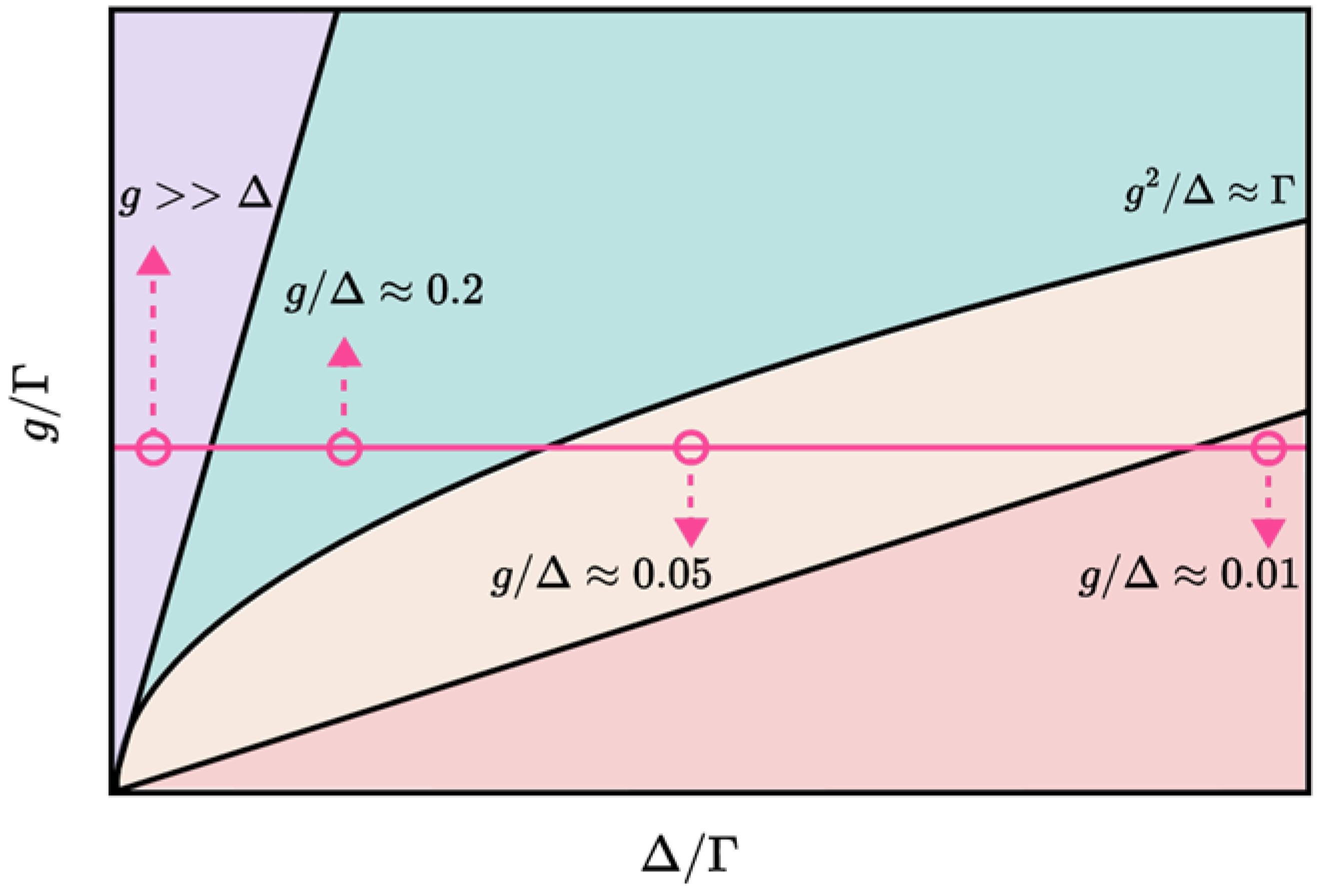
-1 $+1$ $\langle\hat{D}(\beta)\hat{P}\hat{D}(\text{-}\beta)\rangle$

Wigner function picture

Operations on bosonic states - an overview



Operations on bosonic states – different coupling regimes



Dispersive Interaction:

$$\boldsymbol{H} = -\frac{\chi}{2}\boldsymbol{a}^{\dagger}\boldsymbol{a}\boldsymbol{\sigma}_z$$

Strong dispersive

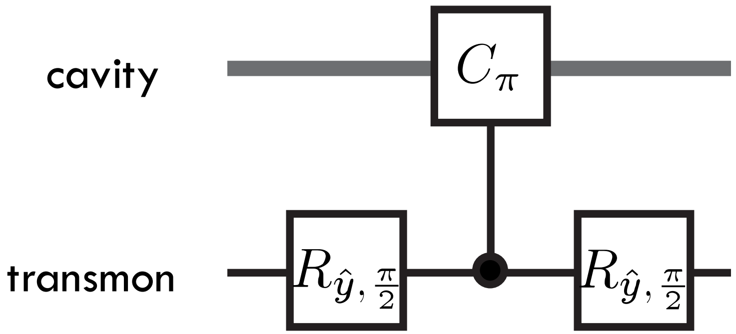
- Fast controlled phase shifts (CPS)
- Distortion due to higher order dynamics

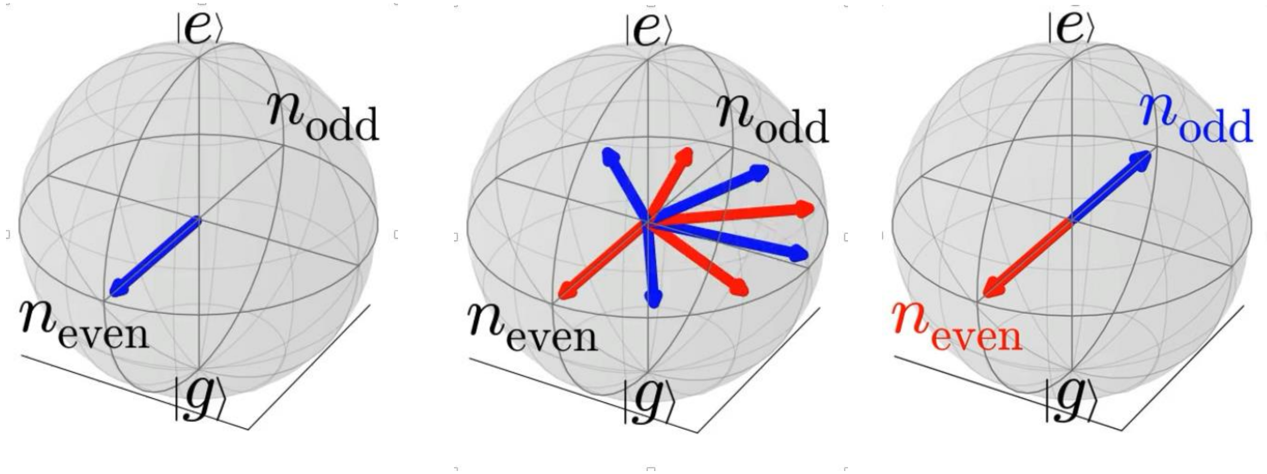
Weak dispersive

- Conditional displacement gates
- Suppression of spurious inherited non-linearity

Single cavity gate: controlled phase shifts on bosonic

$$H/\hbar = (\omega_q - \chi_s a^\dagger a) |e\rangle \langle e|$$





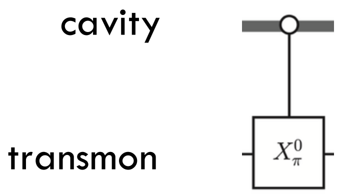
- Qubit state dependent cavity frequency shift
- Effective unitary operator $\hat{U} = e^{i\hat{a}^\dagger \hat{a}\theta}$
- Experimentally, it is similar to Ramsey interference
- Key part of parity mapping
- Simple to implement (just wait) but gate speed limited fully by χ

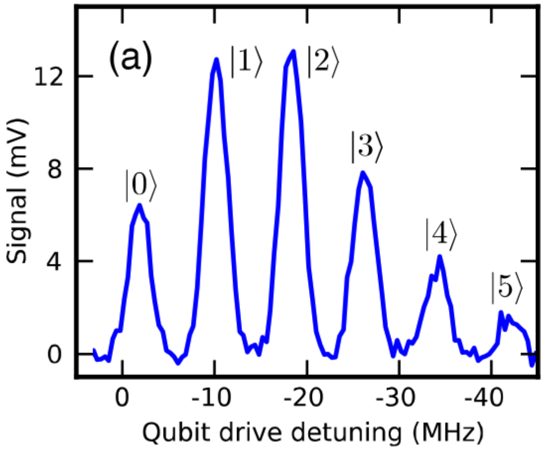
Single cavity gate: cavity photon dependent transmon rotation

$$H/\hbar = (\omega_q - \chi_s a^\dagger a) |e\rangle \langle e|$$

Selective pi-pulse on a qubit

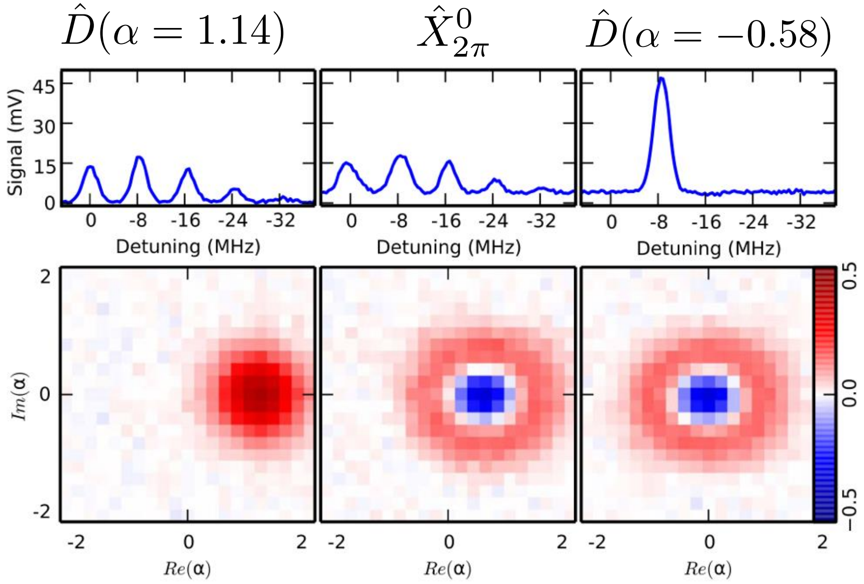
- Most common: flip the qubit if $n = 0$
- Useful for Q-function measurements





Selective number-dependent arbitrary phase (SNAP) gate

- a geometric phase applied **selectively** to the n-th Fock state via rotations on the transmon



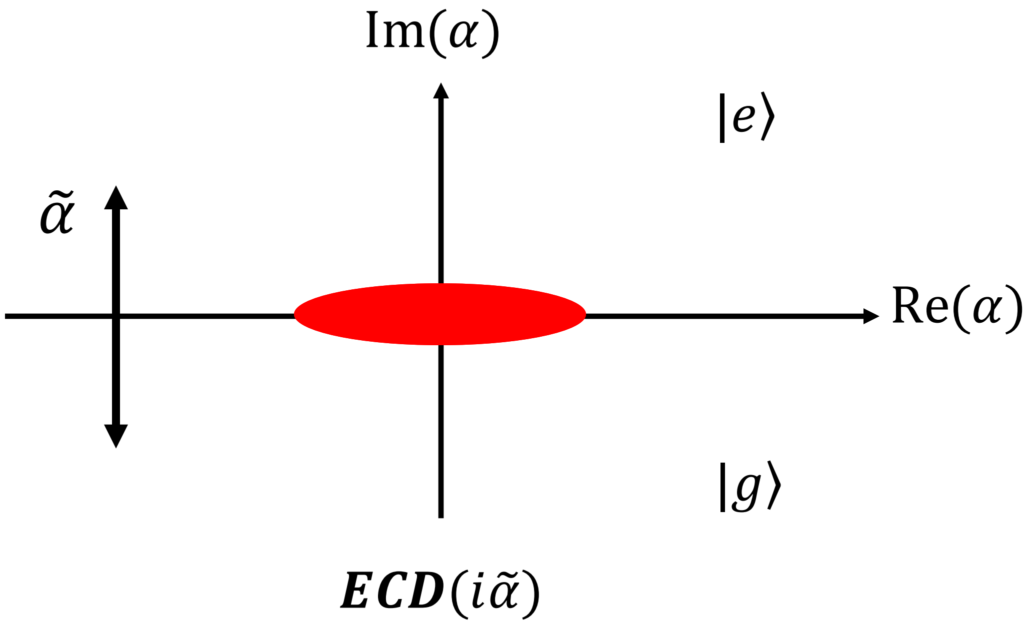
Heeres PRL 2015, Krastanov PRA 2015, …

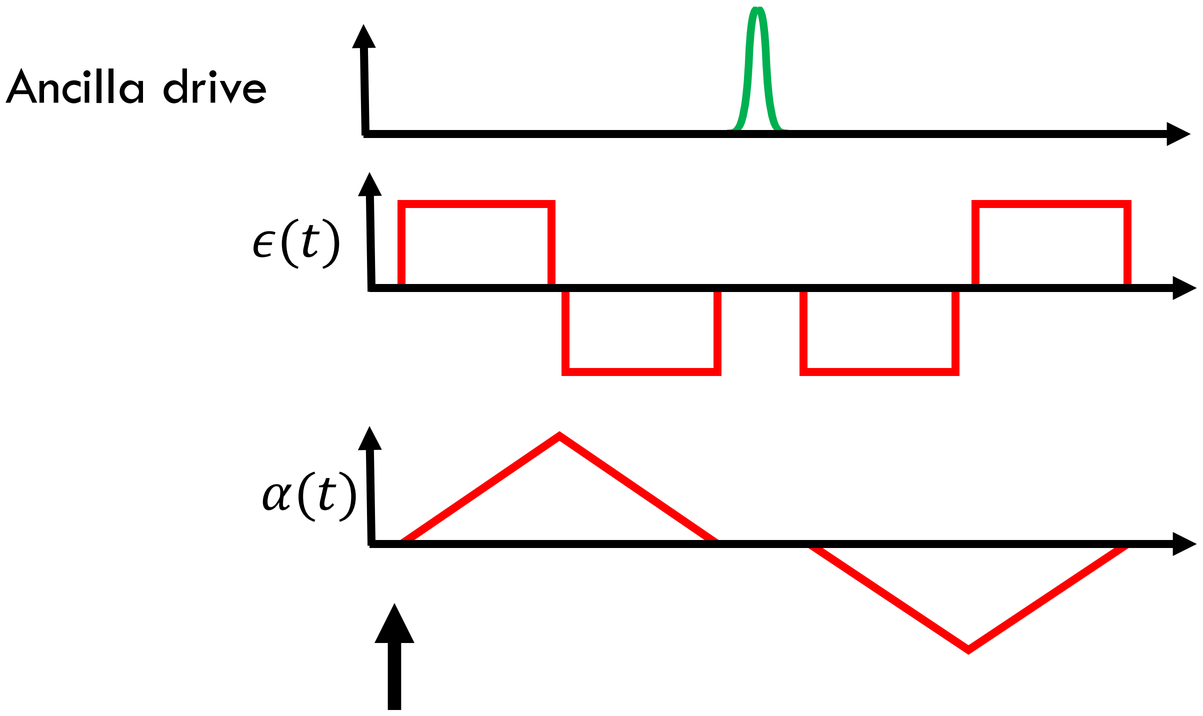
Single cavity gate: controlled displacements

$$\boldsymbol{H} = -\frac{\chi}{2}\boldsymbol{a}^\dagger\boldsymbol{a}\boldsymbol{\sigma}_z + \epsilon(t)\boldsymbol{a}^\dagger + \epsilon^*(t)\boldsymbol{a} \qquad \boldsymbol{a} \to \boldsymbol{a} + \alpha(t)$$

$$\widetilde{\boldsymbol{H}} = -\frac{\chi}{2}\boldsymbol{a}^\dagger\boldsymbol{a}\boldsymbol{\sigma}_z - \frac{\chi}{2}|\alpha(\mathrm{t})|^2\boldsymbol{\sigma}_z - \frac{\chi}{2}\left(\alpha(t)\boldsymbol{a}^\dagger + \alpha^*(t)\boldsymbol{a}\right)\boldsymbol{\sigma}_z$$

Conditional Displacement





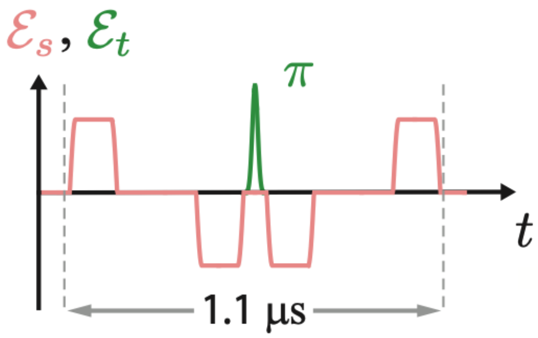
Viola *et al.*, PRL (1999), Eddins *et al.*, PRL (2018)

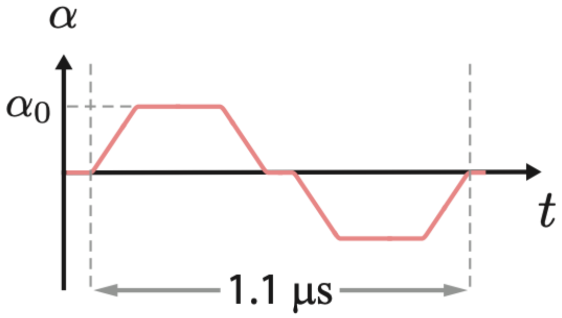
Single cavity gate: controlled displacements

$$\mathrm{CD} = \hat{D}(\alpha)|e\rangle\langle e| + \hat{D}(\text{-}\alpha)|g\rangle\langle g|$$

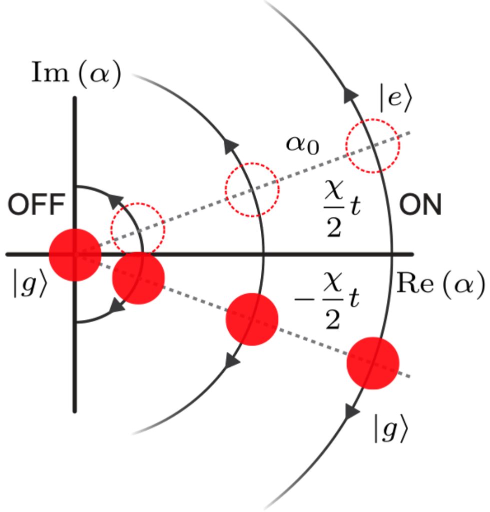
$$\mathrm{ECD} = \hat{D}(\alpha)|e\rangle\langle g| + \hat{D}(\text{-}\alpha)|g\rangle\langle e|$$

- Key component for making GKP states, cats, and offers universal control
- Requires large photon population during the gate
- Need small dispersive shift, minimal higher order nonlinearity

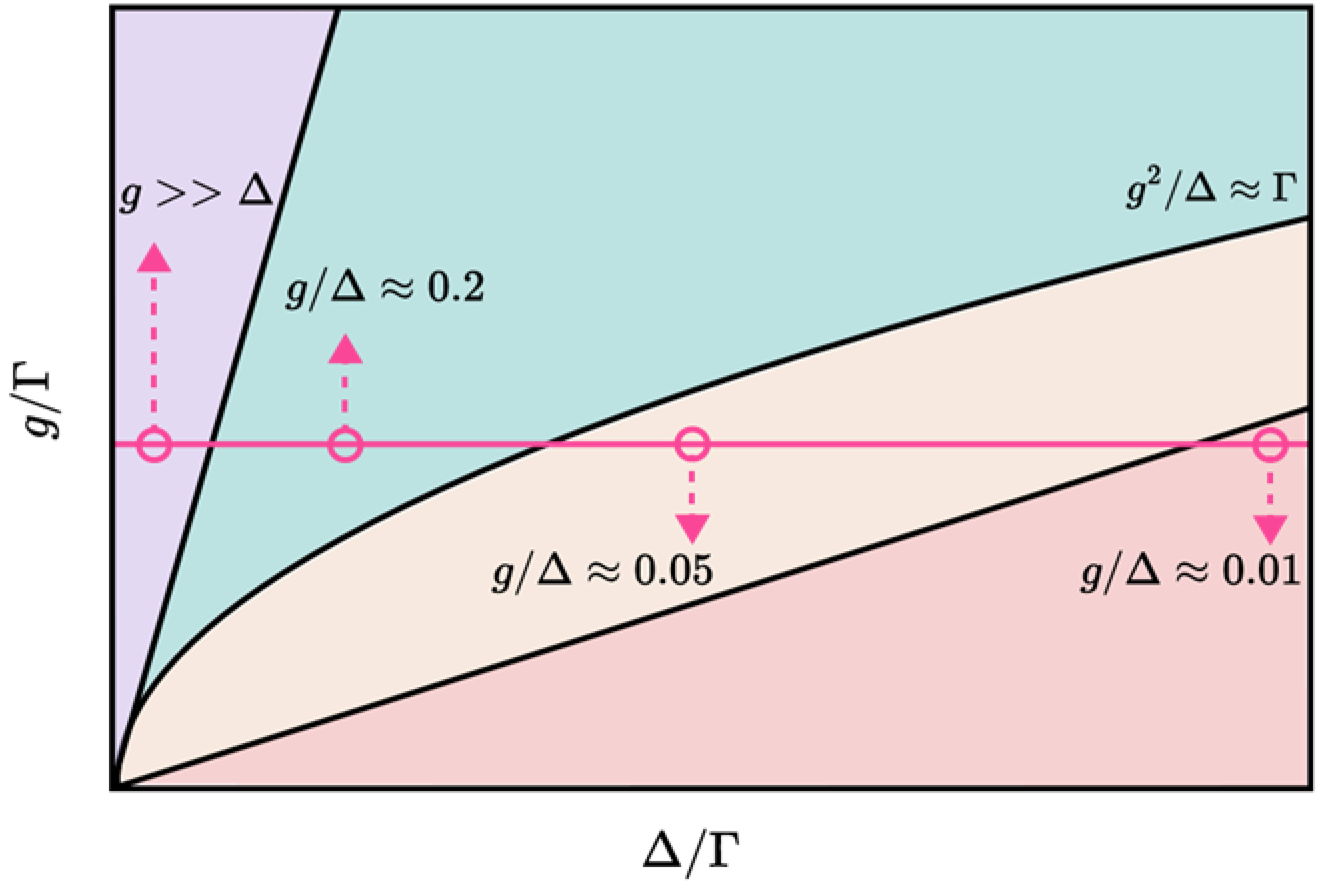


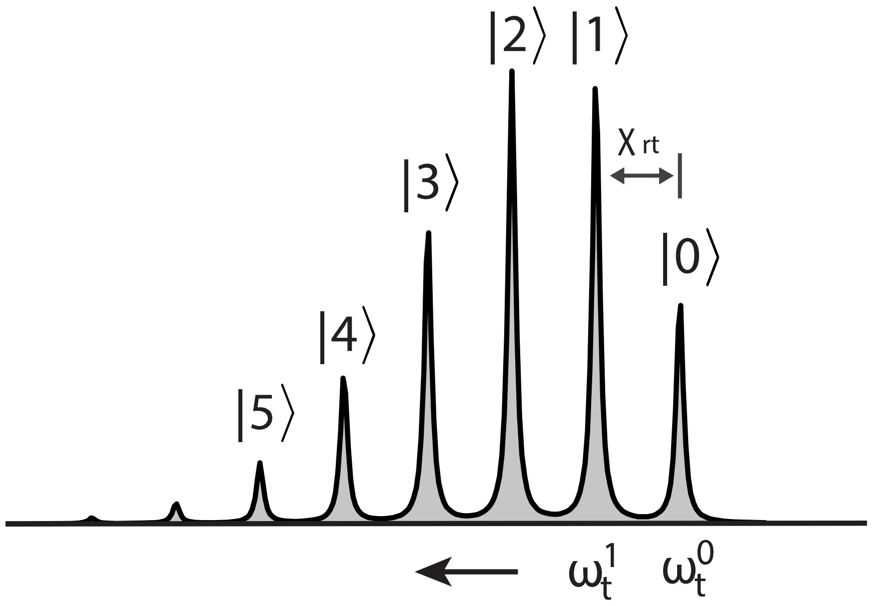


$|\alpha_0|^2 \sim 1000$ photons



How to make practical choices on which operations to use?

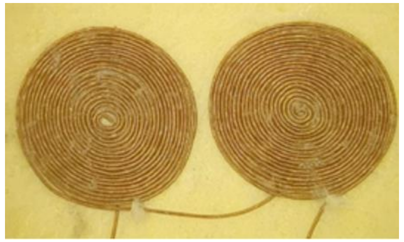


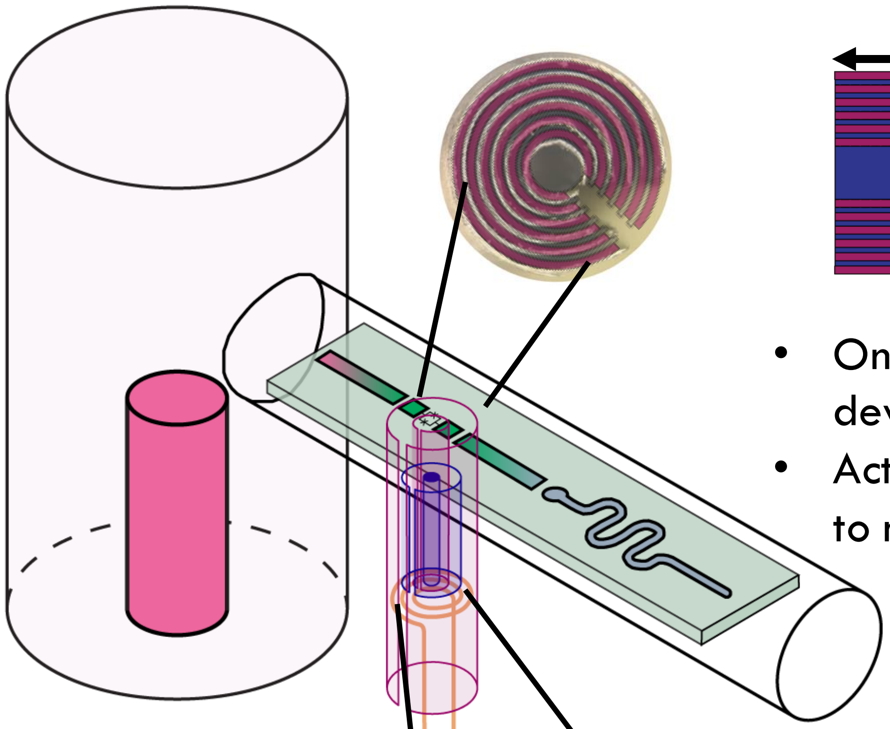


Achieve tunable coupling via a magnetic hose

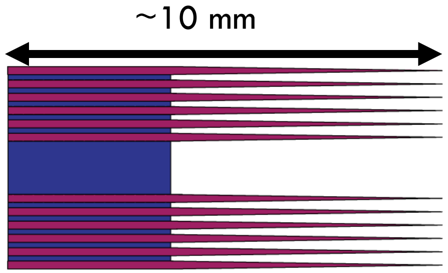
- Cavity T_1 :150-250 µs
- Effective field ~ 80 nT/mA
- DC tunability: ~ 1 flux quanta
- AC flux bandwidth: ~ 100 MHz

- a double-layer coil
- ~20 turns each
- Epoxied to a dielectric handle and thermally anchored to the main device with a clamp





3mm

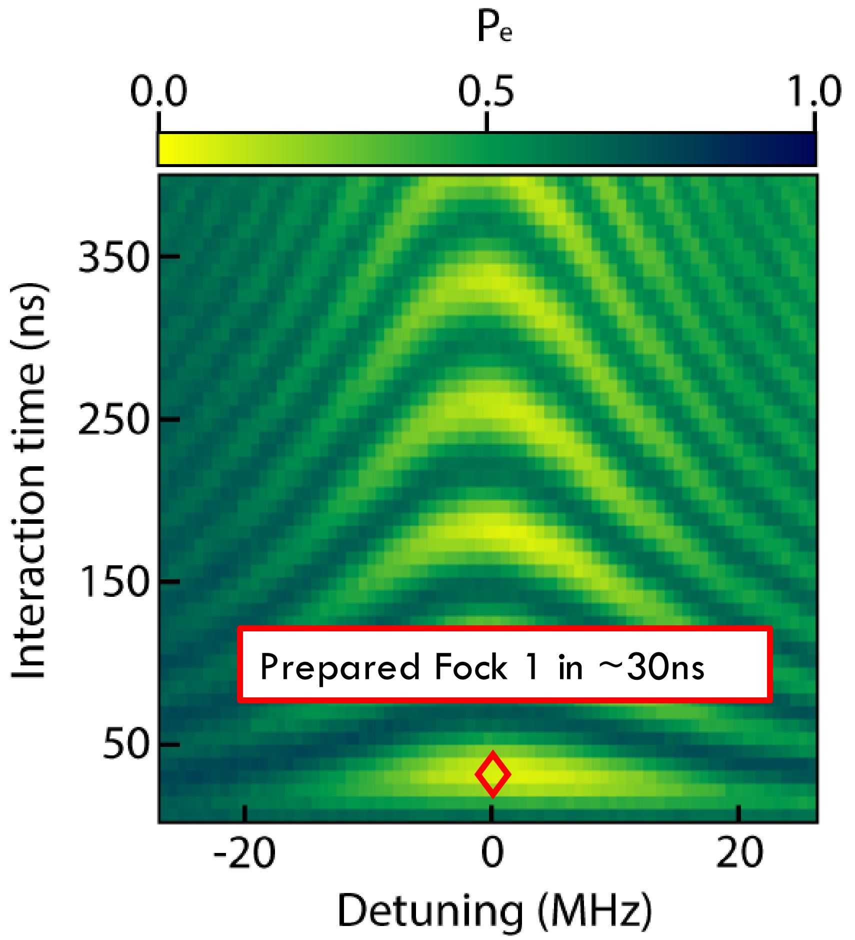


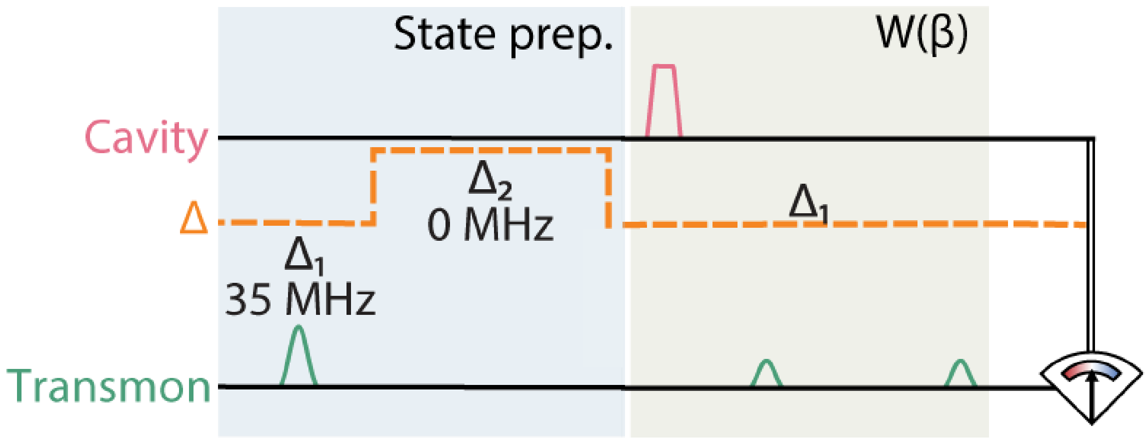
- Only Al layers protrude into device enclosure
- Acts as filter to reduce loss due to m-metal layers

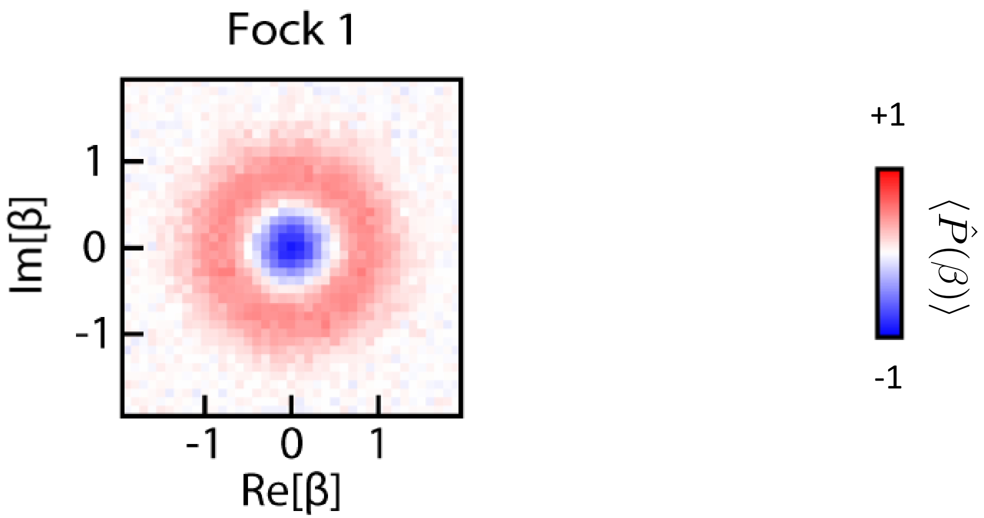
- 6 aluminium layers (0.15mm)
- 5 m-metal layers (0.1mm)
- cut prevents formation of supercurrent loops

Valadares, Nature Comm (2025), inspired by Gargiulo et al, APL (2021); Navau et al, PRL (2014)

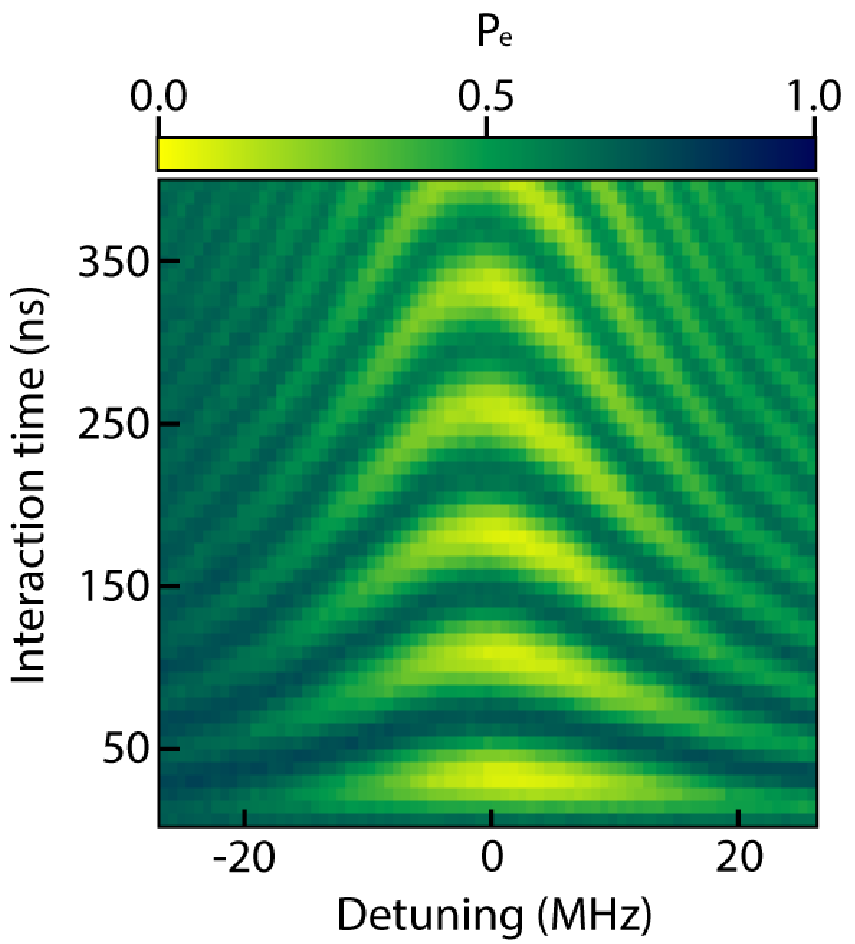
From resonant control to dispersive interactions

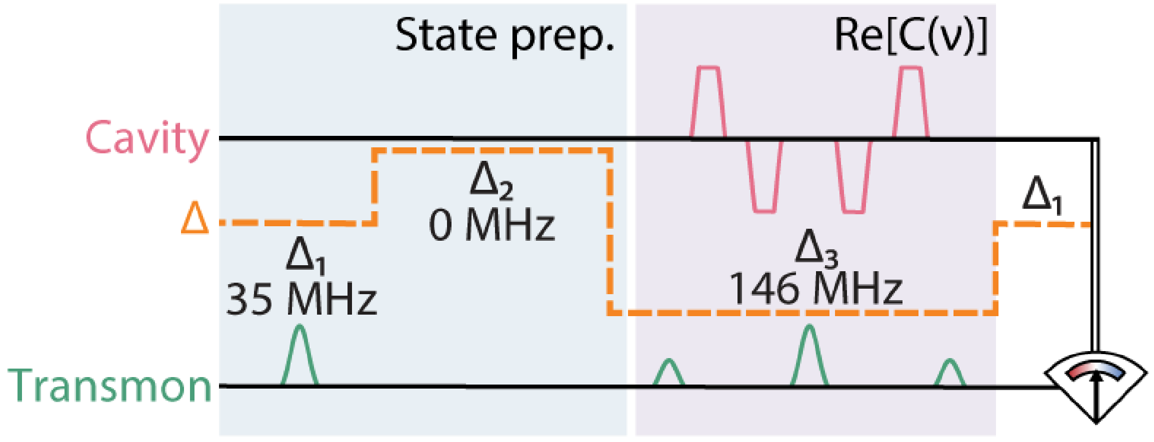


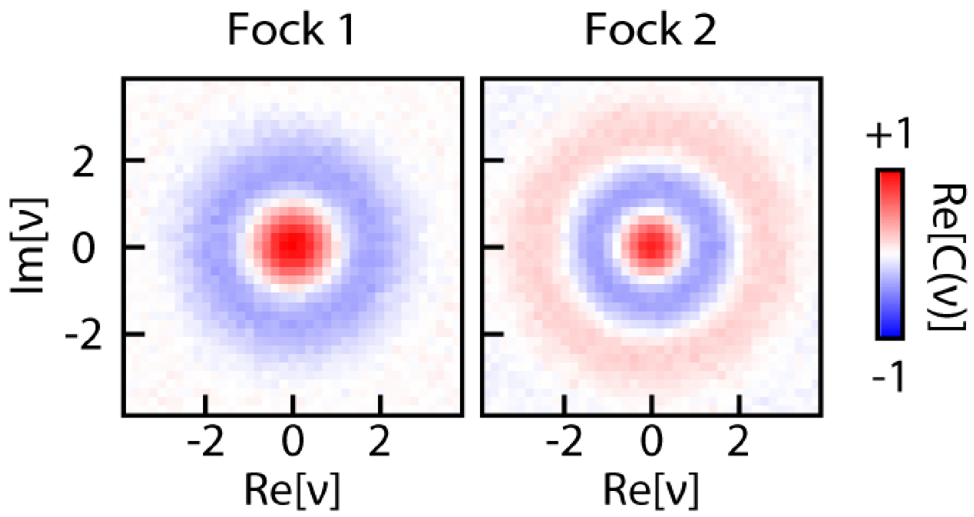




From resonant control to dispersive interactions

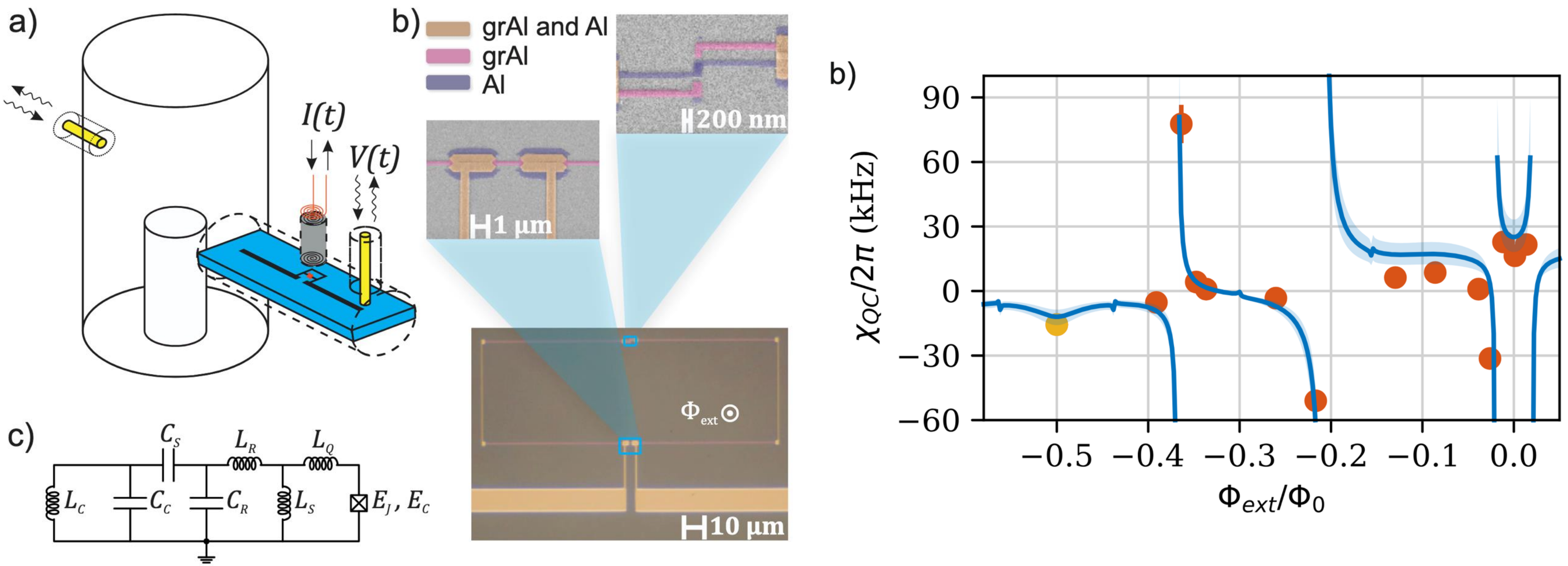






Valadares, Nature communications (2024)

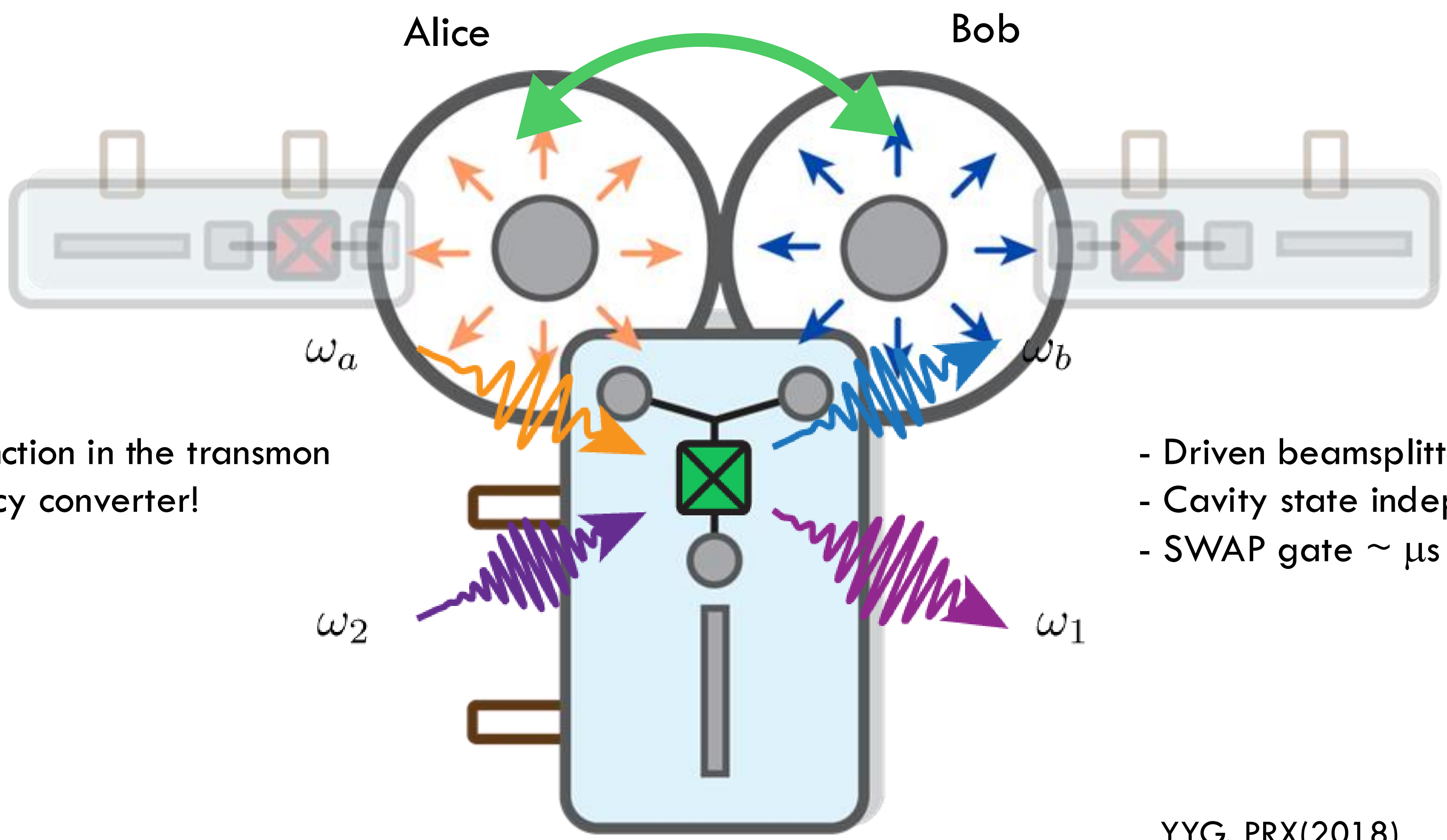
Tunable coupling between high-Q cavity and fluxonium



https://arxiv.org/pdf/2409.07612

Two cavity gate: a bosonic beamsplitter

Detuned excitations do not mix!

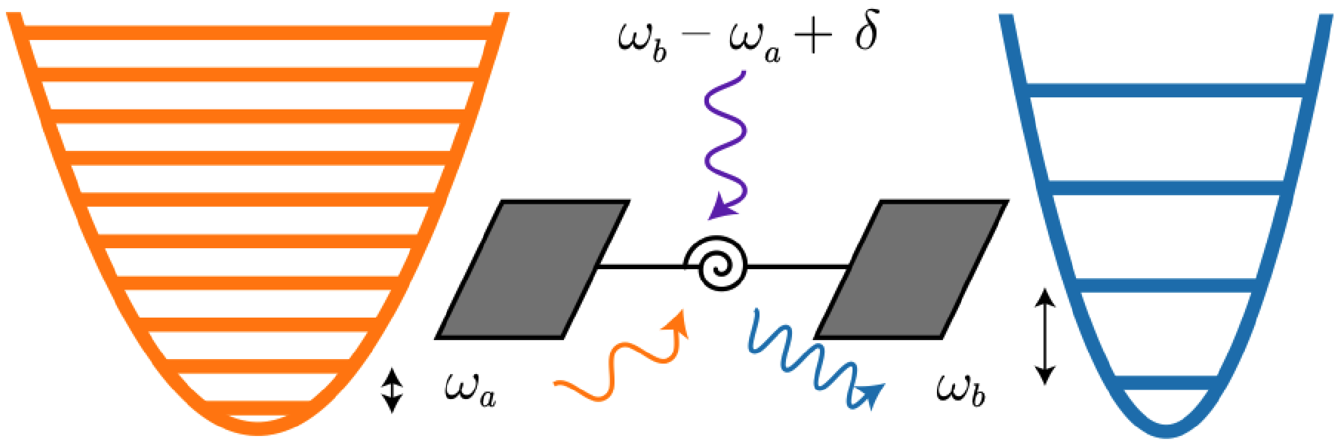


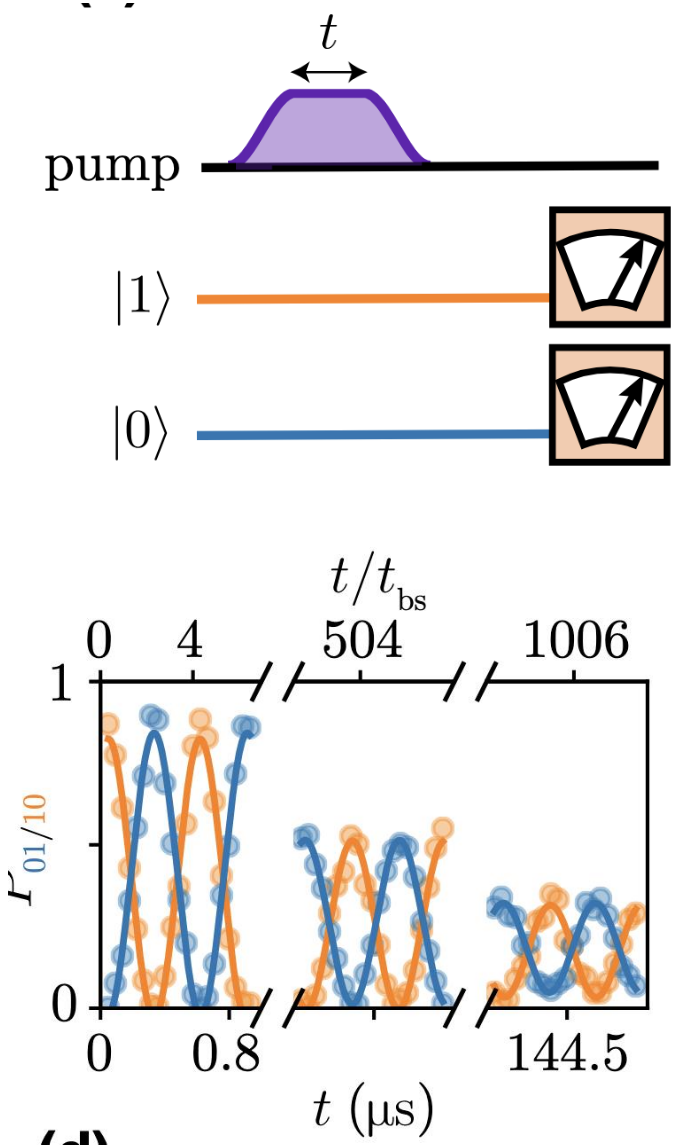
Josephson junction in the transmon
as a frequency converter!

- Driven beamsplitter
- Cavity state independent
- SWAP gate ~ µs

YYG, PRX(2018)

SNAIL-mediated beamsplitter

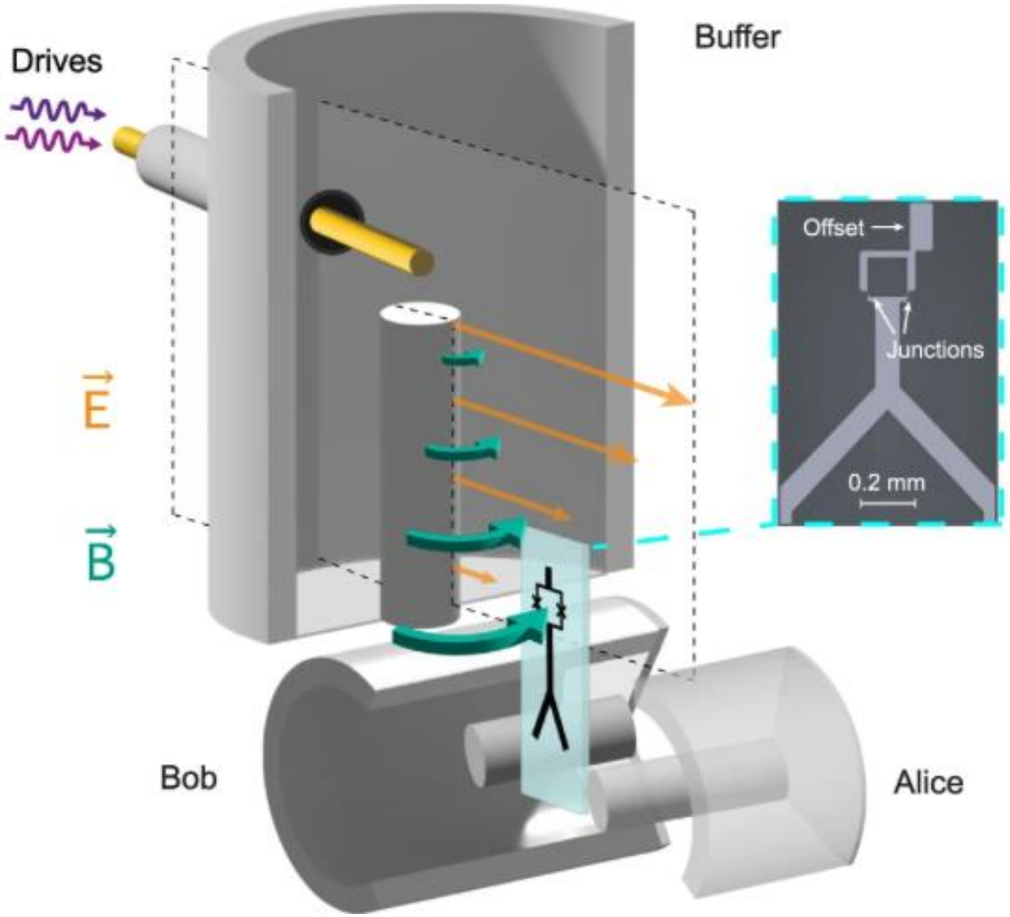


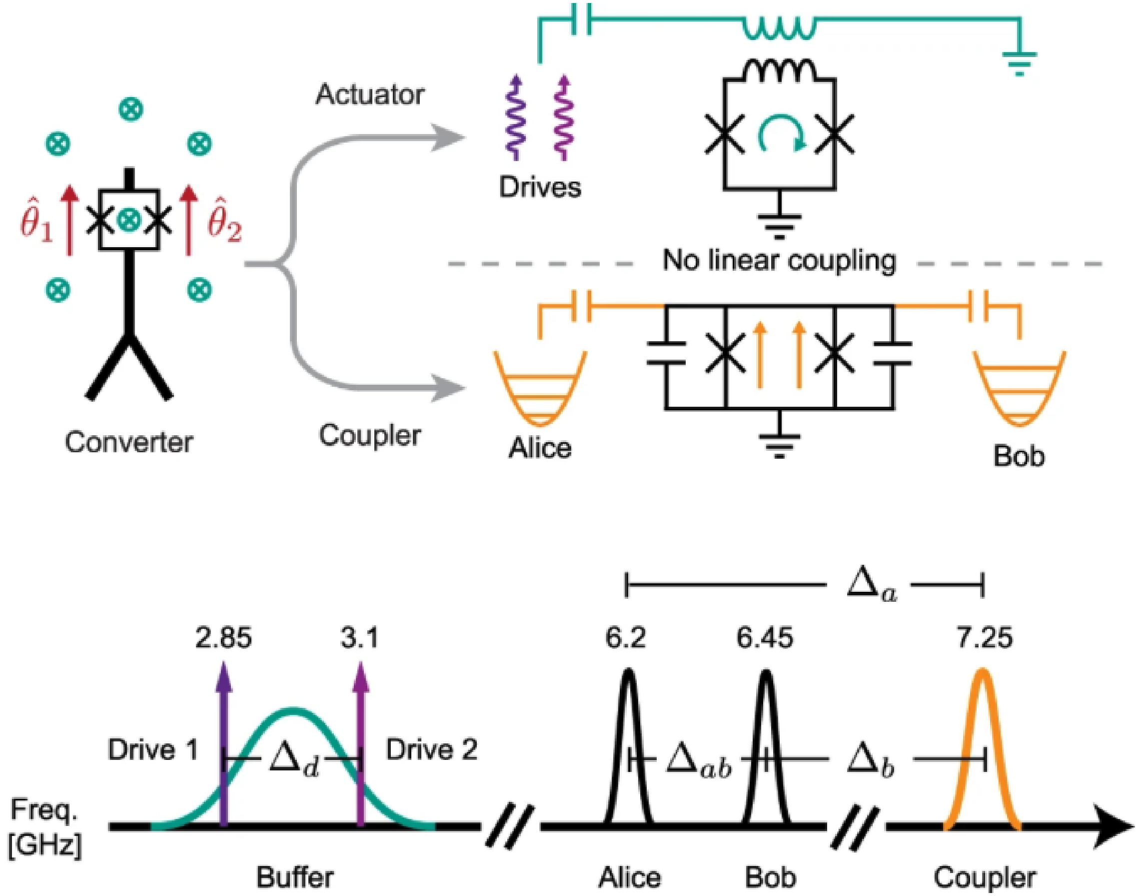


Use SNAIL as a three-wave mixing element

Faster beamsplitting with minimal unwanted higher order terms

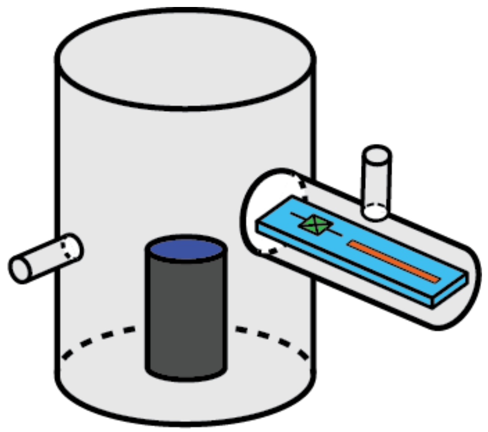
Flux-pumped beamsplitter



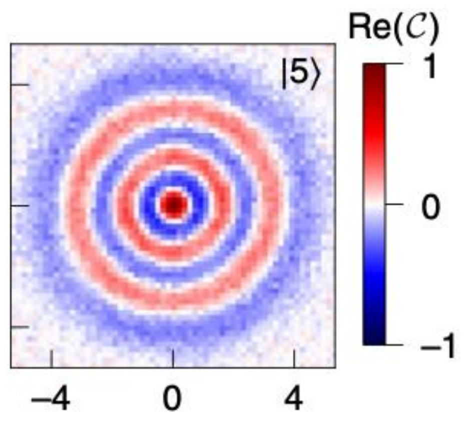


https://www.nature.com/articles/s41467-023-41104-0

Summary for today



Simple bosonic cQED devices require
- a good, relatively long-lived quantum harmonic oscillator
- a non-linear element with good control and readout
- a sensible choice of coupling parameters based on the task



- Quantum resource states can be created on-demand in the oscillator
- They are controlled via engineered dynamics achieved by driving physical or virtual transitions on the nonlinear element
- Bosonic gates are slow but lifetime is also long

Plans for tomorrow

1. Brief introduction to cQED
2. What goes into a minimal bosonic cQED hardware module
3. Bosonic cQED as a playground for good old quantum optics concepts
   - States
   - Gates
   - **Measurements**
4. **Leverage bosonic cQED devices for quantum information processing**
   - **Continuous-variable logical qubits encoded in bosonic modes**
   - **Non-Gaussian bosonic resources for simulation and metrology**
5. **Looking ahead: challenges and exciting developments**