High-Impedance Circuits, Superinductances and Fluxonium Qubits

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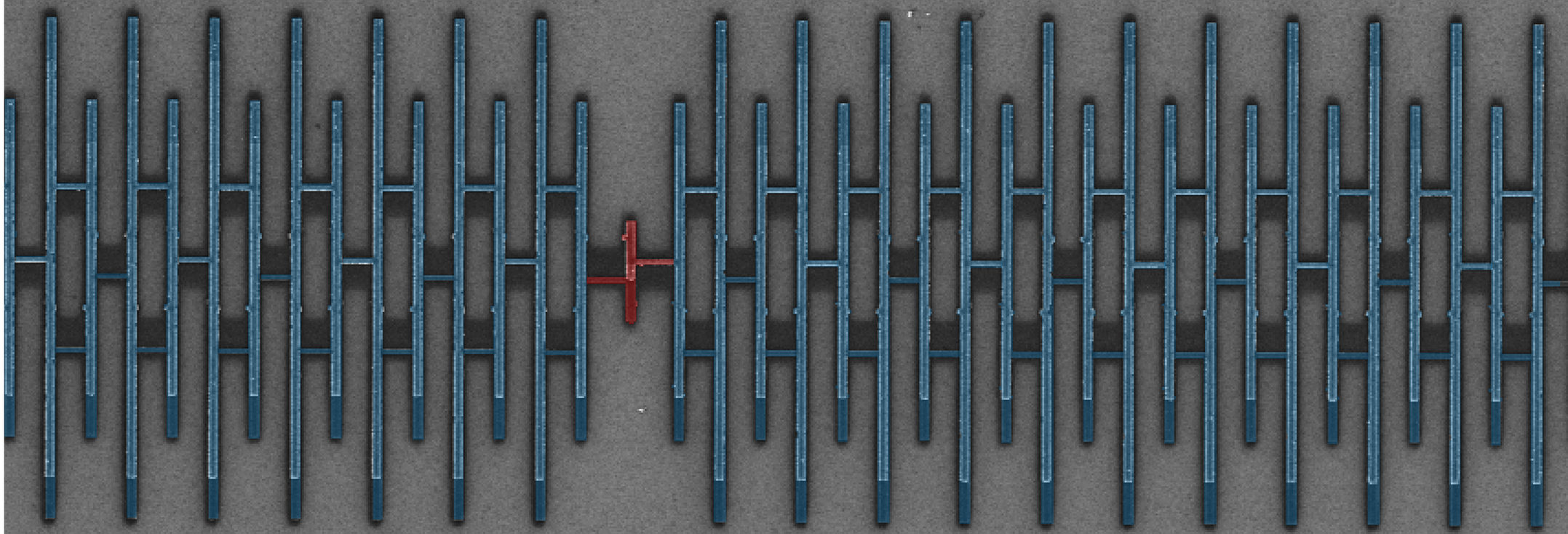
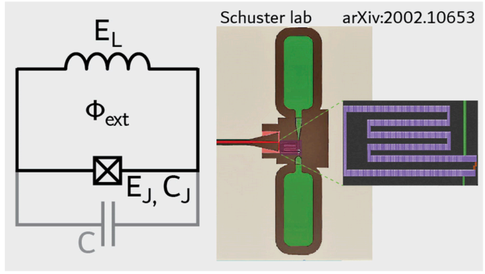


2025 Spring School on
Superconducting Qubit Technology

Diagonalising the Fluxonium Hamiltonian: scQubits

Fluxonium



The Hamiltonian of the fluxonium qubit [Manucharyan2009] in phase basis representation is given by

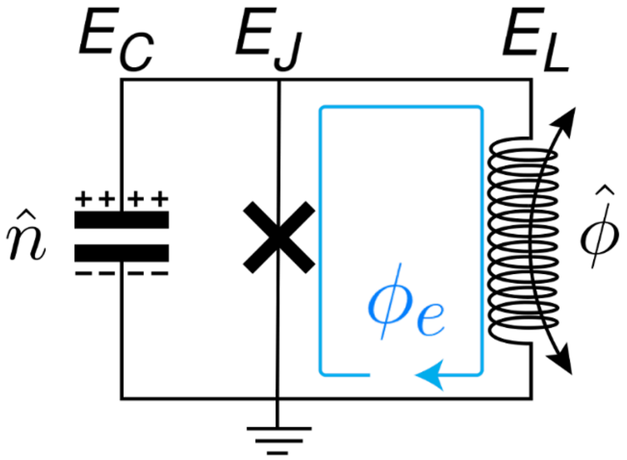
$$H = -4E_C\partial_\phi^2 - E_J\cos(\phi - \varphi_{ext}) + \frac{1}{2}E_L\phi^2.$$

Here, E_C is the charging energy, E_J the Josephson energy, E_L the inductive energy, and $\varphi_{ext} = 2\pi\Phi_{ext}/\Phi_0$ the external flux in dimensionless form. The **Fluxonium** class internally uses the E_C-E_L harmonic-oscillator basis [Zhu2013] with truncation level specified by **cutoff**.

An instance of the fluxonium qubit is created as follows:

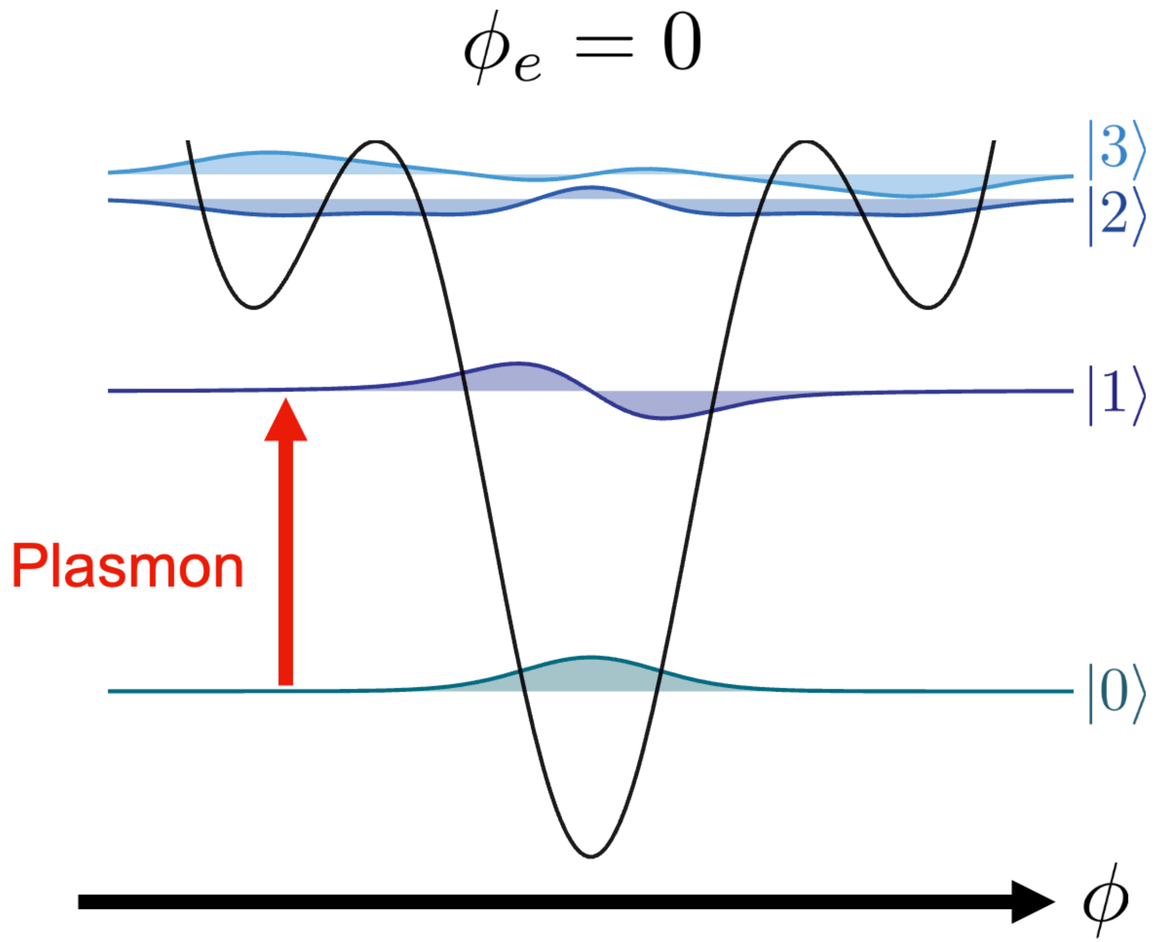
```
fluxonium = scqubits.Fluxonium(EJ = 8.9,
                               EC = 2.5,
                               EL = 0.5,
                               flux = 0.33,
                               cutoff = 110)
```

Fluxonium: standard parameters



$$E_L \ll E_J$$
$$1 \lesssim E_J/E_C \lesssim 10$$

$$E_C/h = 1.08 \text{ GHz}$$
$$E_L/h = 0.64 \text{ GHz}$$
$$E_J/h = 5.57 \text{ GHz}$$



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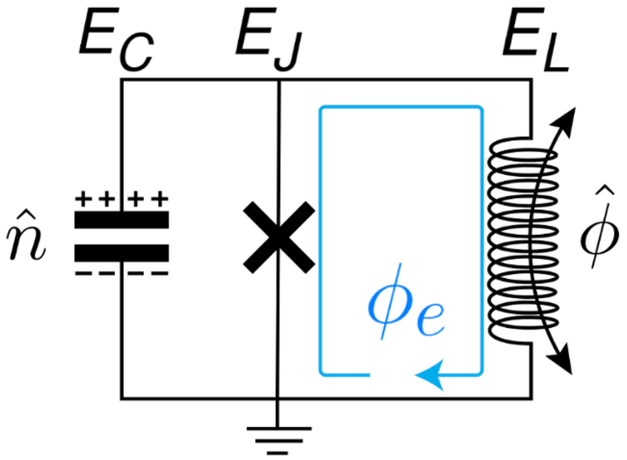

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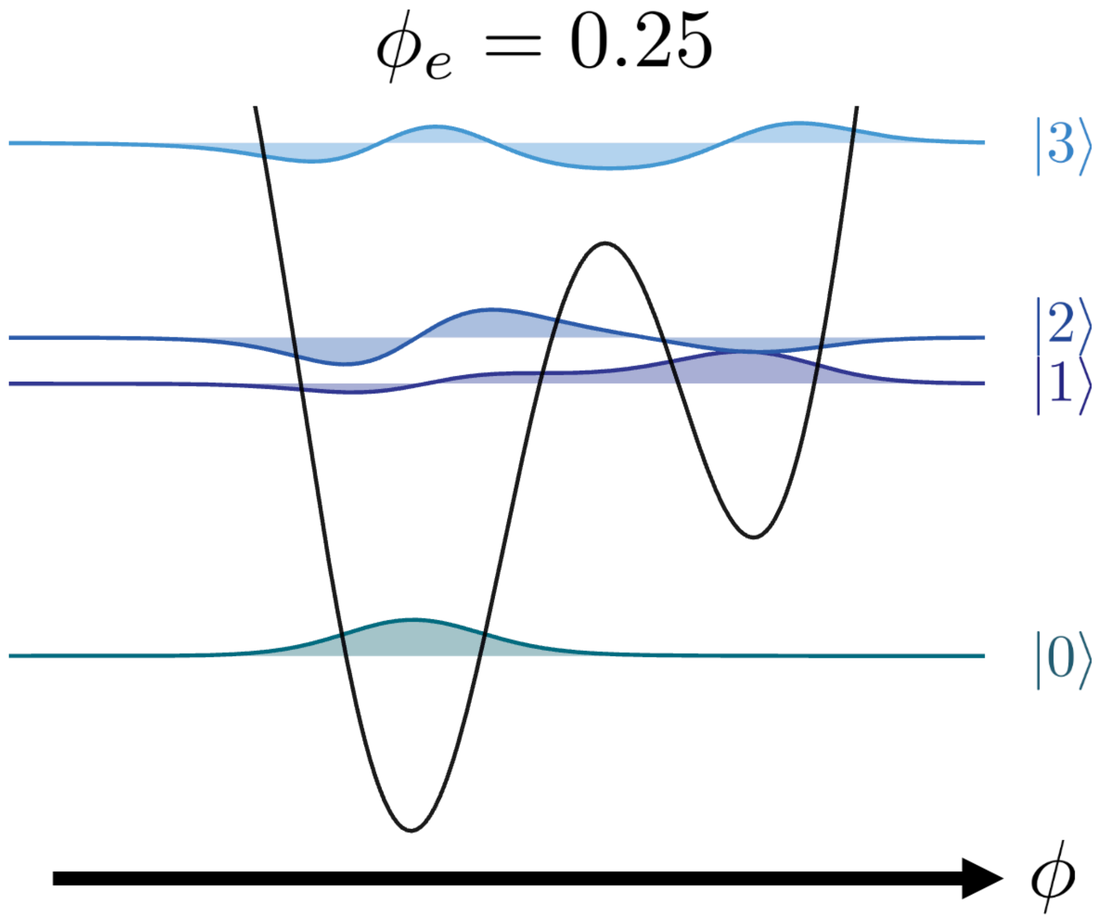

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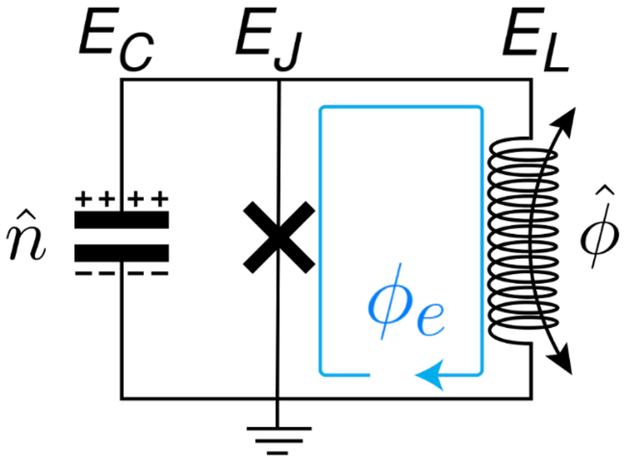

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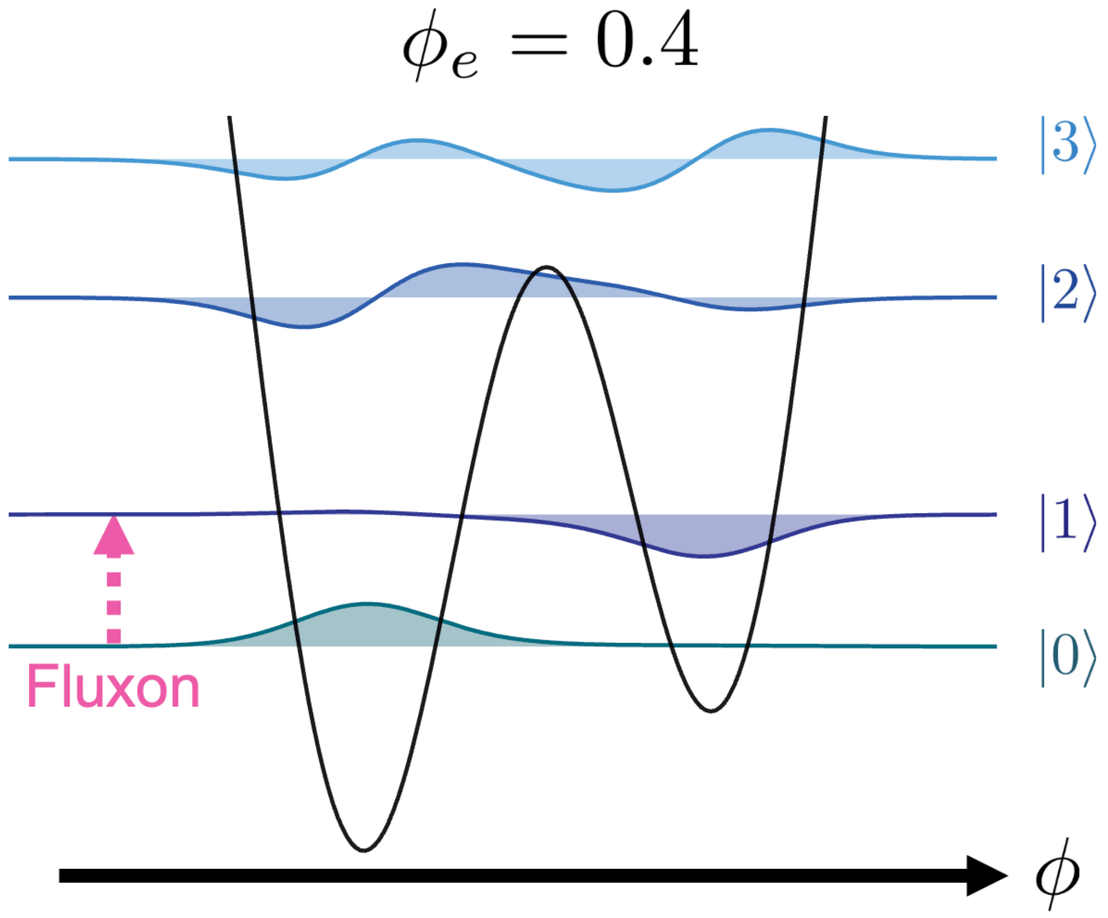

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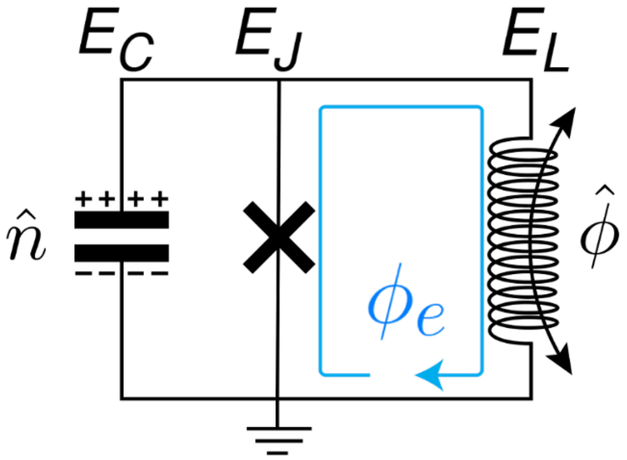

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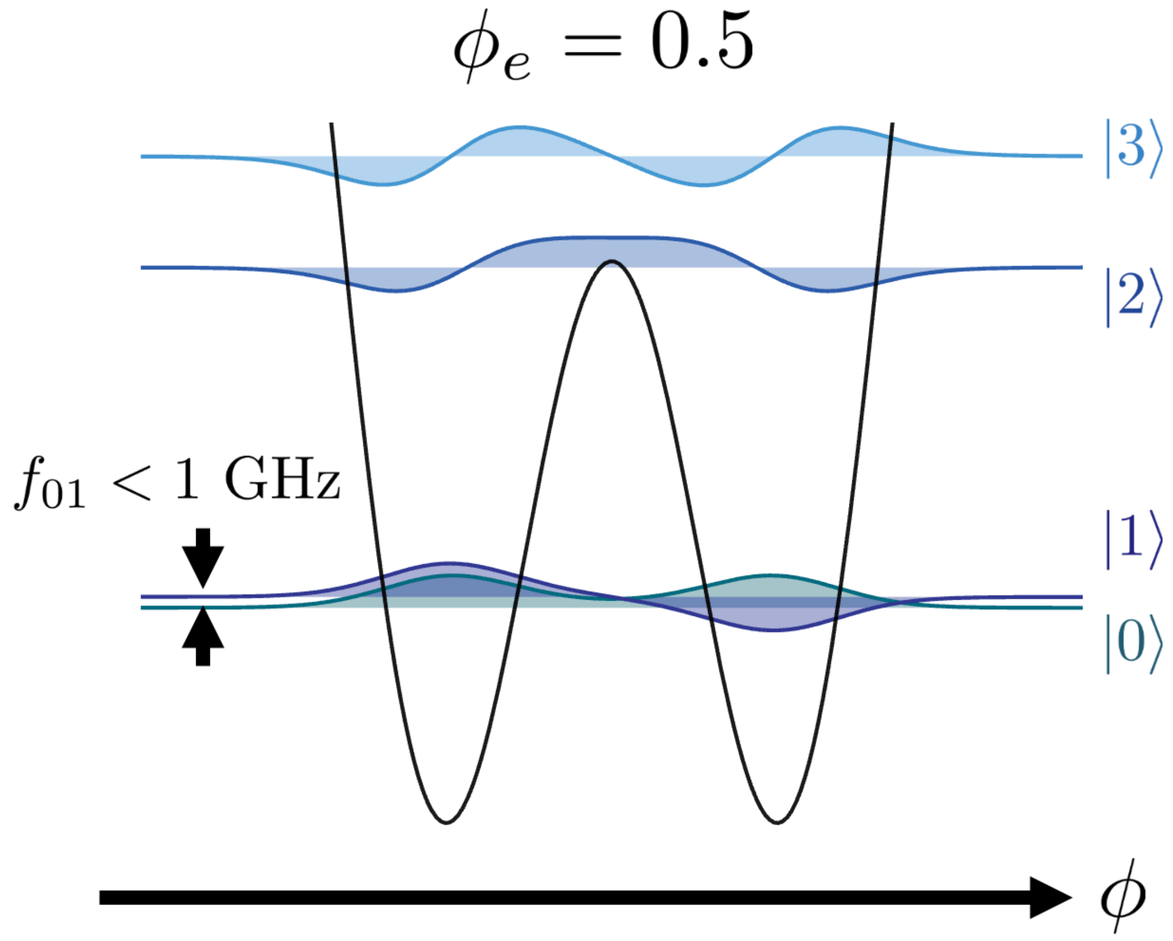

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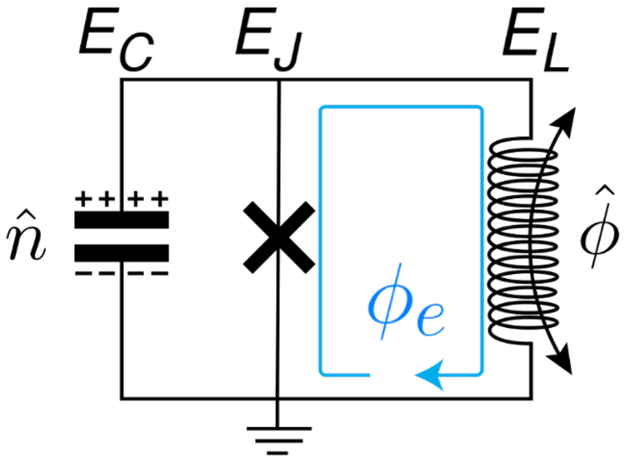

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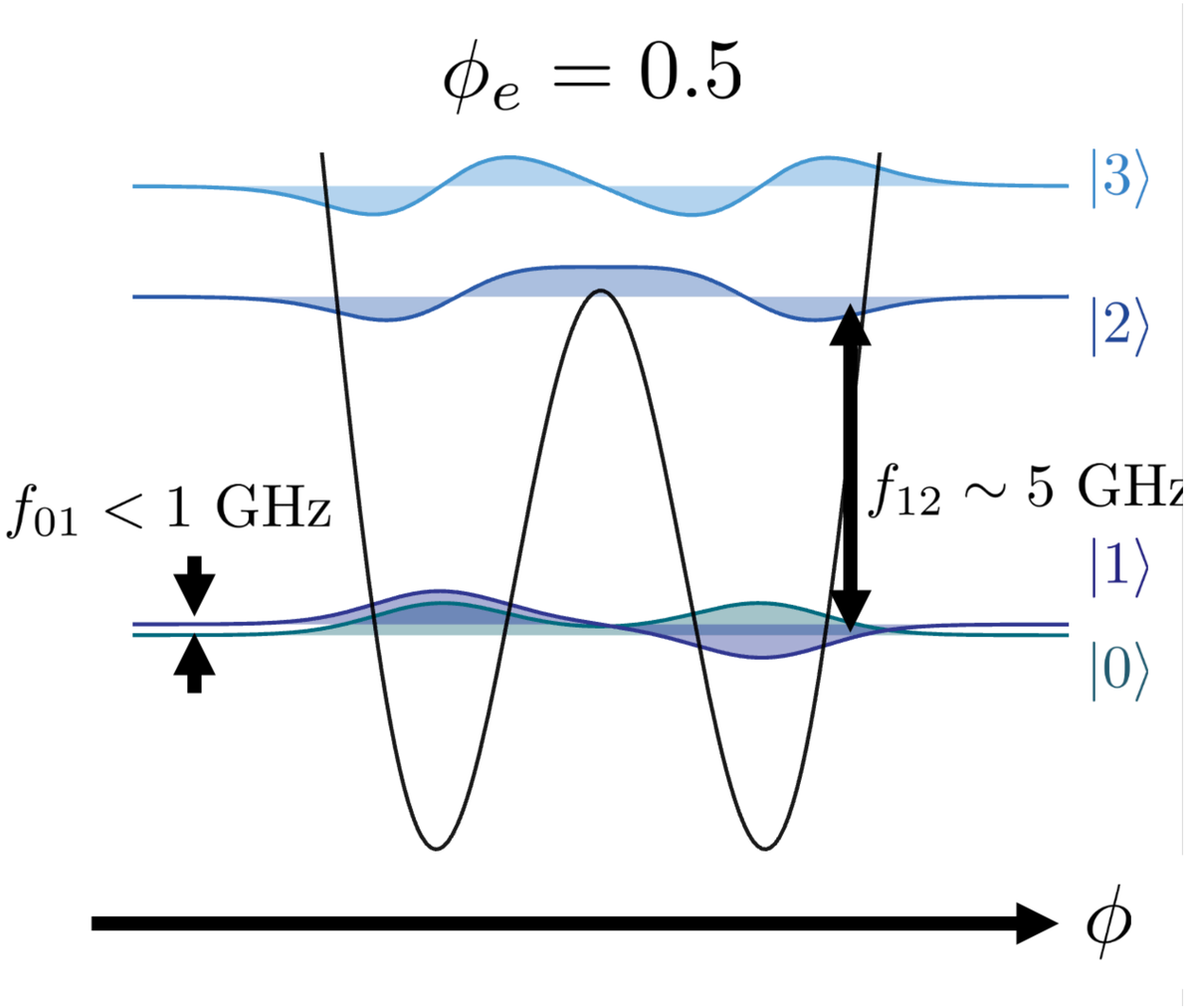

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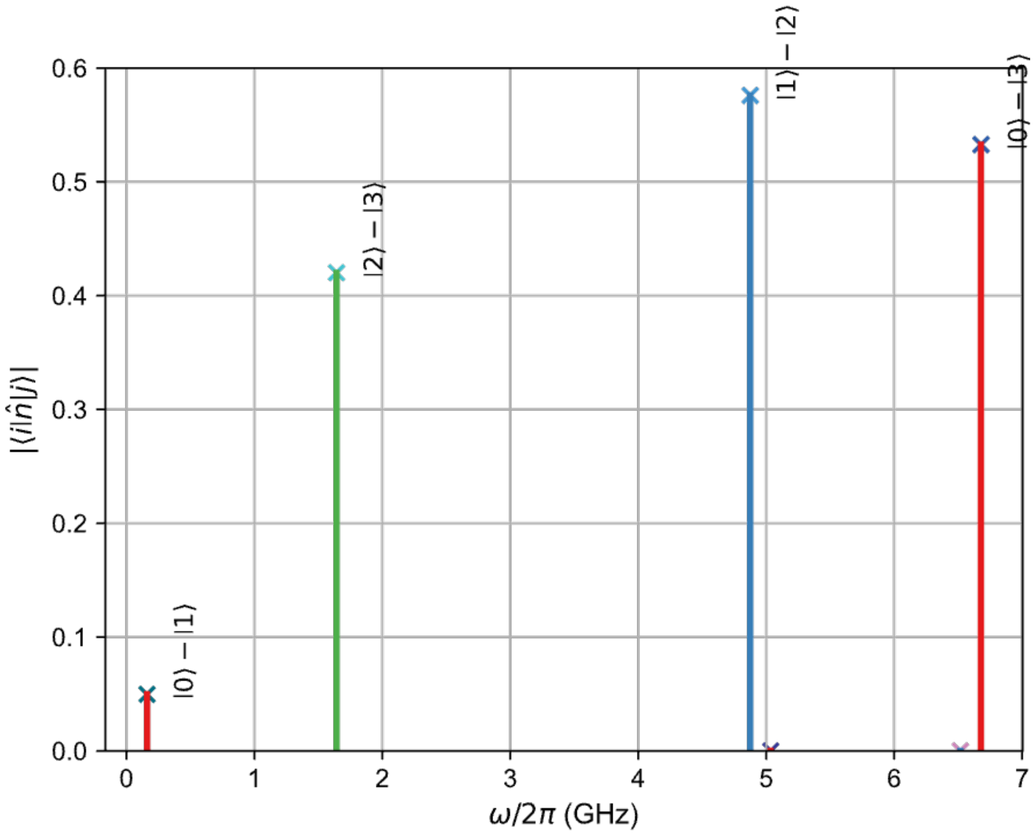

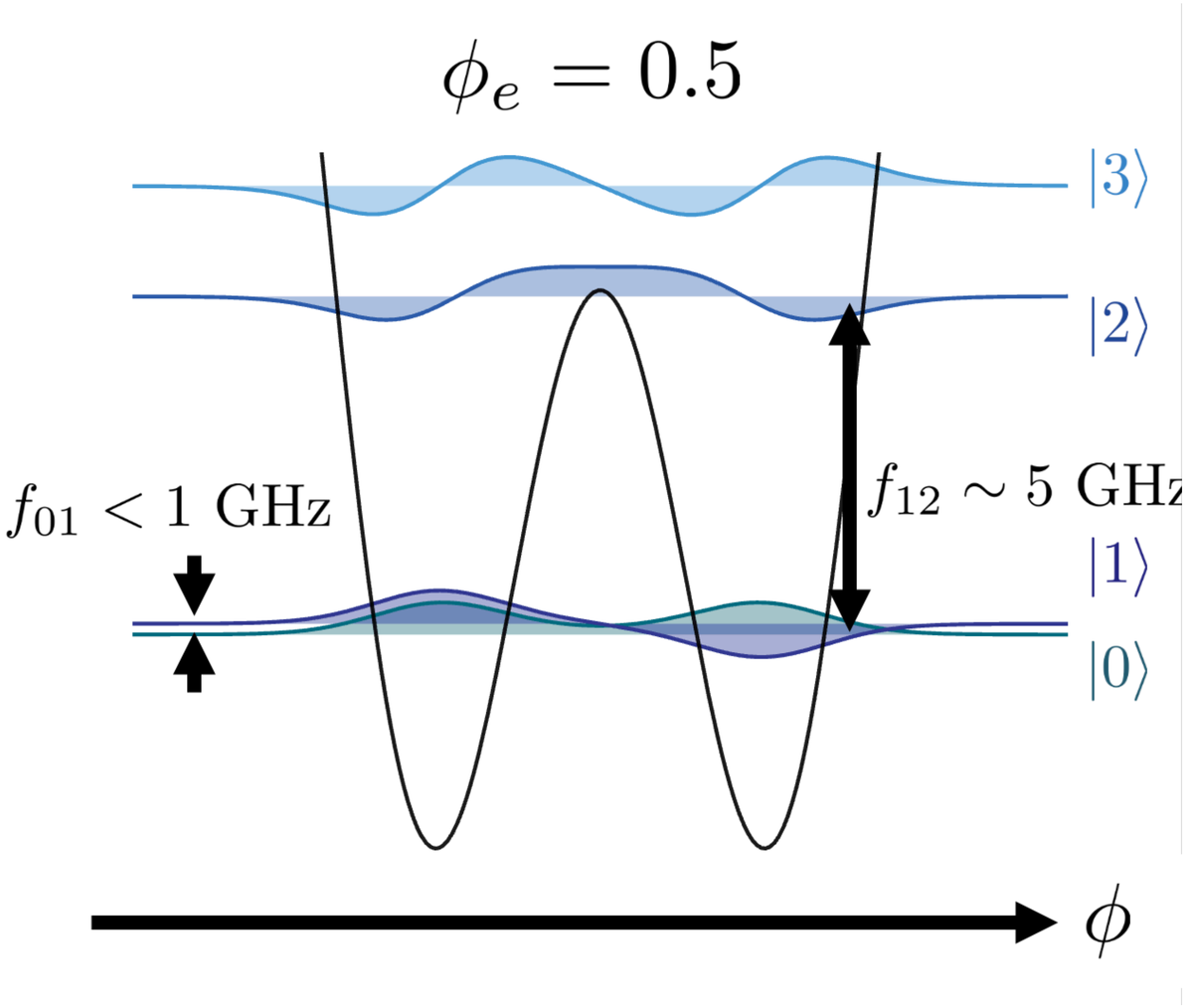

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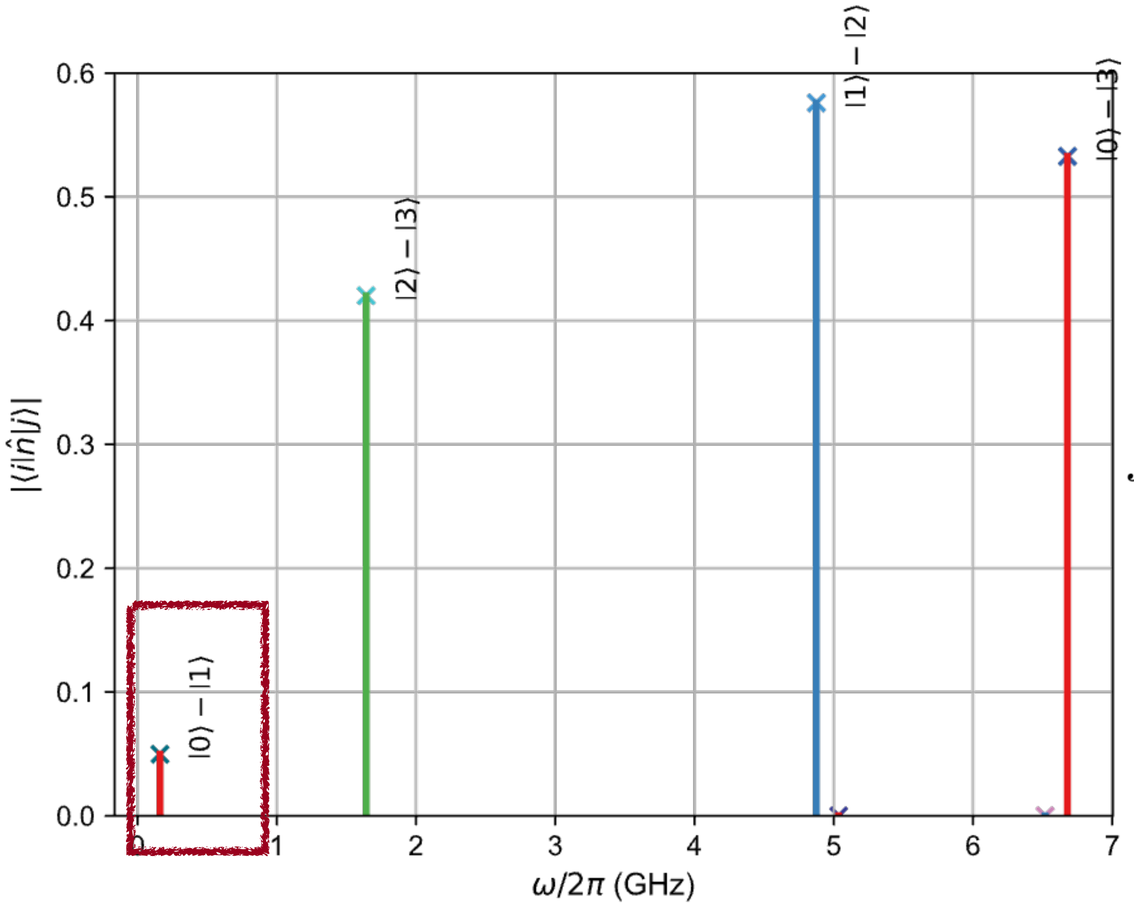
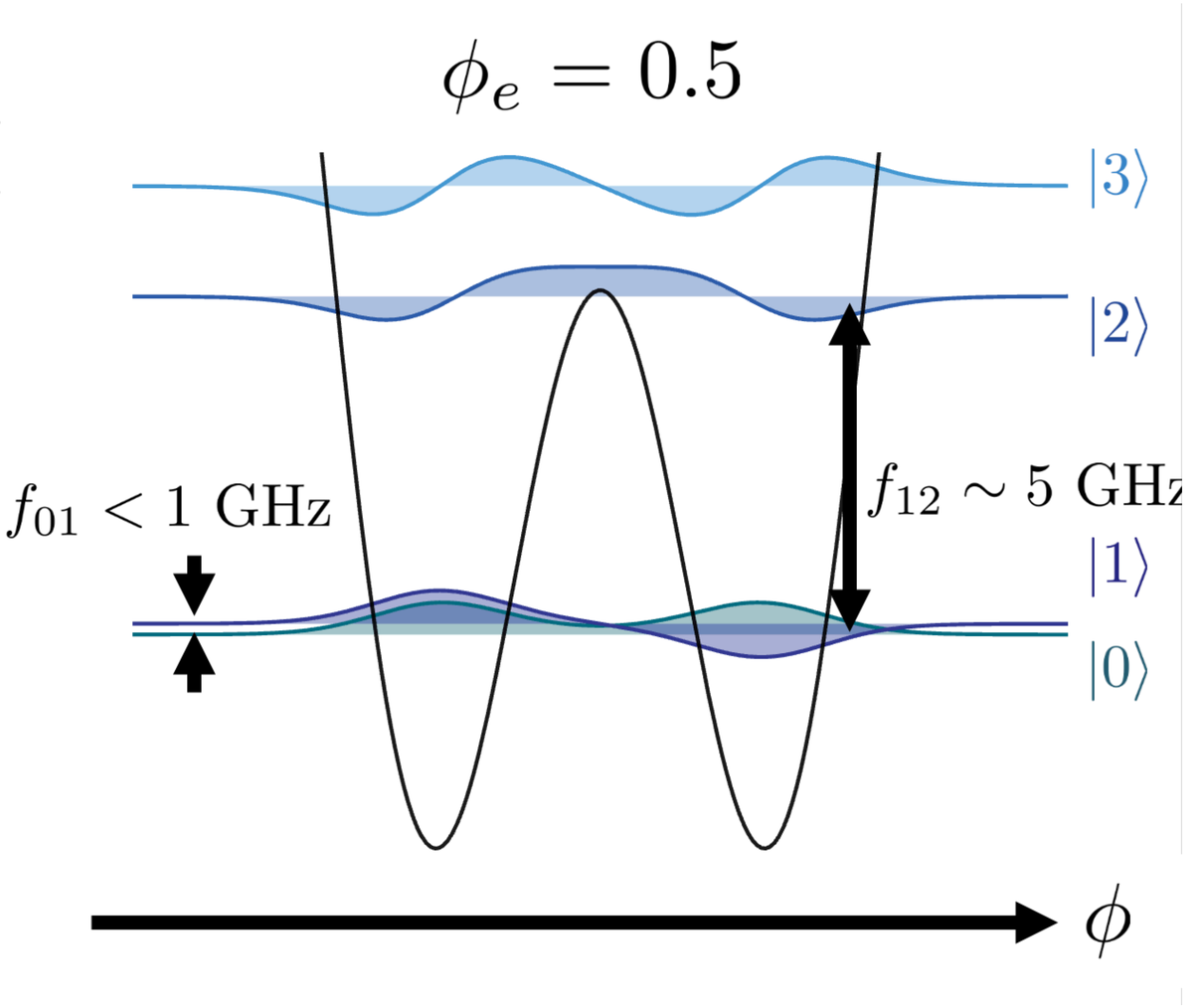


Qubit transition



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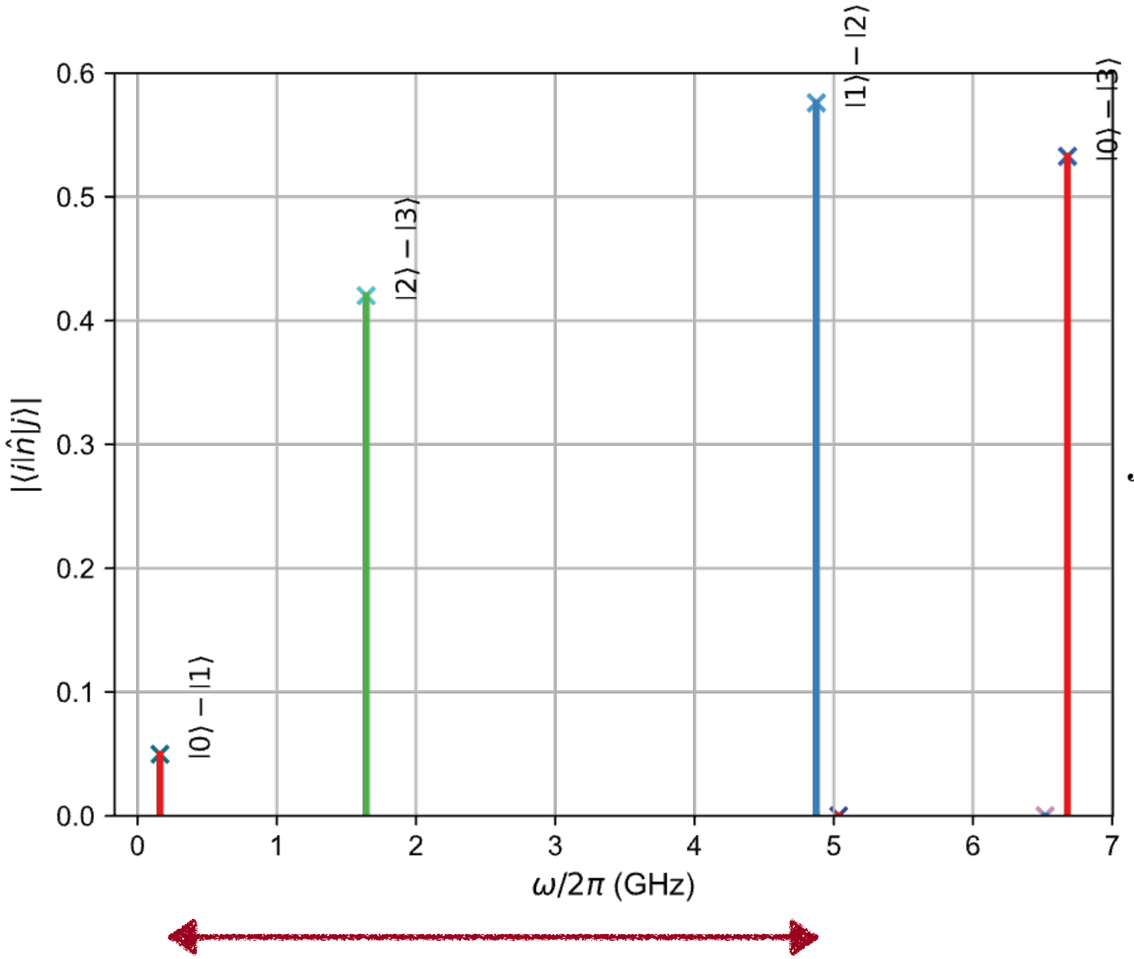
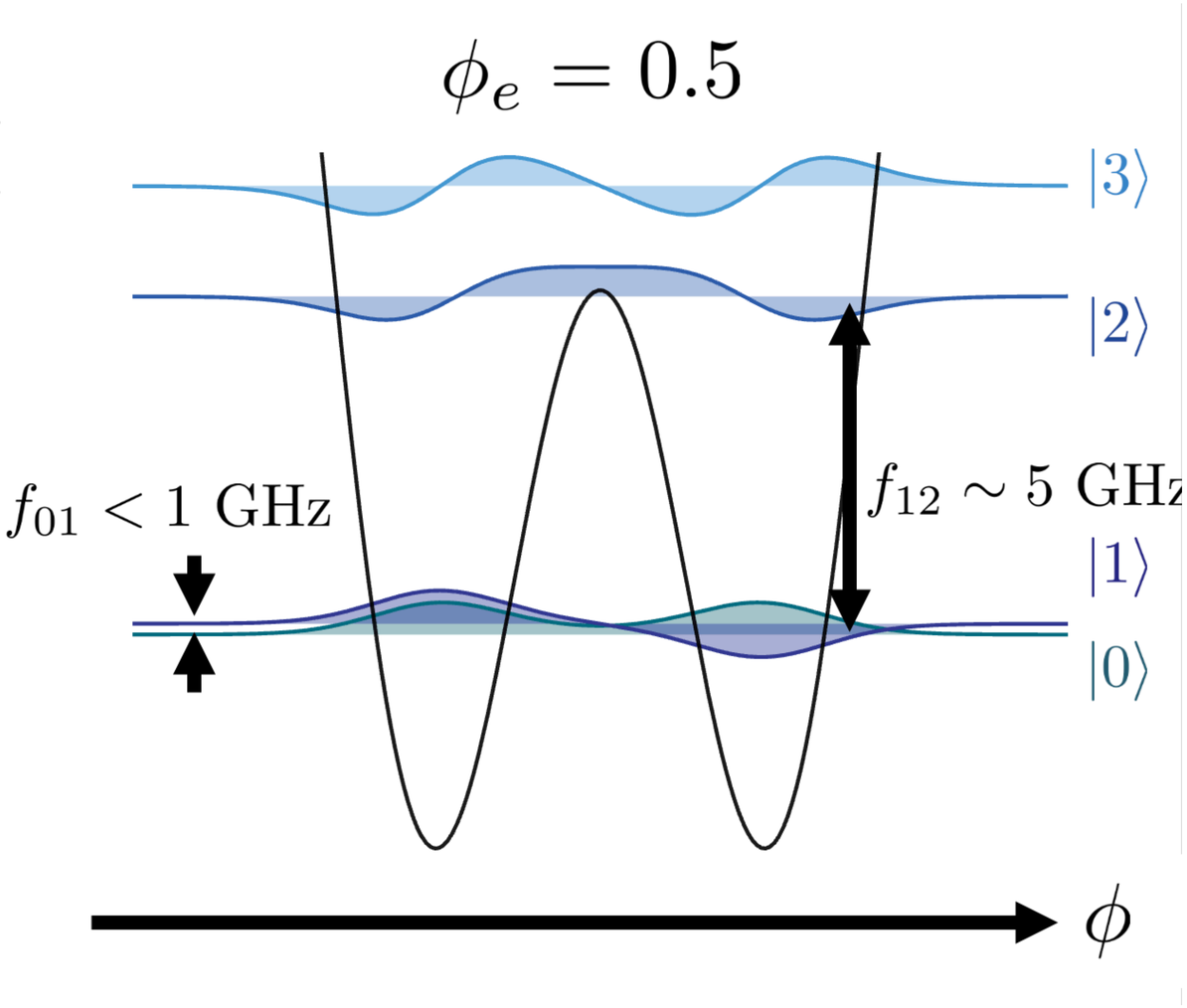


Large anharmonicity



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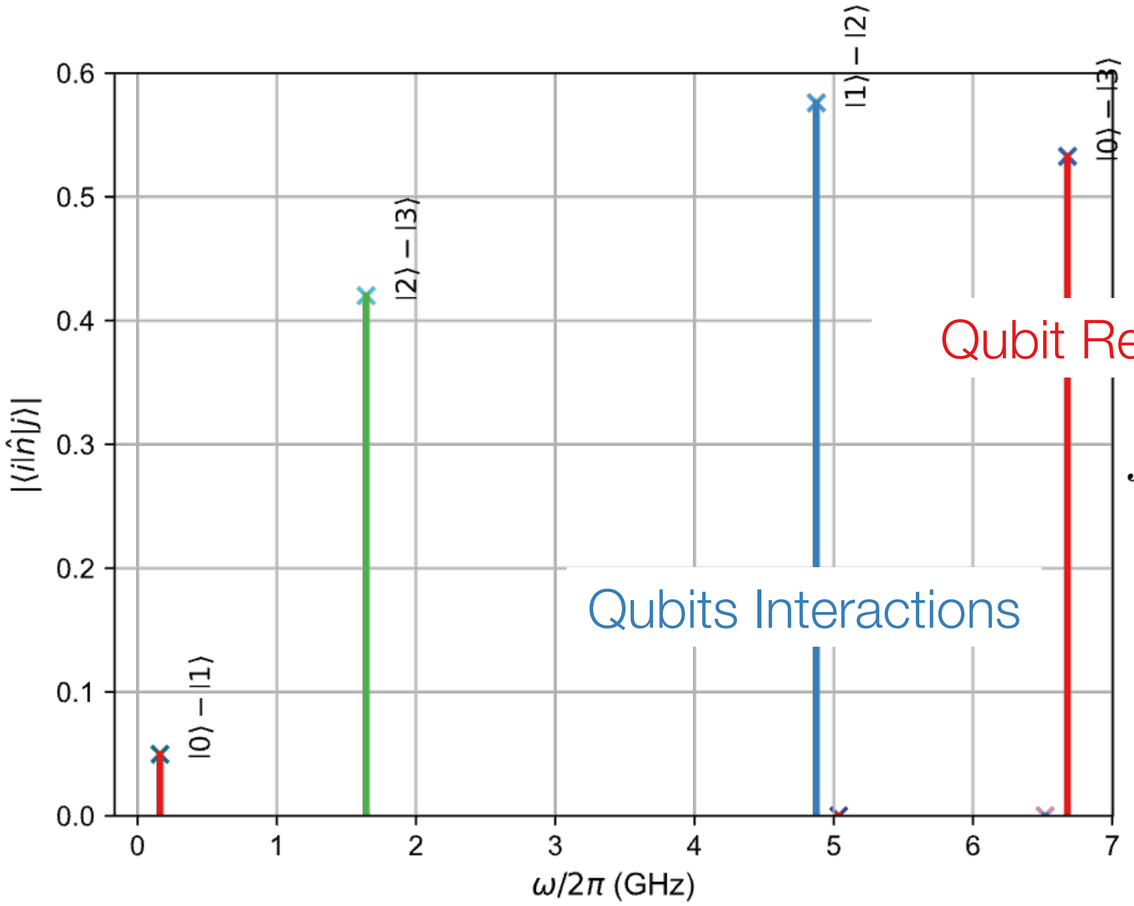
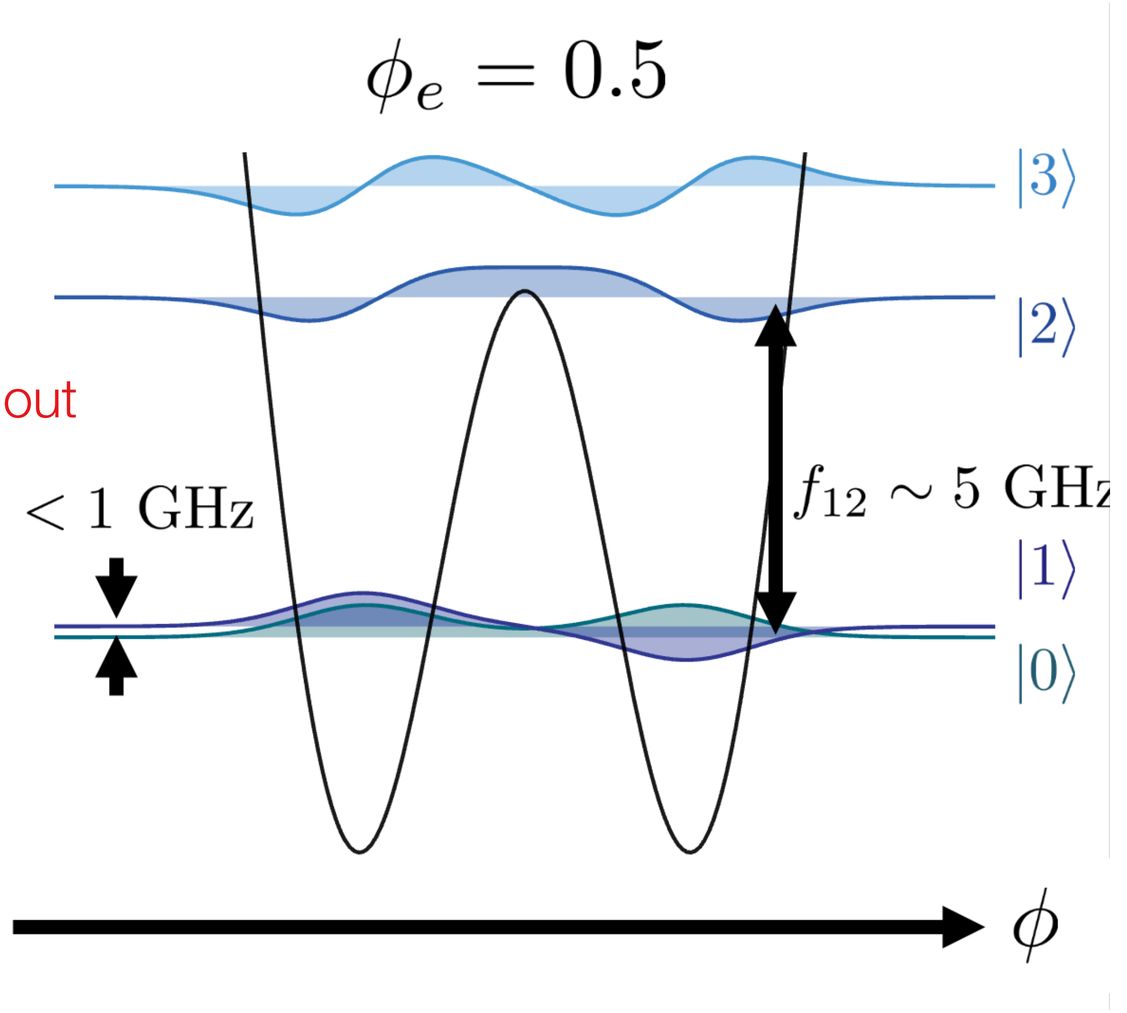




Fluxonium: different flavours

Light Fluxonium

$E_\mathrm{J}/E_\mathrm{C} \ll 1$

$E_\mathrm{J}/E_\mathrm{L} \gg 1$

Heavy Fluxonium

$E_\mathrm{J}/E_\mathrm{C} \gg 1$

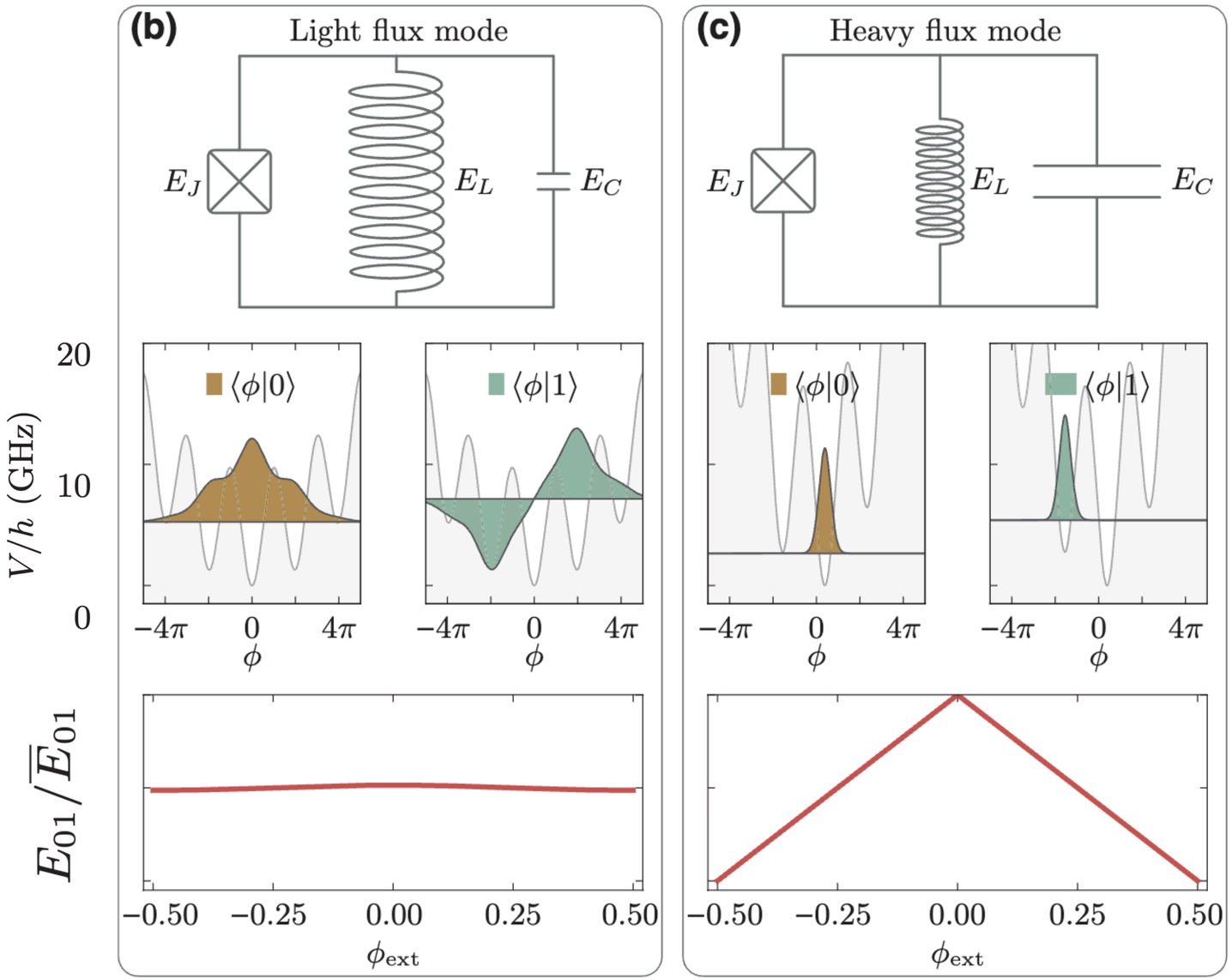
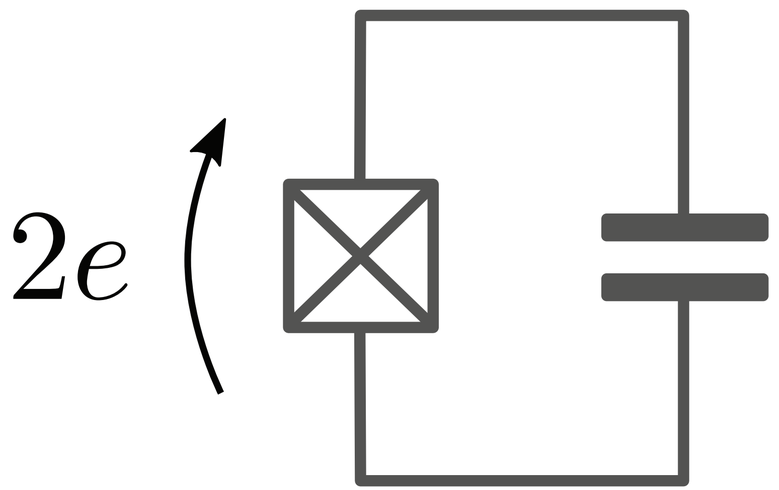
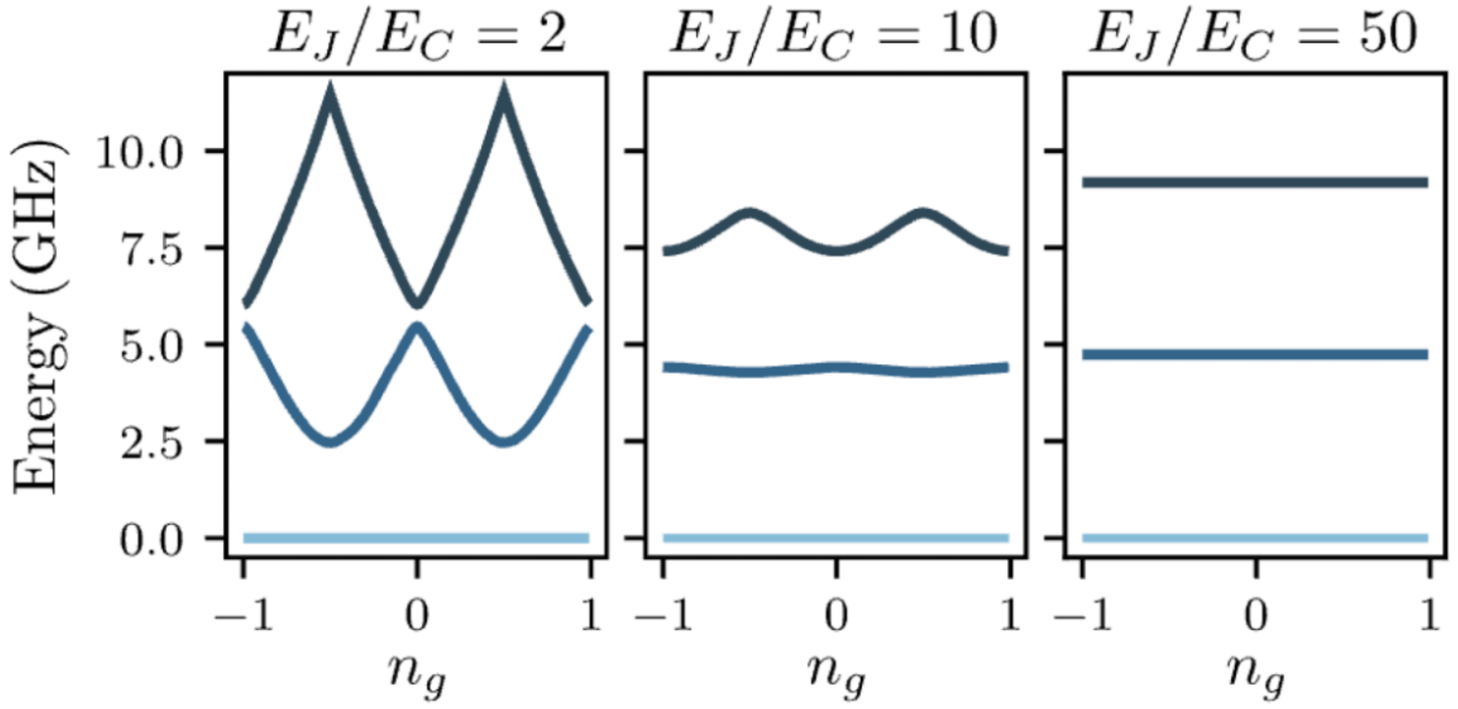


Figure from Gyenis et al. PRX Quantum (2021)

Reducing dephasing (T_2-protection): transmon case

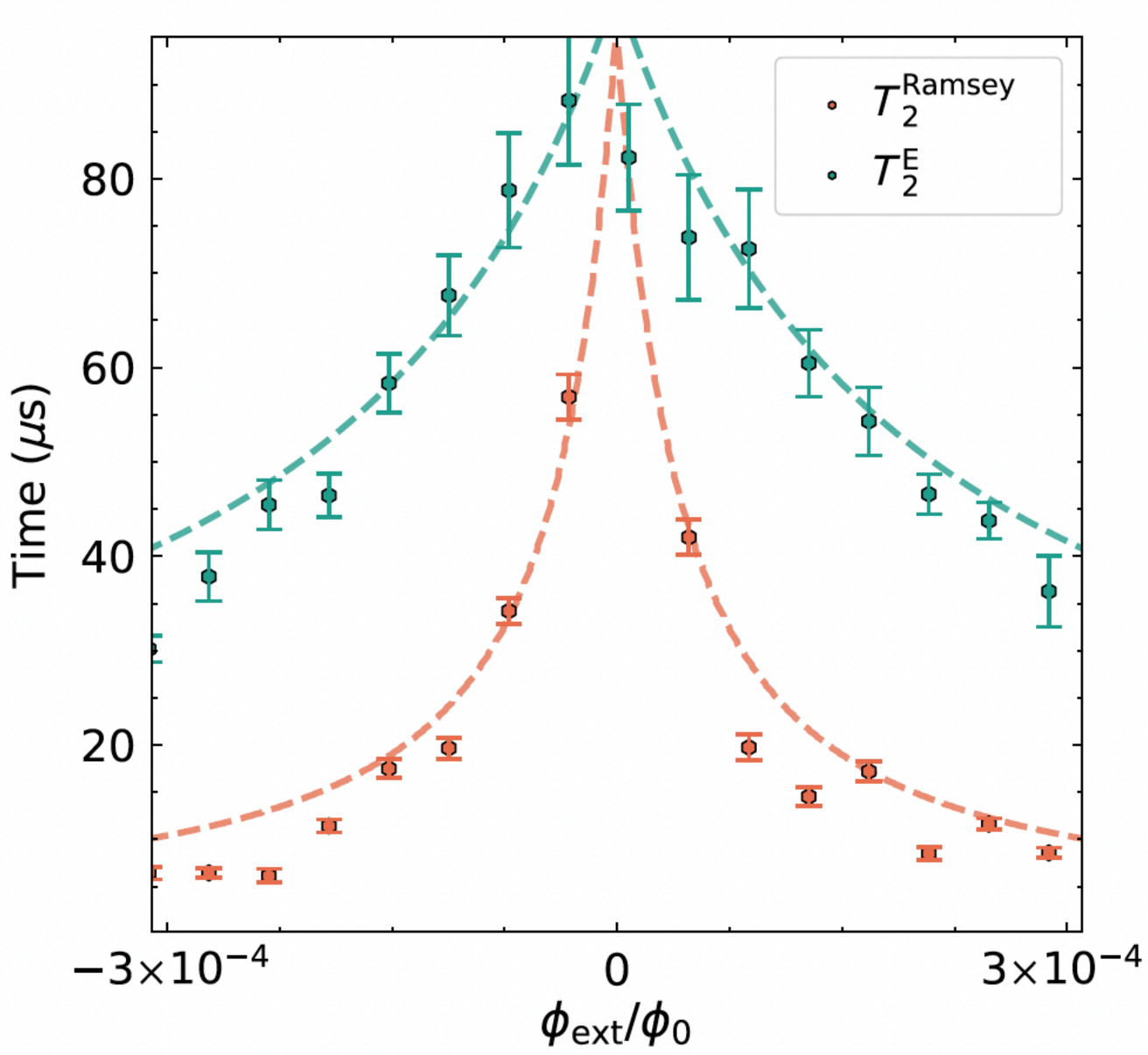


$$\hat{H}_{\mathrm{T}} = 4E_{\mathrm{C}}\left(\hat{n} - n_{\mathrm{g}}\right)^2 - E_{\mathrm{J}}\cos$$



Exponential protection against dephasing (T_2-protection)

Dephasing in fluxonium qubits: flux noise

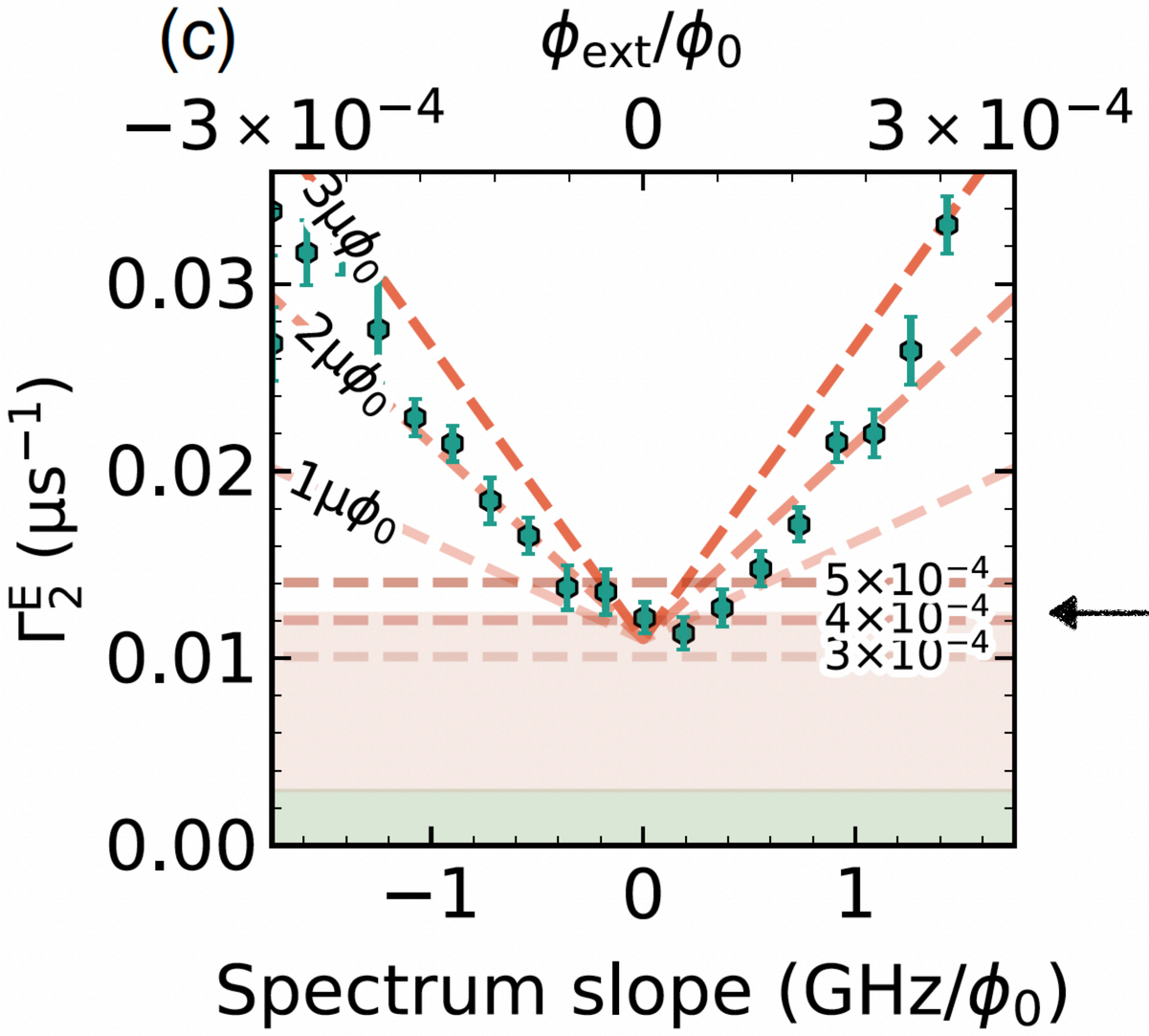


$$\Gamma_\phi^{\mathrm{R}}(\Phi_{\mathrm{ext}}) = \sqrt{A^{\mathrm{R}} \ln\left(\frac{\Gamma_\phi^{\mathrm{R}}(\Phi_{\mathrm{ext}})}{2\pi f_{\mathrm{IR}}}\right)} \left|\frac{\partial\omega}{\partial\Phi_{\mathrm{ext}}}\right|,$$

$$\Gamma_\phi^{\mathrm{E}}(\Phi_{\mathrm{ext}}) = \sqrt{A^{\mathrm{E}} \ln(2)} \left|\frac{\partial\omega}{\partial\Phi_{\mathrm{ext}}}\right|,$$

Figure from Ardati et al. PRX (2024)

Dephasing in fluxonium qubits: photon shot noise



$$\Gamma_\phi^{\mathrm{th}} = \frac{\bar{n}_{\mathrm{th}}\kappa\chi_{01}^2}{\kappa^2+\chi_{01}^2}.$$

Figure from Ardati et al. PRX (2024)

Reducing depolarisation (T_1-protection)

Fermi golden rule

$$1/T_1^{\lambda} \propto \left|\langle 0|\hat{\mathcal{O}}|1\rangle\right|^2 S_{\lambda}(\omega = E_{01}/\hbar)$$

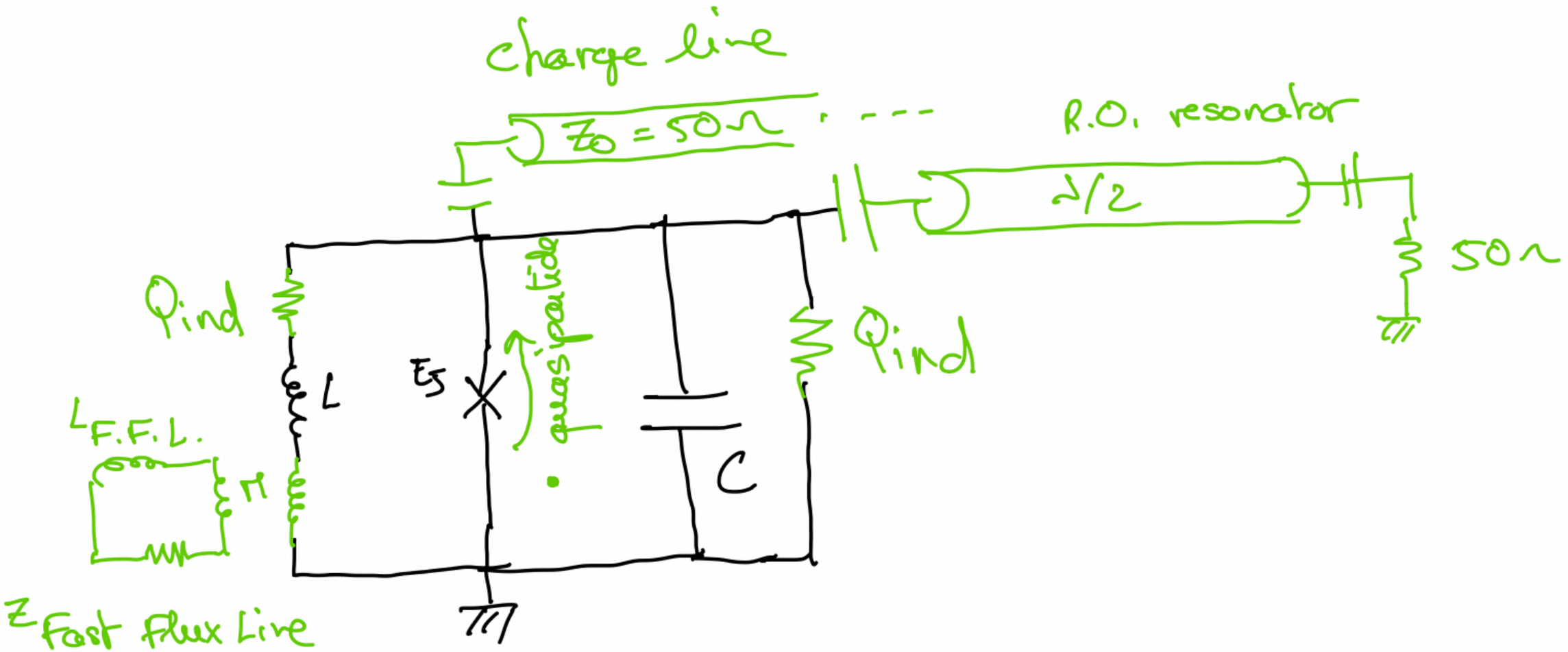
Matrix element
(qubit design)

Noise PSD
(material, fab, environnement…)

λ : a given noise channel

$\hat{\mathcal{O}}$: relevant operator for a given noise channel

Various depolarisation channels



Various depolarisation channels

loss mechanism	$\langle 0\vert \hat{C} \vert 1\rangle$	$\mathrm{Re}\left[Y_X(\omega_{01})\right]$	quality factor
capacitive	$\Phi_0 \langle 0\vert \hat{\varphi} \vert 1\rangle$	$\frac{\omega_{01} C}{Q_{cap}}$	$Q_{cap} > 3 \cdot 10^6$
inductive	$\Phi_0 \langle 0\vert \hat{\varphi} \vert 1\rangle$	$\frac{1}{\omega_{01} L Q_{ind}}$	$Q_{ind} > 500 \cdot 10^6$
quasiparticle	$\langle 0\vert \sin(\hat{\varphi}/2) \vert 1\rangle$	$\frac{G_t}{2Q_{qp}} \left(\frac{2\Delta}{\hbar\omega_{01}}\right)^{3/2}$	$Q_{qp} \approx x_{qp}^{-1} > 0.3 \cdot 10^6$
radiative (Purcell)	$\Phi_0 \langle 0\vert \hat{\varphi} \vert 1\rangle$	HFSS numerical simulations	$Q_{out} = 2000$

Table from Pop et al. Nature (2014)

See also: Nguyen et al. Phys. Rev. X (2019)
Smith et al. npj Quant. Inf. (2020)

Bonus track: dephasing caused by QPS in the superinductance

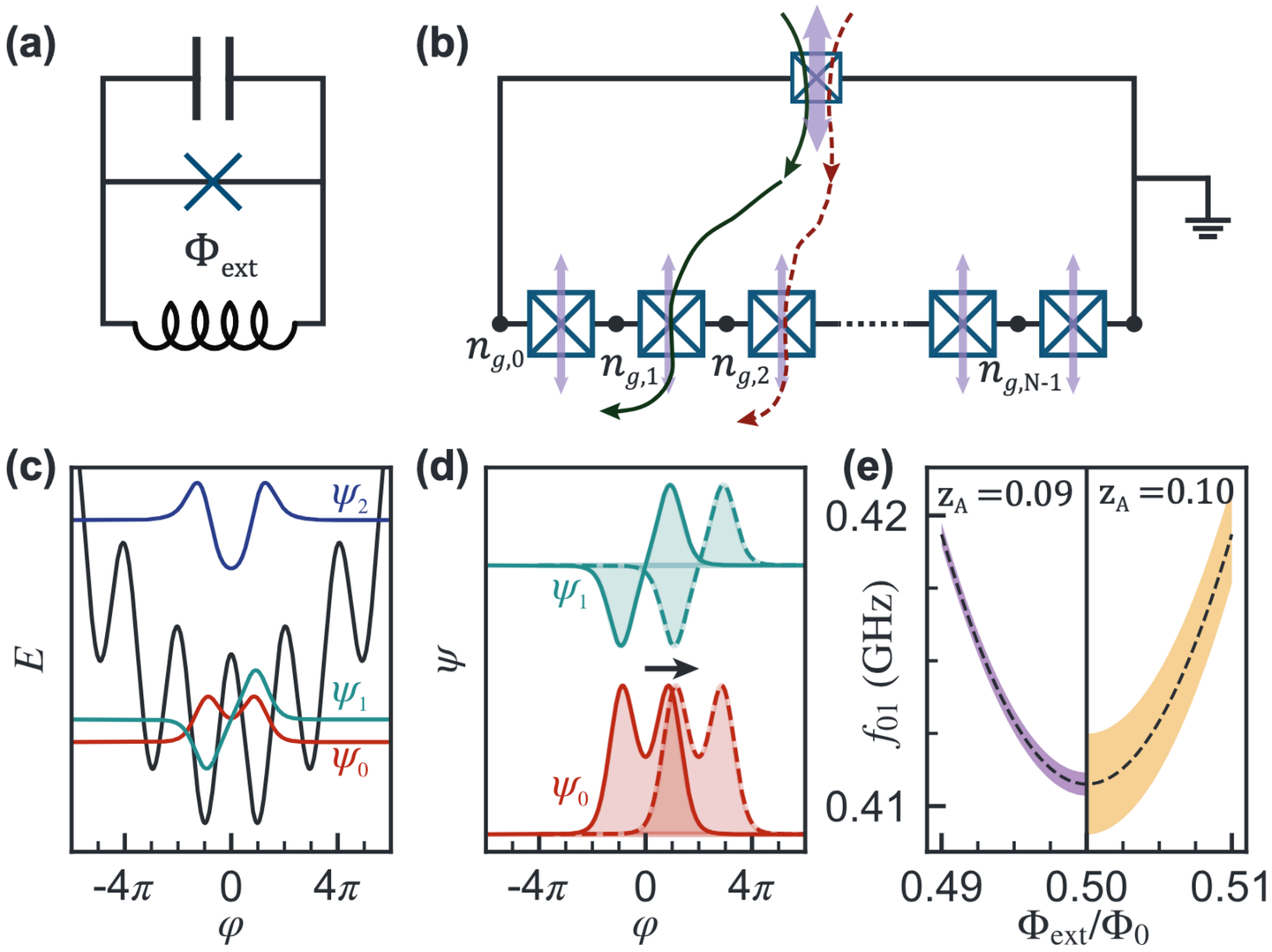
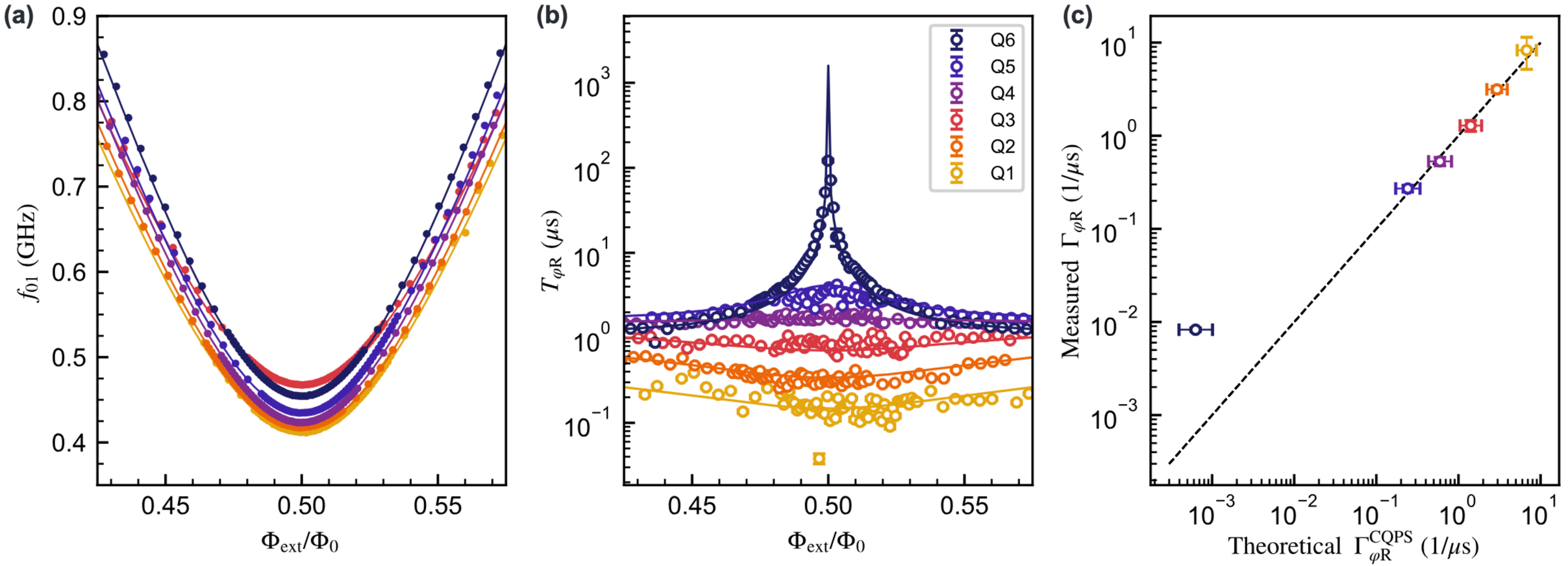


Figure from Randeria et al. PRX Quantum (2024)

Bonus track: dephasing caused by QPS in the superinductance



Take home message $E_J/E_C > 60$ (In the superinductance)

Figure from Randeria et al. PRX Quantum (2024)