

Tensor Network Approaches for Simulations of Larger Superconducting Circuits

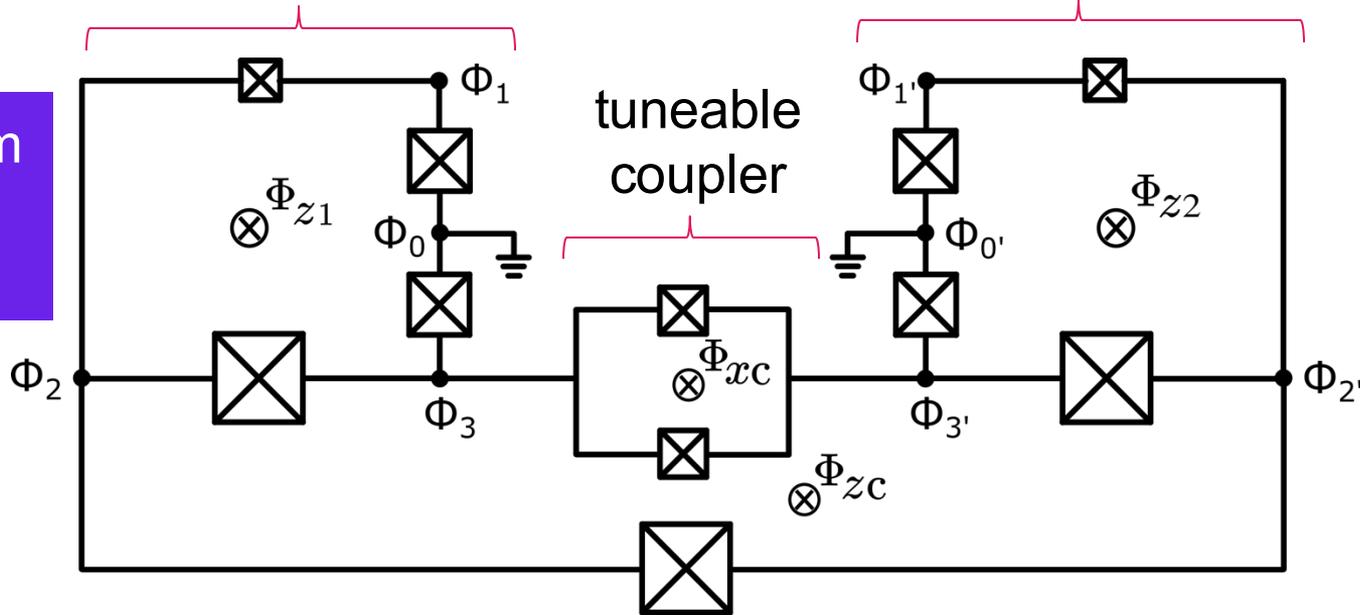
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Problem: Simulation of a “larger” circuit

persistent current qubit: A

persistent current qubit: B

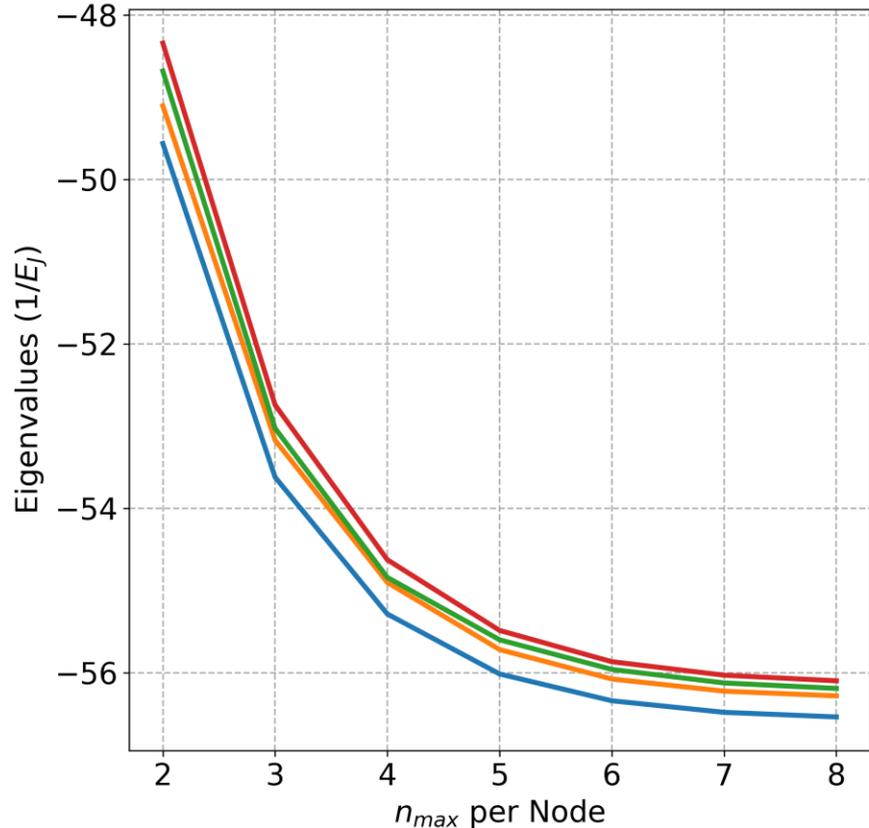
Circuit from
Andrea's
Poster



explore strong coupling regime (e.g. for non-stoquastic effects)

Problem: Simulation of a “larger” circuit

- matrix representation must converge on a suitable basis (here charges per node)
- **But it does not converge**



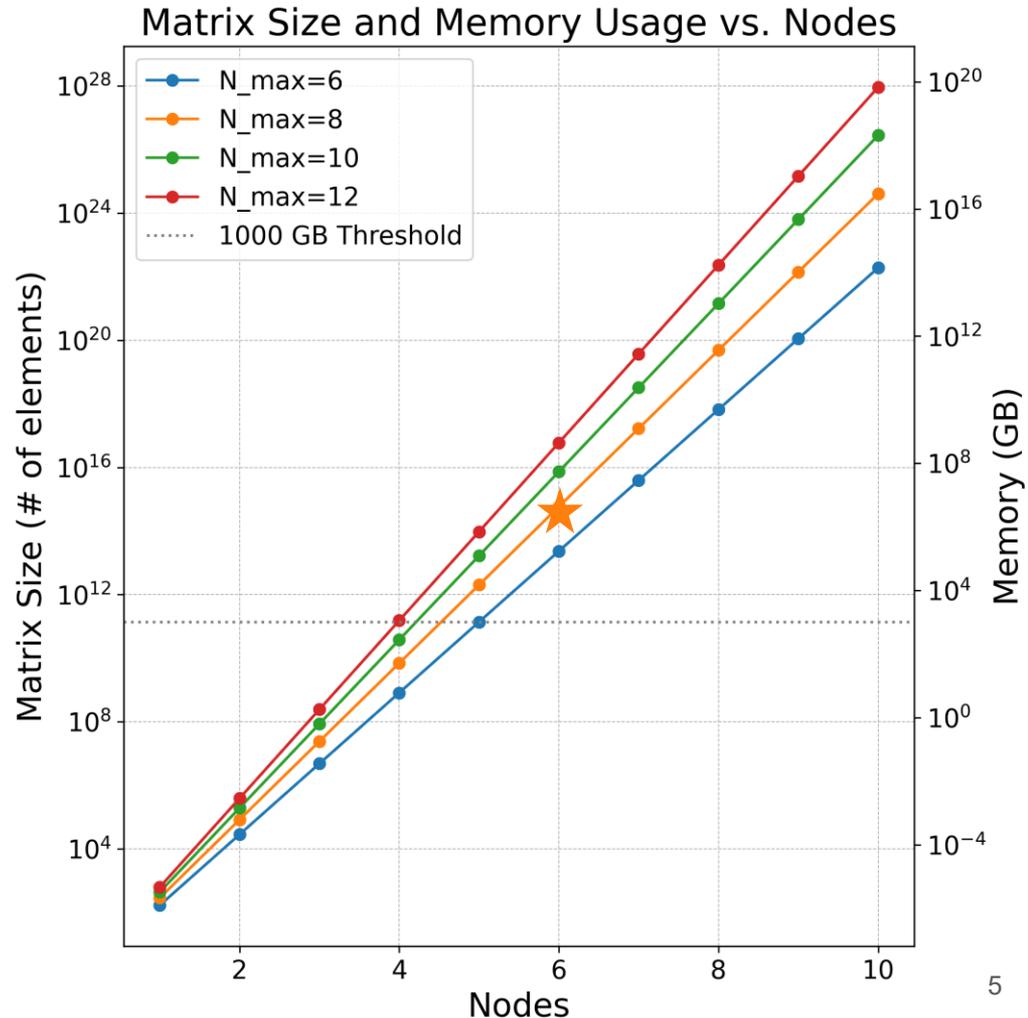
Bottleneck in exact diagonalisation

Our example run with 1TB of RAM

- orange star: maximum sparse matrix size reached

General

- **Exponential** scaling
- Limitation already for modest circuits



Discarded approximations

Born-Oppenheimer

- only linear interaction of subsystems
- need assumptions of the interaction mechanisms
- typical inductive interaction but not capacitance interaction

Hierarchical diagonalisation [1]

- Hamiltonians of the subsystems can get very large when capturing more coupling mechanisms

Do not capture strong coupling and all coupling terms.

Goal

Explore tensor networks techniques aiming to **accurately simulate superconducting circuits** of two qubits and **beyond**, considering **strong coupling and revealing all coupling terms.**

Method

Why are we using Tensor Networks?

Generic entangled state

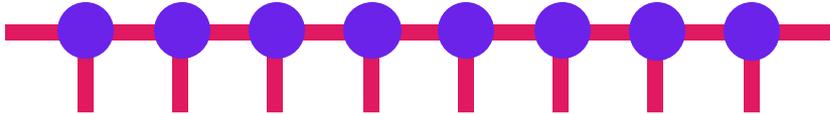
$$|\psi\rangle = \sum_{i_1, \dots, i_N} \psi_{i_1, \dots, i_N} |i_1\rangle \cdots |i_N\rangle$$

Specified by parameters

$$\mathcal{O}(\exp(N))$$

Tensor Networks: Product of simple objects

Some (very well tuneable) entanglement but still efficient



$$\mathcal{O}(\text{poly}(N))$$

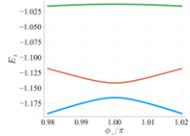
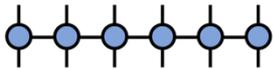
Product state (or unentangled state)

$$|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_N\rangle$$

$$\mathcal{O}(N)$$

Workflow

$$\mathcal{H}(Q, \phi)$$



$$\mathcal{H}_{eff}$$

$$J_{\mu\nu}$$

Lumped Element Hamiltonian

Matrix Product Operator (MPO)

Eigenvalues

Effective Hamiltonian

Coupling terms

MPO for each term in the Hamiltonian:

OpSum in ITensors

Density matrix renormalization group algorithm (DMRG)

Schrieffer Wolf Transformation (in tensor network framework)

Expansion in Pauli coefficients

Work in Progress

High Performance Computing

- in Julia using **ITensors** library [3]
- executing on CPUs, with GPU support
- deployment multiple nodes on the Mare Nostrum 5 supercomputer for **large-scale parallel simulations**

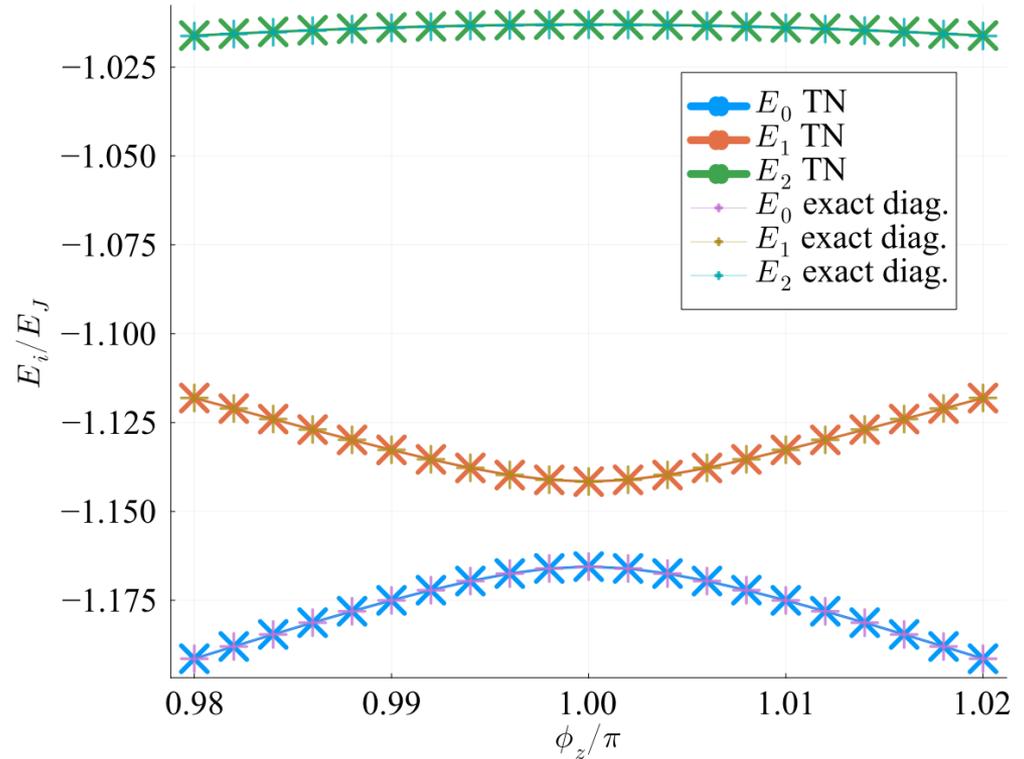




Preliminary Results

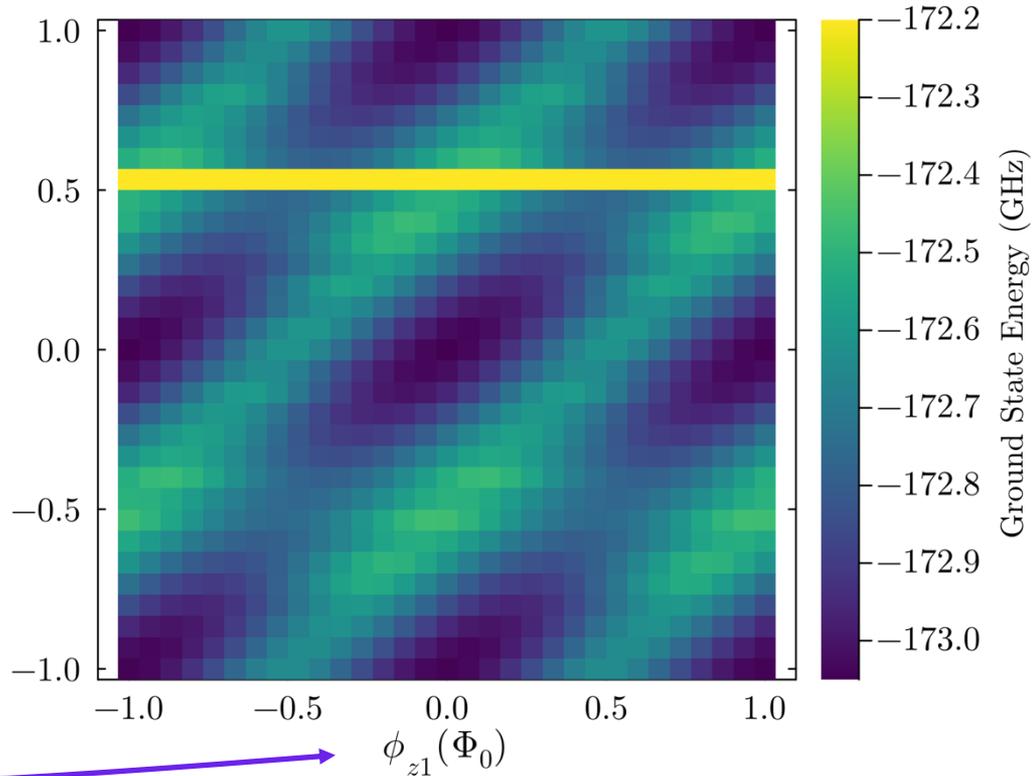
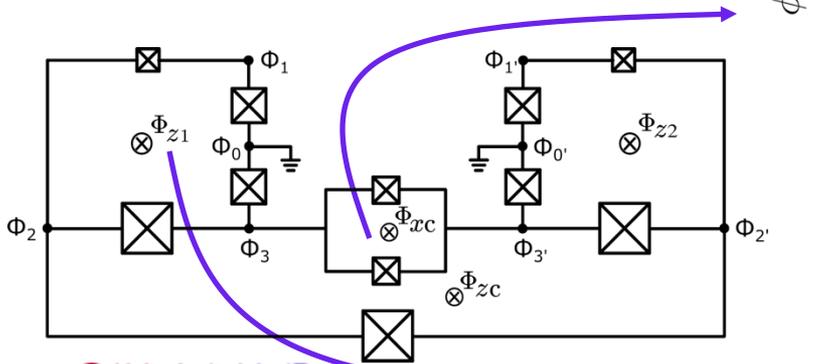
Single persistent current qubit

Spectra of TN exactly and exact diagonalisation **match** exactly



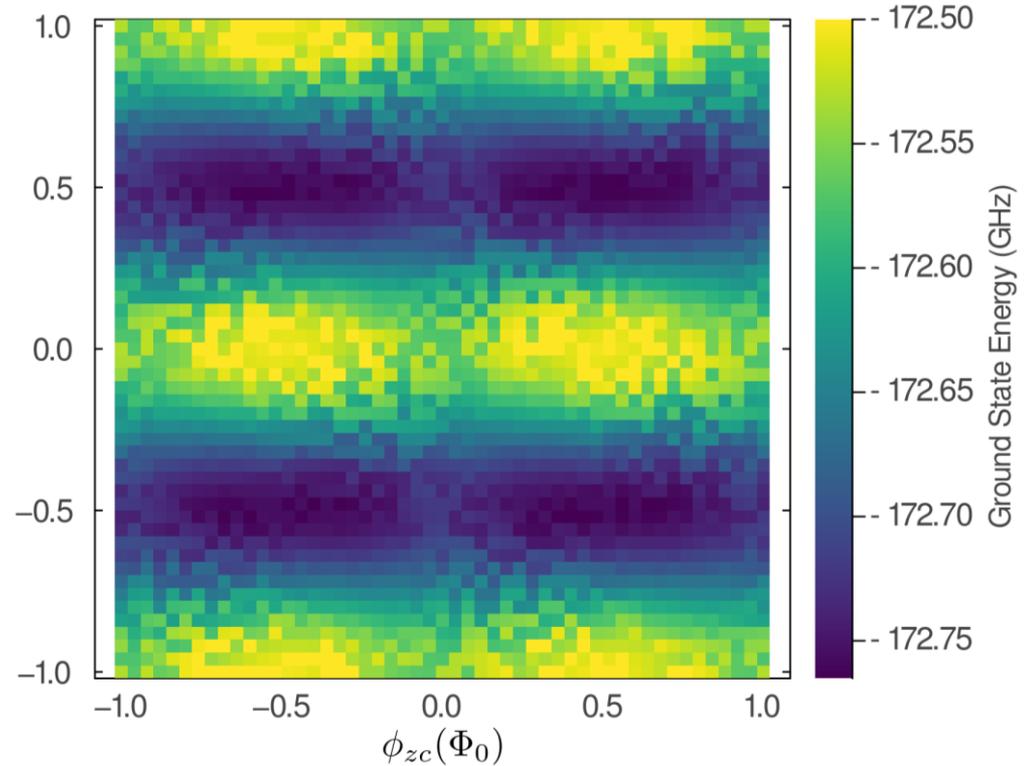
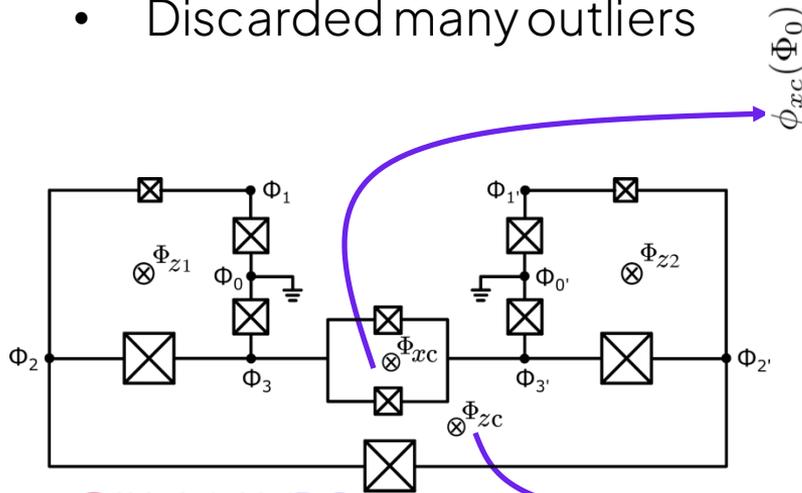
Two tuneable coupled persistent current qubits

- Whole System Energy: SWT was not ready
- **Periodic Structure**
- Tilt Effect from crosstalk
- Discontinuities: yellow line from a local minimum
- Approx. 30 s per datapoint



Two tuneable coupled persistent current qubits

- Whole System Energy
- **Periodic Structure**
- wavefront propagation for increased parallelism.
- Discarded many outliers



Outlook and Conclusion

Outlook

- **Refining to match with experimental results**
- Aid cross talk simulations (Andrea's poster)
- **Schrieffer-Wolff transformation** in the TN framework — **likely reveal coupling terms that approximate methods miss** [5]
- Machine-learning strategies to mitigate **local minima**
- Computational: improve **efficiency** and conclusive benchmark
- Harnessing **tensor network perspective** (e.g. study entanglement, fine tune bond dimensions)

Conclusion

- **Scalable Framework:** Designed, implemented, and are currently validating a **methodology** to **break through computational bottlenecks in larger, strongly coupled circuits.**
- **Preliminary Proof of Concept:** Our method **surpass the computational limits of exact diagonalization.**

Acknowledgment and References

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1. Kerman, Andrew J. “Efficient Numerical Simulation of Complex Josephson Quantum Circuits.” 2020 <http://arxiv.org/abs/2010.14929>.
2. Di Paolo, Agustin, Thomas E. Baker, Alexandre Foley, David Sénéchal, and Alexandre Blais. “**Efficient Modeling of Superconducting Quantum Circuits with Tensor Networks.**” 2021 <https://doi.org/10.1038/s41534-020-00352-4>.
3. “The **ITensor** Software Library for Tensor Network Calculations”, Matthew Fishman, Steven R. White, E. Miles Stoudenmire, 2022
4. Tensors.Net, <https://www.tensors.net/intro>. Accessed 15 May 2025.
5. Hita-Pérez, María, Gabriel Jaumà, Manuel Pino, and Juan José García-Ripoll. “**Three-Josephson Junctions Flux Qubit Couplings.**” 2021, <https://doi.org/10.1063/5.0069530>.



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Previous Work

“Efficient Modeling of Superconducting Quantum Circuits with Tensor Networks.”

Di Paolo et al. 2011 [2]

- Applied to Fluxonia

