



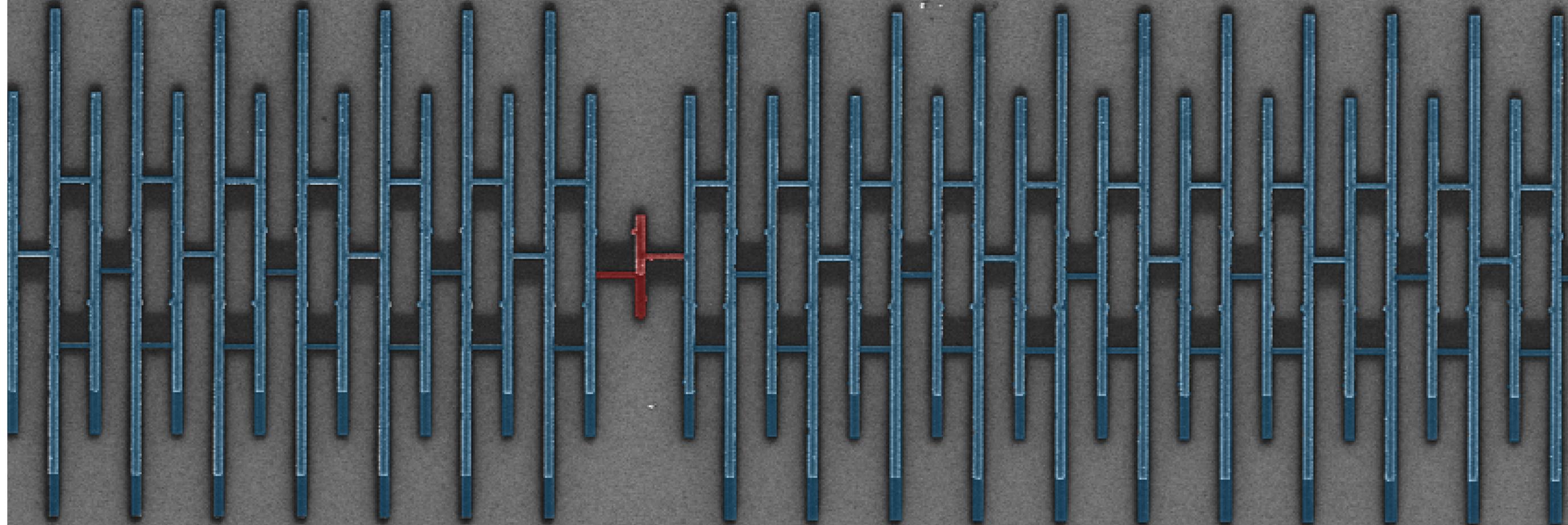
European Research Council  
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SILENT  
WAVES

# High-Impedance Circuits, Superinductances and Fluxonium Qubits

Nicolas Roch, Institut Néel, Grenoble, France



2025 Spring School on  
Superconducting Qubit Technology

10µm

SQC

[www.sqc.cnrs.fr](http://www.sqc.cnrs.fr)



SQC

# Understanding the quantum behaviour of superconducting circuits

## Quantum computing



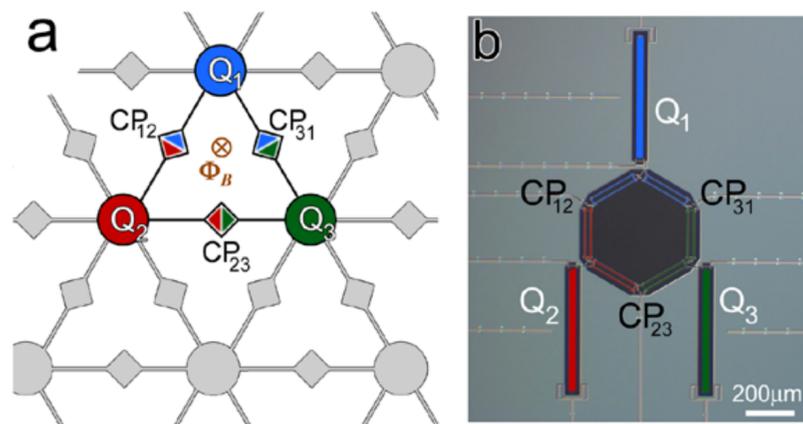
Credit: Google Quantum

## Quantum communications



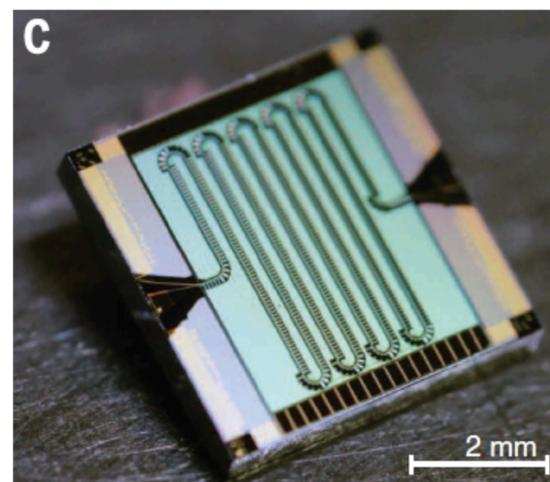
Credit: ETH Zurich Department of Physics

## Quantum simulation



Roushan et al. 16'

## Quantum sensing



Macklin et al. 15'

# Understanding the quantum behaviour of superconducting circuits

## Quantum computing



Credit: Google Quantum

## Quantum communications



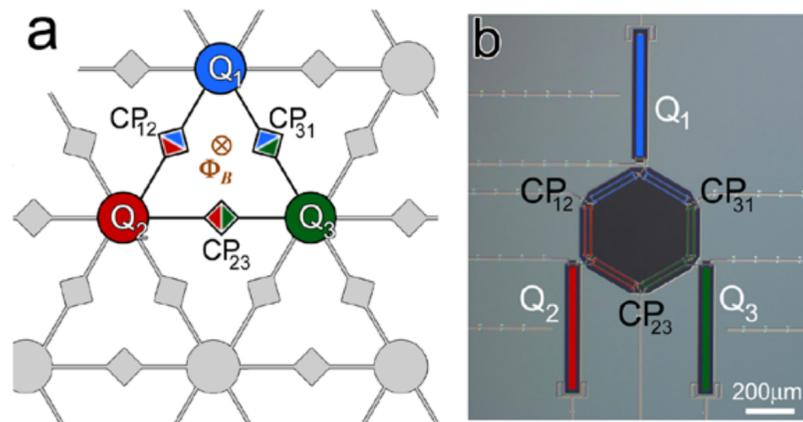
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## Still many open questions!

Ultimate limit of decoherence  
(quasi-particles, cosmic rays,  
materials..)

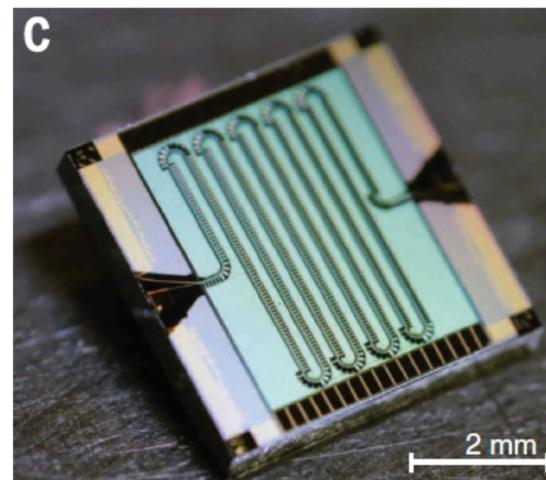
Large scale (many-body) system

## Quantum simulation



Roushan et al. 16'

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Macklin et al. 15'

Best way to protect coherence  
(a huge computer with decent  
qubits, a smaller computer with  
outstanding qubits...)

# Understanding the quantum behaviour of superconducting circuits

## Quantum computing



Credit: Google Quantum

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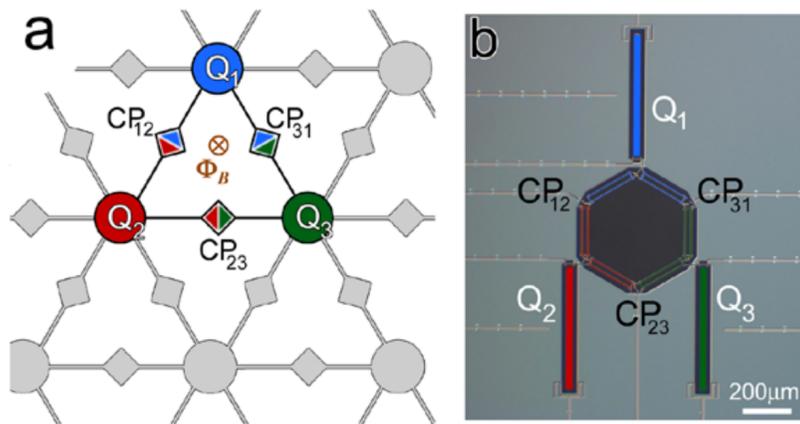
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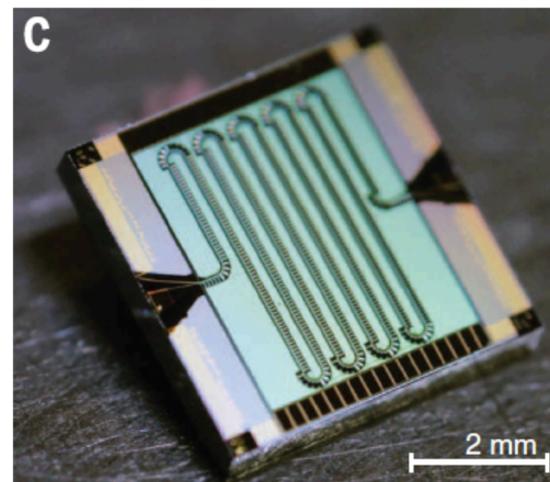
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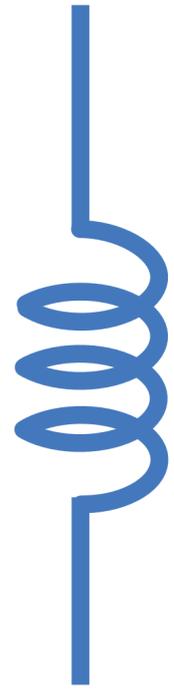
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Best way to protect coherence  
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**High impedance  
quantum circuits**

# Nature prefers low impedance/low inductance

(passive) electric dipoles



Inductance

$$L = \frac{\mu_0 l}{2\pi}$$

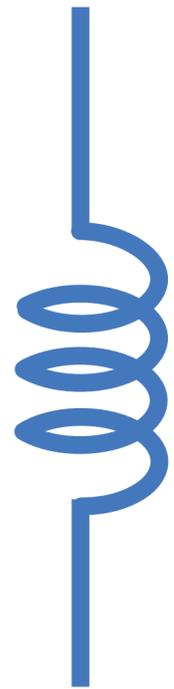


Capacitance

$$C = \frac{\epsilon_0 S}{d}$$

# Nature prefers low impedance/low inductance

(passive) electric dipoles



Inductance

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Capacitance

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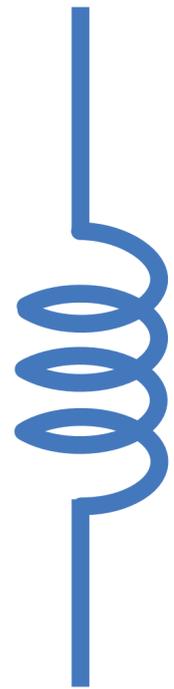
Conductance quantum

Fine structure constant:

$$\alpha = \frac{e^2}{2\epsilon_0 hc} = \frac{1}{4} \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{2e^2}{h} \approx \frac{1}{137}$$

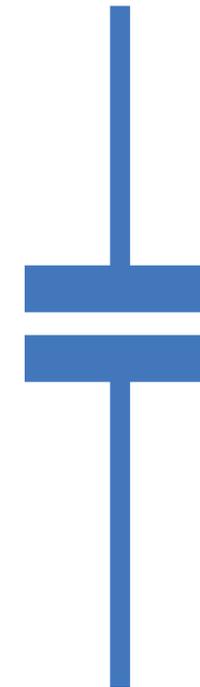
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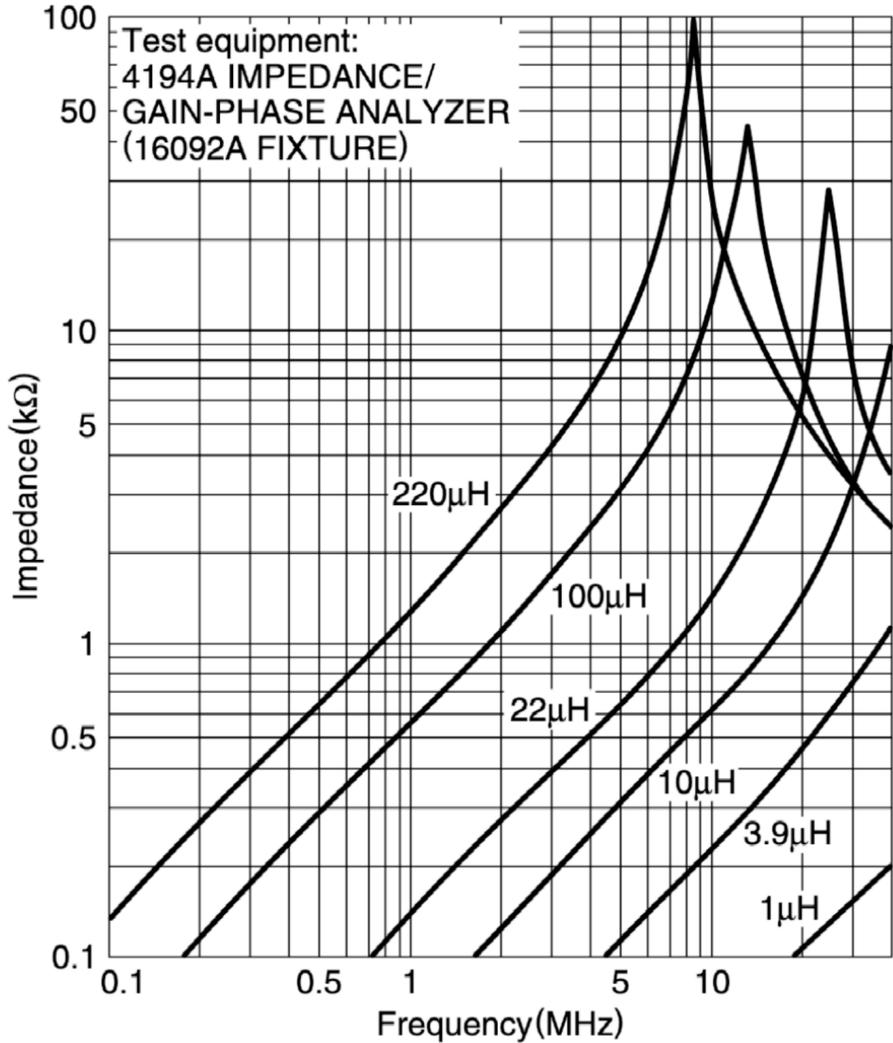
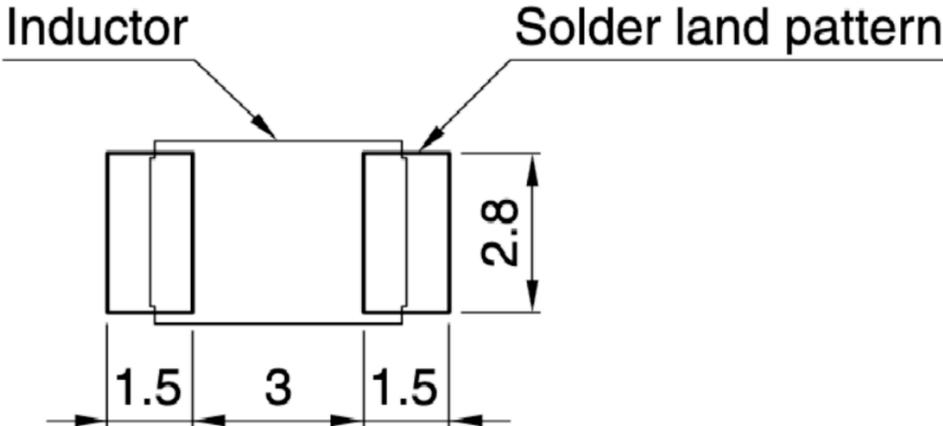
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Can we build large inductance (quantum) circuits?

# Large inductance (quantum) circuits: why is it difficult ?



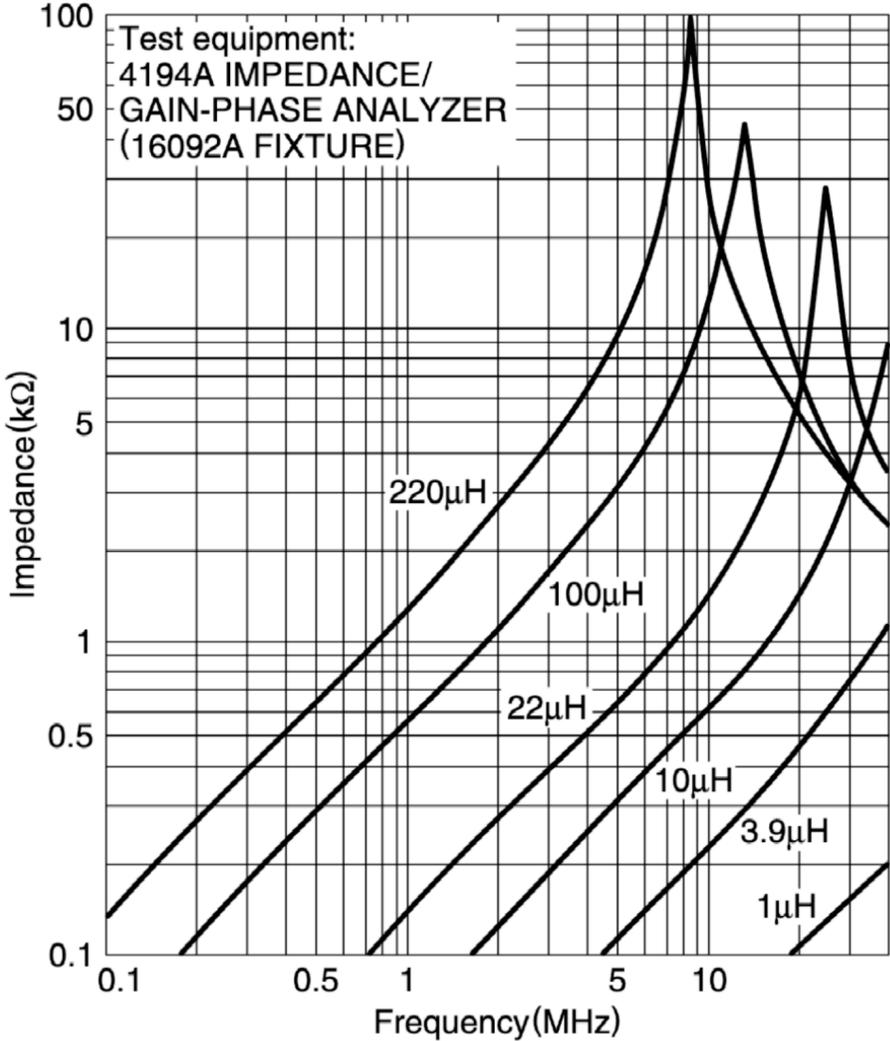
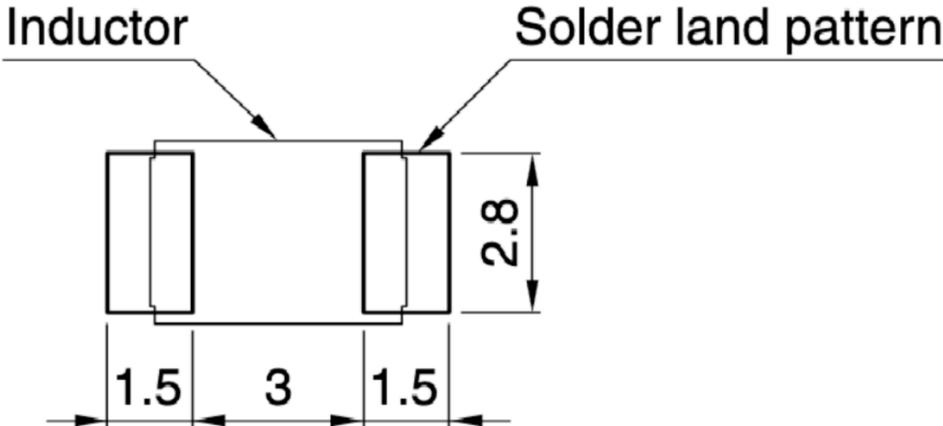
NLC4532



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NLC4532



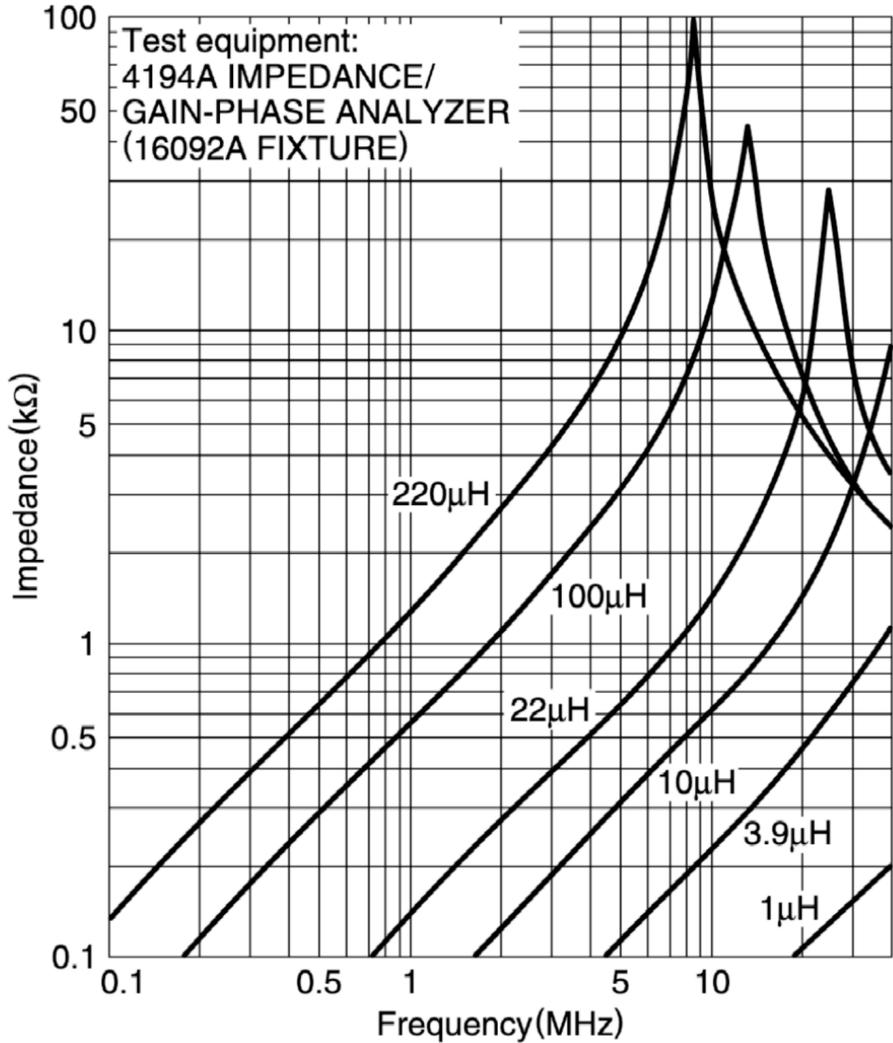
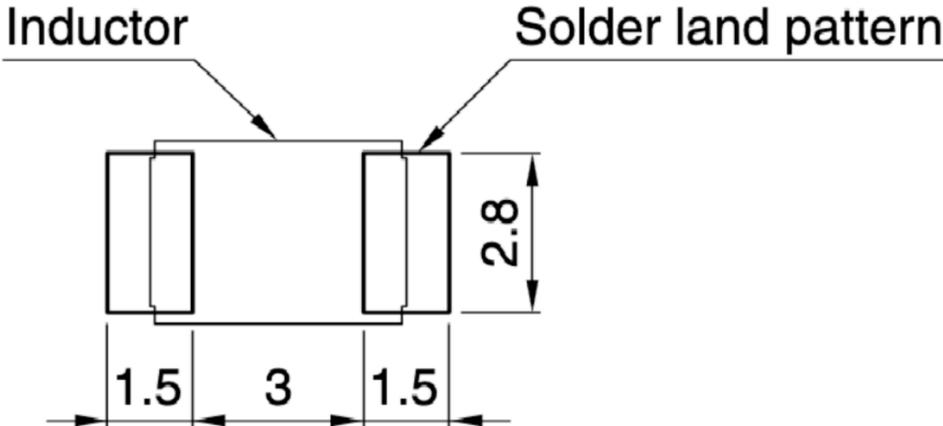
same problem for any commercial inductor



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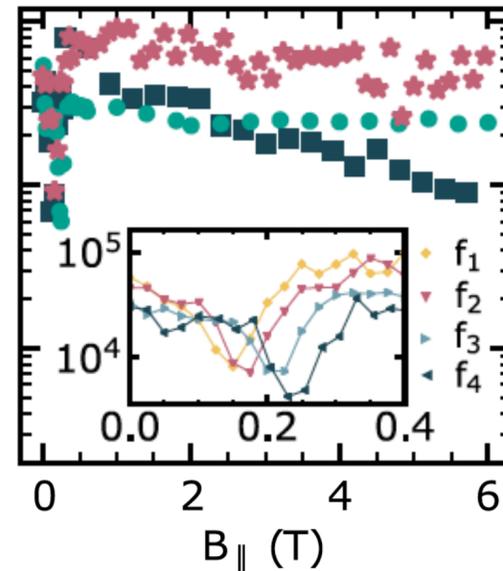
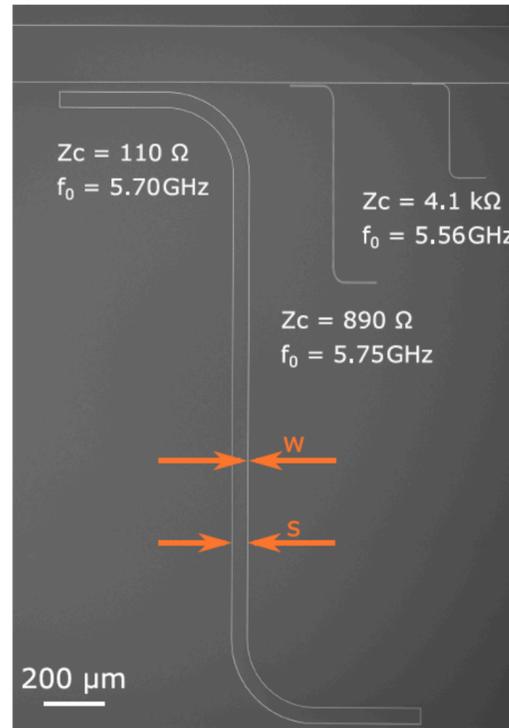
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**Wanted: parasitic capacitance under control AND Low loss**

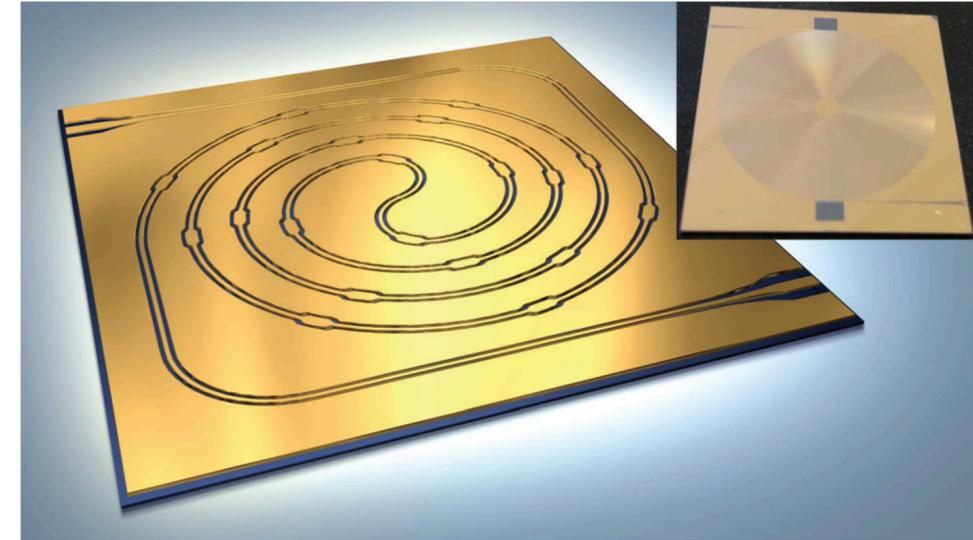
# Large inductance quantum circuit: why should you care ?

Small footprint/Magnetic field resilience



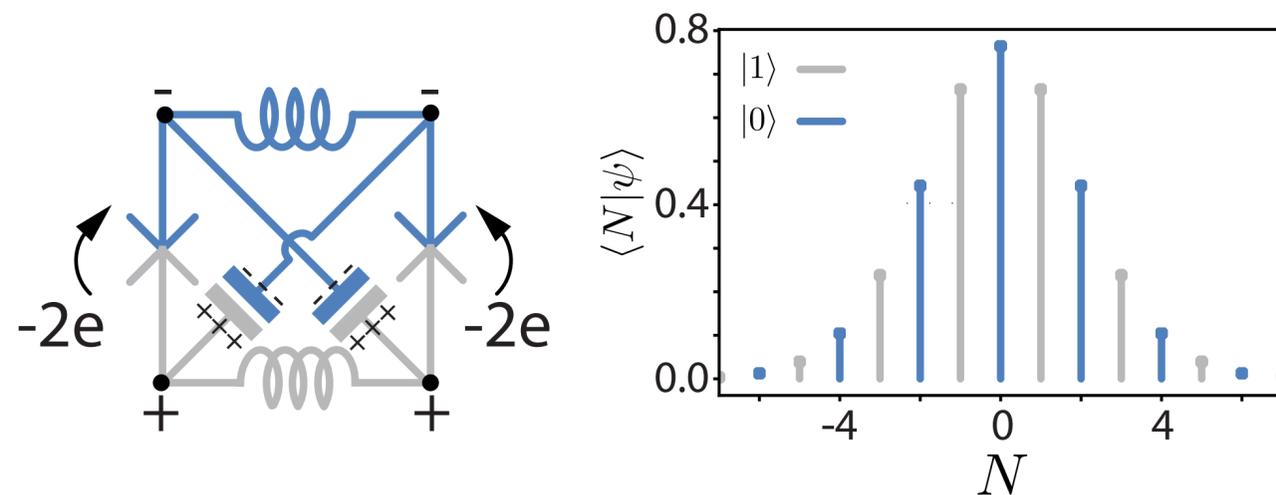
C. Yu et al., 21'

Slow light ( $v = 1/\sqrt{lc}$ )



M. R. Vissers et al., 16'

Large inductance: new devices



A. Kitaev, 06'

Large phase fluctuations:  
fundamental questions

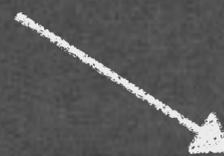
$$Z = jL\omega$$

What about  $L \rightarrow \infty$  ?

# Large inductance quantum circuit: Josephson junctions

Top Superconducting Electrode

$$\Psi_1 = |\Psi_1| e^{i\theta_1}$$



Bottom Superconducting Electrode

$$\Psi_2 = |\Psi_2| e^{i\theta_2}$$



300 nm



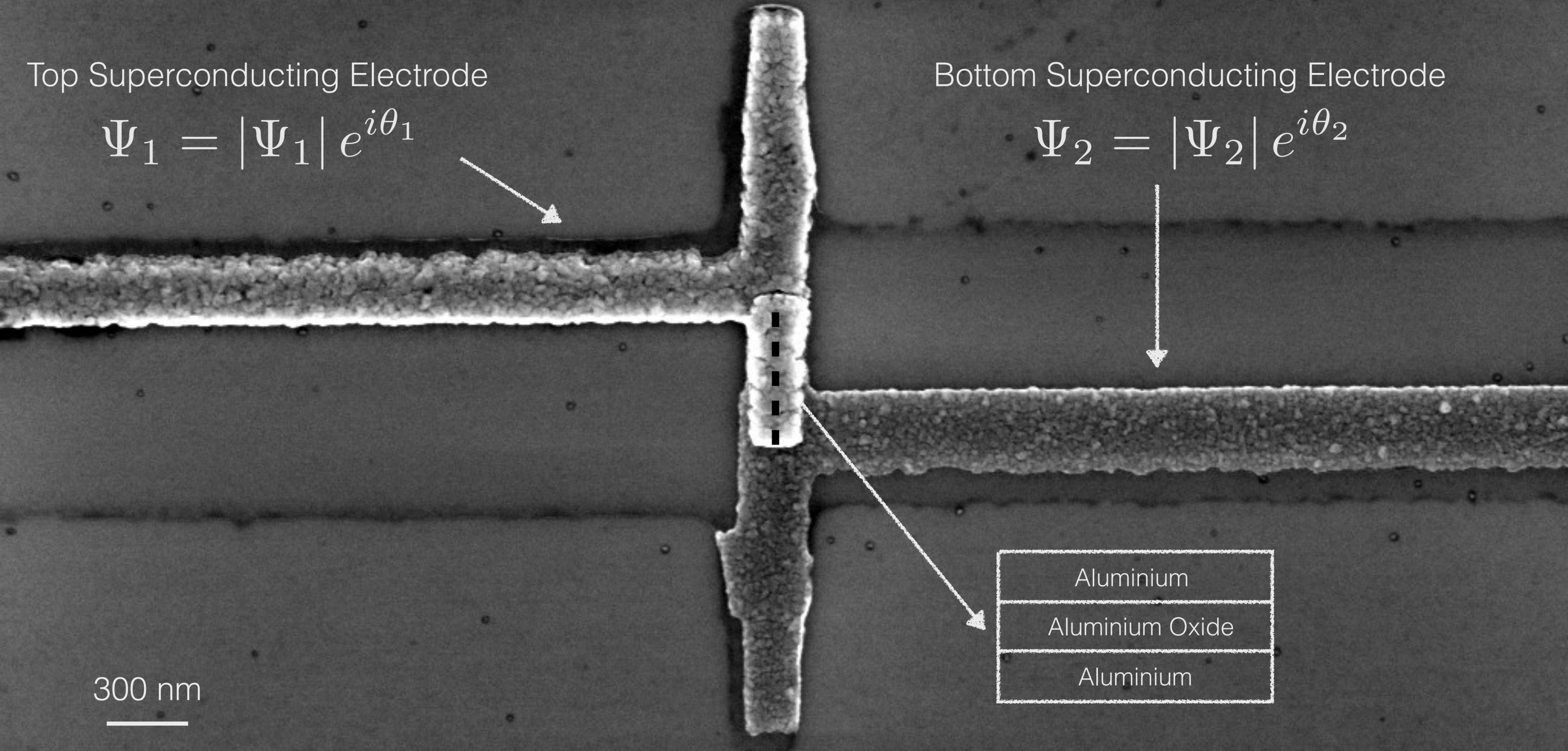
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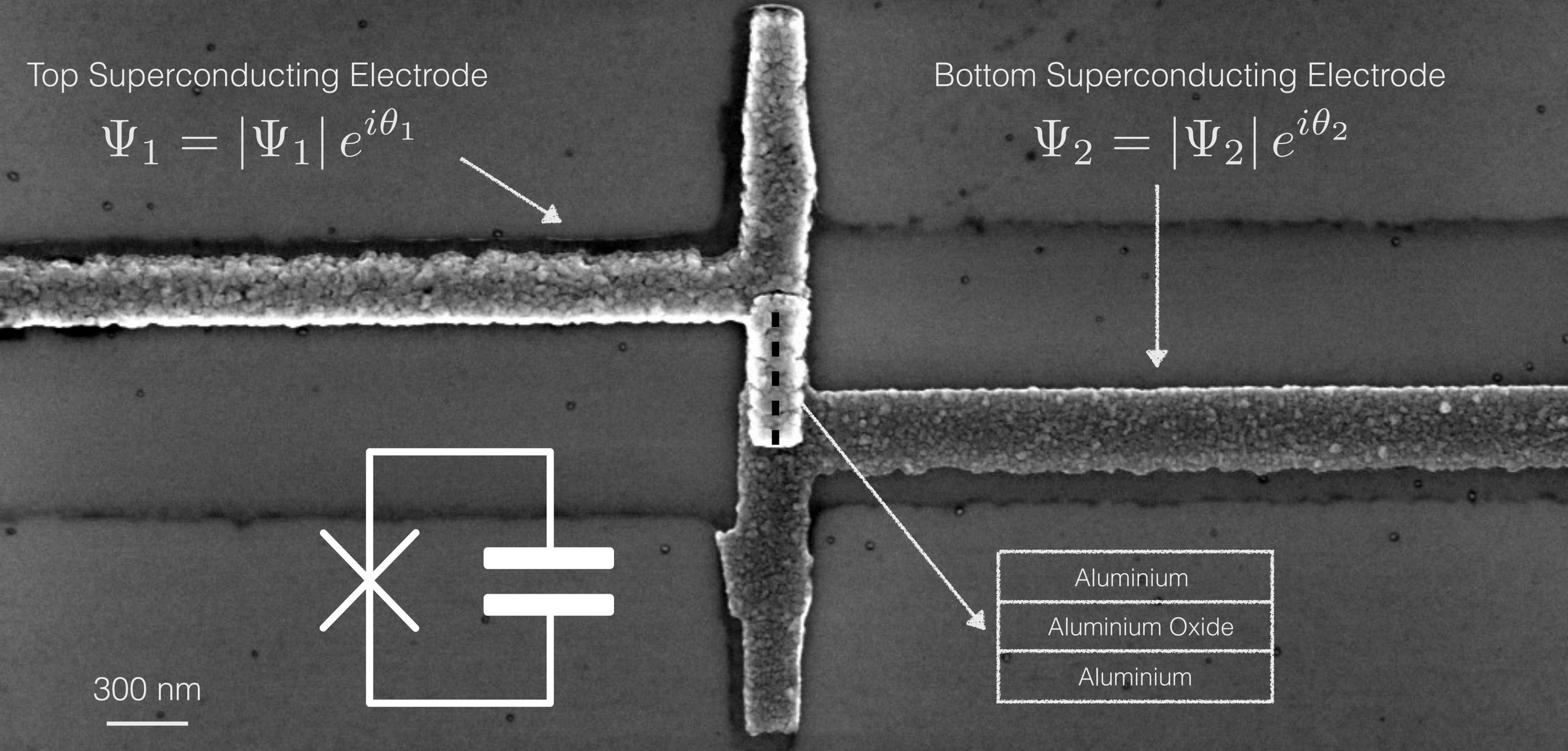
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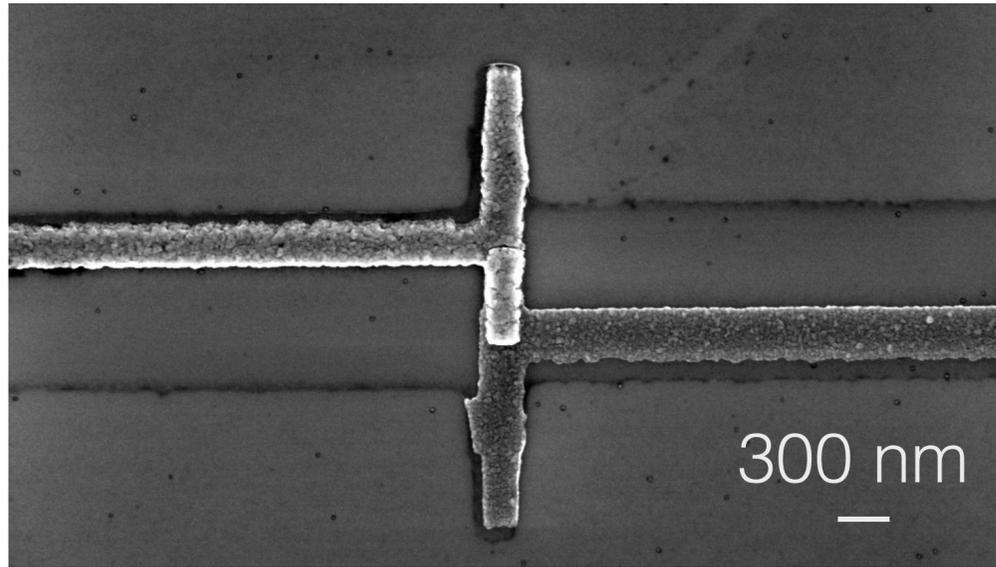
$$\Psi_2 = |\Psi_2| e^{i\theta_2}$$



300 nm



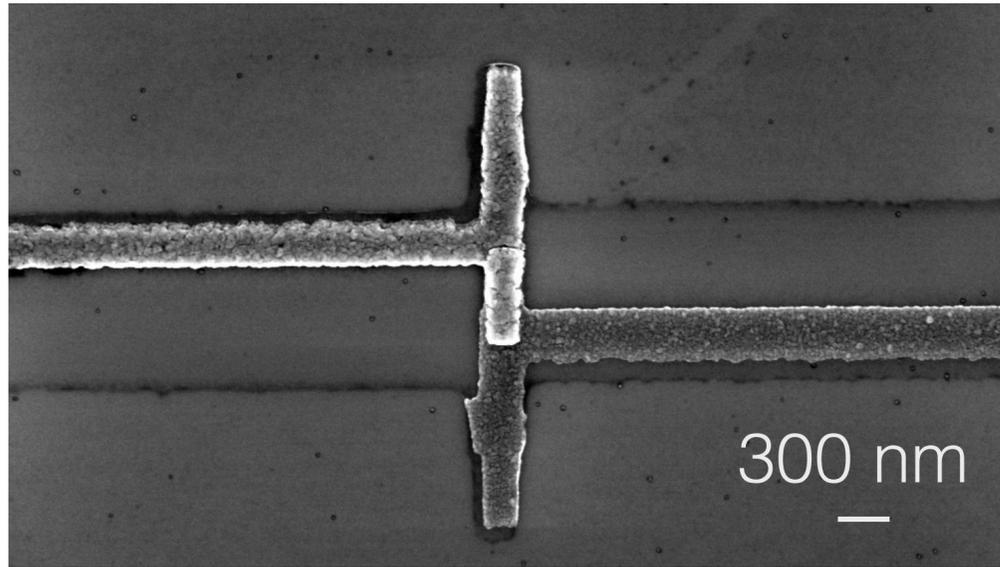
# Large inductance quantum circuit: Josephson junctions



Current-phase relation (first Josephson relation)

$$i(t) = I_c \sin(\varphi(t)) \quad \text{with} \quad \varphi(t) = \theta_1 - \theta_2$$

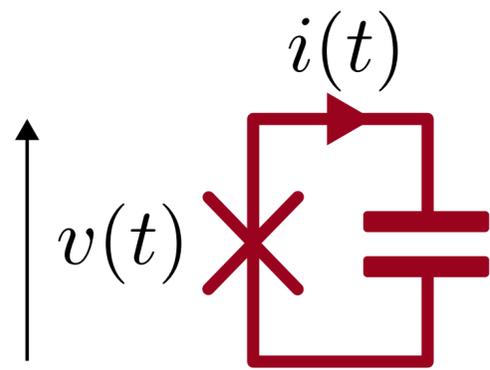
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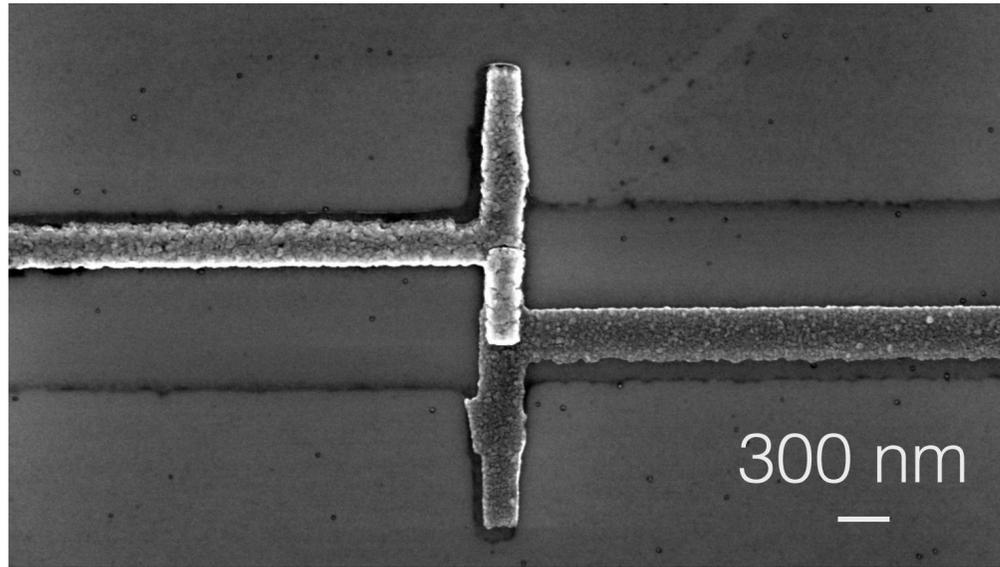
$$i(t) = I_c \sin(\varphi(t)) \quad \text{with} \quad \varphi(t) = \theta_1 - \theta_2$$

Superconducting phase evolution (second Josephson relation)



$$\frac{\partial \varphi}{\partial t} = \frac{v(t)}{\varphi_0} \quad \text{with} \quad \varphi_0 = \hbar/2e$$

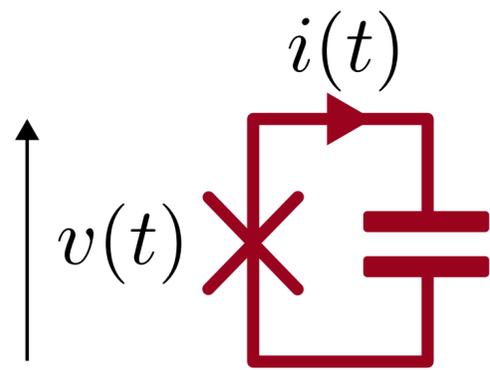
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Superconducting phase evolution (second Josephson relation)



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Josephson (kinetic) inductance versus geometric inductance

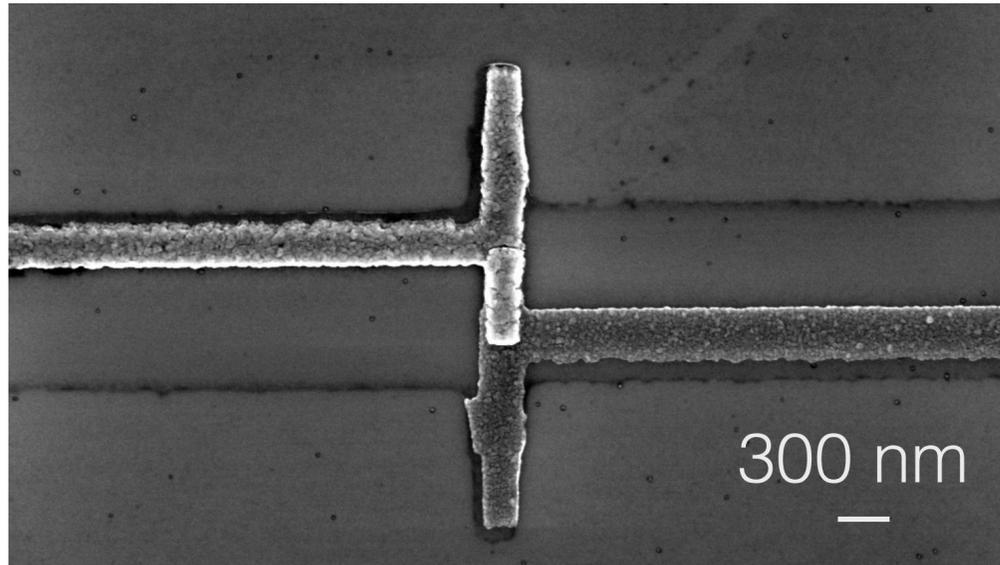
Geometric inductance: **0.2 pH/ $\mu\text{m}$**

Josephson inductance: **1 nH/ $\mu\text{m}$**

$$L = \frac{\mu_0 l}{2\pi}$$

$$L_J = \varphi_0 \left( \frac{\partial i}{\partial \varphi} \right)^{-1} = \frac{L_{J,0}}{\cos \varphi} \quad \text{with} \quad L_J = \frac{\varphi_0}{I_c}$$

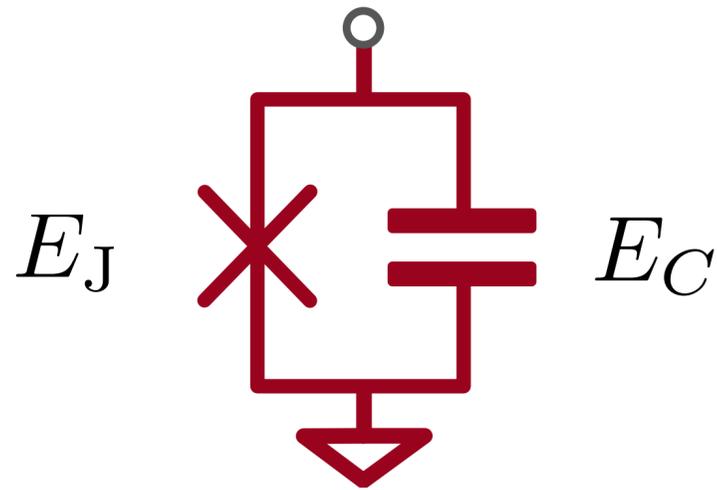
# Non-linearity and Josephson junction



Current-phase relation

$$i(t) = I_c \sin(\varphi(t)) \quad \text{with} \quad \varphi(t) = \theta_1 - \theta_2$$

Energy scales



Charging energy

$$E_C = \frac{(2e)^2}{2C_J}$$

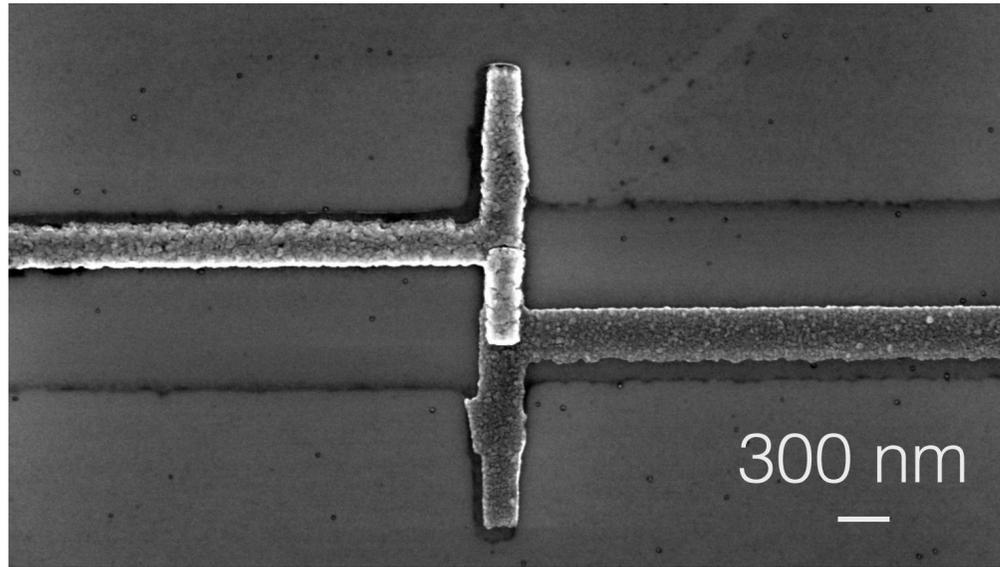
Josephson energy

$$E_J = \varphi_0 I_c$$

Characteristic impedance

$$Z_J = \frac{Z_q}{2\pi} \sqrt{\frac{2E_C}{E_J}} \quad \text{with} \quad Z_q = \frac{h}{(2e)^2} \simeq 6.5\text{k}\Omega$$

# Non-linearity and Josephson junction

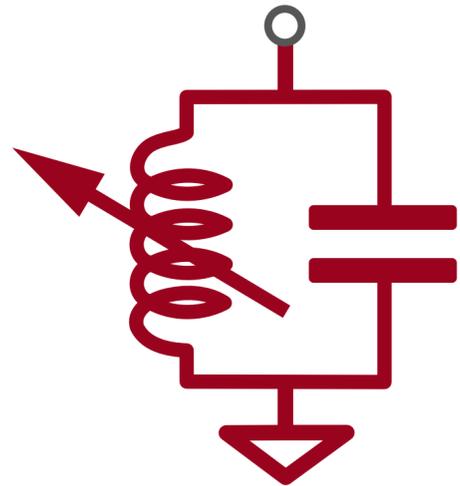


Current-phase relation

$$i(t) = I_c \sin(\varphi(t)) \quad \text{with} \quad \varphi(t) = \theta_1 - \theta_2$$

Two limits  $Z_J \ll Z_q$        $Z_J \gtrsim Z_q$

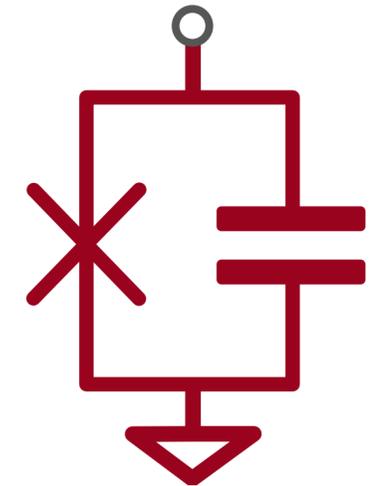
$$\langle \varphi(t)^2 \rangle \ll 1$$



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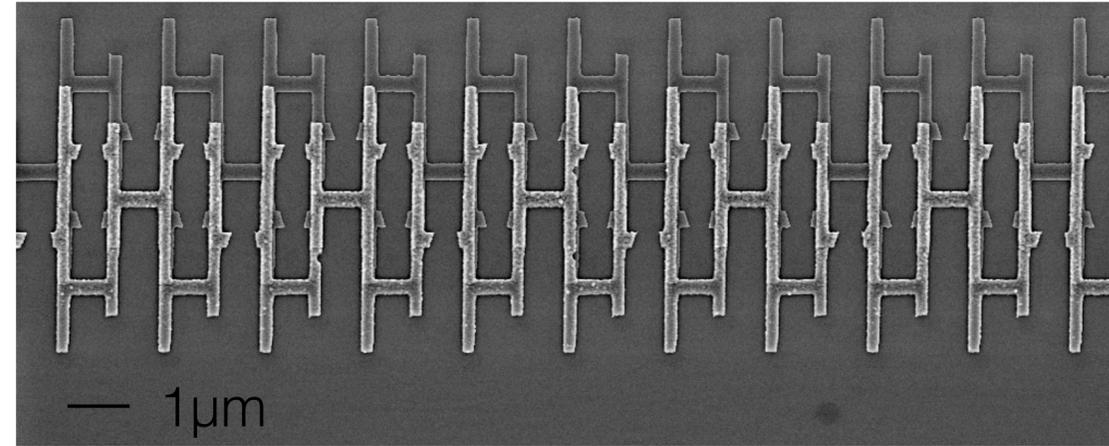
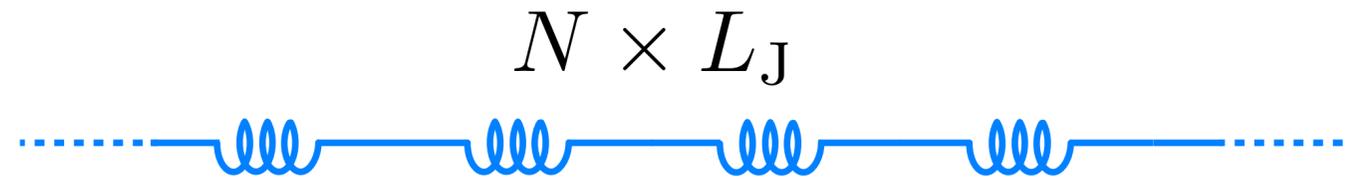
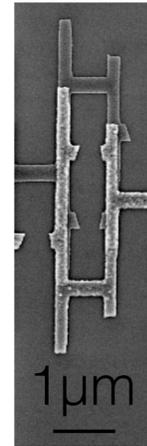
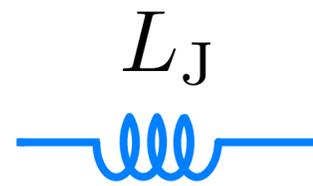
$$\langle \varphi(t)^2 \rangle \gtrsim 1$$



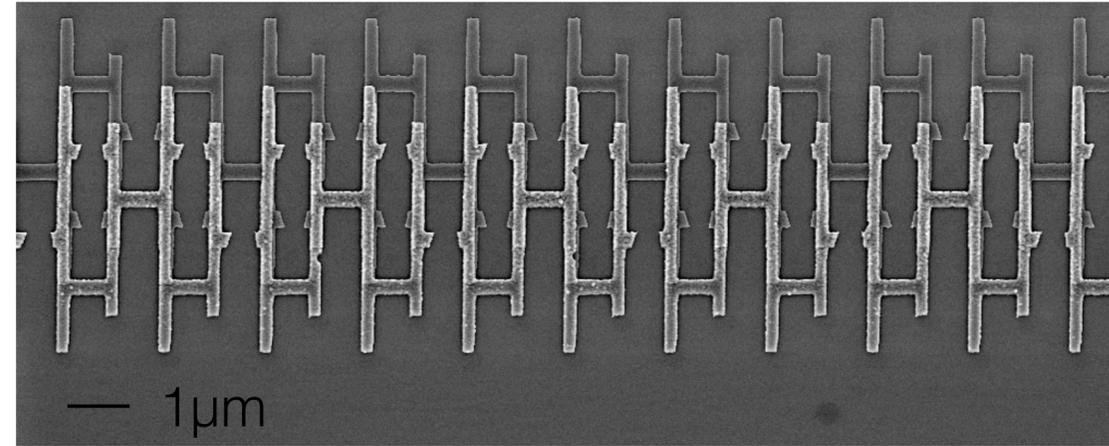
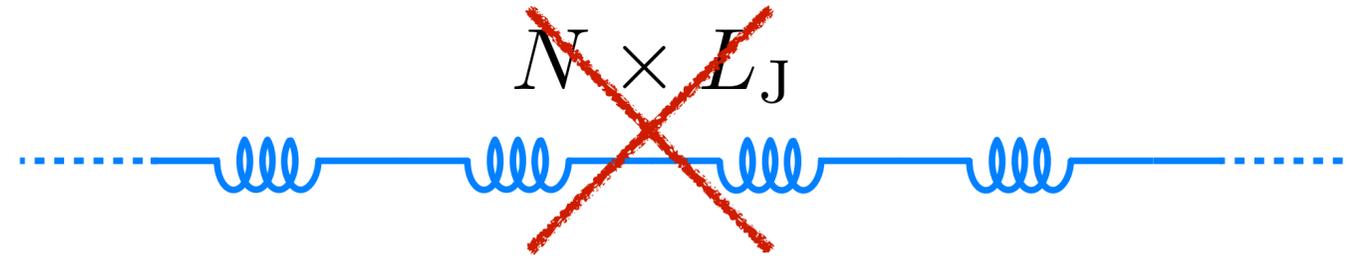
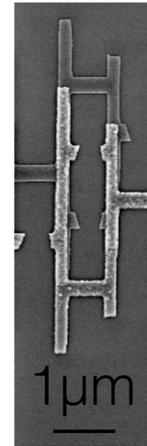
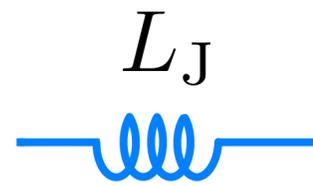
$$i(t) = I_c \varphi(t) + I_c \frac{\varphi(t)^3}{3!} + \dots$$

$$i(t) = I_c \sin(\varphi(t))$$

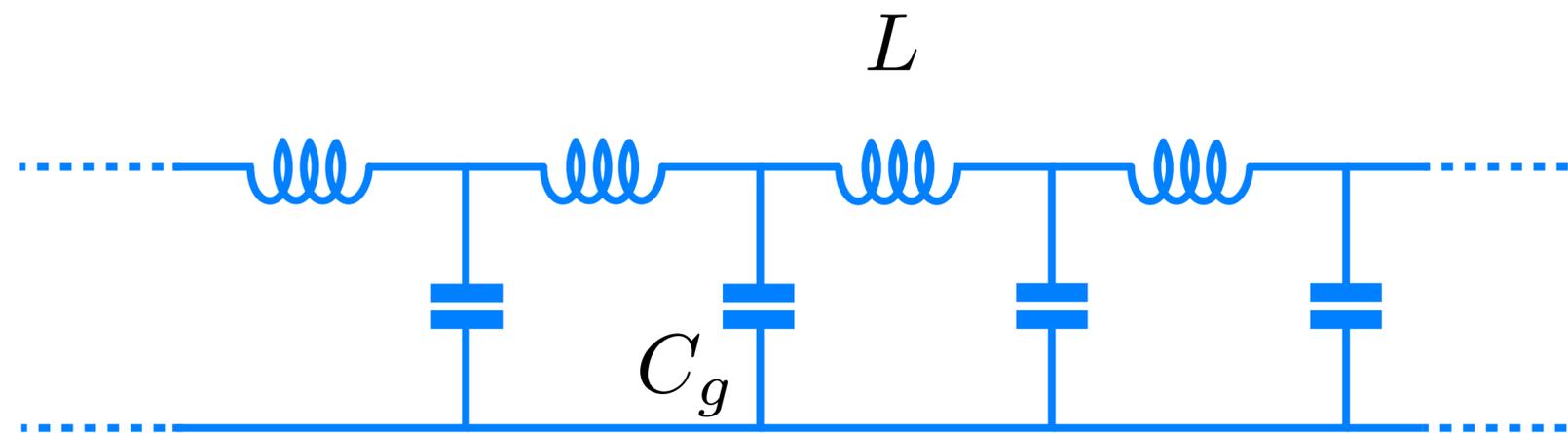
# Josephson junction meta-material



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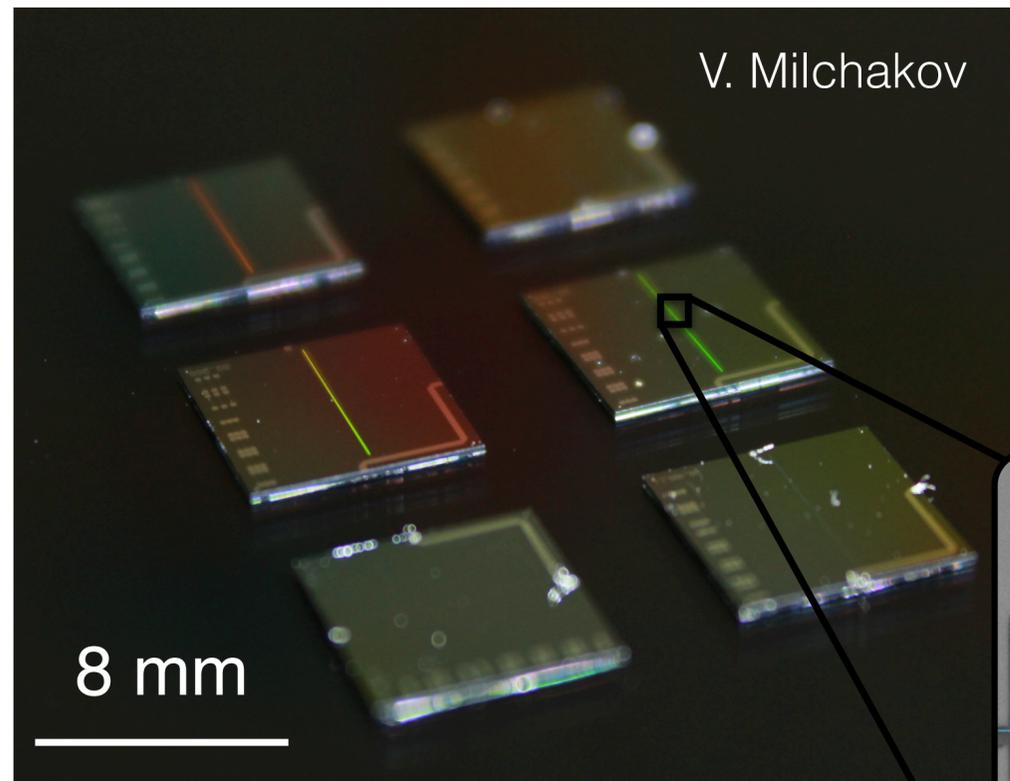
“Plasma” modes or transmission line modes J. E. Mooij and G. Schön, 85’



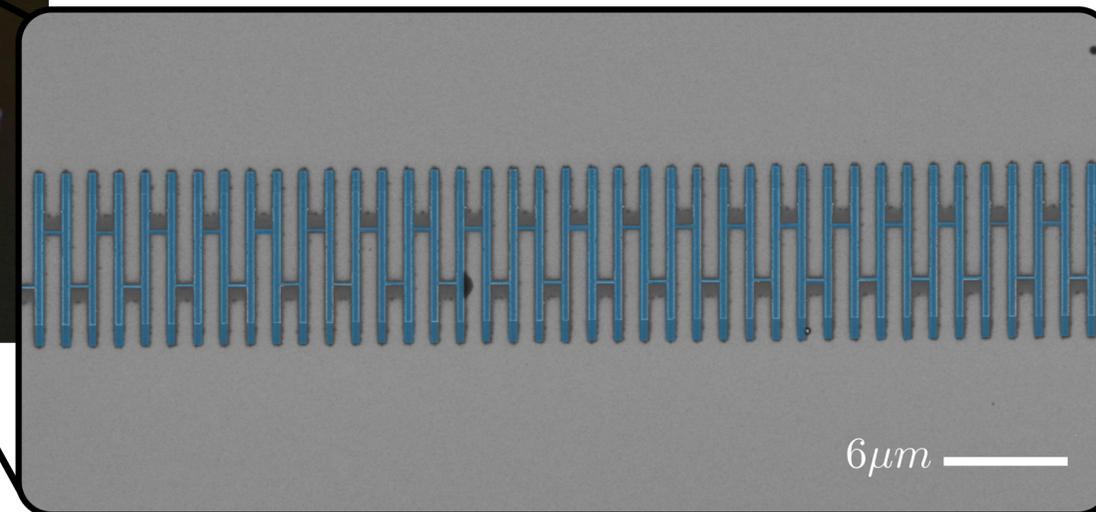
Plasma modes in JJ meta-material: N. Masluk et al 12’, Bell et al 12’, S. Butz et al. 13’, C. Altimiras et al. 13’, R. Kuzmin et al 18’....

Observation in disordered superconductors (granular aluminium): O. Buisson et al., 94’

# Josephson junction meta-material: Fabrication



1D array of Josephson junctions:  
up to 10 000 cells



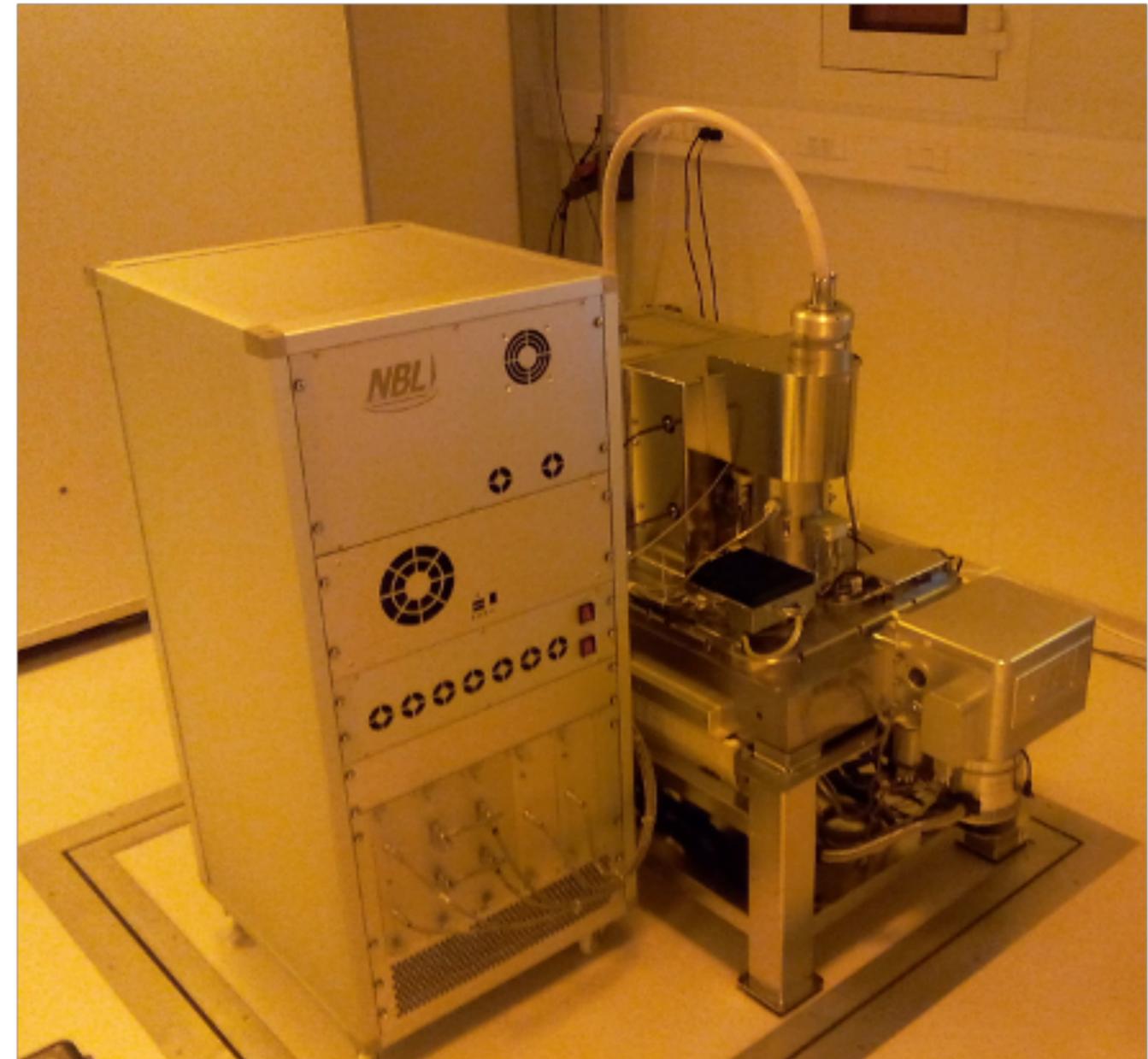
Challenges faced: stitching errors, resist homogeneity, focus homogeneity, proximity effect....

# Josephson junction meta-material: Fabrication

Pôle Nanofab

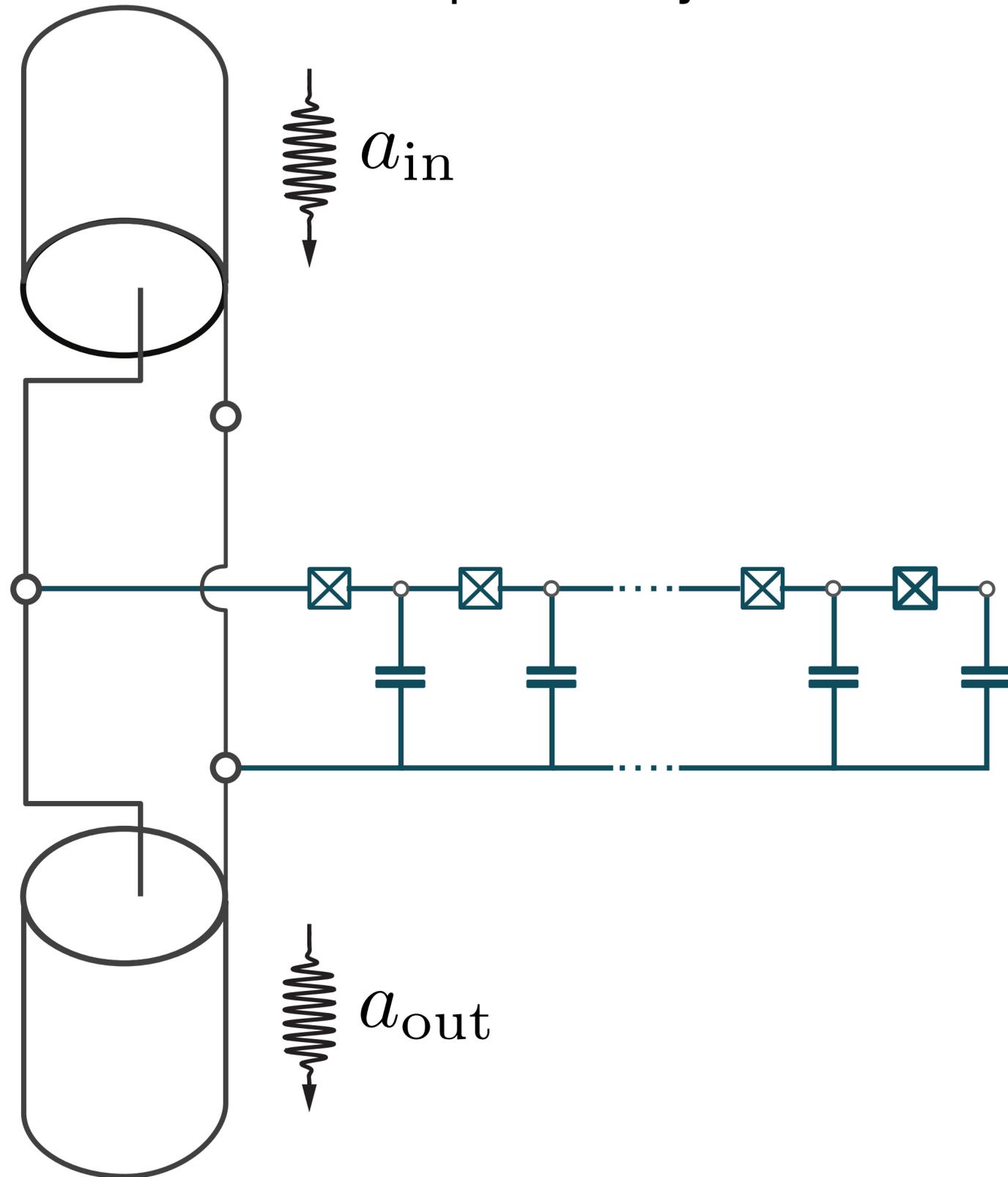


E-beam evaporator "Suprafab"



E-beam writer "Nanobeam"

# Josephson junction meta-material

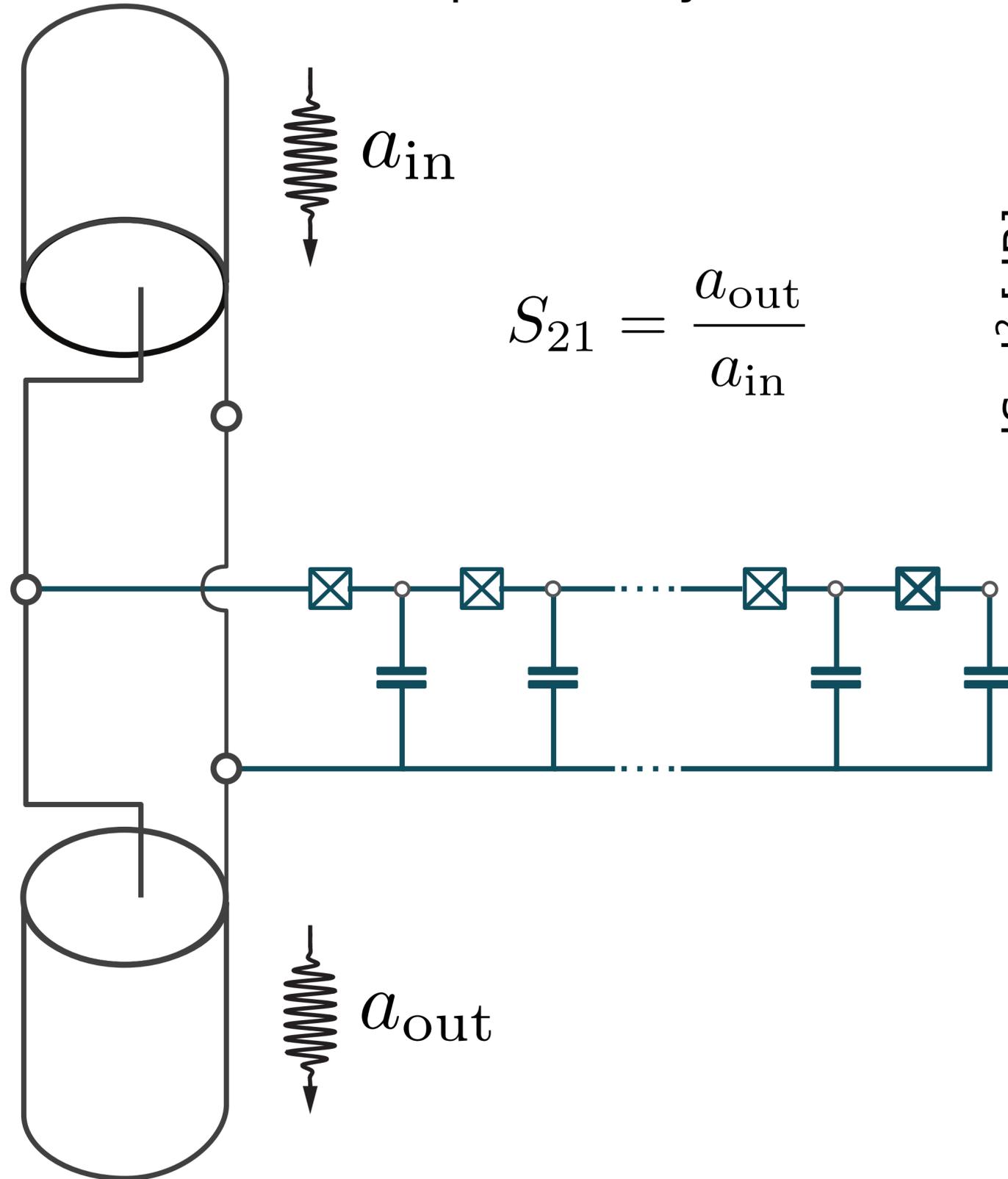


high frequency  
& low temperature

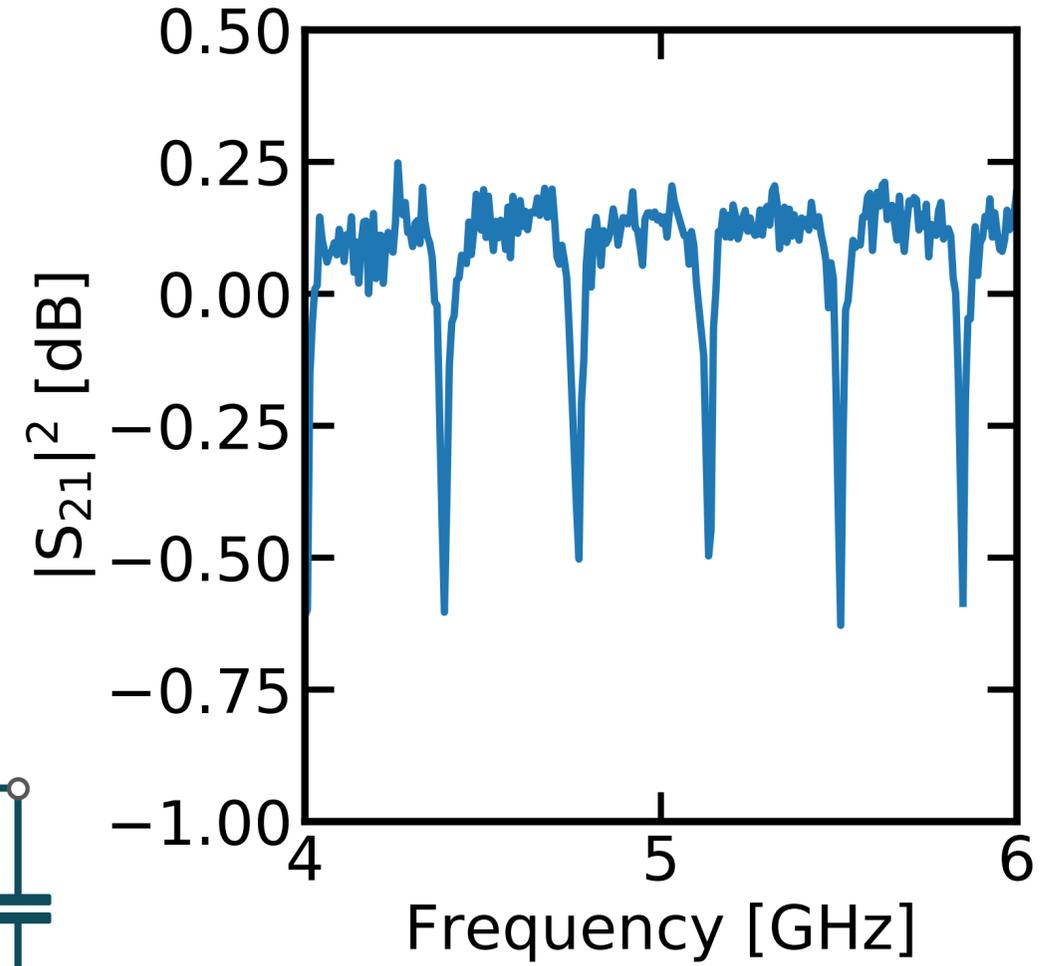
$$\hbar\omega \gg k_B T$$

( $T = 20$  mK)

# Josephson junction meta-material



$$S_{21} = \frac{a_{out}}{a_{in}}$$

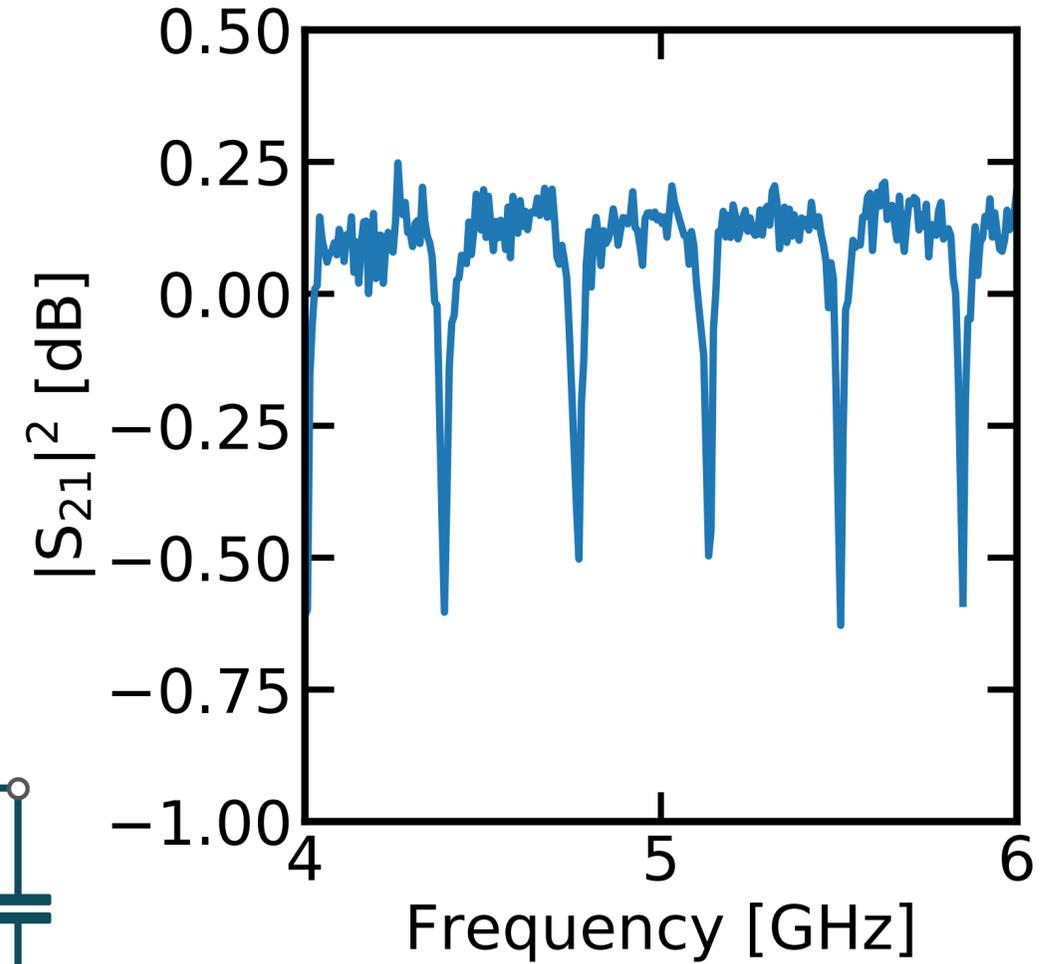
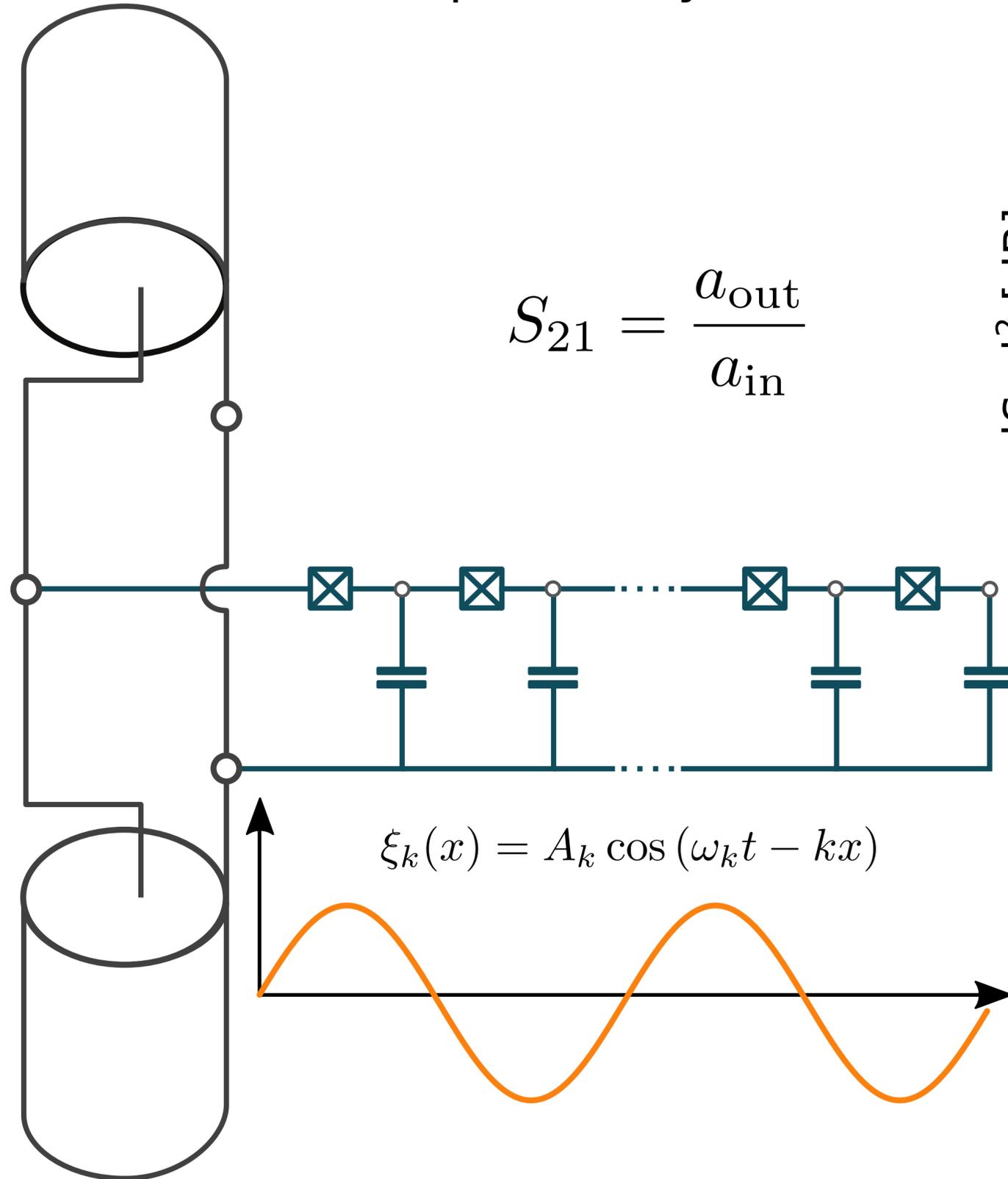


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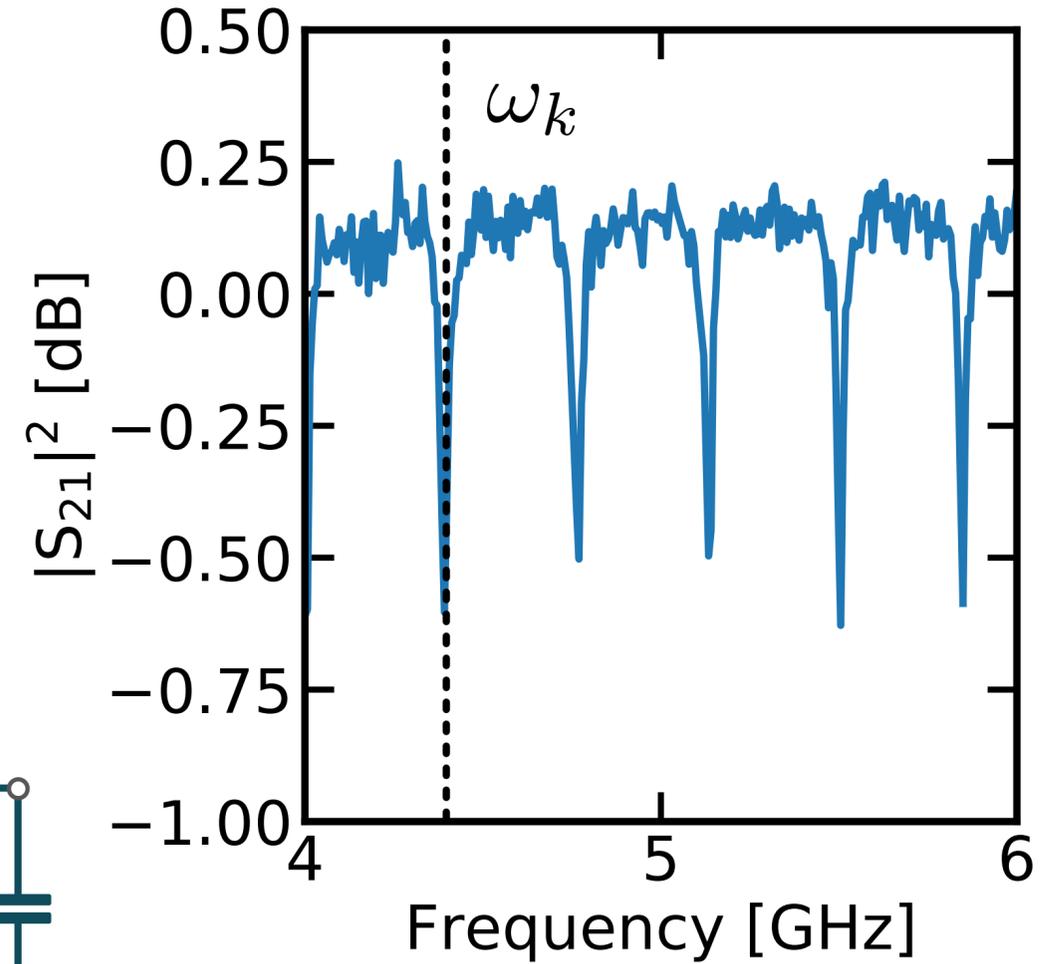
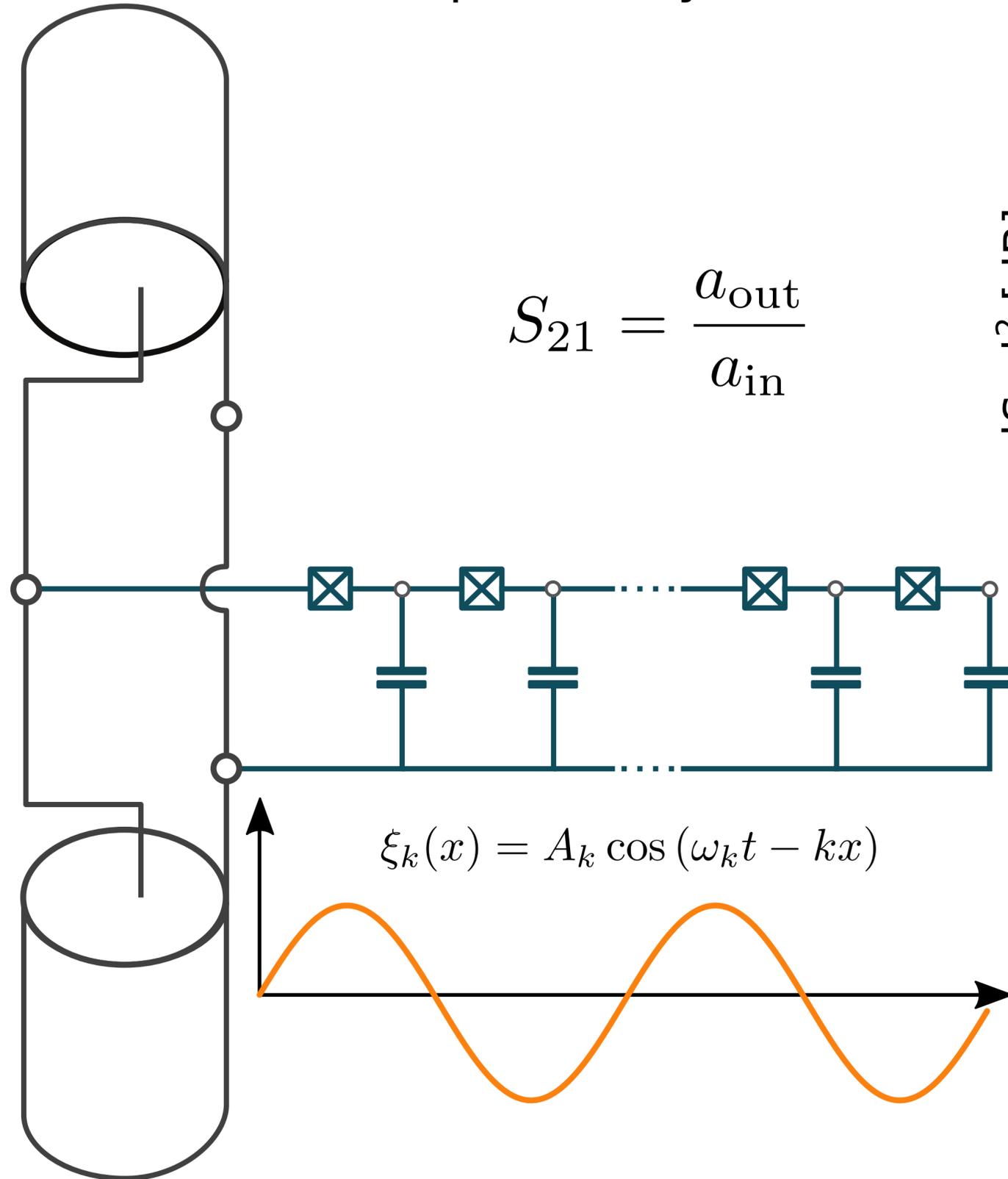


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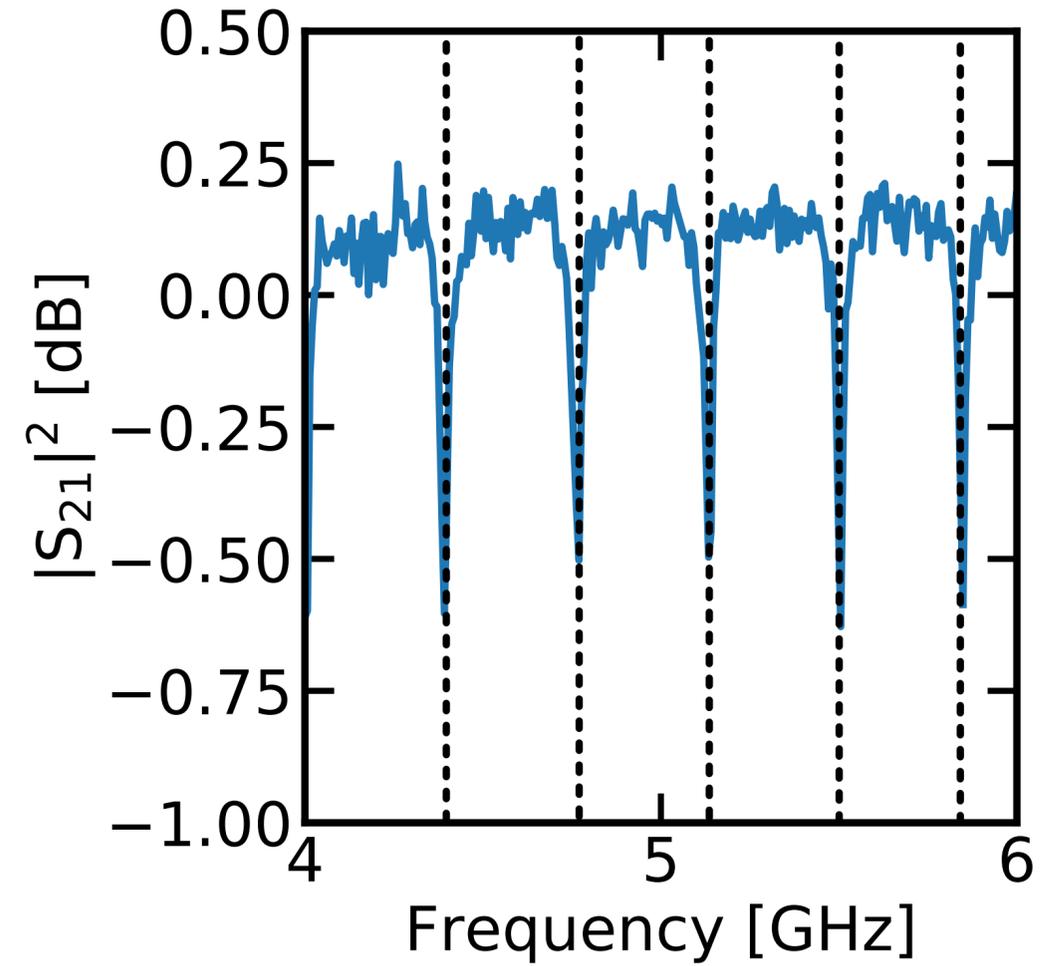
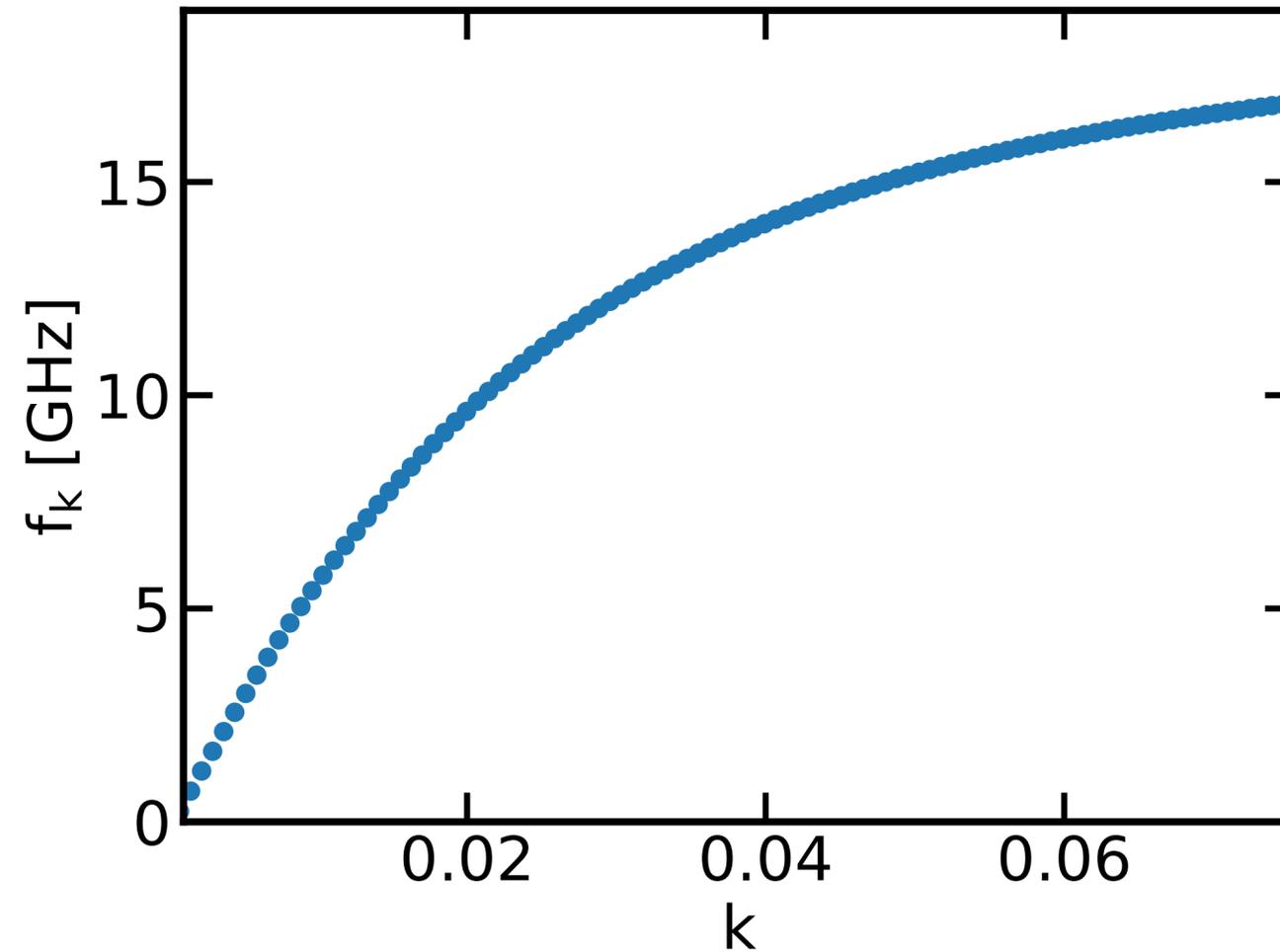


high frequency  
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# JJ meta-material: dispersion relation

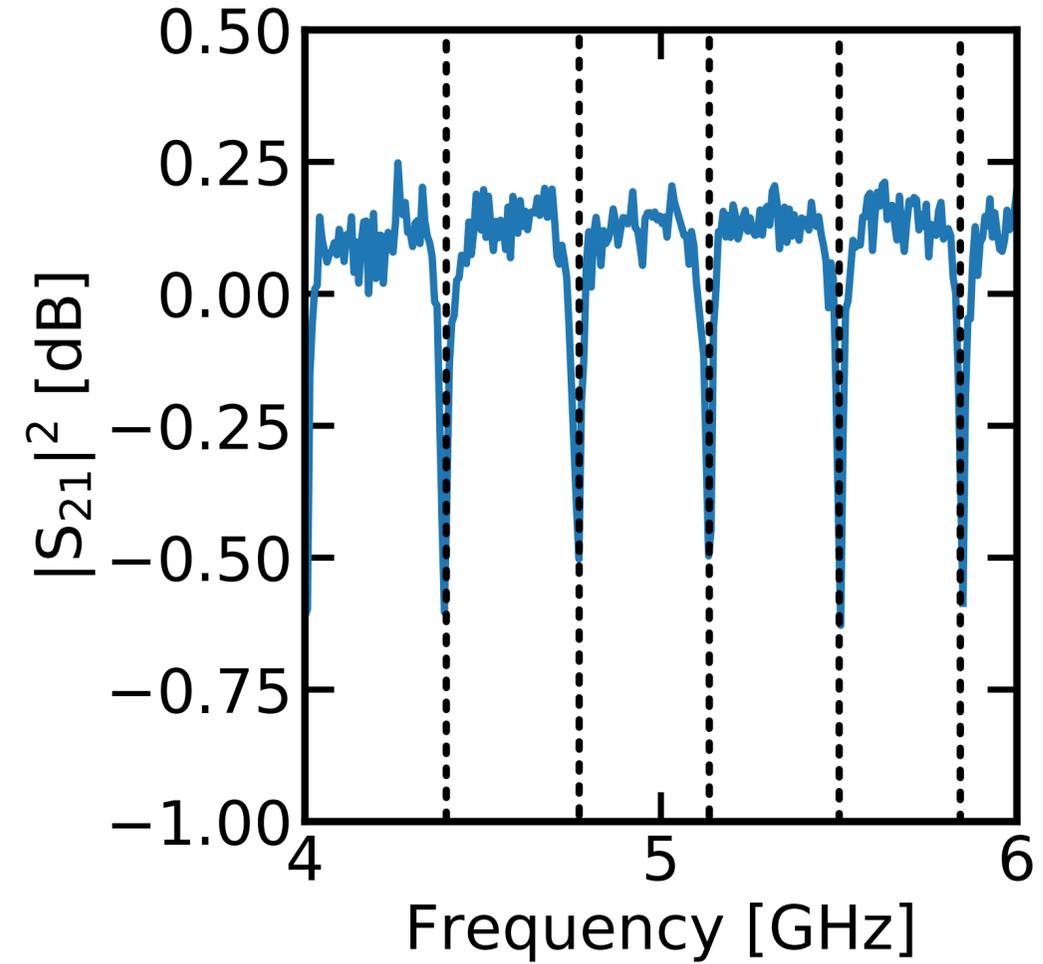
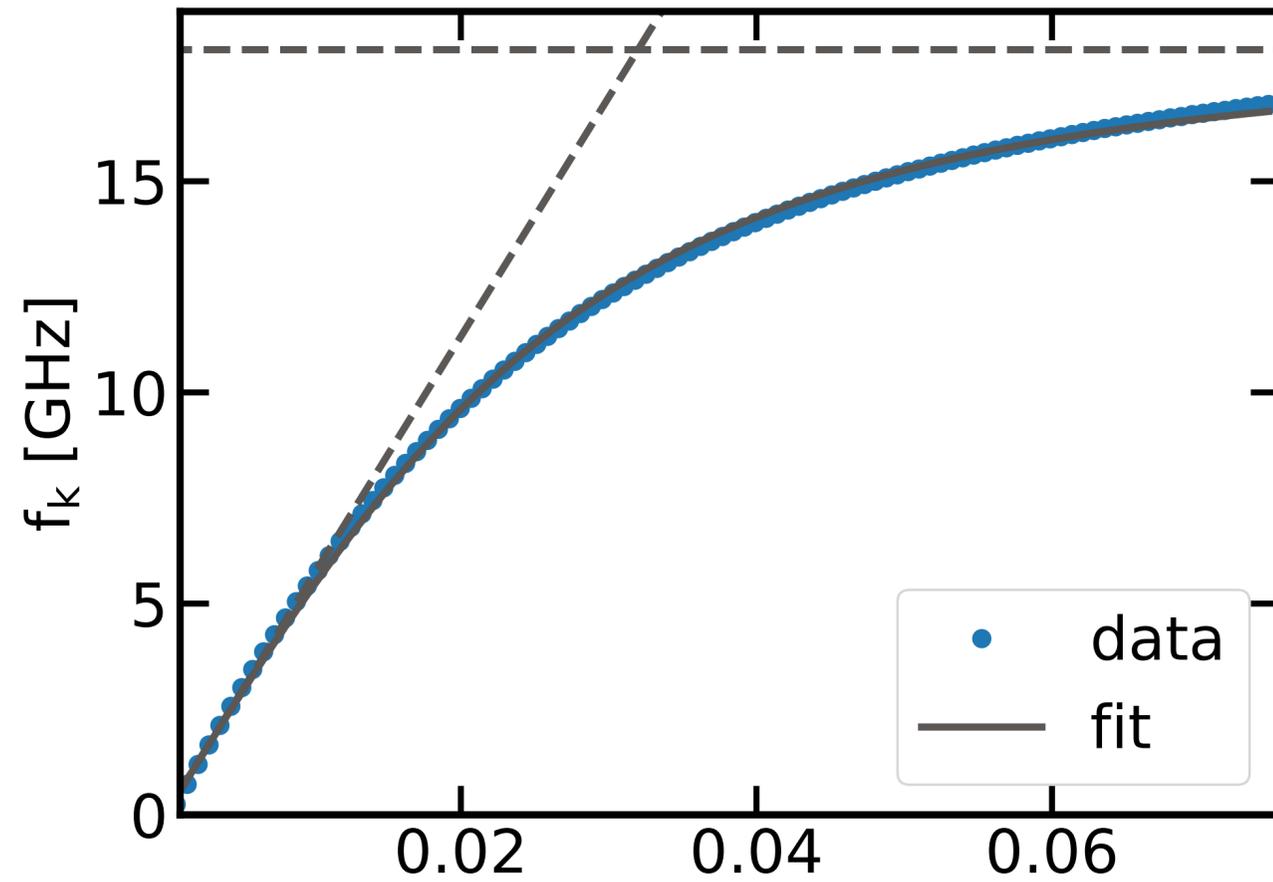


high frequency  
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$$\hbar\omega \gg k_B T$$

$$(T = 20 \text{ mK})$$

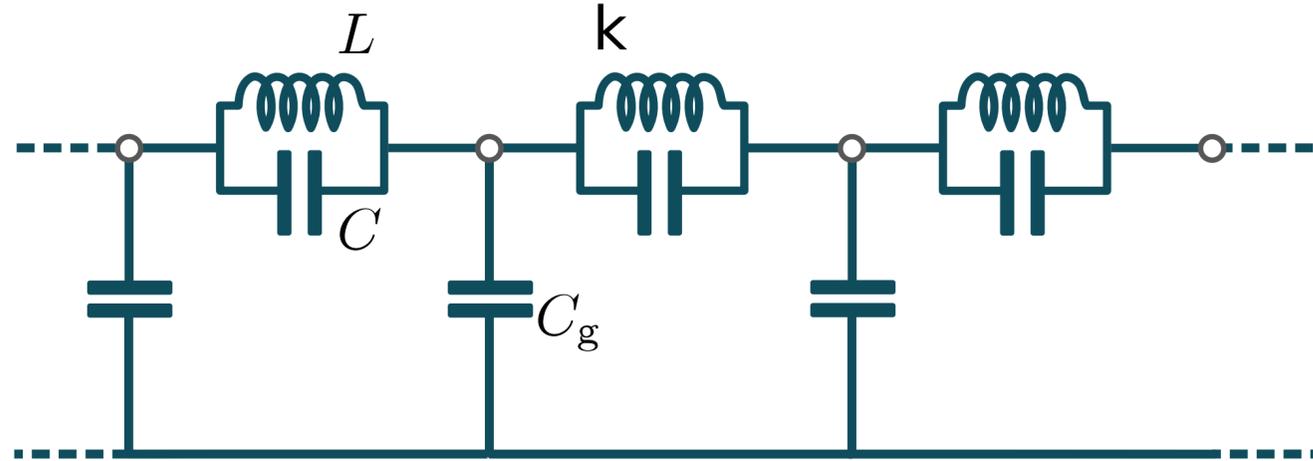
# JJ meta-material: dispersion relation



$$L = 0.53 \text{ nH}$$

$$C = 144 \text{ fF}$$

$$C_g = 0.147 \text{ fF}$$



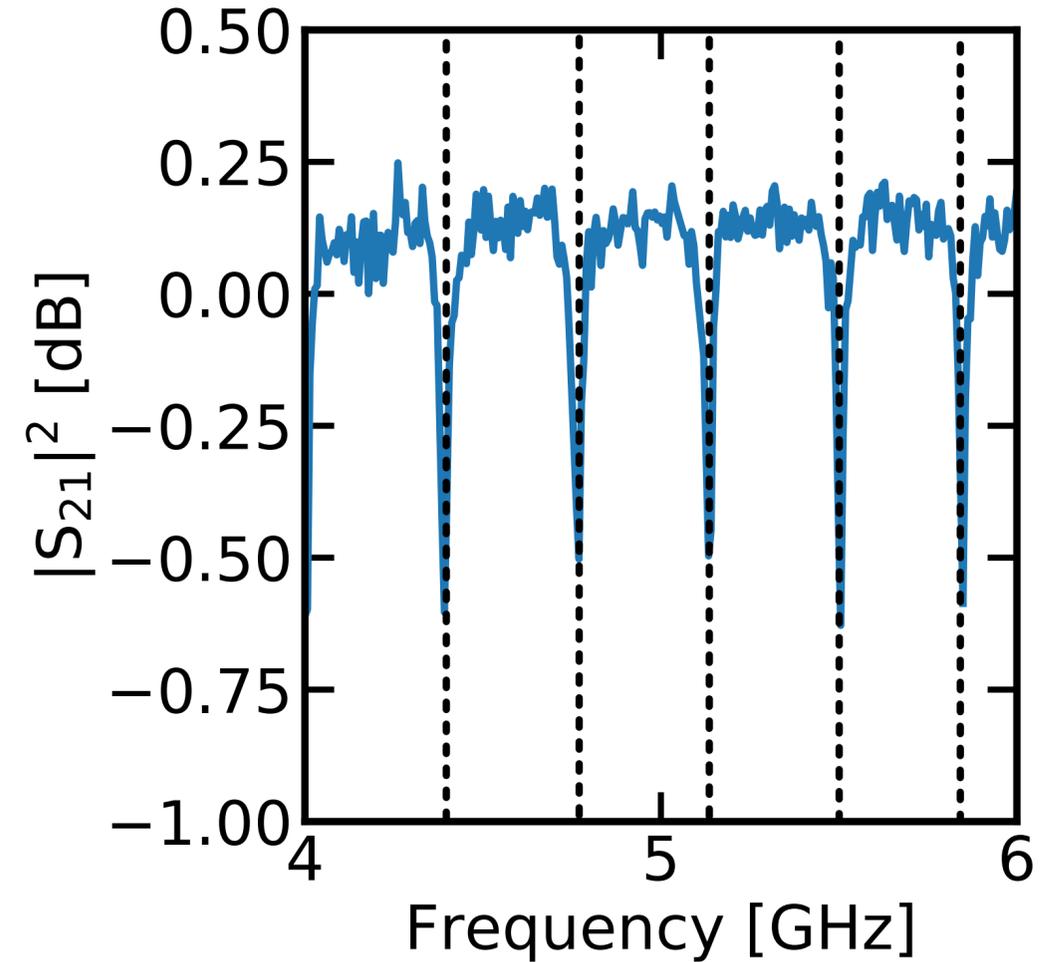
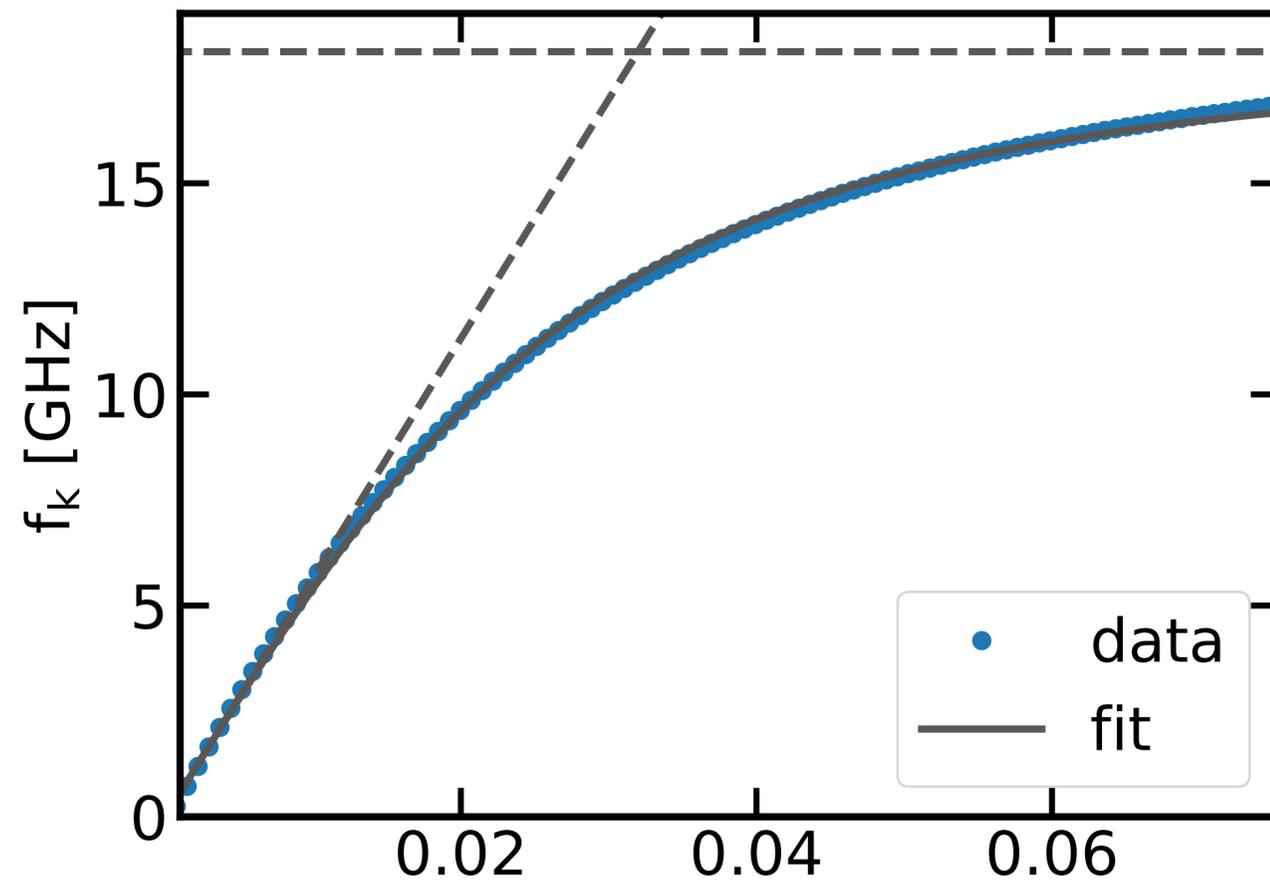
Characteristic impedance

$$Z_c = \sqrt{\frac{L}{C_g}} \sim 1.9 \text{ k}\Omega$$

Quality factor

$$Q_{\text{int}} \sim 10^4$$

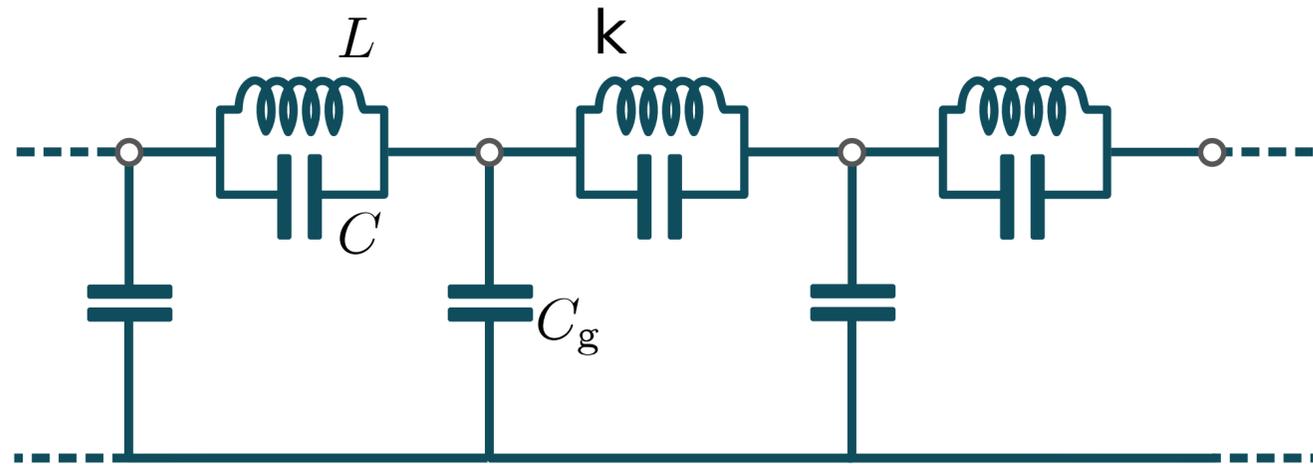
# JJ meta-material: dispersion relation



$$L = 0.53 \text{ nH}$$

$$C = 144 \text{ fF}$$

$$C_g = 0.147 \text{ fF}$$



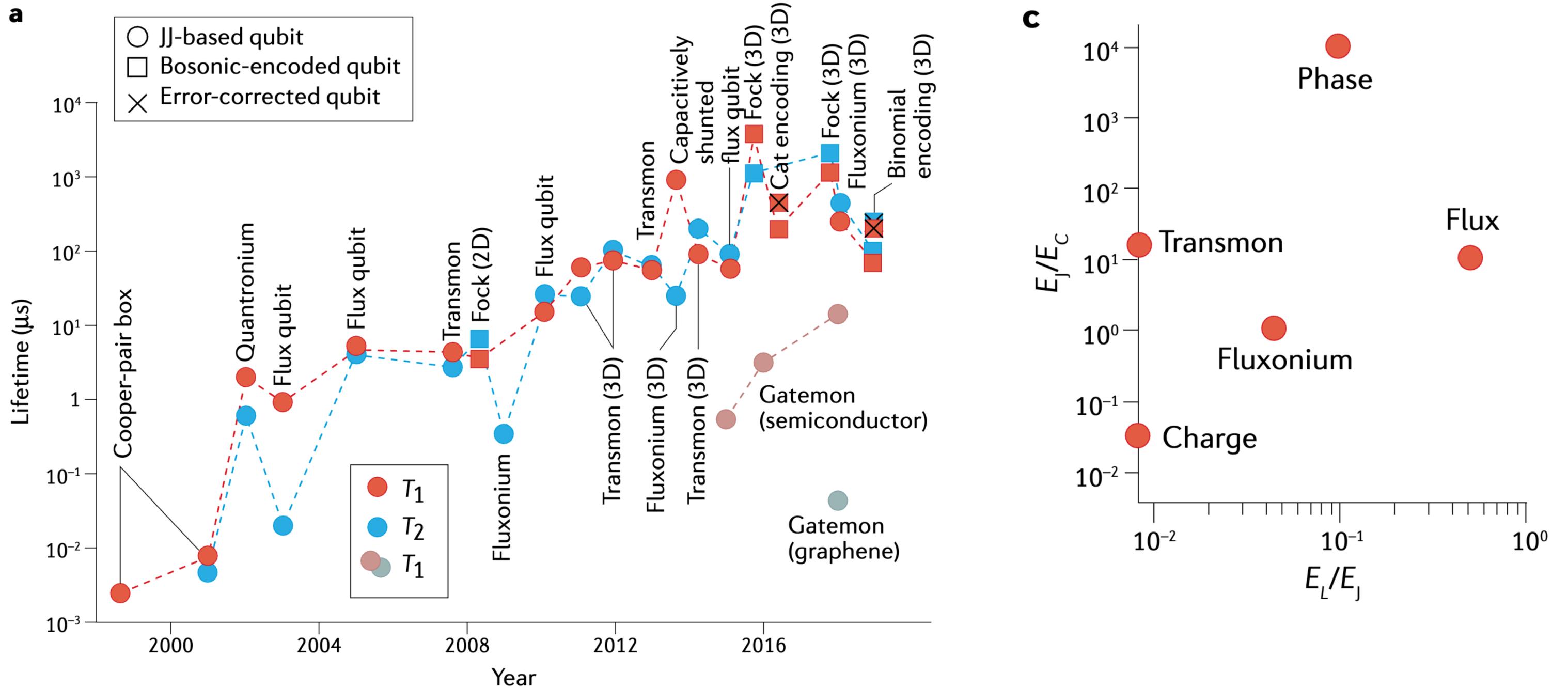
Characteristic impedance  $Z_c = \sqrt{\frac{L}{C_g}} \sim 1.9 \text{ k}\Omega$

Quality factor  $Q_{\text{int}} \sim 10^4$

**Wanted: parasitic capacitance under control AND Low loss**

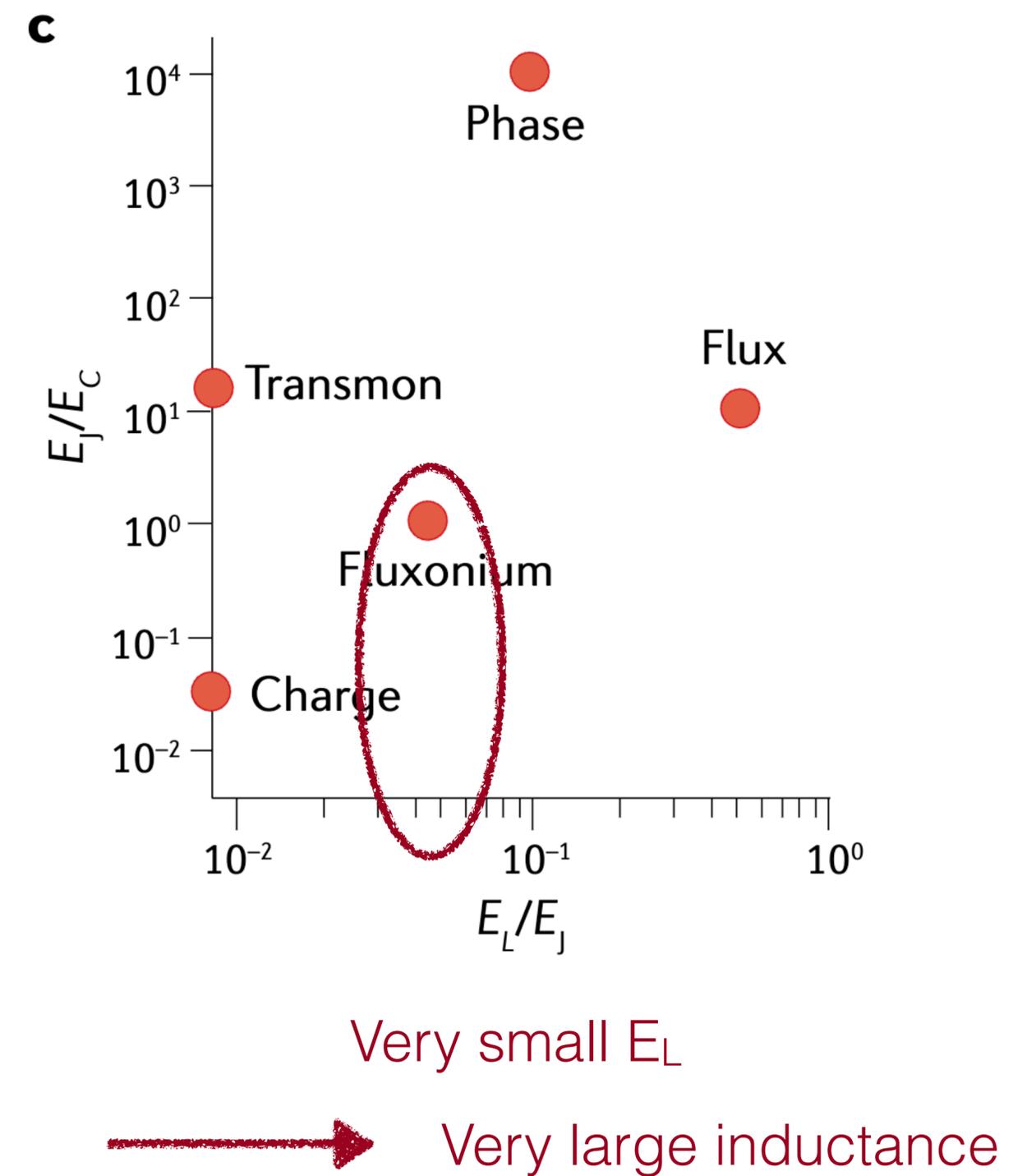
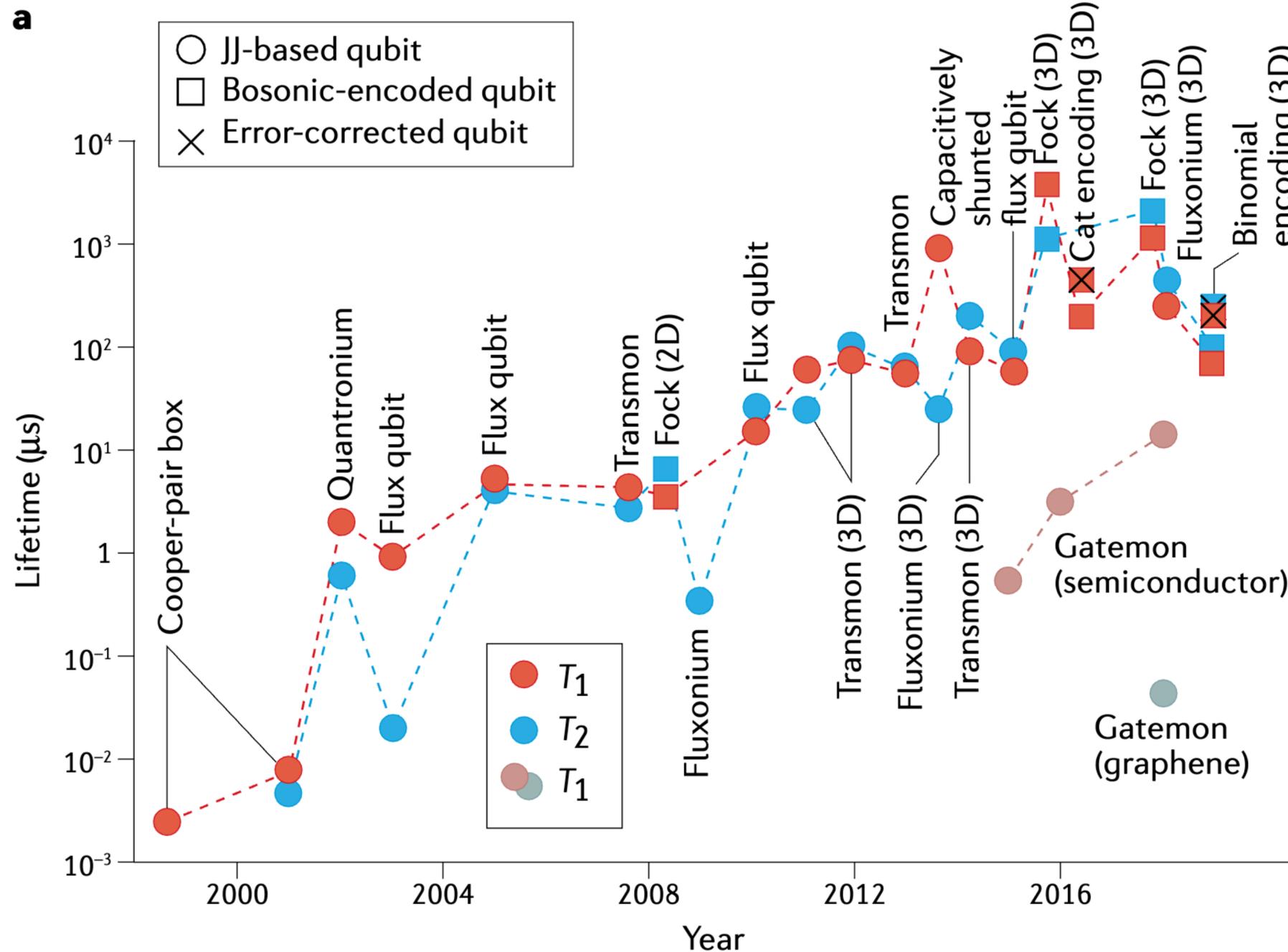
# Fluxonium devices:

Comparison with other Josephson artificial atoms



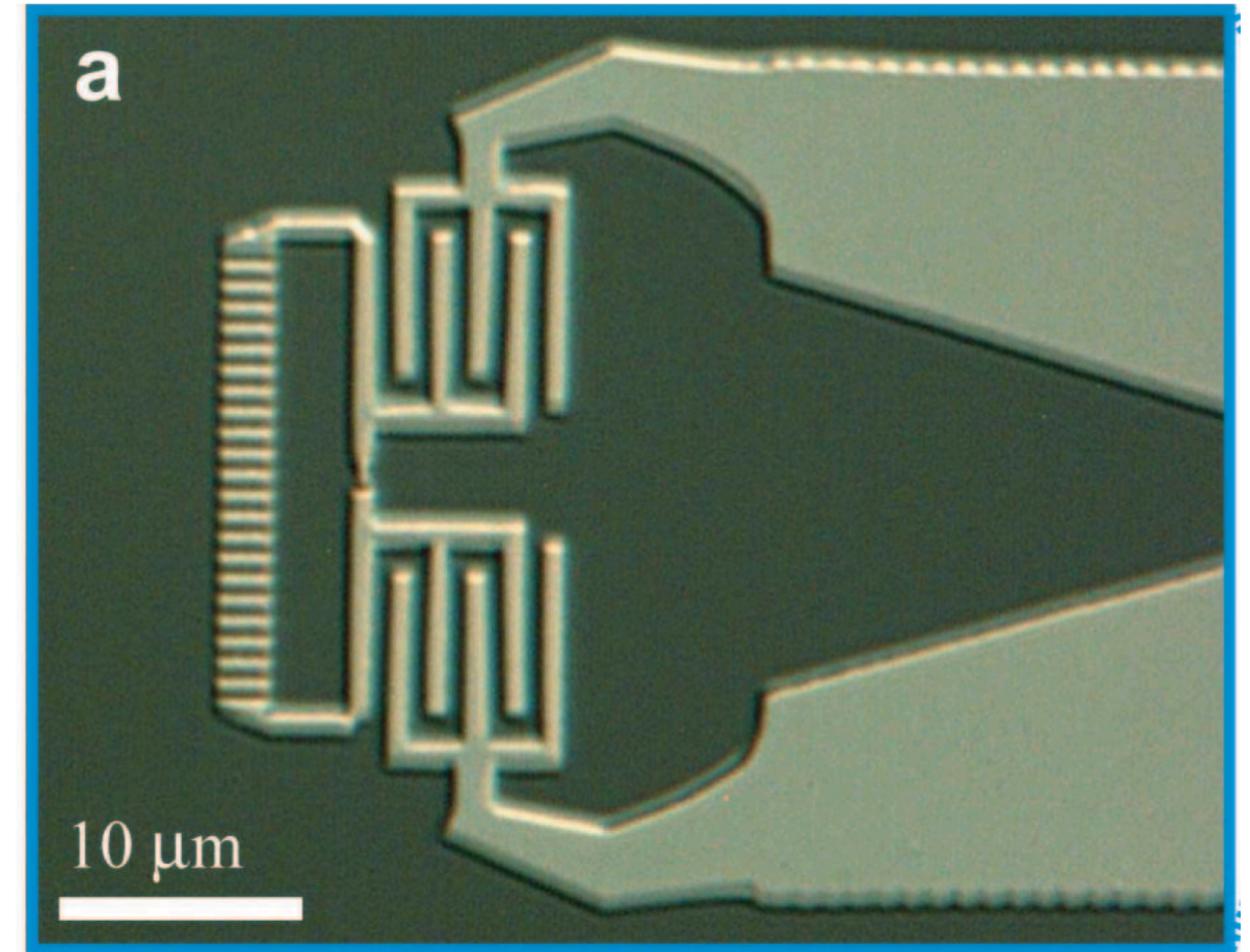
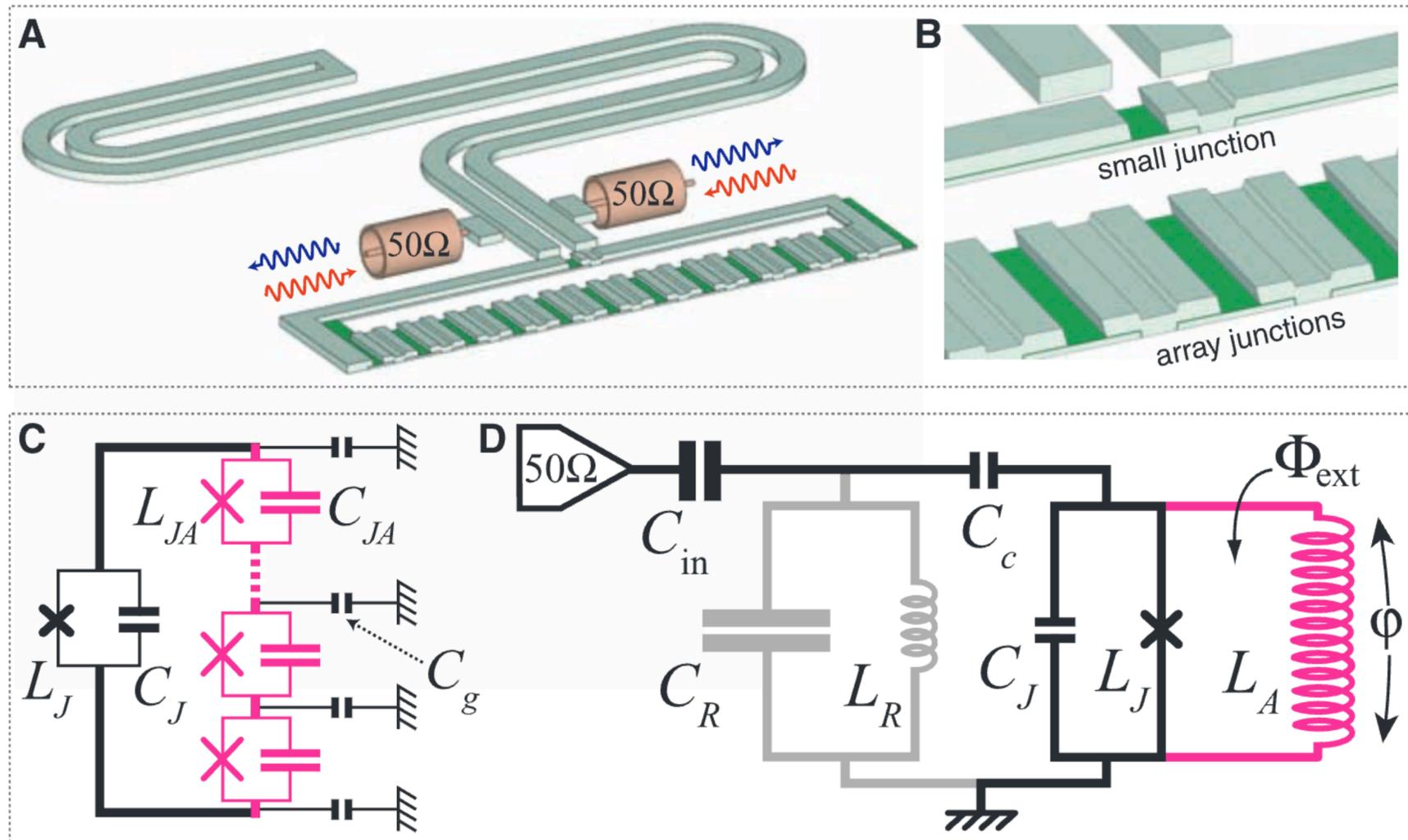
# Fluxonium devices:

Comparison with other Josephson artificial atoms



# Fluxonium devices:

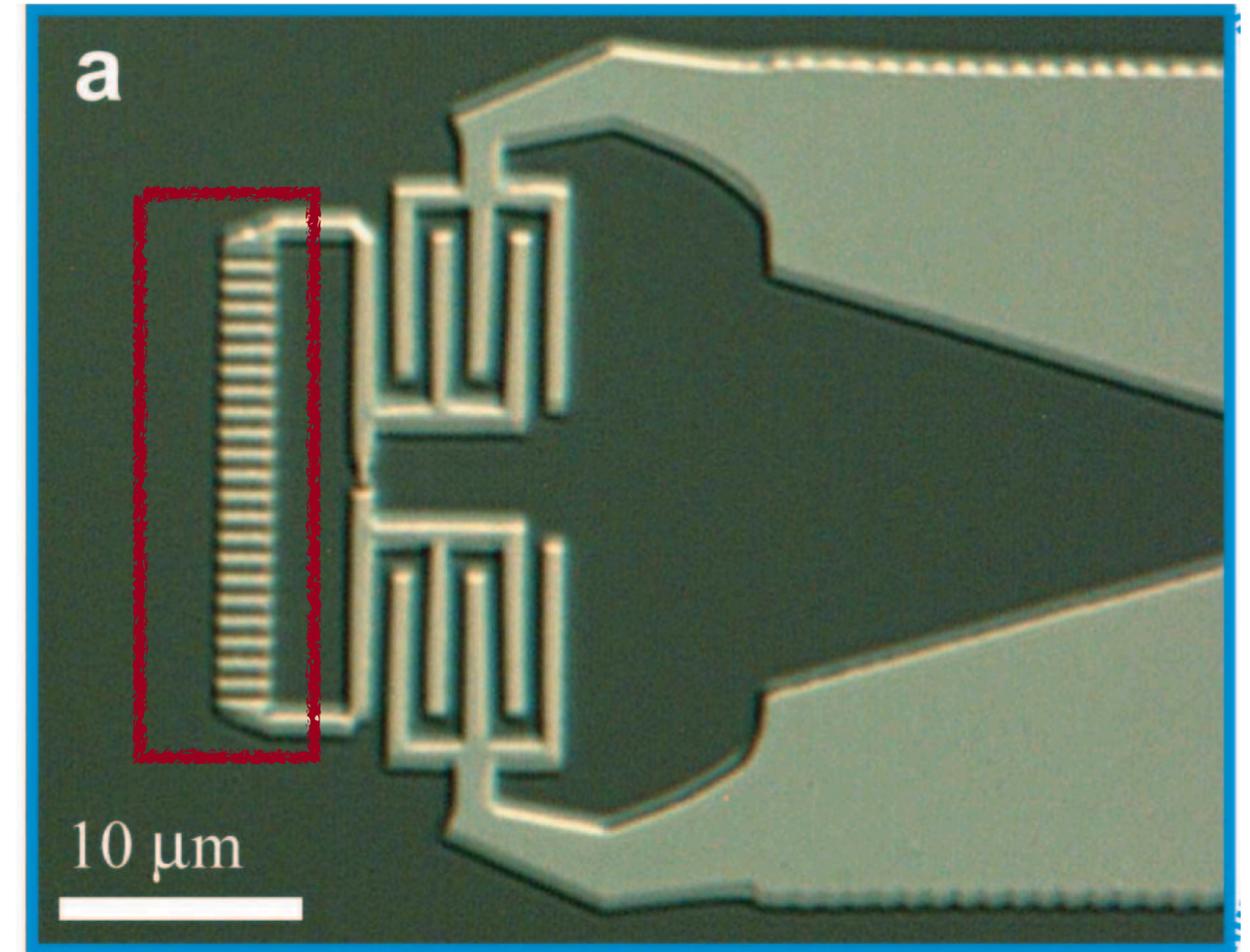
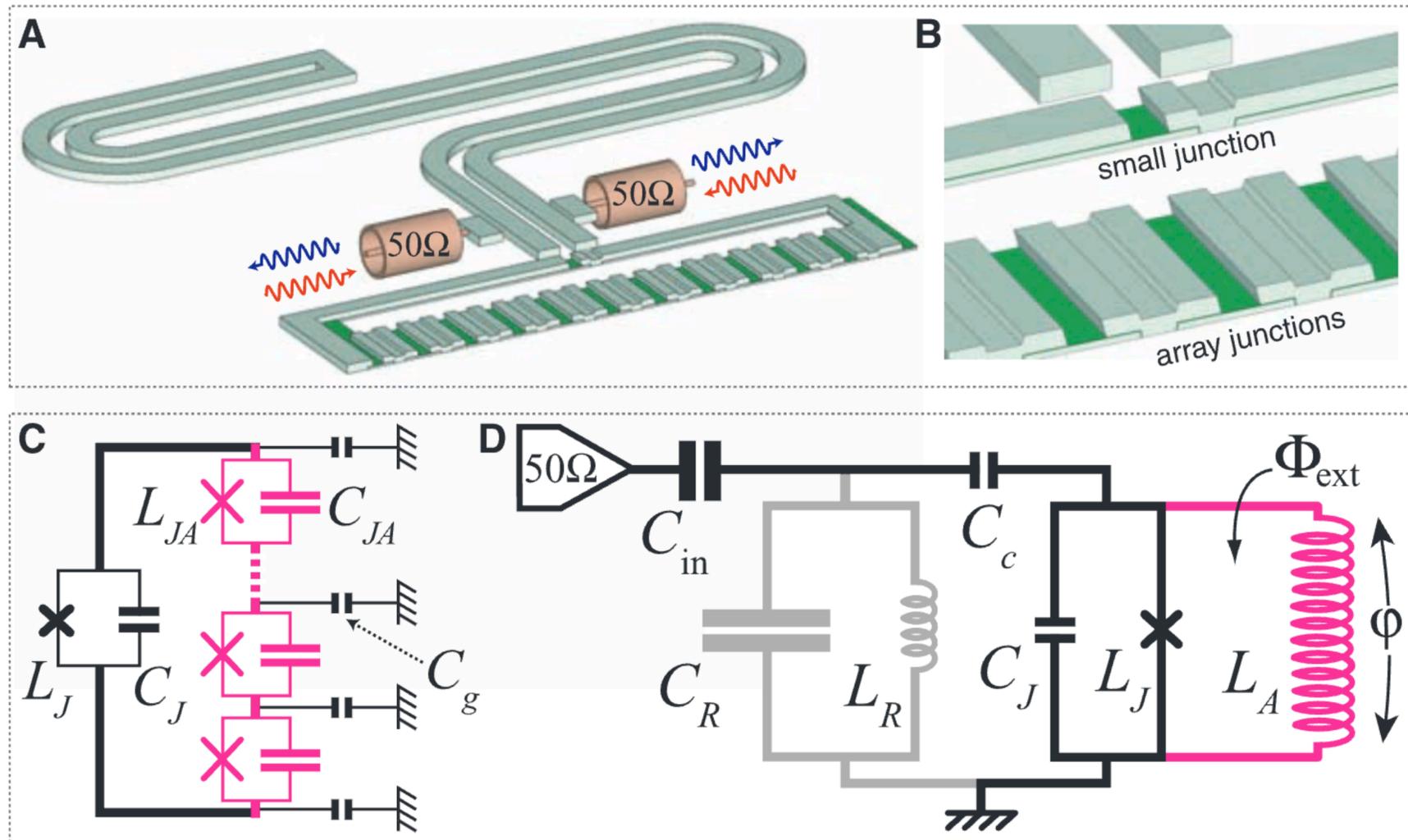
First device:



- V. Manucharyan et al., Science (2009)
- V. Manucharyan et al., arXiv 0910:3039 (2009)
- V. Manucharyan et al., Phys. Rev. B (2012)

# Fluxonium devices:

First device:



Very large inductance a.k.a “Superinductance”

Made from a JJ array

V. Manucharyan et al., Science (2009)

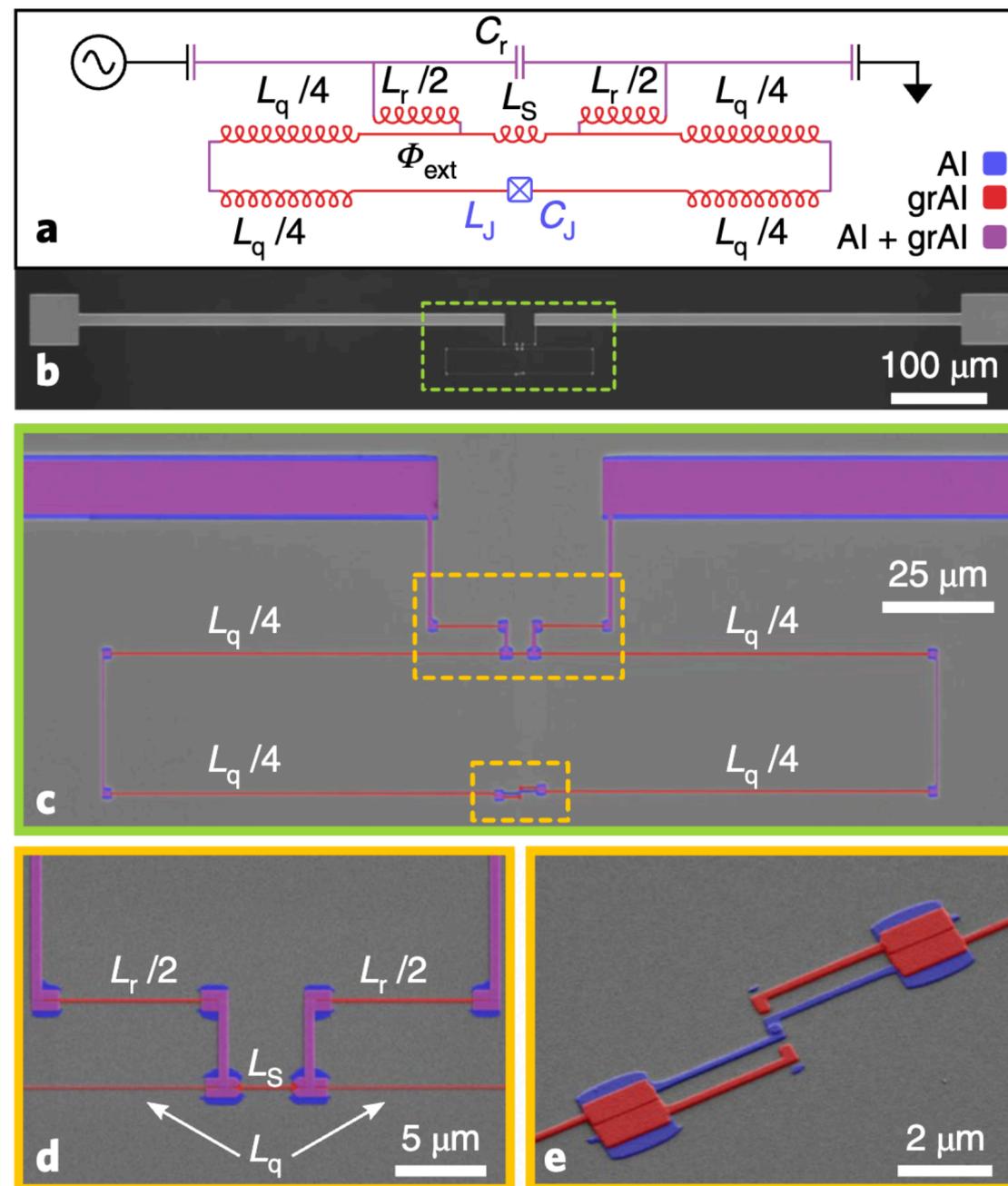
V. Manucharyan et al., arXiv 0910:3039 (2009)

V. Manucharyan et al., Phys. Rev. B (2012)

# Fluxonium devices:

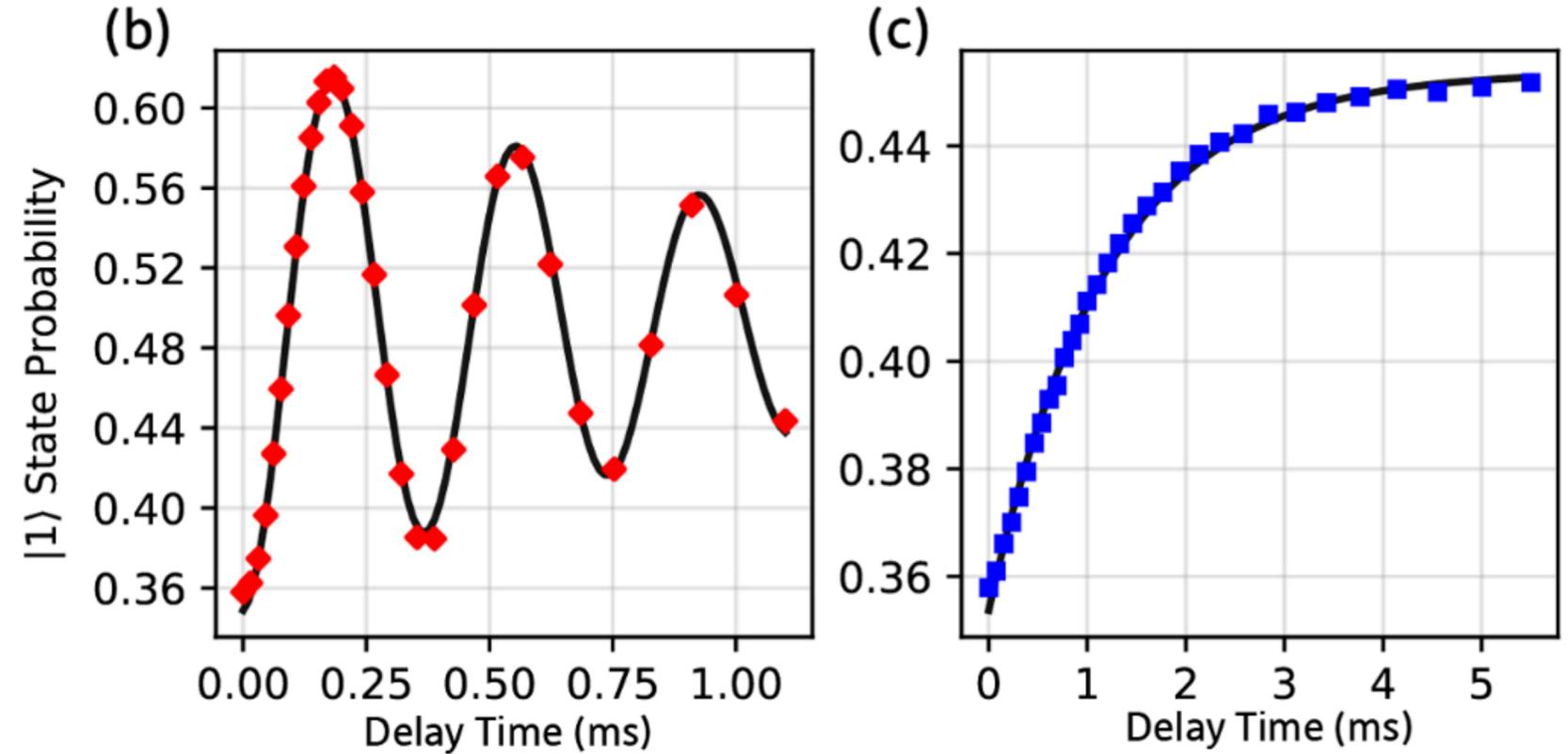
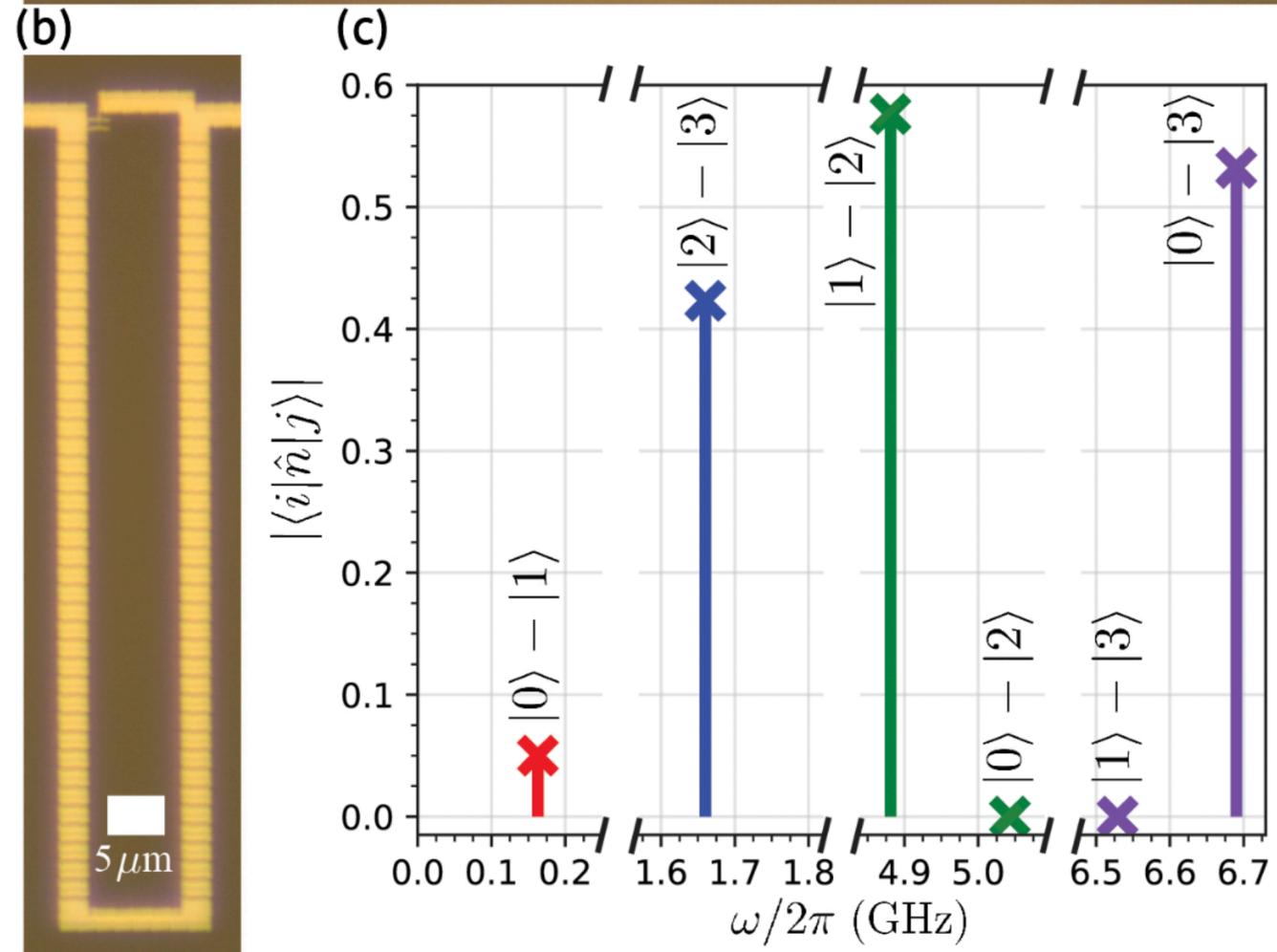
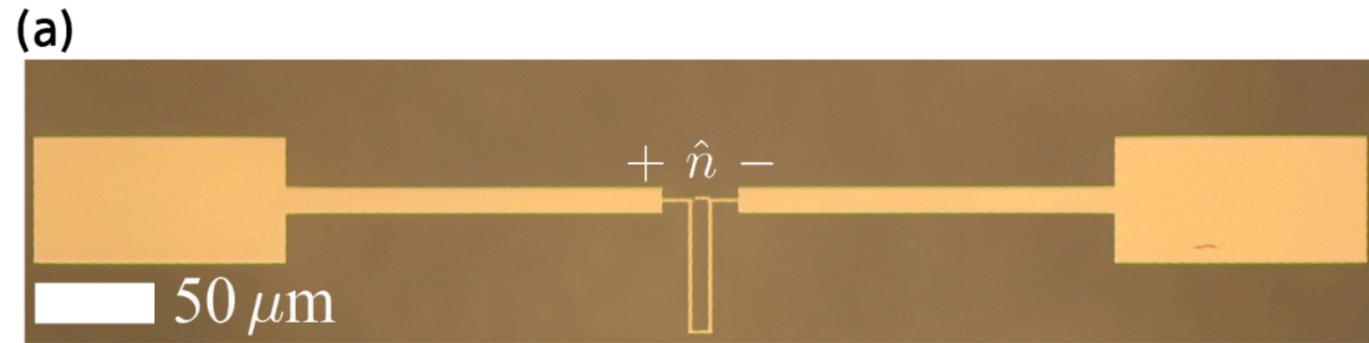
More recent devices

(inside a 3D cavity + super inductance made of granular aluminum):



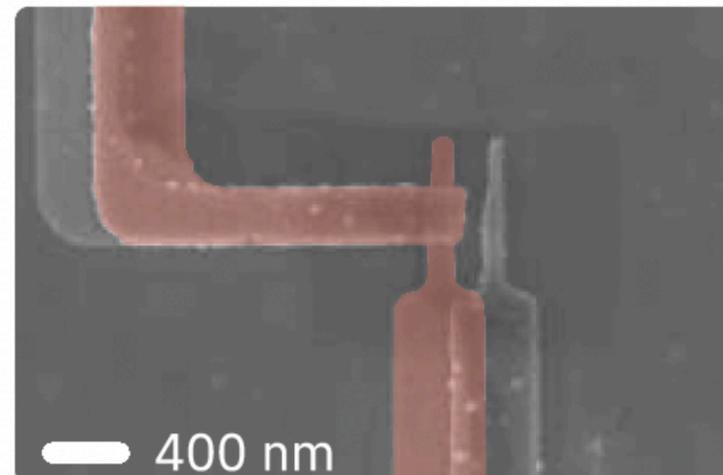
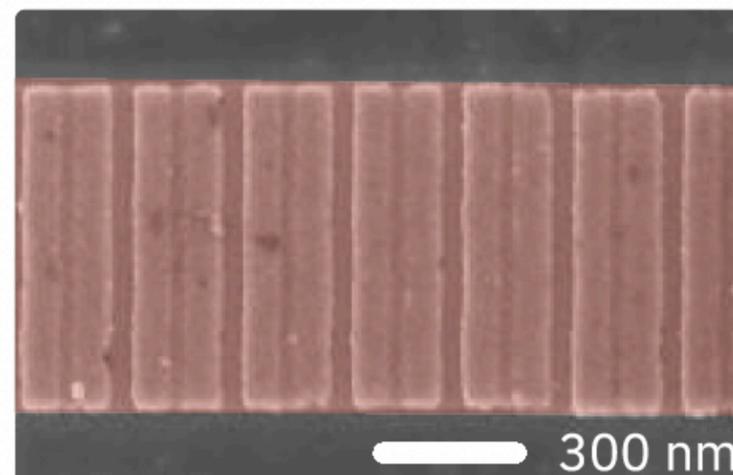
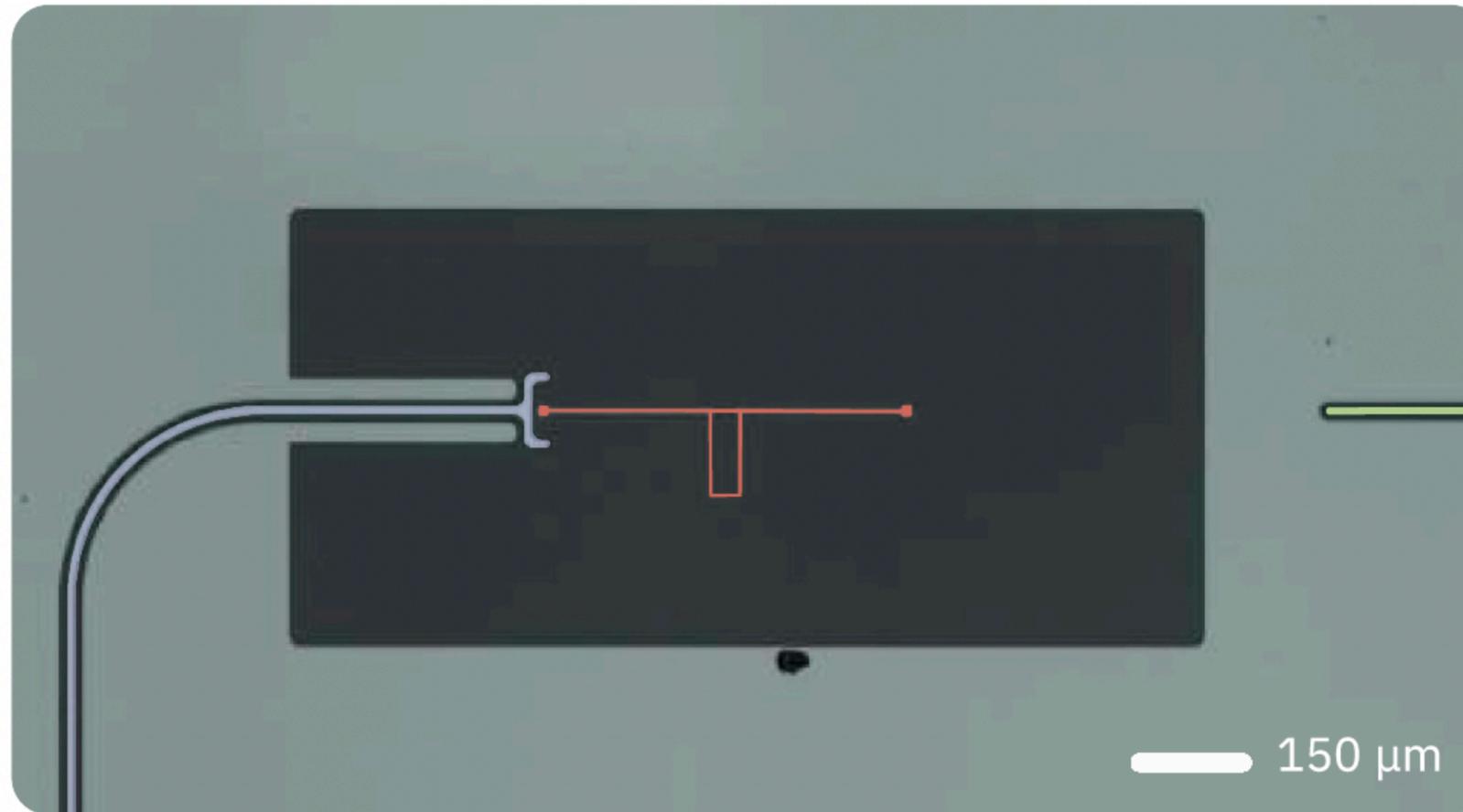
# Fluxonium devices:

More recent devices (inside a 3D cavity):



# Fluxonium devices:

More recent devices (in a 2D architecture):



# Why the Fluxonium?

Argument #1: in superconducting circuits there are “broken Cooper pairs” (a.k.a. quasiparticles) but there is no “broken Fluxons”.

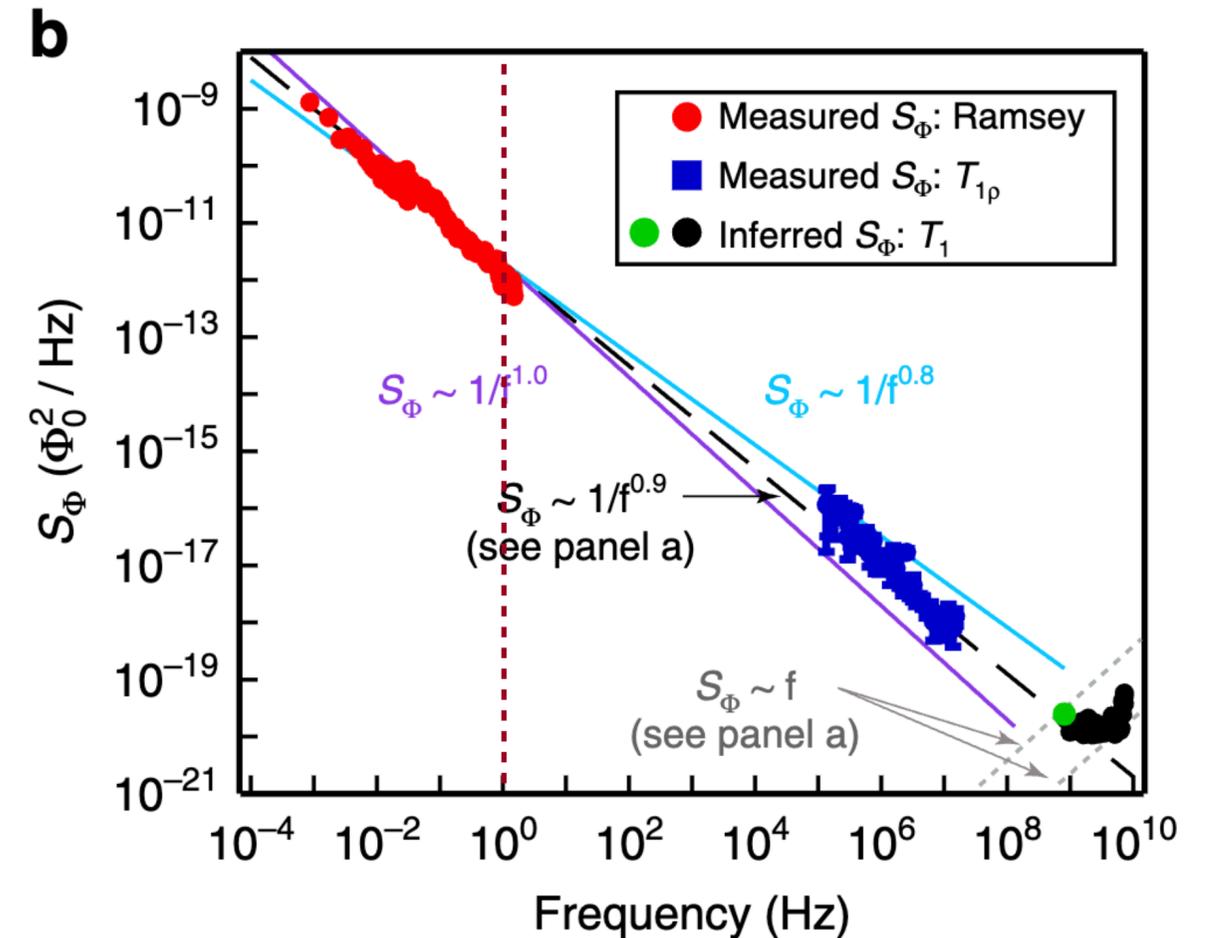
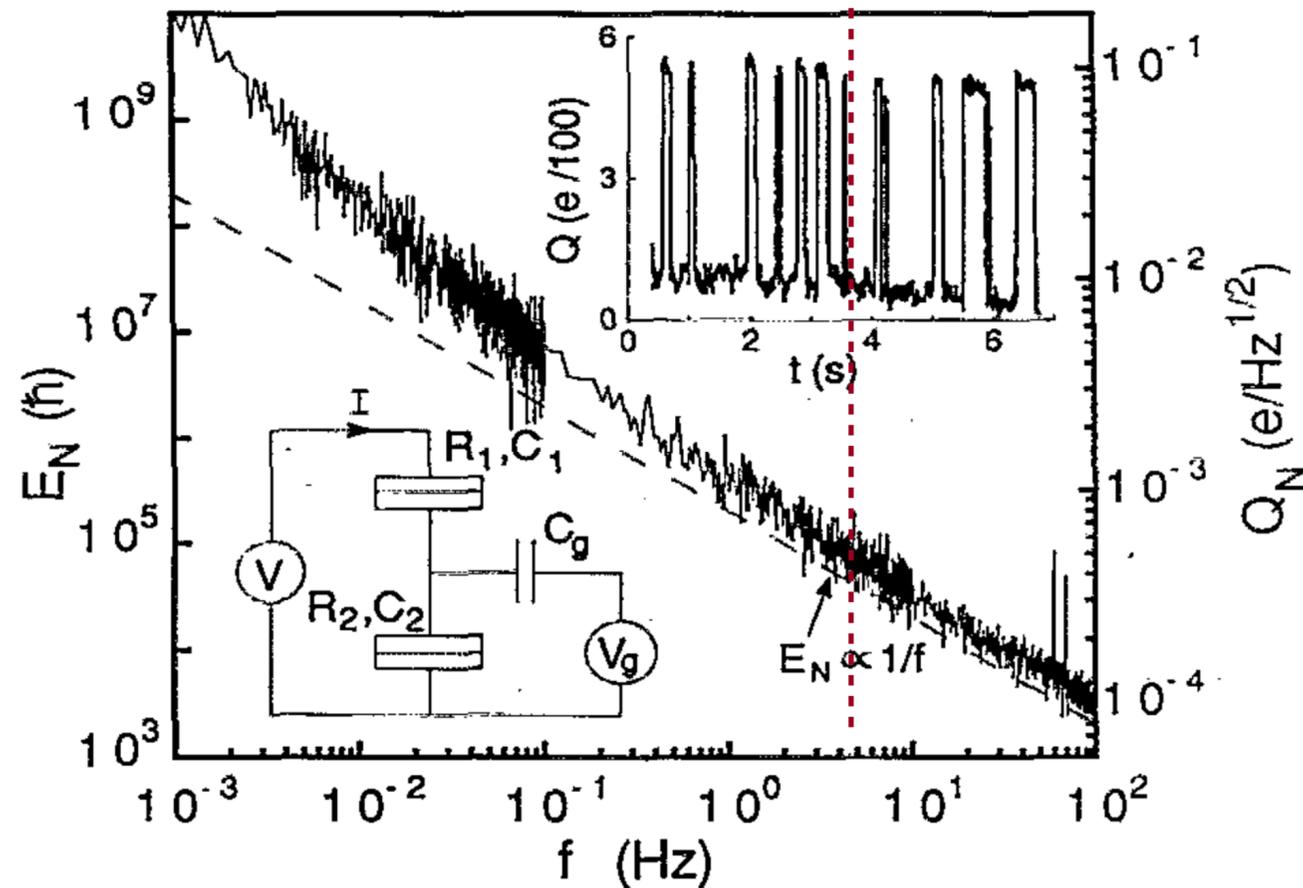
# Why the Fluxonium?

Argument #2: charge noise amplitude is two orders of magnitude higher than flux noise:

Charge noise  
 $S_N \sim 10^{-4} e/\sqrt{\text{Hz}}$

Flux noise  
 $S_\Phi \sim 10^{-6} \Phi_0/\sqrt{\text{Hz}}$

Slope of the 1/f noise PSD



# Outline

1. Large inductance: Josephson meta-materials
2. Traveling Wave Parametric Amplifiers
3. Quantum impurities