
Robust multi-mode superconducting qubit design with evolutionary algorithms

arXiv:2407.18895

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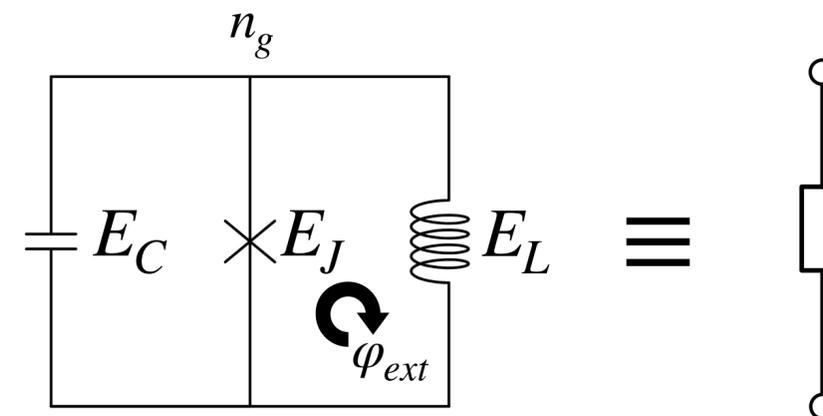




Flexibility of the platform and motivation

Single-mode

$$\hat{H}_m = 4 E_C (\hat{n}_m + n_g)^2 + \frac{E_L}{2} (\hat{\varphi}_m + \varphi_{ext})^2 - E_J \cos \hat{\varphi}_m$$

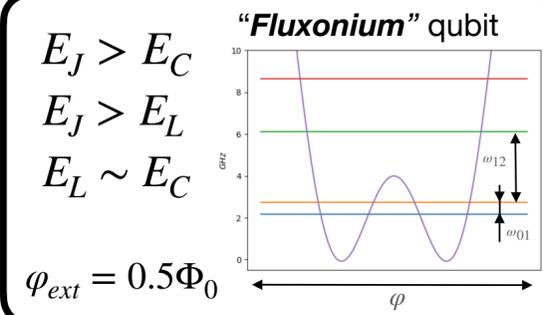
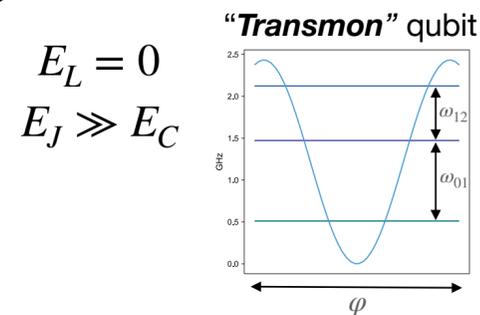
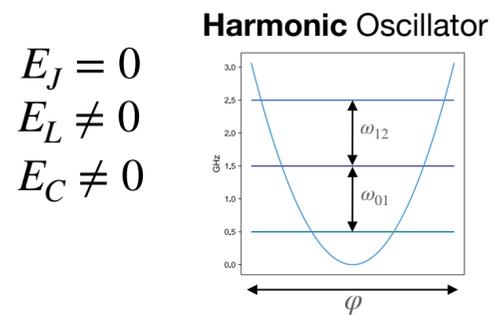
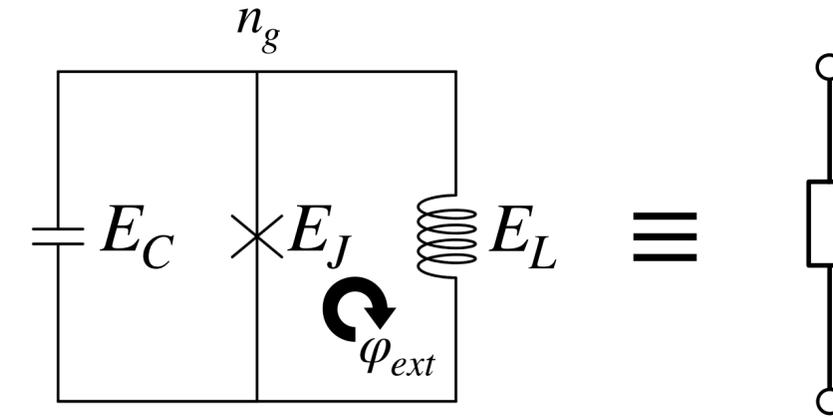




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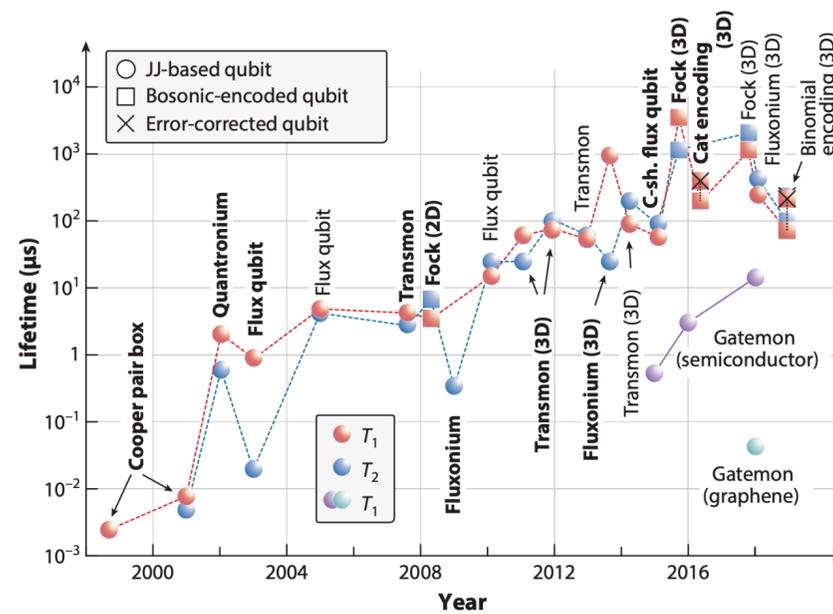
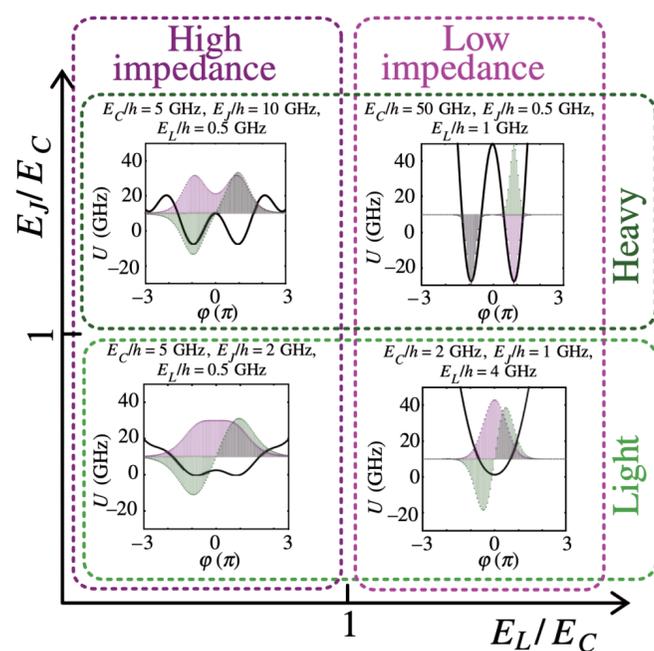
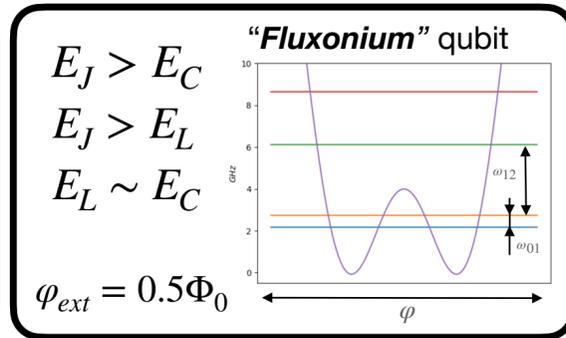
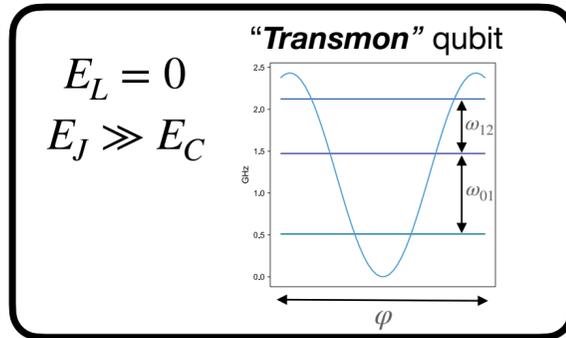
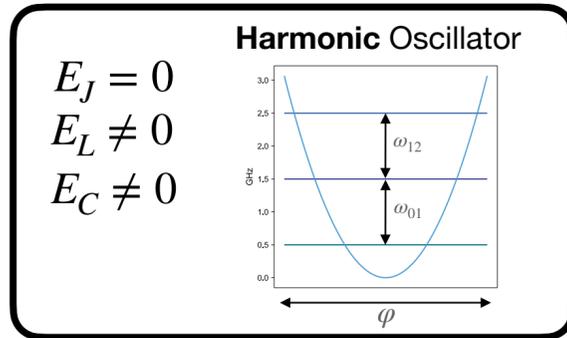
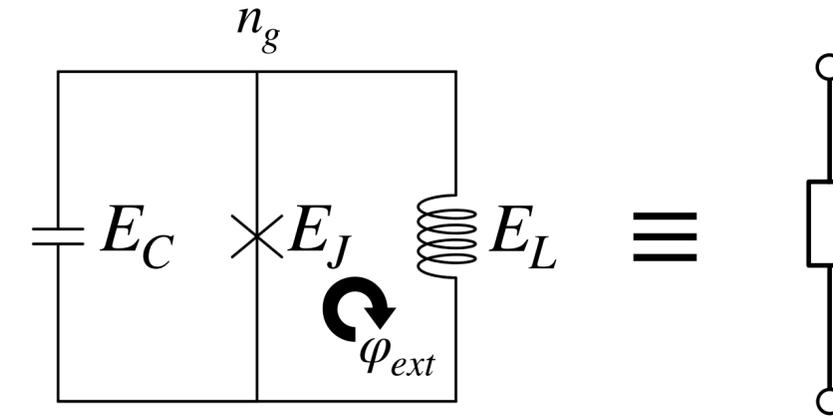




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*M. Kjaergaard et al., (2020).

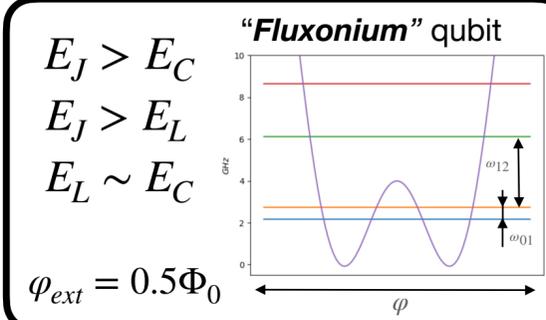
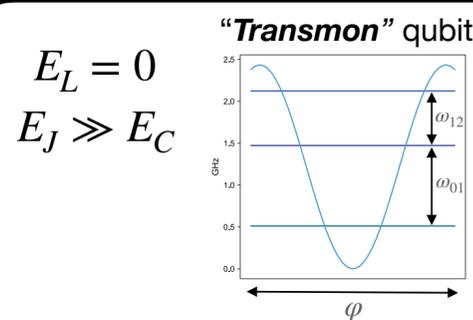
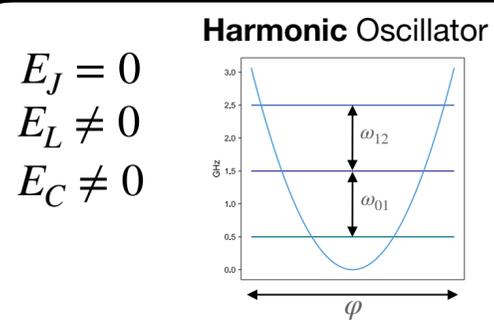
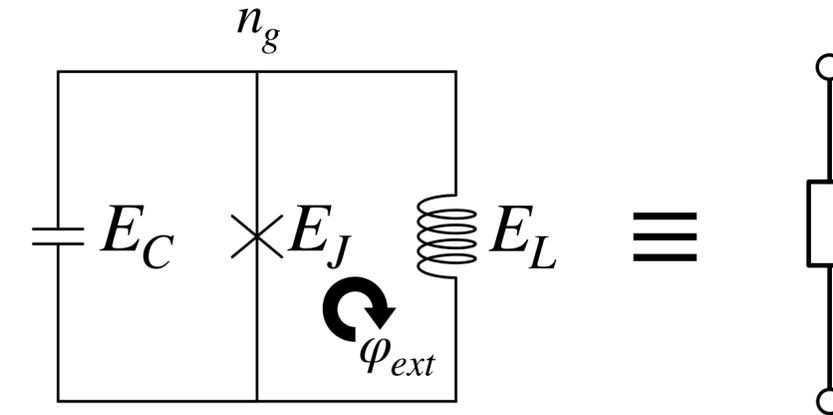
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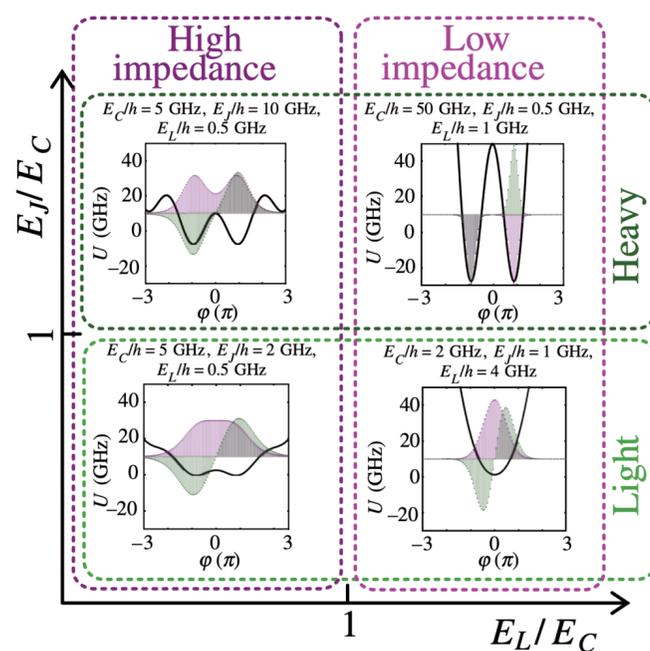
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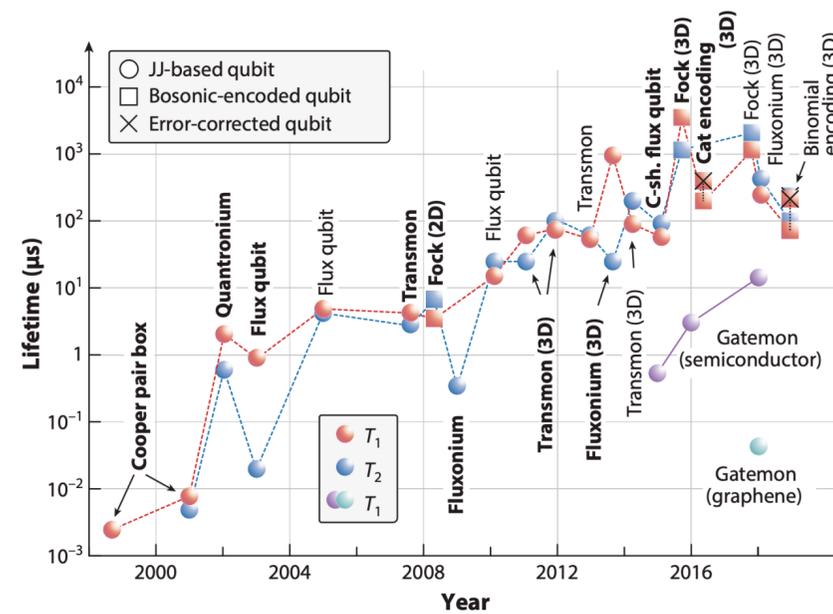
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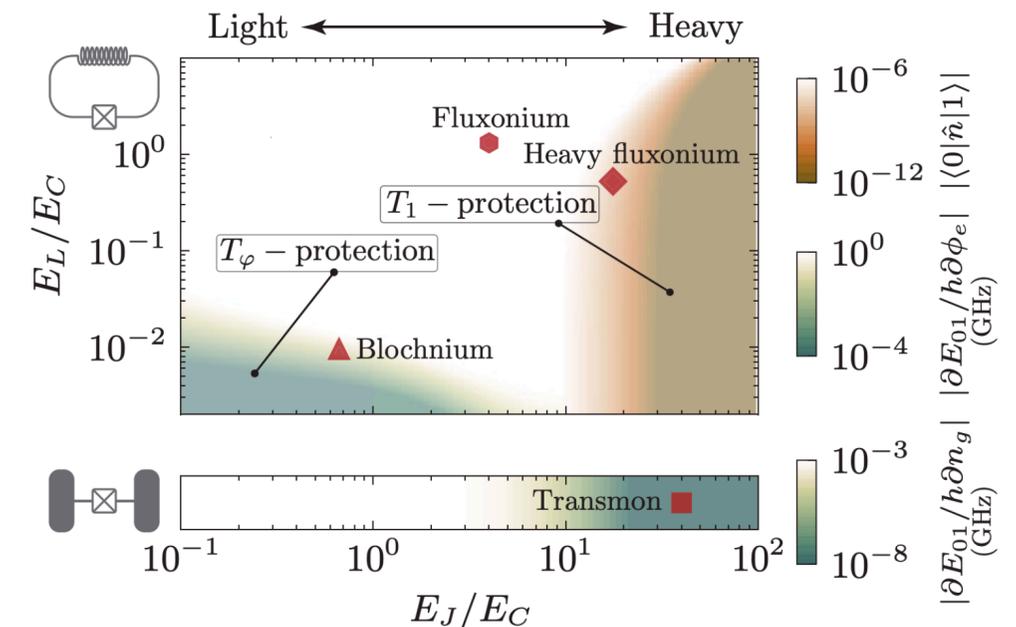
In single-mode devices certain protection characteristics **conflict***



*Peruzzo, M., et. al. (2021).



*M. Kjaergaard et al., (2020).



*A. Gyenis et al., (2021).

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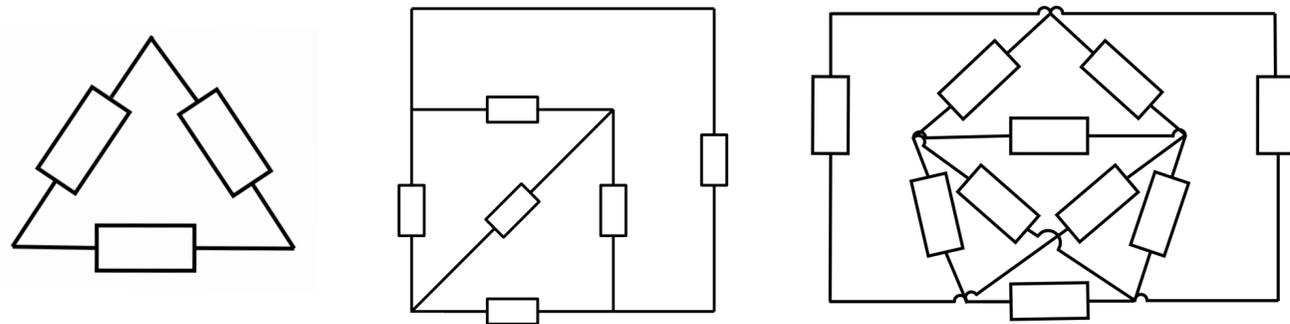
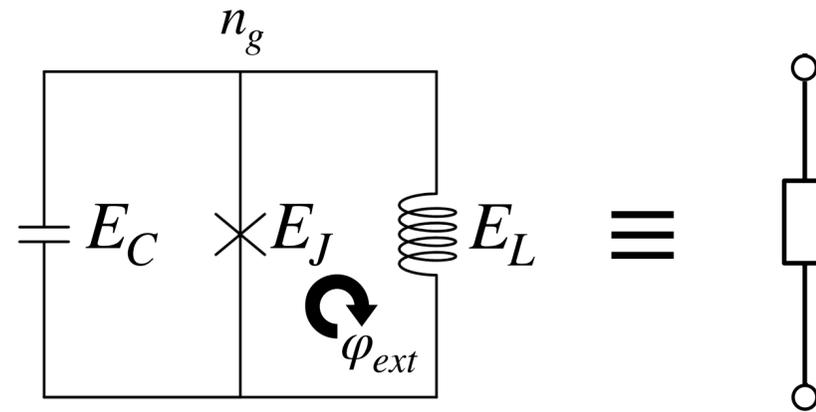


What if the information is encoded in **multiple modes**?



Flexibility of the platform and motivation

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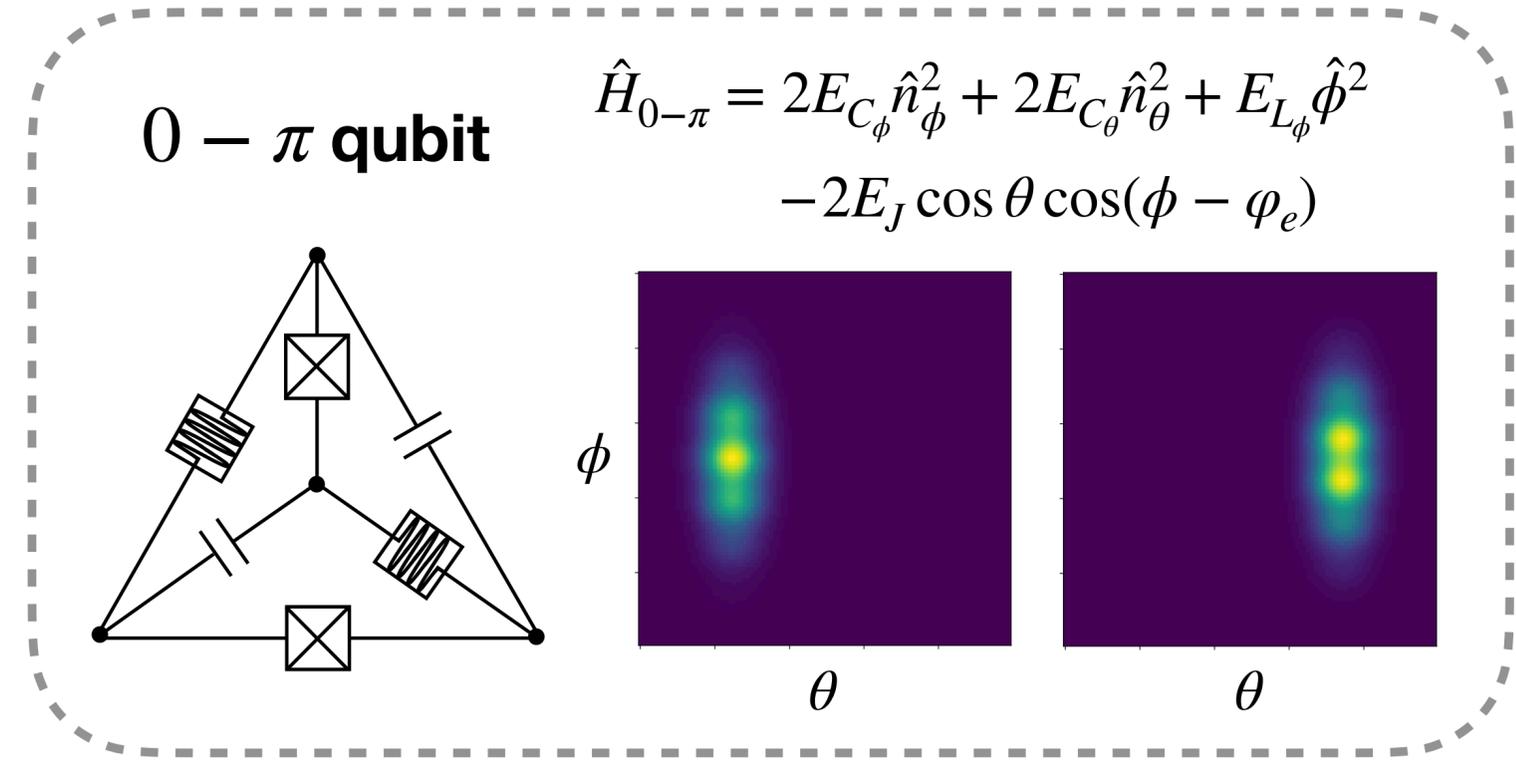
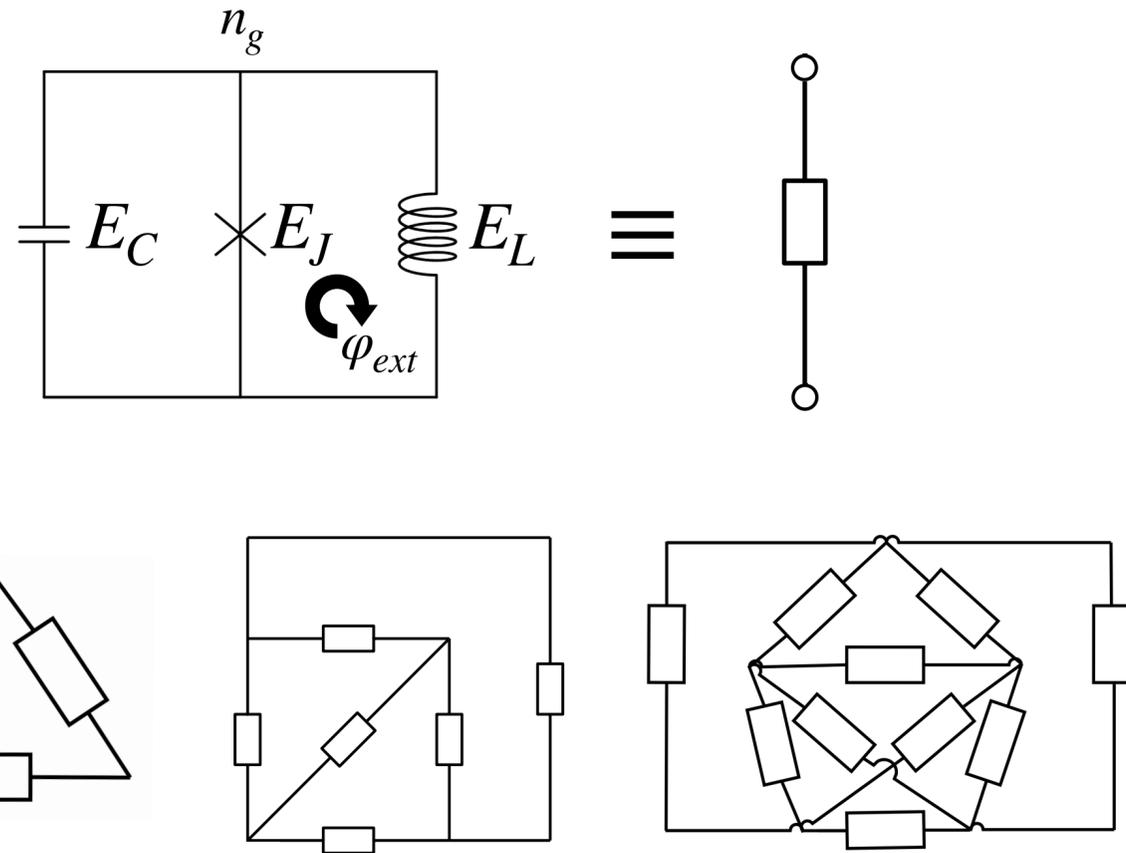
Multi-mode

$$\hat{H} = \sum_m \hat{H}_m(\hat{n}_m, \hat{\phi}_m) + \sum_{nm} \hat{H}_{nm}(\hat{n}_n, \hat{n}_m, \hat{\phi}_n, \hat{\phi}_m)$$



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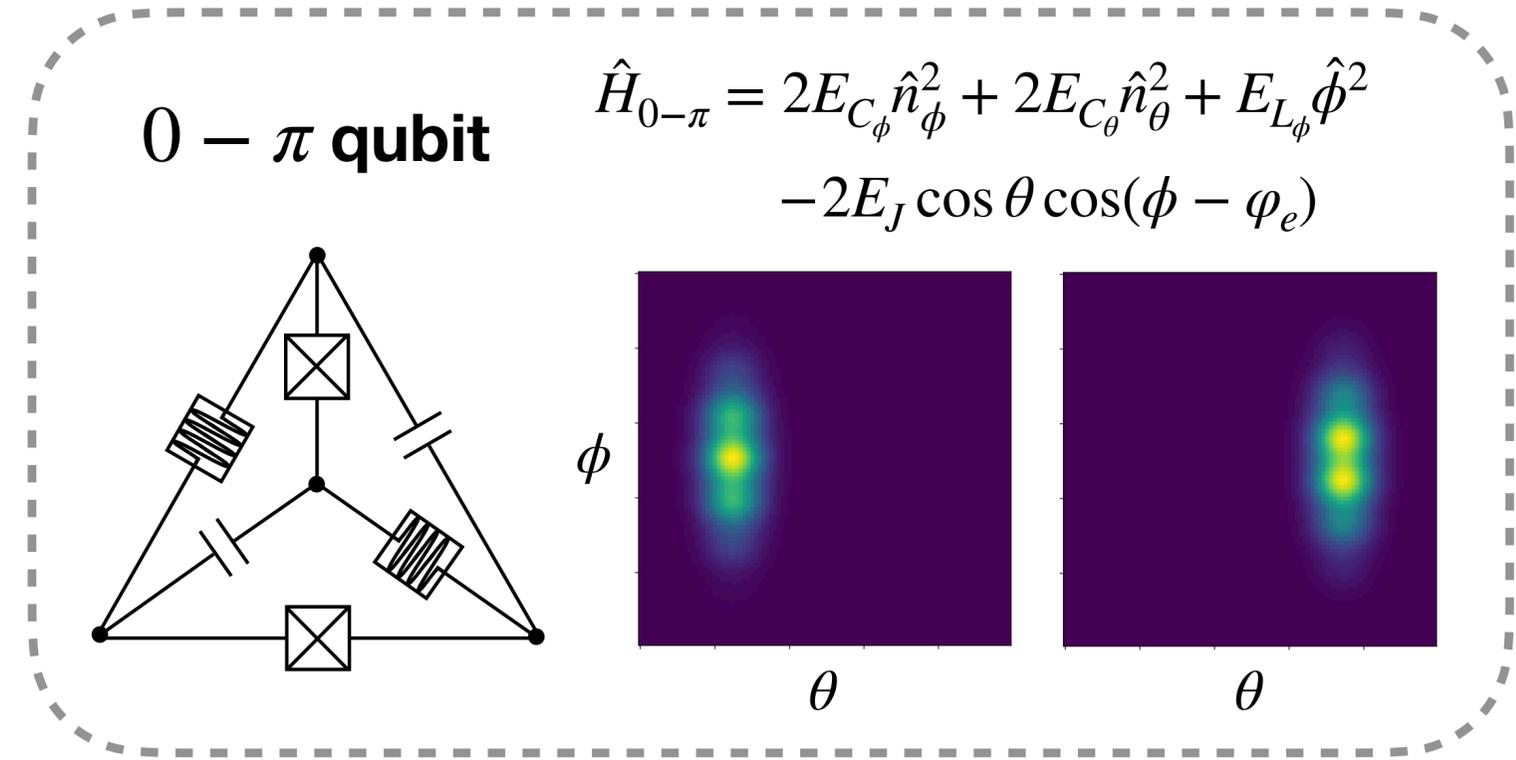
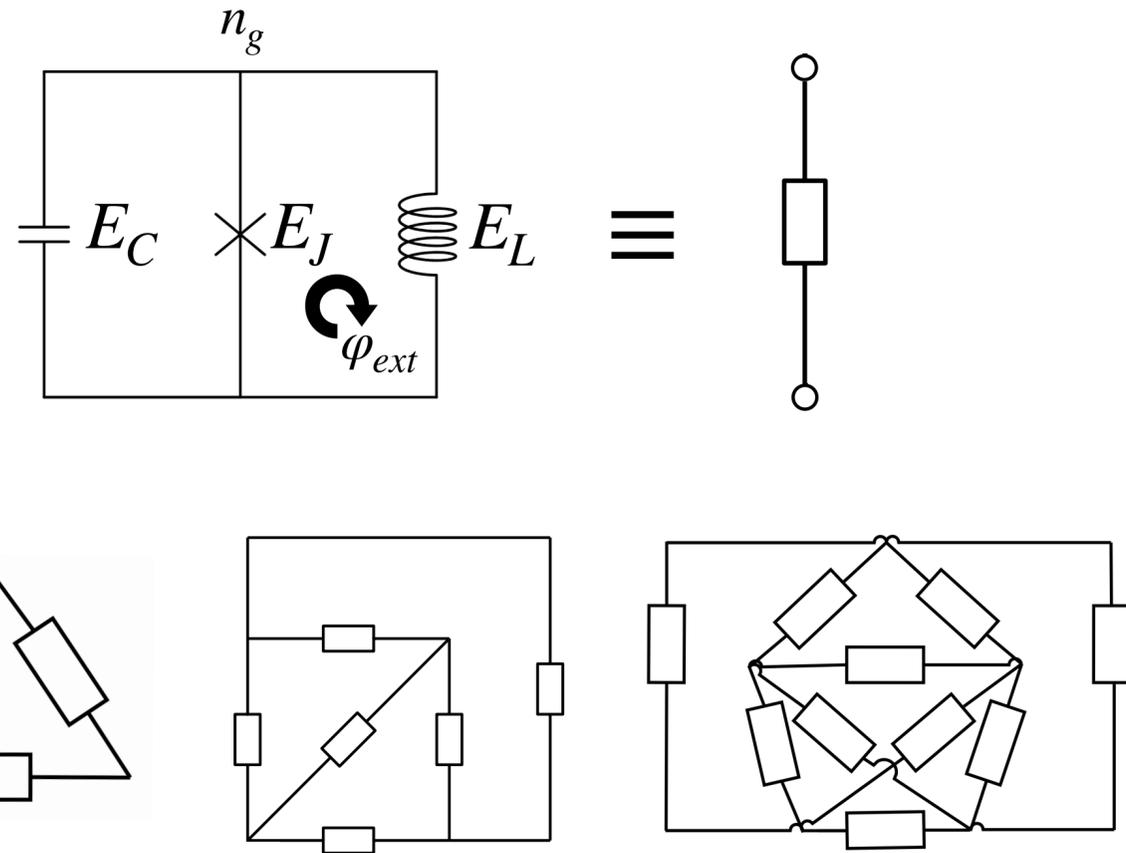
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Problems

- The space of possible configurations grows (**large parameter space**)
- Computational **cost**



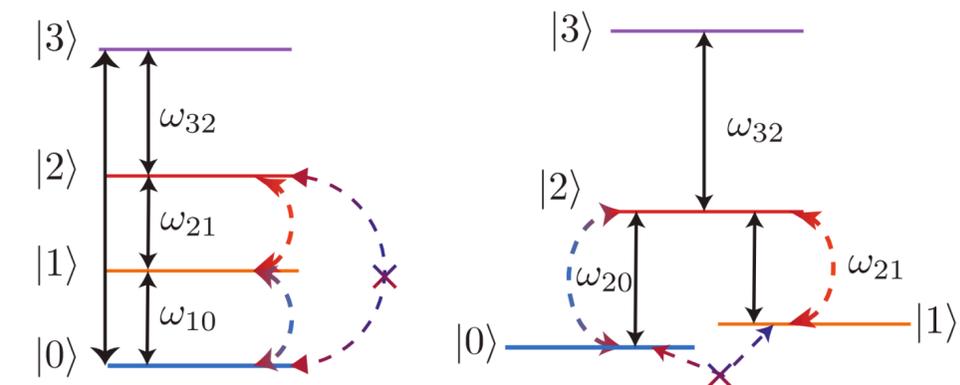
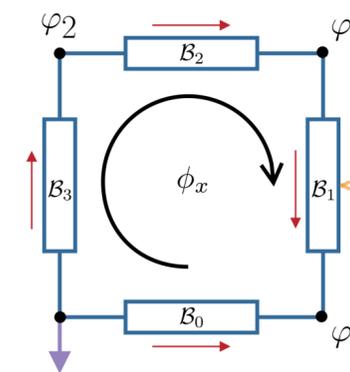
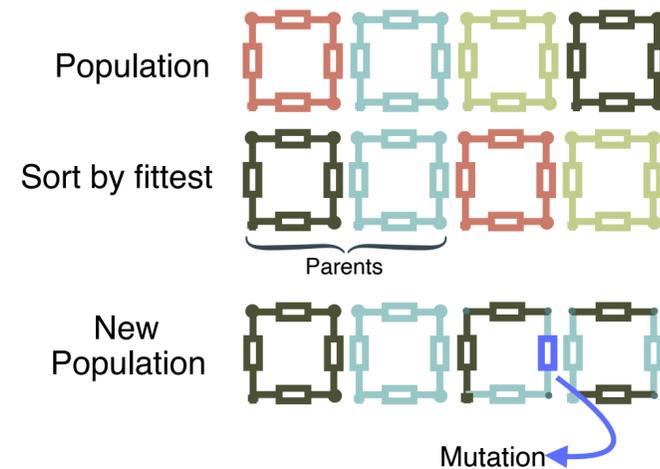
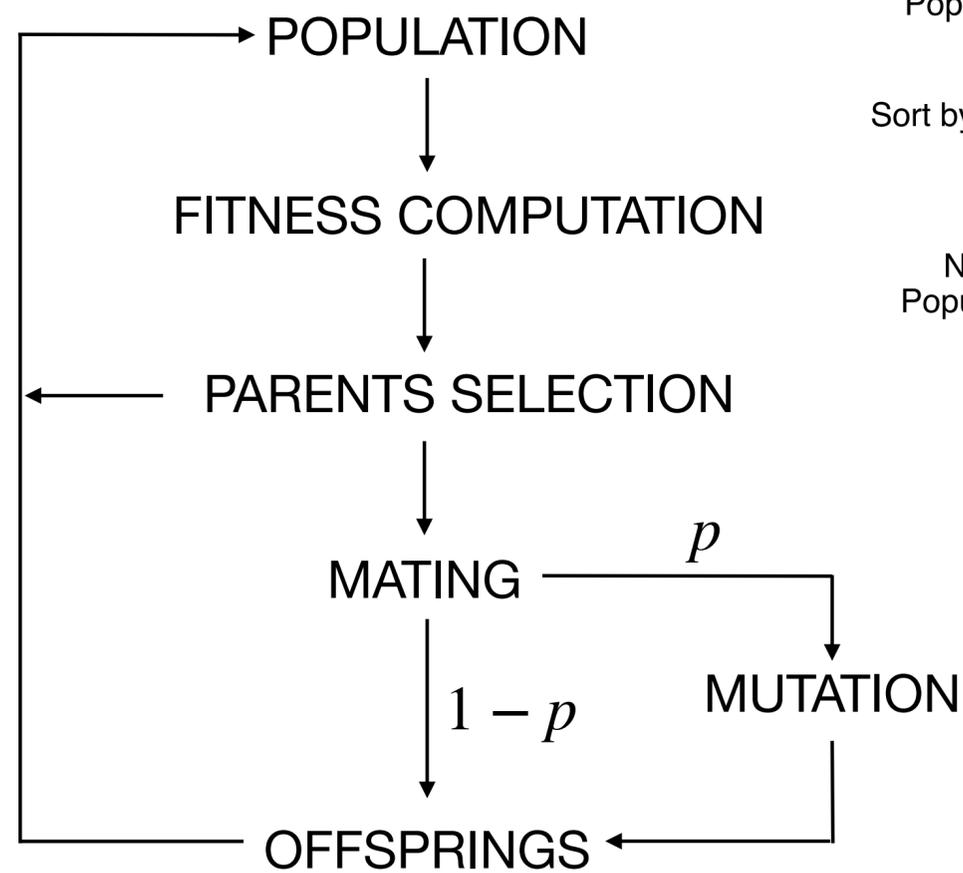
Flexibility of the platform and motivation

How do we find configurations with **desired characteristics**?

Evolutionary algorithms (Genetic Algorithms)



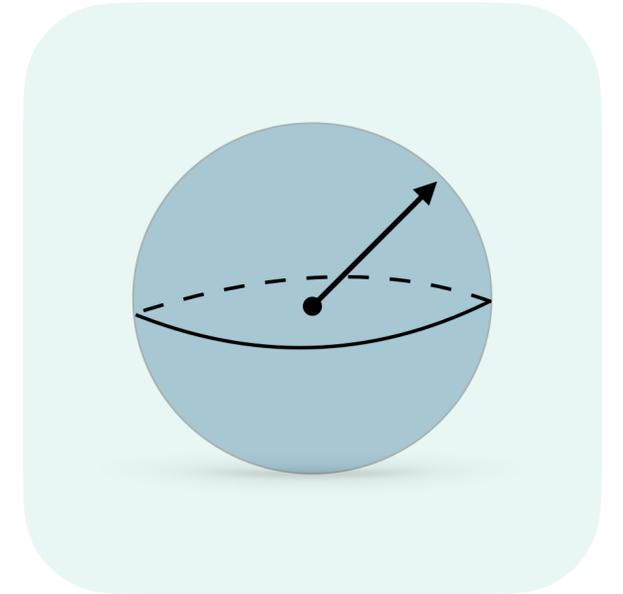
2006 NASA ST5 spacecraft antenna



*Cárdenas-López, F. A., Retamal, J. C., Chen, X., Romero, G., & Sanz, M. (2023). Resilient superconducting-element design with genetic algorithms. arXiv:2302.01837

Building a good performance qubit out of a multi-mode device

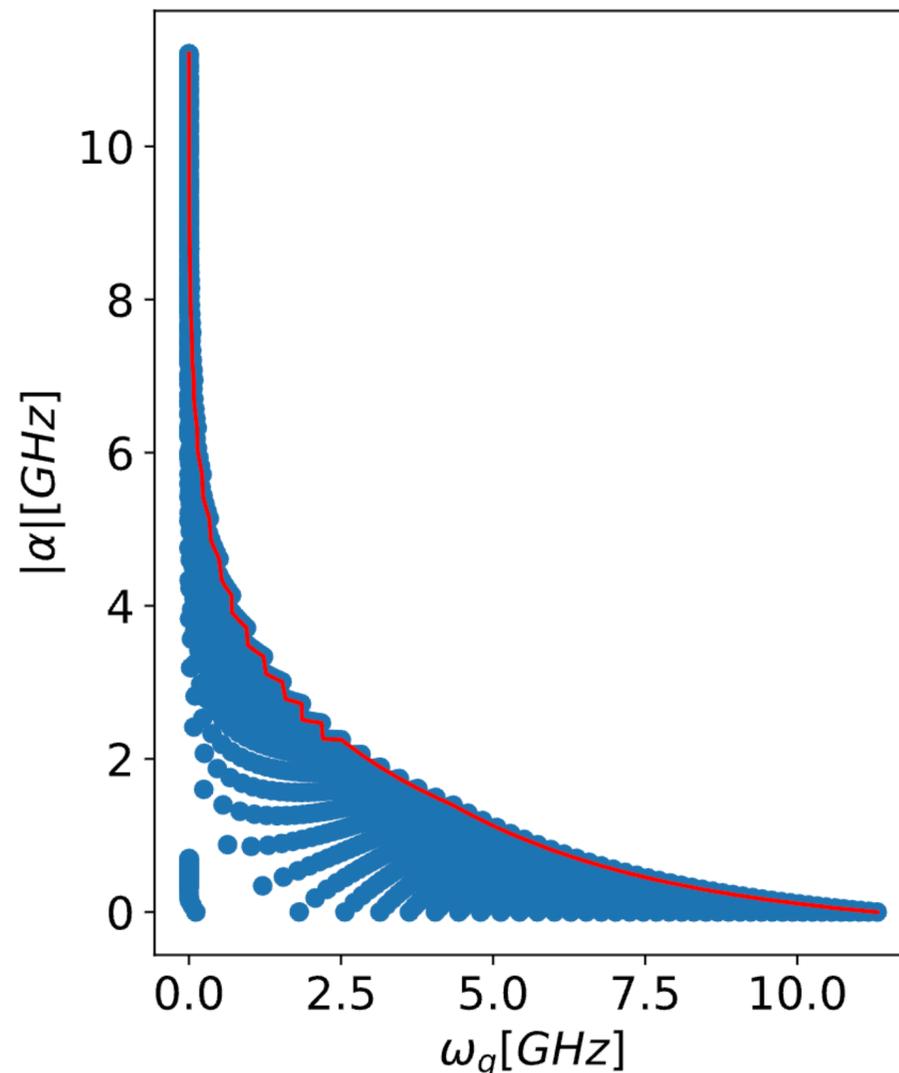
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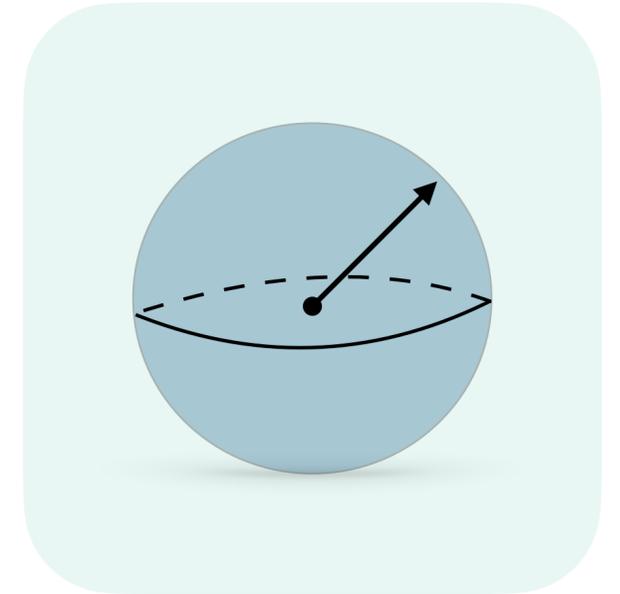
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Large anharmonicity and qubit frequency for **fast manipulation and readout**

$$|\omega_{01} + \omega_d|, |\omega_{ij} - \omega_d| \gg |\Omega_{01}|$$

$$\omega_d \approx \omega_{01}$$

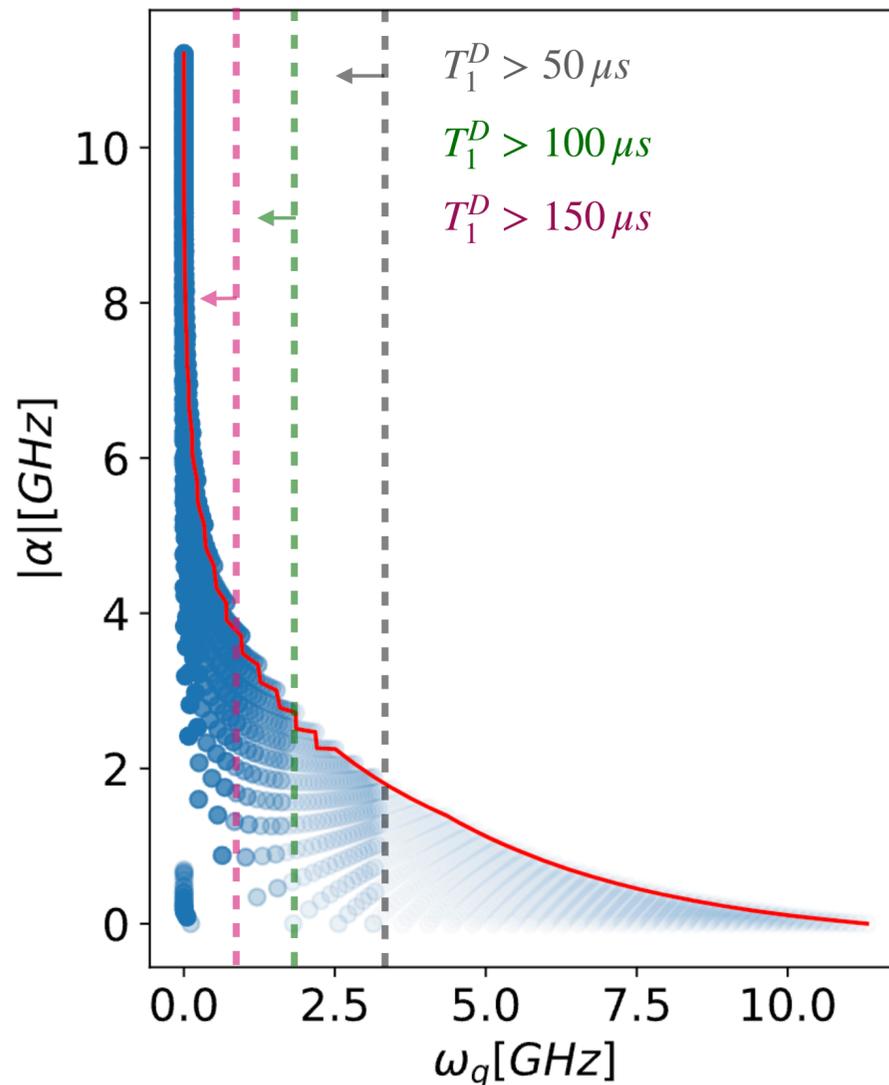
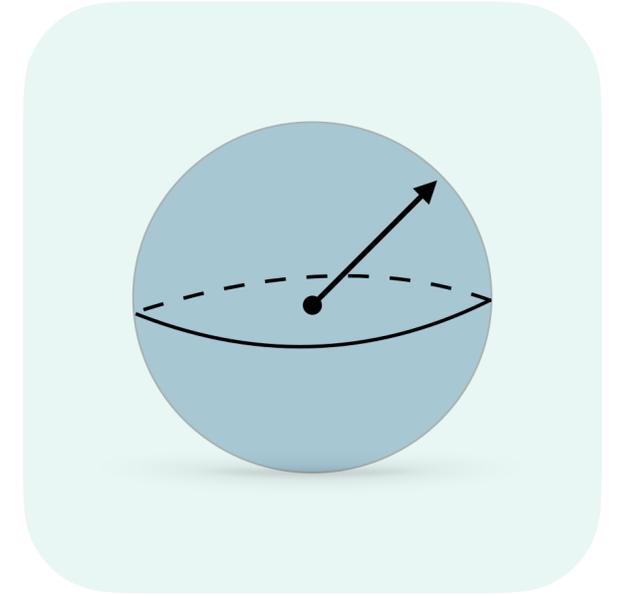


*Taking $E_C/h = 0.25$ GHz as the energy scale.
The Ratios of $E_L/E_J \sim 0.01 - 100$ and $\varphi_e = 0.5$.



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Physical constraints such as **coherence times and max power**

$$T_2 = \left(1/2T_1 + 1/T_\varphi \right)^{-1}$$

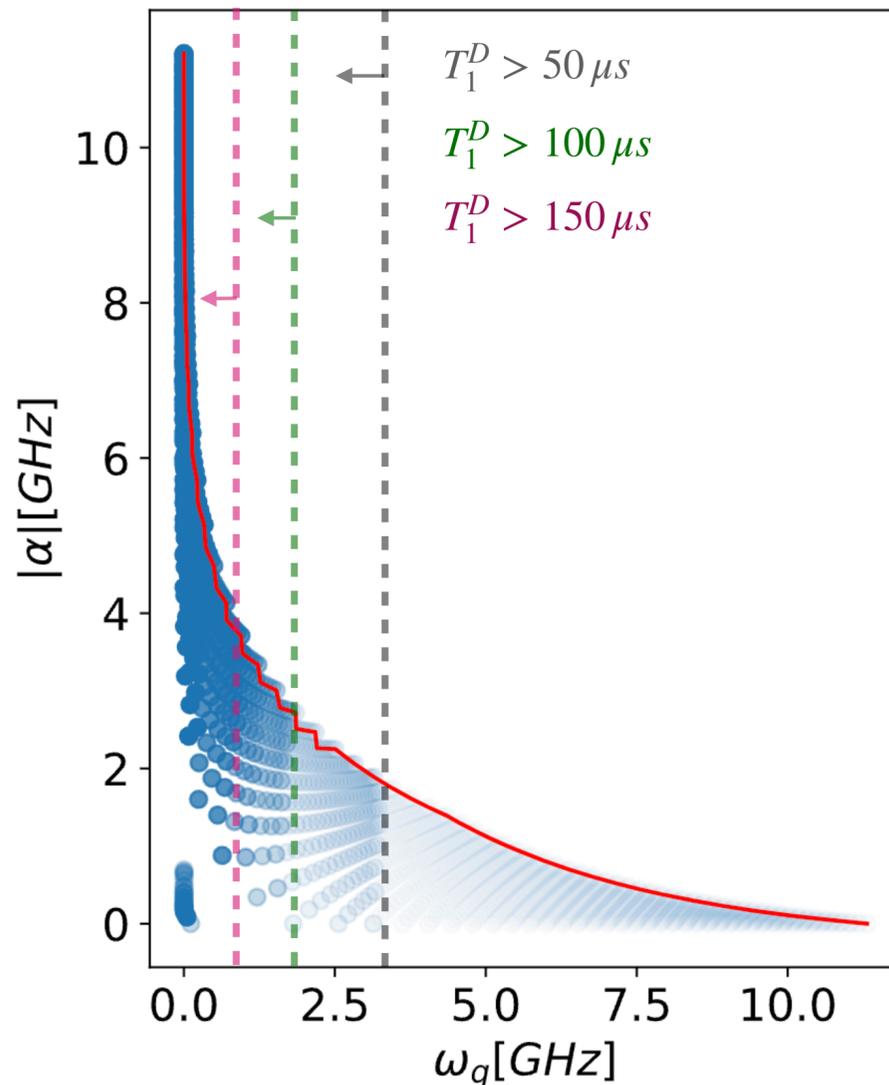
$$|\hbar\Omega_{01}|^{\max} = 2e \frac{C_g}{C + C_g} V_g^{\max} |\langle 0 | \hat{n} | 1 \rangle|$$

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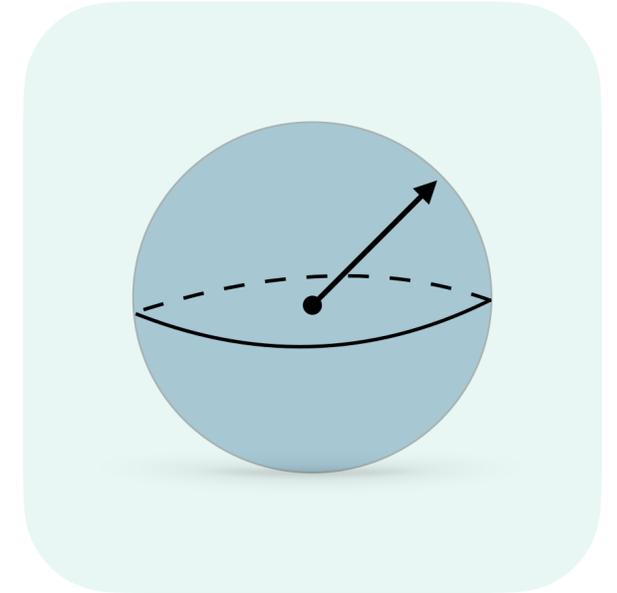
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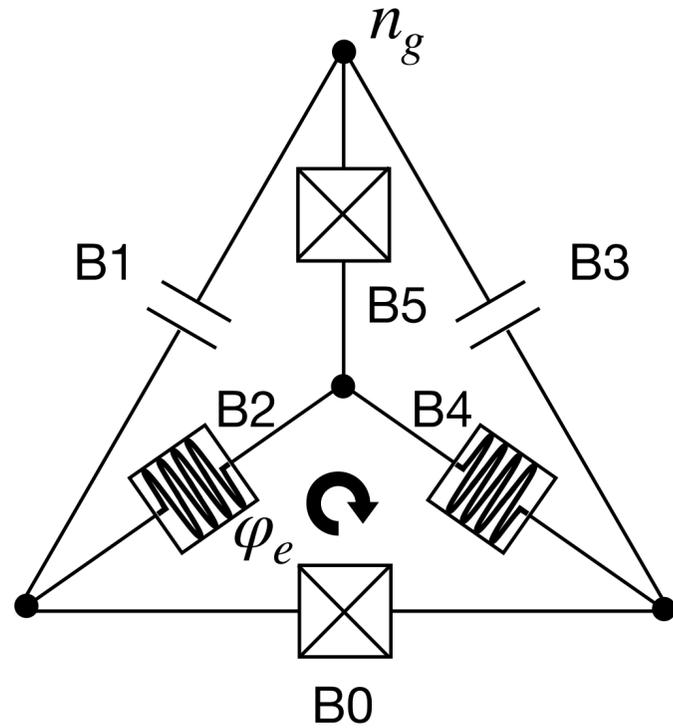
Metric to be optimized

$$T_2/t_g$$

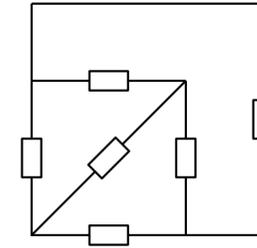
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Multi-mode **Difluxmon Qubit**

arXiv:2407.18895

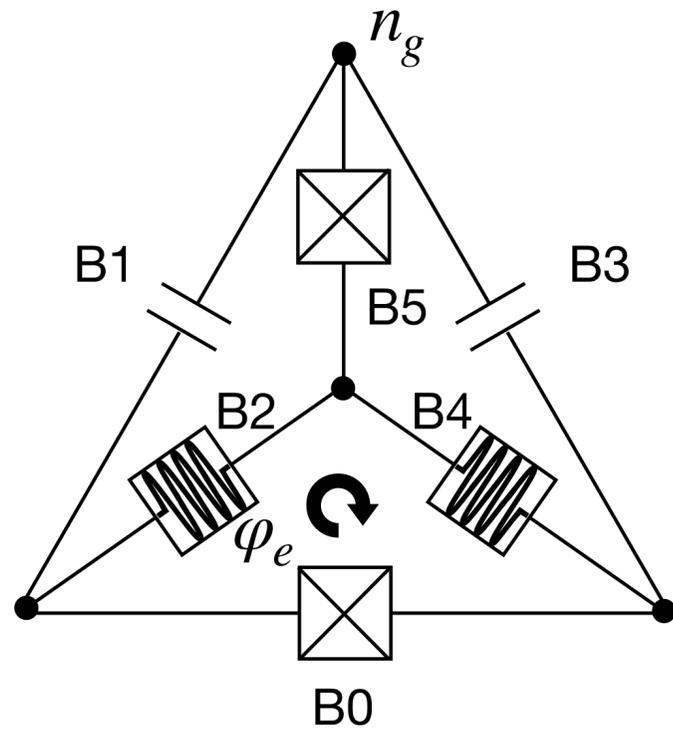


Structure used in the optimization

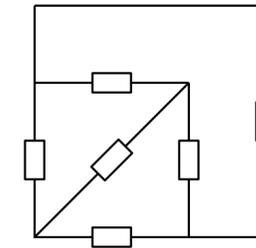


Multi-mode **DiFluxmon Qubit**

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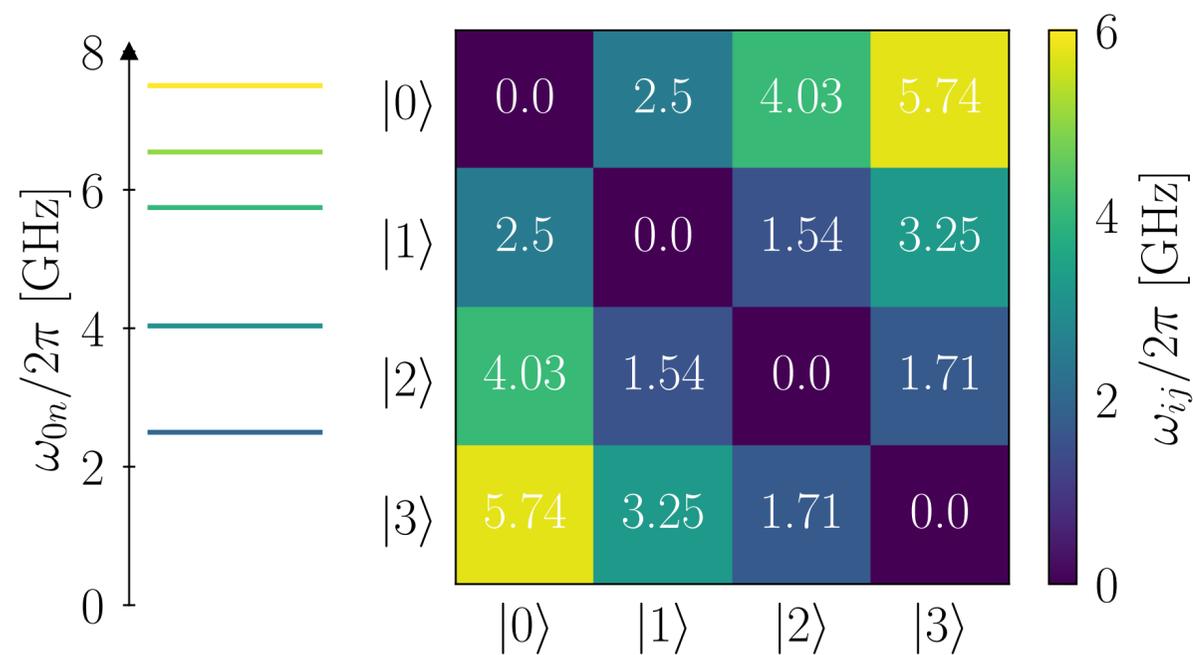


Structure used in the optimization



Result: Three mode system of **non-linearly** coupled modes with anharmonicity 1/3 of the qubit frequency at $\varphi_e = 0.5$

Subjected to both external **charge** and **flux** bias

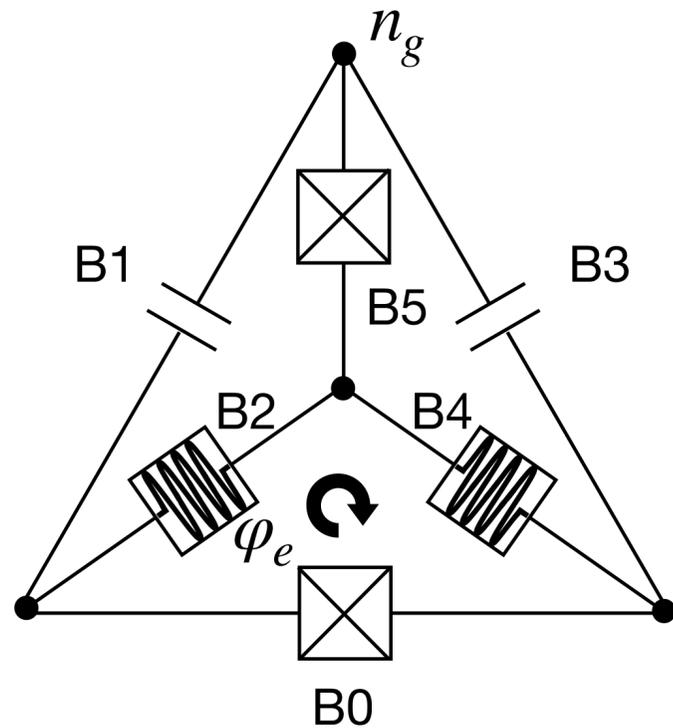


$$\omega_q \approx 2.5 \text{ GHz} \quad \alpha = \omega_{13} - \omega_q \approx 750 \text{ MHz}$$

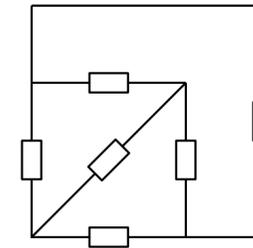
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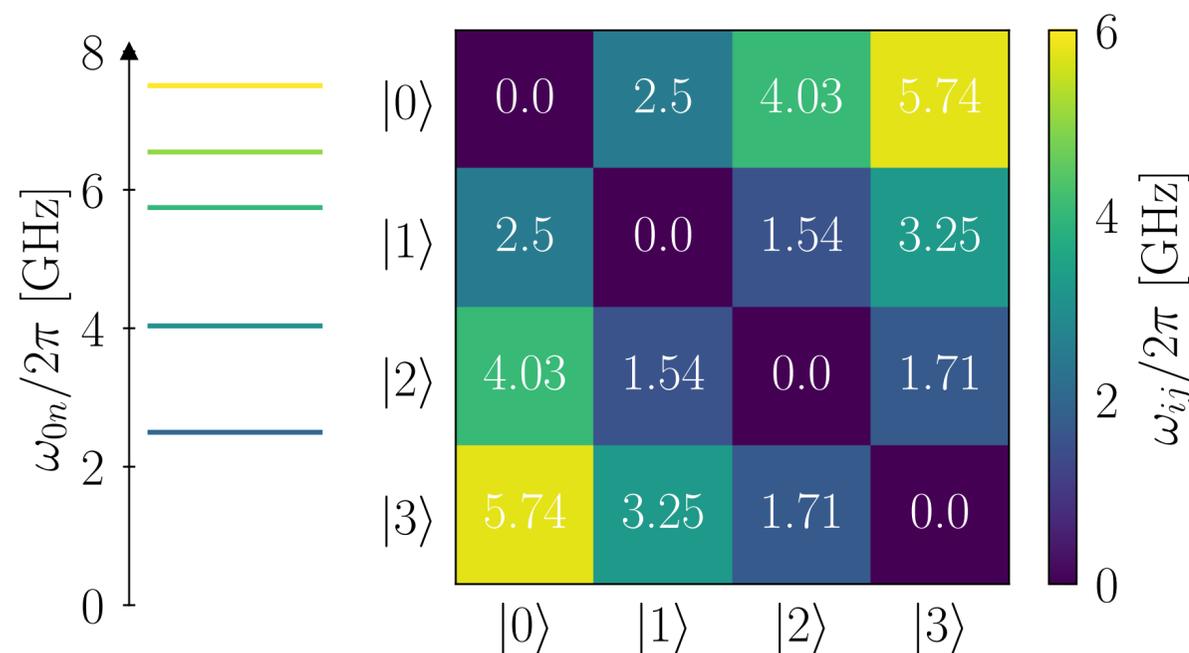


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Branches	EC	EL	EJ
B0	1.67	0	2.5
B1	1.55	0	0
B2	1.27	4.64	0
B3	1.58	0	0
B4	1.87	4.98	0
B5	1.77	0	6.85

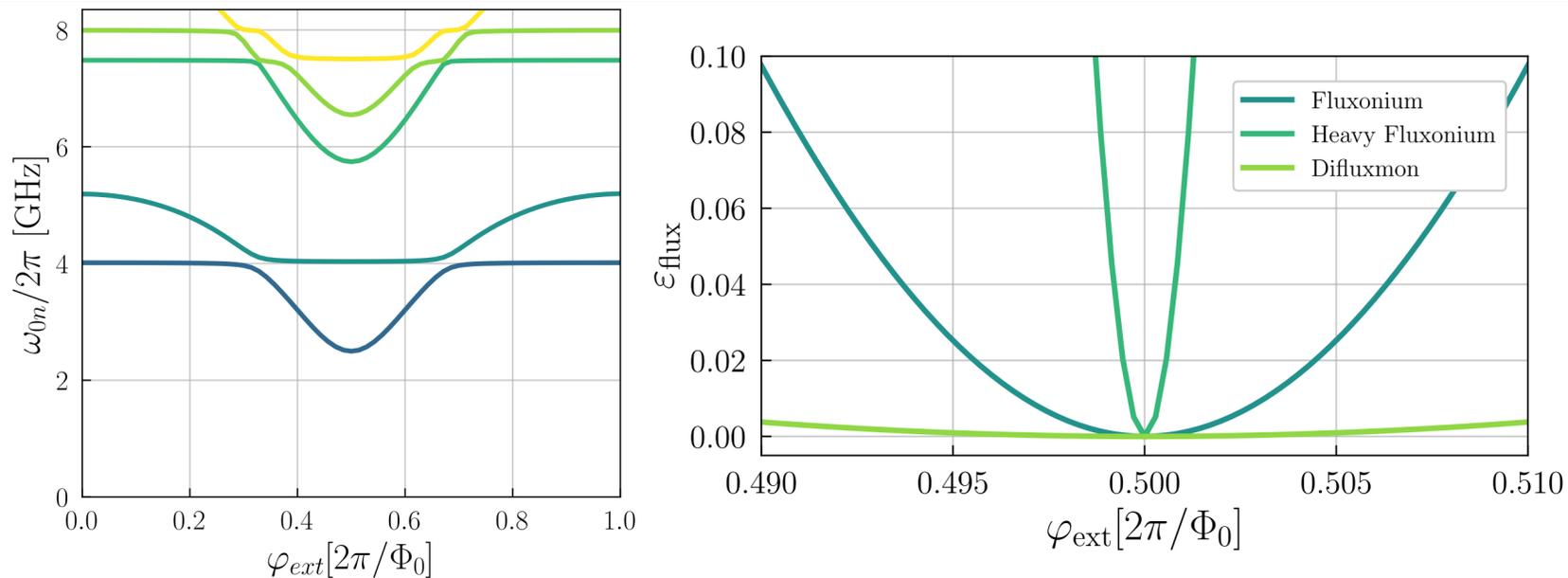
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*in [GHz]

Noise resilience

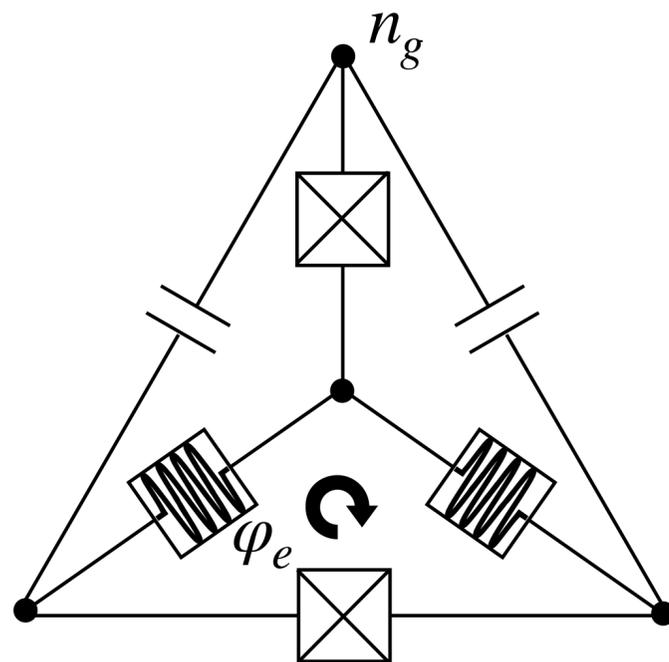


External Flux dispersion

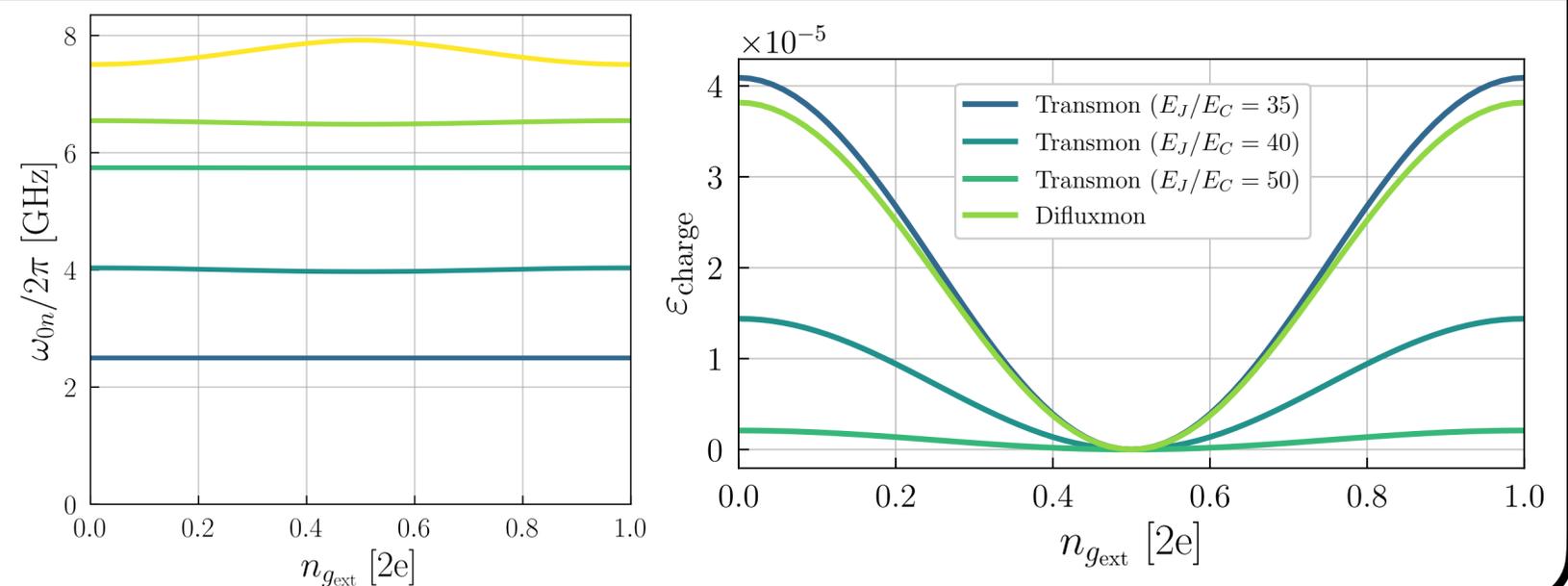


- **Two** extended and **one** periodic mode
- **Strongly coupled** modes
- Susceptible to both charge and flux but **reduced dispersion** around the operating point

Fluxonium $\{E_J = 4, E_C = 1, E_L = 1\}$ GHz
 Heavy Fluxonium $\{E_J = 3.395, E_C = 0.479, E_L = 0.132\}$ GHz

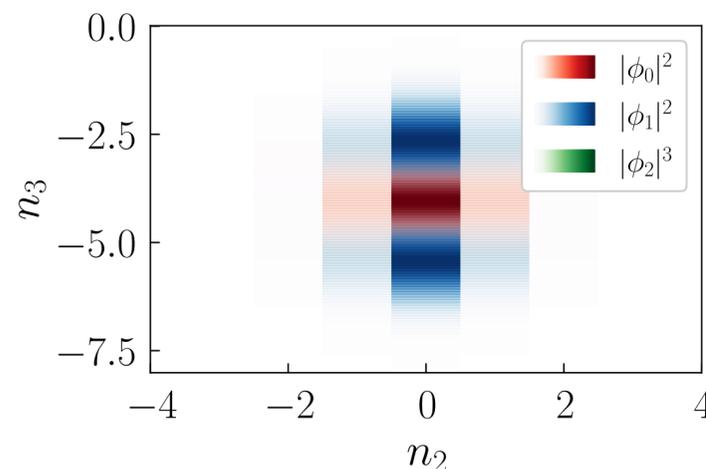
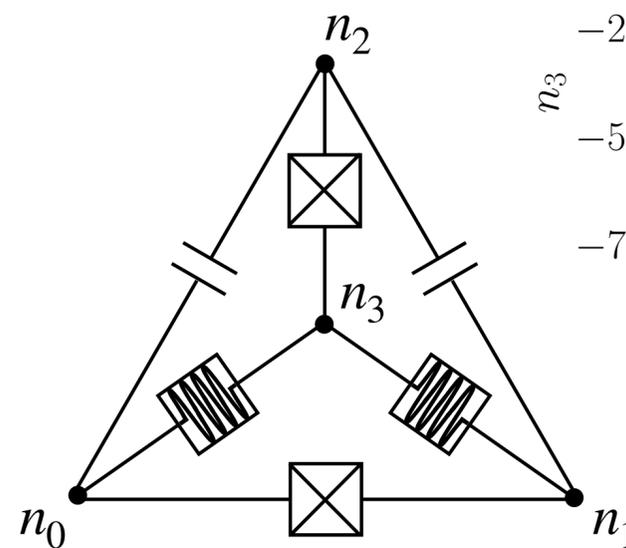
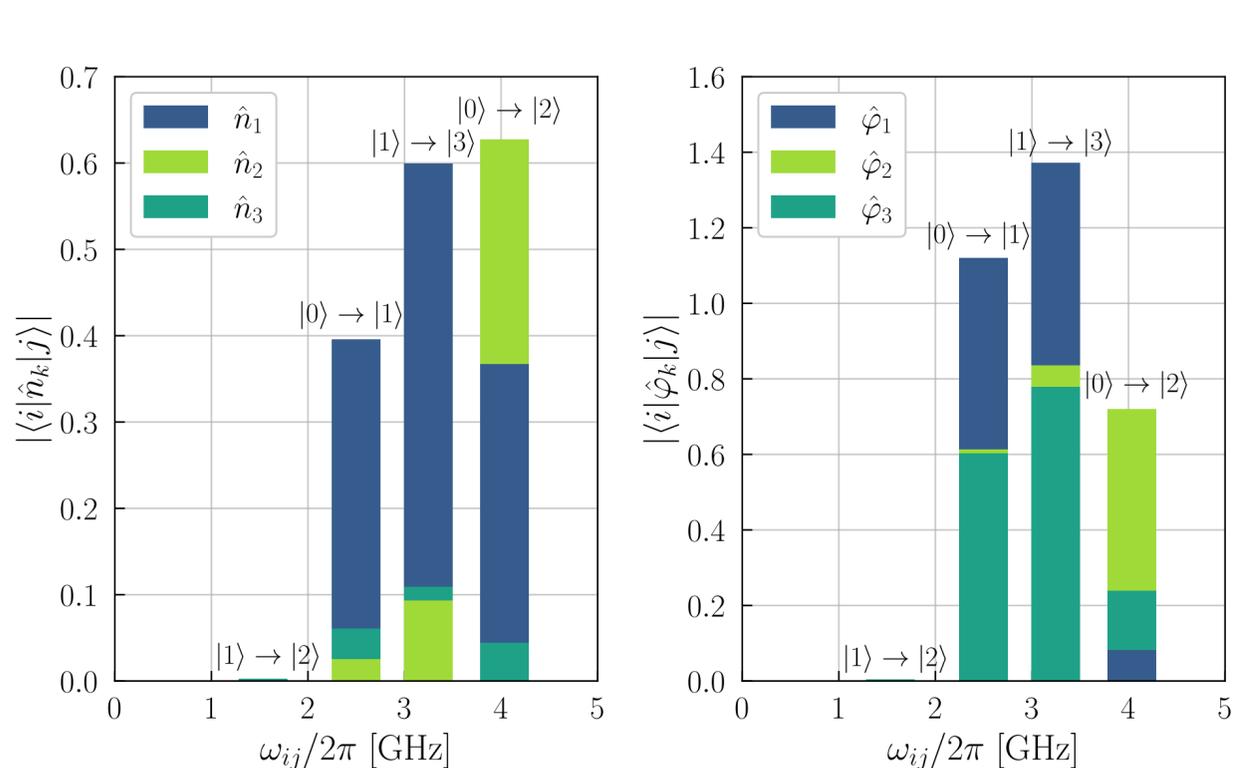


External charge dispersion

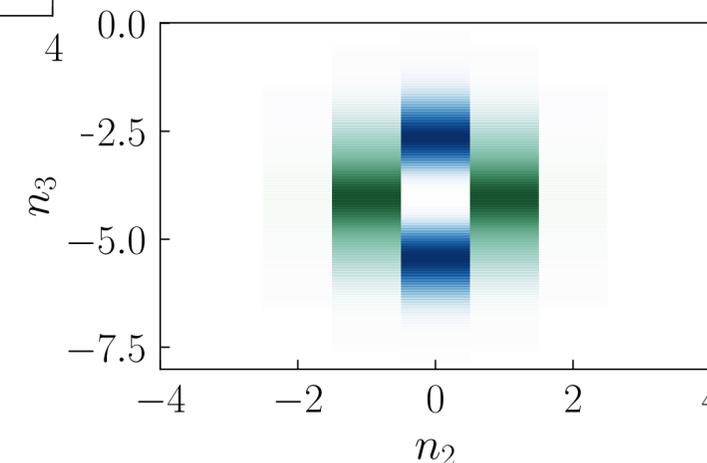




Matrix elements and wavefunctions



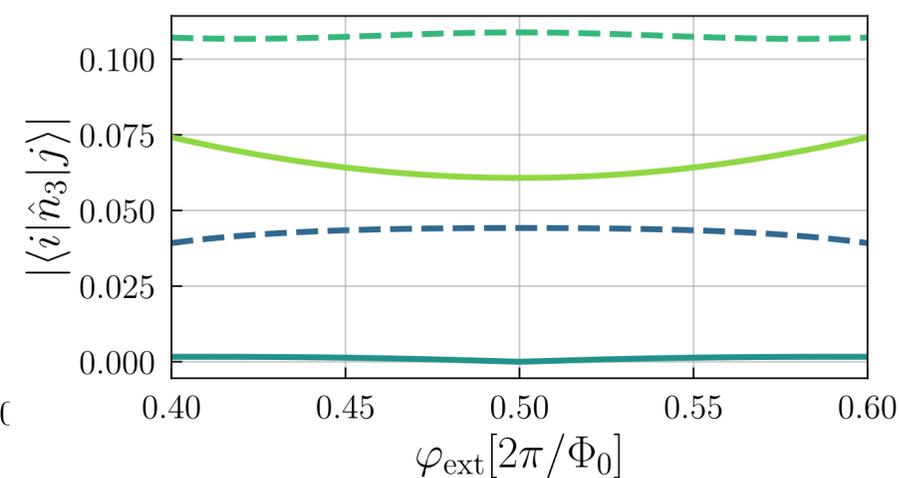
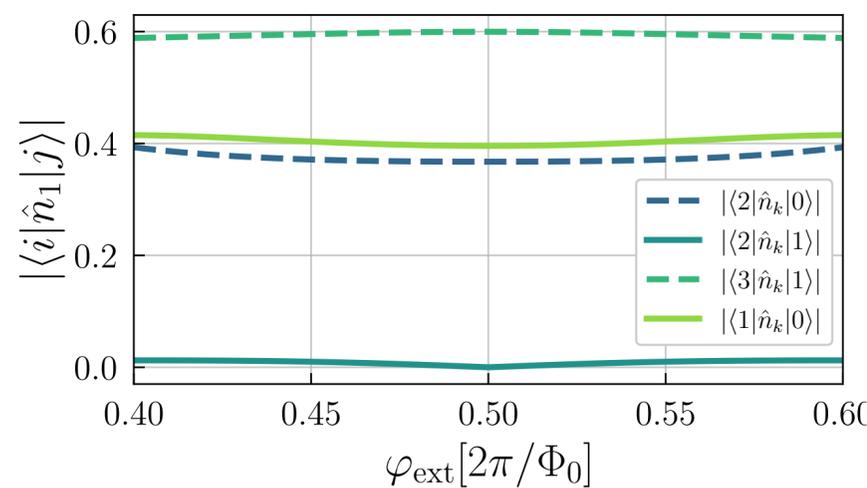
*2D projections onto 2 and 3 dimension in charge basis



- **Balanced charge dipole matrix elements**

$$\Omega \propto |\langle n \rangle| \quad g_{q_1 q_2} \propto |\langle n \rangle|^2$$

- **1-2 transition cancelled by parity**
- **Reduced support in 2 and 3 dimensions**



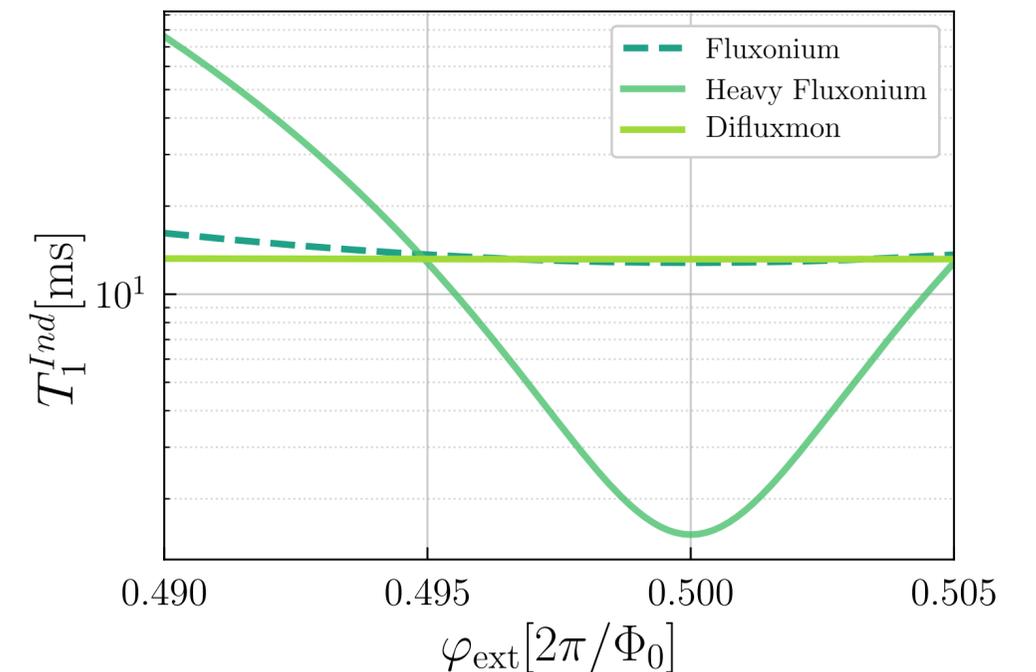
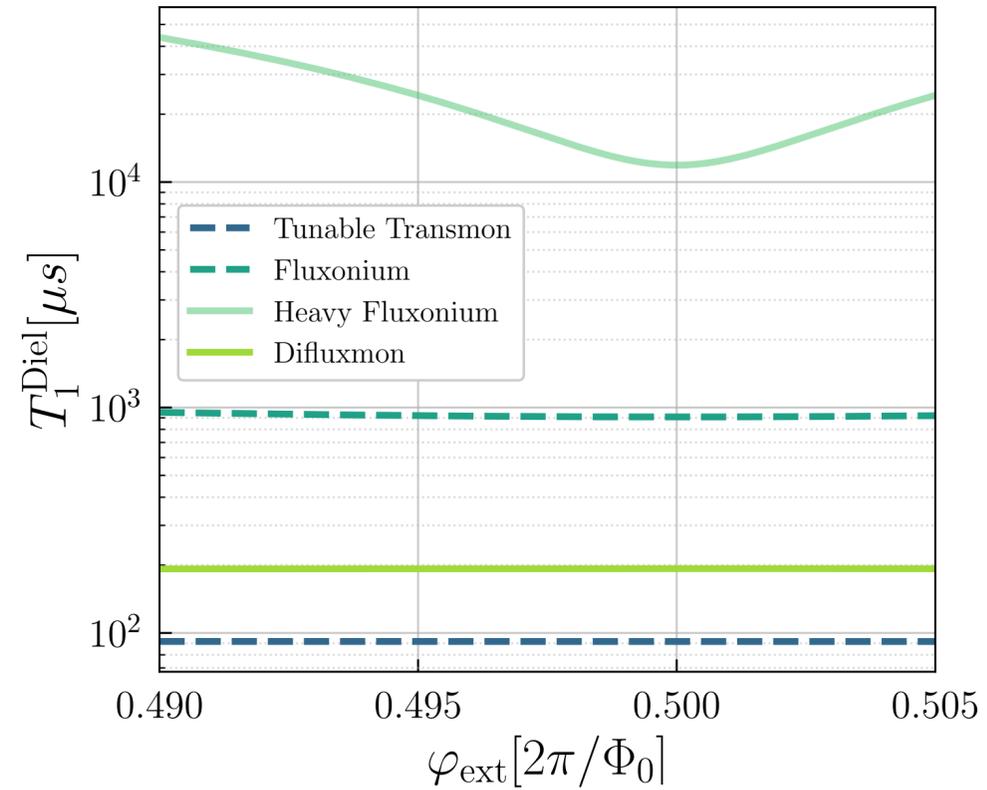


Coherence time estimation

Depolarization errors

- **Depolarisation channels considered:** Dielectric losses, Inductive losses and QP tunneling
- All possible transitions into or out from the **computational space** considered
- Estimation for a device with **multiple branches**

$$\Gamma_1^\lambda = \sum_{b_n \in B} \sum_{ij} \frac{1}{\hbar^2} \left| \langle i | \hat{O}_\lambda^{b_n} | j \rangle \right|^2 S_\lambda^{b_n}(\omega_{ij})$$



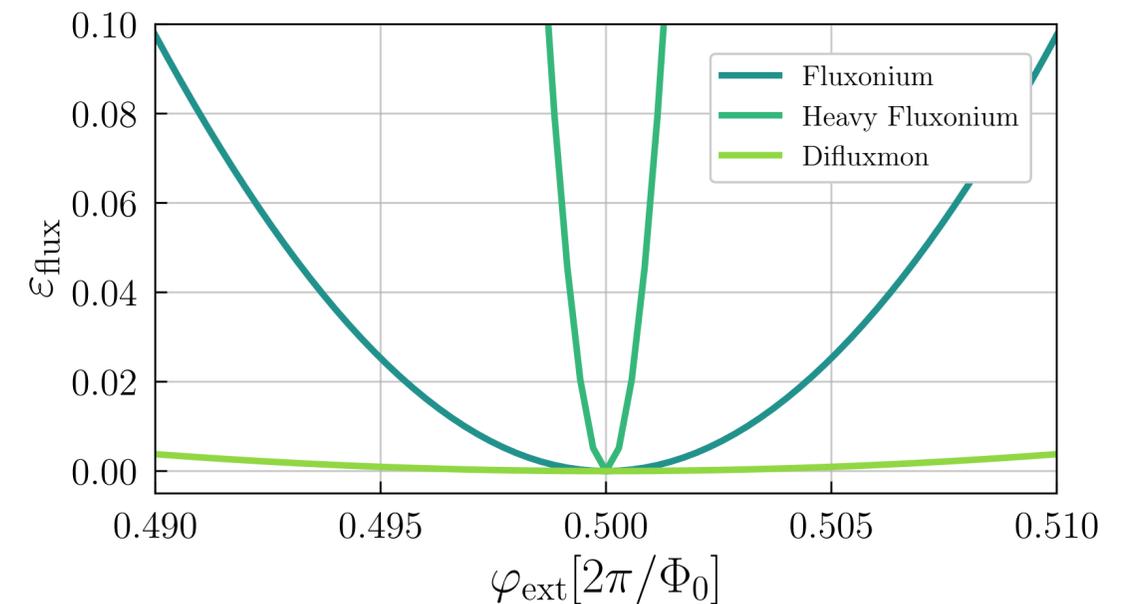
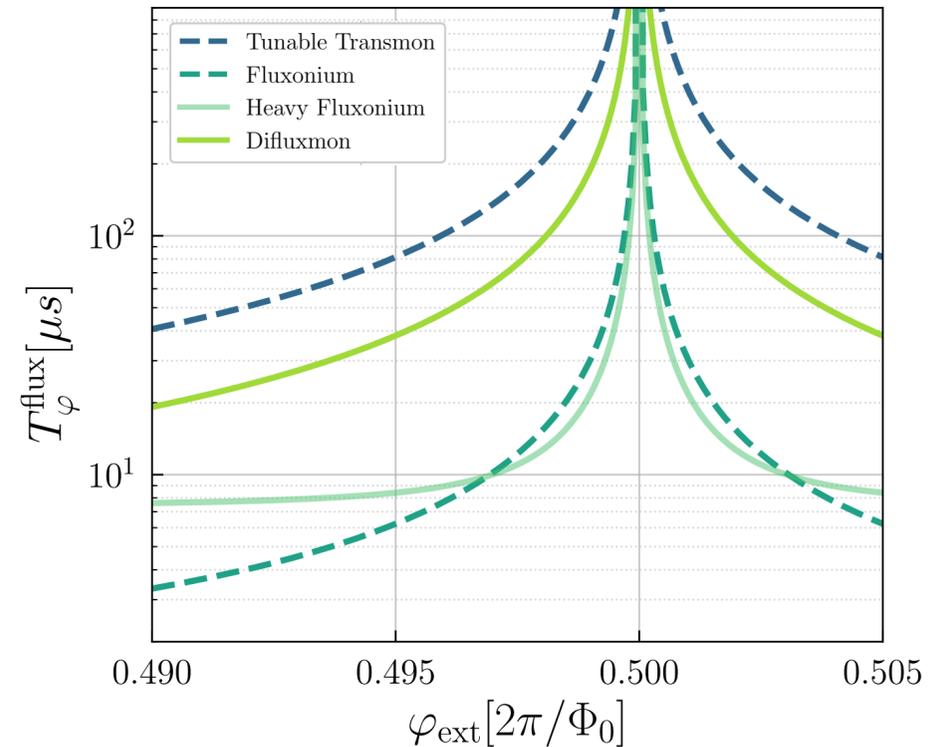


Coherence time estimation

Dephasing errors

- **Dephasing channels considered:** 1/f noise sources
- **Two** external effects inducing dephasing: charge and magnetic flux
- Estimation for a device with **multiple** branches

$$\Gamma_{1/f}^\lambda = \sqrt{2} A_\lambda \left| \frac{\partial \omega_{01}}{\partial \lambda} \right| \sqrt{|\ln \omega_{ir} t|}$$



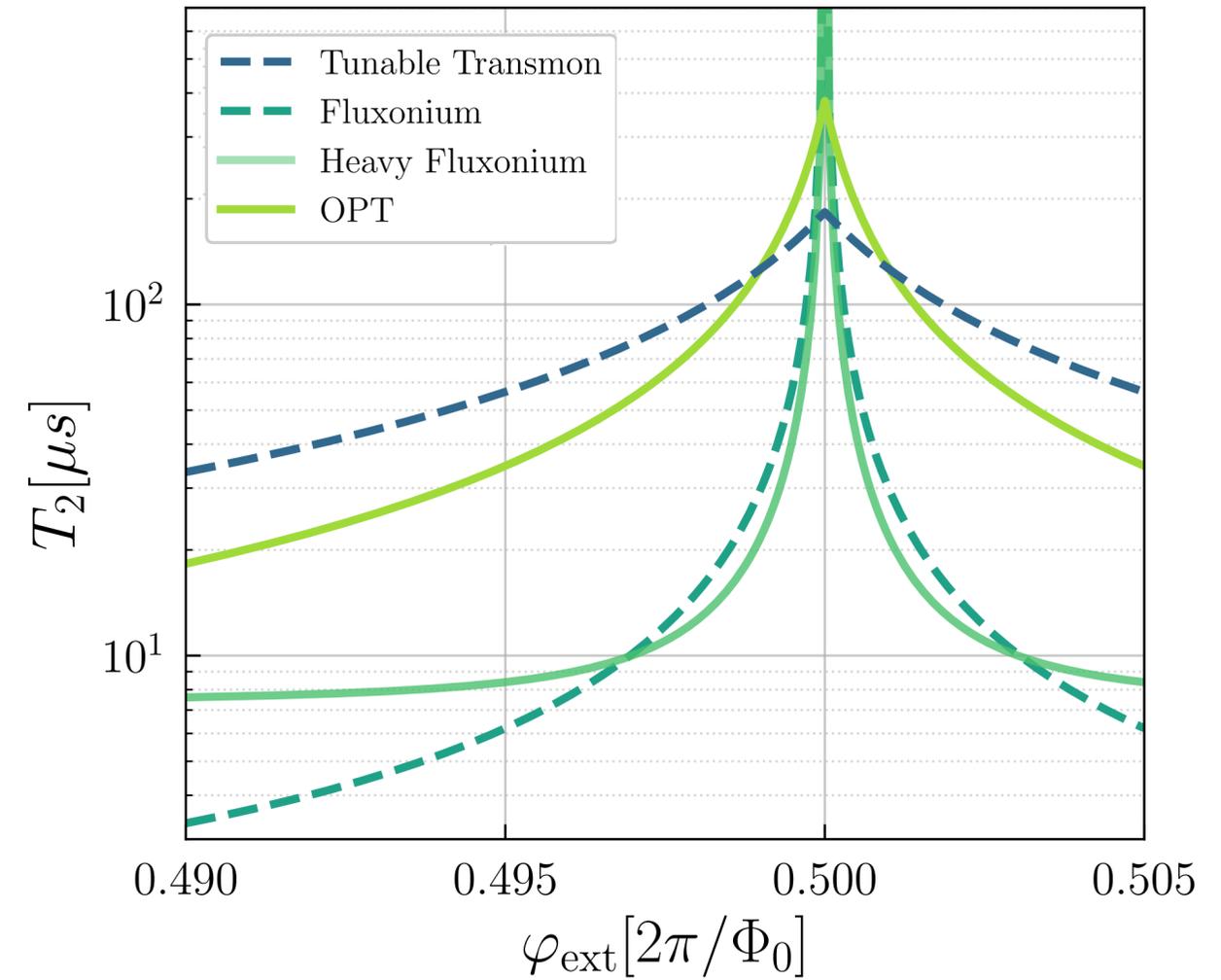


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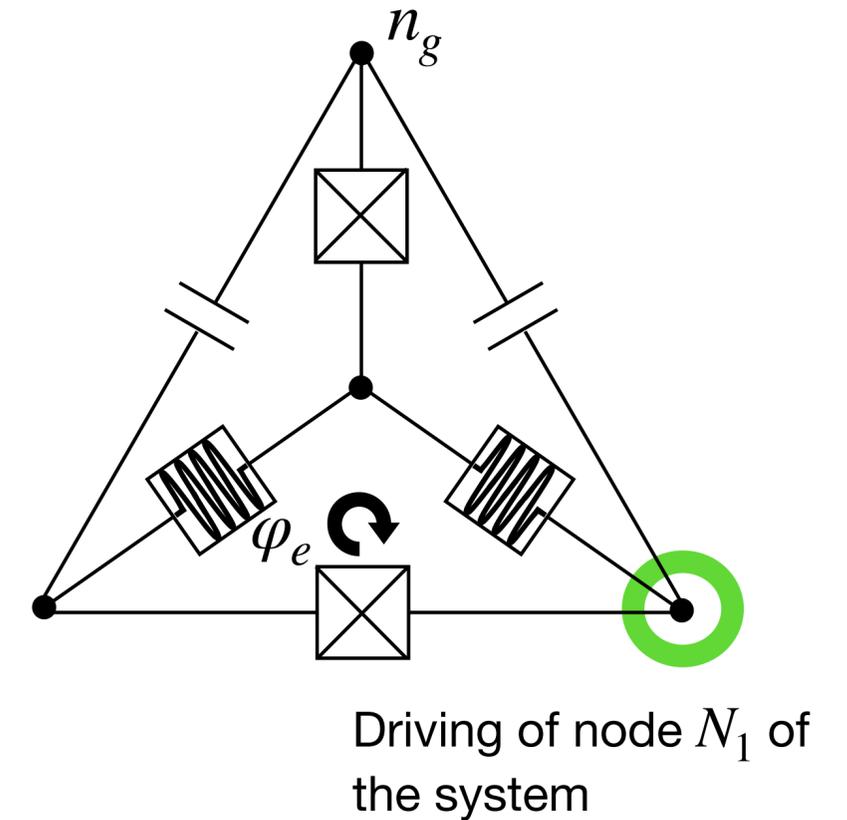
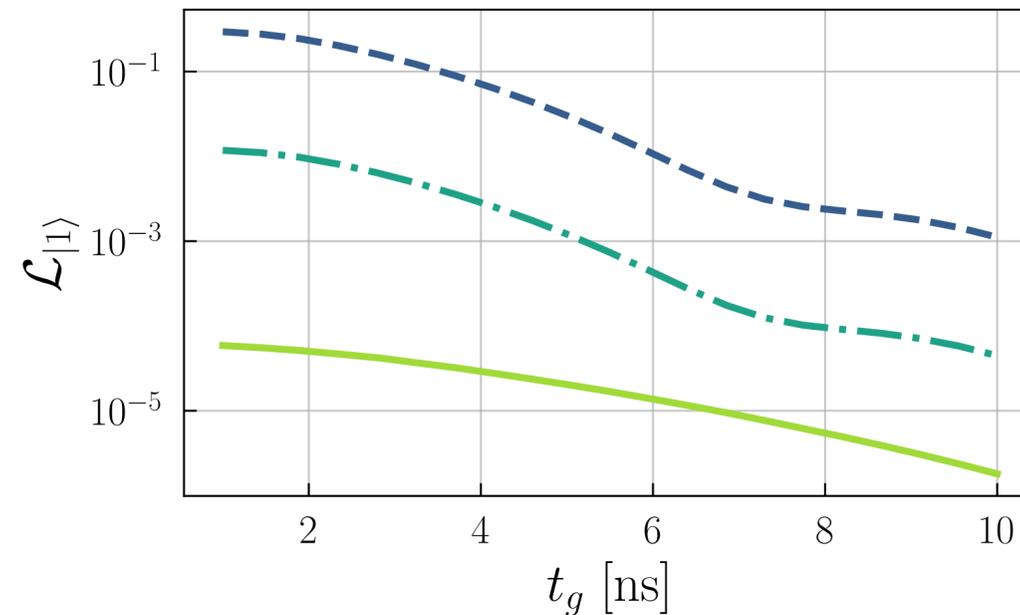
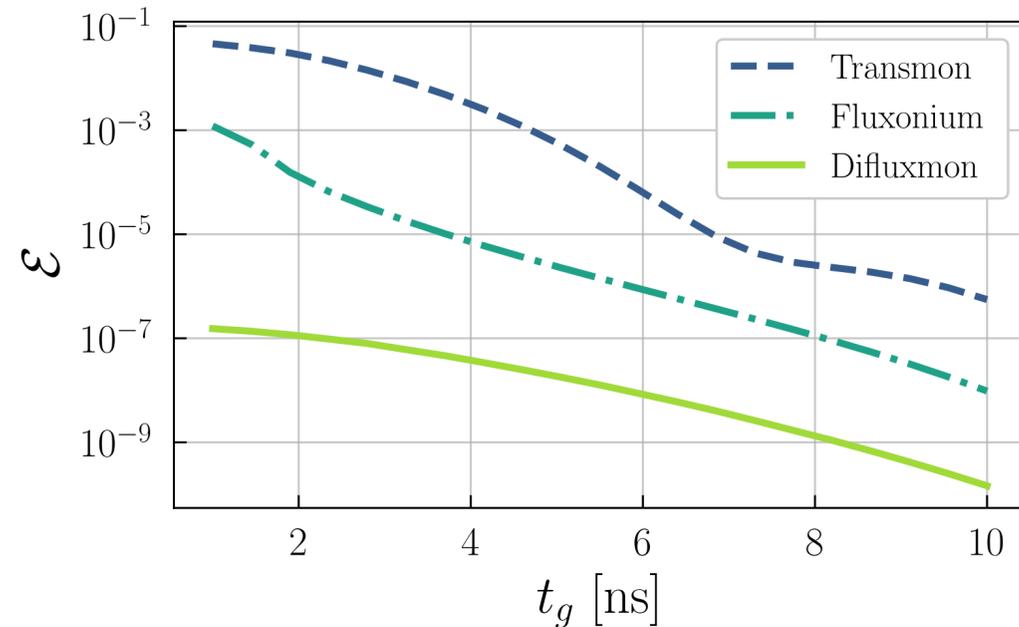
Total coherence time

- Coherence mainly limited by **dielectric losses**
- Improved **sensitivity** to external magnetic flux
- Improvement accounting the 3 times larger α than **Transmon** and the 3 times larger $|\langle 1 | \hat{n} | 0 \rangle|$ than **Fluxonium**

$$T_2 = \left(\frac{1}{2T_1} + \frac{1}{T_\varphi} \right)^{-1}$$



Gate simulations



- Increased **anharmonicity** and **qubit frequency** allows faster gates without breaking the RWA ($\omega_q = 2.5$ GHz and $\alpha_{13} \sim 750$ MHz)
- Suppressed usual **leakage** channel $\langle 2 | \hat{n}_1 | 1 \rangle = 0$
- **DRAG** protocol to suppress dephasing and 1-3 leakage
- Detuning, amplitude and DRAG **parameter optimization** for lower error rates

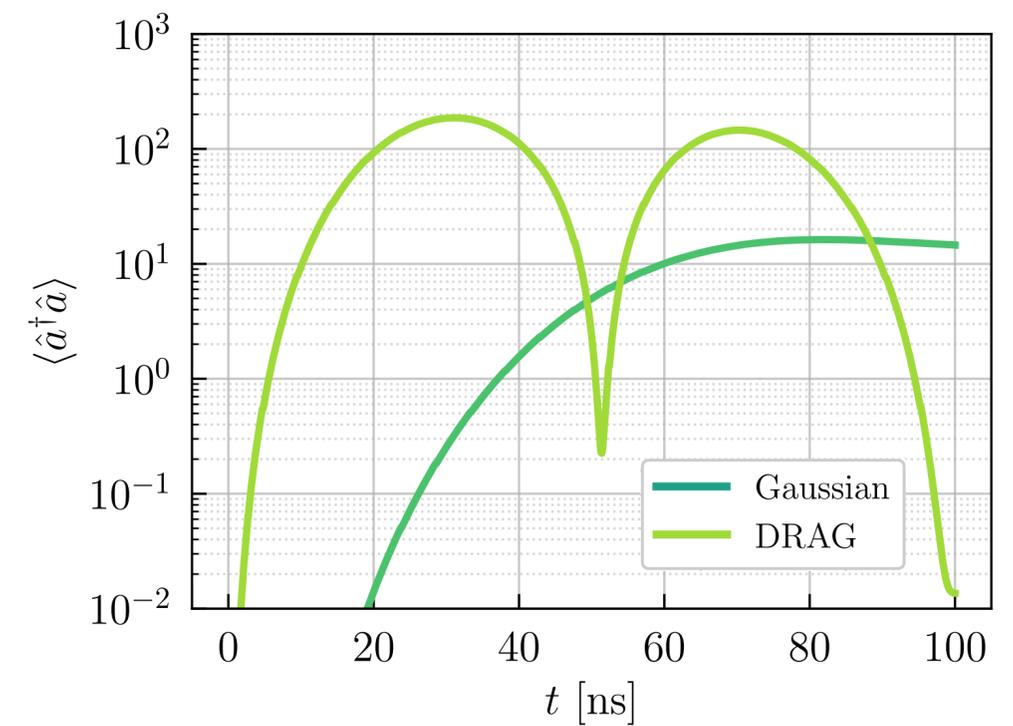
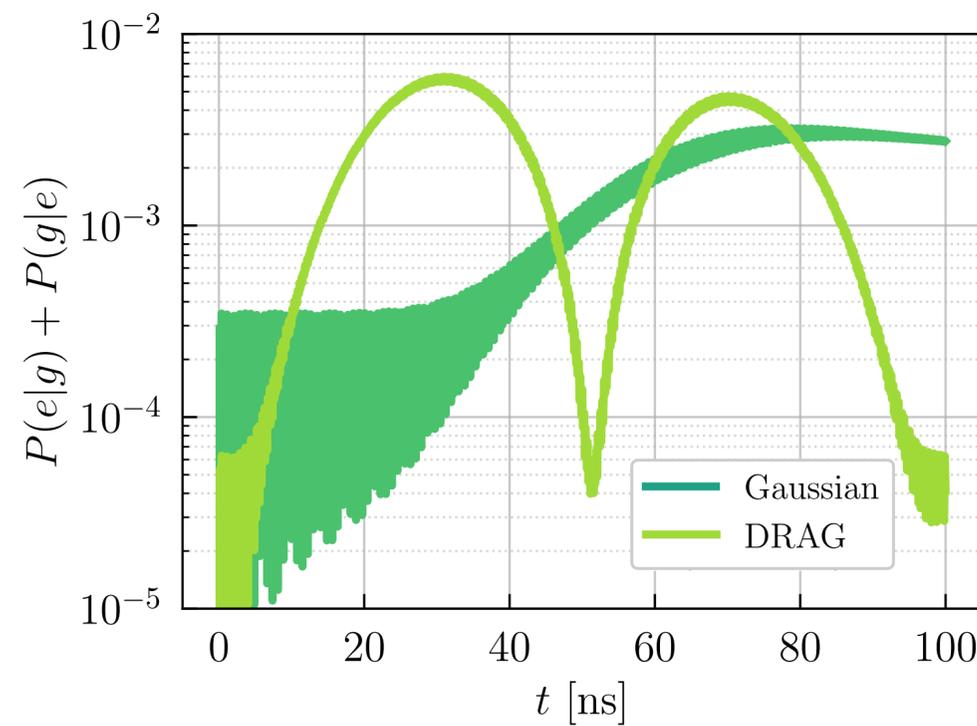
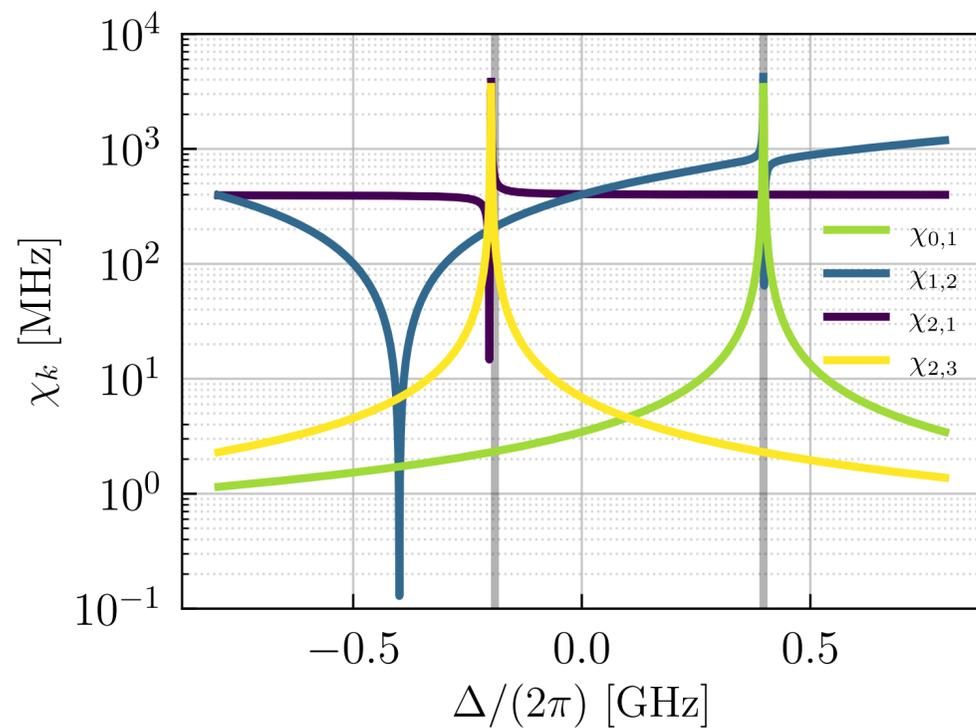
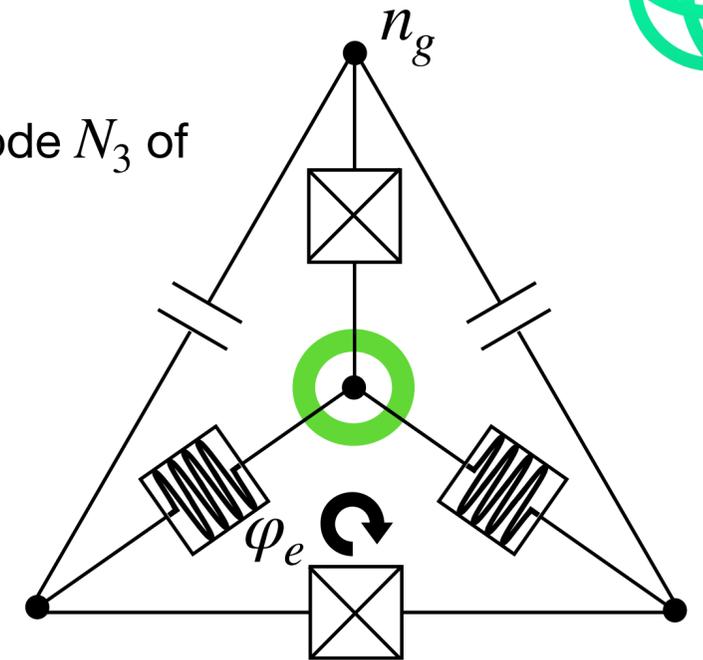
Nelder-Mead method for the gate parameters optimization of $\{\Delta, \Omega, \beta\}$

Readout and active reset



- **Driving** and **readout** points in different nodes of the system
- **Dispersive** readout with reduced charge matrix elem. (reduced Purcell effect)
- **Active reset** scheme to maximize I-Q plane state separation and minimize resonator population at the end of the measurement process

Readout of node N_3 of the system



Resilience to fabrication fluctuations



Procedure

Generate **multiple circuits** where parameters are **chosen** based on a normal distribution of certain standard deviation σ around the optimal point

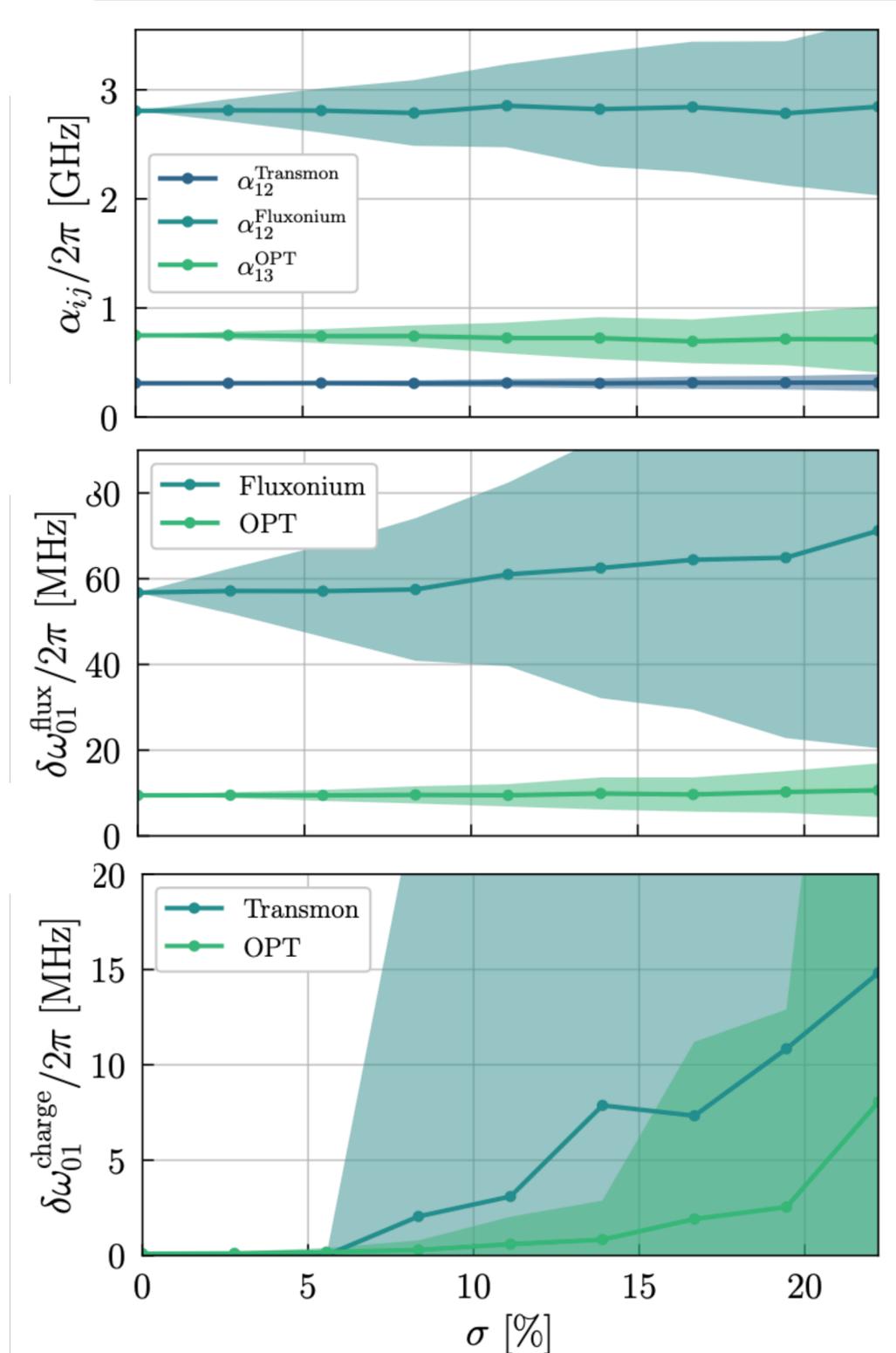
$$x = x_{optimal} \pm \delta x \quad \delta x = D \cdot x_{optimal}$$

Numerically compute their characteristics

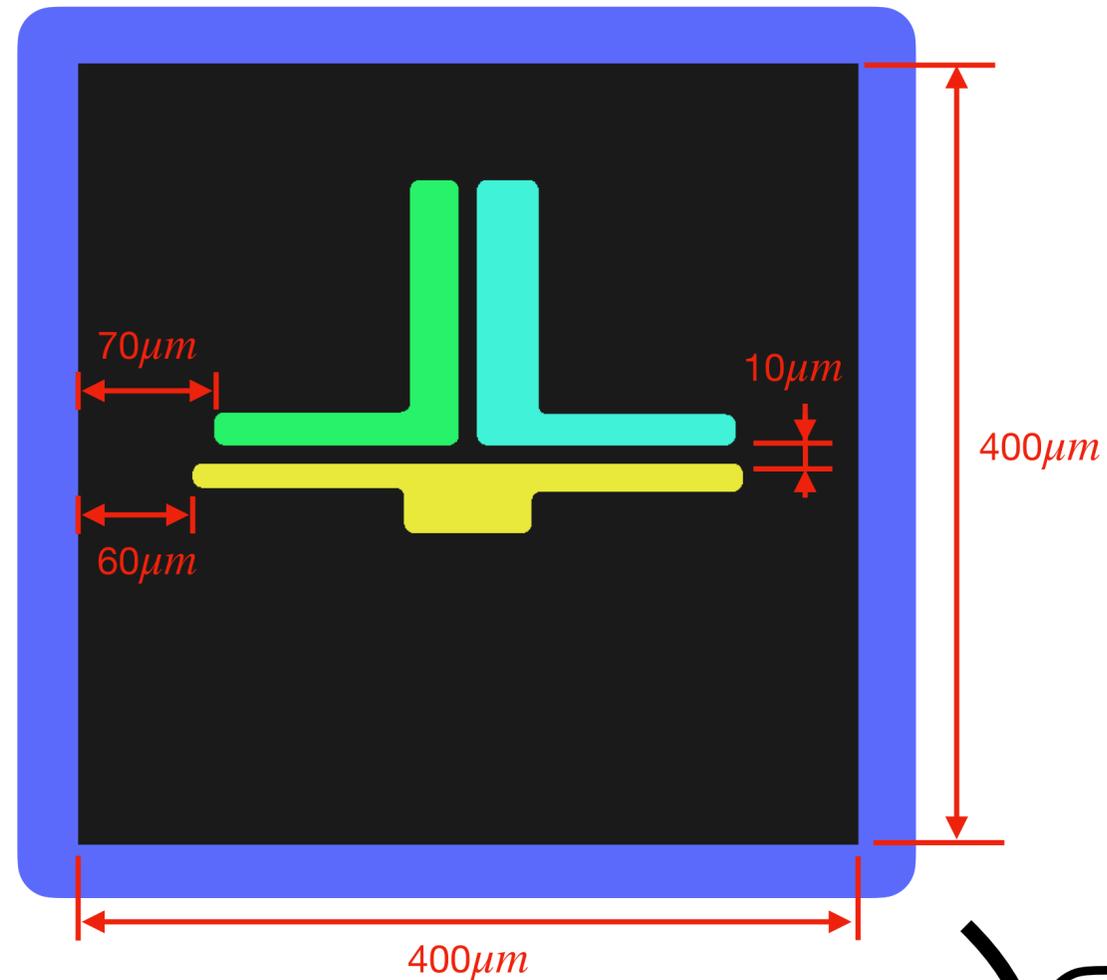
Compute the **mean** and **standard deviation**

Characteristics computed: anharmonicity, energy dispersion due to charge and flux fluctuations

Computed for **500 circuits** deviated from optimal solution

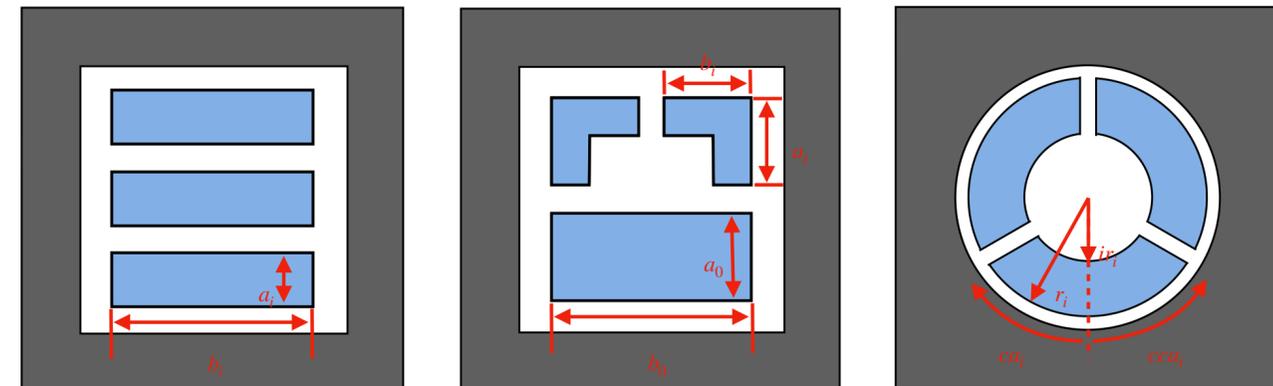


Lithography optimization



Method

- Generate **parametrized geometries** and **optimize** their parameters
- Constrain** to chip size and fabrication limitations
- Optimize **footprint**



Fast field solver

Approximated Capacitance Matrix

$$C_{\text{approx}} = \begin{pmatrix} 32.3 & -9.9 & -9.3 \\ -9.9 & 33.7 & -9.7 \\ -9.3 & -9.7 & 35.1 \end{pmatrix} [fF]$$

Expected **small extra C**
between islands connected
with JJ or JJ-arrays

Target Capacitance Matrix

$$C_{\text{target}} = \begin{pmatrix} 34.2 & -12.3 & -10.3 \\ -12.3 & 35.7 & -10.9 \\ -10.3 & -10.9 & 36.5 \end{pmatrix} [fF]$$



arXiv:2407.18895

Final remarks

- **Evolutionary algorithms** are a nice tool to explore the large landscape of possible devices and find approximate solutions
- Optimized **multi-mode** device offering **large anharmonicity**, **balanced charge matrix elements** and **reduced dispersion**

Transmon

$$T_2 \sim 200 \mu s$$
$$t_g \sim 20 ns$$

$$T_2/t_g \sim 10^4$$

Fluxonium

$$T_2 \sim 800 \mu s$$
$$t_g \sim 8 ns$$

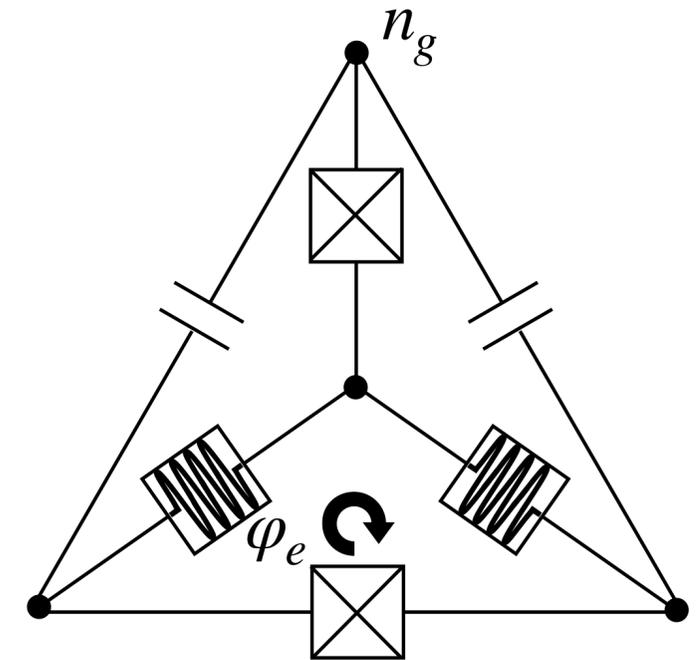
$$T_2/t_g \sim 10 \cdot 10^4$$

Difluxmon

$$T_2 \sim 400 \mu s$$
$$t_g \sim 2 ns$$

$$T_2/t_g \sim 20 \cdot 10^4$$

- **In the future:** get a better understanding of the limitations of single-mode devices and characteristics needed for improved T_2 , t_g and readout
- Check if the **increase in complexity** is worth it



Collaborators:



Francisco Cardenas-López



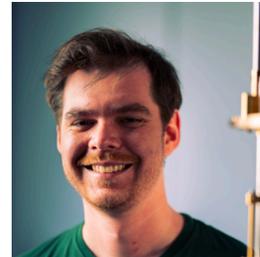
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Thank you for your attention!

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