Gluon dipole factorization diffractive heavy quarks

Patricia Gimeno-Estivill
University of Jyväskylä, Finland
In collaboration with Paul Caucal, Edmond Iancu, Tuomas Lappi and Farid Salazar

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Outline

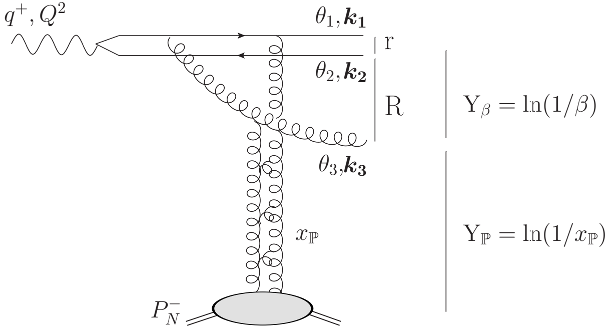
Iancu, Mueller, Triantafyllopoulos, Wei (2022)

Cross section diffractive heavy quark pair + gluon

0. Coherent diffraction 2+1

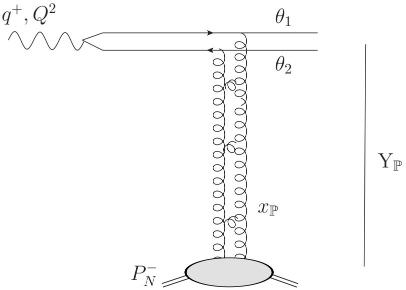
1. Small β
   - Light-Cone Perturbation Theory
   - Scattering off the CGC
   - Correlation limit
   - TMD factorization cross section

2. Generic β
   - Corrections to $q\bar{q}g$ wavefunction
   - TMD factorization cross section



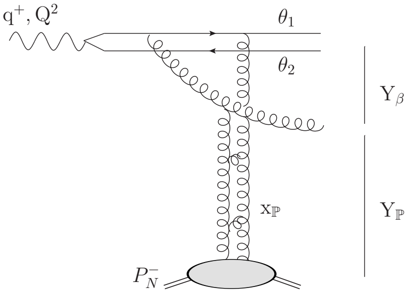
0. Motivation

- Exclusive pair production at LO



$$\boxed{\frac{d\sigma}{d\theta_1 d\theta_2 d^2\boldsymbol{P}} \sim \frac{1}{P_\perp^6}}$$ → More suppressed!

Inclusive diffractive 2+1

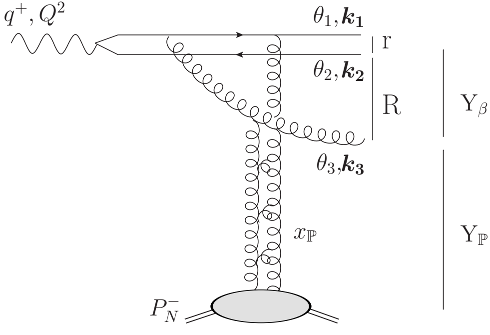


$$\boxed{\frac{d\sigma}{d\theta_1 d\theta_2 d^2\boldsymbol{P} d^2\boldsymbol{K} dY_{\mathbb{P}}} \sim \frac{1}{P_\perp^4}}$$

- New experimental data on inclusive heavy quark production $J/\psi, D, \Upsilon$

0. Coherent 2+1 diffraction

- 2+1
  - $\rightarrow$ back-to-back $\mathrm{q\bar{q}}$ ($k_{1\perp} \simeq k_{2\perp}$) + soft g ($k_{3\perp} \sim Q_\mathrm{s}$)
  - $\rightarrow$ Strong scattering effective gg dipole: $R \sim \frac{1}{k_{3\perp}} \gg r$
  - (!) Massive quarks $\Rightarrow$ small r: no need for $\boldsymbol{k}_1 \gg Q_\mathrm{s}$



- Elastic coherent: target does not break
  - $\rightarrow$ Imbalance $K_\perp = |\boldsymbol{k}_1 + \boldsymbol{k}_2| \simeq k_{3\perp}$
  - $\rightarrow$ Colorless exchange: $\mathbb{P}$omeron

- Diffractive: large rapidity gap $Y_\mathbb{P} = \ln\left(\frac{1}{x_\mathbb{P}}\right)$

$$\beta = \frac{Q^2}{Q^2 + M_{q\bar{q}g}^2} \qquad x_\mathbb{P} = \frac{Q^2 + M_{q\bar{q}g}^2}{2q \cdot P_N} \qquad M_{q\bar{q}g}^2 = \frac{\boldsymbol{k}_1^2}{\theta_1} + \frac{\boldsymbol{k}_2^2}{\theta_2} + \frac{\boldsymbol{k}_3^2}{\theta_3} + \frac{m^2}{\theta_1\theta_2}$$

1. Cross section diffractive 2+1:
Small β (large $M_{q\bar{q}g}$)

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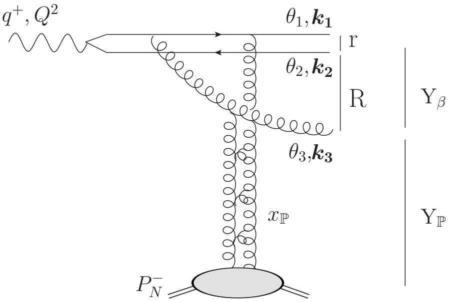
Kinematics

- Correlation limit: $k_{3\perp} \sim Q_s \ll k_{1\perp}, k_{2\perp} \sim Q$
- Very soft gluon: $\theta_3 \ll \frac{Q_s^2}{Q^2} \ll 1$

  $\rightarrow$ Emission in the eikonal approximation

  $\rightarrow M_{q\bar{q}g}^2 \simeq \frac{\boldsymbol{k}_3^2}{\theta_3} \gg Q^2 \sim M_{q\bar{q}}^2$

  $\rightarrow$ Small β



$$M_{q\bar{q}g}^2 = \frac{\boldsymbol{k}_1^2}{\theta_1} + \frac{\boldsymbol{k}_2^2}{\theta_2} + \frac{\boldsymbol{k}_3^2}{\theta_3} + \frac{m^2}{\theta_1\theta_2} \qquad M_{q\bar{q}}^2 = \frac{\boldsymbol{k}_1^2}{\theta_1} + \frac{\boldsymbol{k}_2^2}{\theta_2} + \frac{m^2}{\theta_1\theta_2} - \boldsymbol{K}^2 \qquad \beta = \frac{Q^2}{Q^2 + M_{q\bar{q}g}^2} \qquad x_{\mathbb{P}} = \frac{Q^2 + M_{q\bar{q}g}^2}{2q \cdot P_N}$$

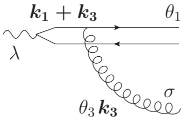
Light-cone perturbation theory

- Fock state expansion in *momentum space*

$$\left|\gamma^*\right\rangle_{\rm D}^{in} \sim \left|\gamma^*\right\rangle_0 + \sum \Psi^{\gamma^*\to q\bar{q}}\left|q\bar{q}\right\rangle_0 + \sum \Psi^{\gamma^*\to q\bar{q}g}\left|q\bar{q}g\right\rangle_0 + \dots$$

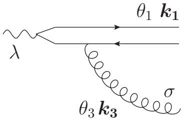
$$\underbrace{\left|q\bar{q}g\right\rangle_{\rm D,s}^{out}}_{\text{color singlet}} \sim \left|q\bar{q}g\right\rangle_0 + \sum \underbrace{\Psi^{q\bar{q}g\to q\bar{q}}}_{\substack{-\Psi^{\gamma^*\to q\bar{q}g} \\ \text{orthonormalization states}}} \left|q\bar{q}g\right\rangle_0 + \cdots$$

- Light-cone wavefunctions



$$\Psi^{\gamma_T^*\to q\bar{q}}(\theta_1, \boldsymbol{k}_1) \sim \delta_{\alpha_1\alpha_2} \frac{\boldsymbol{\epsilon}_\lambda\left(\boldsymbol{k}_1\varphi(\theta_1)\delta_{h_1,-h_2} - \sqrt{2}m\boldsymbol{\epsilon}_{h_1}\delta_{h_1,h_2}\right)}{\bar{Q}^2 + k_{1\perp}^2}$$

$$\Psi^{\gamma_T^*\to q\bar{q}g}(\theta_1, \theta_3, \boldsymbol{k}_1, \boldsymbol{k}_3) \sim t^a_{\alpha_1\alpha_2}\left(\frac{\boldsymbol{k}_3\boldsymbol{\epsilon}_\sigma^*}{k_{3\perp}^2}\right)\left[\Psi^{\gamma^*\to q\bar{q}}(\theta_1+\theta_3, \boldsymbol{k}_1+\boldsymbol{k}_3) - \Psi^{\gamma^*\to q\bar{q}}(\theta_1, \boldsymbol{k}_1)\right]$$



$$\varphi^{ij}(\theta_1) = (2\theta_1 - 1)\delta^{ij} + i\varepsilon^{ij}h_1 \quad \text{and} \quad \bar{Q}^2 = Q^2\theta_1\theta_2 + m^2$$

Scattering off the CGC

- Invariant amplitude from scattering matrix element:

$$\boxed{\mathcal{M}^{\gamma_T^*\to q\bar{q}g} \sim {}_{\mathrm{D,s}}^{\;out}\langle q\,\bar{q}\,g|\,\hat{S}-1|\gamma^*\rangle_{\mathrm{D}}^{in}}$$

Eikonal interaction

- Color Glass Condensate theory

  Many gluons in the target $\to$ described as a classical gluon field $A(x)$

- Scattering operator $\hat{S} \longrightarrow$ color rotation in mixed space:

$$\hat{S}\,|q(k^+,\boldsymbol{x}_\perp,i)\rangle = \sum_j V(\boldsymbol{x})_{ji}\,|q(k^+,\boldsymbol{x},j)\rangle$$

  Wilson Line: path ordered exponential, resums multiple scatterings

$$V(\boldsymbol{x}) = \mathcal{P}\exp\left(-ig\int dx^+ A_a^-(x^+,\boldsymbol{x})t^a\right)$$

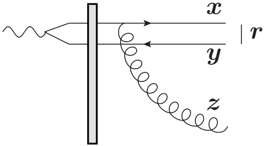
Scattering off the CGC

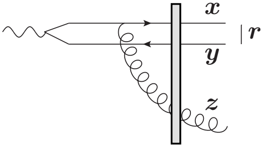
- Scattering amplitude:

$$\bar{Q}^2 = Q^2\theta_1\theta_2 + m^2$$

$$\mathcal{M}^{\gamma_T^*\to q\bar{q}g}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) \sim \int_{\boldsymbol{xyz}} e^{-i\boldsymbol{k}_1\boldsymbol{x}-i\boldsymbol{k}_2\boldsymbol{y}-i\boldsymbol{k}_3\boldsymbol{z}} \Big[\underbrace{i\varphi^{ij}(\theta_1)\bar{Q}K_1\big(\bar{Q}r_\perp\big)\frac{\epsilon_\lambda^i r^j}{r_\perp}\delta_{h_1-h_2} - \sqrt{2}mK_0\big(\bar{Q}r_\perp\big)\epsilon^i_{h_2}\epsilon^i_\lambda\delta_{h_1h_2}}_{\gamma^*\to q\bar{q}}\Big]$$

$$\times \Big[\underbrace{\epsilon_\sigma^{*m}\Big(\frac{(\boldsymbol{x}-\boldsymbol{z})^m}{|\boldsymbol{x}-\boldsymbol{z}|^2} - \frac{(\boldsymbol{y}-\boldsymbol{z})^m}{|\boldsymbol{y}-\boldsymbol{z}|^2}\Big)}_{q\bar{q}\to q\bar{q}g}\Big(S_{q\bar{q}g}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) - S_{q\bar{q}}(\boldsymbol{x},\boldsymbol{y})\Big)\Big]$$





$$S_{q\bar{q}}(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{N_c}\big\langle \mathrm{Tr}\,\big[V(\boldsymbol{x})V^\dagger(\boldsymbol{y})\big]\big\rangle \qquad S_{q\bar{q}g}(\boldsymbol{z},\boldsymbol{x},\boldsymbol{y}) = \frac{1}{N_c C_F}\big\langle U^{ba}(\boldsymbol{z})\,\mathrm{Tr}\,\big[t^bV(\boldsymbol{x})t^aV^\dagger(\boldsymbol{y})\big]\big\rangle$$

Correlation limit

- To obtain factorization we use correlation limit ($k_{3\perp} \sim Q_s \ll k_{1\perp}, k_{2\perp} \sim Q$) + change of variables:

$$\text{relative momentum:} \quad \boldsymbol{P} \equiv \theta_2 \boldsymbol{k}_1 - \theta_1 \boldsymbol{k}_2 \qquad \boldsymbol{k}_1 = \boldsymbol{P} + \theta_1 \boldsymbol{K}$$

$$\text{total momentum:} \quad \boldsymbol{K} \equiv \boldsymbol{k}_1 + \boldsymbol{k}_2 \qquad \boldsymbol{k}_2 = -\boldsymbol{P} + \theta_2 \boldsymbol{K}$$

$$\longrightarrow$$

$$\text{center of mass:} \quad \boldsymbol{b} \equiv \theta_1 \boldsymbol{x} + \theta_2 \boldsymbol{y} \qquad \boldsymbol{x} = \boldsymbol{b} + \theta_2 \boldsymbol{r}$$

$$q\bar{q} \text{ size:} \quad \boldsymbol{r} = \boldsymbol{x} - \boldsymbol{y} \qquad \boldsymbol{y} = \boldsymbol{b} - \theta_1 \boldsymbol{r}$$

and $\boldsymbol{R} = \boldsymbol{z} - \boldsymbol{b}$ is the separation between the gluon and the $q\bar{q}$ pair.

- The exponent in the Fourier transform of the amplitude is now $\boldsymbol{P} \cdot \boldsymbol{r} + (\boldsymbol{K} + \boldsymbol{k}_3) \cdot \boldsymbol{b} + \boldsymbol{k}_3 \cdot \boldsymbol{R}$ which gives $\delta^{(2)}(\boldsymbol{K} + \boldsymbol{k}_3)$ for an homogeneous target.

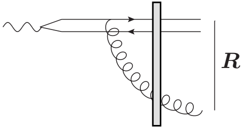
  $\longrightarrow$ Correlation limit: $P_\perp \sim Q \gg K_\perp = k_{3\perp} \sim Q_s$

Correlation limit

- Correlation limit in momentum space $P_\perp \sim Q \gg K_\perp = k_{3\perp} \sim Q_s$ or heavy quarks
  $\Longrightarrow$ $\boxed{\mathrm{r} \ll R}$ in coordinate space

$$S_{q\bar{q}g}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) \simeq S_{q\bar{q}g}(\boldsymbol{b},\boldsymbol{b},\boldsymbol{R}+\boldsymbol{b}) \sim \left\langle U^{ba}(\boldsymbol{R}+\boldsymbol{b})\,\mathrm{Tr}\left[t^b V(\boldsymbol{b}) t^a V^\dagger(\boldsymbol{b})\right]\right\rangle \sim \left\langle \mathrm{Tr}\left[U(\boldsymbol{R}+\boldsymbol{b})U^\dagger(\boldsymbol{b})\right]\right\rangle = S_g(\boldsymbol{R})$$

$S_g(\boldsymbol{R})$: Effective gluon-gluon dipole of transverse separation $\boldsymbol{R}$



$$S_{q\bar{q}g}(\boldsymbol{b},\boldsymbol{b},\boldsymbol{R}+\boldsymbol{b}) - S_{q\bar{q}}(\boldsymbol{b},\boldsymbol{b}) \simeq S_g(\boldsymbol{R}) - 1 = -\mathcal{T}_g(\boldsymbol{R})\,.$$

Independent of the size $\boldsymbol{r}$ of the $q\bar{q}$ pair.

TMD factorization

- With the change of variables, the amplitude factorizes:

$$\mathcal{M}^{\gamma_T^*\to q\bar{q}g}(\boldsymbol{P},\boldsymbol{K}) = \int \mathrm{d}^2\boldsymbol{r}\mathrm{d}^2\boldsymbol{R}e^{-i\boldsymbol{P}\cdot\boldsymbol{r}+i\boldsymbol{K}\boldsymbol{R}}\Big[\underbrace{i\varphi^{ij}(\theta_1)\sqrt{\bar{Q}}K_1\big(\sqrt{\bar{Q}}r\big)\frac{r^j}{r}\delta_{h_1-h_2} - \sqrt{2}mK_0\big(\sqrt{\bar{Q}}r\big)\epsilon^i_{h_2}\delta_{h_1h_2}}_{\gamma_T^*\to q\bar{q}}\Big]$$

$$\times\left\{\underbrace{\frac{r^n}{R^2}\left(\delta^{nm}-\frac{2R^mR^n}{R^2}\right)}_{q\bar{q}\to q\bar{q}g}\Big(S_g(\boldsymbol{R})-1\Big)\right\}$$

TMD factorization

- Factorized amplitude

$$\mathcal{M}^{\gamma_T^*\to q\bar{q}g} = \epsilon_\lambda^i \epsilon_\sigma^{*m} \underbrace{\mathcal{H}^{in}(\boldsymbol{P},\bar{Q})}_{\text{hard factor}} \underbrace{\mathcal{G}^{nm}(\boldsymbol{K},Y_{\mathbb{P}})}_{\text{semi-hard factor}}$$

Hard factor:

$$\mathcal{H}^{in}(\boldsymbol{P},\bar{Q}) = i(2\pi)\left[\varphi^{ij}(\theta)\frac{1}{P_\perp^2+\bar{Q}^2}\left(\delta^{jn}-\frac{2P^jP^n}{P_\perp^2+\bar{Q}^2}\right)\delta_{h_1,-h_2} - \epsilon_{h_1}^i 2\sqrt{2}m\frac{P^n}{\left(P_\perp^2+\bar{Q}^2\right)^2}\delta_{h_1,h_2}\right]$$

$$\bar{Q}^2 = Q^2\theta_1\theta_2 + m^2$$

Semi-hard tensorial distribution:

$$\mathcal{G}^{nm}(\boldsymbol{K},Y_{\mathbb{P}}) = \int \mathrm{d}^2\boldsymbol{R}e^{i\boldsymbol{K}\boldsymbol{R}}\left(\delta^{nm}-\frac{2R^mR^n}{R^2}\right)\frac{\mathcal{T}_{gg}(R,Y_{\mathbb{P}})}{R^2} = \left(\frac{K^nK^m}{K_\perp^2}-\frac{\delta^{nm}}{2}\right)\mathcal{G}(\boldsymbol{K},Y_{\mathbb{P}})$$

where

$$\mathcal{G}(\boldsymbol{K},Y_{\mathbb{P}}) = 2(2\pi)\int_0^\infty \frac{\mathrm{d}R}{R}\mathcal{T}_g(R,Y_{\mathbb{P}})J_2(K_\perp R)$$

TMD factorization

- Cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\theta_1\mathrm{d}\theta_2\mathrm{d}\theta_3\mathrm{d}^2\boldsymbol{K}\mathrm{d}^2\boldsymbol{P}\mathrm{d}^2\boldsymbol{k}_3} = \delta(1-\theta_1-\theta_2)\delta^2(\boldsymbol{K}+\boldsymbol{k}_3)\frac{S_\perp\alpha_{\mathrm{em}}\alpha_{\mathrm{s}}e_f^2}{4\pi^4\theta_3}C_{\mathrm{F}}N_{\mathrm{c}}$$

$$\times\left[\left(\theta_1^2+\theta_2^2\right)\frac{P_\perp^4+\bar{Q}^4}{\left[P_\perp^2+\bar{Q}^2\right]^4}+2m^2\frac{P_\perp^2}{\left[P_\perp^2+\bar{Q}^2\right]^4}\right][\mathcal{G}(K_\perp,Y_{\mathbb{P}})]^2$$

$$\bar{Q}^2=\theta_1\theta_2Q^2+m^2$$

Same hard factor as in inclusive DIS (Dominguez, Marquet, Xiao, Yuan (2011))
Same result as in Iancu, Mueller, Triantafyllopoulos, Wei (2022), now with heavy quarks

2. Cross section diffractive 2+1: Generic β

2. Generic β

Kinematics

- Correlation limit: $K_\perp = k_{3\perp} \sim Q_s \ll k_{1\perp}, k_{2\perp} \sim Q$ or heavy quarks.
- Relatively soft gluon: $\theta_3 \sim \frac{Q_s^2}{Q^2} \ll 1$.
- Generic values of θ_1 and θ_2.

$$M_{q\bar{q}}^2 \simeq \frac{P_\perp^2 + m^2}{\theta_1\theta_2} \qquad \beta \simeq \frac{\bar{Q}^2}{\bar{Q}^2 + P_\perp^2 + \theta_1\theta_2\frac{K_\perp^2}{\theta_3}} \qquad x \simeq \frac{\bar{Q}^2 + P_\perp^2}{\bar{Q}^2 + P_\perp^2 + \theta_1\theta_2\frac{K_\perp^2}{\theta_3}}$$

$$\bar{Q}^2 = \theta_1\theta_2 Q^2 + m^2$$

$\rightarrow$ When $K_\perp^2/Q^2 \sim \theta_3$ then $\beta \sim \mathcal{O}(1)$

and $\theta_3 \ll 1$ so still $1 - \theta_3 = 1$ and $\theta_3 + \theta_{1,2} = \theta_{1,2}$.

$\rightarrow$ We keep the terms in the LCPT vertices that are linear in θ_3: regular and instantaneous diagrams. Omit terms of $\mathcal{O}(\theta_3^2 P_\perp, \theta_3 K_\perp, ..)$

Corrections to $\Psi^{q\bar{q}g}$ wavefunction

- Regular diagram

$$\Psi^{\gamma_T^*\to q\bar{q}g}(\boldsymbol{k}_1,\boldsymbol{k}_3)\sim\left(\frac{\epsilon_\sigma^{*m}\epsilon_\lambda^i}{\mathcal{M}^2+k_{3\perp}^2}\right)\left[\frac{\left(k_3+\frac{\theta_3}{\theta_1}k_2\right)^m\left(\varphi^{ij}(\theta_1)(k_1+k_3)^j\delta_{h_1-h_2}-\sqrt{2}m\epsilon_{h_2}^i\delta_{h_1h_2}\right)}{\bar{Q}^2+(k_{1\perp}+k_{3\perp})^2}\right.$$

$$\left.-\frac{\left(k_3+\frac{\theta_3}{\theta_2}k_1\right)^m\left(\varphi^{ij}(\theta_1)k_1^j\delta_{h_1-h_2}-\sqrt{2}m\epsilon_{h_2}^i\delta_{h_1h_2}\right)}{\bar{Q}^2+k_{1\perp}^2}\right]$$

$$\bar{Q}^2=\theta_1\theta_2Q^2+m^2$$

where in the gluon emission energy denominator

$$\mathcal{M}^2=\theta_3\left[Q^2+\frac{k_{1\perp}^2+m^2}{\theta_1}+\frac{k_{2\perp}^2+m^2}{\theta_2}\right]$$
$$\simeq\frac{\theta_3}{\theta_1\theta_2}(P_\perp^2+\bar{Q}^2)$$

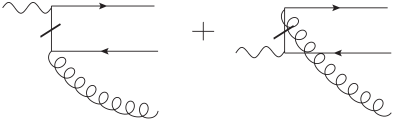
Corrections to $\Psi^{q\bar{q}g}$ wavefunction

- Regular diagrams

$$\Psi^{im}_{reg}(\boldsymbol{P},\boldsymbol{k}_3) \sim \underbrace{\left[\frac{\varphi^{ij}(\theta_1)\delta^{jn}\delta_{h_1,-h_2}}{\bar{Q}^2+P_\perp^2} - \frac{\varphi^{ij}(\theta_1)2P^jP^n\delta_{h_1,-h_2} - 2\sqrt{2}m\epsilon^i_{h_2}P^n\delta_{h_1h_2}}{(\bar{Q}^2+P_\perp^2)^2}\right]}_{\text{Hard factor} \quad \mathcal{H}^{in}(\boldsymbol{P})} \frac{k_3^n k_3^m}{k_{3\perp}^2+\mathcal{M}^2}$$

$$- \frac{\mathcal{M}^2}{k_{3\perp}^2+\mathcal{M}^2}\frac{\varphi^{ij}(\theta_1)P^jP^m\delta_{h_1,-h_2} - \sqrt{2}m\epsilon^i_{h_2}P^m\delta_{h_1h_2}}{\left[\bar{Q}^2+P_\perp^2\right]^2}$$

- Instantaneous diagrams



$$\Psi^{im}_{inst}(\boldsymbol{P},\boldsymbol{k}_3) \sim \frac{\mathcal{M}^2}{\mathcal{M}^2+k_{3\perp}^2}\frac{\varphi^{ij}(\theta_1)\delta^{jm}}{P_\perp^2+\bar{Q}^2}\delta_{h_1,-h_2}$$

Corrections to $\Psi^{q\bar{q}g}$ wavefunction

- Total wavefunction: regular + instantaneous diagrams

$$
\begin{aligned}
\Psi^{im}_{tot}(\boldsymbol{P},\boldsymbol{k}_3) &= \Psi^{im}_{reg}(\boldsymbol{P},\boldsymbol{k}_3) + \Psi^{im}_{inst}(\boldsymbol{P},\boldsymbol{k}_3) \\
&= \mathcal{H}^{in}(\boldsymbol{P})\frac{k_3^n k_3^m + \delta^{nm}\mathcal{M}^2/2}{k_{3\perp}^2+\mathcal{M}^2}
\end{aligned}
$$

(!) This can be decomposed into a traceless 2D tensor and a diagonal part:

$$
\Psi^{im}_{tot}(\boldsymbol{P},\boldsymbol{k}_3) = \underbrace{\mathcal{H}^{in}(\boldsymbol{P})\frac{k_3^n k_3^m - \delta^{nm}\frac{k_{3\perp}^2}{2}}{k_{3\perp}^2+\mathcal{M}^2}}_{\Psi_{\mathbb{P}}(\boldsymbol{P},\boldsymbol{k}_3)} + \underbrace{\frac{\mathcal{H}^{in}(\boldsymbol{P})}{2}\delta^{nm}}_{\Psi_{\text{diag}}(\boldsymbol{P})}
$$

Corrections to $\Psi^{q\bar{q}g}$ wavefunction

- For $P_\perp^2 \gg Q_s^2$, we neglect the change due to the scattering of the relative momentum $\boldsymbol{P}$. The Fourier transform of the traceless 2D tensor w.r.t $k_{3\perp}$:

$$\begin{aligned}
\Psi_{\mathbb{P}}^{im}(\boldsymbol{P}, \boldsymbol{R}) &= \mathcal{H}^{in}(\boldsymbol{P}) \int \frac{\mathrm{d}^2 \boldsymbol{k}_3}{(2\pi)^2} e^{i\boldsymbol{k}_3 \cdot \boldsymbol{R}} \psi_{\mathbb{P}}^{nm} \\
&= \mathcal{H}^{in}(\boldsymbol{P}) \int \frac{\mathrm{d}^2 \boldsymbol{k}_3}{(2\pi)^2} e^{i\boldsymbol{k}_3 \cdot \boldsymbol{R}} \frac{k_3^n k_3^m - \delta^{nm} \frac{k_{3\perp}^2}{2}}{k_{3\perp}^2 + \mathcal{M}^2} \\
&= \mathcal{H}^{in}(\boldsymbol{P}) \frac{1}{2\pi} \left[\frac{\delta^{nm}}{2} - \left(\frac{R^n R^m}{R^2} \right) \right] \mathcal{M}^2 K_2(\mathcal{M} R)
\end{aligned}$$

The Fourier transform of $\Psi_{diag}(\boldsymbol{P})$ gives a $\delta^2(\boldsymbol{R})$. When multiplying it with the eikonal scattering amplitude $\mathcal{T}_g(R)$ it gives zero by color transparency. We keep only the $\Psi_{\mathbb{P}}$ wavefunction.

TMD factorization

The pomeron term gives the amplitude

$$\mathcal{M}^{\gamma^* \to q\bar{q}g} = \epsilon^i_\lambda \epsilon^{*m}_\sigma \mathcal{H}^{in}(\boldsymbol{P}) \mathcal{G}^{nm}(\boldsymbol{K}, \mathcal{M}, Y_{\mathbb{P}})$$

where

$$\mathcal{G}^{nm}(\boldsymbol{K}, \mathcal{M}, Y_{\mathbb{P}}) = \int \mathrm{d}^2\boldsymbol{R} e^{i\boldsymbol{K}\boldsymbol{R}} \left(\delta^{nm} - \frac{2R^n R^m}{R_\perp^2}\right) \mathcal{M}^2 K_2(\mathcal{M}R) \mathcal{T}_g(R, Y_{\mathbb{P}}) = \left(\frac{K^n K^m}{K^2} - \frac{\delta^{nm}}{2}\right) \mathcal{G}(K, \mathcal{M}, Y_{\mathbb{P}})$$

and

$$\mathcal{G}(K, \mathcal{M}, Y_{\mathbb{P}}) = 2(2\pi)\mathcal{M}^2 \int_0^\infty \mathrm{d}R R J_2(KR) K_2(\mathcal{M}R) \mathcal{T}_g(R, Y_{\mathbb{P}})$$

- In collinear factorization: semi-hard gluon TMD is a target distribution. But

$$\mathcal{M}^2 \simeq \frac{\theta_3}{\theta_1\theta_2}(P_\perp^2 + \bar{Q}^2)$$

TMD factorization

- To factorize the amplitude, we need a change of variables from "plus" to "minus" longitudinal momentum x or $x_{\mathbb{P}}$. Using

$$\mathcal{M}^2 \simeq \frac{\theta_3}{\theta_1\theta_2}(\bar{Q}^2 + P_\perp^2) \qquad x \simeq \frac{\bar{Q}^2 + P_\perp^2}{\bar{Q}^2 + P_\perp^2 + \theta_1\theta_2\frac{K_\perp^2}{\theta_3}}$$

we get

$$\boxed{\mathcal{M}^2 = \frac{x}{1-x}K_\perp^2}$$

Using also $x_{\mathbb{P}}$:

$$x_{\mathbb{P}} = \frac{Q^2 + M_{q\bar{q}g}^2}{2q\cdot P_N} \qquad x \equiv \frac{x_{q\bar{q}}}{x_{\mathbb{P}}} \simeq \frac{Q^2 + M_{q\bar{q}}^2 + K_\perp^2}{Q^2 + M_{q\bar{q}}^2 + K_\perp^2 + k_{3\perp}^2/\theta_3}$$

we have the change of variables:

$$\frac{x_{\mathbb{P}} - x_{q\bar{q}}}{x_{q\bar{q}}} = \frac{x_{\mathbb{P}}}{x_{q\bar{q}}} - 1 = \frac{Q^2 + M_{q\bar{q}}^2 + K_\perp^2 + k_{3\perp}^2/\theta_3}{Q^2 + M_{q\bar{q}}^2 + K_\perp^2} - 1 = \frac{k_{3\perp}^2/\theta_3}{Q^2 + M_{q\bar{q}}^2 + K_\perp^2}$$

Taking the logarithmic differential:

$$\boxed{\frac{\mathrm{d}\theta_3}{\theta_3} = \frac{\mathrm{d}x_{\mathbb{P}}}{x_{\mathbb{P}} - x_{q\bar{q}}} = \frac{\mathrm{d}x_{\mathbb{P}}}{x_{\mathbb{P}}}\frac{1}{1-x}}$$

TMD factorization

Then the cross section can be written in the factorized form:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\theta_1 \mathrm{d}\theta_2 \mathrm{d}^2\boldsymbol{K}\mathrm{d}^2\boldsymbol{P}\mathrm{d}Y_{\mathbb{P}}} = H(\theta_1, \theta_2, Q^2, P_\perp^2)\frac{\mathrm{d}xG_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}}$$

with the hard factor:

$$H(\theta_1, \theta_2, Q^2, P_\perp^2) \equiv \delta(1-\theta_1-\theta_2)\alpha_{\mathrm{em}}\alpha_{\mathrm{s}}e_f^2\left[\left(\theta_1^2+\theta_2^2\right)\frac{P_\perp^4+\bar{Q}^4}{\left[P_\perp^2+\bar{Q}^2\right]^4}+2m^2\frac{P_\perp^2}{\left[P_\perp^2+\bar{Q}^2\right]^4}\right]$$

$$\bar{Q}^2 = \theta_1\theta_2 Q^2 + m^2$$

and the semi-hard factor (diffractive gluon TMD):

$$\frac{\mathrm{d}xG_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}} \equiv \frac{S_\perp(N_c^2-1)}{4\pi^3}\frac{\left[\mathcal{G}(K_\perp, x, Y_{\mathbb{P}})\right]^2}{2\pi(1-x)}$$

Summary

- Diffractive 2+1 with massive quarks:

  → Same hard factor as in inclusive DIS (Dominguez, Marquet, Xiao, Yuan (2011))

  → Same result as in Iancu, Mueller, Triantafyllopoulos, Wei (2022), now with heavy quarks

- Outlook

  → Diffractive 2+1 with soft quark (massive)
  → Predictions