Scaling of the elastic pp cross-section

Michał Praszałowicz
(Jagiellonian University, Krakow)

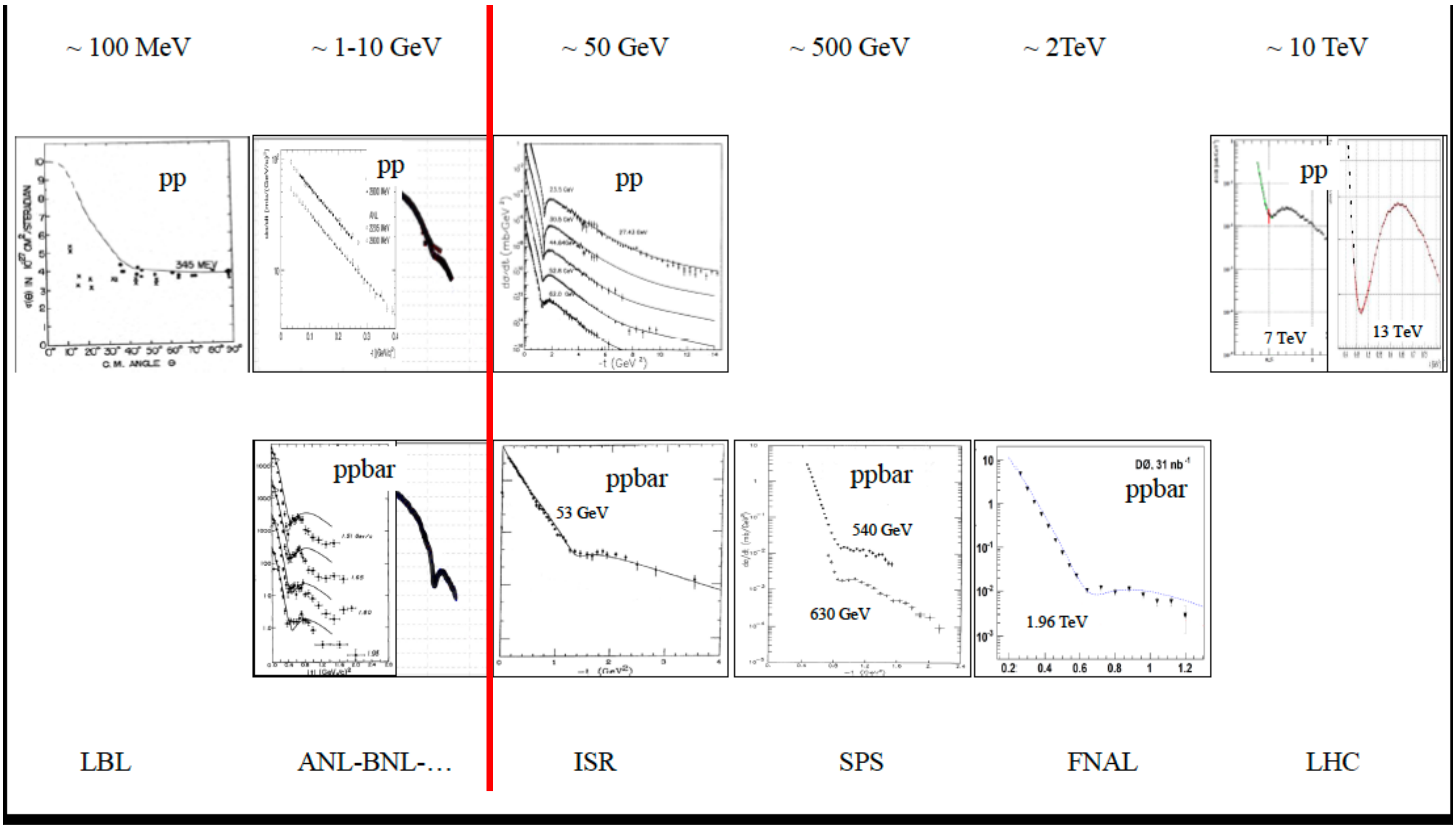
Benasque,
August 12, 2025

Scaling laws of elastic proton-proton scattering differential cross sections
Cristian Baldenegro (Rome U.),
Michal Praszalowicz (Jagiellonian U.),
Christophe Royon (Kansas U.),
Anna M. Stasto (Penn State U.) (Jun 3, 2024)
Phys.Lett.B 856 (2024) 138960 • e-Print: 2406.01737 [hep-ph]

Geometric scaling of elastic pp cross section at the LHC
Michal Praszalowicz (Jagiellonian U.) • e-Print: 2504.18841 [hep-ph]

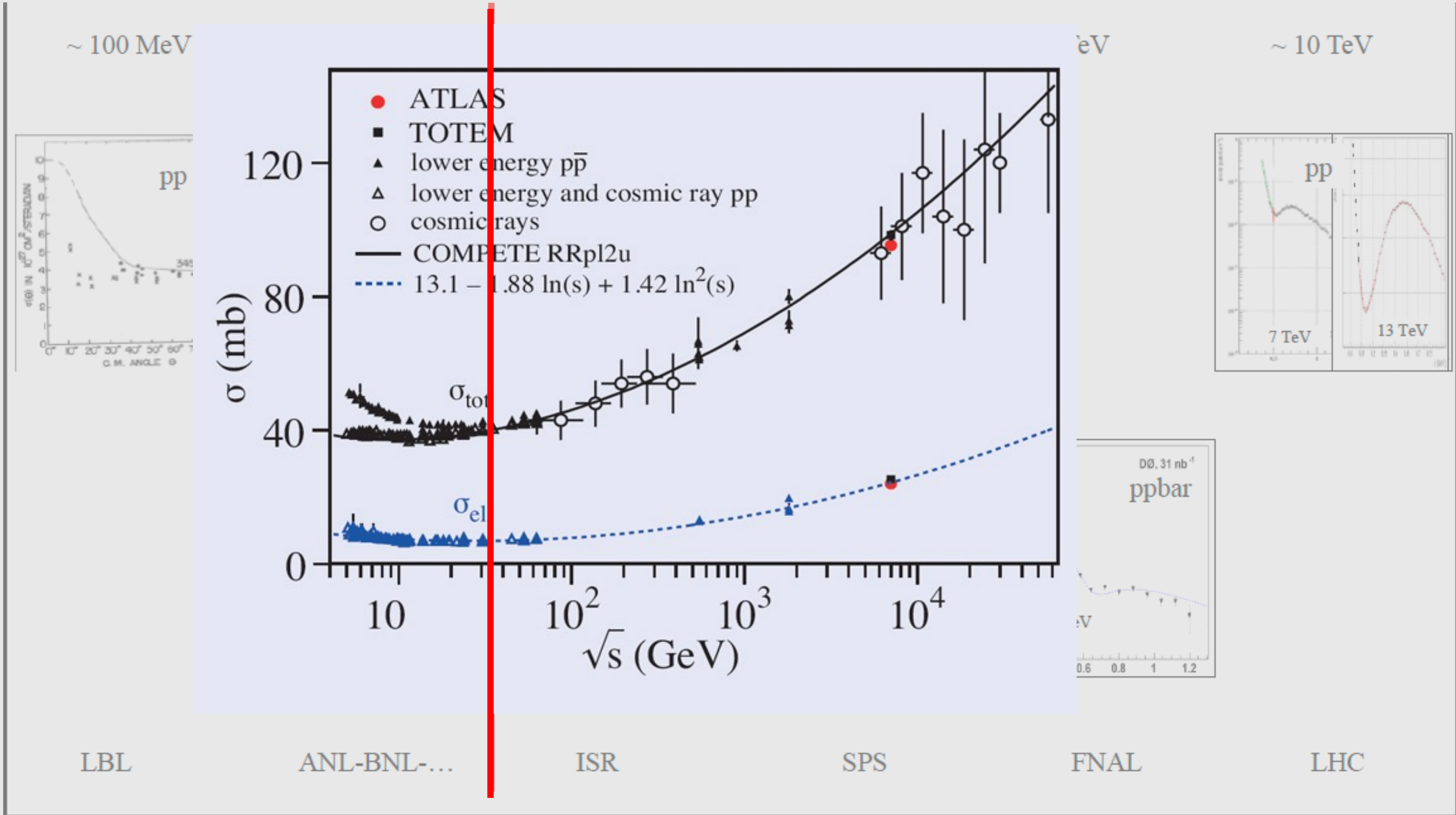
Differential elastic cross section

Talk by S. Giani, PP elastic nuclear elastic scattering, …

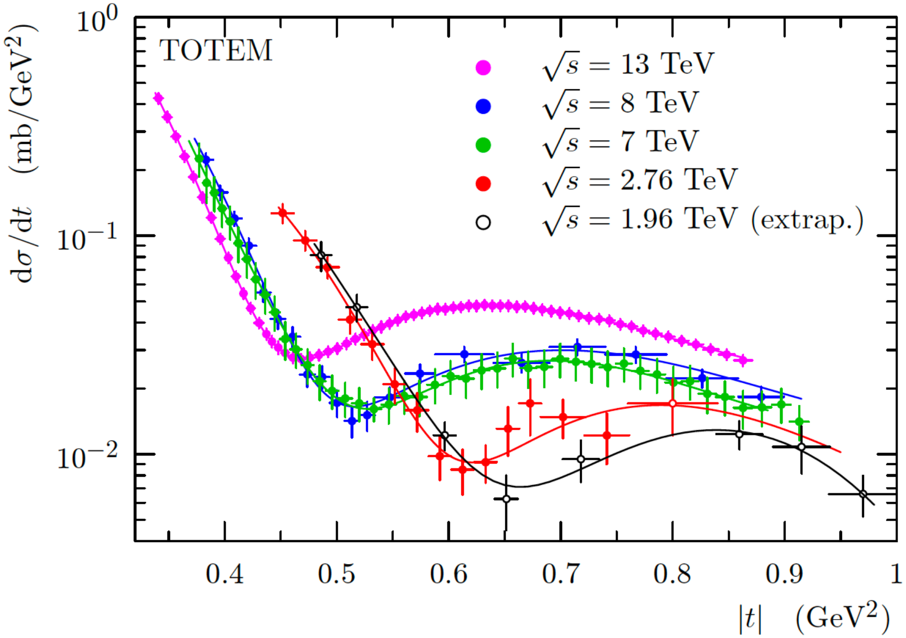


Differential elastic cross section

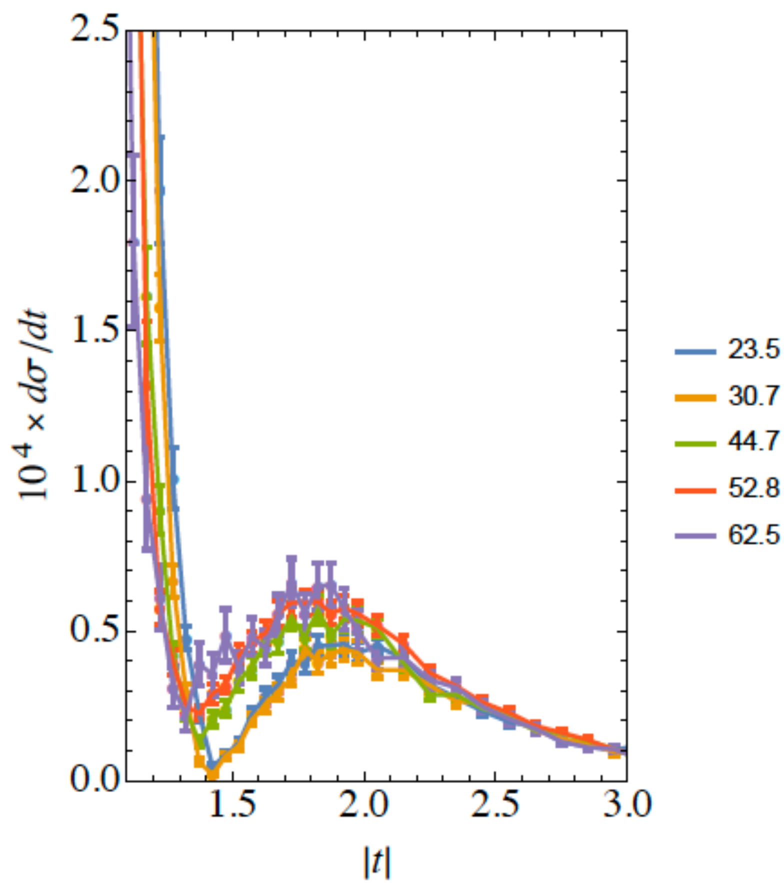
S. Giani, PP elastic nuclear elastic scattering, …



Differential elastic cross-sections



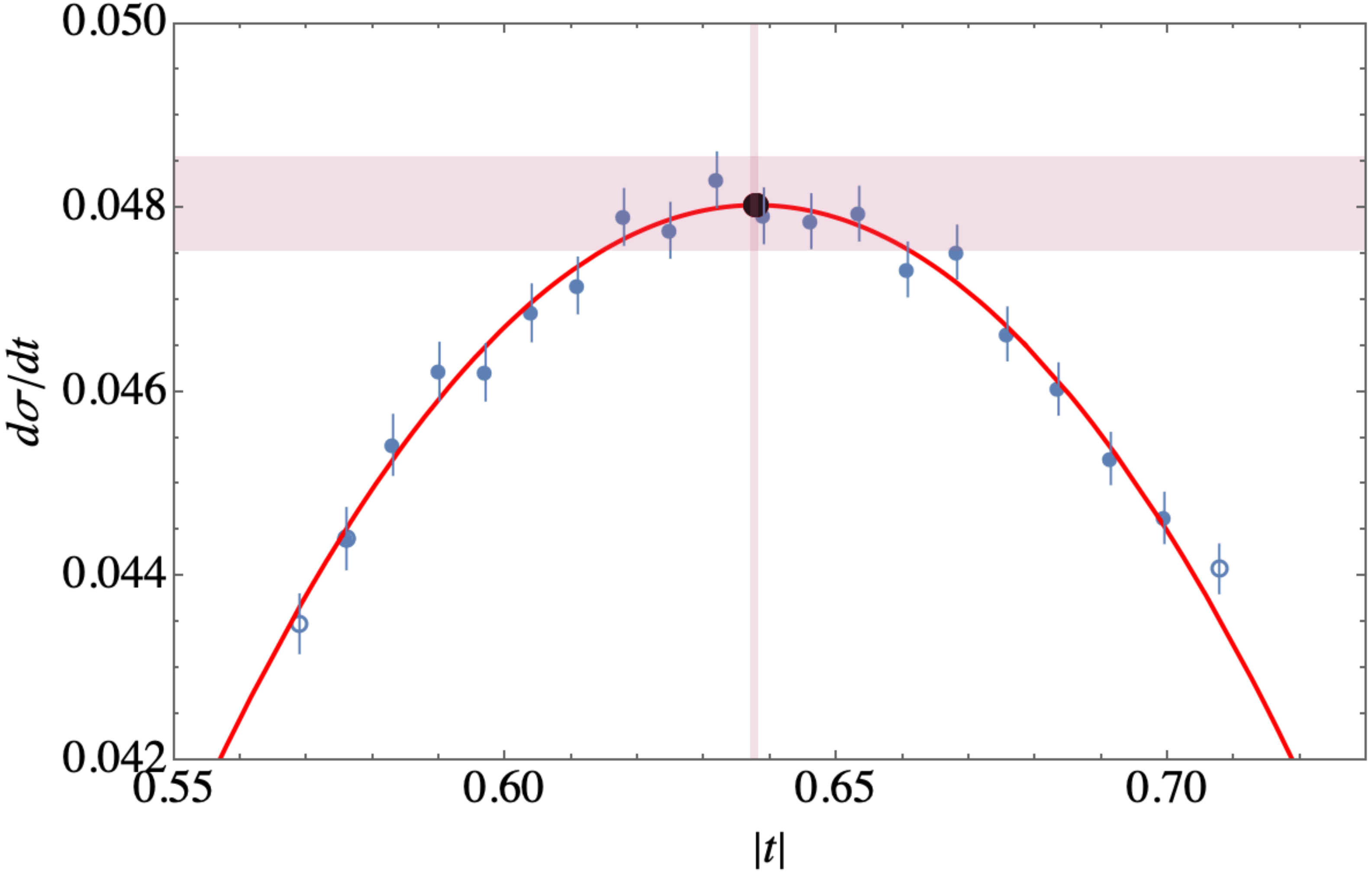
V.M. Abazov [TOTEM and D0]
PRL 102 (2020) 062003
(Royon odderon paper)



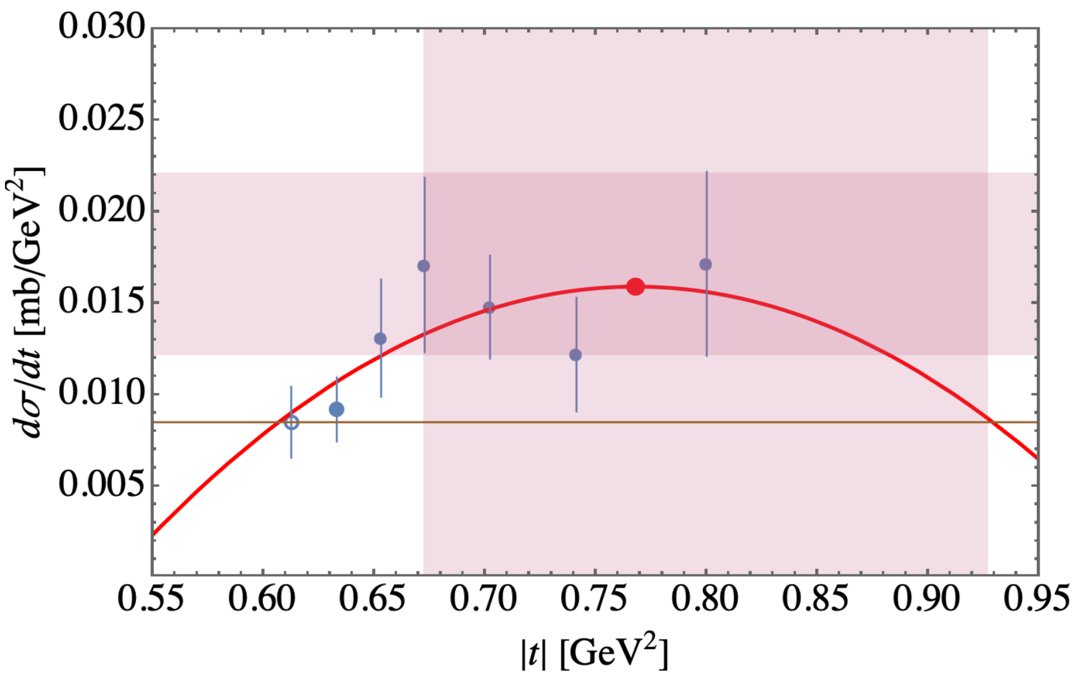
LHC

ISR

13 TeV

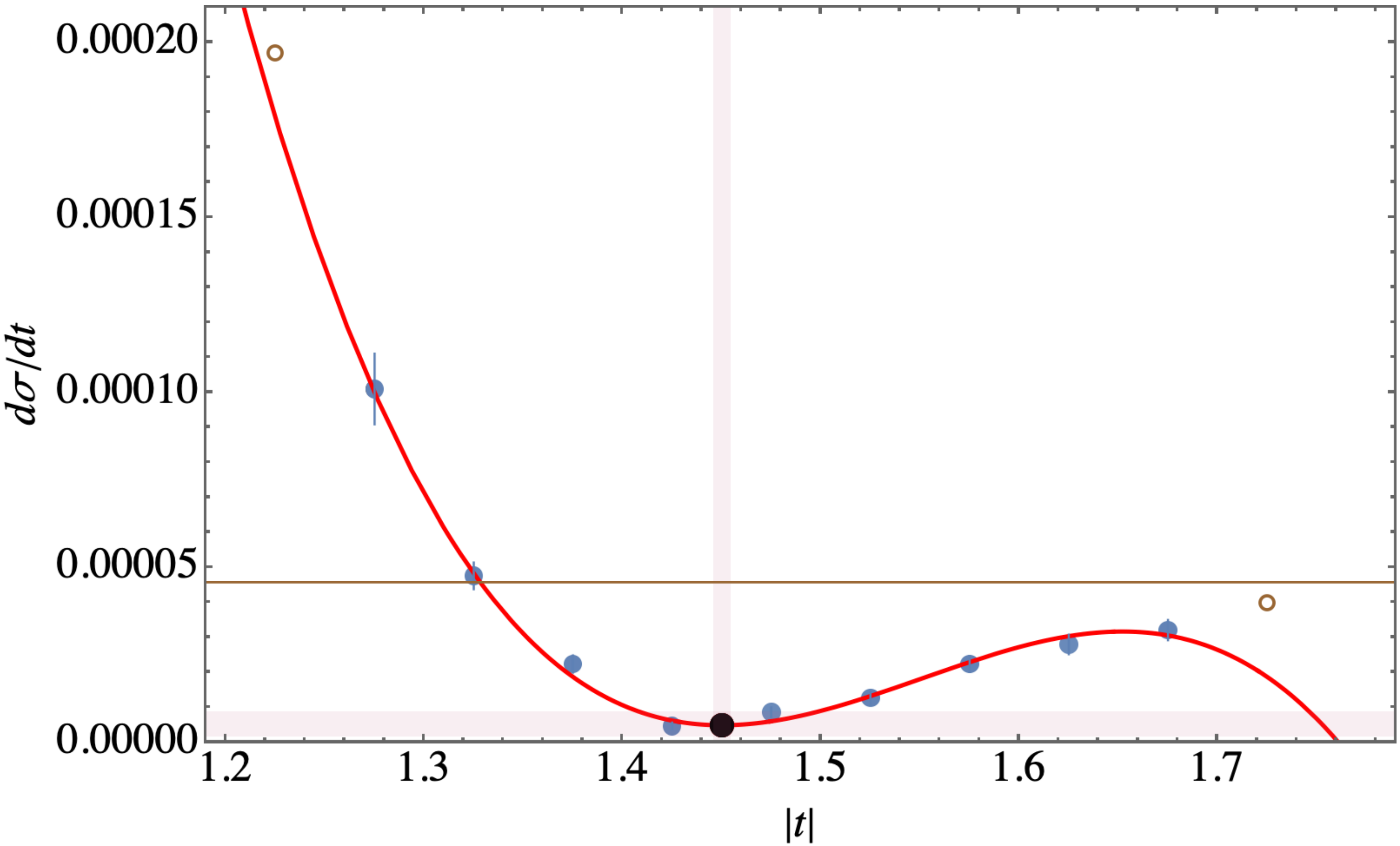


2.76 TeV



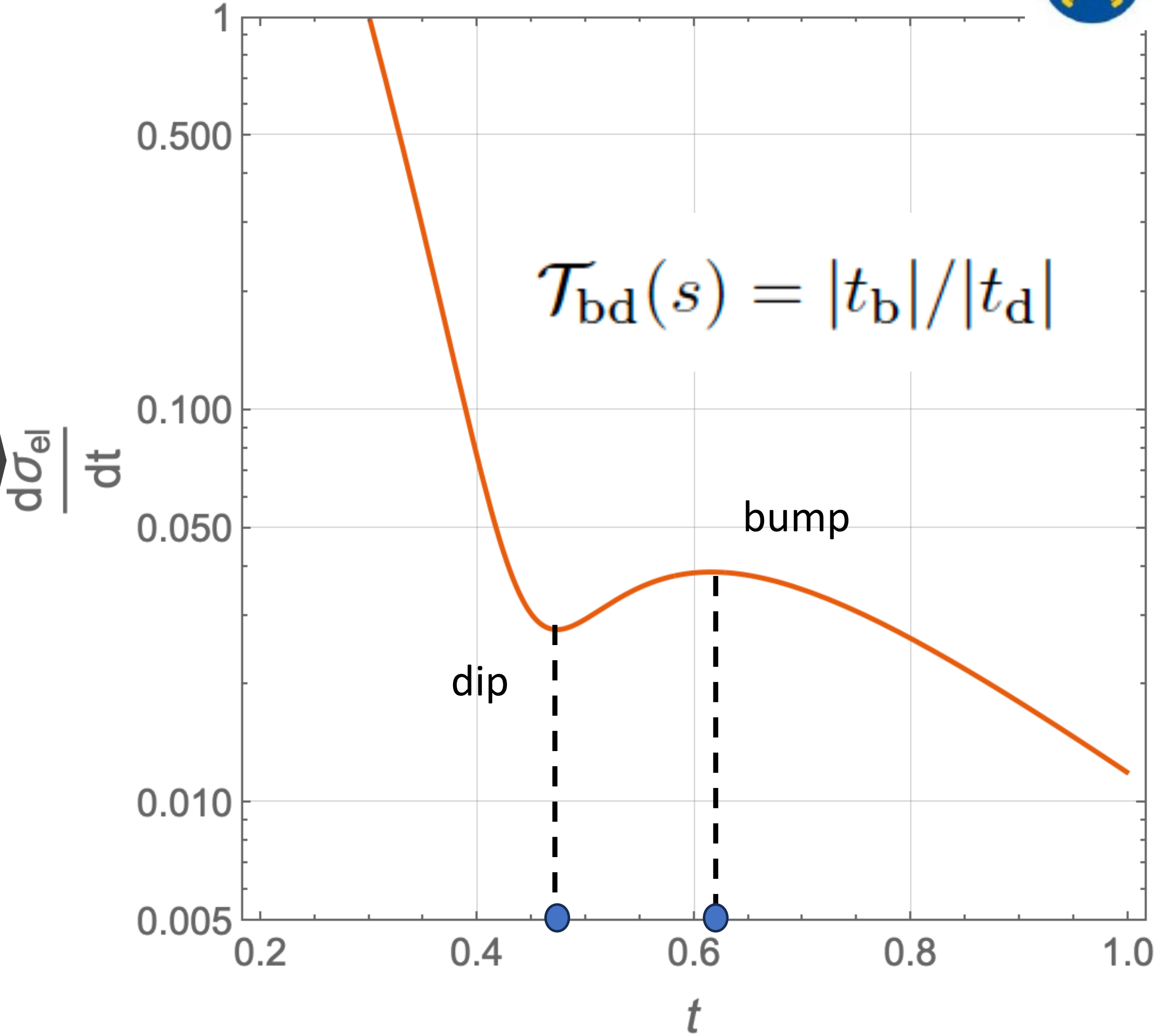
0.030
0.025
0.020
0.015
0.010
0.005
dσ/dt [mb/GeV²]
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
|t| [GeV²]

23 GeV



0.00020
0.00015
0.00010
0.00005
0.00000
$d\sigma/dt$
1.2
1.3
1.4
1.5
1.6
1.7
$|t|$

Bump/Dip behaviour



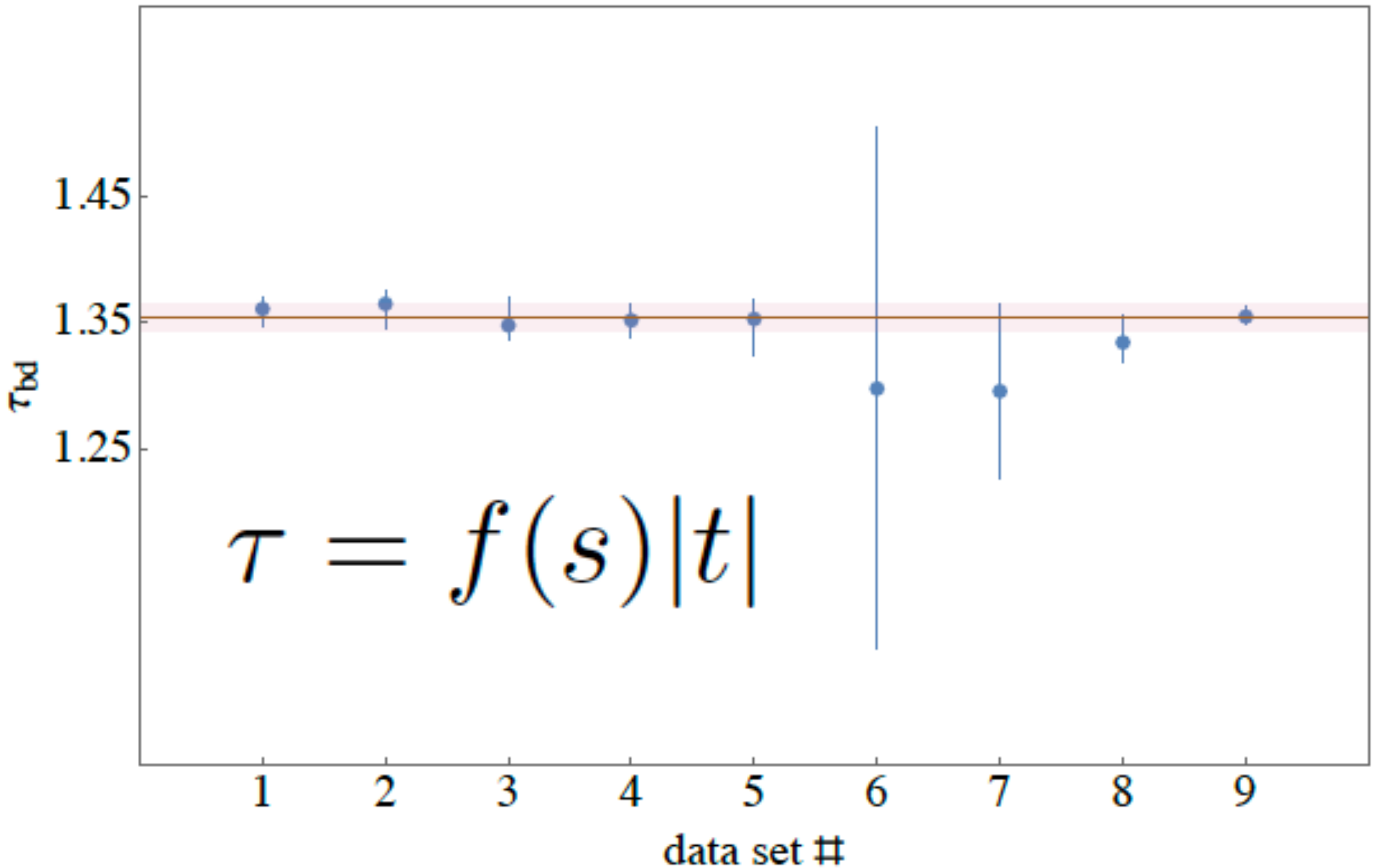
An observation

Phys.Lett.B 856 (2024) 138960

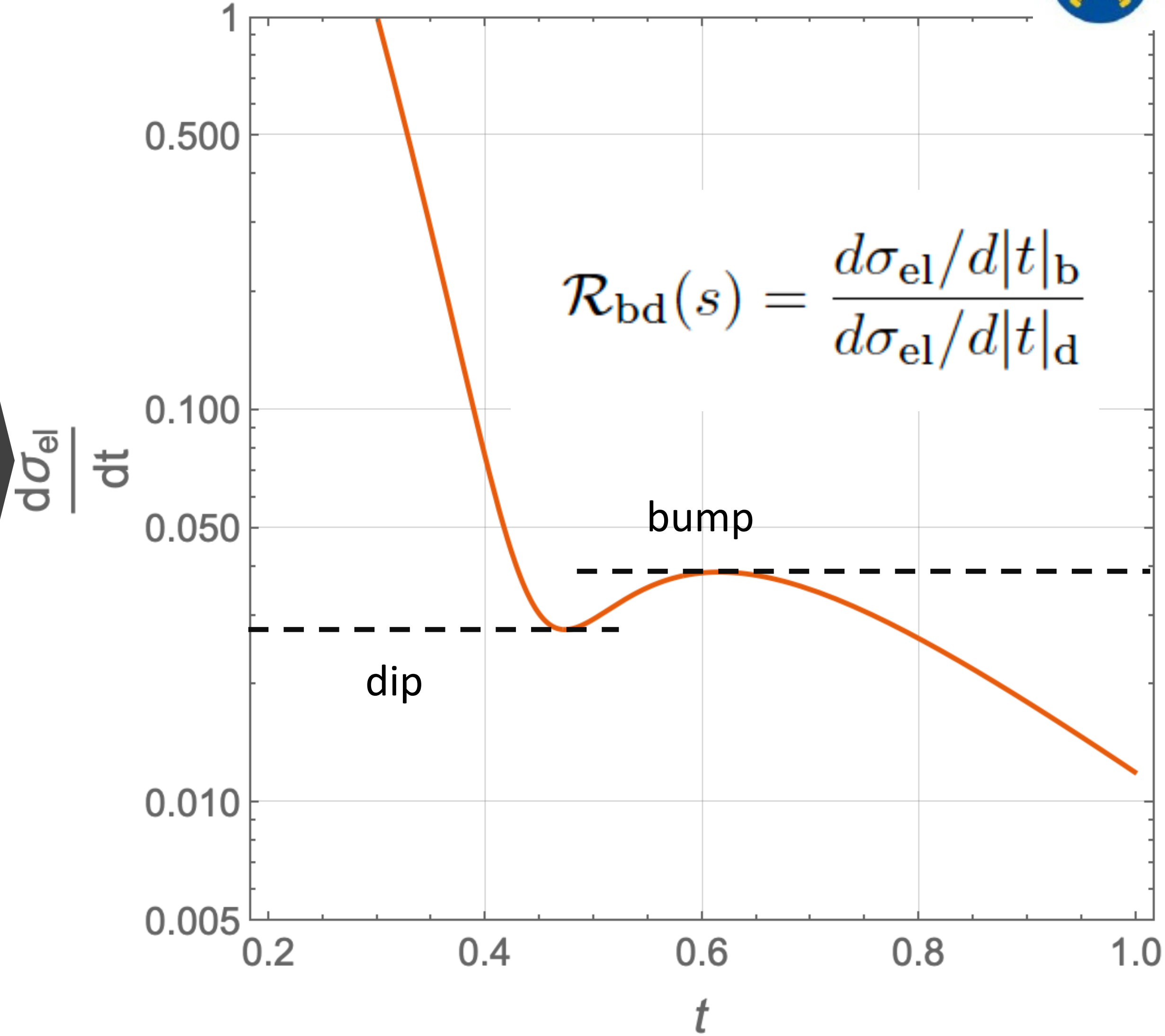
$$\mathcal{T}_{\rm bd}(s) = |t_{\rm b}|/|t_{\rm d}|$$

	#	W	dip		bump		ratios	
			$\|t\|_{\rm d}$	error	$\|t\|_{\rm b}$	error	$t_{\rm b}/t_{\rm d}$	error
LHC [TeV]	9	13.00	0.471	$^{+0.002}_{-0.003}$	0.6377	$^{+0.0006}_{-0.0006}$	1.355	$^{+0.008}_{-0.005}$
	8	8.00	0.525	$^{+0.002}_{-0.004}$	0.700	$^{+0.010}_{-0.008}$	1.335	$^{+0.021}_{-0.016}$
	7	7.00	0.542	$^{+0.012}_{-0.013}$	0.702	$^{+0.034}_{-0.034}$	1.296	$^{+0.069}_{-0.069}$
	6	2.76	0.616	$^{+0.001}_{-0.002}$	0.800	$^{+0.127}_{-0.127}$	1.298	$^{+0.206}_{-0.206}$
ISR [GeV]	5	62.50	1.350	$^{+0.011}_{-0.011}$	1.826	$^{+0.016}_{-0.039}$	1.353	$^{+0.016}_{-0.029}$
	4	52.81	1.369	$^{+0.006}_{-0.006}$	1.851	$^{+0.014}_{-0.018}$	1.352	$^{+0.012}_{-0.014}$
	3	44.64	1.388	$^{+0.003}_{-0.007}$	1.871	$^{+0.031}_{-0.015}$	1.348	$^{+0.023}_{-0.011}$
	2	30.54	1.434	$^{+0.001}_{-0.004}$	1.957	$^{+0.013}_{-0.028}$	1.365	$^{+0.010}_{-0.020}$
	1	23.46	1.450	$^{+0.005}_{-0.004}$	1.973	$^{+0.011}_{-0.018}$	1.361	$^{+0.009}_{-0.013}$

An observation

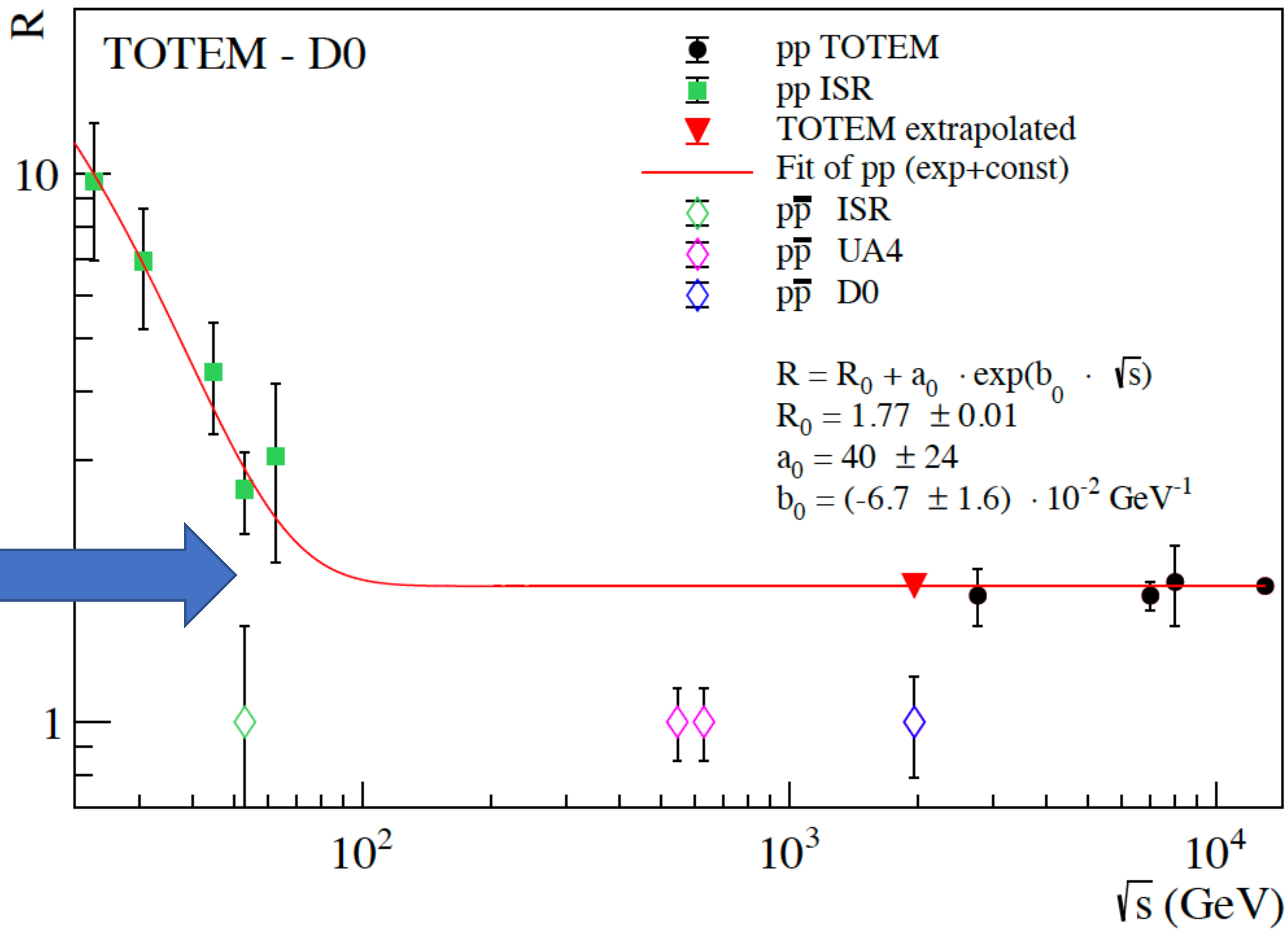


Bump/Dip behaviour



Bump/Dip behaviour

$$\mathcal{R}_{\mathrm{bd}}(s) = \frac{d\sigma_{\mathrm{el}}/d|t|_{\mathrm{b}}}{d\sigma_{\mathrm{el}}/d|t|_{\mathrm{d}}}$$



Hope for scaling at the LHC

V.M. Abazov [TOTEM and D0] PRL 102 (2020) 062003 (Royon odderon paper)

ISR - a bit of history

Nuclear Physics B59 (1973) 231–236 North-Holland Publishing Company

GEOMETRIC SCALING, MULTIPLICITY DISTRIBUTIONS AND CROSS SECTIONS

J DIAS DE DEUS
The Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark

Received 8 March 1973

Abstract From a geometric picture of hadrons as extended objects we arrive at some universal features of high energy collisions In this approach the mean multiplicity, as a function of s and the KNO scaling function are universal and asymptotically the ratio $\sigma_{elastic}/\sigma_{total}$ is expected to be the same for all processes

Cross-sections

Impact parameter space (Barone, Predazzi):

$$\sigma_{\rm el} = \int d^2\boldsymbol{b}\, \underbrace{\left|1 - e^{-\Omega(s,b)+i\chi(s,b)}\right|}_{\text{-}i\,A_{\rm el}}{}^2 ,$$

$$\sigma_{\rm tot} = 2\int d^2\boldsymbol{b}\, {\rm Re}\left[1 - e^{-\Omega(s,b)+i\chi(s,b)}\right],$$

$$\sigma_{\rm inel} = \int d^2\boldsymbol{b}\, \left[1 - \left|e^{-\Omega(s,b)}\right|^2\right].$$

Geometric scaling

$$\Omega(s,b) = \Omega\left(b/R(s)\right)$$

Opacity is a function of one varible,
and $R(s)$ grows with energy. Changing variable

$$\boldsymbol{b} \to \boldsymbol{B} = \boldsymbol{b}/R(s)$$

$$\sigma_{\text{inel}} = R^2(s) \underbrace{\int d^2\boldsymbol{B}\left[1 - \left|e^{-\Omega(B)}\right|^2\right]}_{\text{constant}}$$

Immediate consequences

$$
\begin{aligned}
\sigma_{\rm el} &= \int d^2\boldsymbol{b}\, \left|1 - e^{-\Omega(s,b) + i\chi(\cancel{s,b})}\right|^2 , \\
\sigma_{\rm tot} &= 2\int d^2\boldsymbol{b}\, {\rm Re}\left[1 - e^{-\Omega(s,b) + i\chi(\cancel{s,b})}\right] , \\
\sigma_{\rm inel} &= \int d^2\boldsymbol{b}\, \left[1 - \left|e^{-\Omega(s,b)}\right|^2\right] .
\end{aligned}
$$

Immediate consequences

$$\sigma_{\text{el}} = R^2(s) \int d^2\boldsymbol{B} \left|1 - e^{-\Omega(B)}\right|^2$$

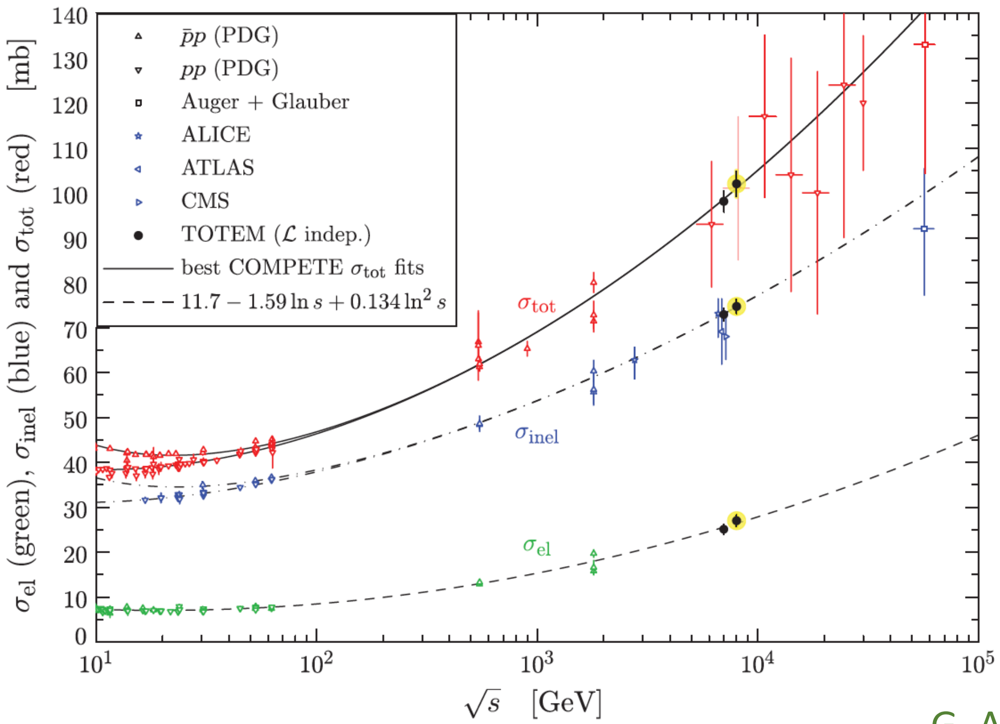
$$\sigma_{\text{tot}} = 2R^2(s) \int d^2\boldsymbol{B} \,\text{Re}\left[1 - e^{-\Omega(B)}\right]$$

$$\sigma_{\text{inel}} = R^2(s) \int d^2\boldsymbol{B} \left[1 - \left|e^{-\Omega(B)}\right|^2\right]$$

If we neglect χ (indeed ρ parameter is small), then all cross-sections have the same energy dependence.

Scaling at the LHC?

	elastic	inelastic	total	ρ
ISR	$W^{0.1142\pm0.0034}$	$W^{0.1099\pm0.0012}$	$W^{0.1098\pm0.0012}$	$0.02-0.095$
LHC	$W^{0.2279\pm0.0228}$	$W^{0.1465\pm0.0133}$	$W^{0.1729\pm0.0163}$	$0.15-0.10$



G. Antchev [TOTEM] PRL 111 (2013) 012001

Momentum space

$$T_{\text{el}}(s,t) = \int d^2\boldsymbol{b}\, e^{-i\,\boldsymbol{bq}} T_{\text{el}}(s,b)$$

$$= \frac{1}{2}\int\limits_0^\infty db^2 T_{\text{el}}(s,b) \int\limits_0^{2\pi} d\varphi e^{-ibq\cos\varphi}$$

$$= \pi \int\limits_0^\infty db^2 T_{\text{el}}(s,b) J_0(bq).$$

generates dips

Momentum space

$$s\sigma_{\text{tot}}(s) = 2\,\text{Im}\,\tilde{T}_{\text{el}}(s,0)$$

$$\sigma_{\text{el}}(s) = \frac{1}{4\pi s^2}\int dt\,\left|\tilde{T}_{\text{el}}(s,t)\right|^2$$

Construct amplitude that exhibits GS, gives correct energy dependence of σ_{tot}

Momentum space

$$s\sigma_{\mathrm{tot}}(s) = 2\,\mathrm{Im}\,\tilde{T}_{\mathrm{el}}(s,0)$$

$$\sigma_{\mathrm{el}}(s) = \frac{1}{4\pi s^2}\int dt\,\left|\tilde{T}_{\mathrm{el}}(s,t)\right|^2$$

Construct amplitude that exhibits GS, gives correct energy dependence of σ_{tot}

$$\tilde{T}_{\mathrm{el}}(s,\tau) \sim isR^2(s)\Phi(\tau)$$

$$\tau = |t|\,R^2(s)$$

$$\sigma_{\mathrm{tot}}(s) \sim R^2(s)$$

Geometric scaling at the ISR

Nuclear Physics B71 (1974) 481–492

SCALING LAW FOR THE ELASTIC DIFFERENTIAL CROSS SECTION IN pp SCATTERING FROM GEOMETRIC SCALING*

A.J. BURAS and J. DIAS de DEUS
The Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen Ø, Denmark

Received 6 December 1973

Abstract: Plots of $(1/\sigma_{in}^2)d\sigma_{el}/d|t| \equiv \Phi(\tau, s)$ as a function of $\tau \equiv |t|\sigma_{in}$ are shown to scale in the NAL-ISR energy region. Such scaling is shown to be a consequence of geometric scaling for the inelastic overlap function $G_{in}(\beta = \pi b^2/\sigma_{in})$ in the limit $\rho = \mathrm{Re}A/\mathrm{Im}A \to 0$ and in the case of $\sigma_{in} \sim (\ln s)^2$ is equivalent to the scaling proposed by Auberson, Kinoshita and Martin. A possible relation to the KNO multiplicity scaling is indicated.

$$\tau = \sigma_{\text{inel}}(s)\,|t| = R^2(s)|t| \times \text{const.}$$

Geometric scaling at the ISR

Vol. B9 (1978) ACTA PHYSICA POLONICA No 2

DIPS, ZEROS AND LARGE $|t|$ BEHAVIOUR OF THE ELASTIC AMPLITUDE

By J. Dias de Deus*

Physics Department, University of Wuppertal, Germany and CFMC-Instituto Nacional de Investigação Cientifica, Lisboa, Portugal

and P. Kroll

Physics Department, University of Wuppertal

(Received September 9, 1977)

$$\sigma_{\text{tot}}(s) \sim R^2(s)$$

Geometric scaling at the ISR

$$\frac{d\sigma_{\rm el}}{dt} = \frac{1}{4\pi s^2}\left|\tilde{T}_{\rm el}(s,t)\right|^2$$

$$= \frac{1}{4}\left|\int_0^\infty db^2\, T_{\rm el}(b/R(s))J_0(b\sqrt{-t})\right|^2 \qquad B = b/R(s)$$

$$= \frac{1}{4}R^4(s)\left|\int_0^\infty dB^2\, T_{\rm el}(B)J_0(B\sqrt{\tau})\right|^2 \qquad \tau = -tR^2(s)$$

$$= R^4(s)\Phi^2(\tau)$$

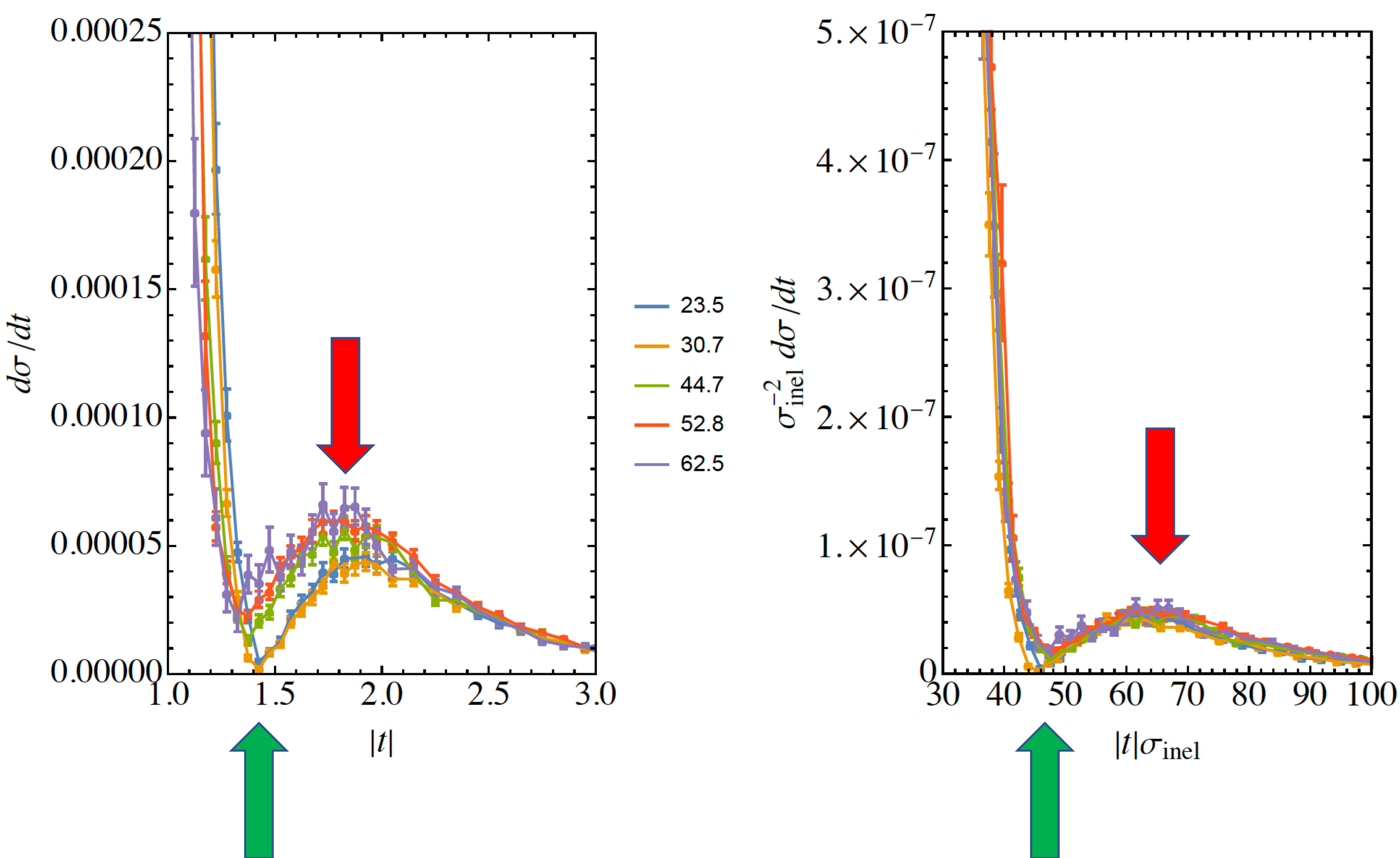
$$R^2(s) = \sigma_{\rm tot}(s) \sim \sigma_{\rm inel}(s)$$

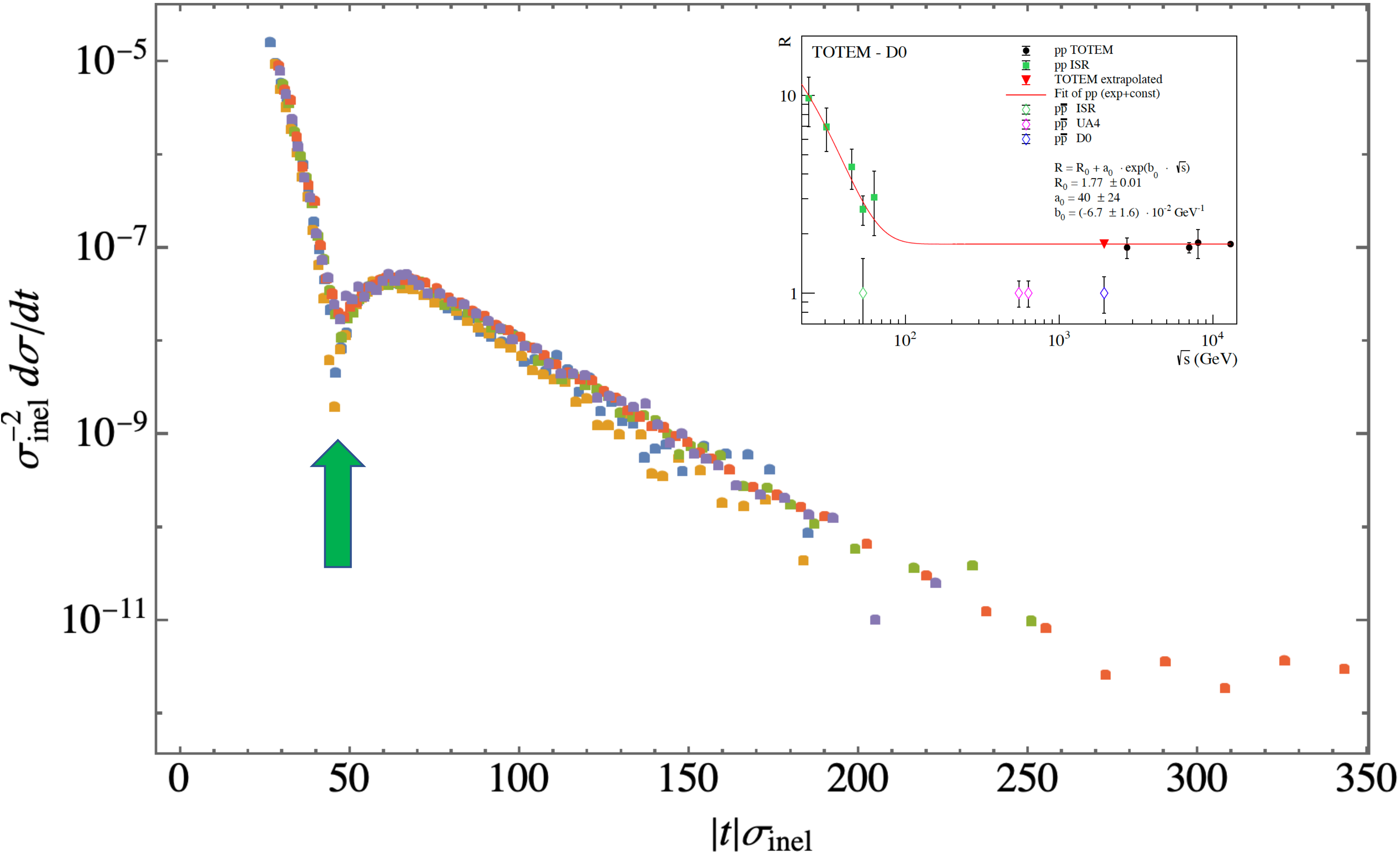
Geometric scaling at the ISR

$$\tau = \sigma_{\rm inel}(s)\,|t| = R^2(s)|t| \times {\rm const.}$$

$$\frac{1}{\sigma^2_{\rm inel}(s)}\frac{d\sigma_{\rm el}}{d|t|}(s,t) = \Phi^2(\tau)$$

Geometric scaling at the ISR





10^{-5}
10^{-7}
10^{-9}
10^{-11}
$\sigma_{\rm inel}^{-2}\, d\sigma/dt$
0
50
100
150
200
250
300
350
$|t|\sigma_{\rm inel}$
R
TOTEM - D0
pp TOTEM
pp ISR
TOTEM extrapolated
Fit of pp (exp+const)
$p\bar{p}$ ISR
$p\bar{p}$ UA4
$p\bar{p}$ D0
$R = R_0 + a_0 \cdot \exp(b_0 \cdot \sqrt{s})$
$R_0 = 1.77 \pm 0.01$
$a_0 = 40 \pm 24$
$b_0 = (-6.7 \pm 1.6) \cdot 10^{-2}$ GeV^{-1}
10
1
10^2
10^3
10^4
$\sqrt{s}$ (GeV)

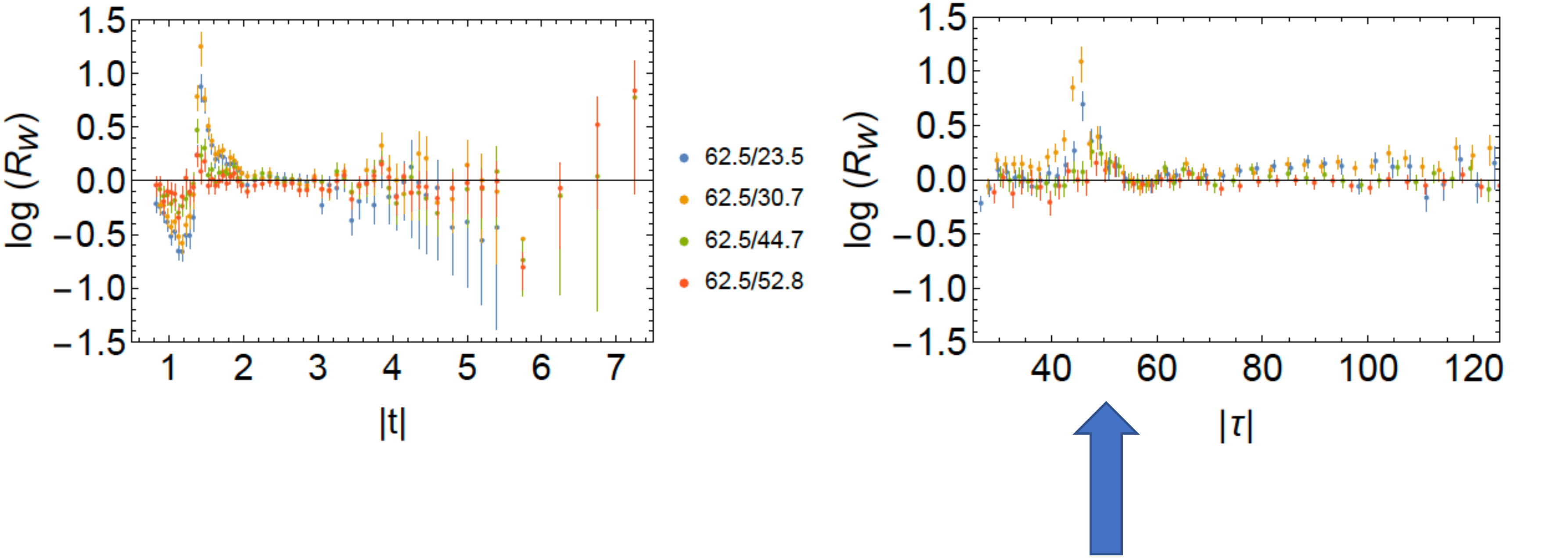
Ratio method

$$\frac{d\tilde{\sigma}_{\rm el}}{d|t|}(W,\tau_i) = \frac{1}{\sigma^2_{\rm inel}(W)}\,\frac{d\sigma_{\rm el}}{d|t|}(W,\tau_i)$$

$$R_W(\tau_i) = \frac{d\tilde{\sigma}_{\rm el}/d|t|(W_{\rm ref},\tau_i)}{d\tilde{\sigma}_{\rm el}/d|t|(W,\tau_i)}$$

$$W_{\rm ref} = 62.5\ {\rm GeV}$$

Ratio method

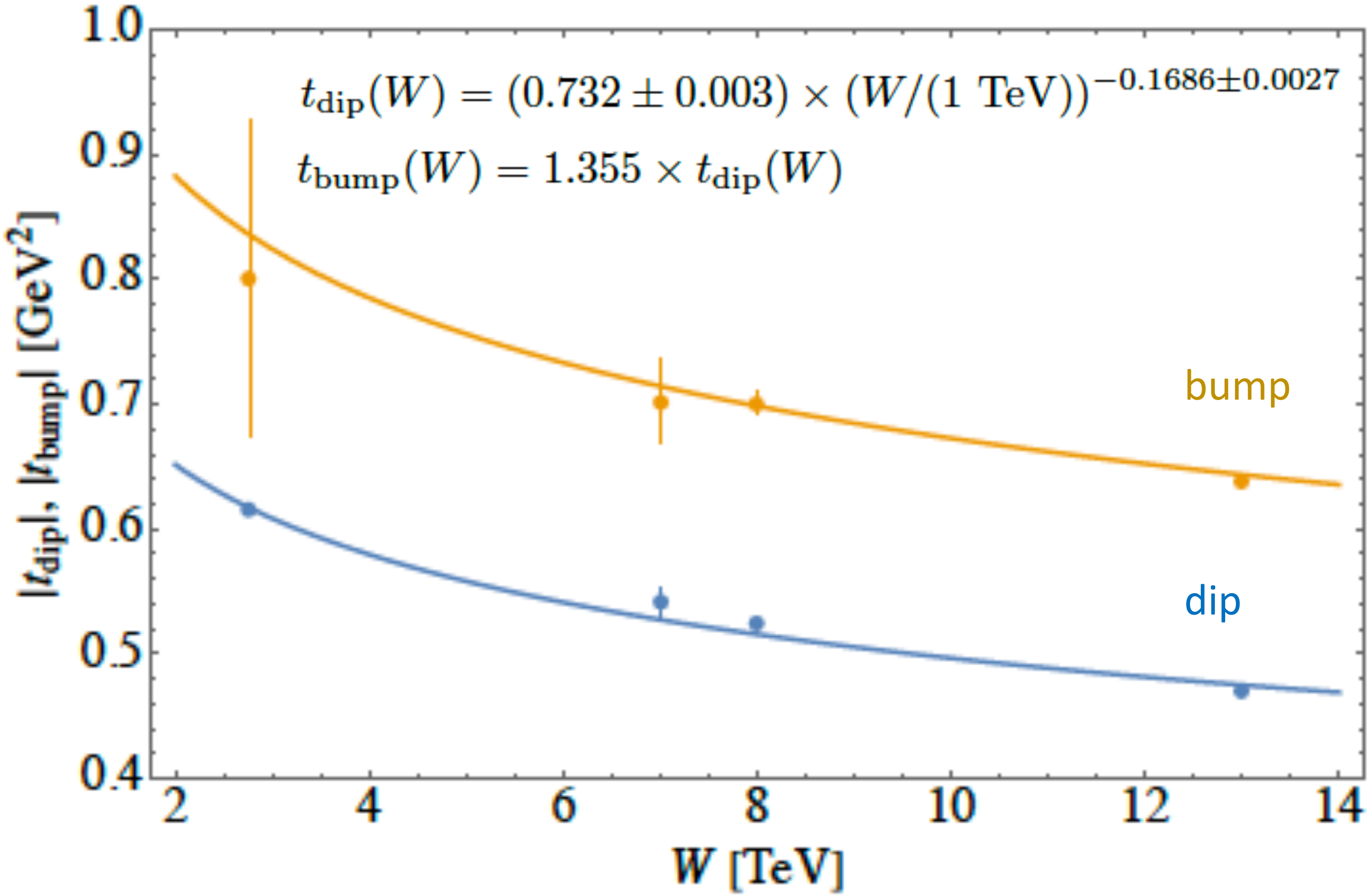


The fact that geometrical scaling is violated in the dip region has been attributed to the vanishing of the imaginary part of the scattering amplitude at t_d. Whatever small the real part of the amplitude is, it takes over in the vicinity of $t_\mathrm{d} \pm \Delta t$. For $|t - t_\mathrm{d}| > |\Delta t|$ it is the imaginary part that dominates, and geometrical scaling is restored [14].

[14] J. Dias de Deus and P. Kroll, Acta Phys. Polon. B **9**, 157 (1978)

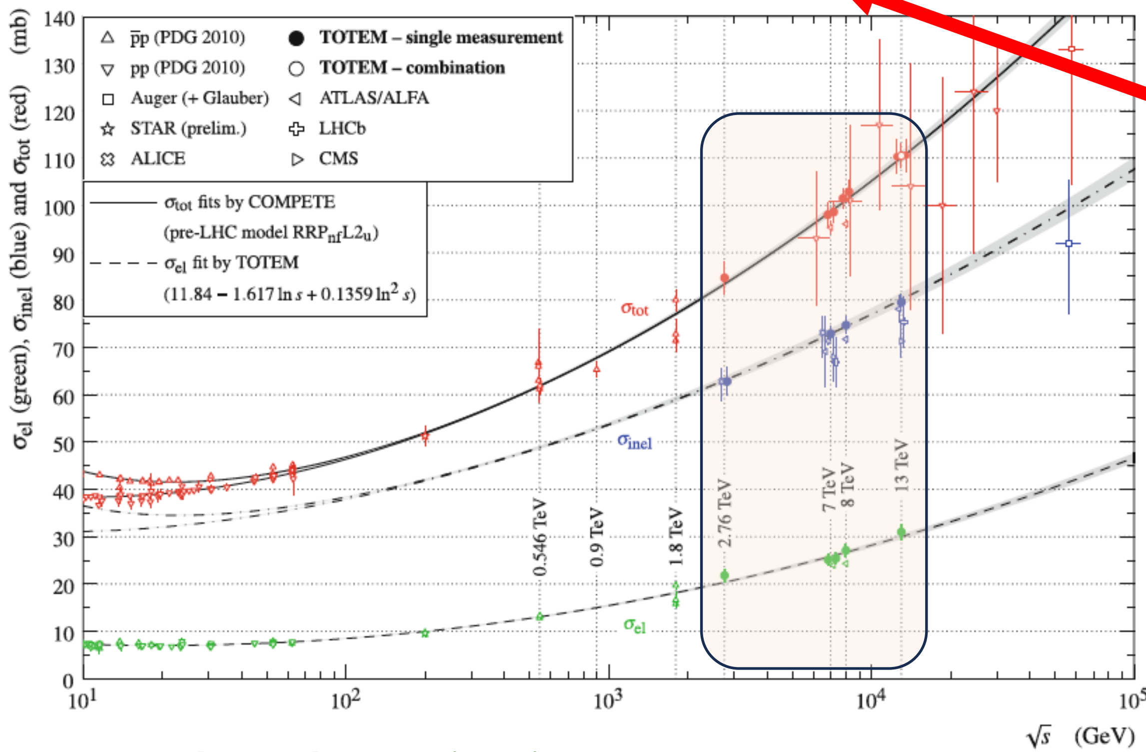
Scaling variable at the LHC

The fact that t_{bump}/t_{dip} = const. implies: $\tau = f(s)|t|$



$\beta = 0.1686$

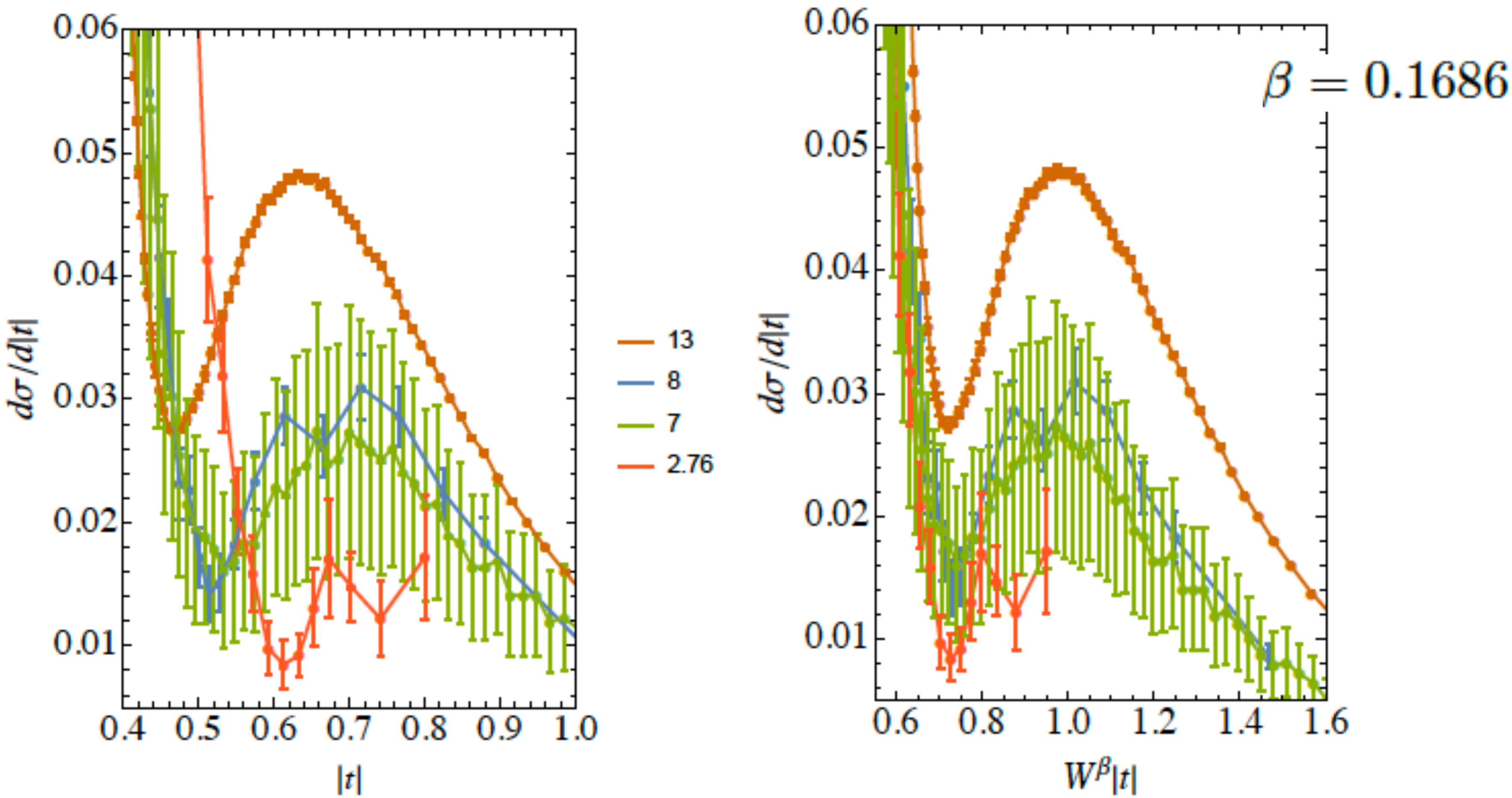
	elastic	inelastic	total	$\frac{\text{elastic}}{\text{inelastic}}$	ρ
ISR	$W^{0.1142\pm0.0034}$	$W^{0.1099\pm0.0012}$	$W^{0.1098\pm0.0012}$	$W^{0.0043\pm0.0036}$	$0.02 - 0.095$
LHC	$W^{0.2279\pm0.0228}$	$W^{0.1465\pm0.0133}$	$W^{0.1729\pm0.0163}$	$W^{0.0814\pm0.0264}$	$0.15 - 0.10$



$$\beta = 0.1686$$

G. Antchev [TOTEM] EPJ C79 (2019) 785

Scaling at the LHC – first step

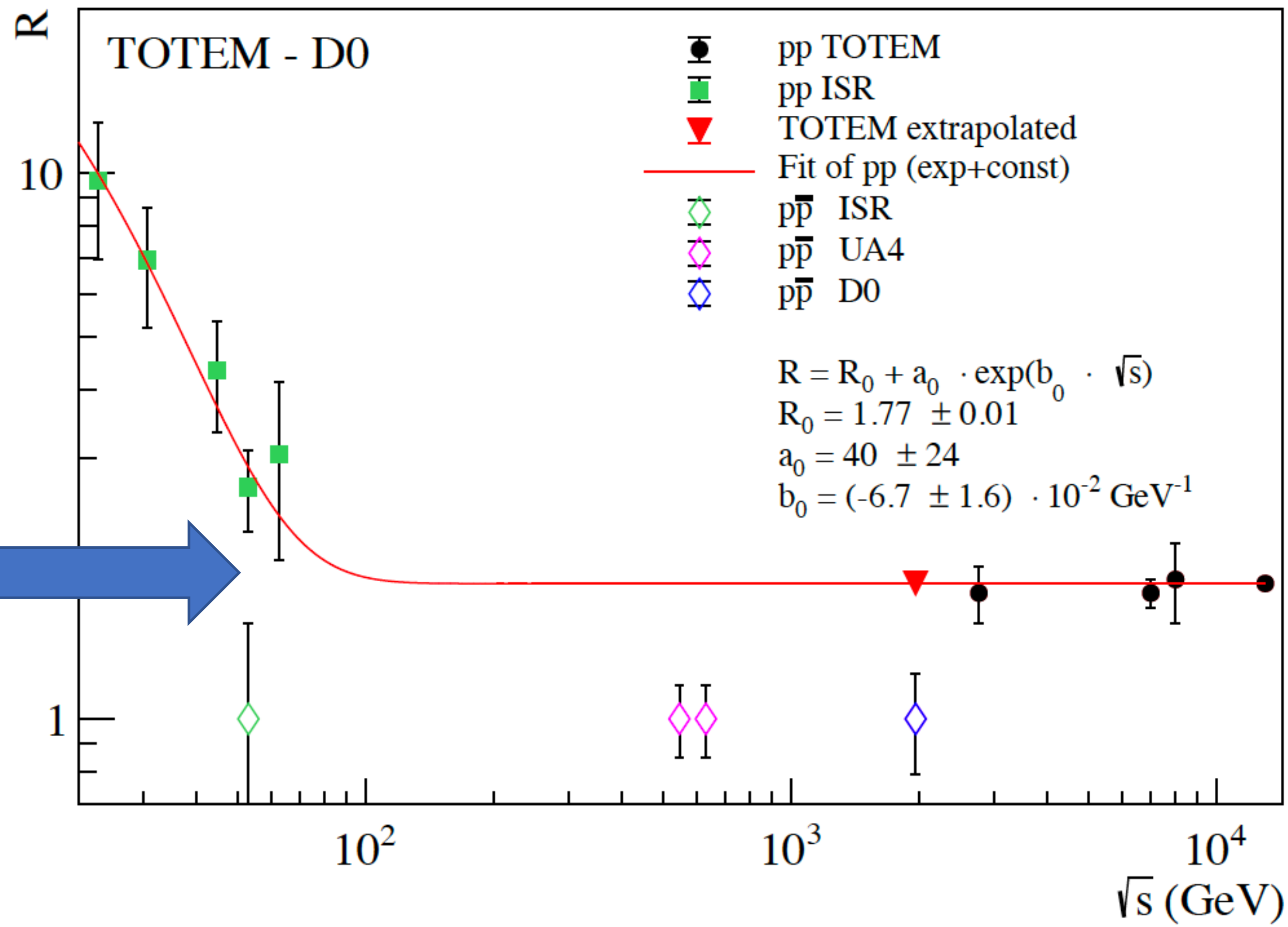


Bump and dip positions are superimposed. Now we have to superimpose bump and dip values.

Bump/Dip behaviour

$$\mathcal{R}_{\rm bd}(s) = \frac{d\sigma_{\rm el}/d|t|_{\rm b}}{d\sigma_{\rm el}/d|t|_{\rm d}}$$

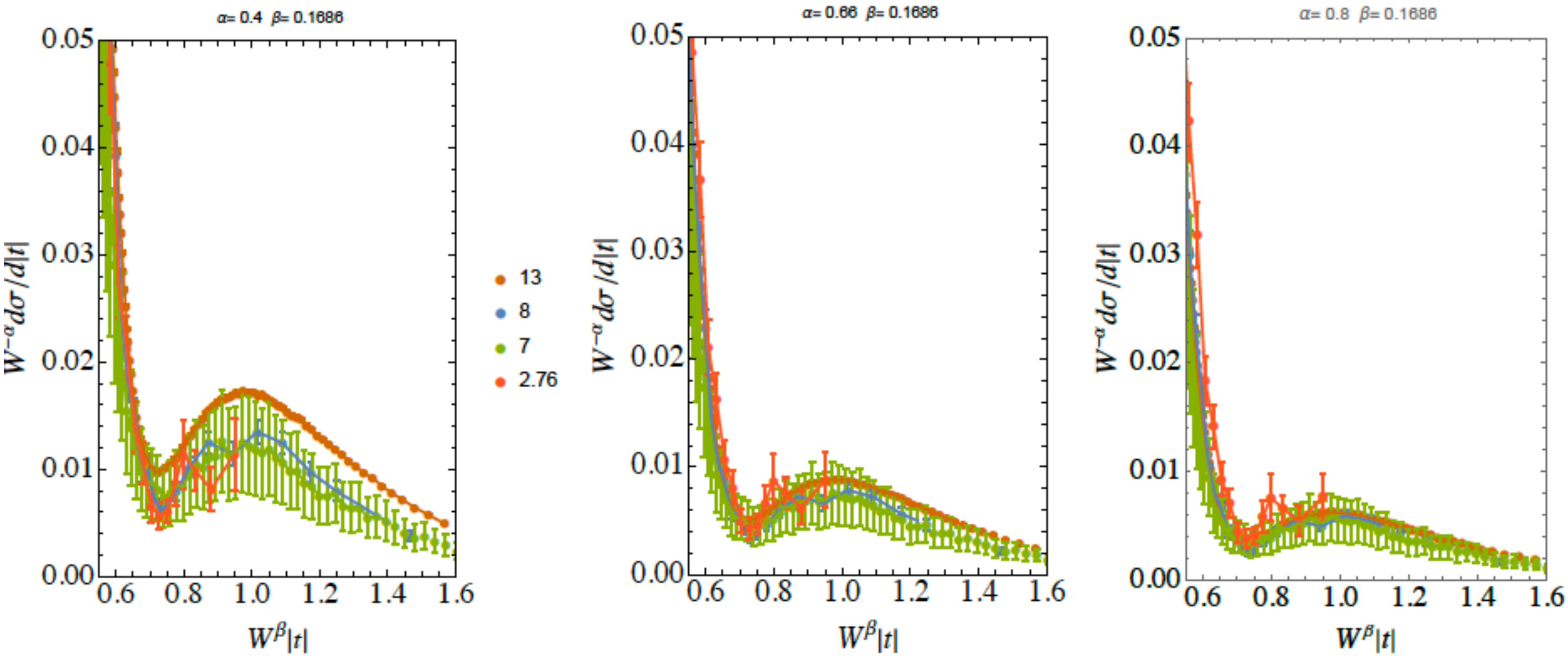
Hope for scaling at the LHC



V.M. Abazov [TOTEM and D0] PRL 102 (2020) 062003 (Royon odderon paper)

Scaling at the LHC – second step

$\alpha = 0.4$ 0.66 0.8



$$\frac{d\sigma_{\rm el}}{d|t|}(\tau) \rightarrow \left(\frac{W}{1\mathrm{TeV}}\right)^{-\alpha} \frac{d\sigma_{\rm el}}{d|t|}(\tau)$$

Momentum space

$$s\sigma_{\rm tot}(s) = 2\,{\rm Im}\,\tilde{T}_{\rm el}(s,0)$$

$$\sigma_{\rm el}(s) = \frac{1}{4\pi s^2}\int dt\,\left|\tilde{T}_{\rm el}(s,t)\right|^2$$

Construct amplitude that exhibits GS, gives correct energy dependence of $\sigma_{\rm tot}$

$$\tilde{T}_{\rm el}(s,\tau) \sim isR^2(s)\Phi(\tau)$$

$$\tau = |t|\,R^2(s)$$

$$\sigma_{\rm tot}(s) \sim R^2(s)$$

Momentum space

$$s\sigma_{\mathrm{tot}}(s) = 2\,\mathrm{Im}\,\tilde{T}_{\mathrm{el}}(s,0)$$

$$\sigma_{\mathrm{el}}(s) = \frac{1}{4\pi s^2}\int dt\left|\tilde{T}_{\mathrm{el}}(s,t)\right|^2$$

Construct amplitude that exhibits GS, gives correct energy dependence of σ_{tot} and satisfies crossing

$$\tilde{T}_{\mathrm{el}}(u,t) \simeq \tilde{T}_{\mathrm{el}}(-s,t) = \tilde{T}^{*}_{\mathrm{el}}(s,t)$$

$$\tilde{T}_{\mathrm{el}}(s,\tau) = isR^2(-is)\Phi\left[|t|\,R^2(-is)\right]$$

Identifying Real and Imaginary parts

Use rapidity: $y = \ln s$ observe $-is = e^{y - i\pi/2}$ and expand

$$R^2(-is) \to R^2\left(y - i\frac{\pi}{2}\right) \simeq R^2(y) - i\frac{\pi}{2}\frac{dR^2(y)}{dy}$$

As a result, one gets:

$$\mathrm{Im}\,\tilde{T}_{\mathrm{el}}(s,\tau) = sR^2(y)\Phi[\tau]$$

$$\mathrm{Re}\,\tilde{T}_{\mathrm{el}}(s,\tau) = s\frac{\pi}{2}\frac{dR^2(y)}{dy}\frac{d}{d\tau}\left(\tau\Phi[\tau]\right)$$

J. Dias de Deus, "On the Real Part of a Geometrical Pomeron," Nuovo Cim. A 28, 114 (1975)
J. Dias de Deus and P. Kroll, "Dips, Zeros and Large |t| Behavior of the Elastic Amplitude," Acta Phys.Pol. B 9 (1978), 157

Identifying Real and Imaginary parts

Use rapidity: $y = \ln s$ observe $-is = e^{y-i\pi/2}$ and expand

$$R^2(-is) \to R^2\left(y - i\frac{\pi}{2}\right) \simeq R^2(y) - i\frac{\pi}{2}\frac{dR^2(y)}{dy}$$

As a result, one gets:

$$\mathrm{Im}\,\tilde{T}_{\mathrm{el}}(s,\tau) = sR^2(y)\Phi[\tau]$$

$$\mathrm{Re}\,\tilde{T}_{\mathrm{el}}(s,\tau) = s\frac{\pi}{2}\frac{dR^2(y)}{dy}\frac{d}{d\tau}(\tau\Phi[\tau])$$

$$\rho(y) = \frac{\pi}{2}\frac{dR^2(y)/dy}{R^2(y)}$$

parameter free prediction!

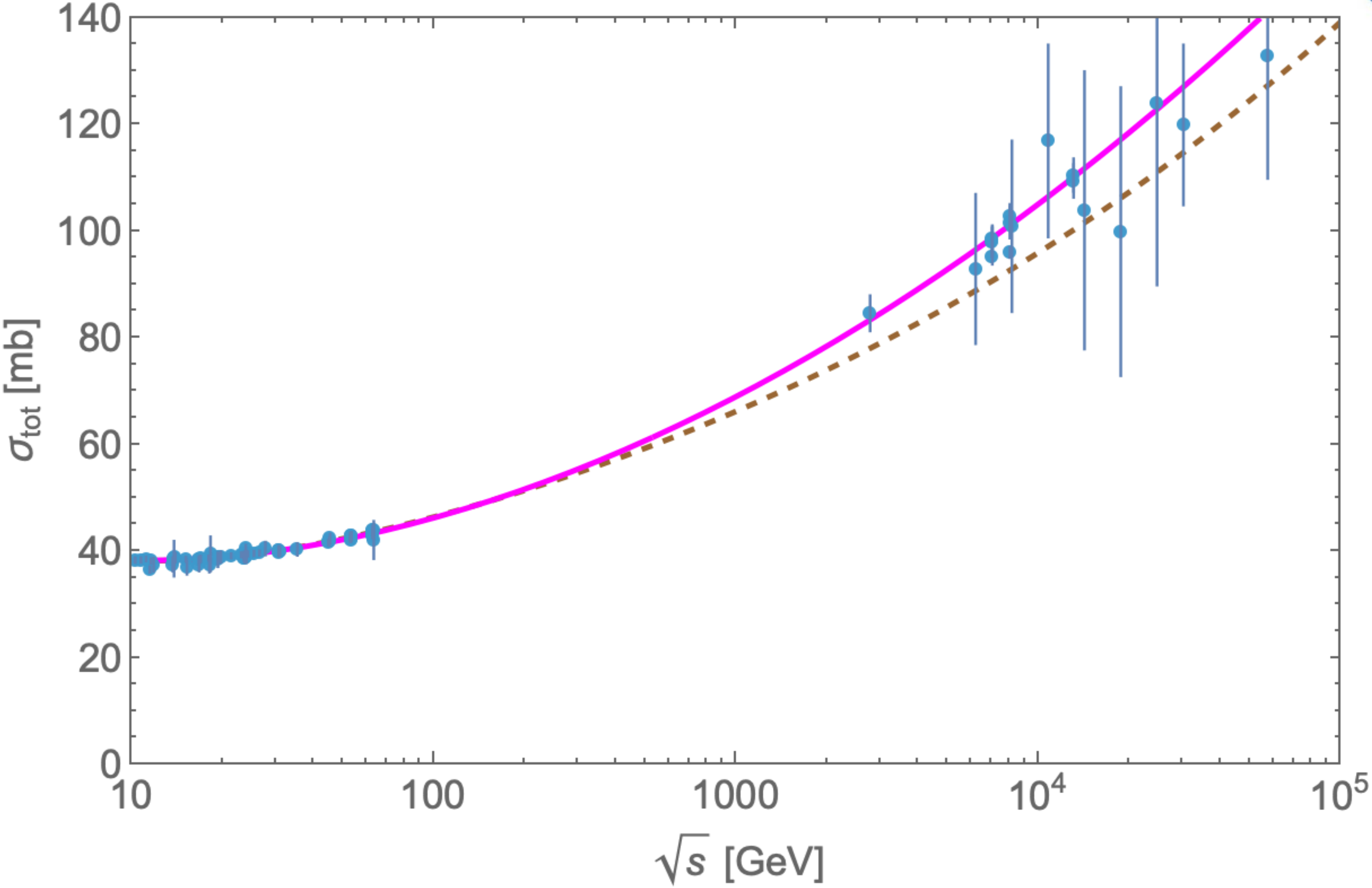
Parametrizations of sigma_tot

COMPETE@PDG2010

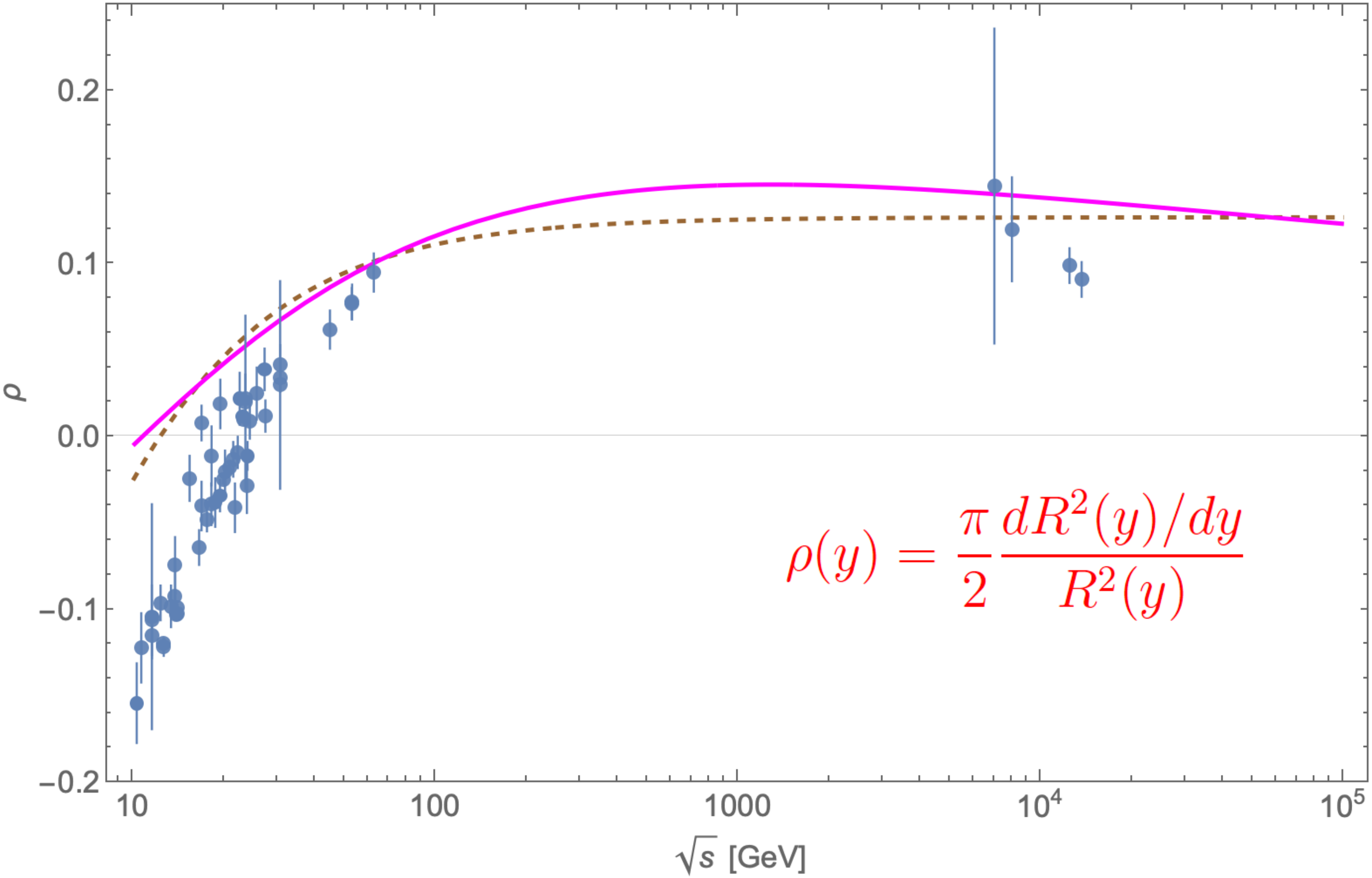
$$\sigma_{\text{tot}}^{\text{PDG}}(s) = Z + C\ln^2\left(\frac{s}{s_0}\right) + Y_1\left(\frac{s}{s_1}\right)^{-\eta_1} - Y_2\left(\frac{s}{s_1}\right)^{-\eta_2}$$

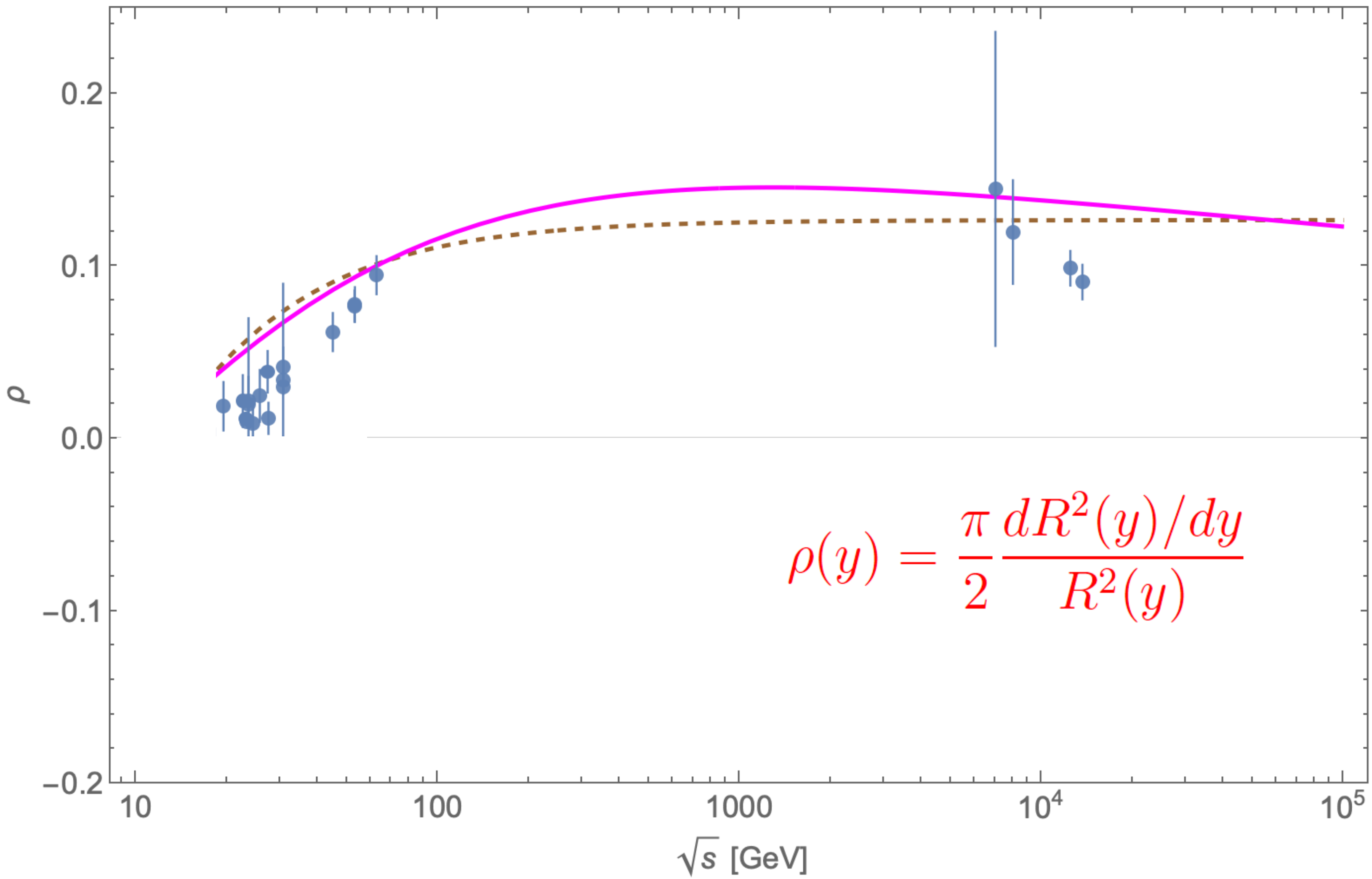
Donnachie & Landshoff

$$\sigma_{\text{tot}}^{\text{DL}}(s) = A\left(\frac{s}{s_1}\right)^{\alpha} + B\left(\frac{s}{s_1}\right)^{\beta}$$



140
120
100
80
60
40
20
0
σ_{tot} [mb]
10
100
1000
10^4
10^5
$\sqrt{s}$ [GeV]


0.2
0.1
0.0
−0.1
−0.2
ρ
10
100
1000
10^4
10^5
$\sqrt{s}$ [GeV]
$\rho(y) = \frac{\pi}{2}\frac{dR^2(y)/dy}{R^2(y)}$



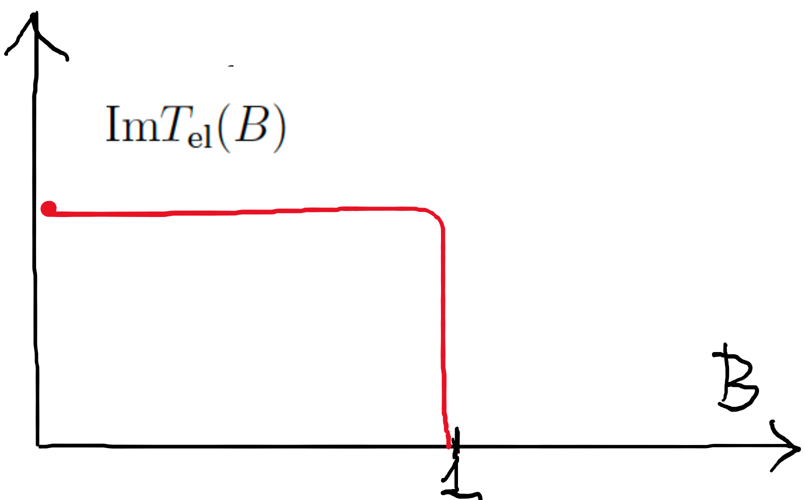
$$\rho(y) = \frac{\pi}{2}\frac{dR^2(y)/dy}{R^2(y)}$$

Dips and bumps

Function $\Phi[\tau]$ has a zero, which corresponds to a dip

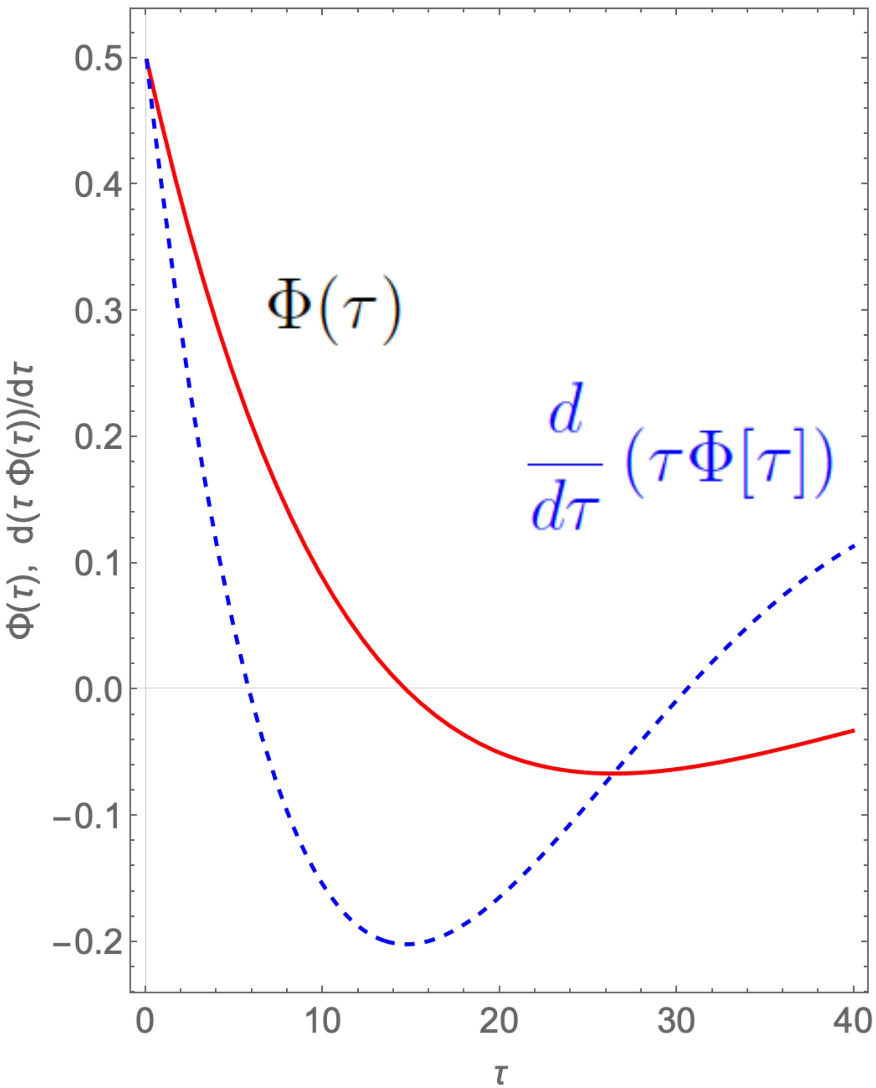
$$\mathrm{Im}\tilde{T}_{\mathrm{el}}(\tau) = 2\pi s R^2(s) \int_0^\infty dB^2 \mathrm{Im}T_{\mathrm{el}}(B) J_0(B\sqrt{\tau}) \sim sR^2(s)\Phi(\tau)$$

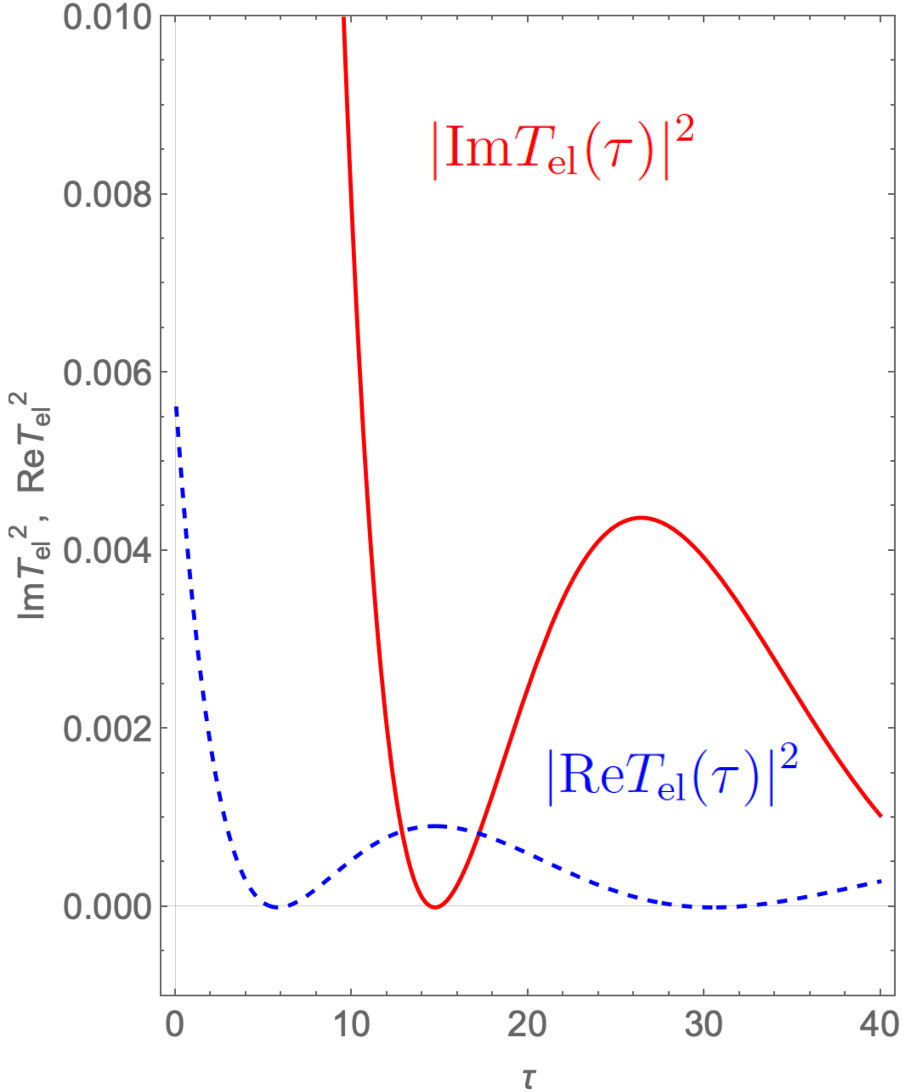
For a hard disc one can compute this integral analytically



$$\Phi(\tau) = 2\pi \frac{J_1(\sqrt{\tau})}{\sqrt{\tau}}$$

Dips and Bumps





Dips and bumps

$$\Phi[\tau_{\rm dip}] = 0 \to {\rm Im}\,\tilde{T}_{\rm el}(s,\tau_{\rm dip}) = 0$$

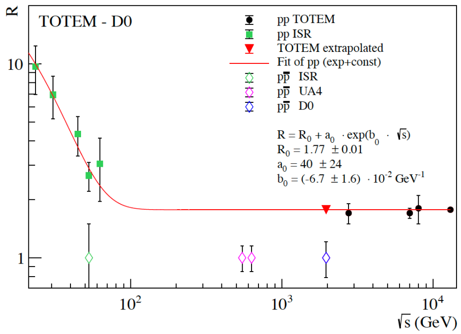
$$ {\rm Re}\,\tilde{T}_{\rm el}(s,\tau_{\rm dip}) = s\frac{\pi}{2}\frac{dR^2(y)}{dy}\frac{d}{d\tau}\Phi[\tau_{\rm dip}]$$

$$\frac{d}{d\tau}\Phi[\tau_{\rm bump}] = 0 \to {\rm Im}\,\tilde{T}_{\rm el}(s,\tau_{\rm dip}) = sR^2(y)\Phi[\tau_{\rm bump}]$$

$$ {\rm Re}\,\tilde{T}_{\rm el}(s,\tau_{\rm dip}) = s\frac{\pi}{2}\frac{dR^2(y)}{dy}\Phi[\tau_{\rm bump}]$$

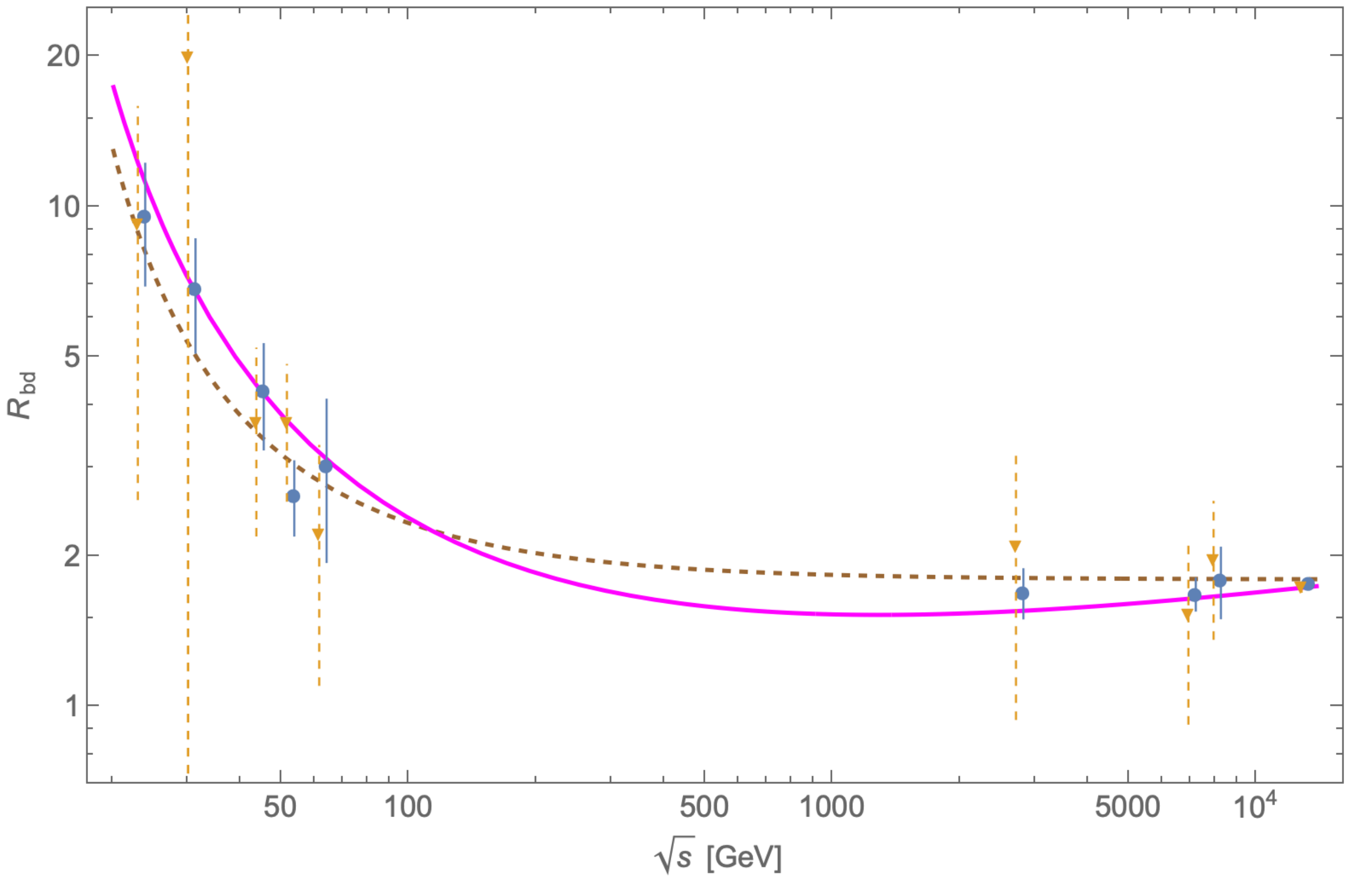
$$\frac{d\sigma/dt(t_{\rm bump})}{d\sigma/dt(t_{\rm dip})} = c_0\frac{1+\rho^2(y)}{\rho^2(y)}$$

$$c_0 = \frac{\Phi^2[\tau_{\rm bump}]}{\left(\tau_{\rm dip}\frac{d}{d\tau}\Phi[\tau_{\rm dip}]\right)^2}$$



Ratio bump to dip

$$\frac{d\sigma/dt(t_{\text{bump}})}{d\sigma/dt(t_{\text{dip}})}$$



COMPETE

A. Donnachie(Manchester U.), P.V. Landshoff(CERN) Phys.Lett.B 296 (1992) 227-232

Total elastic cross section

Assuming GS holds everywhere

$$\sigma_{\rm el}(s) = \frac{1}{4\pi R^2(y)}\left[R^4(y)\int d\tau\Phi^2[\tau] + \left(\frac{\pi}{2}\frac{dR^2(y)}{dy}\right)^2\int d\tau\left(\frac{d}{d\tau}(\tau\Phi[\tau])\right)^2\right]$$

$$= \frac{R^2(y)}{4\pi}\left(1 + c_1\rho^2(y)\right)\times\int d\tau\Phi^2[\tau]$$

$$c_1 = \frac{\int d\tau\left(\frac{d}{d\tau}(\tau\Phi[\tau])\right)^2}{\int d\tau\Phi^2[\tau]}$$

- ISR: rho is very small, does not influence energy behavior
- LHC: rho is larger but almost constant, does not change energy behavior either

Total elastic cross section

Assuming exponential diffractive peak (no dips and bumps)

$$\frac{\sigma_{\mathrm{el}}(s)}{\sigma_{\mathrm{tot}}(s)} \sim \frac{\sigma_{\mathrm{tot}}(s)}{B(s)} \left(1 + \rho^2(s)\right)$$

Works within a few %. However, if $\sigma_{\mathrm{tot}}(s) \neq B(s)$
GS is violated.
Asymptotically (M.M. Block, Phys. Rept. (2006))

$$\sigma_{\mathrm{tot}}(s)/B(s) \rightarrow \mathrm{const.}$$

Summary

- Bump to dip position ratio is constant from ISR to LHC
- Universal scaling variable $\tau \sim \sigma_{\rm tot}(s)\,|t| = R^2(y)\,|t|$
- Crossing and GS and expansion

$$R^2\left(y - i\frac{\pi}{2}\right) \simeq R^2(y) - i\frac{\pi}{2}\frac{dR^2(y)}{dy}$$

- Parameter free prediction for rho parameter
- Dip and bump structure understood in terms of sig_tot and its derivative
- Main properties of total and differential cross-sections at all energies in the dip – bump region explained from a simple and intuitive picture based on GS
- But still approximate, total elastic x-section is not reproduced → GSV at small t