INSTITUTO GALEGO DE FÍSICA DE ALTAS ENERXÍAS IGFAE

CENTRO DE CIENCIAS DE BENASQUE PEDRO PASCUAL

High energy QCD: from the LHC to the EIC
Benasque Science Center, August 11th 2025

Exploring the DGLAP resummation in the JIMWLK Hamiltonian

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arXiv:2501.19250 [hep-ph]

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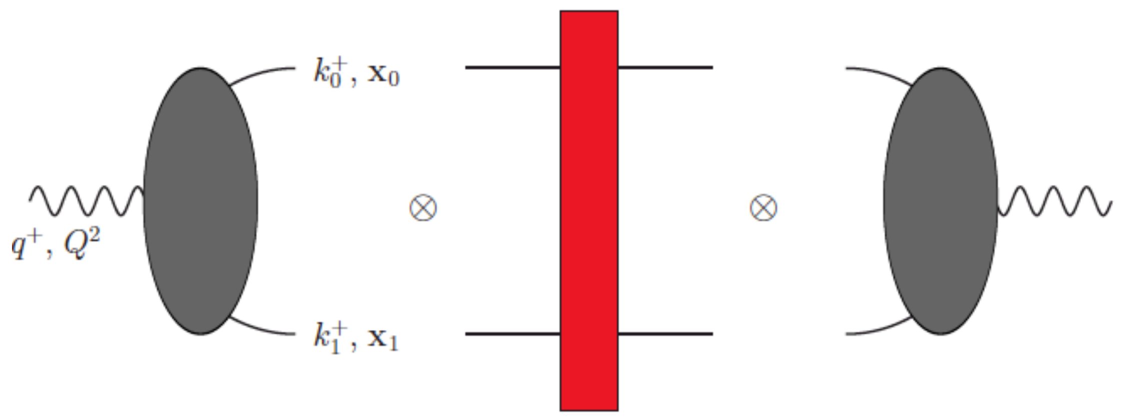
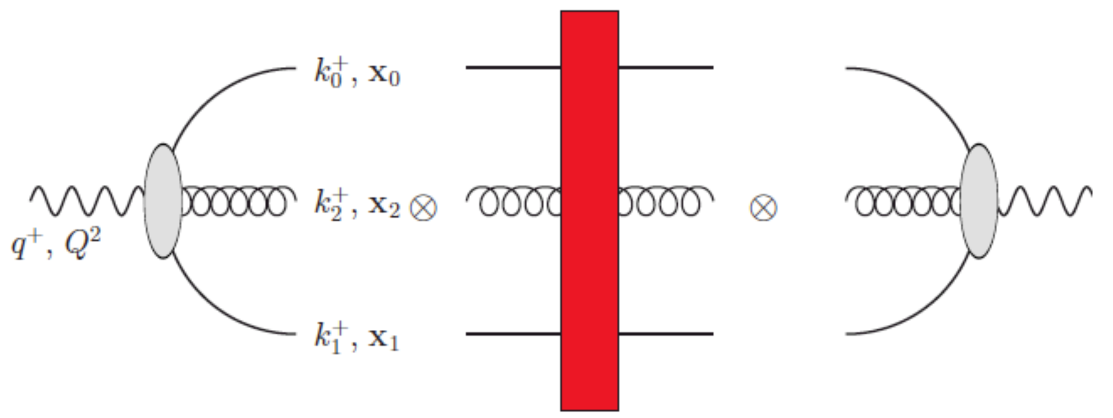
NLO evolution equations:

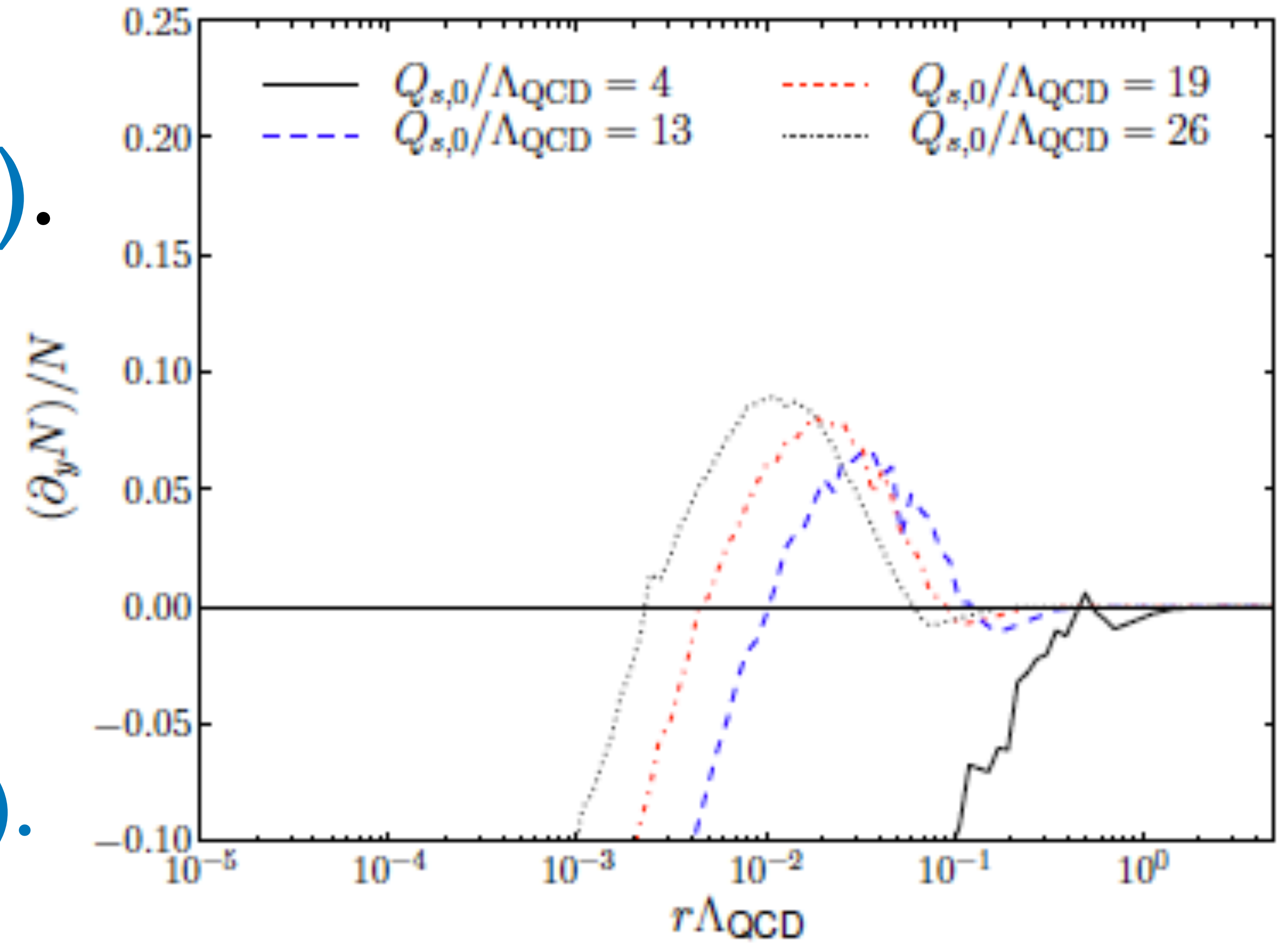
- NLO evolution equations available:
  - ➜ NLO BK (0710.4330, 1309.7644).
  - ➜ NLO JIMWLK (1310.0378, 1610.03453).





- Instabilities appeared (akin to those in NLO BFKL, late 90's):
  - ➜ Kinematic constrains (1401.0313, 1902.06637).
  - ➜ Collinear improvements (1502.05642,1507.03651).

1502.02400



- Good fits to HERA data (~ rcBK - LO impact factor) (1507.07120).

- Recent discussions on scales (several choices possible):
  - ➜ Large transverse logs (from typical momenta of projectile to target) assigned to DGLAP instead of running coupling (**2308.15545**).
  - ➜ No general Langevin implementation for NLO JIMWLK (2310.10738).

- Burst of activity on unifying JIMWLK with CSS/DGLAP (NLO DIS dijet papers; 2406.04238, 2407.15960, 2412.05085, 2412.05097, 2412.10160).

NLO JIMWLK:

- For the projectile probability of a color ensemble $W_P[S]$, at LO:

$$\frac{d}{dY}\mathcal{W}_P[S] = H_{\rm JIMWLK}[S,J]\mathcal{W}_P[S]$$

$$H^{\rm LO}_{\rm JIMWLK} = \frac{\alpha_s}{2\pi^2}\int_{\mathbf{x},\mathbf{y},\mathbf{z}} \frac{(\mathbf{x}-\mathbf{z})\cdot(\mathbf{y}-\mathbf{z})}{(\mathbf{x}-\mathbf{z})^2(\mathbf{y}-\mathbf{z})^2} \times \left[J^a_L(\mathbf{x})\,J^a_L(\mathbf{y}) + J^a_R(\mathbf{x})\,J^a_R(\mathbf{y}) - 2J^a_L(\mathbf{x})\,S^{ab}(\mathbf{z})\,J^b_R(\mathbf{y})\right]$$

with S the eikonal scattering matrix of a projectile gluon (Wilson line in the adjoint representation) and $J_{L(R)}$ left (right) color rotation operators.

- The target correlation length, Q_T^{-1}, is smaller than that of the projectile Q_P^{-1}.
- At **NLO:**

$$H^{\rm NLO}_{\rm JIMWLK} = \int_{\mathbf{x},\mathbf{y},\mathbf{z}} K_{JSJ}(\mathbf{x},\mathbf{y},\mathbf{z})\left[J^a_L(\mathbf{x})J^a_L(\mathbf{y}) + J^a_R(\mathbf{x})J^a_R(\mathbf{y}) - 2J^a_L(\mathbf{x})S^{ab}(\mathbf{z})J^b_R(\mathbf{y})\right]$$
$$+\int K_{JSSJ}(\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{z}')\left[J^a_L(\mathbf{x})D^{ad}(\mathbf{z},\mathbf{z}')J^d_R(\mathbf{y}) - \frac{N_c}{2}\left[J^a_R(\mathbf{x})J^a_R(\mathbf{y}) + J^a_L(\mathbf{x})J^a_L(\mathbf{y})\right]\right]$$
$$+\int K_{q\bar{q}}(\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{z}')\left[2\,J^a_L(\mathbf{x})\,D^{ab}_F(\mathbf{z},\mathbf{z}')\,J^b_R(\mathbf{y}) - \frac{1}{2}\left[J^a_R(\mathbf{x})J^a_R(\mathbf{y}) + J^a_L(\mathbf{x})J^a_L(\mathbf{y})\right]\right]$$
$$+\,\text{terms without }\beta_0$$

$$D^{ab}(\mathbf{z}_1,\mathbf{z}_2) = {\rm Tr}[T^a S(\mathbf{z}_1) T^b S^+(\mathbf{z}_2)]$$
$$D^{ab}_F(\mathbf{z}_1,\mathbf{z}_2) = {\rm Tr}[\tau^a V(\mathbf{z}_1)\tau^b V^+(\mathbf{z}_2)]$$

NLO JIMWLK:

- Several types of large logarithms in $K_{JSJ}, K_{JSSJ}, K_{q\bar{q}}$:
  1. UV logarithms.
  2. Logarithms associated with two partons emitted at the same transverse position.
  3. Others.

- The first two types are usually treated in the running coupling: divergence subtracted from $K_{JSSJ}, K_{q\bar{q}}$ and introduced into K_{JSJ} which leads to the running of the coupling in the LO.

- Different prescriptions: Balitsky, Kovchegov-Weigert,…

- **2308.15545**: express H_{JIMWLK} in terms of dressed gluon states ($\mathbb{S}_Q$, resolution Q^{-1}) and introduce the second type of logs into the LO with naive, parent parton scale choice in the r.c., $g^2 \to g(|\mathbf{x}-\mathbf{z}|)g(|\mathbf{y}-\mathbf{z}|)$. At $\mathcal{O}(\alpha_s)$ and LL, DGLAP-like expression (only $g \to gg$)

$$D^{ab}(\mathbf{z}_1,\mathbf{z}_2) = \mathrm{Tr}[T^a S(\mathbf{z}_1) T^b S^+(\mathbf{z}_2)]$$

$$\mathbb{S}_Q^{ab}(\mathbf{z}) = [1+\frac{\alpha_s\beta_0^g}{4\pi}\ln\frac{\mu^2}{Q^2}]S^{ab}(\mathbf{z}) - \frac{\alpha_s\beta_0^g}{4\pi^2 N_c}\int_{|Z|<Q^{-1}}\frac{d^2Z}{Z^2}D^{ab}(\mathbf{z}+Z/2,\mathbf{z}-Z/2)$$

DGLAP resummation:

- After resummation of these large logs (i.e., independence of Q), KLSZ get a DGLAP equation excluding the pole contribution already contained in JIMWLK:

$$\boxed{\frac{\partial}{\partial \ln Q^2}\mathbb{S}_Q^{ab}(\mathbf{z}) = -\frac{\alpha_s \beta_0^g}{4\pi}\left[\mathbb{S}_Q^{ab}(\mathbf{z}) - \frac{1}{N_c}\int \frac{d\phi}{2\pi}\left(\mathbb{D}_Q^{ab}(\mathbf{z}+\frac{1}{2}Q^{-1}\mathbf{e}_\phi, \mathbf{z}-\frac{1}{2}Q^{-1}\mathbf{e}_\phi)\right)\right]} \quad \mathbb{S}_{Q_T}(\mathbf{z}) = S(\mathbf{z})$$

$$\mathbb{D}_Q^{ab}(\mathbf{z}_1, \mathbf{z}_2) = \mathrm{Tr}[T^a \mathbb{S}_Q(\mathbf{z}_1) T^b \mathbb{S}_Q^{\dagger}(\mathbf{z}_2)] \qquad \beta_0^g = \frac{11 N_c}{3} \qquad \mathbf{e}_\phi = \text{unit radial vector from } \mathbf{z}$$

- This state should be evolved and then substituted into JIMWLK.
- Solved analytically in the dilute and dense regimes:

$$\mathbb{S}_Q(\mathbf{x}) = 1 + iT^a \alpha_Q^a(\mathbf{x}) \longrightarrow \alpha_Q(\mathbf{z}) = \left(\frac{Q_T^2}{Q^2}\right)^{\frac{\alpha_s \beta_0^g}{4\pi}} \alpha(\mathbf{z}) + \int_{1/Q_T<|\mathbf{x}-\mathbf{z}|<1/Q} d^2\mathbf{x}\, \frac{\alpha_s \beta_0^g}{4\pi}\frac{Q^2}{\pi}\left[1 - \left(\frac{1}{Q^2(\mathbf{x}-\mathbf{z})^2}\right)^{1+\frac{\alpha_s \beta_0^g}{4\pi}}\right]$$

$$\mathbb{S}_Q(\mathbf{z}) = \left[\frac{Q_T^2}{Q^2}\right]^{\frac{\alpha_s \beta_0}{4\pi}} S(\mathbf{z})$$

- **This work: one step beyond the dilute limit** (which requires numerics).

General equations for $SU(2)$:

- We take the simplified setup of $SU(2)$:

$$\mathbb{S}^{ab}(\mathbf{z}) = A(\mathbf{z})\delta^{ab} + \lambda_c(\mathbf{z})\epsilon^{abc} - 2B^{ab}(\mathbf{z})$$

with A, λ^a, B^{ab} scalar, vector and traceless rank-2 tensor, respectively.

- Using ϵ_{abc} identities,

$$\mathrm{Tr}\left[\epsilon^a\epsilon^b\epsilon^c\right] = \epsilon^{abc} = \epsilon^a_{bc},$$

$$\epsilon^a_{fc}\epsilon^b_{de} = \delta^{ab}\delta^{fd}\delta^{ce} - \delta^{ab}\delta^{fe}\delta^{cd} - \delta^{ad}\delta^{fb}\delta^{ce} + \delta^{ad}\delta^{fe}\delta^{cb} - \delta^{ae}\delta^{fd}\delta^{cb} + \delta^{ae}\delta^{ae}\delta^{fb}\delta^{cd},$$

$$[\epsilon^a\epsilon^b]_{ef} = \delta^{af}\delta^{eb} - \delta^{ab}\delta^{ef}.$$

we get ($A_1 \equiv A(\mathbf{z}_1)$, etc.)

$$\mathrm{Tr}\left[T^a\mathbb{S}_1T^b\mathbb{S}_2^\dagger\right] = \delta^{ab}\left[2A_1A_2 + \frac{2}{3}\lambda_1\cdot\lambda_2 - \frac{4}{3}Tr[B_1B_2]\right] + \epsilon^s_{ab}\left[A_1\lambda_2^s + A_2\lambda_1^s - 2(\lambda_1\cdot B_2)^s - 2(\lambda_2\cdot B_1)^s\right]$$
$$+ \left[\lambda_1^a\lambda_2^b + \lambda_1^b\lambda_2^a - \frac{2}{3}\delta^{ab}\lambda_1\cdot\lambda_2\right] + 2A_1B_2^{ab} + 2A_2B_1^{ab} + 4\left[(B_1B_2)^{ab} + (B_2B_1)^{ab} - \frac{2}{3}Tr(B_1B_2)\right]$$

General equations for $SU(2)$:

- We get the following set of coupled non-linear integro-differential equations:

$$\mathbb{S}^{ab} = A\delta_{ab} + \lambda^c \epsilon_{abc} - 2B^{ab}$$

$$\frac{\partial}{\partial \ln Q^2} A_Q(\mathbf{z}) = -\frac{\alpha_s \beta_0^g}{4\pi} \Bigg[A_Q(\mathbf{z}) + \frac{1}{2} \int \frac{d\phi}{2\pi} \Big(-2A_Q(\mathbf{z}_1) A_Q(\mathbf{z}_2) - \frac{2}{3} \lambda_Q^c(\mathbf{z}_1) \lambda_Q^c(\mathbf{z}_2) + \frac{4}{3} B_Q^{pq}(\mathbf{z}_1) B_Q^{qp}(\mathbf{z}_2) \Big) \Bigg],$$

$$\frac{\partial}{\partial \ln Q^2} \lambda_Q^c(\mathbf{z}) = -\frac{\alpha_s \beta_0^g}{4\pi} \Bigg[\lambda_Q^c(\mathbf{z}) - \frac{1}{2} \int \frac{d\phi}{2\pi} \Big(A_Q(\mathbf{z}_1) \lambda_Q^c(\mathbf{z}_2) + \lambda_Q^c(\mathbf{z}_1) A_Q(\mathbf{z}_2) - 2\lambda_Q^d(\mathbf{z}_1) B_Q^{dc}(\mathbf{z}_2) - 2B_Q^{cd}(\mathbf{z}_1) \lambda_Q^d(\mathbf{z}_2) \Big) \Bigg],$$

$$\frac{\partial}{\partial \ln Q^2} B_Q^{cd}(\mathbf{z}) = -\frac{\alpha_s \beta_0^g}{4\pi} \Bigg[B_Q^{cd}(\mathbf{z}) + \frac{1}{2} \int \frac{d\phi}{2\pi} \Big(A_Q(\mathbf{z}_1) B_Q^{cd}(\mathbf{z}_2) + B_Q^{cd}(\mathbf{z}_1) A_Q(\mathbf{z}_2) + \lambda_Q^c(\mathbf{z}_1) \lambda_Q^d(\mathbf{z}_2) - \frac{1}{3} \delta_{cd} \lambda_Q^a(\mathbf{z}_1) \lambda_Q^a(\mathbf{z}_2) + 2B_Q^{dp}(\mathbf{z}_1) B_Q^{pc}(\mathbf{z}_2) + 2B_Q^{cp}(\mathbf{z}_1) B_Q^{pd}(\mathbf{z}_2) - \frac{4}{3} \delta_{cd} B_Q^{pq}(\mathbf{z}_1) B_Q^{qp}(\mathbf{z}_2) \Big) \Bigg]$$

Second-order expressions:

• Close to unitarity (dilute limit), the only equation is that of λ, all other terms are of higher order. Here we work at $\mathcal{O}(\lambda^2)$ but still $\lambda \propto \alpha_s \ll 1$:

$$\mathbb{S}^{ab} = A\delta_{ab} + \lambda^c \epsilon_{abc} - 2B^{ab} \qquad A_Q = 1 + \Delta_Q \qquad \Delta, B \propto \lambda^2$$

$$\frac{\partial}{\partial \ln Q^2} \Delta_Q(\mathbf{z}) = -\frac{\alpha_s \beta_0^g}{4\pi} \left[1 + \Delta_Q(\mathbf{z}) + \frac{1}{2} \int \frac{d\phi}{2\pi} \left(-2 - 2\Delta_Q(\mathbf{z}_1) - 2\Delta_Q(\mathbf{z}_2) - \frac{2}{3} \lambda_Q^c(\mathbf{z}_1) \lambda_Q^c(\mathbf{z}_2) \right) \right],$$

$$\frac{\partial}{\partial \ln Q^2} \lambda_Q^c(\mathbf{z}) = -\frac{\alpha_s \beta_0^g}{4\pi} \left[\lambda_Q^c(\mathbf{z}) - \frac{1}{2} \int \frac{d\phi}{2\pi} \left(\lambda_Q^c(\mathbf{z}_2) + \lambda_Q^c(\mathbf{z}_1) \right) \right],$$

$$\frac{\partial}{\partial \ln Q^2} B_Q^{cd}(\mathbf{z}) = -\frac{\alpha_s \beta_0^g}{4\pi} \left[B_Q^{cd}(\mathbf{z}) + \frac{1}{2} \int \frac{d\phi}{2\pi} \left(B_Q^{cd}(\mathbf{z}_2) + B_Q^{cd}(\mathbf{z}_1) + \lambda_Q^c(\mathbf{z}_1) \lambda_Q^d(\mathbf{z}_2) - \frac{1}{3} \delta_{cd} \lambda_Q^a(\mathbf{z}_1) \lambda_Q^a(\mathbf{z}_2) \right) \right].$$

• In this way, the equation for λ decouples, and λ acts as a source term for the decoupled equations for Δ, B.

Solutions:

- Transforming to momentum space, solutions can be written:

$$\mathbb{S}^{ab} = A\delta_{ab} + \lambda^c\epsilon_{abc} - 2B^{ab}$$

$$A_Q = 1 + \Delta_Q$$

$$\lambda^c_Q(\mathbf{p}) = \exp\left[-\int_Q^{Q_T} \frac{dQ'^2}{Q'^2} R(p,Q')\right] \lambda^c_{Q_T}(\mathbf{p})$$

$$R(p,Q) = \frac{\alpha_s\beta_0^g}{4\pi}\left[J_0\left(\frac{p}{2Q}\right) - 1\right]$$

$$\Delta_Q(\mathbf{p}) = \exp\left[-\int_Q^{Q_T} \frac{dQ'^2}{Q'^2} R_\Delta(p,Q')\right] \Delta_{Q_T}(\mathbf{p}) - \int_Q^{Q_T} \frac{dQ'^2}{Q'^2} \exp\left[-\int_Q^{Q'} \frac{dQ''^2}{Q''^2} R_\Delta(p,Q'')\right] F(\mathbf{p},Q')$$

$$F(\mathbf{p},Q) = \frac{\alpha_s\beta_0^g}{12\pi}\int \frac{d^2\mathbf{k}}{(2\pi)^2} J_0(Q^{-1}k)\lambda^c_Q(\mathbf{p}/2+\mathbf{k})\lambda^c_Q(\mathbf{p}/2-\mathbf{k})$$

$$R_\Delta(p,Q) = \frac{\alpha_s\beta_0^g}{4\pi}\left[2J_0\left(\frac{p}{2Q}\right) - 1\right]$$

$$B^{cd}_Q(\mathbf{p}) = \exp\left[-\int_Q^{Q_T} \frac{dQ'^2}{Q'^2} R_B(p,Q')\right] B^{cd}_{Q_T}(\mathbf{p}) - \int_Q^{Q_T} \frac{dQ'^2}{Q'^2} \exp\left[-\int_Q^{Q'} \frac{dQ''^2}{Q''^2} R_B(p,Q'')\right] G^{cd}(\mathbf{p},Q')$$

$$G^{cd}(\mathbf{p},Q) = -\frac{\alpha_s\beta_0^g}{4\pi N_c}\int \frac{d^2\mathbf{k}}{(2\pi)^2} J_0(Q^{-1}k)\left[\lambda^c_Q(\mathbf{p}/2+\mathbf{k})\lambda^d_Q(\mathbf{p}/2-\mathbf{k}) - \frac{1}{3}\delta_{cd}\lambda^a_Q(\mathbf{p}/2+\mathbf{k})\lambda^a_Q(\mathbf{p}/2-\mathbf{k})\right],$$

$$R_B(p,Q) = -\frac{\alpha_s\beta_0^g}{4\pi}\left[J_0\left(\frac{p}{2Q}\right) + 1\right]$$

Initial condition:

- As initial condition at $Q = Q_T$, we take a dipole with legs at x_1 and x_2:

$$\lambda^a_{Q_T}(\mathbf{p}) \propto \delta^{3a}\frac{1}{p^2}\left(e^{i\mathbf{p}\mathbf{x}_1} - e^{i\mathbf{p}\mathbf{x}_2}\right) \longrightarrow \lambda^a_{Q_T}(\mathbf{p}) = \lambda\delta^{3a}\frac{1}{p^2}\sin(\mathbf{p}\cdot\mathbf{x}) \qquad \text{for } x_1 = -x_2 = x$$

- At $Q = Q_T$, unitarity is fulfilled at $\mathcal{O}(\lambda^2)$:

$$\mathbb{S}^{ab} = A\delta_{ab} + \lambda^c\epsilon_{abc} - 2B^{ab} \qquad A_Q = 1 + \Delta_Q \qquad \mathbb{S}^{ab}_{Q_T}(\mathbf{z})\mathbb{S}^{bc\,\dagger}_{Q_T}(\mathbf{z}) = \delta^{ac} + \mathcal{O}(\lambda^3)$$

leading to

$$\Delta_{Q_T}(\mathbf{z}) = -\frac{1}{2}\lambda^c_{Q_T}(\mathbf{z})\lambda^c_{Q_T}(\mathbf{z}),$$
$$B^{ab}_{Q_T}(\mathbf{z}) = -\frac{1}{4}\left(\lambda^a_{Q_T}(\mathbf{z})\lambda^b_{Q_T}(\mathbf{z}) - \frac{1}{3}\delta^{ab}\lambda^c_{Q_T}(\mathbf{z})\lambda^c_{Q_T}(\mathbf{z})\right)$$

$$\Delta_{Q_T}(\mathbf{p}) = -\frac{1}{2}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\lambda^c_{Q_T}(\mathbf{k})\lambda^c_{Q_T}(\mathbf{p}-\mathbf{k}),$$
$$B^{ab}_{Q_T}(\mathbf{p}) = -\frac{1}{4}\left(\int\frac{d^2\mathbf{k}}{(2\pi)^2}\lambda^a_{Q_T}(\mathbf{k})\lambda^b_{Q_T}(\mathbf{p}-\mathbf{k}) - \frac{1}{3}\delta^{ab}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\lambda^c_{Q_T}(\mathbf{k})\lambda^c_{Q_T}(\mathbf{p}-\mathbf{k})\right)$$

Deviation from unitarity:

- For our initial condition, for any Q $\lambda^c_Q \propto \delta^{c3}$ $B^{ab}_Q(\mathbf{p}) = B_Q(\mathbf{p})\left(\delta^{a3}\delta^{b3} - \frac{1}{3}\delta^{ab}\right)$

$$B_{Q_T}(\mathbf{p}) = -\frac{1}{4}\int \frac{d^2\mathbf{k}}{(2\pi)^2}\lambda^a_{Q_T}(\mathbf{k})\lambda^a_{Q_T}(\mathbf{p}-\mathbf{k})$$

- If unitarity were fulfilled at $\mathcal{O}(\lambda^2)$ at any Q, we would have:

$$\Delta^U_Q(\mathbf{p}) = -\frac{1}{2}\int \frac{d^2\mathbf{k}}{(2\pi)^2}\lambda^c_Q(\mathbf{k})\lambda^c_Q(\mathbf{p}-\mathbf{k}) \qquad B^U_Q(\mathbf{p}) = -\frac{1}{4}\int \frac{d^2\mathbf{k}}{(2\pi)^2}\lambda^a_Q(\mathbf{k})\lambda^a_Q(\mathbf{p}-\mathbf{k})$$

- We define quantities sensitive to evolution and to the deviation from unitarity during evolution:

$$\frac{\Delta_{Q_T}(\mathbf{p}) - \Delta_Q(\mathbf{p})}{\Delta_{Q_T}(\mathbf{p})} = 1 - \frac{\Delta_Q(\mathbf{p})}{\Delta_{Q_T}(\mathbf{p})} \qquad \mathcal{R}_\Delta(Q,\mathbf{p}) = \frac{1 - \Delta^U_Q(\mathbf{p})/\Delta_Q(\mathbf{p})}{1 - \Delta_Q(\mathbf{p})/\Delta_{Q_T}(\mathbf{p})}$$

$$\frac{B_{Q_T}(\mathbf{p}) - B_Q(\mathbf{p})}{B_{Q_T}(\mathbf{p})} = 1 - \frac{B_Q(\mathbf{p})}{B_{Q_T}(\mathbf{p})} \qquad \mathcal{R}_B(Q,\mathbf{p}) = \frac{1 - B^U_Q(\mathbf{p})/B_Q(\mathbf{p})}{1 - B_Q(\mathbf{p})/B_{Q_T}(\mathbf{p})}$$

- If deviation from unitarity is small while evolution is sizeable, $\mathcal{R}_{\Delta,B} \simeq 0$; if deviation from unitarity is of the same order of evolution, $|\mathcal{R}_{\Delta,B}| \simeq 1$.

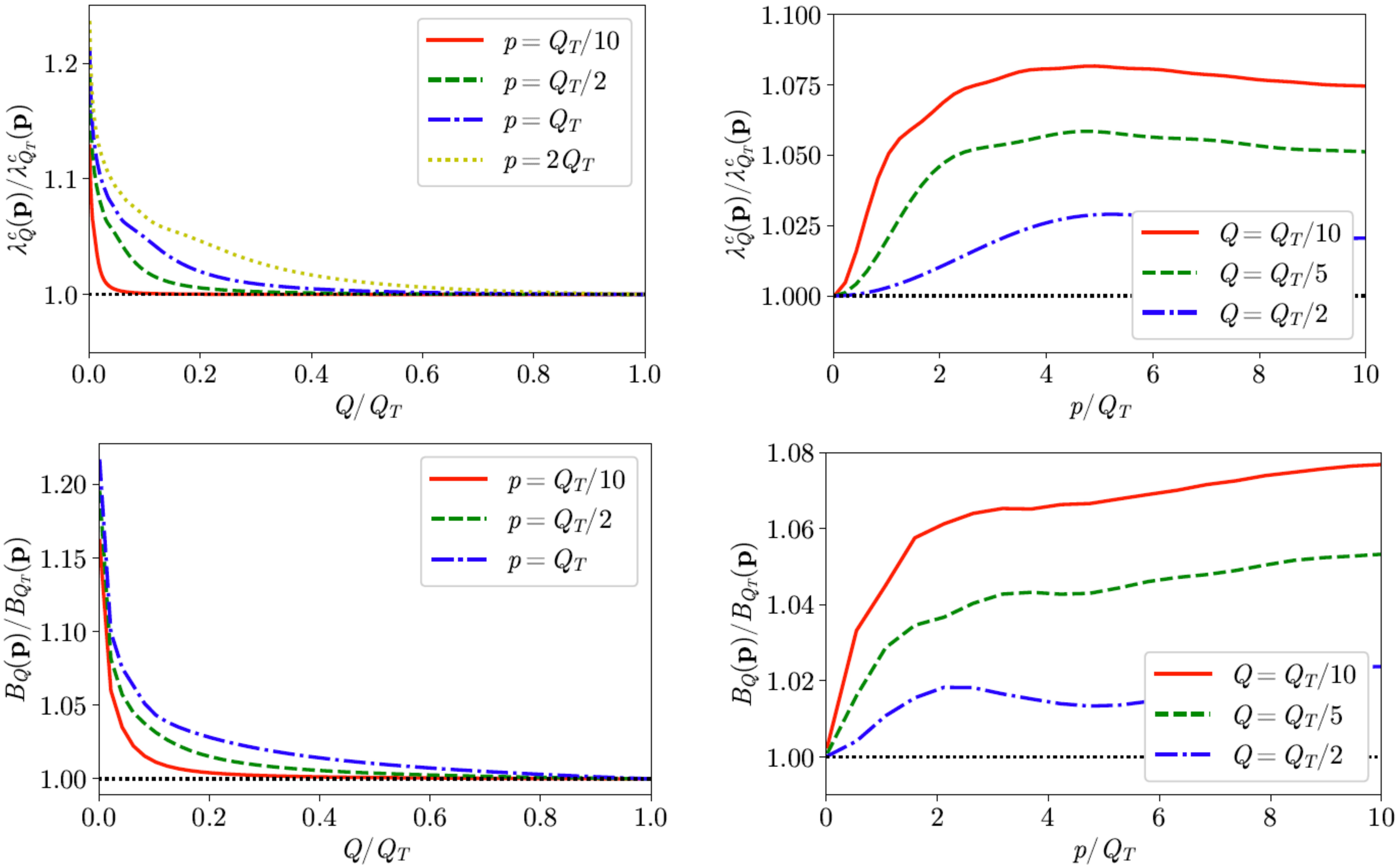
Numerical results: λ, B

- Modest deviation from the initial condition, smaller for smaller p.

$$\mathbb{S}^{ab} = A\delta_{ab} + \lambda^c \epsilon_{abc} - 2B^{ab}$$

$$A_Q = 1 + \Delta_Q$$

$$\alpha_s = 0.1,\ \phi = \widehat{p,x} = \pi/2$$



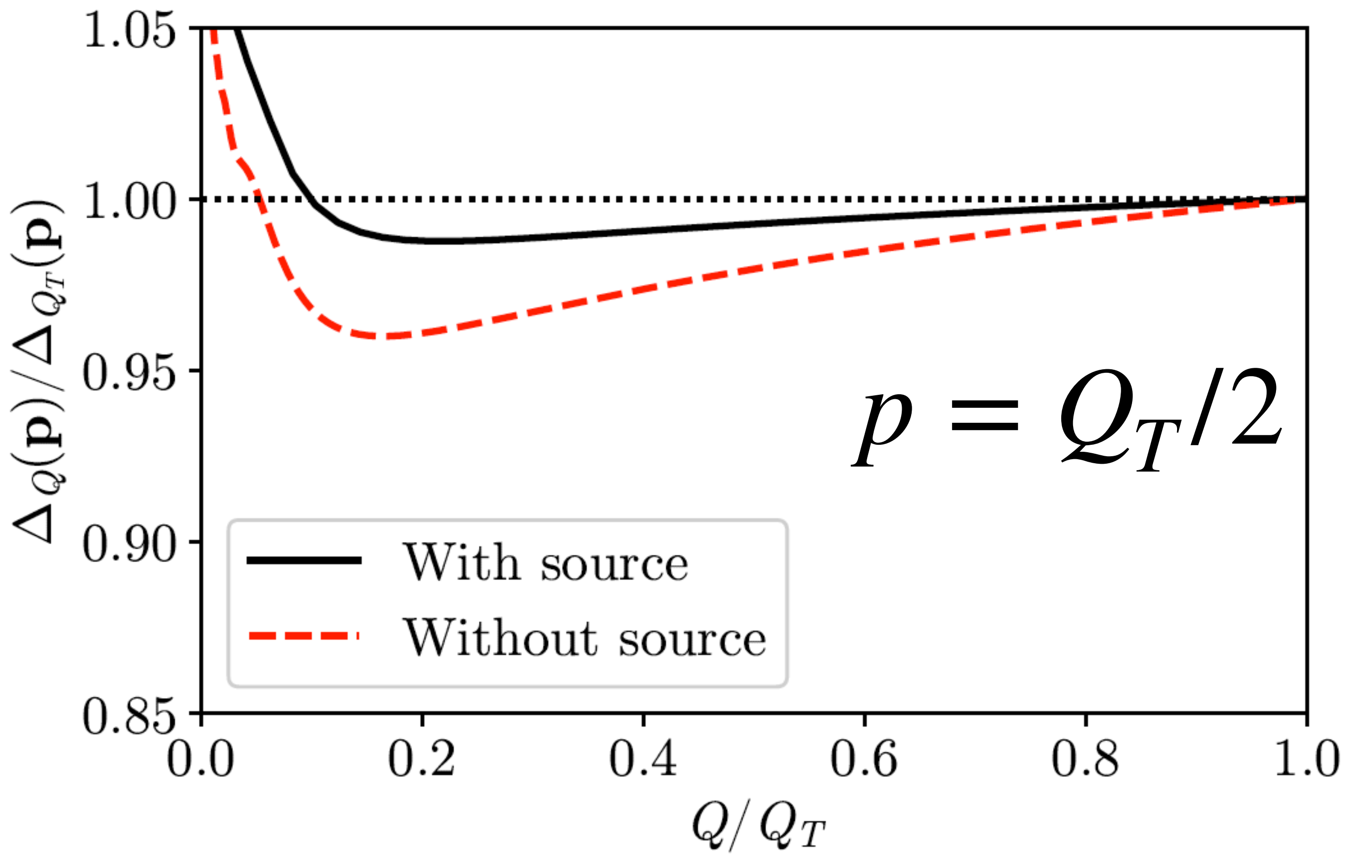
Numerical results: Δ

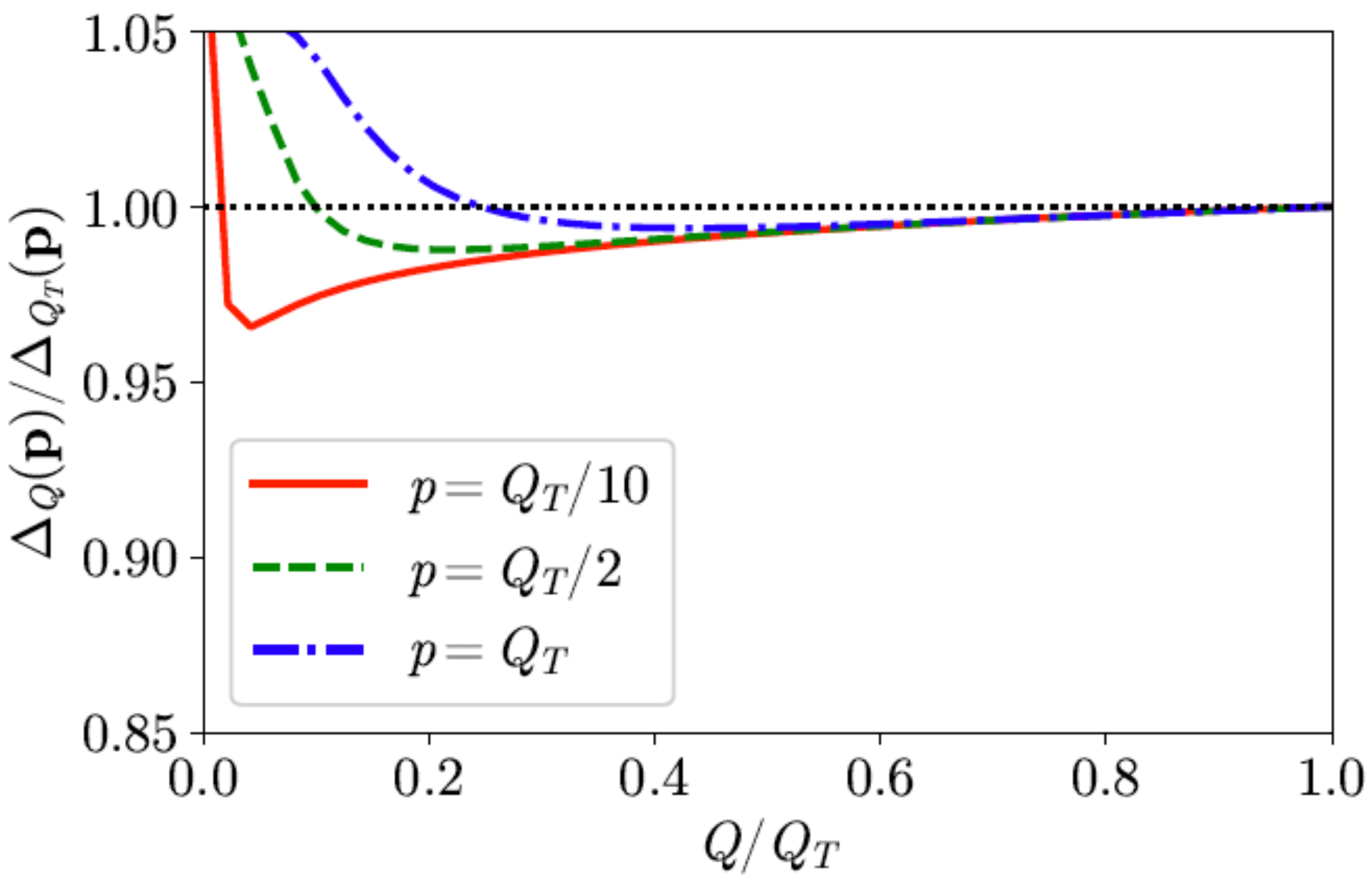
- Modest deviation from i.c..
- Source tames evolution.
- Same qualitative picture independent of angle.

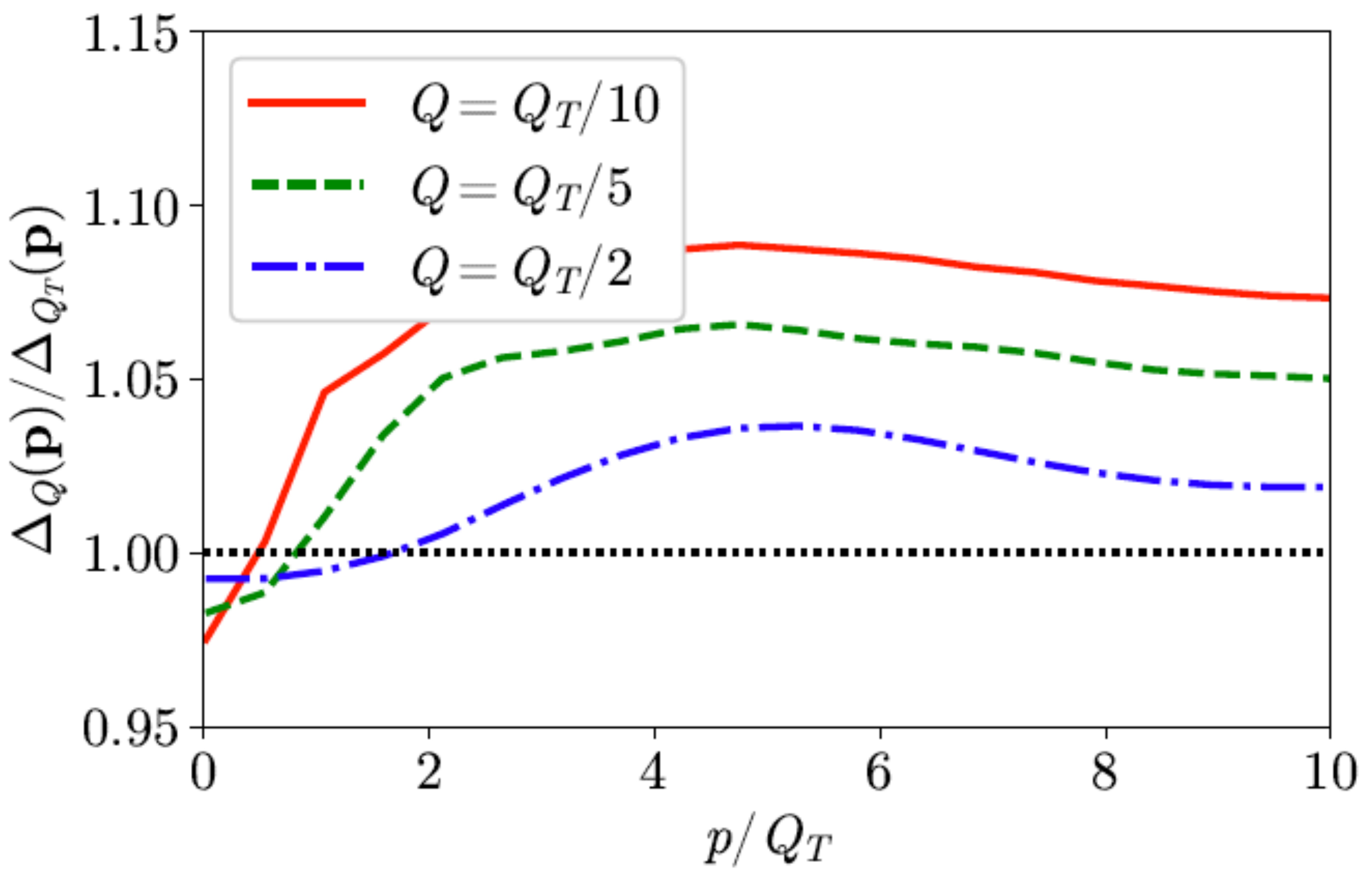
$$\mathbb{S}^{ab} = A\delta_{ab} + \lambda^c \epsilon_{abc} - 2B^{ab}$$

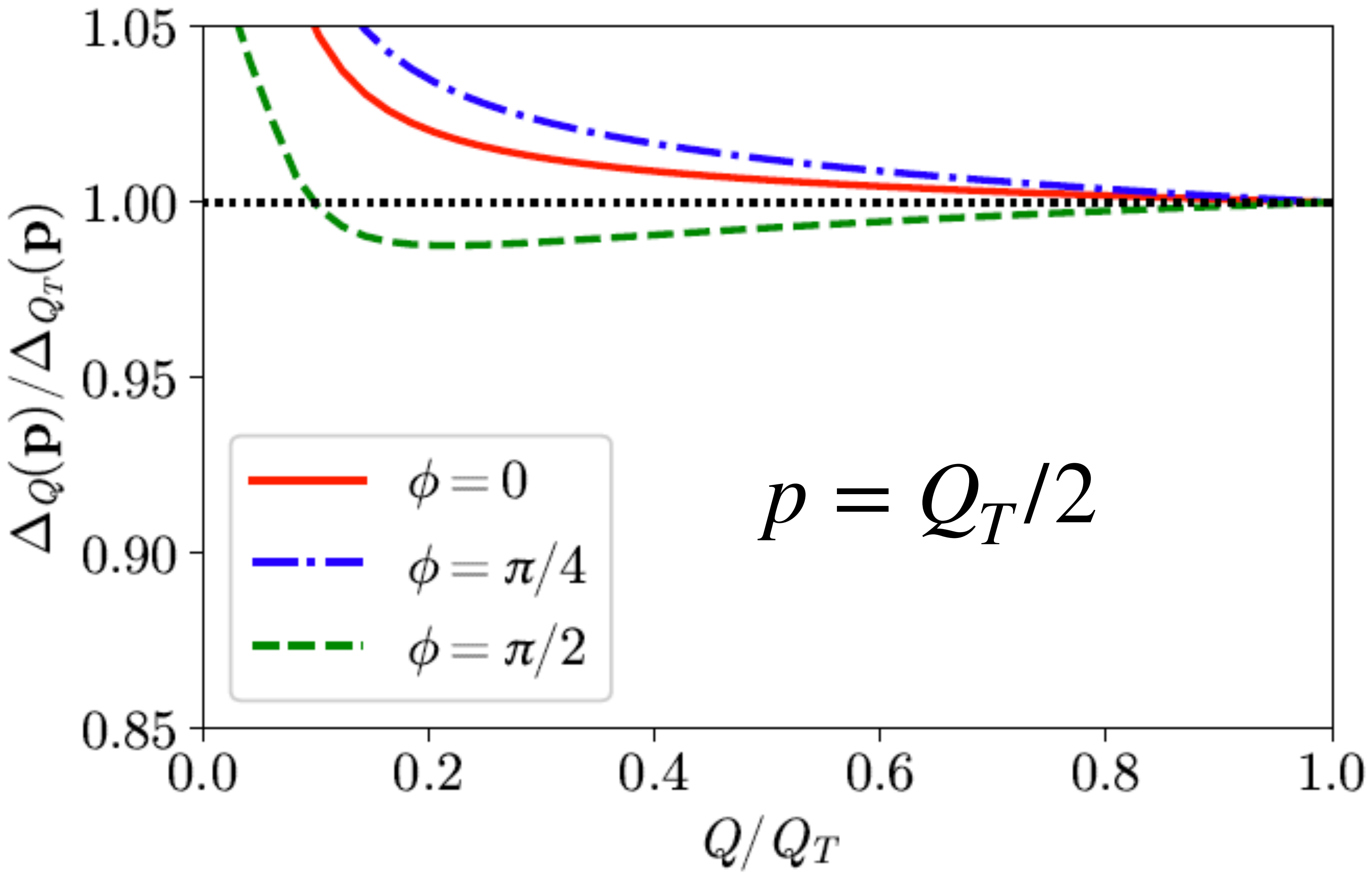
$$A_Q = 1 + \Delta_Q$$

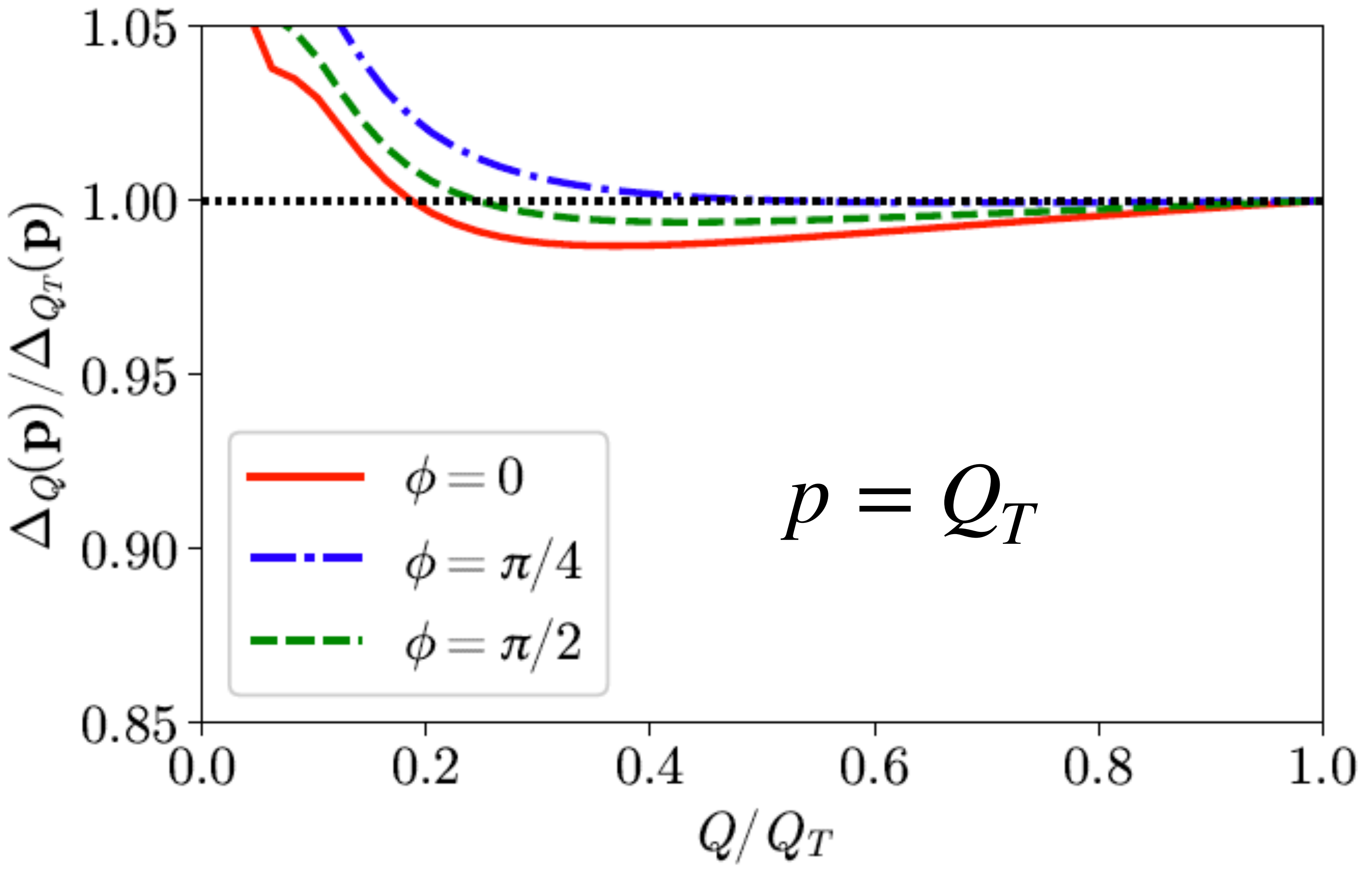
$\alpha_s = 0.1,\ \phi = \widehat{p,x} = \pi/2$









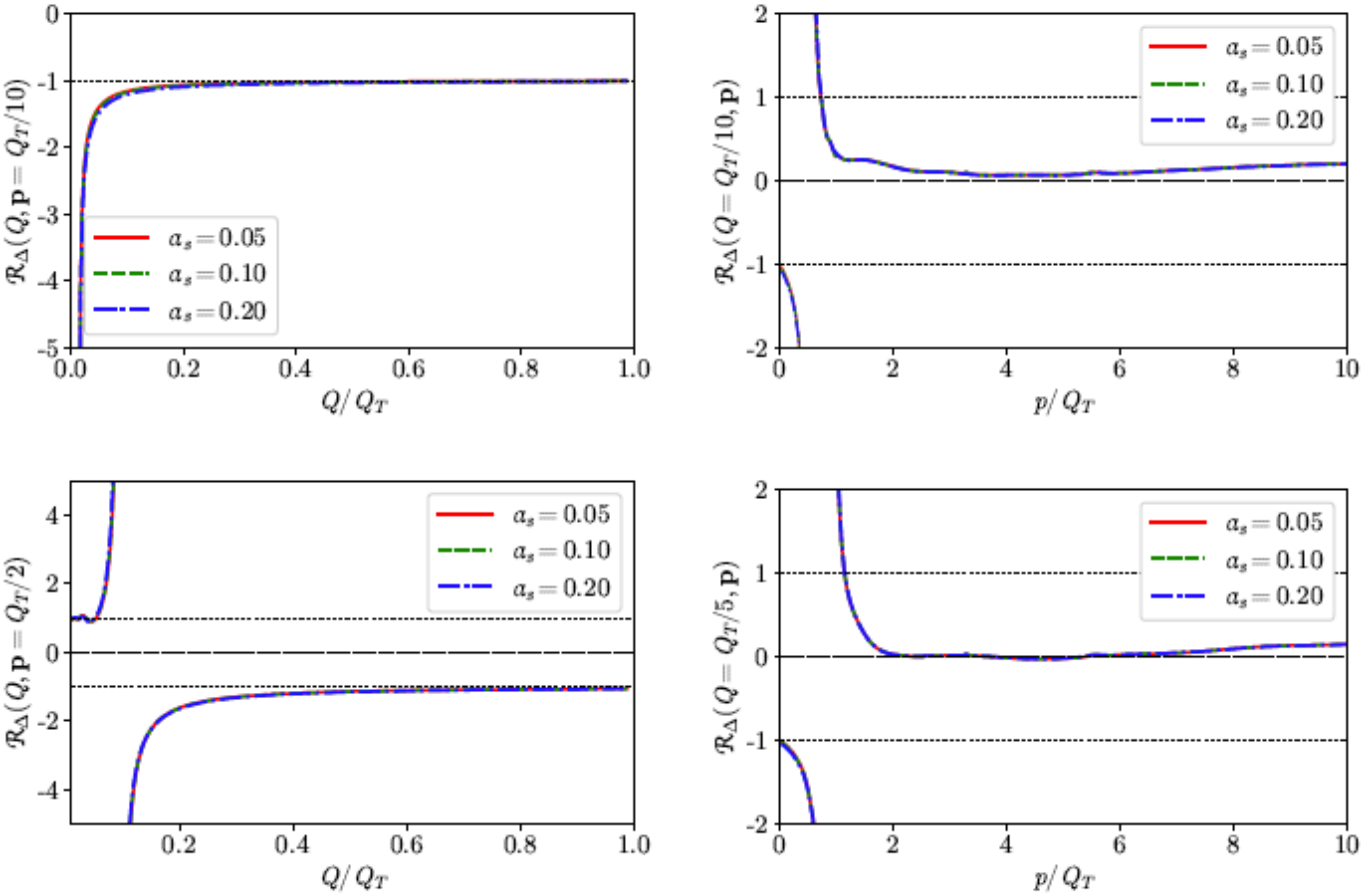


Numerical results: unitarity for Δ

- Deviation from unitarity similar to evolution (order 1) for $p < Q_T$ and for not too large evolution $Q > Q_T/3$.

- Small deviation from unitarity for large p.

- Independence of α_s.



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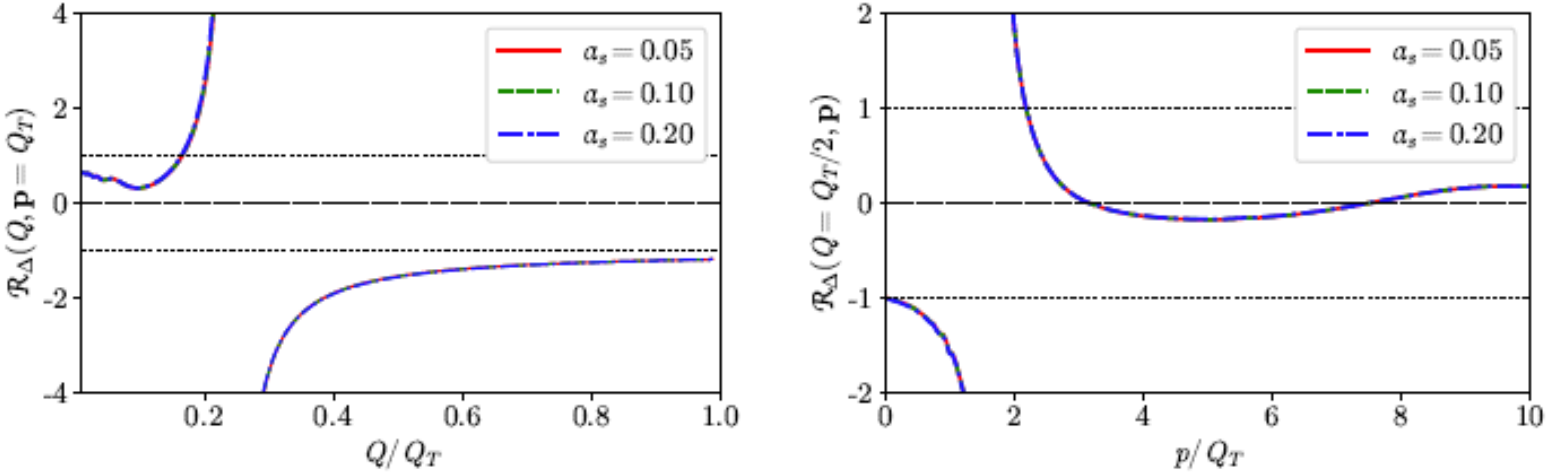
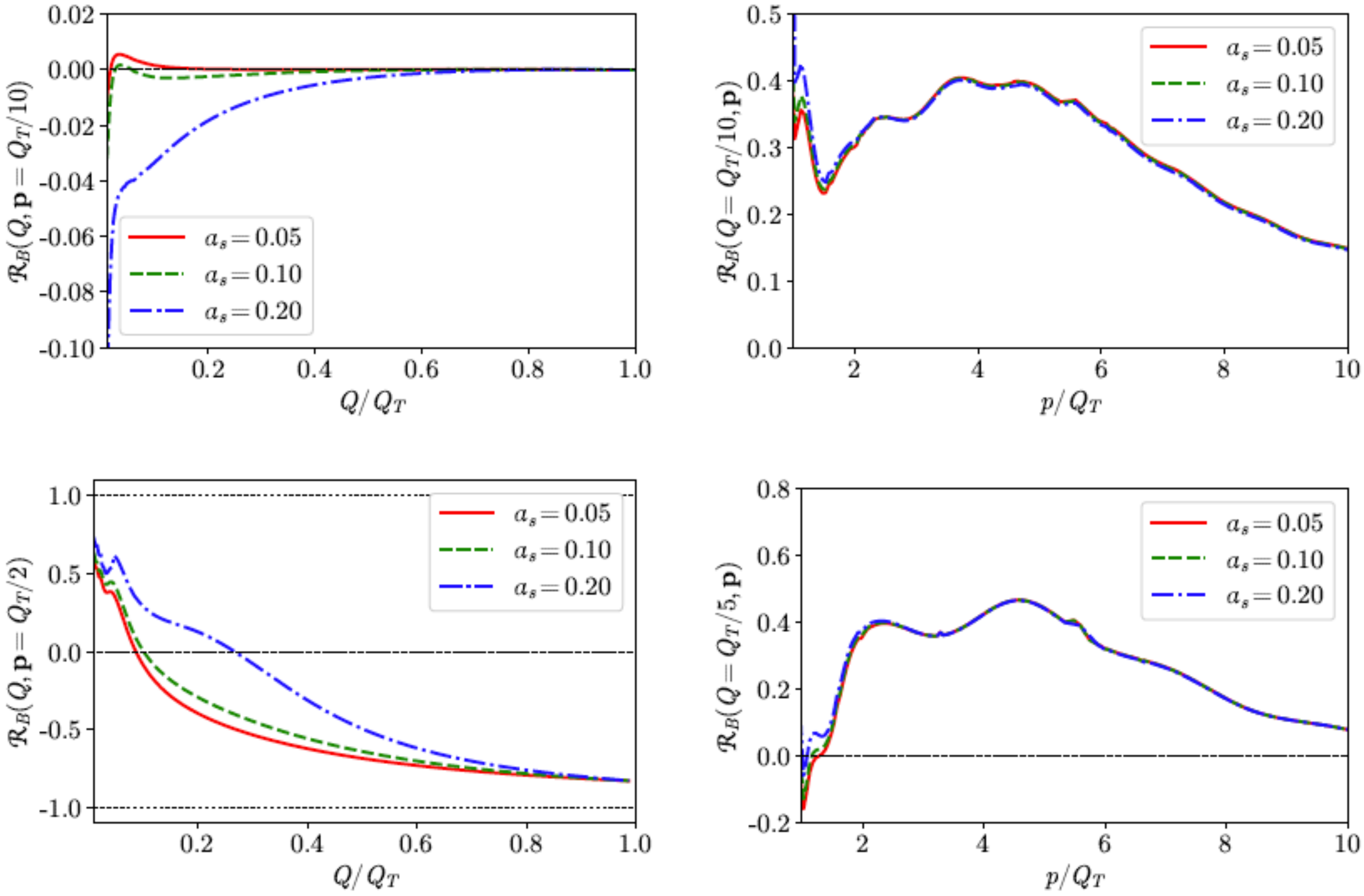


Numerical results: unitarity for B

- Deviation from unitarity for small and relatively large p.

- These deviations are somewhat complementary to those of Δ.

- Not so good independence of α_s.



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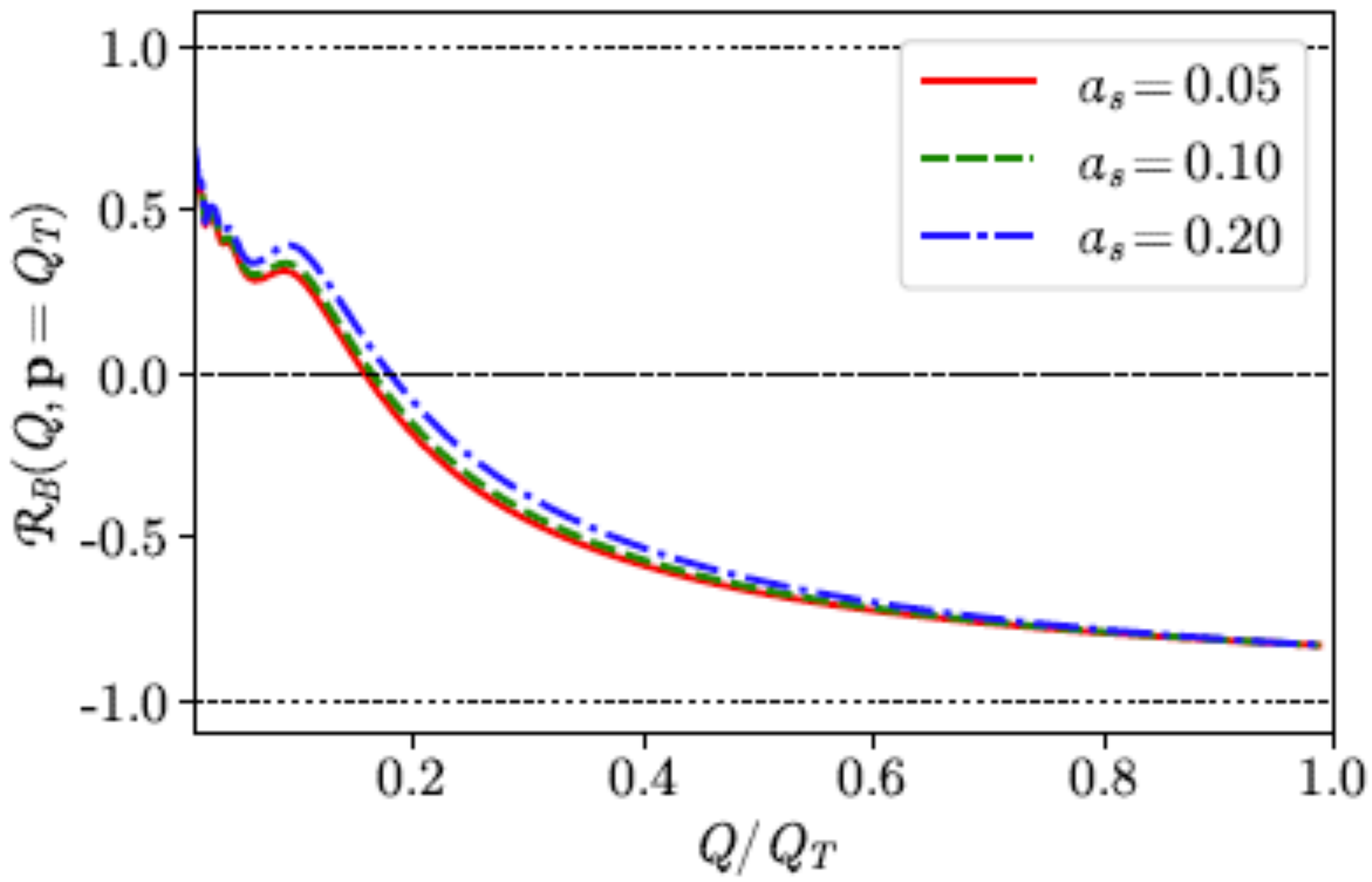
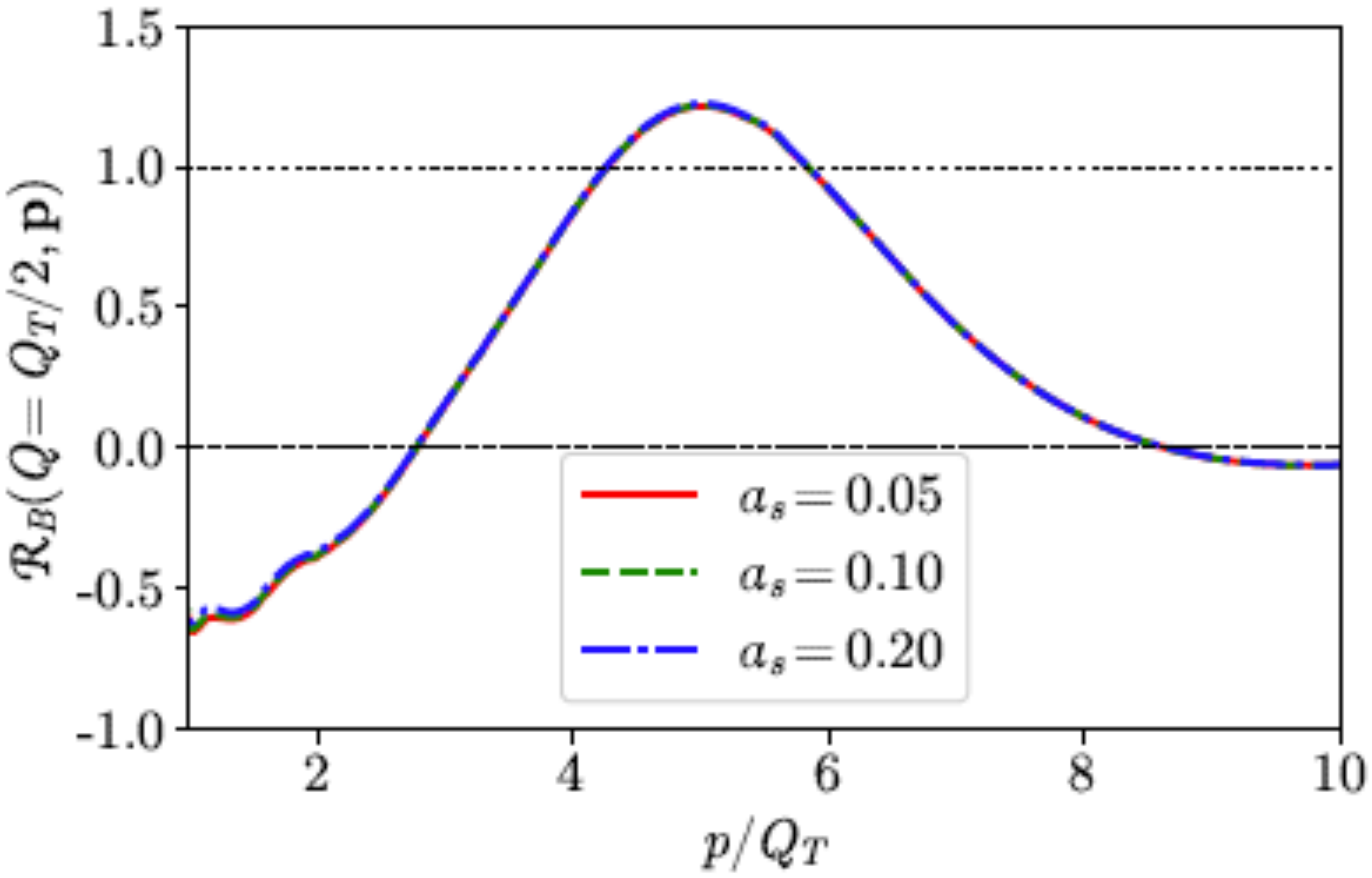
- These deviations are somewhat complementary to those of Δ.

- Not so good independence of α_s.



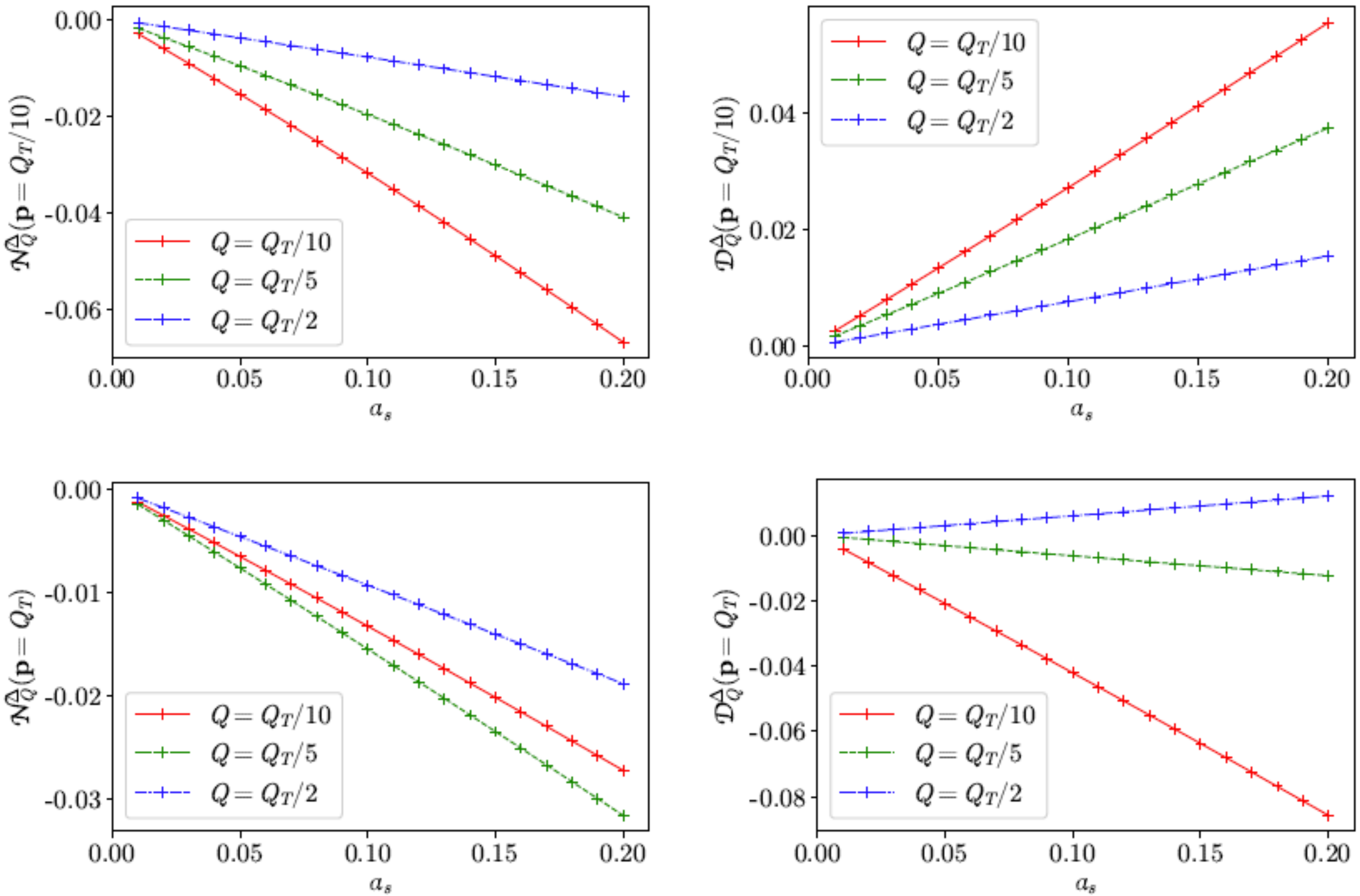


Independence of α_s:

- We plot numerator and denominator versus α_s:

$$\mathcal{R}_\Delta(Q, \mathbf{p}) = \frac{\mathcal{N}_Q^\Delta(\mathbf{p})}{\mathcal{D}_Q^\Delta(\mathbf{p})}$$

- Both linear with α_s up to large values of α_s and quite large evolution.



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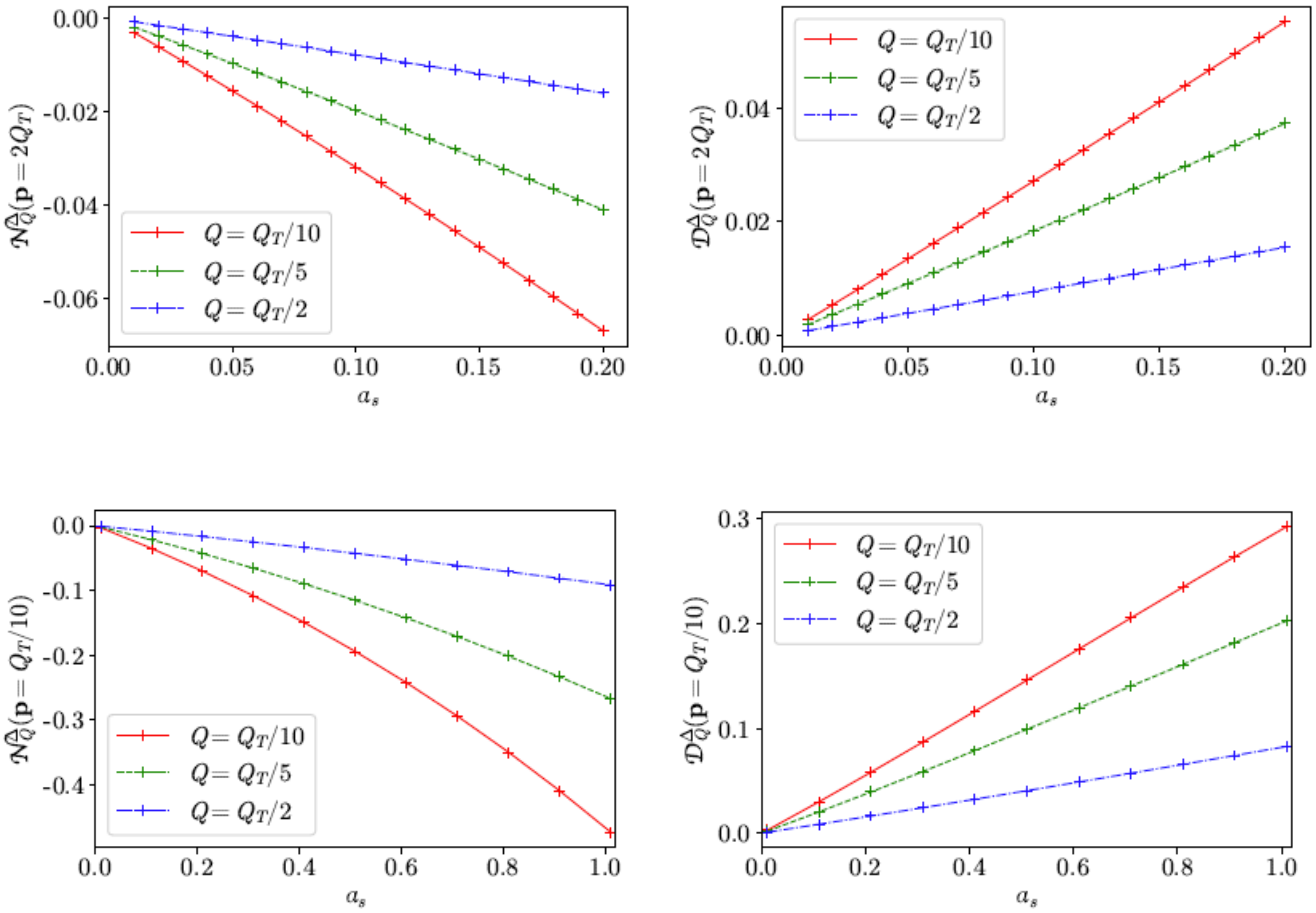

Summary and outlook:

• We have examined numerically the impact of the DGLAP resummation of large transverse logarithms in JIMWLK (2308.15545) on the scattering matrix of a dressed gluon state:

➜ We have restricted ourselves to the $SU(2)$ pure gauge theory.

➜ We have taken the weak field limit but at second order, larger than previously.

➜ We take a single dipole as initial condition.

• We have focused on the deviations from unitarity, and found them significant, as large as the effects of evolution from the initial condition in most of the studied kinematic range.

➜ Their dependence of α_s is quite weak, even when $\alpha_s \ln \frac{Q_T^2}{Q^2} \sim 1$.

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➜ Extension to $SU(3)$.

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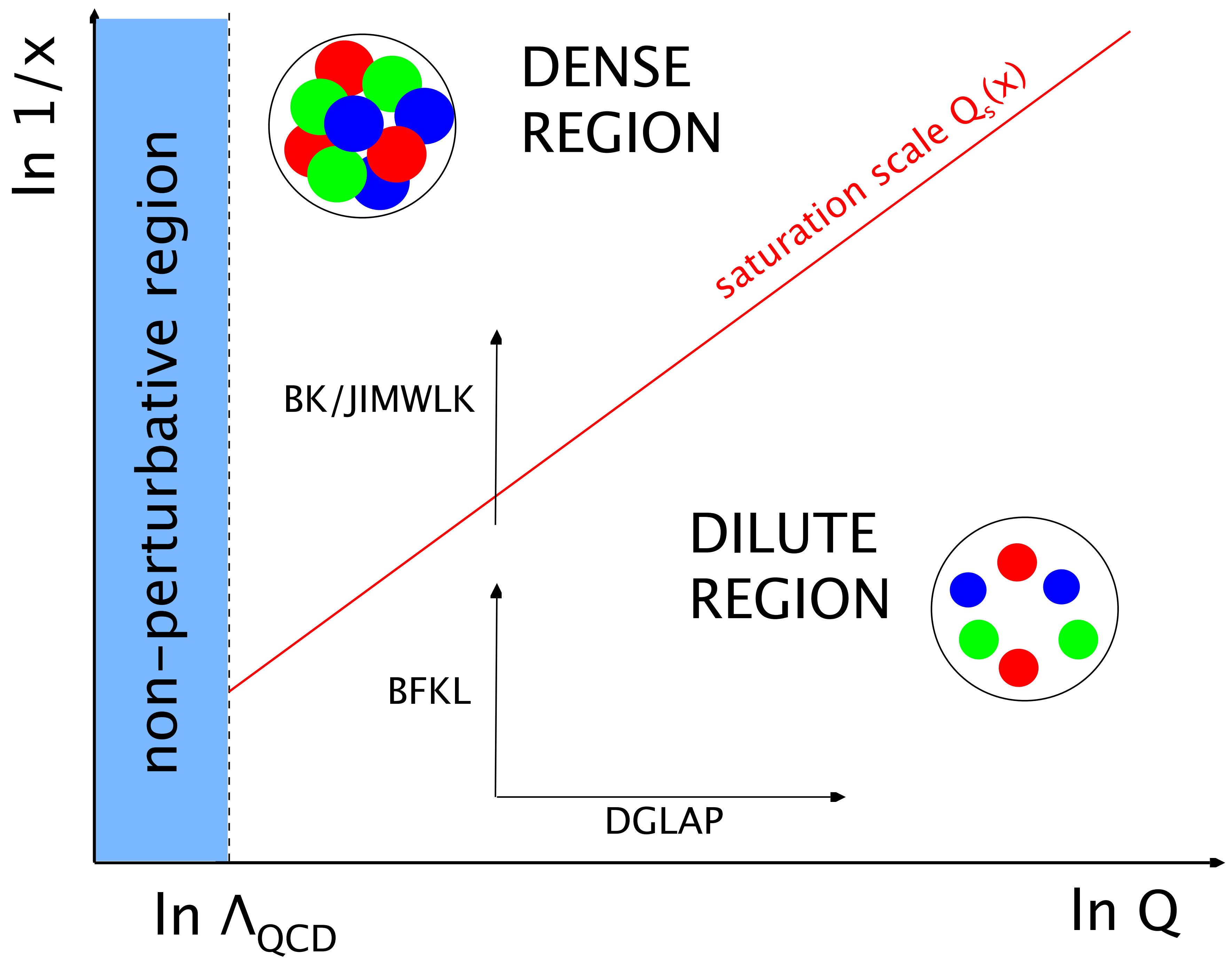
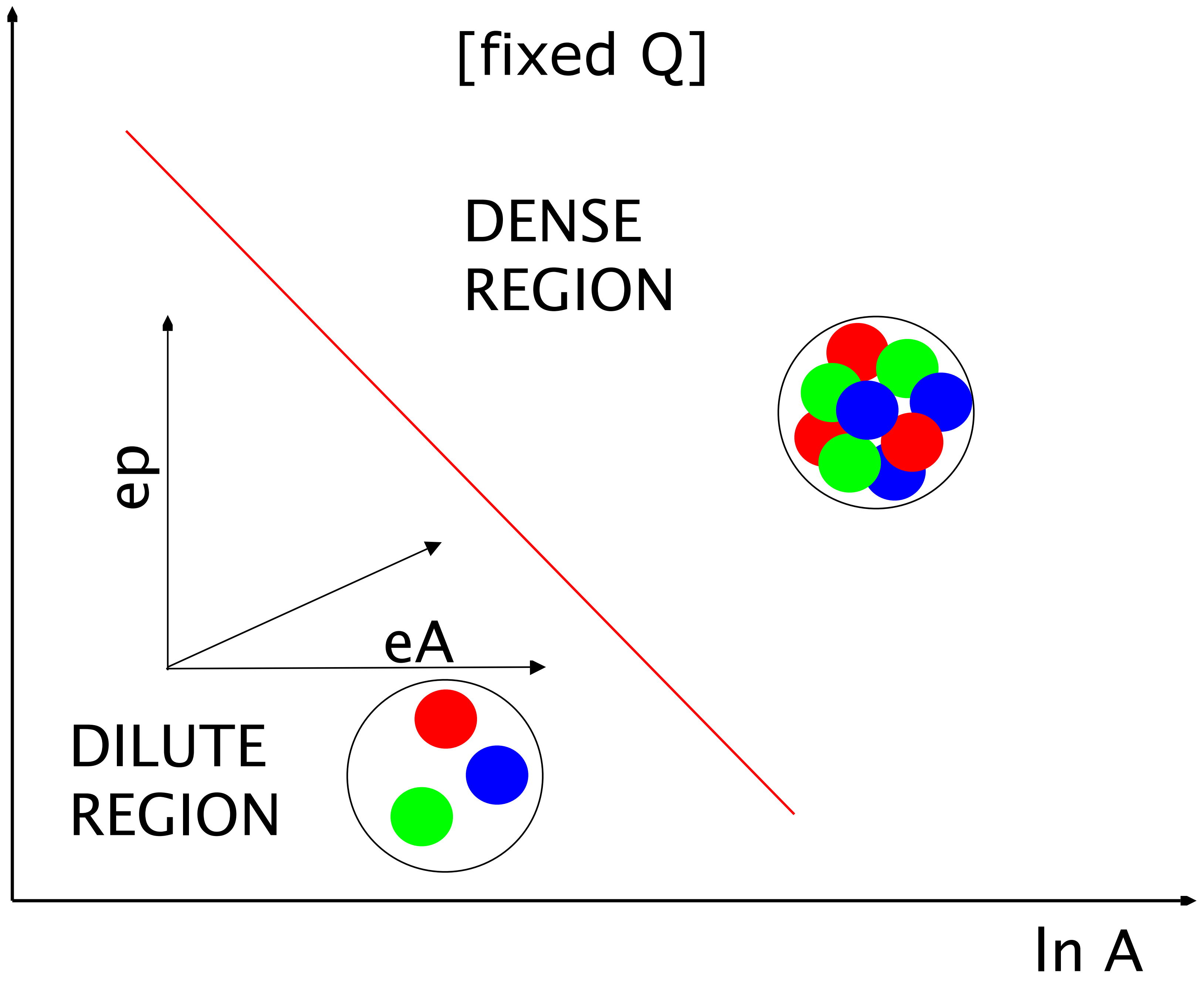
Thank you very much to you all for your attention!!!

Backup

Small x:

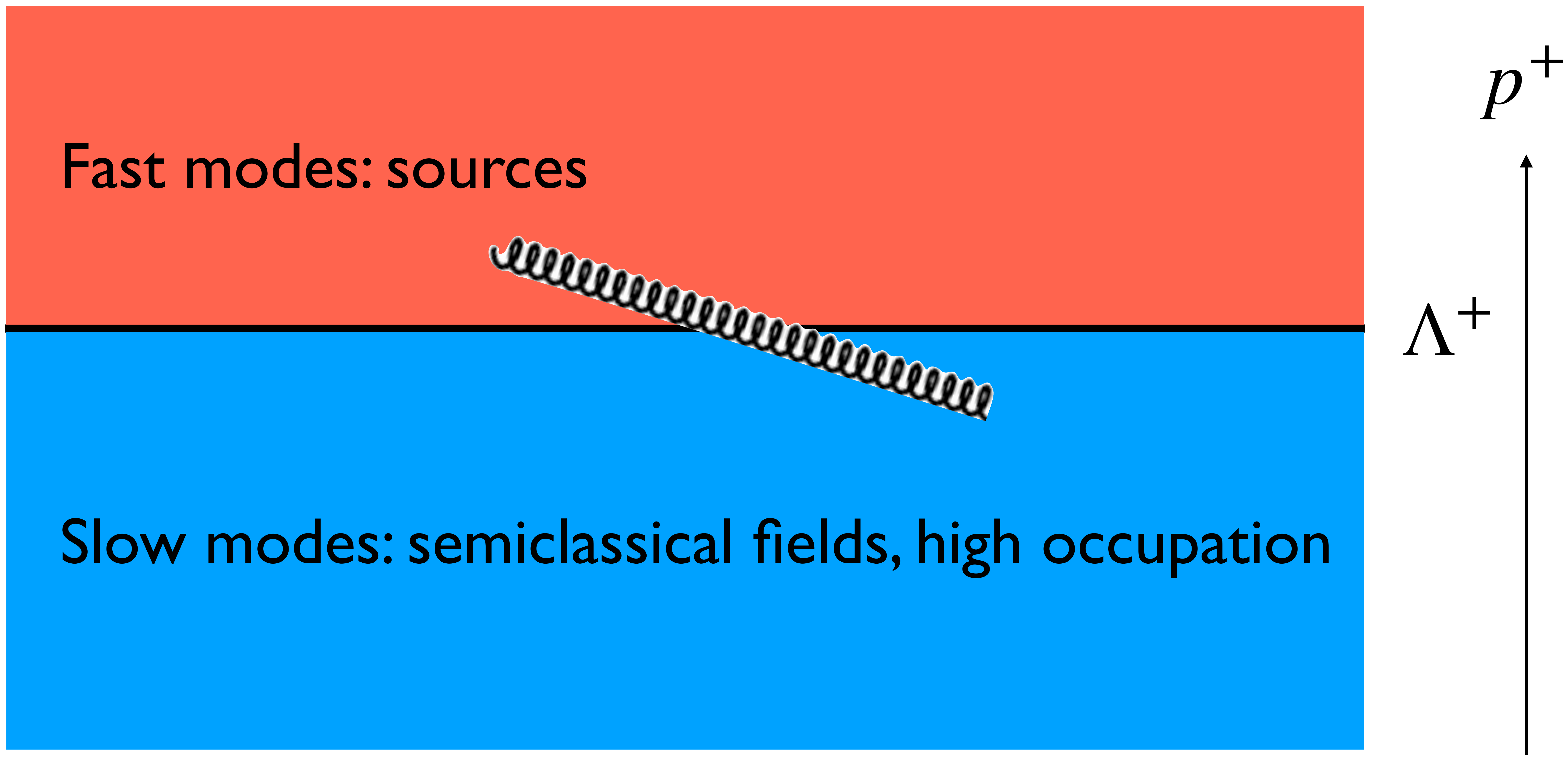
- Standard fixed-order perturbation theory (DGLAP, linear evolution) must eventually fail:
  - ➜ Large logs, e.g., $\alpha_s \ln 1/x \sim 1$: resummation (BFKL,CCFM,ABF,CCSS).
  - ➜ High density ⇒ linear evolution cannot hold: saturation, either perturbative (CGC) or non-perturbative. $$\frac{xG_A(x,Q_s^2)}{\pi R_A^2 Q_s^2} \sim 1 \Longrightarrow Q_s^2 \propto A^{1/3} x^{\sim -0.3}$$
- Non-linear effects driven by density ⇒ 2-pronged approach: ↓x/↑A.





The CGC:

- The CGC is the effective field theory that describes high-energy scattering in QCD in the Regge-Gribov limit (fixed $Q^2, x \to 0$), at weak coupling but non-perturbatively.



- Independence of the physical observables on the cut-off separating fast and slow modes leads to an RG-type equation which, considering scattering of a dilute projectile on a dense target, is JIMWLK, and for ensembles of Wilson lines describing the target results in Balitsky's hierarchy, BK for dipoles at large N_c.