

Maximal Entanglement and Bell Nonlocality at an Electron-Ion Collider

Bo-Wen Xiao

School of Science and Engineering, CUHK-Shenzhen

Wei Qi, Zijing Guo, and BX, [arXiv:2506.12889v1 \[hep-ph\]](#)



High energy QCD: from the LHC to the EIC



The Einstein-Podolsky-Rosen Paradox (1935)

- **The EPR Paper** [Phys. Rev. 47, 777 (1935)]
 - Can Quantum-Mechanical Description of Physical Reality be Considered Complete?
 - Challenge to Copenhagen orthodox interpretation
- **Key Arguments:**
 - Reality criterion: If we can predict with certainty, there must be an element of reality
 - Locality: No instantaneous action at a distance
 - Completeness: QM might be incomplete; Suggested hidden variable theories
- Einstein's famous phrase: **"God does not play dice"**
 - To which Bohr replied: **"Einstein, stop telling God what to do"**

"The EPR paradox revealed the profound nature of quantum entanglement"



Einstein, Podolsky, and Rosen



Quantum Entanglement

■ Quantum Entanglement

- The quintessential phenomenon of QM introduced by Schrödinger in response to the EPR paper.
- Non-local correlations between particles
- Violates local realism assumptions

■ Mathematical Description

- Nonseparability: Entangled states cannot be written as product states
- Example: Spin singlet $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

■ Key Properties

- Measurement of one particle **instantly** affects the outcome of the other
- Cannot be explained by classical physics
- Foundation of quantum information theory



Erwin Schrödinger

**"Spooky action
at a distance"**

- Albert Einstein



ER=EPR Conjecture: Entanglement as Wormhole Geometry

The ER=EPR Conjecture

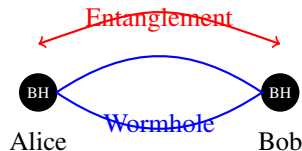
[Maldacena & Susskind, 2013]

**Einstein-Rosen Bridge =
Einstein-Podolsky-Rosen Pair**
Entanglement \Leftrightarrow Wormhole

- **EPR correlations** create geometric connections
- **Wormhole Geometry** is holographic manifestation of entanglement
- **Non-traversable wormhole** - no superluminal signaling
- **Bridge between QM and GR:** unifying general relativity and quantum mechanics into string theory.

Supporting Evidence: Holographic Realization: [Jensen & Karch, 2013]

- EPR pair in AdS_5 space [Xiao, 2008]
- The holographic dual of the EPR pair has two horizons and a string (wormhole) connecting them.



"Entanglement weaves the fabric of spacetime"



Separable vs Entangled States: Two-Qubit Systems

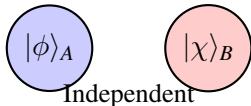
Separable States

- Can be written as: $|\psi\rangle = |\phi\rangle_A \otimes |\chi\rangle_B$
- No quantum correlations

Examples:

$$|00\rangle = |0\rangle_A \otimes |0\rangle_B, \quad |01\rangle = |0\rangle_A \otimes |1\rangle_B$$

$$\begin{aligned} |\psi_{\text{sep}}\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_A \otimes |0\rangle_B \\ &= \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) \end{aligned}$$



Entangled States

- **Cannot** be written as product
- Genuine quantum correlations

Bell States (Maximally Entangled):

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$



Bell's Theorem

■ Quantum Indeterminacy

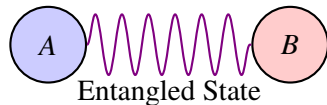
- **Realism:** Quantum indeterminacy reflects our ignorance of hidden variables; outcomes are determined but unknown.
- **Copenhagen:** Indeterminacy is fundamental; outcomes are truly probabilistic until measured.
- **Agnosticism:** The reality behind quantum events is unknowable; only predictive power of the theory matters.

■ Bell Nonlocality [Bell, 1964]

- Bell inequality: It makes an observable difference for Realism vs Copenhagen, and eliminates Agnostic view.
- Decisive evidence supporting QM (Copenhagen).

■ CHSH Inequality [Clauser et al., 1969]

- Generalized Bell inequality
- Foundation for quantum information theory



$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

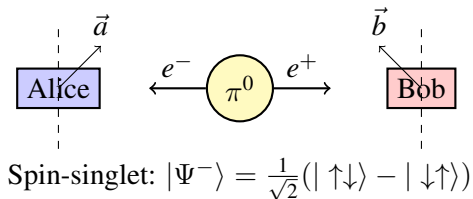


John Stewart Bell



EPRB Experiment: Testing Bell Nonlocality

Einstein-Podolsky-Rosen-Bohm Experiment



Correlation: $E(\vec{a}, \vec{b}) = \langle A(\vec{a}) \cdot B(\vec{b}) \rangle$

Bell CHSH Inequality:

$$\mathbb{B}_{HT} = |E(a, b) - E(a, b') + E(a', b) + E(a', b')| \leq 2$$

$$\mathbb{B}_{QM} = |\cos \theta_{ab} - \cos \theta_{ab'} + \cos \theta_{a'b} + \cos \theta_{a'b'}| \leq 2\sqrt{2}$$

QM violates Bell inequality \Rightarrow Nature is nonlocal!

Local Hidden Variable Theory

- Pre-existing density $P(\lambda)$ for λ
- $A(\vec{a}, \lambda) = \pm 1$ predetermined
- $E(\vec{a}, \vec{b}) = \int P(\lambda) A(\vec{a}, \lambda) B(\vec{b}, \lambda) d\lambda$
- Local realism: $\mathbb{B}_{HT} \leq 2$

Quantum Mechanics

- No predetermined values
- $E(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b} = -\cos \theta_{ab}$
- **Nonlocality:** $2 < \mathbb{B}_{QM} \leq 2\sqrt{2}$
Elementary proof with:
 $\alpha \cos \theta + \beta \sin \theta \leq \sqrt{\alpha^2 + \beta^2}$



Concurrence: Measuring the Degree of Entanglement (Pure States)

Time Reversal Operation flips spins:

- $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$
- $\mathcal{C}(|\psi\rangle) \equiv |\langle\tilde{\psi}|\psi\rangle|$, $|\tilde{\psi}\rangle = -\sigma_y \otimes \sigma_y |\psi^*\rangle$
- [Wootters, 98] flip spins with $\hat{T} = -i\sigma^y \hat{K}$ (Anti-Unitary)
- $|\tilde{\psi}\rangle = \delta^*|00\rangle - \gamma^*|01\rangle - \beta^*|10\rangle + \alpha^*|11\rangle$, the spin-flipped complex conjugate.
- $\mathcal{C}(|\psi\rangle) = 2|\alpha\delta - \beta\gamma|$ measures overlap with time-reversed state.
- $\mathcal{C} = 0$: Separable (no entanglement)
- $0 < \mathcal{C} < 1$: Partially entangled
- $\mathcal{C} = 1$: Maximally entangled

\mathcal{C} = invariance under time reversal

Separable State

$|\psi_1\rangle = |00\rangle$ and $\alpha = 1, \beta = \gamma = \delta = 0$
 $\mathcal{C} = 2|1 \cdot 0 - 0 \cdot 0| = 0$

Bell State (Maximally Entangled)

$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
 $\mathcal{C} = 2|\frac{1}{2} - 0| = 1$

Partially Entangled

$|\psi_2\rangle = \frac{1}{\sqrt{3}}|00\rangle + \sqrt{\frac{2}{3}}|11\rangle$
 $\mathcal{C} = 2|\frac{1}{\sqrt{3}} \cdot \sqrt{\frac{2}{3}}| = \frac{2\sqrt{2}}{3} \approx 0.94$



Spin Density Matrix for Spin-1/2 Particles

Density Matrix Formalism

For a spin-1/2 particle, the density matrix is:

$$\rho = \frac{\mathbb{1}_2 + \vec{n} \cdot \vec{\sigma}}{2}$$

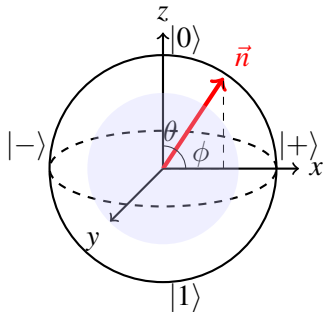
Bloch Vector: $n_i = \langle \sigma_i \rangle = \text{Tr}(\rho \sigma_i)$

- $|\vec{n}| = 1$: Pure state
- $|\vec{n}| < 1$: Mixed state
- $\vec{n} = 0$: Maximally mixed state

For a heavy quark:

- **Production mechanism:** QCD processes determine initial Bloch vectors
- **Experimental access:** Weak decay measures spin projections $\langle \vec{n} \cdot \vec{\sigma} \rangle$

Bloch Sphere Representation



Geometry encodes quantum information

- $\vec{B} = (0, 0, 1)$: $\rho = |0\rangle\langle 0|$
- $\vec{B} = (1, 0, 0)$: $\rho = |+\rangle\langle +|$
- $\vec{B} = (0, 0, 0)$: $\rho = \frac{1}{2}\mathbb{1}_2$ (classical)



Density Matrix and Concurrence for Two-Qubit Systems

Extending to mixed states

Density Matrix Representation:

- Pure state: $\rho = |\psi\rangle\langle\psi|$
- Mixed state: $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$
- General form in computational basis:

$$\rho = \begin{pmatrix} \rho_{00,00} & \rho_{00,01} & \rho_{00,10} & \rho_{00,11} \\ \rho_{01,00} & \rho_{01,01} & \rho_{01,10} & \rho_{01,11} \\ \rho_{10,00} & \rho_{10,01} & \rho_{10,10} & \rho_{10,11} \\ \rho_{11,00} & \rho_{11,01} & \rho_{11,10} & \rho_{11,11} \end{pmatrix}$$

Properties:

- Hermitian: $\rho^\dagger = \rho$
- Trace $\text{Tr}(\rho) = 1$; Non-negative.

Concurrence in general:

[Hill, Wootters, 97; Wootters, 98]

- Define: $\tilde{\rho} = (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y)$
- Compute: $\mathcal{R} = \sqrt{\sqrt{\rho} \tilde{\rho} \sqrt{\rho}}$
- Eigenvalues of \mathcal{R} : $\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ (descending order)

$$\mathcal{C}(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

Example - Werner State:

$$\rho_W = p |\Psi^-\rangle\langle\Psi^-| + \frac{1-p}{4} \mathbb{1}_4$$

- $p = 1$: Pure Bell state
- $\mathcal{C}(\rho_W) = \max\{0, \frac{3p-1}{2}\}$
- Entangled when $p > 1/3$



Separable vs Entangled States and Classical Communication

1. Separable (Unentangled) States:

$$\rho = \sum_{i,j} P(a_i, b_j) \rho_a^i \otimes \rho_b^j$$

- $P(a_i, b_j) \geq 0$ are probabilities and $\sum_{i,j} P(a_i, b_j) = 1$
- ρ_a^i, ρ_b^j are local density matrices
- Peres Horodecki criterion (PPT):
 $\rho^{T_b} = \sum_{i,j} P(a_i, b_j) \rho_a^i \otimes (\rho_b^j)^T$

2. Entangled States:

Cannot be written in separable form

$$\rho \neq \sum_{i,j} P(a_i, b_j) \rho_a^i \otimes \rho_b^j$$

Classical Correlations:

- Preparable by LOCC
"Alice prepares ρ_a^i , tells Bob classically to prepare ρ_b^j "
- No violation of Bell inequalities
- Correlation \neq Entanglement

Quantum Correlations:

- Require entangled resource states
- Can violate Bell inequalities

$$\rho_{\text{ent}} = |\Psi^-\rangle\langle\Psi^-|$$

- Special Quantum correlation.



Spin Density Matrix: Physical Interpretation

The most general two-qubit density matrix:

$$\rho = \frac{1}{4} \left(\mathbb{1}_4 + B_i^+ \sigma^i \otimes \mathbb{1}_2 + B_j^- \mathbb{1}_2 \otimes \sigma^j + C_{ij} \sigma^i \otimes \sigma^j \right)$$

Physical Quantities:

- $B_i^+ = \text{Tr } \rho(\sigma_i \otimes \mathbb{1}_2)$
- $B_j^- = \text{Tr } \rho(\mathbb{1}_2 \otimes \sigma_j)$
- $C_{ij} = \text{Tr } \rho(\sigma_i \otimes \sigma_j)$
Spin correlation /NB: Not $[C]$

Special Case:

For Bell states: $B_i^+ = B_j^- = 0$
(No individual spin polarization)

Bell States & Correlation Matrices:

State	Correlation Matrix
$ \Psi^-\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow\rangle - \downarrow\uparrow\rangle)$	$C_{ij} = \text{diag}(-1, -1, -1)$
$ \Psi^+\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow\rangle + \downarrow\uparrow\rangle)$	$C_{ij} = \text{diag}(1, 1, -1)$
$ \Phi^+\rangle = \frac{1}{\sqrt{2}}(\uparrow\uparrow\rangle + \downarrow\downarrow\rangle)$	$C_{ij} = \text{diag}(1, -1, 1)$
$ \Phi^-\rangle = \frac{1}{\sqrt{2}}(\uparrow\uparrow\rangle - \downarrow\downarrow\rangle)$	$C_{ij} = \text{diag}(-1, 1, 1)$

For singlet state:

$C_{ij} = -\delta_{ij}$ means spins are always anti-parallel.

Correlation matrix C_{ij} fully characterizes entanglement structure for Bell states



Entanglement and Bell Nonlocality Conditions

Starting from the spin density matrix: $\rho_{\alpha\alpha',\beta\beta'} = \frac{1}{4} \left(\mathbb{1}_{\alpha\alpha',\beta\beta'} + C_{ii} \sigma_{\alpha\beta}^i \otimes \sigma_{\alpha'\beta'}^i \right)$

- Anti-correlated spins: $C_{xx}, C_{yy}, C_{zz} < 0$
- Def: $D = (C_{xx} + C_{yy} + C_{zz})/3 = \text{tr}C/3$
- $D = -1$: Perfect anti-correlation

Four eigen values of $\mathcal{R} = \rho$ (since $\tilde{\rho} = \rho$)

$$\begin{aligned}\lambda_1 &= \frac{1}{4}(1 - C_{xx} - C_{yy} - C_{zz}), \\ \lambda_2 &= \frac{1}{4}(1 + C_{xx} + C_{yy} - C_{zz}), \\ \lambda_3 &= \frac{1}{4}(1 + C_{xx} - C_{yy} + C_{zz}), \\ \lambda_4 &= \frac{1}{4}(1 - C_{xx} + C_{yy} + C_{zz}).\end{aligned}$$

Entanglement Condition

- Concurrence $\mathcal{C}[\rho] = \frac{1}{2}(-3D - 1) > 0$:

$$D < -\frac{1}{3}$$

Bell Nonlocality Condition

- For CHSH violation $\mathbb{B} > 2$:
[Horodecki, et al, 95]

$$D < -\frac{1}{\sqrt{2}} \approx -0.707$$

Hierarchy: Bell Nonlocality \subset Entanglement \subset All Quantum States



Top Quark Weak Decay and Spin Transfer

Top Quark Decay: Choose its rest frame

$$t \rightarrow W^+ b \rightarrow \ell^+ \nu_\ell b, \bar{t} \rightarrow W^- \bar{b} \rightarrow \ell^- \bar{\nu}_\ell \bar{b}$$

Decay Spin Density Matrix:

$$\Gamma_\pm = \frac{\mathbb{1}_2 + \kappa_\pm \vec{\sigma}_t \cdot \hat{l}_\pm}{2}$$

Parity Violating Angular Distribution:

$$\frac{d\Gamma}{d\cos\theta} \propto 1 + \kappa_\pm \cos\theta$$

- Weak decay (parity violation) provides **Spin-momentum correlation**
- $\kappa_\pm = \pm 1$ ($t\bar{t}$) **spin analyzing power**
- $\sigma_{l_+ l_-} \propto \text{tr}[\Gamma_+ \otimes \Gamma_- \rho]$ NB $\text{tr}[\sigma^i \sigma^j] = 2\delta^{ij}$

Correlation between di-leptons

$$\frac{d^2\sigma}{\sigma d\Omega_+ d\Omega_-} = \frac{1}{(4\pi)^2} \left[1 - \hat{l}_+ \cdot C \cdot \hat{l}_- \right]$$

Entanglement Signature

$$\langle \cos\varphi \rangle = -\frac{1}{3}D = -\frac{1}{9}\text{Tr}(C)$$

Experimental Reach:

- Extract $D = \text{Tr}(C)/3$ parameter directly
- **Quantum Tomography**: all elements of ρ can be measured. [Bernreuther, Heisler, Si, 15; ATLAS, 1612.07004; CMS, 1907.03729]



First Observation of Quark Entanglement at the LHC

[ATLAS (Nature 2024):] First observation of entanglement in quarks at the highest-energy.

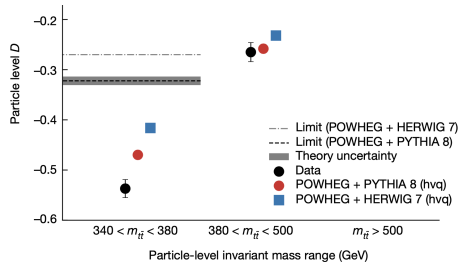
Entanglement Marker:

$$D = \text{tr}[C]/3 = -3\langle \cos \phi \rangle$$

where ϕ is the angle between charged leptons in their parent top/antitop rest frames

Key Features:

- Spin transferred to decay products
- Measured near $t\bar{t}$ threshold
- From atomic physics to high-energy collisions: **A new frontier!**
- **CMS, STAR, BES-III** more to come.



$\sqrt{s} = 13 \text{ TeV}, 140 \text{ fb}^{-1}$ data (2015-2018)

Measured: $D < -1/3$ (Entanglement criterion)
 $D = -0.547 \pm 0.002$ (stat.) ± 0.021 (syst.)

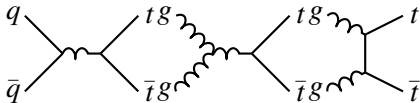
- Observed: $> 5\sigma$ from no entanglement
- **Yet, Bell Nonlocality:** $D < -1/\sqrt{2}$



Theory vs Experiment: Top Quark Entanglement

Quantum State Tomography $\rho_{\alpha\alpha',\beta\beta'} = R_{\alpha\alpha',\beta\beta'} / \text{tr} R$ [Afik, de Nova, 2022]

Top quark pair production



$$R_{\alpha\alpha',\beta\beta'} = \frac{1}{N} \sum \mathcal{M}_{t_\alpha \bar{t}_{\alpha'}}^* \mathcal{M}_{t_\beta \bar{t}_{\beta'}}$$

- **Measured $D \approx -0.54$** near threshold
- **Gluon fusion dominance** at LHC
- **Angular momentum conservation** determines spin correlations
- **Statistical mixture** of $q\bar{q}$ and gg

"Observation of Entanglement but not Bell Nonlocality due to Quark channel mixture"

Near Threshold ($\beta \rightarrow 0$):

- $q\bar{q}$: **Separable state** ($\mathcal{C} = 0$), since $t\bar{t}$ spin (± 1) is equally mixed along beam.
- gg : **Maximally entangled** singlet Ψ^-

High Energy ($\beta \rightarrow 1$) with $\theta = \pi/2$:

- Both channels: Maximally entangled triplet Ψ^+ along \hat{n} with nonzero OAM.

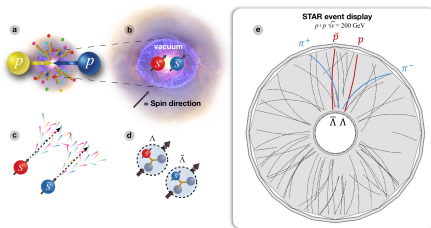
Mixed State at LHC

$$\rho = w_{q\bar{q}} \rho_{q\bar{q}} + w_{gg} \rho_{gg}$$



First evidence of spin correlation in $\Lambda\bar{\Lambda}$ hyperon pairs

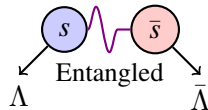
STAR Collaboration [arXiv:2506.05499] with data from $p + p$ collisions at $\sqrt{s} = 200$ GeV



- Relative polarization (same as D): $P_{\Lambda\bar{\Lambda}} = (18 \pm 4)\%$
- Parallel: $1/3$; Antiparallel: -1 ; no spin correlation 0 .
- Short-range pairs show maximal entanglement
- Long-range pairs: correlation vanishes (decoherence)
- Evidence for quantum entanglement in QCD vacuum

"Entanglement: A new paradigm for exploring QCD phenomena"

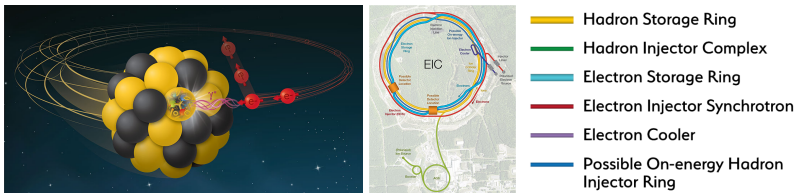
Entanglement as a Tool



- QCD Confinement
- Chiral Symmetry
- Spin Dynamics
- Decoherence
- Bell Nonlocality
- Nuclear Medium !?



EIC Status Update



Aiming to the **start of operation in 2031**, EIC has reached several milestones:

- Five stages of project **Critical Decision** approvals:

- 1 CD-0 Approve Mission Need ✓

January 9, 2020: **EIC CD-0 and site selection** [▶ Link](#)

- 2 CD-1 Approve Alternative Selection and Cost Range ✓

June 29, 2021: **EIC CD1 and start of project execution** [▶ Link](#)

- 3 CD-2 Approve Performance Baseline

- 4 CD-3 Approve Start of Construction

- 5 CD-4 Approve Start of Operations or Project Completion

- RHIC → eRHIC; Energy: 20 → 141 GeV; Luminosity: $10^{34} \text{ cm}^{-2}/\text{s}$; Polarized electron and hadron beams



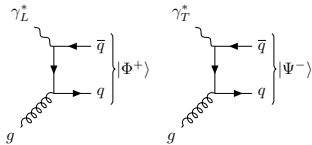
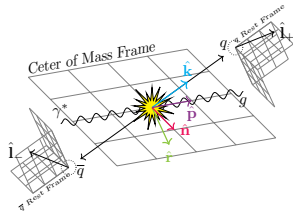
Quark Pair Production in Photon-Gluon Fusion: Longitudinal case

[Qi, Guo, Xiao] [arXiv:2506.12889v1 \[hep-ph\]](https://arxiv.org/abs/2506.12889v1)

Photon-Gluon Fusion Process

$$\gamma_{\lambda=\pm,0}^* + g \rightarrow q + \bar{q}$$

$$\rho_L = \frac{1}{4} (\mathbb{1}_4 + C_{ij} \sigma^i \otimes \sigma^j)$$



For $q\bar{q}$ with $\beta \rightarrow 0$ and $\theta = \frac{\pi}{2}$

Longitudinal photons contribution:

$$C_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\chi_1 & -\chi_2 \\ 0 & -\chi_2 & \chi_1 \end{pmatrix}$$

with
$$\chi_1 = \frac{1 - 2z^2 + z^2\beta^2}{1 - z^2\beta^2}, \quad \chi_2 = \sqrt{1 - \chi_1^2}.$$

■ ρ_L is given by a pure state $= |\Psi\rangle \langle\Psi|$, with

$$|\Psi\rangle = \frac{1}{2} (\sqrt{1 + \chi_1}, i\sqrt{1 - \chi_1}, i\sqrt{1 - \chi_1}, \sqrt{1 + \chi_1}).$$

■ **Near Threshold** ($\beta \rightarrow 0$): $|\Phi^+\rangle$.

■ **High Energy** ($\beta \rightarrow 1$): $|\Phi^+\rangle$.

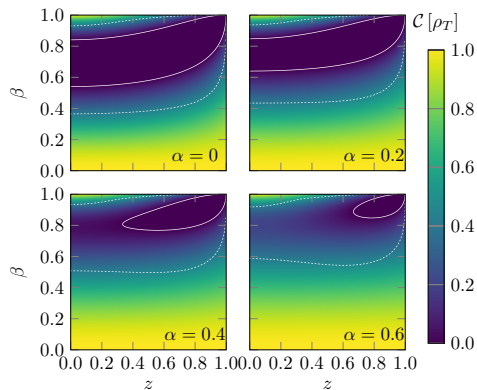
■ $q\bar{q}$ has spin 1 with nonzero OAM and $\mathcal{C}[\rho_L] \equiv 1!$

Always Maximally Entangled! Very Special!



Quark Pair Production in Photon-Gluon Fusion: Transverse case

[Qi, Guo, Xiao] [arXiv:2506.12889v1 \[hep-ph\]](#) **Transverse photons:** similar to $gg \rightarrow q\bar{q}$ channel.



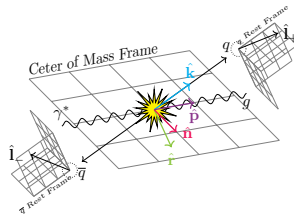
- **Density plots of the concurrence** for transverse photon at EIC as functions of β and $z = \cos \theta$ at given $\alpha \equiv Q^2/\hat{s}$.
- Solid lines (boundaries for entanglement ($C[\rho_T] = 0$)) and dashed lines (boundaries for Bell nonlocality ($\mathcal{N}[\rho_T] = 0$)).
- **Near Threshold ($\beta \rightarrow 0$):**
Maximally entangled singlet Ψ^-
- **High Energy ($\beta \rightarrow 1$) with $\theta = \pi/2$:**
Maximally entangled triplet Φ^- .

Experimental Reach at EIC: Better to have LT separation!

- **Low background** and **Maximal signal** at EIC (including ultra-peripheral collisions).
- Possible measurements: $b\bar{b}$ or $c\bar{c}$ or hyperon $\Lambda\bar{\Lambda}$.



Summary and Outlook



- **Entanglement** and **Bell Nonlocality** are measurable at high energy collisions.
- **EIC** offers a unique and clean experimental environment for measuring entanglement and Bell Nonlocality.
- Using entanglement as a tool to probe **nuclear environment** and other QCD effects.
- **New opportunities** to explore the interplay of quantum information phenomena and high energy and hadronic physics in the years to come.



Celebrating Ian Balitsky's 70th Birthday



Celebrating Ian Balitsky's 70th Birthday

and Ian's Pioneering Contributions to High Energy QCD:

BFKL equation, LO and NLO

BK equation, LO and NLO

A toast to Quantum Chromodynamics and its pioneer!

