Dipole picture diffractive structure function at NLO

T. Lappi

University of Jyväskylä, Finland

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JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

Research Council of Finland

HELSINKI INSTITUTE OF PHYSICS

Outline

Outline of this talk

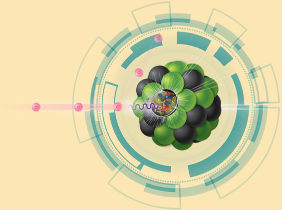
- High energy collisions as eikonal scattering
- Diffractive structure function at NLO

  G. Beuf, T. L., H. Mäntysaari, R. Paatelainen, J. Penttala, arXiv:2401.17251 [hep-ph]
- The $q\bar{q}g$ contribution and the Wüsthoff limit

  G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari, arXiv:2206.13161 [hep-ph]

Process of interest

DIS at high energy

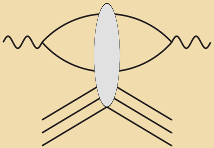


High energy collisions as eikonal scattering

Dipole picture of DIS

Limit of small x, i.e. high γ^*-target energy

Leading order

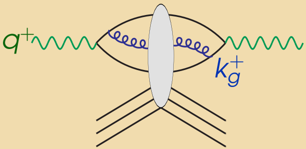


- $\gamma^* \to q\bar{q}$ in vacuum
- $q\bar{q}$ interacts eikonally with target
- σ^{tot} is $2\times$Im-part of amplitude

"Dipole model": Nikolaev, Zakharov 1991

Many fits to HERA data, starting with Golec-Biernat, Wüsthoff 1998

Leading Log: add **soft** gluon



- Soft gluon: large logarithm

$$\int_{x_{Bj}} \frac{dk_g^+}{k_g^+} \sim \ln \frac{1}{x_{Bj}}$$

Absorb into renormalization of target: **BK equation** Balitsky 1995, Kovchegov 1999

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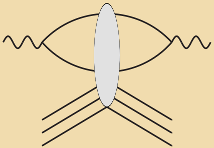

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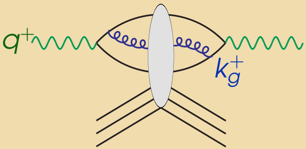

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NLO: the same gluon with full kinematics

Idea of LCPT calculation

- Know free particle Fock states: $|\gamma^*\rangle_0, \quad |q\bar{q}\rangle_0, \quad |q\bar{q}g\rangle_0$ etc.
- **Interacting** states are superpositions of these:

$$|\gamma^*\rangle = (1 + \dots)|\gamma^*\rangle_0 + \psi^{\gamma^* \to q\bar{q}} \otimes |q\bar{q}\rangle_0 + \psi^{\gamma^* \to q\bar{q}g} \otimes |q\bar{q}g\rangle_0 + \dots$$

- Calculate in QM perturbation theory, e.g. ground state $|0\rangle$ wavefunction:

$$\psi^{0 \to n} = \sum_n \frac{\langle n| \hat{V} |0\rangle}{E_n - E_0} + \dots$$

  - Here $1/\Delta E$ is $\sim$ the lifetime of the quantum fluctuation from 0 to n
- In "Energy" E is conjugate to "time", LC time is x^+ $\Longrightarrow$ LC energy k^-
- Note: energy not "conserved"!

Connection to Feynman perturbation theory

- Matrix elements $\langle n| \hat{V} |m\rangle$ are vertices in Feynman rules
- LC energy denominators from propagators, integrating over k^- pole

DIS at NLO: Fock state expansion

$$\left|\gamma^*_\lambda(q^+, \mathbf{q}; Q^2)\right\rangle_D = \sqrt{Z_{\gamma^*_\lambda}} \Bigg\{ \text{Non-QCD Fock states} + \widetilde{\sum_{q_0\bar{q}_1 \text{ F. s.}}} \widetilde{\Psi}_{\gamma^*_\lambda \to q_0\bar{q}_1} \left|q_0\bar{q}_1\right\rangle$$

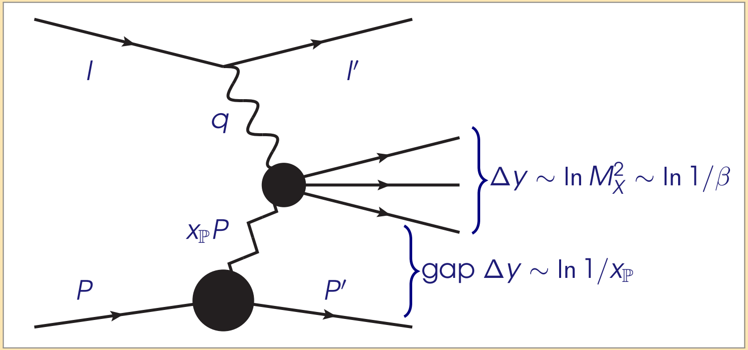
$$+ \widetilde{\sum_{q_0\bar{q}_1 g_2 \text{ F. s.}}} \widetilde{\Psi}_{\gamma^*_\lambda \to q_0\bar{q}_1 g_2} \left|q_0\bar{q}_1 g_2\right\rangle + \cdots \Bigg\},$$

- Non-QCD Fock states: EW corrections, not needed
- Tree level $\widetilde{\Psi}_{\gamma^*_\lambda \to q_0\bar{q}_1}$: LO
- $\widetilde{\Psi}_{\gamma^*_\lambda \to q_0\bar{q}_1}$: known to 1 loop
- $\widetilde{\Psi}_{\gamma^*_\lambda \to q_0\bar{q}_1 g_2}$: tree level

Diffractive DIS

Inclusive diffraction, kinematics

$\gamma^* + A \to X + A$, differential in M_X



- Momentum transfer $t = (P - P')^2$
- Gap size $x_{\mathbb{P}}$, target evolution rapidity $\sim \ln 1/x_{\mathbb{P}}$
- Diffractive system mass M_X^2, $\beta = Q^2/(Q^2 + M_X^2)$
- Virtuality Q^2
- Lower $x_{\mathbb{P}}$ than dijets (e.g. at EIC)

$$x_{Bj} = x_{\mathbb{P}} \beta$$

This talk: $x_{\mathbb{P}}$ small, β not.

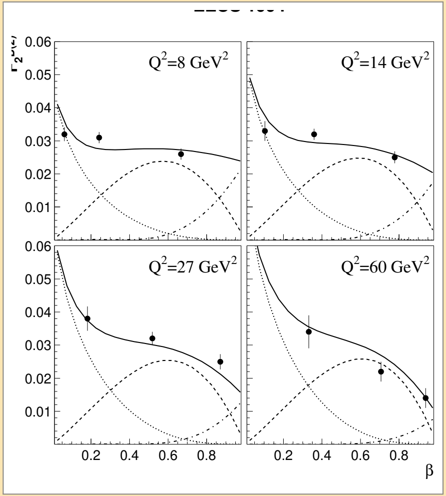
Dependence on β, i.e. M_X

${M_X}^2$ = photon remnants.

Essential regimes:

LO+NLO
- Large $\beta \to 1$ — small M_X: longitudinal $\gamma^* \to q\bar{q}$
- Medium $\beta \sim 0.5$ — ${M_X}^2 \sim Q^2$: transverse $\gamma^* \to q\bar{q}$

NLO
- Small $\beta \ll 1$ — large ${M_X}^2$: higher Fock states ($q\bar{q}g$ etc.)



LO $q\bar{q}$ and leading ln Q^2 $q\bar{q}g$

Golec-Biernat& Wüsthoff hep-ph/9903358

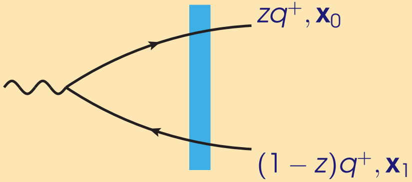
LO diffractive DIS

Diffractive DIS at leading order

- Full kinematics and impact parameter dependence

  G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari, arXiv:2206.13161

$$\frac{d\sigma^{D}_{\lambda, q\bar{q}}}{dM_X{}^2 \, d|t|} = \frac{N_c}{4\pi} \int_0^1 dz \int_{\mathbf{x}_0\mathbf{x}_1\bar{\mathbf{x}}_0\bar{\mathbf{x}}_1} \mathcal{I}^{(2)}_{\mathbf{\Delta}} \mathcal{I}^{(2)}_{M_X}$$
$$\times \sum_{f,h_0,h_1} \left(\widetilde{\psi}_{\gamma^*_\lambda \to q_{\bar{0}}\bar{q}_{\bar{1}}}\right)^\dagger \left(\widetilde{\psi}_{\gamma^*_\lambda \to q_0\bar{q}_1}\right) \boxed{\left[S^\dagger_{\bar{0}\bar{1}} - 1\right]\left[S_{01} - 1\right]}$$



(Blue: shockwave target)

- $q\bar{q}$ crossing shockwave: dipole S_{01}
- "Transfer functions:" relate coordinates at shockwave to:
  - Momentum transfer $t = -\mathbf{\Delta}^2$: $\mathcal{I}^{(2)}_{\mathbf{\Delta}} = \frac{1}{4\pi} J_0\left(\sqrt{|t|}\, \|z\mathbf{x}_{\bar{0}0} - (1-z)\mathbf{x}_{\bar{1}1}\|\right)$
  - Invariant mass $\mathcal{I}^{(2)}_{M_X} = \frac{1}{4\pi} J_0\left(\sqrt{z(1-z)} M_X \|\bar{\mathbf{r}} - \mathbf{r}\|\right)$

LO, recovering known result

We can recover known results Golec-Biernat, Wüsthoff, Marquet et al :

$$x_{\mathbb{P}} F^{\mathrm{D}}_{L,q\bar{q}}\left(\beta, x_{\mathbb{P}}, Q^2\right) = \frac{N_c Q^4}{2\pi^3 \beta} \sum e_f^2 \int \mathrm{d}^2\mathbf{b} \int_0^1 \mathrm{d}z\, z^3 (1-z)^3 Q^2 \Phi_0(z, \beta, Q, \mathbf{b}),$$

$$x_{\mathbb{P}} F^{\mathrm{D}}_{T,q\bar{q}}\left(\beta, x_{\mathbb{P}}, Q^2\right) = \frac{N_c Q^4}{8\pi^3 \beta} \sum e_f^2 \int \mathrm{d}^2\mathbf{b} \int_0^1 \mathrm{d}z\, z^2 (1-z)^2 \left(z^2 - (1-z)^2\right) Q^2 \Phi_1(z, \beta, Q, \mathbf{b}),$$

$$\Phi_n(z, \beta, Q, \mathbf{b}) = \left[\int \mathrm{d}r\, r J_n\left(\sqrt{z(1-z)} M_X r\right) K_n\left(\sqrt{z(1-z)} Q\, r\right) (S_{rb} - 1)\right]^2 .$$

Requires approximations

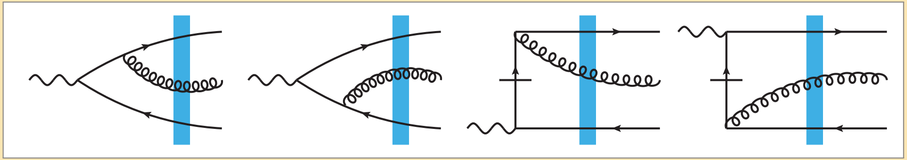
- Dependence on center-of-mass impact parameter $z_0\mathbf{x}_0 + z_1\mathbf{x}_1$ factorizes
- Dipole amplitude does not depend on $\mathbf{b}$, $\mathbf{r}$-angle:
  $\Longrightarrow$ Bessel function index from angular structure of $\gamma^*_\lambda \to q\bar{q}$ vertex

Now to NLO

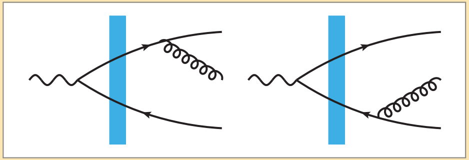
NLO diffractive DIS

NLO radiative corrections

- Emission before target

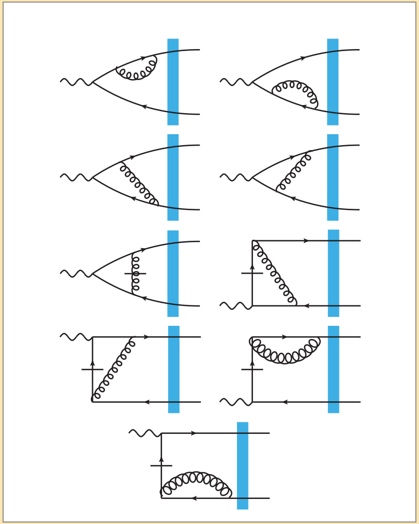


  - Squares already in G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari arXiv:2206.13161
  - Contain leading $\ln Q^2$ contribution
- Emission after target



- Interferences $\Longrightarrow$ simplify with some of the virtual corrections

NLO virtual

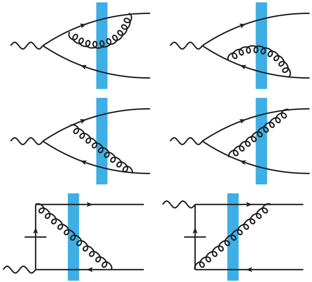


- Vertex corrections:
  known 1-loop $\gamma \to q\bar{q}$ wavefunction

See also Boussarie et al 2014: diffractive jets,

also Caucal et al 2021 inclusive

NLO virtual



- Vertex corrections:
  known 1-loop $\gamma \to q\bar{q}$ wavefunction
- Gluon crosses shockwave, but not the cut:
  - Loop corrections to amplitude, tree level wavefunctions
  - 3-point operator of Wilson lines
  - BK/JIMWLK evolution of LO amplitude

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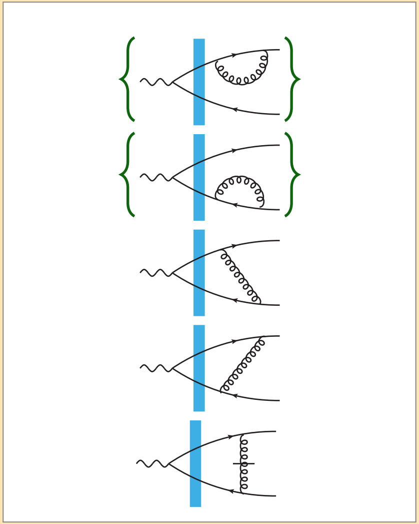
NLO virtual



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- Gluon crosses shockwave, but not the cut:
  - Loop corrections to amplitude,
    tree level wavefunctions
  - 3-point operator of Wilson lines
  - BK/JIMWLK evolution of LO amplitude
- Final state interactions
  (Propagator corrections $\{\}$ $\to$
  State renormalization, in fact = 0 in dim. reg.)

See also Boussarie et al 2014: diffractive jets,
also Caucal et al 2021 inclusive

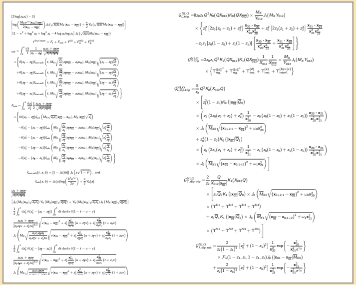
NLO diffractive DIS cross section

Calculation in `2401.17251 [hep-ph]`

Beuf, T.L. , Paatelainen, Mäntysaari, Penttala

We have calculated all these contributions

- Diffractive structure function:
  clean IR-safe, [perturbative = experimental] final state definition M_X!
  (No fragmentation function, jet definition)
  $\Longrightarrow$ Divergences must cancel



(Result: logs, Bessel fn's, polynomials in z_i's)

Features of the calculation:

- Divergence structure
- Treatment of energy denominators
- Collinear factorization limit (Wüsthoff limit)

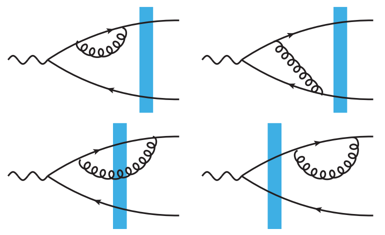
Regularization and divergences

Regularization procedure

- Transverse momentum in $2-2\varepsilon$ dimensions $\Longrightarrow$ $\frac{1}{\varepsilon}$ divergences, collinear or UV
- Longitudinal k^+: cutoff $k^+ > \alpha$, $\alpha \to 0$ $\Longrightarrow$ $1/\alpha$, $\ln^2 \alpha$, $\ln \alpha$ divergences

1. UV $\frac{1}{\varepsilon}$ and $\frac{1}{\varepsilon}\ln\alpha$ divergences:
   $\gamma^* \to q\bar{q}$ vertex, gluon crossing shock, wavefunction renormalization

1. UV $\frac{1}{\varepsilon}$ e.g. in



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2. Collinear $\frac{1}{\varepsilon}$:
   wavef. renormalization, final state emission

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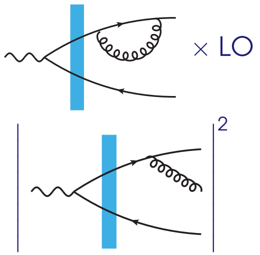

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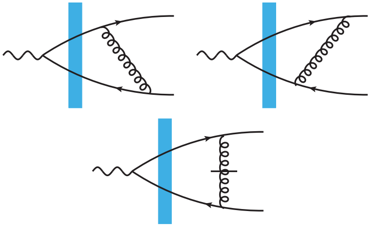
3. $1/\alpha$ e.g. in



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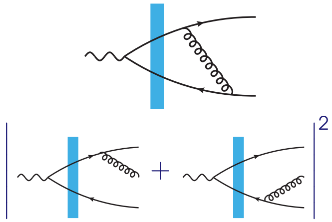
4. $\ln^2 \alpha$ from final state exchange and emission
   (M_X restriction matters here!)

4. $\ln^2 \alpha$ e.g. in



Regularization and divergences

Regularization procedure

- Transverse momentum in $2-2\varepsilon$ dimensions $\implies \frac{1}{\varepsilon}$ divergences, collinear or UV
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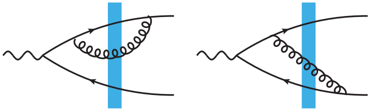
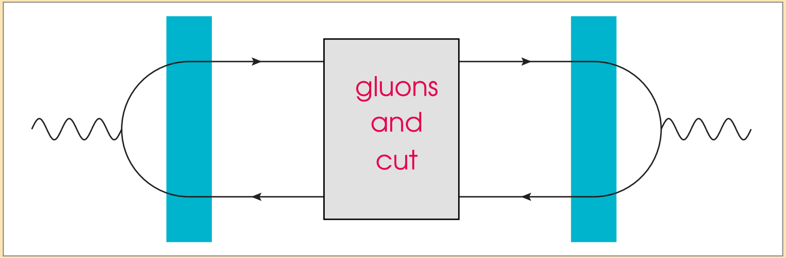
2. Collinear $\frac{1}{\varepsilon}$:
   wavef. renormalization, final state emission
3. $1/\alpha$ cancels between normal and instantaneous exchange
4. $\ln^2\alpha$ from final state exchange and emission
   (M_X restriction matters here!)
5. Remaining $\ln\alpha$ absorbed into BK/JIMWLK

5. $\ln\alpha$ e.g. in



Purely final state corrections

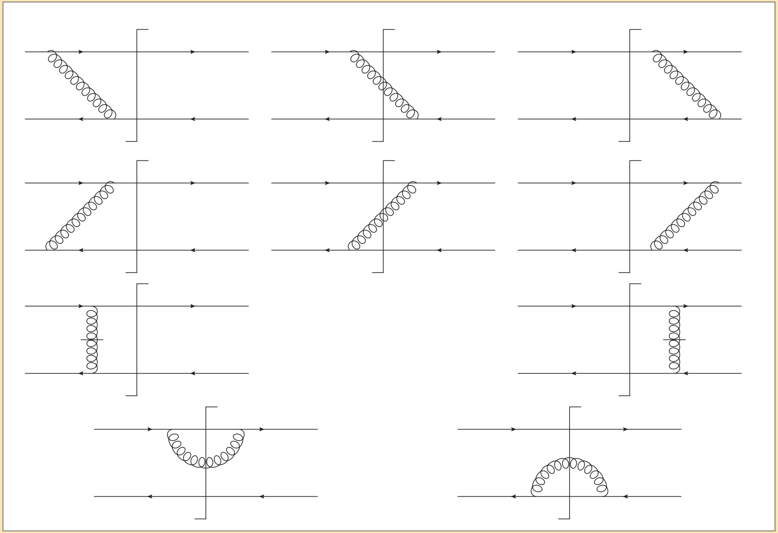
Easier to treat at cross section level



- Only $q\bar{q}$ at shockwave $\Longrightarrow$ dipole S_{01} and $S_{\bar{0}\bar{1}}$
- If fully inclusive, these would cancel, now M_X restriction
- $\mathcal{M} \sim \langle f|(\hat{S}-1)|i=\gamma^*\rangle$, final state $|f\rangle = |q\bar{q}\rangle\,, |q\bar{q}g\rangle$ with $M_{q\bar{q}} = M_X, M_{q\bar{q}g} = M_X$
  LCWF's $\Psi^*_{q\bar{q}\to q\bar{q}}$ and $\Psi^*_{q\bar{q}g\to q\bar{q}}$ $\Longrightarrow$ **Energy denominators from the cut = M_X!**

How to dig out different types of divergences?

(only drawing part between shockwaves)



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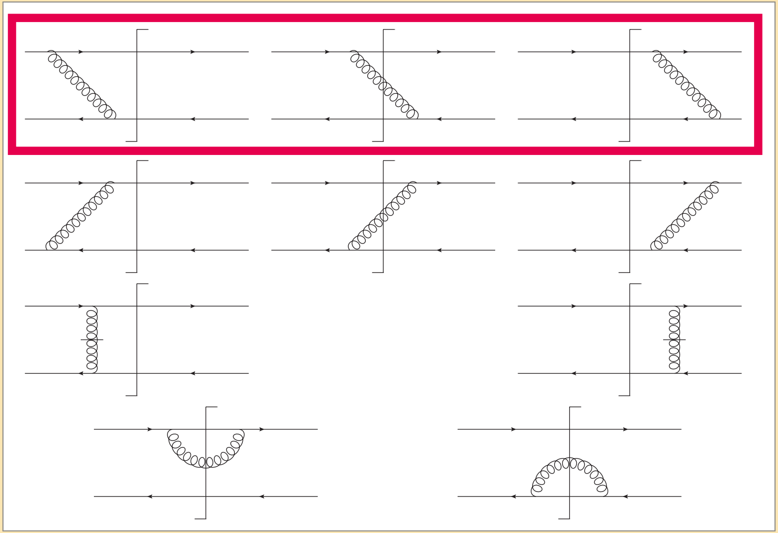
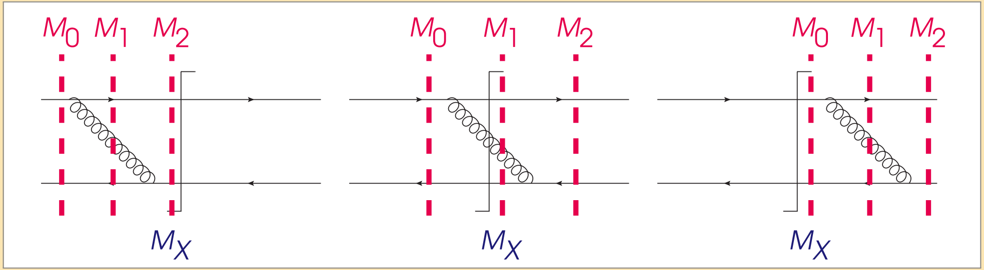


As an example: consider 1st row

Combine energy denominators

"Beuf trick": write M_X delta function as imaginary part of "propagator"



$$\frac{\delta({M_X}^2 - M_2^2)}{(M_2^2 - M_1^2 + i\delta)(M_2^2 - M_0^2 + i\delta)} + \frac{\delta({M_X}^2 - M_1^2)}{(M_1^2 - M_0^2 + i\delta)(M_1^2 - M_2^2 - i\delta)} + \frac{\delta({M_X}^2 - M_0^2)}{(M_0^2 - M_1^2 - i\delta)(M_0^2 - M_2^2 - i\delta)}$$

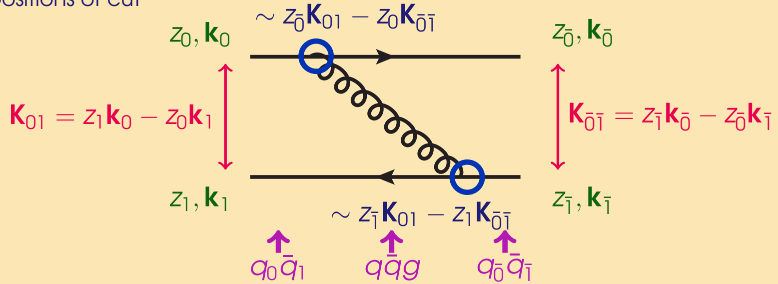
$$= \frac{1}{2\pi i}\left[\frac{1}{({M_X}^2 - M_0^2 - i\delta)({M_X}^2 - M_1^2 - i\delta)({M_X}^2 - M_2^2 - i\delta)} - \text{c.c.}\right]$$

(Note: sign of $i\delta$ essential)

Looks even more complicated than before, but . . .

Numerator

Same for all 3 positions of cut

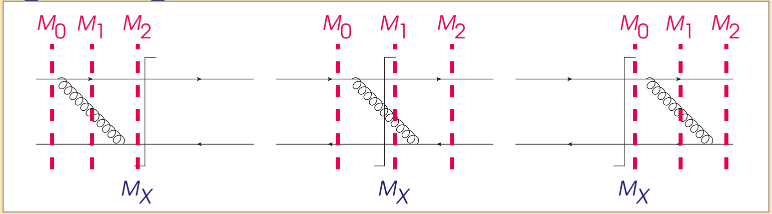


- $\mathbf{K}_{01}, \mathbf{K}_{\bar{0}\bar{1}}$ are conjugate to the dipole size $\Longrightarrow$ integration variables
- Numerator $\sim (z_{\bar{0}}\mathbf{K}_{01} - z_0\mathbf{K}_{\bar{0}\bar{1}}) \cdot (z_{\bar{1}}\mathbf{K}_{01} - z_1\mathbf{K}_{\bar{0}\bar{1}})$
- Invariant masses

$$M^2(q_0\bar{q}_1) = \frac{\mathbf{K}_{01}^2}{z_0 z_1} \quad M^2(q\bar{q}g) = \frac{1}{z_0 - z_{\bar{0}}}\left[\frac{z_{\bar{1}}}{z_1}\mathbf{K}_{01}^2 + \frac{z_0}{z_{\bar{0}}}\mathbf{K}_{\bar{0}\bar{1}}^2 - 2\mathbf{K}_{01}\cdot\mathbf{K}_{\bar{0}\bar{1}}\right] \quad M^2(q_{\bar{0}}\bar{q}_{\bar{1}}) = \frac{\mathbf{K}_{\bar{0}\bar{1}}^2}{z_{\bar{0}} z_{\bar{1}}},$$

- Express numerator as linear combination of invariant masses

Separating divergences



$$\frac{\text{numerator}}{2\pi i}\left[\frac{1}{({M_X}^2 - M_0^2 - i\delta)({M_X}^2 - M_1^2 - i\delta)({M_X}^2 - M_2^2 - i\delta)} - \text{c.c.}\right]$$

Numerator $\sim [\ldots]({M_X}^2 - M_0^2) + [\ldots]({M_X}^2 - M_2^2) + [\ldots]({M_X}^2 - M_1^2) + [\ldots]{M_X}^2$

- Numerator $\sim ({M_X}^2 - M_0^2), \sim ({M_X}^2 - M_2^2)$: divergence $\sim \ln^2 \alpha$
- Numerator $\sim ({M_X}^2 - M_1^2)$: divergence $\sim 1/\alpha$
- Numerator $\sim {M_X}^2$: finite (but most complicated, 3 ED's)

With Beuf trick have separated divergences into different terms

Wüsthoff limit

Large Q^2

G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari, arXiv:2206.13161

Recover "Wüsthoff result" (origin somewhat mysterious)

$$x_{\mathbb{P}} F^{\mathrm{D\,(GBW)}}_{T,q\bar{q}g} = \frac{\alpha_s \beta}{8\pi^4} \sum_f e_f^2 \int_{\mathbf{b}} \int_\beta^1 \mathrm{d}z \left[\left(1 - \frac{\beta}{z}\right)^2 + \left(\frac{\beta}{z}\right)^2 \right] \int_0^{Q^2} \mathrm{d}k^2 k^4 \ln \frac{Q^2}{k^2}$$

$$\times \left[\int_0^\infty \mathrm{d}r r K_2(\sqrt{z} k r) J_2(\sqrt{1-z} k r) \frac{\mathrm{d}\tilde{\sigma}_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}} (\mathbf{b}, \mathbf{r}, x_{\mathbb{P}}) \right]^2 .$$

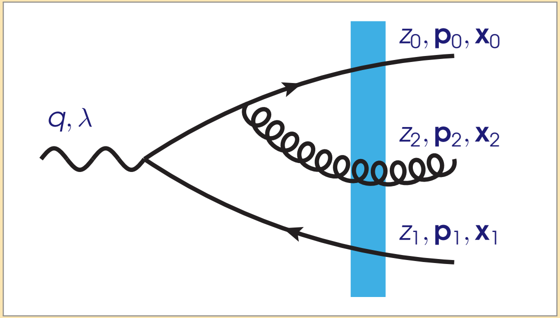
- Explicit $\ln Q^2$
- $g \to q\bar{q}$ DGLAP splitting function: target evolution
- Color-octet small-size $q\bar{q}$ is "effective gluon" $\tilde{g}$ $\Longrightarrow$ adjoint dipole $\Longrightarrow$ diffractive gluon PDF
- J_2, K_2 from curious "effective gluon wavefunction"

$$\psi^{\gamma \to g\tilde{g}} \sim k^i k^j - \frac{1}{2} \mathbf{k}^2 \delta^{ij}$$

Deriving large Q^2 limit: aligned jet limit

Leading large Q^2 from:

- $z_0 \gg z_1 \gg z_2$
- $\mathbf{p}_0^2 \sim \mathbf{p}_1^2 \gg \mathbf{p}_2^2$
- $p_0^- \sim p_1^- \sim p_2^-$
  $\Longrightarrow$ Wüsthoff momentum fractions

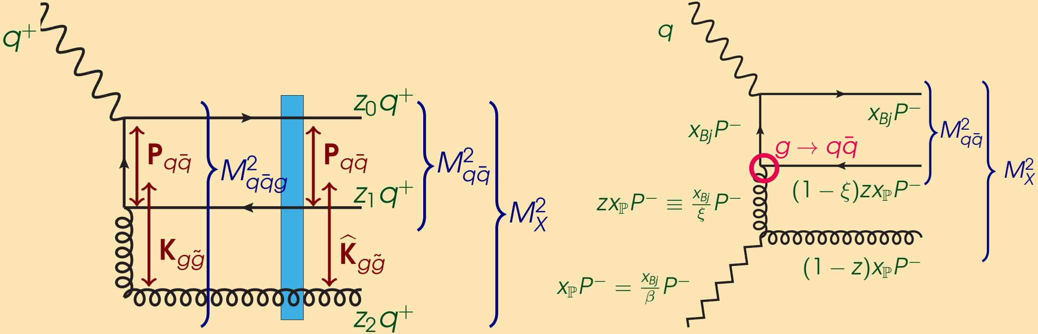


Consistently taking this limit derivation is straightforward:

- q and $\bar{q}$ close: not resolved by target $\Longrightarrow$ point-like "effective gluon" $\tilde{g}$
- Consequently relative momentum of $q\bar{q}$ pair does not change in shockwave only relative momentum of $g\tilde{g}$
- Explicit log from $q\bar{q}$ internal dynamics, $g \to q\bar{q}$ (!) splitting function
- Rank-2 tensor for $\gamma^* \to g\tilde{g}$ Edmond and Dionysios had a much more elegant way!

Deriving large Q^2 limit: matching

Identification with collinear variables via invariant masses



- $\mathbf{K}_{g\tilde{g}}$: before shock, Fourier-transform
- $\widehat{\mathbf{K}}_{g\tilde{g}}$: final state, fixed

$$M^2_{q\bar{q}} \approx \mathbf{P}^2_{q\bar{q}}/z_1 \approx (1/\xi - 1)Q^2$$

$$M^2_{q\bar{q}g} \approx M^2_{q\bar{q}} + \mathbf{K}^2_{g\tilde{g}}/z_2 \qquad M^2_X = (1/\beta - 1)Q^2 \approx M^2_{q\bar{q}} + \widehat{\mathbf{K}}^2_{g\tilde{g}}/z_2$$

Conclusions

- High energy scattering of dilute probe off strong color fields:
  - Target: classical color field
  - Probe: virtual photon,
    develop in a Fock state expansion in Light Cone Perturbation Theory
- Inclusive diffraction at one loop: key piece of EIC physics
- Connection to diffractive PDF/TMD from dipole picture

Thank you