Dijet production in DIS beyond eikonal accuracy

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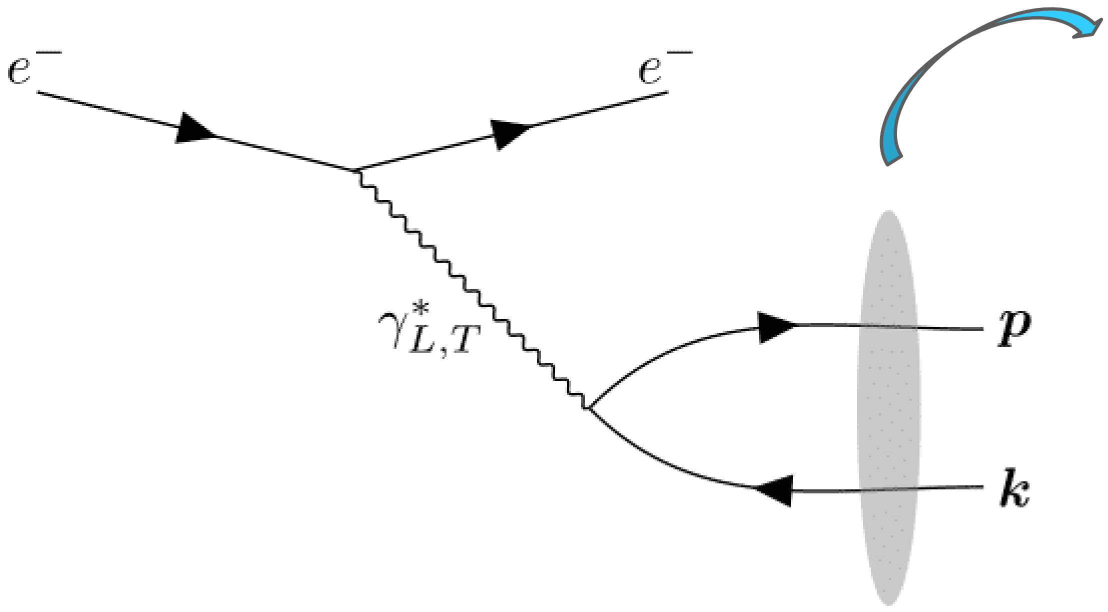
Introduction

- Dijet production in Deep Inelastic Scattering:



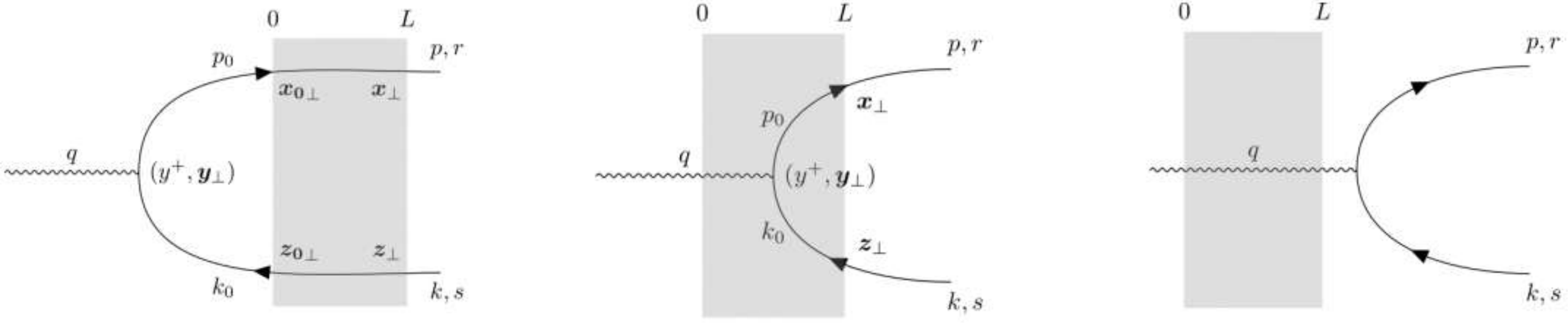
<u>Eikonal approximation:</u>

- Lorentz contraction: Shockwave
- Time dilatation: $A^\mu \neq A^\mu\left(x^-\right)$
- No transverse fields

We focus on the longitudinal photon, whose calculation contains all the ingredients.

Beyond Eikonal

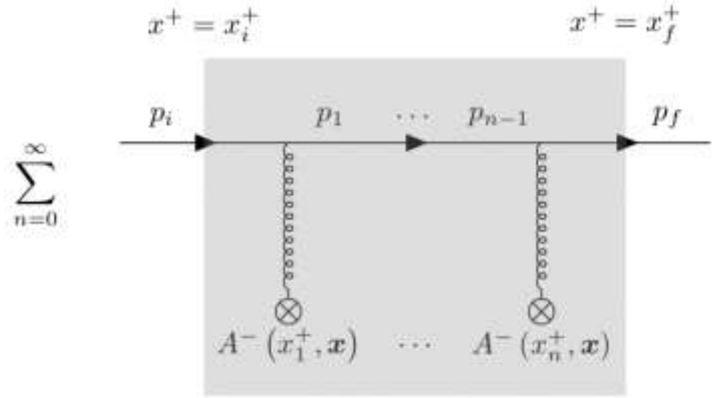
1. The photon can split inside the medium



$$\mathcal{M}_{\gamma^*_{L,T} A \to q\bar{q}+X} = \mathcal{M}_{bef} + \mathcal{M}_{in} + \mathcal{M}_{aft}$$

Beyond Eikonal

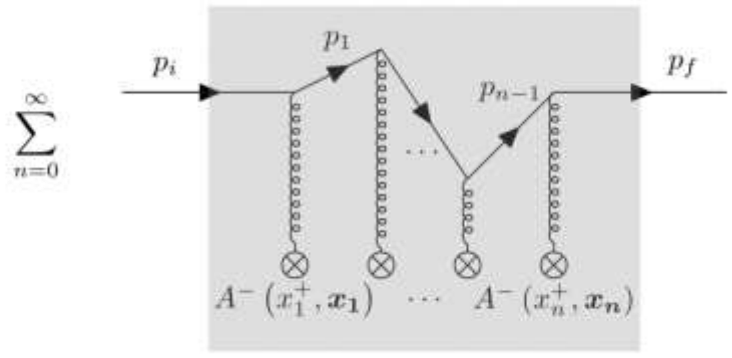
2. We cannot neglect the transverse propagation of the quark or antiquark inside the medium



$$\propto U\left(\mathbf{x}; x_f^+, x_i^+\right) = \mathcal{P}_+ \exp\left[-ig\int_{x_i^+}^{x_f^+} dx^+ A^-\left(x^+, \mathbf{x}\right)\right]$$

B.G. Zakharov.
JETP Lett. 65 (1997) 615-620

R. Baier, Yuri L. Dokshitzer,
Alfred H. Mueller, D. Schiff.
Nucl.Phys.B 531 (1998) 403-425



$$\propto G\left(x_f^+, \mathbf{x_f}; x_i^+, \mathbf{x_i}\right) = \int_{\mathbf{r}\left(x_i^+\right)=\mathbf{x_i}}^{\mathbf{r}\left(x_f^+\right)=\mathbf{x_f}} D\mathbf{r}\left(s^+\right)\ \exp\left(\frac{ip^+}{2}\int_{x_i^+}^{x_f^+} ds^+ \left[\frac{d\mathbf{r}}{ds^+}\right]^2\right) U\left(\mathbf{r}\right)$$

CGC average of the medium charges

1. MV model: $\left\langle A^{-a}\left(x^{+},\mathbf{x}\right)A^{-b}\left(y^{+},\mathbf{y}\right)\right\rangle = g^2 n\left(x^{+}\right)\delta^{ab}\delta\left(x^{+}-y^{+}\right)\gamma\left(\mathbf{x}-\mathbf{y}\right)$
2. Harmonic oscillator approximation (analytical): $C_F g^4 n\left(x^{+}\right)\left[\gamma\left(\mathbf{0}\right)-\gamma\left(\mathbf{r}\right)\right] \approx \frac{Q_s^2}{4L}r^2$

- Dipole: $S^{(2)}_{\mathbf{xy}}\left(t_f^{+},t_i^{+}\right) = \frac{1}{N_c}Tr\left\langle U\left(\mathbf{x}\right)U^{\dagger}\left(\mathbf{y}\right)\right\rangle_{\left(t_f^{+},t_i^{+}\right)} = \exp\left[-\frac{Q_s}{4L}\int_{t_i^{+}}^{t_f^{+}} d\xi^{+}\ \left|\mathbf{x}\left(\xi^{+}\right)-\mathbf{y}\left(\xi^{+}\right)\right|^2\right]$

- Quadrupole: $S^{(4)}_{\mathbf{12\bar{2}\bar{1}}}\left(t_f^{+},t_i^{+}\right) = \frac{1}{N_c}Tr\left\langle U\left(\mathbf{r_1}\right)U^{\dagger}\left(\mathbf{r_2}\right)U\left(\bar{\mathbf{r}}_\mathbf{2}\right)U^{\dagger}\left(\bar{\mathbf{r}}_\mathbf{1}\right)\right\rangle_{\left(t_f^{+},t_i^{+}\right)} = S^{(2)}_{\mathbf{1\bar{1}}}\left(t_f^{+},t_i^{+}\right)S^{(2)}_{\mathbf{2\bar{2}}}\left(t_f^{+},t_i^{+}\right)$

$$+\int_{t_i^{+}}^{t_f^{+}} d\xi^{+}\ S^{(2)}_{\mathbf{1\bar{1}}}\left(t_f^{+},\xi^{+}\right)S^{(2)}_{\mathbf{2\bar{2}}}\left(t_f^{+},\xi^{+}\right)T\left(\xi^{+}\right)S^{(2)}_{\mathbf{12}}\left(\xi^{+},t_i^{+}\right)S^{(2)}_{\mathbf{\bar{1}\bar{2}}}\left(\xi^{+},t_i^{+}\right)+\mathcal{O}\left(1/N_c\right)$$

$$T\left(\xi^{+}\right) = \frac{1}{2}\frac{Q_s^2}{\left(t_f^{+}-t_i^{+}\right)}\left(\mathbf{r_1}-\bar{\mathbf{r}}_\mathbf{1}\right)\cdot\left(\mathbf{r_2}-\bar{\mathbf{r}}_\mathbf{2}\right)$$

Note: HO identical to the GBW model in this context (P. Agostini, T. Altinoluk, N. Armesto, F. Domínguez, J. Guilherme. *Eur.Phys.J.C* 82 (2022) 1001).

L. Apolinário, N. Armesto, J. Guilherme, C. A. Salgado. *JHEP* 02 (2015) 119

J. Blaizot, F. Domínguez, E. Iancu, Y. Mehtar-Tani. *JHEP* 01 (2013) 143

Path Integral Solutions

- $\mathcal{J}\left(\mathbf{u}_f, \mathbf{u}_i; \Delta t^+\right) = \int_{\mathbf{u}_i}^{\mathbf{u}_f} D\mathbf{u} \exp\left\{\int_{t_i^+}^{t_f^+} dt^+ \left[\frac{iq^+}{2}\zeta\left(1-\zeta\right)\dot{u}^2 - \frac{Q_s^2}{4L}u^2\right]\right\} = \frac{1}{2\pi i}\frac{q^+\zeta\left(1-\zeta\right)\Omega}{\sin\left(\Omega\Delta t^+\right)} \exp\left\{i\frac{q^+\zeta\left(1-\zeta\right)\Omega}{2\sin\left(\Omega\Delta t^+\right)}\left[\cos\left(\Omega\Delta t^+\right)\left(u_f^2+u_i^2\right) - 2\mathbf{u}_f\mathbf{u}_i\right]\right\}$

- $\int_{\mathbf{u_i}}^{\mathbf{u_f}} D\mathbf{u} \int_{\mathbf{v_i}}^{\mathbf{v_f}} D\mathbf{v} \ \exp\left\{\int_{t_i^+}^{t_f^+} dt^+ \left[ik^+\dot{\mathbf{u}}\cdot\dot{\mathbf{v}} - \frac{Q_s^2}{4L}u^2\right]\right\} = \left(\frac{k^+}{2\pi i\Delta t^+}\right)^2 \exp\left\{\frac{ik^+}{\Delta t^+}\Delta\mathbf{u}\cdot\Delta\mathbf{v}\right\} \exp\left\{-\frac{Q_s^2}{12L}\Delta t^+\left(u_f^2+u_i^2+\mathbf{u}_f\cdot\mathbf{u}_i\right)\right\}$

After integration of $\mathbf{v_f}, \mathbf{u_i}$ ⇩

$$\mathcal{P}\left(\mathbf{u_f}; \Delta t^+\right) = \exp\left\{-\frac{Q_s^2}{4L}\Delta t^+ u_f^2\right\}$$

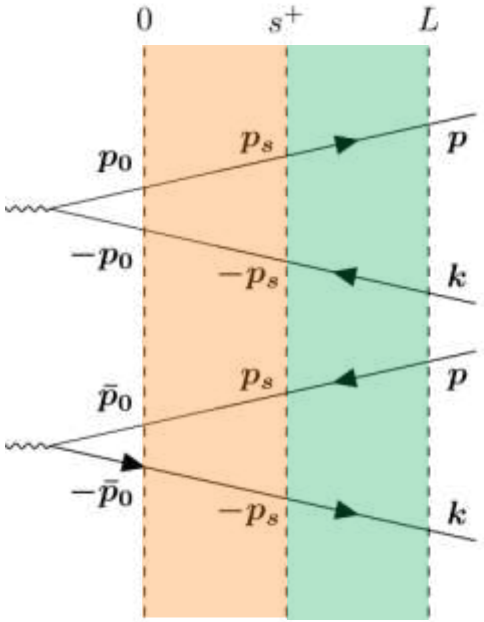
$\Omega^2 = \frac{-iQ_s}{2Lq^+\zeta\left(1-\zeta\right)}$ $\qquad \zeta = \frac{p^+}{q^+}$

Before-Before Contribution

$$\sum_{spin}\sum_{colour}|\mathcal{M}_{bef}|^2 = 2N_c e^2 \frac{\zeta(1-\zeta)}{Q^2} S_\perp \left[\int \frac{d^2\mathbf{p_0}}{(2\pi)^2}\left(\frac{p_0^2-\epsilon^2}{p_0^2+\epsilon^2}\right)^2 \tilde{\mathcal{P}}(\mathbf{p}-\mathbf{p_0};L)\,\tilde{\mathcal{P}}(\mathbf{k}+\mathbf{p_0};L)\right.$$

$$-\frac{Q_s^2}{2L}4\int \frac{d^2\mathbf{p_s}}{(2\pi)^2}(\mathbf{p}-\mathbf{p_s})(\mathbf{k}+\mathbf{p_s})\int_0^L ds^+ \frac{L^2}{\left[Q_s^2(L-s^+)\right]^2}\tilde{\mathcal{P}}(\mathbf{p}-\mathbf{p_s};L-s^+)\,\tilde{\mathcal{P}}(\mathbf{k}+\mathbf{p_s};L-s^+)$$

$$\left.\times\int \frac{d^2\mathbf{p_0}}{(2\pi)^2}\frac{d^2\bar{\mathbf{p}}_0}{(2\pi)^2}\left(\frac{p_0^2-\epsilon^2}{p_0^2+\epsilon^2}\right)\left(\frac{\bar{p}_0^2-\epsilon^2}{\bar{p}_0^2+\epsilon^2}\right)\tilde{\mathcal{J}}(\mathbf{p_s},\mathbf{p_0};s^+)\,\tilde{\mathcal{J}}^*(\mathbf{p_s},\bar{\mathbf{p}}_0;s^+)\right]$$



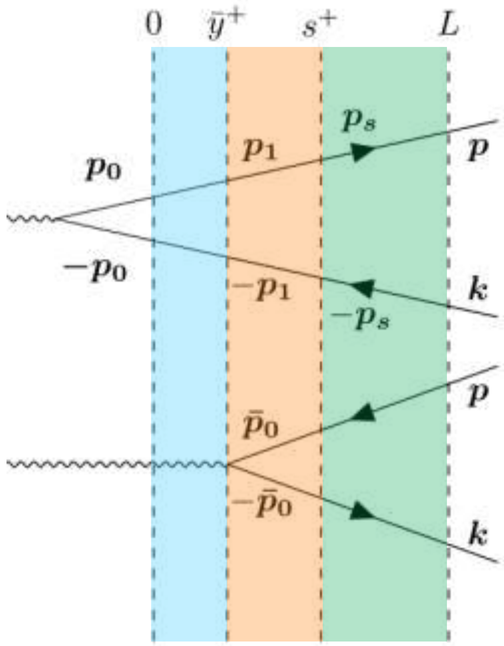
$$\epsilon^2 = Q^2\zeta(1-\zeta)$$

Before-Inside Contribution

$$\sum_{spin}\sum_{colour}\mathcal{M}_{bef}\mathcal{M}^*_{in} = -\frac{2iN_c e^2}{2q^+}\frac{1}{Q^2}S_\perp\int_0^L d\bar{y}^+\; e^{iq^-\bar{y}^+}\int\frac{d^2\bar{\mathbf{p}}_\mathbf{0}}{(2\pi)^2}\left(\bar{p}_0^2-\epsilon^2\right)\int\frac{d^2\mathbf{p_0}}{(2\pi)^2}\frac{p_0^2-\epsilon^2}{p_0^2+\epsilon^2}\Bigg[\tilde{\mathcal{J}}\left(\bar{\mathbf{p}}_\mathbf{0},\mathbf{p_0};\bar{y}^+\right)$$

$$\times\tilde{\mathcal{P}}\left(\mathbf{p}-\bar{\mathbf{p}}_\mathbf{0};L-\bar{y}^+\right)\tilde{\mathcal{P}}\left(\mathbf{k}+\bar{\mathbf{p}}_\mathbf{0};L-\bar{y}^+\right)-\frac{Q_s^2}{2L}4\int\frac{d^2\mathbf{p_s}}{(2\pi)^2}\left(\mathbf{p}-\mathbf{p_s}\right)\left(\mathbf{k}+\mathbf{p_s}\right)\int\frac{d^2\mathbf{p_1}}{(2\pi)^2}\tilde{\mathcal{J}}\left(\mathbf{p_1},\mathbf{p_0};\bar{y}^+\right)$$

$$\times\int_{\bar{y}^+}^L ds^+\frac{L^2}{\left[Q_s^2\left(L-s^+\right)\right]^2}\tilde{\mathcal{P}}\left(\mathbf{p}-\mathbf{p_s};L-s^+\right)\tilde{\mathcal{P}}\left(\mathbf{k}+\mathbf{p_s};L-s^+\right)\tilde{\mathcal{J}}\left(\mathbf{p_s},\mathbf{p_1};s^+-\bar{y}^+\right)\tilde{\mathcal{J}}^*\left(\mathbf{p_s},\bar{\mathbf{p}}_\mathbf{0};s^+-\bar{y}^+\right)\Bigg]$$

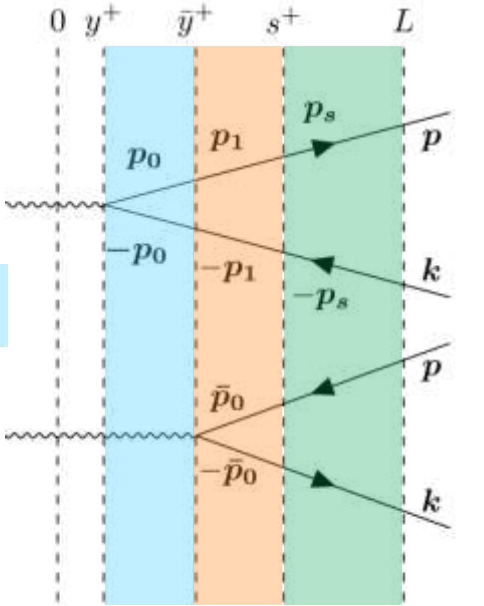


Inside-Inside Contribution

$$\sum_{spin}\sum_{colour}|\mathcal{M}_{in}|^2 = \frac{2N_c e^2}{(2q^+)^2 Q^2\zeta(1-\zeta)} S_\perp 2\mathfrak{Re}\int_0^L dy^+ \int_{y^+}^L d\bar{y}^+ \; e^{-iq^-(y^+-\bar{y}^+)} \int \frac{d^2\mathbf{p_0}}{(2\pi)^2}\frac{d^2\bar{\mathbf{p}}_\mathbf{0}}{(2\pi)^2}\left(p_0^2-\epsilon^2\right)\left(\bar{p}_0^2-\epsilon^2\right)$$

$$\times\Bigg[\tilde{\mathcal{P}}\left(\mathbf{p}-\bar{\mathbf{p}}_\mathbf{0};L-\bar{y}^+\right)\tilde{\mathcal{P}}\left(\mathbf{k}+\bar{\mathbf{p}}_\mathbf{0};L-\bar{y}^+\right)\tilde{\mathcal{J}}\left(\bar{\mathbf{p}}_\mathbf{0},\mathbf{p_0};\bar{y}^+-y^+\right) - \frac{Q_s^2}{2L}4\int\frac{d^2\mathbf{p_s}}{(2\pi)^2}(\mathbf{p}-\mathbf{p_s})(\mathbf{k}+\mathbf{p_s})\int\frac{d^2\mathbf{p_1}}{(2\pi)^2}\tilde{\mathcal{J}}\left(\mathbf{p_1},\mathbf{p_0};\bar{y}^+-y^+\right)$$

$$\times\int_{\bar{y}^+}^L ds^+ \frac{L^2}{\left[Q_s^2(L-s^+)\right]^2}\tilde{\mathcal{P}}\left(\mathbf{p}-\mathbf{p_s};L-s^+\right)\tilde{\mathcal{P}}\left(\mathbf{k}+\mathbf{p_s};L-s^+\right)\tilde{\mathcal{J}}\left(\mathbf{p_s},\mathbf{p_1};s^+-\bar{y}^+\right)\tilde{\mathcal{J}}^*\left(\mathbf{p_s},\bar{\mathbf{p}}_\mathbf{0};s^+-\bar{y}^+\right)\Bigg]$$



After Contributions

$$\sum_{s,r}\sum_{color}\mathcal{M}_{bef}\mathcal{M}^*_{aft} = -\left(2\pi\right)^2\delta^{(2)}\left(\mathbf{p}+\mathbf{k}\right)S_\perp 2N_c e^2\frac{\zeta\left(1-\zeta\right)}{Q^2}e^{-i\frac{Q^2}{2q^+}L}\frac{p^2-\epsilon^2}{p^2+\epsilon^2}\int\frac{d^2\mathbf{p_0}}{\left(2\pi\right)^2}\frac{p_0^2-\epsilon^2}{p_0^2+\epsilon^2}\tilde{\mathcal{J}}\left(\mathbf{p},\mathbf{p_0};L\right)$$

$$\sum_{s,r}\sum_{color}\mathcal{M}_{in}\mathcal{M}^*_{aft} = -\left(2\pi\right)^2\delta^{(2)}\left(\mathbf{p}+\mathbf{k}\right)S_\perp\frac{iN_c e^2}{2q^+}\frac{2}{Q^2}e^{-i\frac{Q^2}{2q^+}L}\frac{p^2-\epsilon^2}{p^2+\epsilon^2}\int\frac{d^2\mathbf{p_0}}{\left(2\pi\right)^2}\left(p_0^2-\epsilon^2\right)\int_0^L dy^+ e^{-iq^-y^+}\tilde{\mathcal{J}}\left(\mathbf{p},\mathbf{p_0};L-y^+\right)$$

$$\sum_{s,r}\sum_{color}\left|\mathcal{M}_{aft}\right|^2 = \left(2\pi\right)^2\delta^{(2)}\left(\mathbf{p}+\mathbf{k}\right)S_\perp N_c\frac{2e^2\zeta\left(1-\zeta\right)}{Q^2}\left(\frac{p^2-\epsilon^2}{p^2+\epsilon^2}\right)^2$$

‘Shockwave ’ expansion

- $Q^2 \frac{L}{q^+} \ll 1$
- $Q_s^2 \frac{L}{q^+} \ll 1$

$$\sum |\mathcal{M}|^2 = \sum |\mathcal{M}|^2_{(0)} + \sum |\mathcal{M}|^2_{(1)} + \sum |\mathcal{M}|^2_{(2)} + \mathcal{O}\left(\frac{L^3}{q^{+3}}\right)$$

+ Correlation limit (hard, back-to-back Jets)

- Relative momentum: $\mathbf{P} = \frac{\mathbf{p} - \mathbf{k}}{2}$
- Imbalance: $\mathbf{q} = \mathbf{p} + \mathbf{k}$

$$q \ll P, \; Q_s \ll P$$

‘Shockwave’ expansion: 0^{th} – Order
(Eikonal)

$$\sum |\mathcal{M}|^2_{(0)} = \frac{2N_c e^2 \zeta(1-\zeta)}{Q^2} 16 \frac{\epsilon^2 P^2}{(P^2+\epsilon^2)^4} \epsilon^2 \left\{ S_\perp \int d^2\mathbf{\Delta_b} \, \frac{e^{-i\mathbf{q}\mathbf{\Delta_b}}}{\Delta_b^2} \left[1 - \exp\left(-\frac{\Delta_b^2 Q_s^2}{2} \right) \right] + \mathcal{O}\left(\frac{Q_s^2}{P^2} \right) \right\}$$

- Hard partonic cross section: $H_{\gamma_L^* g \to q\bar{q}}$
- Weizsäcker-Williams gluon distribution:

Fabio Domínguez, Cyrille Marquet, Bo-Wen Xiao, Feng Yuan. *Phys.Rev.D* 83 (2011) 105005

$$xG^{(1)}(x, q_\perp) = 2 \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P_h^+} e^{ixP_h^+\xi^- - iq_\perp \cdot \xi_\perp} \, Tr \left\langle P_h \left| F^{+i}(\xi^-, \xi) \mathcal{U}^{[+]\dagger} F^{+i}(0) \mathcal{U}^{[+]} \right| P_h \right\rangle$$

‘Shockwave’ expansion: 1^{st} – Order (Next-to-Eikonal)

$$\sum |\mathcal{M}|^2_{(1)} = 0$$

Before taking the Correlation Limit!

Reasons:

- Gaussian distribution of colour sources in the MV model
- Transverse rotational symmetry

Single inclusive gluon production in pA:
(T. Altinoluk, N. Armesto, M. Martínez, C. Salgado. *JHEP* 07 (2014) 068)

‘Shockwave’ expansion: 2nd – Order (Next(2)-to-Eikonal)

$$\sum |\mathcal{M}|^2_{(2)} = \frac{4N_c e^2 \zeta(1-\zeta)}{Q^2} \frac{P^2}{(P^2+\epsilon^2)^4} \frac{P^2}{Q_s^2} \left(\frac{L}{q^+}\right)^2 \left\{ \frac{Q^2}{Q_s^2\zeta(1-\zeta)} S_\perp \int d^2\Delta_b \frac{e^{-iq\Delta_b}}{\Delta_b^{10}} \times \left[1920 - 1920\exp\left(-\frac{\Delta_b^2 Q_s^2}{2}\right) - 960\Delta_b^2Q_s^2 + 240\Delta_b^4Q_s^4 - 40\Delta_b^6Q_s^6 + 5\Delta_b^8Q_s^8 - \frac{1}{2}\Delta_b^{10}Q_s^{10} - \frac{1}{24}\zeta(1-\zeta)\Delta_b^{10}Q_s^8Q^2\right] + \mathcal{O}\left(\frac{Q_s^2}{P^2}\right)\right\}$$

Summary

- We have relaxed the shockwave approximation **to all orders in the medium length** thanks to the path integral formalism.
- We have employed the **MV model** for the medium in order to obtain explicit results.
- Explicit expressions are given in the harmonic oscillator approximation.
- We have studied the correlation limit.
- We recover the known results for the eikonal case.
- We need to go beyond next-to-eikonal when we relax the shockwave approximation to obtain non-zero contributions.

Thank you for your attention!