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DIS Dijets Production at Next-to-eikonal Order

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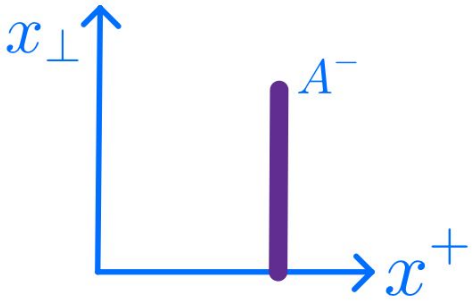
Introduction

- Dijets production in deep inelastic scattering (DIS) is **powerful tool to probe QCD** and study internal structure of hadrons, specially at small-x.
- clean process to study upcoming measurements from Electron-ion Collider (EIC).
- Next-to-eikonal (NEik) corrections generally enhance cross section. NEik corrections to inclusive DIS dijets were computed by *T. Altinoluk et al.(2212.10484)* , *P.Agostini et al. (2403.04603)* (dilute limit).
- Next-to-leading Order (NLO) corrections bring suppression to cross section due to sudakov logarithms. NLO corrections in back to back limit were studied by *P. Caucal et al. (2304.03304)*.
- Furthermore, DIS dijets in back to back limit **allows connecting to TMDs** (*T. Altinoluk (2410.00612)*).
- Hence, there is need to compare NLO and NEik corrections to **address their relative importance.**
- This Work: Next-to-eikonal corrections in the dense limit **:** New target averages of Wilson Lines

Approximations in CGC

Generally in saturation physics in Color Glass Condensate (CGC) framework 2 approximations:

- **Semi-classical approximation:**
  Dense target given by **Strong semi-classical gluon field $A_\mu(x)$~1/g>>1**
- **Eikonal approximation :**
  - Limit of infinite boost of $A_\mu(x)$
  - Only gluons contributes to small-x medium



- Taking into account **only leading power in** terms of high energy : (here, leading order component w.r.t. γ_t)
- Good enough approximation to describe physics at very high energy accelerators.

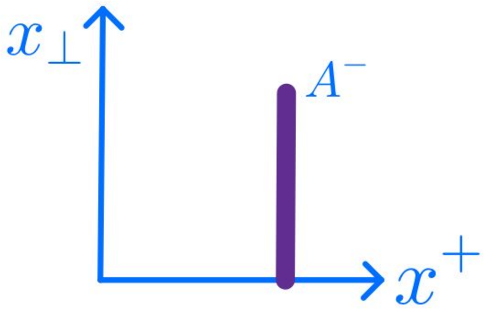
Eikonal Order: For $A_\mu(x)$

Eikonal Order

1. Shockwave approx.: target is localised in the longitudinal direction $x^+ = 0$ (**zero width**).
2. **Only leading - component of target considered**, subleading components are neglected (suppressed by γ_t)
3. Time dilation and static approximation: **x^- dependence of target neglected**

Due to large boost of the target along x^-,
In light-cone coordinate, w.r.t. Lorentz boost factor of target (γ_t)

$$A^- = \mathcal{O}(\gamma_t) >> A^j = \mathcal{O}(1) >> A^+ = \mathcal{O}(1/\gamma_t)$$

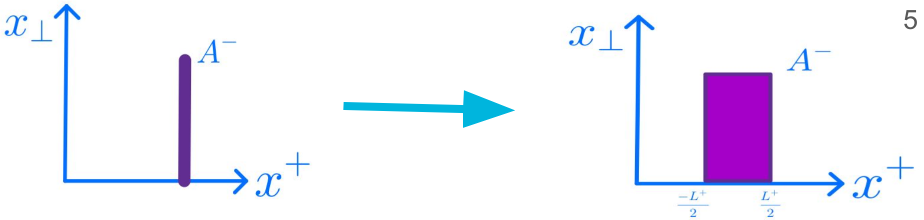


MV model, Gaussian approximation : Gluon Distribution is given as $\langle A^- A^- \rangle$

w.r.t. Lorentz boost factor of target (γ_t)

$$A^- = \mathcal{O}(\gamma_t) >> A^j = \mathcal{O}(1) >> A^+ = \mathcal{O}(1/\gamma_t)$$

Going Beyond Eikonal Order: For $A_\mu(x)$



Eikonal Order

1. Shockwave approx.: target is localised in the longitudinal direction $x^+ = 0$ (**zero width**).
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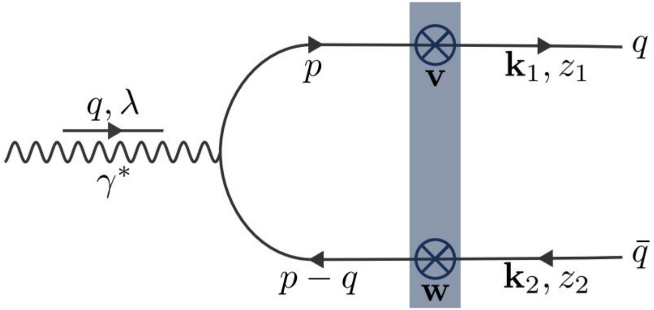
Next-to-eikonal Order

1. Instead of infinite thin shockwave as a target, we consider **finite width** of a target (considered in jet quenching long ago).
2. Include **transverse component** of background field(target), similar to spin physics (*T. Altinoluk et al.*)
3. Consider background field is x^- dependent: **dynamics of the target are considered** (attempts to consider it now in jet quenching by *A. Sadofyev et al.*).

Computations in : *T. Altinoluk et al. (2212.10484), P. Agostini et al. (2403.04603)* etc.

DIS Dijets Production: Eikonal Order

- Virtual photon splits into quark-antiquark pair:
  - Depends upon the polarization of photon (λ):
    - Longitudinal
    - Transverse
  - Interacts with medium eikonally
- The cross section at eikonal order is given as:



P.Agostini et al. (2403.04603 [hep-ph])

$$\left.\frac{d\sigma^{\gamma^*_\lambda + A \to q\bar{q} + X}}{d^2\mathbf{k}_1 d^2\mathbf{k}_2 d\eta_1 d\eta_2}\right|_{\text{Eik.}} = \int_{\mathbf{v},\mathbf{v}',\mathbf{w},\mathbf{w}'} e^{i\mathbf{k}_1\cdot(\mathbf{v}'-\mathbf{v}) + i\mathbf{k}_2\cdot(\mathbf{w}'-\mathbf{w})} \mathcal{C}_\lambda(\mathbf{w}'-\mathbf{v}', \mathbf{w}-\mathbf{v}) \times \left[Q(\mathbf{w}',\mathbf{v}',\mathbf{v},\mathbf{w}) - d(\mathbf{w}',\mathbf{v}') - d(\mathbf{v},\mathbf{w}) + 1 \right],$$

$$\mathcal{C}_L(\mathbf{r}_1,\mathbf{r}_2) = \sum_f \frac{8N_c\alpha_{\text{em}}e_f^2 Q^2}{(2\pi)^6} \delta_z z_1^3 z_2^3 K_0(\epsilon_f|\mathbf{r}_1|) K_0(\epsilon_f|\mathbf{r}_2|),$$

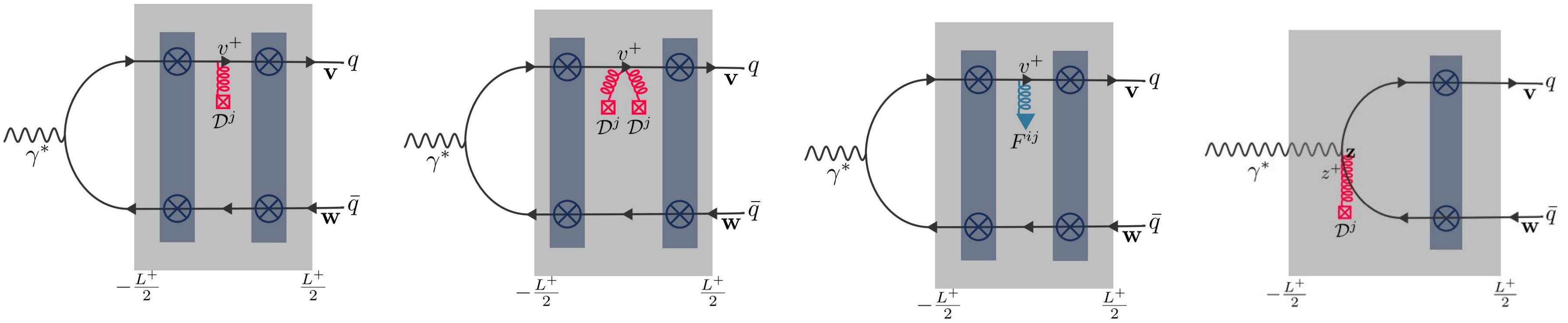
$$\mathcal{C}_T(\mathbf{r}_1,\mathbf{r}_2) = \sum_f \frac{2N_c\alpha_{\text{em}}e_f^2}{(2\pi)^6} \delta_z z_1 z_2 \left[m_f^2 + (z_1^2 + z_2^2)\partial_{\mathbf{r}_1^j}\partial_{\mathbf{r}_2^j} \right] K_0(\epsilon_f|\mathbf{r}_1|) K_0(\epsilon_f|\mathbf{r}_2|).$$

$Q(\mathbf{w}',\mathbf{v}',\mathbf{v},\mathbf{w})$ and $d(\mathbf{w}',\mathbf{v}')$ are quadrupole and dipoles of Wilson lines

$$d(\mathbf{v},\mathbf{w}) = \frac{1}{N_c}\left\langle \text{Tr}[\mathcal{U}(\mathbf{v})\mathcal{U}^\dagger(\mathbf{w})] \right\rangle,$$

$$Q(\mathbf{w}',\mathbf{v}',\mathbf{v},\mathbf{w}) = \frac{1}{N_c}\left\langle \text{Tr}[\mathcal{U}(\mathbf{w}')\mathcal{U}^\dagger(\mathbf{v}')\mathcal{U}(\mathbf{v})\mathcal{U}^\dagger(\mathbf{w})] \right\rangle$$

DIS Dijets Production: Next-to-Eikonal Order



- Two kinds of contributions:
  - When photon splits *before* the medium
  - When photon splits *inside* the medium
- Next-to-eikonal contribution coming due to x^- dependence is not considered

T. Altinoluk et al.(2212.10484 [hep-ph]), P.Agostini et al. (2403.04603 [hep-ph]).

DIS Dijets Production: Next-to-Eikonal Order

- Cross-section is given as:

T. Altinoluk et al.(2212.10484 [hep-ph]), P.Agostini et al. (2403.04603 [hep-ph]).

$$\frac{d\sigma^{\gamma^*_\lambda+A\to q\bar{q}+X}}{d^2\mathbf{k}_1 d^2\mathbf{k}_2 d\eta_1 d\eta_2}\bigg|_{\text{NEik.}} = \frac{1}{q^+}\text{Re}\int_{\mathbf{v},\mathbf{w},\mathbf{v}',\mathbf{w}'} e^{i\mathbf{k}_1\cdot(\mathbf{v}'-\mathbf{v})+i\mathbf{k}_2\cdot(\mathbf{w}'-\mathbf{w})}\mathcal{C}_\lambda(\mathbf{w}'-\mathbf{v}',\mathbf{w}-\mathbf{v})$$
$$\times\Bigg\{\frac{1}{z_1}\left[\frac{\mathbf{k}_2^j-\mathbf{k}_1^j}{2}+\frac{i}{2}\partial_{\mathbf{w}^j}\right]\left[Q_j^{(1)}(\mathbf{w}',\mathbf{v}',\mathbf{v}_*,\mathbf{w})-d_j^{(1)}(\mathbf{v}_*,\mathbf{w})\right]-\frac{i}{z_1}\left[Q^{(2)}(\mathbf{w}',\mathbf{v}',\mathbf{v}_*,\mathbf{w})-d^{(2)}(\mathbf{v}_*,\mathbf{w})\right]$$
$$-\frac{1}{z_2}\left[\frac{\mathbf{k}_2^j-\mathbf{k}_1^j}{2}-\frac{i}{2}\partial_{\mathbf{v}^j}\right]\left[Q_j^{(1)}(\mathbf{v}',\mathbf{w}',\mathbf{w}_*,\mathbf{v})^\dagger-d_j^{(1)}(\mathbf{w}_*,\mathbf{v})^\dagger\right]-\frac{i}{z_2}\left[Q^{(2)}(\mathbf{v}',\mathbf{w}',\mathbf{w}_*,\mathbf{v})^\dagger-d^{(2)}(\mathbf{w}_*,\mathbf{v})^\dagger\right]\Bigg\}$$
$$+\,\delta^{\lambda T}\frac{d\sigma^{\text{trans.}}}{d\Pi},$$

Where,

$$d_j^{(1)}(\mathbf{v}_*,\mathbf{w}) = \frac{1}{N_c}\left\langle\text{Tr}\left[\mathcal{U}_j^{(1)}(\mathbf{v})\mathcal{U}^\dagger(\mathbf{w})\right]\right\rangle,\qquad Q_j^{(1)}(\mathbf{w}',\mathbf{v}',\mathbf{v}_*,\mathbf{w}) = \frac{1}{N_c}\left\langle\text{Tr}\left[\mathcal{U}(\mathbf{w}')\mathcal{U}^\dagger(\mathbf{v}')\mathcal{U}_j^{(1)}(\mathbf{v})\mathcal{U}^\dagger(\mathbf{w})\right]\right\rangle,$$
$$d^{(2)}(\mathbf{v}_*,\mathbf{w}) = \frac{1}{N_c}\left\langle\text{Tr}\left[\mathcal{U}^{(2)}(\mathbf{v})\mathcal{U}^\dagger(\mathbf{w})\right]\right\rangle,\qquad Q^{(2)}(\mathbf{w}',\mathbf{v}',\mathbf{v}_*,\mathbf{w}) = \frac{1}{N_c}\left\langle\text{Tr}\left[\mathcal{U}(\mathbf{w}')\mathcal{U}^\dagger(\mathbf{v}')\mathcal{U}^{(2)}(\mathbf{v})\mathcal{U}^\dagger(\mathbf{w})\right]\right\rangle,$$

and,

$$\mathcal{U}_j^{(1)}(\mathbf{z}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\,\mathcal{U}_{[\frac{L^+}{2},v^+]}(\mathbf{z})\overleftrightarrow{D_{\mathbf{z}^j}}(v^+)\mathcal{U}_{[v^+,-\frac{L^+}{2}]}(\mathbf{z}),$$
$$\mathcal{U}^{(2)}(\mathbf{z}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\,\mathcal{U}_{[\frac{L^+}{2},v^+]}(\mathbf{z})\overleftarrow{D_{\mathbf{z}^j}}(v^+)\overrightarrow{D_{\mathbf{z}^j}}(v^+)\mathcal{U}_{[v^+,-\frac{L^+}{2}]}(\mathbf{z}).$$

Decorated Wilson lines

$$\overrightarrow{D_{\mathbf{z}}^j}(z^+) = \overrightarrow{\partial_{\mathbf{z}^j}} - ig\mathbf{A}^j(z^+,\mathbf{z}),\qquad \overleftarrow{D_{\mathbf{z}^j}}(z^+) = \overleftarrow{\partial_{\mathbf{z}^j}} + ig\mathbf{A}^j(z^+,\mathbf{z})$$
$$\overleftrightarrow{D_{\mathbf{z}^j}}(z^+) = \partial_{\mathbf{z}^j} - \overleftarrow{\partial_{\mathbf{z}^j}} - 2ig\mathbf{A}^j(z^+,\mathbf{z})$$

Going to numbers : Dilute Limit, Eikonal

- The case when $\mathbf{k}_1$, $\mathbf{k}_2$ >> Qs
- Physics is perturbative and Wilson lines are approximated as

$$\mathcal{U}_{[x^+,y^+]}(\mathbf{z}) = 1 - ig\int_{y^+}^{x^+} dz^+ A^-(z^+,\mathbf{z}) - g^2\int_{y^+}^{x^+} dz_1^+ \int_{y^+}^{z_1^+} dz_2^+ A^-(z_1^+,\mathbf{z})A^-(z_2^+,\mathbf{z}) + \mathcal{O}(g^3),$$

- Only two gluon exchanges

P.Agostini et al. (2403.04603 [hep-ph])

- Dipole can be expressed as $d(\mathbf{x},\mathbf{y}) = 1 - \frac{g^2}{N_c}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} z^+ \int_{-\frac{L^+}{2}}^{z^+} dz_1^+$

$$\times \mathrm{Tr}\Big[\langle A^-(z^+,\mathbf{x})A^-(z_1^+,\mathbf{x})\rangle + \langle A^-(z_1^+,\mathbf{y})A^-(z^+,\mathbf{y})\rangle - 2\langle A^-(z^+,\mathbf{x})A^-(z_1^+,\mathbf{y})\rangle\Big].$$

- Homogeneous target, correlators are given as: $\frac{g^2}{N_c}\langle \mathrm{Tr}\left[A^\mu(\vec{x})\, A^\nu(\vec{y})\right]\rangle = 2\pi Q_s^2 \frac{\delta(x^+ - y^+)}{L^+} G^{\mu\nu}(\mathbf{x}-\mathbf{y})$
- Dipole is given as : $d(\mathbf{x},\mathbf{y}) = 1 - 2\pi Q_s^2\left[G^{--}(0) - G^{--}(\mathbf{x}-\mathbf{y})\right]$, where, $G^{--}(\mathbf{r}) = \int_{\mathbf{P}} e^{i\mathbf{P}\cdot\mathbf{r}}\frac{1}{\mathbf{P}^4}$
- At Eik order,

$$\frac{d\sigma^{\gamma_L^*+A\to q\bar{q}+X}}{d^2\mathbf{k}_1 d\eta_1 d^2\mathbf{k}_2 d\eta_2} = \frac{8N_c\alpha_{\mathrm{em}}e_f^2 Q_s^2 S_\perp}{(2\pi)^3}\delta_z z_1^3 z_2^3 \frac{Q^2(\mathbf{k}_1^2 - \mathbf{k}_2^2)^2}{(\mathbf{k}_1+\mathbf{k}_2)^4(\mathbf{k}_1^2+\epsilon_f^2)^2(\mathbf{k}_1^2+\epsilon_f^2)^2}$$

Similar results for Longitudinal photon

Going to numbers : Dilute Limit, Next-to-Eikonal

- Single covariant insertion is

$$\mathcal{U}_j^{(1)}(\mathbf{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+ \Big[-ig \int_{-\frac{L^+}{2}}^{v^+} dz_1^+ \left(\partial_{\mathbf{v}^j} A^-(z_1^+, \mathbf{v}) - \partial_{\mathbf{v}^j} A^-(v^+, \mathbf{v}) \right) - 2ig\mathbf{A}^j(v^+, \mathbf{v})$$
$$+ g^2 \int_{-\frac{L^+}{2}}^{v^+} dz_1^+ \int_{-\frac{L^+}{2}}^{z_1^+} dz_2^+ \partial_{\mathbf{v}^j} \left(A^-(v^+, \mathbf{v}) A^-(z_1^+, \mathbf{v}) - A^-(z_1^+, \mathbf{v}) A^-(z_2^+, \mathbf{v}) \right)$$
$$- 2g^2 \int_{-\frac{L^+}{2}}^{v^+} dz_1^+ \left(\mathbf{A}^j(v^+, \mathbf{v}) A^-(z_1^+, \mathbf{v}) + A^-(v^+, \mathbf{v}) \mathbf{A}^j(z_1^+, \mathbf{v}) \right) \Big] + \mathcal{O}(g^3).$$

- Next-to-eikonal correction arises from:

$$Q_j^{(1)}(\mathbf{w}', \mathbf{v}', \mathbf{v}_*, \mathbf{w}) - d_j^{(1)}(\mathbf{v}_*, \mathbf{w}) = 4\pi Q_s^2 \left[G^{-j}(\mathbf{v}' - \mathbf{v}) - G^{-j}(\mathbf{w}' - \mathbf{v}) \right]$$

- Where, $G^{i-}(\mathbf{r}) = \frac{1}{2P_q^-} \int_{\mathbf{P}} e^{i\mathbf{P}\cdot\mathbf{r}} \frac{\mathbf{P}^i}{\mathbf{P}^4}$ *Energy suppressed*

P.Agostini et al. (2403.04603 [hep-ph])

- Double covariant contribution doesn't contribute.
- At next-to-eikonal order,

$$\frac{d\sigma^{\gamma_L^* + A \to q\bar{q} + X}}{d^2\mathbf{k}_1 d\eta_1 d^2\mathbf{k}_2 d\eta_2} = \frac{8N_c \alpha_{\text{em}} e_f^2 Q_s^2 S_\perp}{(2\pi)^3} \delta_z z_1^3 z_2^3$$
$$\frac{Q^2(\mathbf{k}_1^2 - \mathbf{k}_2^2)^2}{(\mathbf{k}_1 + \mathbf{k}_2)^4 (\mathbf{k}_1^2 + \epsilon_f^2)^2 (\mathbf{k}_1^2 + \epsilon_f^2)^2} \left[1 + \frac{N_c}{W^2} \left(\frac{\mathbf{k}_1^2 + \epsilon_f^2}{z_1} - \frac{\mathbf{k}_2^2 + \epsilon_f^2}{z_2} \right) \right]$$

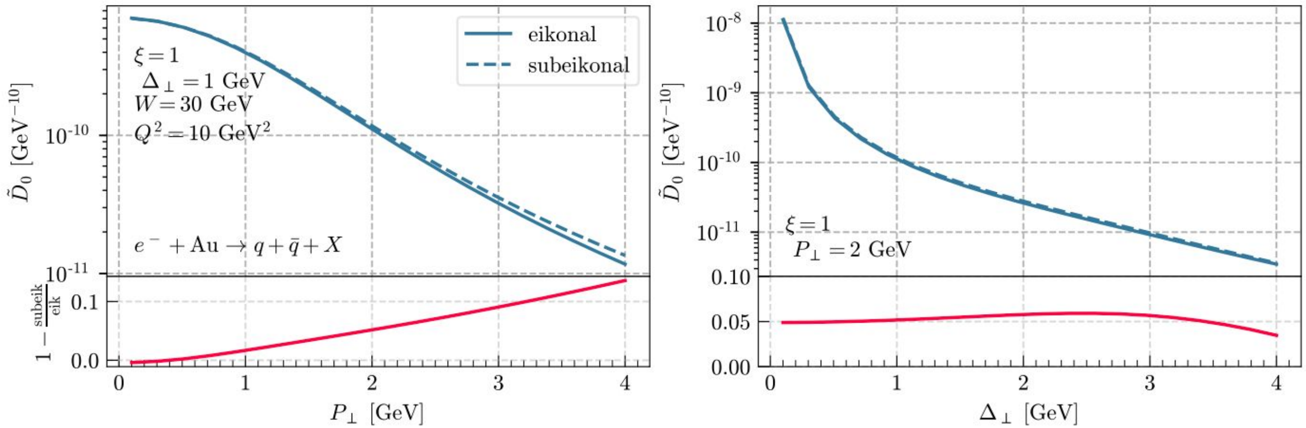
Similar results for Longitudinal photon

Numerical Results: Dilute Limit

P.Agostini et al. (2403.04603 [hep-ph])

- Momentum imbalance: $\mathbf{\Delta} = \mathbf{k_1} + \mathbf{k_2}$
- Relative momentum: $\mathbf{P} = z_2\mathbf{k_1} - z_1\,\mathbf{k_2}$
- Cross section can be expressed as harmonic expansion w.r.t ϕ between **Δ and P**

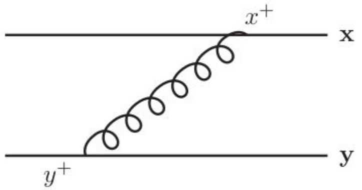
$$\frac{d\sigma_\lambda^{\gamma^* A\to q\bar{q}X}}{d\Pi} = D_{0,\lambda}(P_\perp,\Delta_\perp)\left[1+2\sum_{n=1}^{\infty} v_{n,\lambda}(P_\perp,\Delta_\perp)\cos\phi\right], \qquad D_{n,\lambda}(P_\perp,\Delta_\perp) = \int\frac{d\phi}{2\pi}e^{in\phi}\frac{d\sigma^{\gamma_\lambda^* A\to q\bar{q}X}}{d\Pi}, \qquad v_{n,\lambda} = \frac{D_{n,\lambda}}{D_{0,\lambda}}.$$



- W= 30 GeV, Q_s = 0.6 GeV
- EIC energy (√s = 90 GeV)
- At relatively large **P**, 10% corrections
- Valid when $P_\perp > \Delta_\perp > Q_s$

Going to numbers: Dense Limit, Eikonal order

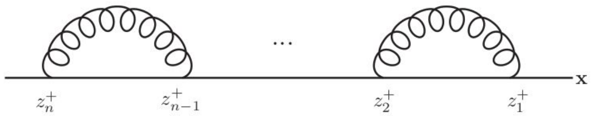
- Compute the average of different number of Wilson lines : dipoles and quadrupoles
- For that we assume that:
  - Target is composed of large number of nuclei A >> 1.
  - Hence, distribution of color sources is *Gaussian.*
- Express average of gluon fields as two point correlator: in terms of kinetic term and color structure



$$\langle A^-(x^+,\mathbf{x})A^-(y^+,\mathbf{y})\rangle = g^2\delta(x^+ - y^+)\mu^2(x^+)G^{--}(\mathbf{x}-\mathbf{y})$$

where, $G^{--}(\mathbf{r}) = \int_{\mathbf{P}} e^{i\mathbf{P}\cdot\mathbf{r}}\frac{1}{\mathbf{P}^4}$ *MV model*

- Sum over multiple correlator function to obtain average



Following F. Dominguez et al. (1101.0715)

Going to numbers: Eikonal order

Dipoles: $d_{[x^+,y^+]}(\mathbf{x},\mathbf{y}) \equiv \frac{1}{N_c}\left\langle \mathrm{Tr}\,\mathcal{U}_{[x^+,y^+]}(\mathbf{x})\mathcal{U}^{\dagger}_{[x^+,y^+]}(\mathbf{y})\right\rangle$

- Express dipoles in terms of Tadpole and non-tadpole contribution

$$d_{[x^+,y^+]}(\mathbf{x},\mathbf{y}) = \mathcal{T}\mathcal{N}$$

- Tadpole part factorizes, given as:

$$\mathcal{T} = \left\langle \mathcal{U}_{[x^+,y^+]}(\mathbf{x})\right\rangle \left\langle \mathcal{U}_{[x^+,y^+]}(\mathbf{y})\right\rangle = e^{-2\pi Q_s^2 G^{--}(0)}$$

- Interaction between two quark (antiquark) lines is given as:

$$\mathcal{N} = \frac{1}{N_c}\sum_{n=0}^{\infty}\int_{z_1^+>\dots>z_n^+}\prod_{i=1}^{n}[g^2\mu^2(z_i^+)G^{--}(\mathbf{x}-\mathbf{y})]$$

Using Fierz identity it simplifies

$$\frac{1}{N_c}\sum_{n=0}^{\infty}\frac{1}{n!}\left[C_F g^2\int dz^+\mu^2(z^+)G^{--}(\mathbf{x}-\mathbf{y})\right]^n = e^{2\pi Q_s^2 G^{--}(\mathbf{x}-\mathbf{y})}$$

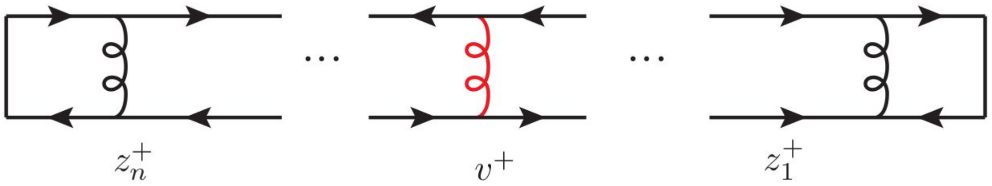
Hence dipole, similar to MV model :

$$d_{[x^+,y^+]}(\mathbf{x},\mathbf{y}) = \exp\left\{-\frac{Q_s^2}{4}(\mathbf{x}-\mathbf{y})^2 \ln\frac{1}{|\mathbf{x}-\mathbf{y}|\Lambda_{\mathrm{QCD}}}\right\}$$

Going to numbers: Next-to-eikonal order

Dipoles: $$d_j^{(1)}(\mathbf{v}_*, \mathbf{w}) = \frac{1}{N_c}\left\langle \mathrm{Tr}[\mathcal{U}_j^{(1)}(\mathbf{v})\mathcal{U}^\dagger(\mathbf{w})]\right\rangle$$

- Similar to eikonal, but extra connection from transverse gluon field $\left\langle \mathbf{A}^j \mathbf{A}^- \right\rangle$



- Kinetic term factorizes, hence: write in terms of eikonal dipole

$$d_j^{(1)}(\mathbf{v}_*, \mathbf{w}) = 2g^2\tilde{\mu}^2 C_F d_{[\frac{L^+}{2}, -\frac{L^+}{2}]}(\mathbf{v}, \mathbf{w}) G^{j-}(\mathbf{v}-\mathbf{w})$$

where, $$G^{i-}(\mathbf{r}) = \frac{1}{2P_q^-}\int_{\mathbf{P}} e^{i\mathbf{P}\cdot\mathbf{r}}\frac{\mathbf{P}^i}{\mathbf{P}^4}$$ *Energy suppressed*

$$d_j^{(1)}(\mathbf{v}_*, \mathbf{w}) = \frac{iQ_s^2}{P_q^-}(\mathbf{v}-\mathbf{w})^j \ln\left(\frac{1}{|\mathbf{v}-\mathbf{w}|\Lambda_{\mathrm{QCD}}}\right)\mathrm{Exp}\left\{-\frac{Q_s^2}{4}(\mathbf{v}-\mathbf{w})^2 \ln\frac{1}{|\mathbf{v}-\mathbf{w}|\Lambda_{\mathrm{QCD}}}\right\}$$

Similar treatment is done for $d^{(2)}$ decorated dipole

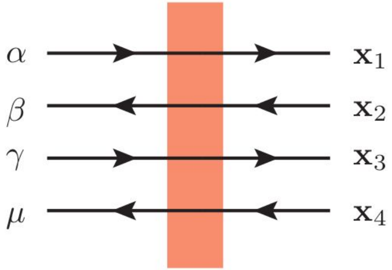
$$d^{(2)}(\mathbf{v}_*, \mathbf{w}) = \left[\frac{Q_s^4 L^+}{6}(\mathbf{v}-\mathbf{w})^2 \ln^2\left(\frac{1}{|\mathbf{v}-\mathbf{w}|\Lambda_{\mathrm{QCD}}}\right) + \frac{Q_s^2}{4(P_q^-)^2 L^+}\ln\frac{\Lambda_{\mathrm{UV}}}{\Lambda_{\mathrm{QCD}}}\right]\exp\left\{-\frac{Q_s^2}{4}(\mathbf{v}-\mathbf{w})^2 \ln\frac{1}{|\mathbf{v}-\mathbf{w}|\Lambda_{\mathrm{QCD}}}\right\}.$$

Going to numbers: eikonal Order

Quadrupole: $Q_{[x^+,y^+]}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4) = \frac{1}{N_c}\left\langle \mathrm{Tr}[\mathcal{U}_{[x^+,y^+]}(\mathbf{x}_1)\mathcal{U}^\dagger_{[x^+,y^+]}(\mathbf{x}_2)\mathcal{U}_{[x^+,y^+]}(\mathbf{x}_3)\mathcal{U}^\dagger_{[x^+,y^+]}(\mathbf{x}_4)]\right\rangle$

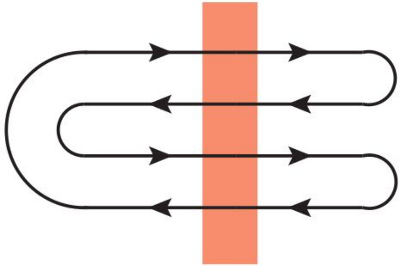
- Shockwave between y^+ and x^+ is given as:

$$\mathcal{Q}_{[x^+,y^+]} = \left\langle \mathcal{U}_{[x^+,y^+]}(\mathbf{x}_1)_{\alpha_1\alpha_2}\mathcal{U}^\dagger_{[x^+,y^+]}(\mathbf{x}_2)_{\beta_1\beta_2}\mathcal{U}_{[x^+,y^+]}(\mathbf{x}_3)_{\gamma_1\gamma_2}\mathcal{U}^\dagger_{[x^+,y^+]}(\mathbf{x}_4)_{\mu_1\mu_2}\right\rangle$$



- By projecting Wilson lines we obtain quadrupole:

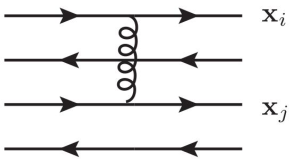
$$Q_{[x^+,y^+]}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4) = \langle e_2|\,\mathcal{Q}_{[x^+,y^+]}\,|e_1\rangle$$



In general,

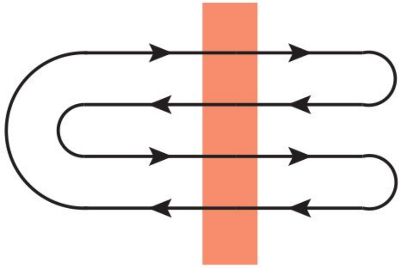
$$\mathcal{H}_{[x^+,y^+]_{ij}} = \langle e_i|\,\mathcal{Q}_{[x^+,y^+]}\,|e_j\rangle$$

- Next, all possible combinations where we can have two-gluon exchanges at a longitudinal point x^+ is defined as : $\Sigma(x^+) = \mu^2(x^+)\sum_{i=1,j>i}^{4} W_{ij}$
where, $W_{ij} = (-1)^{i+j+1}g^2G^{--}(\mathbf{x}_i - x_j)$ *(MV model)*



Going to numbers: eikonal Order

Quadrupole: $Q_{[x^+,y^+]}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4) = \frac{1}{N_c}\left\langle \mathrm{Tr}[\mathcal{U}_{[x^+,y^+]}(\mathbf{x}_1)\mathcal{U}^\dagger_{[x^+,y^+]}(\mathbf{x}_2)\mathcal{U}_{[x^+,y^+]}(\mathbf{x}_3)\mathcal{U}^\dagger_{[x^+,y^+]}(\mathbf{x}_4)]\right\rangle$



- Similar dipole, we then write:

$$Q_{[x^+,y^+]} = \mathcal{T}\mathcal{N}$$

and tadpole factorizes as $\mathcal{T} = e^{-4\pi Q_s^2 G^{--}(0)}$

- After simplifying color structure, we obtain quadruple as:

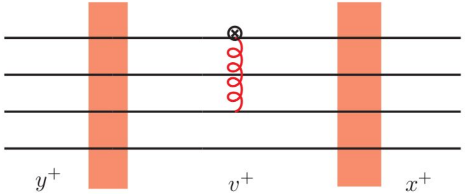
$$Q(\mathbf{w}',\mathbf{v}',\mathbf{v},\mathbf{w}) = d(\mathbf{w},\mathbf{v})d(\mathbf{w}',\mathbf{v}')\Bigg[\left(\frac{F(\mathbf{w}',\mathbf{v},\mathbf{v}',\mathbf{w})+\sqrt{\gamma}}{2\sqrt{\gamma}} - \frac{F(\mathbf{w}',\mathbf{v}',\mathbf{v},\mathbf{w})}{\sqrt{\gamma}}\right)e^{\frac{N_c}{4}\sqrt{\gamma}} - \left(\frac{F(\mathbf{w}',\mathbf{v},\mathbf{v}',\mathbf{w})-\sqrt{\gamma}}{2\sqrt{\gamma}} - \frac{F(\mathbf{w}',\mathbf{v}',\mathbf{v},\mathbf{w})}{\sqrt{\gamma}}\right)e^{-\frac{N_c}{4}\sqrt{\gamma}}\Bigg] e^{-\frac{N_c}{4}F(\mathbf{w}',\mathbf{v},\mathbf{v}',\mathbf{w})+\frac{1}{2N_c}F(\mathbf{w}',\mathbf{v}',\mathbf{v},\mathbf{w})}$$

and, $F(\mathbf{w}',\mathbf{v}',\mathbf{v},\mathbf{w}) = \frac{1}{C_F}\ln\frac{d(\mathbf{w}'-\mathbf{v})d(\mathbf{v}'-\mathbf{w})}{d(\mathbf{w}'-\mathbf{w})d(\mathbf{v}'-\mathbf{v})}$

Quadrupole Computation: Next-to-eikonal Order

Quadrupole: $Q_j^{(1)}(\mathbf{w}', \mathbf{v}', \mathbf{v}_*, \mathbf{w}) = \frac{1}{N_c}\left\langle \mathrm{Tr}[\mathcal{U}(\mathbf{w}')\mathcal{U}^\dagger(\mathbf{v}')\mathcal{U}_j^{(1)}(\mathbf{v})\mathcal{U}^\dagger(\mathbf{w})] \right\rangle$

- Decorated quadruple with single insertion is



$$\equiv \int_{v^+} \mathcal{Q}_{[x^+,v^+]} \Sigma^{\mathrm{NEik}}(v^+) \mathcal{Q}_{[v^+,y^+]}$$

where, $\Sigma_{\mathrm{NEik}}(v^+) = \mu^2(v^+) \sum_{i=2}^{4} \mathcal{W}_{1i} = \mu^2(v^+) \left| e_i \right\rangle \mathcal{V}_{it} \left\langle e_t \right|$

and $\tilde{\mathcal{V}} = \mathcal{V} c^{-1} = \begin{pmatrix} -\frac{\mathcal{W}_{12}(1-N_c^2)+\mathcal{W}_{13}+\mathcal{W}_{14}}{2N_c} & \frac{\mathcal{W}_{12}+\mathcal{W}_{13}}{2} \\ \frac{\mathcal{W}_{13}+\mathcal{W}_{14}}{2} & -\frac{\mathcal{W}_{12}+\mathcal{W}_{13}-\mathcal{W}_{14}(N_c^2-1)}{2N_c} \end{pmatrix}$ $\quad \mathcal{W}_{ij}^J = (-i)^{i+j+1}\frac{g^2}{2\pi}\left[\frac{i}{P_q^-} + 2v^+\right](\mathbf{x}_i - \mathbf{x}_j)^J \ln\frac{1}{|\mathbf{x}_i - \mathbf{x}_j|\Lambda_{\mathrm{QCD}}}$ *Energy suppressed*

- Simplifying, we obtain quadrupole for single insertion: $Q^j = N^j \mathcal{T}$

with $N^j = \int_{v^+} \mu^2(v^+)(\mathcal{H}_{x^+,v^+}\tilde{\mathcal{V}}\mathcal{H}_{[v^+,y^+]})_{21}$

- Similarly, we obtain quadrupole with double insertion:

$$Q_2^j = \int_{v_1^+ > v_2^+} \mu^2(v_1^+)\mu^2(v_2^+)\left(\mathcal{H}_{[x^+,v_1^+]}\tilde{\mathcal{V}}^{(1)}\mathcal{H}_{[v_1^+,v_2^+]}\tilde{\mathcal{V}}^{(2)}\mathcal{H}_{[v_2^+,y^+]}\right)_{21}\mathcal{T}$$

Numerical Results: Eikonal order

- Momentum imbalance: $\mathbf{\Delta} = \mathbf{k_1} + \mathbf{k_2}$
- Relative momentum: $\mathbf{P} = z_2\mathbf{k_1} - z_1\,\mathbf{k_2}$
- Cross section can be decomposed as

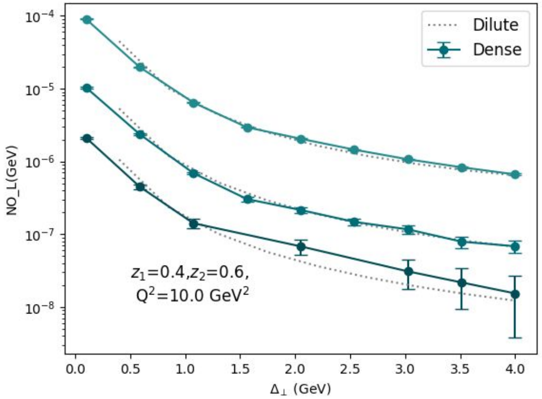
$$\frac{dN^{\gamma^*_\lambda + A \to q\bar{q} + X}}{d^2\Delta d^2\mathbf{P} d\eta_1 d\eta_2} = N_0^\lambda + 2\sum_{n=1}^{\infty} N_n^\lambda(\mathbf{P}, \Delta)\cos(n\phi)$$

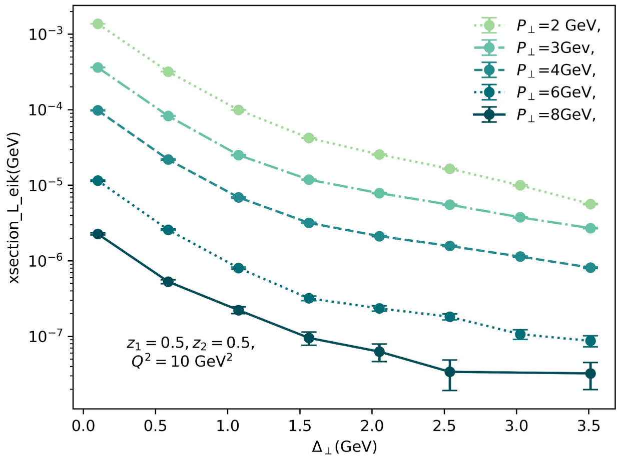
Where modes are given as:

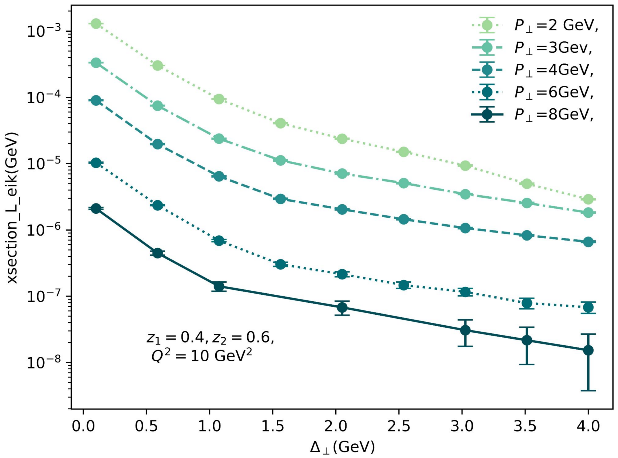
$$N_n^\lambda(\mathbf{P}, \Delta) = \frac{1}{S_\perp}\int \frac{d\phi_{P_\perp}}{2\pi}\frac{d\phi_{\Delta_\perp}}{2\pi} e^{in(\phi_{P_\perp} - \phi_{\Delta_\perp})}\frac{d\sigma^{\gamma^*_\lambda + A \to q\bar{q} + X}}{d^2\Delta d^2\mathbf{P} d\eta_1 d\eta_2}$$

Numerical Results: Eikonal order

- Momentum imbalance: $\mathbf{\Delta} = \mathbf{k_1} + \mathbf{k_2}$
- Relative momentum: $\mathbf{P} = z_2\mathbf{k_1} - z_1\mathbf{k_2}$
- W= 30 GeV, Q_s = 0.6 GeV
- Compared with Dilute results from *P.Agostini et al. (2403.04603 [hep-ph]),* Valid when $\Delta_\perp$, $P_\perp > Q_s$







Next-to-eikonal plots in dense limit coming soon….

Summary

- We are computing the DIS dijet cross section at NEik accuracy. This demands **computing new averages, dipoles and quadrupoles**, never before considered in the dense limit.
- We have computed all field correlators at next-to-eikonal order using Gaussian approximation.
  - Next-to-eikonal quadrupoles and dipoles
  - Two point correlation between neighbouring fields considered
- To analyze upcoming measurement, there is **need to compare Next-to-eikonal corrections with Next-to-leading order** corrections to address their respective importance.
- There is still room to improve model:
  - By including correction coming from x^- dependence
  - By considering inhomogeneous medium

Thank You!

Backup

Angular Momentum Decomposition

$$\left.\frac{d\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X}}{d^2\Delta d^2\mathbf{P} d\eta_1 d\eta_2}\right|_{\text{NEik}}^{\text{dip}} = \text{Re}\frac{2iQ_s^2 N_c S_\perp}{W^2}\left(\frac{z_1-z_2}{z_1 z_2}\right)\int_{\mathbf{r},\mathbf{r}',\mathbf{B}'} e^{i\Delta\cdot\mathbf{B}'}e^{i\mathbf{P}\cdot(\mathbf{r}-\mathbf{r}')}\mathcal{C}_\lambda(\mathbf{r},\mathbf{r}')$$

$$\times\ln\left(\frac{1}{|\mathbf{r}'|\Lambda_{\text{QCD}}}\right)d(-\mathbf{r}')\left[-\frac{(z_2-z_1)\Delta\cdot\mathbf{r}'}{2}+\mathbf{P}\cdot\mathbf{r}'\right]$$

Cross section can decomposed as:

$$\frac{dN^{\gamma_\lambda^*+A\to q\bar{q}+X}}{d^2\Delta d^2\mathbf{P} d\eta_1 d\eta_2} = N_0^\lambda + 2\sum_{n=1}^{\infty} N_n^\lambda(\mathbf{P},\Delta)\cos(n\phi)$$

Using Bessel function identity : $e^{iA\cos\alpha} = \sum_{n=-\infty}^{\infty}(-i)^n J_n(A)e^{-in\alpha}$

Cross section is expressed as: $$\left.N_0^L\right|_{\text{NEik}}^{\text{dip}} = \text{Re}\frac{2Q_s^2 N_c}{W^2}\left(\frac{z_1-z_2}{z_1 z_2}\right)\int_{\mathbf{r},\mathbf{r}',\mathbf{B}'}\mathcal{C}_L(\mathbf{r},\mathbf{r}')\ln\left(\frac{1}{|\mathbf{r}'|\Lambda_{\text{QCD}}}\right)d(-\mathbf{r}')$$

$$\left[\frac{(z_1-z_2)}{2}\Delta_\perp|\mathbf{r}'|J_0(P_\perp|\mathbf{r}-\mathbf{r}'|)J_1(\Delta_\perp|\mathbf{B}'|)\cos(\phi_{\mathbf{B}'}-\phi_{\mathbf{r}'})\right.$$

$$\left.+P_\perp|\mathbf{r}'|J_0(\Delta_\perp|\mathbf{B}'|)J_1(P_\perp|\mathbf{r}-\mathbf{r}'|)\cos(\phi_{\mathbf{r}-\mathbf{r}'}-\phi_{\mathbf{r}'})\right]$$