Probing Weizsäcker-Williams Gluon *Helicity* Distribution at Small x

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Yuri Kovchegov and Ming Li, arXiv: 2504.12979.

Outline

- (Unpolarized) Weizsäcker-Williams (WW) gluon distribution.
- A tale of two transverse-momentum-dependent gluon helicity distributions.
- Double-spin asymmetry for inclusive dijet production in longitudinally polarized DIS.
- Small-x evolution equation for WW gluon helicity distribution .
- Summary and outlook

Review: A Tale of Two (unpolarized) Gluon Distributions

- **General gauge invariant gluon-gluon correlation function**

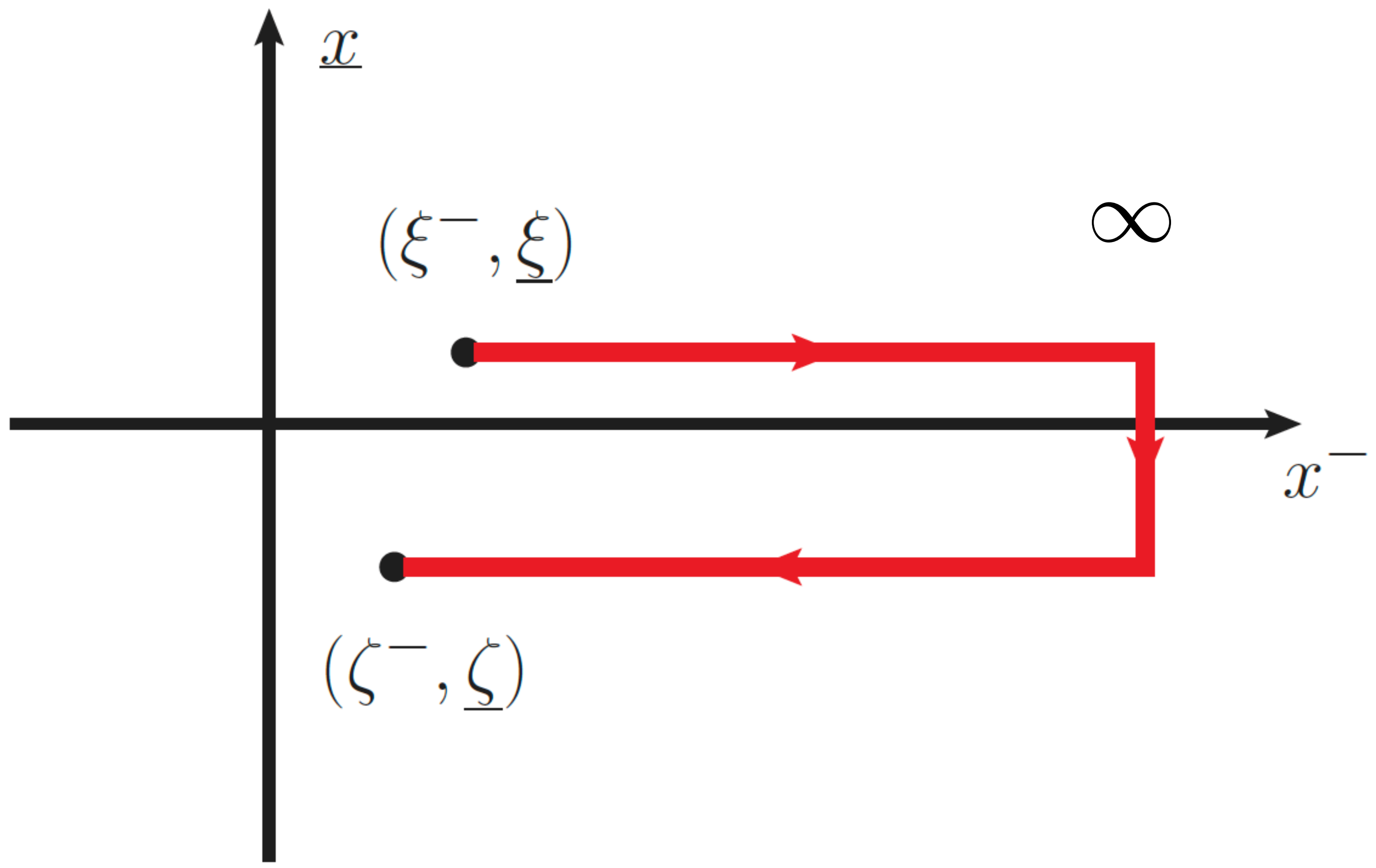
Mulders and Rodrigues (2001)

$$\Gamma^{\mu\nu;\rho\sigma}(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \langle P,S|\mathrm{Tr}\left[F^{\mu\nu}(0)\,\mathcal{U}(0,\xi)\,F^{\rho\sigma}(\xi)\,\widetilde{\mathcal{U}}(\xi,0)\right]|P,S\rangle$$

Proton wavefunction

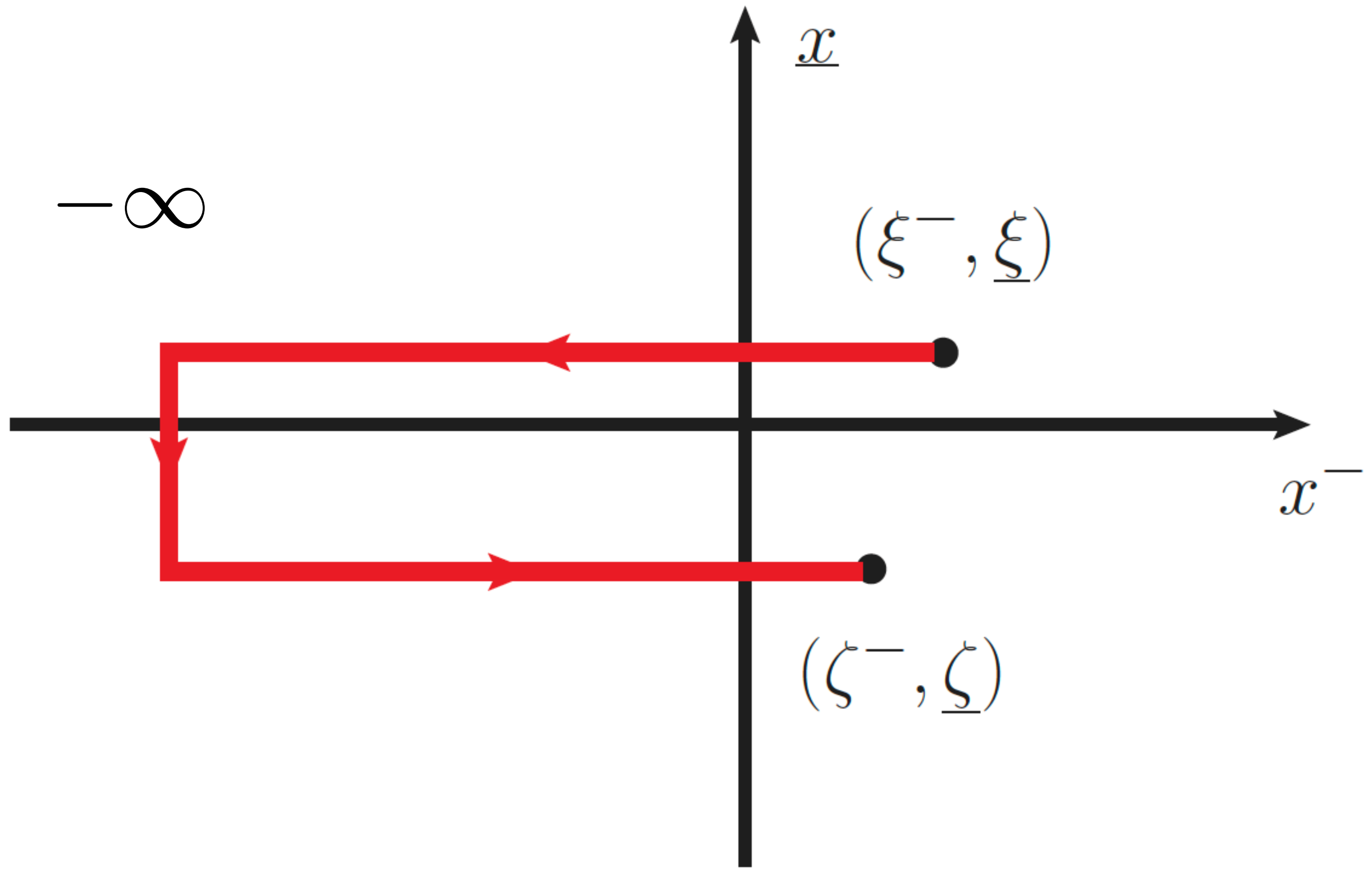
➡ **The gauge links, however, are path-dependent (process-dependent).**

Future-pointing gauge link



$$\mathcal{U}^{[+]}[\zeta,\xi] = V_{\underline{\zeta}}[\zeta^-,\infty]V_{\underline{\xi}}[\infty,\xi^-],$$

Past-pointing gauge link



$$\mathcal{U}^{[-]}[\zeta,\xi] = V_{\underline{\zeta}}[\zeta^-,-\infty]V_{\underline{\xi}}[-\infty,\xi^-].$$

$$V_{\underline{\xi}}[\xi_f^-,\xi_i^-] = \mathcal{P}\exp\left[ig\int_{\xi_i^-}^{\xi_f^-} d\xi^- A^+(0^+,\xi^-,\underline{\xi})\right].$$

Notation: $\underline{\xi} = (\xi^x,\xi^y)$.

Transverse gauge links at spatial infinities are ignored.

Review: A Tale of Two (unpolarized) Gluon Distributions

- **(Unpolarized) transverse-momentum-dependent (TMD) gluon distributions**

Mulders and Rodrigues (2001)

$$\Gamma^{ij}(x,\underline{k}) \equiv \frac{2}{xP^+}\int \frac{d\xi^- d^2\xi}{(2\pi)^3} e^{ixP^+\xi^-} e^{-i\underline{k}\cdot\underline{\xi}} \left\langle P \middle| \mathrm{Tr}\left[F^{+i}(0)\mathcal{U}[0,\xi]F^{+j}(\xi)\widetilde{\mathcal{U}}[\xi,0]\right] \middle| P\right\rangle \Big|_{\xi^+=0}$$

$$=\delta^{ij} f_1^G(x,k_T^2) + \left(\frac{k^i k^j}{k_T^2} - \frac{1}{2}\delta^{ij}\right) h_1^{\perp G}(x,k_T^2).$$

Gluon TMD

Linearly polarized gluon TMD

There are two types of gluon TMDs. *In the small x regime, see Kharzeev, Kovchegov and Tuchin (2003)*

Dipole Gluon TMD:

$$f^G_{1,dip}(x,k_T^2) \quad \longleftarrow \quad \mathcal{U}=\mathcal{U}^{[+]},\ \widetilde{\mathcal{U}}=\mathcal{U}^{[-]},$$

Weizsäcker-Williams Gluon TMD:

$$f^G_{1,WW}(x,k_T^2) \quad \longleftarrow \quad \mathcal{U}=\widetilde{\mathcal{U}}=\mathcal{U}^{[+]},$$

Review: A Tale of Two (unpolarized) Gluon Distributions

- **Operator expressions in the small x limit:** *Kharzeev, Kovchegov and Tuchin (2003)*

Dipole Gluon TMD:

$$\boxed{x f^{G}_{1,dip}(x, k_T^2) = \frac{1}{g^2(2\pi)^3}\, k_T^2 \int d^2\xi d^2\zeta\, e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})} \left\langle \mathrm{tr}\left[V_{\underline{\xi}} V^{\dagger}_{\underline{\zeta}}\right] \right\rangle.}$$

Weizsäcker-Williams Gluon TMD:

$$\boxed{x f^{G}_{1,WW}(x, k_T^2) = -\frac{1}{g^2(2\pi)^3} \int d^2\xi d^2\zeta\, e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})} \left\langle \mathrm{tr}\left[V_{\underline{\xi}} \partial^i V^{\dagger}_{\underline{\xi}} V_{\underline{\zeta}} \partial^i V^{\dagger}_{\underline{\zeta}}\right] \right\rangle.}$$

- **Small-x evolution equations.**

$f^{G}_{1,dip}(x, k_T^2)$ → **BK equation.**

$f^{G}_{1,WW}(x, k_T^2)$ → **JIMWLK equation or BK equation for quadrupole.**

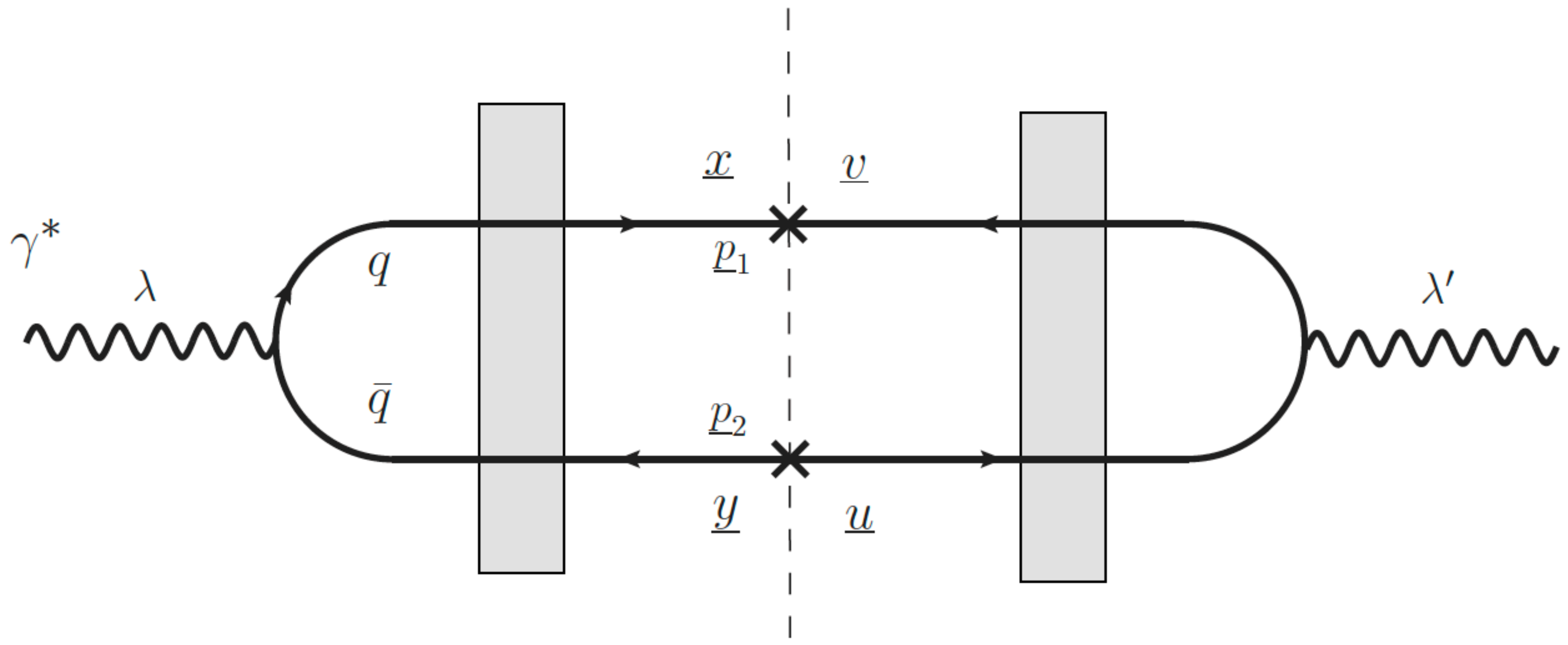
Dominguez, Mueller, Munier and Xiao (2011)
Balitsky and Tarasov (2015)

Review: A Tale of Two (unpolarized) Gluon Distributions

◆ **Probing Weizsäcker-Williams Gluon TMD**

Dominguez, Marquet, Xiao and Yuan (2011)

Inclusive quark-antiquark dijet production in unpolarized DIS



$$\left\langle \mathrm{tr}\left[(V_{\underline{x}}V_{\underline{y}}^{\dagger}-1)(V_{\underline{u}}V_{\underline{v}}^{\dagger}-1)\right]\right\rangle$$

$$= -r^i r'^j \left\langle \mathrm{tr}\left[V_{\underline{R}}\partial^i V_{\underline{R}}^{\dagger} V_{\underline{R}'}\partial^j V_{\underline{R}'}^{\dagger}\right]\right\rangle + \ldots$$

The back-to-back limit:

$$\underline{\Delta} = \underline{p}_1 + \underline{p}_2, \quad \underline{p} = z_2\underline{p}_1 - z_1\underline{p}_2.$$

$$|\underline{\Delta}| \ll |\underline{p}|.$$

$$\underline{r} = \underline{x} - \underline{y}, \quad \underline{R} = z_1\underline{x} + z_2\underline{y},$$
$$\underline{r}' = \underline{v} - \underline{u}, \quad \underline{R}' = z_1\underline{v} + z_2\underline{u}.$$

In the back-to-back limit, the dijet production probes the WW gluon TMD.

A Tale of Two Gluon Helicity Distributions

- **Consider longitudinally polarized proton**

Mulders and Rodrigues (2001)

$$\Gamma_L^{ij}(x,\underline{k}) \equiv \frac{4}{xP^+}\boxed{\frac{1}{2}\sum_{S_L} S_L} \int \frac{d\xi^- d^2\xi}{(2\pi)^3} e^{ixP^+\xi^-} e^{-i\underline{k}\cdot\underline{\xi}} \left\langle P, S_L\right|\mathrm{Tr}\left[F^{+i}(0)\,\mathcal{U}[0,\xi]\,F^{+j}(\xi)\,\widetilde{\mathcal{U}}[\xi,0]\right]\left|P, S_L\right\rangle\Big|_{\xi^+=0}$$

$$= i\epsilon^{ij} g_{1L}^{G}(x,k_T^2) + \frac{(\epsilon^{li}k^j + \epsilon^{lj}k^i)k^l}{k_T^2} h_{1L}^{\perp G}(x,k_T^2)$$

Gluon helicity TMD

Linearly polarized gluon TMD in longitudinally polarized proton

Longitudinally polarized Proton wavefunction

Two types of gluon helicity TMDs. *Kovchegov, Pitonyak and Sievert (2017)*

Dipole Gluon Helicity TMD:

$$g_{1L}^{G,dip}(x,k_T^2) \longleftarrow \mathcal{U} = \mathcal{U}^{[+]},\ \ \widetilde{\mathcal{U}} = \mathcal{U}^{[-]},$$

Weizsäcker-Williams Gluon Helicity TMD:

This talk is about →

$$\boxed{g_{1L}^{G,WW}(x,k_T^2)} \longleftarrow \mathcal{U} = \widetilde{\mathcal{U}} = \mathcal{U}^{[+]},$$

Operator Expressions in the Small x Limit

Dipole Gluon Helicity TMD: *Cougoulic, Kovchegov, Tarasov and Tawabutr (2022)*

$$g_{1L}^{G,dip}(x, k_T^2) = \frac{1}{2g^2\pi^3} i\epsilon^{ij} k^j \int d^2\xi d^2\zeta e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})} \Big\langle\Big\langle \mathrm{tr}\left[V_{\underline{\xi}}^{i\,\mathrm{G}[2]} V_{\underline{\zeta}}^{\dagger}\right] \Big\rangle\Big\rangle + c.c.$$

Weizsäcker-Williams Gluon Helicity TMD:

$$g_{1L}^{G,WW}(x, k_T^2) = -\frac{1}{2g^2\pi^3} \epsilon^{ij} \int d^2\xi d^2\zeta \, e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})} \Big\langle\Big\langle \mathrm{tr}\left[V_{\underline{\zeta}} \partial^i V_{\underline{\zeta}}^{\dagger} V_{\underline{\xi}}^{j\,\mathrm{G}[2]} V_{\underline{\xi}}^{\dagger}\right] \Big\rangle\Big\rangle + c.c.$$

- **Transverse Chromoelectrically Polarized Wilson Line**

$$\Big\langle\Big\langle \ldots \Big\rangle\Big\rangle \equiv s \frac{1}{2} \sum_{S_L} S_L \frac{1}{2P^+V^-} \langle P, S_L | \ldots | P, S_L \rangle.$$

$$V_{\underline{\xi}}^{i,G[2]} = \frac{igP^+}{s} \int_{-\infty}^{+\infty} d\xi^- \xi^- V_{\underline{\xi}}[\infty, \xi^-] F^{+i}(\xi^-, \underline{\xi}) V_{\underline{\xi}}[\xi^-, -\infty],$$

Operator Expressions in the Small x Limit

$$
\begin{aligned}
\Gamma^{ij}_{WW}(x,\underline{k}) &= \frac{4}{xP^+V^-}\frac{1}{(2\pi)^3}\frac{1}{2}\sum_{S_L} S_L \int d\xi^- d^2\xi\, d\zeta^- d^2\zeta\, e^{ixP^+(\xi^- -\zeta^-)} e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})} \\
&\quad \times \left\langle P, S_L \left| \mathrm{tr}\left[F^{+i}(\zeta)\mathcal{U}^{[+]}[\zeta,\xi]F^{+j}(\xi)\mathcal{U}^{[+]}[\xi,\zeta]\right]\right| P, S_L \right\rangle_{\xi^+=\zeta^+=0} \\
&= \frac{4}{xP^+V^-}\frac{1}{(2\pi)^3}\frac{1}{2}\sum_{S_L} S_L \int d^2\xi\, d^2\zeta\, e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})} \langle P, S_L|\mathrm{tr}\left[[E^i(x,\underline{\zeta})]^\dagger E^j(x,\underline{\xi})\right]|P, S_L\rangle
\end{aligned}
$$

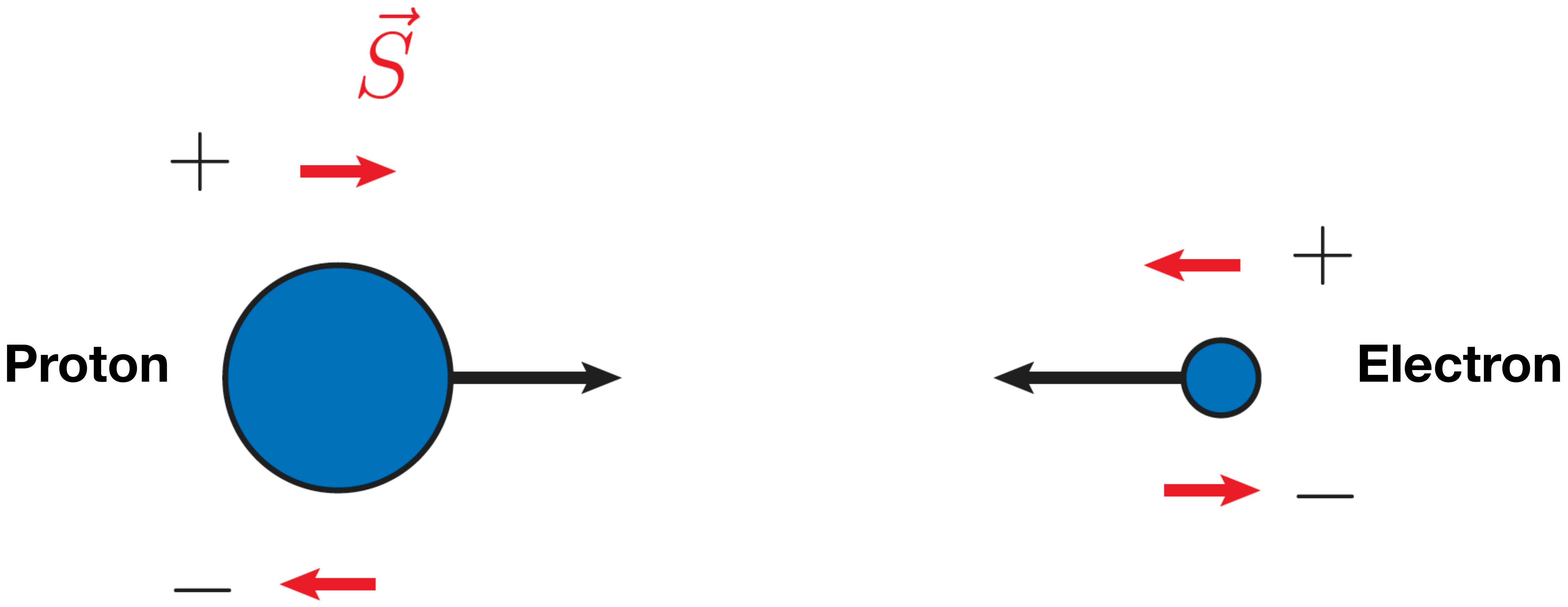
$$
\boxed{
\begin{aligned}
E^j(x,\underline{\xi}) &= \int\limits_{-\infty}^{\infty} d\xi^- e^{ixP^+\xi^-} V_{\underline{\xi}}[\infty,\xi^-]F^{+j}(\xi^-,\underline{\xi})V_{\underline{\xi}}[\xi^-,\infty] \\
&= -\int\limits_{-\infty}^{\infty} d\xi^- e^{ixP^+\xi^-} V_{\underline{\xi}}[\infty,\xi^-]\left(ixP^+A^j + \partial^j A^+\right)V_{\underline{\xi}}[\xi^-,\infty].
\end{aligned}
}
$$

$$
\boxed{
E^j(x,\underline{\xi}) = \int\limits_{-\infty}^{\infty} d\xi^- \;\; V_{\underline{\xi}}[\infty,\xi^-]\;\left[\partial^j A^+ + ixP^+\left(\xi^-\,\partial^j A^+ + A^j\right) + \mathcal{O}(x^2)\right]\; V_{\underline{\xi}}[\xi^-,\infty].
}
$$

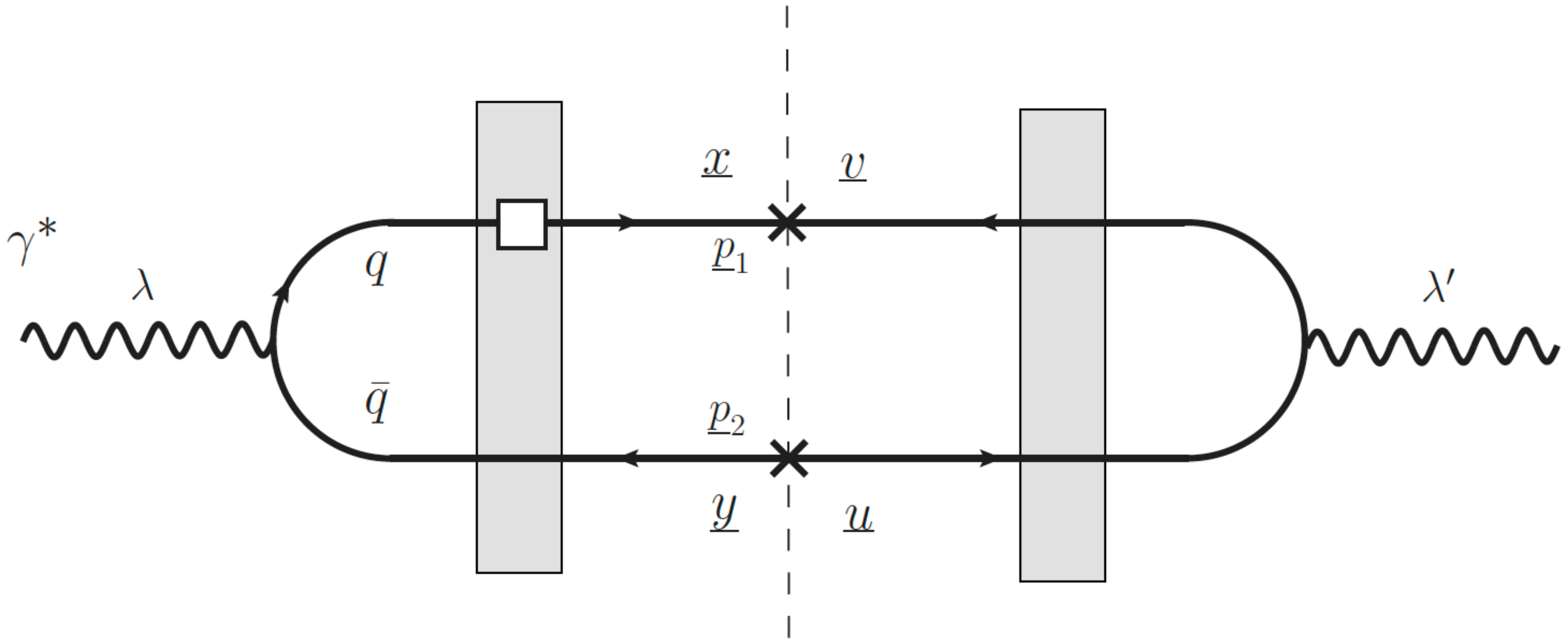
$$
\boxed{
E^j(x,\underline{\xi}) = \frac{1}{ig}\left(\partial^j V_{\underline{\xi}}\right) V_{\underline{\xi}}^\dagger - \frac{xs}{g} V_{\underline{\xi}}^{j\,\mathrm{G}[2]}\, V_{\underline{\xi}}^\dagger + \mathcal{O}(x^2)
}
$$

How to probe the WW gluon helicity TMD?

Double-spin asymmetry for inclusive quark-antiquark dijet production in longitudinally polarized DIS.



$$\boxed{A_{\mathrm{LL}} \equiv \frac{d\Delta\sigma}{d\sigma} \equiv \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}}$$



To be sensitive to the helicity state of the proton, subeikonal order interactions are needed.

In the back-to-back limit, the A_{LL} for inclusive dijet production probes the WW gluon helicity TMD.

Transversely Polarized Virtual Photon States

$$E'\frac{d\sigma^{DSA}}{d^3k'}=\frac{2M_p\alpha_{EM}^2}{sQ^2y^2}\frac{1}{2}\sum_{S_L}S_L\Big\{\left[1-(1-y)^2\right]\sum_{\lambda=\pm1}\lambda W^{\lambda\lambda}-2\sqrt{2(1-y)}y\cos(\phi)\sum_{\lambda=\pm1}\mathrm{Re}W^{\lambda,0}$$
$$-2\sqrt{2(1-y)}y\sin(\phi)\sum_{\lambda=\pm1}\lambda\mathrm{Im}W^{\lambda,0}\Big\}.$$

$$W^{\lambda\lambda'}\equiv W^{\alpha\beta}\,\epsilon_\alpha^{\lambda*}\,\epsilon_\beta^{\lambda'}$$

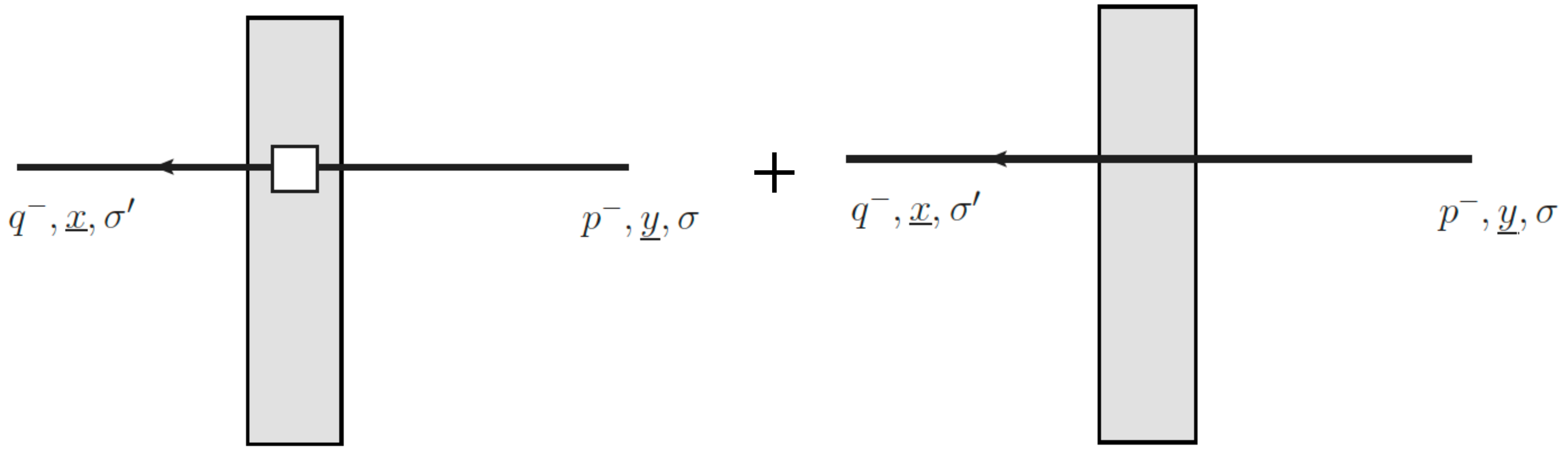
$$2M_p\,W^{\alpha\beta}(P,q)=\frac{1}{2\pi}\sum_X\langle P,S_L|\,j^\alpha(0)\,|X\rangle\,\langle X|\,j^\beta(0)\,|P,S_L\rangle\,(2\pi)^4\,\delta^4(P+q-p_X).$$

Only transversely polarized virtual photon contribute to A_{LL} after integrating out the azimuthal angle of outgoing electron.

$$\int_0^{2\pi}\frac{d\phi}{2\pi}\,E'\,\frac{d\sigma^{DSA}}{d^3k'}=\frac{2M_p\alpha_{EM}^2}{s\,Q^2\,y^2}\,\frac{1}{2}\sum_{S_L}S_L\,\left[1-(1-y)^2\right]\sum_{\lambda=\pm1}\lambda\,W^{\lambda\lambda}.$$

Wilson Lines at Sub-Eikonal Order

- Single quark scattering amplitude up to subeikonal order



$$M^{q\to q}(q^-,\underline{x},\sigma';p^-,\underline{y},\sigma)$$

$$\boxed{\begin{aligned}=&(2\pi)2p^-\delta(p^--q^-)\delta_{\sigma,\sigma'}\Big[V_{\underline{x}}\,\delta^2(\underline{x}-\underline{y})\\ &+\sigma\,V^{\mathrm{pol}[1]}_{\underline{x}}\,\delta^2(\underline{x}-\underline{y})+V^{\mathrm{pol}[2]}_{\underline{x},\underline{y}}\Big]\\ &+\delta_{\sigma,\sigma'}(2\pi)(p^-+q^-)\delta'(p^--q^-)\delta^2(\underline{x}-\underline{y})V^{\mathrm{G}[3]}_{\underline{x}}\\ &+\mathcal{O}(1/s^2)\end{aligned}}$$

$$\boxed{\begin{aligned}&V^{\mathrm{pol}[1]}_{\underline{x}}=V^{\mathrm{G}[1]}_{\underline{x}}+V^{\mathrm{q}[1]}_{\underline{x}},\qquad V^{\mathrm{pol}[2]}_{\underline{x},\underline{y}}=V^{\mathrm{G}[2]}_{\underline{x},\underline{y}}+V^{\mathrm{q}[2]}_{\underline{x}}\delta^2(\underline{x}-\underline{y}),\\ &V^{\mathrm{pol}[3]}_{\underline{x}}=V^{\mathrm{G}[3]}_{\underline{x}}.\end{aligned}}$$

$$V_{\underline{x}}[x_f^-,x_i^-]=\mathcal{P}\exp\left[ig\int\limits_{x_i^-}^{x_f^-}dx^-A^+(0^+,x^-,\underline{x})\right].$$

Eikonal Wilson Line

$$V^{\mathrm{G}[1]}_{\underline{x}}=\frac{i\,g\,P^+}{s}\int\limits_{-\infty}^{\infty}dx^-V_{\underline{x}}[\infty,x^-]\,F^{12}(x^-,\underline{x})\;V_{\underline{x}}[x^-,-\infty].$$

Longitudinal Chromomagnetic Field

$$\sigma V^{\mathrm{q}[1]}_{\underline{x}}+V^{\mathrm{q}[2]}_{\underline{x}}=\frac{g^2P^+}{2\,s}\int\limits_{-\infty}^{\infty}dx_1^-\int\limits_{x_1^-}^{\infty}dx_2^-V_{\underline{x}}[\infty,x_2^-]t^b\psi(x_2^-,\underline{x})U^{ba}_{\underline{x}}[x_2^-,x_1^-]\left[\sigma\gamma^+\gamma^5-\gamma^+\right]\bar{\psi}(x_1^-,\underline{x})t^aV_{\underline{x}}[x_1^-,-\infty].$$

Transverse Chromoelectric Field

$$V^{\mathrm{G}[2]}_{\underline{x},\underline{y}}=-\frac{i\,P^+}{s}\int\limits_{-\infty}^{\infty}dz^-d^2z\;V_{\underline{x}}[\infty,z^-]\,\delta^2(\underline{x}-\underline{z})\,\overleftarrow{D}^i(z^-,\underline{z})\,\overrightarrow{D}^i(z^-,\underline{z})\,V_{\underline{y}}[z^-,-\infty]\,\delta^2(\underline{y}-\underline{z}).$$

$$\longrightarrow\quad V^{i,G[2]}_{\underline{x}}=\frac{igP^+}{s}\int_{-\infty}^{+\infty}dx^-x^-V_{\underline{x}}[\infty,x^-]F^{+i}(x^-,\underline{x})V_{\underline{x}}[x^-,-\infty],$$

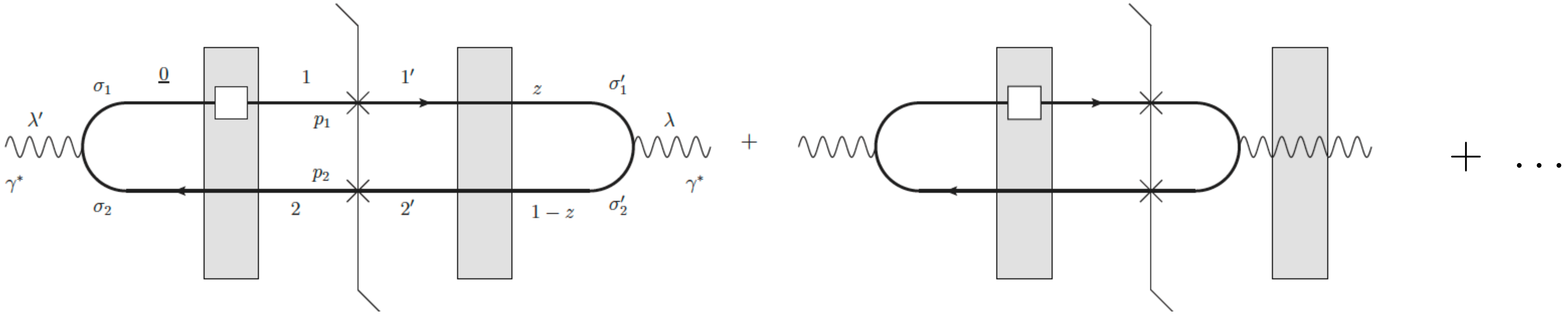
$$V^{\mathrm{G}[3]}_{\underline{x}}=-g\int\limits_{-\infty}^{\infty}dx^-V_{\underline{x}}[\infty,x^-]\,F^{+-}(x^-,\underline{x})\;V_{\underline{x}}[x^-,-\infty].$$

Longitudinal Chromoelectric Field

Kovchegov et al. (2015-2022), Chirilli (2019)
Altinoluk, Beuf, et al (2022, 2024), M. Li (2023)

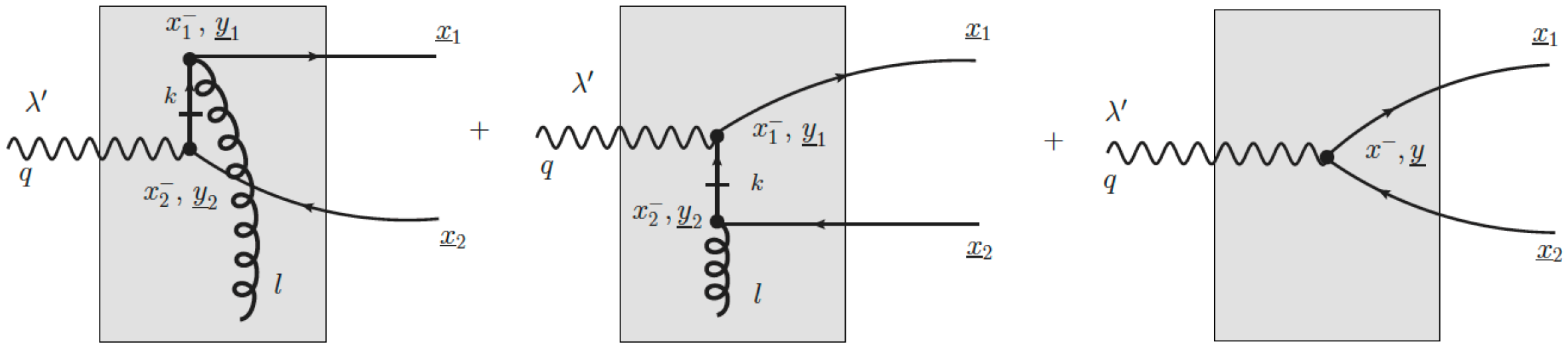
The Diagrams

- Virtual photon splitting before the shockwave



The subeikonal interactions can be associated with either the quark or the antiquark.

- Virtual photon splitting inside the shockwave



$$\sim V^{j,G[2]}_{\underline{x}_1} V^{\dagger}_{\underline{x}_2}\, \delta^2(\underline{x}_1 - \underline{x}_2)$$

Contributing as zero size quark-antiquark dipoles.

Effective photon splitting wavefunction $\Psi^{j,\gamma^*\to q\bar{q}}_{\lambda',\sigma_1,\sigma_2}(\underline{x}_{12}, z) = -2\, eZ_f \sqrt{z\,(1-z)}\, \delta_{\sigma_1,-\sigma_2}\, \delta_{\sigma_1,\lambda'}\, \epsilon^j_{\lambda'}\, \delta^2(\underline{x}_{12})$

The Differential Cross Section

$$
\begin{aligned}
&\sum_{\lambda=\pm 1} \lambda z(1-z) \frac{d\sigma_{\lambda\lambda}^{\gamma^* p \to q\bar{q} X}}{d^2 p_1 \, d^2 p_2 \, dz} = \frac{1}{2(2\pi)^5} \int d^2 x_1 \, d^2 x_{1'} \, d^2 x_2 \, d^2 x_{2'} \, e^{-i \underline{p}_1 \cdot \underline{x}_{11'} - i \underline{p}_2 \cdot \underline{x}_{22'}} \sum_{\lambda=\pm 1} \lambda \Bigg\{ \int d^2 x_0 \sum_{\sigma_1, \sigma_2, \sigma_1', \sigma_2', i} \\
&\times \Bigg[\Psi^{\gamma^* \to q\bar{q}}_{\lambda, \sigma_1, \sigma_2; i, i}(\underline{x}_{02}, z) \left[\Psi^{\gamma^* \to q\bar{q}}_{\lambda, \sigma_1', \sigma_2'; i, i}(\underline{x}_{1'2'}, z) \right]^* \frac{1}{N_c} \Bigg\langle \mathrm{tr} \Bigg[\mathrm{T} \left(V^{\mathrm{pol}}_{\underline{x}_1, \underline{x}_0; \sigma_1', \sigma_1} V^{\dagger}_{\underline{x}_2} \right) \bar{\mathrm{T}} \left(V_{\underline{x}_{2'}} V^{\dagger}_{\underline{x}_{1'}} - 1 \right) \Bigg] \Bigg\rangle (zs) \, \delta_{\sigma_2, \sigma_2'} \\
&- \Psi^{\gamma^* \to q\bar{q}}_{\lambda, \sigma_1, \sigma_2; i, i}(\underline{x}_{10}, z) \left[\Psi^{\gamma^* \to q\bar{q}}_{\lambda, \sigma_1', \sigma_2'; i, i}(\underline{x}_{1'2'}, z) \right]^* \frac{1}{N_c} \Bigg\langle \mathrm{tr} \Bigg[\mathrm{T} \left(V_{\underline{x}_1} V^{\mathrm{pol}\dagger}_{\underline{x}_2, \underline{x}_0; -\sigma_2', -\sigma_2} \right) \bar{\mathrm{T}} \left(V_{\underline{x}_{2'}} V^{\dagger}_{\underline{x}_{1'}} - 1 \right) \Bigg] \Bigg\rangle ((1-z)s) \, \delta_{\sigma_1, \sigma_1'} \\
&+ \Psi^{\gamma^* \to q\bar{q}}_{\lambda, \sigma_1, \sigma_2; i, i}(\underline{x}_{12}, z) \left[\Psi^{\gamma^* \to q\bar{q}}_{\lambda, \sigma_1', \sigma_2'; i, i}(\underline{x}_{02'}, z) \right]^* \frac{1}{N_c} \Bigg\langle \mathrm{tr} \Bigg[\mathrm{T} \left(V_{\underline{x}_1} V^{\dagger}_{\underline{x}_2} - 1 \right) \bar{\mathrm{T}} \left(V_{\underline{x}_{2'}} V^{\mathrm{pol}\dagger}_{\underline{x}_{1'}, \underline{x}_0; \sigma_1, \sigma_1'} \right) \Bigg] \Bigg\rangle (zs) \, \delta_{\sigma_2, \sigma_2'} \\
&- \Psi^{\gamma^* \to q\bar{q}}_{\lambda, \sigma_1, \sigma_2; i, i}(\underline{x}_{12}, z) \left[\Psi^{\gamma^* \to q\bar{q}}_{\lambda, \sigma_1', \sigma_2'; i, i}(\underline{x}_{1'0}, z) \right]^* \frac{1}{N_c} \Bigg\langle \mathrm{tr} \Bigg[\mathrm{T} \left(V_{\underline{x}_1} V^{\dagger}_{\underline{x}_2} - 1 \right) \bar{\mathrm{T}} \left(V^{\mathrm{pol}}_{\underline{x}_{2'}, \underline{x}_0; -\sigma_2, -\sigma_2'} V^{\dagger}_{\underline{x}_{1'}} \right) \Bigg] \Bigg\rangle ((1-z)s) \, \delta_{\sigma_1, \sigma_1'} \Bigg] \\
&+ \sum_{\sigma_1, \sigma_2, i} \Bigg[\Psi^{j, \gamma^* \to q\bar{q}}_{\lambda, \sigma_1, \sigma_2}(\underline{x}_{12}, z) \left[\Psi^{\gamma^* \to q\bar{q}}_{\lambda, \sigma_1, \sigma_2; i, i}(\underline{x}_{1'2'}, z) \right]^* \frac{1}{N_c} \Bigg\langle \mathrm{tr} \Bigg[\mathrm{T} \left(V^{j \, \mathrm{G}[2]}_{\underline{x}_1} V^{\dagger}_{\underline{x}_1} \right) \bar{\mathrm{T}} \left(V_{\underline{x}_{2'}} V^{\dagger}_{\underline{x}_{1'}} - 1 \right) \Bigg] \Bigg\rangle (zs) \\
&+ \Psi^{j, \gamma^* \to q\bar{q}}_{\lambda, \sigma_2, \sigma_1}(\underline{x}_{12}, z) \left[\Psi^{\gamma^* \to q\bar{q}}_{\lambda, \sigma_1, \sigma_2; i, i}(\underline{x}_{1'2'}, z) \right]^* \frac{1}{N_c} \Bigg\langle \mathrm{tr} \Bigg[\mathrm{T} \left(V_{\underline{x}_1} V^{j \, \mathrm{G}[2]\dagger}_{\underline{x}_1} \right) \bar{\mathrm{T}} \left(V_{\underline{x}_{2'}} V^{\dagger}_{\underline{x}_{1'}} - 1 \right) \Bigg] \Bigg\rangle ((1-z)s) \\
&+ \Psi^{\gamma^* \to q\bar{q}}_{\lambda, \sigma_1, \sigma_2; i, i}(\underline{x}_{12}, z) \left[\Psi^{j, \gamma^* \to q\bar{q}}_{\lambda, \sigma_1, \sigma_2}(\underline{x}_{1'2'}, z) \right]^* \frac{1}{N_c} \Bigg\langle \mathrm{tr} \Bigg[\mathrm{T} \left(V_{\underline{x}_1} V^{\dagger}_{\underline{x}_2} - 1 \right) \bar{\mathrm{T}} \left(V_{\underline{x}_{1'}} V^{j \, \mathrm{G}[2]\dagger}_{\underline{x}_{1'}} \right) \Bigg] \Bigg\rangle (zs) \\
&+ \Psi^{\gamma^* \to q\bar{q}}_{\lambda, \sigma_1, \sigma_2; i, i}(\underline{x}_{12}, z) \left[\Psi^{j, \gamma^* \to q\bar{q}}_{\lambda, \sigma_2, \sigma_1}(\underline{x}_{1'2'}, z) \right]^* \frac{1}{N_c} \Bigg\langle \mathrm{tr} \Bigg[\mathrm{T} \left(V_{\underline{x}_1} V^{\dagger}_{\underline{x}_2} - 1 \right) \bar{\mathrm{T}} \left(V^{j \, \mathrm{G}[2]}_{\underline{x}_{1'}} V^{\dagger}_{\underline{x}_{1'}} \right) \Bigg] \Bigg\rangle ((1-z)s) \Bigg] \Bigg\}.
\end{aligned}
$$

The Differential Cross Section

$$\sum_{\lambda=\pm1} \lambda\, z(1-z) \frac{d\sigma_{\lambda\lambda}^{\gamma^* p\to q\bar{q} X}}{d^2p_1\, d^2p_2\, dz} = \int_{\underline{x}_1,\underline{x}_2,\underline{x}_1',\underline{x}_2'} e^{-i\underline{p}_1\cdot(\underline{x}_1-\underline{x}_1')} e^{-i\underline{p}_2\cdot(\underline{x}_2-\underline{x}_2')} \mathcal{H}(x_{12},x_{12}';z)\otimes \mathcal{Q}^{\text{pol}}(x_1,x_2,x_1',x_2';zs).$$

- **Contains polarized quadrupoles.**

$$\mathcal{Q}^{\text{pol}}(x_1,x_2,x_1',x_2';zs) \supset \frac{1}{N_c}\Big\langle\!\Big\langle \text{tr}\left[V_{\underline{x}_1}^{\text{pol}[1]} V_{\underline{x}_2}^{\dagger}\left(V_{\underline{x}_2'} V_{\underline{x}_1'}^{\dagger} - 1\right)\right]\Big\rangle\!\Big\rangle.$$

- **Power suppressed in the back-to-back limit $\sim \Delta_\perp/p_T$**

$$\frac{1}{N_c}\Big\langle\!\Big\langle \text{tr}\left[V_{\underline{x}_1}^{q[2]} V_{\underline{x}_2}^{\dagger}\left(V_{\underline{x}_2'} V_{\underline{x}_1'}^{\dagger} - 1\right)\right]\Big\rangle\!\Big\rangle.$$

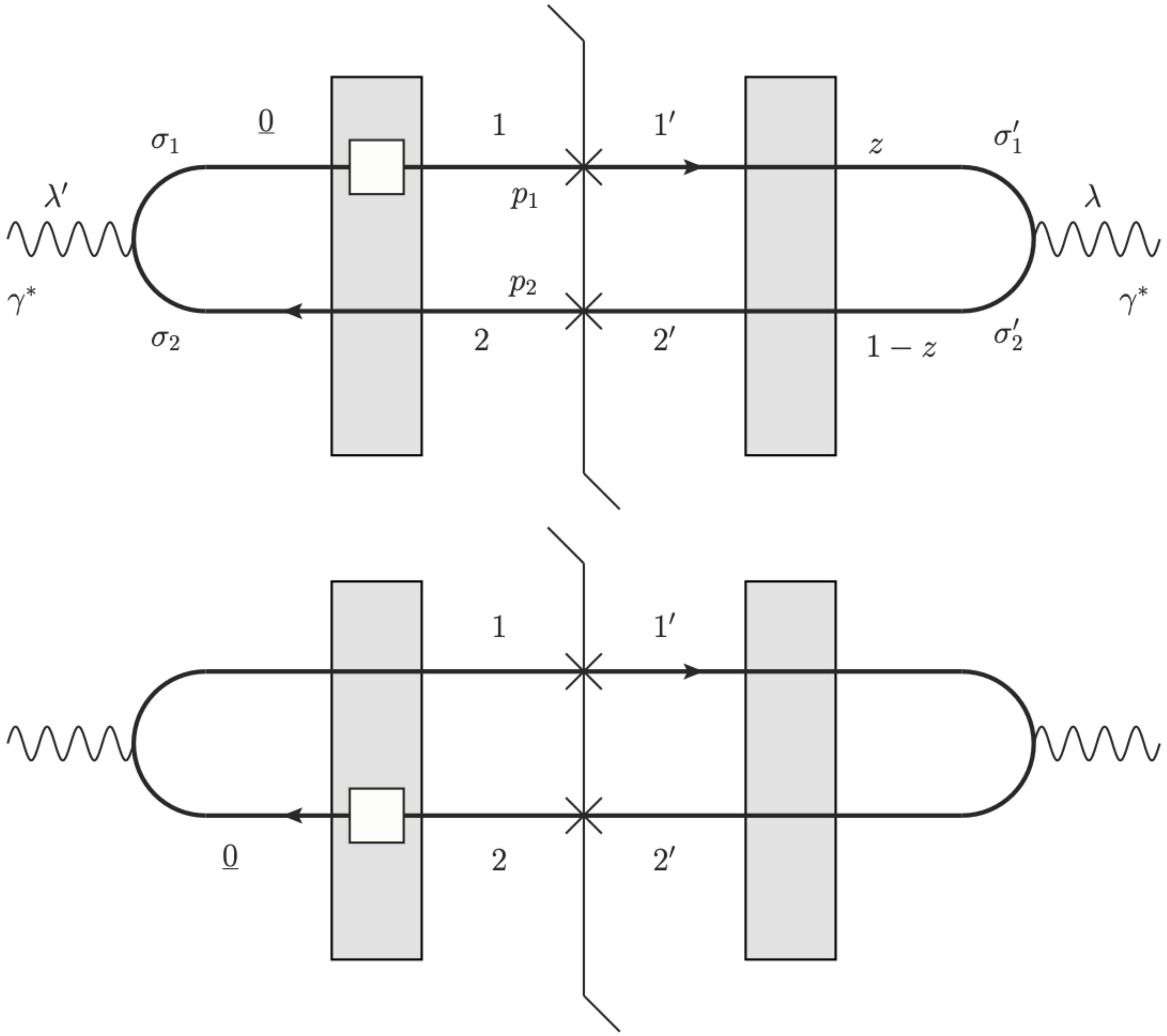
$$\boxed{\frac{1}{N_c}\Big\langle\!\Big\langle \text{tr}\left[V_{\underline{x}_1}^{j,G[2]} V_{\underline{x}_2}^{\dagger}\left(V_{\underline{x}_2'} V_{\underline{x}_1'}^{\dagger} - 1\right)\right]\Big\rangle\!\Big\rangle.}$$

- **Contributes to the leading order in the back-to-bak limit.**

$$\frac{1}{N_c}\Big\langle\!\Big\langle \text{tr}\left[V_{\underline{x}_1}^{G[3]} V_{\underline{x}_2}^{\dagger}\left(V_{\underline{x}_2'} V_{\underline{x}_1'}^{\dagger} - 1\right)\right]\Big\rangle\!\Big\rangle.$$

- **Insensitive to spin, none double-log contribution**

Subtleties on the Quark and Antiquark Longitudinal Momenta



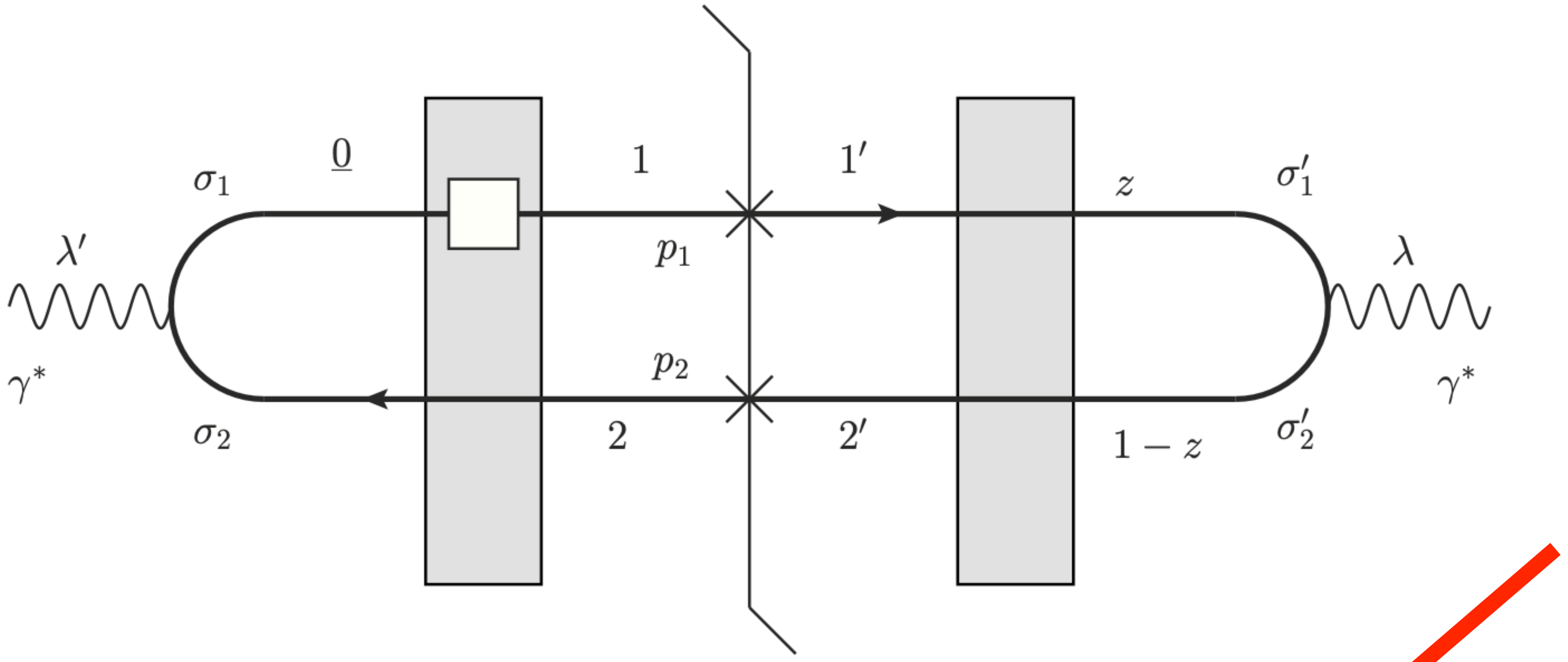
$$\frac{1}{N_c}\left\langle \mathrm{tr}\left[\mathrm{T}\left(V_{\underline{x}_1}^{j\,\mathrm{G}[2]}\, V_{\underline{x}_2}^{\dagger}\right)\bar{\mathrm{T}}\left(V_{\underline{x}_{2'}},\, V_{\underline{x}_{1'}}^{\dagger},\, -1\right)\right]\right\rangle(zs)$$

$$\frac{1}{N_c}\left\langle \mathrm{tr}\left[\mathrm{T}\left(V_{\underline{x}_1}\, V_{\underline{x}_2}^{j\,\mathrm{G}[2]\,\dagger}\right)\bar{\mathrm{T}}\left(V_{\underline{x}_{2'}},\, V_{\underline{x}_{1'}}^{\dagger},\, -1\right)\right]\right\rangle((1-z)s)$$

- **Different dependences on energy, cannot add up naively.**

$$\boxed{z \sim 1 - z \sim \mathcal{O}(1)}$$

Subtleties on Quark and Antiquark Longitudinal Momenta



- Vanishes when $z \sim 1 - z \sim \frac{1}{2}$

$$\sum_{\lambda=\pm1} \lambda z(1-z) \frac{d\sigma_{\lambda\lambda}^{[1]\gamma^* p\to q\bar{q}X}}{d^2p d^2\Delta dz} \approx \frac{\alpha_{EM} Z_f^2}{(2\pi)^6 s} 4\underline{(1-2z)}z(1-z) \int d^2b d^2b' d^2x_{12} d^2x_{1'2'} e^{-i\underline{p}\cdot(\underline{x}_{12}-\underline{x}_{1'2'})-i\underline{\Delta}\cdot(\underline{b}-\underline{b}')}$$
$$\times a_f^2 \frac{\underline{x}_{12}\cdot\underline{x}_{1'2'}}{x_{12}x_{1'2'}} K_1(x_{12}a_f) K_1(x_{1'2'}a_f) \Bigg\{ \frac{x^i_{2'1'}}{z} \Big\langle\Big\langle \mathrm{tr}\Big[\mathrm{T}\Big(V_{\underline{b}}^{\mathrm{pol}[1]} V_{\underline{b}}^{\dagger}\Big) \bar{\mathrm{T}}\Big(V_{\underline{b}'}\Big(\partial^i V_{\underline{b}'}^{\dagger}\Big)\Big)\Big]\Big\rangle\Big\rangle(zs)$$
$$-\frac{x^i_{2'1'}}{1-z} \Big\langle\Big\langle \mathrm{tr}\Big[\mathrm{T}\Big(V_{\underline{b}} V_{\underline{b}}^{\mathrm{pol}[1]\dagger}\Big) \bar{\mathrm{T}}\Big(V_{\underline{b}'}\Big(\partial^i V_{\underline{b}'}^{\dagger}\Big)\Big)\Big]\Big\rangle\Big\rangle((1-z)s) + \frac{x^i_{12}}{z} \Big\langle\Big\langle \mathrm{tr}\Big[\mathrm{T}\Big(V_{\underline{b}}\Big(\partial^i V_{\underline{b}}^{\dagger}\Big)\Big) \bar{\mathrm{T}}\Big(V_{\underline{b}'} V_{\underline{b}'}^{\mathrm{pol}[1]\dagger}\Big)\Big]\Big\rangle\Big\rangle(zs)$$
$$-\frac{x^i_{12}}{1-z} \Big\langle\Big\langle \mathrm{tr}\Big[\mathrm{T}\Big(V_{\underline{b}}\Big(\partial^i V_{\underline{b}}^{\dagger}\Big)\Big) \bar{\mathrm{T}}\Big(V_{\underline{b}'}^{\mathrm{pol}[1]} V_{\underline{b}'}^{\dagger}\Big)\Big]\Big\rangle\Big\rangle((1-z)s) \Bigg\},$$

Probing WW Gluon Helicity Distribution

- **Expanding around the back-to-back limit:**

$$\boxed{\begin{aligned} \underline{x}_1 &= \underline{b} + (1-z)\,\underline{x}_{12}, \quad & \underline{x}_2 &= \underline{b} - z\,\underline{x}_{12}. \\ \underline{x}_1' &= \underline{b}' + (1-z)\,\underline{x}_{12}', \quad & \underline{x}_2' &= \underline{b}' - z\,\underline{x}_{12}'. \end{aligned}}$$

$$\underline{\Delta} \equiv \underline{p}_1 + \underline{p}_2, \quad \underline{p} \equiv (1-z)\,\underline{p}_1 - z\,\underline{p}_2.$$

$$\frac{1}{N_c}\left\langle\left\langle \mathrm{tr}\left[V_{\underline{x}_1} V_{\underline{x}_2}^{j,\mathrm{G}[2]\dagger}\left(V_{\underline{x}_2'} V_{\underline{x}_1'}^{\dagger} - 1\right)\right]\right\rangle\right\rangle \simeq -x_{12}'^{i}\frac{1}{N_c}\left\langle\left\langle \mathrm{tr}\left[V_{\underline{b}} V_{\underline{b}}^{j,\mathrm{G}[2]\dagger}\left(V_{\underline{b}'}\partial^i V_{\underline{b}'}^{\dagger}\right)\right]\right\rangle\right\rangle + \ldots$$

$$\boxed{\sum_{\lambda=\pm 1}\lambda\, z(1-z)\,\frac{d\sigma_{\lambda\lambda}^{\gamma^* p\to q\bar{q}X}}{d^2p\, d^2\Delta\, dz} \approx -\frac{\alpha_s}{2\pi\, s}\,(eZ_f)^2\,\left[z^2+(1-z)^2\right]\,\frac{p_T^2 - a_f^2}{(p_T^2+a_f^2)^2}\; g_{1L}^{G,\,WW}\left(x \approx \frac{p_T^2}{s}, \Delta_T^2\right).}$$

In the back-to-back limit, the A_{LL} for inclusive
dijet production probes the WW gluon helicity TMD.

$$\boxed{g_{1L}^{G,\,WW}(x,\Delta_T^2) = \frac{1}{\alpha_s 4\pi^4}\int d^2b\, d^2b'\, e^{-i\underline{\Delta}\cdot(\underline{b}'-\underline{b})}\,\epsilon^{ji}\,\mathrm{Re}\left\langle\left\langle \mathrm{tr}\left[V_{\underline{b}}\partial^i V_{\underline{b}}^{\dagger}\, V_{\underline{b}'}^{j\mathrm{G}[2]}\, V_{\underline{b}'}^{\dagger}\right]\right\rangle\right\rangle}$$

Linearly Polarized Gluon Distribution

We did not find linearly polarized gluon distribution in longitudinally polarized proton in the back-to-back limit.

$$\Gamma_L^{ij}(x,\underline{k}) \equiv \frac{4}{xP^+}\frac{1}{2}\sum_{S_L} S_L \int \frac{d\xi^- d^2\xi}{(2\pi)^3} e^{ixP^+\xi^-} e^{-i\underline{k}\cdot\underline{\xi}} \left\langle P,S_L\right| \mathrm{Tr}\left[F^{+i}(0)\mathcal{U}[0,\xi]F^{+j}(\xi)\widetilde{\mathcal{U}}[\xi,0]\right]\left|P,S_L\right\rangle\Big|_{\xi^+=0}$$

$$= i\epsilon^{ij} g_{1L}^G(x,k_T^2) + \frac{(\epsilon^{li}k^j+\epsilon^{lj}k^i)k^l}{k_T^2} h_{1L}^{\perp G}(x,k_T^2)$$

$$\underline{-\frac{\epsilon^{ij}}{p_T^2+a_f^2} - \frac{2\epsilon^{jl}p^ip^l}{(p_T^2+a_f^2)^2}} + \underline{\frac{2\epsilon^{il}p^jp^l}{(p_T^2+a_f^2)^2}} = \frac{p_T^2-a_f^2}{(p_T^2+a_f^2)^2}\epsilon^{ij}$$

Photon splitting inside shockwave **Photon splitting outside shockwave**

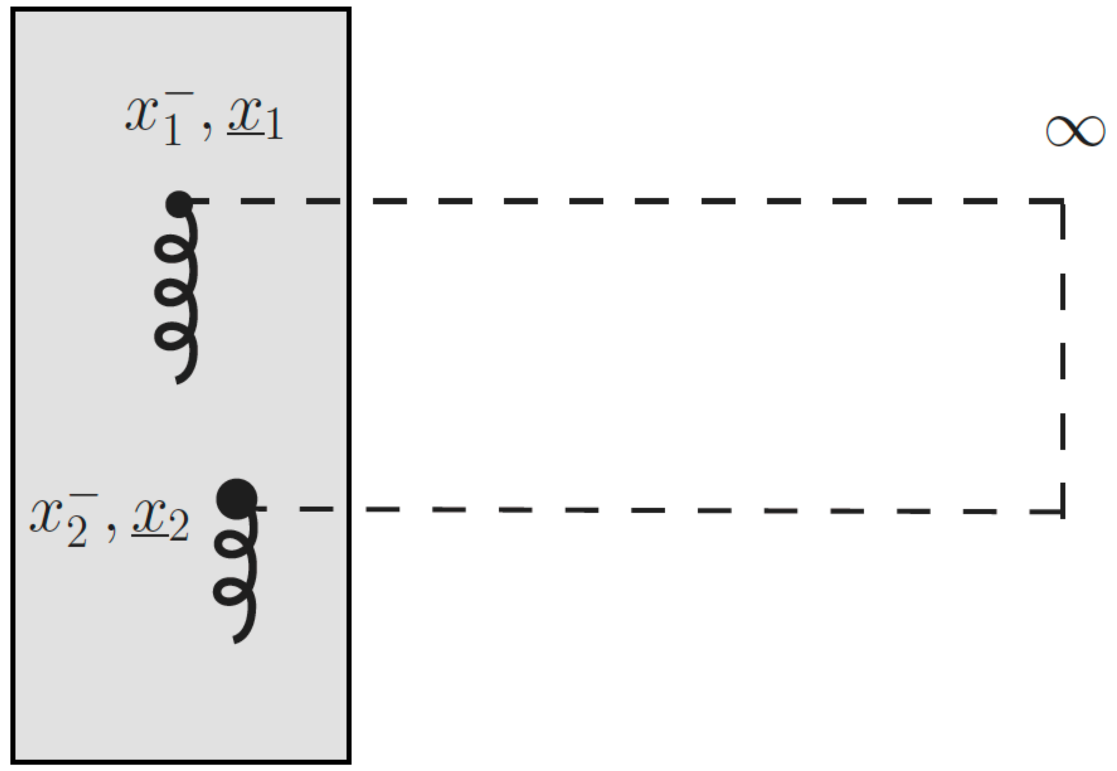
$$(\epsilon^{li}p^j - \epsilon^{lj}p^i)p^l = -p_T^2\epsilon^{ij}.$$

Small x Evolution Equation for WW Gluon Helicity Distribution

- **The operator characterizing WW gluon helicity distribution:**

$$\boxed{O(\underline{x}_1,\underline{x}_2) = \frac{1}{2N_c}\epsilon^{ji}\left\langle\!\left\langle \mathrm{tr}\left[V_{\underline{x}_1}\partial^i V^\dagger_{\underline{x}_1} V^{j\,\mathrm{G}[2]}_{\underline{x}_2} V^\dagger_{\underline{x}_2}\right]\right\rangle\!\right\rangle + c.c.}$$

➡ **Diagrammatic representation of the operator:**



$$\boxed{V_{\underline{x}_1}\partial^i V^\dagger_{\underline{x}_1} = -ig\int_{-\infty}^{\infty} dx_1^-\, \partial^i A_a^+(x_1^-,\underline{x}_1)\, U^{\dagger ab}_{\underline{x}_1}[\infty, x_1^-] t^b,}$$

$$\boxed{V^{j\,\mathrm{G}[2]}_{\underline{x}_2} V^\dagger_{\underline{x}_2} = \frac{-igP^+}{s}\int_{-\infty}^{\infty} dx_2^- \,\left(x_2^-\partial^j A_c^+(x_2^-,\underline{x}_2) + A_c^j(x_2^-,\underline{x}_2)\right)\, U^{\dagger cd}_{\underline{x}_2}[\infty, x_2^-] t^d,}$$

- **Wilsonian approach to renormalization group equation:**

➡ **Averaging out quantum fluctuating fields.**

$$O[A^+ + a^+, A^i + a^i] = O[A^+, A^i] + a^+(x)\frac{\delta O}{\delta A^+(x)} + a^i(x)\frac{\delta O}{\delta A^i(x)} + \frac{1}{2}a^+(x)a^+(y)\frac{\delta^2 O}{\delta A^+(x)\delta A^+(y)}$$
$$+ a^+(x)a^i(y)\frac{\delta^2 O}{\delta A^+(x)\delta A^i(y)} + \frac{1}{2}a^i(x)a^j(y)\frac{\delta^2 O}{\delta A^i(x)\delta A^j(y)} \cdots$$

Kovchegov and Sievert (2019)

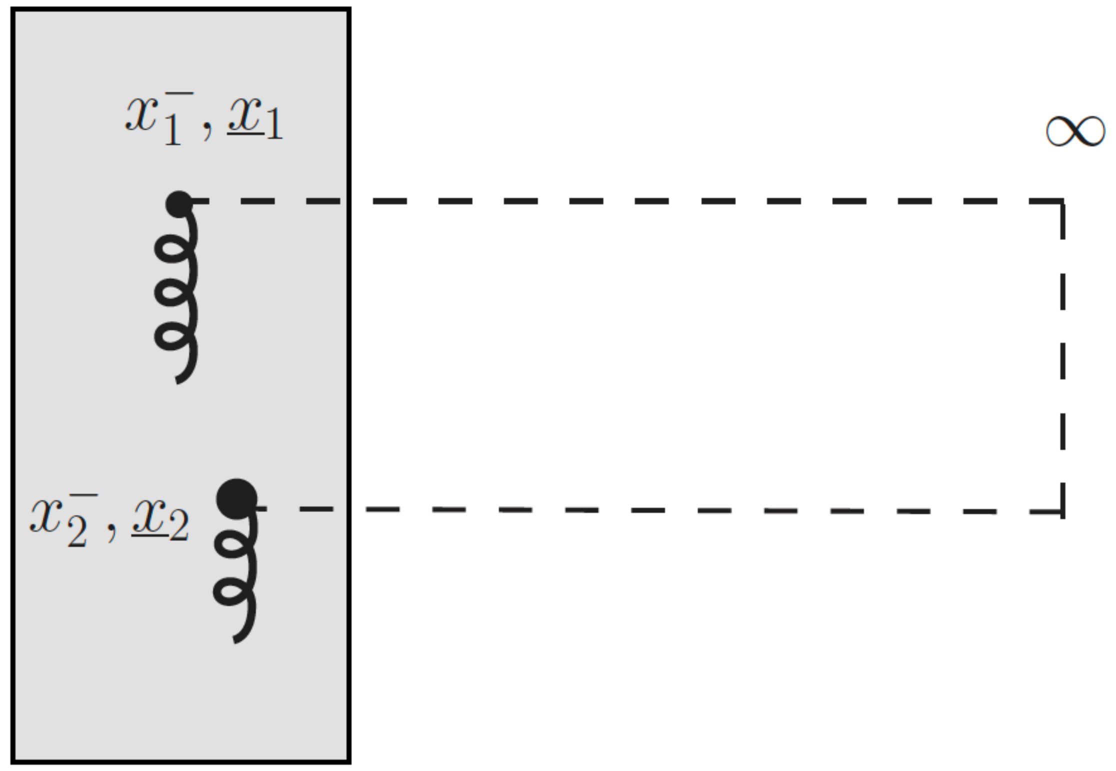
Balitsky (1996)

Deriving the Small x Evolution Equation for WW Gluon Helicity Distribution

- **The operator characterizing WW gluon helicity distribution:**

$$\boxed{O(\underline{x}_1,\underline{x}_2) = \frac{1}{2N_c}\epsilon^{ji}\Big\langle\Big\langle \mathrm{tr}\Big[V_{\underline{x}_1}\partial^i V^\dagger_{\underline{x}_1} V^{j\,\mathrm{G}[2]}_{\underline{x}_2} V^\dagger_{\underline{x}_2}\Big]\Big\rangle\Big\rangle + c.c.}$$

➡ **Diagrammatic representation of the operator:**



$$\boxed{V_{\underline{x}_1}\partial^i V^\dagger_{\underline{x}_1} = -ig\int_{-\infty}^{\infty} dx_1^-\;\partial^i A_a^+(x_1^-,\underline{x}_1)\,U^{\dagger ab}_{\underline{x}_1}[\infty,x_1^-]t^b,}$$

$$\boxed{V^{j\,\mathrm{G}[2]}_{\underline{x}_2}V^\dagger_{\underline{x}_2} = \frac{-igP^+}{s}\int_{-\infty}^{\infty} dx_2^-\;\left(x_2^-\partial^j A_c^+(x_2^-,\underline{x}_2) + A_c^j(x_2^-,\underline{x}_2)\right)\,U^{\dagger cd}_{\underline{x}_2}[\infty,x_2^-]t^d,}$$

- **Wilsonian approach to renormalization group equation:**

➡ **Averaging out quantum fluctuating fields.**

$$O[A^+ + a^+, A^i + a^i] = O[A^+, A^i] + \cancel{a^+(x)\frac{\delta O}{\delta A^+(x)}} + \cancel{a^i(x)\frac{\delta O}{\delta A^i(x)}} + \frac{1}{2}a^+(x)a^+(y)\frac{\delta^2 O}{\delta A^+(x)\delta A^+(y)}$$

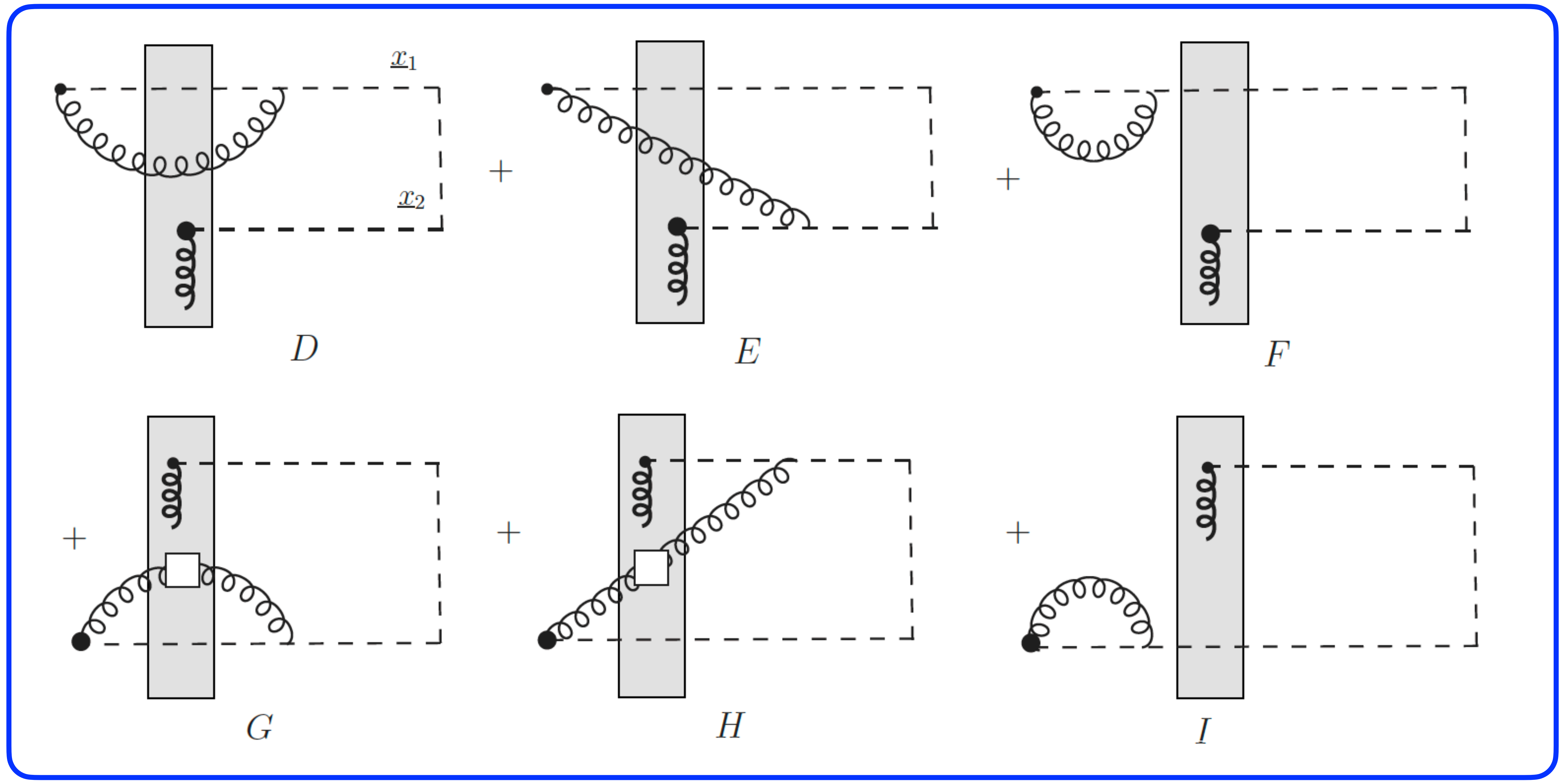
$$+\, a^+(x)a^i(y)\frac{\delta^2 O}{\delta A^+(x)\delta A^i(y)} + \frac{1}{2}a^i(x)\cancel{a^j(y)\frac{\delta^2 O}{\delta A^i(x)\delta A^j(y)}}\;\cdots$$

Kovchegov and Sievert (2019)

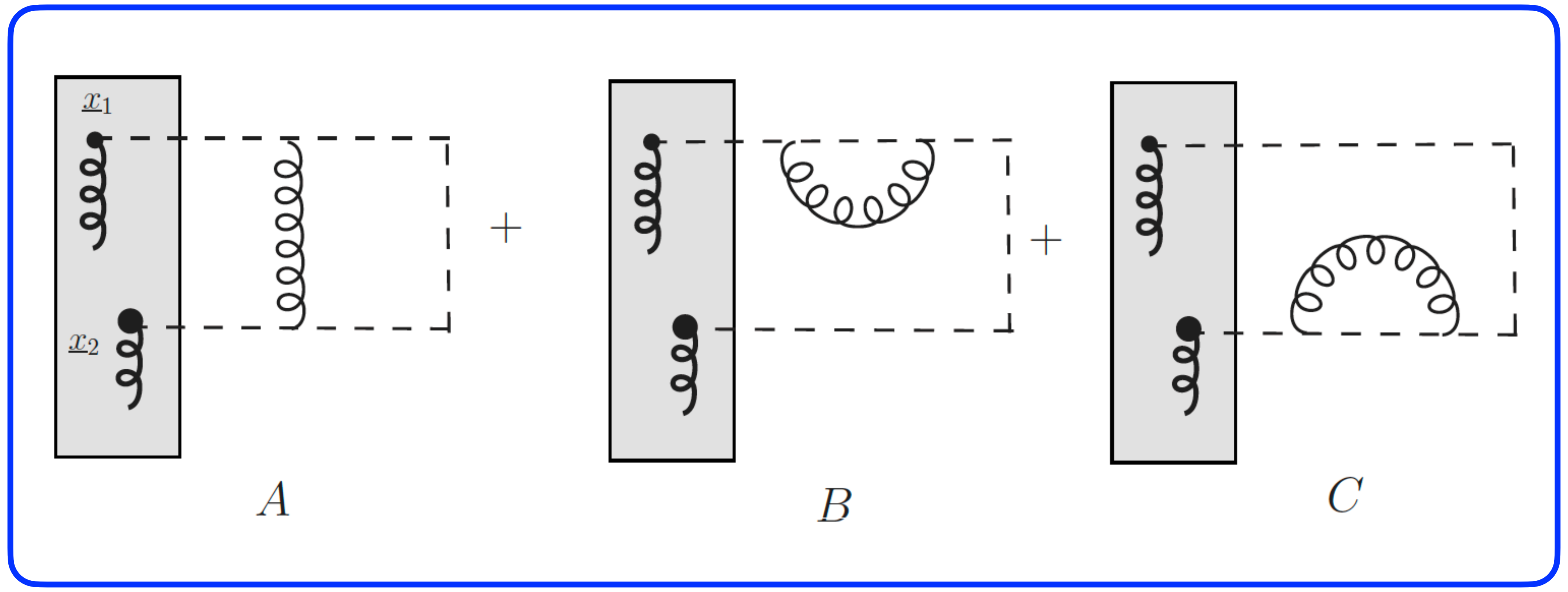
Balitsky (1996)

The Diagrams for Small x Evolution Equations

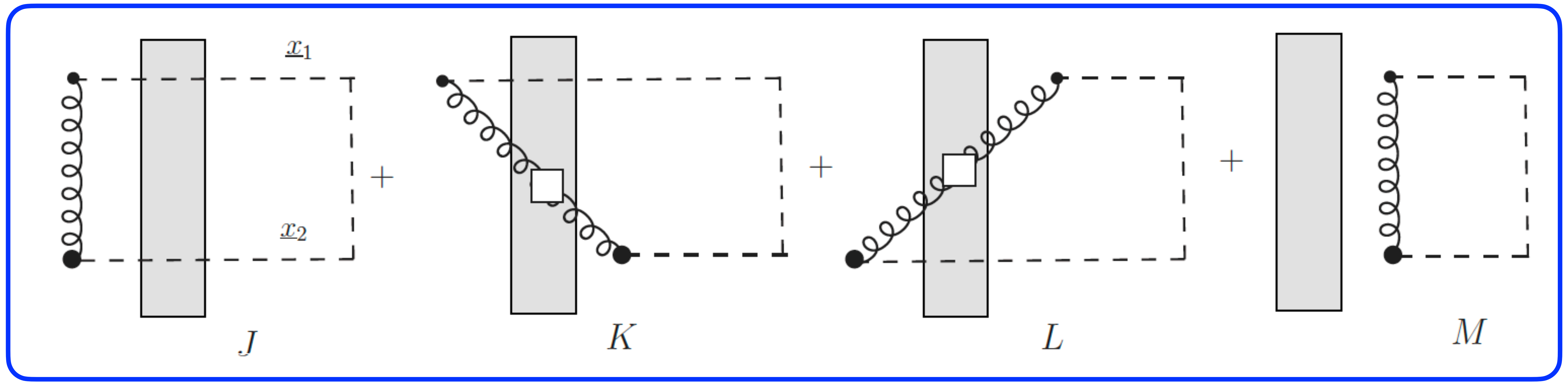
- **Real Diagrams I**



- **Virtual diagrams**



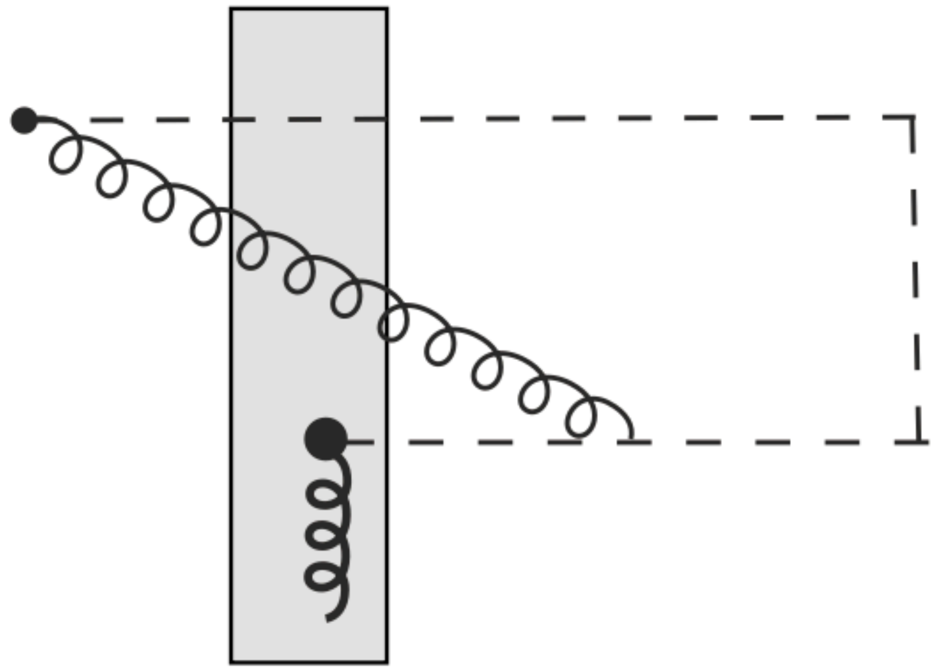
- **Real Diagrams II**



Two-Point Correlation Functions in the Background Fields

- **Eikonal Order**

$$\int_{-\infty}^{0} dx^-_{2'} \int_{0}^{\infty} dx^-_2 \; \overbracket{a^{+\,a}(x^-_{2'},\underline{x}_1)\; a}{}^{+\,b}(x^-_2,\underline{x}_0) = -\frac{1}{4\pi^3}\int_{\Lambda^2/s}^{z}\frac{dz'}{z'}\int d^2x_2\, U^{ba}_{\underline{x}_2}\,\frac{\underline{x}_{21}\cdot\underline{x}_{20}}{x^2_{21}\,x^2_{20}}.$$



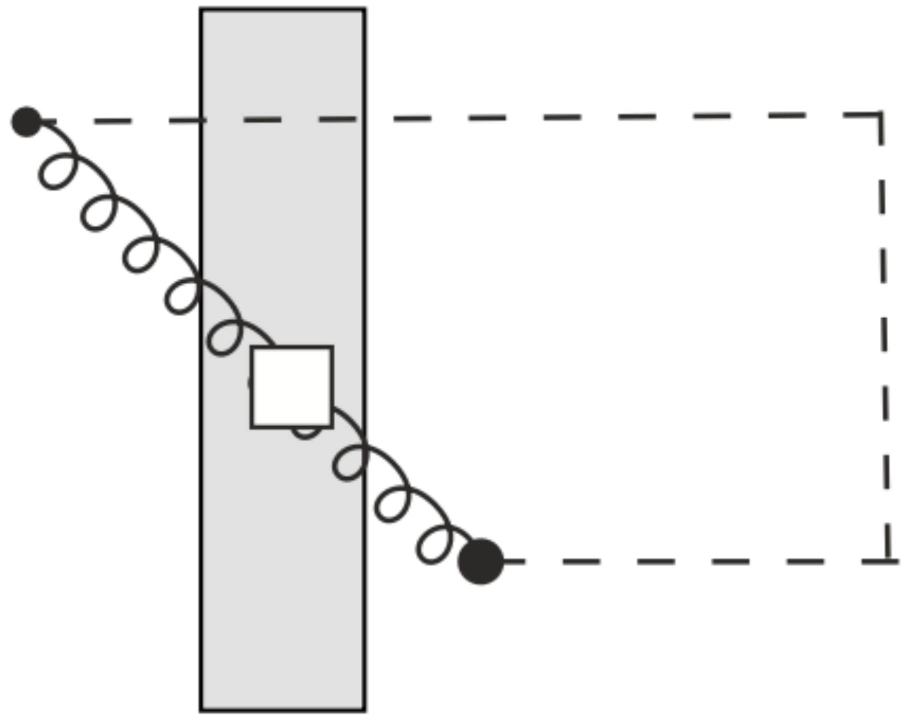
E

- **Subeikona Order**

$$\int_{-\infty}^{0} dx^-_{2'} \int_{0}^{\infty} dx^-_2 \,\Big[x^-_2\,\partial^i a^{+b}(x^-_2,\underline{x}_1) + \overbracket{a^{ib}(x^-_2,\underline{x}_1)\Big]\, a}{}^{+\,a}(x^-_{2'},\underline{x}_0)$$

$$= \frac{1}{(2\pi)^3}\int_0^{p_2^-} dk^- \Bigg\{\int d^2x_2\left[\frac{\epsilon^{ij}x^j_{20}}{x^2_{20}} - 2x^i_{21}\,\frac{\underline{x}_{21}\times\underline{x}_{20}}{x^2_{21}\,x^2_{20}}\right]\left(U^{\mathrm{pol}[1]}_{\underline{x}_2}\right)^{ba} + i\int d^2x_2\, d^2x_{2'}\left[\frac{x^i_{2'0}}{x^2_{2'0}} - 2x^i_{21}\,\frac{\underline{x}_{21}\cdot\underline{x}_{2'0}}{x^2_{21}\,x^2_{2'0}}\right]\left(U^{\mathrm{pol}[2]}_{\underline{x}_2,\underline{x}_{2'}}\right)^{ba}$$

$$-\int d^2x_2\,\frac{x^j_{20}}{x^2_{20}}\left(\delta^{ij} - 2\,\frac{x^i_{21}\,x^j_{21}}{x^2_{21}}\right)\left(U^{\mathrm{G}[3]}_{\underline{x}_2}\right)^{ba}\Bigg\}.$$



K

Small x Evolution Equation in the Double-Logarithmic-Approximation (DLA)

- **General form of the Small x Evolution Equation**

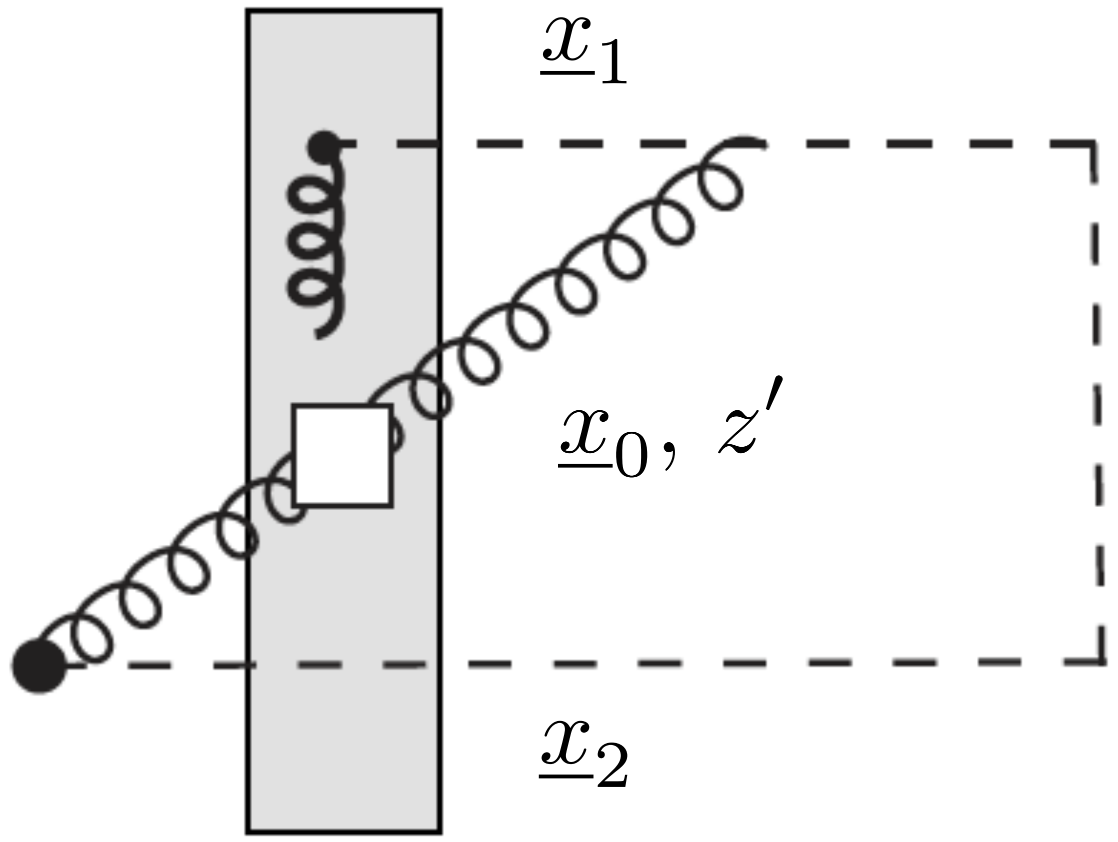
Kernel functions

New operators

$$O(\underline{x}_1, \underline{x}_2; zs) = O(\underline{x}_1, \underline{x}_2; z_0 s) - \frac{\alpha_s N_c}{\pi^2} \int_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int d^2 x_0 \sum_i \mathcal{K}_{[i]}(\underline{x}_{10}, \underline{x}_{20})\, O_{[i]}(\underline{x}_1, \underline{x}_2, \underline{x}_0; z's).$$

Integrations over transverse coordinates are constrained by life-time ordering and UV cut-off.

$$\boxed{\theta(z x_{12}^2 - z' x_{20}^2), \quad \min\left\{x_{10}^2,\, x_{20}^2\right\} > 1/z's.}$$



$$O_{[i]}(\underline{x}_1, \underline{x}_2, \underline{x}_0; z's) \supset \frac{1}{N_c}\left\langle\left\langle \mathrm{tr}\left[V_{\underline{x}_2}^{j,G[2]} V_{\underline{x}_2}^{\dagger} V_{\underline{x}_1} V_{\underline{x}_0}^{\dagger}\right]\right\rangle\right\rangle \frac{1}{N_c}\left\langle \mathrm{tr}\left[V_{\underline{x}_0} V_{\underline{x}_1}^{\dagger}\right]\right\rangle$$

$$\frac{1}{N_c}\left\langle \mathrm{tr}\left[V_{\underline{x}_1} \partial^i V_{\underline{x}_1}^{\dagger} V_{\underline{x}_0} V_{\underline{x}_2}^{\dagger}\right]\right\rangle \frac{1}{N_c}\left\langle\left\langle \mathrm{tr}\left[V_{\underline{x}_0}^{j,\mathrm{G}[2]\dagger} V_{\underline{x}_2}\right]\right\rangle\right\rangle$$

$$\frac{1}{N_c}\left\langle\left\langle \mathrm{tr}\left[V_{\underline{x}_0}^{j,\mathrm{G}[2]} V_{\underline{x}_1}^{\dagger}\right]\right\rangle\right\rangle \frac{1}{N_c}\left\langle \mathrm{tr}\left[V_{\underline{x}_1} V_{\underline{x}_0}^{\dagger}\right]\right\rangle$$

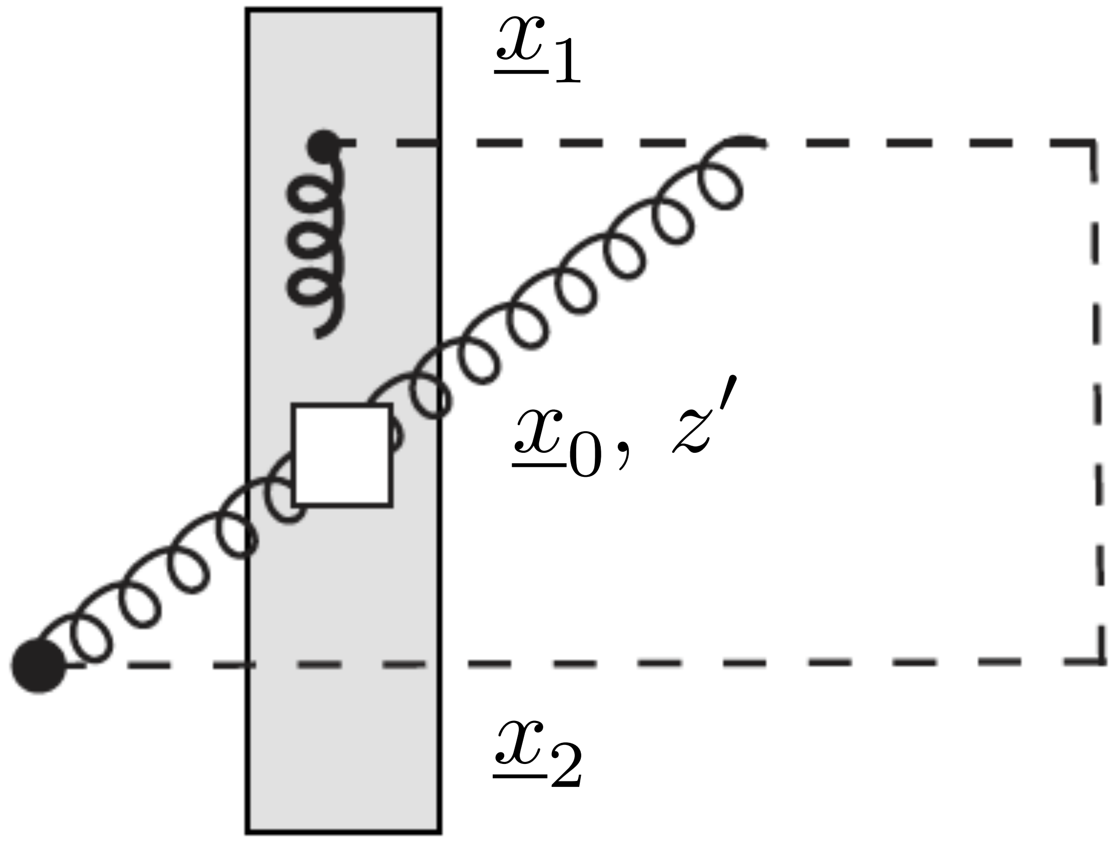
$$\ldots$$

➡ **In the large-N_c limit, the general small x evolution equation for WW gluon helicity distribution is not closed.**

Small x Evolution Equation in the Double-Logarithmic-Approximation (DLA)

- **General form of the Small x Evolution Equation**

$$\boxed{O(\underline{x}_1,\underline{x}_2;zs) = O(\underline{x}_1,\underline{x}_2;z_0 s) - \frac{\alpha_s N_c}{\pi^2}\int_{\Lambda^2/s}^{z}\frac{dz'}{z'}\int d^2x_0 \sum_i \mathcal{K}_{[i]}(\underline{x}_{10},\underline{x}_{20})\, O_{[i]}(\underline{x}_1,\underline{x}_2,\underline{x}_0;z's).}$$



- **Further Simplifications**

1. Take large-N_c limit.
2. Integrate over impact parameter.
3. Consider the dilute limit, no gluon saturation effect.
4. Only consider terms dominated in the DLA.

Resuming Double-Logs

$$\alpha_s \ln^2\frac{1}{x},\ \ \alpha_s \ln\frac{1}{x}\ln\frac{Q^2}{\Lambda^2}.$$

Three regions in the transverse coordinate integrations:

$\underline{x}_0 \to \underline{x}_1,$ → UV Logs.

$\underline{x}_0 \to \underline{x}_2,$ → UV Logs.

$x_{20} \sim x_{10} \gg x_{12}.$ → IR Logs.

Small x Evolution Equation in the Double-Logarithmic-Approximation (DLA)

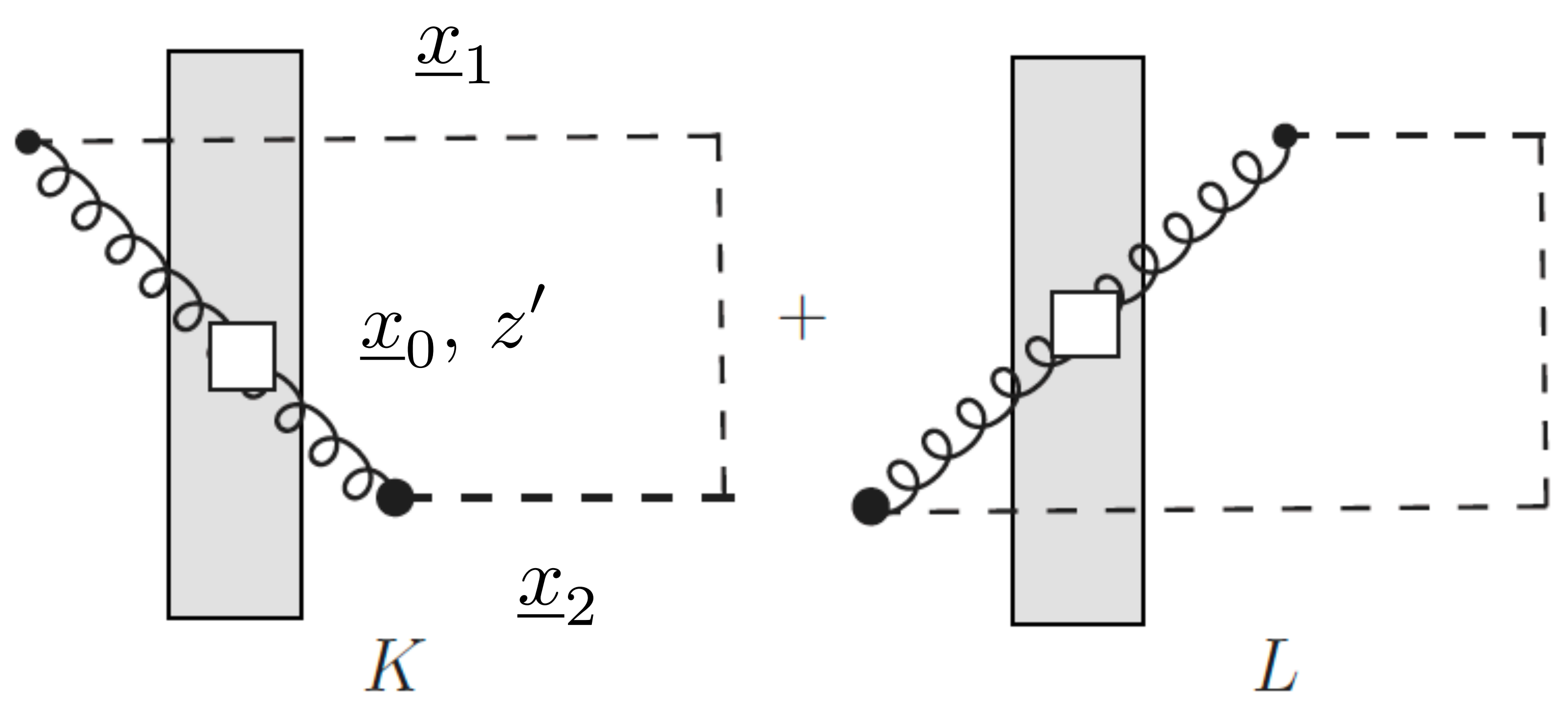
- **Diagrammatically speaking, the DLA of the diagrams *A, B, C, D, E, G, H* cancel out. Only the diagrams *K, L* contribute in the DLA.**

$$G_W(x_{12}^2, zs) = G_W(x_{12}^2, z_0 s) + \frac{\alpha_s N_c}{\pi} \int\limits_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int\limits_{\max\left\{x_{12}^2, \frac{1}{z's}\right\}}^{\min\left\{\frac{z}{z'}x_{12}^2, \frac{1}{\Lambda^2}\right\}} \frac{dx_{10}^2}{x_{10}^2} \Big[G(x_{10}^2, z's) + 2G_2(x_{10}^2, z's)\Big].$$

➡ **Exactly the same equation as** $G_2(x_{12}^2, zs)$.

Cougoulic, Kovchegov, Tarasov and Tawabutr (2022)

➡ **In the DLA, the small x evolution equations for WW gluon helicity distribution and the dipole gluon helicity distribution are the same.**



Polarized Wilson Line Dipole

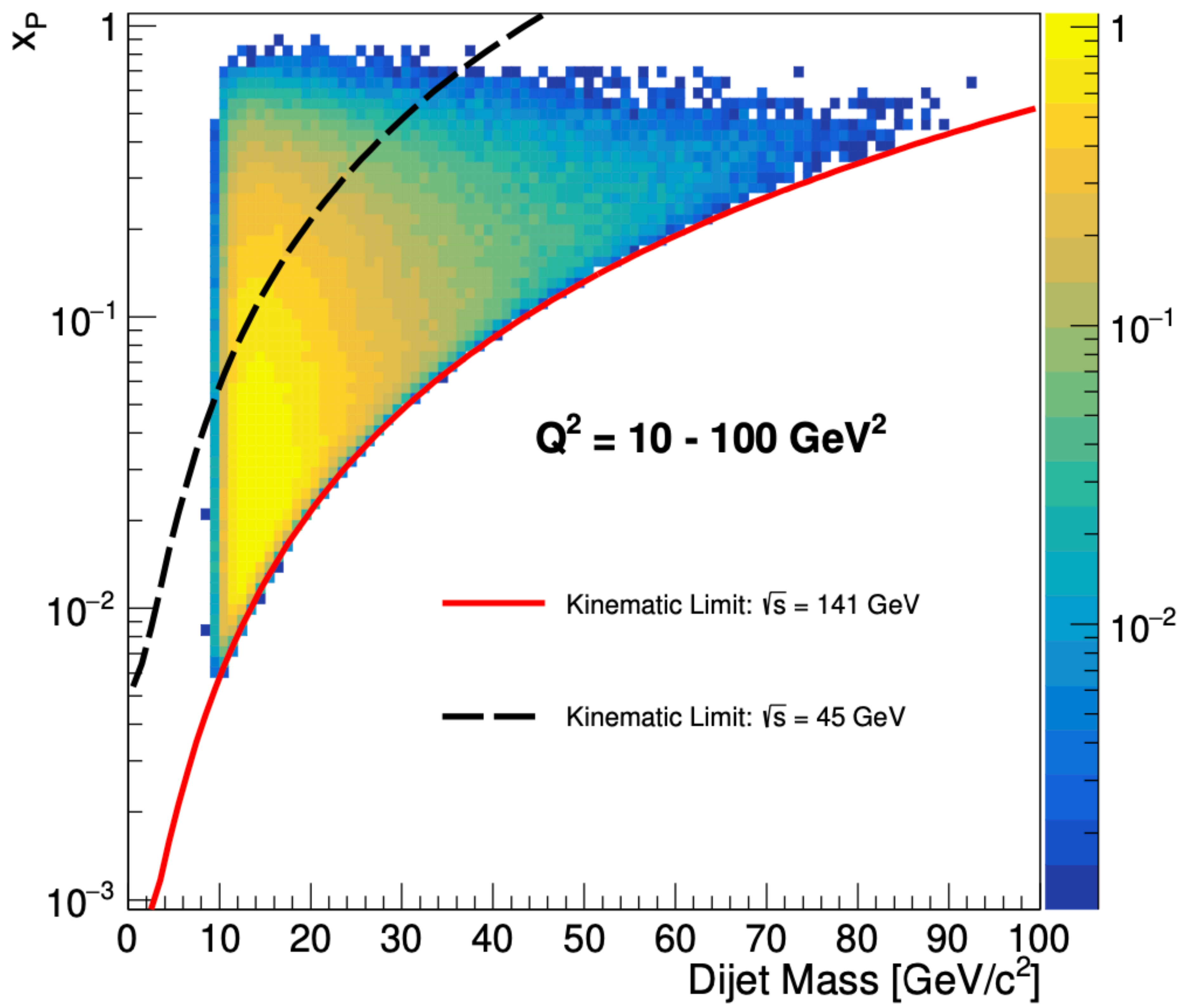
$$G_{10}^i(zs) = \frac{1}{2N_c}\Big\langle\Big\langle \mathrm{tr}\left[V_{\underline{x}_0}^{\dagger} V_{\underline{x}_1}^{i,G[2]}\right] + \mathrm{tr}\left[V_{\underline{x}_1}^{i,G[2]\dagger} V_{\underline{x}_0}\right] \Big\rangle\Big\rangle(zs).$$

$$\int d^2\left(\frac{\underline{x}_0 + \underline{x}_1}{2}\right) G_{10}^i(zs) = x_{10}^i\, G_1(x_{10}^2, zs) + \epsilon^{ij} x_{10}^j\, G_2(x_{10}^2, zs).$$

Feasibility at EIC

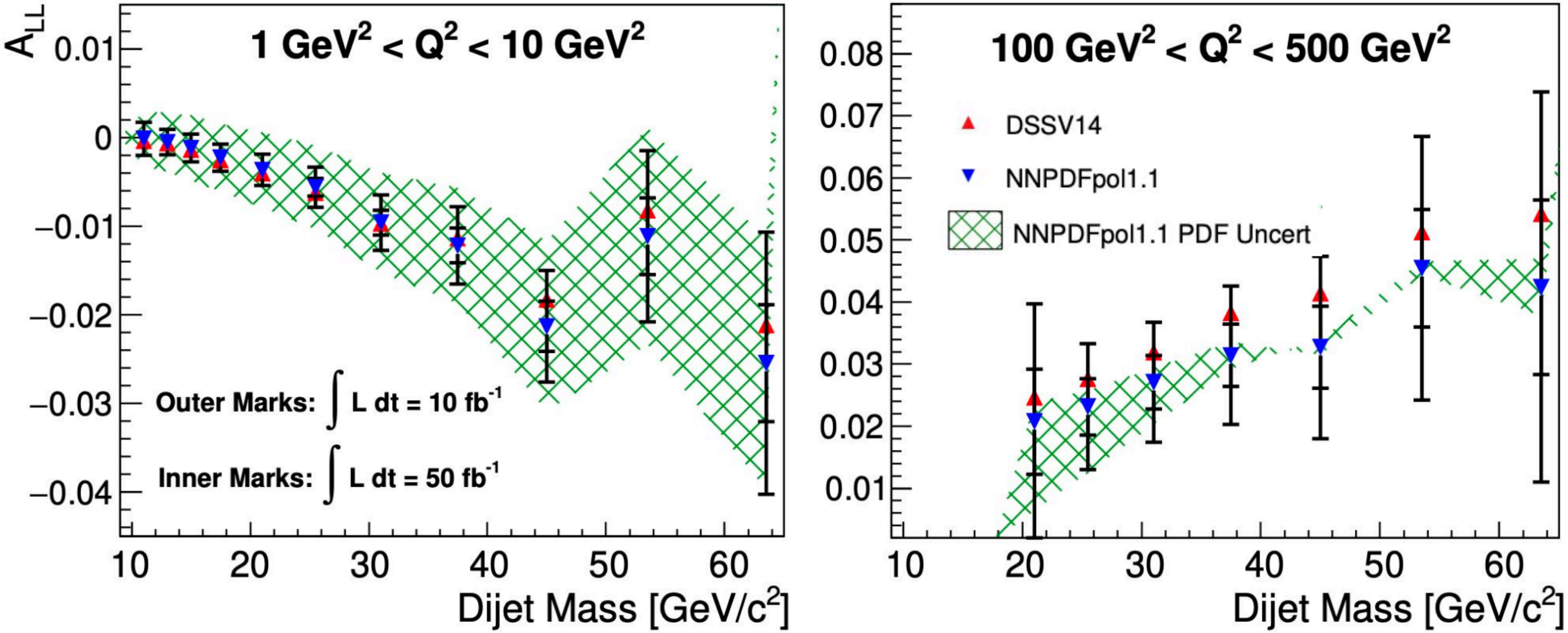
The theory is based on the well-established collinear formalism, different from the small x TMD framework presented in this talk.

Page, Chu and Aschenauer (2020), (Chapter 7 in the EIC Yellow Report)



Feasibility at EIC

Page, Chu and Aschenauer (2020),
(Chapter 7 in the EIC Yellow Report)



Feasibility at EIC

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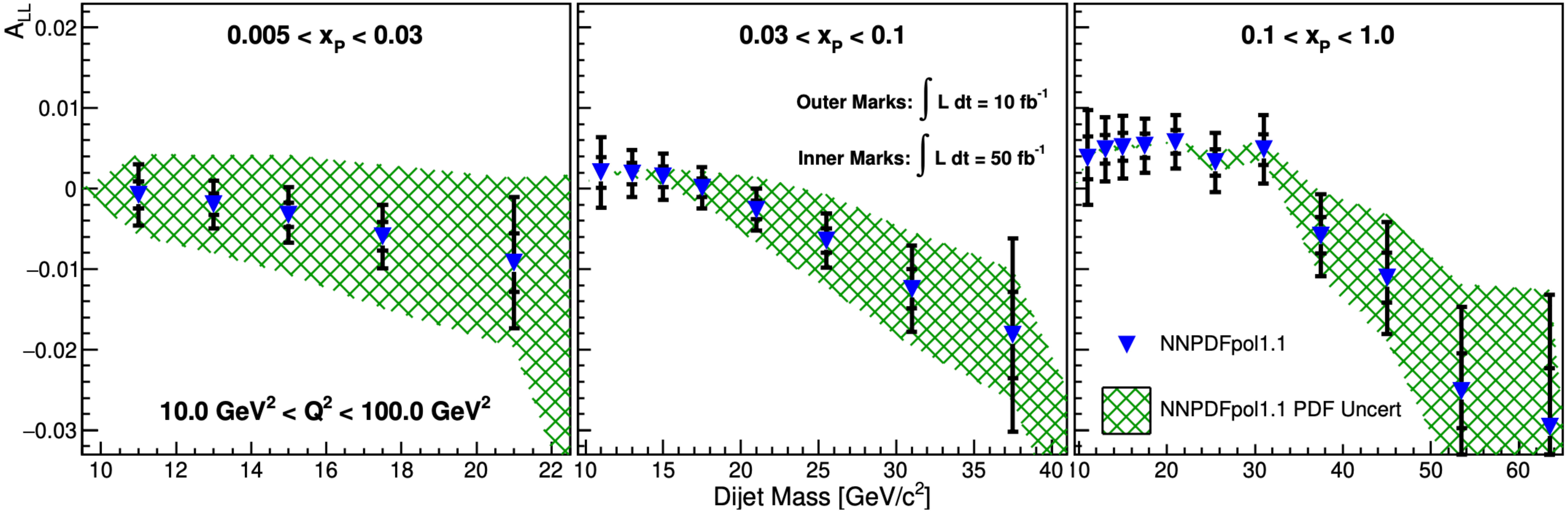
(Chapter 7 in the EIC Yellow Report)



"…substantial integrated luminosities will be needed in order for the dijet measurements to improve on our current knowledge of $\Delta G(x, Q^2)$, meaning it will be unlikely the dijet measurement can compete directly with $g_1(x, Q^2)$ in constraining the gluon contribution to the proton spin. The benefit of the dijet measurement will likely be in its complementarity to $g_1(x, Q^2)$ as the dijets arise from different subprocesses and will have different associated systematics than inclusive observables."

Summary and Outlooks

- We showed that the double-spin asymmetry for inclusive quark-antiquark dijet production in longitudinally polarized DIS can probe the WW gluon helicity TMD.
- We derived the small-x helicity evolution equation for the **WW gluon helicity TMD** and found that, in the DLA, it follows the same equation as the **dipole gluon helicity TMD**.
- The A_{LL} for inclusive dijet production in DIS can be measured at the upcoming EIC and will provide further constraint on the initial conditions for small-x helicity evolution equations and ultimately constraining proton's spin distribution at small x.
- Power suppressed terms in the back-to-back limit, potential nonlinear effects in the small-x helicity evolution equations, double-spin asymmetry for SIDIS.

Thank you for your time!