Cold nuclear matter effects on azimuthal decorrelation in heavy-ion collisions

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"High energy QCD: from the LHC to the EIC", Benasque, Spain

Based on BW, JHEP 07 (2021) 002 [arXiv:2102.12916]; Armesto, Cougoulic and BW,JHEP **11**, 081 (2024) [arXiv:2407.19243 [hep-ph]].

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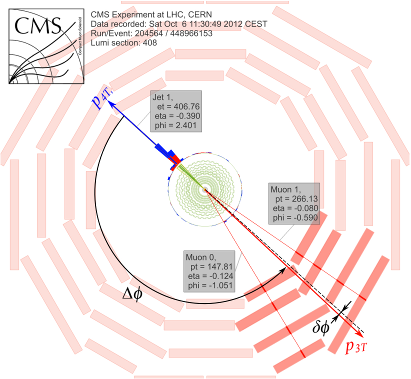
Azimuthal decorrelation

The observable: azimuthal decorrelation

Definition: $\Delta\phi \equiv |\phi_V - \phi_J|$ $(\delta\phi \equiv \pi - \Delta\phi)$



$$\delta\phi \sim Q_T/p_T \equiv |\mathbf{p}_3 + \mathbf{p}_4|/p_T \text{ with } p_{3T} \approx p_{4T} = p_T$$

In the back to back limit: $Q_T \ll p_T$

TMD factorization in *pp*

- **Logarithms in p_T/Q_T need to be resummed!**
- **In the Drell-Yan process:** Proved factorization theorem

Collins, Soper and Sterman, Nucl. Phys. B **250**, 199-224 (1985).

- **In the jet processes:** assumed factorization. Is it broken by Glauber modes?

J. Collins and J. W. Qiu, Phys. Rev. D **75**, 114014 (2007) [arXiv:0705.2141 [hep-ph]].

T. C. Rogers and P. J. Mulders, Phys. Rev. D **81**, 094006 (2010) [arXiv:1001.2977 [hep-ph]]. · · ·

- **Resummation of logarithms using Renormalization Group Equation**
  - Collins-Soper-Sterman formalism (CSS)

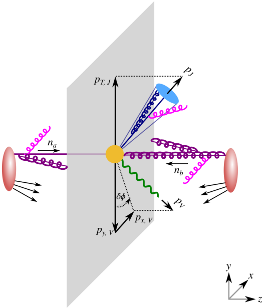
    J. Collins, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **32**, 1-624 (2011) Cambridge University Press, 2011.
  - Soft-Collinear Effective Theory (SCET)

    T. Becher, A. Broggio and A. Ferroglia, Lect. Notes Phys. **896**, pp.1-206 (2015) Springer, 2015, [arXiv:1410.1892 [hep-ph]].

Ignore Glauber modes and use SCET.

Factorization formula of boson-jet in pp using SCET

An all-order resummation formula using the Winner-Take-All (WTA) axis:



Hard function: $\mathcal{H}_{ij\to Vk} \leftarrow$ parton-level $\hat{\sigma}$

Beam functions: $\mathcal{B}_i, \mathcal{B}_j \leftarrow$ TMDs (with different gauge links)

Soft function: $S_{ijk} \leftarrow$ soft radiation

Jet function: $\mathcal{J}_k$ does NOT contain NGLs!

$$\frac{d\sigma}{dq_x\, dp_{T,V}\, dy_V\, d\eta_J} = \int \frac{db_x}{2\pi} e^{b_x q_x} \sum_{ijk} \mathcal{H}_{ij\to Vk}(p_{T,V}, y_V - \eta_J)\mathcal{B}_i(x_a, b_x)B_j(x_b, b_x)\mathcal{J}_k(b_x)S_{ijk}(b_x, \eta_J)$$

Chien, Rahn, Schrijnder van Velzen, Shao, Waalewijn and BW, Phys. Lett. B **815**, 136124 (2021) [arXiv:2005.12279 [hep-ph]].

Chien, Rahn, Shao, Waalewijn and BW, JHEP 02 (2023), 256 [arXiv:2205.05104 [hep-ph]].

Rapidity divergences

- $\mathcal{B}$, $\mathcal{J}_k(b_x)$ and S contain additional divergences not regularized by dim. reg.!

$$\text{divergences} \propto \int \frac{dk^+}{k^+}$$

- Rapidity divergences cancel in the whole factorized cross section!

- Method 1. collinear anomaly: one simply makes the following replacement

$$\int d^d k \to \int d^d k \left(\frac{\nu}{n_1 \cdot k}\right)^{\alpha},$$

where n_1 is the right-moving beam direction, ν and α are regulators.

T. Becher and G. Bell, Phys. Lett. B **713**, 41-46 (2012) [arXiv:1112.3907 [hep-ph]].

Rapidity divergences

- Method 2. η-regulator:

$$W_n = \sum_{\text{perms}} \exp\left[-\frac{gw^2}{\bar{n}\cdot\mathcal{P}} \frac{\mid \bar{n}\cdot\mathcal{P}_g \mid^{-\eta}}{\nu^{-\eta}} \cdot A_n\right]$$

$$S_n = \sum_{\text{perms}} \exp\left[-\frac{gw}{n\cdot\mathcal{P}} \frac{\mid 2\mathcal{P}_{g3} \mid^{-\eta/2}}{\nu^{-\eta/2}} n\cdot A_s\right]$$

where n is the collinear direction, $\bar{n} = (1, -\vec{n})$, ν and η are regulators, the bookkeeping parameter w will be set to one.

J. y. Chiu, A. Jain, D. Neill and I. Z. Rothstein, Phys. Rev. Lett. **108**, 151601 (2012) [arXiv:1104.0881 [hep-ph]];

J. Y. Chiu, A. Jain, D. Neill and I. Z. Rothstein, JHEP **05**, 084 (2012) [arXiv:1202.0814 [hep-ph]].

- The RG equations for a function F:

$$\frac{d}{d\ln\mu} F(\mu) = \Gamma^F_\mu F(\mu), \quad \frac{d}{d\ln\nu} F(\mu,\nu) = \Gamma^F_\nu F(\mu,\nu),$$

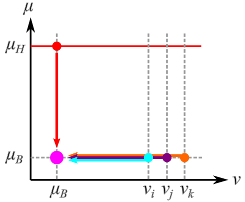
where Γ^F_μ and Γ^F_ν denote the standard and rapidity anomalous dimensions, respectively.

Resummation

All-order resummation formula by RG running

$$\frac{d\sigma_{\text{resum}}}{dq_x\, dp_{T,V}\, dy_V} = \sum_{ijk} \int_0^\infty \frac{db_x}{\pi} \cos(b_x q_x) \prod_{a=ijk} \left(\frac{\nu_S}{\nu_a}\right)^{\Gamma_\nu^{B_a}(\mu_B)} \exp\left(\int_{\mu_H}^{\mu_B} \frac{d\mu}{\mu} \Gamma_\mu^{\mathcal{H}_{ij\to Vk}}(\alpha_s)\right)$$
$$\times \mathcal{H}_{ij\to kV}(p_{T,V}, y_V - \eta_J, \mu_H) \mathcal{B}_i(x_1, b_x, \mu_B, \nu_i) \mathcal{B}_j(x_2, b_x, \mu_B, \nu_j)$$
$$\times \mathcal{J}_k(b_x, \mu_B, \nu_k) S_{ijk}(b_x, \mu_B, \nu_S)$$

with $\Gamma_\mu^{\mathcal{H}_{ij\to Vk}}$ anomalous dimension of the hard function.



Natural momentum scales:

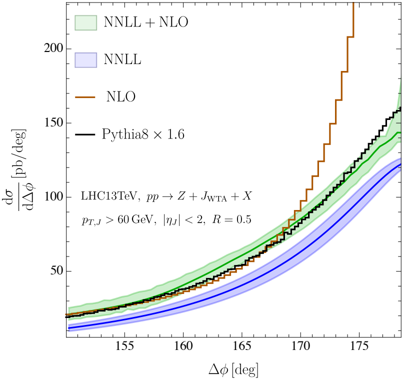
$$\mu_H = \sqrt{m_V^2 + p_{T,V}^2}\,, \quad \nu_a = \bar{n}_a \cdot p_a\,,$$
$$\mu_B = \nu_S = 2e^{-\gamma_E}\sqrt{1 + b_x^2/b_{\max}^2}/|b_x|$$

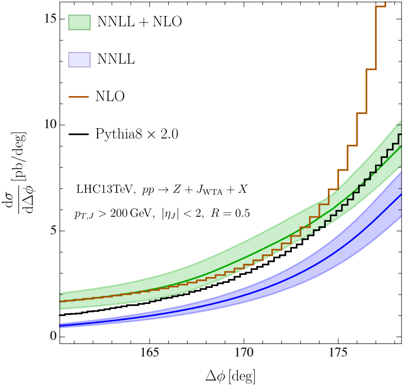
with $b_{\max} = 1.5\ \text{GeV}^{-1}$.

Uncertainties are estimated by varying μ_H, μ_B and ν_S.

Theoretical predictions

At NNLL + NL0 accuracy





Chien, Rahn, Schrijnder van Velzen, Shao, Waalewijn and BW, Phys. Lett. B **815**, 136124 (2021) [arXiv:2005.12279 [hep-ph]].

Chien, Rahn, Shao, Waalewijn and BW, JHEP 02 (2023), 256 [arXiv:2205.05104 [hep-ph]].

In heavy-ion collisions (AA)

Jets are subject to energy loss and momentum broadening:

- **Jet quenching parameter $\hat{q}$**

$$\langle p_\perp^2 \rangle = \hat{q} L$$

with L the path length.

- **Medium-induced energy loss**

$$\Delta E = \frac{\alpha_s N_c}{12} \hat{q} L^2 = \frac{\alpha_s N_c}{12} \langle p_\perp^2 \rangle L.$$

Baier, Dokshitzer, Mueller, Peigne and Schiff, Nucl. Phys. B **484**, 265-282 (1997) [arXiv:hep-ph/9608322 [hep-ph]].

- **$\hat{q}$ in AA collisions is mostly measured via jet quenching phenomena.**

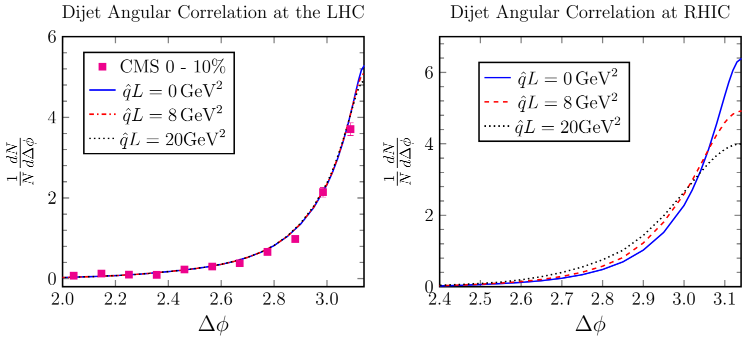
$$\hat{q} = (2-4) T^3$$

S. Cao *et al.* [JETSCAPE], Phys. Rev. C **104**, no.2, 024905 (2021) [arXiv:2102.11337 [nucl-th]].

- **An alternative way to measure $\hat{q}$:** $\delta\phi \sim \frac{\sqrt{\hat{q}L}}{p_{T,J}}$

Measurement of $\hat{q}$ via $\Delta\phi$

Using dijets: NLL resummation of Sudakov logs + tree-level medium effects



A. H. Mueller, BW, B. W. Xiao and F. Yuan, Phys. Lett. B **763**, 208-212 (2016) [arXiv:1604.04250 [hep-ph]]. .

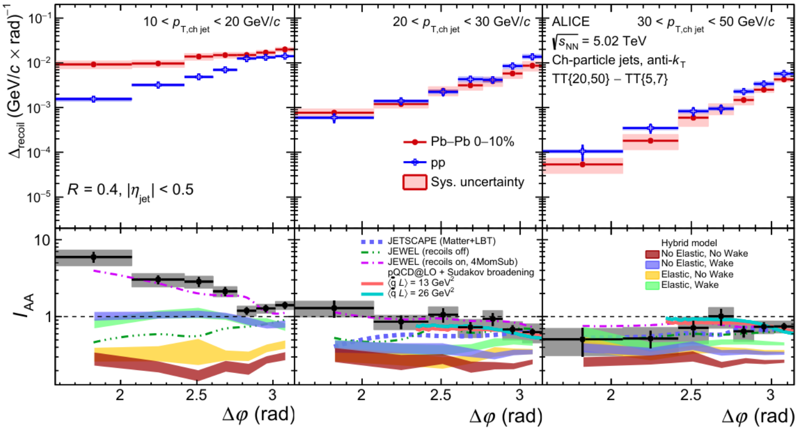
Using boson-jet:

$$\hat{q}_0 = (4-8)\ \mathrm{GeV}^2/\mathrm{fm}\ \text{for}\ T_0 = 509\ \mathrm{MeV}\ \text{at}\ \sqrt{s_{NN}} = 5.02\ \mathrm{TeV}.$$

L. Chen, S. Y. Wei and H. Z. Zhang, PoS **HardProbes2020**, 031 (2021).

$\hat{q}$ measurement: momentum broadening $\Delta\phi$

Using $\Delta\varphi$ in hadron-jet:

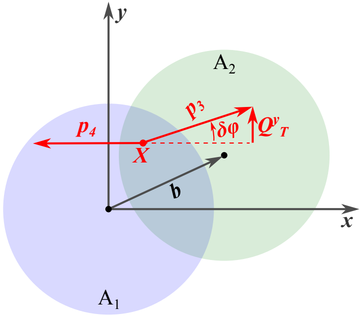


S. Acharya *et al.* [ALICE], [arXiv:2308.16131 [nucl-ex]].

Theory: L. Chen, G. Y. Qin, S. Y. Wei, B. W. Xiao and H. Z. Zhang, Phys. Lett. B **773** (2017), 672-676 [arXiv:1607.01932 [hep-ph]].

Going beyond phenomenological studies in AA?

The observable: $Q \gg |Q_T^y| \gg \Lambda_{QCD}$



$$\frac{1}{p_T}\frac{d\sigma_{A_1A_2}}{d^2\mathbf{b}d\eta_3 d\eta_4 dp_T d\delta\varphi} \approx \frac{d\sigma_{A_1A_2}}{d^2\mathbf{b}d\eta_3 d\eta_4 dp_T dQ_T^y}$$

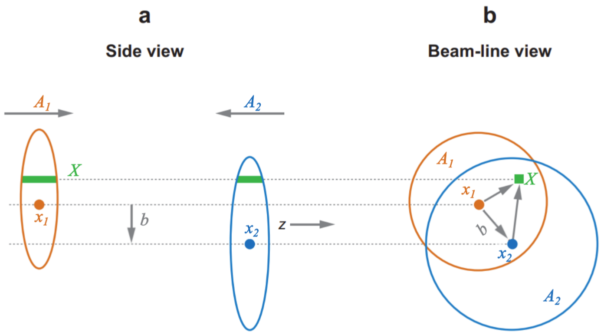
with $\delta\varphi = \arcsin(Q_T^y/|\mathbf{p}_3|)$, $\mathbf{Q}_T \equiv \mathbf{p}_3 + \mathbf{p}_4$, $Q = p_T \equiv |\mathbf{p}_4| \approx |\mathbf{p}_3|$.

The focus of this talk: the first step toward verifying factorization in AA.

Impact-parameter dependent cross sections

The classical picture

The CLASSICAL picture of collision geometry in the Glauber model:

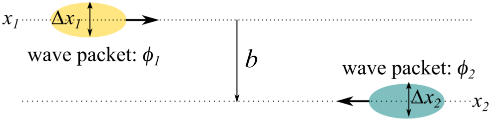


Miller, Reygers, Sanders and Steinberg, *Ann. Rev. Nucl. Part. Sci.* **57** (2007) 205 [arXiv:nucl-ex/0701025].

Impact-parameter dependent cross section in QFT?

The quantum picture

In quantum theory, the probability for producing any observable O:



$$\frac{dP}{dO} \equiv \int \prod_f [d\Gamma_{p_f}]\, \delta(O - O(\{p_f\}))\langle \phi_1\phi_2|\hat{S}^\dagger|\{p_f\}\rangle\langle\{p_f\}|\hat{S}|\phi_1\phi_2\rangle$$

$$= \int \prod_f [d\Gamma_{p_f}]\, \delta(O - O(\{p_f\}))\mathrm{Tr}[\hat{S}^\dagger|\{p_f\}\rangle\langle\{p_f\}|\hat{S}|\phi_1\phi_2\rangle\langle\phi_1\phi_2|] \to \frac{d\sigma}{d^2\mathbf{b}dO}$$

where the wave packages of the colliding particles with $\mathbf{x}_i$ the transverse location of nucleus i:

$$|\phi_i\rangle = \int \frac{d^3\vec{p}}{(2\pi)^3}\frac{e^{-i\mathbf{p}\cdot\mathbf{x}_i}}{\sqrt{2E_p}} \underbrace{\phi_i(\vec{p})}_{\text{unknown}} |\vec{p}\rangle \qquad \int d\Gamma_p \equiv \int \frac{d^4p}{(2\pi)^4}(2\pi)\delta(p^2 - m^2)\theta(p^0).$$

BW, JHEP 07 (2021) 002 [arXiv:2102.12916].

Conditions for defining $\frac{d\sigma}{d^2\mathbf{b}dO}$

$\frac{dP}{dO} \to \frac{d\sigma}{d^2\mathbf{b}dO}$ defines cross section only if ϕ_i's can be integrated out!

ϕ_i's depend on how the beams are prepared and the cross section should be independent of it.

M.E. Peskin and D.V. Schroeder, Addison-Wesley, Reading, USA (1995).

For this, one needs to impose one more condition:

i) high energy: $|P_{iz}| \gg |\mathbf{P}_i|, \Delta p_T, \Delta p_z$;

ii) localization: $|\mathbf{b}| \gg \Delta x_T$.

It would be questionable if one extrapolates the CLASSICAL picture into low-energy or small collision systems.

Impact-parameter dependent cross sections

After integrating the two wave packets:

$$\frac{d\sigma}{d^2\mathbf{b}dO} = \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} \int \prod_f [d\Gamma_{p_f}]\,\delta(O - O(\{p_f\}))$$
$$\times \frac{1}{2s} M(p_1, p_2 \to \{p_f\}) M^*(\bar{p}_1, \bar{p}_1 \to \{p_f\})(2\pi)^4 \delta^{(4)}(p_1 + p_2 - \sum p_f)$$

where

$$p_1^\mu = p_1^+ \frac{n_1^\mu}{2} + \frac{q_T^\mu}{2} - \frac{q_T^2}{4p_1^+}\frac{\bar{n}_1^\mu}{2}, \qquad p_2^\mu = p_2^- \frac{n_2^\mu}{2} - \frac{q_T^\mu}{2} - \frac{q_T^2}{4p_2^-}\frac{\bar{n}_2^\mu}{2},$$
$$\bar{p}_2^\mu = p_1^+ \frac{n_1^\mu}{2} - \frac{q_T^\mu}{2} - \frac{q_T^2}{4p_1^+}\frac{\bar{n}_1^\mu}{2}, \qquad \bar{p}_2^\mu = p_2^- \frac{n_2^\mu}{2} + \frac{q_T^\mu}{2} - \frac{q_T^2}{4p_2^-}\frac{\bar{n}_2^\mu}{2}.$$

with $n_{1,2} = (1, 0, 0, \pm 1)$ and $\bar{n}_{1,2} = (1, 0, 0, \mp 1)$.

Factorization using SCET (leading twist)

For the Drell-Yan process: BW, JHEP 07 (2021) 002 [arXiv:2102.12916].

$$\frac{d\sigma}{d^2\mathbf{b}dO} \equiv \int \prod_f [d\Gamma_{p_f}]\,\delta(O - O(\{p_f\})\langle\phi_1\phi_2|\hat{S}^\dagger|\{p_f\}\rangle\langle\{p_f\}|\hat{S}|\phi_1\phi_2\rangle$$

where

$$\langle p_C, \{p_X\}|\hat{S}|\phi_A\phi_B\rangle = \int d^4x e^{ip_C\cdot x}\langle\{p_X\}|i\hat{M}(x)|\phi_A\phi_B\rangle.$$

with

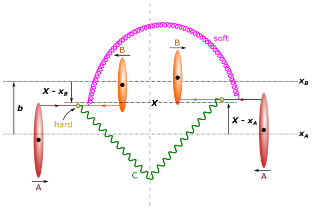
$$i\hat{M}(x) = \overbrace{\int dt_A dt_B \underbrace{C^{a_A a_B}_{\alpha_A\alpha_B}(\epsilon, t_A, t_B)}_{\text{hard}} \underbrace{[S_{n_A}\phi_{n_A}(x + t_A\bar{n}_A)]^{\alpha_A}_{a_A}[S_{n_B}\phi_{n_B}(x + t_B\bar{n}_B)]^{\alpha_B}_{a_B}}_{\text{soft-collinear factorization}}}^{\text{hard-collinear factorization}},$$

S_{n_i}: the soft Wilson line along the collinear direction n_i

ϕ_{n_i} : $\chi_{n_i}(x) \equiv W^\dagger_{n_i}(x)\frac{n_i\bar{n}_i}{4}\psi_{n_i}(x)$, $\quad \bar{\chi}_{n_i}(x)$, $\quad \mathcal{B}^\mu_{n_i T} = \frac{1}{g_s}W^\dagger_{n_i}(x)iD^\mu_{n_i T}W_{n_i}(x)$

with $D^\mu_{n_i T} \equiv \partial^\mu_T - ig_s A^\mu_{n_i T}$ and W_{n_i} the n_i-collinear Wilson line.

Factorization using SCET (leading twist)



$$\frac{d\sigma}{d^2\mathbf{b}\,dy_C\,d^2\mathbf{p}_C} = \frac{1}{4\pi s}\sum_{j,k}\int d^2\mathbf{X}\int d^2\mathbf{x}\,e^{i\mathbf{p}_C\cdot\mathbf{x}}\int_0^1 \frac{dz_A}{z_A}\frac{dz_B}{z_B}$$

$$\times \int \prod_f [d\Gamma_{p_f}] \prod_{i=A,B} \delta\Big(z_i \bar{n}_i\cdot P_i - \bar{n}_i\cdot p_C - \sum \bar{n}_i\cdot p_f\Big)$$

$$\times \underbrace{\mathcal{T}_{j/A}(\mathbf{X}, z_A, \mathbf{x})\,\mathcal{T}_{k/B}(\mathbf{X}-\mathbf{b}, z_B, \mathbf{x})}_{\text{Thickness beam functions}}$$

$$\times \underbrace{H^{\bar{a}_A\bar{a}_B}_{a_A a_B}(z_A P_A, z_B P_B \to p_C, \{p_f\})}_{\text{hard function}}\underbrace{\mathcal{S}^{\bar{a}_A\bar{a}_B}_{a_A a_B}(\mathbf{x})}_{\text{soft function}}.$$

Thickness beam functions

Definition:

$$\mathcal{T}_{q/i}(\mathbf{r}, z, \mathbf{x}) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{r}} \int \frac{dt}{2\pi} e^{-izt\bar{n}\cdot P} \times \left\langle \bar{n}\cdot P, -\frac{\mathbf{q}}{2} \right| \bar{\chi}_n \left(\frac{t\bar{n}}{2} + \frac{x_T}{2} \right) \frac{\bar{n}}{2} \chi_n \left(-\frac{t\bar{n}}{2} - \frac{x_T}{2} \right) \left| \bar{n}\cdot P, \frac{\mathbf{q}}{2} \right\rangle$$

$$\mathcal{T}_{g/i}(\mathbf{r}, z, \mathbf{x}) = z\bar{n}\cdot P(-g_{T\alpha'\alpha}) \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{r}} \int \frac{dt}{2\pi} e^{-izt\bar{n}\cdot P} \times \left\langle \bar{n}\cdot P, -\frac{\mathbf{q}}{2} \right| \mathcal{B}^{a\alpha'}_{nT} \left(\frac{t\bar{n}}{2} + \frac{x_T}{2} \right) \mathcal{B}^{a\alpha}_{nT} \left(-\frac{t\bar{n}}{2} - \frac{x_T}{2} \right) \left| \bar{n}\cdot P, \frac{\mathbf{q}}{2} \right\rangle .$$

where

$$\text{For } q/\bar{q} : \chi_{n_i}(x) \equiv W^\dagger_{n_i}(x) \frac{n_i \bar{n}_i}{4} \psi_{n_i}(x), \qquad \bar{\chi}_{n_i}(x),$$

$$\text{For } g : \mathcal{B}^\mu_{n_i T} = \frac{1}{g_s} W^\dagger_{n_i}(x) i D^\mu_{n_i T} W_{n_i}(x)$$

with $D^\mu_{n_i T} \equiv \partial^\mu_T - i g_s A^\mu_{n_i T}$ and W_{n_i} the n_i-collinear Wilson line.

The operators are the same as beam functions!

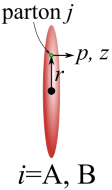
I. W. Stewart, F. J. Tackmann and W. J. Waalewijn, Phys. Rev. D **81**, 094035 (2010) [arXiv:0910.0467 [hep-ph]].

Thickness beam functions

Physical interpretation:

Fourier transform of transverse phase space (TPS) PDF with respect to $\mathbf{x}$

$$f_{j/i}(\mathbf{r}, z, \mathbf{p}) =$$



$$= \int d^2\mathbf{x} e^{i\mathbf{p}\cdot\mathbf{x}} \mathcal{T}_{j/i}(\mathbf{r}, z, \mathbf{x}).$$

TMD PDF at r! (up to gauge link prescription etc)

In the limit $\mathbf{x} \to 0$: $\mathcal{T}_{j/i}(\mathbf{r}, z, \mathbf{0})$ admits the interpretation as the PDF at $\mathbf{r}$.

Given experimental uncertainties, we shall use the Glauber model below.

The Glauber model

Nuclei are modelled as uncorrelated nucleons:

the nucleons are distributed according to the charge distribution

$$\rho_A(r)/\rho_0 = \frac{1}{1+e^{\frac{r-R_{ws}}{a}}} \qquad \text{with } R_{ws} = 6.62 \text{ fm and } a = 0.55 \text{ fm for } {}^{208}\text{Pb}.$$

where ρ_0 is the density at the center.

C. W. De Jager, H. De Vries and C. De Vries, Atom. Data Nucl. Data Tabl. **14** (1974), 479-508.

The thickness function

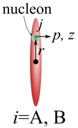
$$T_i(\mathbf{r}_i) \equiv \int dz \rho_i(\mathbf{r}_i, z).$$

Miller, Reygers, Sanders and Steinberg, *Ann. Rev. Nucl. Part. Sci.* **57** (2007) 205 [arXiv:nucl-ex/0701025].

Thickness beam functions in the Glauber model

In terms of the thickness functions

$$\mathcal{T}_{j/i}(\mathbf{r}_i, z, \mathbf{x}) \rightarrow$$



$$= T_i(\mathbf{r}_i)\left[\frac{Z_i}{Z_i+N_i}B_{j/p}(z,\mathbf{x}) + \frac{N_i}{Z_i+N_i}B_{j/n}(z,\mathbf{x})\right]$$

$$= \text{the thickness functions} \times \text{the beam functions}$$

where $B_{j/p}$ and $B_{j/n}$ are the beam functions for protons and neutrons, respectively, and the nucleus is assumed to contain Z_i protons and N_i neutrons.

The factorization formula gives

$$R_{AA} \equiv \frac{\frac{d\sigma_{AB}}{d^2\mathbf{b}dy_C d^2\mathbf{p}_C}}{T_{AB}(\mathbf{b})\frac{d\sigma_{nn}}{dy_C d^2\mathbf{p}_C}} = 1,$$

with $T_{AB}(\mathbf{b}) \equiv \int d^2\mathbf{X}\, T_A(\mathbf{X}) T_B(\mathbf{X}-\mathbf{b})$.

To see nuclear effects, one needs to include terms enhanced by nuclear size.

Formulating soft and hard processes

The Glauber modelling of heavy nuclei

Neglecting correlations between nucleons, for the right-moving A_1:

$$|\phi_1\rangle\langle\phi_1| = \prod_{i=1}^{A_1} \int \frac{dP_i^+ d^2\mathbf{P}_i}{(2\pi)^3 2P_i^+} \frac{1}{2} \int db_i^- d^2\mathbf{b}_i W_{A_1}(P_i, b_i)$$
$$\times \int \frac{dq_i^+ d^2\mathbf{q}_i}{(2\pi)^3} e^{\frac{i}{2} q_i^+ b_i^- - i\mathbf{q}_i\cdot\mathbf{b}_i} |P_i + q_i/2\rangle\langle P_i - q_i/2|,$$

where the Wigner distribution function for one nucleon is defined as

$$W_{A_1}(P, b) \equiv \int \frac{dq^+ d^2\mathbf{q}}{(2\pi)^3 2P^+} e^{-\frac{i}{2} q^+ b^- + i\mathbf{q}\cdot\mathbf{b}} \langle P + q/2|\phi_1\rangle\langle\phi_1|P - q/2\rangle.$$

Replacing W_{A_1} with the following form:

$$W_{A_1}(p, b) = \hat{\rho}_{A_1}(b^-, \mathbf{b}) 2(2\pi)^3 \delta(p^+ - P_1^+)\delta^{(2)}(\mathbf{p}),$$

where P_1^+ denotes the "+" momentum of the nucleon, and $\hat{\rho}_{A_1} = \rho_{A_1}/A_1$ with $\rho_{A_1}(b^-, \mathbf{b})$ the distribution of nucleons (say a Woods-Saxon functional form).

Kovchegov and Sievert, Phys. Rev. D **89**, no.5, 054035 (2014) [arXiv:1310.5028 [hep-ph]];

BW and Kovchegov, JHEP **03**, 158 (2018)[arXiv:1709.02866 [hep-ph]].

AA collisions ⇒ binary nucleon collisions

Expressing the density matrices in terms of the Wigner functions:

$$\frac{d\sigma}{d^2\mathbf{b}dO} = \int \prod_f [d\Gamma_{p_f}]\,\delta(O - O(\{p_f\}))$$
$$\times \prod_{i=1}^{A_1} \frac{1}{2P_1^+} \int d^2\mathbf{b}_i\, db_i^-\, \hat{\rho}_{A_1}(b_i^-, \mathbf{b}_i) \int \frac{dq_i^+ d^2\mathbf{q}_i}{(2\pi)^3} e^{\frac{i}{2}q_i^+ b_i^- - i\mathbf{q}_i\cdot\mathbf{b}_i}$$
$$\times \prod_{j=1}^{A_2} \frac{1}{2P_2^-} \int d^2\mathbf{b}_j'\, db_j'^+\, \hat{\rho}_{A_2}(b_j'^+, \mathbf{b}_j' - \mathbf{b}) \int \frac{dq_j'^- d^2\mathbf{q}_j'}{(2\pi)^3} e^{\frac{i}{2}q_j'^- b_j'^+ - i\mathbf{q}_j'\cdot\mathbf{b}_j'}$$
$$\times \langle \{P_1 - q_i/2\}, \{P_2 - q_j'/2\}|\hat{S}^\dagger|\{p_f\}\rangle\langle\{p_f\}|\hat{S}|\{P_1 + q_i/2\}, \{P_2 + q_j'/2\}\rangle,$$

where the momenta of the constituent nucleons within the two nuclei are respectively given by $P_1^\mu = P_1^+ n_1^\mu/2$ and $P_2^\mu = P_2^- n_2^\mu/2$, with the two beam directions defined as $n_1^\mu = (1,0,0,1)$ and $n_2^\mu = (1,0,0,-1)$.

N. Armesto, F. Cougoulic and BW, JHEP **11**, 081 (2024) [arXiv:2407.19243 [hep-ph]].

We deal with soft (semi-hard) and hard prcesses at the same footing!

AA collisions ⇒ partonic processes

Expression in terms of nucleon PDFs:

$$
\frac{d\sigma}{d^2\mathbf{b}dO} = \int \prod_f [d\Gamma_{p_f}]\,\delta(O - O(\{p_f\}))
$$

$$
\times \sum_{\{a_i, b_j\}} \left(\prod_{i=1}^{A_1} \frac{1}{P_1^+} \int \frac{d\xi_i}{\xi_i} f_{a_i}(\xi_i) \right) \left(\prod_{j=1}^{A_2} \frac{1}{P_2^-} \int \frac{d\xi_j'}{\xi_j'} f_{b_j}(\xi_j') \right)
$$

$$
\times \langle \{\xi_i P_1\}, \{\xi_j' P_2\} | \hat{S}^\dagger | \{p_f\} \rangle \langle \{p_f\} | \hat{S} | \{\xi_i P_1\}, \{\xi_j' P_2\} \rangle
$$

$$
\otimes \left(\prod_{i=1}^{A_1} \hat{\rho}_{A_1}(X_i^-, \mathbf{X}_i) \right) \left(\prod_{j=1}^{A_2} \hat{\rho}_{A_2}(Y_j^+, \mathbf{Y}_j - \mathbf{b}) \right),
$$

where a_i and b_j iterate over all the parton species, and the operator $\otimes$ indicates that the incoming partons i and j enter the diagrams in the amplitude (the conjugate amplitude) at x_i (x_i') and y_j (y_j') respectively with $X_i = (x_i + x_i')/2$ and $Y_j = (y_j + y_j')/2$. Here, the initial- and final-state spins and colors are respectively averaged and summed over in the square of the partonic S-matrix element.

We will start from here!

Checking: zeroth order in AA collisions

Each nucleon in nucleus *i* has an equal chance to participate the hard collision:

$$\frac{d\sigma^{(0)}}{d^2\mathbf{b}dO} = \int \prod_f [d\Gamma_{p_f}]\, \delta(O - O(\{p_f\})\frac{1}{s_{NN}} \sum_{ij} \int \frac{d\xi}{\xi}\frac{d\xi'}{\xi'} f_i(\xi) f_j(\xi')$$
$$\times \langle \xi P_1, \xi' P_2 | \hat{S}^\dagger | \{p_f\}\rangle \langle \{p_f\}|\hat{S}|\xi P_1, \xi' P_2\rangle \otimes \rho_{A_1}(X^-, \mathbf{X}) \rho_{A_2}(Y^+, \mathbf{Y} - \mathbf{b})$$

with $s_{NN} = P_1^+ P_2^-$. In the expansion at large Q, offshell propagators shrink to one single spacetime point, and we get

$$\frac{d\sigma^{(0)}}{d^2\mathbf{b}dO} = 2\int d^4X \rho_{A_1}(X^-, \mathbf{X})\rho_{A_2}(X^+, \mathbf{X} - \mathbf{b}) \frac{d\sigma^{(0)}_{nn\to kl}}{dO},$$
$$= T_{A_1A_2}(\mathbf{b}) \frac{d\sigma^{(0)}_{nn\to kl}}{dO} \Leftrightarrow R_{AA} = 1,$$

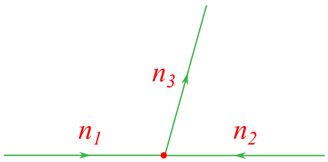
where the nucleon-nucleon cross section is defined as

$$\frac{d\sigma^{(0)}_{nn\to kl}}{dO} \equiv \sum_{ij} \int d\xi d\xi' f_i(\xi) f_j(\xi') \frac{d\hat{\sigma}^{(0)}_{ij\to kl}}{dO}.$$

Expansion of Feynman diagrams at high Q

- We expand all diagrams to leading order in $\delta\phi$ and in medium/nuclues size:
  1. $\delta\phi \sim Q_T/Q \ll 1$: $\Leftrightarrow$ expansion at large Q given Q_T^y fixed.
  2. Enhanced by medium size: $\Leftrightarrow$ collinear internal momenta nearly on-shell

- Consider both the Drell-Yan and boson-jet processes: denote distinct collinear directions by n_i ($n_i \cdot n_j \sim 1$ for $i \neq j$).



- Use Feynman gauge: $G_F^{\mu\nu}(p) = -ig^{\mu\nu}/(p^2 + i\epsilon)$.

• A convenient choice of coordinates: for any pair n_i and n_j with $i \neq j$, one can write

$$g^{\mu\nu} = \frac{n_i^\mu n_j^\nu + n_j^\mu n_i^\nu}{n_{ij}} + g_\perp^{\mu\nu} \Leftrightarrow g_{\mu\nu} = \begin{pmatrix} 0 & \frac{1}{n_{ij}} & 0 & 0 \\ \frac{1}{n_{ij}} & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

with $g_\perp^{\mu\nu}$ orthogonal to both n_i and n_j and $n_{ij} \equiv n_i \cdot n_j$. We define the transverse "1" and "2" components using:

$$n_{\perp_1}^\mu = \frac{1}{\sqrt{1 - (\vec{n}_i \cdot \vec{n}_j)^2}} \left(1 + \vec{n}_i \cdot \vec{n}_j, \vec{n}_i + \vec{n}_j\right), \qquad n_{\perp_2}^\mu = \left(0, \frac{\vec{n}_i \times \vec{n}_j}{|\vec{n}_i \times \vec{n}_j|}\right).$$

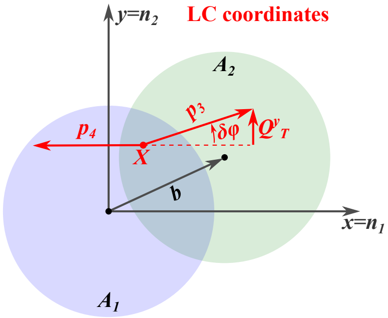
with $n_{\perp_2}^\mu$ aligned with the y-axis.

• Decomposition of any four-vector v^μ:

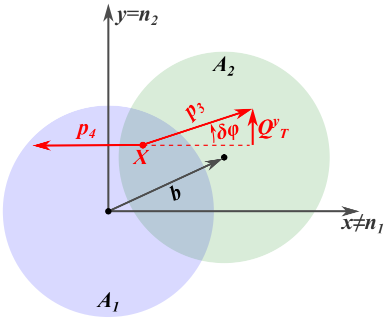
$$v^\mu = \frac{v^{n_i} n_j^\mu + v^{n_j} n_i^\mu}{n_{ij}} + v^1 n_{\perp_1}^\mu + v^2 n_{\perp_2}^\mu \ ,$$

with $v^{n_i} \equiv n_i \cdot v, \qquad v^{n_j} \equiv n_j \cdot v$ and $\mathbf{v}_\perp = (v^1, v^2)$.

Why these coordinates?



$n_i = n_1$, $n_j = n_2$



$n_i = n_{1,2}$, $n_j = n_3$

For p collinear to n_i, $p^2 = 2 n_i \cdot p \, n_j \cdot p / n_i \cdot n_j - |\mathbf{p}_\perp|^2 \simeq 0$:

$$p_\perp \sim \delta\phi Q, \qquad n_i \cdot p \sim \delta\varphi^2 Q, \qquad n_j \cdot p \sim Q \qquad \text{for } j \neq i.$$

• A convenient choice of coordinates: for any pair n_i and n_j with $i \neq j$, one can write

$$g^{\mu\nu} = \frac{n_i^\mu n_j^\nu + n_j^\mu n_i^\nu}{n_{ij}} + g_\perp^{\mu\nu} \Leftrightarrow g_{\mu\nu} = \begin{pmatrix} 0 & \frac{1}{n_{ij}} & 0 & 0 \\ \frac{1}{n_{ij}} & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

with $g_\perp^{\mu\nu}$ orthogonal to both n_i and n_j and $n_{ij} \equiv n_i \cdot n_j$. We define the transverse "1" and "2" components using:

$$n_{\perp_1}^\mu = \frac{1}{\sqrt{1-(\vec{n}_i \cdot \vec{n}_j)^2}} \left(1 + \vec{n}_i \cdot \vec{n}_j, \vec{n}_i + \vec{n}_j\right), \qquad n_{\perp_2}^\mu = \left(0, \frac{\vec{n}_i \times \vec{n}_j}{|\vec{n}_i \times \vec{n}_j|}\right).$$

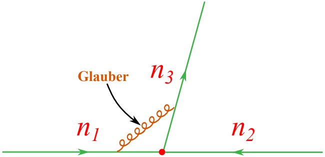
with $n_{\perp_2}^\mu$ aligned with the y-axis.

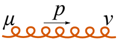
• Decomposition of any four-vector v^μ:

$$v^\mu = \frac{v^{n_i} n_j^\mu + v^{n_j} n_i^\mu}{n_{ij}} + v^1 n_{\perp_1}^\mu + v^2 n_{\perp_2}^\mu \ ,$$

with $v^{n_i} \equiv n_i \cdot v$, $\qquad v^{n_j} \equiv n_j \cdot v$ and $\mathbf{v}_\perp = (v^1, v^2)$.

- Glauber gluons connecting i, j





$$= \frac{ig_{\mu\nu}}{|\mathbf{p}_\perp|^2} \qquad \text{with} \qquad |\mathbf{p}_\perp|^2 = (p^1)^2 + (p^2)^2$$

- Feynman rules for coupling between collinear partons and Glauber modes:

In Feynman gauge (the same as LC gauge in each direction):

$$= D_F(p) \equiv \frac{i}{p^2 + i\epsilon},$$

$$= -ig\,\hat{T}^a 2p_i \cdot A^a,$$

where all collinear partons are represented by solid lines, and $\hat{T}^a$ stands for the color generator: $\hat{T}^a_{bc} = t^a_{bc}$ for quarks, $\hat{T}^a_{bc} = -t^a_{cb}$ for antiquarks, and $\hat{T}^a_{bc} = -if^{abc}$ for gluons.

In coordinate space:

For collinear partons:

$$D_F(x) = \int \frac{d^4p}{(2\pi)^4}\sqrt{|g|}\frac{ie^{-ip\cdot x}}{p^2+i\epsilon} = \frac{1}{n_{ij}}\int \frac{dp^{n_i}dp^{n_j}d^2\mathbf{p}_\perp}{(2\pi)^4}\frac{ie^{-ip\cdot x}}{p^2+i\epsilon}$$
$$\to \int \frac{dp^{n_j}}{2\pi}\frac{1}{2p^{n_j}}e^{-\frac{i}{n_{ij}}p^{n_j}x^{n_i}}\delta^{(2)}(\mathbf{x}_\perp)\theta(x^{n_j}),$$

For Glauber gluons:

$$G_F^{\mu\nu}(x) = ig^{\mu\nu}n_{ij}\delta(x^{n_i})\delta(x^{n_j})F_1(|\mathbf{x}_\perp|),$$

where in dimensional regularization with $d = 2-2\epsilon$, F_1 is defined as

$$F_1(|\mathbf{x}|) \equiv \mu^{2-d}\int \frac{d^d\mathbf{l}}{(2\pi)^d}\frac{e^{i\mathbf{l}\cdot\mathbf{x}}}{|\mathbf{l}|^2} = \frac{1}{4\pi}\frac{\Gamma(-\epsilon)}{(\pi|\mathbf{x}|^2\mu^2)^{-\epsilon}}$$
$$= -\frac{1}{4\pi}\left[\frac{1}{\epsilon}+\gamma_E+\ln(\pi|\mathbf{x}|^2\mu^2)+O(\epsilon)\right].$$

See, e.g.,M. Li, W. Qian, BW and H. Zhang, JHEP **08**, 144 (2023) [arXiv:2304.06557 [hep-ph]].

Why, also for Feynman gauge? Expanding at leading order in Q (=Eikonal approximation):

1. For quarks:

$$= -ig\not{p}_i D_F(p) \not{A} u_{p_i} = -2ig(p_i \cdot A) D_F(p) u_{p_i} \checkmark$$

with u_{p_i} passing through to give the amplitude for the hard process.

2. For gluons:

$$= gf^{abc} A^a_\mu [g^{\mu\nu}(k+p)^\rho + g^{\nu\rho}(-p-p_i)^\mu$$
$$+ g^{\rho\mu}(p_i - k)^\nu] \epsilon_\rho(p_i) D_F(p)$$
$$\approx -gf^{abc}[2p_i \cdot A^a \epsilon^\nu(p) - A^a \cdot \epsilon(p)\ p_i^\nu] D_F(p),$$

but

$$\langle M | T \partial_{\mu_1} A^{\mu_1}(x_1) \cdots \partial_{\mu_n} A^{\mu_n}(x_n) | N \rangle = 0,$$

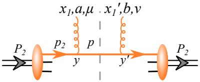
so

$$= -2gf^{abc}(p_i \cdot A^a) \epsilon^\nu(p_i) D_F(p) = -igT^a_{bc} 2p_i \cdot A^a\ \epsilon^\nu(p_i) D_F(p) \checkmark$$

in the expansion at large Q with ϵ^ν pathing through to give the amplitude for the hard scattering.

Ensemble average = MV model

For the left-moving nucleus A_2:



$$\equiv \frac{\delta^{ab}}{N_c^2-1}\langle A^{c\mu}(x_1)A^{c\nu}(x_1')\rangle = \frac{1}{P_2^-}\int \frac{d\xi}{\xi} f_q(\xi)\int d\Gamma_p$$

$$\times \frac{\mathrm{Tr}(t^a t^b)}{2N_c} g^2 \mathrm{Tr}(p\gamma_\alpha p_2 \gamma_\beta)$$

$$\times \int d^4y d^4y' \rho_{A_2}(Y^+, \mathbf{Y}-\mathbf{b}) e^{i\Delta y\cdot(p-p_2)} G_F^{\mu\alpha}(x_1-y) G_F^{\nu\beta}(x_1'-y')$$

with $p_2 = \xi P_2$, $Y = (y+y')/2$ and $\Delta y = y - y'$. In large Q limit:

$$\langle A^{c\mu}(x_1)A^{c\nu}(x_1')\rangle = g^2 C_F n_2^\mu n_2^\nu \delta(x_1^+ - x_1'^+)$$
$$\times \int d^2\mathbf{y}\rho_{A_2}(x_1^+, \mathbf{y}-\mathbf{b}) \int d\xi f_q(\xi) F_1(|\mathbf{x}_1-\mathbf{y}|) F_1(|\mathbf{x}_1'-\mathbf{y}|).$$

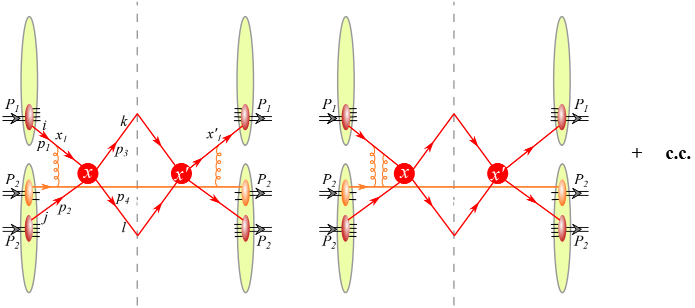
For the right-moving nucleus A_1, one swaps $+$ and $-$ components as well as 1 and 2.

See, e.g., Y. V. Kovchegov, Phys. Rev. D **54**, 5463-5469 (1996) [arXiv:hep-ph/9605446 [hep-ph]].

Initial-state effects

Initial-state single scattering

All the single-scattering diagrams



(a) (b)

$$= -\frac{g^2 C_i}{2(N_c^2-1)} \int d^4X \rho_{A_1}(X^-, \mathbf{X}) \rho_{A_2}(X^+, \mathbf{X}-\mathbf{b}) \int_{-\infty}^{X^+} dX_1^+ d\sigma^{(0)}_{ij\to kl}(X^+ - X_1^+)$$

$$\times \int d^2\mathbf{x} e^{-i\mathbf{x}\cdot\mathbf{Q}_T} \underbrace{\int dt \langle [A^{a-}(X_1^+ + t, \mathbf{X}+\mathbf{x}/2) - A^{a-}(X_1^+ - t, \mathbf{X}-\mathbf{x}/2)]^2 \rangle}_{G(X_1^+, \mathbf{X}, \mathbf{x})} .$$

- It contains a large logarithm $\ln(|\mathbf{x}|/R)$ with R nucleus size:

$$G(X_1^+, \mathbf{X}, \mathbf{x}) \equiv \int dt \langle [A^{a-}(X_1^+ + t, \mathbf{X} + \mathbf{x}/2) - A^{a-}(X_1^+ - t, \mathbf{X} - \mathbf{x}/2)]^2 \rangle$$
$$= 2g^2 C_F \int d^2\mathbf{r} \rho_{A_2}(X_1^+, \mathbf{X} - \mathbf{r}) \int d\xi f_q(\xi) [F_1(|\mathbf{r} + \mathbf{x}/2|) - F_1(|\mathbf{r} - \mathbf{x}/2|)]^2$$
$$= \frac{\alpha_s C_F}{2\pi} \int d^2\mathbf{r} \rho_{A_2}(X_1^+, \mathbf{X} - \mathbf{r}) \int d\xi f_q(\xi) \ln^2 \left(\frac{|\mathbf{r} + \mathbf{x}/2|^2}{|\mathbf{r} - \mathbf{x}/2|^2} \right),$$

where the vector $\mathbf{r} \equiv \mathbf{X} - \mathbf{y}$. This is because the factorization for nuleuon breaks down at scale around Λ_{QCD}.

- By introducing a facotorizaiton scale μ to normalize nucleon PDFs:

$$G(X_1^+, \mathbf{X}, \mathbf{x}) = \pi |\mathbf{x}|^2 \rho_{A_2}(X_1^+, \mathbf{X}) \lim_{x \to 0} xG(x, 1/|\mathbf{x}|^2) + \text{nonlogarithmic terms},$$

where the gluon distribution generated by the quark in a nucleon is given by

$$xG(x, 1/|\mathbf{x}|^2) = \frac{\alpha_s C_F}{\pi} \ln \left(\frac{1}{\mu^2 |\mathbf{x}|^2} \right) \int d\xi f_q(\xi, \mu).$$

The Fourier transform of the transverse-momentum dependent gluon distribution under the same approximation.

• For the Drell-Yan process:

$$\frac{d\sigma_{A_1A_2\to l^+l^-}}{d^2\mathbf{b}d\eta_3 d\eta_4 dp_T d^2\mathbf{Q}_T} = 2\int d^4X \rho_{A_1}(X^-,\mathbf{X})\rho_{A_2}(X^+,\mathbf{X}-\mathbf{b})\sum_{ij}\frac{d\sigma^{(0)}_{ij\to l^+l^-}}{d\eta_3 d\eta_4 dp_T}$$
$$\times\int\frac{d^2\mathbf{x}}{(2\pi)^2}e^{-i\mathbf{x}\cdot\mathbf{Q}_T}\left[1-\frac{|\mathbf{x}|^2}{4}\int_{-\infty}^{X^+}dX_1^+\hat{q}_{i/A_2}(X_1^+,\mathbf{X}-\mathbf{b},|\mathbf{x}|)\right.$$
$$\left.-\frac{|\mathbf{x}|^2}{4}\int_{-\infty}^{X^-}dX_1^-\hat{q}_{j/A_1}(X_1^-,\mathbf{X},|\mathbf{x}|)\right].$$

Here, we have made the replacements such as

$$\frac{2\sin[(X^+-X_1^+)(p_2^- - p_3^- - p_4^-)]}{p_2^- - p_3^- - p_4^-}\to 2\pi\delta(p_2^- - p_3^- - p_4^-)$$

due to confing in a box (classically).

More about $\hat{q}$: the jet quenching parameter

$$\hat{q}_{j/A_i}(X^\pm,\mathbf{X},|\mathbf{x}|) = \frac{4\pi^2\alpha_s C_j}{N_c^2-1}\rho_{A_i}(X^\pm,\mathbf{X})xG(x,1/|\mathbf{x}|^2).$$

The same as that in parton saturation physics: *Baier, Dokshitzer, Mueller, Peigne and Schiff, Nucl. Phys. B* ***484****, 265 (1997).*

Resumming intitial-state multiple scattering

Factorized multiple scattering due to the fact that diagrams with the Glauber gluon connecting two partons collinear to the same direction vanish.

In heavy nuclei, the summation of multiple scattering results can be substituted by the exponentiation of single scattering results, namely:

$$\frac{d\sigma_{A_1A_2\to l^+l^-}}{d^2\mathbf{b}d\eta_3 d\eta_4 dp_T d^2\mathbf{Q}_T} = 2\int d^4X \rho_{A_1}(X^-,\mathbf{X})\rho_{A_2}(X^+,\mathbf{X}-\mathbf{b})\sum_{ij}\frac{d\sigma^{(0)}_{ij\to l^+l^-}}{d\eta_3 d\eta_4 dp_T}$$
$$\times\int\frac{d^2\mathbf{x}}{(2\pi)^2}e^{-i\mathbf{x}\cdot\mathbf{Q}_T-\frac{|\mathbf{x}|^2}{4}\int_{-\infty}^{X^+}dX_1^+\hat{q}_{i/A_2}(X_1^+,\mathbf{X}-\mathbf{b},|\mathbf{x}|)}$$
$$\times\, e^{-\frac{|\mathbf{x}|^2}{4}\int_{-\infty}^{X^-}dX_1^-\hat{q}_{j/A_1}(X_1^-,\mathbf{X},|\mathbf{x}|)}.$$

where $\mathbf{Q}_T = \mathbf{p_3}_T + \mathbf{p_4}_T$.

Could be measured via azimuthal decorrelation of the l^+l^- pair?

N. Armesto, F. Cougoulic and BW, JHEP **11**, 081 (2024) [arXiv:2407.19243 [hep-ph]].

Recall parton saturation in DIS

$$\frac{dN}{d^2bd^2Q_T} = \int\frac{d^2x_\perp}{(2\pi)^2}e^{-iQ_T\cdot x_\perp}\rho_0 xq_N(x)\int_0^L dz e^{-\frac{1}{4}\hat{q}x_\perp^2 z}$$

Kovchegov and Mueller, Nucl. Phys. B **529**, 451 (1998).

Final-state effects

The coordinates for jet production

- For $n_i = n_3^\mu = (1, \sin\theta_3, 0, \cos\theta_3)$ and $n_j^\mu = n_1^\mu = (1, 0, 0, 1)$

$$n_{\perp_1}^\mu = (n_3^+ / \sin\theta_3, 1, 0, n_3^+ / \sin\theta_3), \qquad n_{\perp_2}^\mu = (0, 0, 1, 0),$$

$$D_F(x) = \int \frac{dp^-}{2\pi} \frac{1}{2p^-} e^{-\frac{i}{n_3^-} p^- x^{n_3}} \theta(x^-) \delta^{(2)}(\mathbf{x}_\perp),$$

- Classical trajectory of a classical particle moving in the n_3 direction:

$$\mathbf{x}_T = \frac{x^- \mathbf{n}_3 + x^{n_3} \mathbf{n}_1}{n_3^-} + x^1 \mathbf{n}_{\perp_1} + x^2 \mathbf{n}_{\perp_2} = \left(\frac{x^- \sin\theta_3}{n_3^-}, 0 \right) + \mathbf{x}_\perp,$$

where $\mathbf{x}_T$ are transverse with respect to the beam directions.

$$\delta^{(2)}(\mathbf{x}_\perp) \Rightarrow \mathbf{x}_T = \left(\frac{x^- \sin\theta_3}{n_3^-}, 0 \right),$$

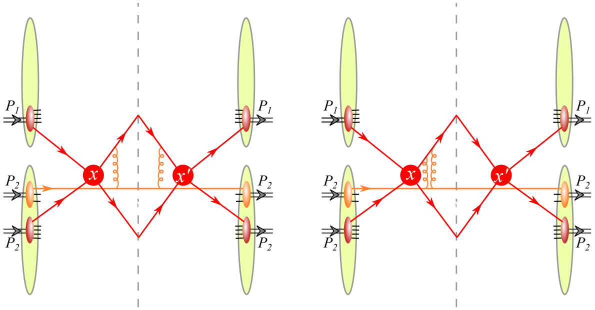
identical to the transverse components of the trajectory of a classical particle moving along n_3:

$$x^\mu(t) = n_3^\mu t = \left(t, \frac{x^-(t) \sin\theta_3}{n_3^-}, 0, t\cos\theta_3 \right)$$

with $x^-(t) = t - v_z t = t - \cos\theta_3 t = n_3^- t$.

Final-state single scattering

All the single-scattering diagrams



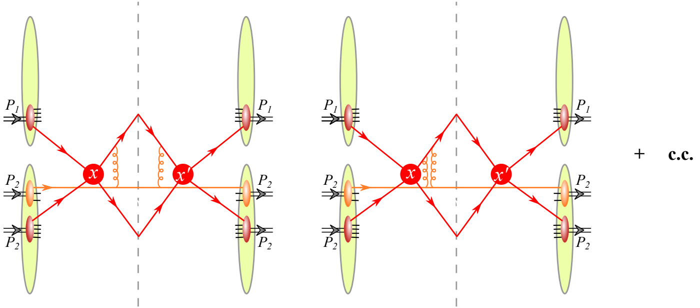
(a) (b)

+ c.c.

$$= \frac{-g^2 C_k}{2(N_c^2-1)} \int d^4X \int_{X^+}^{\infty} dX_1^+ \rho_{A_1}(X^-, \mathbf{X}) \rho_{A_2}(X^+, \mathbf{X}-\mathbf{b}) d\sigma^{(0)}_{ij\to kl}(X_1^+ - X^+) \int d^2\mathbf{x}_\perp$$

$$\times \int dt e^{-i\mathbf{x}_\perp\cdot\mathbf{Q}_\perp} \langle [A^{a-}(X_1^+ + t, \mathbf{X}_\perp + \mathbf{x}_\perp/2) - A^{a-}(X_1^+ - t, \mathbf{X}_\perp - \mathbf{x}_\perp/2)]^2 \rangle.$$

Keeping only the lograithmic terms:



(a) (b)

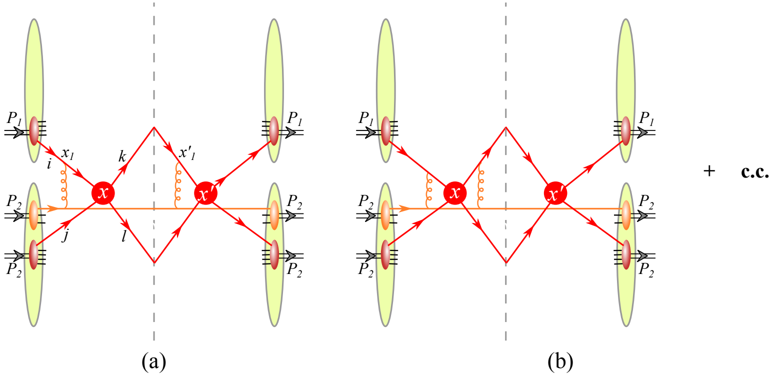
$$= -\frac{1}{2}\int d^4X \int_{X^+}^{\infty} dX_1^+ \rho_{A_1}(X^-,\mathbf{X})\rho_{A_2}(X^+,\mathbf{X}-\mathbf{b})d\sigma^{(0)}_{ij\to kl}(X_1^+-X^+)$$
$$\times \int d^2\mathbf{x}_\perp e^{-i\mathbf{x}_\perp\cdot\mathbf{Q}_\perp}|\mathbf{x}_\perp|^2\hat{q}_{k/A_2}(X_1^+,\mathbf{X}((X_1^+-X^+)/n_3^+)-\mathbf{b},|\mathbf{x}_\perp|),$$

where X^μ is where the hard collision occurs, X_1 is the "time" for Glauber exchange, and

$$\mathbf{X}(t) \equiv \mathbf{X} + (\sin\theta_3, 0)t.$$

Interference between intial- and final-state collisions

Such diagrams are power suppressed! See, i.e., J. C. Collins, D. E. Soper and G. F. Sterman, Phys. Lett. B **134**, 263 (1984).



- Gathering all single scattering contributions for boson-jet production:

$$\frac{d\sigma^{(1)}_{A_1A_2\to JV}}{d^2\mathbf{b}d\eta_3 d\eta_4 dp_T dQ_T^y} = -\frac{1}{2}\int d^4X\rho_{A_1}(X^-,\mathbf{X})\rho_{A_2}(X^+,\mathbf{X}-\mathbf{b})\int\frac{dy}{2\pi}e^{-iyQ_T^y}y^2$$
$$\times\sum_{ijk}\Big[\int_{-\infty}^{X^+}dX_1^+\frac{d\widetilde{\sigma}^{(0)}_{ij\to kV}}{d\eta_3 d\eta_4 dp_T}(X^+-X_1^+)\hat{q}_{i/A_2}(X_1^+,\mathbf{X}-\mathbf{b},|y|)$$
$$+\int_{-\infty}^{X^-}dX_1^-\frac{d\widetilde{\sigma}^{(0)}_{ij\to kV}}{d\eta_3 d\eta_4 dp_T}(X^--X_1^-)\hat{q}_{j/A_1}(X_1^-,\mathbf{X},|y|)$$
$$+\int_{X^+}^{\infty}dX_1^+\frac{d\widetilde{\sigma}^{(0)}_{ij\to kV}}{d\eta_3 d\eta_4 dp_T}(X_1^+-X^+)\hat{q}_{k/A_2}(X_1^+,\mathbf{X}((X_1^+-X^+)/n_3^+)-\mathbf{b},|y|)$$
$$+\int_{X^-}^{\infty}dX_1^-\frac{d\widetilde{\sigma}^{(0)}_{ij\to kV}}{d\eta_3 d\eta_4 dp_T}(X_1^--X^-)\hat{q}_{k/A_1}(X_1^-,\mathbf{X}((X_1^--X^-)/n_3^-),|y|)\Big].$$

Here $\tilde{\sigma}$ reminds us that we have not imposed energy conservation yet.

Resumming multiple scatterings

- For boson-jet production:

$$\frac{d\sigma_{A_1A_2\to JV}}{d^2\mathbf{b}d\eta_3 d\eta_4 dp_T dQ_T^y} = \sum_{ijk} \frac{d\sigma^{(0)}_{ij\to kV}}{d\eta_3 d\eta_4 dp_T} \int dX^+ dX^- d^2\mathbf{X} \rho_{A_1}(X^-, \mathbf{X})\, \rho_{A_2}(X^+, \mathbf{X}-\mathbf{b})$$
$$\times \int \frac{dy}{2\pi} e^{-iyQ_T^y - \frac{y^2}{4}\int_{-\infty}^{X^+} dX_1^+ \hat{q}_{i/A_2}(X_1^+, \mathbf{X}-\mathbf{b}, |y|) - \frac{y^2}{4}\int_{-\infty}^{X^-} dX_1^- \hat{q}_{j/A_1}(X_1^-, \mathbf{X}, |y|)}$$
$$\times\, e^{-\frac{y^2}{4}\int_{X^+}^{\infty} dX_1^+ \hat{q}_{k/A_2}(X_1^+, \mathbf{X}((X_1^+ - X^+)/n_3^+) - \mathbf{b}, |y|) - \frac{y^2}{4}\int_{X^-}^{\infty} dX_1^- \hat{q}_{k/A_1}(X_1^-, \mathbf{X}((X_1^- - X^-)/n_3^-), |y|)}.$$

That is, there are both initial- and final-state broadening effects, characterized by Q_s, referred to as cold nuclear effects. This is complementary to recent many studies for momentum broadening in Glasma back ground, See, i.e., and reference therein: D. Avramescu, C. Lamas, T. Lappi, M. Li and C. A. Salgado, [arXiv:2506.06206 [hep-ph]].

Summary and Perspective

1. We define, and investigate the factorization of, the impact-parameter dependent cross section for heavy-ion collisions.

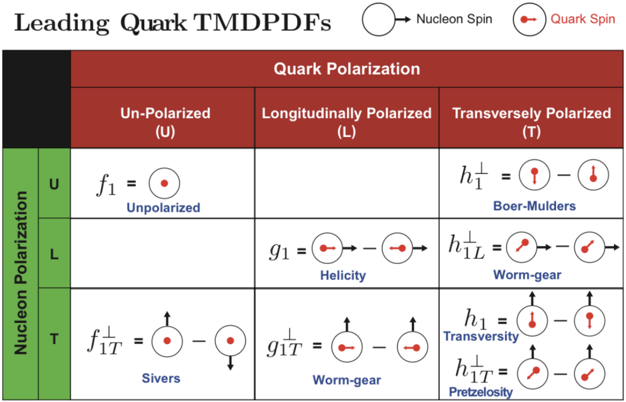
2. Using Glauber modelling of heavy nuclei, we present a formalism to deal with both hard and soft (semi-hard) processes.

3. We discuss the cold nuclear effects, which could be observable in azimuthal decoorelation in the Drell-Yan prcess ($AA \to l^+ l^-$) and are present in jet processes, although expected to be small compared to jet quenching effects.

4. Hopefully, we will learn how to combine TMD evolution and nuclear effects from this workshop even for the Drell-Yan processes.

Quark TMD PDFs

- 8 quark TMDs and 8 gluon TMDs:



Kotzinian, Nucl. Phys. B **441**, 234-248 (1995)[arXiv:hep-ph/9412283 [hep-ph]];
Tangerman and Mulders, Phys. Rev. D **51**, 3357-3372 (1995) [arXiv:hep-ph/9403227 [hep-ph]] ;

Boer and Mulders, Phys. Rev. D **57**, 5780-5786 (1998) [arXiv:hep-ph/9711485 [hep-ph]].

- Splitting of rapidity singularities in the soft factor to define TMDs free of rapidity divergences. See: Schimeni@ REF2018

Soft and hard functions

The soft function:

$$\mathcal{S}^{\bar{a}_A\bar{a}_B}_{a_A a_B}(\mathbf{x}) = \langle 0|\,\bar{T}[S^{\dagger a'_B \bar{a}_B}_{n_B}(\mathbf{x}) S^{\dagger a'_A \bar{a}_A}_{n_A}(\mathbf{x})]\,T[S^{a_A a'_A}_{n_A}(0) S^{a_B a'_B}_{n_B}(0)]|0\rangle$$

The hard function: the partonic cross section

$$H^{\bar{a}_A\bar{a}_B}_{a_A a_B} \equiv \frac{P^{\bar{\alpha}_A \alpha_A}}{d_{c_A}} \frac{P^{\bar{\alpha}_B \alpha_B}}{d_{c_B}} \tilde{\mathcal{C}}^{*\bar{a}_A\bar{a}_B}_{\bar{\alpha}_A\bar{\alpha}_B} \tilde{\mathcal{C}}^{a_A a_B}_{\alpha_A \alpha_B}$$

with $\tilde{\mathcal{C}}$ given by

$$\tilde{\mathcal{C}}(\epsilon, z_A \bar{n}_A \cdot P_A, z_B \bar{n}_B \cdot P_B) = \int dt_A dt_B e^{i(t_A z_A \bar{n}_A \cdot P_A + t_B z_B \bar{n}_B \cdot P_B)} \mathcal{C}(\epsilon, t_A, t_B).$$

The same as for pp collisions!