Evolution of structure functions at NLO without PDFs

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High energy QCD 2025 @ Benasque

JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

Centre of Excellence
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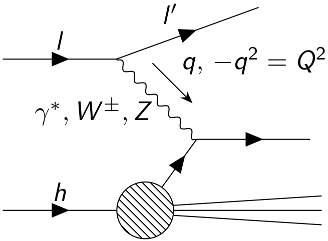
Background

- Deep Inelastic Scattering (DIS):



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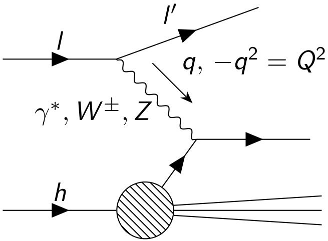

- Measured cross sections expressed in terms of structure functions

- Structure functions expressed in terms of parton distribution functions (PDFs)
  $F_i(x, Q^2) = \sum_j C_{ij}(Q^2, \mu^2) \otimes f_j(\mu^2) \quad j = q, \bar{q}, g \quad \mu =$ factorization scale

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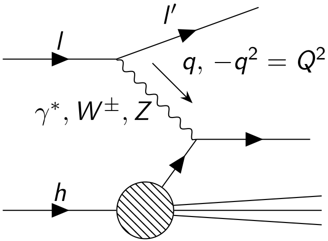

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- The conventional procedure in collinear factorization:
  - PDFs are fitted to DIS data (to structure functions)
  - Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution: PDFs to higher Q^2

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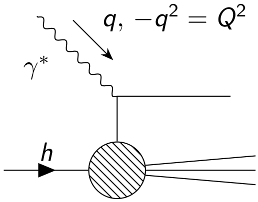
- The novelty of our work:
  - Momentum space
  - Full three-flavor basis at NLO
- NLO physical basis 2412.09589 continuation for LO work 2304.06998

Straightforward example with only two observables



$$F_i(x, Q^2) = \sum_j C_{F_i f_j}(Q^2, \mu^2) \otimes f_j(\mu^2),$$

where $F_i = F_2, F_{\mathrm{L}}/\frac{\alpha_{\mathrm{s}}}{2\pi}$, and $f_j = \Sigma, g$

Quark singlet:
$\Sigma(x, \mu^2) = \sum_q^{n_{\mathrm{f}}} \left[q(x, \mu^2) + \overline{q}(x, \mu^2)\right]$, $n_{\mathrm{f}} = 3$
Gluon PDF: $g(x, \mu^2)$

First step: invert the linear mapping (difficult because $f \otimes g = \int_x^1 \frac{\mathrm{d}z}{z} f(z) g\left(\frac{x}{z}\right)$)

$$f_j(\mu^2) = \sum_i C^{-1}_{F_i f_j}(Q^2, \mu^2) \otimes F_i(Q^2) + \mathcal{O}(\alpha_{\mathrm{s}}^2)$$

DGLAP evolution in physical basis

$$\frac{\mathrm{d} F_i(x, Q^2)}{\mathrm{d}\log(Q^2)} = \sum_j \frac{\mathrm{d} C_{F_i f_j}(Q^2, \mu^2)}{\mathrm{d}\log(Q^2)} \otimes f_j(\mu^2)$$

$$= \sum_j \frac{\mathrm{d} C_{F_i f_j}(Q^2, \mu^2)}{\mathrm{d}\log(Q^2)} \otimes \sum_k C^{-1}_{F_k f_j}(Q^2, \mu^2) \otimes F_k(Q^2) + \mathcal{O}(\alpha_{\mathrm{s}}^3)$$

Scheme and scale dependence at NLO

DGLAP evolution in physical basis:

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$$= \sum_k \mathcal{P}_{ik} \otimes F_k(Q^2) + \mathcal{O}(\alpha_\mathrm{s}^3)$$

Kernels $\mathcal{P}_{ik}$ are independent of the factorization scheme and scale

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$\mathcal{P}_{ij}$'s determined by:

- Splitting functions
- Coefficient functions

  $\longrightarrow$ The scheme and scale dependence exactly cancels between these two

Inverting the gluon PDF at NLO

Simple example without quarks

Invert $g(x)$ from $\widetilde{F}_{\mathrm{L}} = C^{(1)}_{F_{\mathrm{L}}g} \otimes g + \frac{\alpha_{\mathrm{s}}}{2\pi} C^{(2)}_{F_{\mathrm{L}}g} \otimes g$ where $\widetilde{F}_{\mathrm{L}}(x, Q^2) \equiv \frac{2\pi}{\alpha_{\mathrm{s}}} \frac{F_{\mathrm{L}}(x, Q^2)}{x}$

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$$g(x) = \hat{P}(x)\left[\widetilde{F}_\mathrm{L}(x) - \frac{\alpha_\mathrm{s}}{2\pi} C^{(2)}_{F_\mathrm{L}g} \otimes g\right]$$

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$$g(x) = \hat{P}(x)\left[\widetilde{F}_{\mathrm{L}}(x) - \frac{\alpha_s}{2\pi} C^{(2)}_{F_{\mathrm{L}}g} \otimes g\right]$$

Plug in $g(x) = \hat{P}(x)\widetilde{F}_{\mathrm{L}}(x) + \mathcal{O}(\alpha_s)$ to the right hand side

$$g(x) = \hat{P}(x)\widetilde{F}_{\mathrm{L}}(x) - \frac{\alpha_s(Q^2)}{2\pi}\hat{P}(x)\left[C^{(2)}_{F_{\mathrm{L}}g} \otimes \hat{P}\widetilde{F}_{\mathrm{L}}\right] + \mathcal{O}\left(\alpha_s^2\right)$$

Six observable basis

- Full three-flavor basis: $u, \bar{u}, d, \bar{d}, s = \bar{s}$, and g
  $\longrightarrow$ Need six linearly independent DIS structure functions

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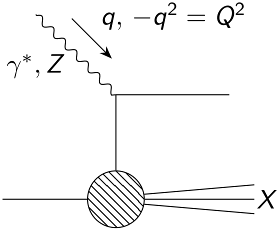
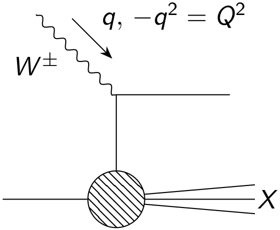
  $\longrightarrow$ Need six linearly independent DIS structure functions
- We choose the NLO structure functions:





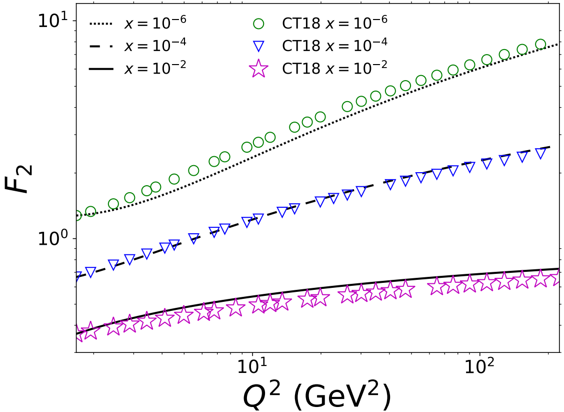
Neutral current γ^*, Z

- γ^* exchange $\rightarrow F_2$ and $F_{\rm L}$
- Z boson exchange $\rightarrow F_3$

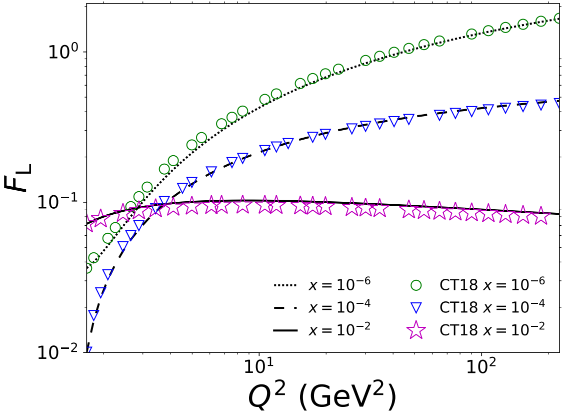
Charged current $W^\pm$

- W^- exchange $\rightarrow F_3^{\rm W^-}$ and $F_{\rm 2c}^{\rm W^-}$
- $\Delta F_2^W = F_2^{\rm W^-} - F_2^{W^+}$

Comparison with conventional DGLAP evolution



F_2



F_{L}

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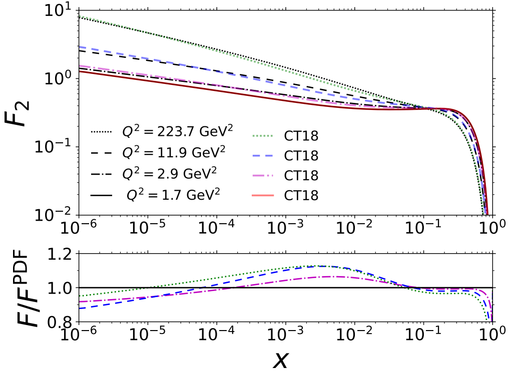
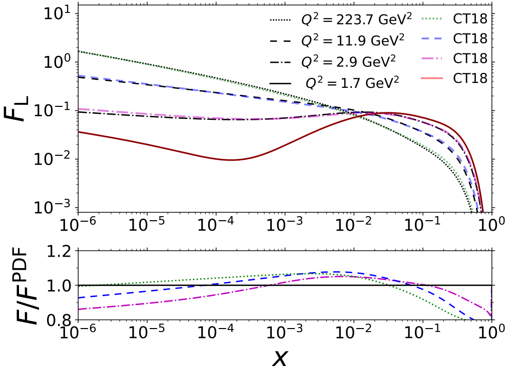


F_2/F_2^{CT18}

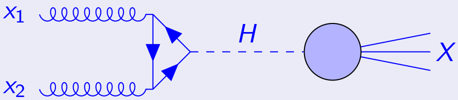


$F_{\mathrm{L}}/F_{\mathrm{L}}^{\mathrm{CT18}}$

- Similar Q^2 evolution
- Differences in values from:
  - uncertainty in PDFs from scheme and scale (error band not shown)
  - perturbative truncation

Cross sections in terms of physical basis

Example of Higgs production by gluon fusion



$$\sigma(p + p \longrightarrow \mathrm{H} + X) = \int \mathrm{d}x_1 \mathrm{d}x_2 g(x_1, \mu) g(x_2, \mu) \hat{\sigma}_{gg \to H+X}(x_1, x_2, \frac{m_H^2}{\mu^2}),$$

where m_H is the Higgs mass, $g(x_1, \mu)$ and $g(x_2, \mu)$ are the gluon PDFs

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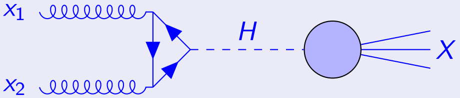

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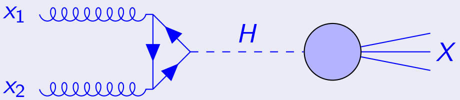

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$$\sigma(p+p\longrightarrow H+X)=$$

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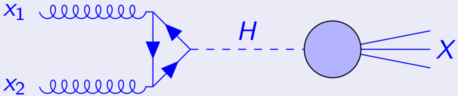

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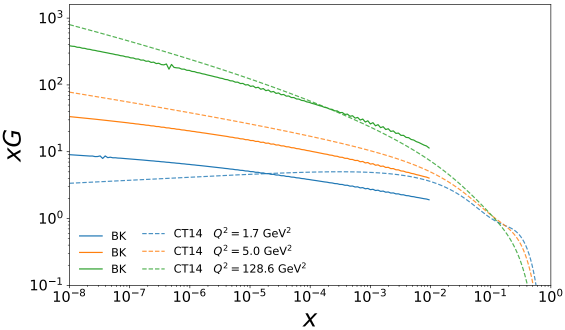
Harland-Lang and Thorne 1811.08434:

explicit μ dependence vanishes and terms $\log\left(Q^2/m_H^2\right)$ are left behind

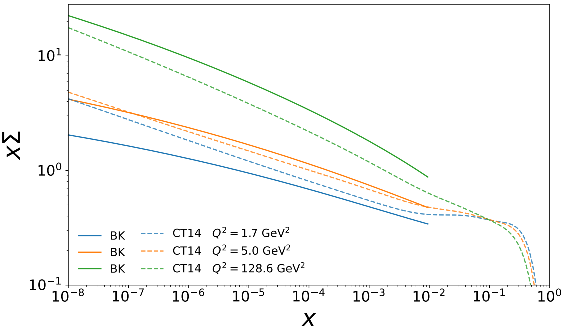
$\longrightarrow$ no need to choose relation between μ and Q or m_H

PDFs from BK-evolved structure functions

Now we have analytical form to calculate gluon PDF and quark singlet from F_2 and F_L in dipole picture



LO gluon



LO quark singlet

- Weaker x-evolution with BK-evolved $F_{2,\mathrm{L}}$
- Bigger difference in gluon than in quark singlet

Comparison to BK-evolved $F_{2,\mathrm{L}}$ (work in preparation)

Goal

Set BK-evolved F_2 and F_L as initial condition for (2-observable) physical basis DGLAP evolution
$\longrightarrow$ compare BK vs. DGLAP dynamics

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- *However..*
  - LO DGLAP evolution (and NLO PDFs) in physical basis includes convolutions e.g.
    $P_{qq} \otimes F_2 = \int_x^1 \frac{\mathrm{d}z}{z} P_{qq}(z) F_2\left(\frac{x}{z}\right)$
    $\longrightarrow$ need $F_{2,\mathrm{L}}$ initial values up to $x = 1$
  - Validity region for BK-evolved $F_{2,\mathrm{L}}$ only up to $x \sim 10^{-2}$

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Quick fix: BK-improved initial condition

- Initial values for $F_{2,\mathrm{L}}$:
  - at $x \leq 10^{-2}$ from BK/dipole picture
  - at $x > 10^{-2}$ from DGLAP/collinear factorization
- Scale collinear factorization $F_{2,\mathrm{L}}$ so that they match dipole picture values

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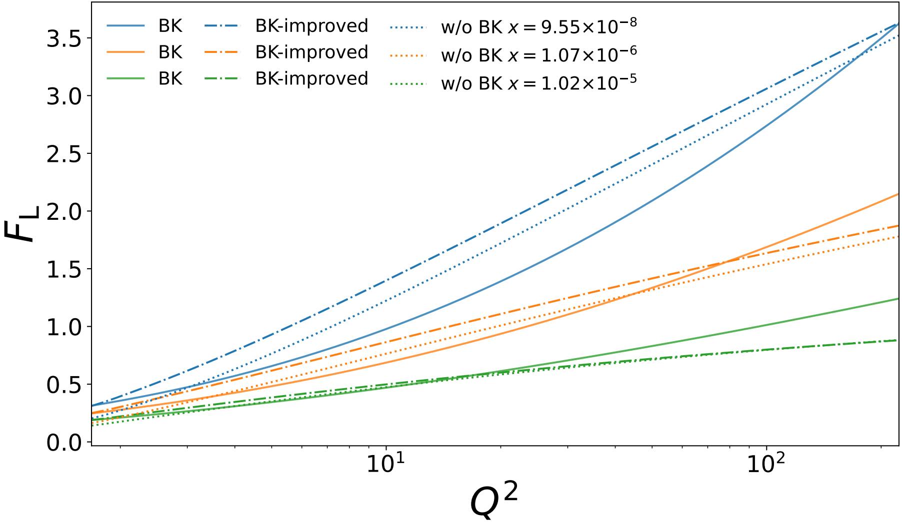
- Scheme dependence of PDFs play a role at NLO in α_{s}
  $\longrightarrow$ Scheme and scale dependence avoided in the physical basis
- What next:
  - BK vs. DGLAP comparison
  - Express LHC cross sections, e.g. Drell-Yan, in physical basis
  - Include heavy quarks

Backup: Comparison to BK-evolved F_{L} (work in preparation)



Backup: Comparison to BK-evolved F_2 (work in preparation)