Sea quark TMDs from the CGC: factorisation and evolution

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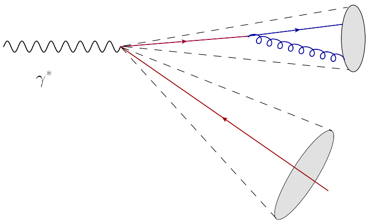
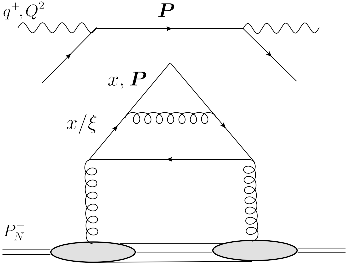
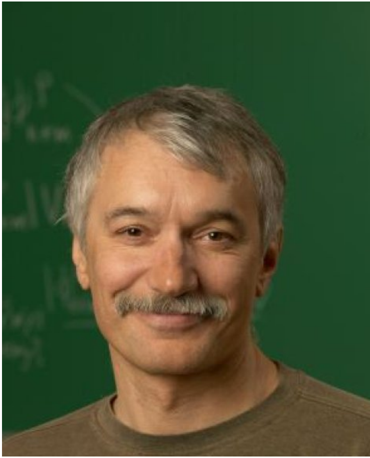
with Paul Caucal, Marcos Morales, Al Mueller, Farid Salazar, Feng Yuan,
arXiv:2408.03129 & 2503.16162





Dedicated to Ian's 70th birthday

- B as in BFKL
- A story about stars $(x_\star, p_\star)$ and dots $(x_\bullet, p_\bullet)$
- Readable of not... but often inspiring
- B as in Balitsky hierarchy (vs. JIMWLK)
- Diamond action, NLO BK ...
- Visiting me in Saclay (several times)
- Trying to understand each other
- The importance of being shy ... and stubborn



- *Happy stars and dots, Ian ! To many unreadable papers !*

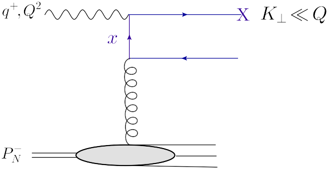
The CGC/TMD correspondence

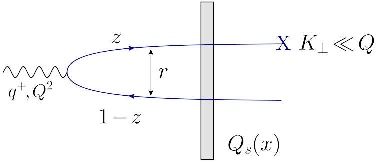
- Deriving TMD factorisation at small x from the CGC effective theory
- Explicit results for the TMD PDFs for small x gluons and sea partons
  - tree-level + quantum evolutions
  - high-energy (BK/JIMWLK) & collinear (DGLAP, CSS)
- Problems with two widely separated transverse momentum scales
  - SIDIS with $Q^2 \gg K_\perp^2$, di-jets (or di-hadrons) with $P_\perp \gg K_\perp$ ...
  - eA DIS, AA UPCs (inclusive & diffractive), pA
- Similar to standard TMD fact at moderate x, but also important differences
  - same hard factors, but different TMDs (valid at small x)
  - saturation effects when $K_\perp \lesssim Q_s$: multi-gluon correlations
  - gauge links fully materialised at leading order: multiple scattering
  - more than just "single-particle" parton distributions !
- High-energy and collinear resummation in a same framework !
- Is that the road towards the Grand Unification (small x and moderate x) ??

Sea quark TMD factorisation for SIDIS

Marquet, Xiao and Yuan, arXiv:0906.1454 [hep-ph]

- Measure a single jet (or hadron) in the "hard" regime $Q^2 \gg K_\perp^2 \gtrsim Q_s^2$
- Extract the leading power in $1/Q^2$ (expansion in both $K_\perp^2/Q^2$ and Q_s^2/Q^2)
- Target picture: photon absorbed by a sea quark ("photon-gluon fusion")





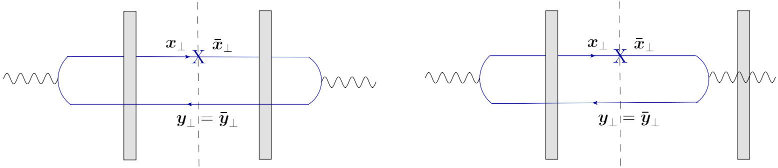
- Colour dipole picture: a quark-antiquark pair of "aligned jets": $z(1-z) \ll 1$

$$r \sim \frac{1}{K_\perp} \gtrsim \frac{1}{Q} \quad \text{and} \quad z(1-z)r^2Q^2 \lesssim 1 \quad \Longrightarrow \quad z(1-z) \sim \frac{K_\perp^2}{Q^2} \ll 1$$

  - relatively large dipole $r \gg 1/Q$: saturation effects when $K_\perp \lesssim Q_s$
  - the struck fermion is aligned with the photon: $z \simeq 1$ or $1 - z \simeq 1$

The sea quark TMD

- $K_\perp \lesssim Q_s$: resum multiple scattering in the eikonal approximation
  - transverse coordinates: $\boldsymbol{x}$, $\bar{\boldsymbol{x}}$ for the quark, $\boldsymbol{y} = \bar{\boldsymbol{y}}$ for antiquark
  - Wilson lines $V_{\boldsymbol{x}}$, $V^\dagger_{\boldsymbol{y}}$ and dipole S-matrices $\mathcal{D}(\boldsymbol{x}, \boldsymbol{y}) = (1/N_c)\langle \mathrm{tr} V_{\boldsymbol{x}} V^\dagger_{\boldsymbol{y}}\rangle$
  - colour structure: $\mathcal{D}(\boldsymbol{x}, \boldsymbol{y}) - \mathcal{D}(\boldsymbol{x}, \bar{\boldsymbol{x}}) - \mathcal{D}(\boldsymbol{x}, \bar{\boldsymbol{y}}) + 1$
  - N.B. the scattering of the unmeasured quark matters as well



$$\frac{\mathrm{d}\sigma^{\gamma^*_{\mathrm{T}}+A\to j+X}}{\mathrm{d}^2\boldsymbol{K}} = \frac{8\pi^2\alpha_{\mathrm{em}}e_f^2}{Q^2}\,\mathcal{F}_q^{(0)}(x, \boldsymbol{K})\left[1 + \mathcal{O}\left(\frac{K_\perp^2}{Q^2}\right)\right]$$

- the standard hard factor for photon absorption by a quark
- the (tree-level) sea quark TMD as given by the CGC

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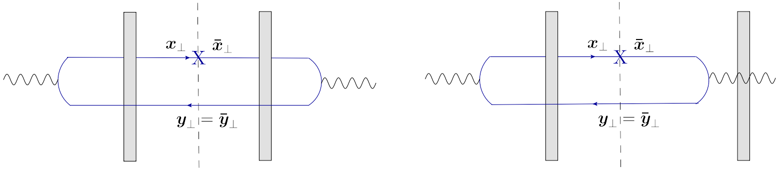
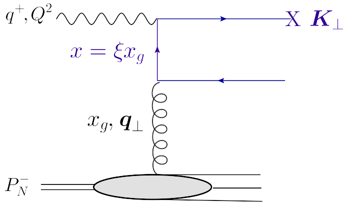


$$\mathcal{F}_q^{(0)}(x,\boldsymbol{K}) = \frac{N_c}{\pi^2}\int \mathrm{d}^2\boldsymbol{b}\int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}\,\mathcal{D}(\boldsymbol{b},\boldsymbol{q})\left[1 - \frac{\boldsymbol{K}\cdot(\boldsymbol{K}-\boldsymbol{q})}{(K_\perp^2 - (\boldsymbol{K}-\boldsymbol{q})^2)}\,\ln\frac{K_\perp^2}{(\boldsymbol{K}-\boldsymbol{q})^2}\right]$$

- $\mathcal{D}(\boldsymbol{b},\boldsymbol{q})$: Fourier transform of $\mathcal{D}(\boldsymbol{x},\boldsymbol{y})$: $\boldsymbol{x}-\boldsymbol{y} \to \boldsymbol{q}$ and $(\boldsymbol{x}+\boldsymbol{y})/2 = \boldsymbol{b}$

Recovering one step in DGLAP

- The sea quark TMD describes a gluon decay $g \to q\bar{q}$ (as it should)
  - the integral over $\boldsymbol{q}$ features the "dipole" gluon TMD



$$\mathcal{G}_D^{(0)}(x, \boldsymbol{q}) \equiv \frac{q_\perp^2 N_c}{2\pi^2 \alpha_s} \int \mathrm{d}^2\boldsymbol{b}\, \mathcal{D}(x, \boldsymbol{b}, \boldsymbol{q})$$

$$xG^{(0)}(x, K_\perp^2) = \int_0^{K_\perp^2} \mathrm{d}^2\boldsymbol{q}\, \mathcal{G}_D^{(0)}(x, \boldsymbol{q})$$

- Assume the jet is relatively hard: $Q^2 \gg K_\perp^2 \gg Q_s^2$
- The integral over $q_\perp^2$ is controlled by $Q_s^2 \ll q_\perp^2 \ll K_\perp^2$ (log enhancement)
- The sea quark TMD is proportional to the gluon PDF $xG^{(0)}(x, K_\perp^2)$

$$\mathcal{F}_q^{(0)}(x, \boldsymbol{K}) \simeq \frac{\alpha_s}{2\pi^2} \frac{1}{K_\perp^2} \frac{1}{3}\, xG^{(0)}(x, K_\perp^2)$$

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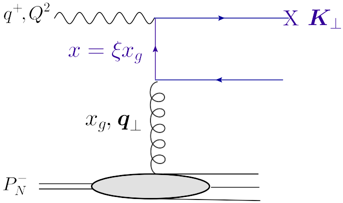


$$\xi \equiv \frac{x}{x_g} = \frac{Q^2}{Q^2 + K_\perp^2/z(1-z)} \sim \mathcal{O}(1/2)$$

$$P_{qg}(\xi) = \frac{1}{2}\left[\xi^2 + (1-\xi)^2\right]$$

$$\int_0^1 \mathrm{d}\xi\, P_{qg}(\xi) = \frac{1}{3}$$

- This can be recognised as one step in the DGLAP splitting $g \to q\bar{q}$
  - introduce the splitting fraction ξ w.r.t. target longitudinal momentum

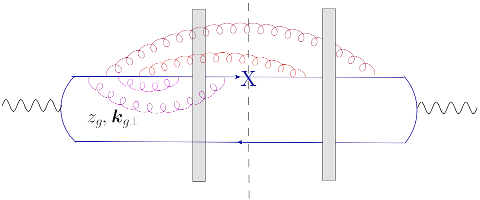
$$\mathcal{F}_q^{(0)}(x, \boldsymbol{K}) \simeq \frac{\alpha_s}{2\pi^2}\frac{1}{K_\perp^2}\int_0^1 \mathrm{d}\xi\, P_{qg}(\xi)\,\frac{x}{\xi}G^{(0)}\left(\frac{x}{\xi}, K_\perp^2\right)$$

- For generic $K_\perp \sim Q_s$: generalised, $K_\perp$-dependent splitting function

(Xiao, Yuan, and Zhou, 2017; Altinoluk, Jalilian-Marian, Marquet, 2024)

High-energy evolution

- Quantum evolution: gluon emissions $(k_g^+ = z_g q^+, \boldsymbol{k}_g)$ by the $q\bar{q}$ dipole
- High-energy evolution: corrections enhanced by rapidity logarithm $\alpha_s \int \frac{\mathrm{d}z_g}{z_g}$
- The only non-trivial aspect: the boundaries on the phase-space in z_g
  - not all the soft gluons $(z_g \ll 1)$ contribute to the high energy evolution



- BK evolution of dipole $\mathcal{D}(\boldsymbol{x}, \boldsymbol{y})$
- gluon emissions by the quark only

- Lower & upper boundaries on the gluon formation time $\tau_g \simeq \frac{2z_g q^+}{k_{g\perp}^2}$
  - larger than the target width, smaller than the photon coherence time

$$\frac{1}{P_N^-} < \frac{2z_g q^+}{k_{g\perp}^2} < \frac{2q^+}{Q^2} \quad \Longrightarrow \quad \frac{k_{g\perp}^2}{2q^+ P_N^-} < z_g < \frac{k_{g\perp}^2}{Q^2}$$

High-energy evolution

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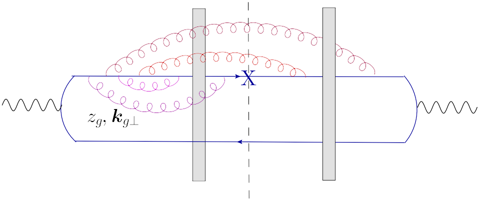

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$$\int_{k_{g\perp}^2/s}^{k_{g\perp}^2/Q^2} \frac{\mathrm{d}z_g}{z_g} = \ln\frac{s}{Q^2} = \ln\frac{1}{x} \quad \Longleftarrow \quad \frac{k_{g\perp}^2}{2q^+P_N^-} < z_g < \frac{k_{g\perp}^2}{Q^2}$$

High-energy evolution

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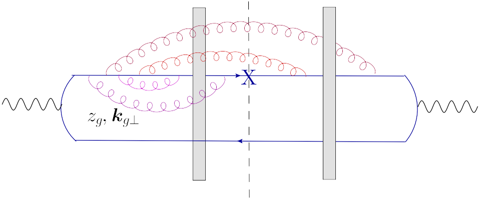
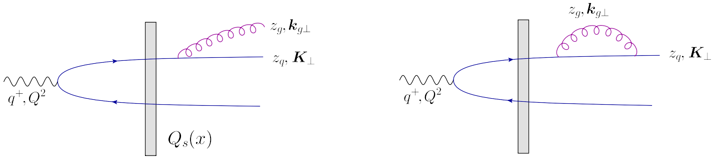


- BK evolution of dipole $\mathcal{D}(\boldsymbol{x}, \boldsymbol{y})$
- gluon emissions by the quark only

- The expected rapidity phase-space $\ln(1/x)$ for the evolution of the target
- The collinearly improved version of the BK equation
  (G. Beuf, 2014, E.I. + Al M., Dionysis T., Bertrand D., Grégory S. 2015–19)
- Remaining logarithmic phase-space at $\frac{k_{g\perp}^2}{Q^2} < z_g < 1$: goes to CSS evolution

The Sudakov double logarithm

- The remaining logarithmic phase-space in z_g refers to final state (FS) emissions by the measured quark
  - emissions by the unmeasured $\bar{q}$ do not change the final state
  - the same is true for emissions by the quark with $k_{g\perp} \ll K_\perp$
  - their effect cancel between real and virtual emissions



- Soft ($z_g \ll 1$) FS emissions factorise: a change in the sea quark TMD

$$\Delta\mathcal{F}_q(x, \boldsymbol{K}) = \frac{\alpha_s C_F}{\pi^2} \int \mathrm{d}^2\boldsymbol{k}_g \int_{k_{g\perp}^2/Q^2}^{1} \frac{\mathrm{d}z_g}{z_g} \, \frac{\mathcal{F}_q^{(0)}(x, \boldsymbol{K}+\boldsymbol{k}_g) - \mathcal{F}_q^{(0)}(x, \boldsymbol{K})}{(z_q\boldsymbol{k}_g - z_g\boldsymbol{K})^2}$$

  - double-log: virtual gluons with $K_\perp^2 \ll k_{g\perp}^2 \ll Q^2$ and $z_g \ll 1$

Jet vs hadron measurement

$$\Delta\mathcal{F}_q(x,\boldsymbol{K}) = -\frac{\alpha_s C_F}{\pi}\mathcal{F}_q^{(0)}(x,\boldsymbol{K})\int_{K_\perp^2}^{Q^2}\frac{\mathrm{d}k_{g\perp}^2}{k_{g\perp}^2}\int_{k_{g\perp}^2/Q^2}^{1}\frac{\mathrm{d}z_g}{z_g}$$

- The overall (negative) coefficient: the Sudakov double log for SIDIS

$$\Delta\mathcal{F}_q(x,\boldsymbol{K},Q^2) = -\frac{\alpha_s C_F}{2\pi}\ln^2\frac{Q^2}{K_\perp^2}\,\mathcal{F}_q^{(0)}(x,\boldsymbol{K})$$

Xiao, Yuan, and Zhou, arXiv:1703.06163 (target picture)

Altinoluk, Jalilian-Marian, and Marquet, 2406.08277 (dipole picture)

- The “penalty” for trying to fix $K_\perp$ (and hence suppress radiation)
- Large, but can be resummed to all orders by solving CSS equations
- The total Sudakov double log refers to the measurement of a hadron

$$S_{\rm had} = -2\times\frac{\alpha_s C_F}{4\pi}\ln^2\left(\frac{Q^2}{K_\perp^2}\right) = S_{\rm jet} + S_{\rm frag}$$

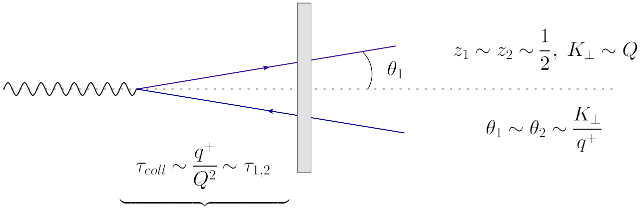
  - two CSS: sea quark TMD in the target & quark fragmentation TMD

The Sudakov for jets in SIDIS

- "Duality" between initial-state evolution of the target & final-state jet production in the projectile picture
  - $S_{\rm jet}$ as computed in the dipole picture should match the cusp anomalous dimension (the coefficient in the dual CSS equation)
  - verified e.g. for di-jet production in DIS, pA... but not also for SIDIS
- Our pupose *(P. Caucal, E.I., Al Mueller & F. Yuan, arXiv:2408.03129, PRL)*
  - compute $S_{\rm jet}$ in the dipole picture... but we instead obtained $S_{\rm had}$ ☹
- $S_{\rm jet}$ receives contributions from gluon emissions outside the jet: $\theta_g > \theta_{\rm jet}$
- What is the correct definition of a "jet" in SIDIS ?
  - why should this be more subtle than, say, for di-jets in DIS ?
- Symmetric di-jets ($z \sim 1 - z \sim 1/2$) have $K_\perp \sim Q$ and low virtualities
  - they are easily put on-shell by the collision
- The aligned jet ($z \simeq 1$) has $K_\perp^2 \sim (1-z)Q^2$ and a large virtuality $Q^2 \gg K_\perp^2$

Formation times & jet virtualities

- Uncertainty principle argument based on formation times
- $\tau_k = \frac{2k^+}{\boldsymbol{k}_\perp^2}$: the typical times it takes a parton created with longitudinal momentum $k^+ = zq^+$ and transverse momentum $\boldsymbol{k}_\perp$ to get on-shell
- The collision time $\tau_{coll} \sim \frac{2q^+}{Q^2}$: the lifetime of the $q\bar{q}$ pair
- Consider symmetric jets first: $z \sim 1 - z \sim \frac{1}{2}, \quad K_\perp \sim Q$



- Both quarks are essentially on-shell by the time of scattering
- After the collision, they propagate along the natural angles $\theta_i \sim \frac{K_\perp}{q^+}$

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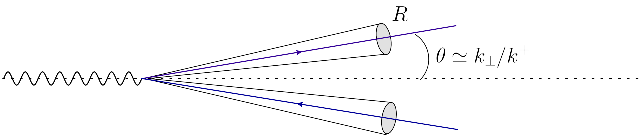
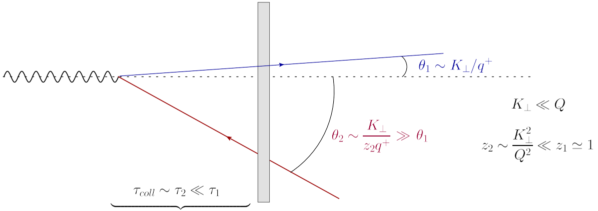


- Jet angular opening proportional to the angle made by the leading parton

$$\Delta\theta \simeq R \frac{K_\perp}{k^+} \quad \text{with} \quad R \sim 0.2 \div 0.4$$

Aligned jets: SIDIS

- Yet, the typical SIDIS events are asymmetric: a fast quark and a slow one
- Assume a slow antiquark: $z_2 \sim \frac{K_\perp^2}{Q^2} \ll 1 \;\;\&\;\; z_1 = 1 - z_2 \simeq 1$



- Both fermions have relatively low transverse momenta $K_\perp \ll Q$
- The quark has a larger longitudinal momentum: $k_1^+ \simeq q^+ \gg k_2^+ = z_2 q^+$
- Hence the quark makes a tiny angle w.r.t. the collision axis:

$$\theta_1 \sim \frac{K_\perp}{q^+} \ll \theta_{sym} = \frac{Q}{q^+} \ll \theta_2 \sim \frac{K_\perp}{z_2 q^+}$$

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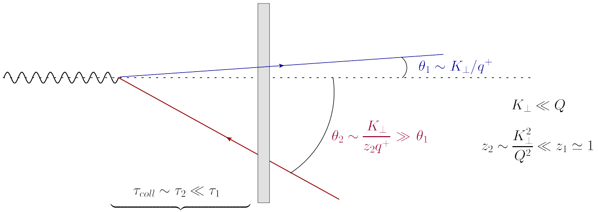

- WRONG ! The fast quark is highly off-shell

$$\tau_1 \simeq \frac{q^+}{K_\perp^2} \;\gg\; \tau_{coll} \simeq \frac{q^+}{Q^2} \simeq \frac{z_2 q^+}{K_\perp^2} \sim \tau_2$$

- The antiquark is put on-shell by the scattering, but the quark is highly virtual

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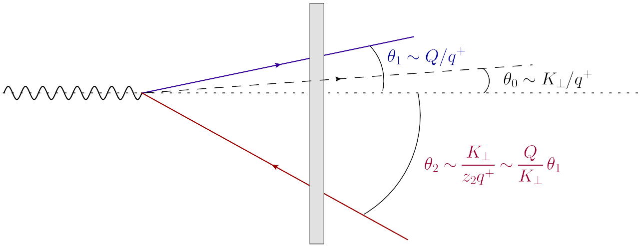

- By the uncertainty principle, the quark makes a much larger angle $\theta_1 \sim \frac{Q}{q^+}$
- The angle of the aligned jet is as large as it would be for symmetric jets:

$$\theta_1 \sim \frac{Q}{q^+} \sim \theta_{sym} \ll \theta_2 \sim \frac{Q}{K_\perp}\,\theta_1$$

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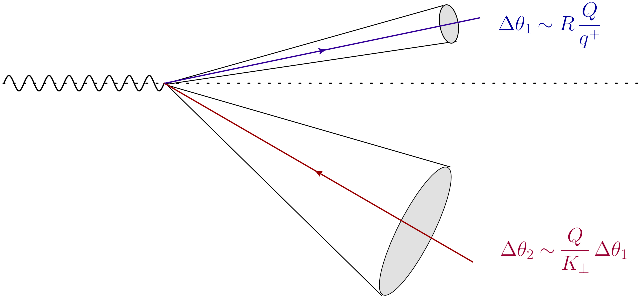


- Two asymmetric jets, but none of them is tiny
- The jet algorithm must properly account for its virtuality $\sim Q^2$
- The precise jet definition starts to matter at NLO

The jet Sudakov in SIDIS

- Recall the expression of the full ("hadronic") Sudakov:

$$\Delta\mathcal{F}_q(x,\boldsymbol{K}) = -\frac{\alpha_s C_F}{\pi}\mathcal{F}_q^{(0)}(x,\boldsymbol{K})\int_{K_\perp^2}^{Q^2}\frac{\mathrm{d}k_{g\perp}^2}{k_{g\perp}^2}\int_{k_{g\perp}^2/Q^2}^{1}\frac{\mathrm{d}z_g}{z_g}$$

- If one measures a jet, only emissions outside the jet should matter: $\theta_g > \theta_{\rm jet}$

$$\theta_g \sim \frac{k_{g\perp}}{z_g q^+} > \theta_{\rm jet} \sim \frac{Q}{q^+} \quad\Longrightarrow\quad z_g < \frac{k_{g\perp}}{Q}$$

- Separate the z_g-integral between
  - "out-of-jet" $(k_{g\perp}^2/Q^2 < z_g < k_{g\perp}/Q)$ emissions
  - ... and "intra-jet" $(k_{g\perp}/Q < z_g < 1)$ emissions
  - each of them contributes half of the total Sudakov

$$S_{\rm jet} = -\frac{\alpha_s C_F}{4\pi}\ln^2\left(\frac{Q^2}{K_\perp^2}\right)$$

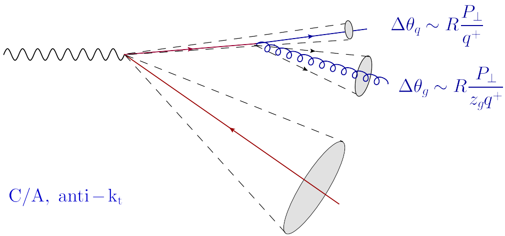
- The requirement of TMD factorisation beyond LO constraints jet definition

Narrow vs fat jets

- The aligned quark ($z \simeq 1$, $K_\perp \ll Q$), emits a hard gluon in the final state
  - the final quark and gluon are back-to-back: $k_{1\perp} \simeq k_{g\perp} \simeq P_\perp$
- Will the gluon generate a third jet ?
- Or will it be a part of a quark jet with transverse momentum $\sim K_\perp$?
- The gluon lies outside the quark jet provided $\Delta\theta_{1g} > R\theta_1$

Naively: $\Delta\theta_{1g} \simeq \dfrac{P_\perp}{z_g q^+} > R\,\dfrac{P_\perp}{z_1 q^+}$ for any $z_g < z_1 \simeq 1$

- All the hard gluon emissions are predicted to be outside the jet



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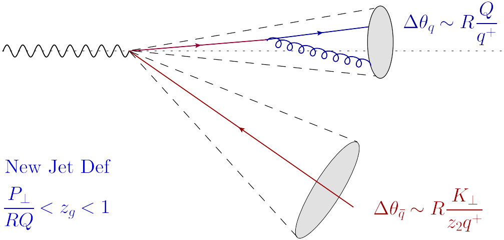
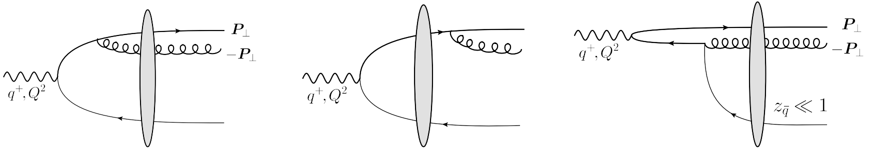
Correctly: $\Delta\theta_{1g} \simeq \dfrac{P_\perp}{z_g q^+} > R\,\dfrac{Q}{q^+}$ when $z_g < \dfrac{P_\perp}{RQ} \ll 1$

- Emissions with larger z_g make small angles & remain inside the “fat” jet



Back-to-back quark-gluon jets in DIS

- Nearly back-to-back di-jets at forward rapidities: $k_{1\perp} \simeq k_{2\perp} \simeq P_\perp \gg K_\perp$
- The paradigm: quark-antiquark dijets, WW gluon TMD factorisation
- Quark-gluon jets are possible as well: sea quark TMD factorisation
- CGC: $q\bar{q}g$ fluctuation of γ^* with a soft antiquark: $z_{\bar{q}} \sim K_\perp^2/Q^2$
  - the soft $\bar{q}$ transferred to the target wavefunction $\Rightarrow$ sea quark TMD



$$\frac{\mathrm{d}\sigma^{\gamma_T^\star + A \to qg + X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{K} \mathrm{d}z_q \mathrm{d}z_g} = H_T(P_\perp, Q, z_q, z_g)\, \mathcal{F}_q^{(0)}(x, \boldsymbol{K})$$

- The same sea quark TMD as in SIDIS *(Caucal et al, arXiv:2503.16162)*
- Same hard factor as for diffractive 2+1 jets *(Hauksson et al, 2402.14748)*

Back-to-back quark-gluon jets in DIS

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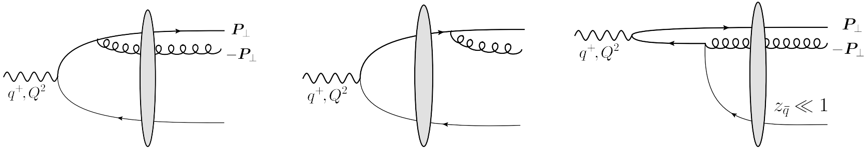

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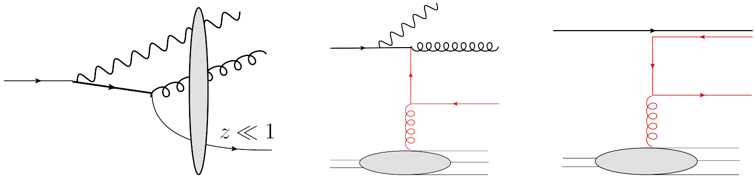
- By integrating out the gluon when $Q^2 \gg P_\perp^2$: real NLO correction to SIDIS
  - gluon must be emitted outside the quark jet *(Caucal et al, 2408.03129)*

More sea quark TMDs

(P. Caucal, M. Guerrero Morales, E. I., F. Salazar, F. Yuan, arXiv:2503.16162)

- Sea quark TMD factorisation also emerges in pA collisions
  - back-to-back di-jets in processes where $\Delta n_q = 1$
- Coloured particles on the incoming lin: new colour operators
- The simplest example: photon-gluon production in quark-nucleus collisions
- Target picture: the quark from p annihilates against a sea anti-quark



$$\mathcal{F}_q^{(2)}(x, \boldsymbol{K}) = \int_{\boldsymbol{b},\boldsymbol{q}} \left\langle \hat{\mathcal{D}}(\boldsymbol{b}, \boldsymbol{q})\, \hat{\mathcal{F}}_q^{(0)}(\boldsymbol{b}, \boldsymbol{K} - \boldsymbol{q}) \right\rangle_x$$

Jet algorithms for SIDIS

- Roughly: 2 partons i and j belong to the same jet provided

$$\Delta\theta_{ij} \le R\theta_{\rm jet}, \qquad \Delta\theta_{ij} \simeq \left| \frac{\boldsymbol{k}_{i\perp}}{z_i q_+} - \frac{\boldsymbol{k}_{j\perp}}{z_j q_+} \right|$$

- The main question: what is $\theta_{\rm jet}$?

- Standard algorithms for jets at the LHC: anti-k_t, Cambridge/Aachen:

$$\theta_{\rm jet} = \frac{k_{\perp,\rm jet}}{z_{\rm jet} q^+}, \qquad \boldsymbol{k}_{\perp,\rm jet} = \boldsymbol{k}_{i\perp} + \boldsymbol{k}_{j\perp}, \qquad z_{\rm jet} = z_i + z_j$$

  - if applied to the aligned jet in SIDIS, it would select $\boldsymbol{k}_{\perp,\rm jet} = P_\perp$

- However, for the aligned jet in SIDIS, $\theta_{\rm jet} = \frac{Q}{q^+}$. The precise condition:

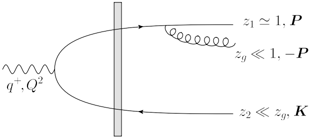
$$M_{ij}^2 \le z_i z_j Q^2 R^2, \quad M_{ij}^2 \equiv (k_i + k_j)^2 \quad : \text{boost invariant}$$

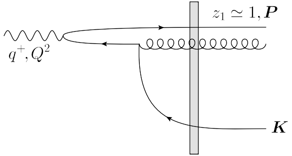
- Roughly equivalent criterion (CENTAURO) by *Arratia, Makris, Neill, Ringer, Sato, 2006.10751*

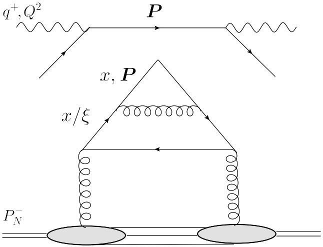
Conclusions

Happy 70th Birthday Ian !

- A hard quark-gluon pair $(P_\perp)$ plus a semi-hard antiquark $(Q_s \lesssim K_\perp \ll P_\perp)$
- The colour dipole calculation reproduces the DGLAP splitting function





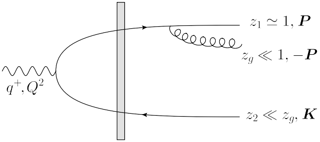


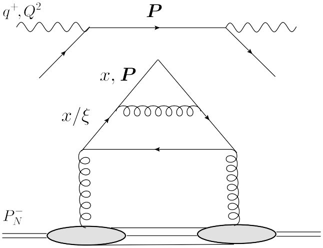
$$x\mathcal{F}_q^{(1)}(x, \boldsymbol{P}_\perp) = \frac{\alpha_s C_F}{2\pi^2} \frac{1}{P_\perp^2} \int_x^{1-\xi_0} \mathrm{d}\xi \, \frac{1+\xi^2}{1-\xi} \, \frac{x}{\xi} f_q^{(0)}\left(\frac{x}{\xi}, P_\perp^2\right)$$

- The rapidity cutoff $\xi_0 \ll 1$ depends upon the jet definition

The "real" Sudakov

- The longitudinal variables z_g and ξ are related: $2k_g^+ k_g^- = P_\perp^2$





$$z_g = \frac{\xi}{1-\xi}\frac{P_\perp^2}{Q^2} \le \frac{P_\perp}{Q}$$

$$x\mathcal{F}_q^{(1)}(x, \boldsymbol{P}_\perp, Q^2)\Big|_R = \frac{\alpha_s C_F}{2\pi^2}\frac{1}{P_\perp^2}\int_x^{1-\xi_0} \mathrm{d}\xi\, \frac{1+\xi^2}{1-\xi}\,\frac{x}{\xi} f_q^{(0)}\left(\frac{x}{\xi}, P_\perp^2\right)$$

- The upper limit on $z_g \Rightarrow$ a lower limit $\xi_0 = \frac{P_\perp}{Q}$ on $1-\xi$
  - very soft gluon emissions with $\xi_g \equiv 1-\xi < \xi_0$ cannot be resolved
- Via ξ_0, the quark TMD is logarithmically dependent to $Q \Rightarrow$ CSS evolution