Tensor Networks: from quantum information to quantum many-body systems and QFT

Mari-Carmen Bañuls

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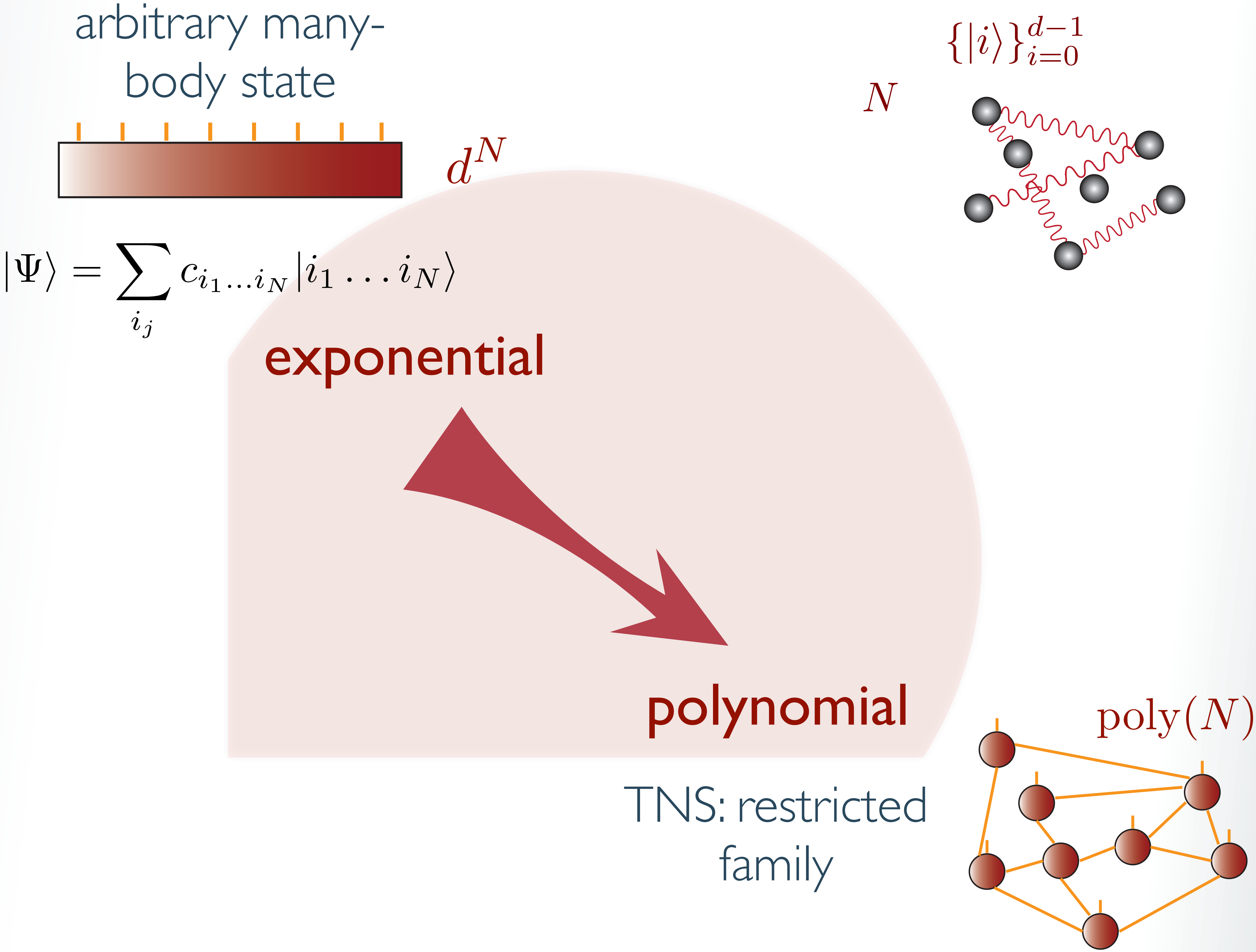
T-NiSQ

3.4.2025

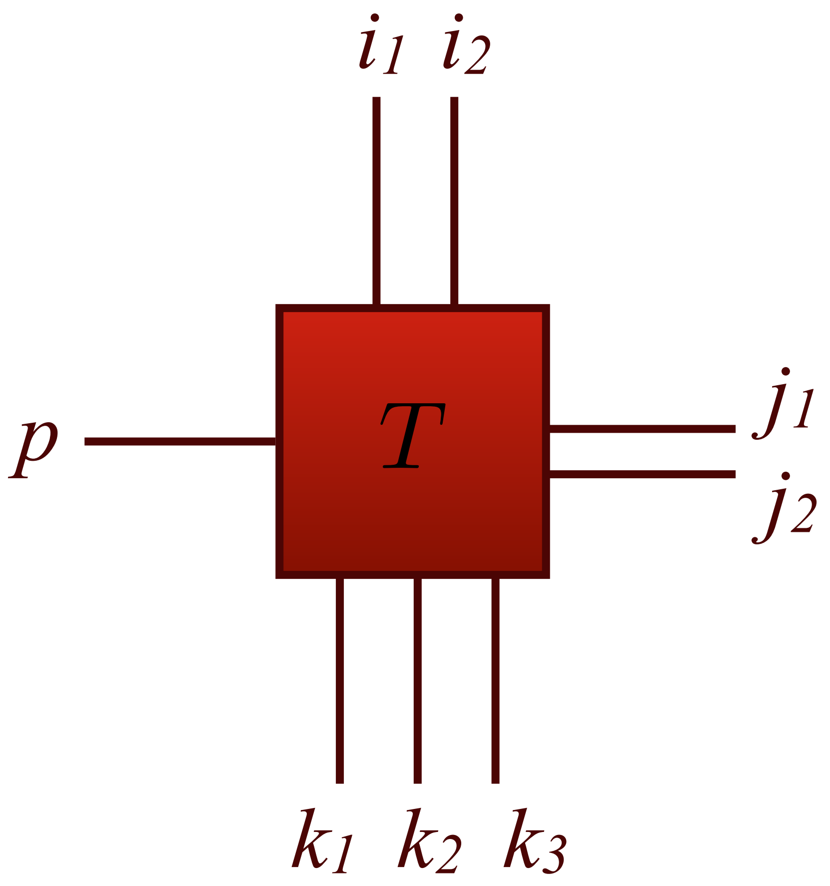
what are Tensor Networks?

TNS: entanglement-based ansatzes for quantum many-body states


arbitrary many-body state
d^N
$|\Psi\rangle = \sum_{i_j} c_{i_1 \dots i_N} |i_1 \dots i_N\rangle$
$\{|i\rangle\}_{i=0}^{d-1}$
N
exponential
polynomial
TNS: restricted family
$\mathrm{poly}(N)$

pictorial representation

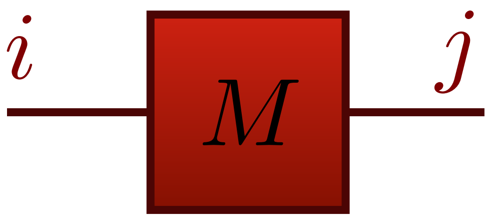
tensor = multidimensional array



pictorial representation

tensor = multidimensional array

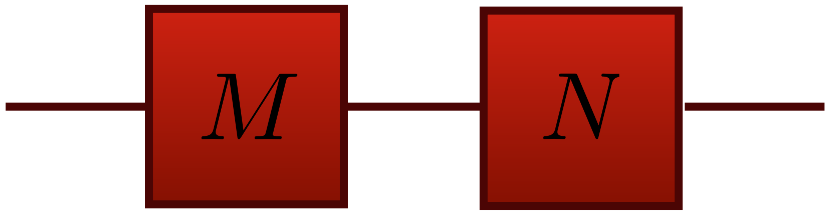
matrix



M_{ij}

pictorial representation

tensor = multidimensional array

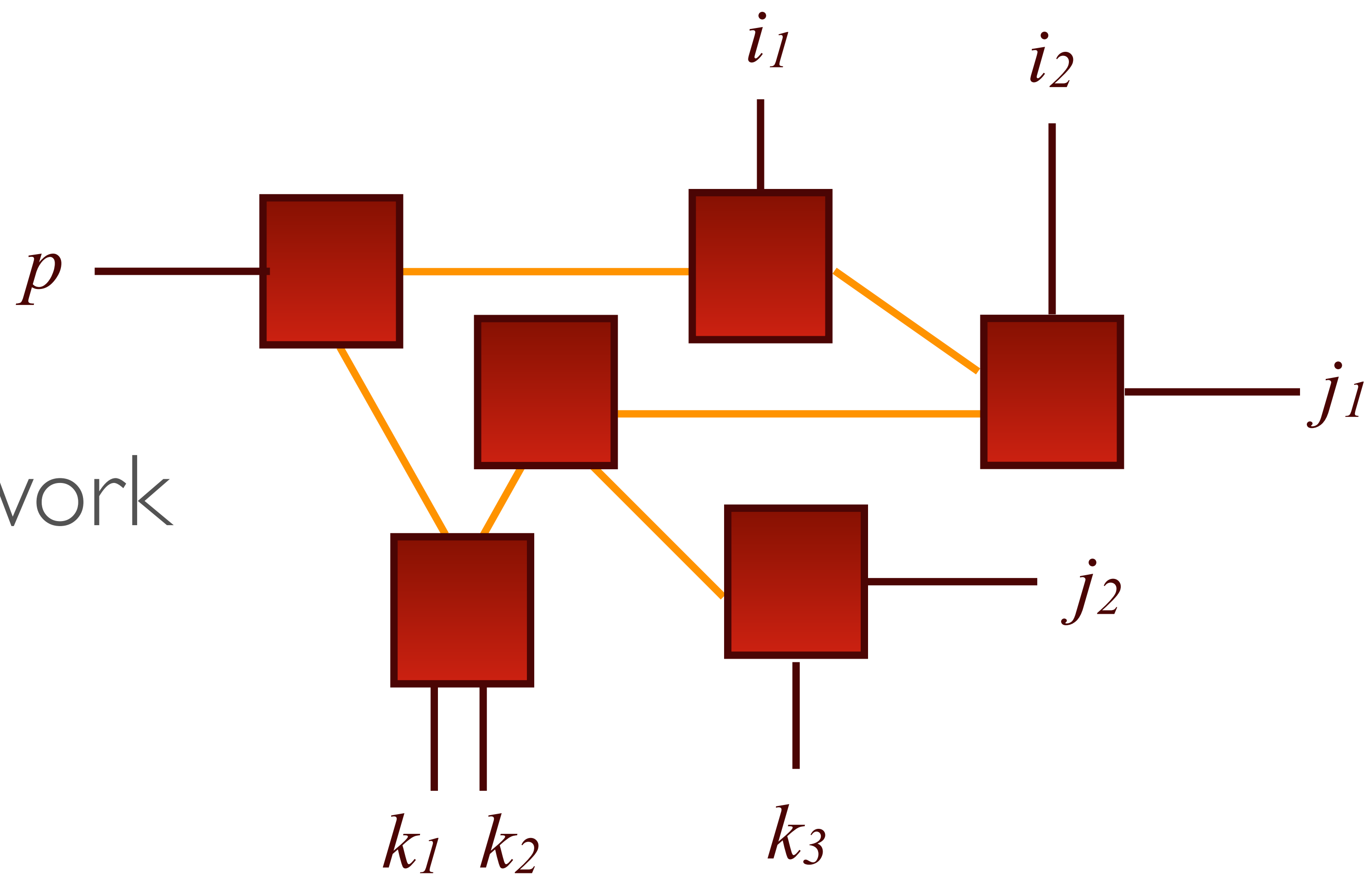


$$M \cdot N = \sum_j M_{ij} N_{jk}$$

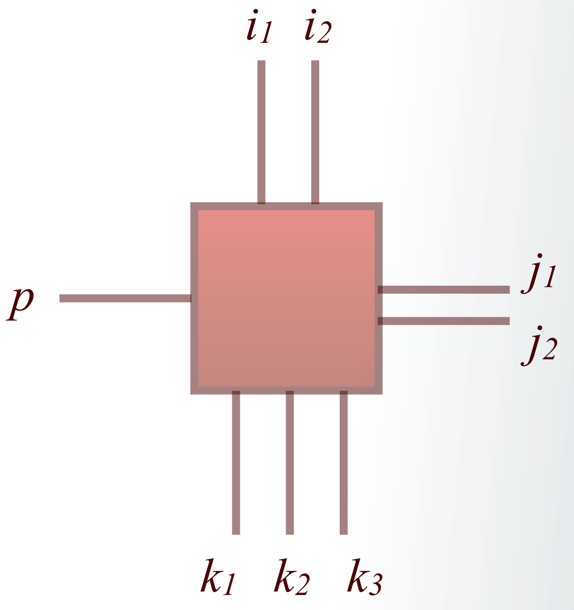
tensor network

tensor = multidimensional array

tensor network (TN)



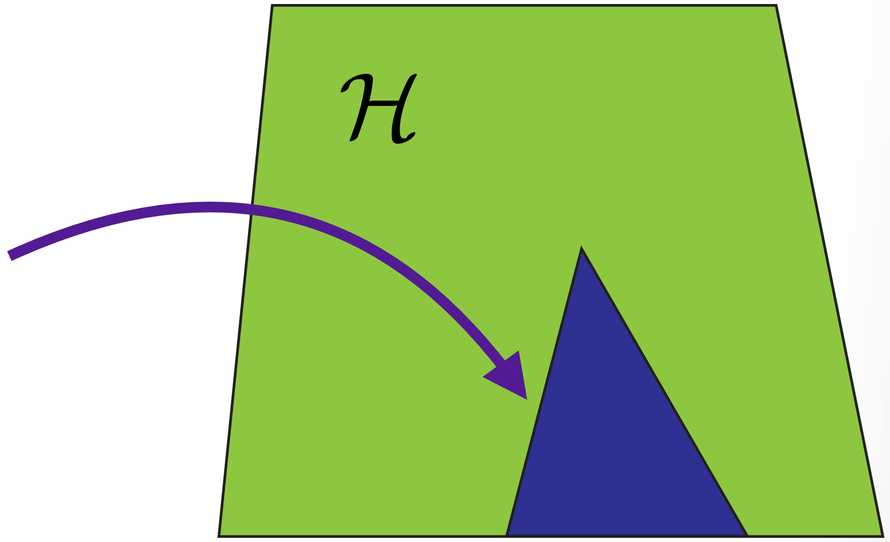
$$\{T_{i_1 i_2, j_1 j_2, k_1 k_2 k_3, p}\}_{\{i,j,k,p\}}$$



why are TNS useful?

we are looking for states with small entanglement

relevant corner of the Hilbert space



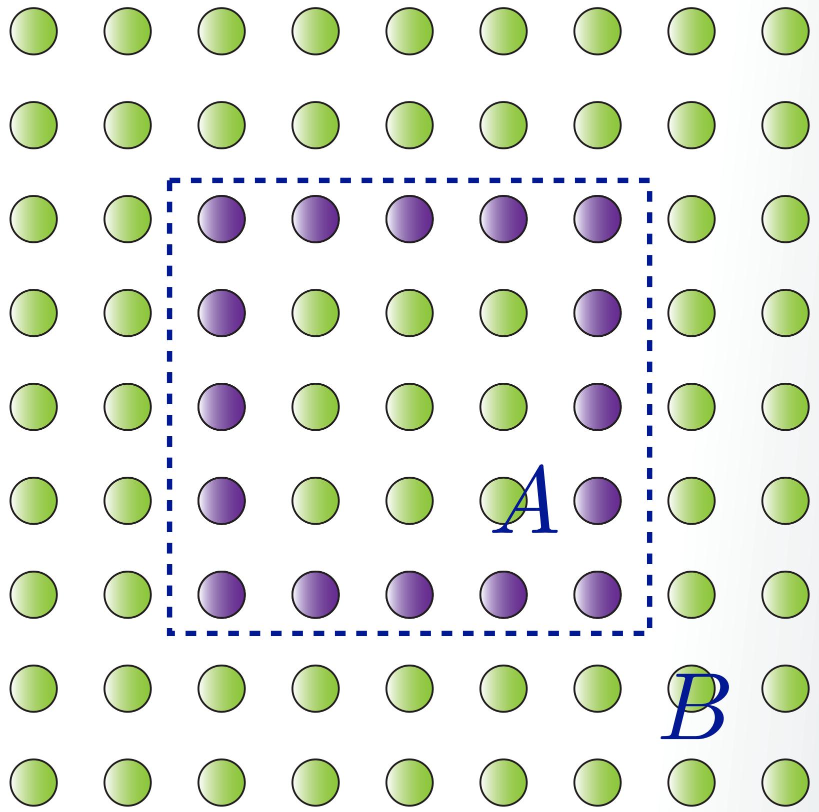
local gapped 1D Hamiltonians
have ground states
with area law of entanglement

$S_{A\,\mathrm{max}} \propto |\delta A|$ Hastings 2007

in 1D critical systems,
logarithmic corrections

$S_{A\,\mathrm{max}} \propto |\delta A| \log A$ Calabrese, Cardy 2004

area law

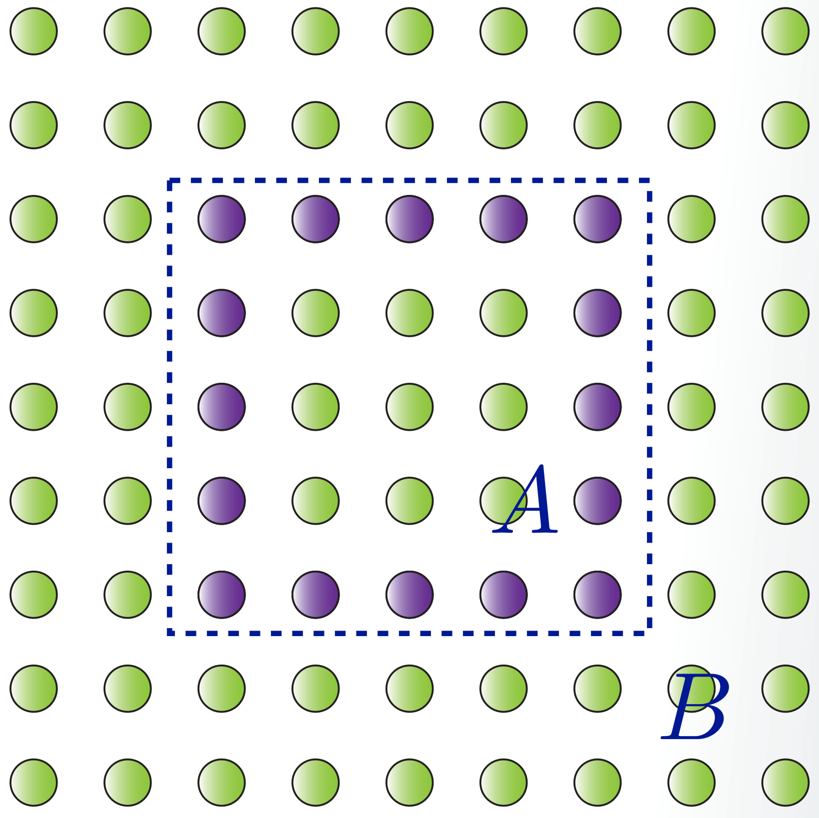


satisfied at finite temperature Wolf, Verstraete, Hastings, Cirac, PRL 2008

TNS come in different forms…

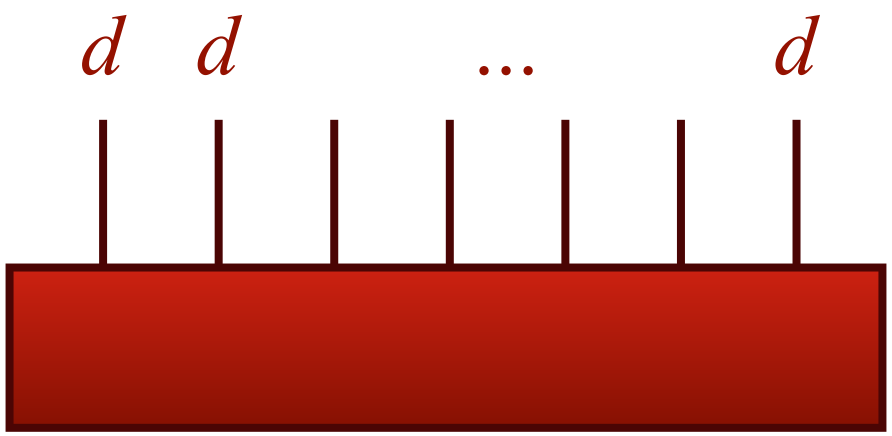
Ansätze satisfying
the area law
by construction

Area law



MPS & PEPS

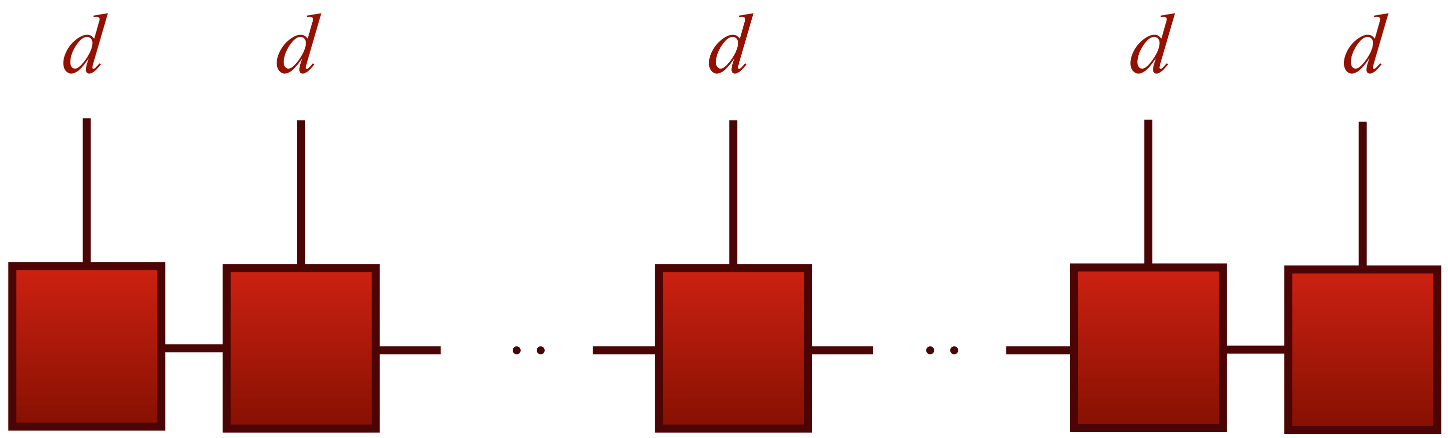
MPS



$$|\Phi\rangle = \sum_{s_1,\ldots,s_N=1}^{d} c_{s_1,\ldots,s_N} |s_1,\ldots,s_N\rangle$$

Fannes, Nachtergaele, Werner, 1992

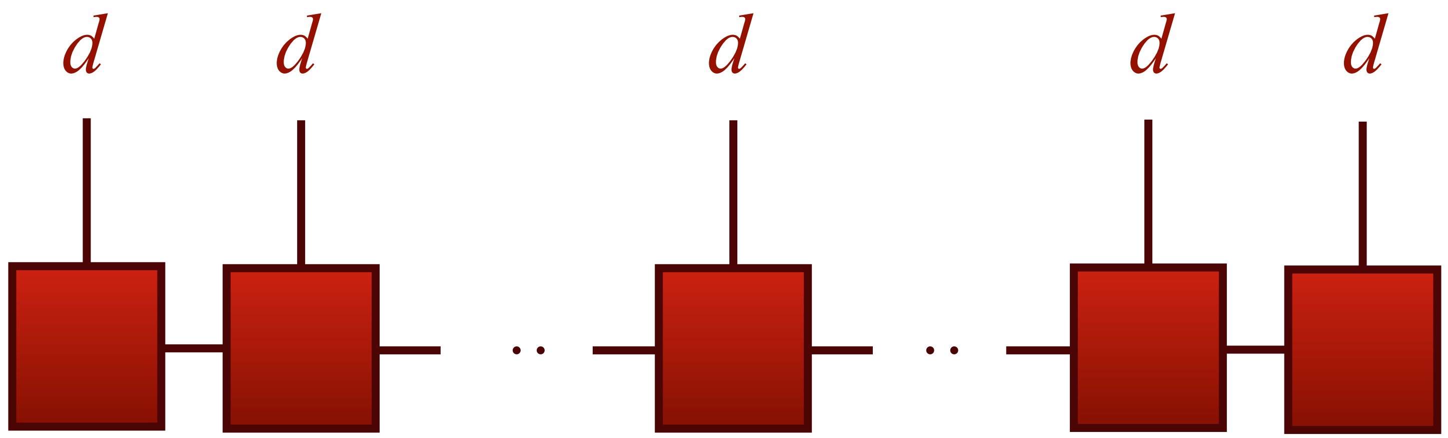
MPS



$$|\Phi_D\rangle = \sum_{s_1,\ldots,s_N=1}^{d} \mathrm{tr}\left(A^{s_1}[1]\ldots A^{s_1}[N]\right)|s_1,\ldots,s_N\rangle$$

Fannes, Nachtergaele, Werner, 1992

MPS



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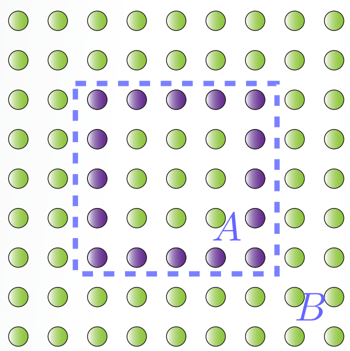
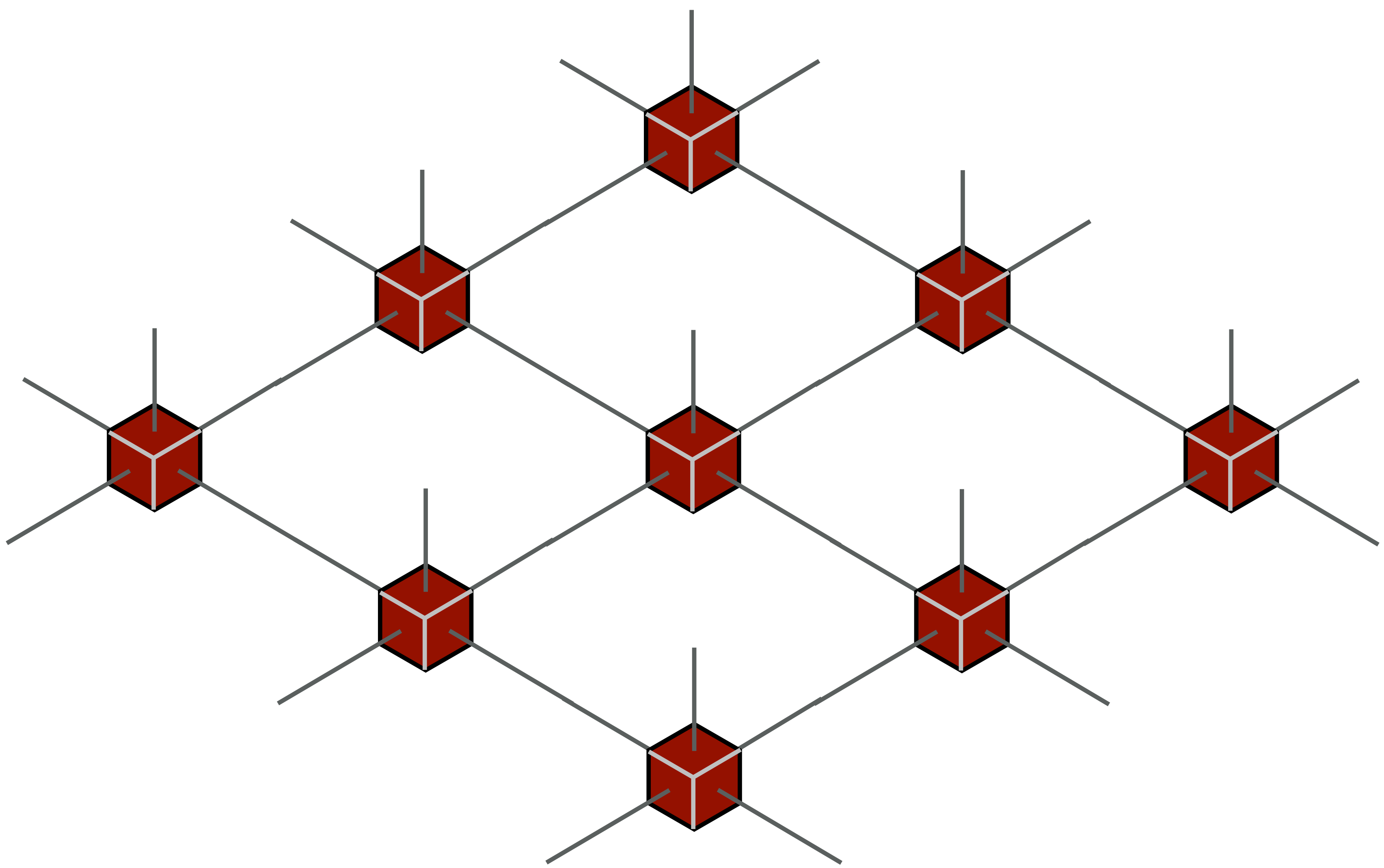


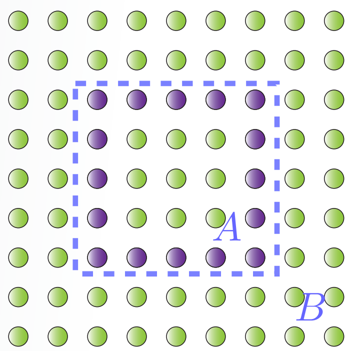
area law by construction

$$S(L/2) \leq \log D$$

Fannes, Nachtergaele, Werner, 1992

peps



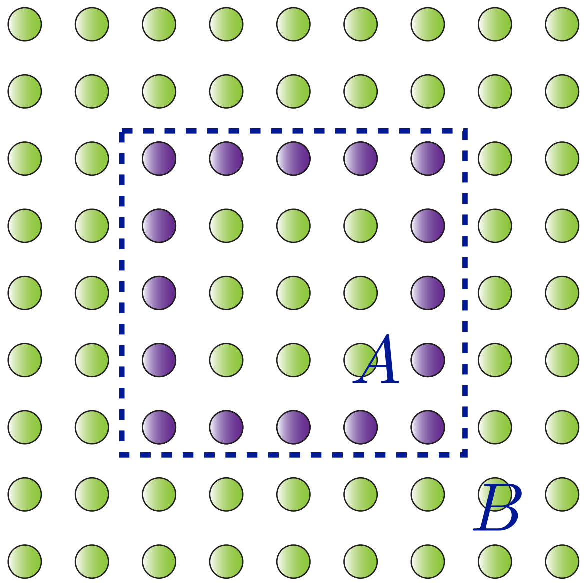


area law by construction

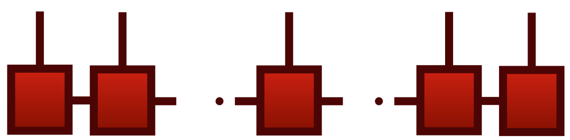
Verstraete, Cirac, 2004

TNS = entanglement based ansatz

area law

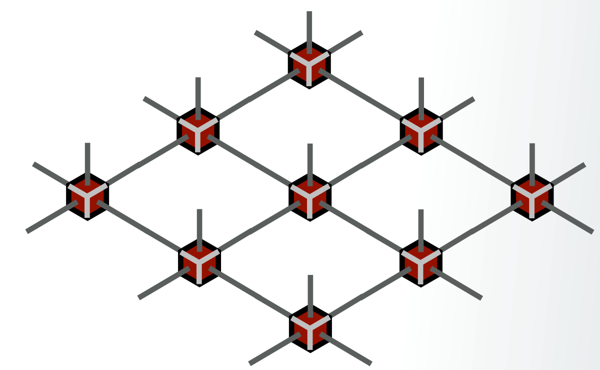


MPS



Schollwöck Ann.Phys.2011

PEPS

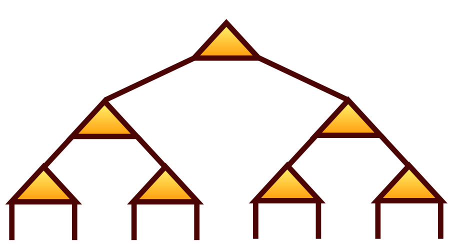


Verstraete et al. Adv. Phys. 2008

other TNS

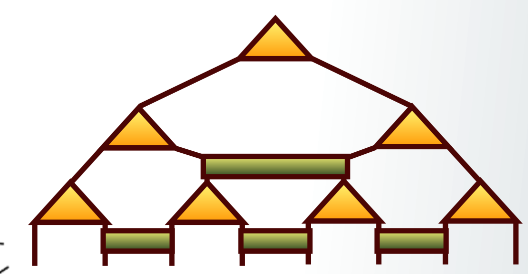
critical 1D correlations

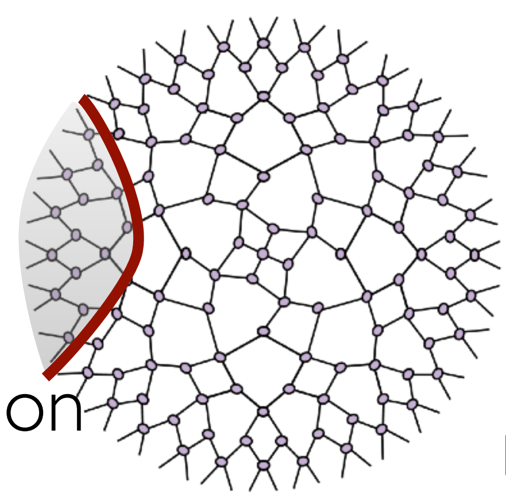
TTN



Shi et al PRA 2006

Vidal PRL 2007 MERA





suggested connection to AdS/CFT

Swingle PRD 2012
Molina JHEP 2013
Nozaki et al JHEP 2012
Bao et al PRD 2015

some interesting properties of MPS & PEPS

complete families

good approximation of ground/thermal states of local Hamiltonians

Verstraete, Cirac, PRB 2006
Hastings J. Stat. Phys 2007

Hastings PRB 2006
Molnar et al PRB 2015

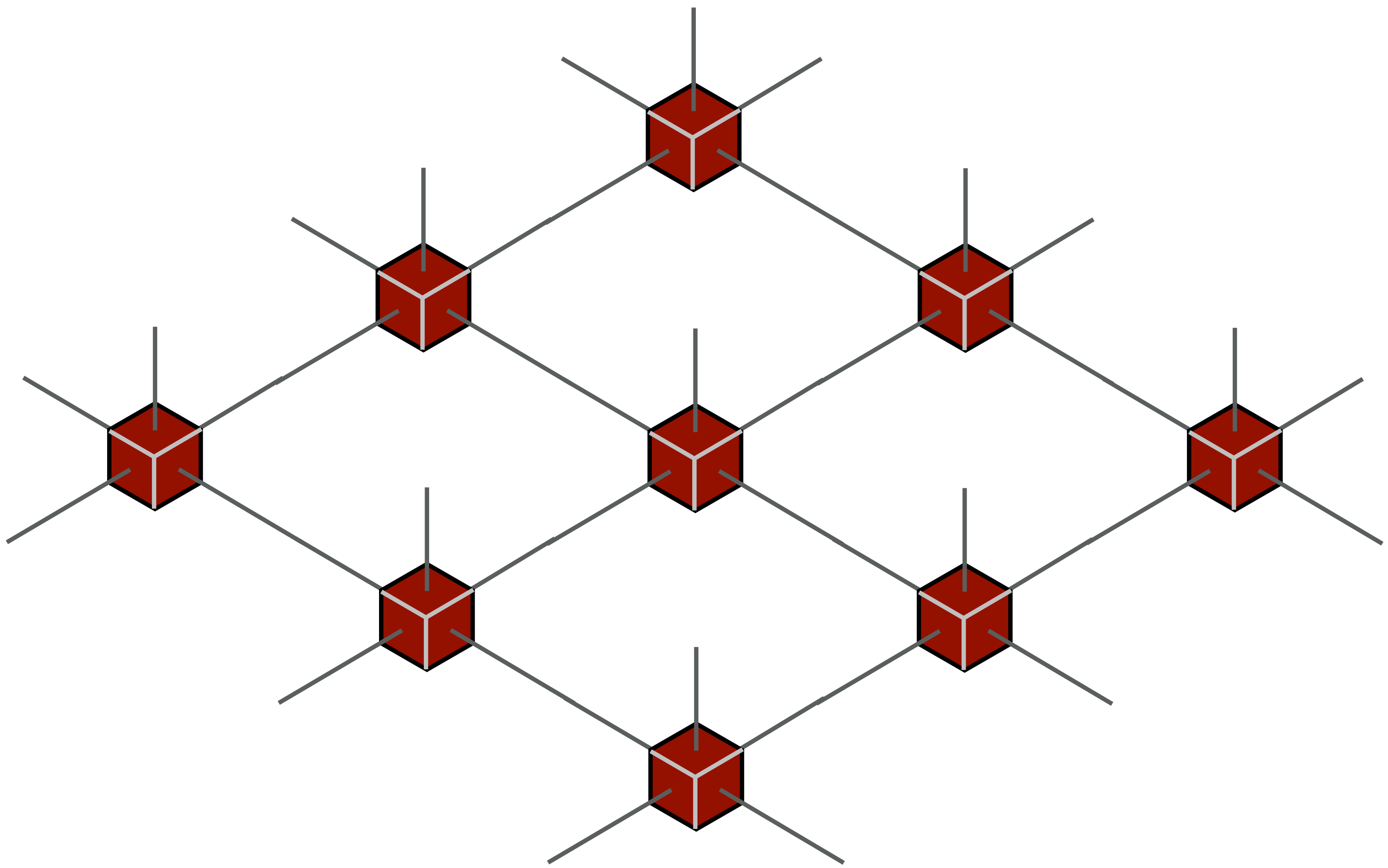
can be defined directly in the thermodynamic limit

ground states of local parent Hamiltonians

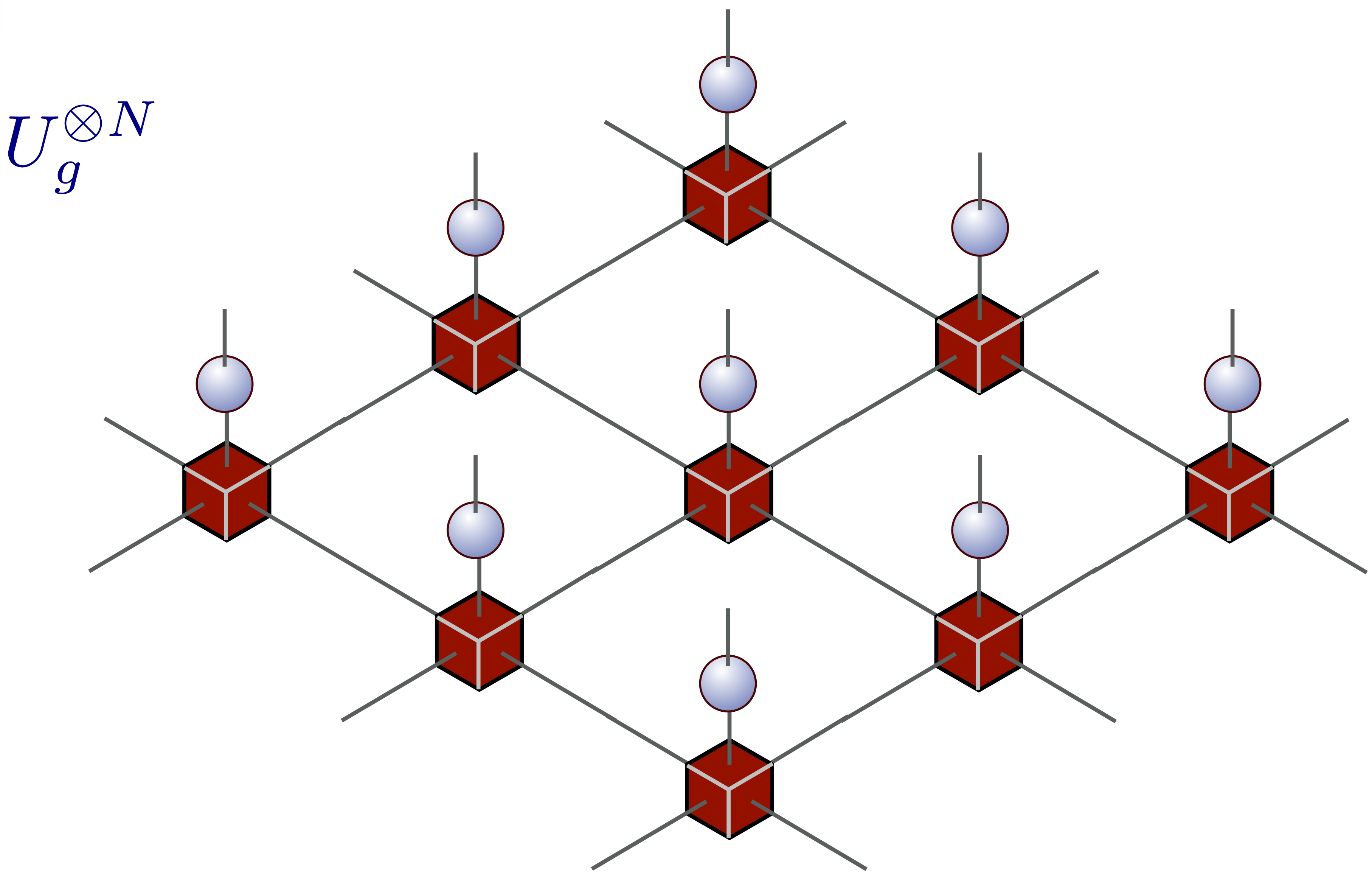
Pérez-García et al., QIC 2007
Schuch et al., Ann. Phys. 2010
Cirac et al RMP 2021

TNS play well with symmetries

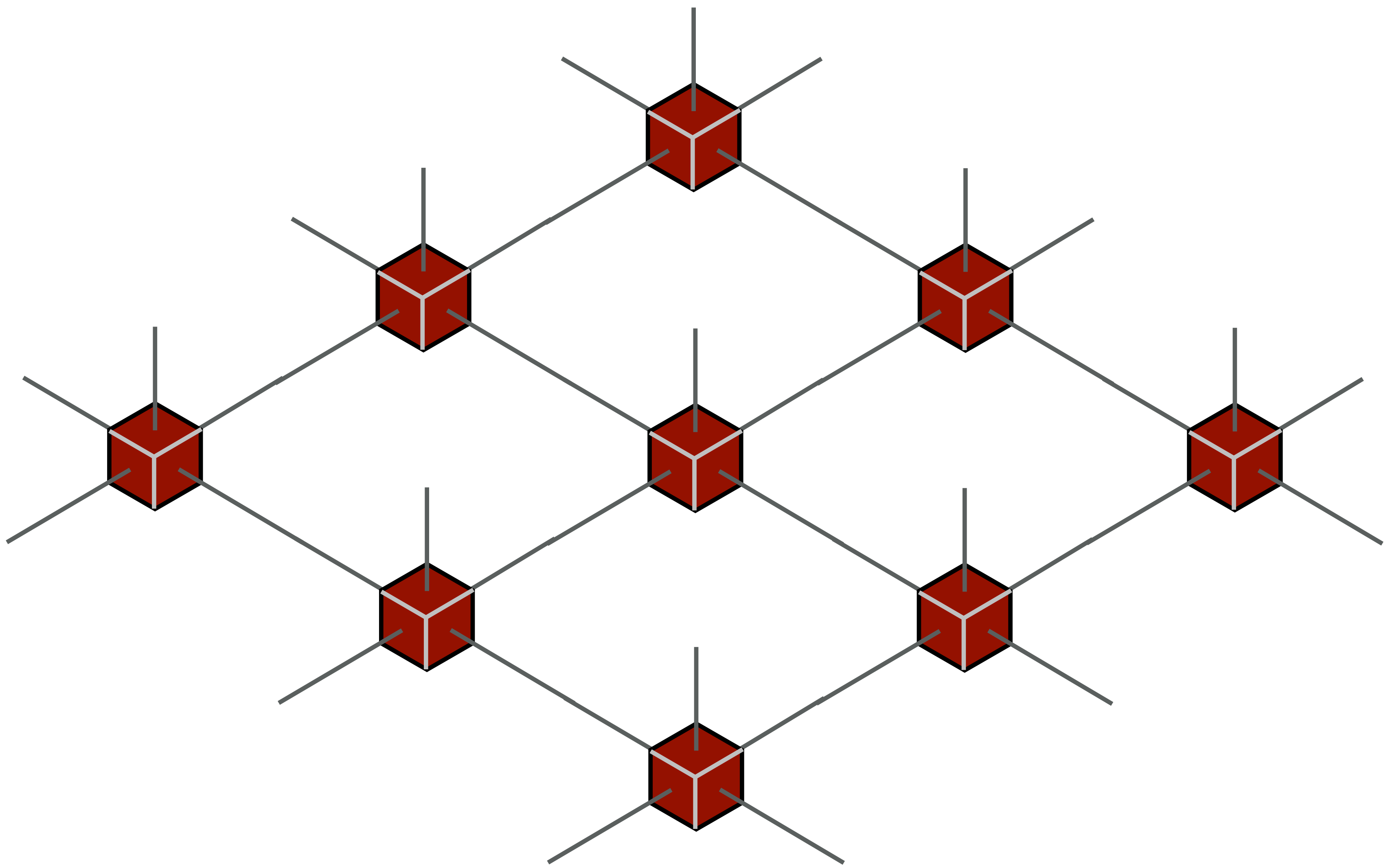
state invariant under global symmetry



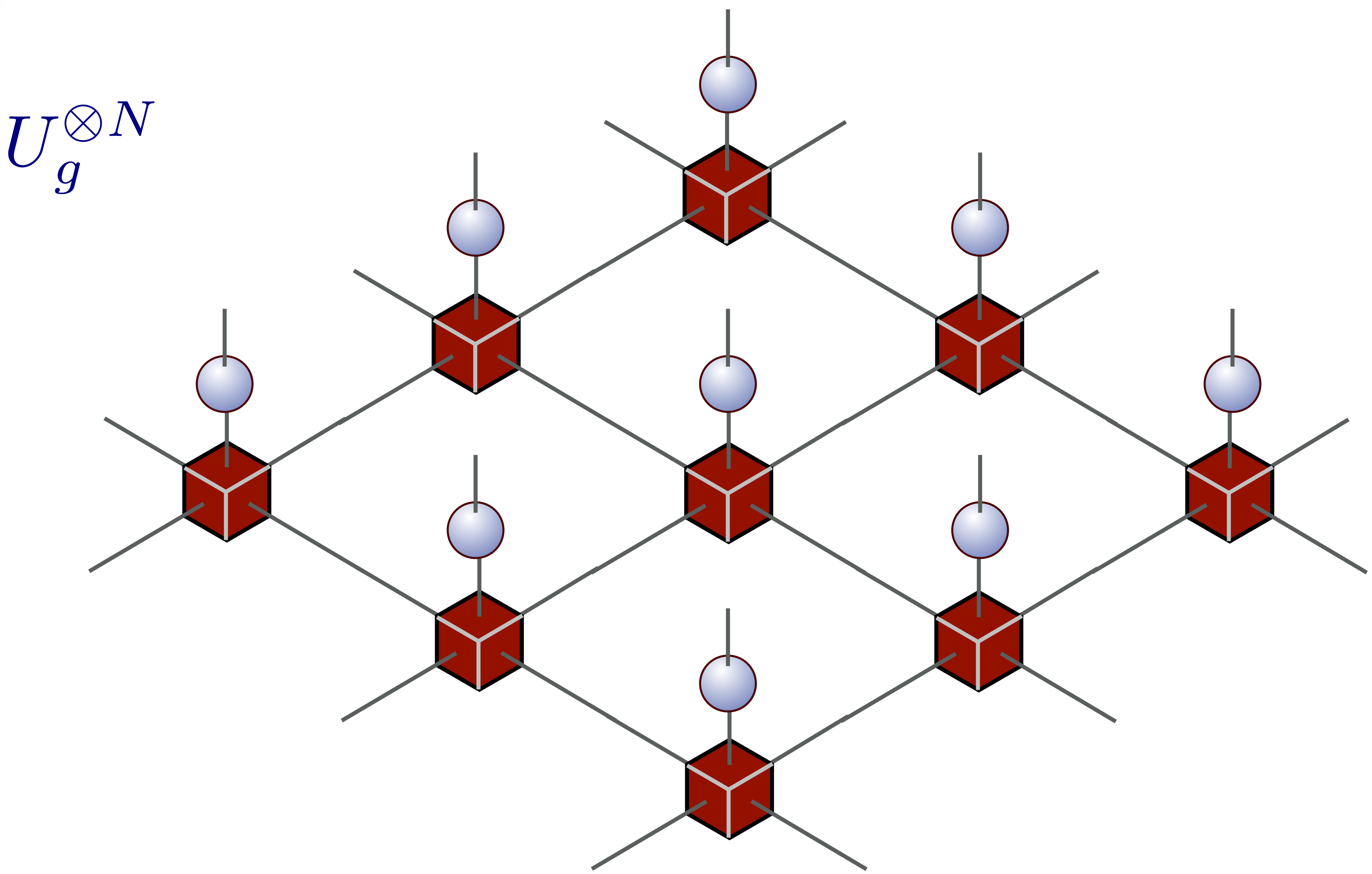
state invariant under global symmetry

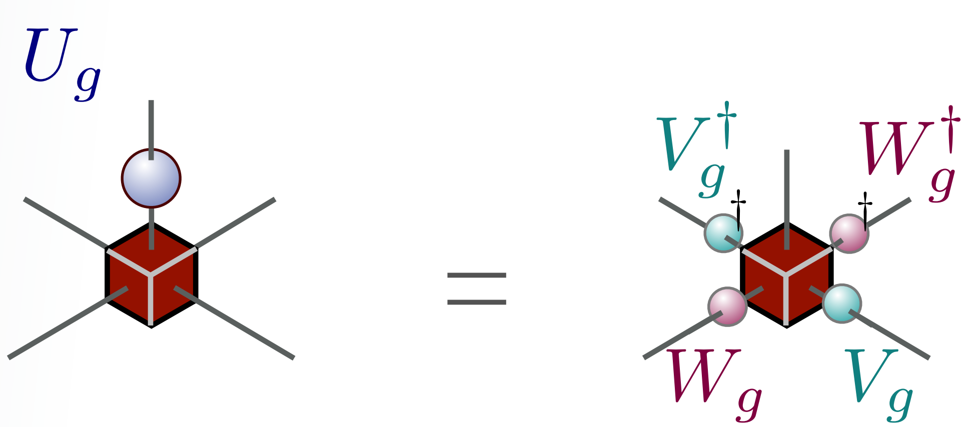


state invariant under global symmetry



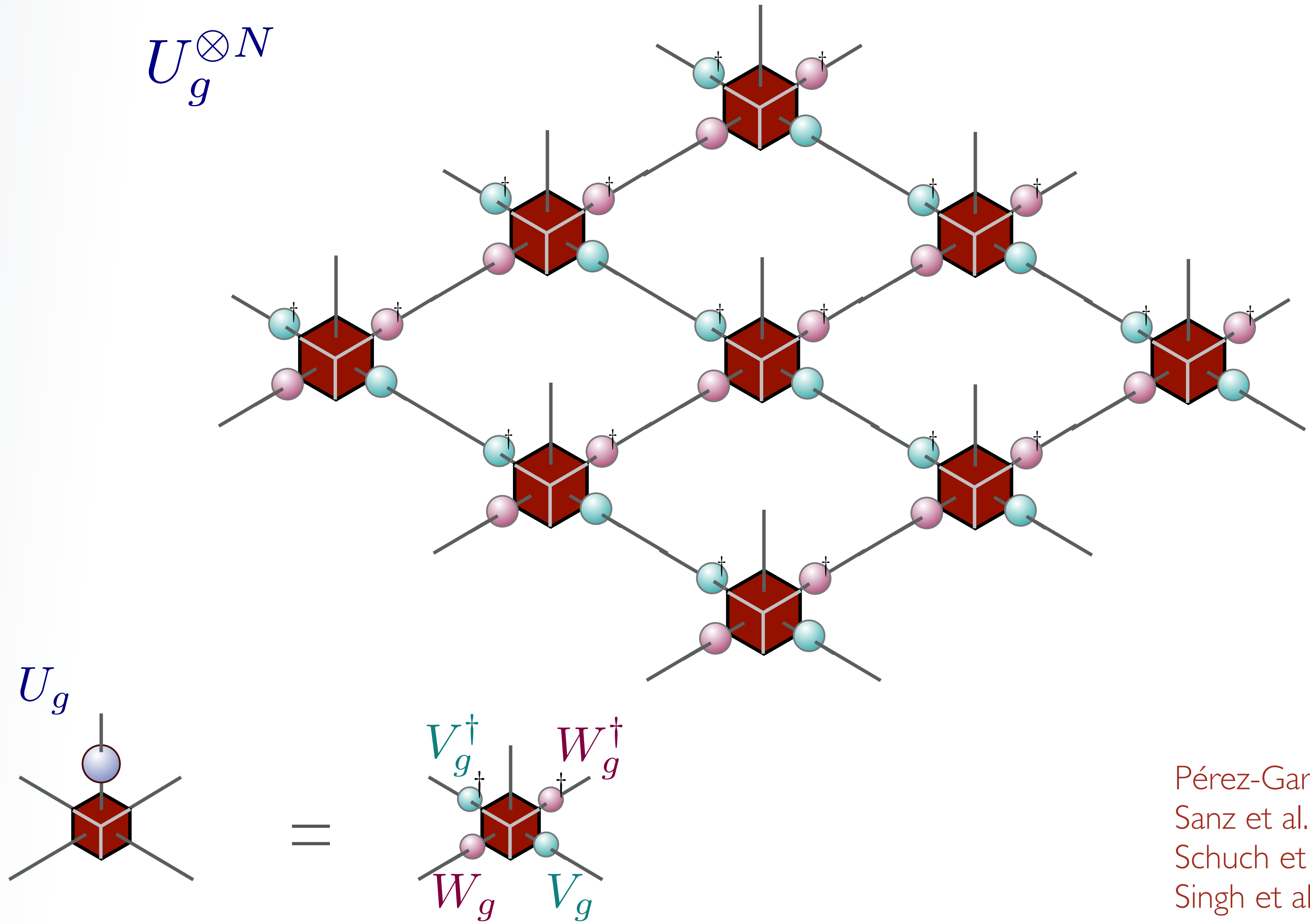
state invariant under global symmetry





Pérez-García et al., PRL 2008
Sanz et al., PRA 2009
Schuch et al., Ann. Phys. 2010
Singh et al., NJP 2007, PRA 2010

state invariant under global symmetry



Pérez-García et al., PRL 2008
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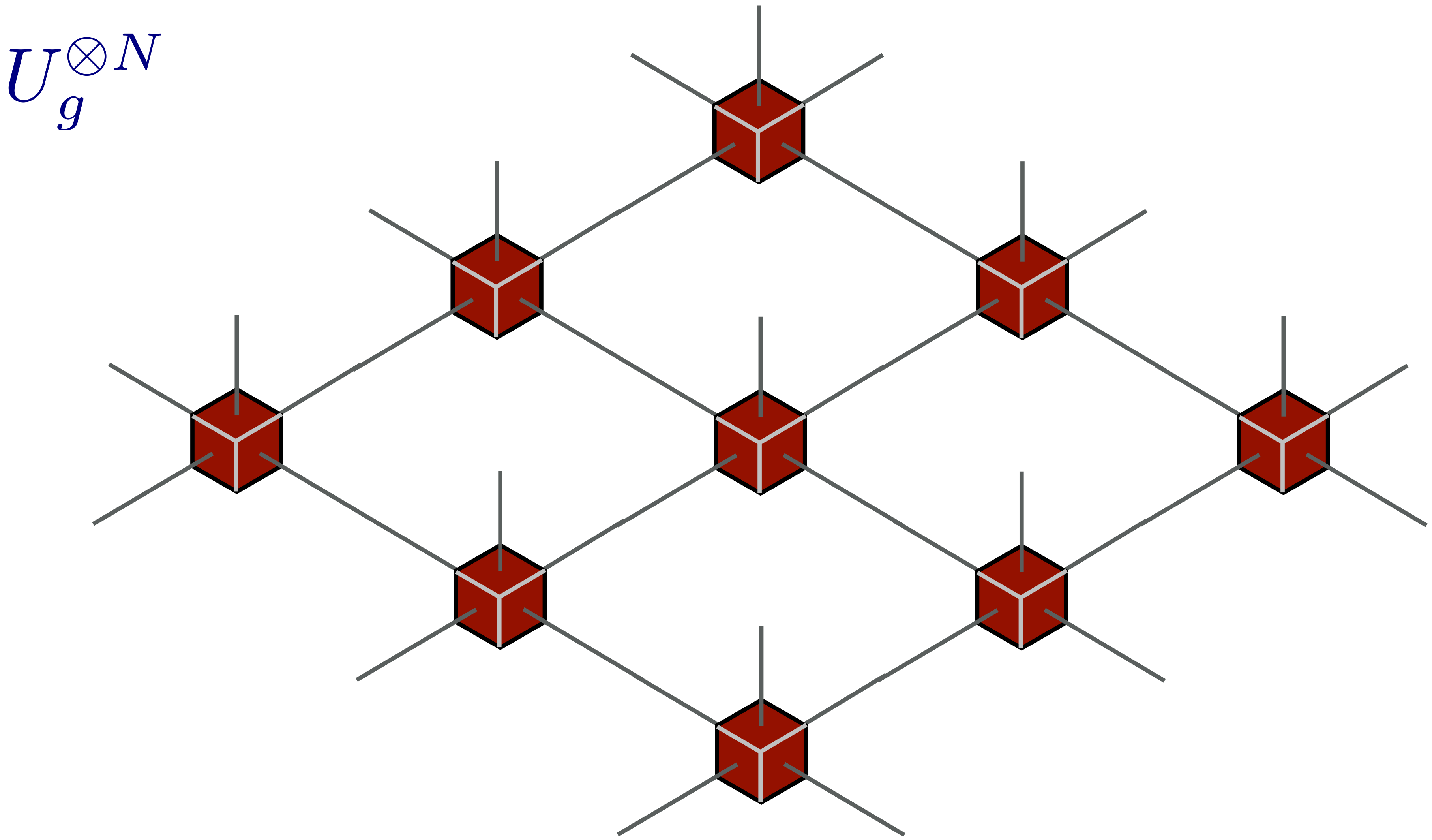
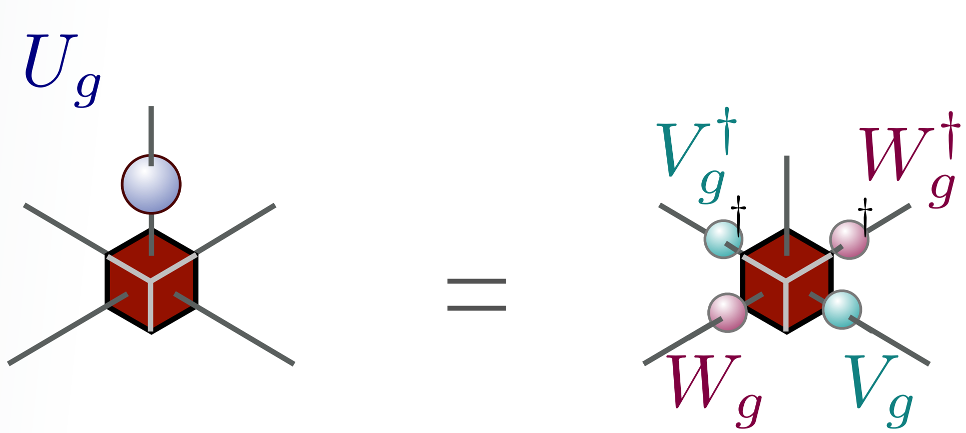
state invariant under global symmetry





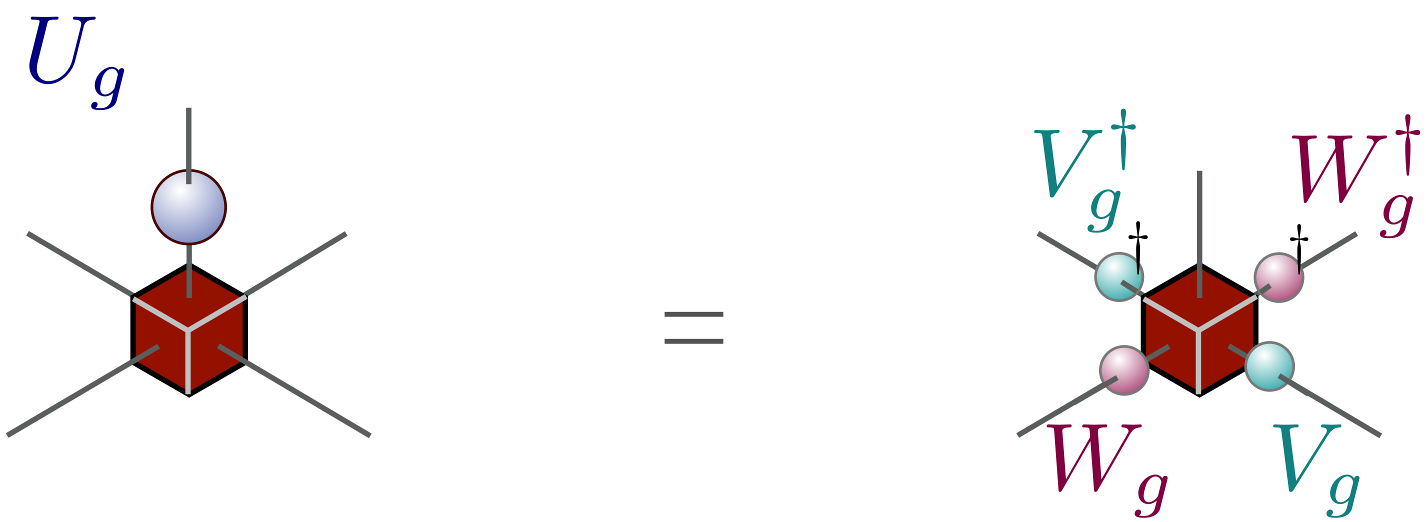
Pérez-García et al., PRL 2008
Sanz et al., PRA 2009
Schuch et al., Ann. Phys. 2010
Singh et al., NJP 2007, PRA 2010

state invariant under global symmetry

constructed with invariant tensors

for MPS & PEPS

state (globally) invariant ⇔



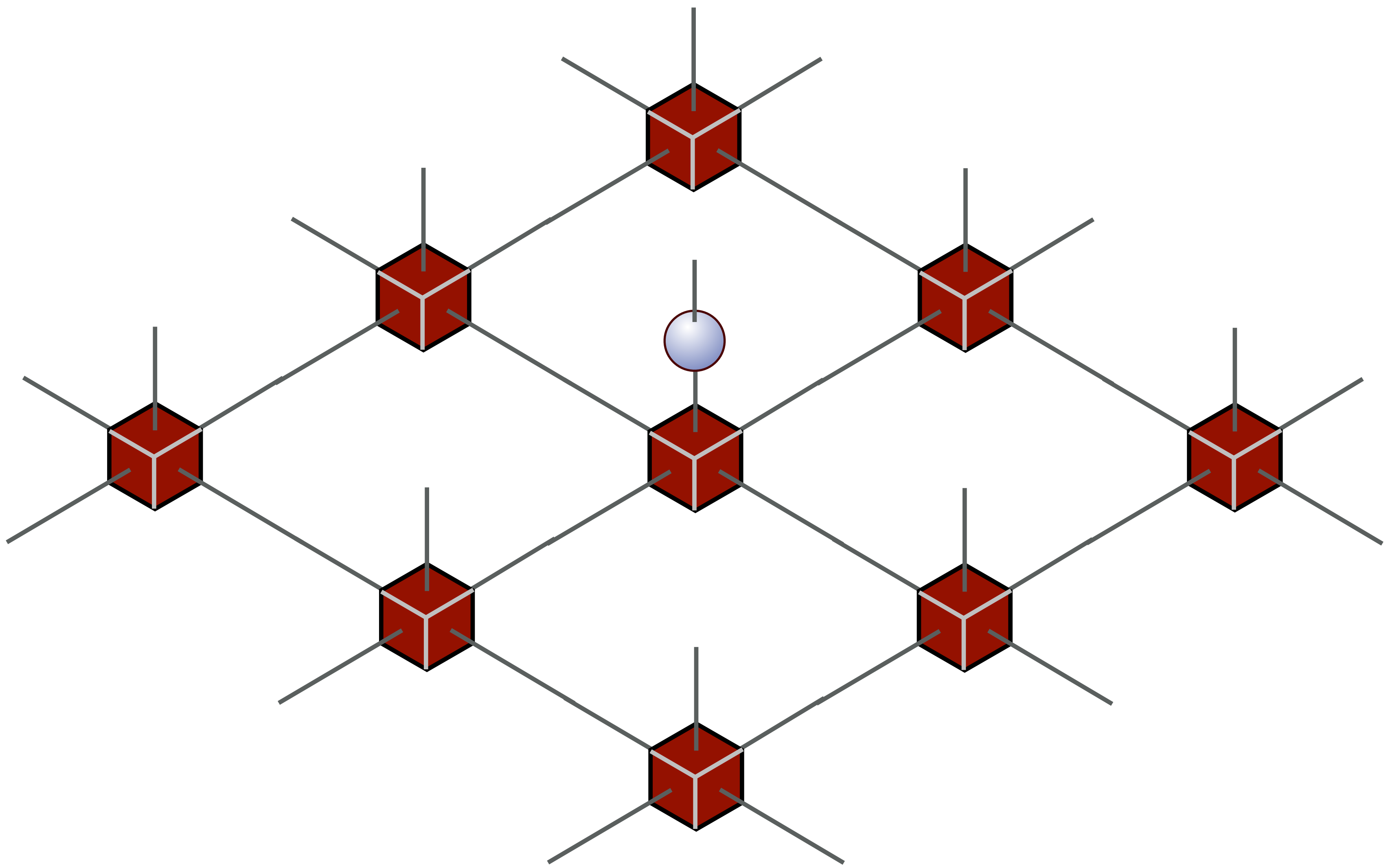
structure of tensors

also gauge symmetries

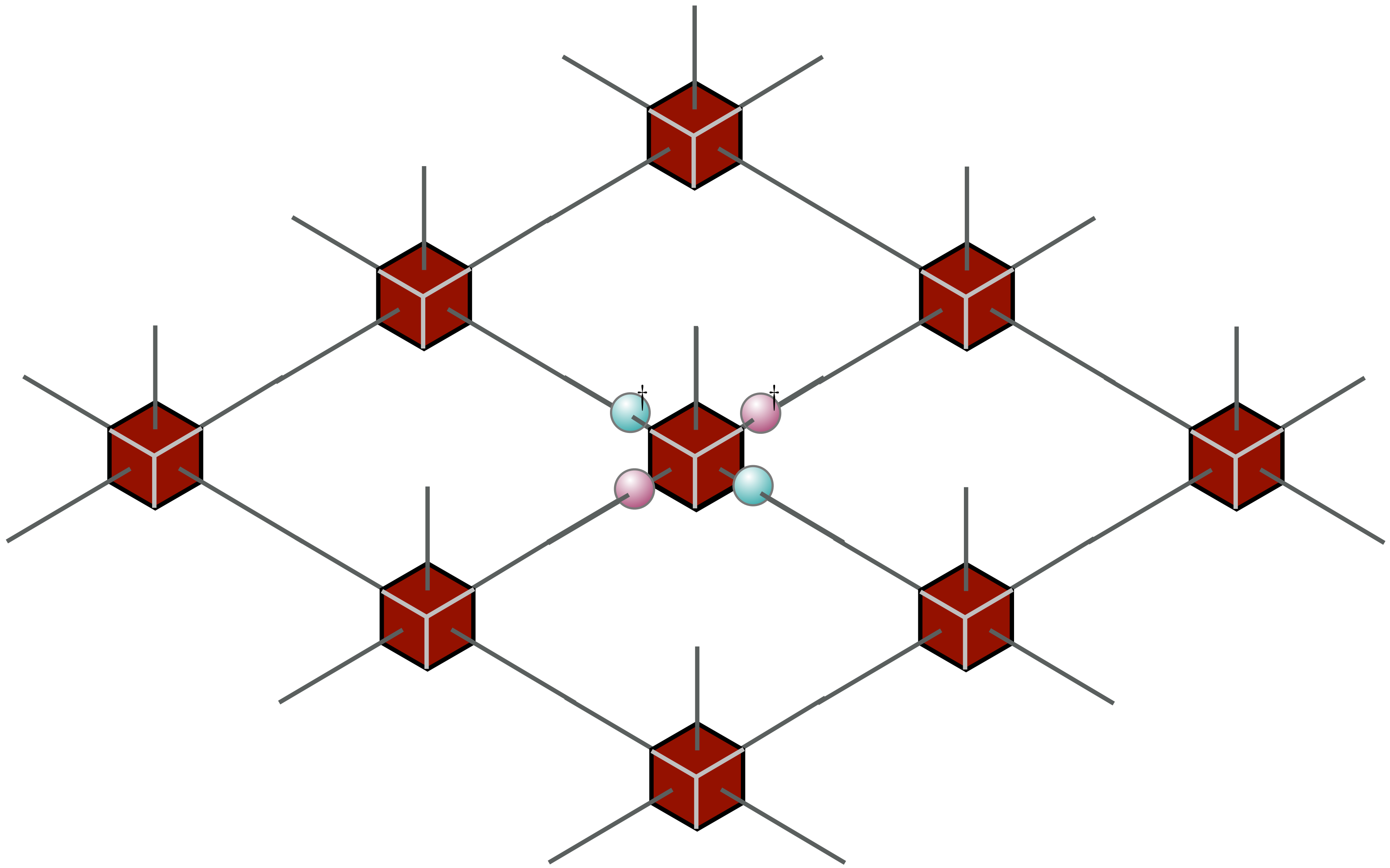
Tagliacozzo et al PRX 2014
Haegeman et al PRX 2014
Zohar et al Ann Phys 2015

Pérez-García et al., PRL 2008
Sanz et al., PRA 2009
Schuch et al., Ann. Phys. 2010
Singh et al., NJP 2007, PRA 2010

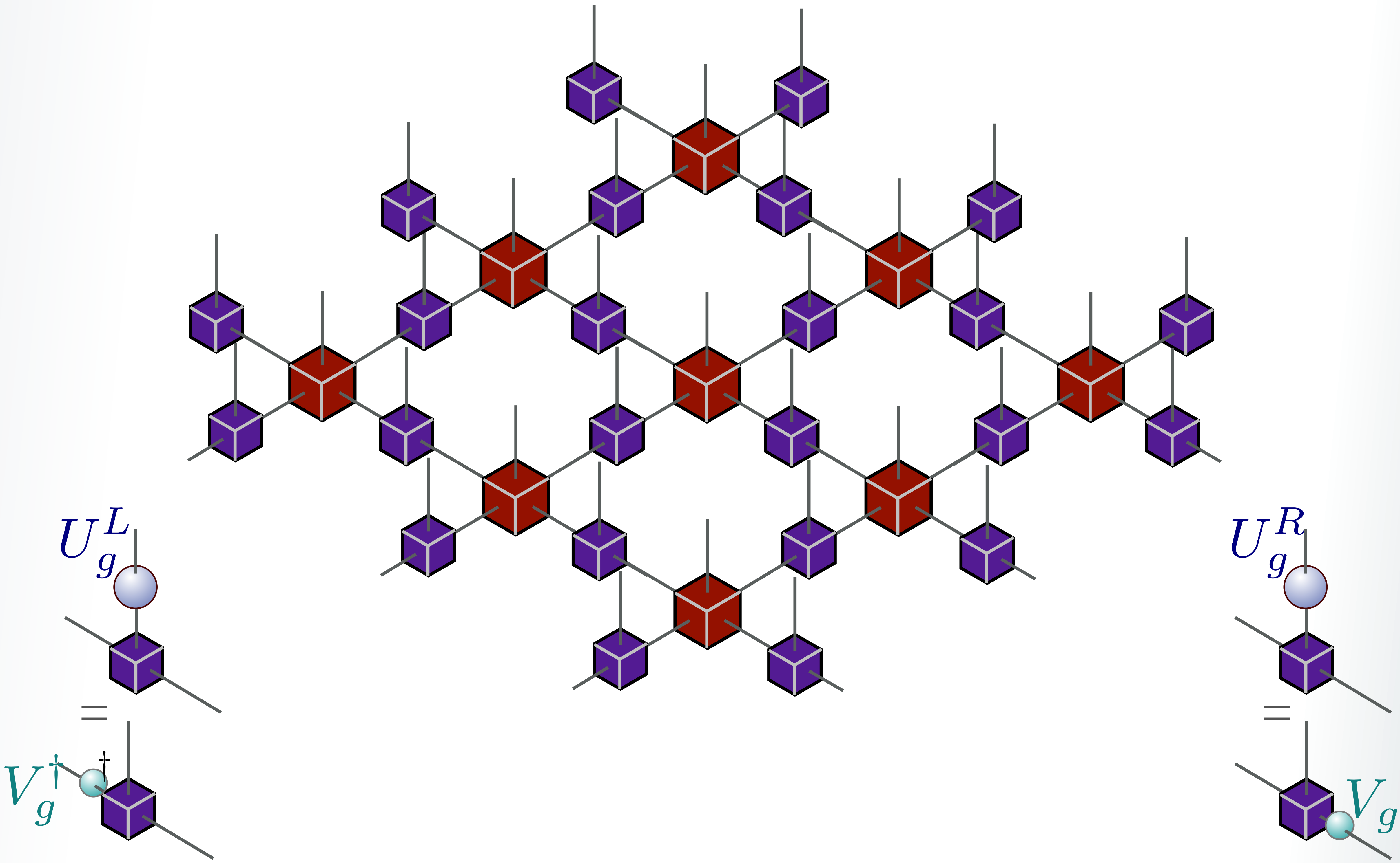
gauging PEPS



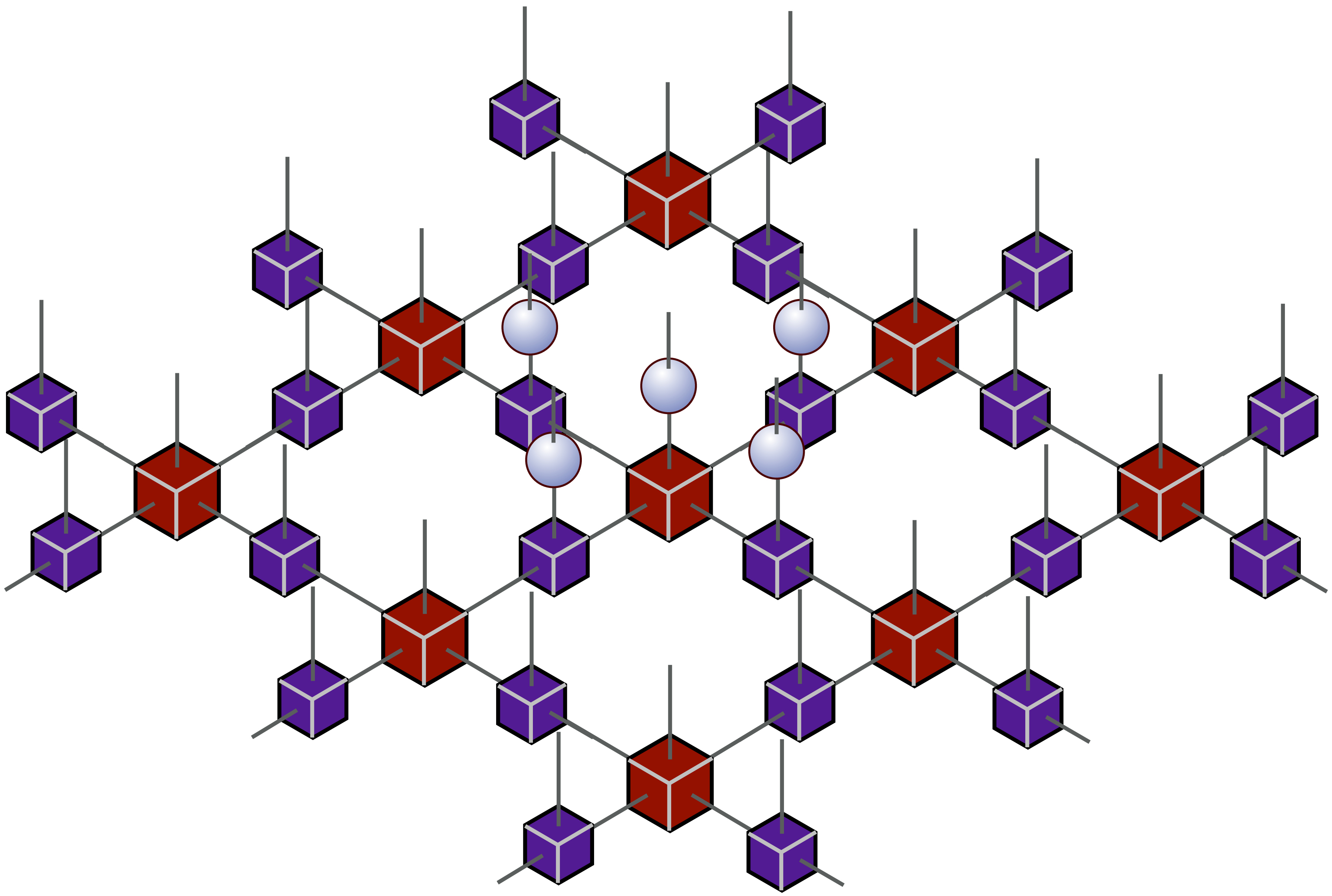
gauging PEPS



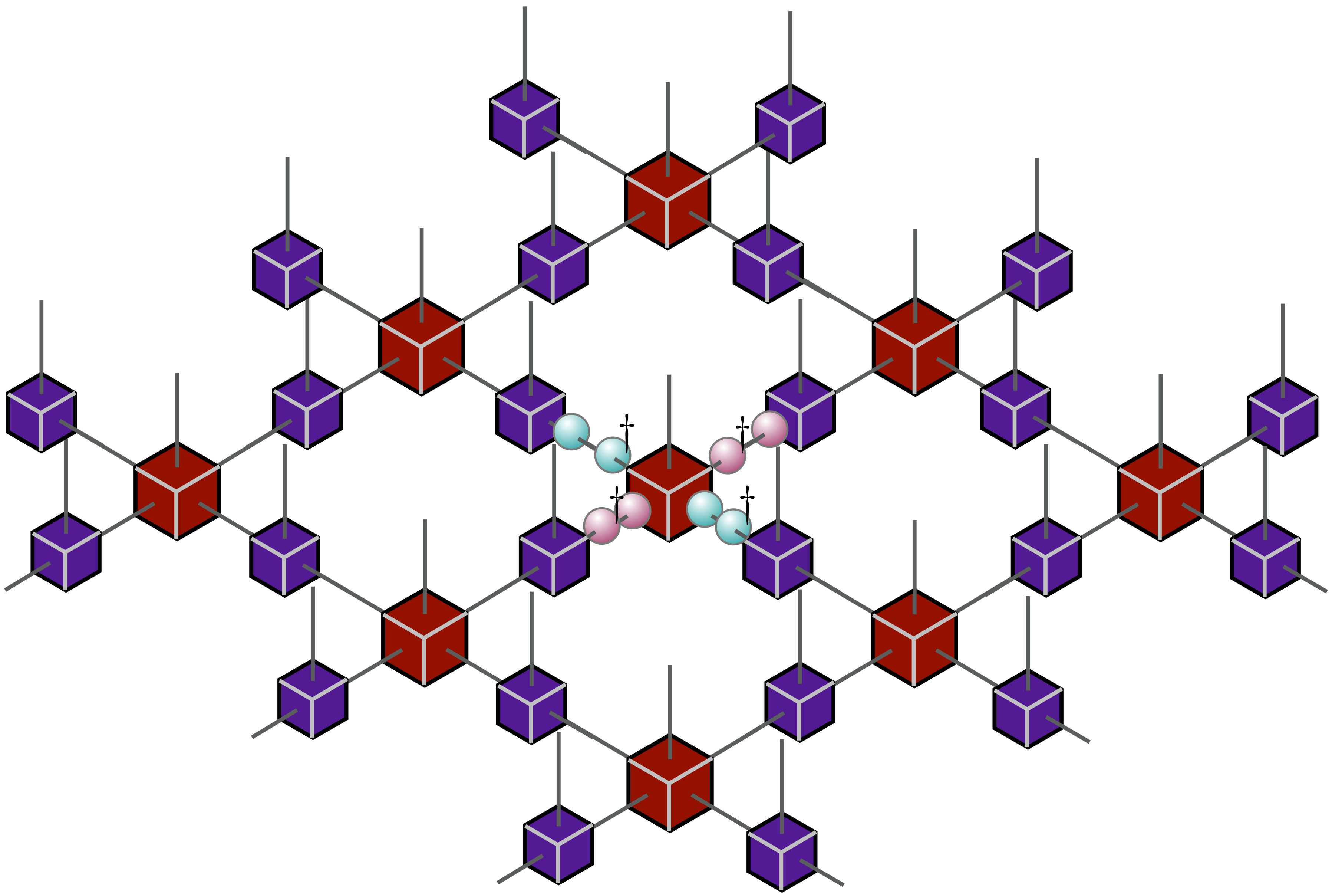
gauging PEPS



gauging PEPS



gauging PEPS

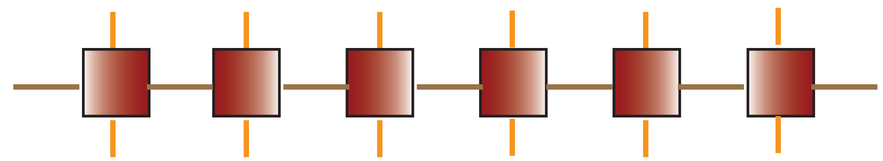


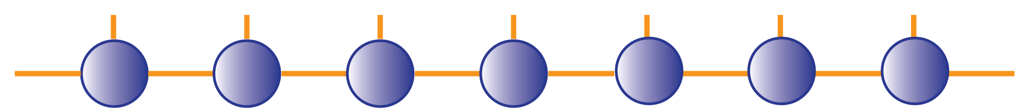
numerics with TNS

basic algorithms

variational minimization of energy

local
Hamiltonian

$H =$ 

$|E_0\rangle \simeq$  ground state
excitations

apply local operators → simulate time evolution

imaginary time → ground state
thermal state

also: evolve using TDVP

White, PRL 1992; Schollwöck, Ann. Phys. 2011;
Vidal, PRL 2003, 2004; Verstraete et al., PRL 2004;
Haegeman et al., PRL 2011

basic algorithms

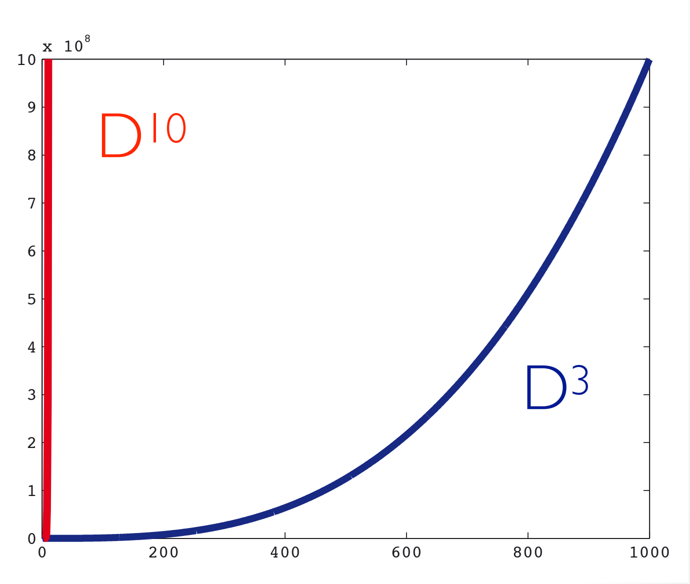
basically work in 1D or 2D

different computational costs

1D → $O(D^3)$

2D → $O(D^{10})$

no exact contractions

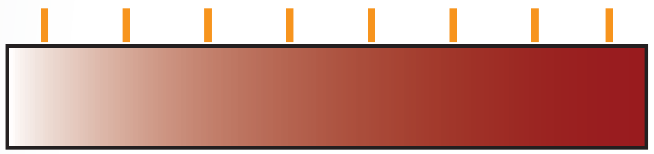


regarding real time

entanglement may grow fast in non-equilibrium scenarios!

up to exponentially growing D

works for close to equilibrium or moderate times arbitrary many-body state



d^N

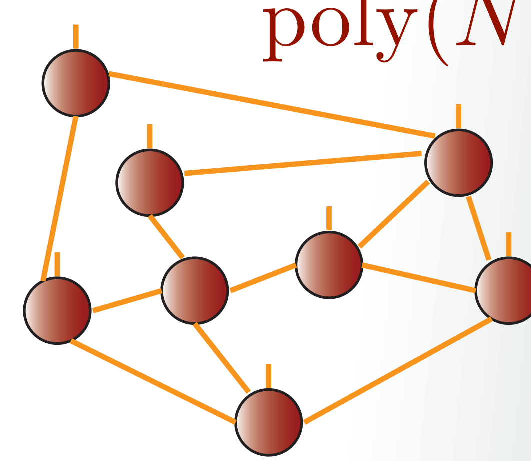
$$|\Psi\rangle = \sum_{i_j} c_{i_1 \ldots i_N} |i_1 \ldots i_N\rangle$$

exponential

polynomial

TNS: restricted family

$\mathrm{poly}(N)$



good ansatz for ground states and thermal equilibrium: area law

entanglement hierarchy

efficient numerics

TNS for QFT

TNS

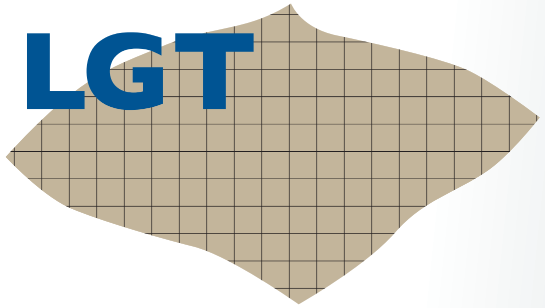
Non-perturbative for Hamiltonian systems

Extremely practical (and successful) for 1D systems (MPS)

Powerful, but more costly for higher dimensions

ground states
low-lying excitations
thermal states
time evolution

apply to



using TNS for LGT

formal approach

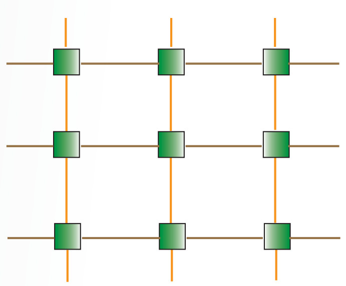
gauging the symmetry

explicitly invariant states

Tagliacozzo et al PRX 2014
Haegeman et al PRX 2014
Zohar et al Ann Phys 2015

general prescriptions, U(1), SU(2)

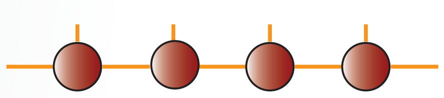
numerical simulations **no sign problem**



TN describe partition functions (observables)

TRG approaches to classical and quantum models

Liu et al PRD 2013; Shimizu, Kuramashi, PRD 2014; Kawauchi, Takeda 2015;
review Meurice et al. 2010.06539



TN describe states

general strategy

Hamiltonian formulation

acting on a Hilbert space

⟶ choose proper basis

Finite dimensional degrees of freedom

fermions

⟶ ✓ no sign problem

gauge bosons require attention

⟶ truncating, integrating out (also QLinks)

★ Common ingredients for quantum simulation

Zohar et al. PRL 2010, 2012 ,
Tagliacozzo et al., Nat. Comm. 2013
Banerjee et al., PRL 2012

Rico et al. PRL 2014
Pichler et al, PRX 2016
Zohar, Burrello, PRD 2015


there is long way to go until LQCD
journey begins with 1+1D steps
Photo by Maria Teneva on Unsplash

early works with DMRG/TNS

Byrnes PRD2002; Sugihara NPB2004
Tagliacozzo PRB2011; Sugihara JHEP2005
Meurice PRB2013

Schwinger model
U(1) in 1D

precise equilibrium simulations, feasibility of QSim

MCB et al JHEP11(2013)158;
Rico et al PRL 2014; Buyens et al. PRL 2014;
Kühn et al., PRA 90, 042305 (2014);
MCB et al PRD 2015, Buyens et al. PRD 2016;
Pichler et al. PRX 2016;
review Dalmonte, Montangero, Cont. Phys. 2016
MCB, Cichy, Cirac, Jansen, Kühn, arXiv:1810.12838

MCB, K. Cichy 1910.00257
QTFLAG Collab.1911.00003

3+1 dimensions

Magnifico et al. Nat. Com.12, 3600 (2021)

2+1 dimensions

Felser et al. PRX10, 041040 (2020)
Robaina et al. PRL126, 050401 (2021)
Emonts et al. PRD102, 074501 (2020)

Non-Abelian in 1D

string breaking dynamics

S. Kühn et al., JHEP 07 (2015) 130;
Silvi et al., Quantum 2017
S. Kühn et al. PRX 2017

SU(3)QLM

Silvi et al, PRD 2019

finite density

S. Kuehn et al, PRL118 (2017) 071601

finite density

spectrum

entropy

1+1D RESULTS

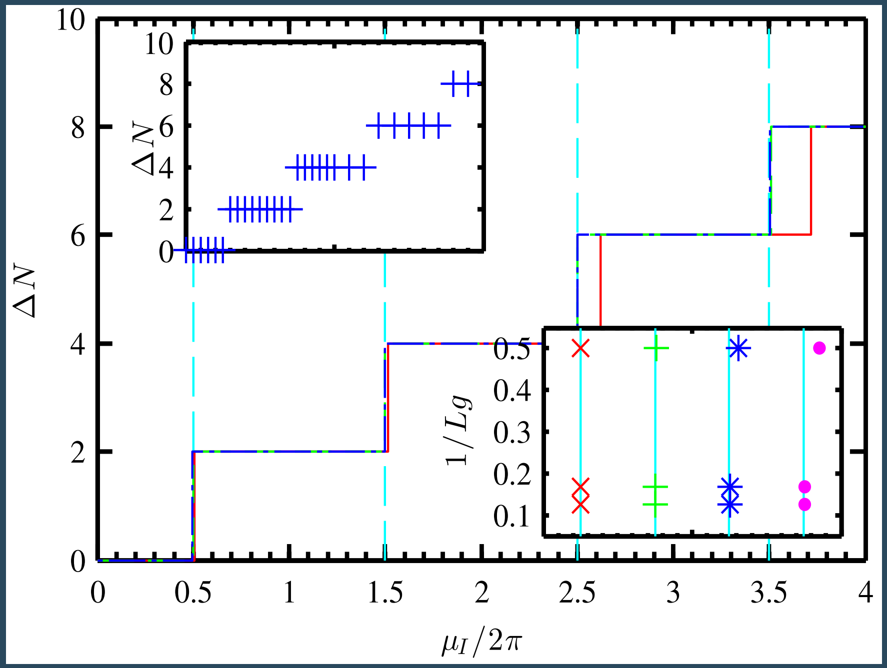
thermal equilibrium

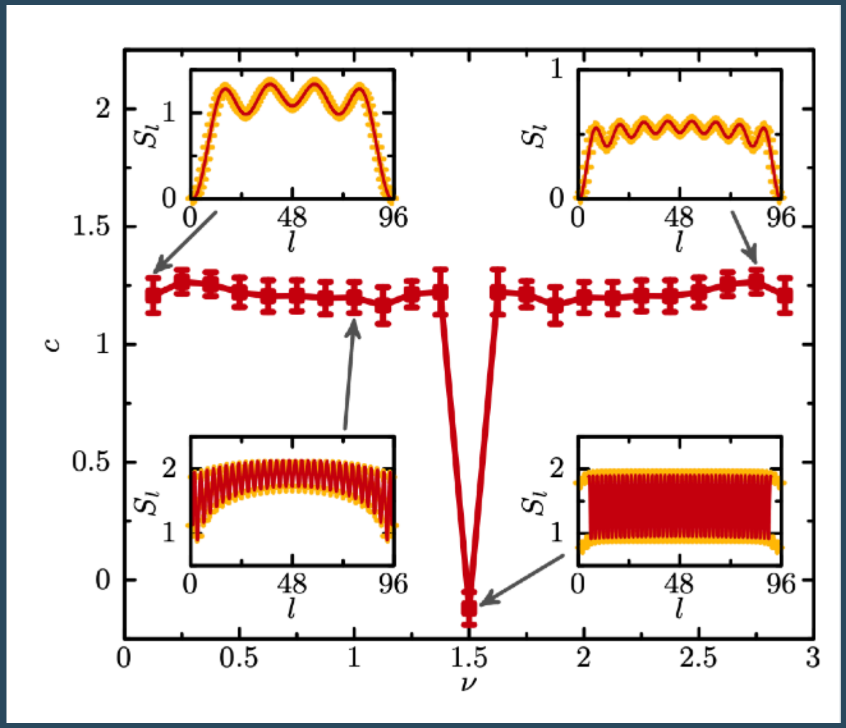
time evolution

finite density

Schwinger model with several flavours and chemical potentials (continuum)

S. Kühn et al, PRL118 (2017) 071601





phase diagram of SU(2) and SU(3) QLM at finite density

Silvi et al, Quantum 1, 9 (2017)
PRD100, 074512 (2019)

some results in 2+1, 3+1 U(1)

finite density

spectrum

entropy

1+1D RESULTS

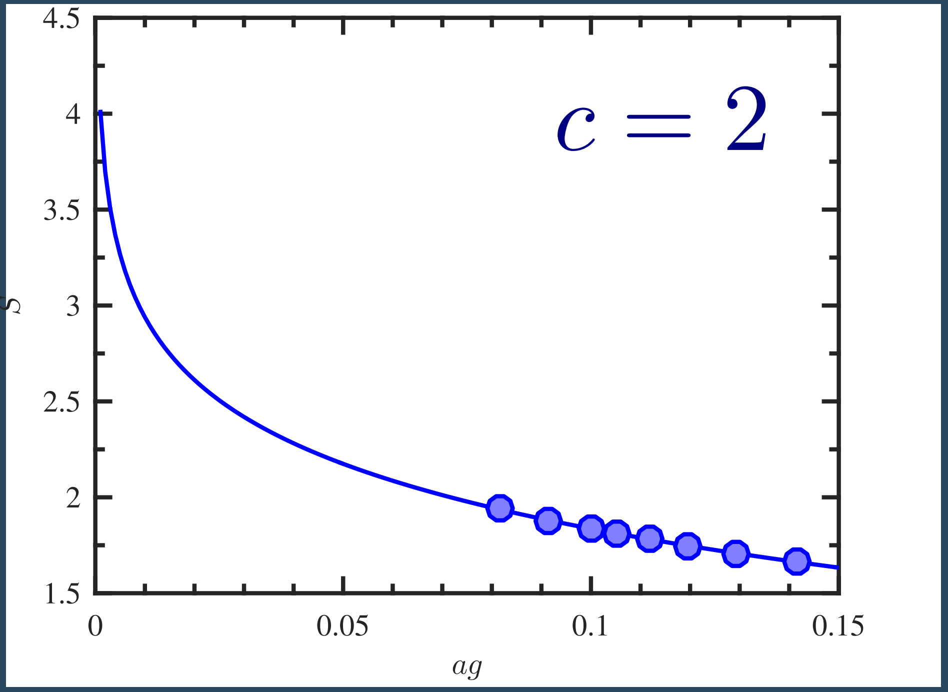
thermal equilibrium

time evolution

gauge constraints not purely local ⇒ not all entropy physical

Casini et al 2014; Gosh et al JHEP 2015
Soni, Trivedi JHEP 2016; van Acoleyen et al PRL 2016

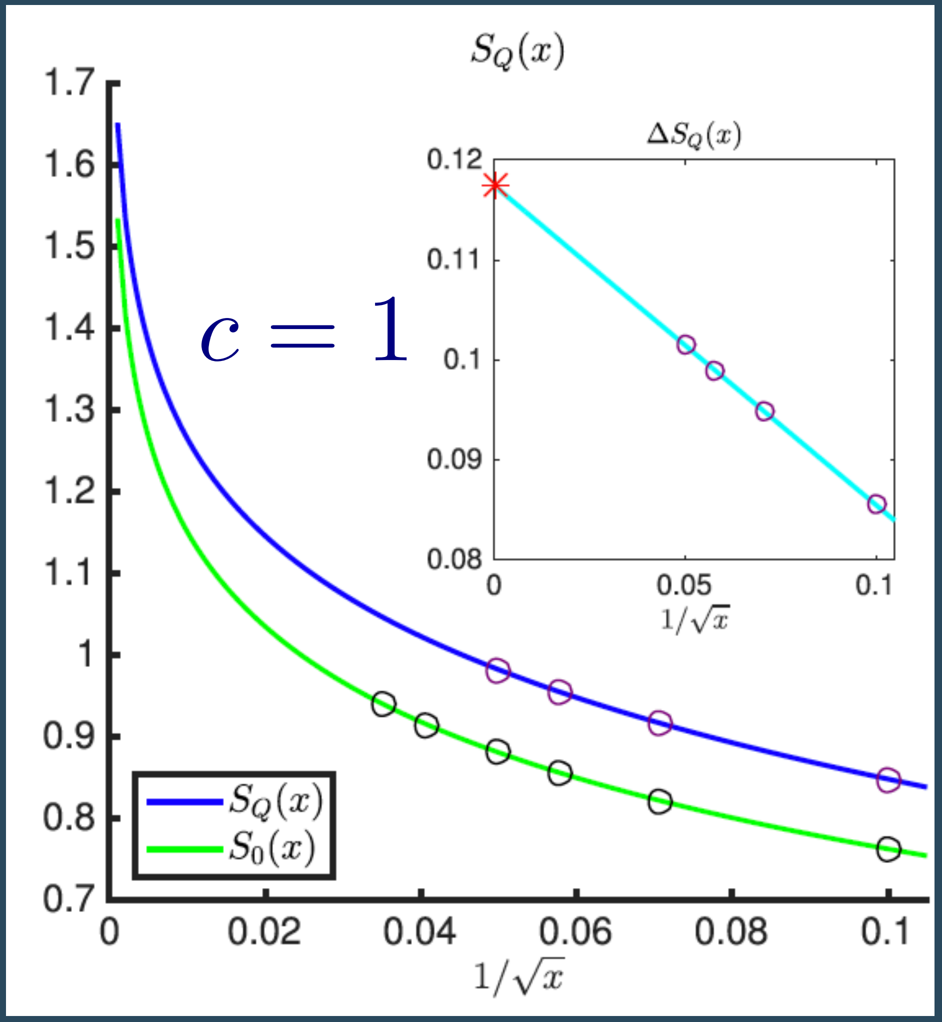
SU(2)



Kühn PRX7, 041046 (2017)

entropy

Schwinger



Buyens PRX6, 041040 (2016)

divergence in the continuum limit

$$S \propto \frac{c}{6} \log_2 \frac{\xi}{a}$$

Calabrese, Cardy JStatMech 2004

finite density

spectrum

entropy

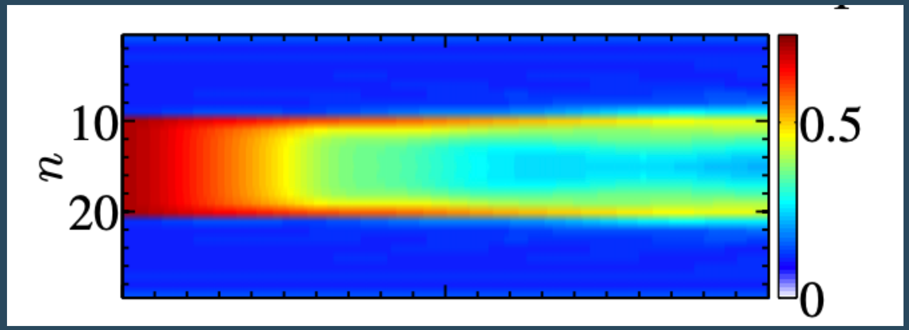
1+1D RESULTS

thermal equilibrium

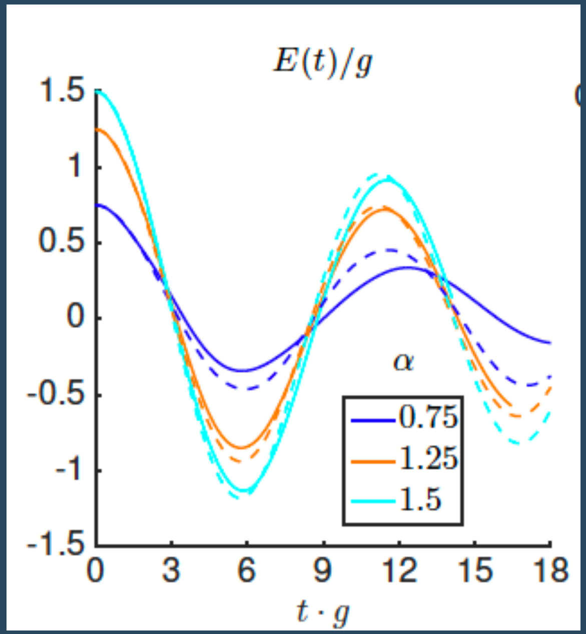
time evolution

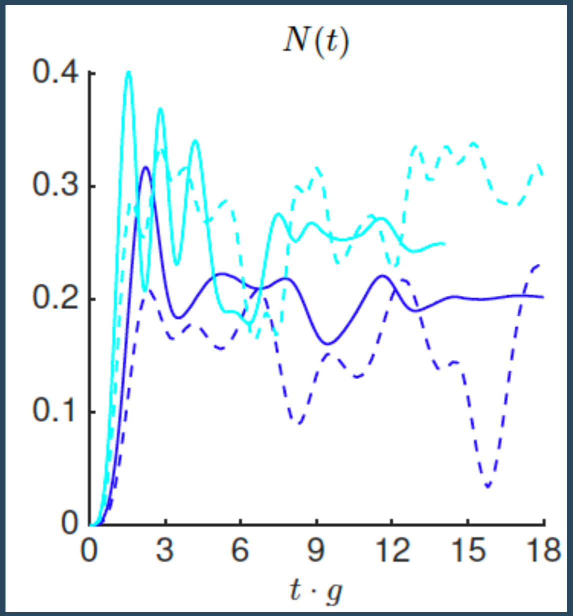
string breaking in lattice models: SU(2), U(1)

Buyens et al. PRL113, 091601(2014)
Kühn et al, JHEP 07 (2015) 130
Pichler et al., PRX 6, 011023 (2016)
PRX 6, 041040 (2016)



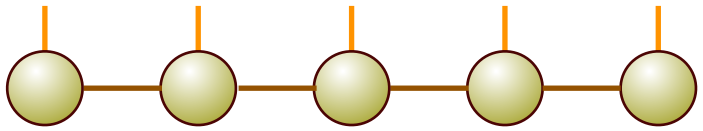
near the continuum limit: Schwinger pair production





time evolution

real time evolution with MPS



TEBD, t-DMRG

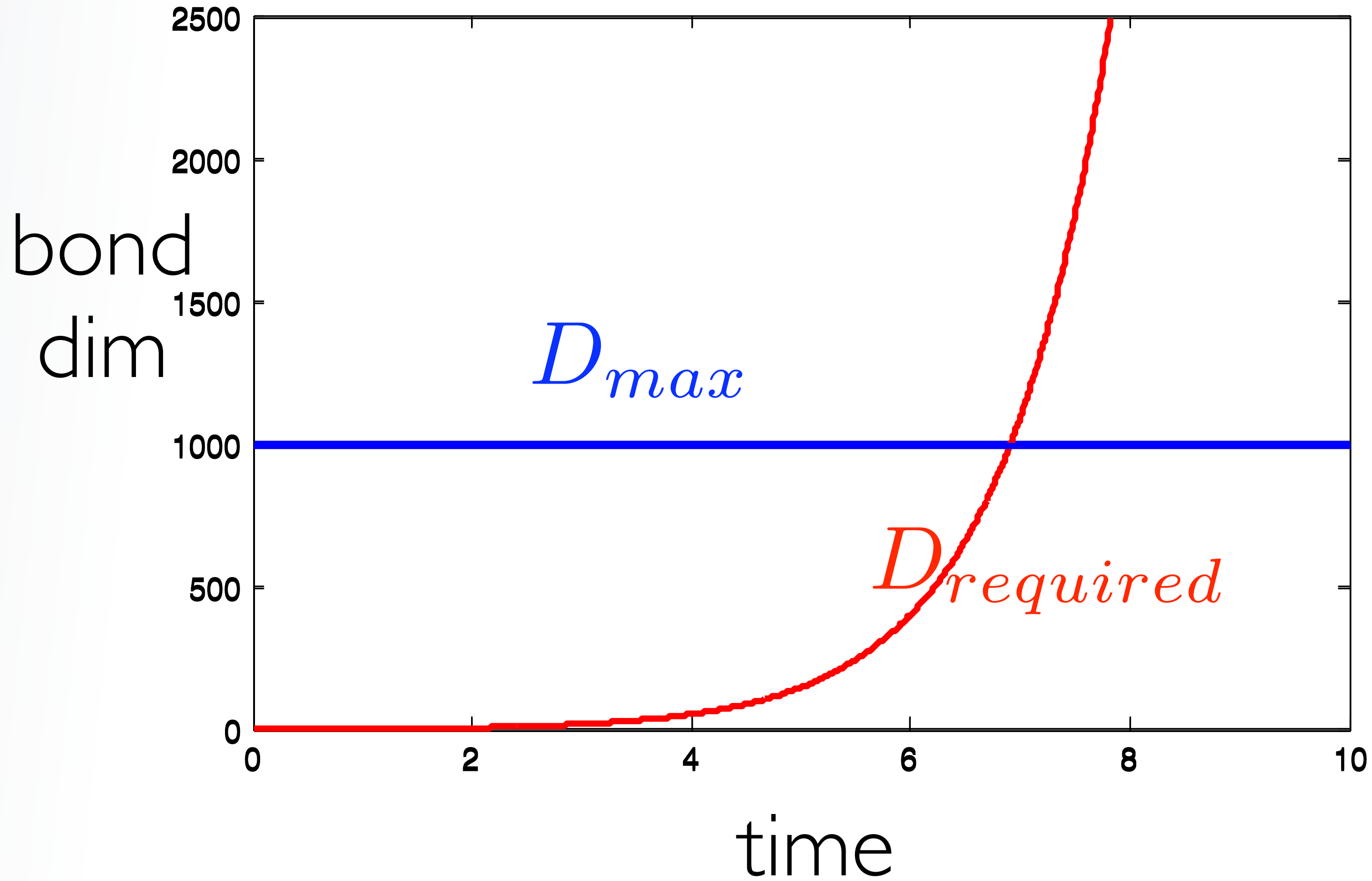
Vidal, PRL 2003, 2004

Verstraete, García-Ripoll, Cirac, PRL 2004

time evolved state approximated by MPS

but entanglement grows

Osborne, PRL 2006
Schuch et al., NJP 2008



required bond for fixed precision

$$D \sim e^{\alpha t}$$

yet many physical situations (in closed and open quantum systems) can be successfully studied!

short times, adiabatic, low energy can work well

García-Ripoll, NJP 2006
Wall, Carr NJP 2012
Paeckel et al arXiv:1901.05824

PDF in Schwinger model

with K. Cichy, C.J. David Lin, M. Schneider,
LATTICE'24 arXiv:2409.16996
2504.XXXXX

fermionic PDF

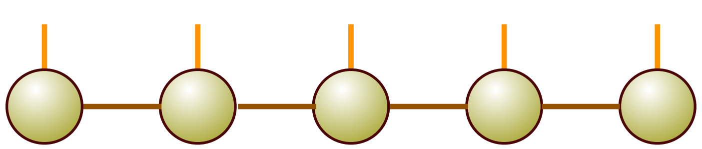
$$f_\Psi(\xi) = \int \frac{dz^-}{4\pi} e^{-i\xi P^+ z^-} \langle P|\overline{\Psi}(z^-) W_{(z^-,0)} \gamma^+ \Psi(0)|P\rangle$$

meson state

Wilson line along lightcone direction

lattice version (staggered fermions)

$$f_\Psi(\xi) = \frac{NM}{8\pi x} \sum_{\Delta z = 0,2,4,\ldots,N} e^{-i\xi \frac{M\Delta z}{2x}} \left[\mathcal{M}_{\mathrm{ee}}(\Delta z) - \mathcal{M}_{\mathrm{eo}}(\Delta z) - \mathcal{M}_{\mathrm{oe}}(\Delta z) + \mathcal{M}_{\mathrm{oo}}(\Delta z)\right]$$



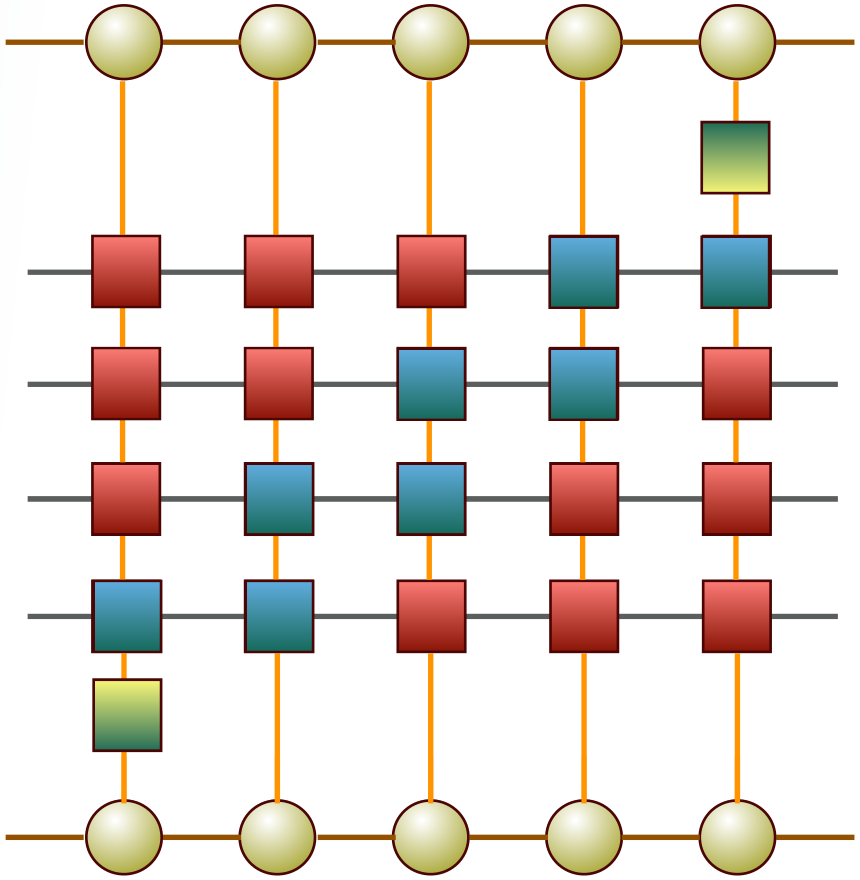
MCB, Cichy, Lin, Schneider, to appear: 2504.XXXXX
preliminary results presented in LATTICE'24 arXiv:2409.16996

fermionic PDF

$$\langle P|\overline{\Psi}(z^-)W_{(z^-,0)}\gamma^+\Psi(0)|P\rangle$$

lattice version (staggered fermions)

$$\langle h|e^{iH\frac{\Delta t}{2}t}\prod_{k<\Delta z}(i\sigma_k^z)\,\sigma_{\Delta z}^+ e^{-iH_0\frac{\Delta t}{2}t}\prod_{k'<0}(-i\sigma_{k'}^z)\,\sigma_0^-|h\rangle$$



Jordan-Wigner spin mapping

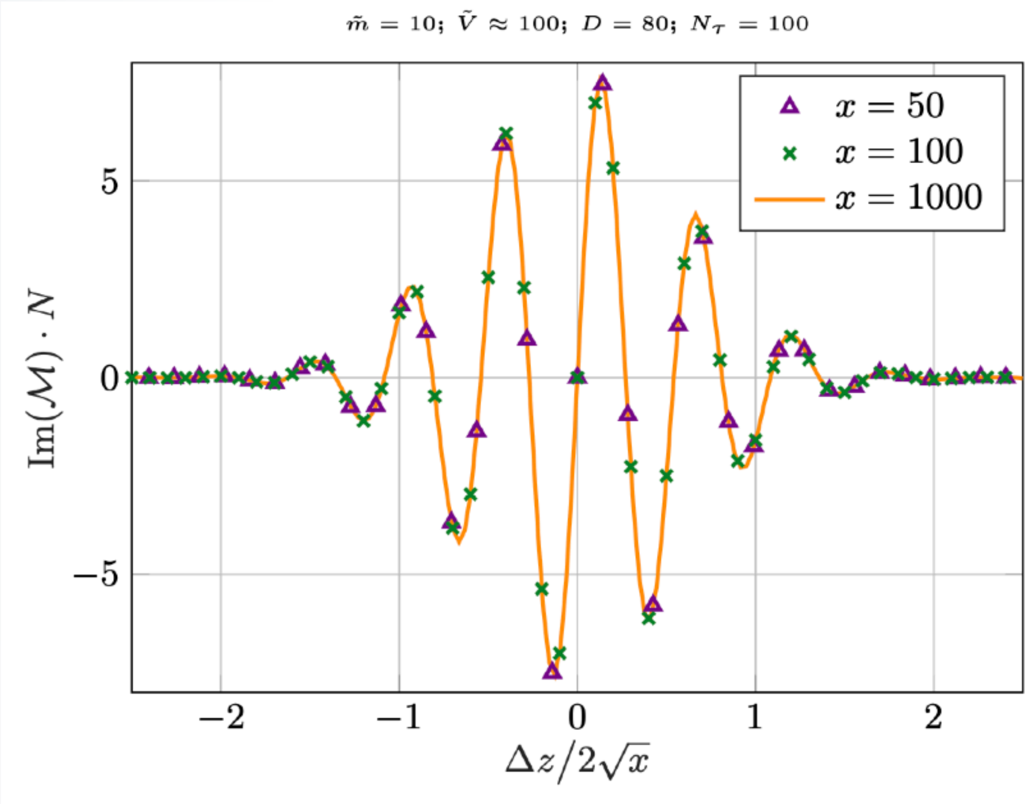
eigenstate

Trotterized evolution

zigzag light cone

MCB, Cichy, Lin, Schneider, to appear: 2504.XXXXX
preliminary results presented in LATTICE'24 arXiv:2409.16996

fermionic PDF

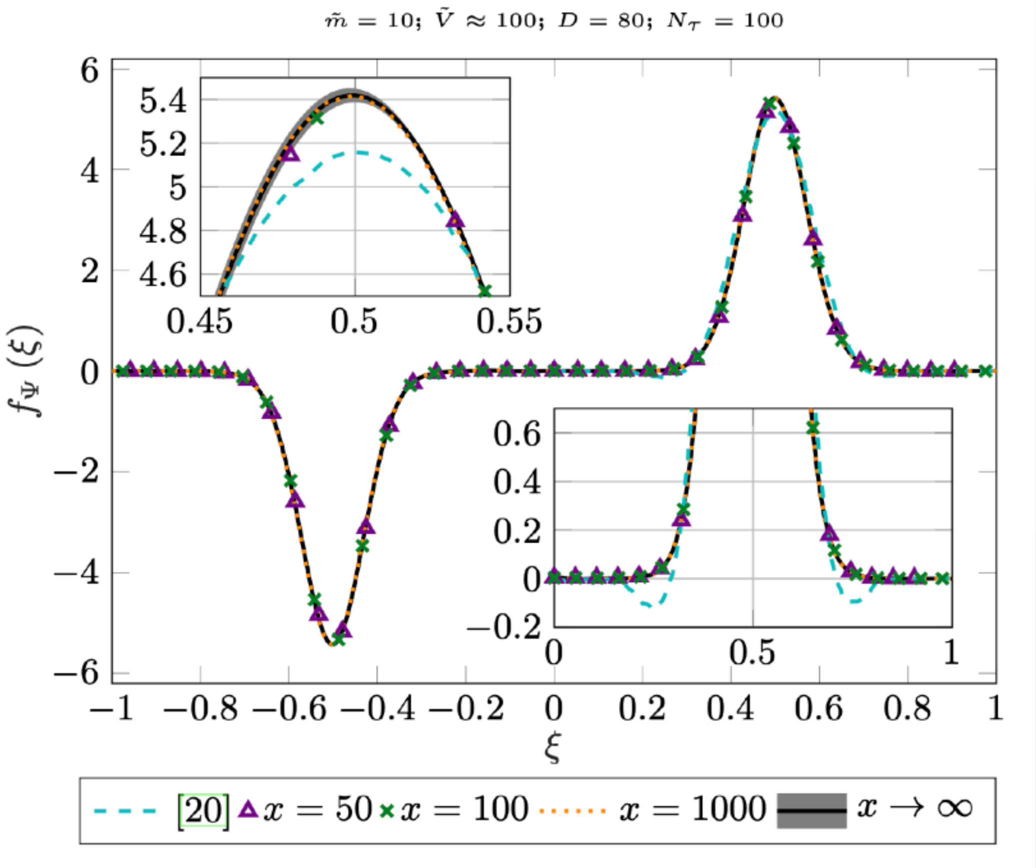


basic matrix element

$$x = \frac{1}{(ga)^2}$$

convergence with very moderate size of tensors

PDF (Fourier transform)



MCB, Cichy, Lin, Schneider, to appear: 2504.XXXXX
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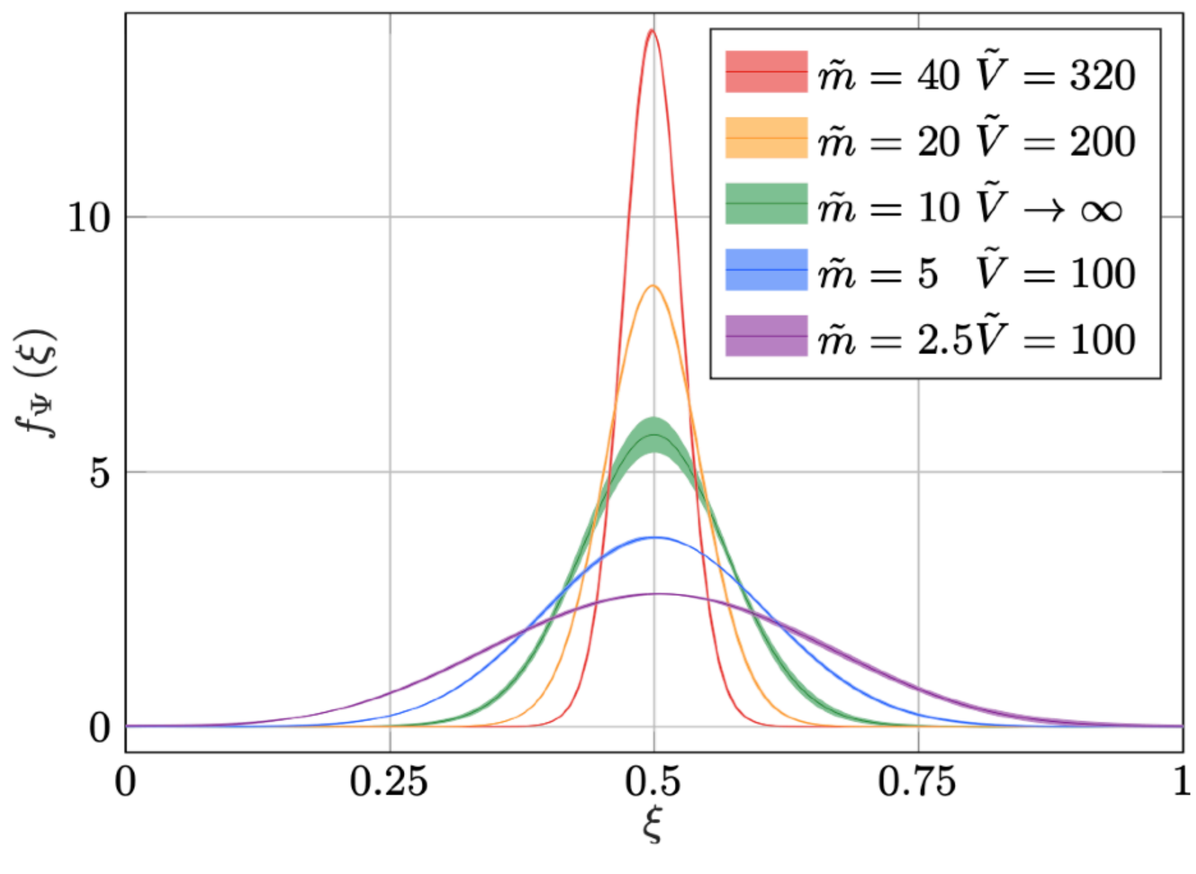
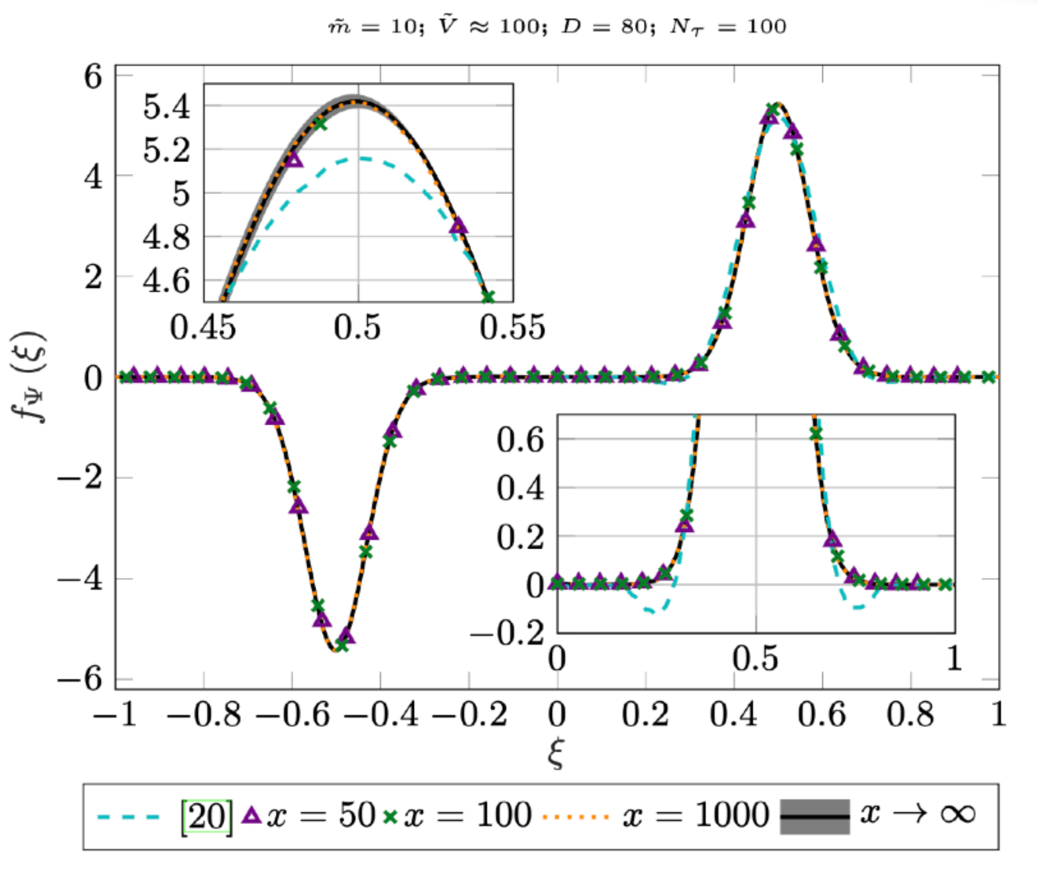
fermionic PDF



different fermion mass, continuum

PDF (Fourier transform)



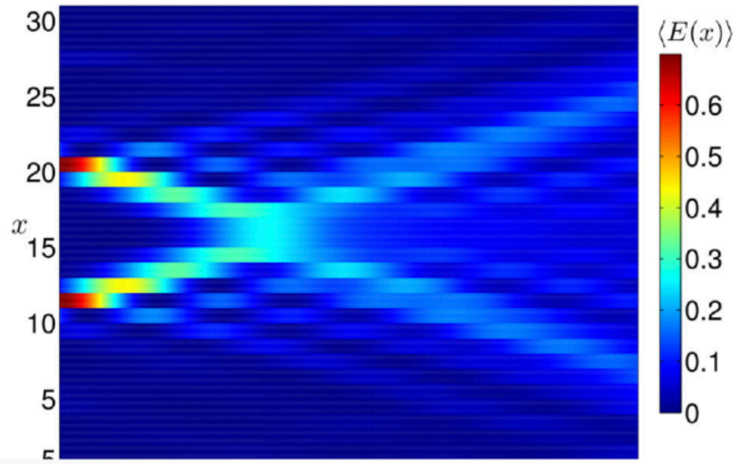
convergence with very moderate size of tensors

MCB, Cichy, Lin, Schneider, to appear: 2504.XXXXX
preliminary results presented in LATTICE'24 arXiv:2409.16996

(inelastic) scattering in the Schwinger model

with I. Papaefstathiou, J. Knolle, PRD111, 014504 (2025)

earlier simulations: elastic scattering in LGT



PHYSICAL REVIEW X **6,** 011023 (2016)

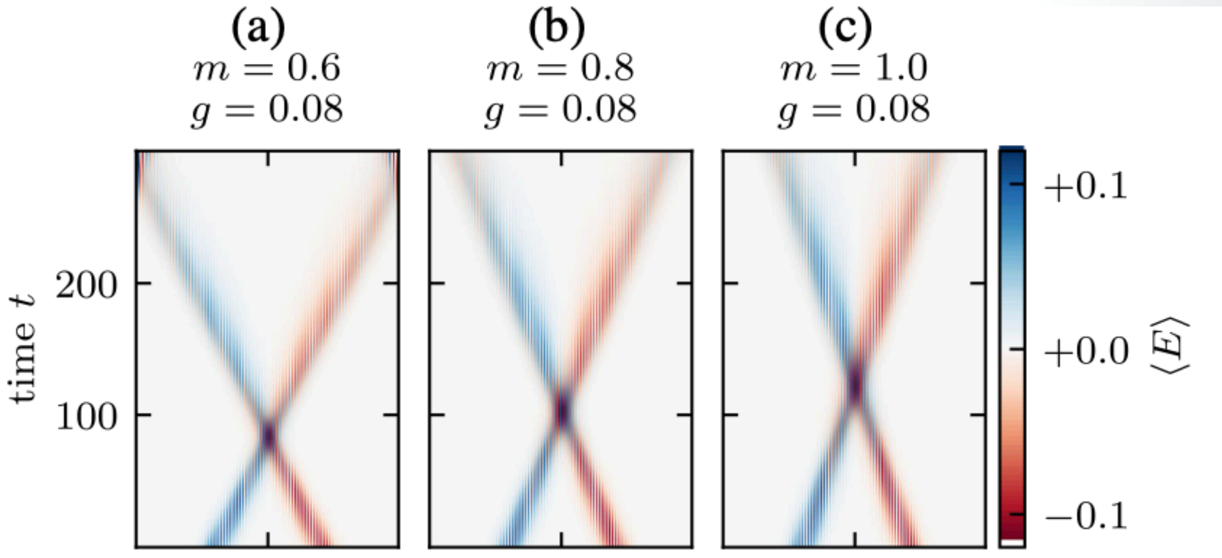
Real-Time Dynamics in U(1) Lattice Gauge Theories with Tensor Networks

T. Pichler,[1] M. Dalmonte,[2,3] E. Rico,[4,5,6] P. Zoller,[2,3] and S. Montangero[1]

PHYSICAL REVIEW D **104,** 114501 (2021)

Entanglement generation in (1+1)D QED scattering processes

Marco Rigobello,[*] Simone Notarnicola, Giuseppe Magnifico, and Simone Montangero

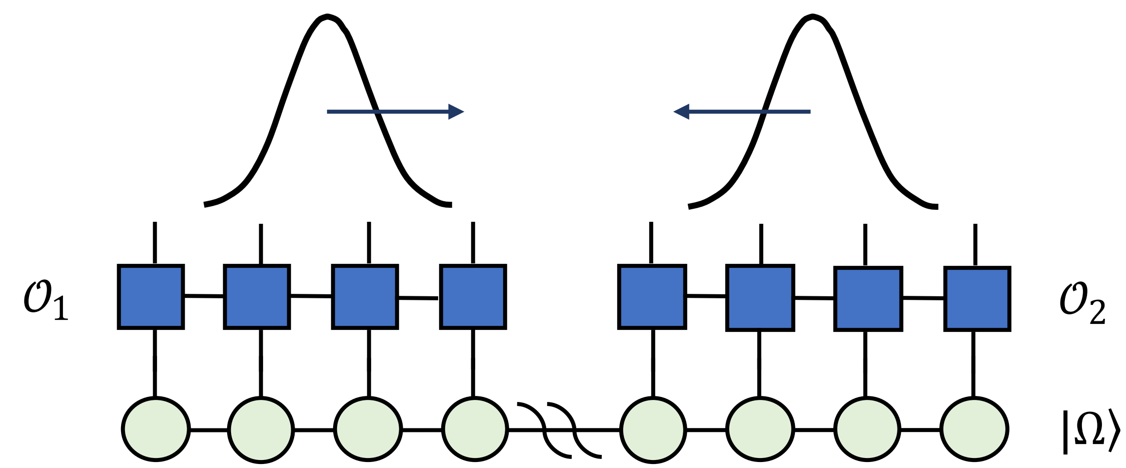


notice also:
Surace, Lerose, New J. Phys. 23 (2021) 062001
Vovrosh et al. PRX Quantum 3, 040309 (2022)
Su, Osborne, Halimeh arXiv:2401.05489

inelastic scattering in the Schwinger model

collision of two vector mesons can produce two scalars

initial setup



$$\sum_n e^{-(n-n_0)^2/(2\sigma^2)} e^{-ikn} \left(\sigma_n^+ \sigma_{n+1}^- - \sigma_{n+1}^+ \sigma_n^-\right) |\Omega\rangle$$

Gaussian wavepacket with momentum k

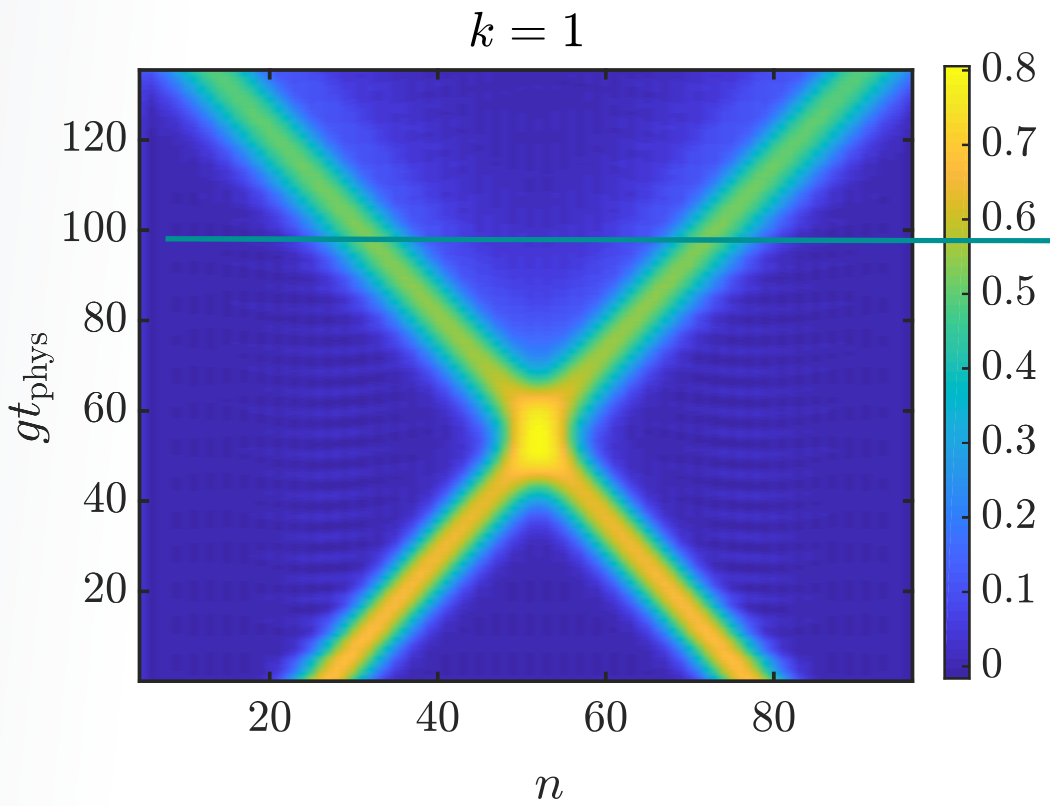
probe the inelastic threshold

strong coupling regime

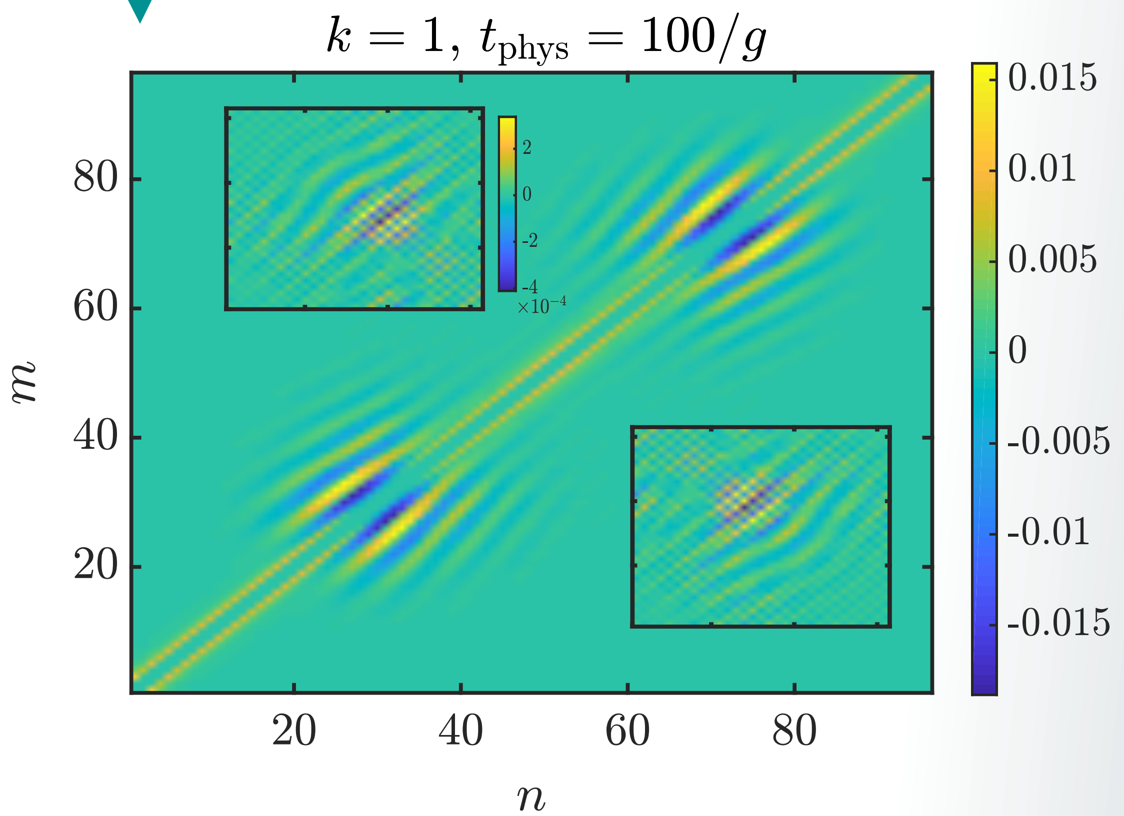
Papaefstathiou, Knolle, MCB, PRD 111, 014504 (2025)

below momentum threshold

entropy of two sites



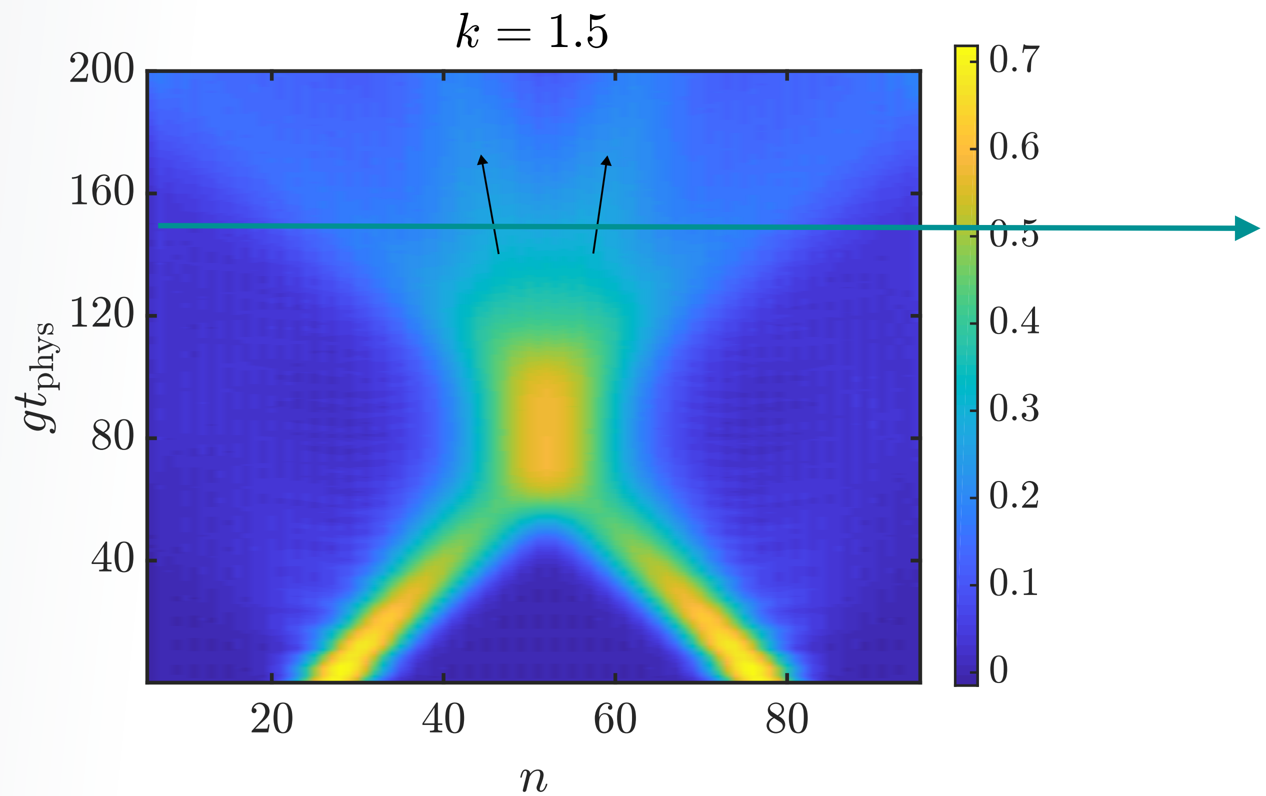
electric flux correlator

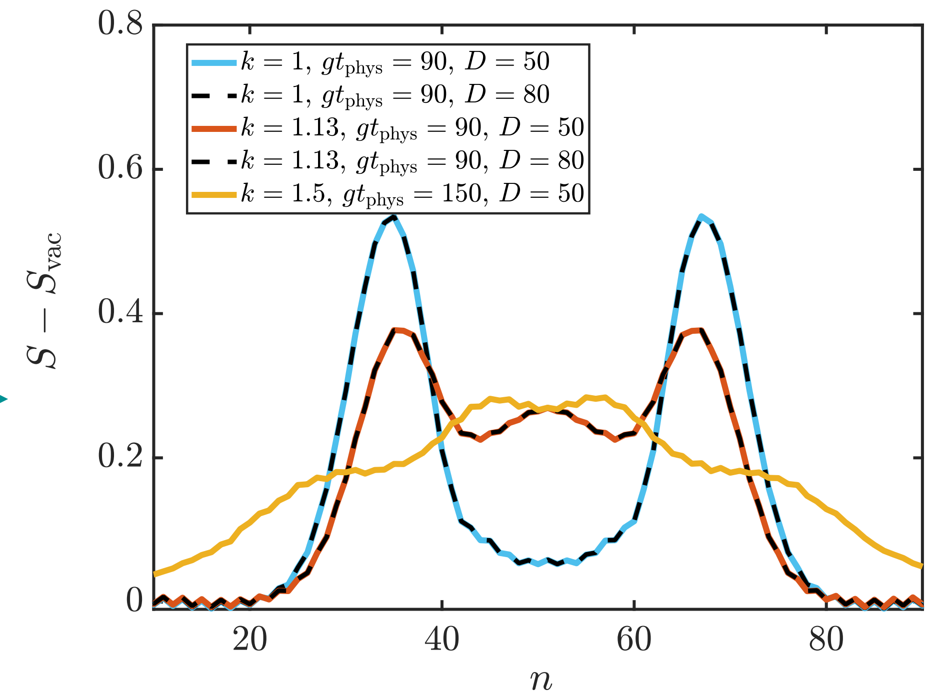


Papaefstathiou, Knolle, MCB, PRD111, 014504 (2025)

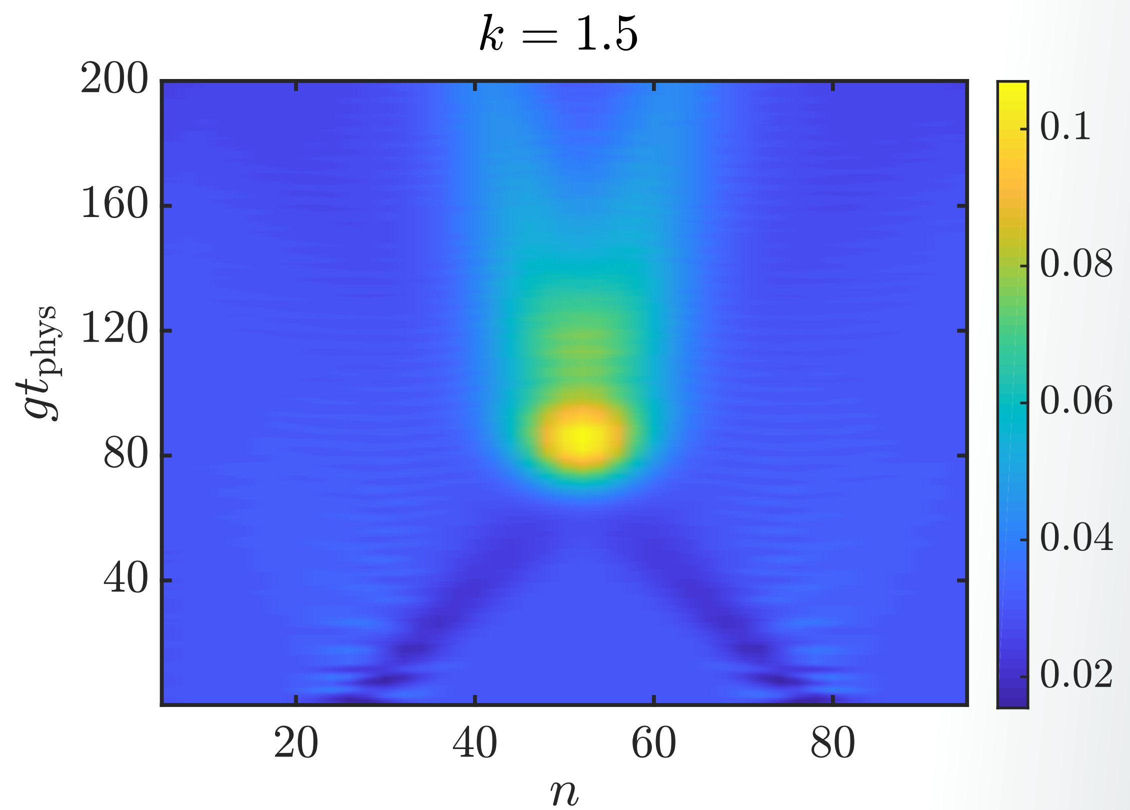
above momentum threshold

entropy of two sites





4-fermion projector



Papaefstathiou, Knolle, MCB, PRD 111, 014504 (2025)

BEYOND ID

active research: PEPS for LGT

explicitly gauge invariant PEPS

Tagliacozzo et al PRX 2014
Haegeman et al PRX 2014
Zohar et al Ann Phys 2015
arXiv:1807.01294

restricted ansatz calculations

also fully Gaussian PEPS — Zohar, Cirac PRD 2018

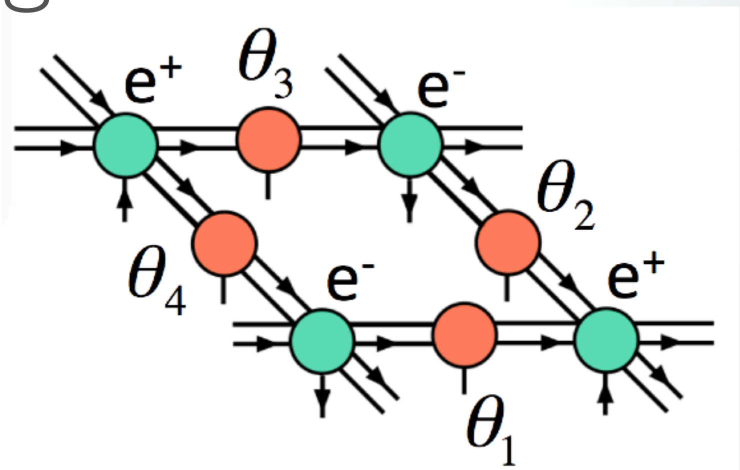
few results with other TNS

Tagliacozzo, Vidal PRB 2011
Felser et al. PRX10, 041040 (2020)

standard PEPS toolbox contains all ingredients for full variational computation

computational cost, required D

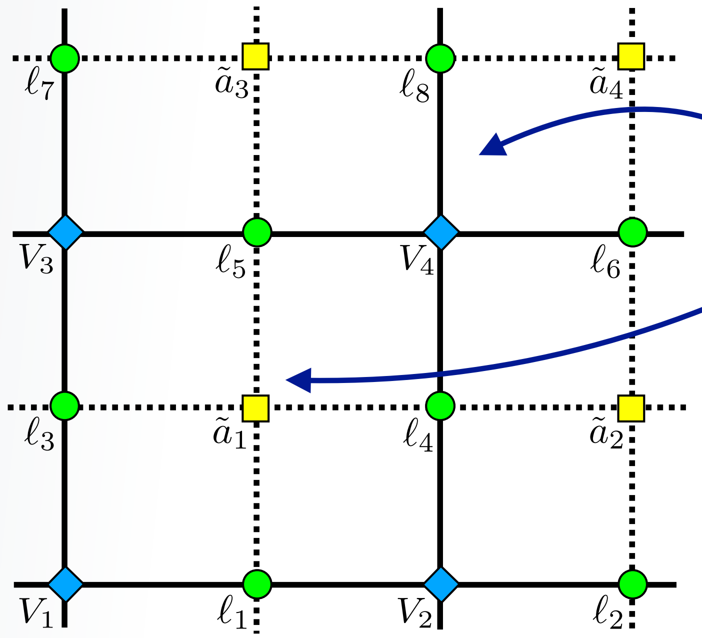
plaquette terms



Zapp, Orús PRD 2017

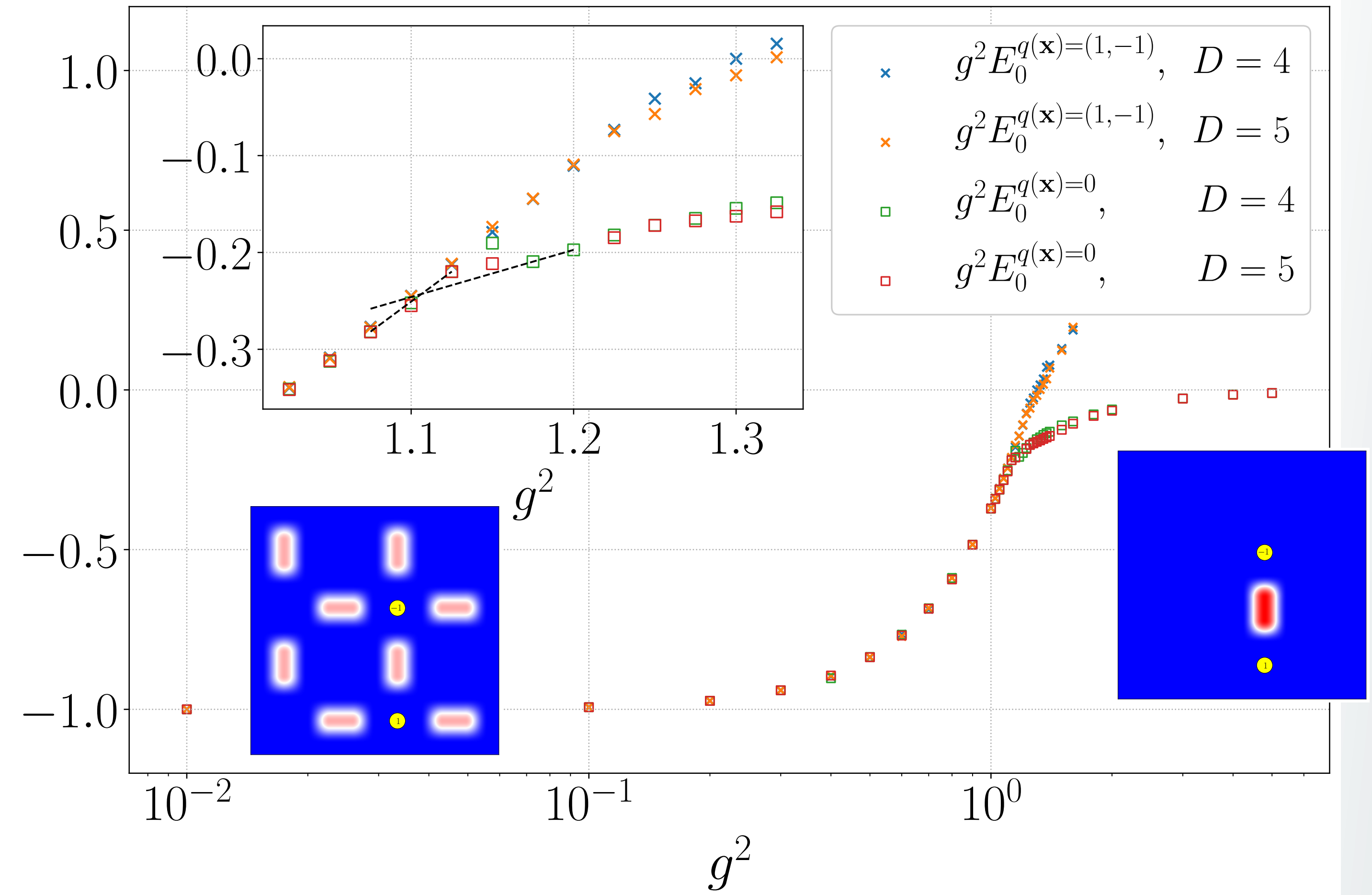
variational iPEPS study of Z3 in 2+1D

$$H_{\mathbb{Z}_3} = \frac{g^2}{2}\sum_{\ell} E_\ell^2 - \frac{1}{2g^2}\sum_P \left(U_P + U_P^\dagger\right)$$



plaquette terms can be implemented as sequence of 2-body gates

Zohar et al, PRA 2017



Robaina, MCB, Cirac, PRL126, 050401 (2021)

Thanks for your attention!

T-NiSQ
Tensor Networks in Simulation of Quantum Matter

DFG FOR 5522

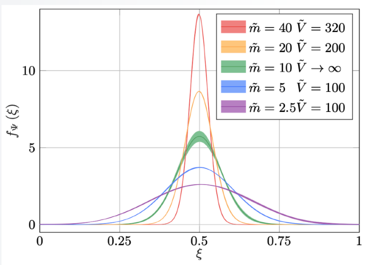
MAX PLANCK INSTITUTE OF QUANTUM OPTICS

MCQST

TNS: entanglement-based ansatz can be a suitable ansatz also for LGT/QFT

real time is more challenging than equilibrium

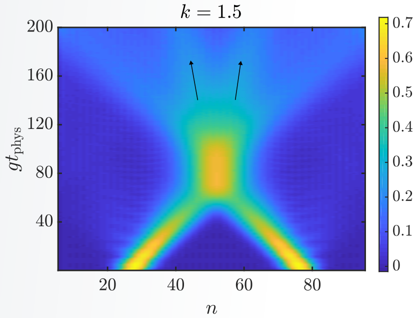
current results: fermion PDF Schwinger model



- ab initio calculation in Minkowski space
- controlled continuum extrapolation

LATTICE'24 arXiv:2409.16996
to appear 2504.XXXXX

other dynamical simulations: inelastic scattering



- momentum threshold observed
- production detected through local observables

Papaefstathiou, arXiv:2402.18429
Belyanski et al PRL132, 091903 (2024)

ongoing progress in higher dimensions

- PEPS, tree TN, MPS

Thanks for your attention!

T-NiSQ

Tensor Networks in Simulation of Quantum Matter

DFG FOR 5522

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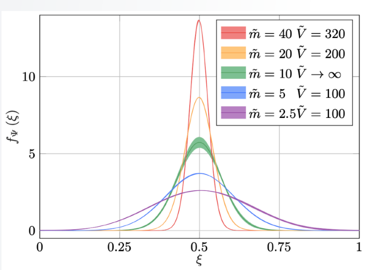
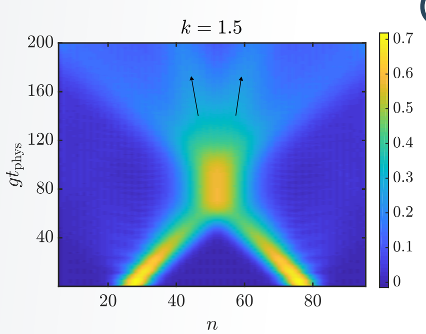


ab initio calculati

controlled conti

other dynamical si



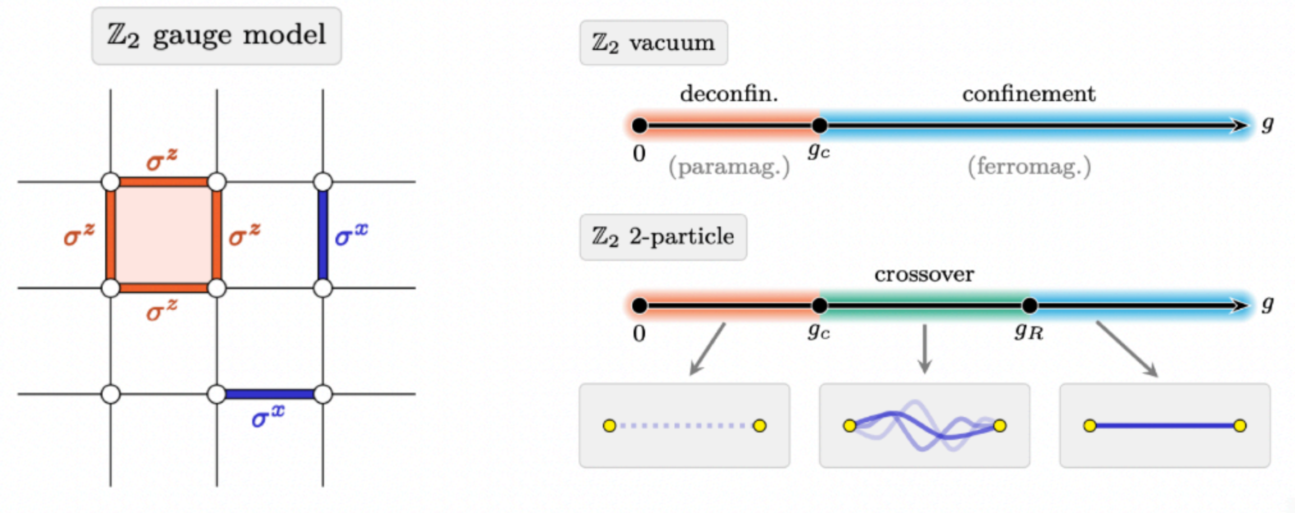
momentum thre

production dete

ongoing progress i

PEPS, tree TN, M

$\mathbb{Z}_2$ roughening transition, and string dynamics



$$H_{\rm LGT}(g) = -g\sum_{(x,\hat{\mu})} \sigma^x_{(x,\hat{\mu})} - \frac{1}{g}\sum_{\square}(\sigma^z\sigma^z\sigma^z\sigma^z)_\square$$

Di Marcantonio, Pradhan, Vallecorsa, Rico, in progress