# Hadronic resonances from lattice QCD

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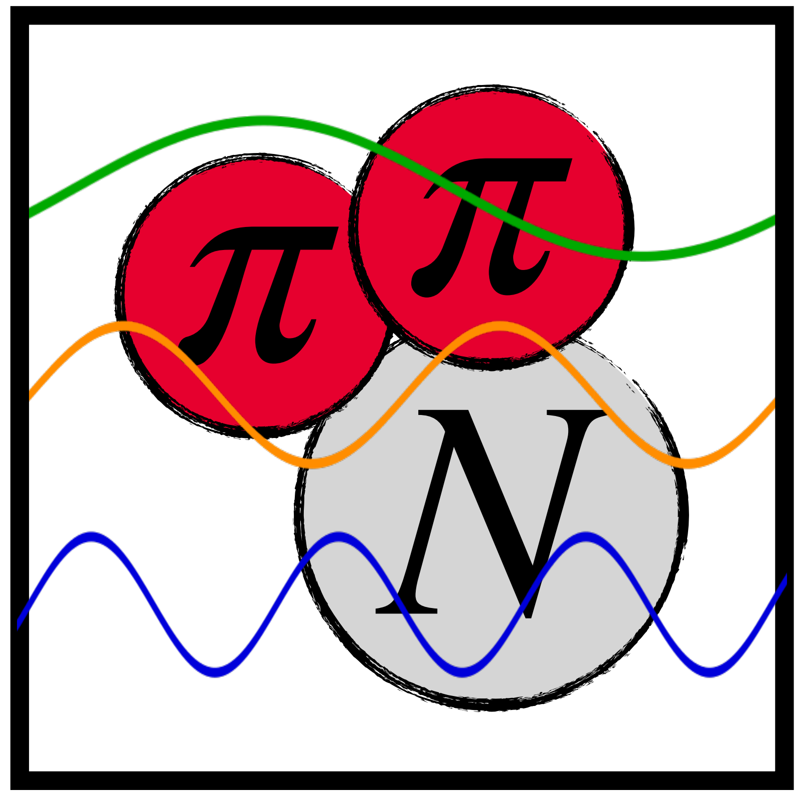
*2nd LatticeNET Workshop*
*Benasque Science Center*
*April 1st*

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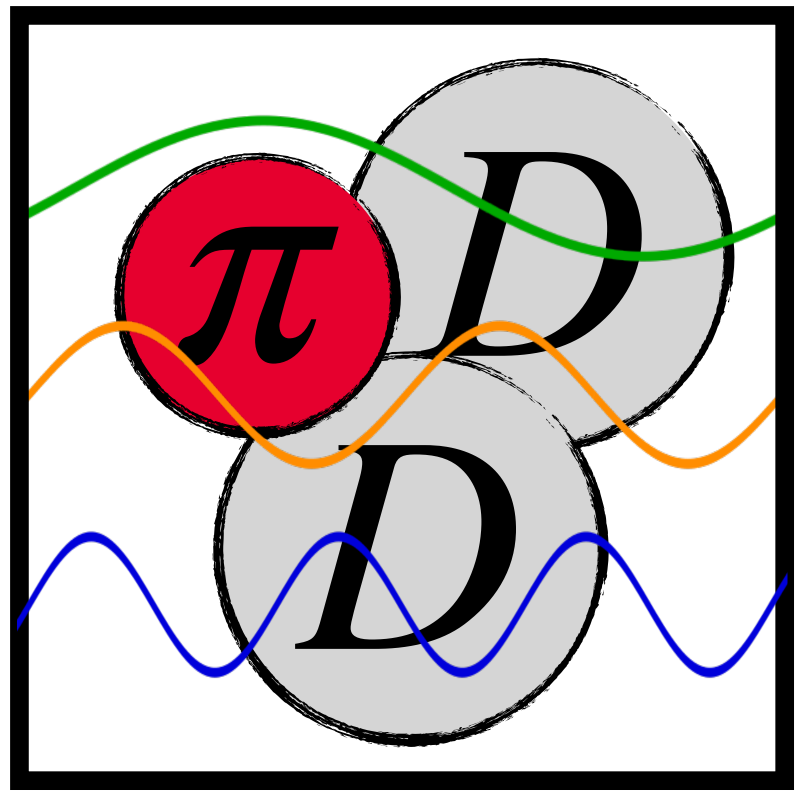

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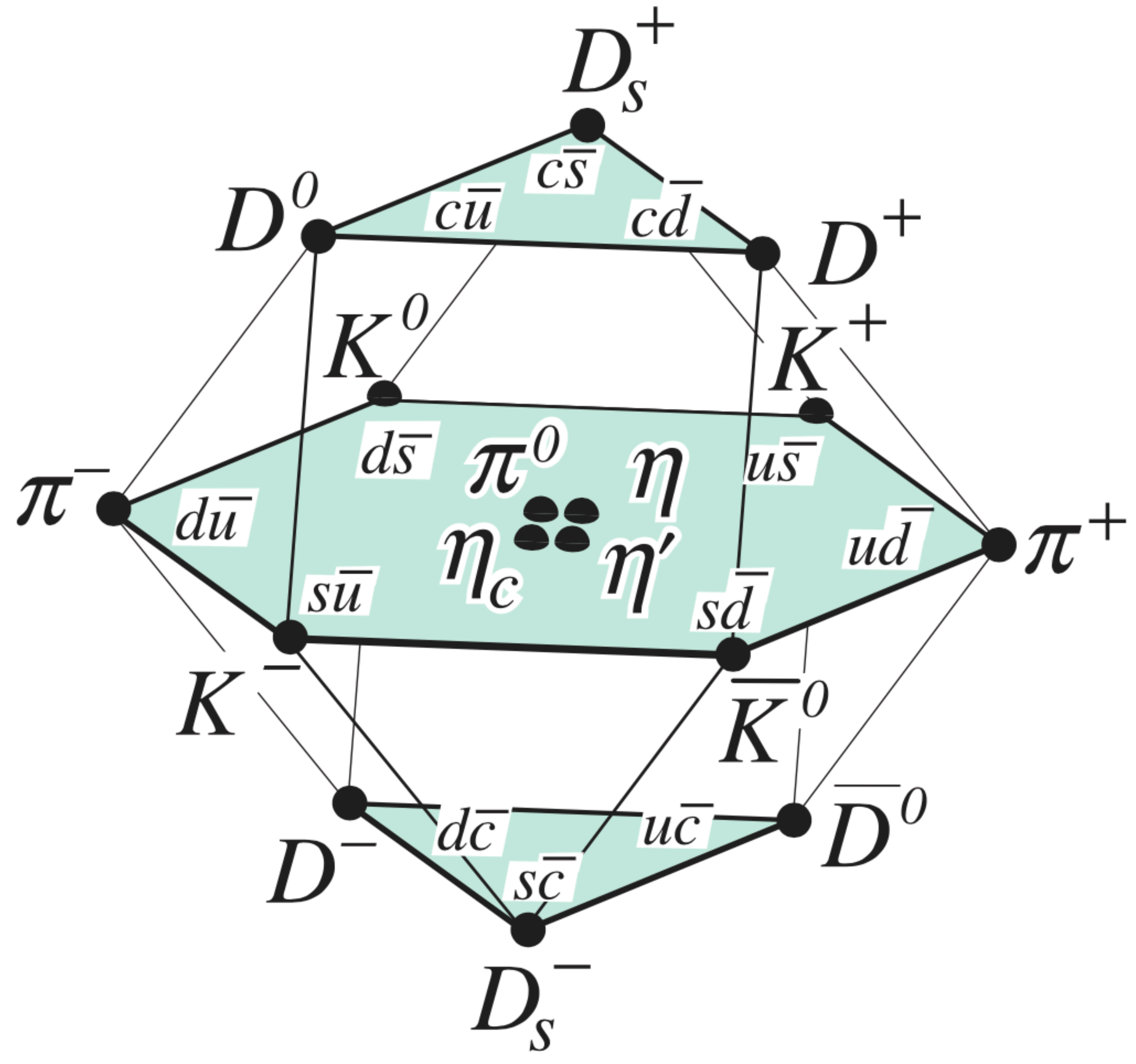


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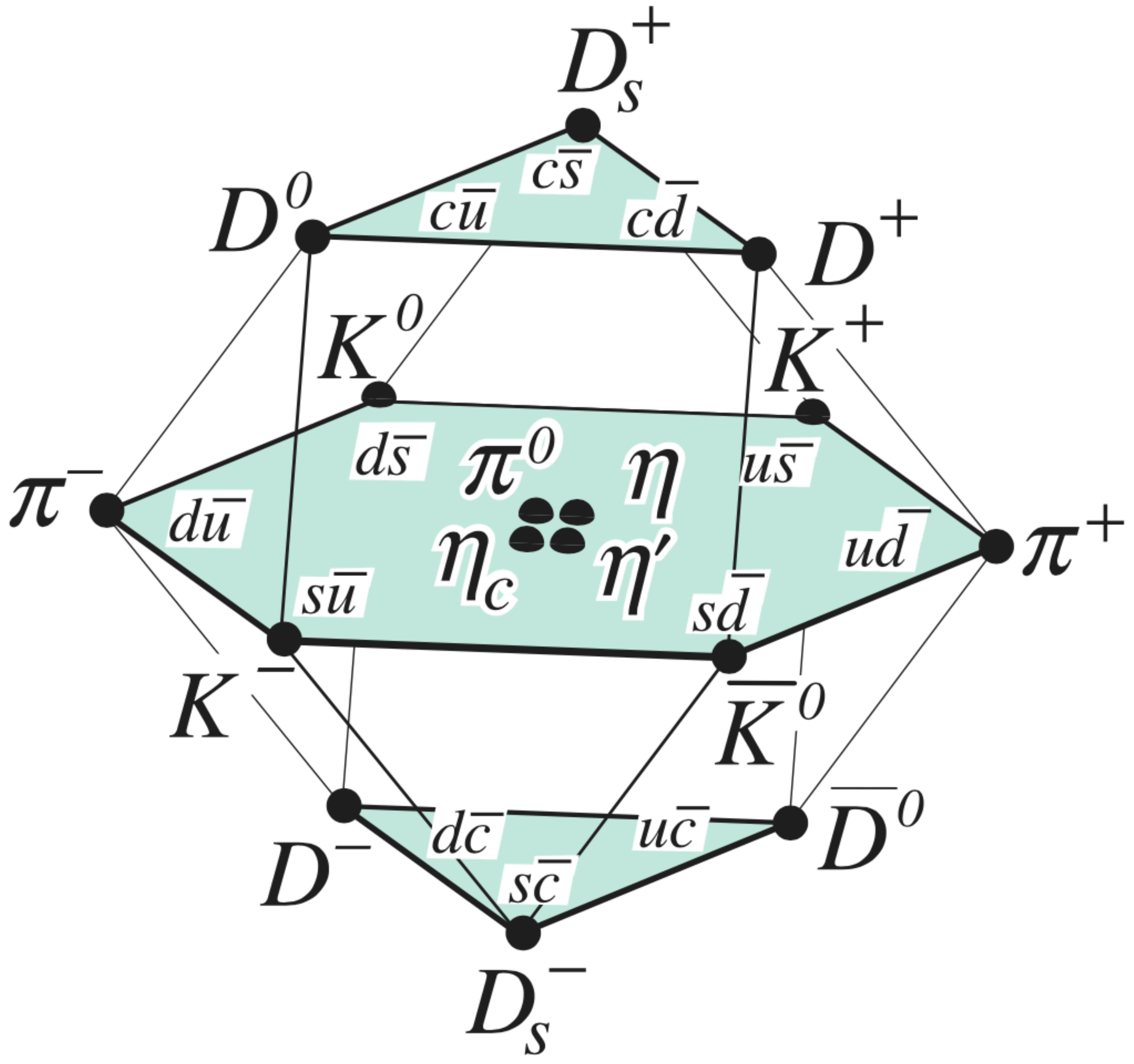


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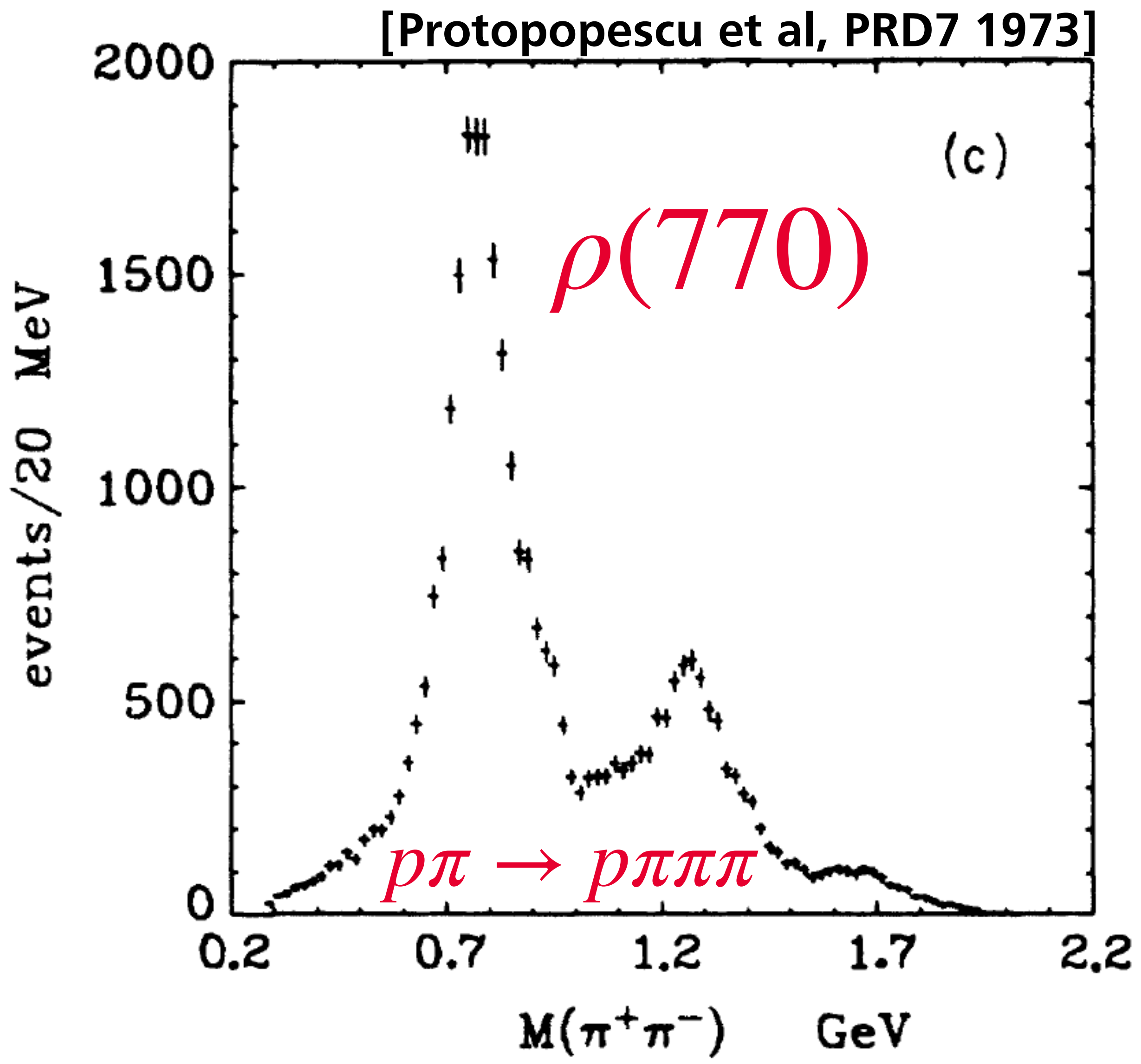
- **Resonances show up in scattering processes**

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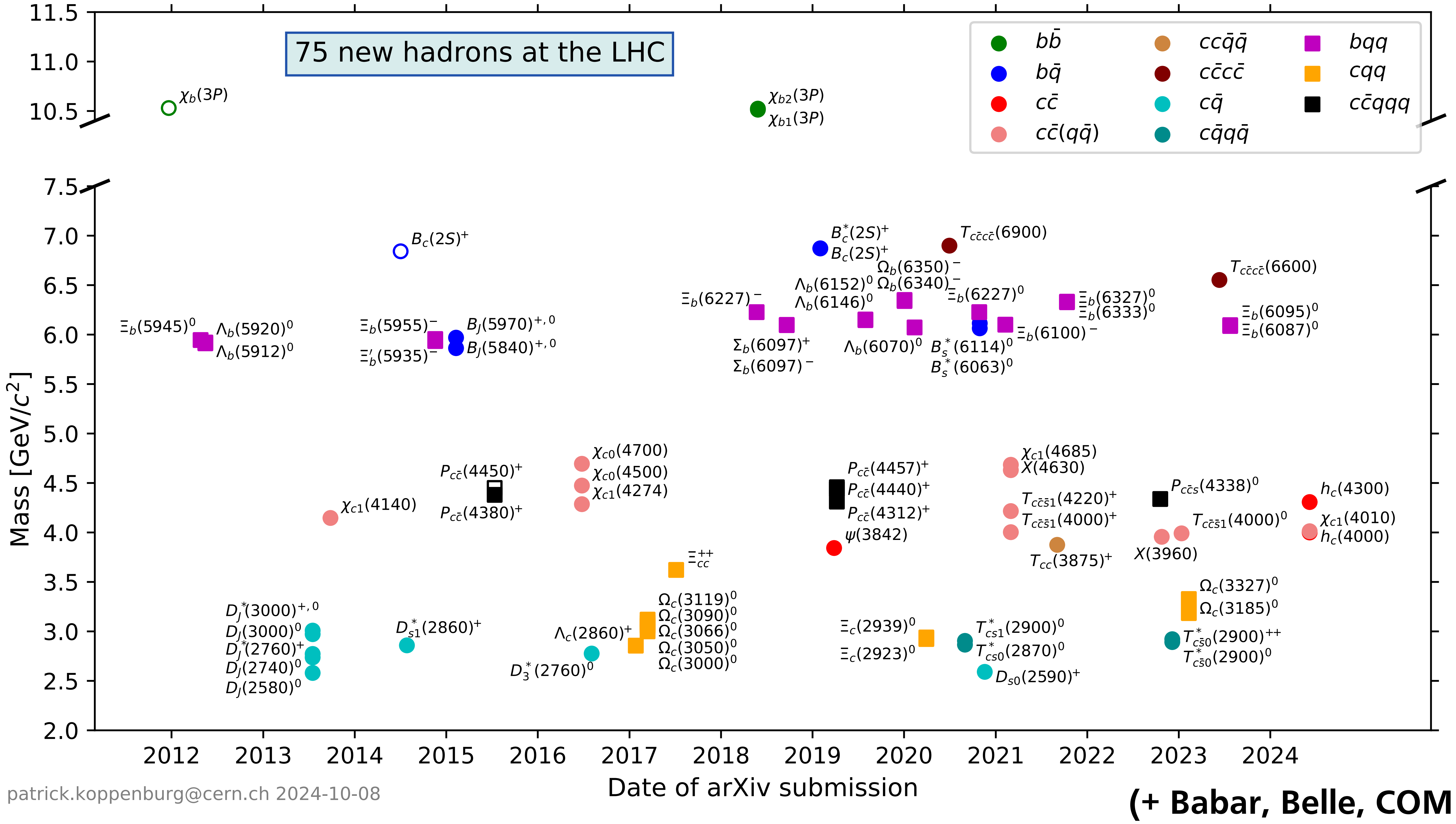
# The Hadron Spectrum

**A growing hadron spectrum still requires first principles understanding**

**[https://www.nikhef.nl/~pkoppenb/particles.html]**

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**Fernando Romero-López, Uni Bern**

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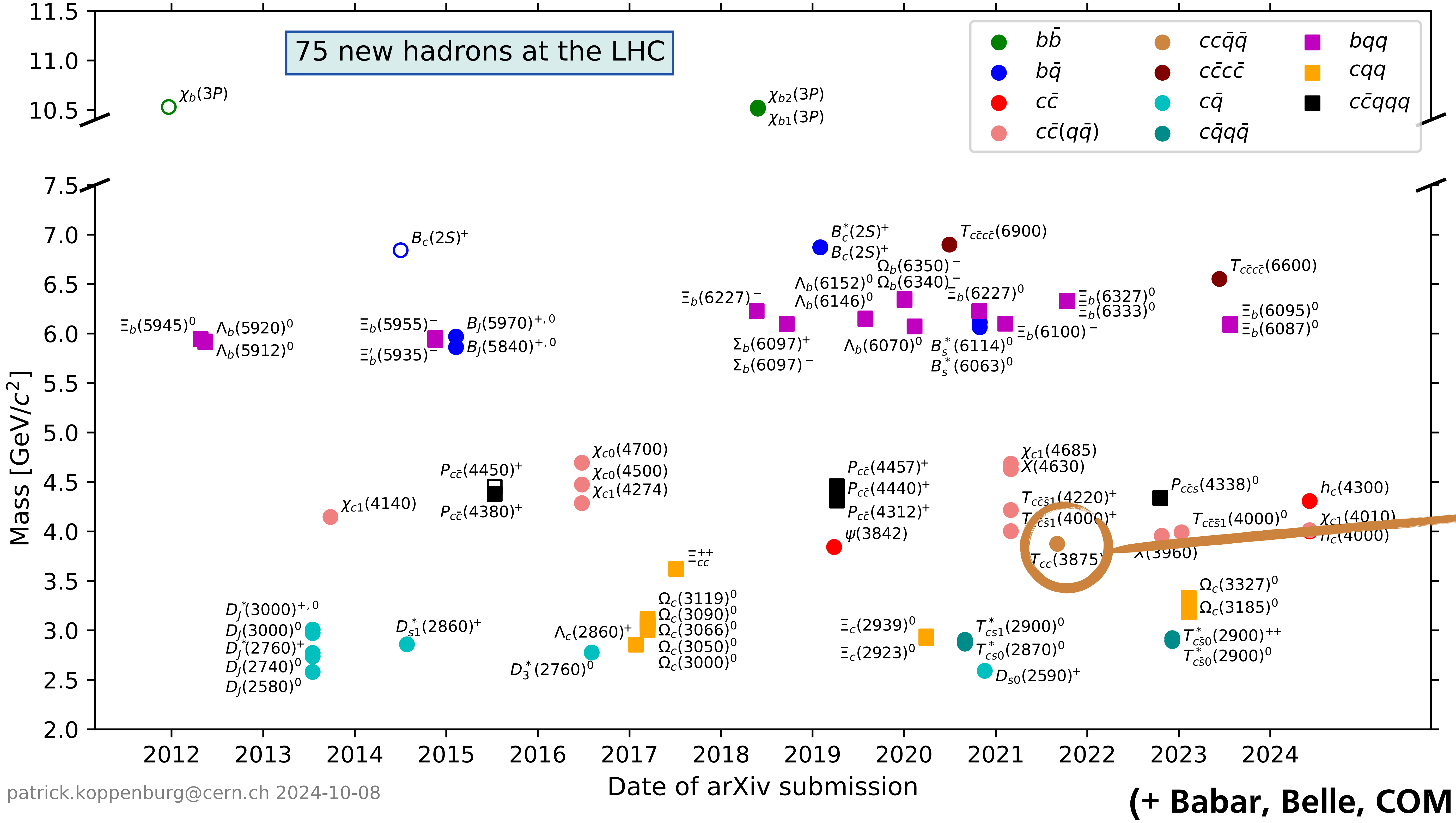


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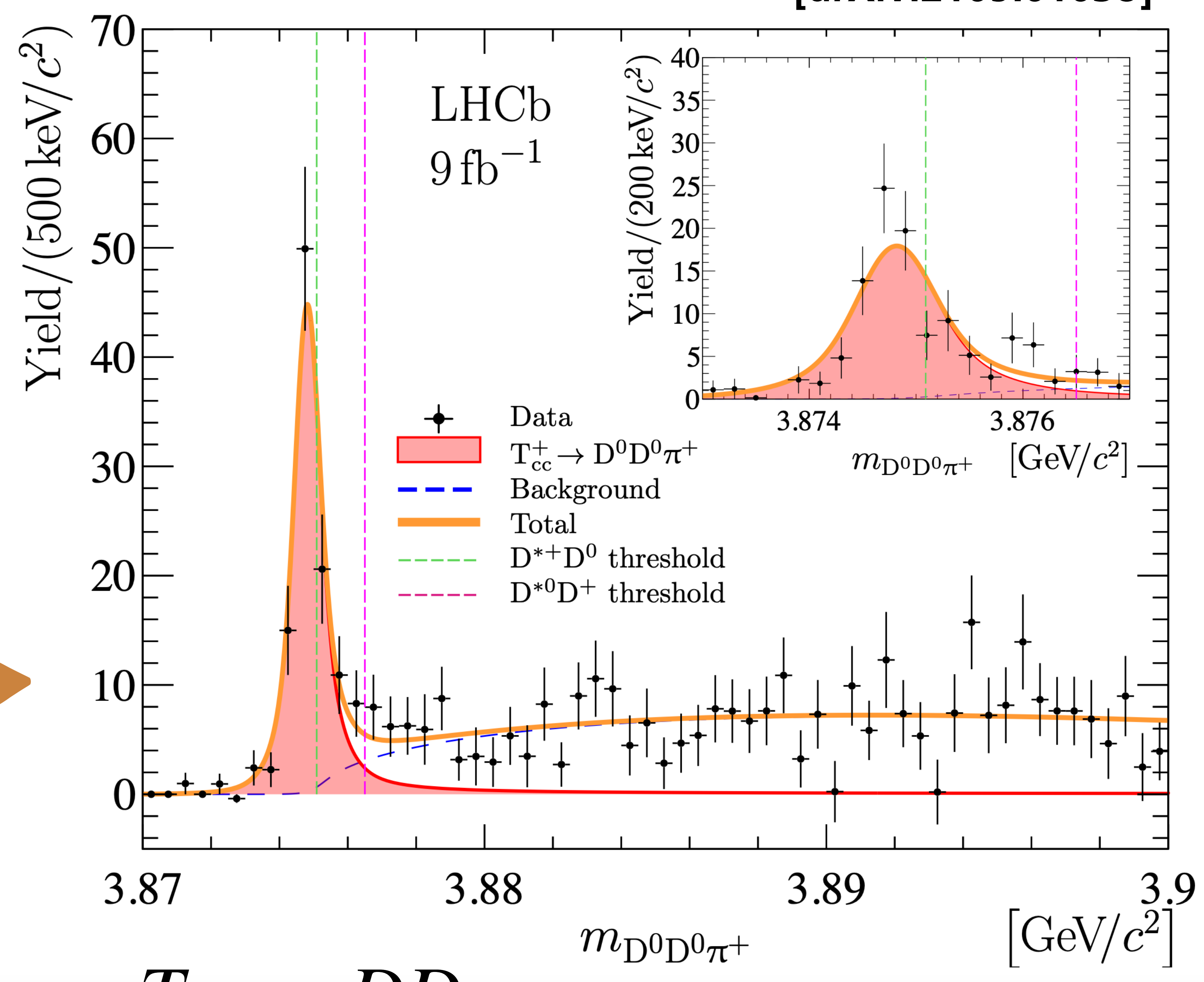
**[arXiv:2109.01038]**




$T_{cc} \to DD\pi$



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# Multi-hadron interactions

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CP violation in $K \to \pi\pi$ weak decays

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**[NA48 & KTeV, 2002 & 2009]**

**$\sigma$ resonance**

**[RBC/UKQCD, 2004.09440]**

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$f_0(1710)$ **resonance**

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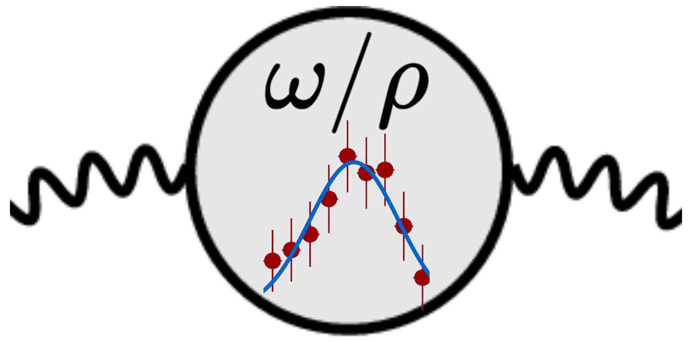

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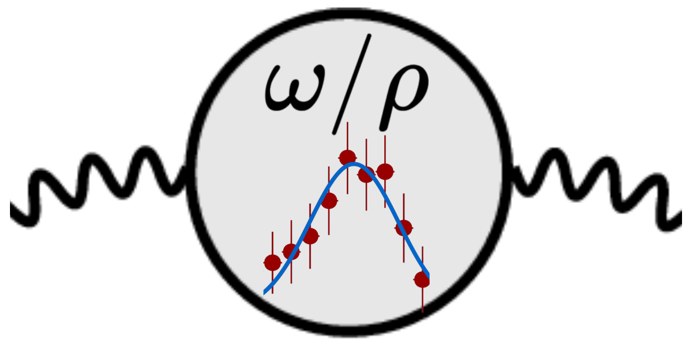
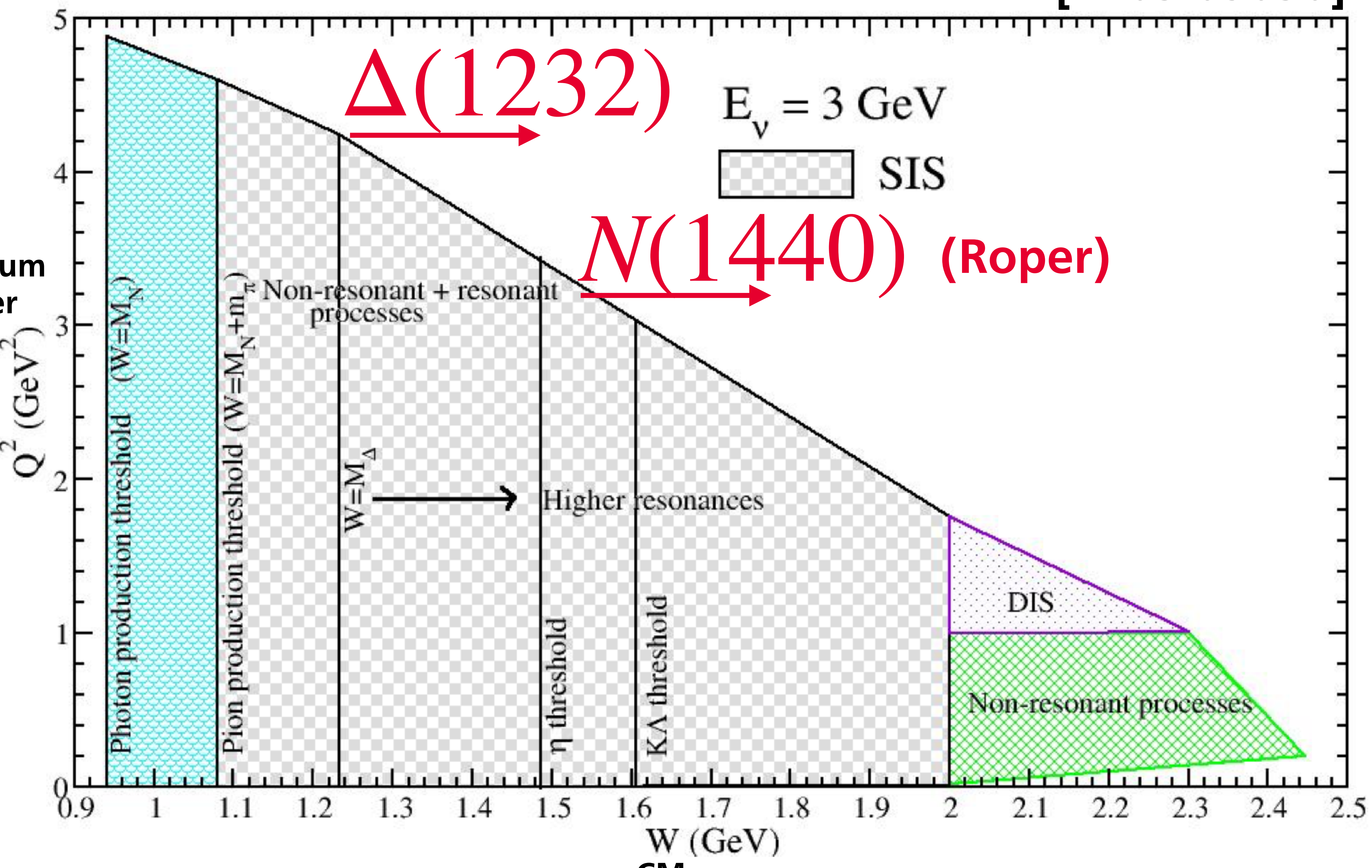



- **Neutrino-nucleus scattering (DUNE, Hyper-K)**

$$\nu N \to \ell N \pi$$

**[2203.09030]**

momentum transfer




CM energy

# Hadronic resonances

- Hadronic resonances typically manifest themselves as enhancements in cross-sections

- The rigorous definition of a hadronic resonance is a pole in the complex plane

coupling

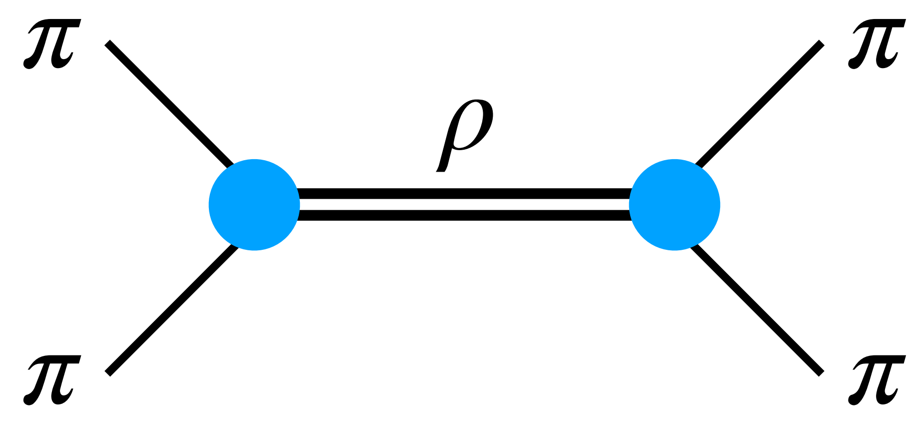
$$\mathcal{M}_\ell \sim -\frac{g^2}{s - s_R}$$

scattering amplitude

$$\sqrt{s_R} = M_R - i\frac{\Gamma}{2}$$

width

mass



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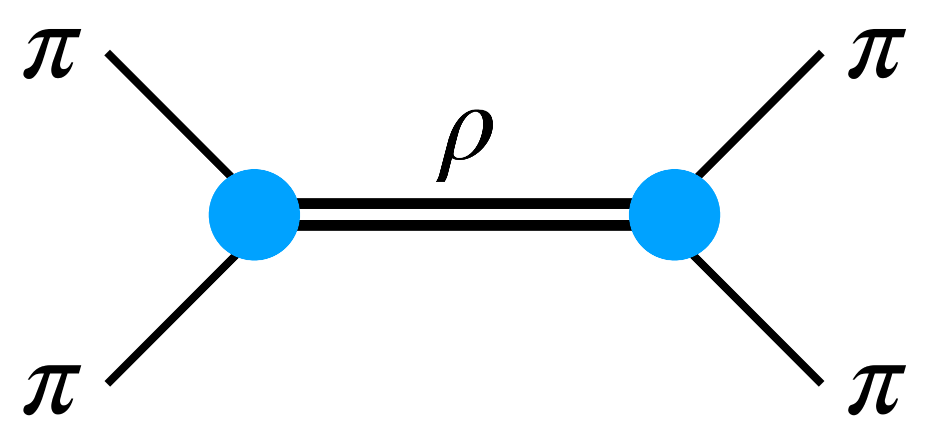
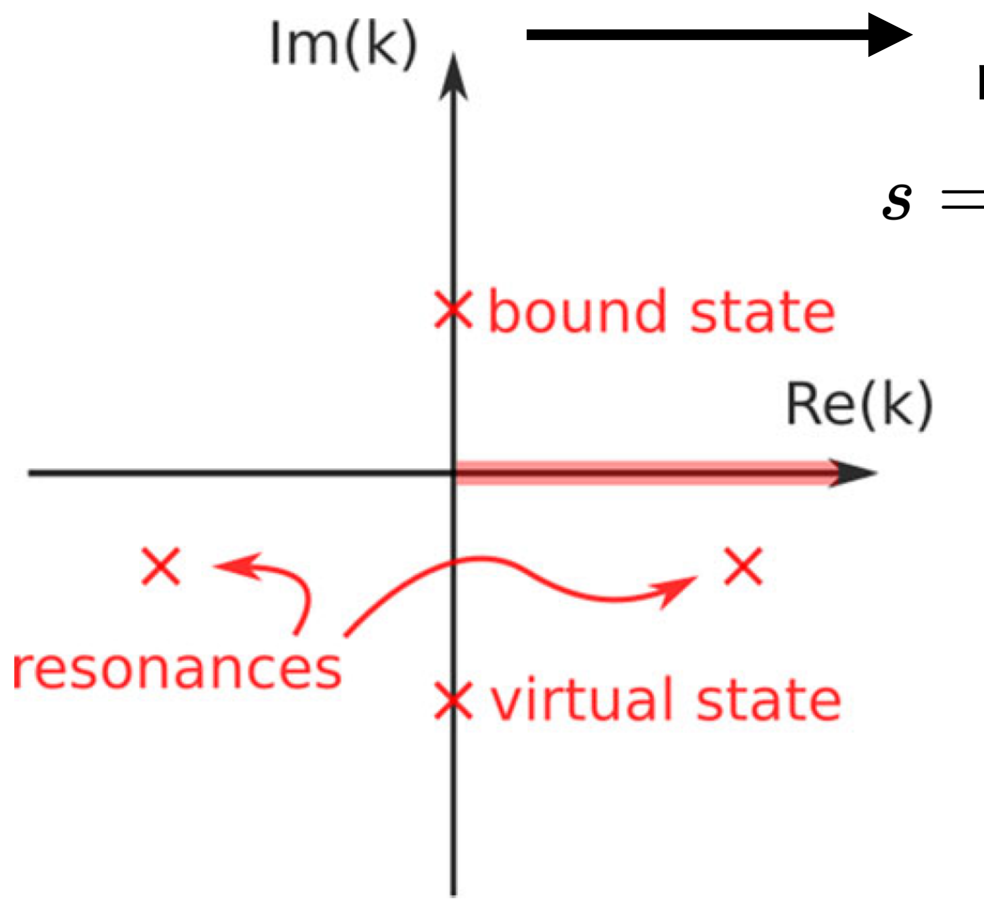
$M_R$: mass; $\Gamma$: width




- Based on the location of the poles, they receive different names
  - Bound states: stable particles, e.g. the deuteron is an NN bound state
  - Resonances: unstable hadrons, e.g. the rho resonance
  - Virtual states: "non-normalizable QM states", e.g. "dineutron"
    [See talk by Andre Walker-Loud]




Im(k) → scattering momentum

$s = 2\sqrt{M^2 + k^2}$

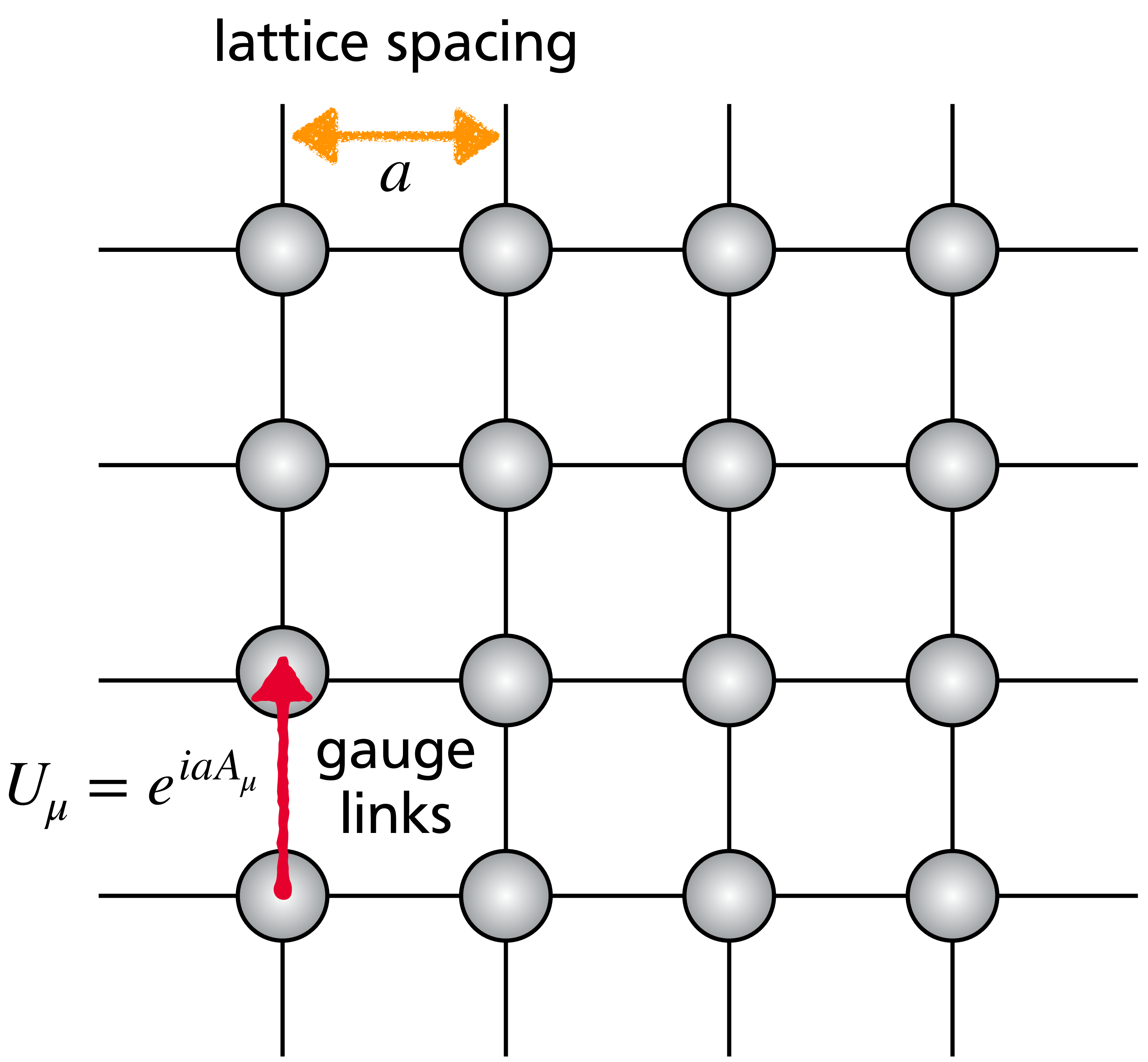
[Matuschek et al, EPJA 2021]

# Lattice QCD

- Lattice QCD is a first-principles numerical approach to the strong interaction

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle = \frac{1}{\mathcal{Z}}\int D\psi D\bar{\psi} DA\, \mathcal{O}(t)\mathcal{O}(0) e^{-S_E(\psi,\bar{\psi},A_\mu)}$$

Euclidean action



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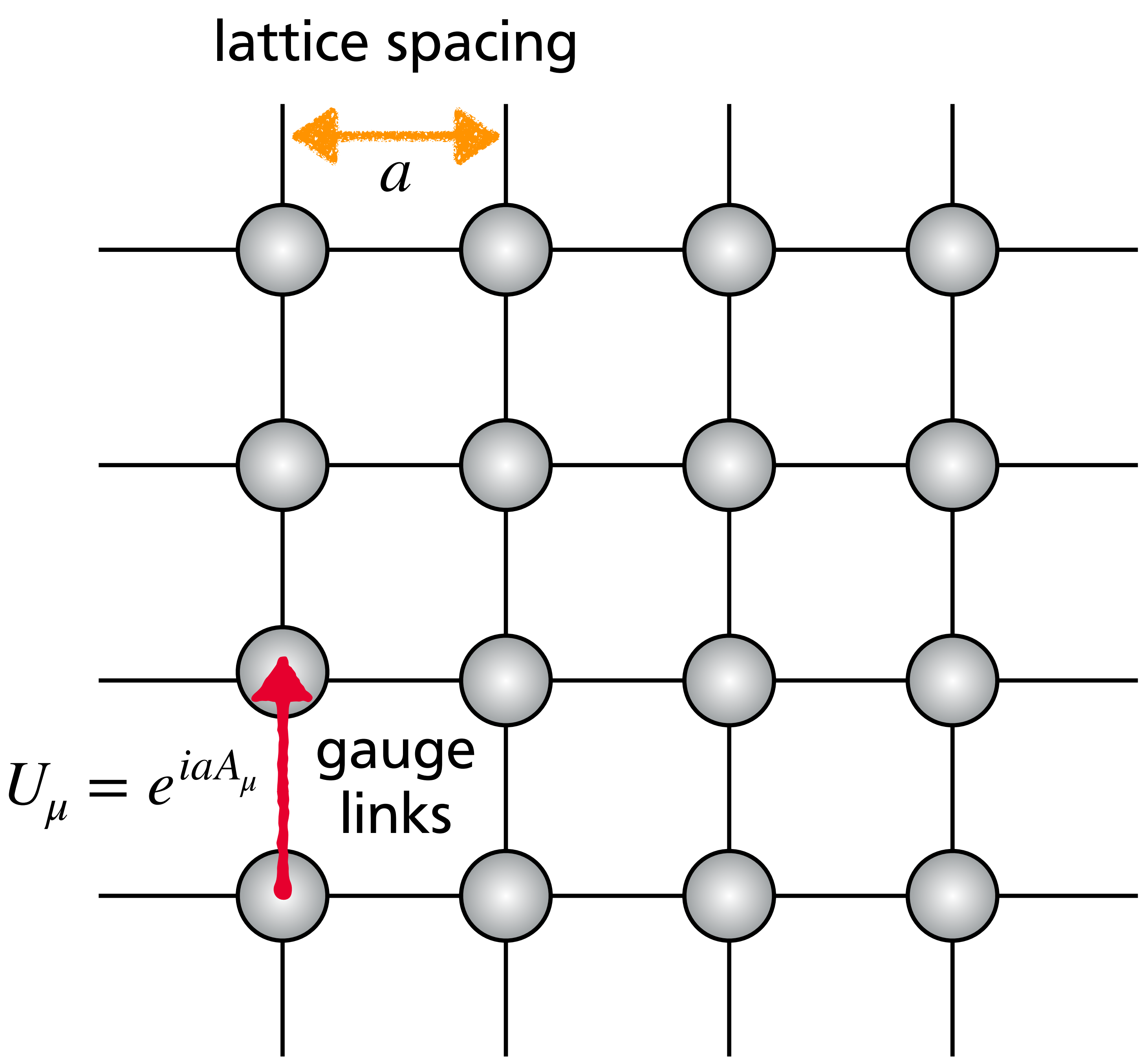

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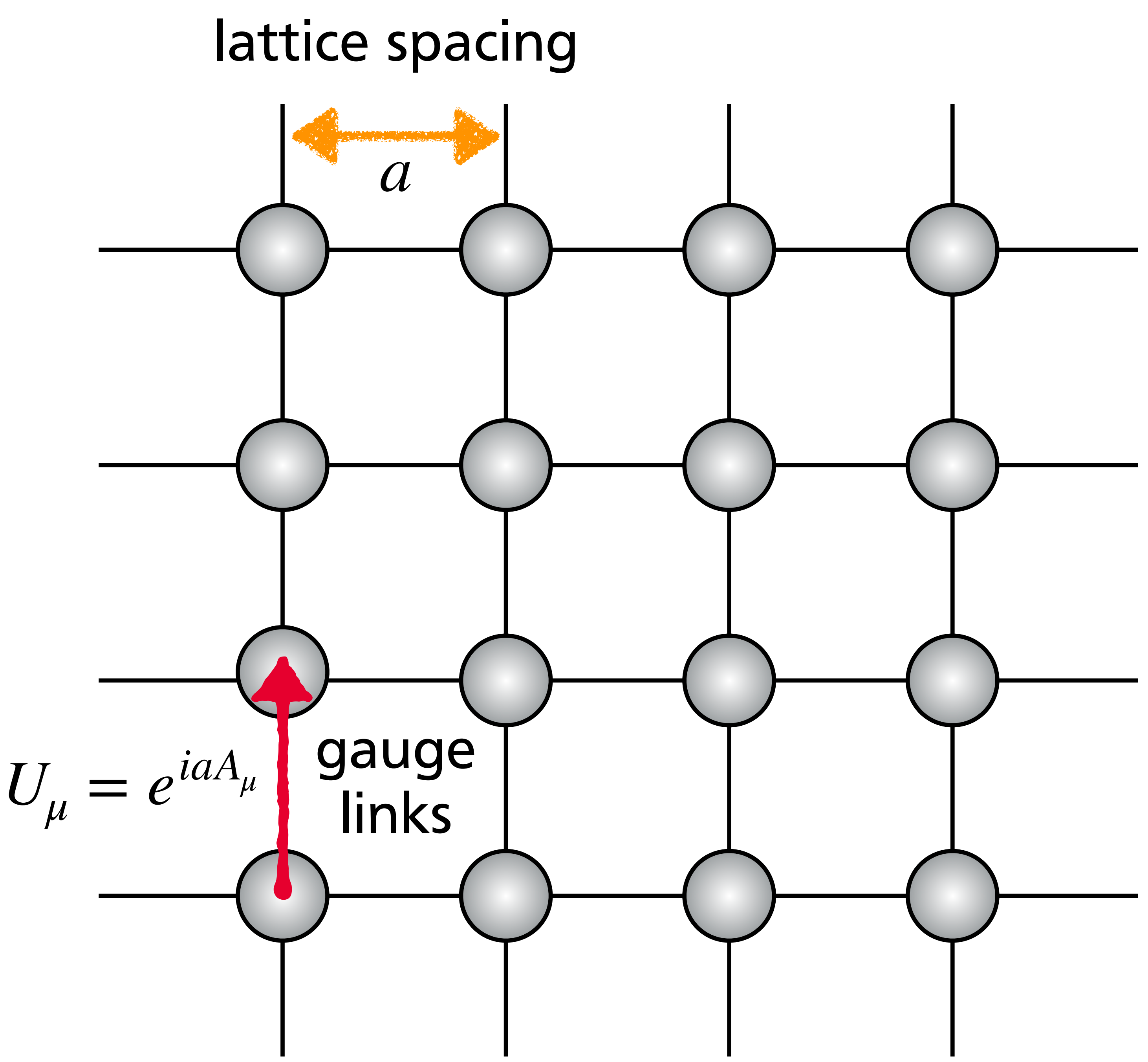


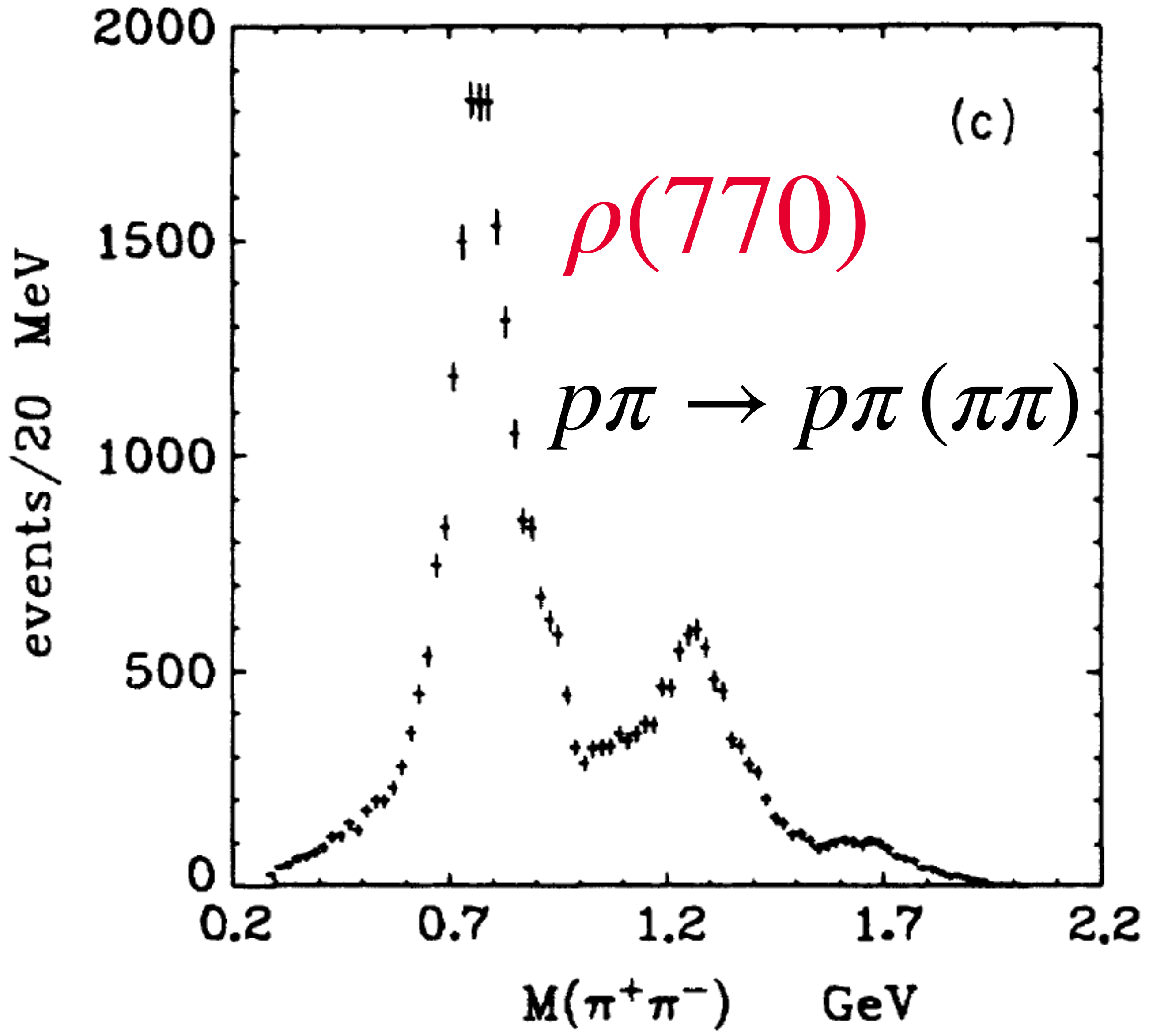

**Can we obtain scattering amplitudes and resonance properties from Euclidean correlation functions?**

→ **Yes, but not that simple!**

# Infinite vs finite volume

## Experiments

- Asymptotic states
- Direct access to scattering amplitudes




[Protopopescu et al, PRD7 1973]

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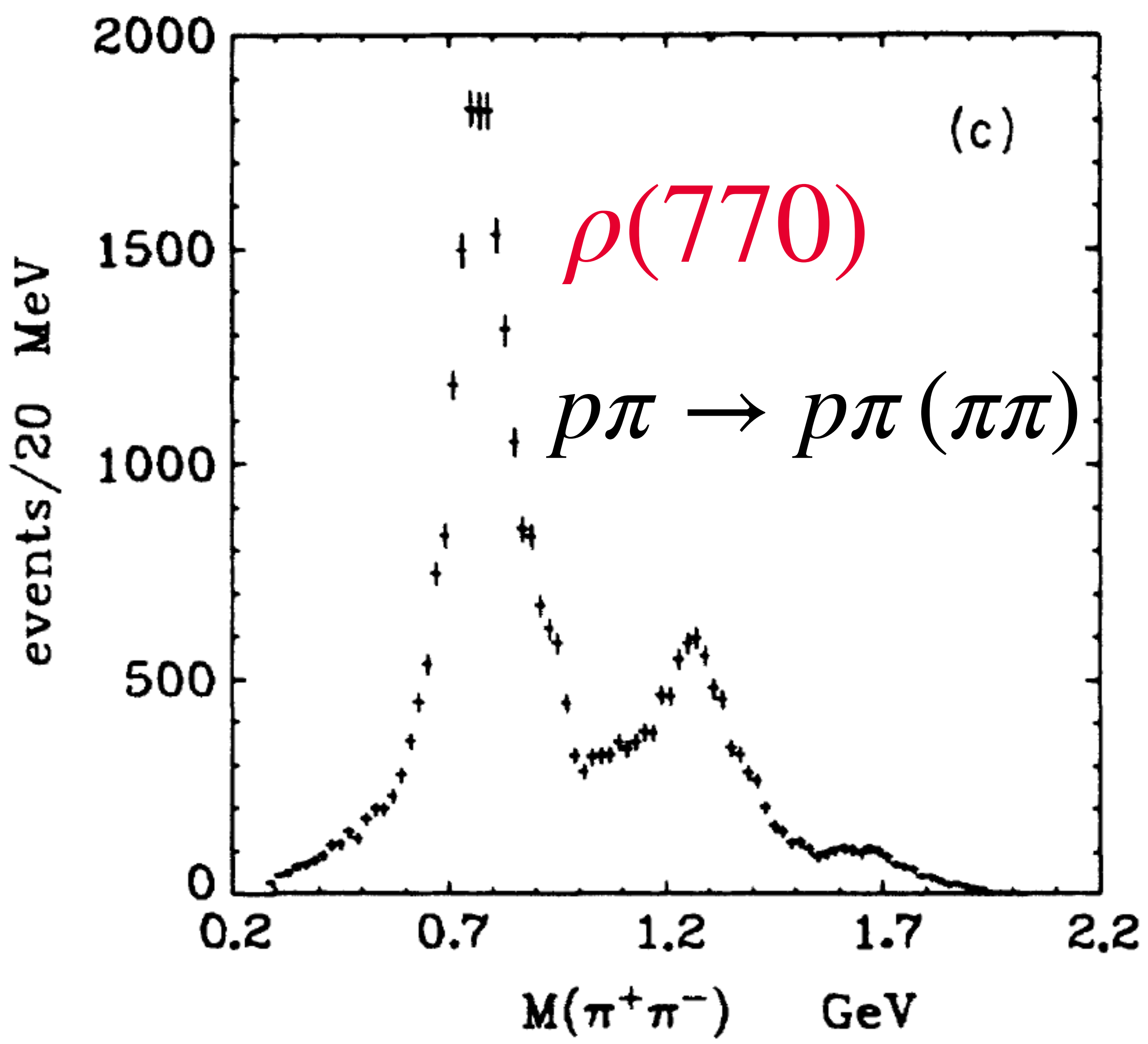


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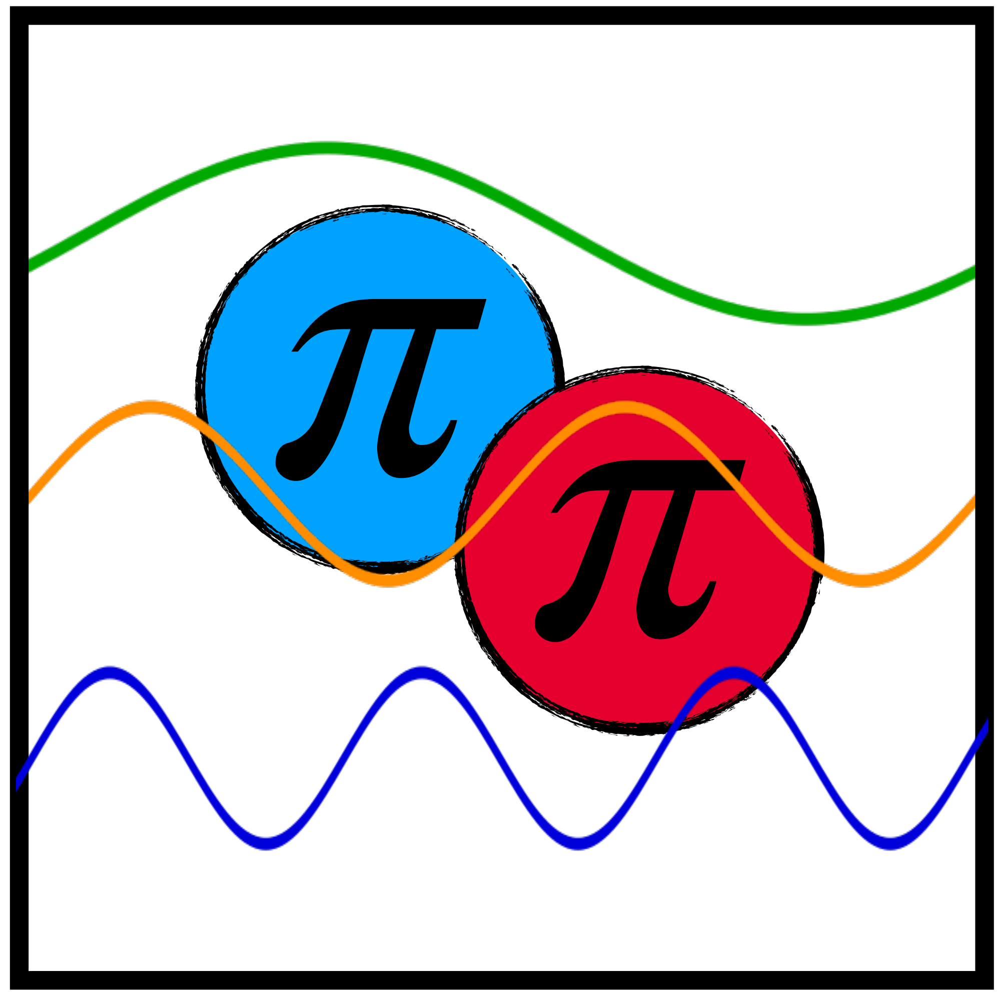
## Lattice QCD

- Euclidean time
- Stationary states in a box



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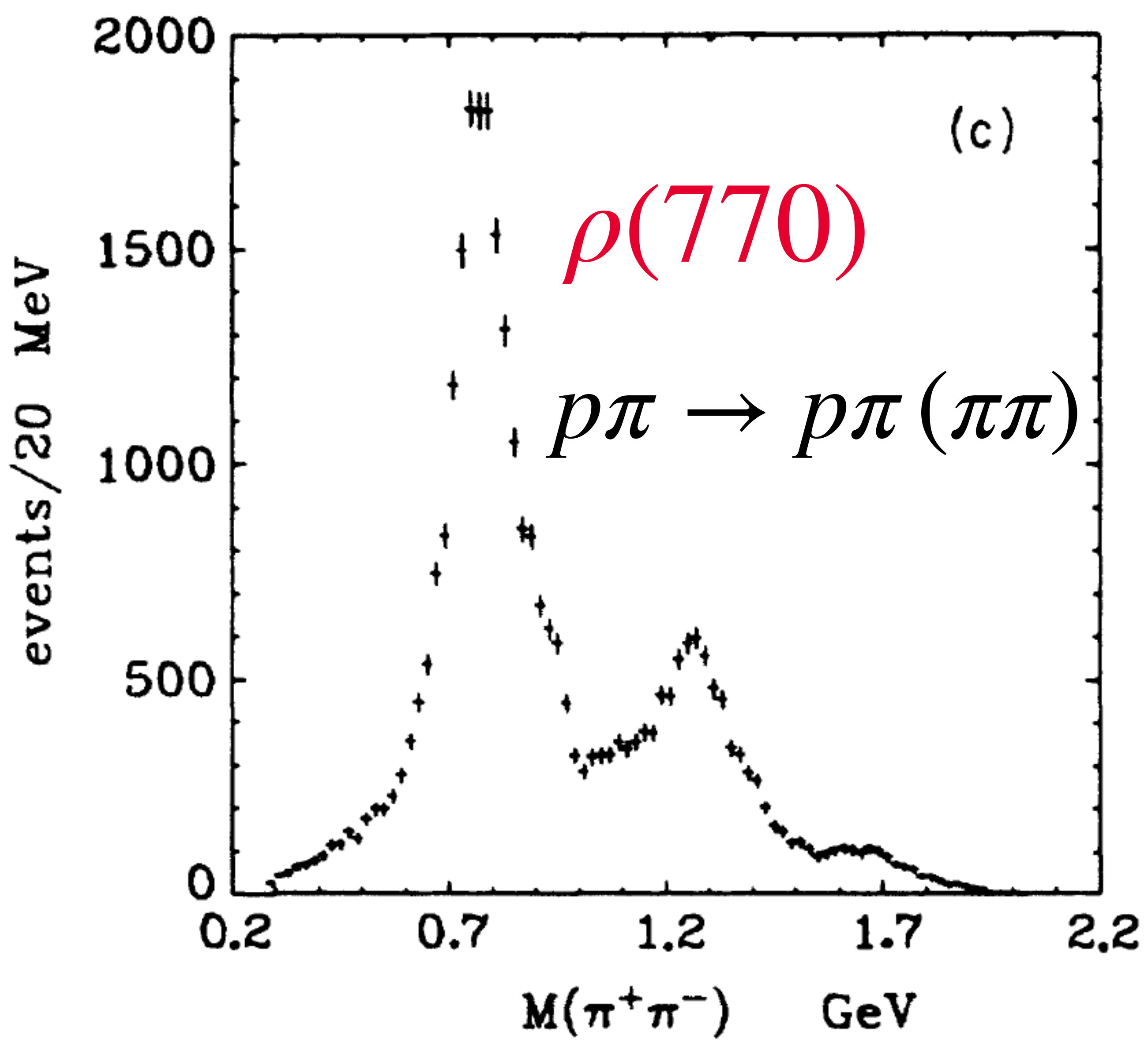


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Finite-volume formalism

[Lüscher, 89']

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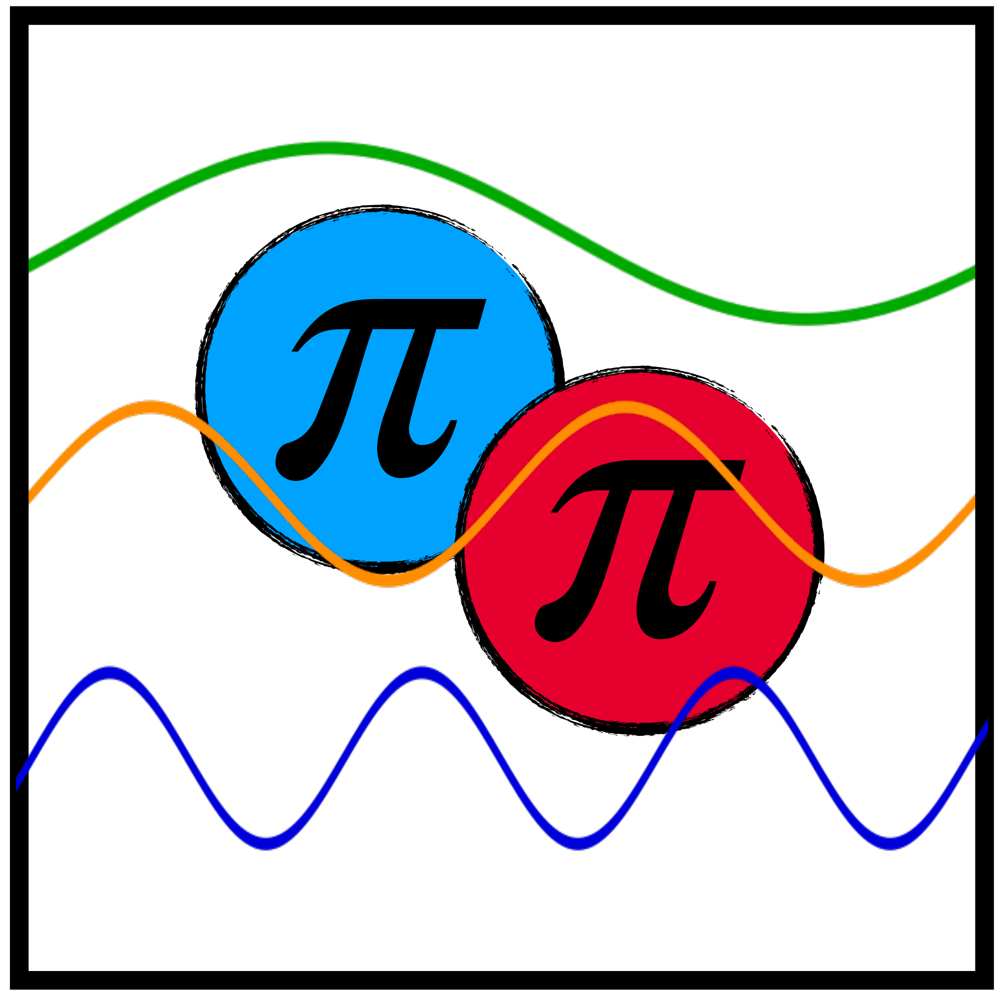

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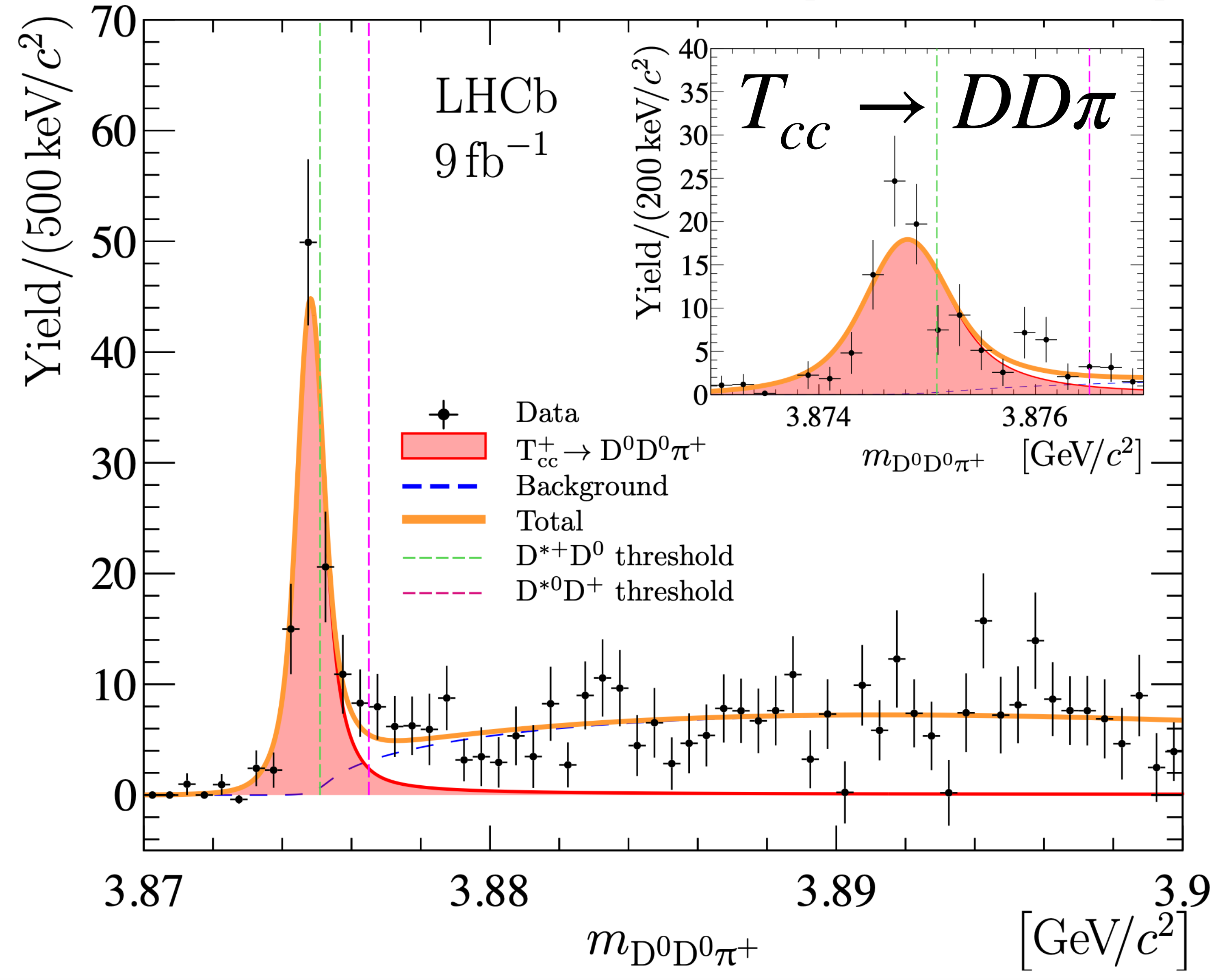


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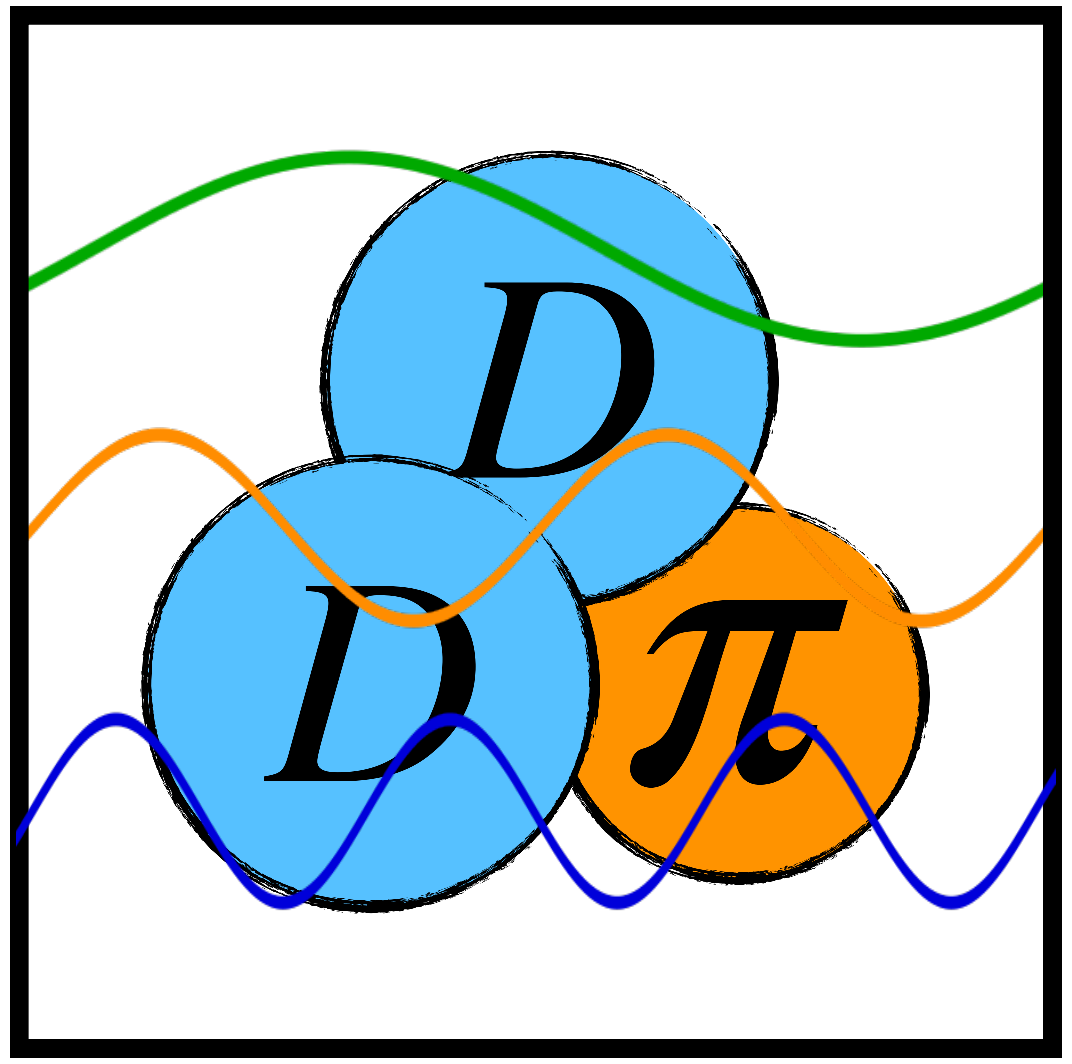
Need to include 3-body effects!

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# Towards the QCD S-Matrix

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**Lattice QCD** → **QCD S-matrix**

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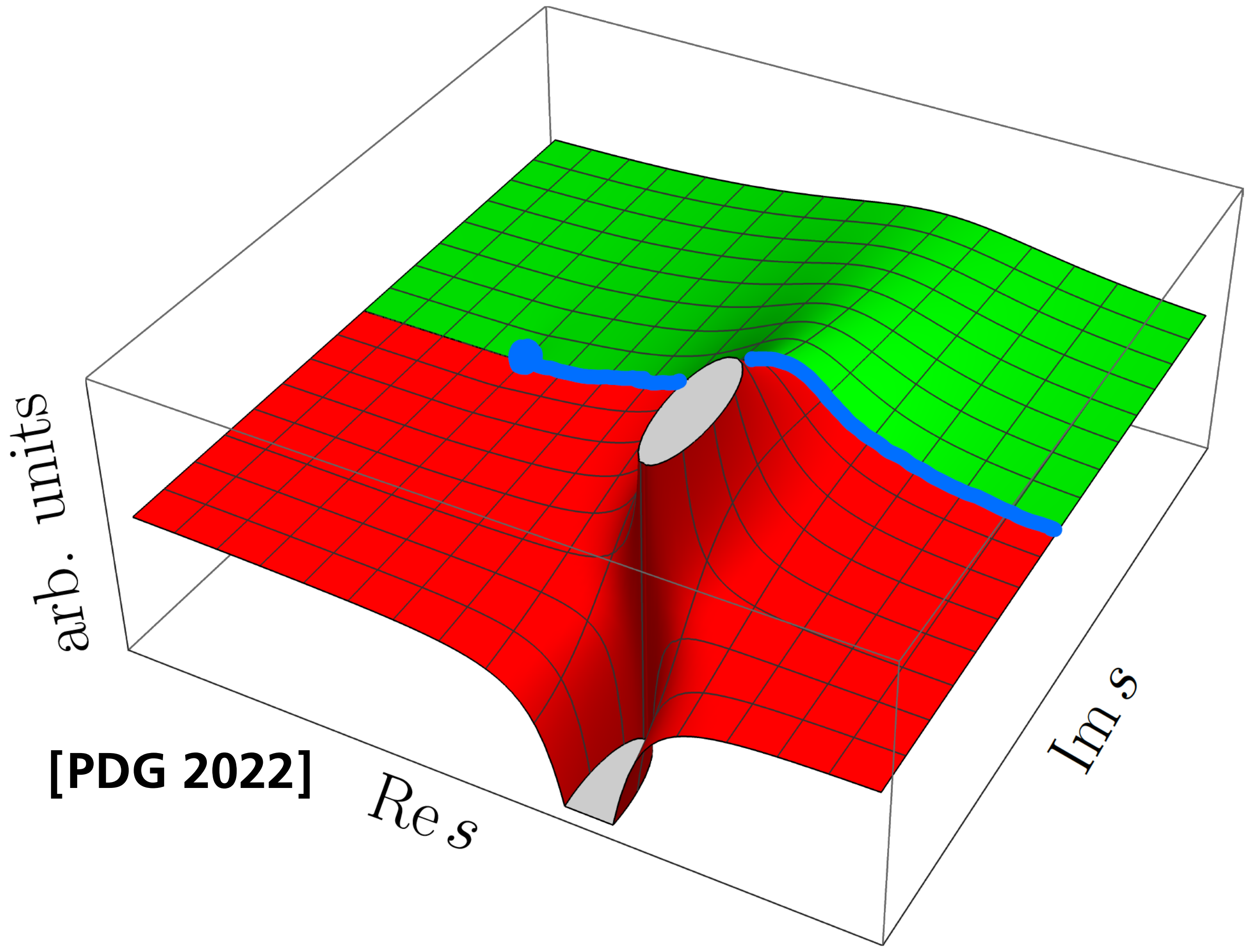


[PDG 2022]

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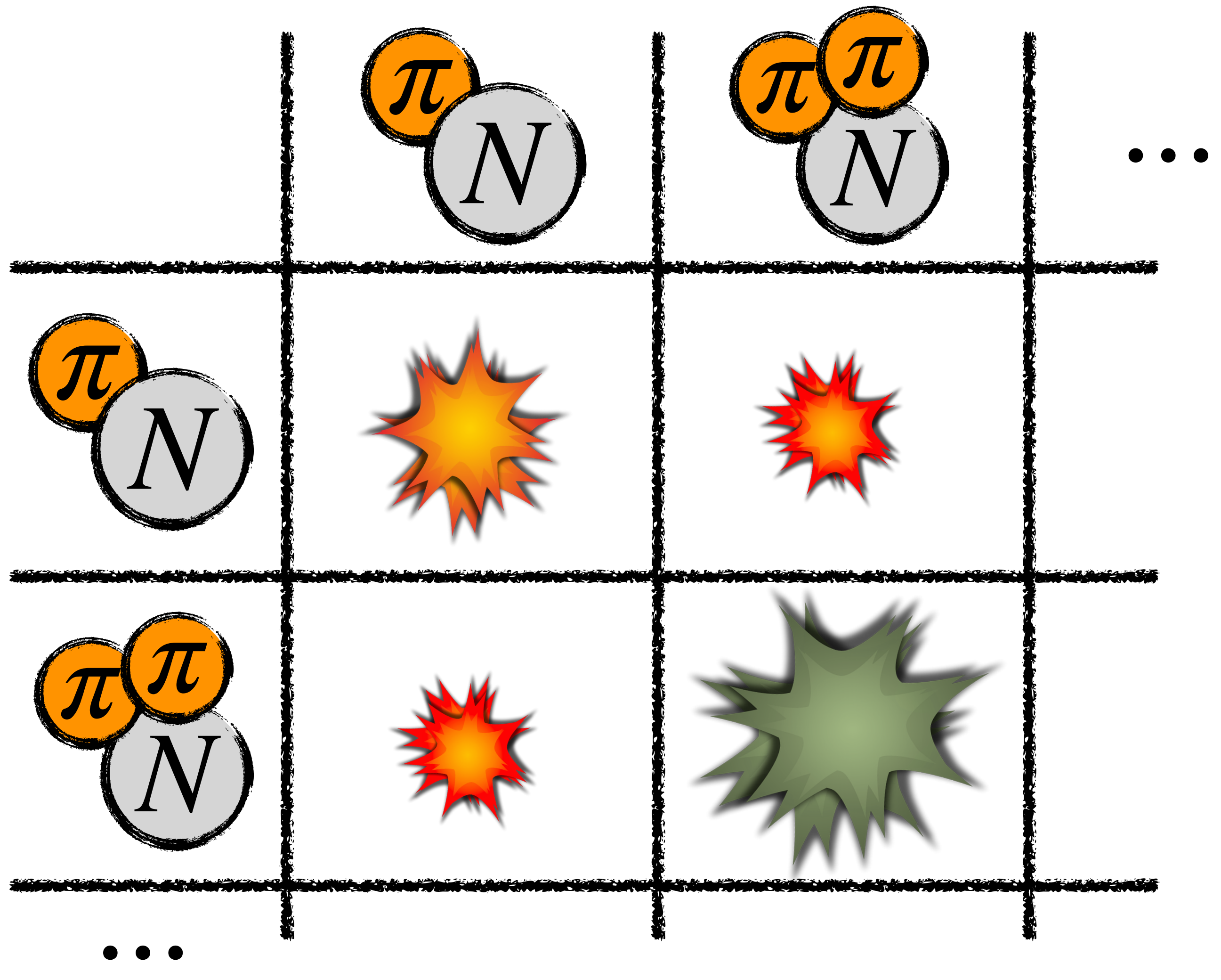


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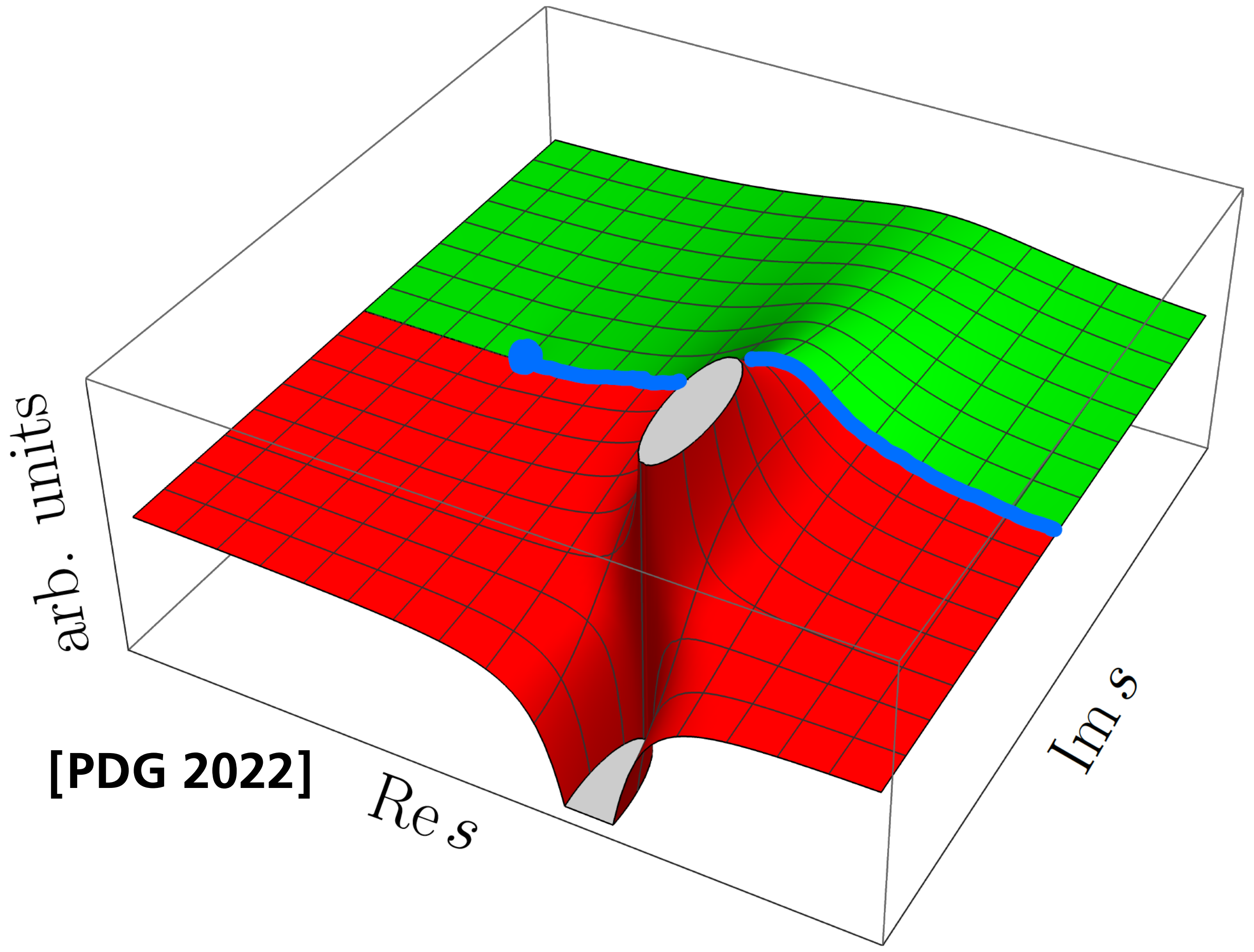


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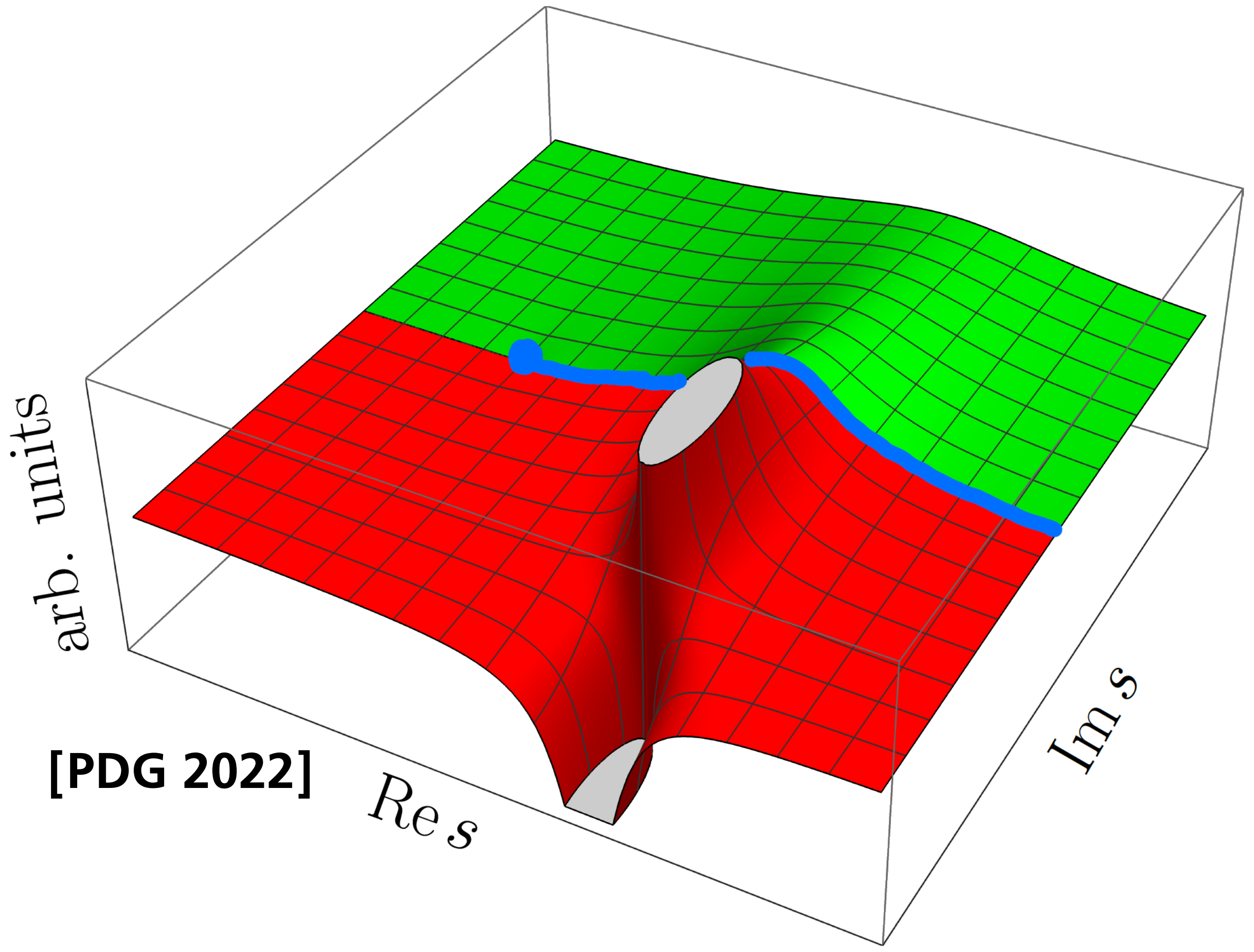


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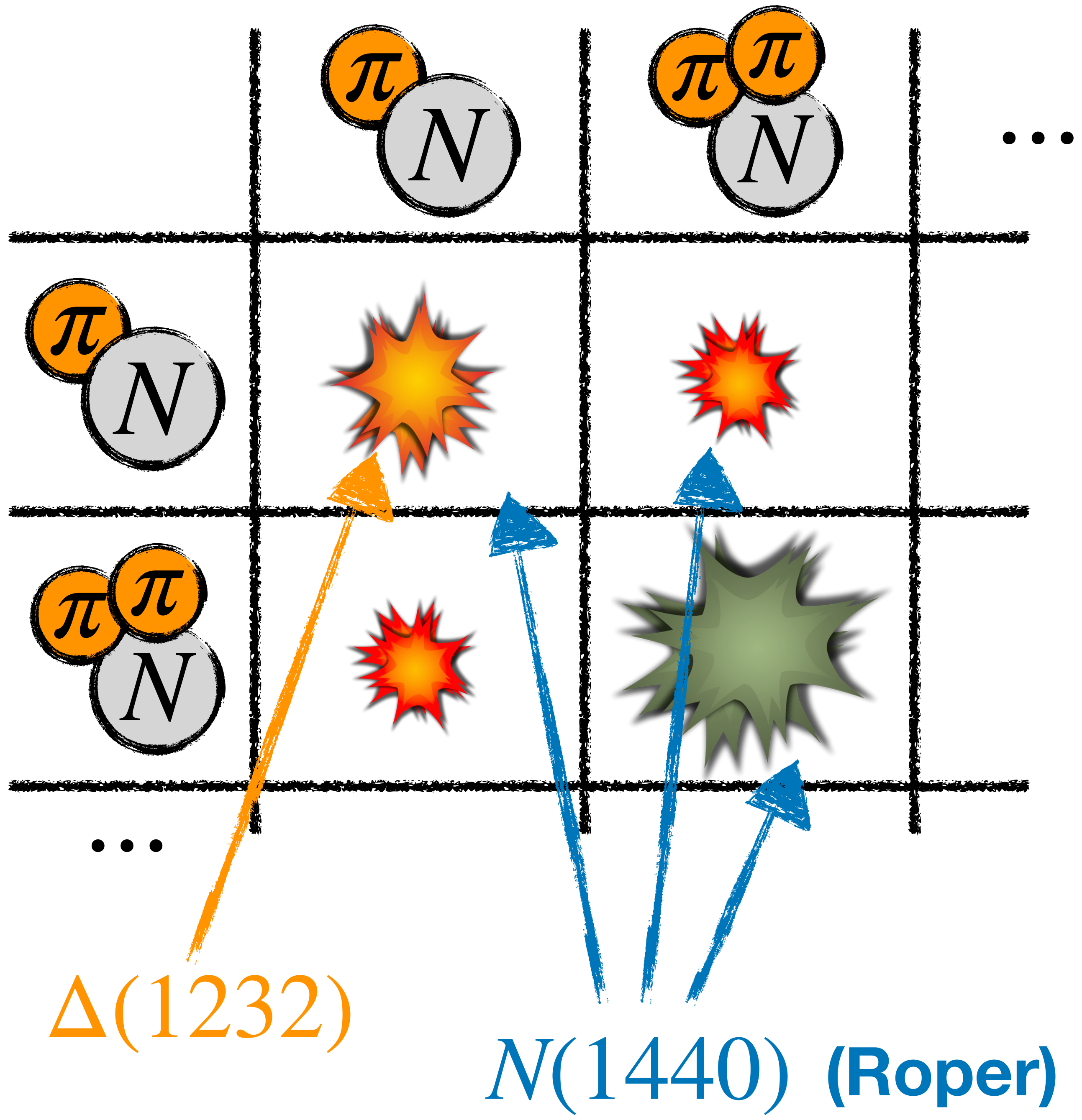

# Outline

1) Scattering amplitudes from lattice QCD

2) Applications to meson-baryon scattering

3) The three-hadron frontier from lattice QCD

4) Towards a three-body description of the $T_{cc}$

# Scattering amplitudes from Lattice QCD

# The general strategy

**Spectrum**

$E_4$
$E_3$
$E_2$
$E_1$
$E_0$

**From Euclidean**

**Correlation functions**

**Quantization condition**

$$\det_{\ell m}\left[F^{-1} + \mathcal{K}\right] = 0$$

**A bridge between:**

finite volume ⟶ infinite volume

**Infinite-volume scattering quantities**

| K matrix | Phase shift | Amplitude |
|---|---|---|
| $\mathcal{K}$ | $\delta_\ell$ | $\mathcal{M}$ |

**Resonance poles**

$$\mathcal{M}^{-1}(E_{\text{pole}}) = 0$$

# Energy levels from lattice QCD

- The energy levels of the theory are measured from Euclidean correlation functions

$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \sum_n \left|\langle 0|\mathcal{O}(0)|n\rangle\right|^2 e^{-E_n t}$$

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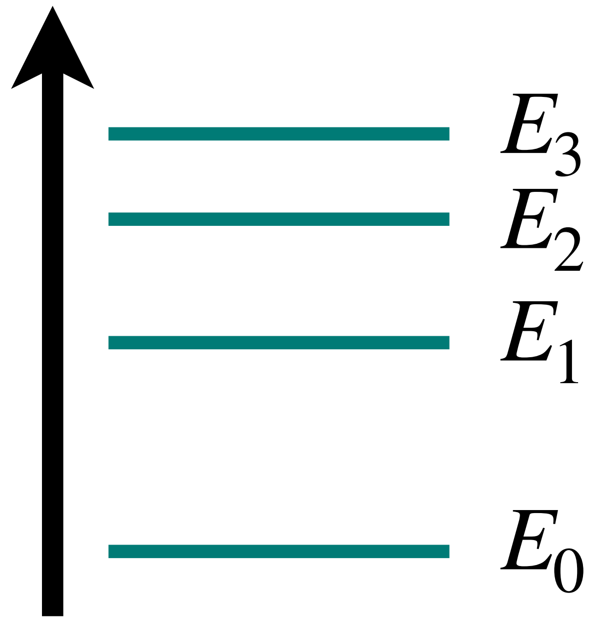
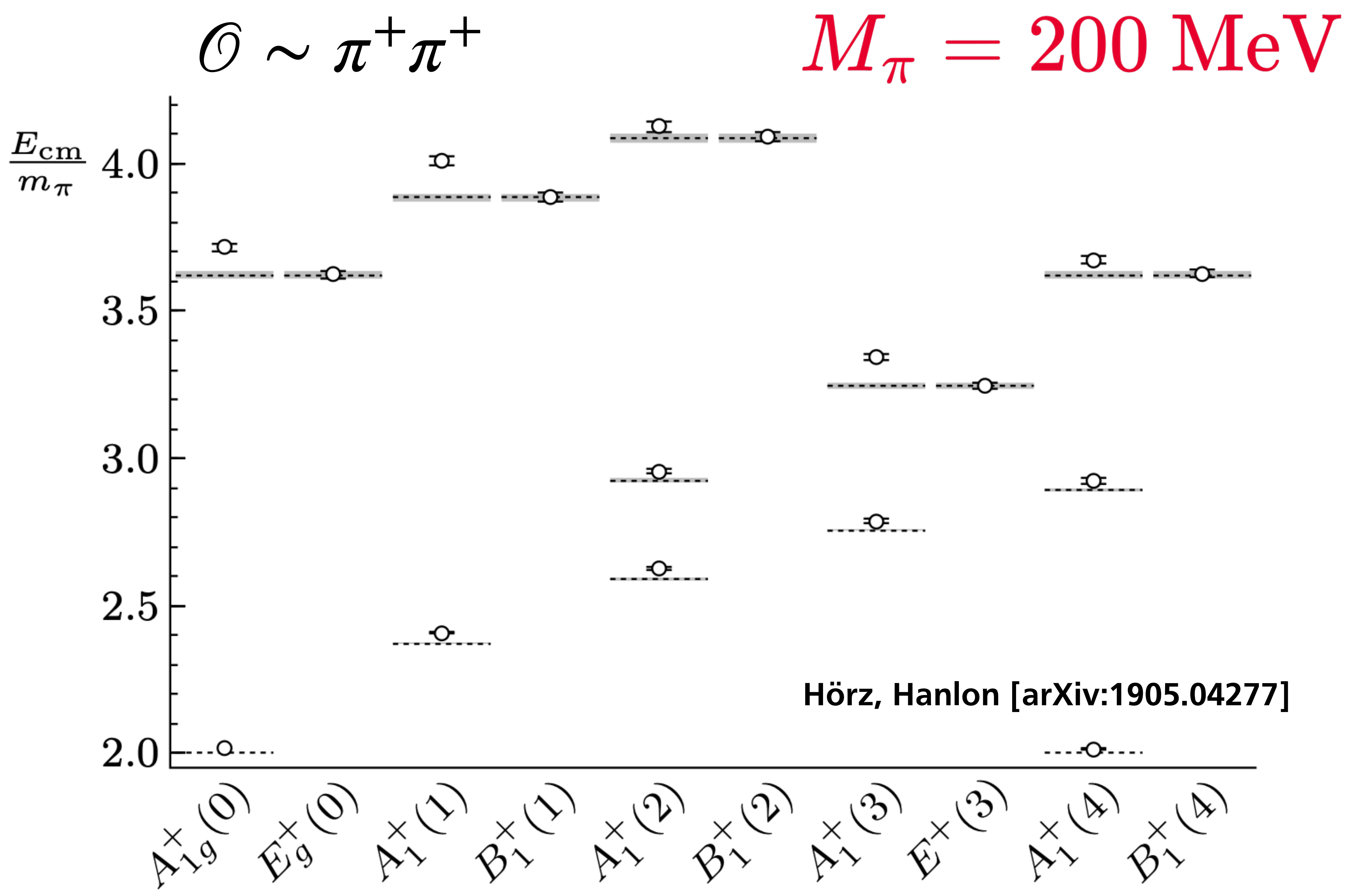
- Multiple operators with the same quantum names to obtain several energy levels

  - **Variational techniques**
    **(Generalized EigenValue Problem, GEVP)**

$$C_{ij}(t) = \langle \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)\rangle$$

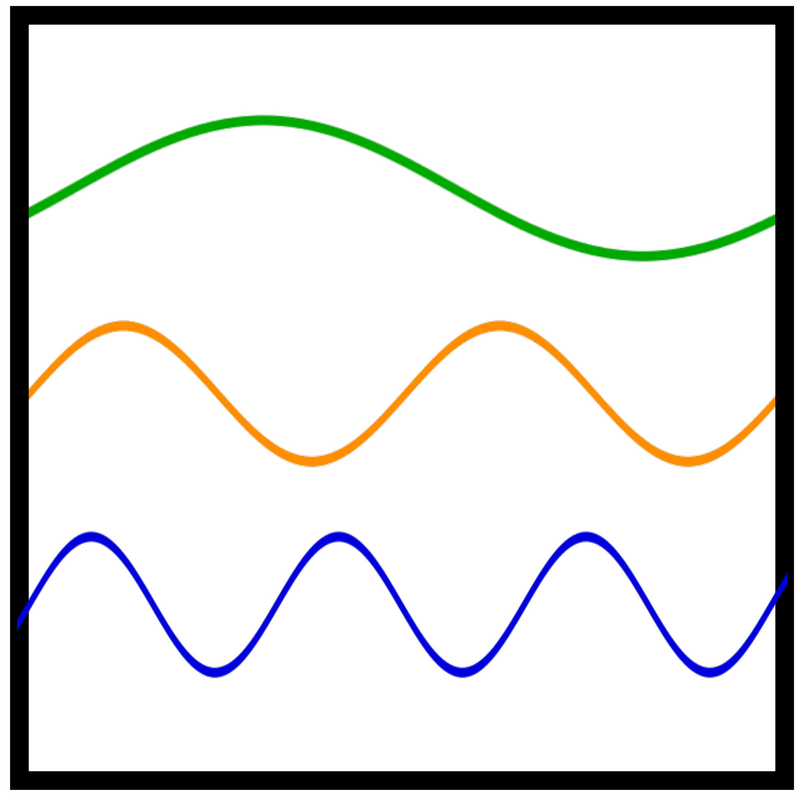
$$C_{ij}(t) = \langle 0|\mathcal{O}_i|n\rangle\langle n|\mathcal{O}_j|0\rangle^* e^{-E_n t}$$



# Finite-Volume Spectrum

**Free scalar particles in finite volume with periodic boundaries**



$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

**Two particles:** $$E = 2\sqrt{m^2 + \frac{4\pi^2}{L^2}\vec{n}^2}$$

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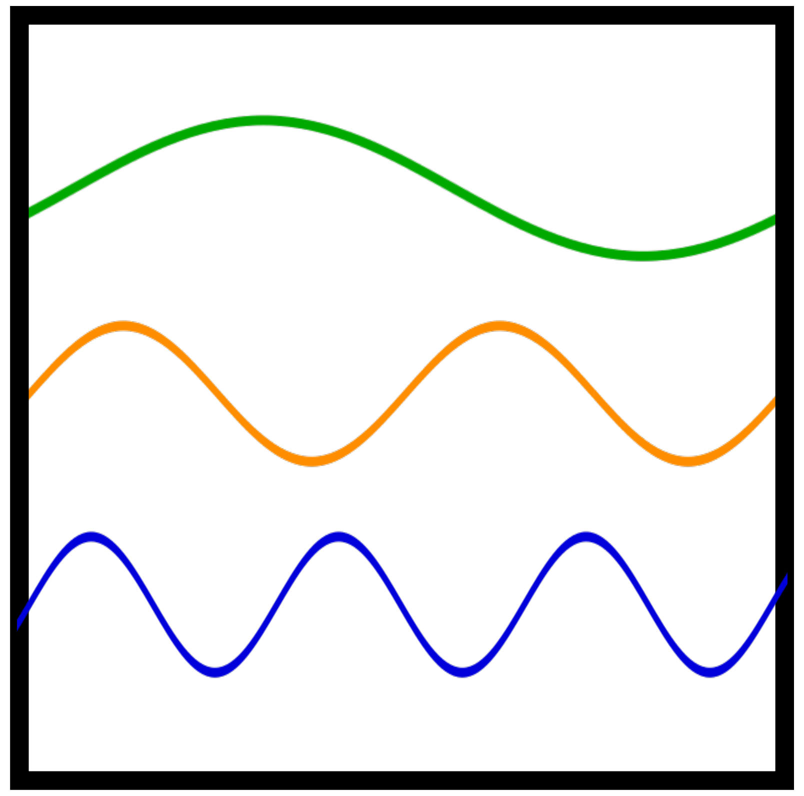

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→

**Interactions change the spectrum: it can be treated as a perturbation**

**Ground state to leading order**

$$E_2 - 2m = \langle \phi(\vec{0})\phi(\vec{0}) \,|\, \mathbf{H_I} \,|\, \phi(\vec{0})\phi(\vec{0}) \rangle$$

$$\Delta E_2 = \frac{\mathcal{M}_2(E=2m)}{8m^2L^3} + O(L^{-4})$$

[Huang, Yang, 1958]

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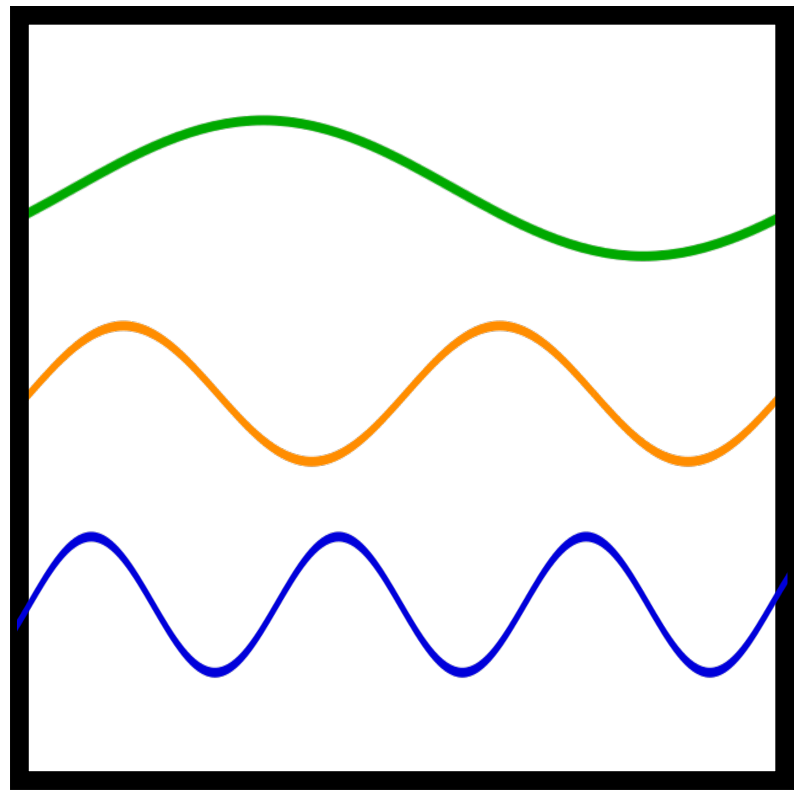

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# Finite-Volume Spectrum

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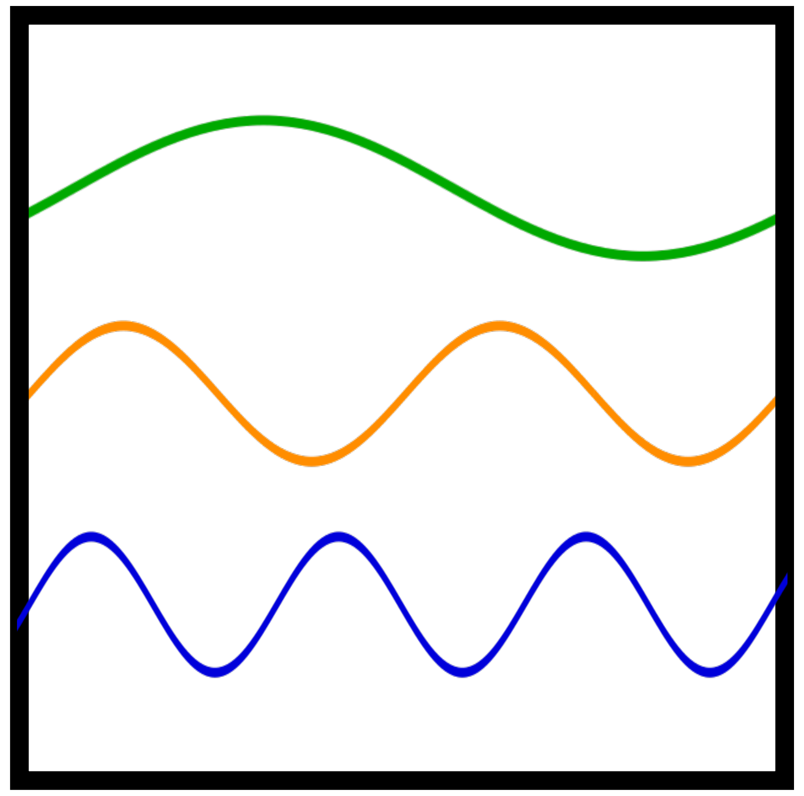

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In general a problem of
Quantum Field Theory
in finite volume

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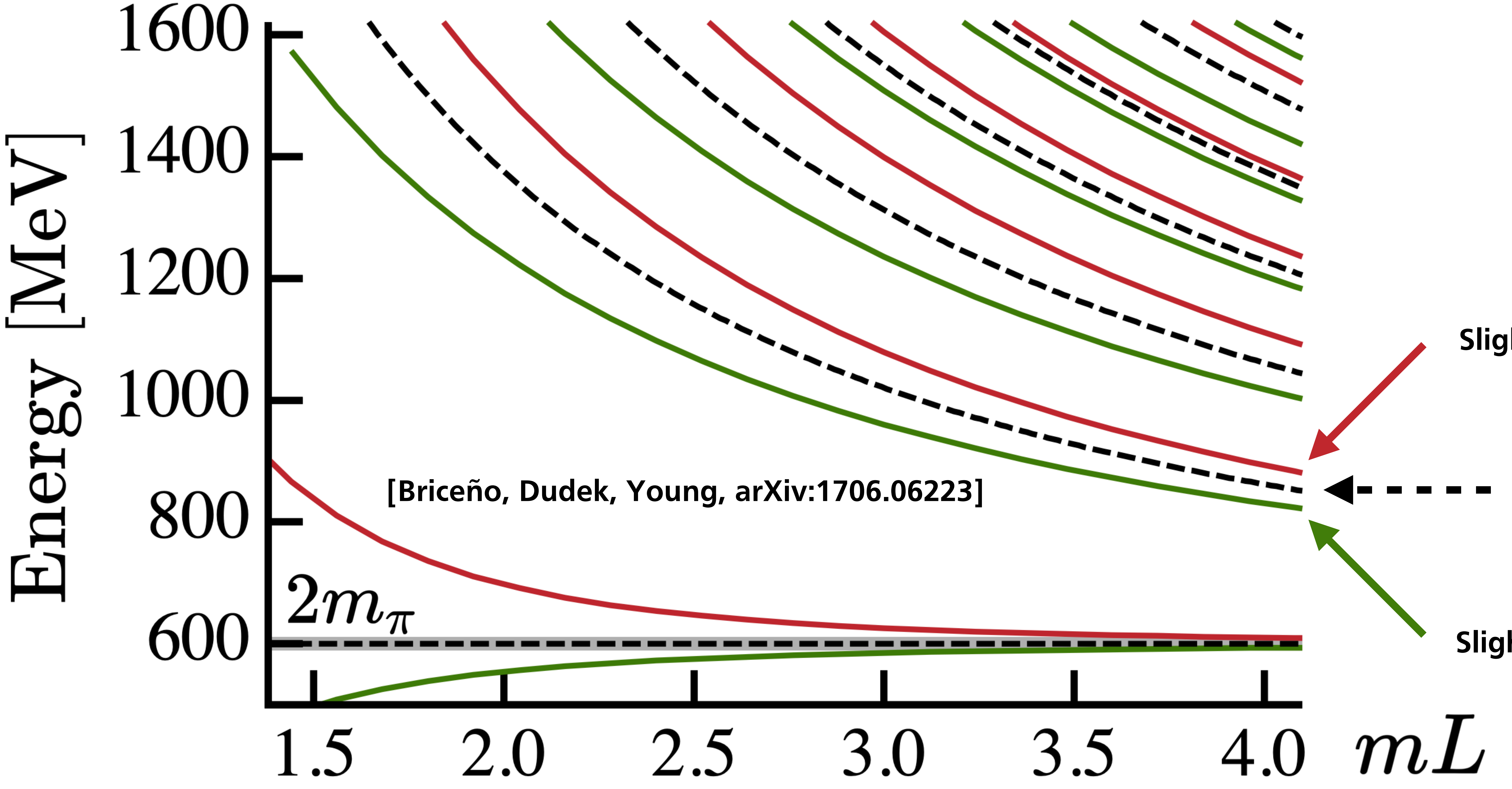
# Key insight

**Volume dependence of finite-volume energy states contains scattering information**

[Lüscher 89']




$$\vec{p} = \frac{2\pi}{L}\vec{n}$$

# Finite-Volume formalism

- Indirect connection between the spectrum and the two-particle scattering amplitude

[Lüscher 89′]

**Two-particle Quantization Condition**

$$\det_{\ell m}\left[\mathcal{K}_2(E) + F^{-1}(E,\vec{P},L)\right]_{E=E_n} = 0$$

Scattering K-Matrix

Known kinematic function

"QC2"

**K-matrix parametrized in terms of phase shift**

$$\mathcal{K}_2^{\ell} = \frac{16\pi\sqrt{s}}{q^{2\ell+1}\cot\delta_\ell}$$

**Finite-volume information**

$$F_{00}\left(q^2\right) \sim \left[\frac{1}{L^3}\sum_{\vec{k}} - \int\frac{d^3k}{(2\pi)^3}\right]\frac{1}{k^2-q^2}$$

**!** Note: only valid for two particles below inelastic thresholds.

# Example: I=0 $\pi\pi$ scattering

**Two pions in s-wave**

$$\mathcal{K}_2^{s-\text{wave}}(E_n) = \frac{-1}{F_{00}(E_n, \vec{P}, L)}$$

$$\mathcal{K}_2^{s-\text{wave}} = \frac{16\pi\sqrt{s}}{q\cot\delta_0}$$

one energy level → a phase shift point

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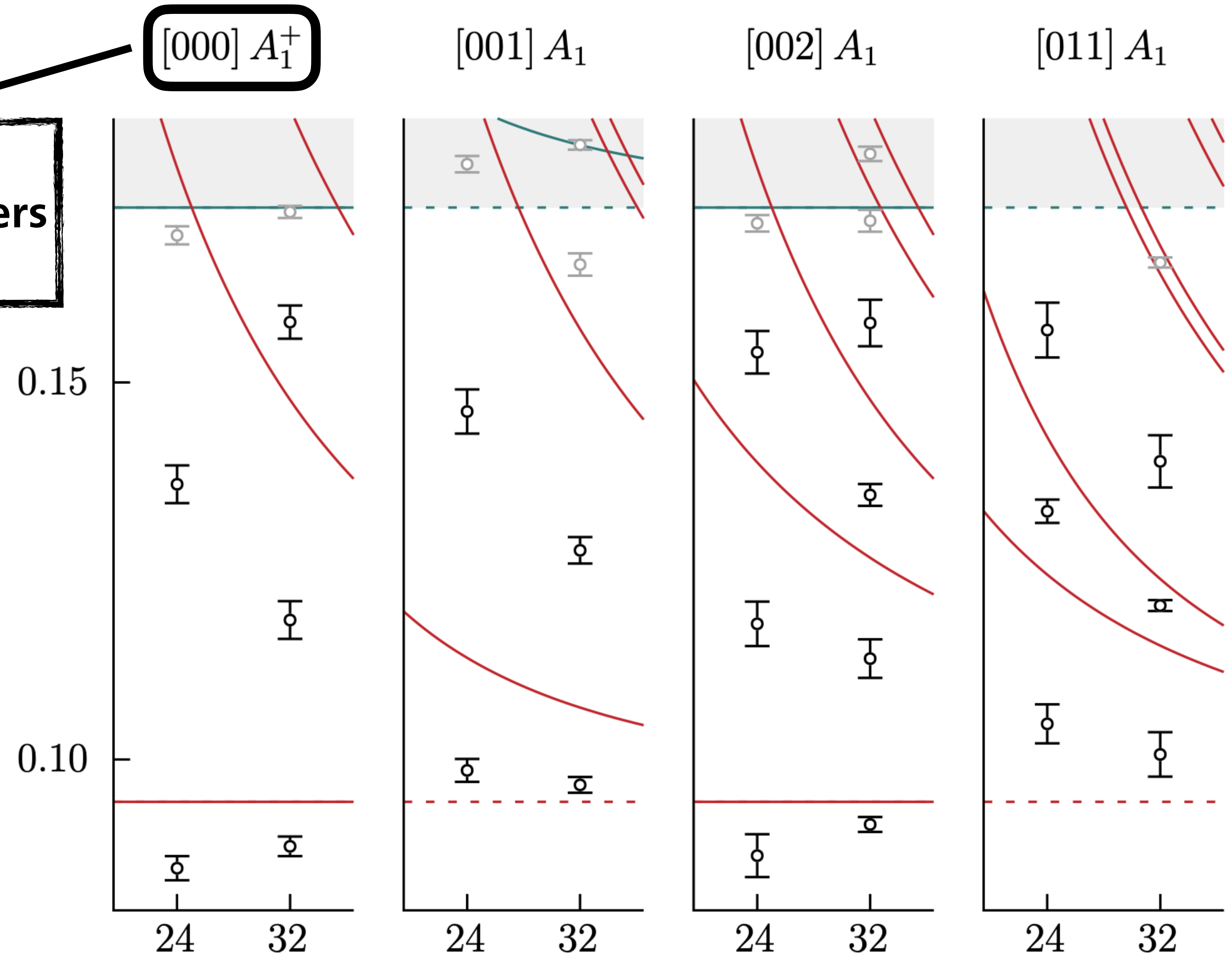


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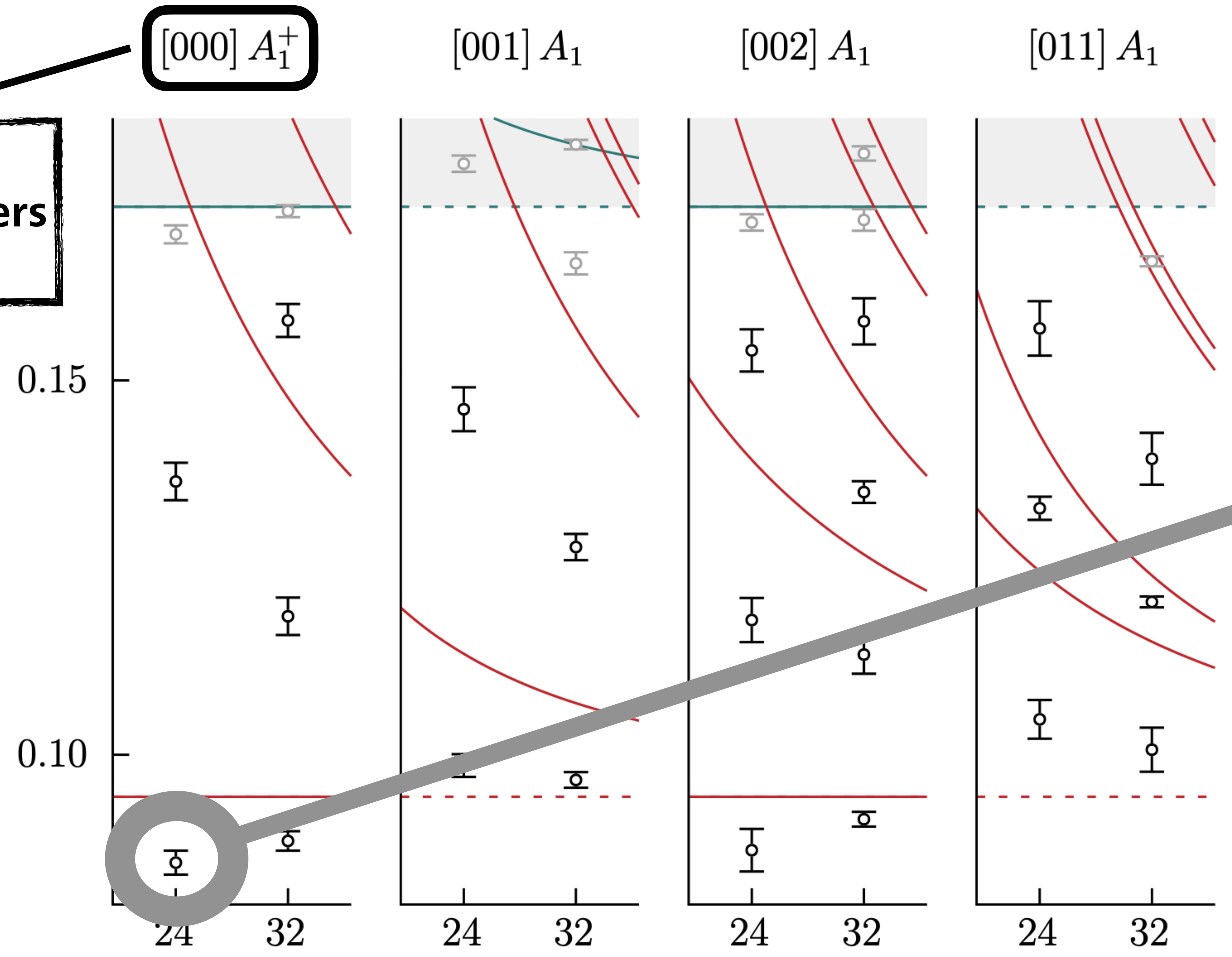


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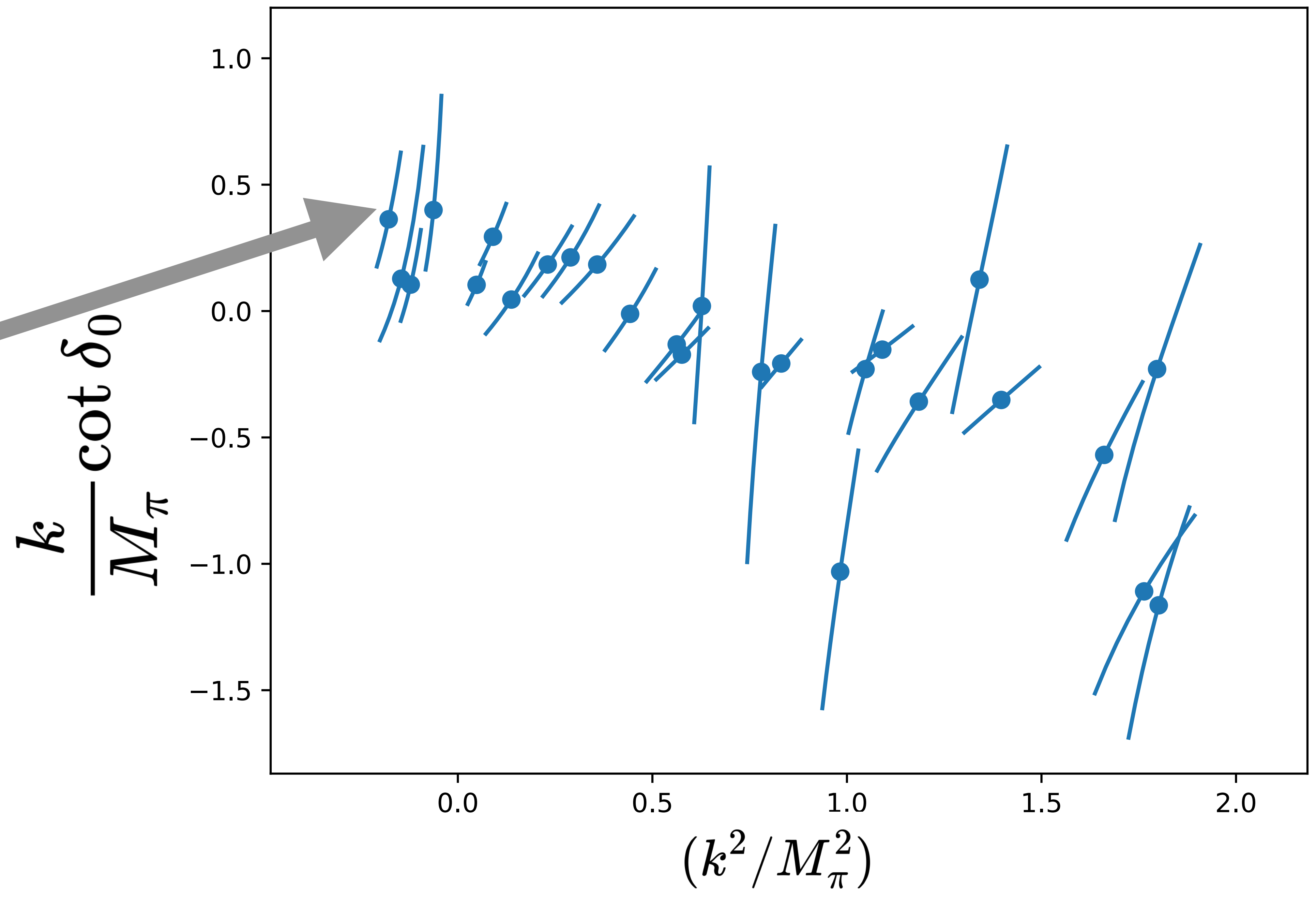


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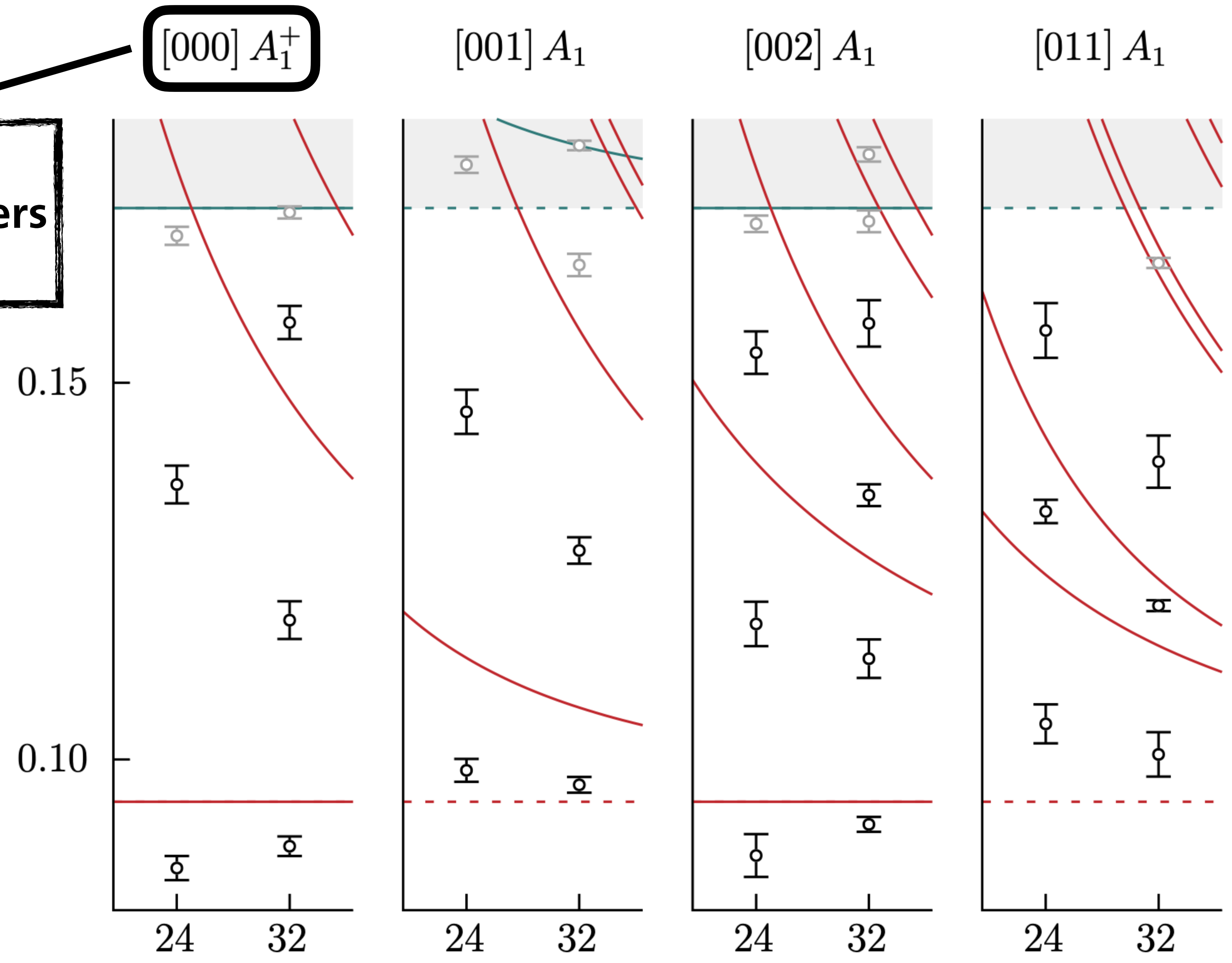


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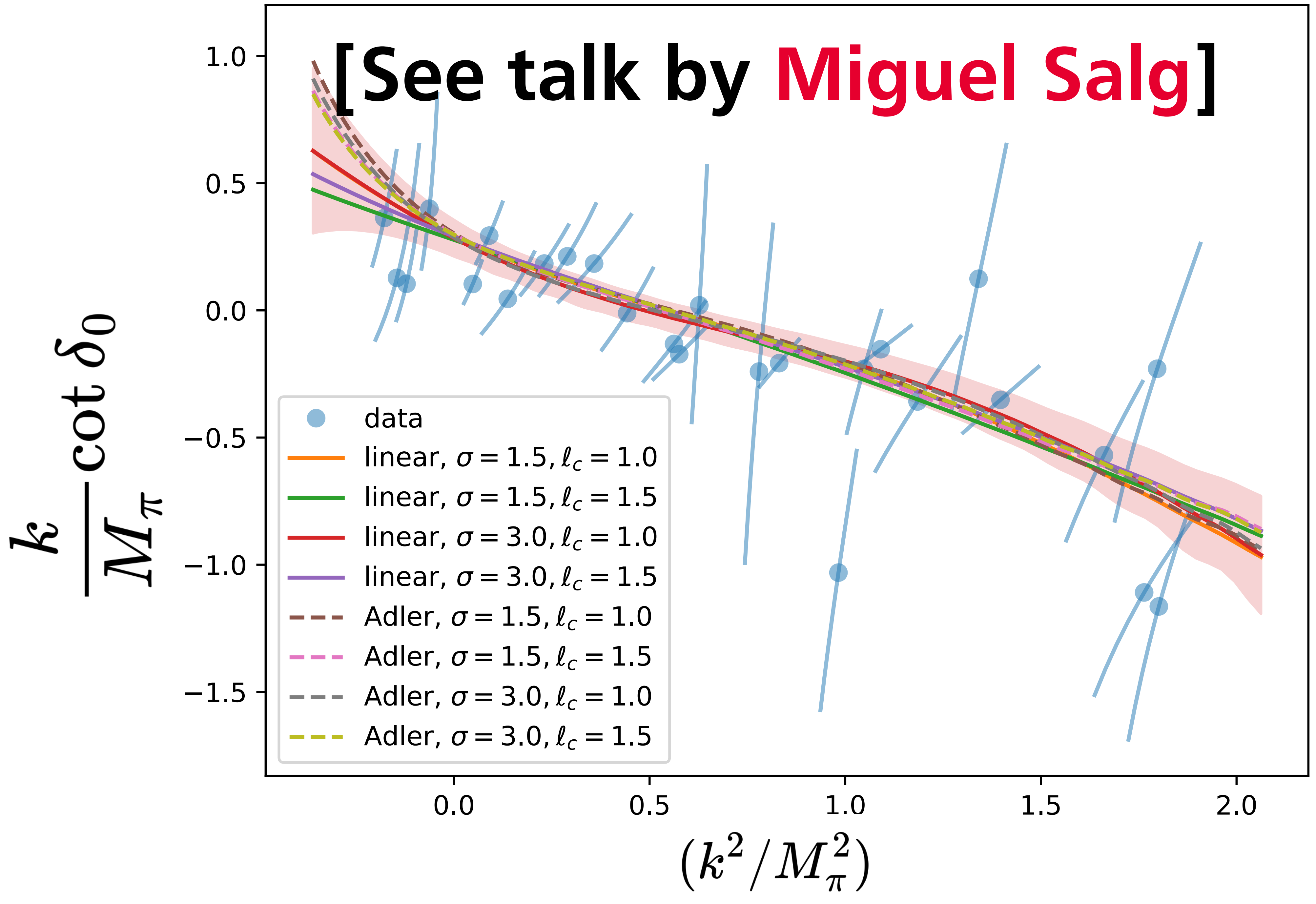
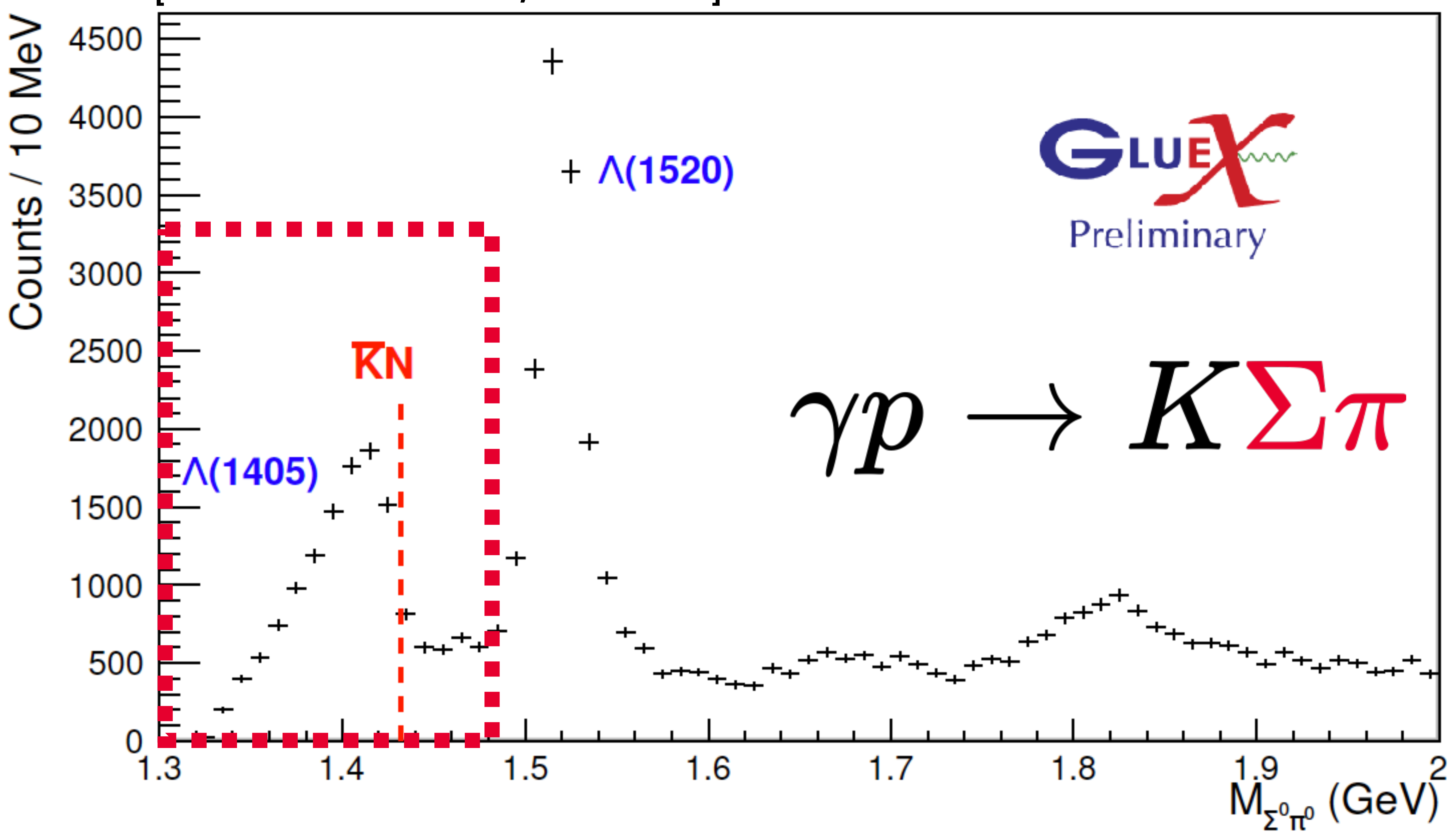


# Coupled Σπ-KN scattering
# &
# the Λ(1405) resonance

# The Λ(1405) resonance

- **Known since 1950s, still under investigation**
  **[Dalitz, Tuan PRL 1959]**

**[Wickramaarachchi et al, 2209.06230]**




- **Appears in coupled-channel scattering**

$$\begin{pmatrix} \pi\Sigma \to \pi\Sigma & \pi\Sigma \to Kp \\ Kp \to \pi\Sigma & Kp \to Kp \end{pmatrix}$$

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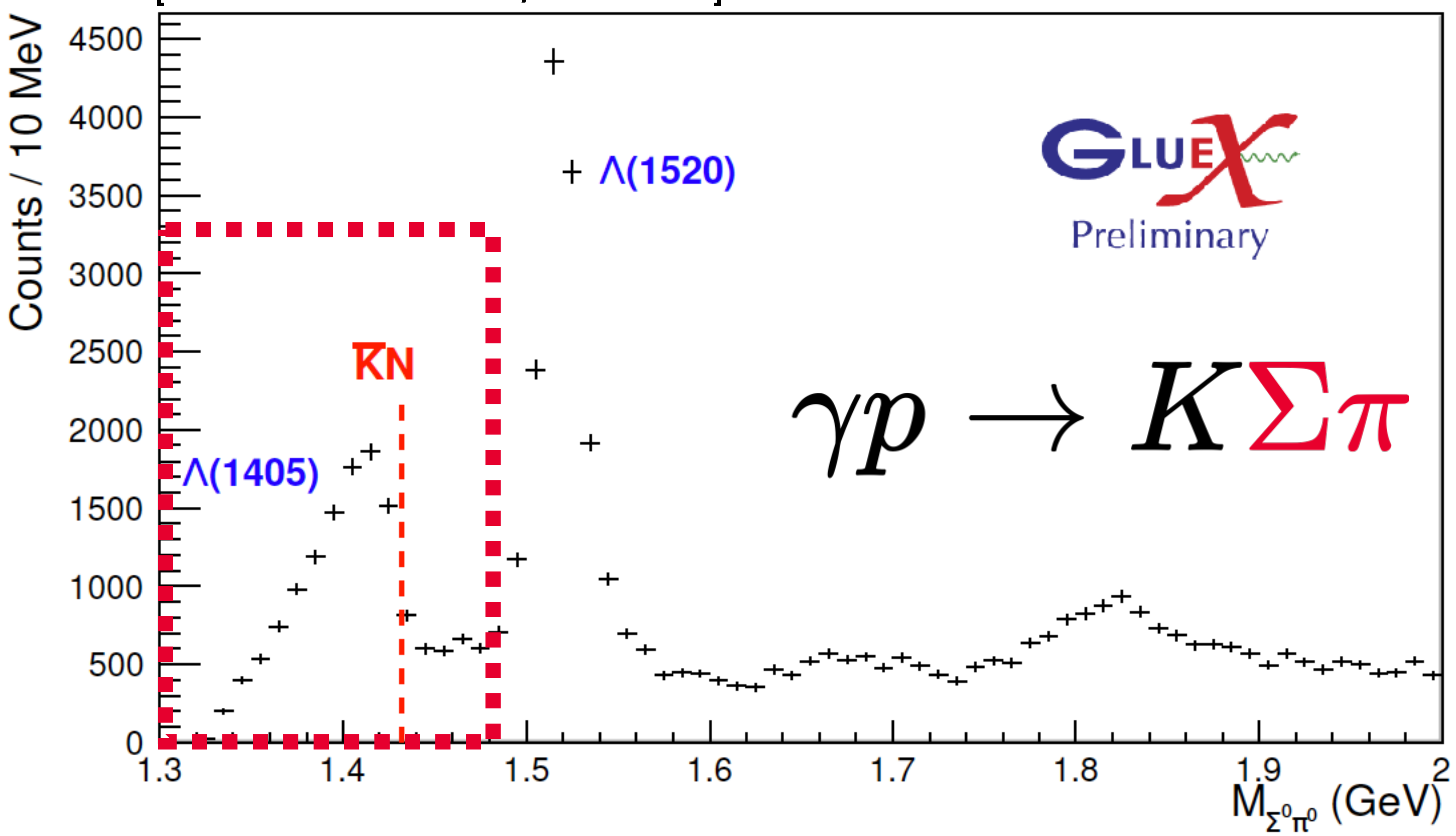


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- **Latest PDG lists two resonances in the energy region**

$\Lambda(1405)\ 1/2^-$ $I(J^P) = 0(\frac{1}{2}^-)$ Status: ∗∗∗∗ 😀

$\Lambda(1380)\ 1/2^-$ $J^P = \frac{1}{2}^-$ Status: ∗∗ 😐

| | |
|---|---|
| ∗∗∗∗ | Existence is certain. |
| ∗∗∗ | Existence is very likely. |
| ∗∗ | Evidence of existence is fair. |
| ∗ | Evidence of existence is poor. |

# One or two resonances?

The nature of the $\Lambda(1405)$ is a theoretical and experimental challenge

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The nature of the $\Lambda(1405)$ is a theoretical and experimental challenge

## Experiment

- Quantum numbers $J^P = 1/2^-$ @ CLAS
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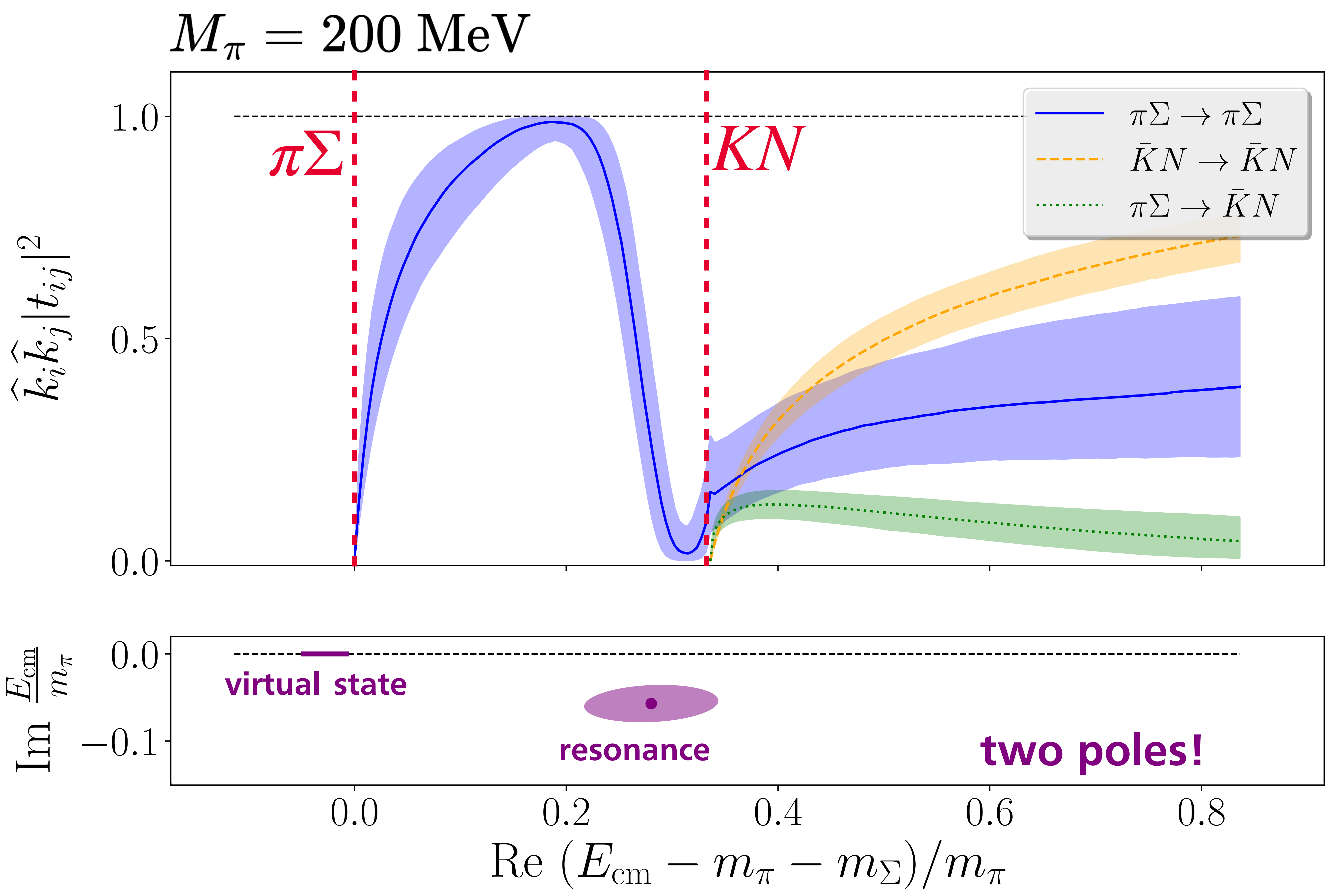
**This work!**

[Bulava, Cid Mora, Hanlon, Hörz, Mohler, Morningstar, Moscoso, Nicholson, FRL, Skinner, Walker-Loud, 2307.13471 & 2307.10413 ]

PRL and PRD Editor's suggestion!

# Amplitudes and poles




[Bulava, Cid Mora, Hanlon, Hörz, Mohler, Morningstar, Moscoso, Nicholson, FRL, Skinner, Walker-Loud, 2307.13471 & 2307.10413 ]

- **Scattering amplitudes for "preferred" fit**
  **i.e. with lowest** $\mathrm{AIC} = \chi^2 - 2\,\mathrm{dof}$

- **Pole positions for "preferred" fit**
- **All other parametrization find two poles!**

# Double-pole picture

**Two poles with** $(\text{sign Im } k_{\pi\Sigma}, \text{ sign Im } k_{KN}) = (-,+)$

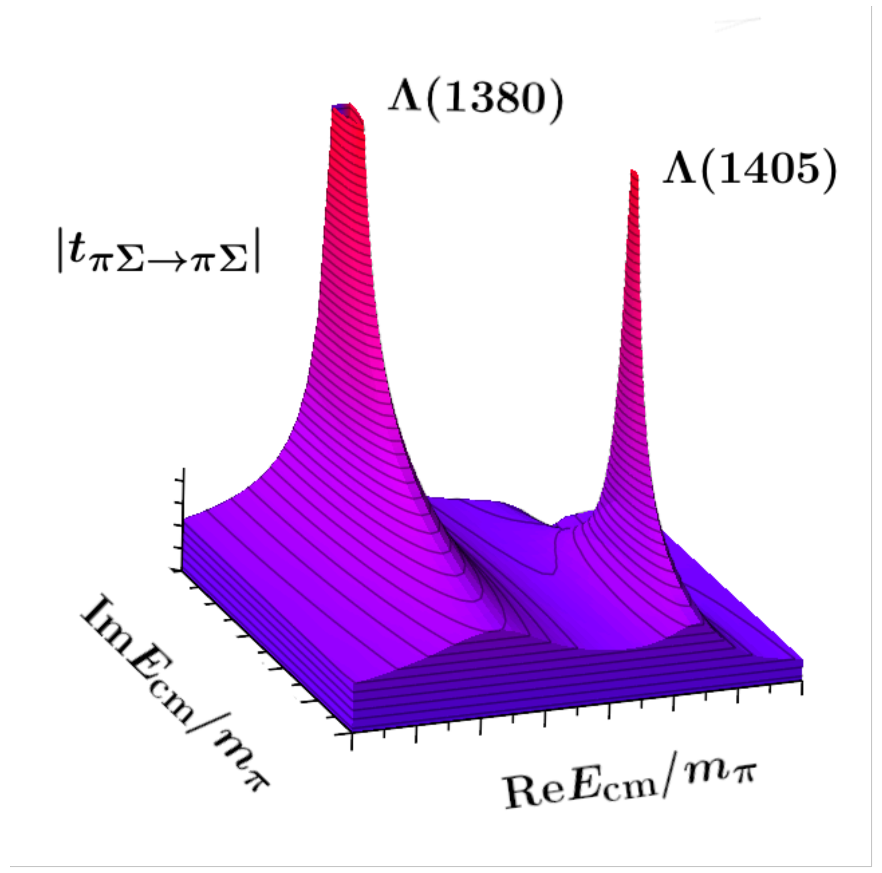
### Virtual bound state

$$E_1 = 1392(9)_{\text{stat}}(2)_{\text{model}}(16)_a \text{ MeV}$$

$$\left| \frac{c^{(1)}_{\pi\Sigma}}{c^{(1)}_{\bar{K}N}} \right| = 1.9(4)_{\text{stat}}(6)_{\text{model}}$$

Stronger coupling to $\pi\Sigma$

ratio of residues of the pole




### Resonance pole

$$E_2 = [1455(13)_{\text{stat}}(2)_{\text{model}}(17)_a - i \times 11.5(4.4)_{\text{stat}}(4.0)_{\text{model}}(0.1)_a] \text{ MeV}$$

$$\left| \frac{c^{(2)}_{\pi\Sigma}}{c^{(2)}_{\bar{K}N}} \right| = 0.53(9)_{\text{stat}}(10)_{\text{model}}$$

Stronger coupling to $KN$

- ☑ Qualitative agreement with chiral approaches [See PDG, section 83]

$$\text{Re } E_1 = 1325 - 1380 \text{ MeV}$$

$$\text{Re } E_2 = 1421 - 1434 \text{ MeV}$$

- Poles are at slightly larger energies
- Lower pole on the real axis
  - Unphysical pion mass effect?

# The three-hadron frontier from LQCD

# Why three particles?

- The two-body formalism is restricted to few interesting resonances
  - **Exotics:** $T_{cc} \to DD^*, DD\pi$
  - **Roper:** $N(1440) \to \Delta\pi \to N\pi\pi$

| Resonance | $I_{\pi\pi\pi}$ | $J^P$ |
|---|---|---|
| $\omega(782)$ | 0 | $1^-$ |
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**(with $\geq 3\pi$ decay modes)**

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- Major developments in the three-particle finite-volume formalism

[Hansen, Sharpe, PRD 2014 & 2015], [Hammer, Pang, Rusetsky, JHEP 2017] x 2
[Mai, Döring, EPJA 2017]
[...]

[Draper, Hansen, FRL, Sharpe, JHEP 2023]: Formalism for three nucleons
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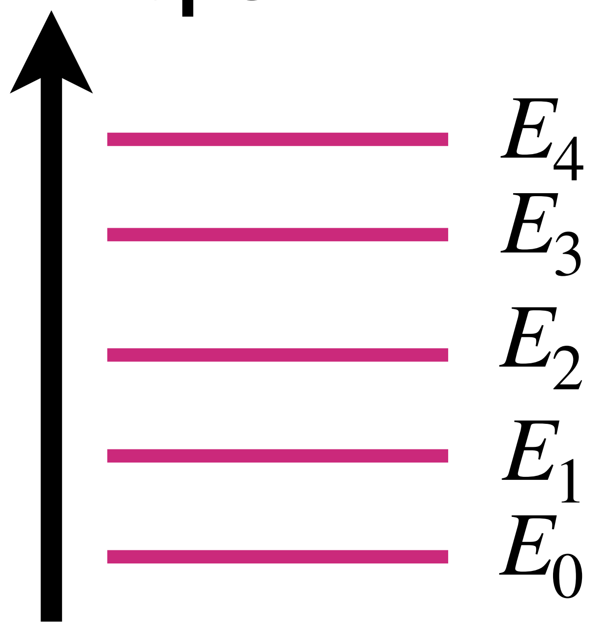
**Two-meson spectrum**

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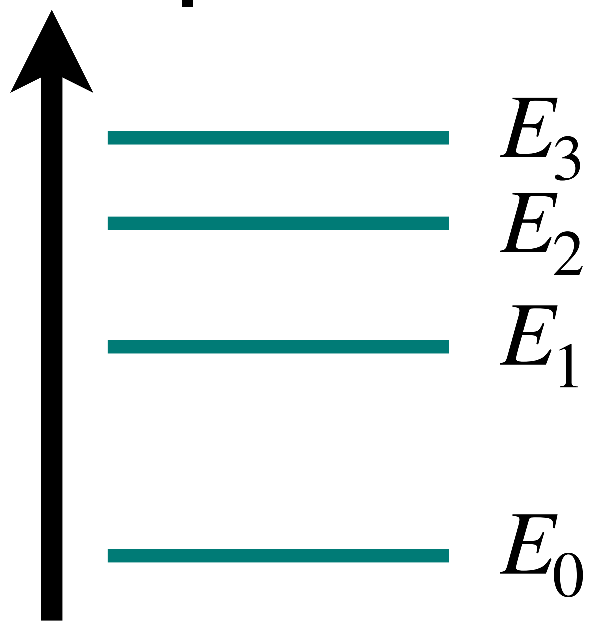





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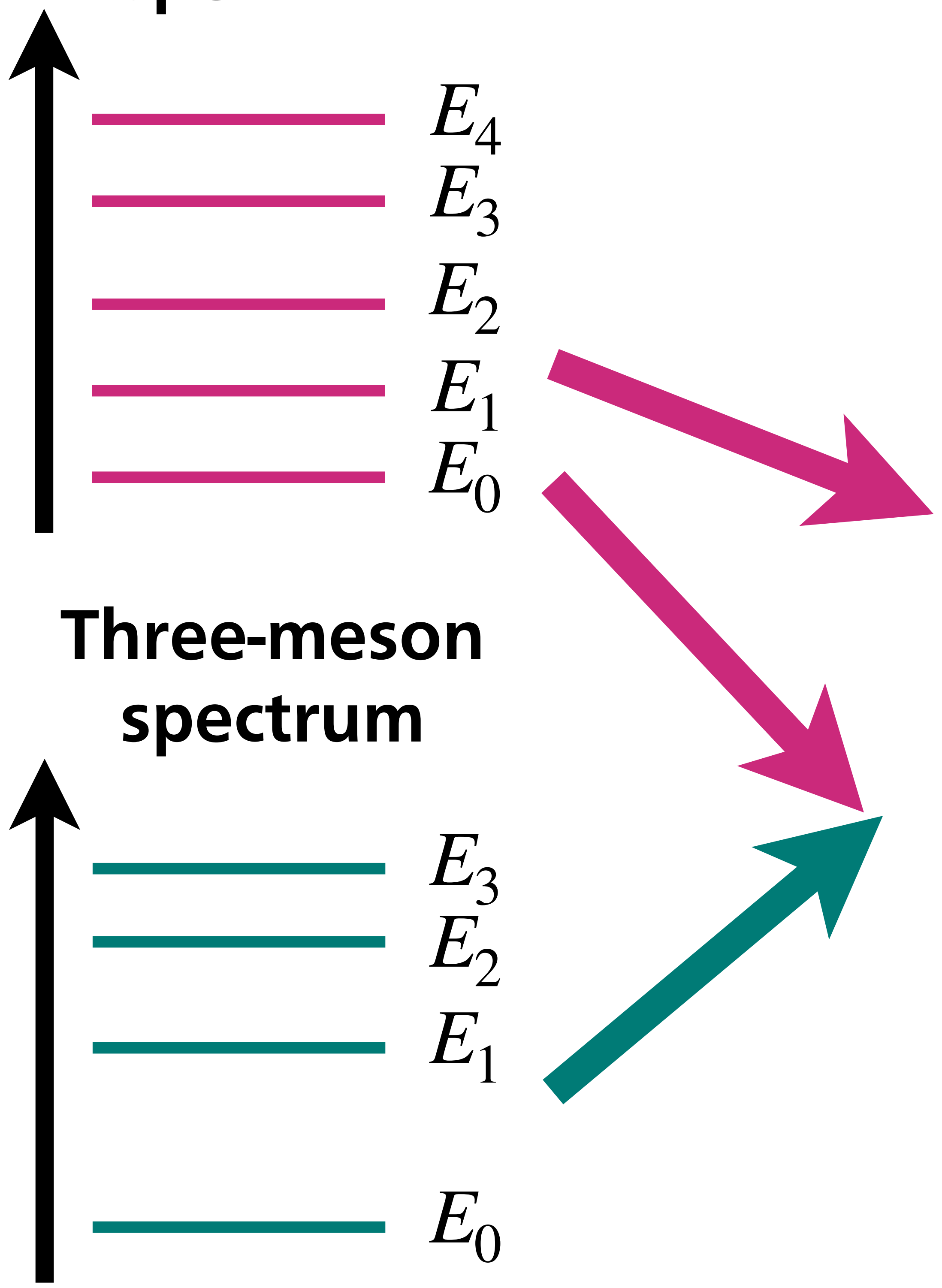


**Quantization conditions**

$$\det_{\ell m}\left[\mathcal{K}_2 + F_2^{-1}\right] = 0$$

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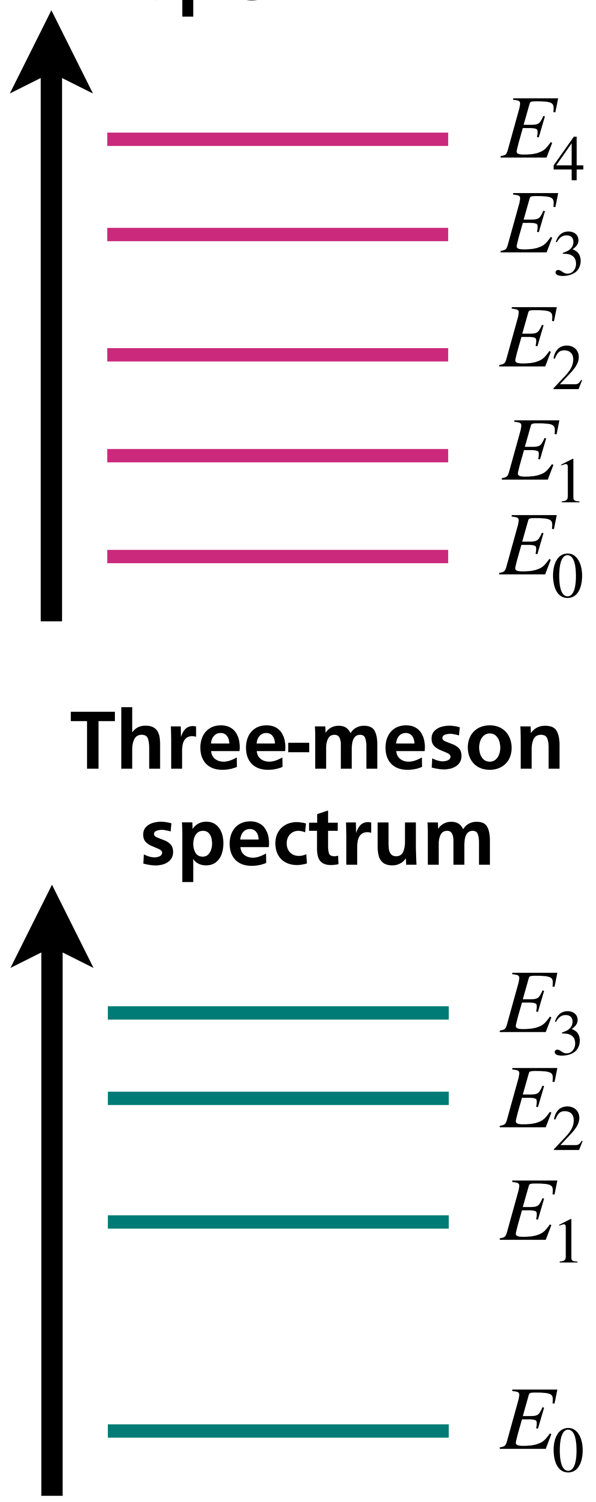


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Fit

**K-matrices**

$$\mathcal{K}_2$$

$$\mathcal{K}_{\mathrm{df},3}$$

**Parametrize:**

$$\mathcal{K}_2 = c_0 + c_1 k^2 + \ldots$$

$$\mathcal{K}_{\mathrm{df},3} = \mathcal{K}_{\mathrm{df},3}^{\mathrm{iso},0} + \mathcal{K}_{\mathrm{df},3}^{\mathrm{iso},1}\left(\frac{s - 9m^2}{9m^2}\right) + \ldots$$

[Blanton, FRL, Sharpe, JHEP 2019]

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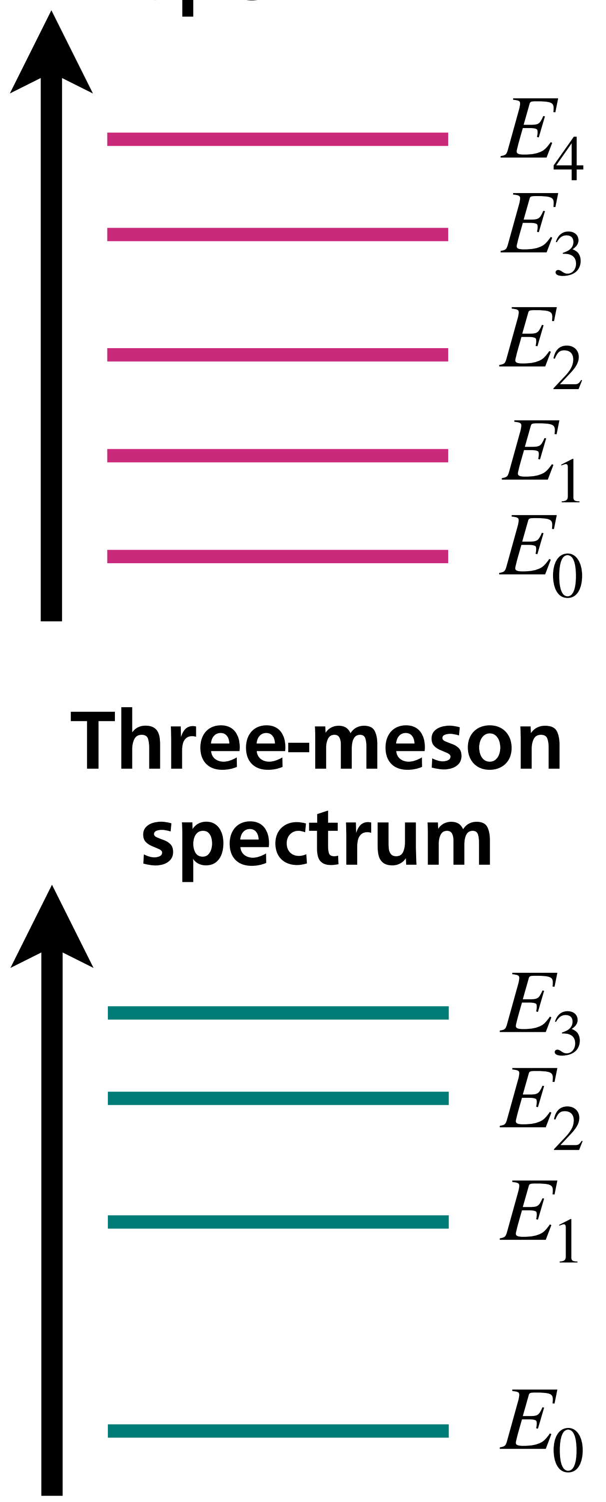


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Fit →

**K-matrices**

$$\mathcal{K}_2$$

$$\mathcal{K}_{\mathrm{df},3}$$

**Unitarity relations** →

**Integral equations**

[Briceño et al., PRD 2018]
[Hansen et al., PRL 2021]
[Jackura et al., PRD 2021]
[Dawid et al., 2303.04394]

**Scattering amplitudes**

$$\mathcal{M}_2$$

$$\mathcal{M}_3$$

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# Three-meson systems

## Important benchmark system: three pseudoscalar mesons at maximal isospin

- Implement formalism and explore its features
- Test fitting strategies to extract three-body K matrix
- Interpret results in combination with EFTs
- Investigate features of scattering amplitudes

$$3\pi^+, \quad 3K^+, \quad \pi^+\pi^+K^+, \quad K^+K^+\pi^+$$

[Blanton ... FRL ... et al., PRL 2020 & JHEP 2021]
[Draper ... FRL ... et al., JHEP 2023],
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## This work:

- Calculations at several pion masses including physical
  [Blanton ... FRL ... et al., PRL 2020 & JHEP 2021]
  [Draper ... FRL ... et al., JHEP 2023]
- Compute physical three-meson scattering amplitudes
  [Hansen et al (HadSpec)]

CLS ensembles

| | $(L/a)^3 \times (T/a)$ | $M_\pi$[MeV] | $M_K$[MeV] | $N_{\text{cfg}}$ |
|---|---|---|---|---|
| N203 | $48^3 \times 128$ | 340 | 440 | 771 |
| N200 | $48^3 \times 128$ | 280 | 460 | 1712 |
| D200 | $64^3 \times 128$ | 200 | 480 | 2000 |
| E250 | $96^3 \times 192$ | 130 | 500 | 505 |

$$a \simeq 0.063 \text{ fm}$$

$$\text{tr } m_q = 2m_{ud} + m_s \simeq \text{const}$$

# Three-pion K matrix

- **Three-particle formalism applied to weakly-interacting (non-resonant) systems, e.g. $\pi^+\pi^+\pi^+$**

[Blanton … FRL … et al., PRL 2020 & JHEP 2021], [Draper … FRL … et al., JHEP 2023], [Fischer … FRL … et al, EPJC 2021]

[Alexandrou et al, Brett et al, Culver et al, Mai et al] [Hansen et al (HadSpec), PRL]

NLO ChPT: [Baeza-Ballesteros, Bijnens, Husek, FRL, Sharpe, Sjö, JHEP 2023]

ETMC: [ Fischer, Kostrzewa, Liu, FRL, Ueding, Urbach, EPJC 2021 ]

This work: [Dawid, Draper, Hanlon, Hörz, Morningstar, FRL, Sharpe, Skinner, arXiv:2502.17976,2502.13978]

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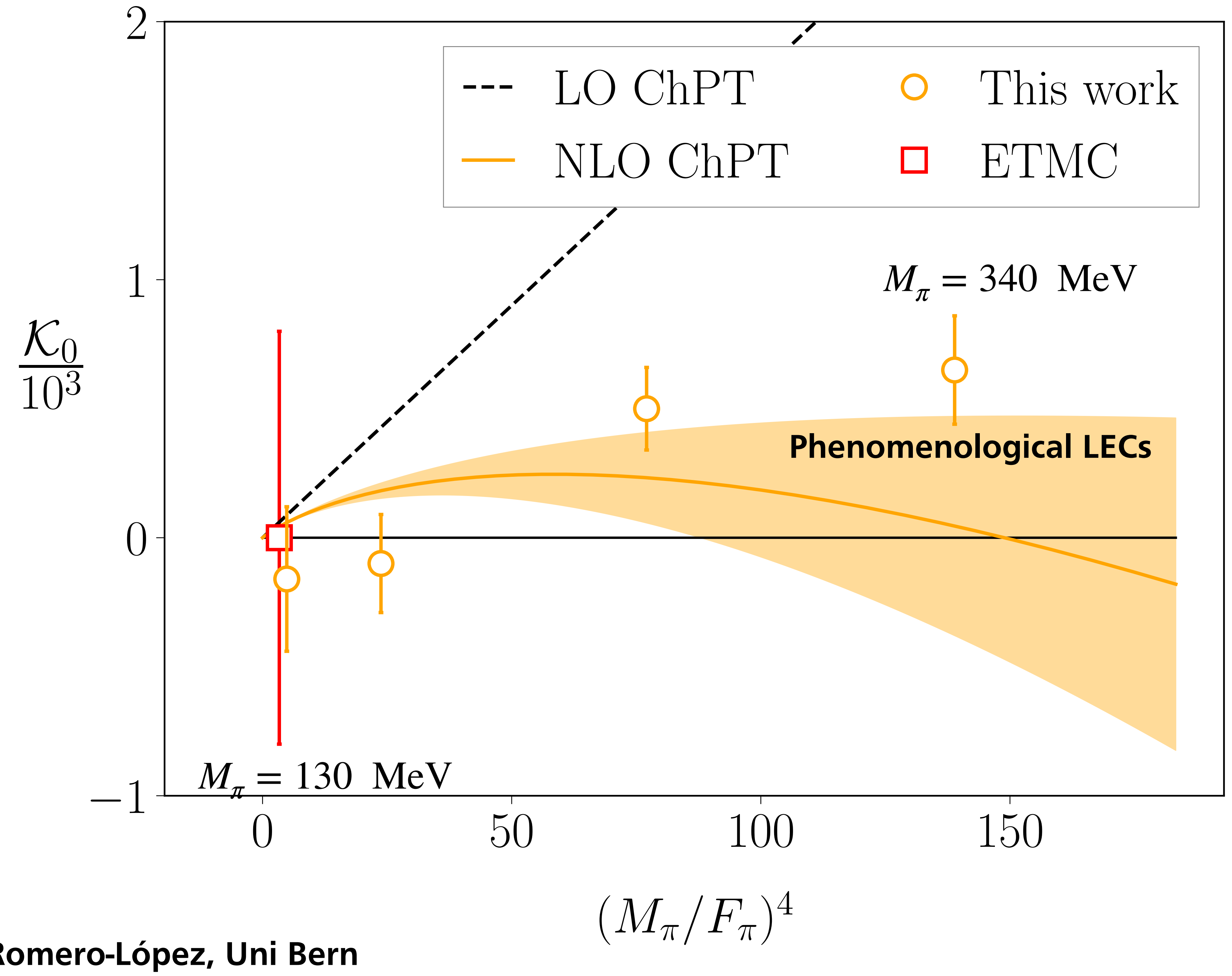
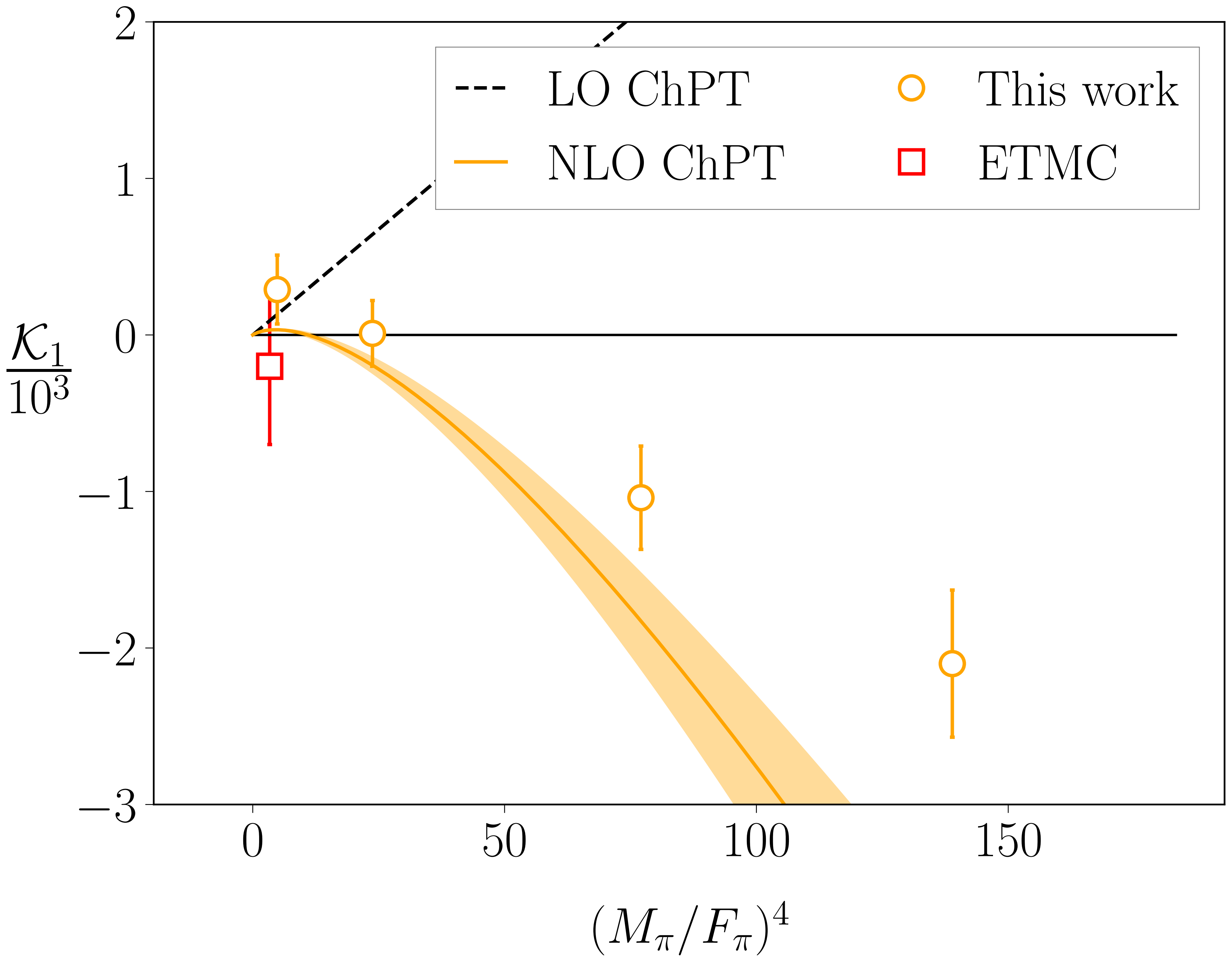


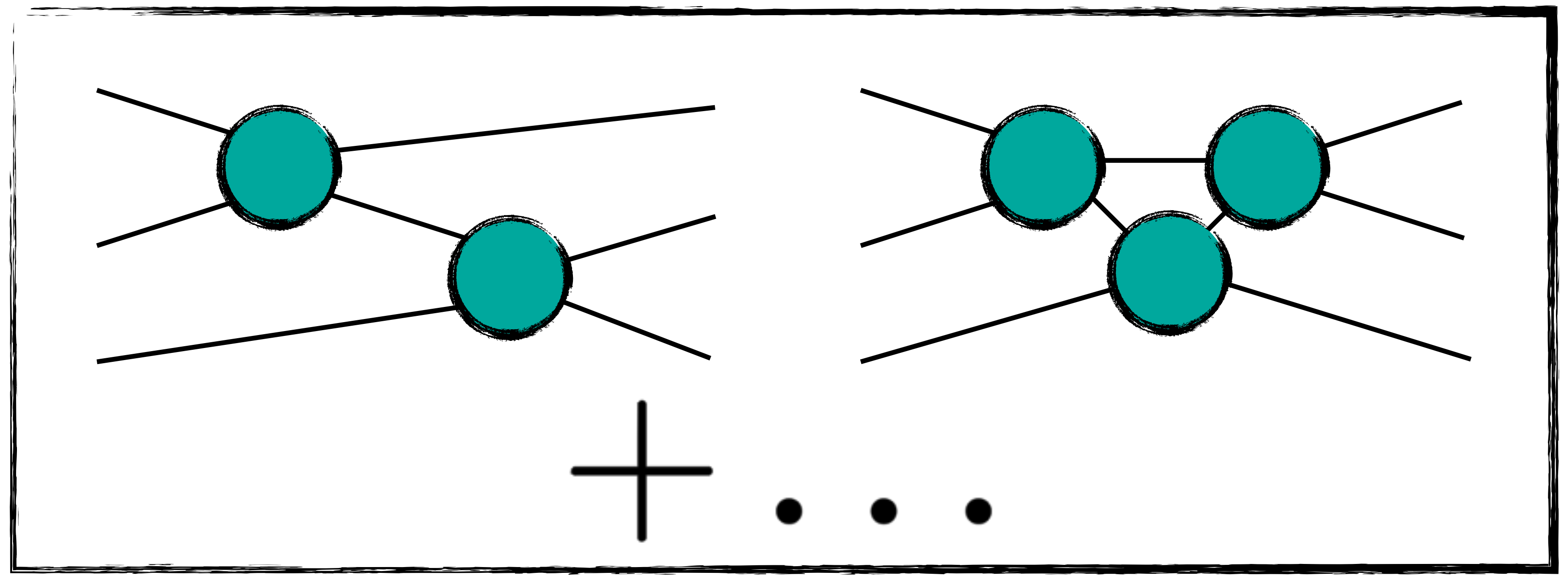



# Scattering amplitudes

- Physical amplitudes that are consistent with unitary are obtained after solving integral equations:

$$\mathcal{M}_3 = \mathcal{D} + \mathcal{M}_{\mathrm{df},3}$$

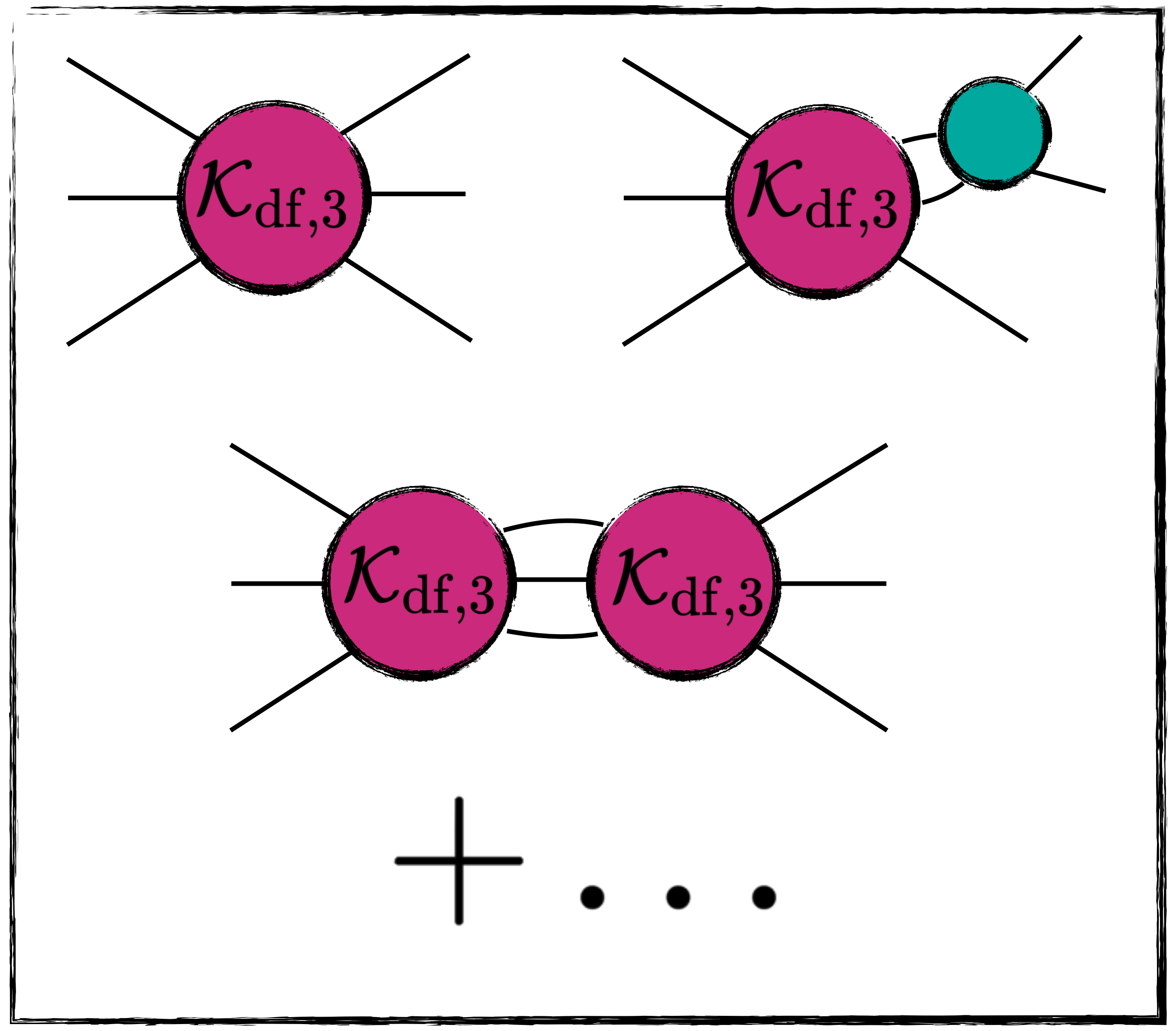
"ladder amplitude"




two-body rescattering

$$\mathcal{D} = -\mathcal{M}_2 G \mathcal{M}_2 - \int \mathcal{M}_2 G \mathcal{D}$$

"divergence-free amplitude"



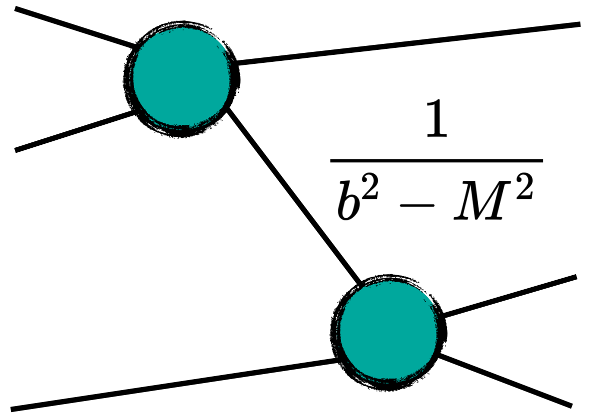

At least one
three-body interaction

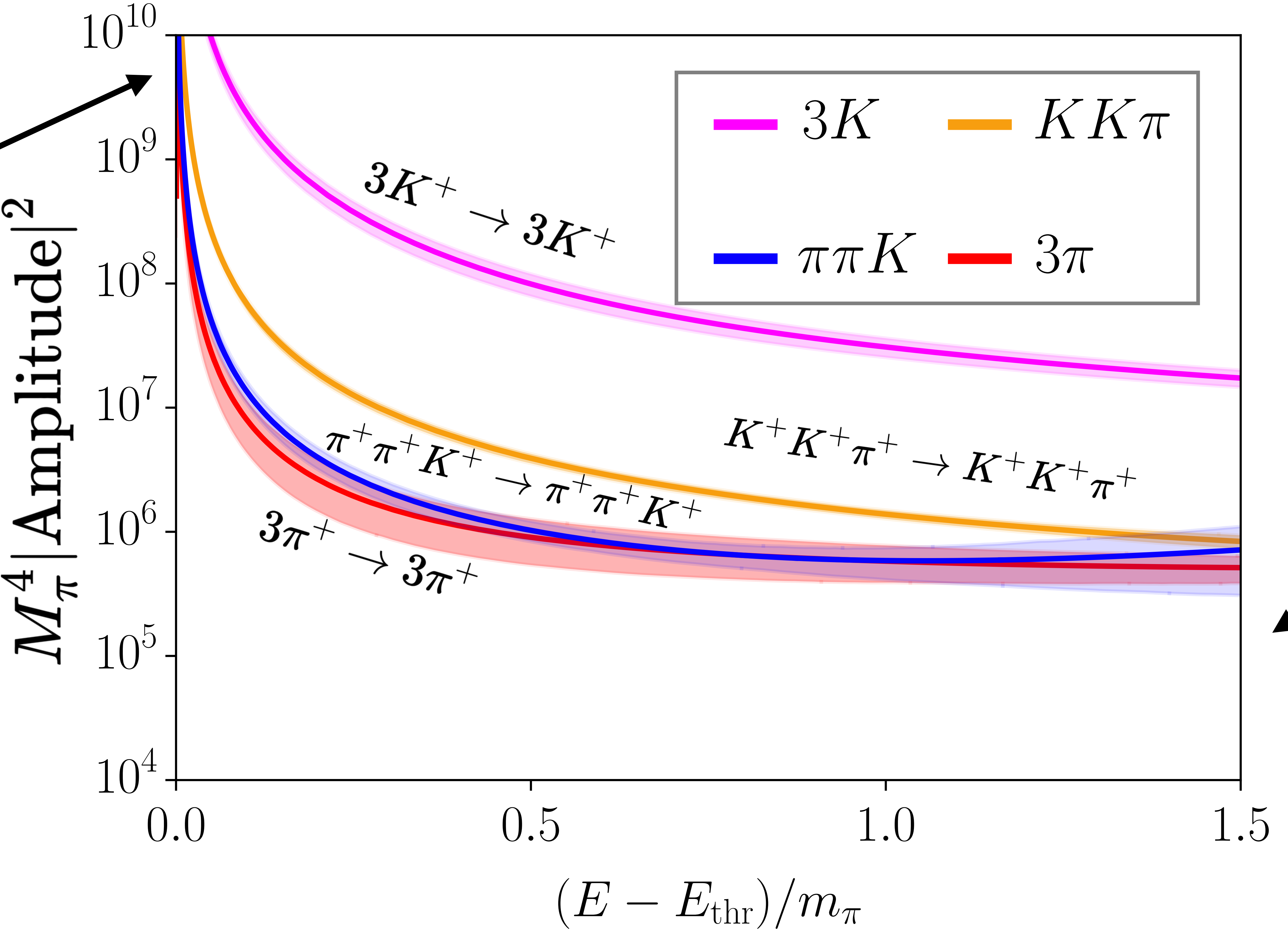
# $J^P=0^-$ three-meson amplitudes

**Lattice QCD predictions for physical three-meson scattering amplitudes**

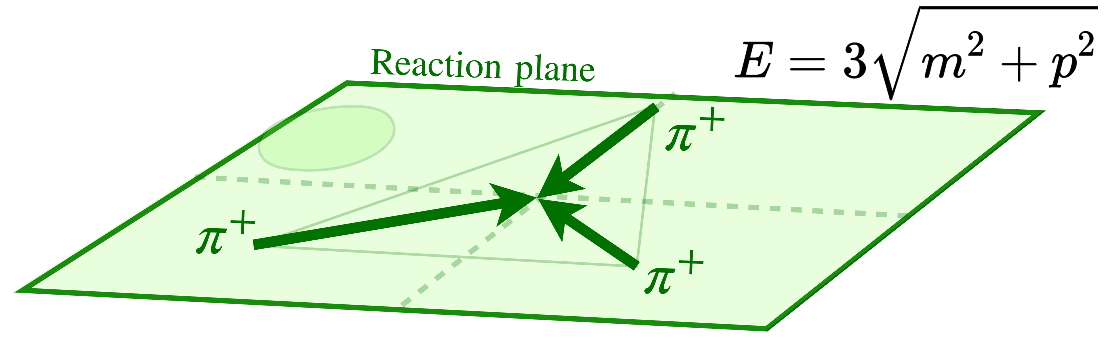
[Dawid, Draper, Hanlon, Hörz, Morningstar, FRL, Sharpe, Skinner, arXiv:2502.17976,2502.13978]

**Divergent at threshold**




**Equilateral kinematic configuration**




**Pion interactions are chirally suppressed**

$$M_\pi^2 \mathcal{M}_3 = O(M_\pi^4/F_\pi^4)$$

# Chiral dependence

**Study chiral dependence of the three-meson amplitudes for the first time!**

[Dawid, Draper, Hanlon, Hörz, Morningstar, FRL, Sharpe, Skinner, arXiv:2502.17976,2502.13978]

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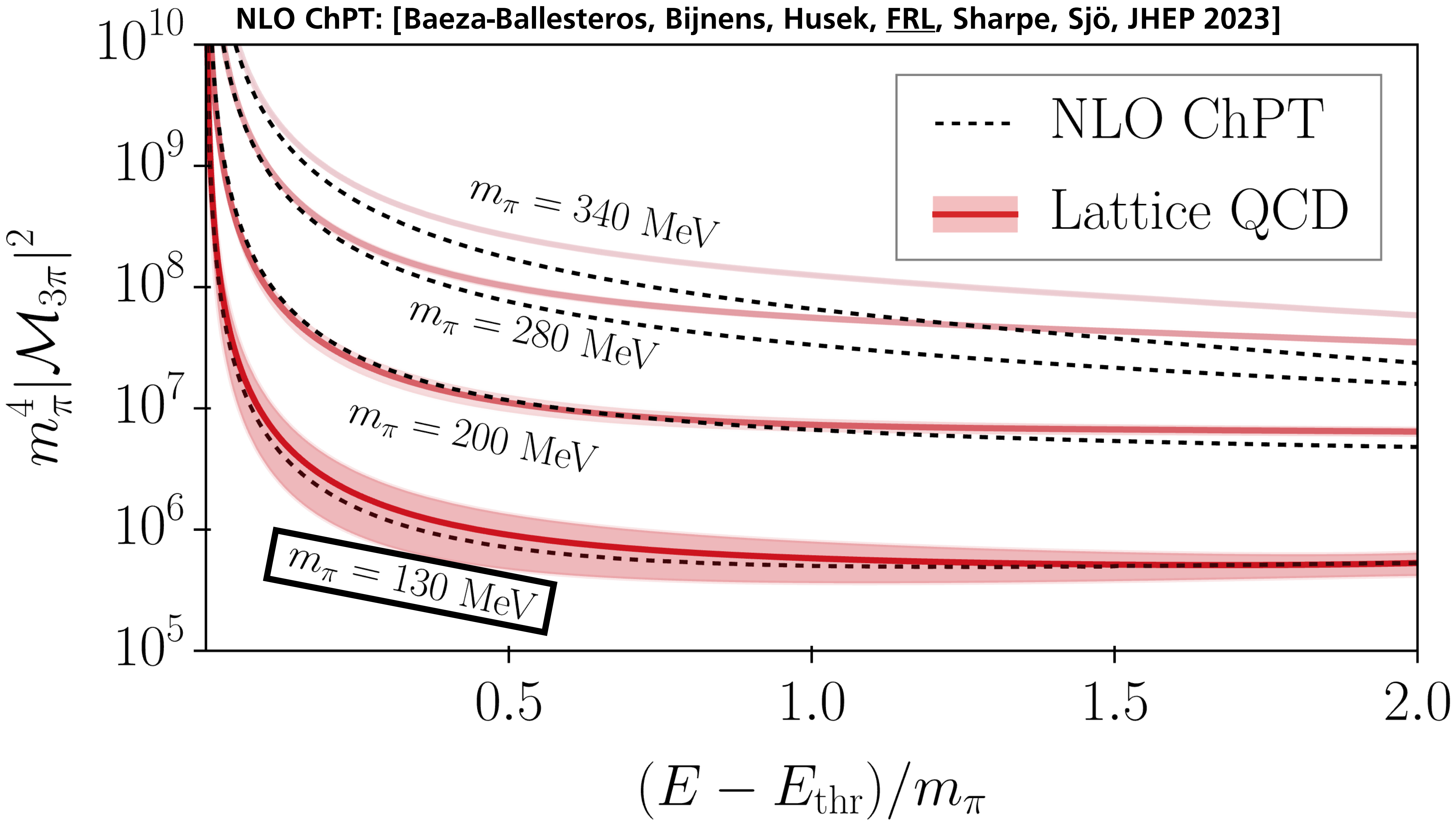


- **Better agreement with ChPT at phys. point**
- **Better agreement al low energies**

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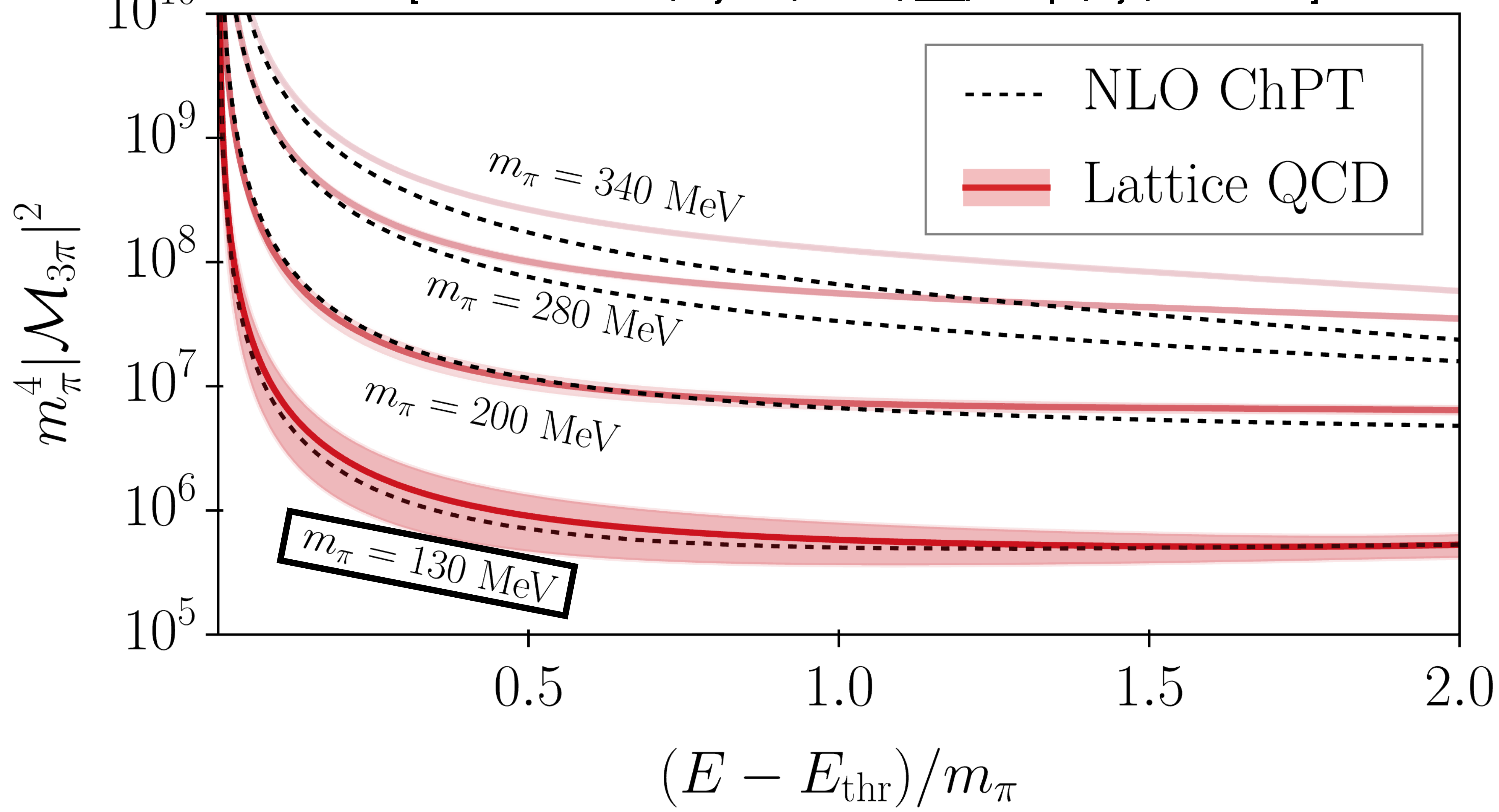
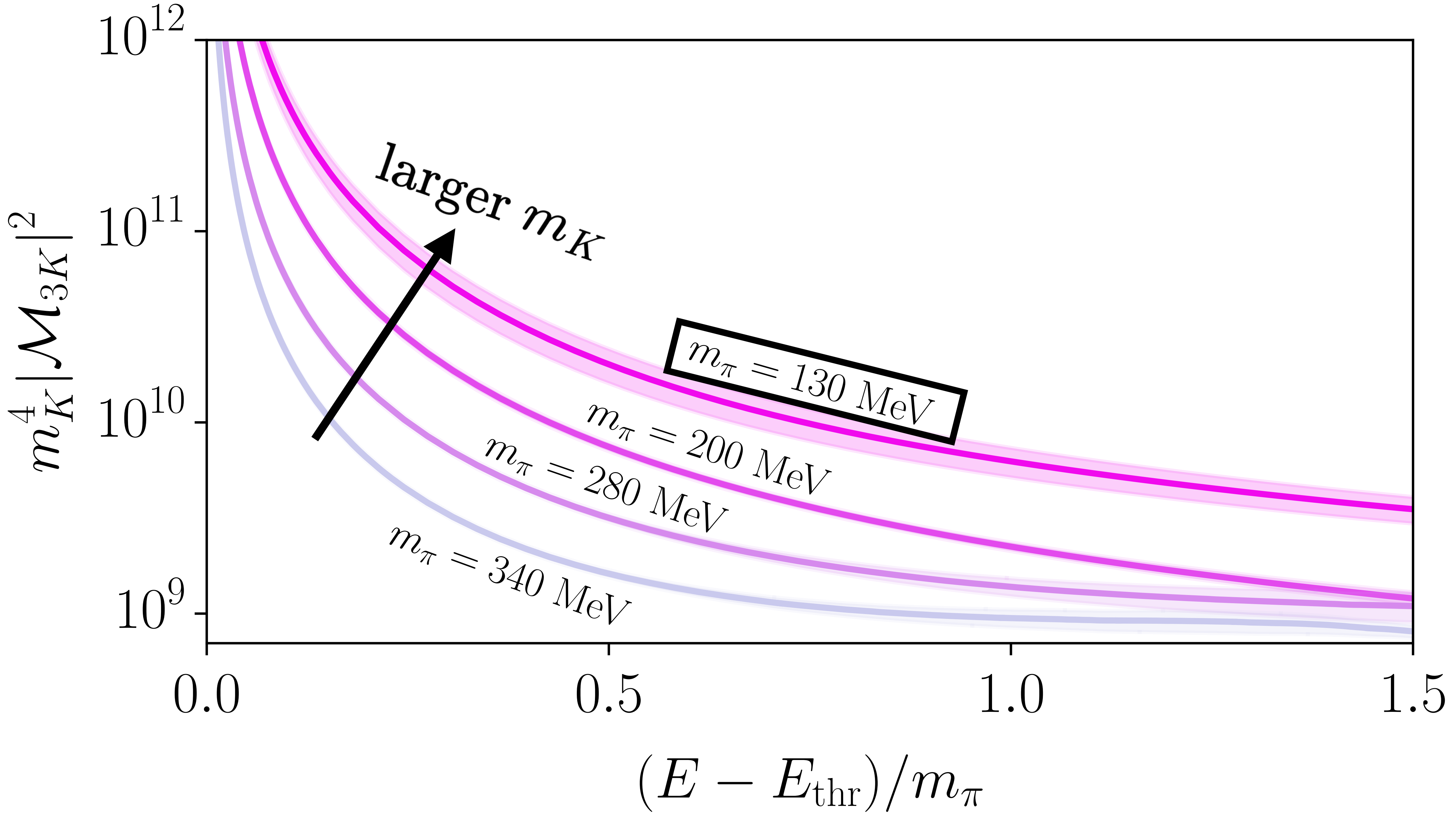


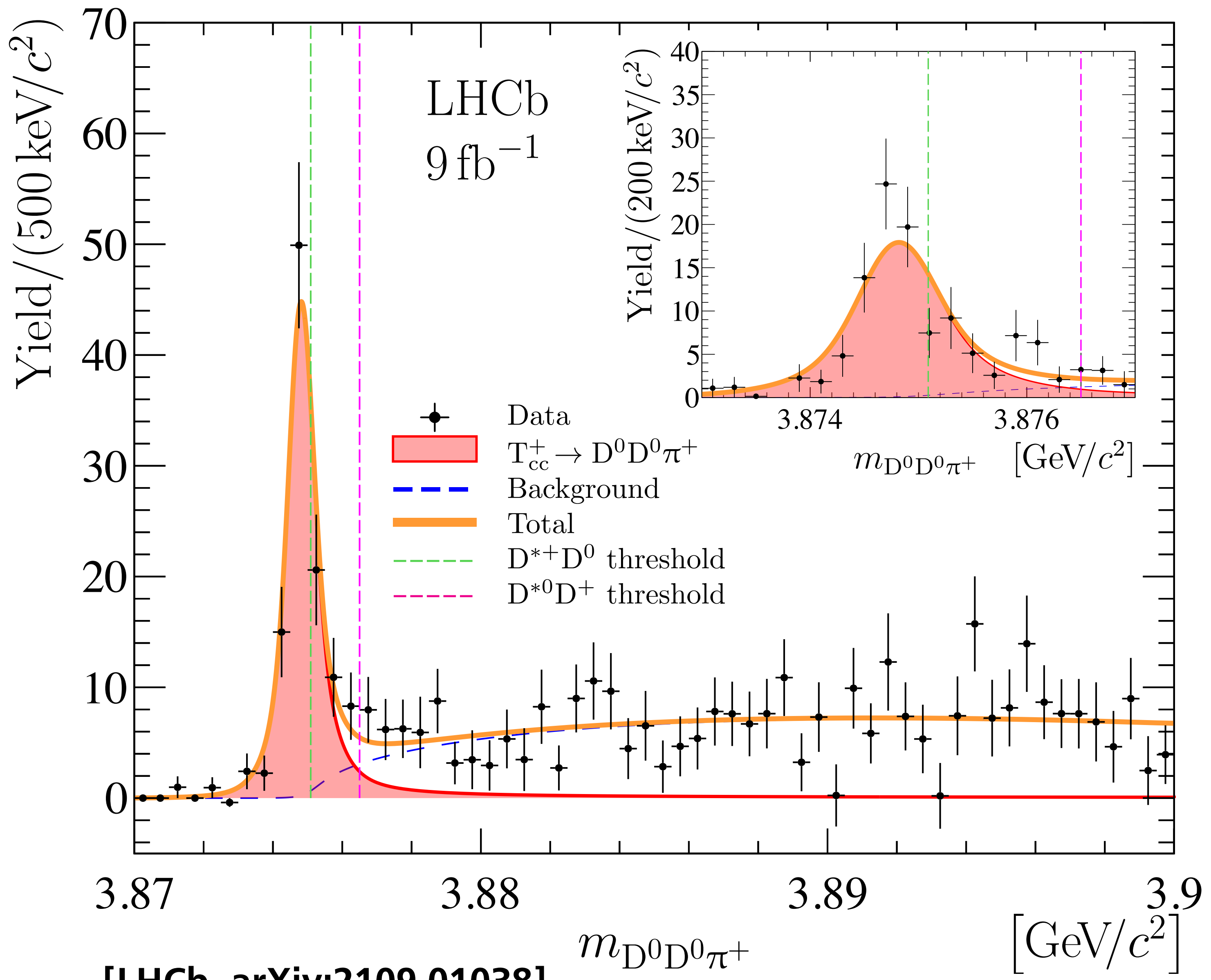




- **Better agreement with ChPT at phys. point**
- **Better agreement al low energies**

- **Stronger Kaon interactions at the phys. point**

$$2m_\ell + m_s = \text{const}$$

# A three-body description of the doubly-charmed tetraquark



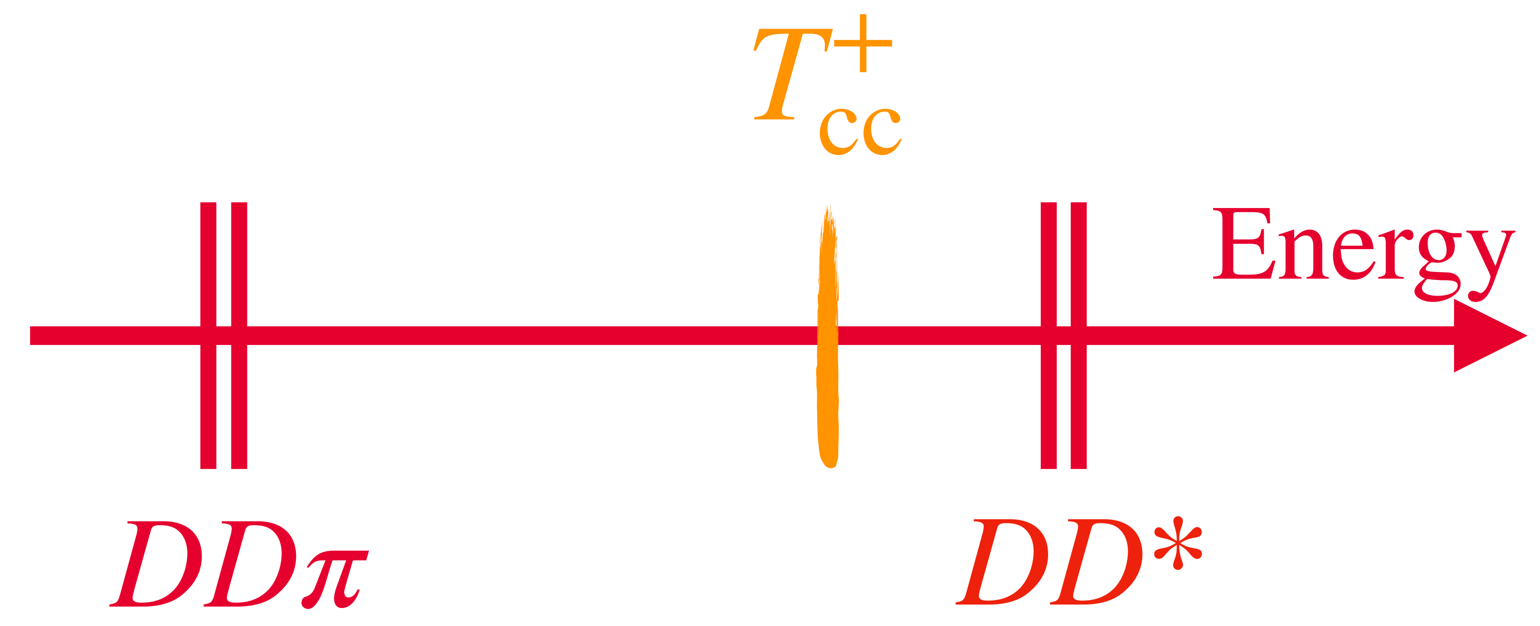

[LHCb, arXiv:2109.01038]



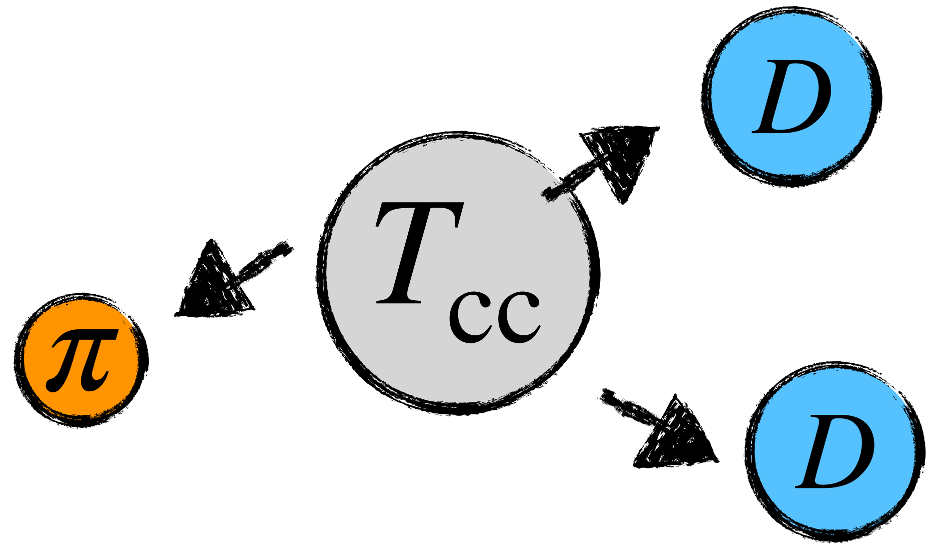
Fernando Romero-López, Uni Bern

# Doubly-charmed tetraquark

**Experiment**

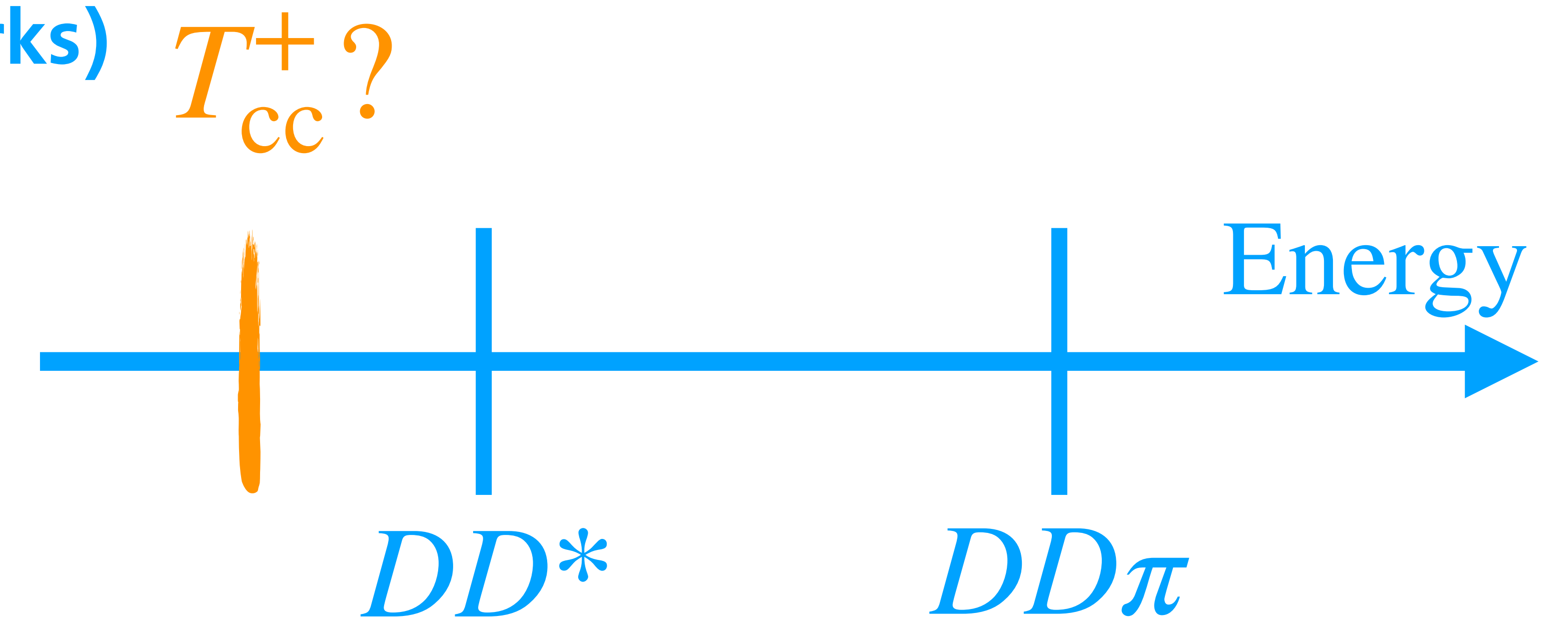



- For physical quark masses is a three-body resonance

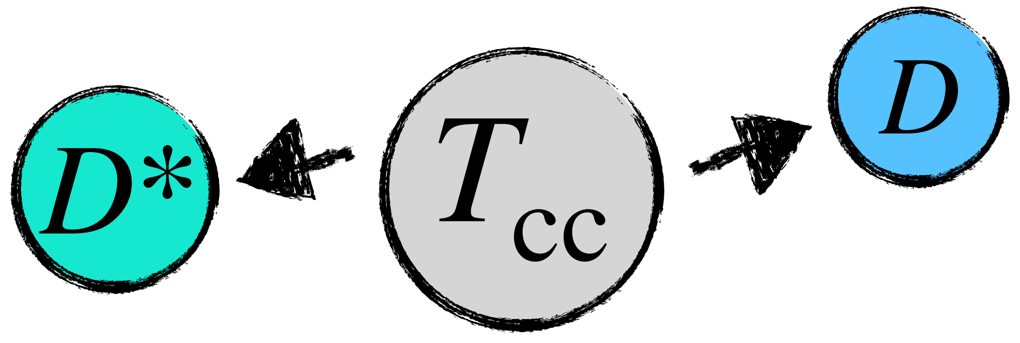



need three-body formalism!

$N_f$=2+1+1 QCD
(heavier quarks)




- Stable D* at slightly heavier-than-physical quark mases

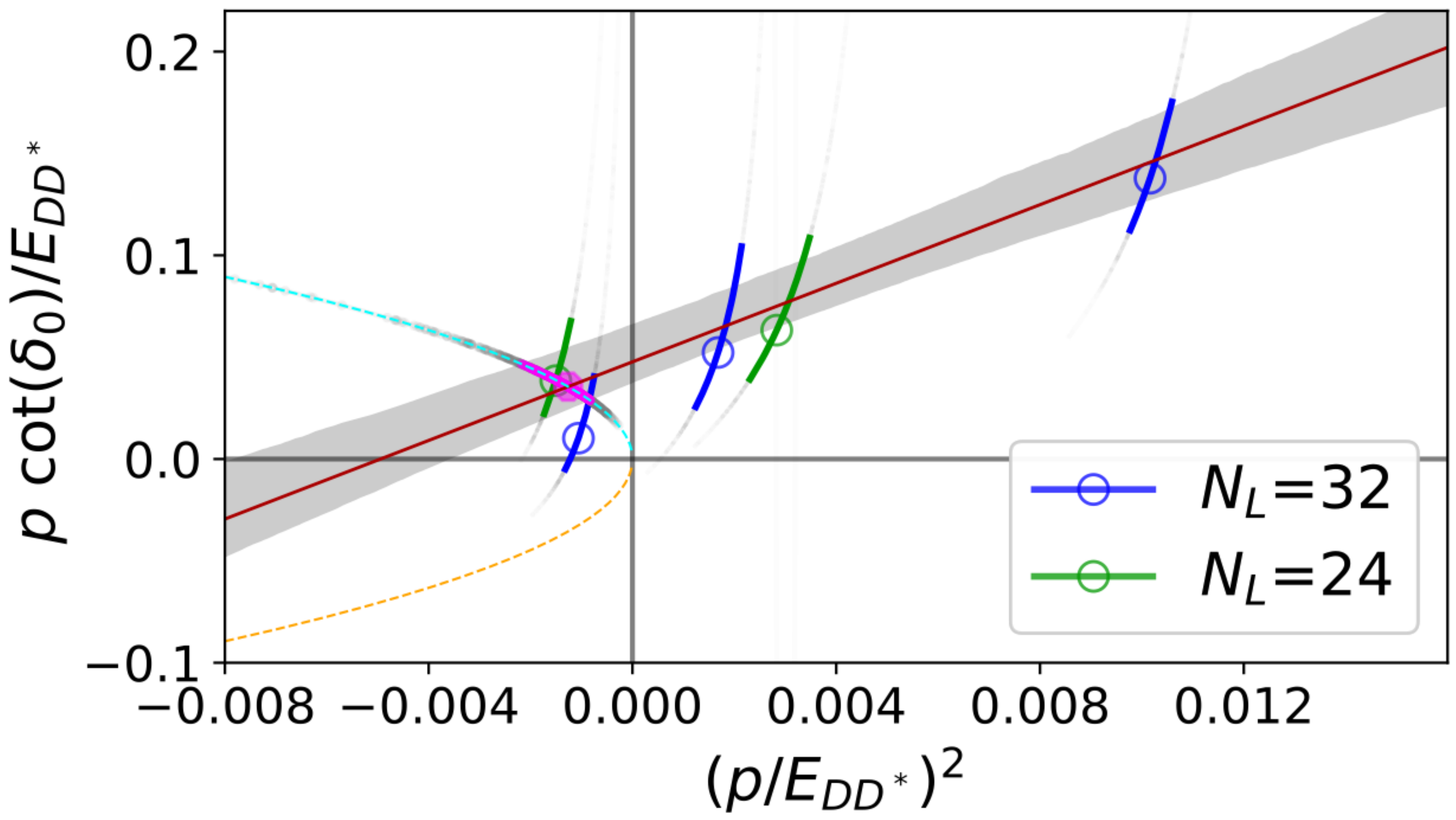



suitable for the two-body finite-volume formalism?

# D-D* scattering

- **Several works study the $T_{cc}$ channel in this setup**
  [Chen et al., 2206.06185] [Lyu et al. (HALQCD), 2302.04505]
  [Padmanath & Prelovsek, 2202.10110] [Collins et al., 2402.14715]
  [Whyte, Thomas, Wilson, 2405.15741]
  - **Signature of virtual bound state?**

[Padmanath, Prelovsek, arXiv:2202.10110]



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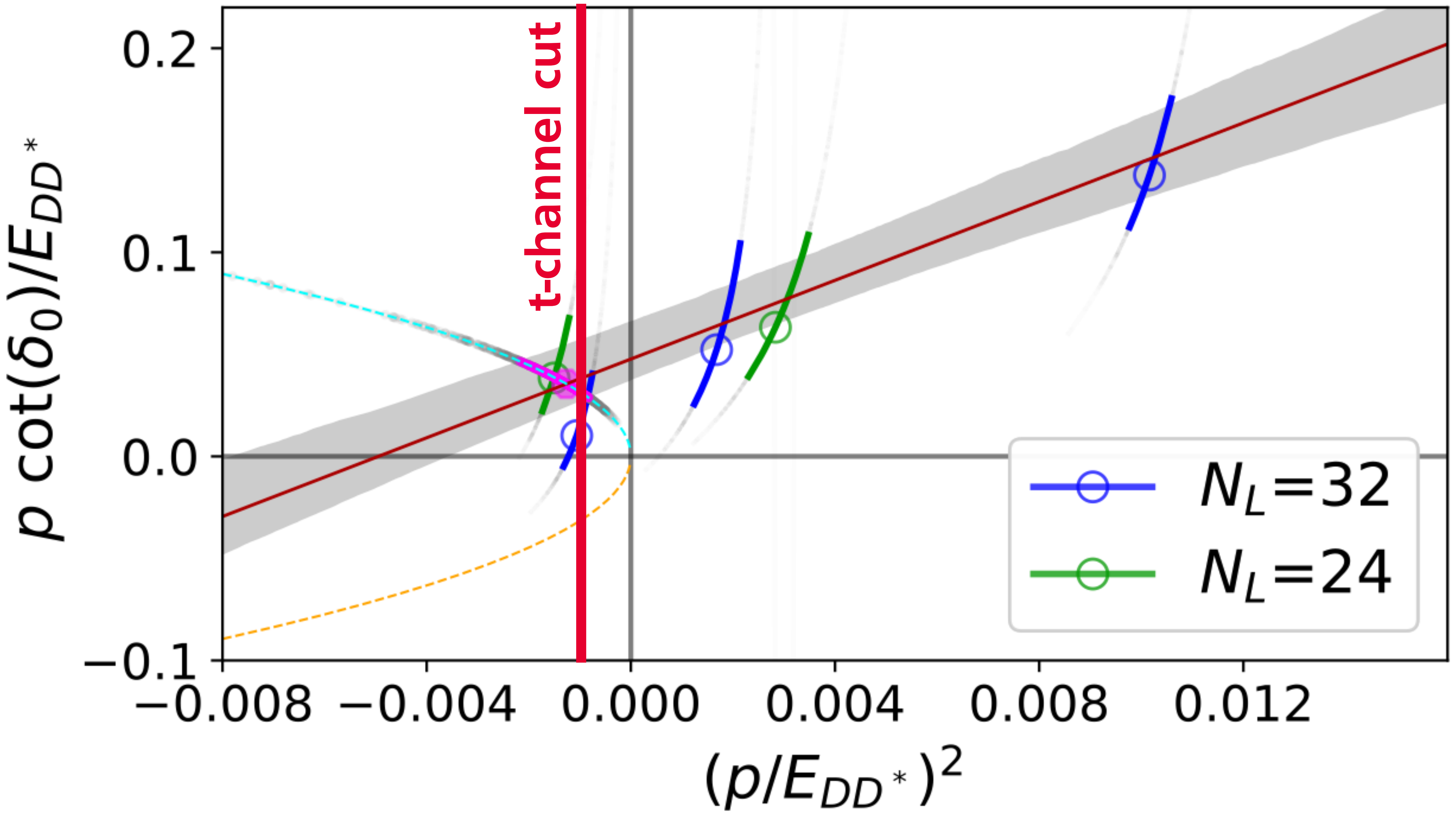
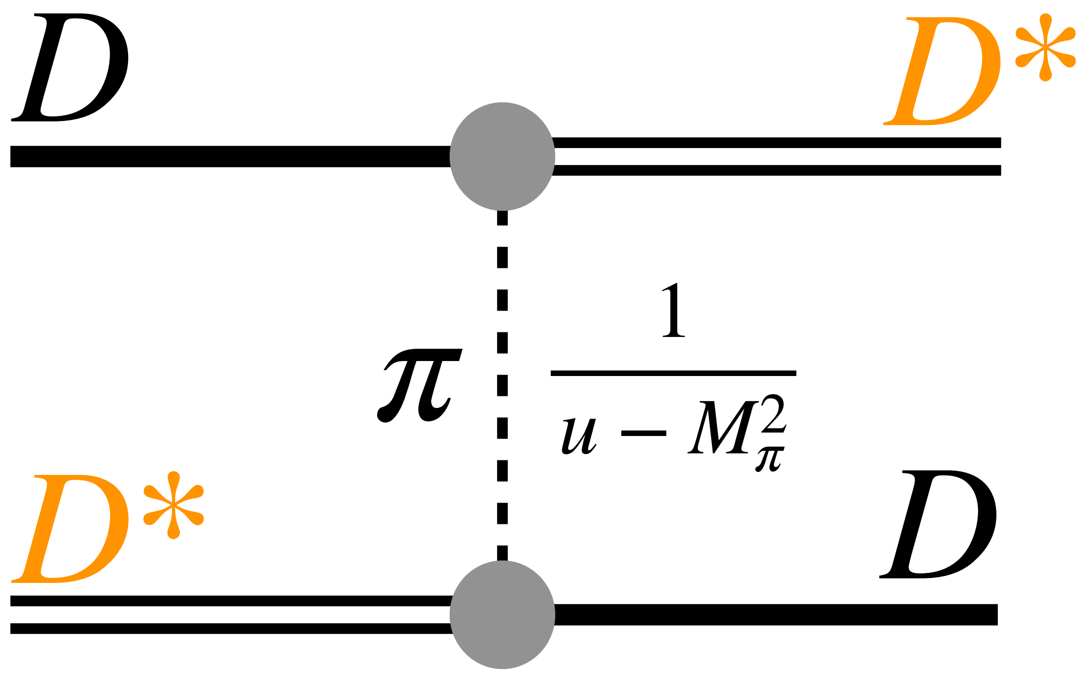



**!** one-pion exchange creates non-analytic behavior:



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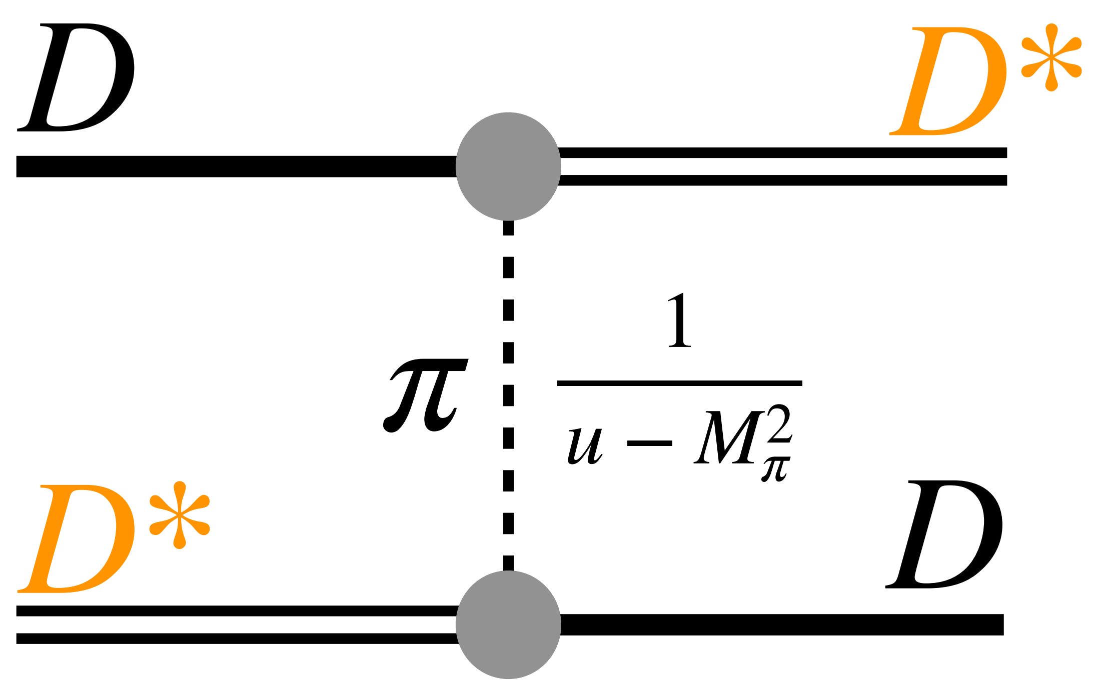
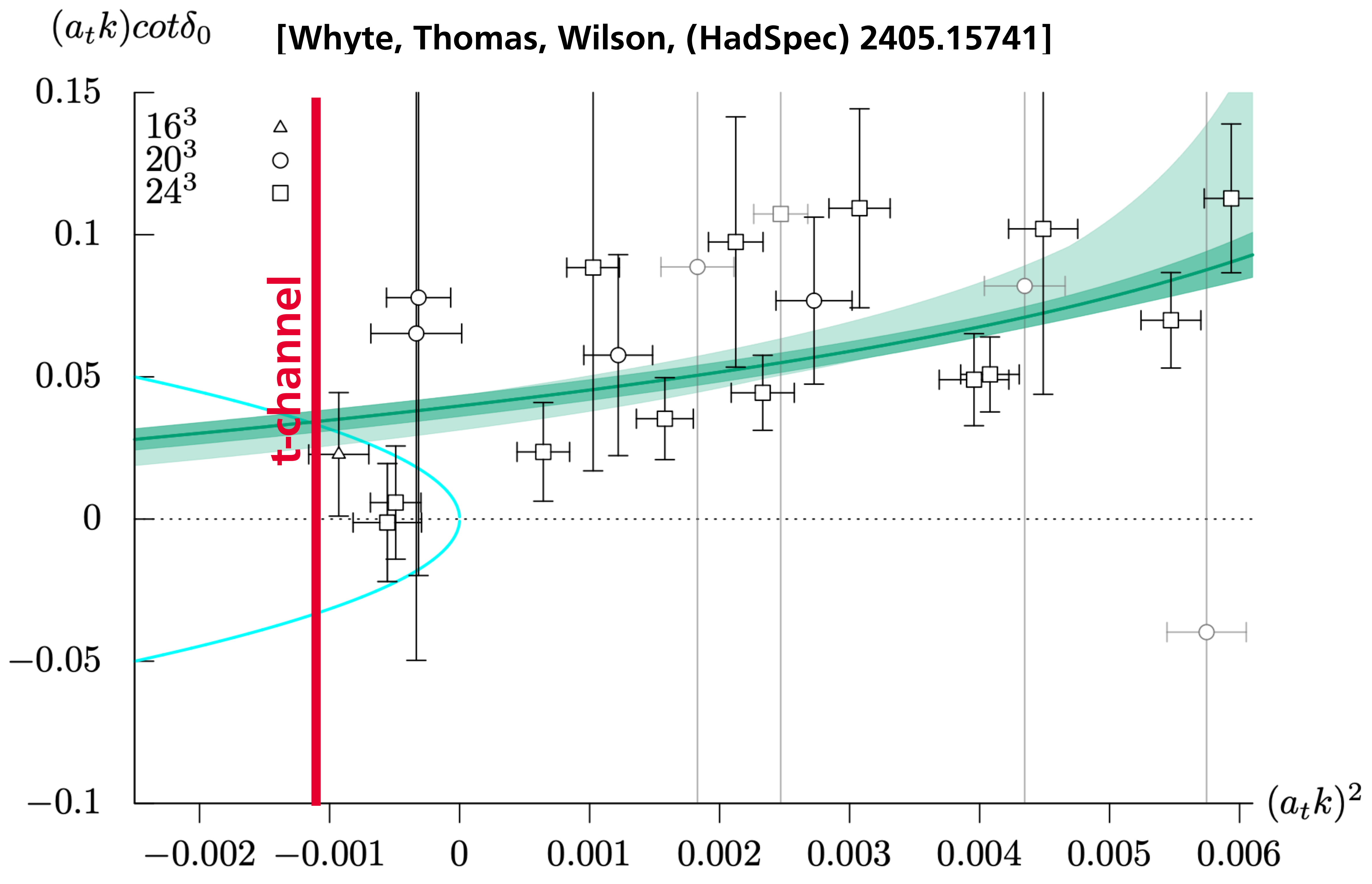



[Whyte, Thomas, Wilson, (HadSpec) 2405.15741]



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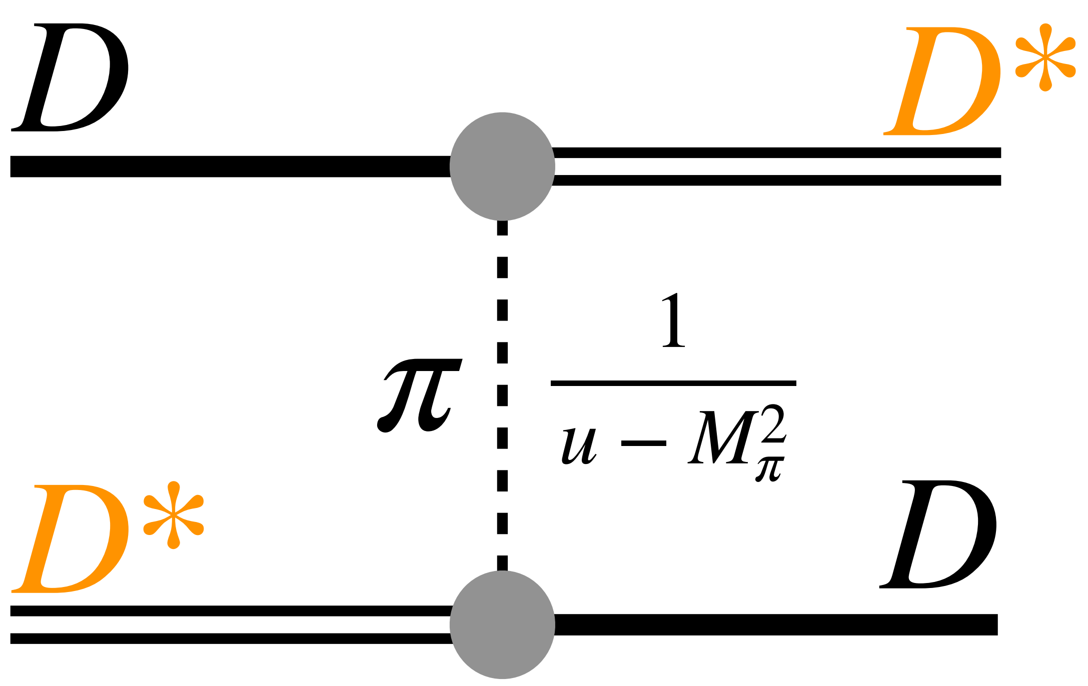
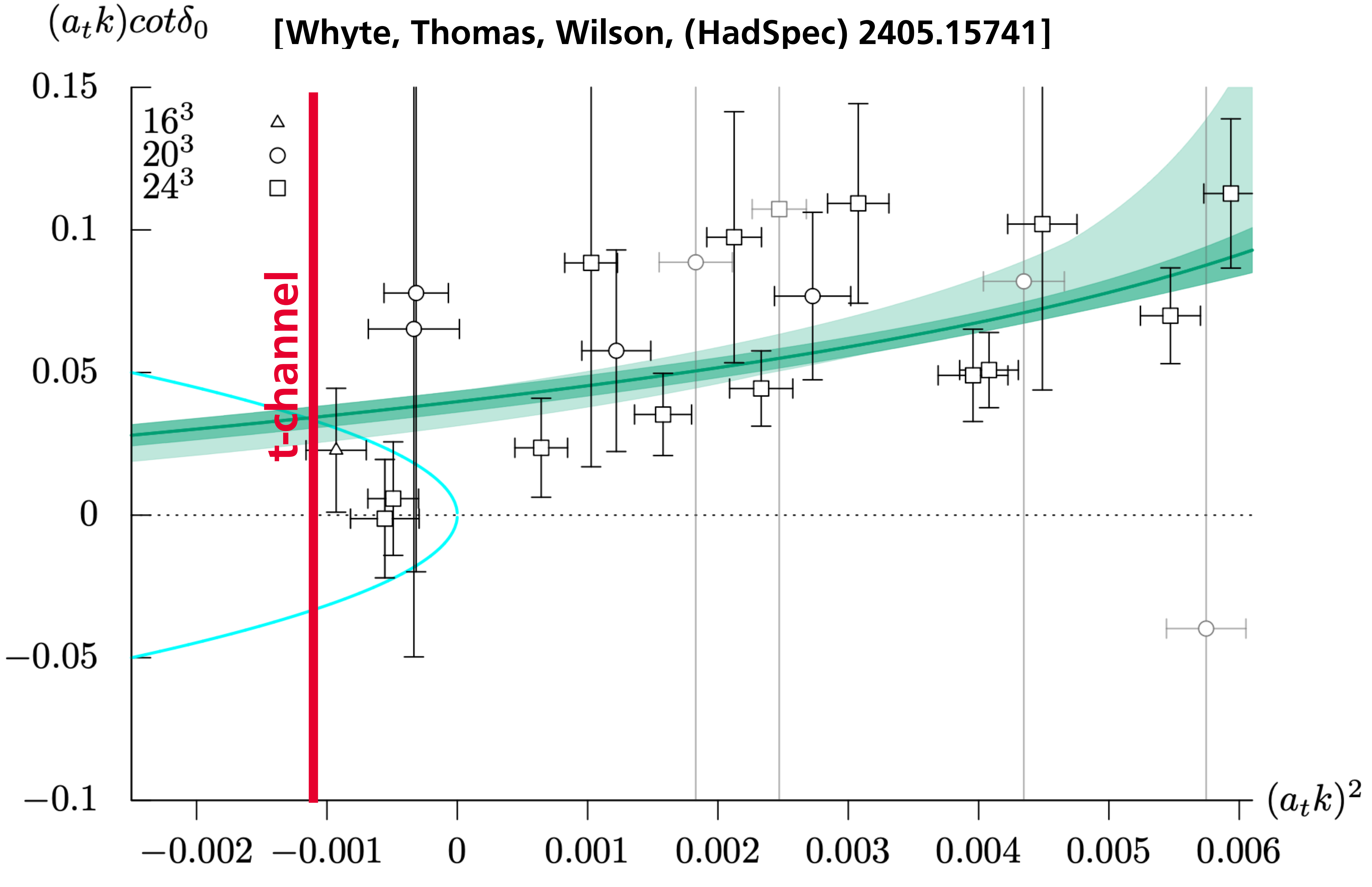



Several solution have been proposed

[Du et al (2408.09375), Abolnikov et al. (2407.04649), Bubna et al. (2402.12985), Meng et al. (2312.01930), Raposo, Hansen (2311.18793)]

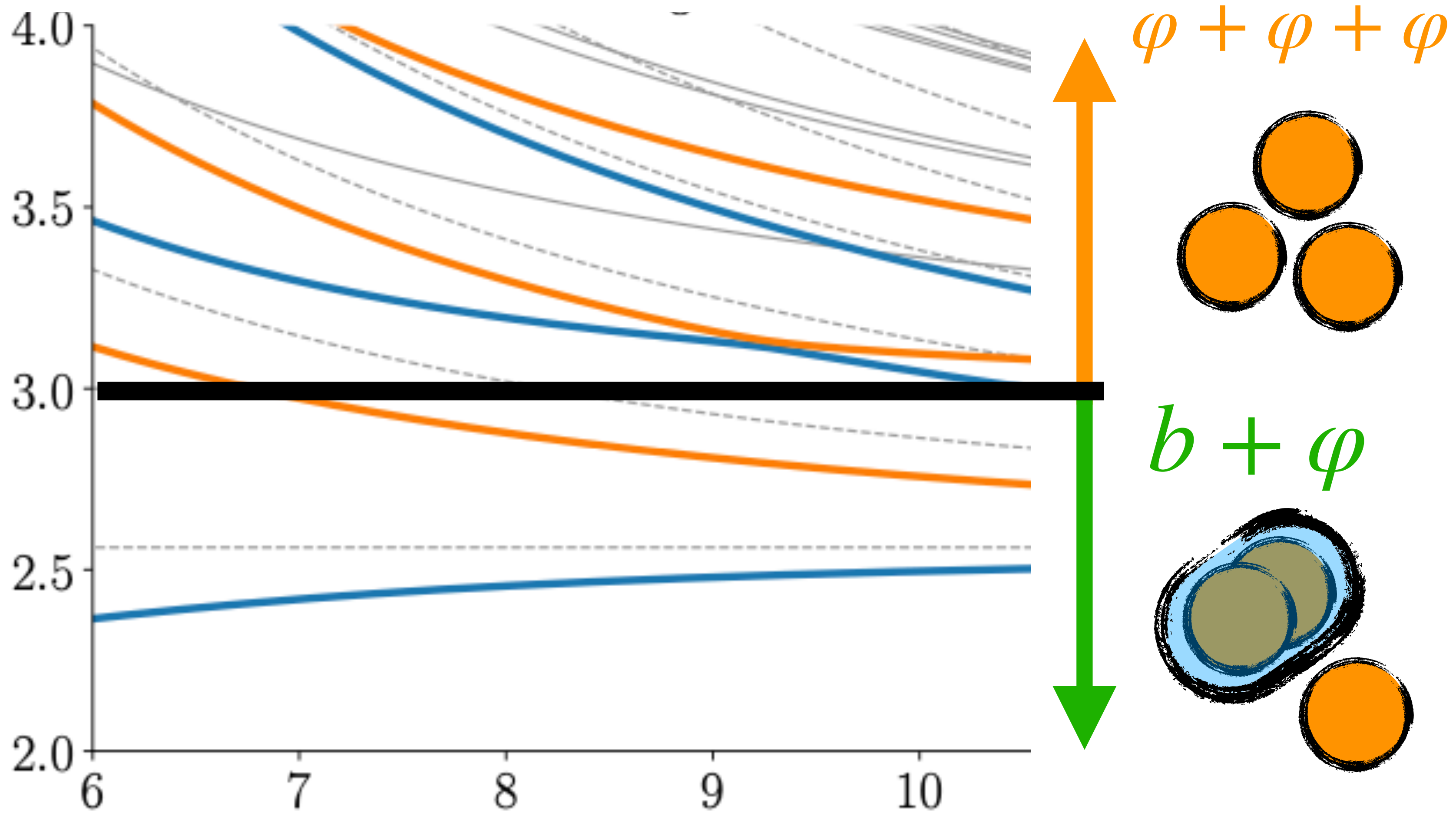
# A three-body solution

- In the presence of a two-body bound state:

  - Below the three-particle threshold, effective "particle-dimer"

    [FRL et al 2302.04505] [Jackura et al 2010.09820]
    [Dawid, Islam, Briceño, 2303.04394]
    [Briceño, Jackura, Pefkou, FRL 2402.12167]

[FRL, Sharpe, Blanton, Briceño, Hansen 1908.02411]



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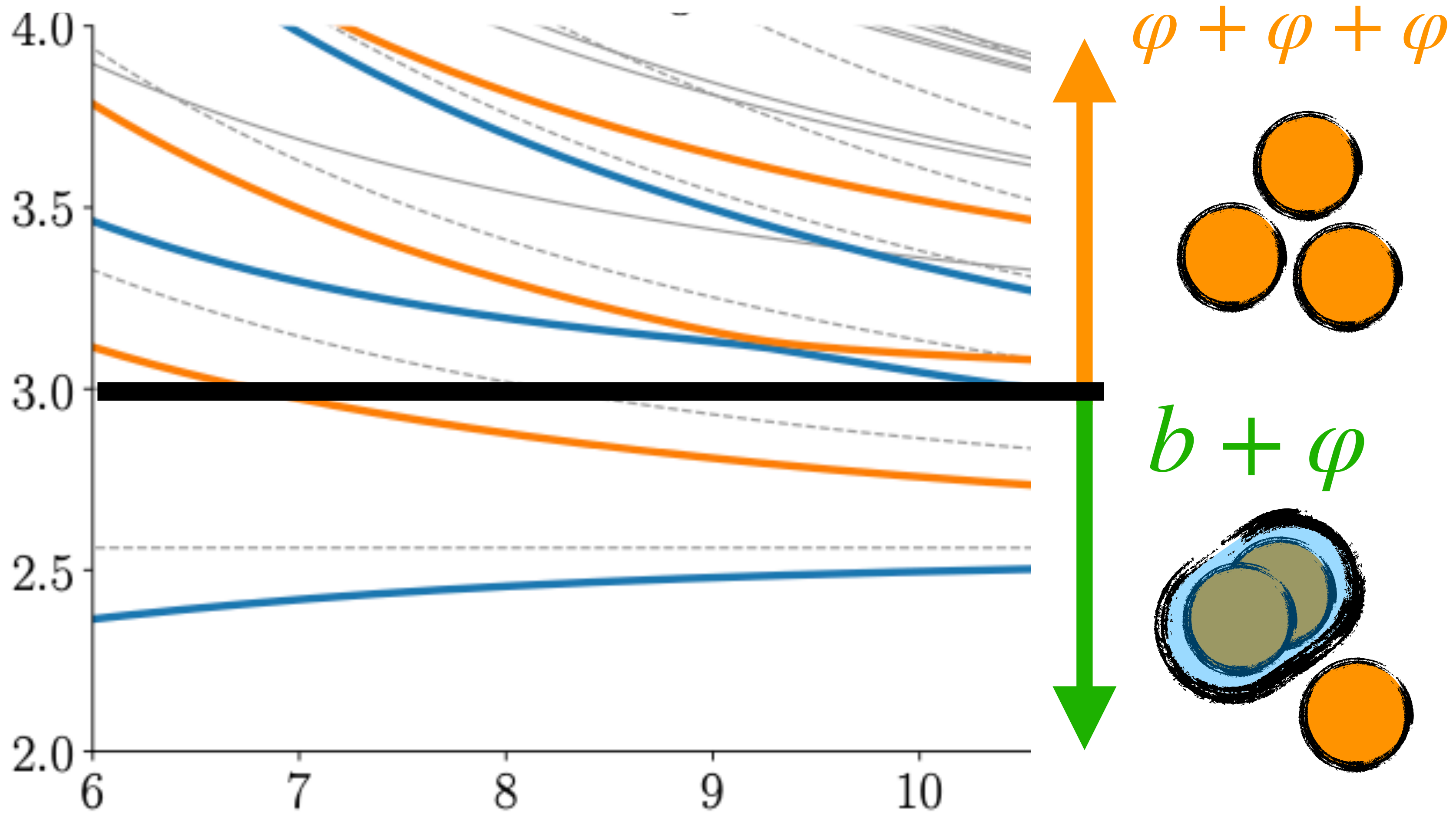
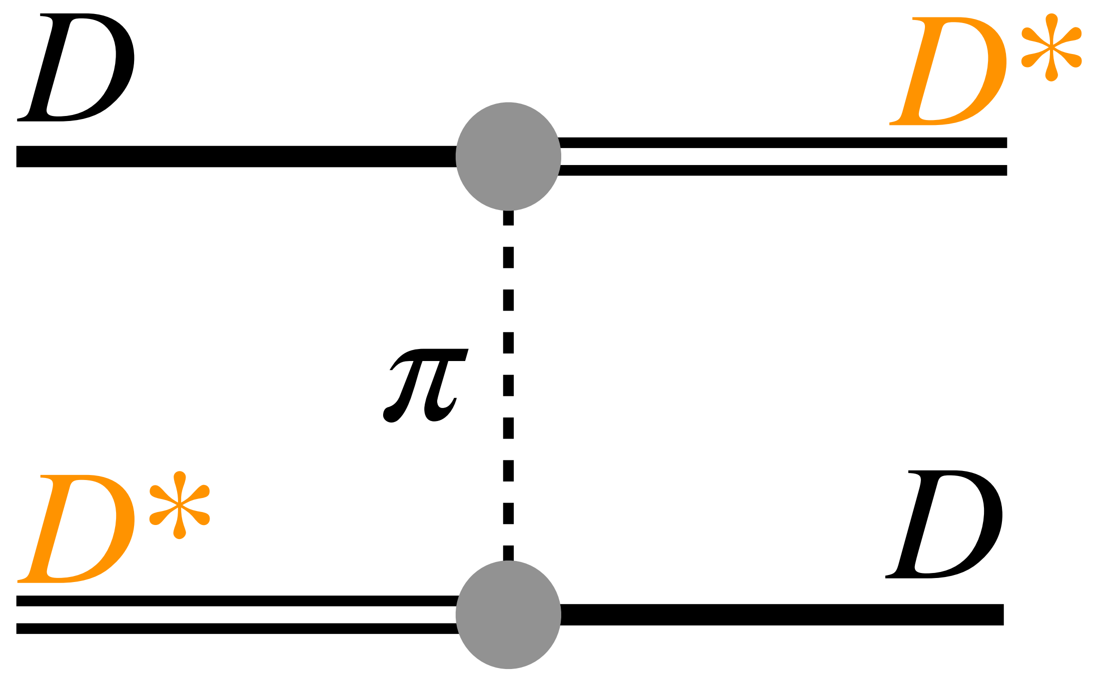
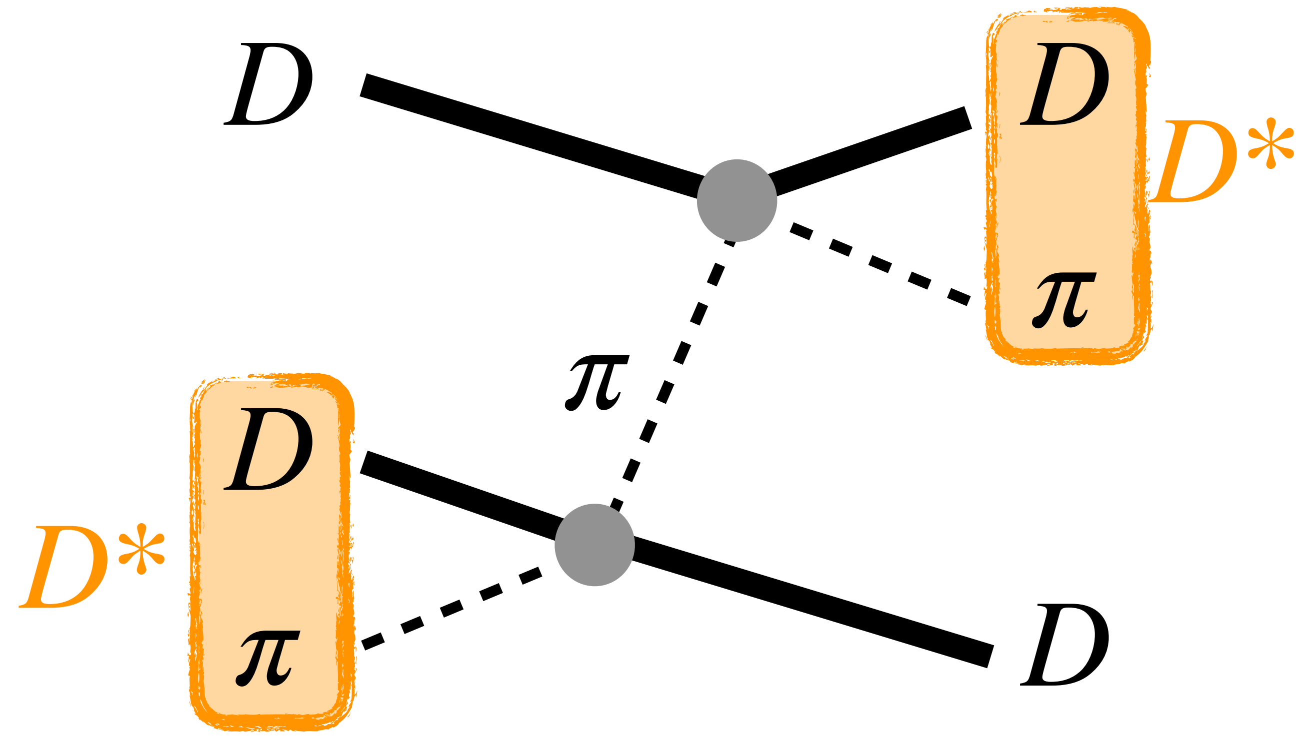



- This solves the left-hand cut problem:
  - Finite-volume effects from one-pion exchange naturally incorporated




[Hansen, FRL, Sharpe, arXiv:2401.06609]

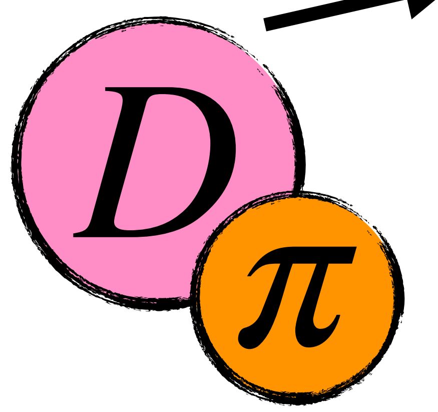


# The strategy for the Tcc

[Hansen, FRL, Sharpe, arXiv:2401.06609]

**Two-meson spectrum**

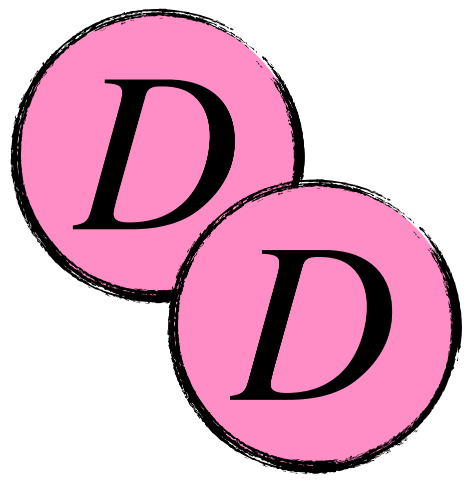
$I_{D\pi} = 1/2$




$D^*$ as a bound state or resonance

s and p waves

$I_{DD} = 1$




s wave

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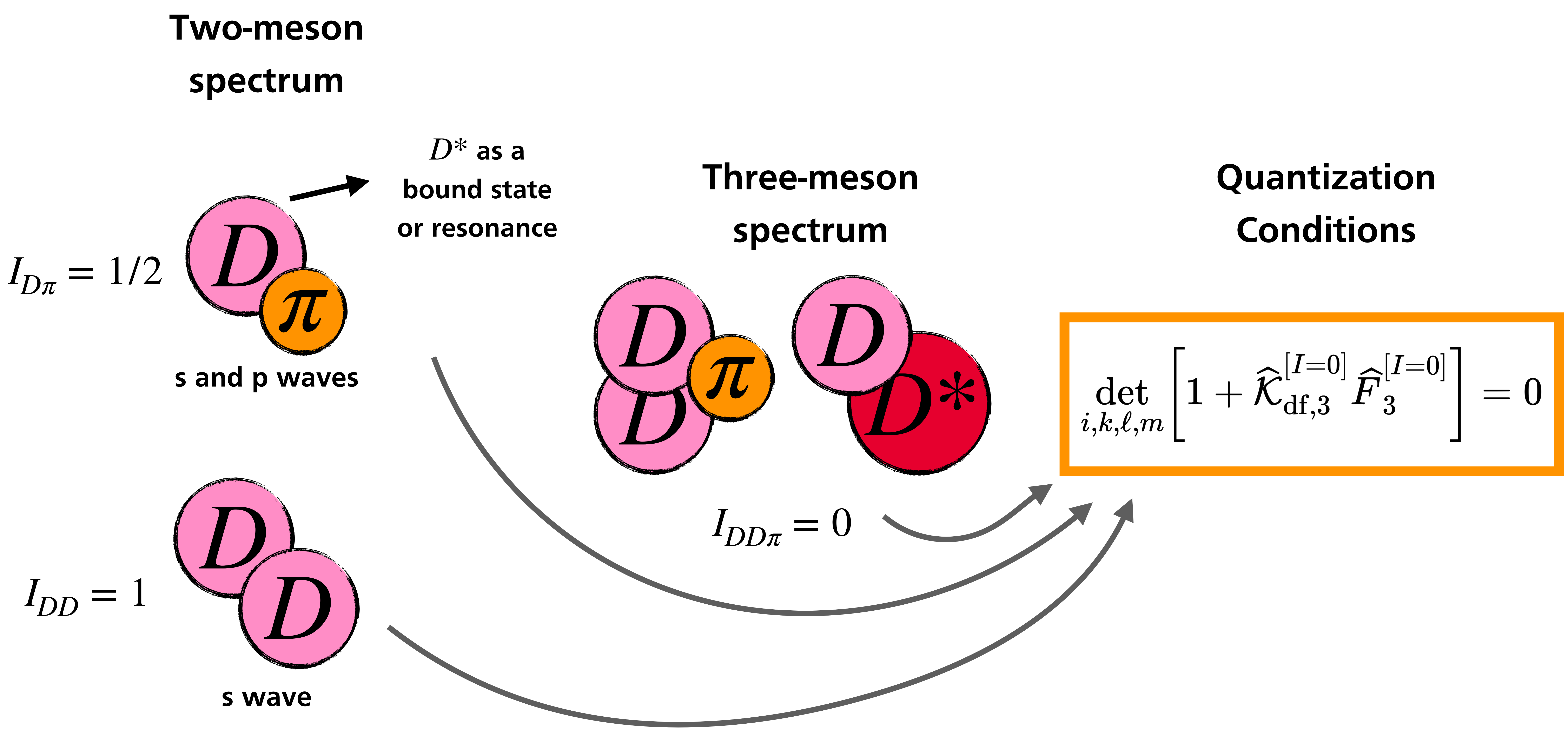

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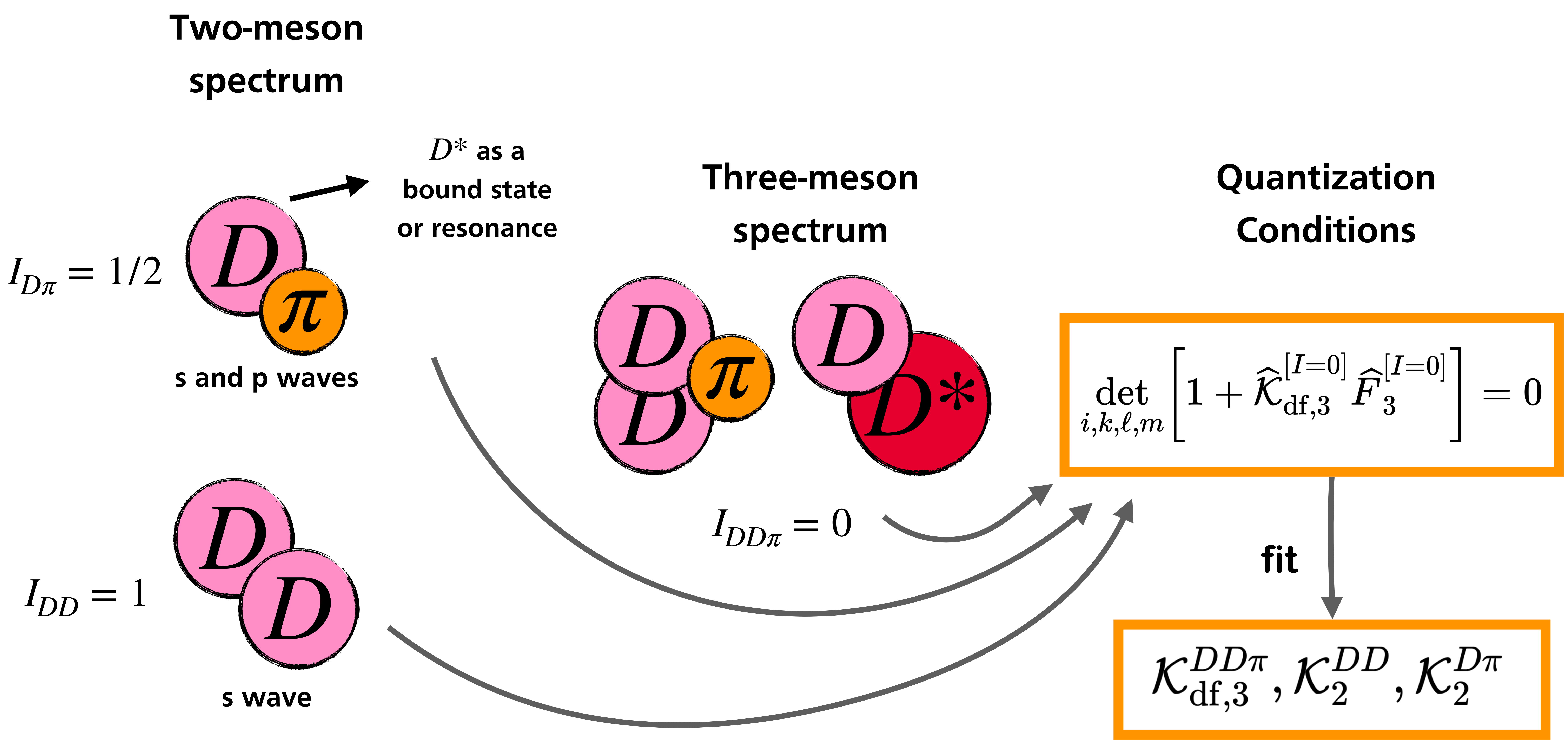

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$I_{D\pi} = 1/2$ — $D\pi$ — s and p waves

$D^*$ as a bound state or resonance

$I_{DD} = 1$ — $DD$ — s wave

**Three-meson spectrum**

$DD\pi$, $DD^*$

$I_{DD\pi} = 0$

**Quantization Conditions**

$$\det_{i,k,\ell,m}\left[1 + \widehat{\mathcal{K}}_{\mathrm{df},3}^{[I=0]}\widehat{F}_3^{[I=0]}\right] = 0$$

**fit**

$$\mathcal{K}_{\mathrm{df},3}^{DD\pi}, \mathcal{K}_2^{DD}, \mathcal{K}_2^{D\pi}$$

**Tetraquark properties**

$$\mathcal{M}_3 \sim \frac{-g^2}{s - M_{T_{cc}}^2}$$

**Integral equations**

[Dawid, FRL, Sharpe, arXiv:2409.17059]

# Analyzing D-D* data

[S. Dawid, FRL, S. Sharpe, arXiv:2409.17059]

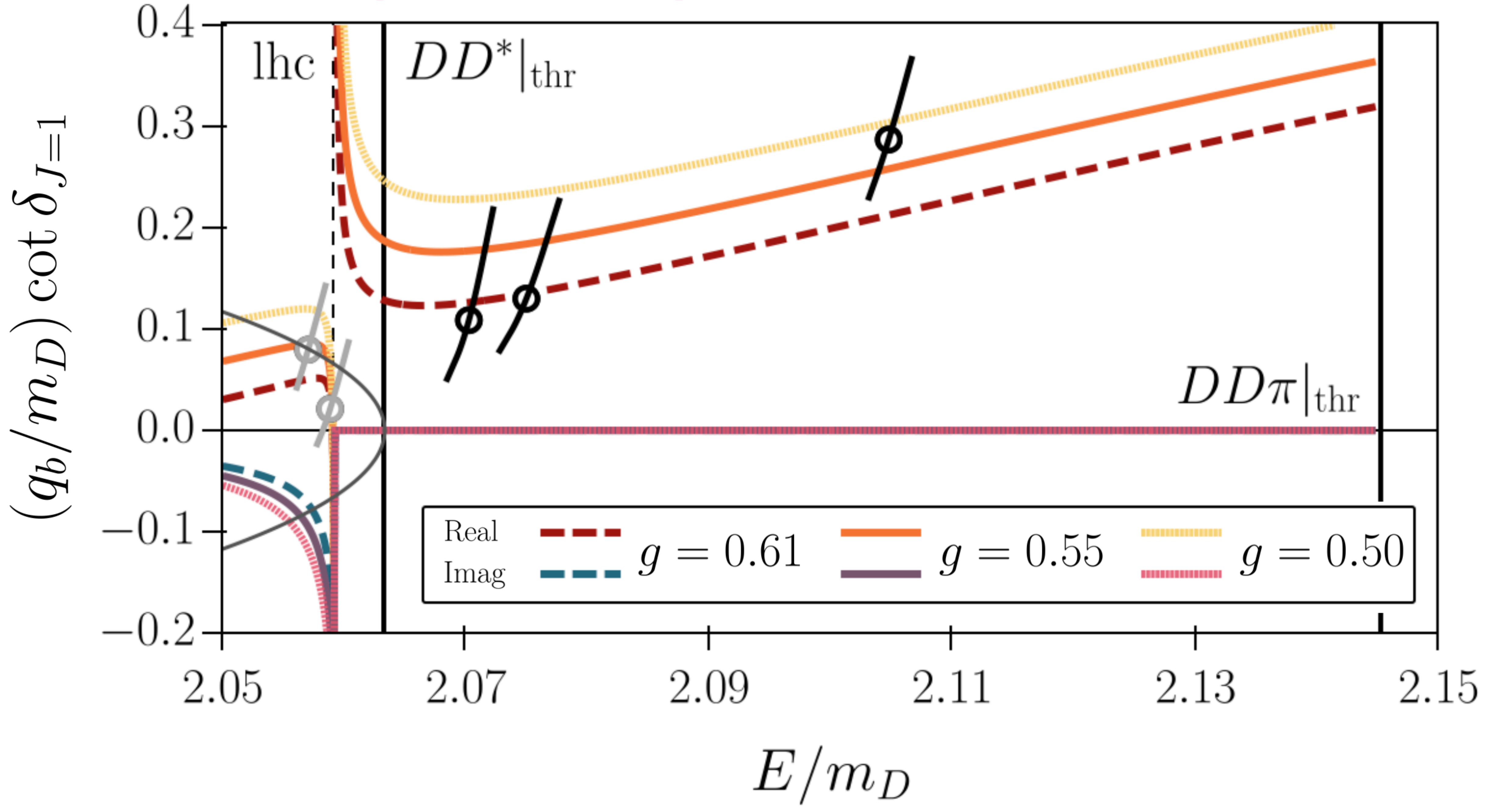
- **Published data only considers DD* energies**
  [Padmanath, Prelovsek, 2202.10110]

- $D\pi$ **and** $DD$ **interactions from "educated guesses"**
  - **HChPT and lattice results**
  - **Neglect** $DD$ **interactions**

- **Only "free" parameter in the three-body K matrix**

$$\mathcal{K}_{\text{df},3} = \mathcal{K}_E (p_\pi - p'_\pi)^2$$

**$J^P=1^+$ ($T_{cc}$ channel)**



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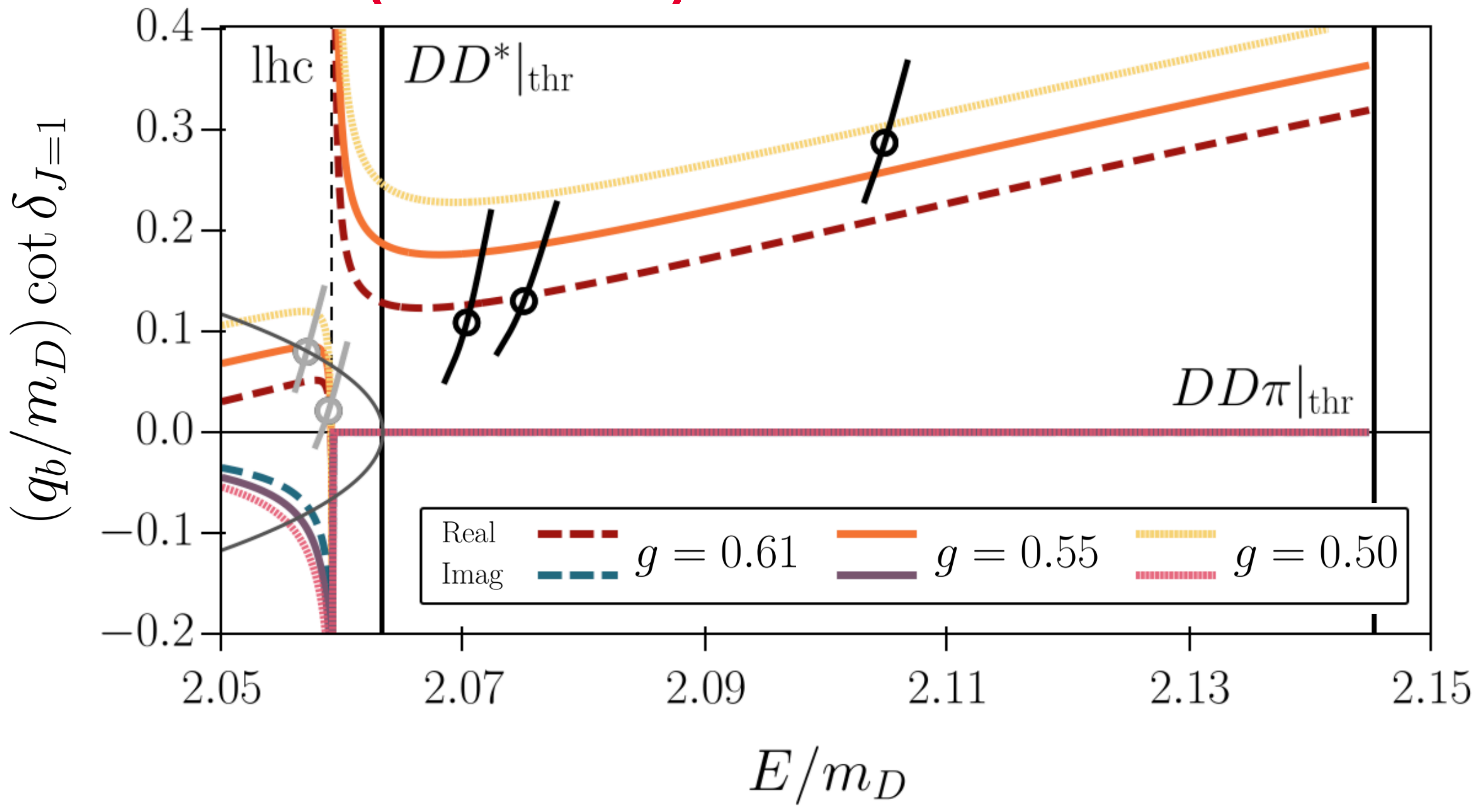


**However, moving forward, we need a genuine three-body study of the $T_{cc}$ !**
- $DD$ and $D\pi$ calculation on the same ensemble
- $DD\pi$ operators in lattice QCD calculations

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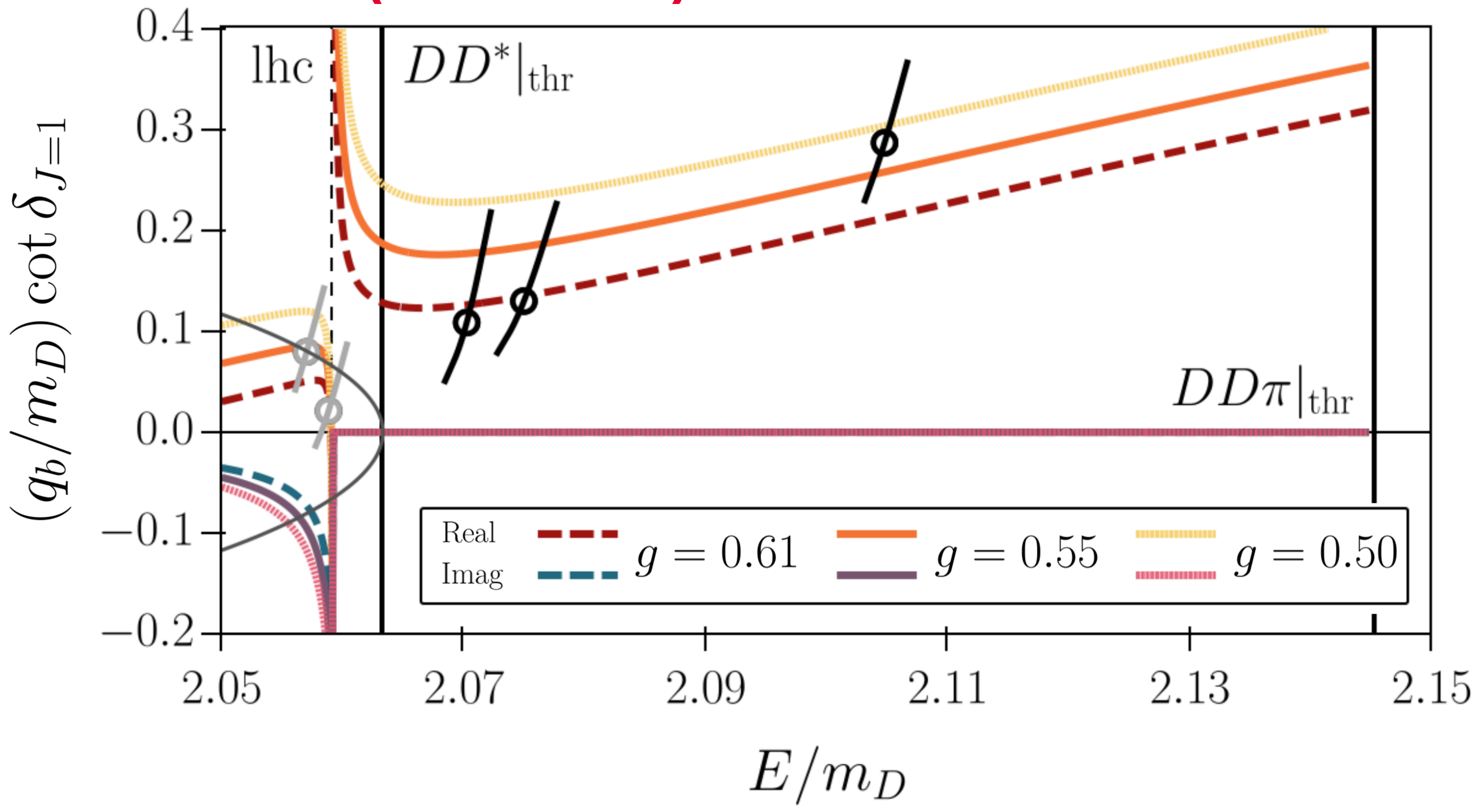


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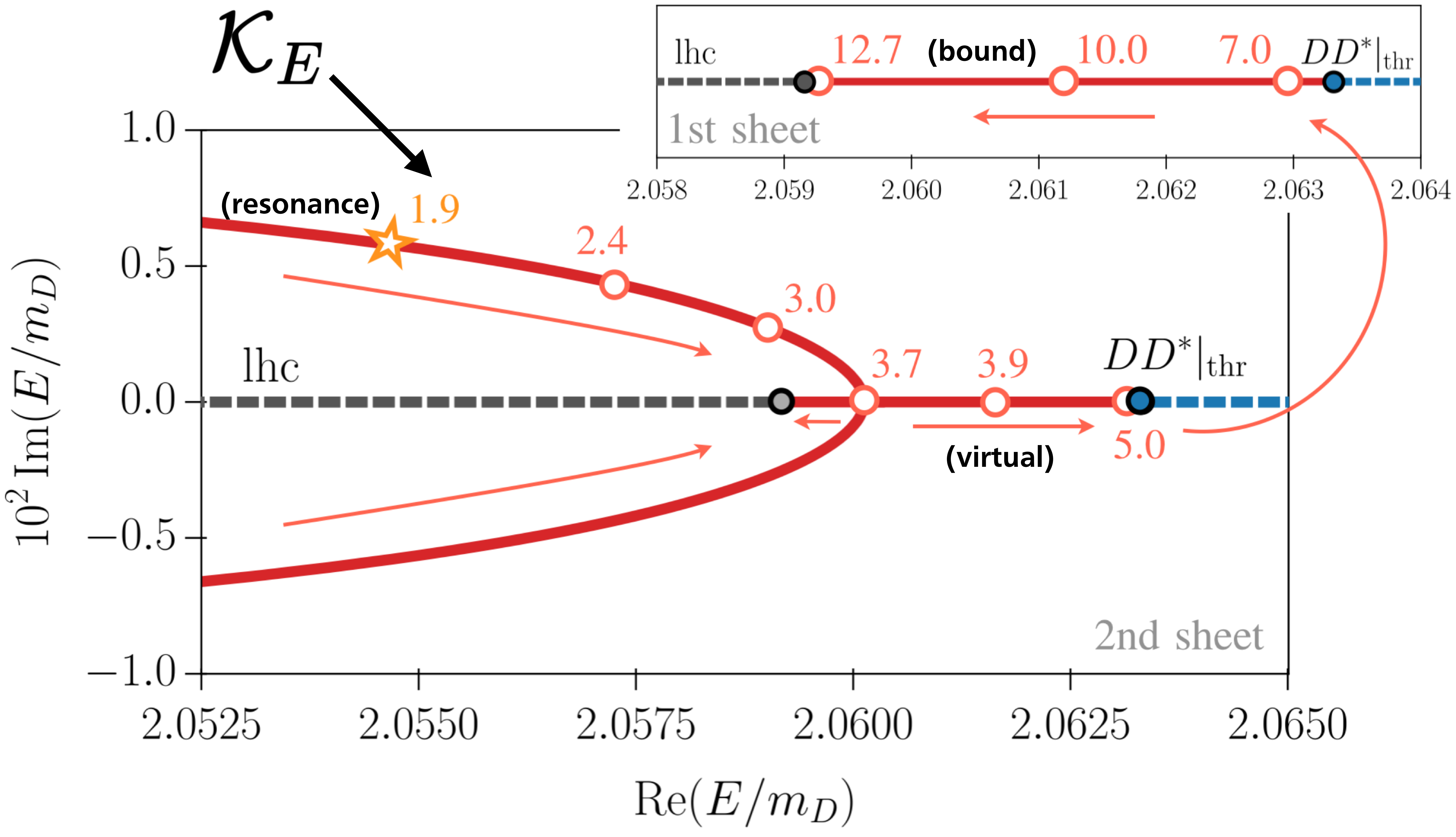
**On-going efforts:**
[Ortiz-Pacheco et al, 2312.13411 LAT23]
[Vujmilovic et al, 2411.08646 LAT24]
[Stumpf, Green, 2412.09246 LAT24]

# $T_{cc}$ pole in the complex plane

[S. Dawid, FRL, S. Sharpe, arXiv:2409.17059]

- The position of the pole as a function of the three-body force:



# Summary and Outlook

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- Lattice QCD provides a first-principle tool to investigate the hadron spectrum
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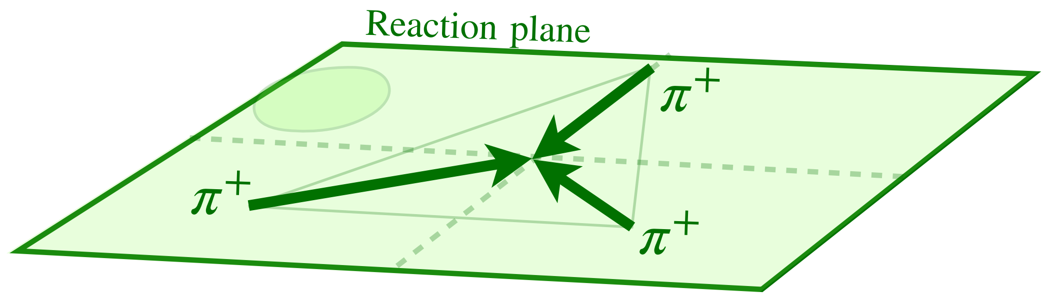
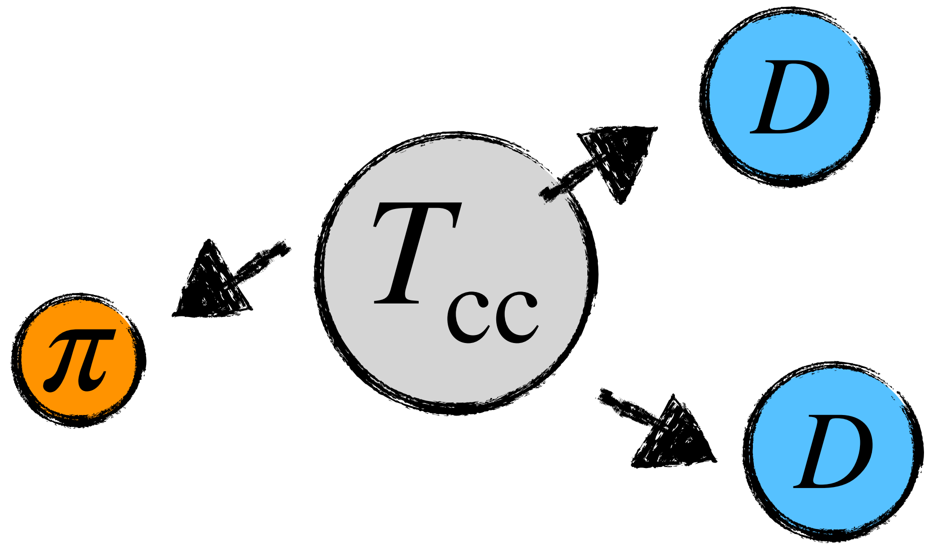
- First results on three-particle scattering amplitudes




- The formalism for DDπ systems, allowing three-body studies of the $T_{cc}$



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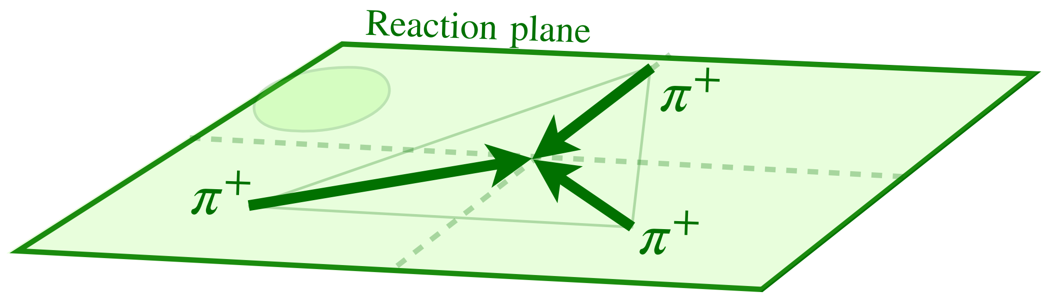


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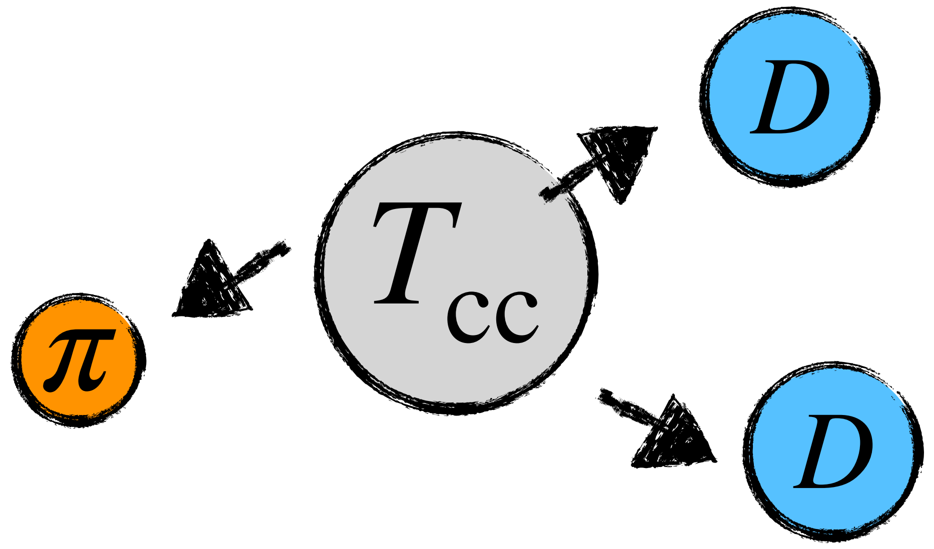


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- ☐ Further theoretical developments & applications are necessary!
  - ➡ Roper resonance $N(1440) \to N\pi, N\pi\pi$
  - ➡ Four or more particle resonances [See talk by Agostino Patella]

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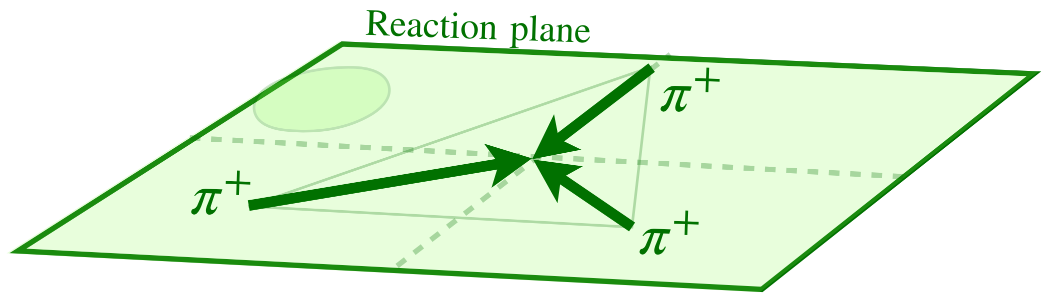


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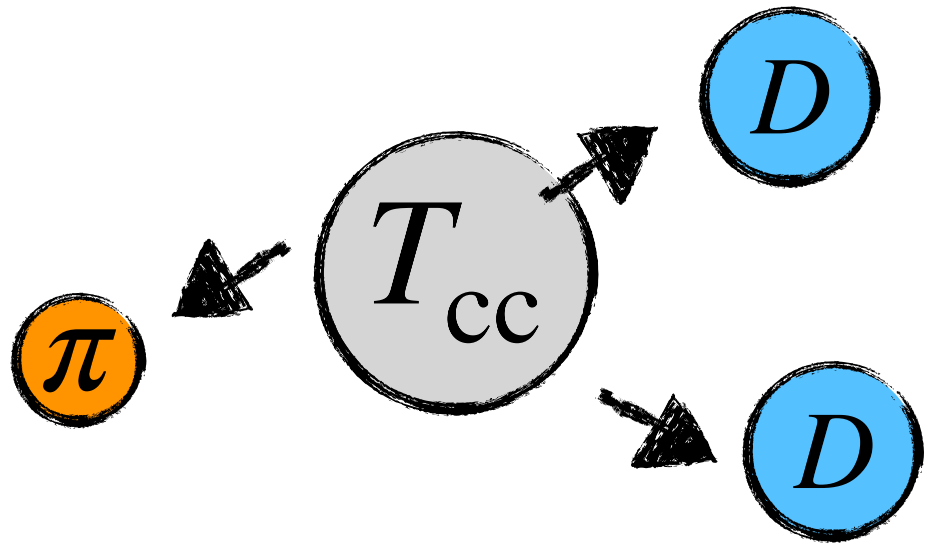


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# Thanks!

# Back-up

# Example: I=2 $\pi\pi$ scattering

**Two pions in s-wave**

$$\mathcal{K}_2^{s-\mathrm{wave}}(E_n) = \frac{-1}{F_{00}(E_n, \vec{P}, L)}$$

$$\mathcal{K}_2^{s-\mathrm{wave}} = \frac{16\pi\sqrt{s}}{q\cot\delta_0}$$

one energy level → a phase shift point

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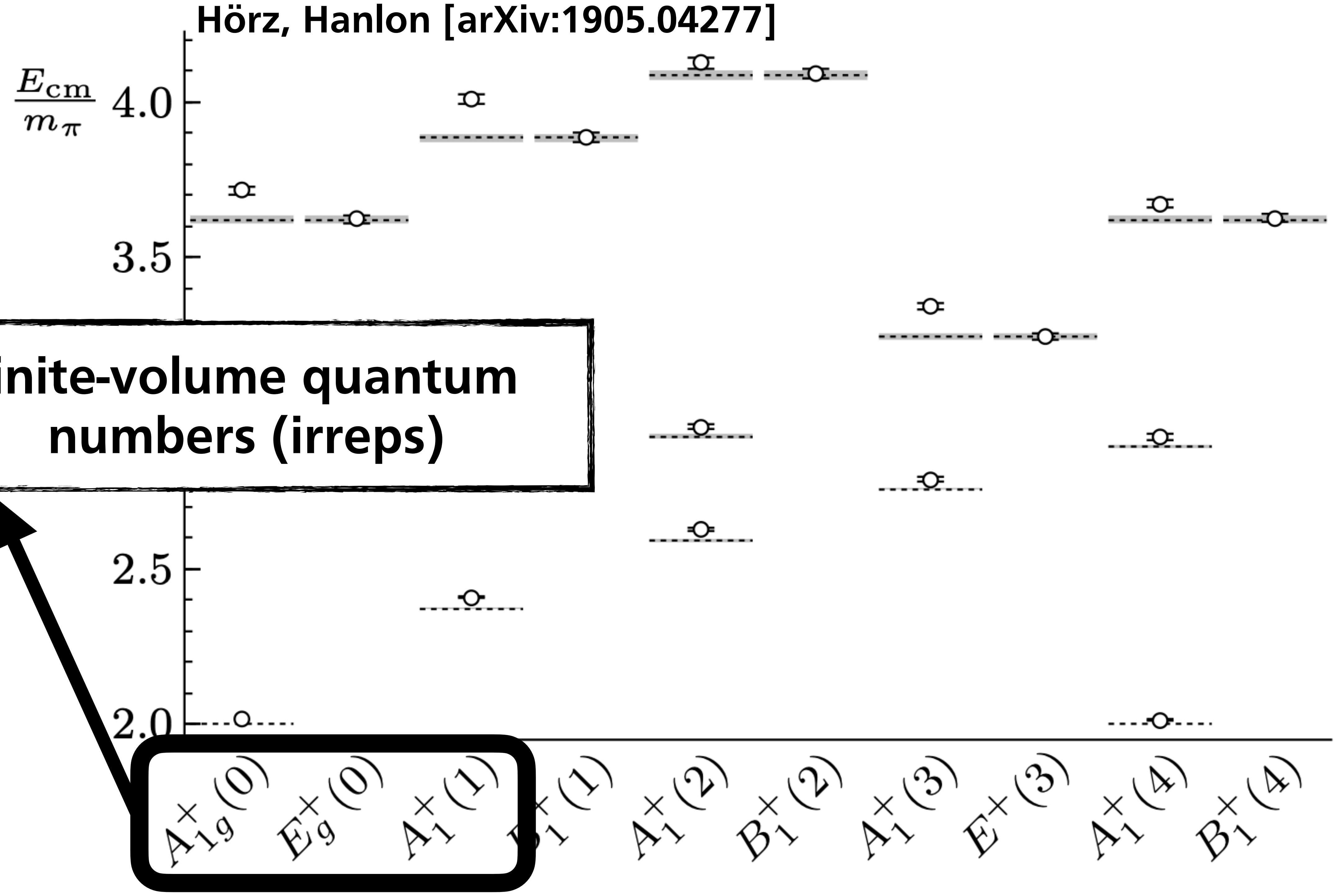
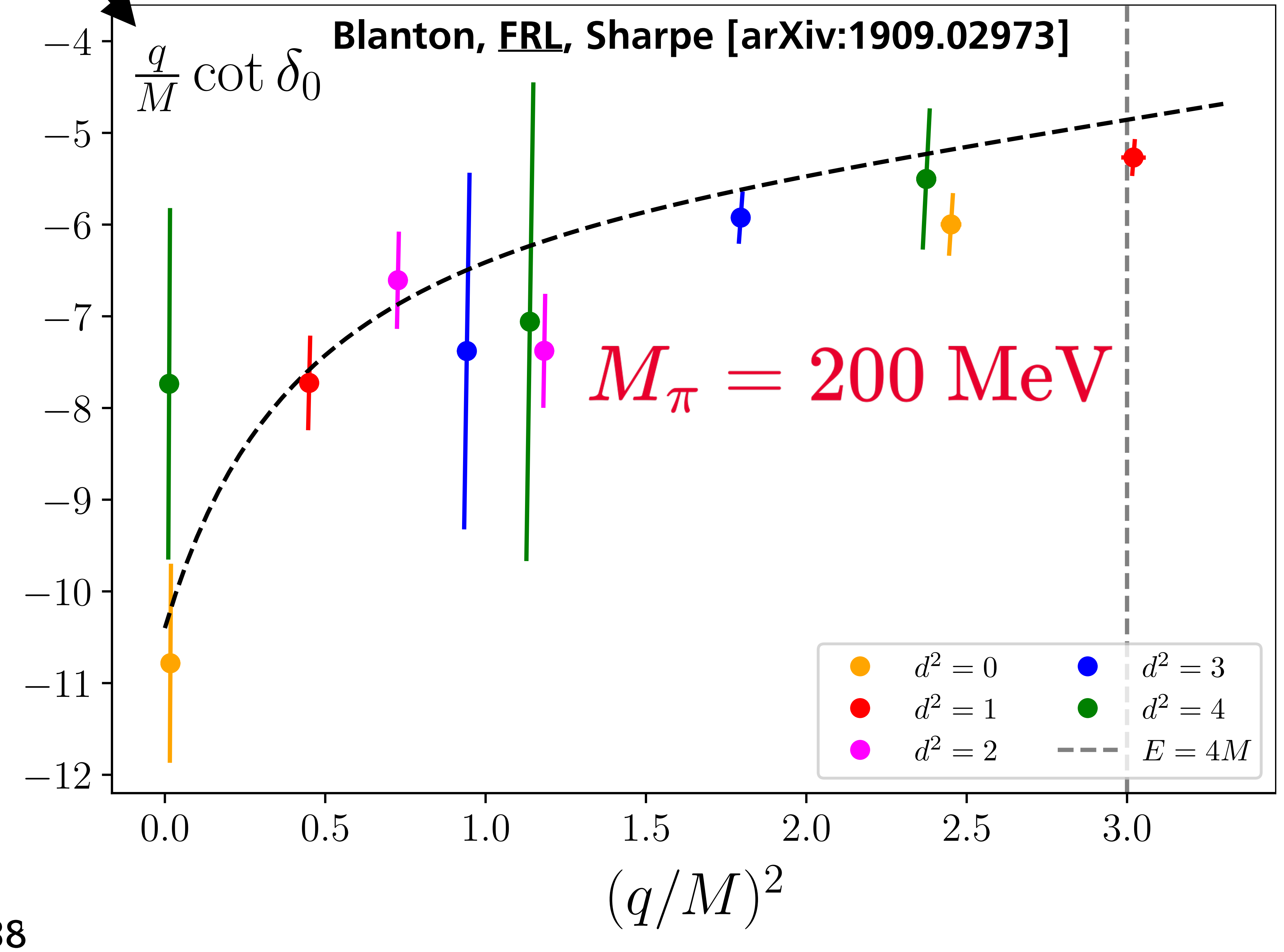



finite-volume quantum numbers (irreps)



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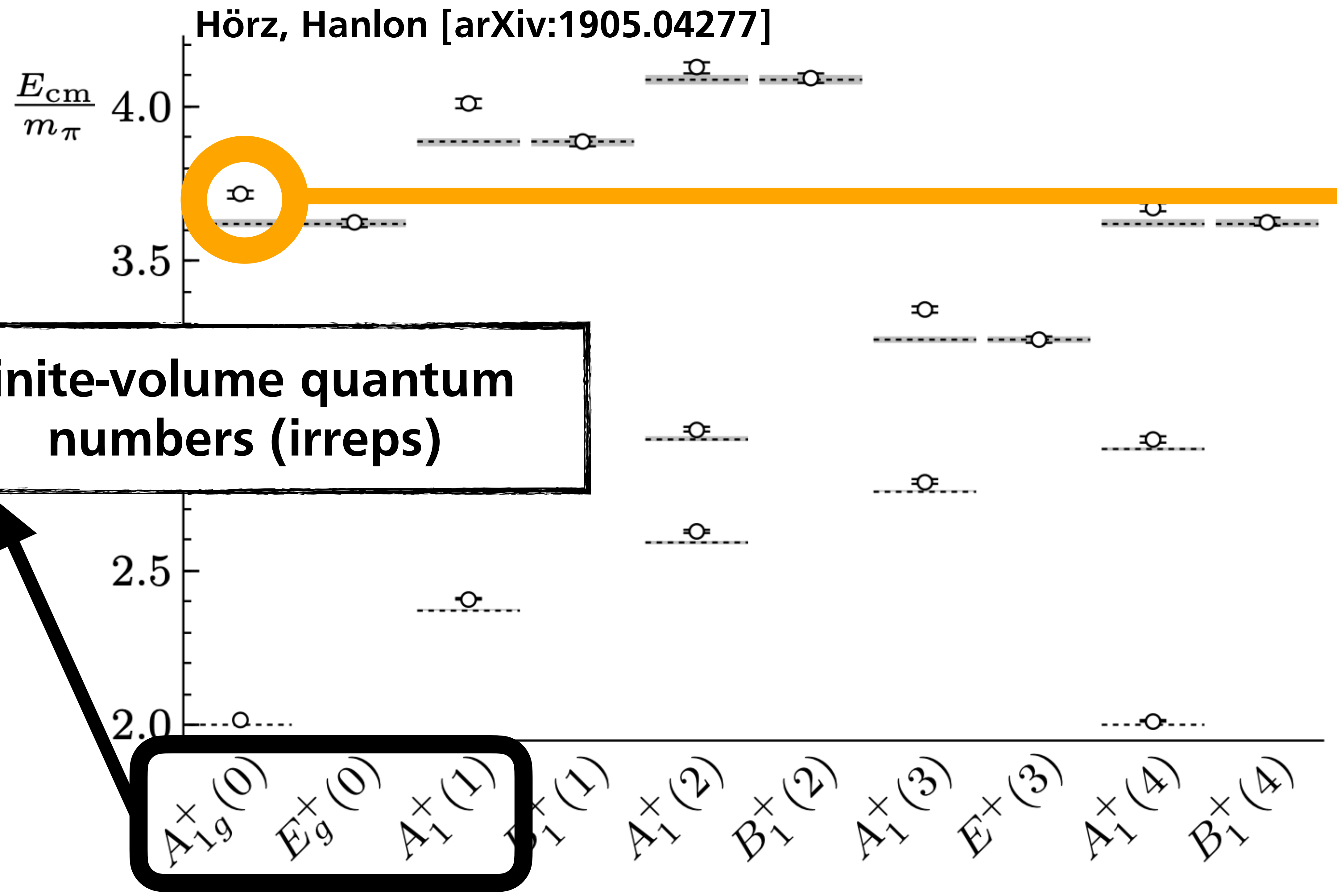


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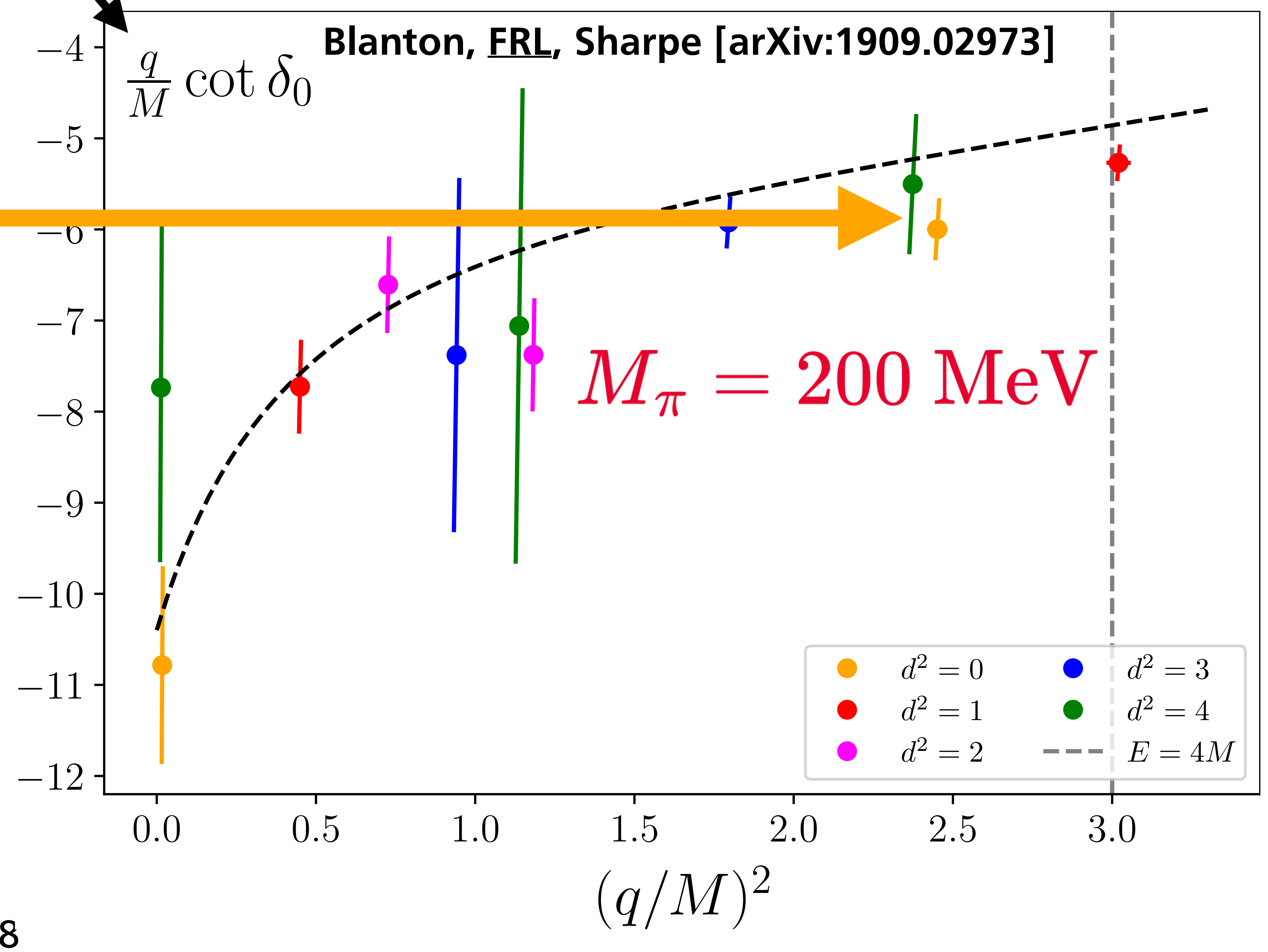

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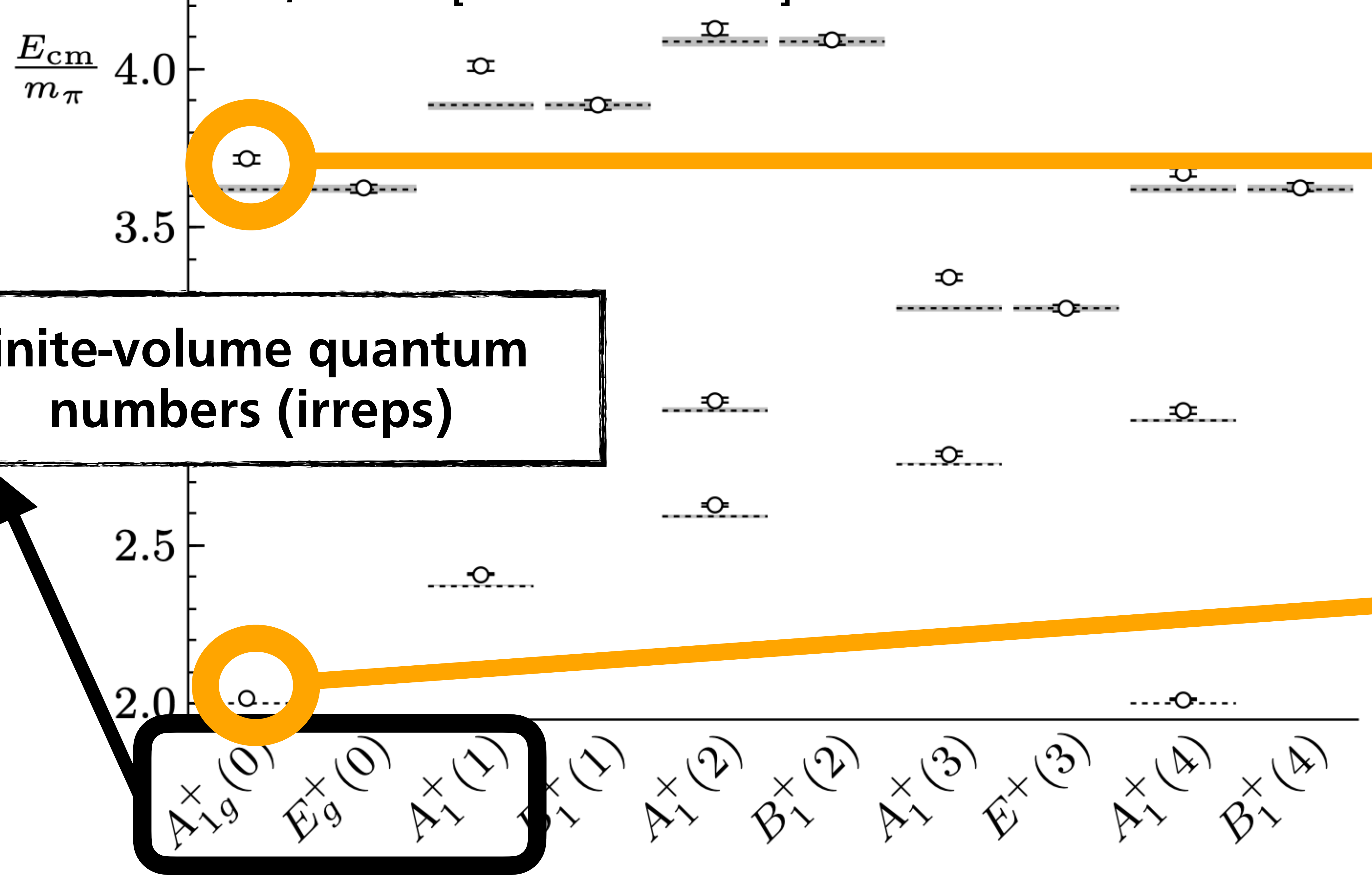
Hörz, Hanlon [arXiv:1905.04277]

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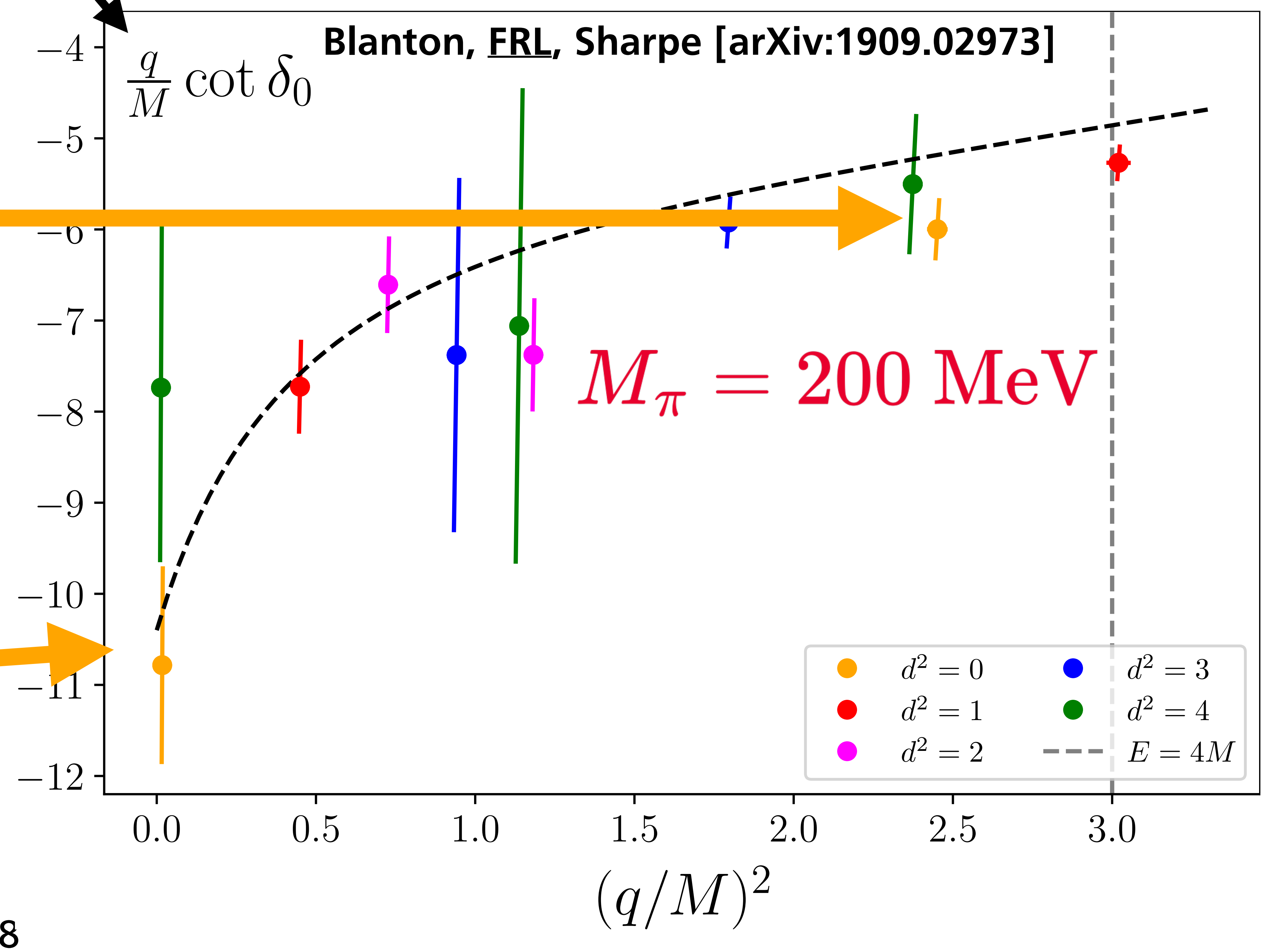

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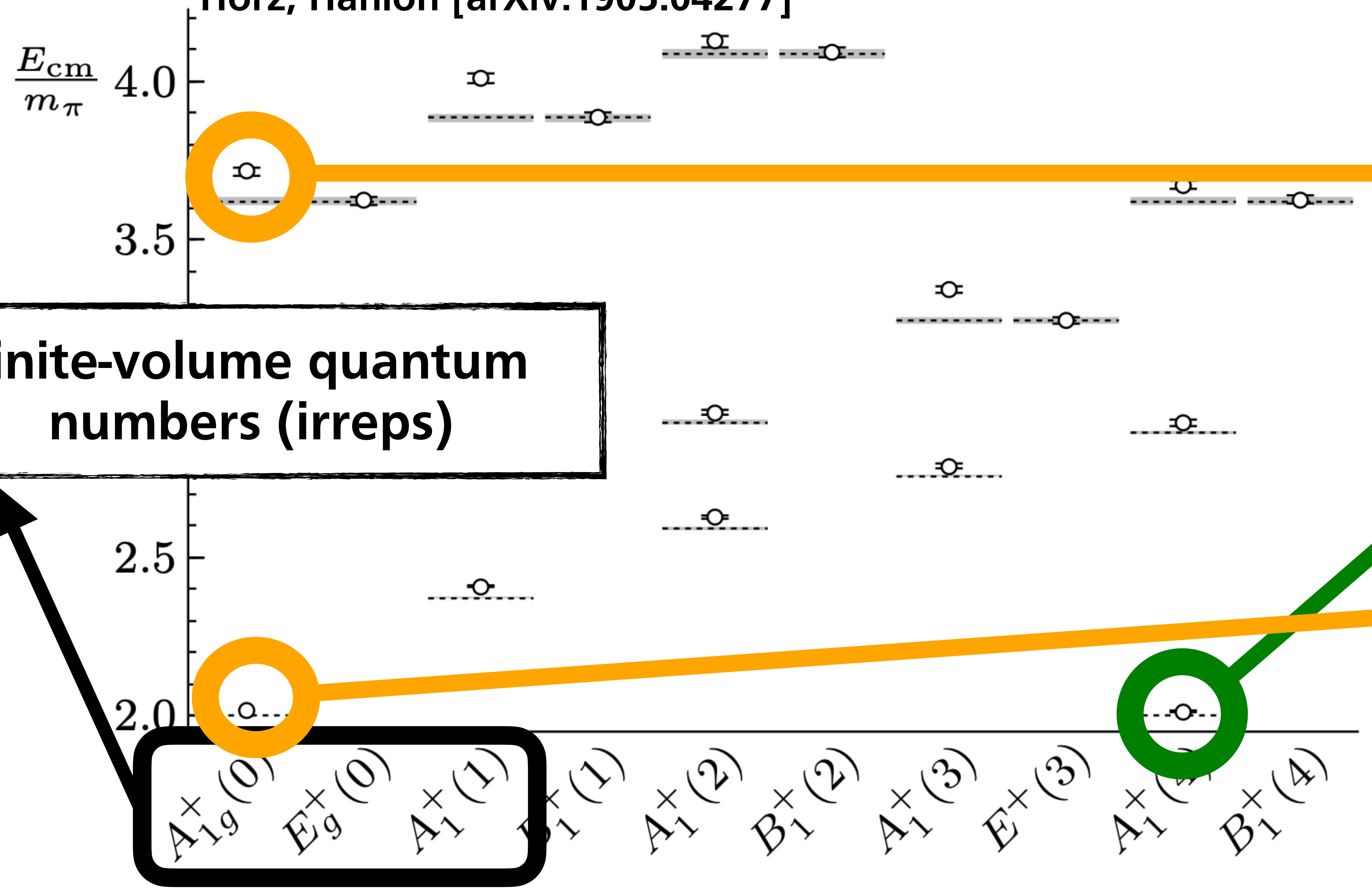


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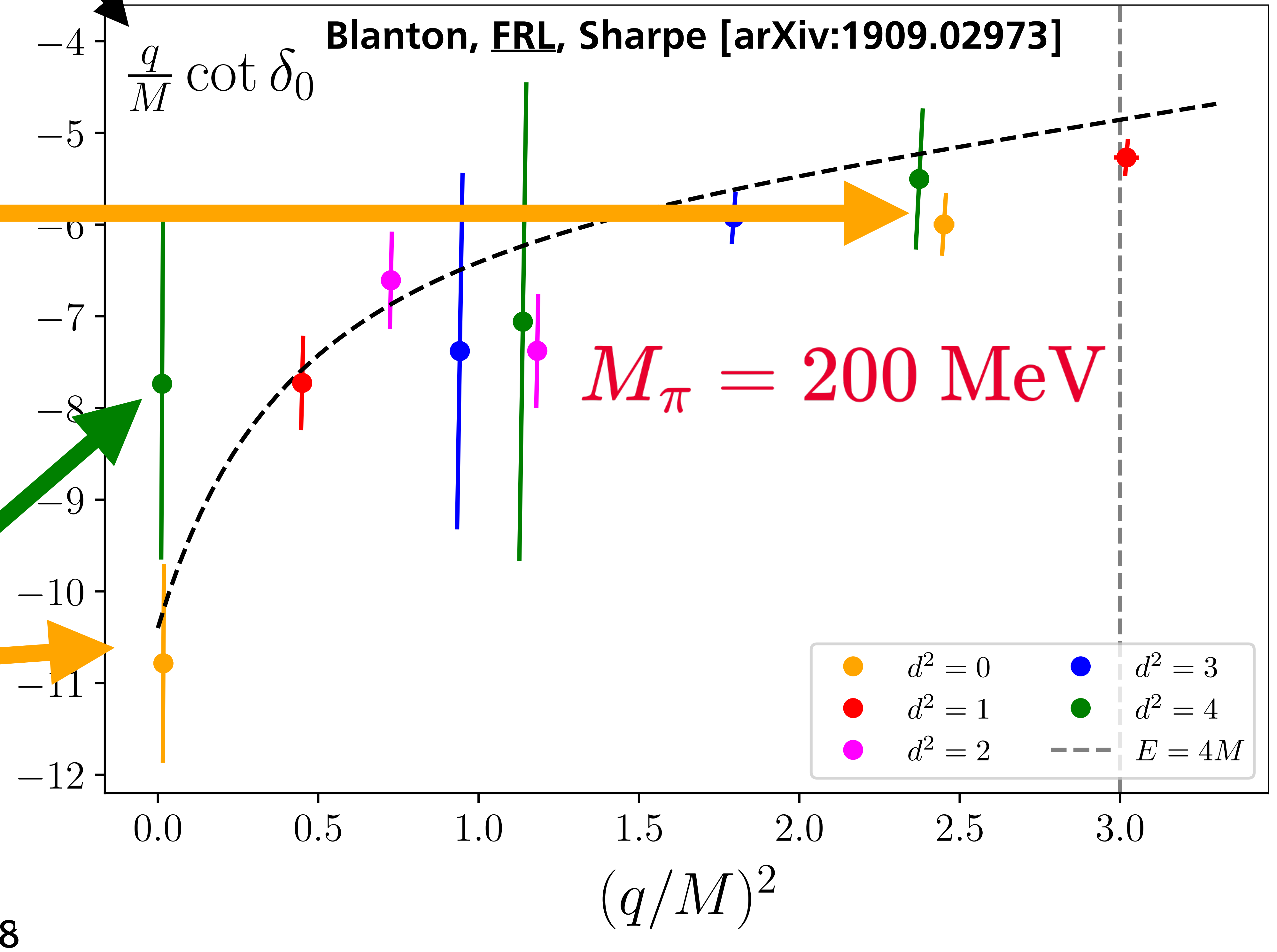
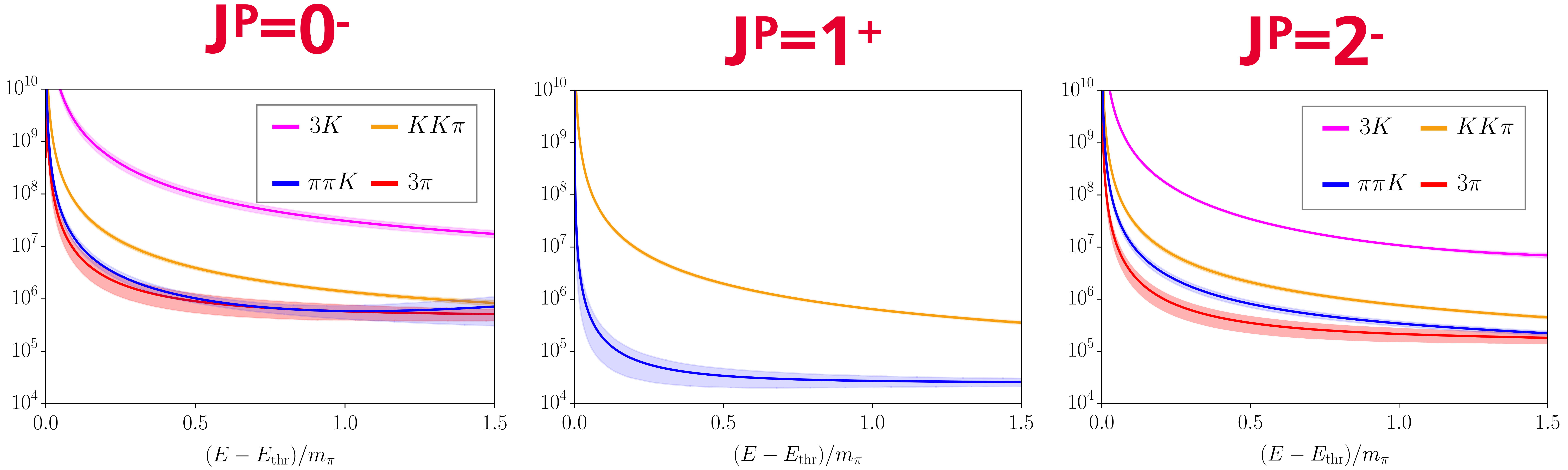





Fernando Romero-López, Uni Bern

# Other scattering amplitudes



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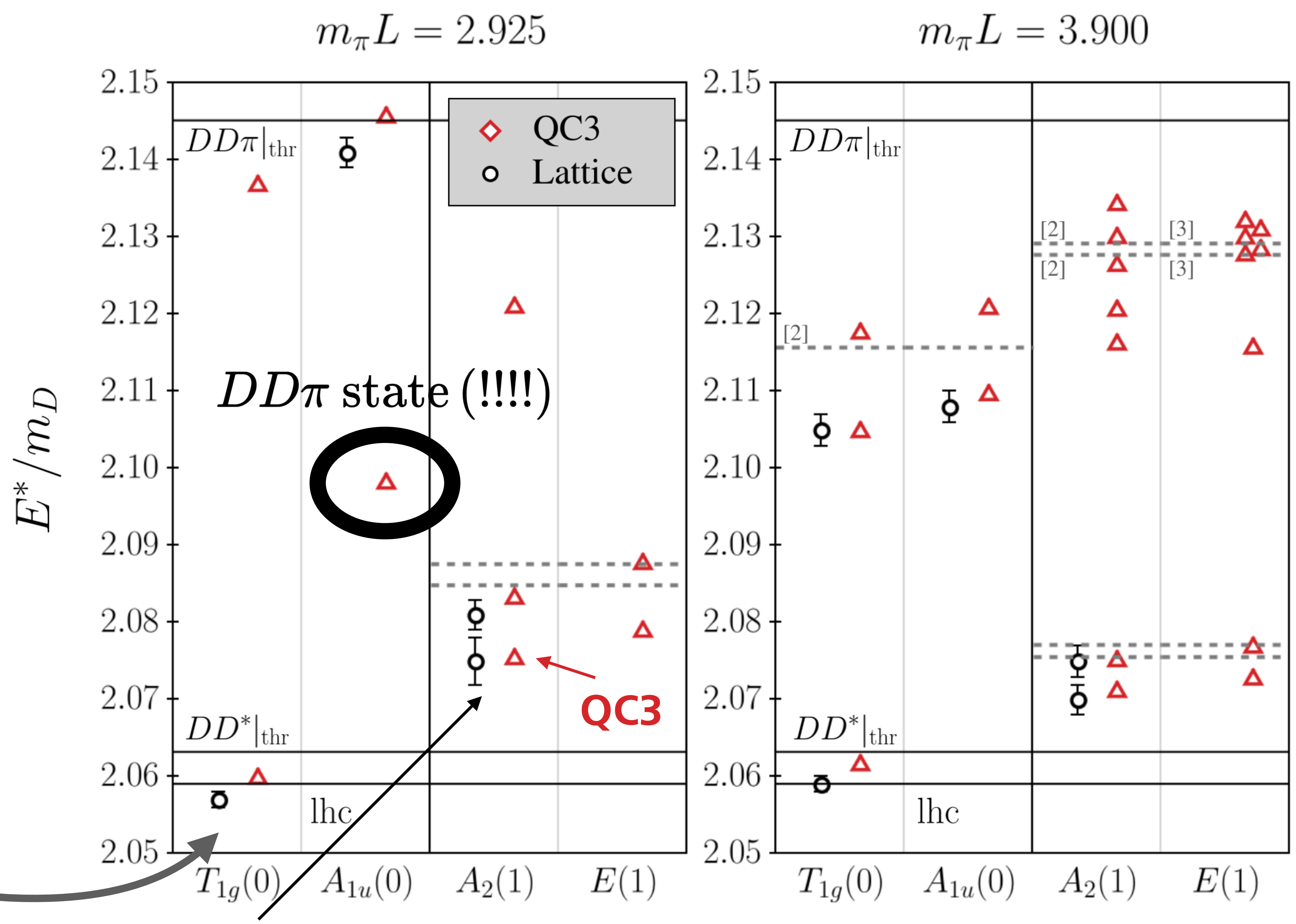
[S. Dawid, FRL, S. Sharpe, arXiv:2409.17059]



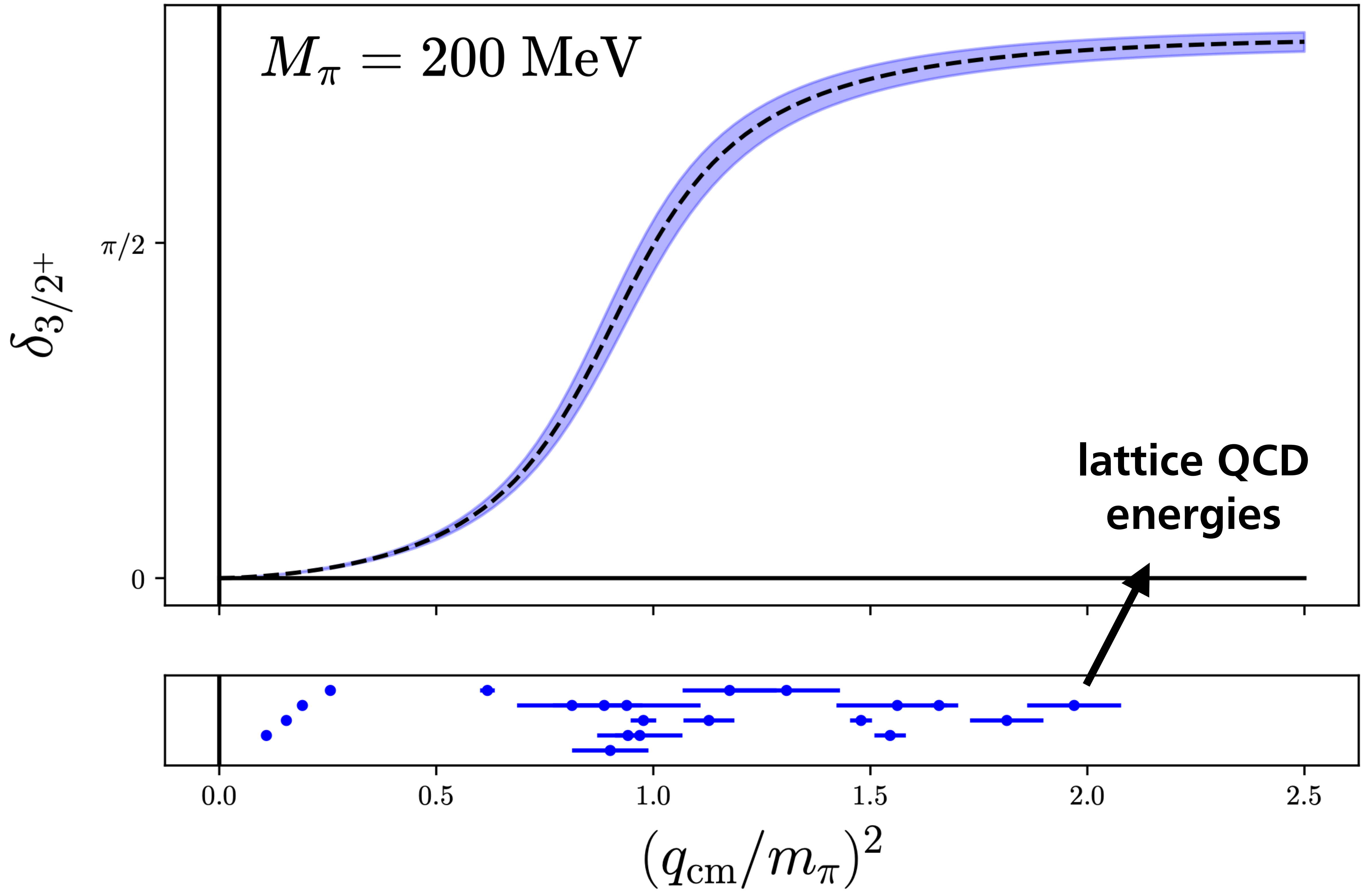

Finite-volume energies near the left-hand cut

Description of several irreps (and thus partial waves)

# The $\Delta(1232)$ from LQCD

## Lattice QCD

[Bulava, Hanlon, Hörz, Morningstar, Nicholson, FRL, Skinner, Vranas, Walker-Loud, 2208.03867]



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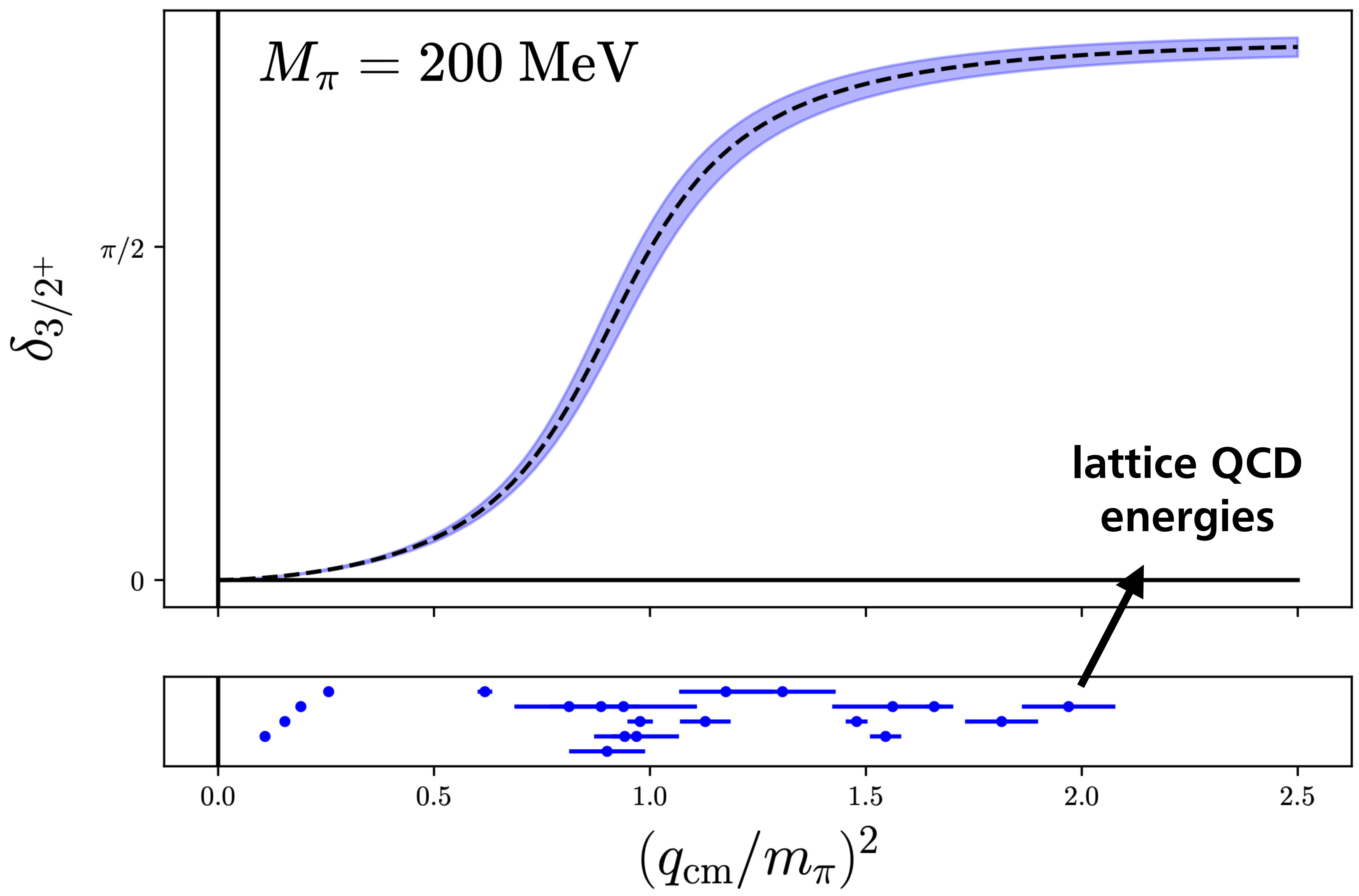
## Lattice QCD

[Bulava, Hanlon, Hörz, Morningstar, Nicholson, FRL, Skinner, Vranas, Walker-Loud, 2208.03867]




## Experiment

scattering phase