Recent developments for proton GPDs from lattice QCD

Martha Constantinou

Temple University

Second LatticeNET workshop on challenges in Lattice field theory

(Benasque Science Center, Mar 30 - Apr 05, 2025)

OUTLINE

A. Methods to access GPDs from lattice QCD

B. Selected results for the proton:
- twist-2 GPDs
- twist-3 GPDs

C. Synergy with phenomenology

D. Concluding remarks

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A. Methods to access GPDs from lattice QCD

B. Selected results for the proton:
- twist-2 GPDs
- twist-3 GPDs

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

C. Synergy with phenomenology

D. Concluding remarks

Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L ($\gamma^+\gamma^5$)	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity

(Selected) **Twist-3** $(f_i^{(1)})$

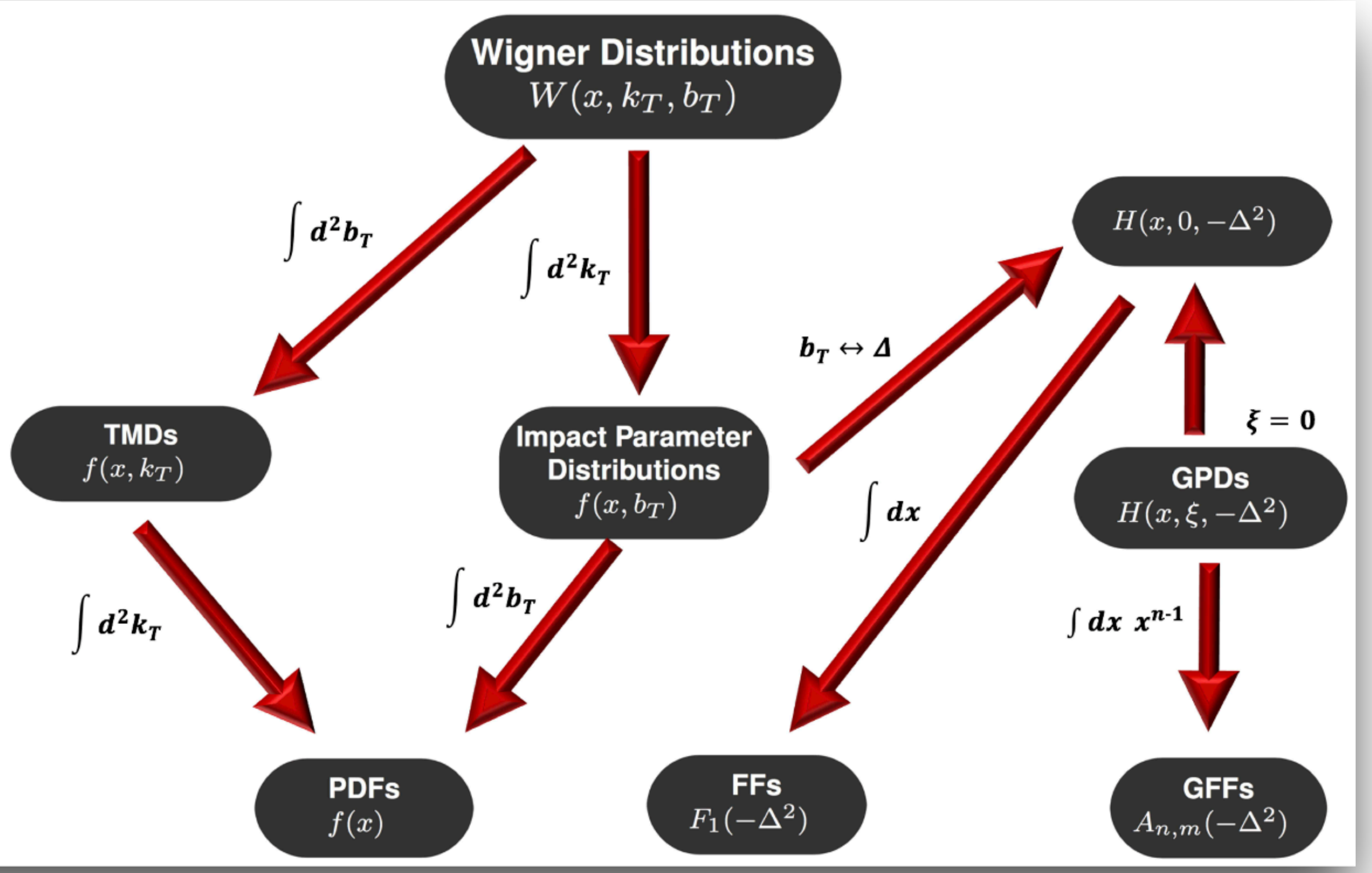
$\mathcal{O}$ / Nucleon	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

Nucleon Characterization

Wigner distributions

★ Fully characterize partonic structure of hadrons

★ Provide multi-dim images of the parton distributions in phase space



★ Correlations between momenta, positions, spins

★ Information on the hadron's mechanical properties (OAM, pressure, etc.)

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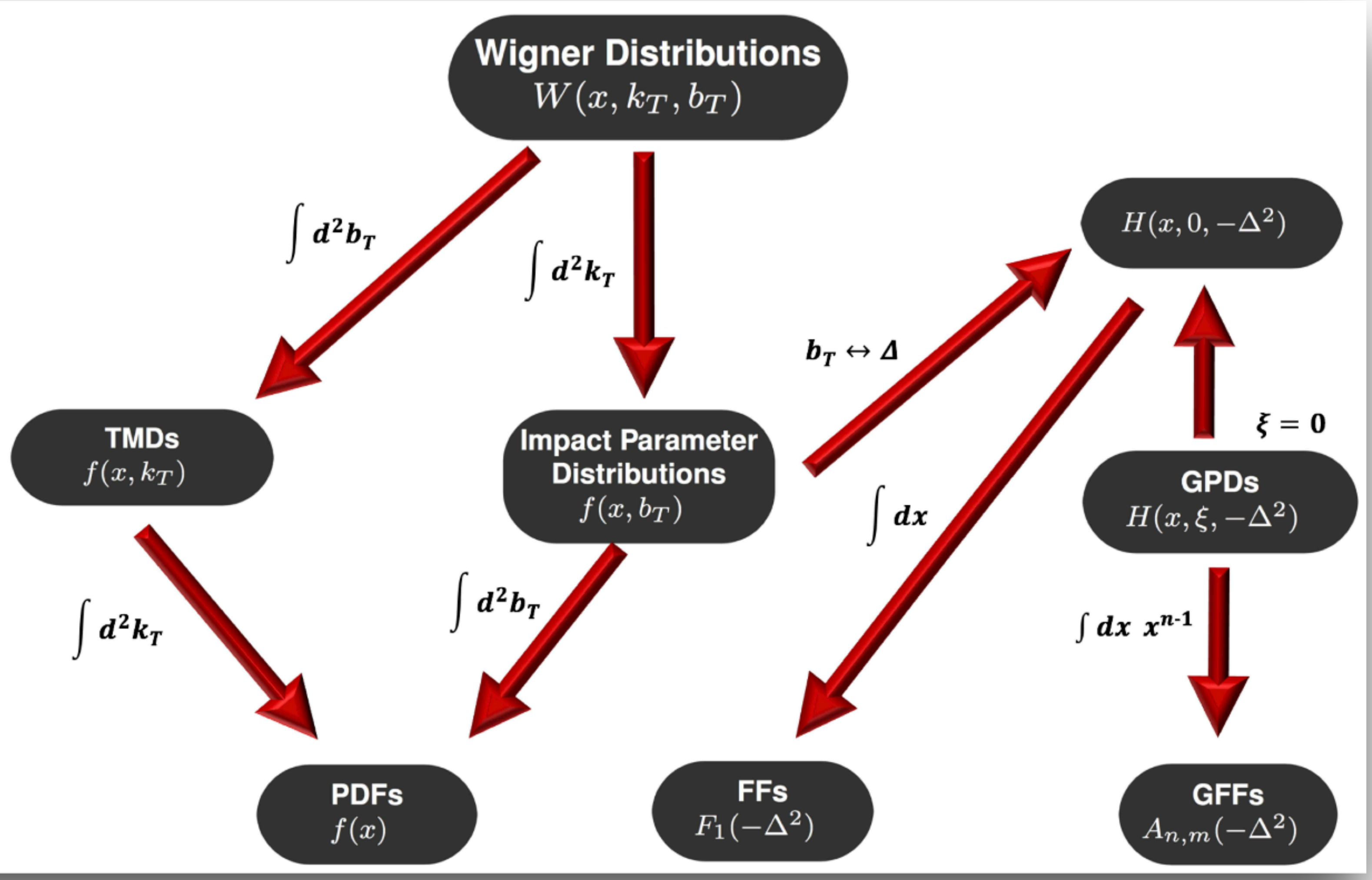
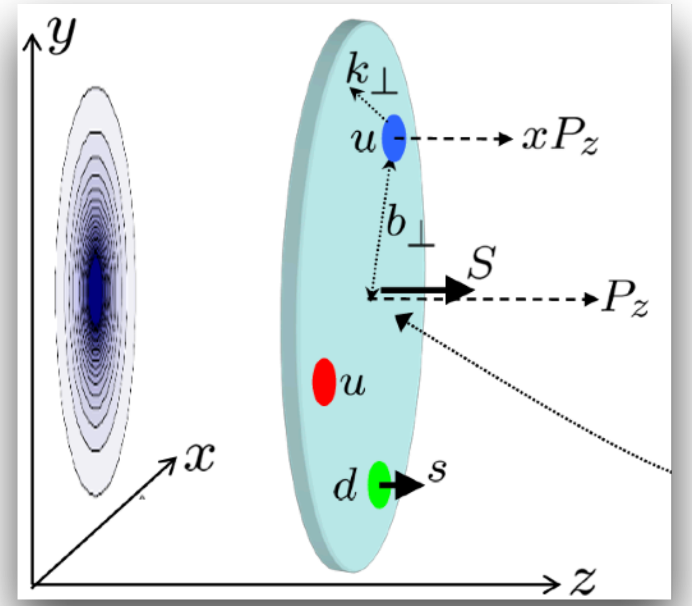


★ Partons contain information on

x: longitudinal momentum fraction

k_T: transverse momentum

$b_\perp$: impact parameter



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Accessing PDFs/GPDs from lattice QCD

Accessing information on PDFs/GPDs

- ★ Parton model: physical picture valid for infinite momentum frame

  [R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969)]

- ★ PDFs via matrix elements of nonlocal light-cone operators ($-t^2 + \vec{r}^2 = 0$)

$$f(x) = \frac{1}{4\pi} \int dy^- e^{-ixP^+y^-} \langle P, S | \bar{\psi}_f \gamma^+ \mathcal{W} \psi_f | P, S \rangle$$

- ★ Light-cone correlations inaccessible from Euclidean lattices ($\tau^2 + \vec{r}^2 = 0$)

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A. Mellin moments (local OPE expansion)

local operators

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z, \tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty} \frac{1}{n!}\, z_{\alpha_1} \ldots z_{\alpha_n} \left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1} \ldots \overleftrightarrow{D}^{\alpha_n} q\right]$$

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1} \left\{\gamma^{\{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t) - i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\} + \frac{\Delta^\mu\Delta^{\mu_1}\cdots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\text{ even}}\Big)\Big\} + \frac{\Delta^\mu\Delta^{\mu_1}\cdots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\text{ even}}\Big]U(P)$$

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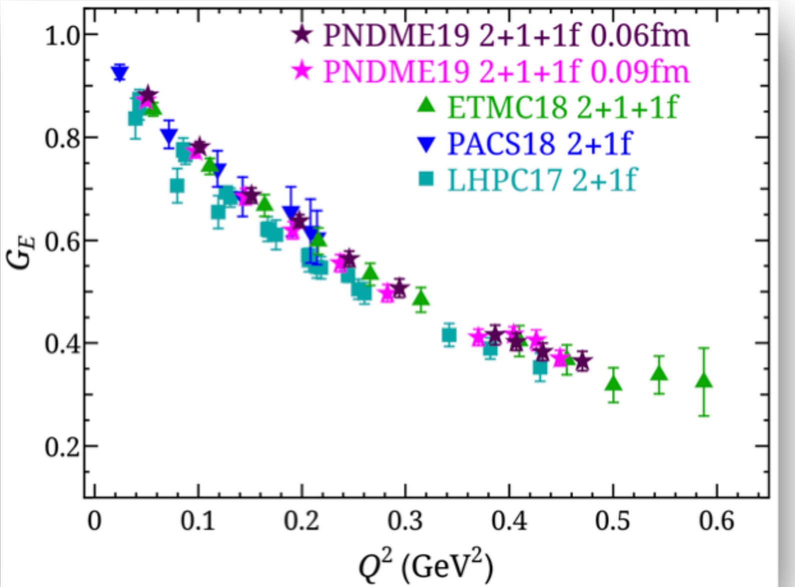
- 👍 Statistical uncertainty can be controlled
- 👍 contain physical information
- 👎 No direct access to x
- 👎 skewness independent
- 👎 Power-divergent mixing for high Mellin moments (derivatives > 3)
- 👎 Signal-to-noise ratio decays with the addition of covariant derivatives
- 👎 Number of GFFs increases with order of Mellin moment

Accessing information on PDFs/GPDs

Computationally efficient extraction of Q^2 dependence



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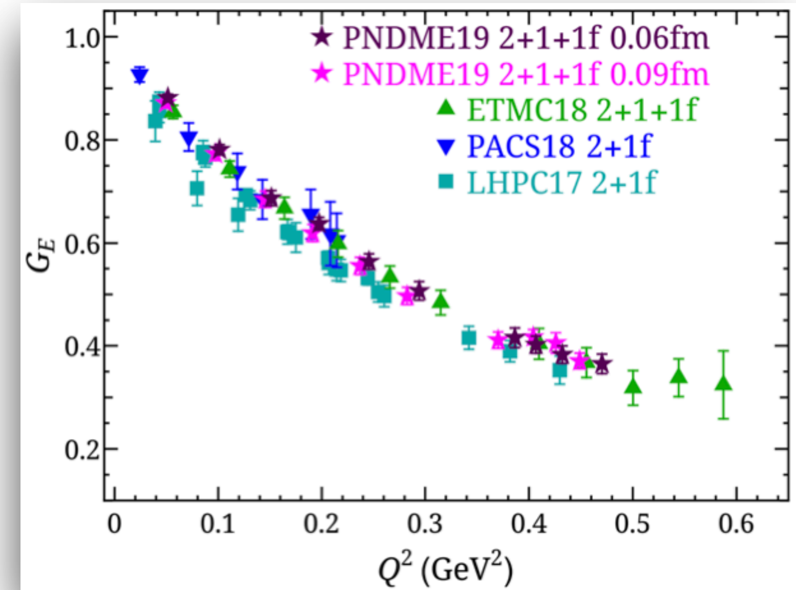

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Reconstruction of PDFs/GPDs very challenging

Accessing information on PDFs/GPDs

B. Matrix elements of nonlocal operators (quasi-GPDs, pseudo-GPDs)

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Nonlocal operator with Wilson line

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

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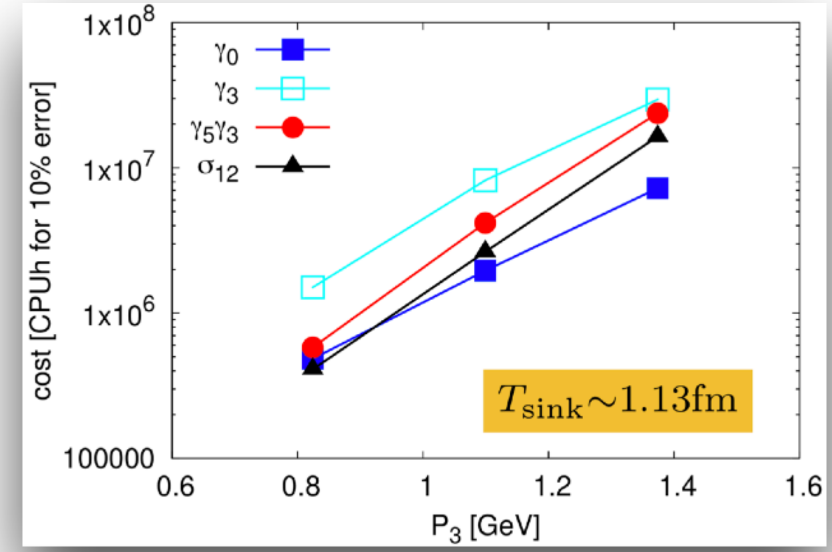
Calculation challenges

- Standard definition of GPDs in symmetric frame
  separate calculations at each t
- Statistical noise increases with P_3, t
  Projection:
  billions of core-hours for physical point at $P_3 = 3\,\mathrm{GeV}$



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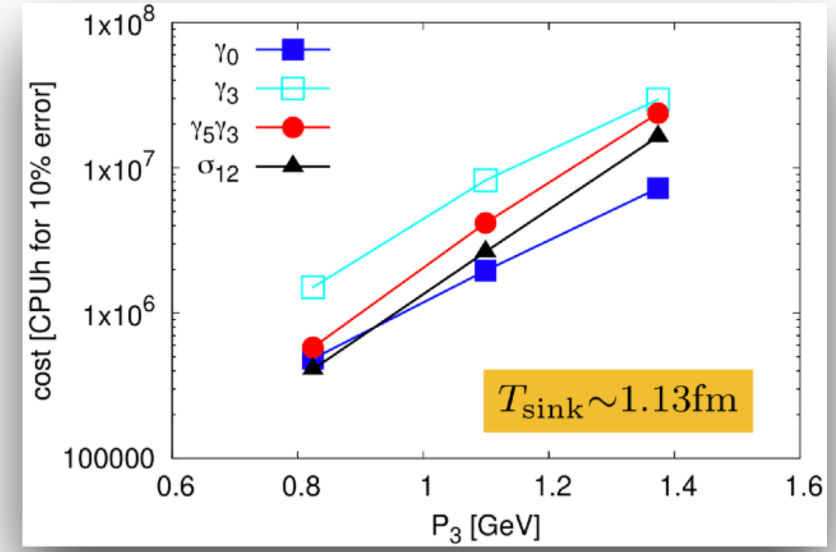

C. Other methods

See next slide

Novel Approaches

★ Hadronic tensor [K.F. Liu, S.J. Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]

Auxiliary scalar quark [U. Aglietti et al., Phys. Lett. B441, 371 (1998), arXiv:hep-ph/9806277]

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Higher moments [Z. Davoudi, M. Savage, Phys. Rev. D86, 054505 (2012)]

Quasi-distributions (LaMET) [X. Ji, PRL 110 (2013) 262002, arXiv:1305.1539; Sci. China PPMA. 57, 1407 (2014)]

Compton amplitude and OPE [A. Chambers et al. (QCDSF), PRL 118, 242001 (2017), arXiv:1703.01153]

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Good lattice cross sections [Y-Q Ma & J. Qiu, Phys. Rev. Lett. 120, 022003 (2018), arXiv:1709.03018]

PDFs without Wilson line [Y. Zhao Phys.Rev.D 109 (2024) 9, 094506, arXiv:2306.14960]

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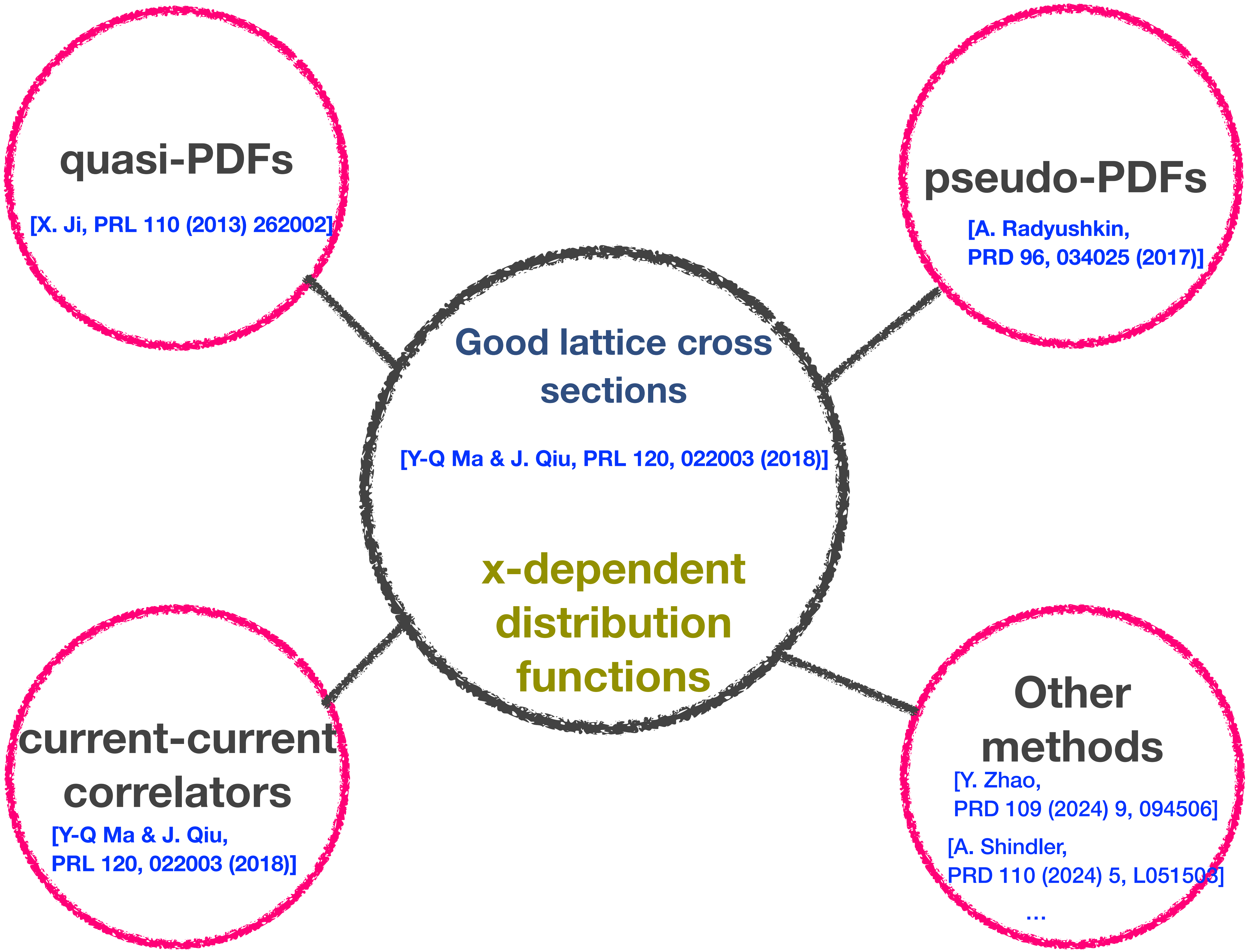
★ Reviews of methods and applications

- ***A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results***
K. Cichy & M. Constantinou (invited review) Advances in HEP 2019, 3036904, arXiv:1811.07248

- ***Large Momentum Effective Theory***
X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang, and Y. Zhao (2020), 2004.03543

- ***The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD***
M. Constantinou (invited review) Eur. Phys. J. A 57 (2021) 2, 77, arXiv:2010.02445

Novel Approaches



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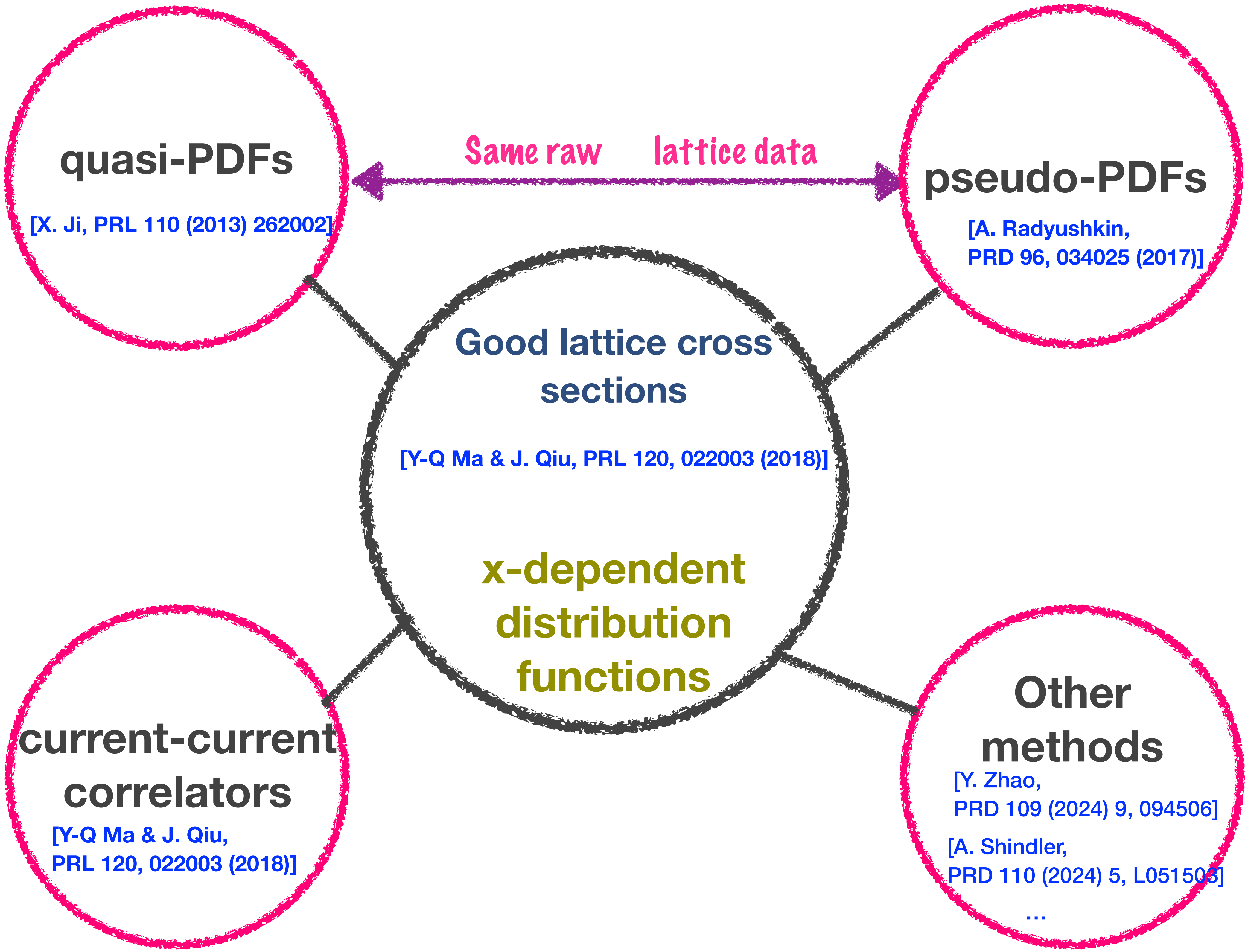

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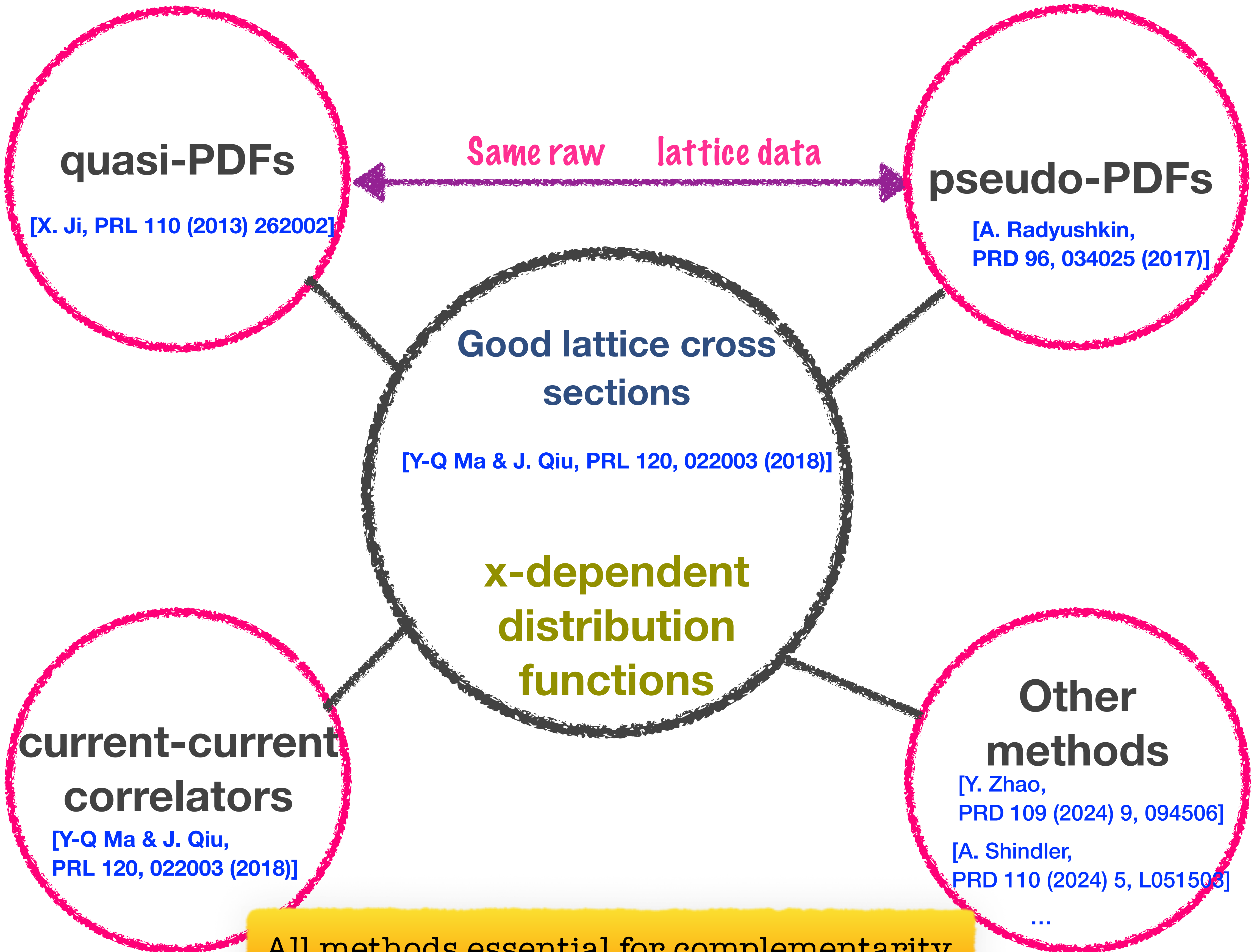

All methods essential for complementarity

Well-studied "novel" methods in LQCD

(Euclidean) Matrix elements of non-local operators with boosted hadrons

$$\mathcal{M}(P_f, P_i, z) = \langle N(P_f) \,|\, \bar{\Psi}(z)\, \Gamma\, \mathcal{W}(z,0)\Psi(0) \,|\, N(P_i) \rangle_\mu$$

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[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]
[X. Ji, Sci. China Phys. M.A. 57 (2014) 1407]

quasi-PDFs

pseudo-ITD [A. Radyushkin, PRD 96, 034025 (2017)]

$$\tilde{q}_\Gamma^{\rm GPD}(x, t, \xi, P_3, \mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z}\, \mathscr{M}(P_f, P_i, z)$$

$$\mathfrak{M}(\nu, \xi, t; z_3^2) \equiv \frac{\mathscr{M}(\nu, \xi, t; z_3^2)}{\mathscr{M}(0,0,0; z^2)} \qquad (\nu = z \cdot p)$$

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Matching in momentum space
(Large Momentum
Effective Theory)

Matching in ν space

$$Q(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{i\nu x} q(x, \mu^2)$$

Light-cone PDFs & GPDs

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Matching resembles factorization:

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Light-cone PDFs & GPDs

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = Q_3/(2P_3)$$

Calculation very taxing!
- length of the Wilson line (z) } PDFs, GPDs
- nucleon momentum boost (P_3) } PDFs, GPDs
- momentum transfer (t) } GPDs
- skewness (ξ) } GPDs

Quasi-GPDs: contact with light-cone quantities

- ★ Non-local operators with Wilson line fully renormalizable to all orders

  [T. Ishikawa et al., Phys. Rev. D 96, no. 9 (2017) 094019] [X. Ji et al., Phys. Rev. Lett. 120, no. 11 (2018) 112001]

- ★ Quasi- & light-cone distributions share the same infrared structure

- ★ Differences in UV region (perturbatively calculable, LaMET)

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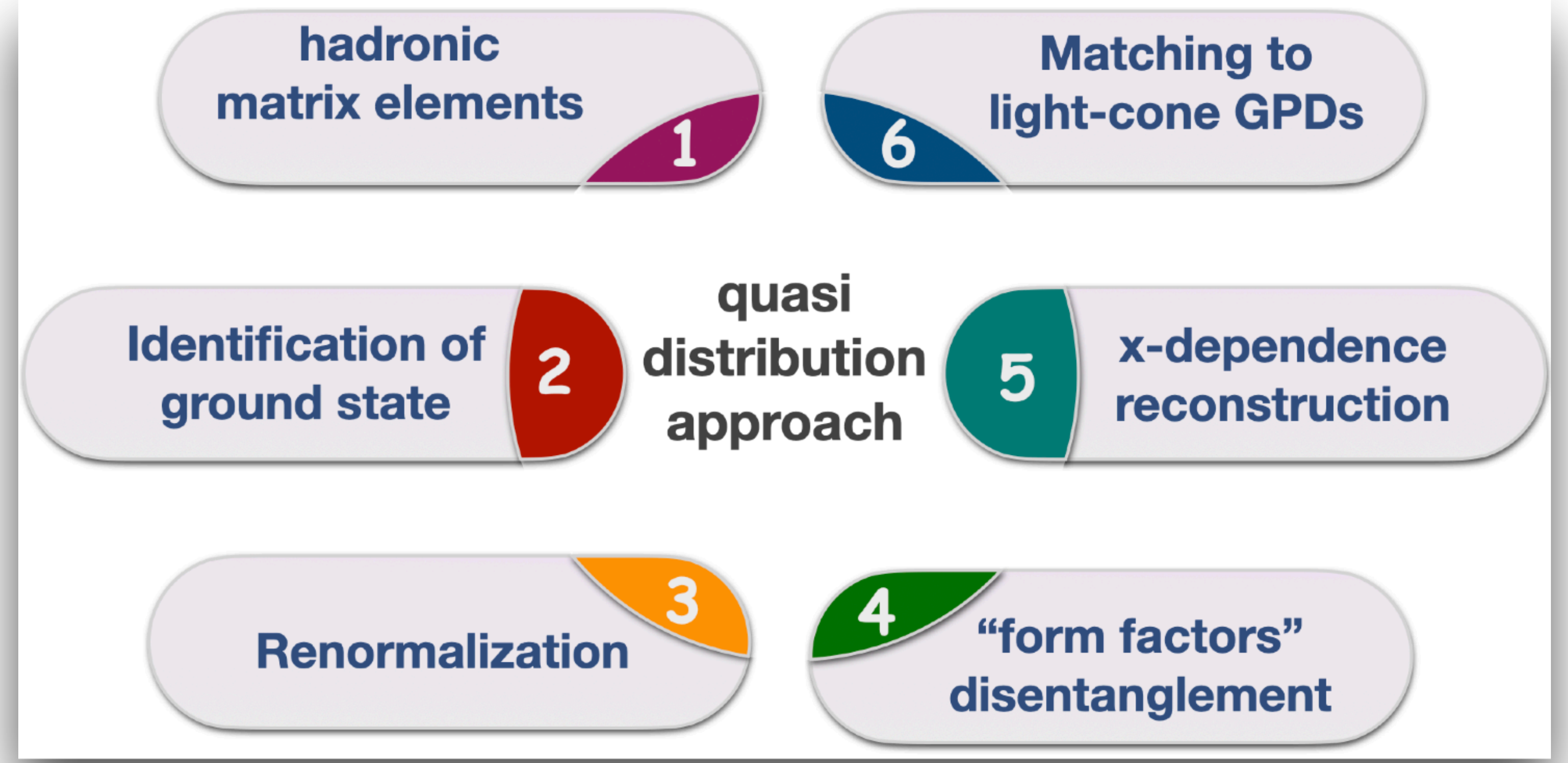
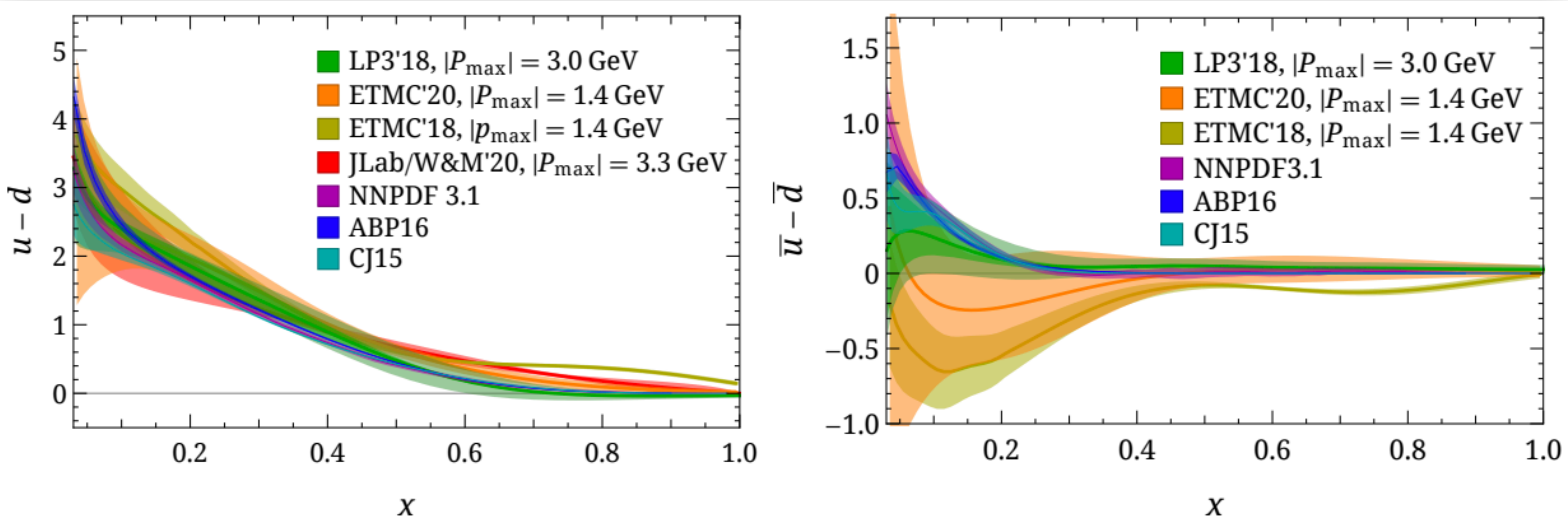
A number of non-trivial steps



Lattice Calculations of GPDs

Collection of results for unpolarized PDF



[M. Constantinou et al. (2020 PDFLattice Report), Prog.Part.Nucl.Phys. 121 (2021) 103908]

★ Several improvements:
- More calculations at physical quark masses
- Ensembles at various lattice spacings
- Addressing systematic uncertainties due to methodologies

★ Progress extended to gluon PDFs, GPDs, TMDs

Disclaimer

The field of GPDs is still developing and sources of systematic uncertainties have not been fully addressed

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- Discretization effects
- physical pion mass
- volume effects
- inverse problem
- matching formalism
- connection to light-cone
- higher twist contaminations
- ...

GPDs

leading twist

GPDs on the lattice: the unpolarized case

- ★ Off-forward matrix elements of non-local light-cone operators

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z}\langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\gamma^+ \mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

- ★ Parametrization in two leading twist GPDs

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E(x,\xi,t)\right]u(p,\lambda)$$

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Let's rethink calculation of GPDs !

Definition of GPDs on Euclidean lattice

★ Parametrization of matrix elements in Lorentz invariant amplitudes

Vector [S. Bhattacharya et al., PRD 106 (2022) 11, 114512]

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}A_1 + z^{\mu} M A_2 + \frac{\Delta^{\mu}}{M}A_3 + i\sigma^{\mu z} M A_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^{\mu} i\sigma^{z\Delta}}{M}A_6 + \frac{z^{\mu} i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^{\mu} i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Axial [S. Bhattacharya et al., PRD 109 (2024) 3, 034508]

$$\widetilde{F}^{\mu} = \bar{u}(p_f,\lambda')\left[\frac{i\epsilon^{\mu P z \Delta}}{m}\widetilde{A}_1 + \gamma^{\mu}\gamma_5\widetilde{A}_2 + \gamma_5\left(\frac{P^{\mu}}{m}\widetilde{A}_3 + m z^{\mu}\widetilde{A}_4 + \frac{\Delta^{\mu}}{m}\widetilde{A}_5\right) + m\not{z}\gamma_5\left(\frac{P^{\mu}}{m}\widetilde{A}_6 + m z^{\mu}\widetilde{A}_7 + \frac{\Delta^{\mu}}{m}\widetilde{A}_8\right)\right]u(p_i,\lambda)$$

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Advantages

- Applicable to any kinematic frame and have definite symmetries
- Lorentz invariant amplitudes A_i can be related to the standard GPDs
- Quasi GPDs may be redefined (Lorentz covariant) to eliminate $1/P_3$ contributions

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Goals

★ Extraction of standard GPDs using A_i obtained from any frame

★ quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone

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$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}A_1 + z^{\mu} M A_2 + \frac{\Delta^{\mu}}{M}A_3 + i\sigma^{\mu z} M A_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^{\mu} i\sigma^{z\Delta}}{M}A_6 + \frac{z^{\mu} i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^{\mu} i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Axial [S. Bhattacharya et al., PRD 109 (2024) 3, 034508]

$$\widetilde{F}^{\mu} = \bar{u}(p_f,\lambda')\left[\frac{i\epsilon^{\mu P z\Delta}}{m}\widetilde{A}_1 + \gamma^{\mu}\gamma_5\widetilde{A}_2 + \gamma_5\left(\frac{P^{\mu}}{m}\widetilde{A}_3 + m z^{\mu}\widetilde{A}_4 + \frac{\Delta^{\mu}}{m}\widetilde{A}_5\right) + m\not{z}\gamma_5\left(\frac{P^{\mu}}{m}\widetilde{A}_6 + m z^{\mu}\widetilde{A}_7 + \frac{\Delta^{\mu}}{m}\widetilde{A}_8\right)\right]u(p_i,\lambda)$$

Advantages

- Applicable to any kinematic frame and have definite symmetries
- Lorentz invariant amplitudes A_i can be related to the standard GPDs
- Quasi GPDs may be redefined (Lorentz covariant) to eliminate $1/P_3$ contributions

Goals

★ Extraction of standard GPDs using A_i obtained from any frame

★ quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone

Light-cone GPDs using lattice correlators in non-symmetric frames

Proof of Concept Calculation

Twisted-mass fermions & clover

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

Test at zero skewness

- symmetric frame: $\vec{p}_f^{\,s} = \overrightarrow{P} + \overrightarrow{Q}/2\,,$ $\vec{p}_i^{\,s} = \overrightarrow{P} - \overrightarrow{Q}/2$ $-t^s = \overrightarrow{Q}^2 = 0.69\,GeV^2$
- asymmetric frame: $\vec{p}_f^{\,a} = \overrightarrow{P}\,,$ $\vec{p}_i^{\,a} = \overrightarrow{P} - \overrightarrow{Q}$ $t^a = -\overrightarrow{Q}^2 + (E_f - E_i)^2 = 0.65\,GeV^2$

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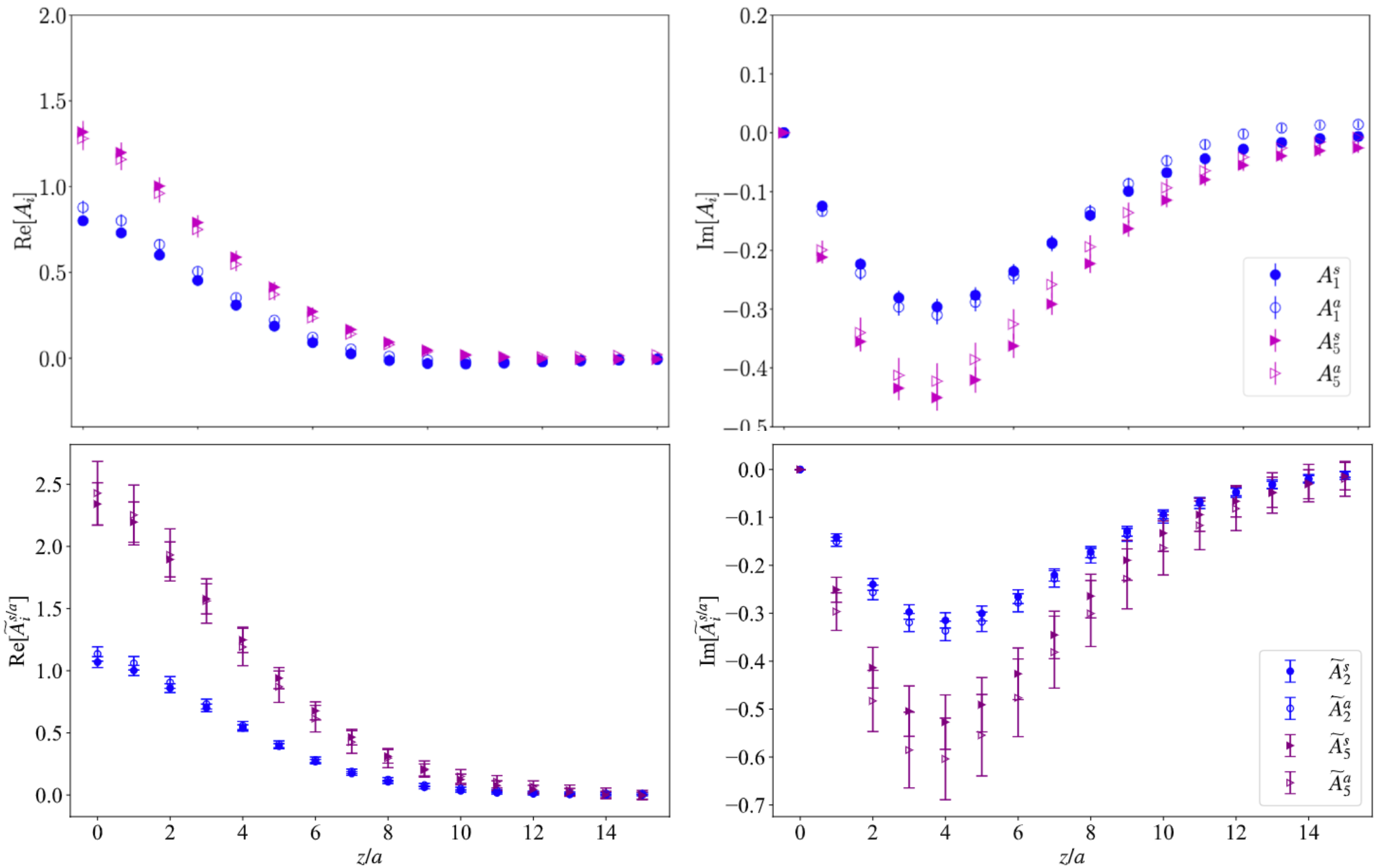
Vector

Axial



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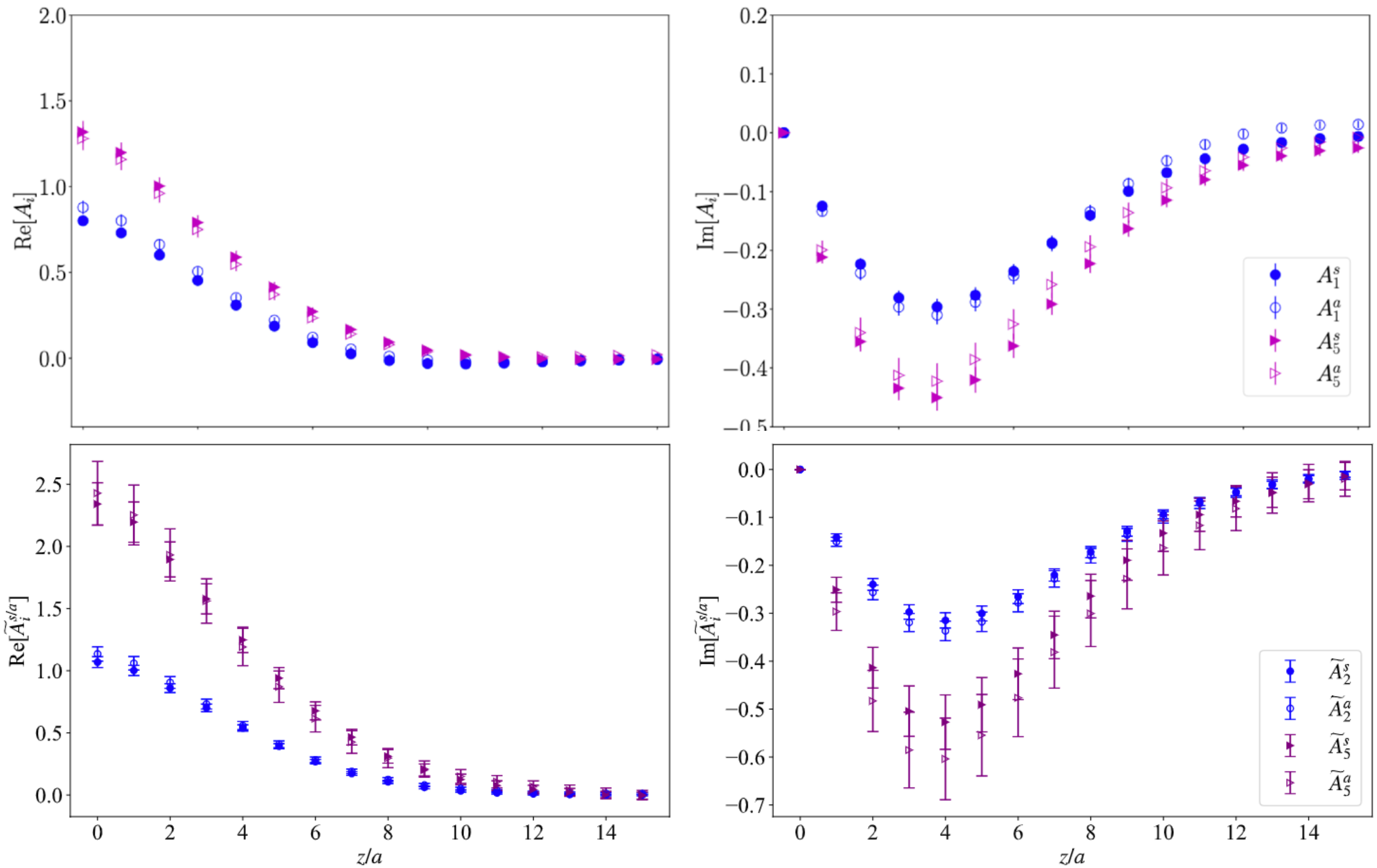

Indeed frame independence

Beyond Exploration

- Symm. frame: separate calculation for each $\vec{Q}$
- Asymm. frame: Two classes of $\vec{Q}$: $(Q_x,0,0)$, $(Q_x, Q_y,0)$

frame	P_3 [GeV]	Δ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	429	8	27456
asymm	±1.25	(±1, ±1, 0)	0.33	0	16	194	8	12416
asymm	±1.25	(±2,0,0), (0,±2,0)	0.64	0	8	429	8	27456
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	429	8	27456
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	429	8	27456

- Momentum transfer range is very optimistic
  (some values have enhanced systematic uncertainties)

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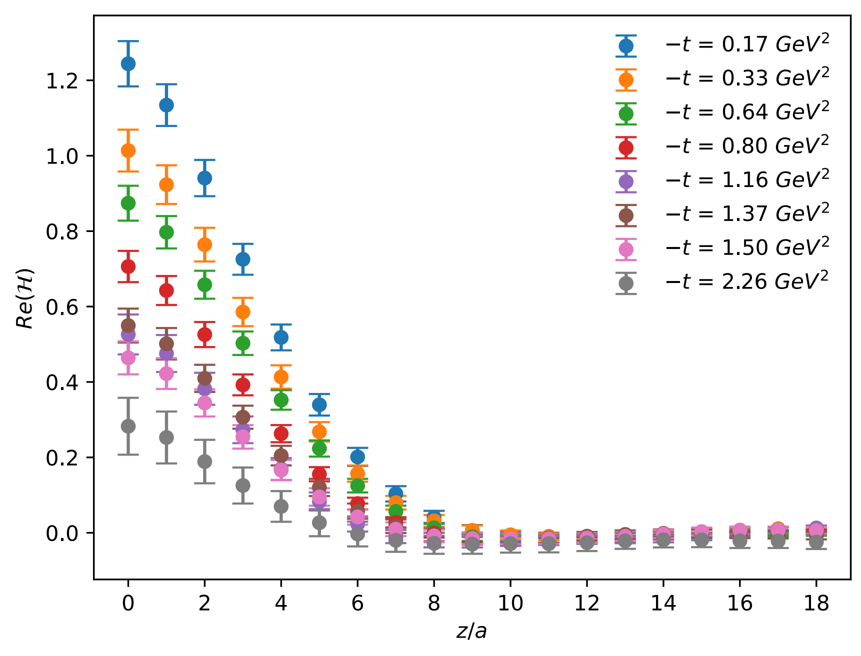
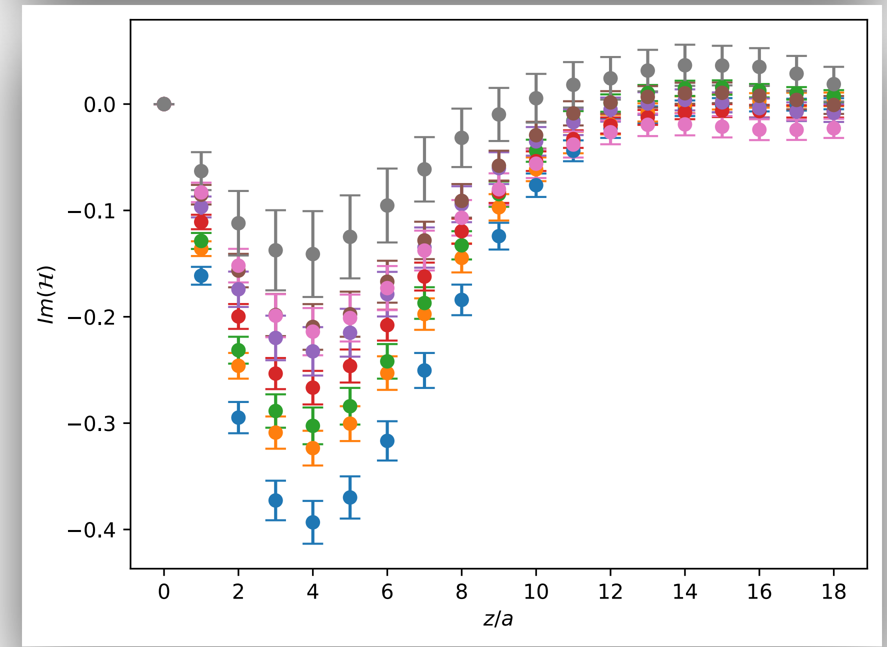
asymmetric frame

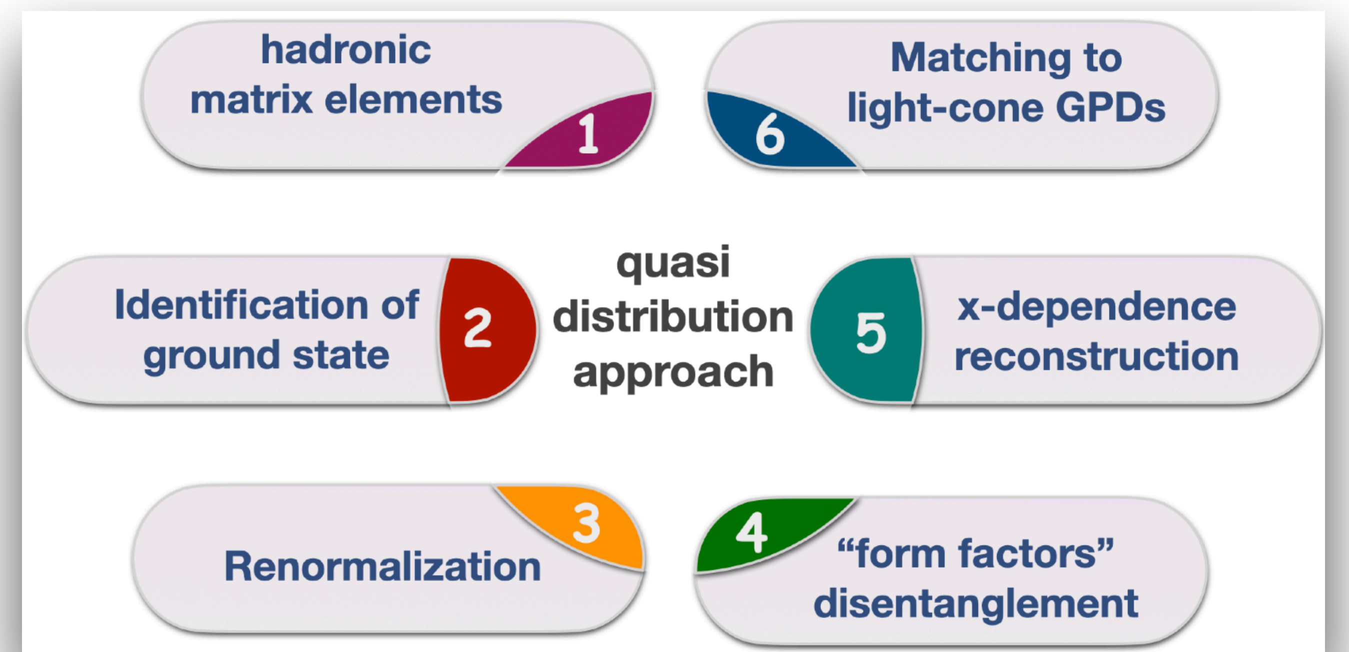




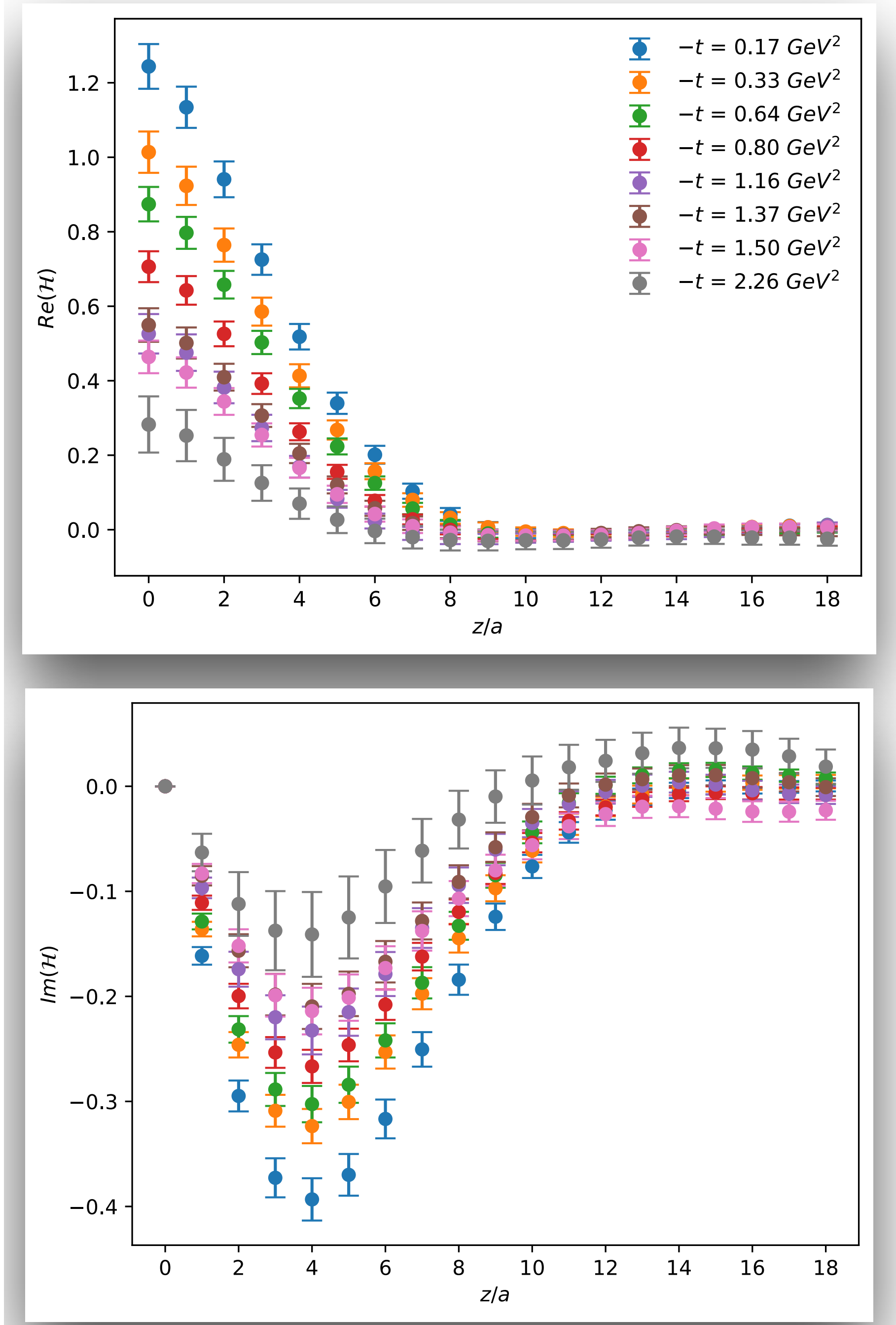
- ★ Momentum transfer range is very optimistic
(some values have enhanced systematic uncertainties)
- ★ Impressive signal quality

Beyond Exploration

Apply non-trivial steps



asymmetric frame



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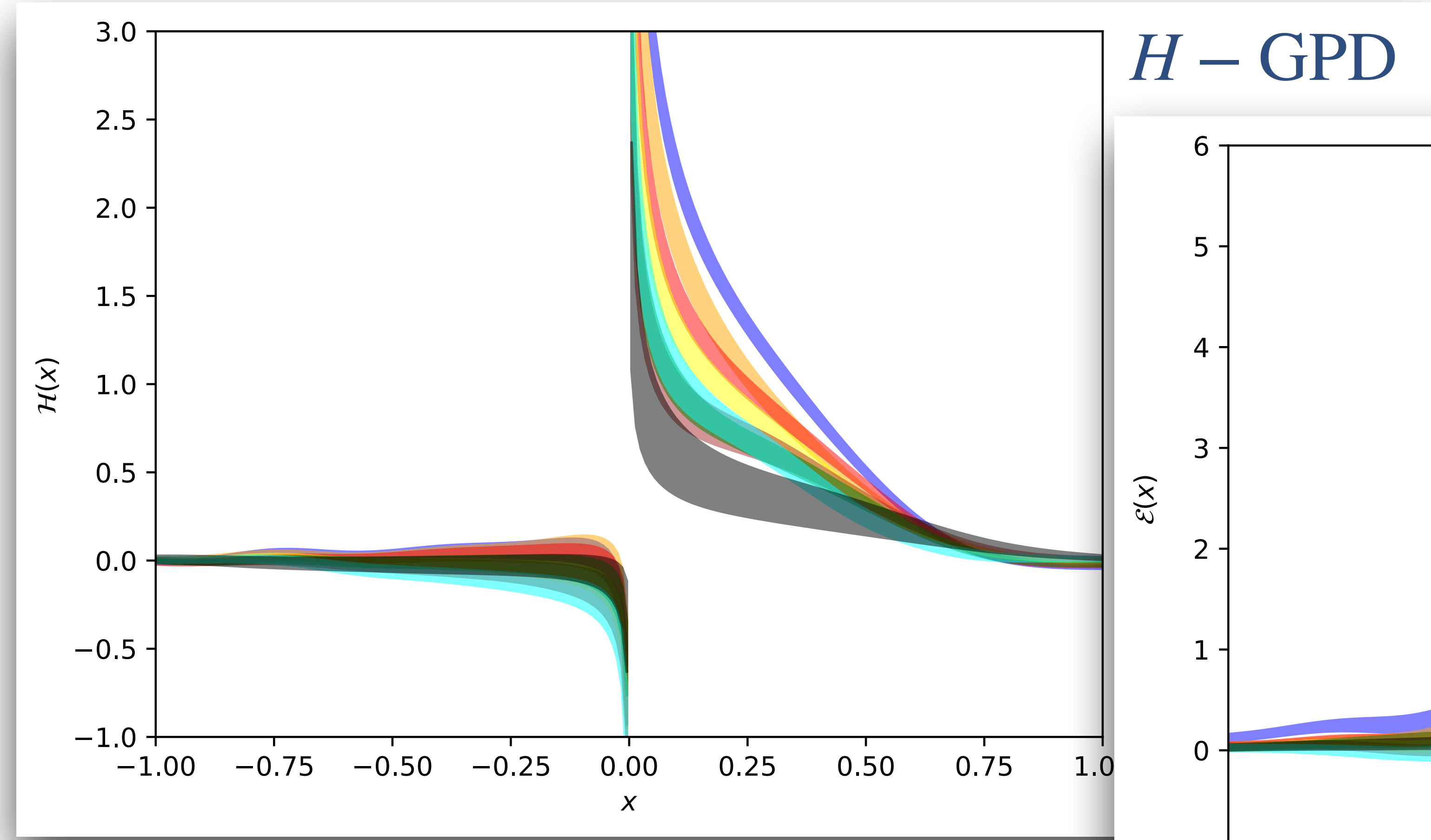
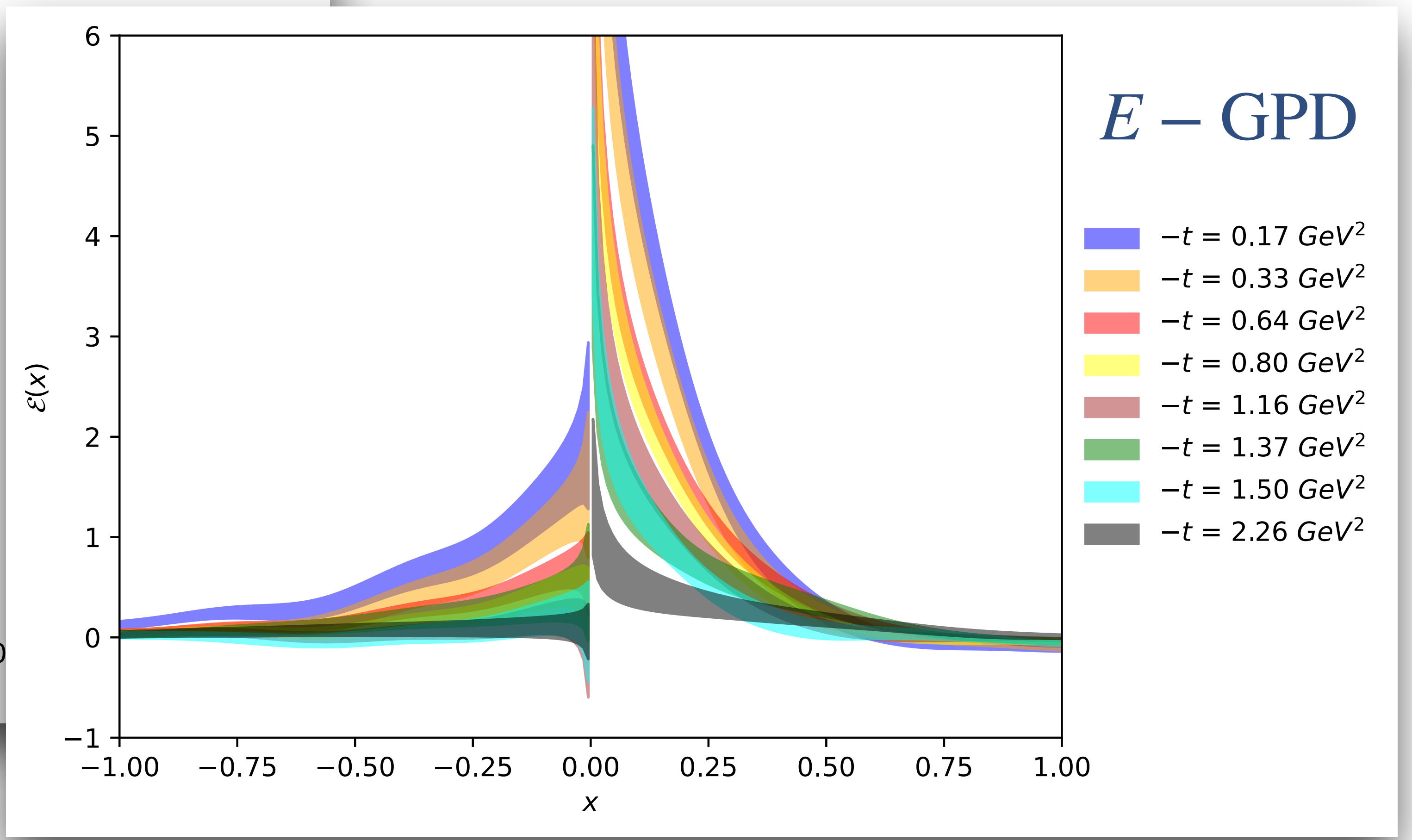
★ Impressive signal quality

Light-cone GPDs





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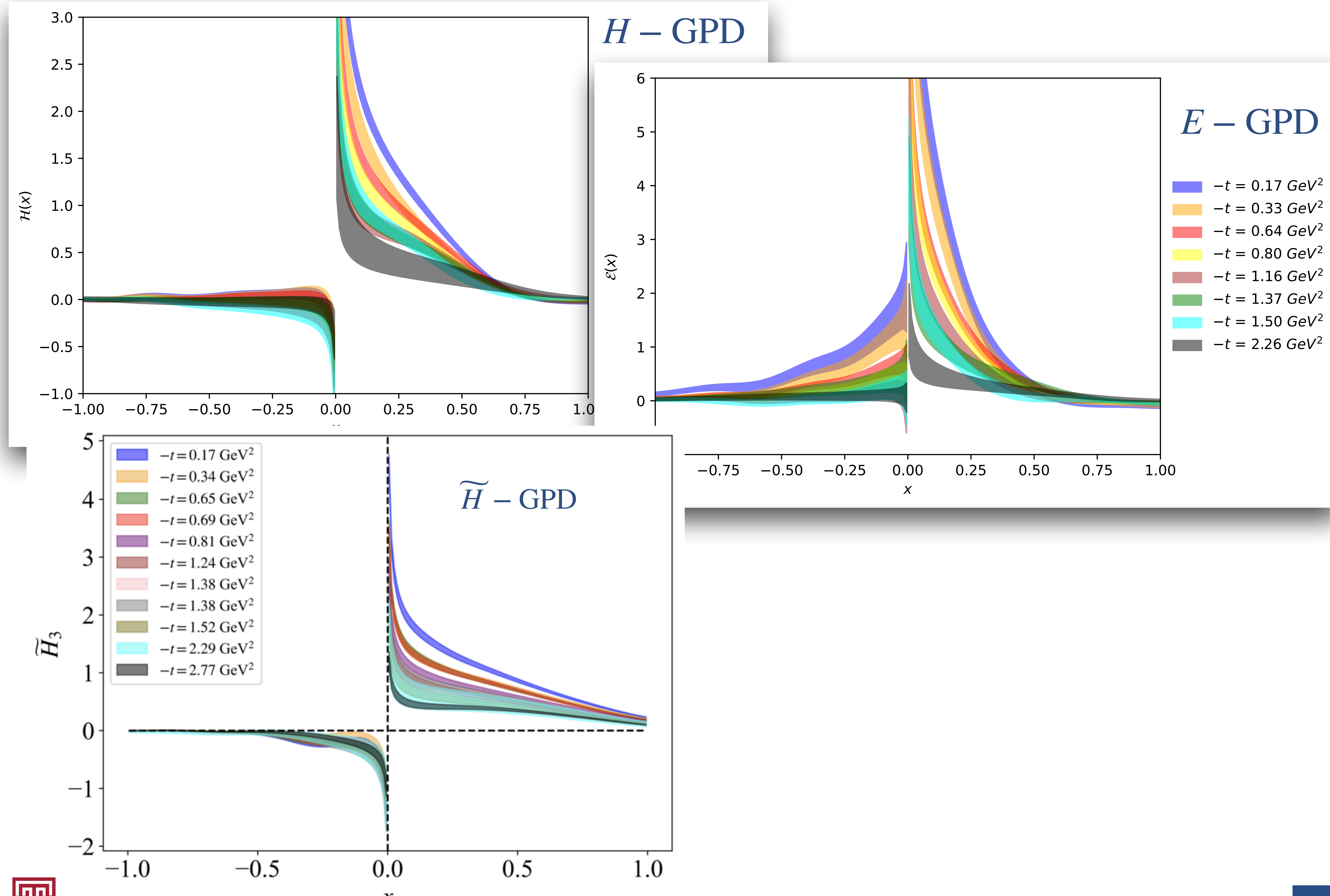

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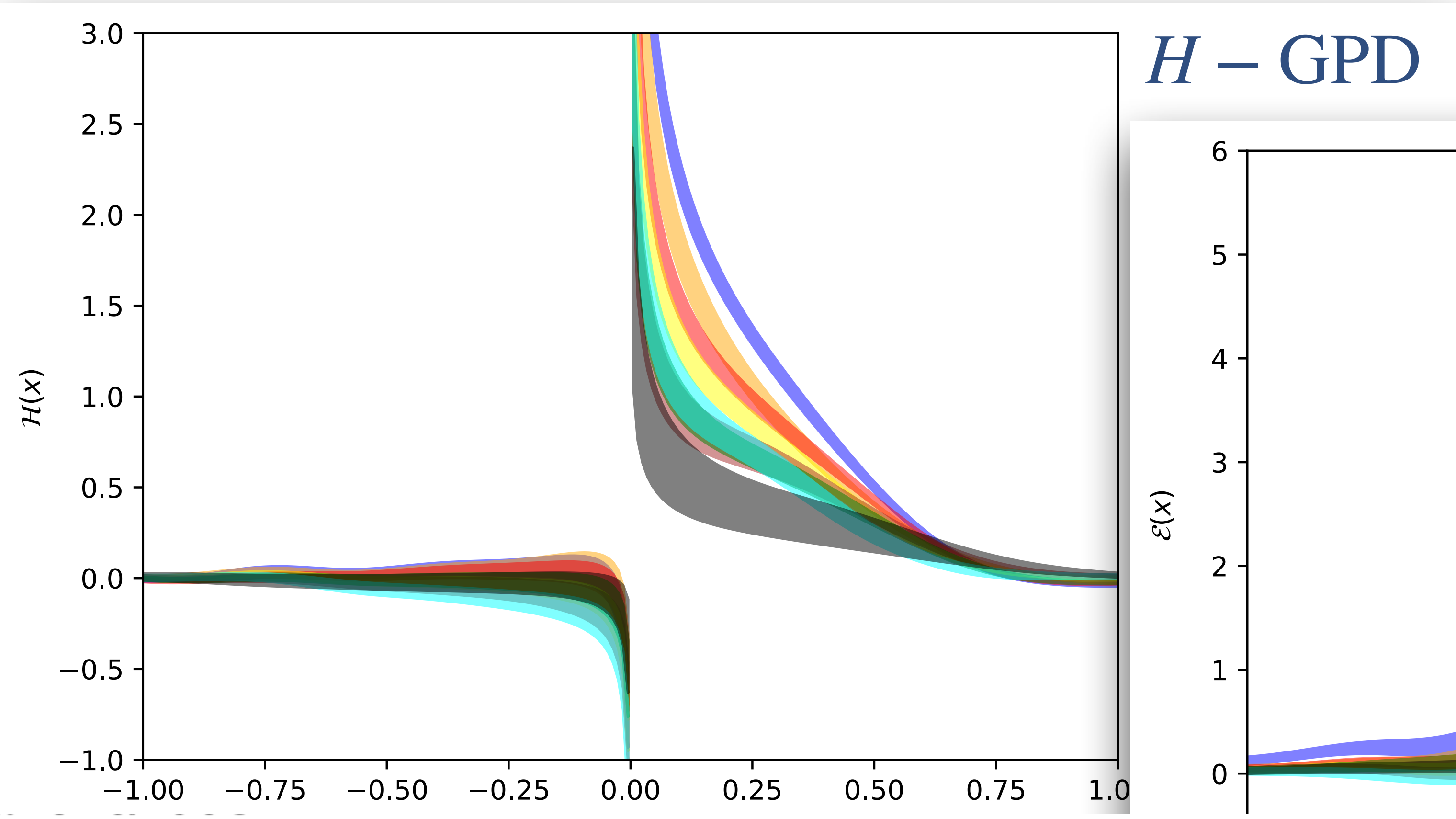

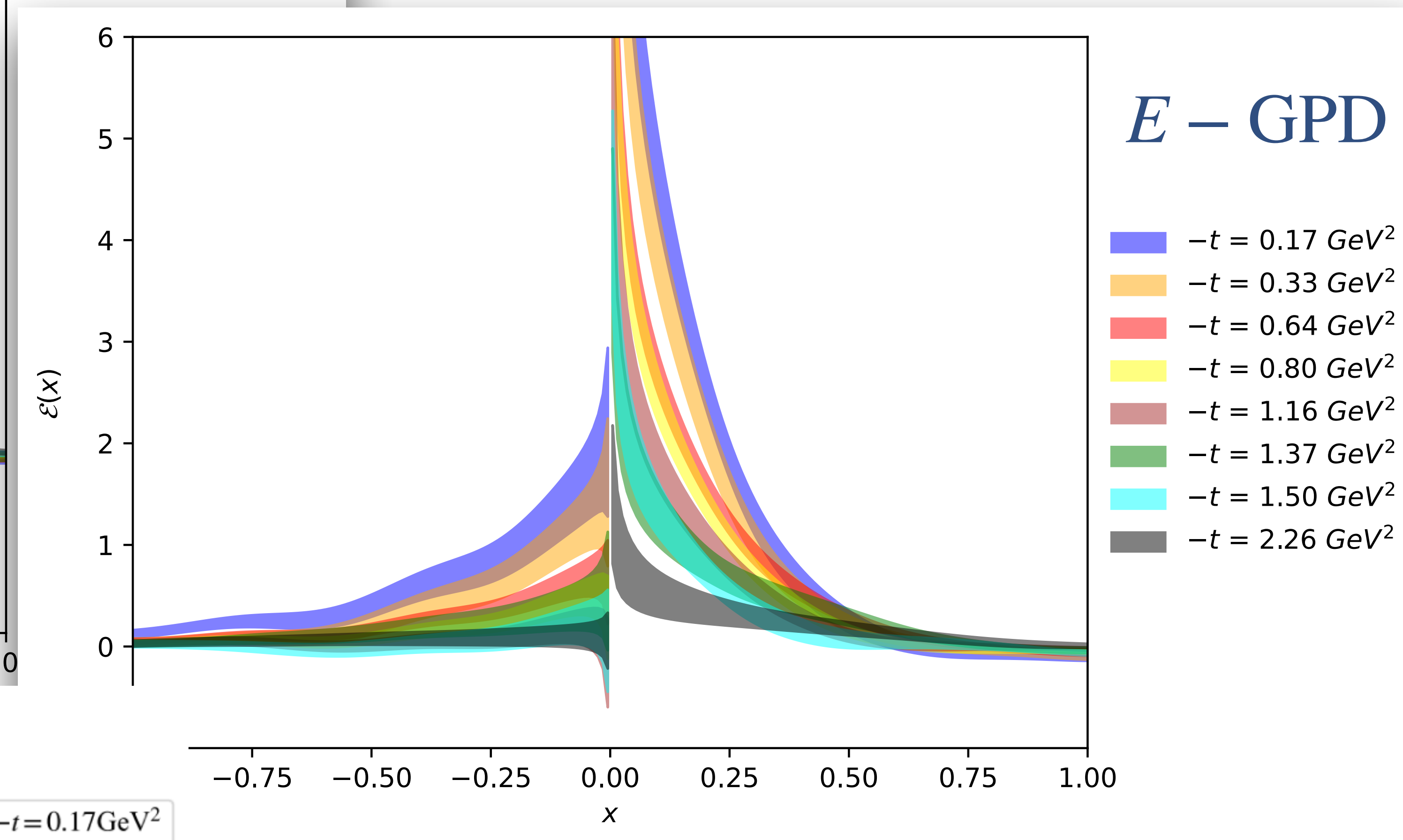

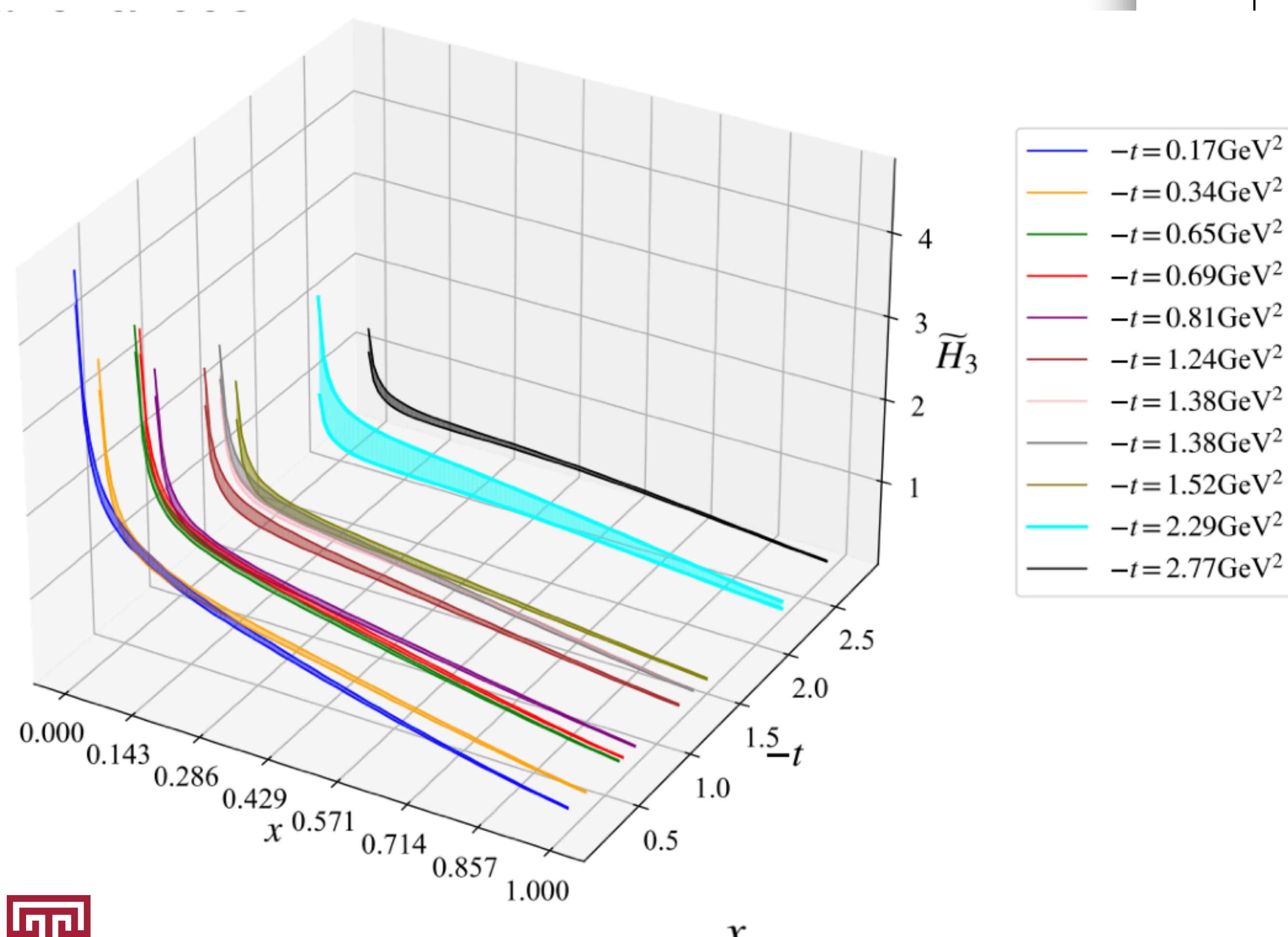

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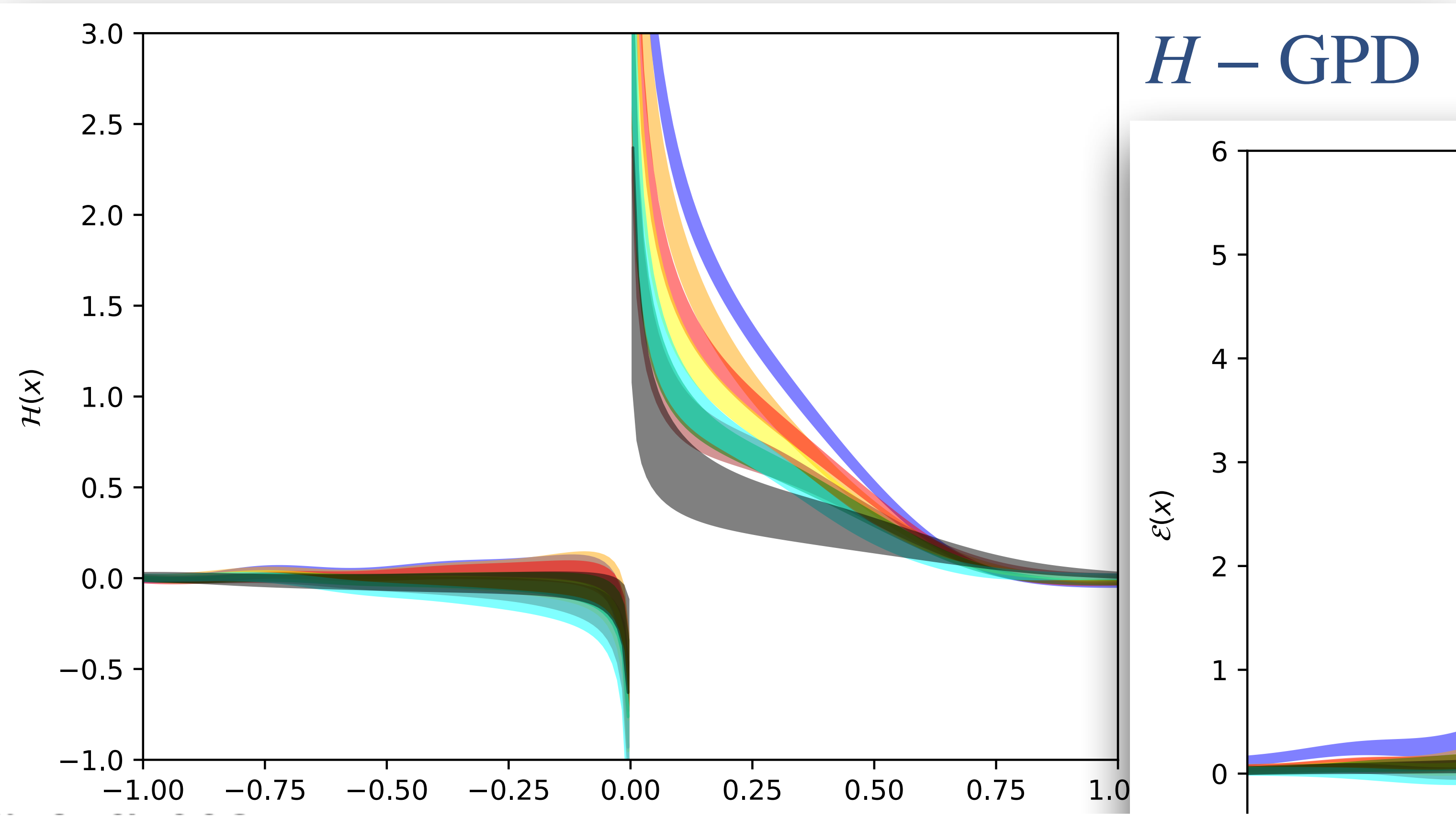
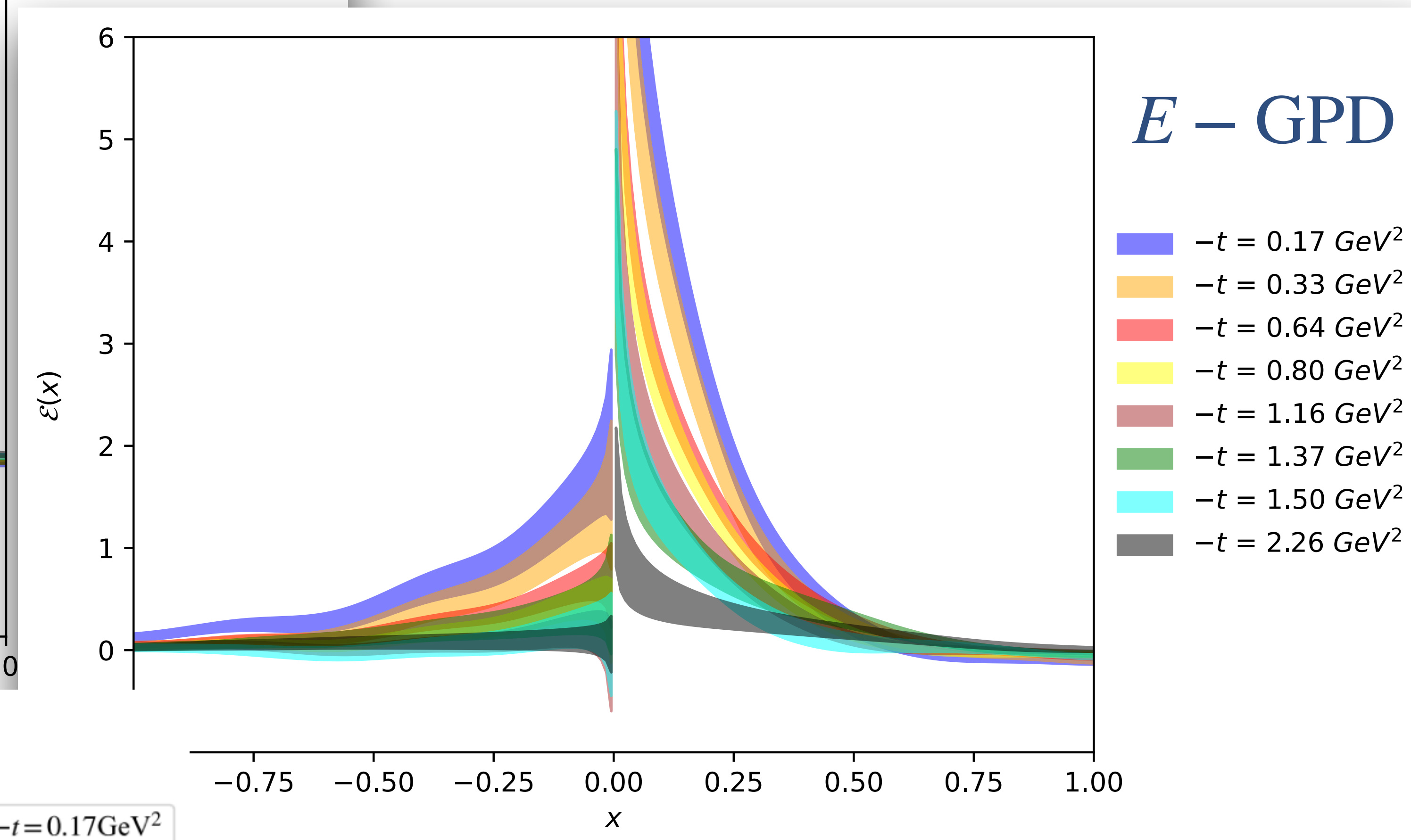
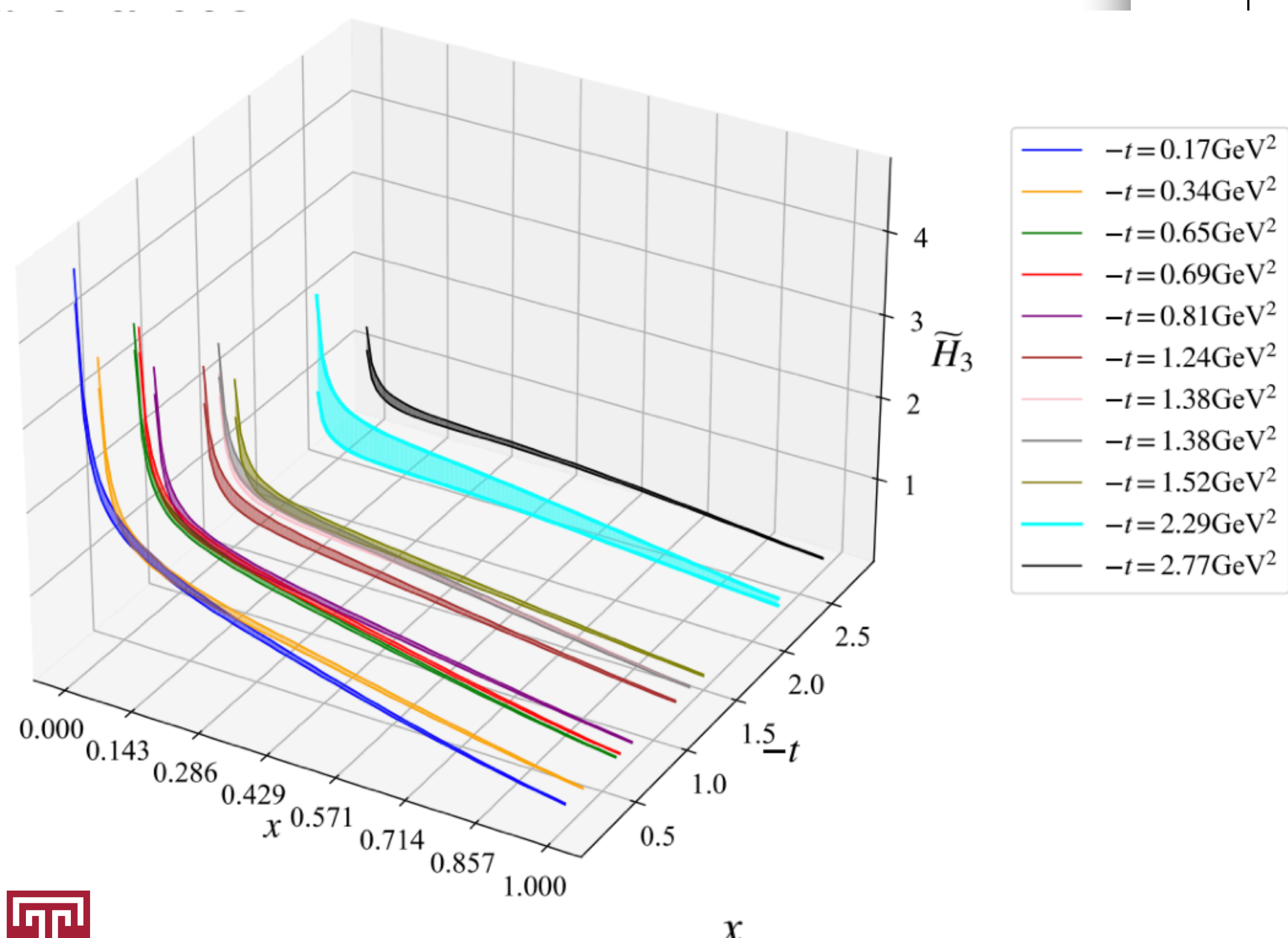
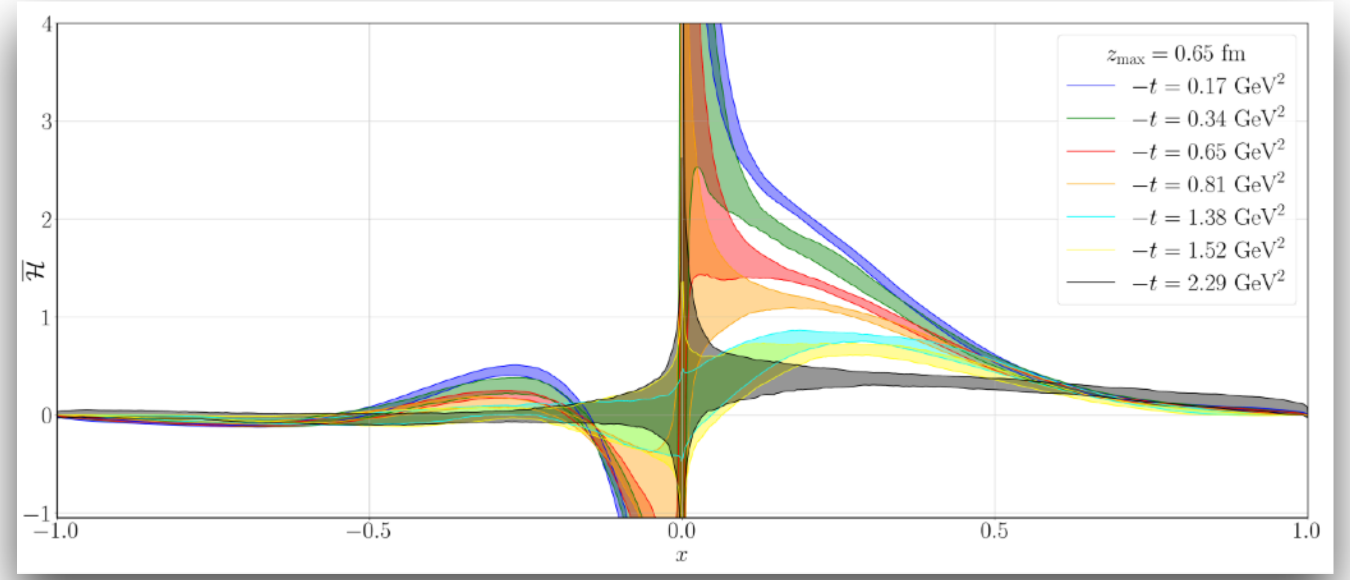






- ★ +x region: quarks
  -x region: anti-quarks
- ★ anti-quark region susceptible
  to more systematic uncertainties
- ★ small- and large-x region
  not reliably extracted
- ★ large -t values unreliable but free

Alternative approach: pseudo-ITD



[Battacharya et al., PRD 110 (2024) 5, 054502]

Different steps between approaches:
- renormalization
- x-dependence reconstruction
- matching formalism

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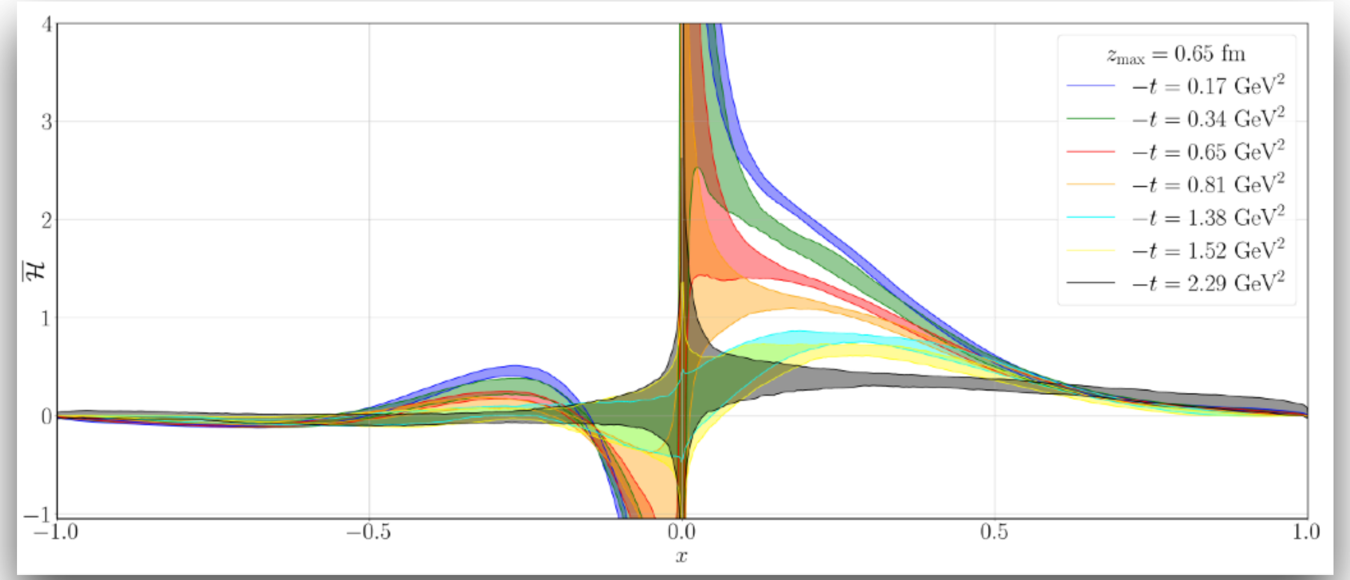

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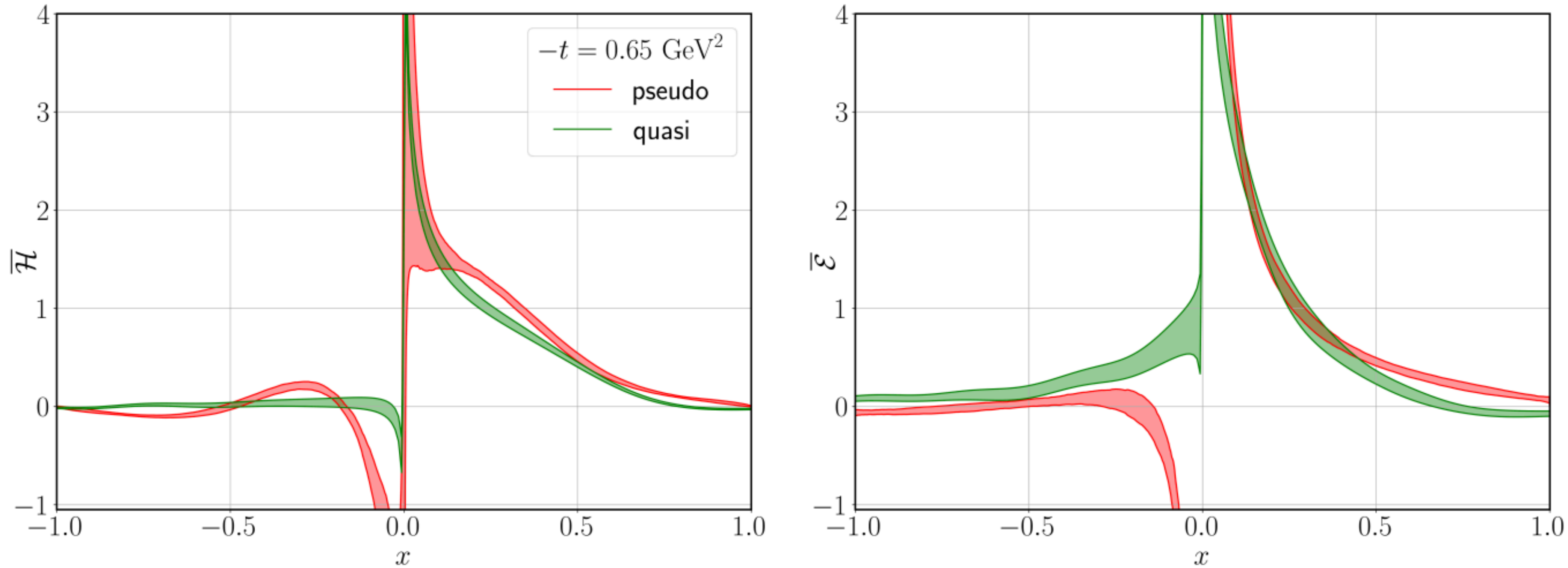
Different steps between approaches:
- renormalization
- x-dependence reconstruction
- matching formalism

★ Comparison between methods helps assess systematic effects



★ x<0 and small-x regions susceptible to systematic effects

★ Comparison only includes systematic uncertainties

Mellin moments from non-local operators

★ Leading-twist factorization formula

$$\mathcal{M}(z, P, \Delta) \equiv \frac{\mathcal{F}(z, P, \Delta)}{\mathcal{F}(z, P=0, \Delta=0)} = \sum_{n=0} \frac{(-izP)^n}{n!} \frac{C_n^{\overline{\rm MS}}(\mu^2 z^2)}{C_0^{\overline{\rm MS}}(\mu^2 z^2)} \langle x^n \rangle + \mathcal{O}(\Lambda_{\rm QCD}^2 z^2)$$

★ Avoid power-divergent mixing of multi-derivative operators

★ Wilson coefficients known to NLO (or NNLO)

★ Both isovector and isoscalar (ignores disconnected; found to be tiny)

[C. Alexandrou et al., PRD 104 (2021) 5, 054503]

[S. Bhattacharya et al., PRD 108 (2023) 1, 014507]

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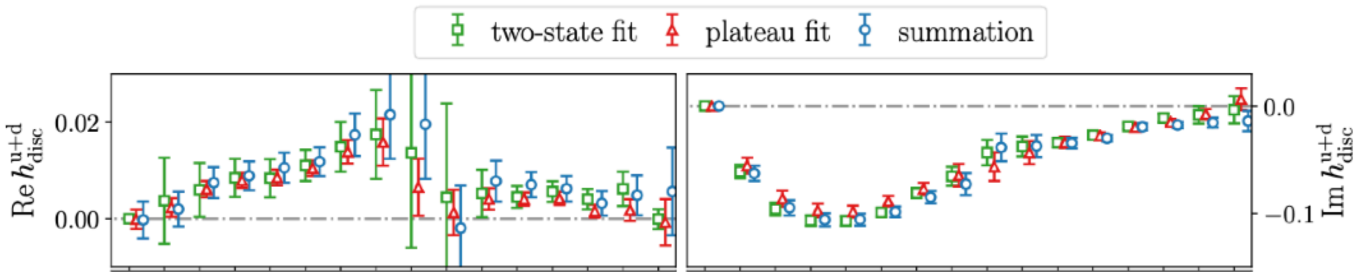

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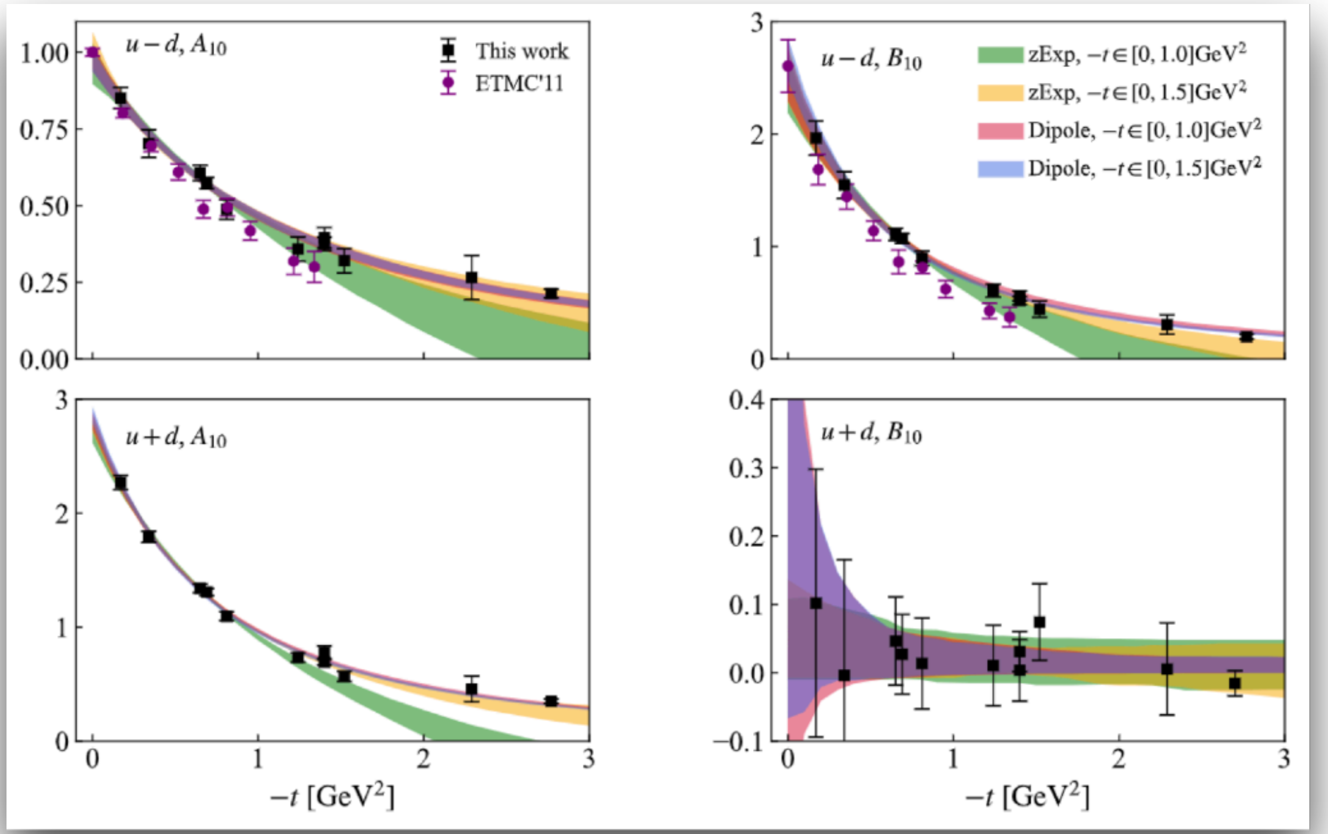

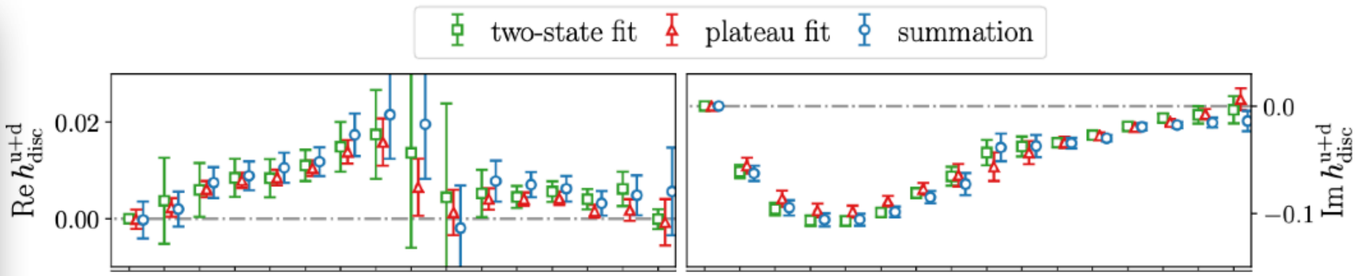

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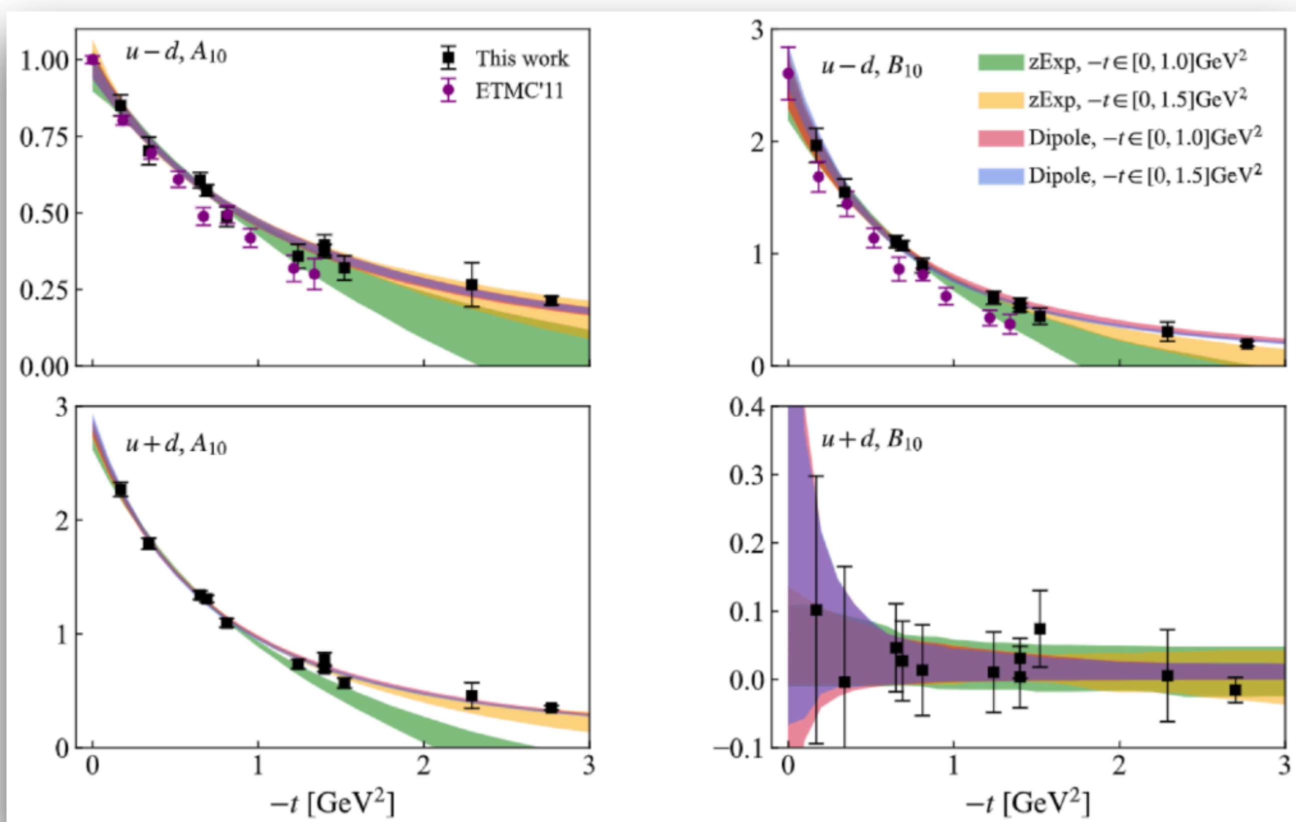

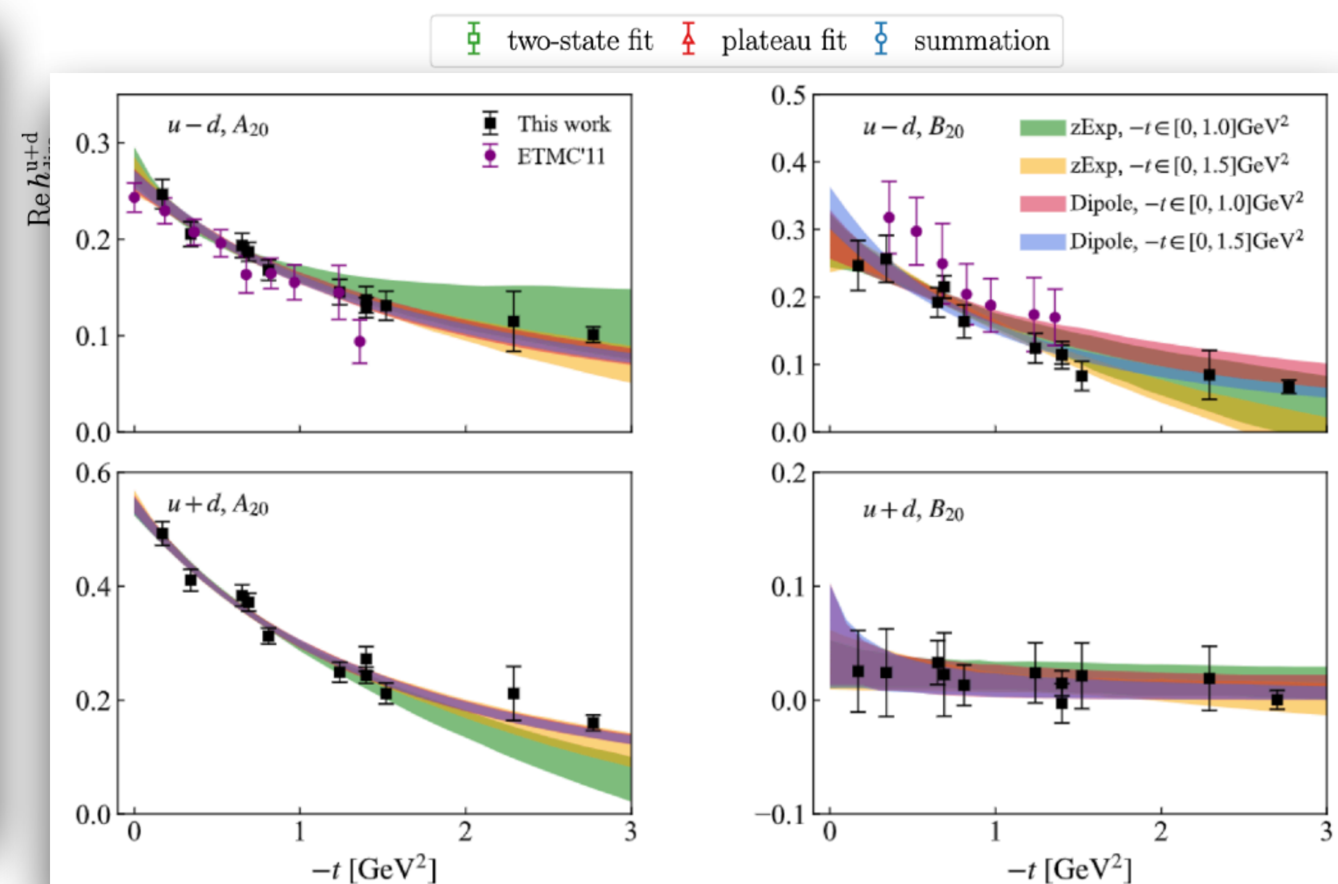

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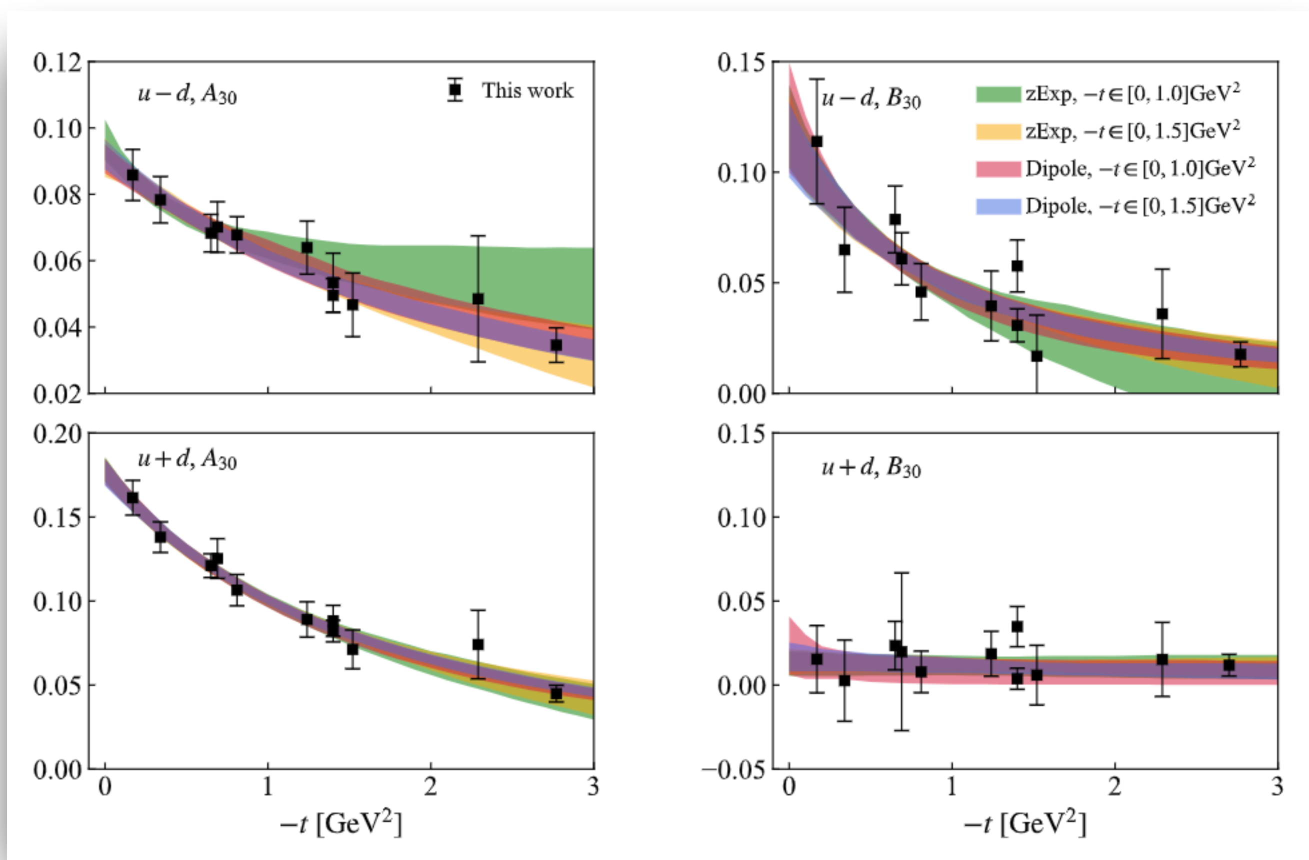

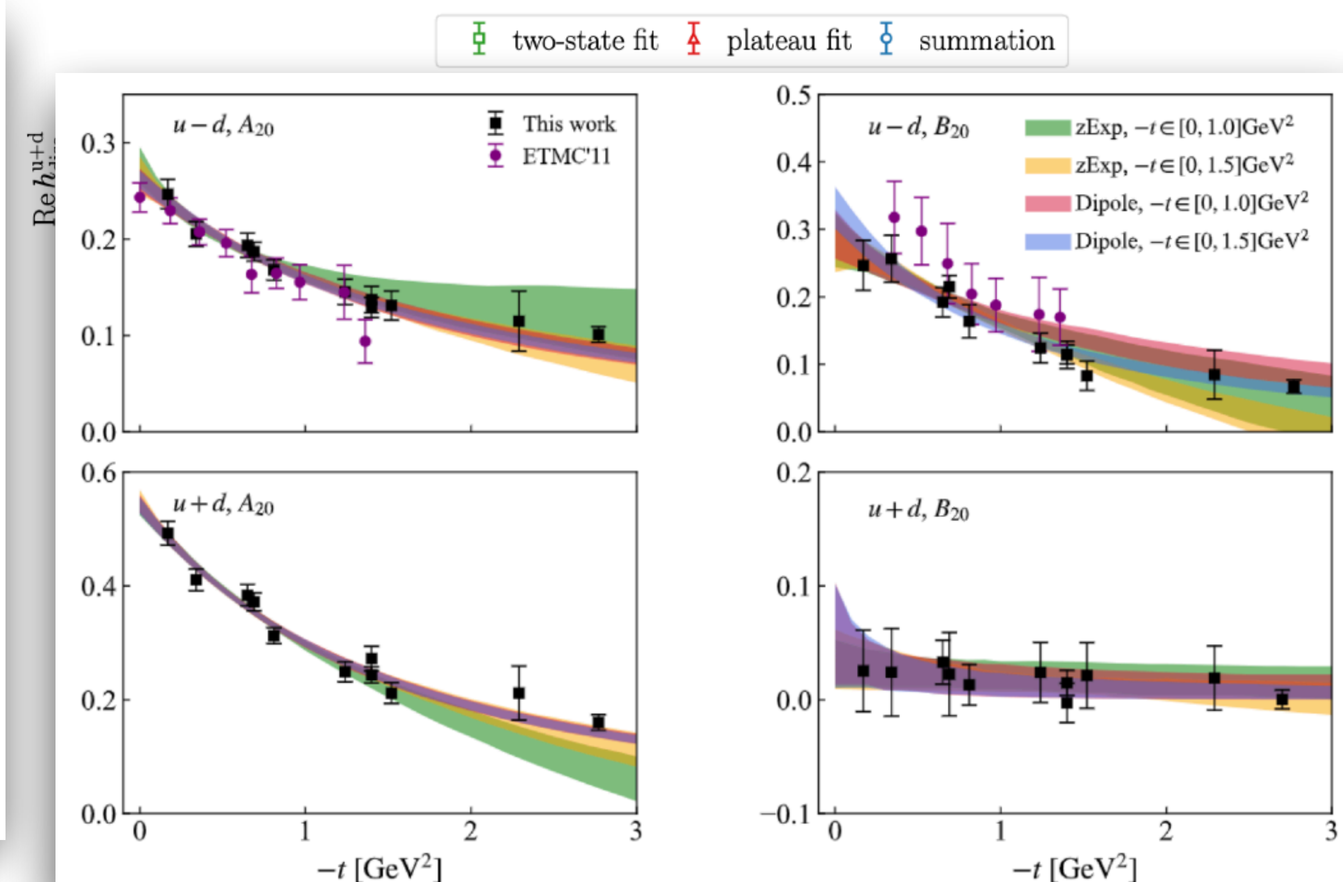

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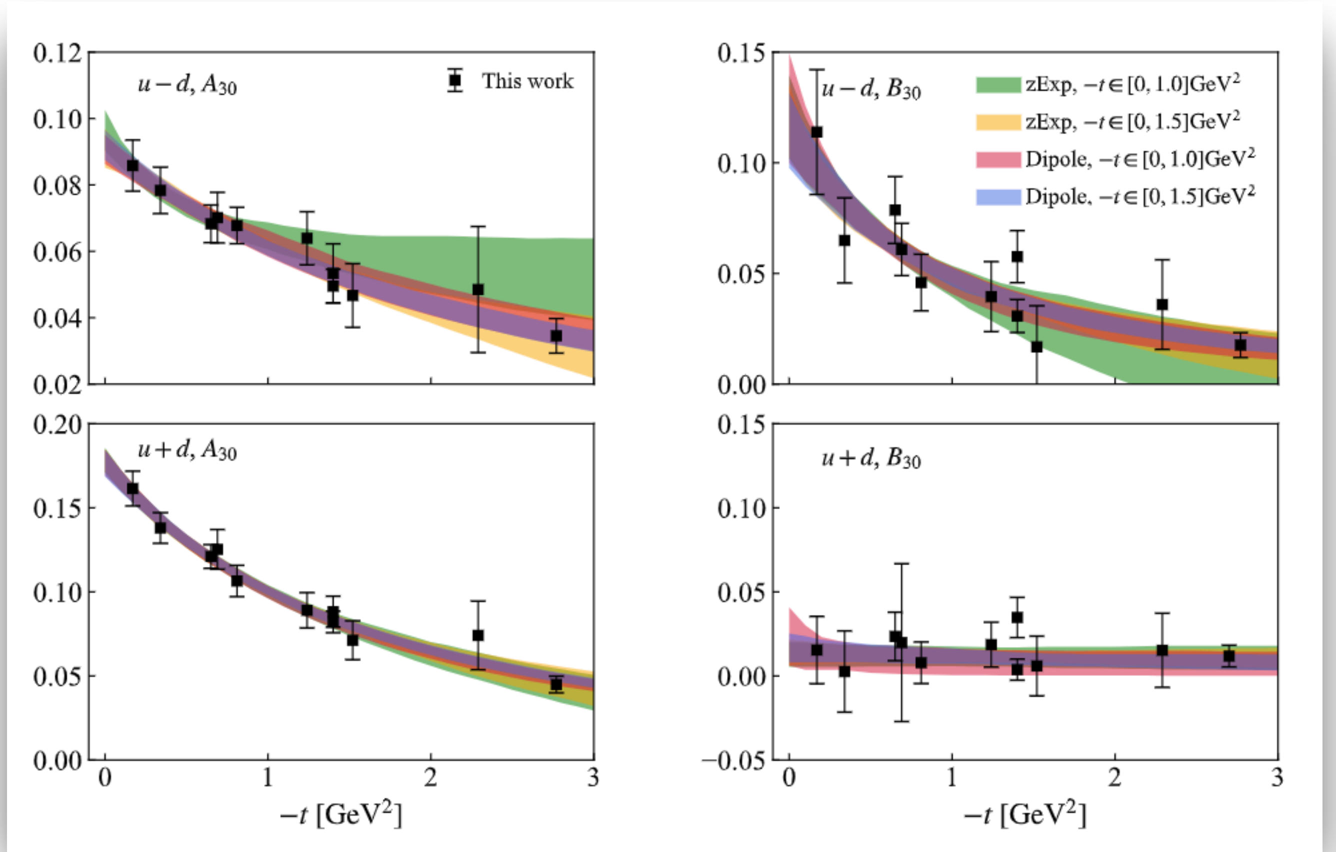

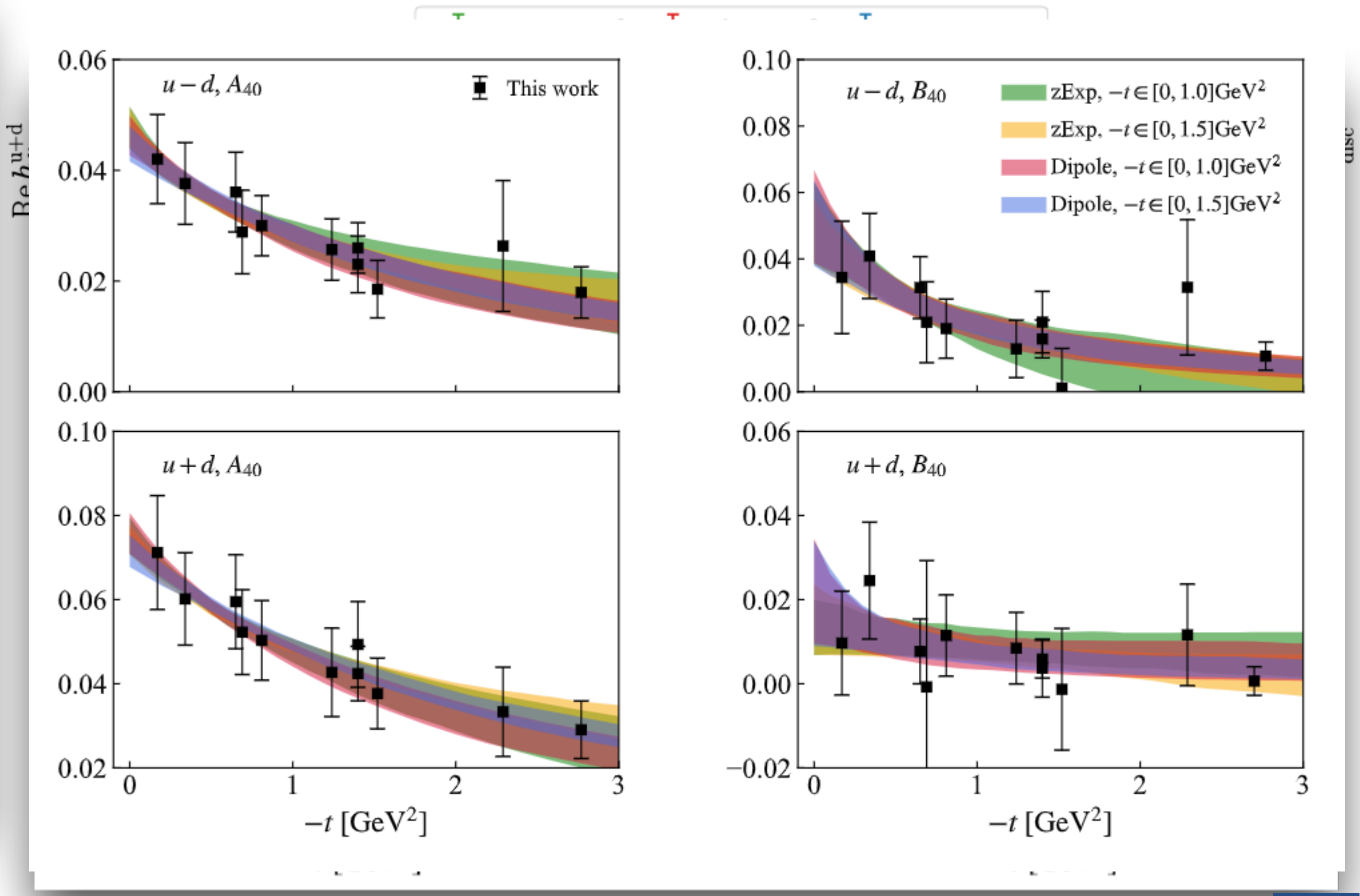

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Access to
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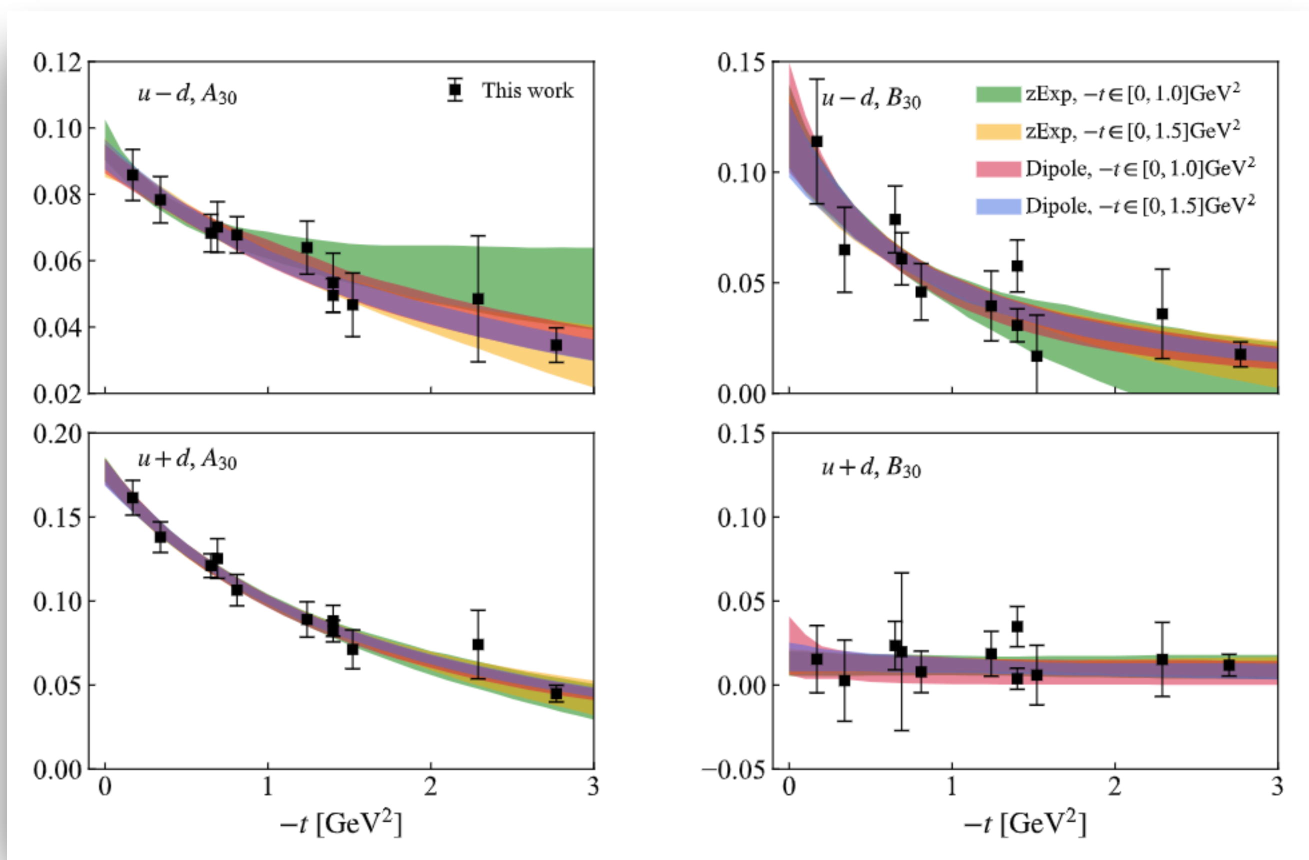

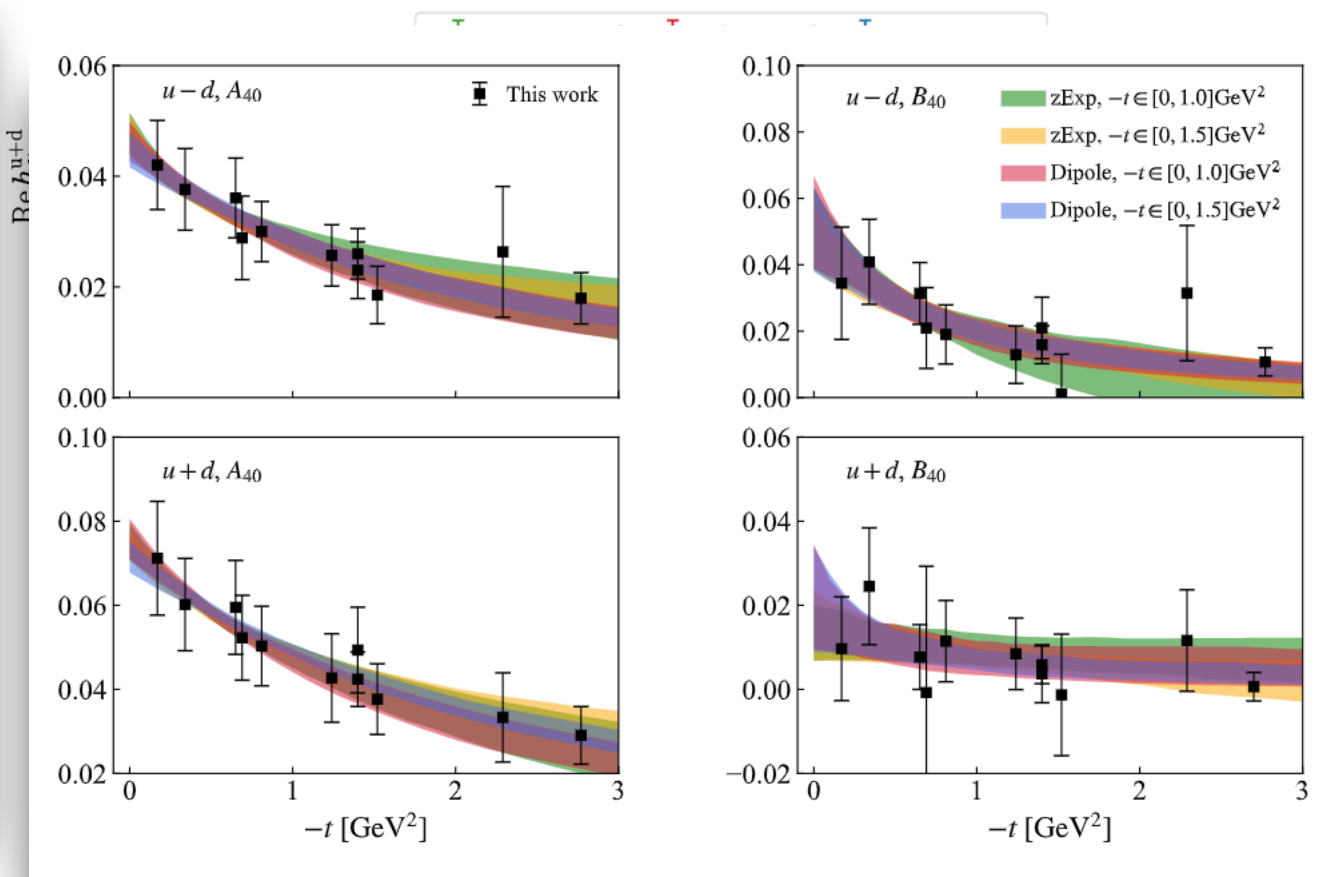

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GPDs

beyond leading twist

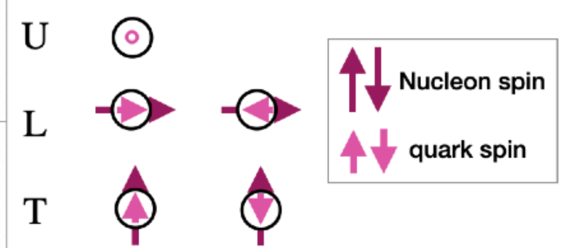
Twist-classification of PDFs, GPDs, TMDs

★ Twist: The order in Q^{-1} entering factorization

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L $(\gamma^+\gamma^5)$	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity



(Selected) Twist-3 $(f_i^{(1)})$

$\mathcal{O}$ / Nucleon	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

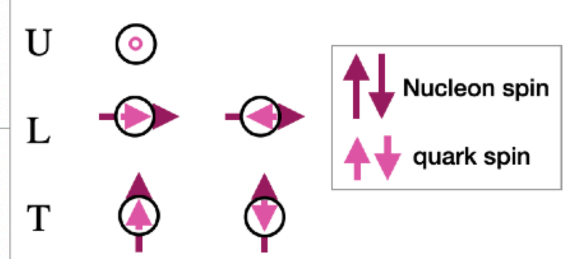
Twist-classification of PDFs, GPDs, TMDs

★ Twist: The order in Q^{-1} entering factorization

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L ($\gamma^+\gamma^5$)	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity



(Selected) Twist-3 $(f_i^{(1)})$

$\mathcal{O}$ / Nucleon	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

★ Twist-2: probabilistic densities - a wealth of information exists (mostly on PDFs)

★ Twist-3: poorly known, but very important and have physical interpretation:
- as sizable as twist-2
- contain information about quark-gluon correlations inside hadrons
- appear in QCD factorization theorems for various observables (e.g. g_2)

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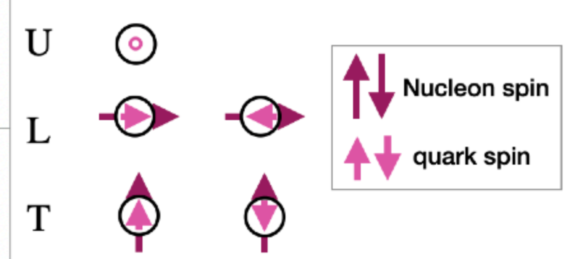

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The extraction of twist-3 is very challenges both experimentally and theoretically

Disclaimer

★ Formalism does not consider mixing with q-g-q correlators

★ Matching formalism with mixing is available

[V. Braun et al., JHEP 05 (2021) 086; JHEP 10 (2021) 087]

★ Nf=2+1+1 twisted mass fermions with a clover term

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

Theoretical setup

★ **Correlation functions in coordinate space**

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ **Parametrization of coordinate-space correlation functions**

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
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★ Twist-3 contributions to helicity GPDs: $\gamma^{1,2}\gamma_5$

Theoretical setup

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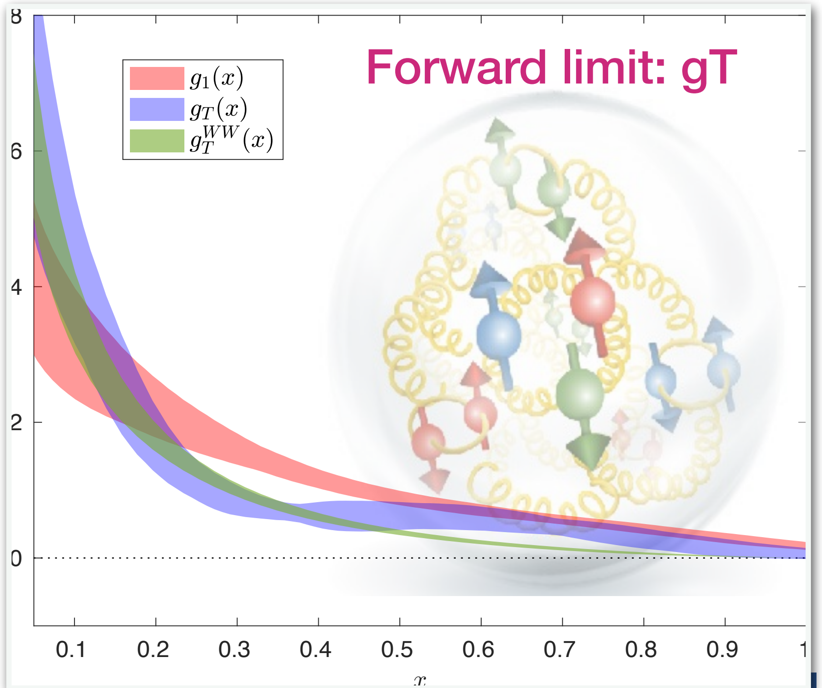

[S. Bhattacharya et al., PRD 102 (2020) 11] (Editors Highlight)

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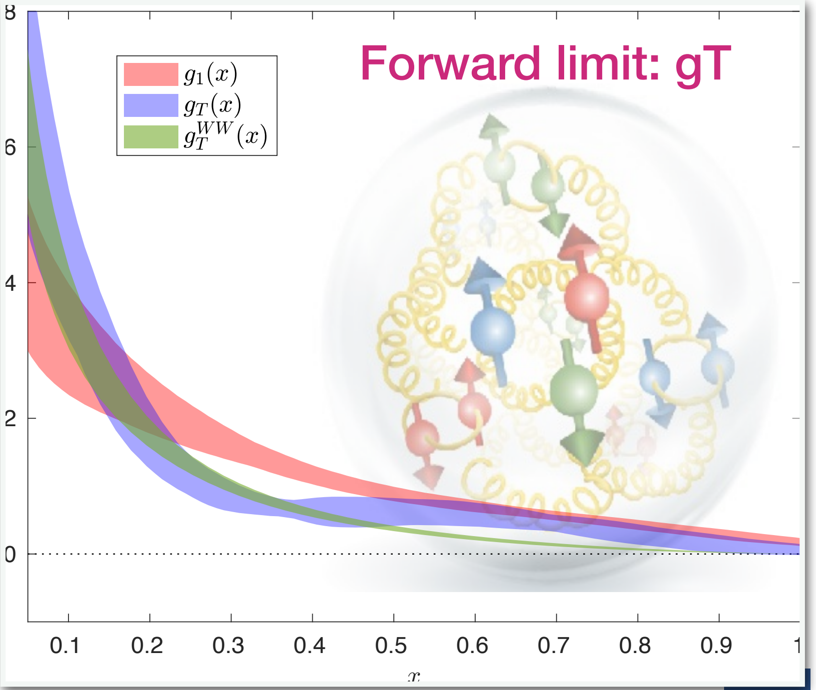
★ Kinematic twist-3 contributions to pseudo- and quasi-GPDs to restore translation invariance

[V. Braun et al., JHEP 10 (2023) 134]



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$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu \frac{\gamma^3\gamma_5}{P^0} F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu \frac{\Delta^3\gamma_5}{2mP^0} F_{\widetilde{E}}(x,\xi,t;P^3) + \Delta_\perp^\mu \frac{\gamma_5}{2m} F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3) + \Delta_\perp^\mu \frac{\gamma^3\gamma_5}{P^3} F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu \frac{\gamma^3}{P^3} F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big] u(p_i,\lambda)$$

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$$\widetilde{F}^\mu(z,P,\Delta) \equiv \langle p_f;\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\gamma^\mu\gamma_5\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i;\lambda\rangle = \bar{u}(p_f,\lambda')\Big[\frac{i\epsilon^{\mu Pz\Delta}}{m}\widetilde{A}_1 + \gamma^\mu\gamma_5\widetilde{A}_2 + \gamma_5\Big(\frac{P^\mu}{m}\widetilde{A}_3 + mz^\mu\widetilde{A}_4 + \frac{\Delta^\mu}{m}\widetilde{A}_5\Big) + m\not{z}\gamma_5\Big(\frac{P^\mu}{m}\widetilde{A}_6 + mz^\mu\widetilde{A}_7 + \frac{\Delta^\mu}{m}\widetilde{A}_8\Big)\Big]u(p_i,\lambda)\,,$$

[S. Bhattacharya et al., 109 (2024) 3, 034508]

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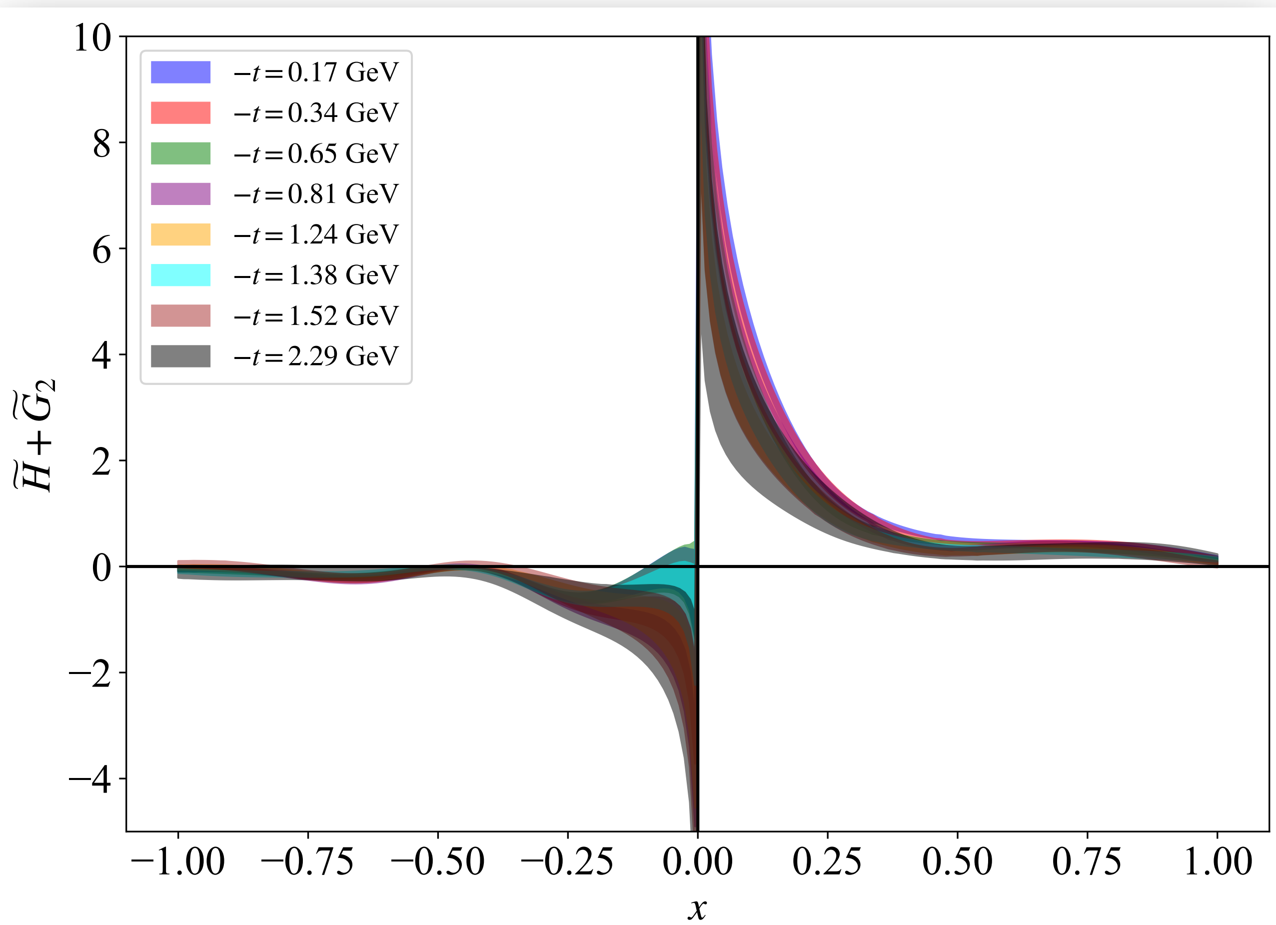
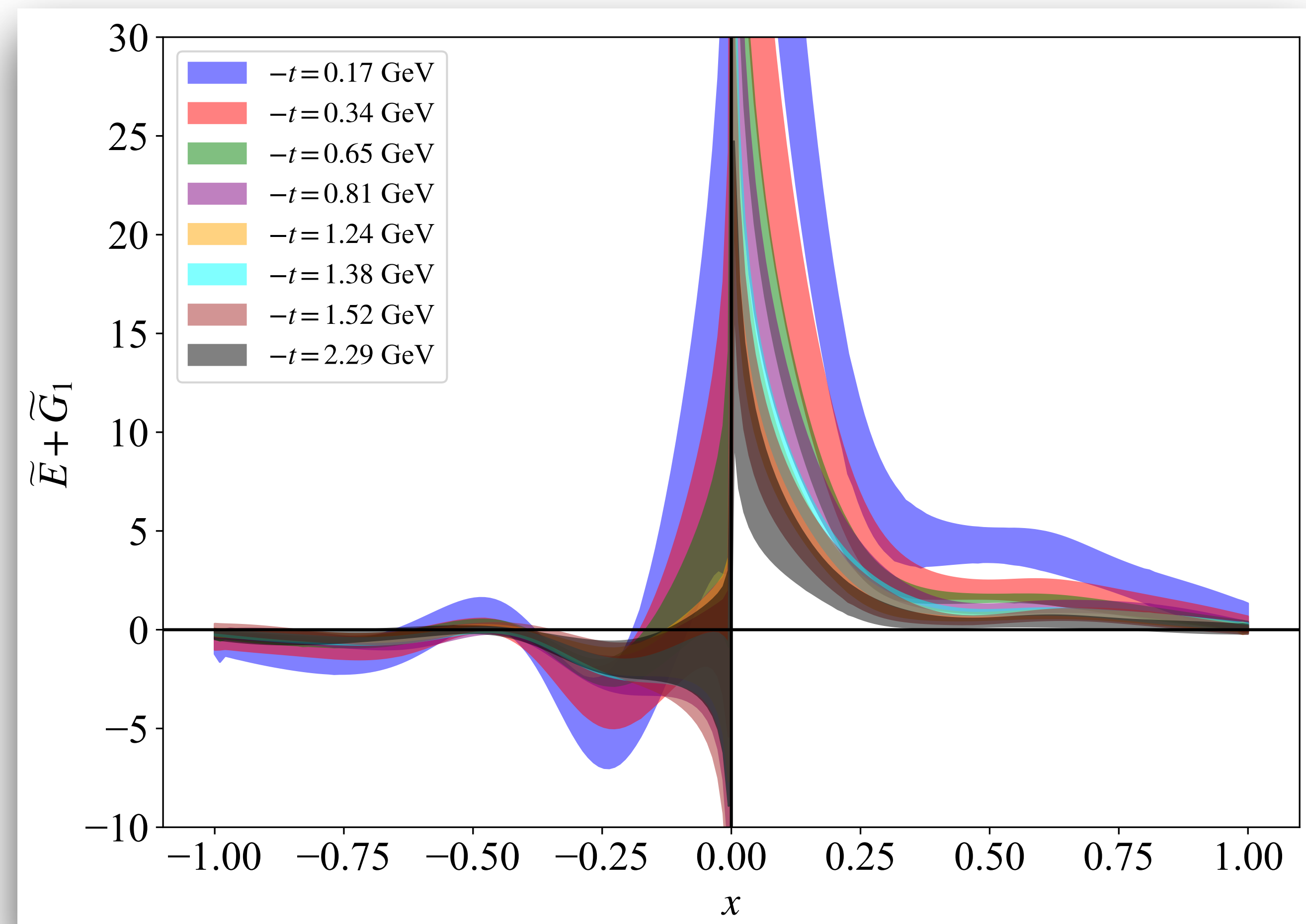
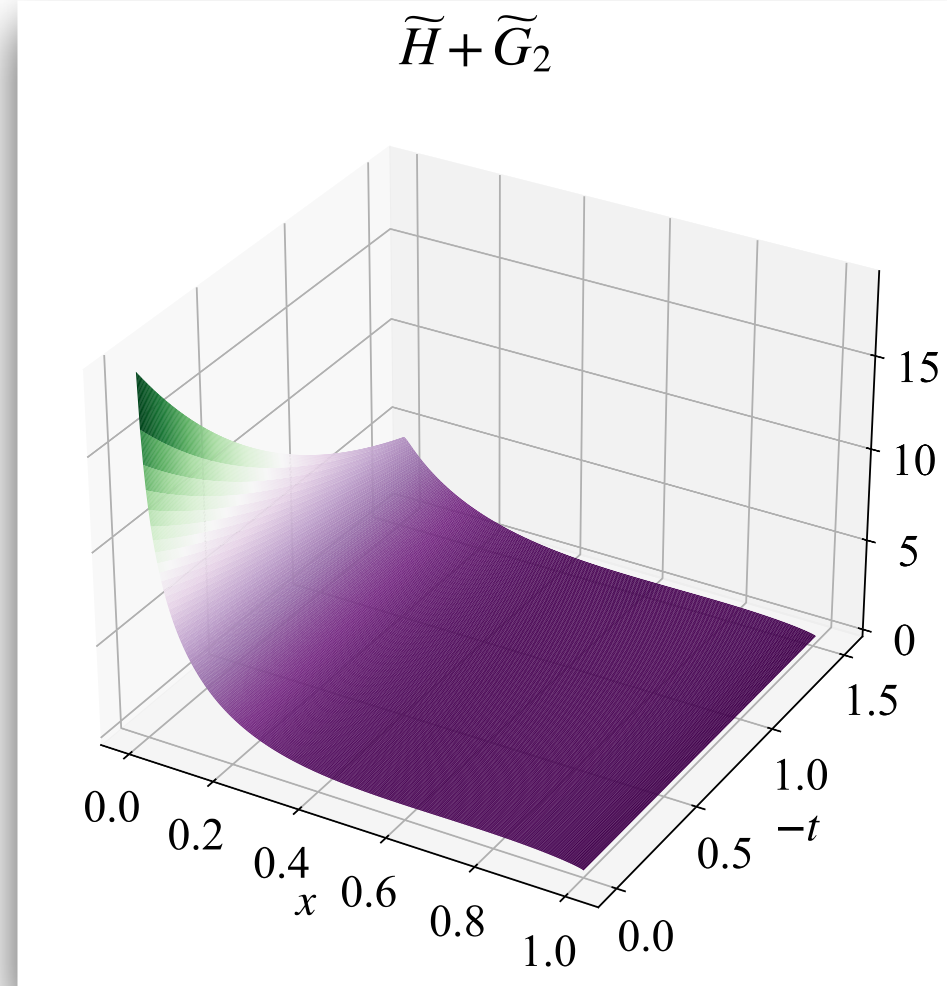


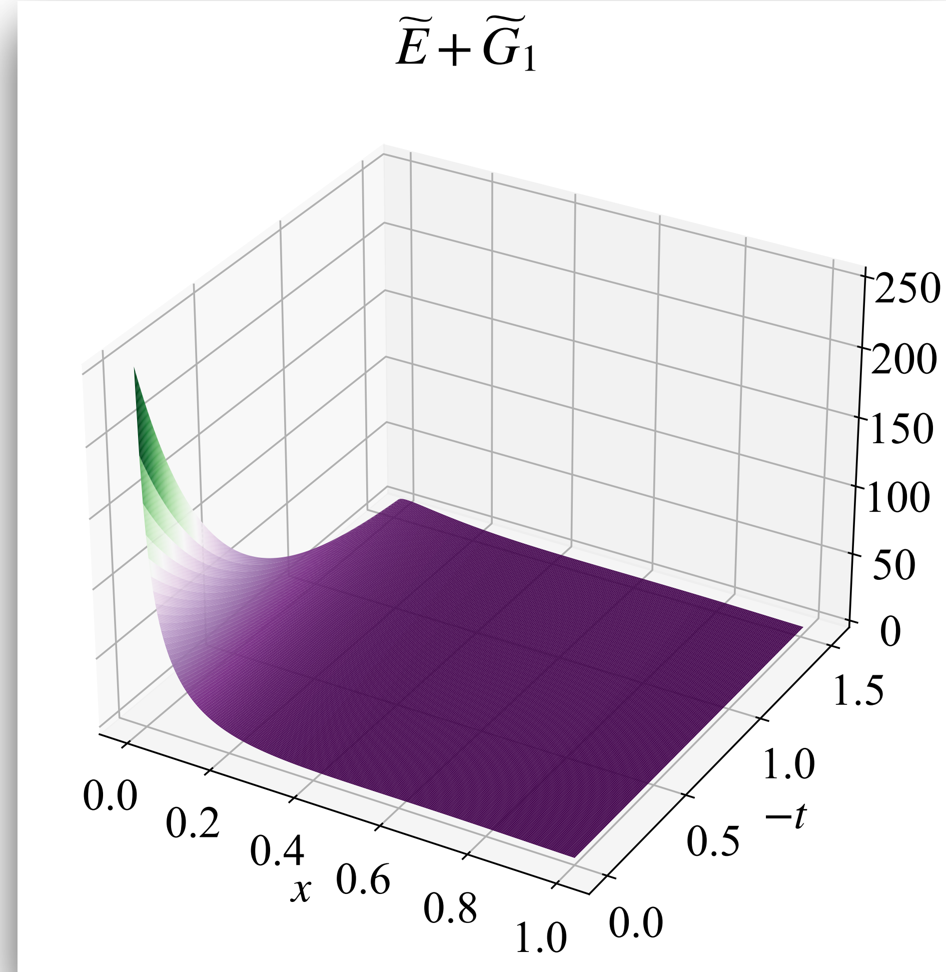


★ Parametrization of -t dependence

$$\mathrm{GPD}(x,-t,0) = A x^{\alpha_0 - \alpha_1 t}(1-x)^\beta$$

Ademollo & Del Giudice Gatto & Preparata





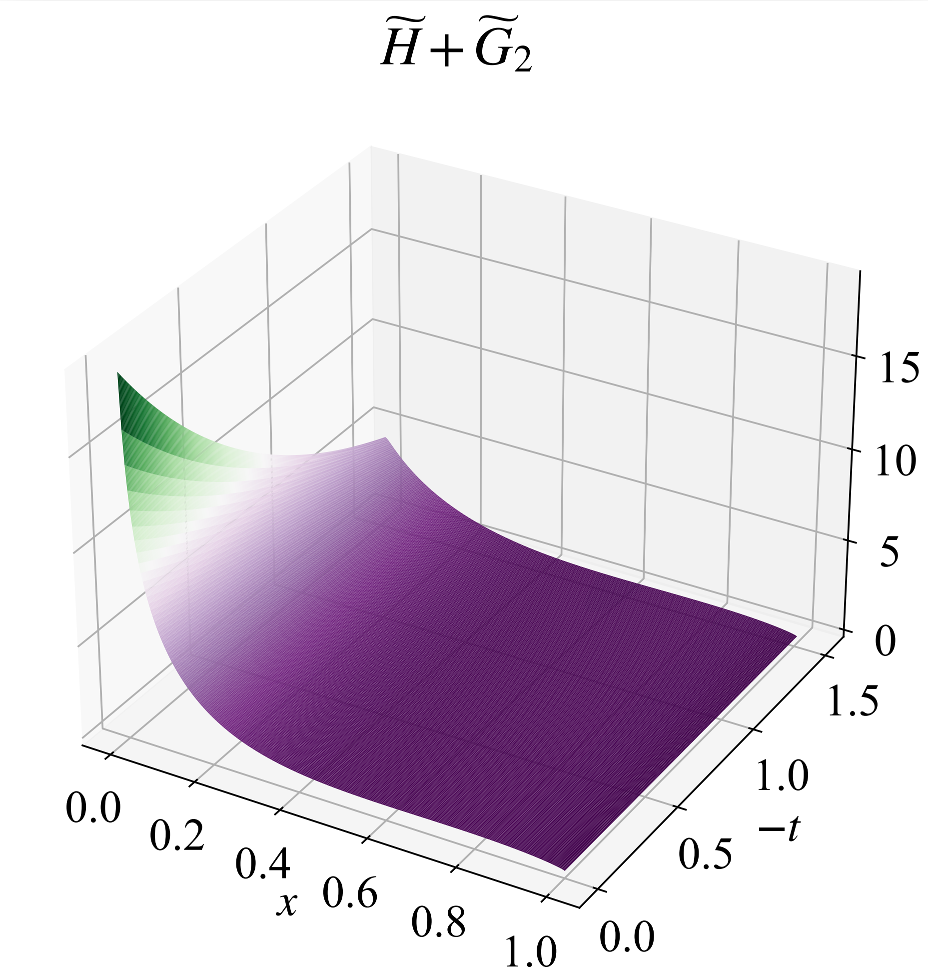
★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness

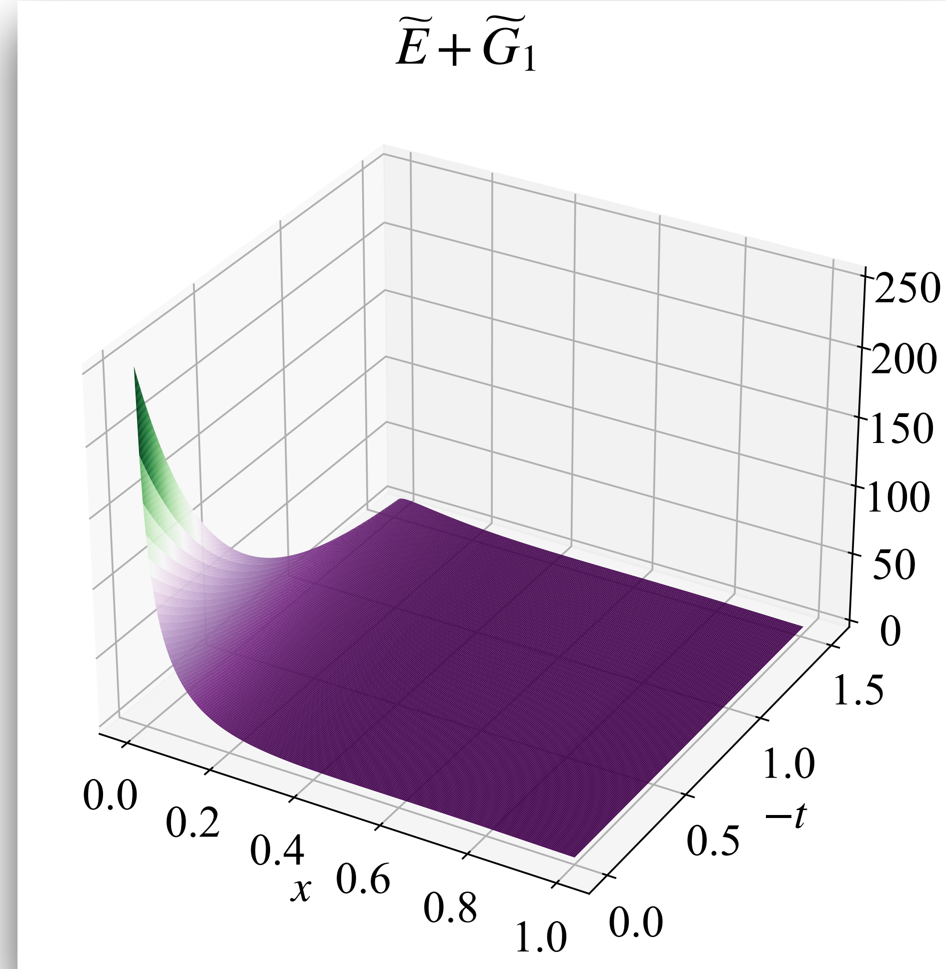
$$P^\mu \frac{\Delta^3 \gamma_5}{2mP^0} F_{\widetilde{E}}(x,\xi,t;P^3)$$

★ Glimpse into $\widetilde{E}$-GPD through twist-3 :

$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i=1,2,3,4$$





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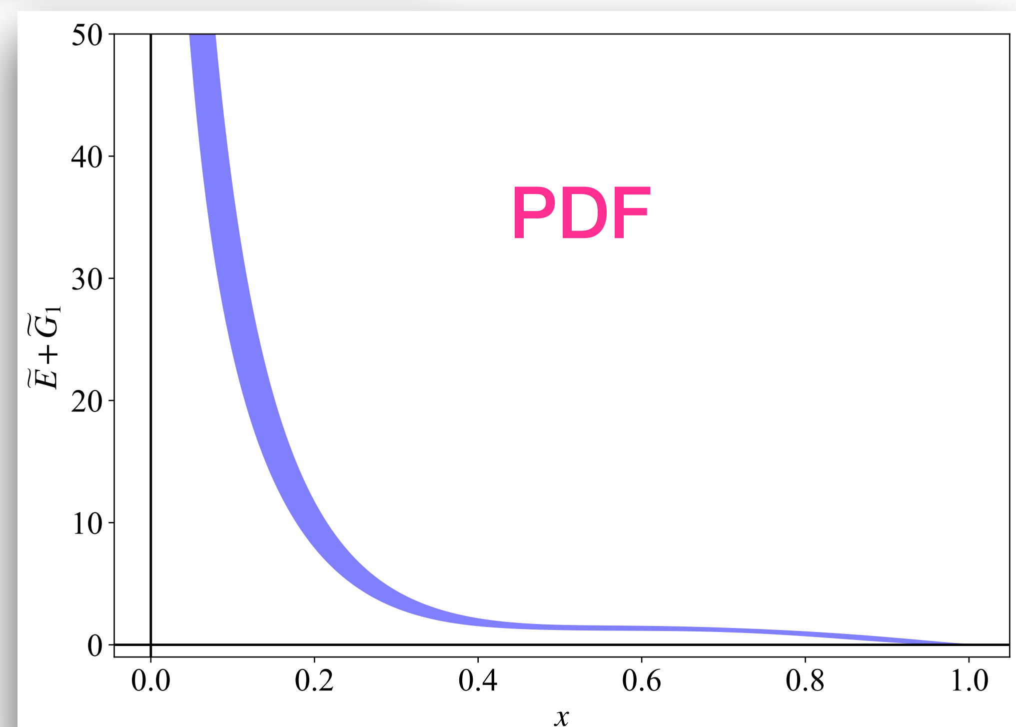

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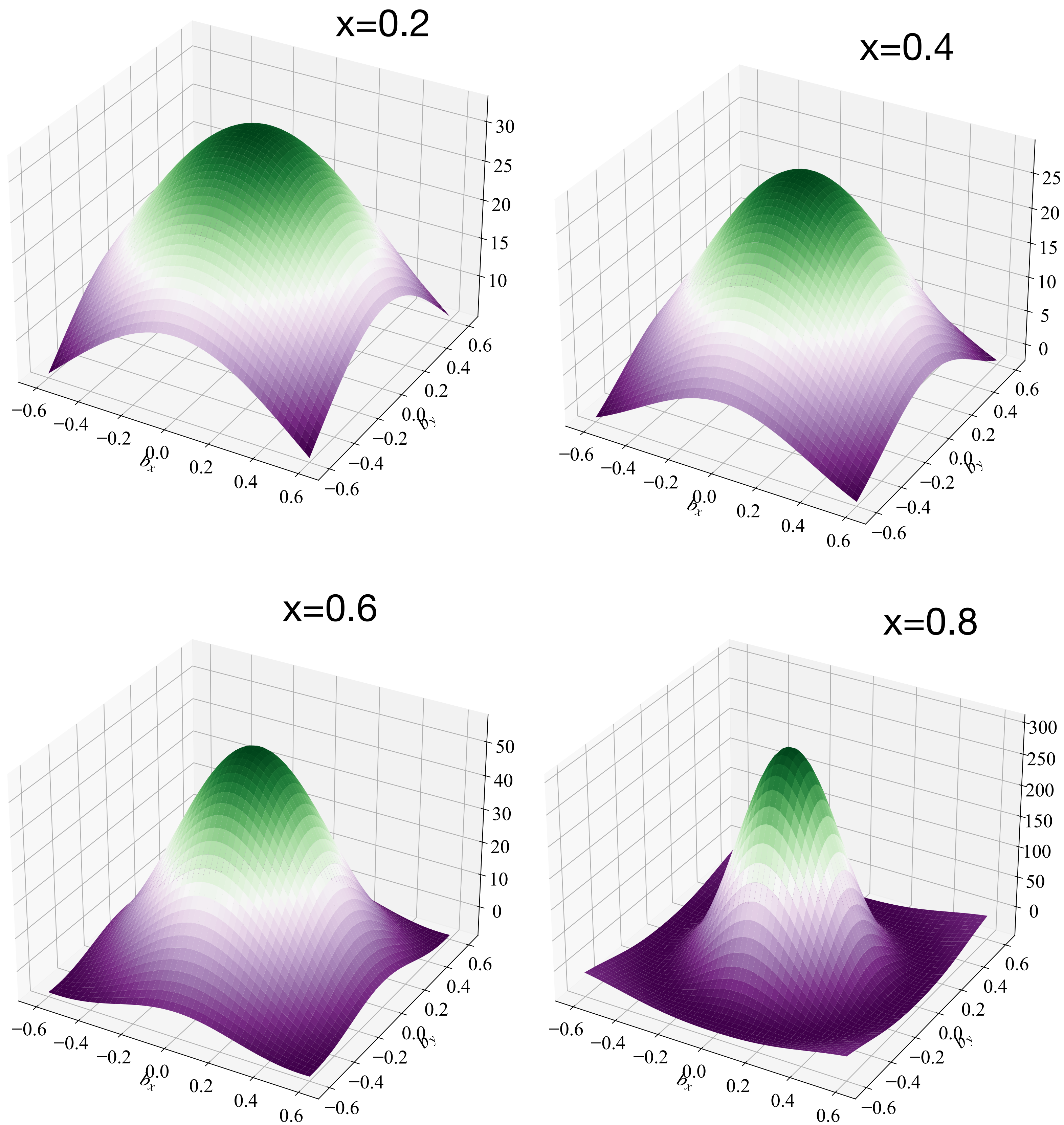
Impact parameter space $\widetilde{H} + \widetilde{G}_2$

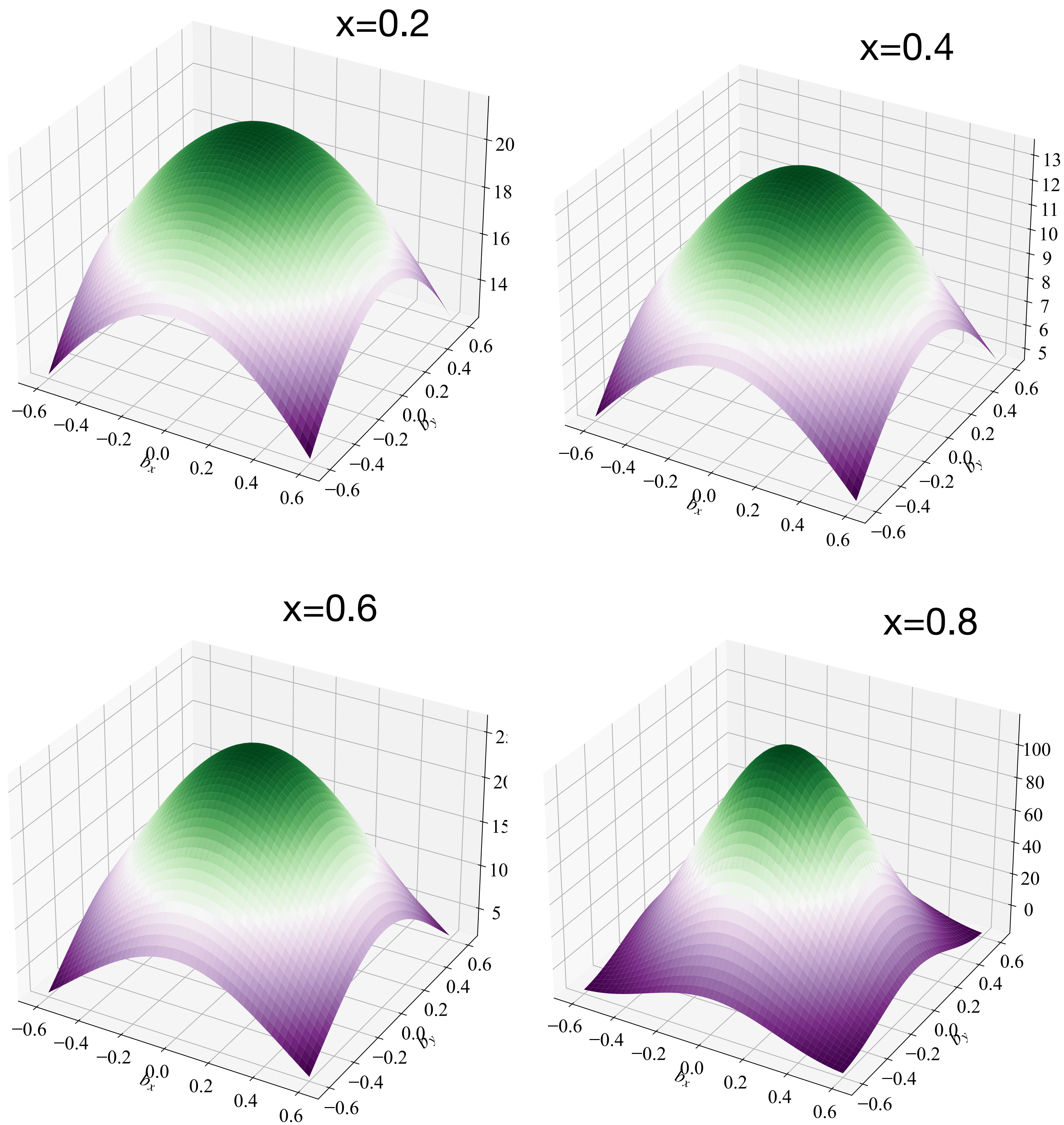


★ GPDs in transverse plane

$$q(x, \mathbf{b}_\perp) = |\mathcal{N}|^2 \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \int \frac{d^2\mathbf{p}'_\perp}{(2\pi)^2} H_q(x, -(\mathbf{p}_\perp - \mathbf{p}'_\perp)^2) e^{i\mathbf{b}_\perp \cdot (\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$
$$= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H_q(x, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp},$$

$b_\perp$: transverse distance from the (transverse) center of momentum

Impact parameter space $\widetilde{E} + \widetilde{G}_1$



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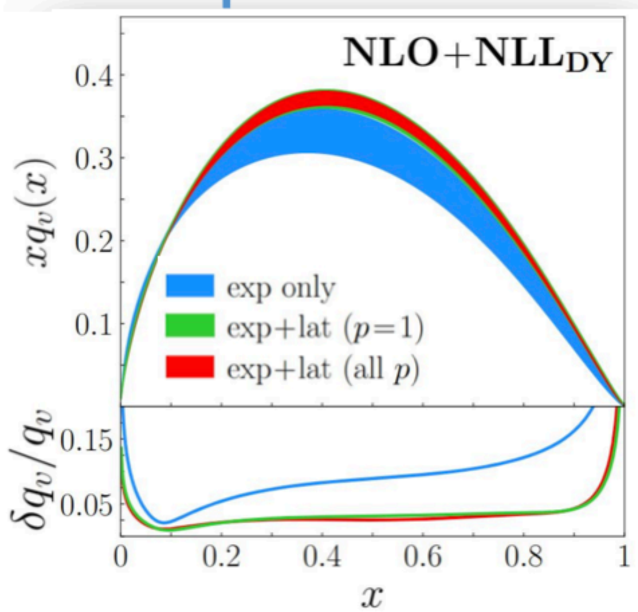
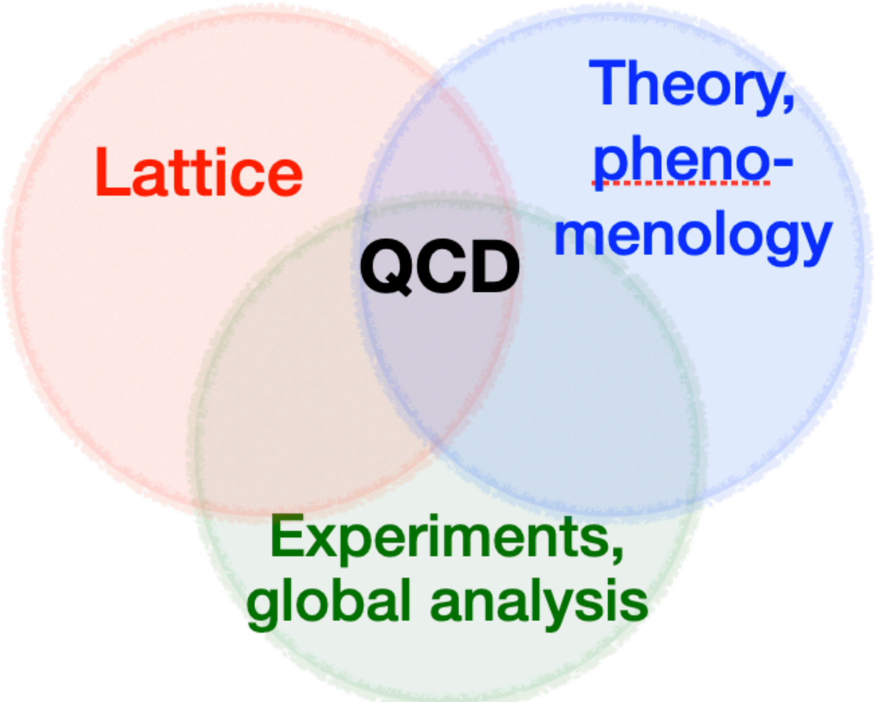
Synergy/Complementarity of lattice and phenomenology

Synergies: constraints & predictive power of lattice QCD

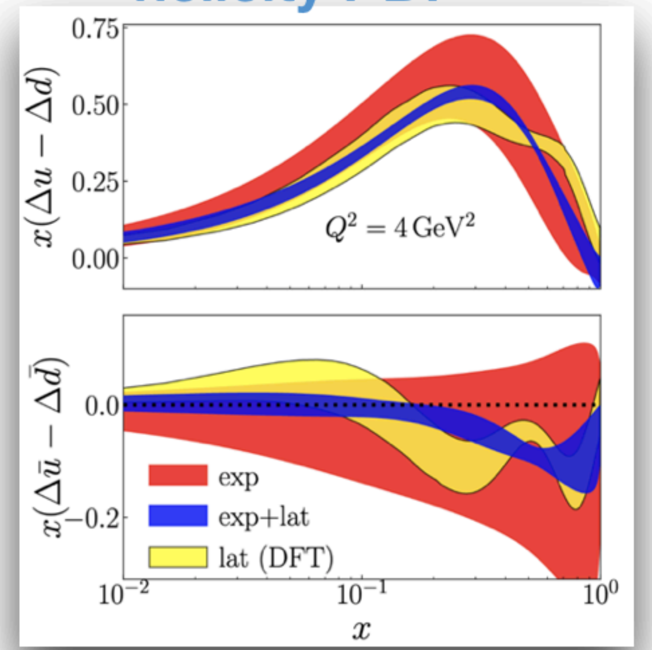
pion PDF



[JAM/HadStruc, PRD105 (2022) 114051]

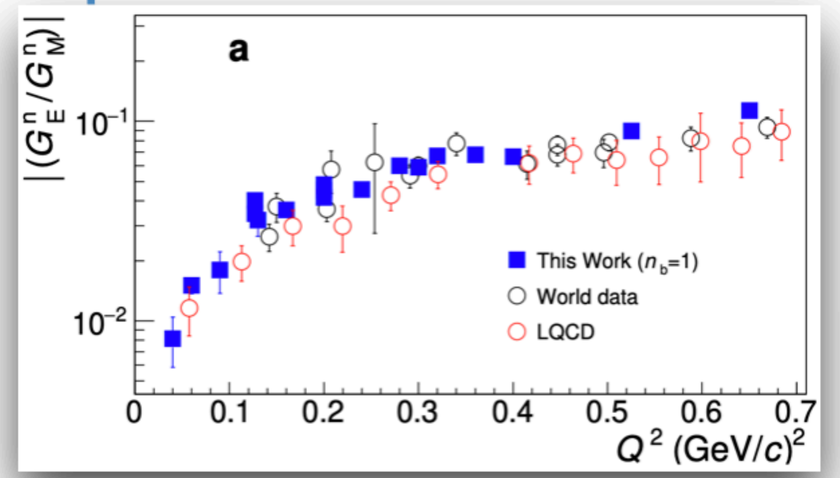


helicity PDF



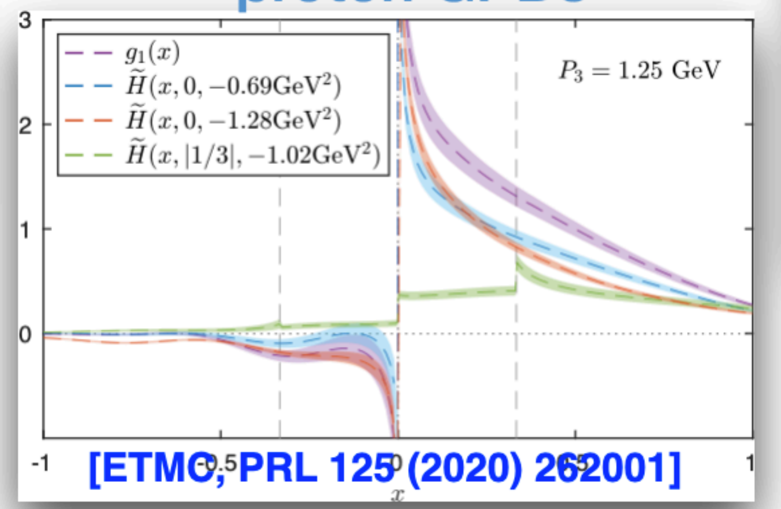
[JAM & ETMC, PRD 103 (2021) 016003]

proton & neutron radius



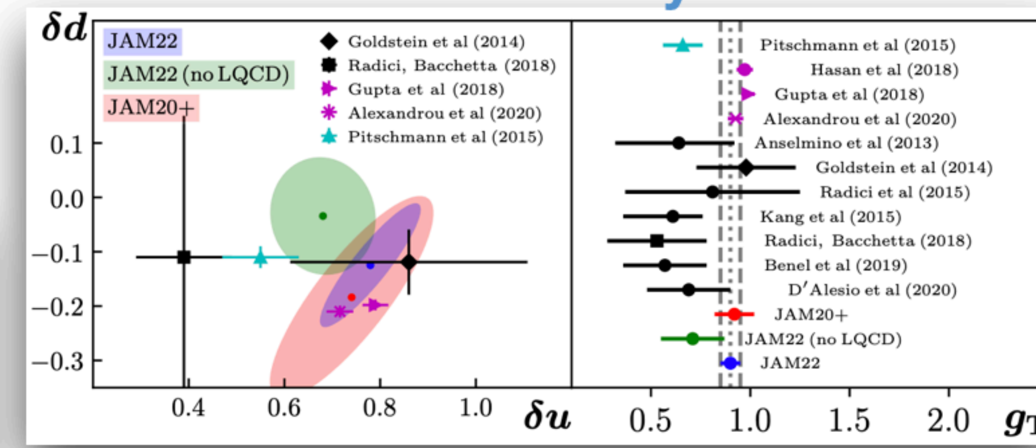
[Atac et al., Nature Comm. 12, 1759 (2021)]

proton GPDs



[ETMC, PRL 125 (2020) 262001]

transversity PDF



[JAM, PRD 106 (2022) 3, 034014]

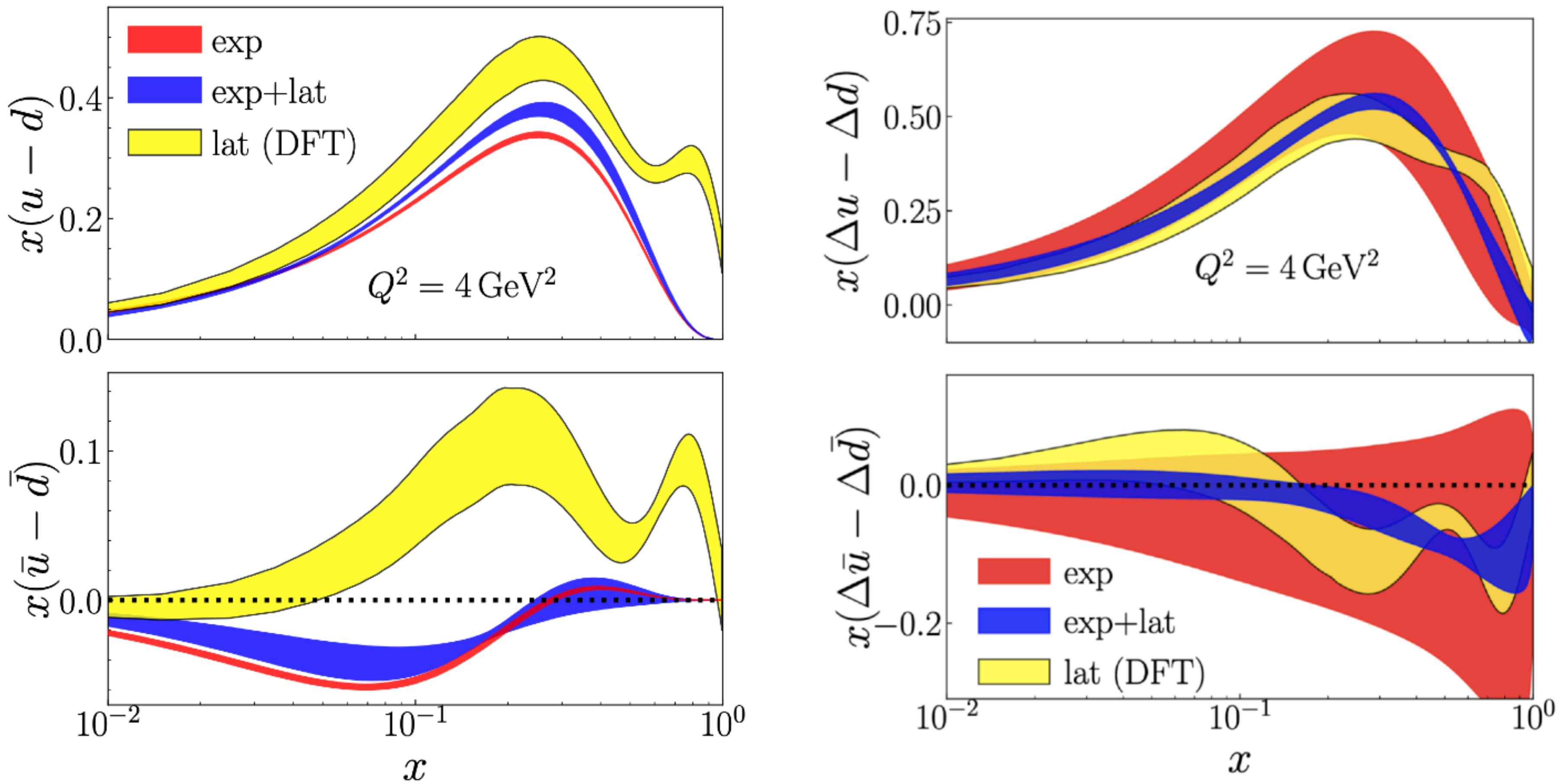
And many more!

Incorporating lattice PDFs in global analyses

Synergy between lattice and phenomenology

★ Lattice and experimental data sets data within the same global analysis (JAM framework)

[J. Bringewatt et al., PRD 103 (2021) 016003, arXiv:2010.00548]



- Consistent picture with JAM for unpolarized PDF

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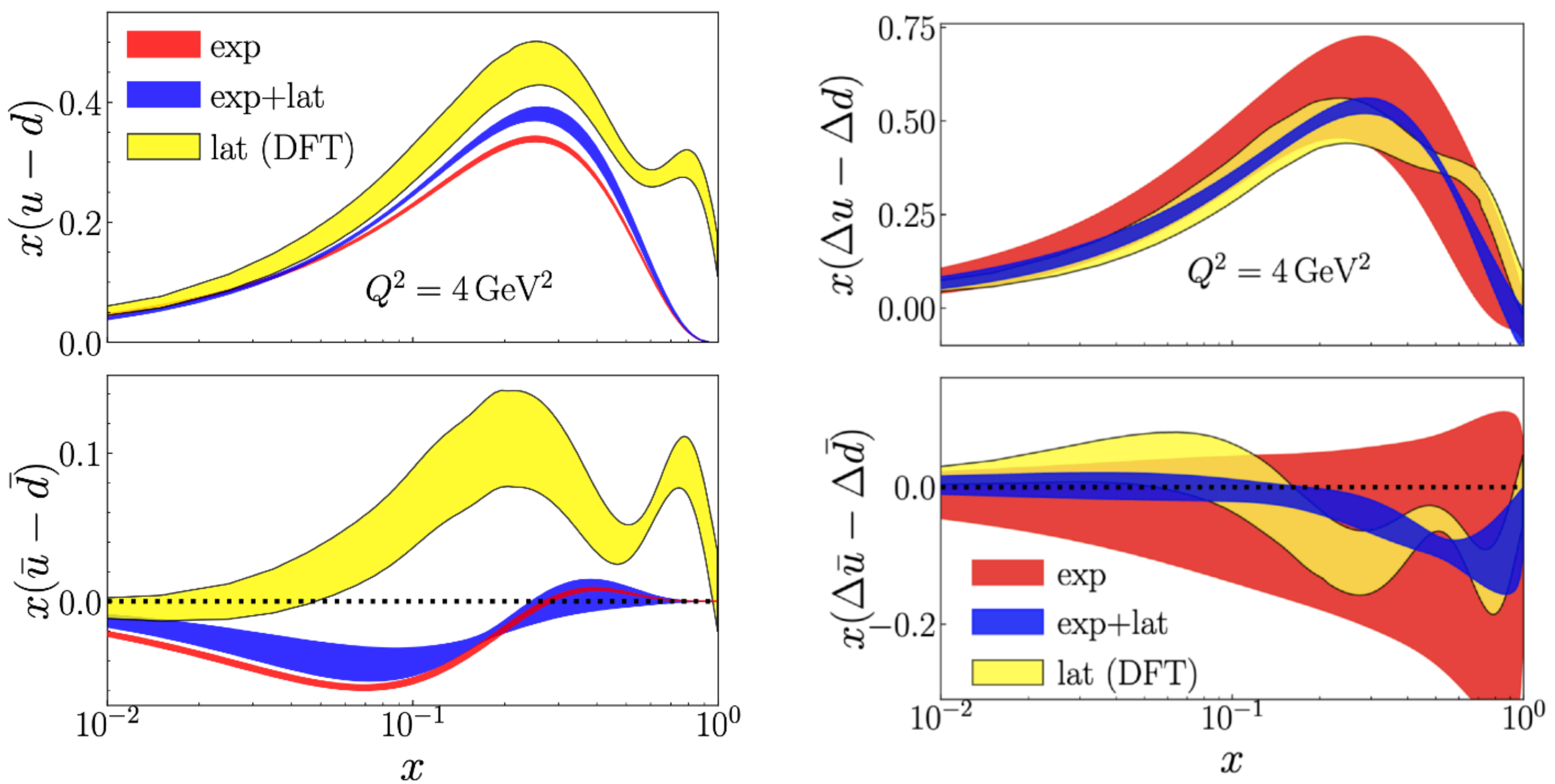

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★ Other efforts within NNPDF framework

[K. Cichy et al., JHEP 10 (2019) 137, arXiv:1907.06037]
[L. Del Debbio et al., JHEP 02 (2021) 138, 2010.03996]

★ Interest in applying similar approach to quantities that are more challenging to extract experimentally (GPDs, twist-3 distributions, …)

Toward synergy for GPDs

★ Forming ratios of GPDs seems to suppress systematic uncertainties

[K. Cichy et al., arXiv:2409.17955]

$$\mathrm{DR}^{\hat{H}^q}_{\mathrm{Re}}(\nu,t) = \frac{\mathrm{Re}\hat{H}^q(\nu,t)}{\mathrm{Re}\hat{H}^q(\nu,0)}\,\frac{\mathrm{Re}\hat{H}^q(0,0)}{\mathrm{Re}\hat{H}^q(0,t)},$$

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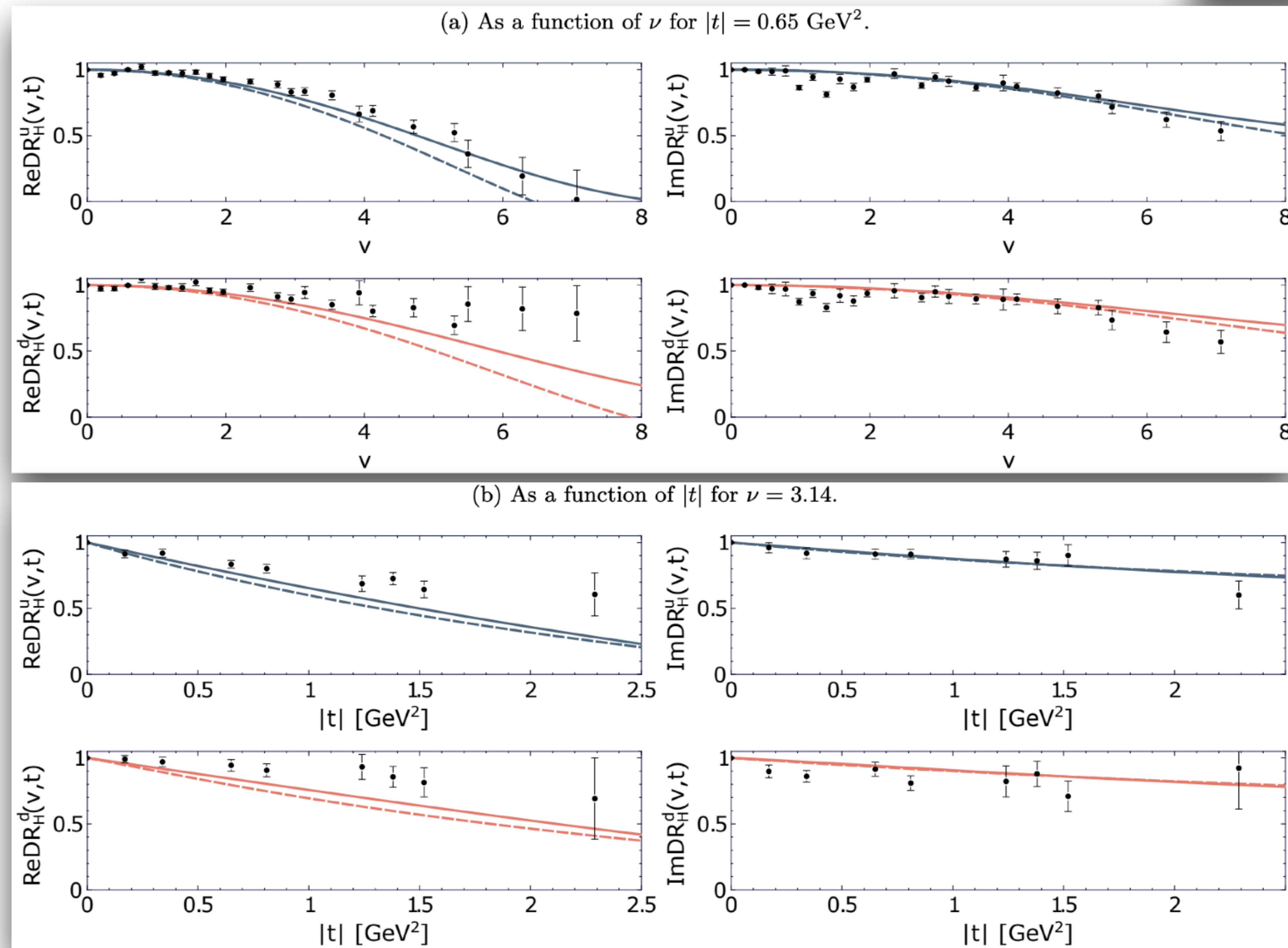

- GK (solid curve)
- VGG (dashed curve)

- Good agreement for up quark
- Reasonable agreement for down quark

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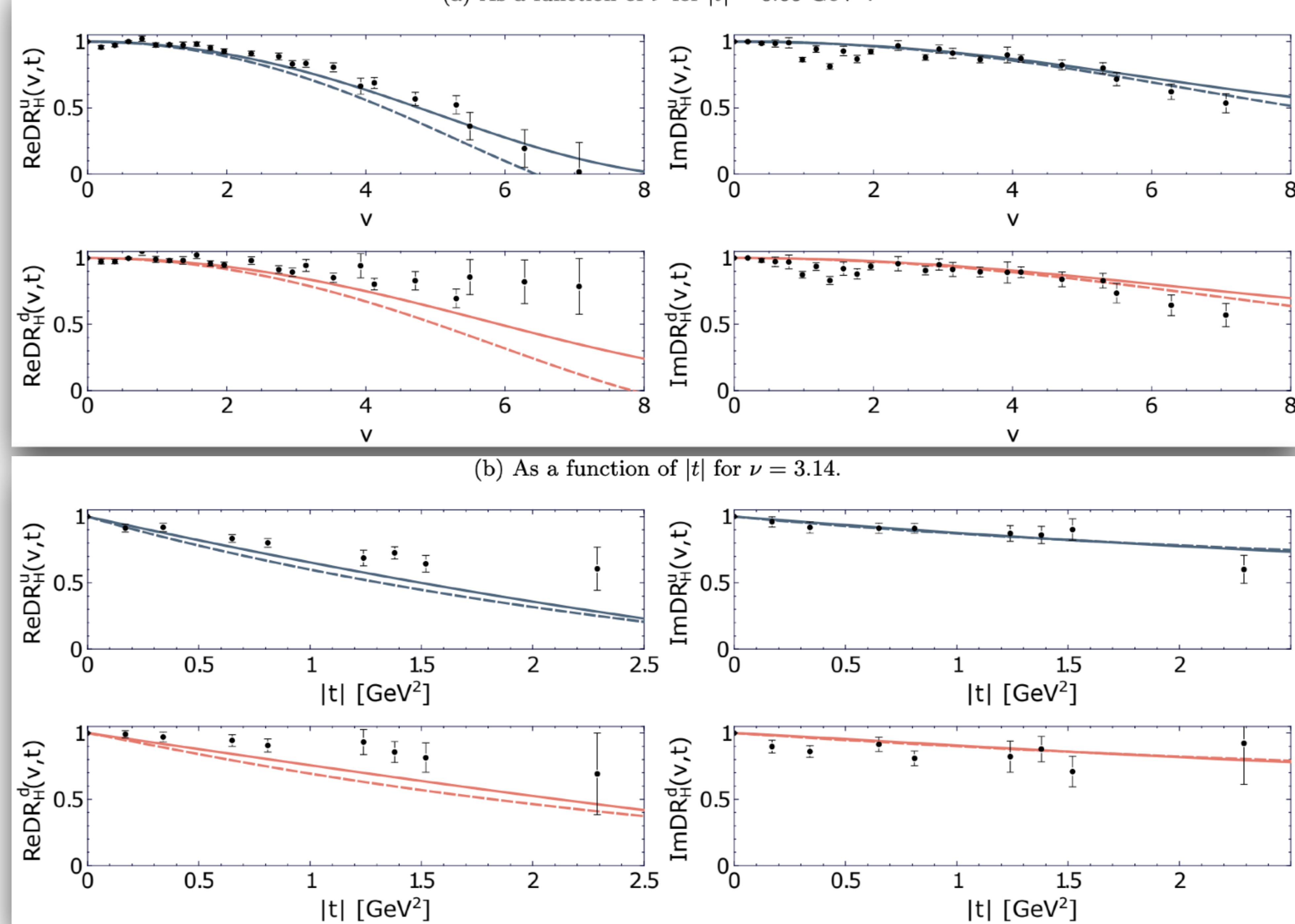

(a) As a function of ν for $|t| = 0.65$ GeV^2.

(b) As a function of $|t|$ for $\nu = 3.14$.

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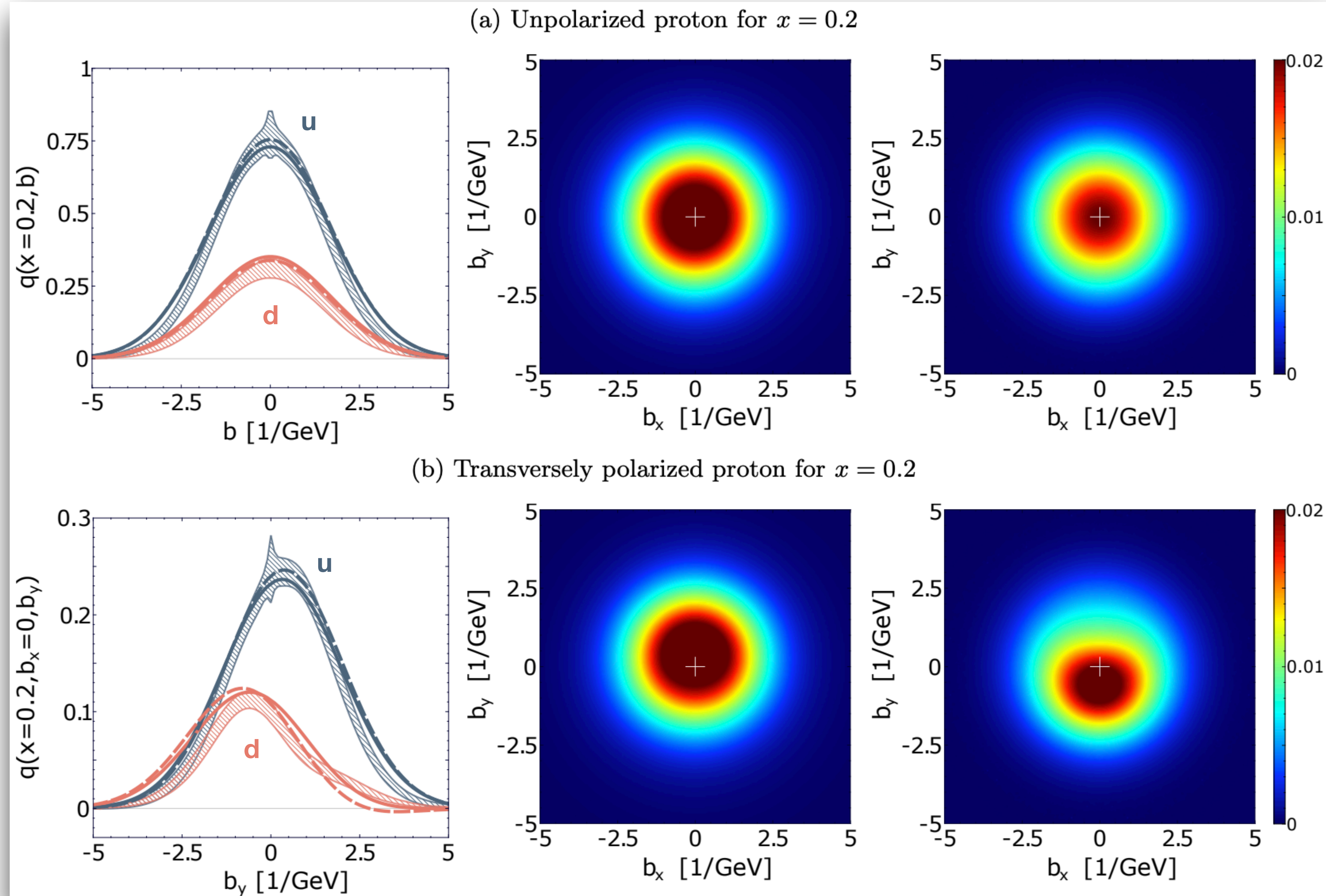
- VGG (dashed curve)

- Good agreement for up quark
- Reasonable agreement for down quark

- Further study needed on how to combine lattice results with data

Tomographic Images



- GK (solid line), VGG (dashed line)

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How to lattice QCD data fit into the overall effort for hadron tomography

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U.S. DEPARTMENT OF ENERGY | Office of Science

Award Number: DE-SC0023646

1. **Theoretical studies** of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties
2. **Lattice QCD** calculations of GPDs and related structures
3. **Global analysis** of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

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Other GPD global analysis efforts:

- Gepard [https://gepard.phy.hr/]
- PARTONS [https://partons.cea.fr]
- EXCLAIM [https://exclaimcollab.github.io/web.github.io/#/]

Concluding remarks

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Thank you

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U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING

NERSC

ASCR Leadership Computing Challenge ALCC

Additional slides

Quasi-GPDs: contact with light-cone quantities

★ Non-local operators with Wilson line fully renormalizable to all orders

[T. Ishikawa et al., Phys. Rev. D 96, no. 9 (2017) 094019] [X. Ji et al., Phys. Rev. Lett. 120, no. 11 (2018) 112001]

★ Quasi- & light-cone distributions share the same infrared structure

★ Differences in UV region (perturbatively calculable, LaMET)

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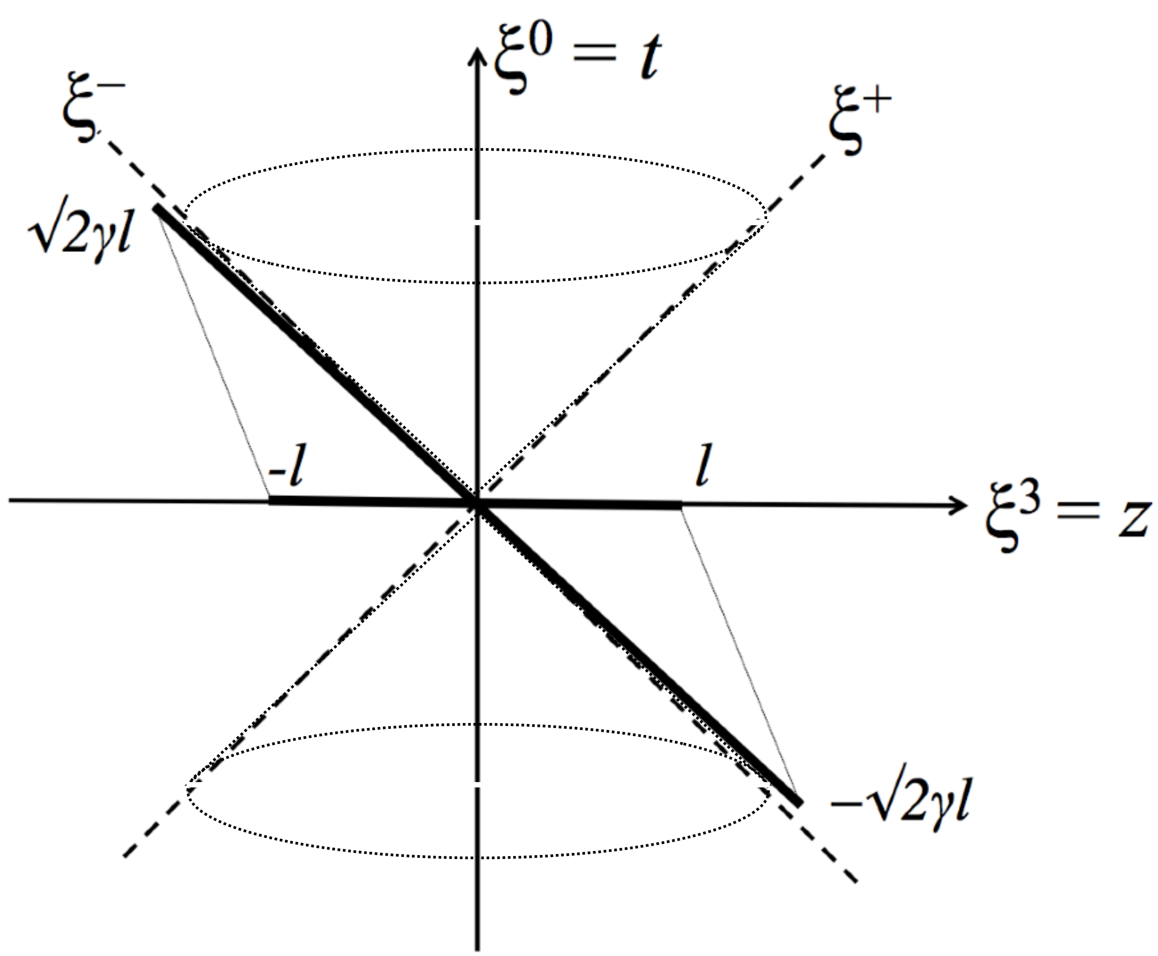

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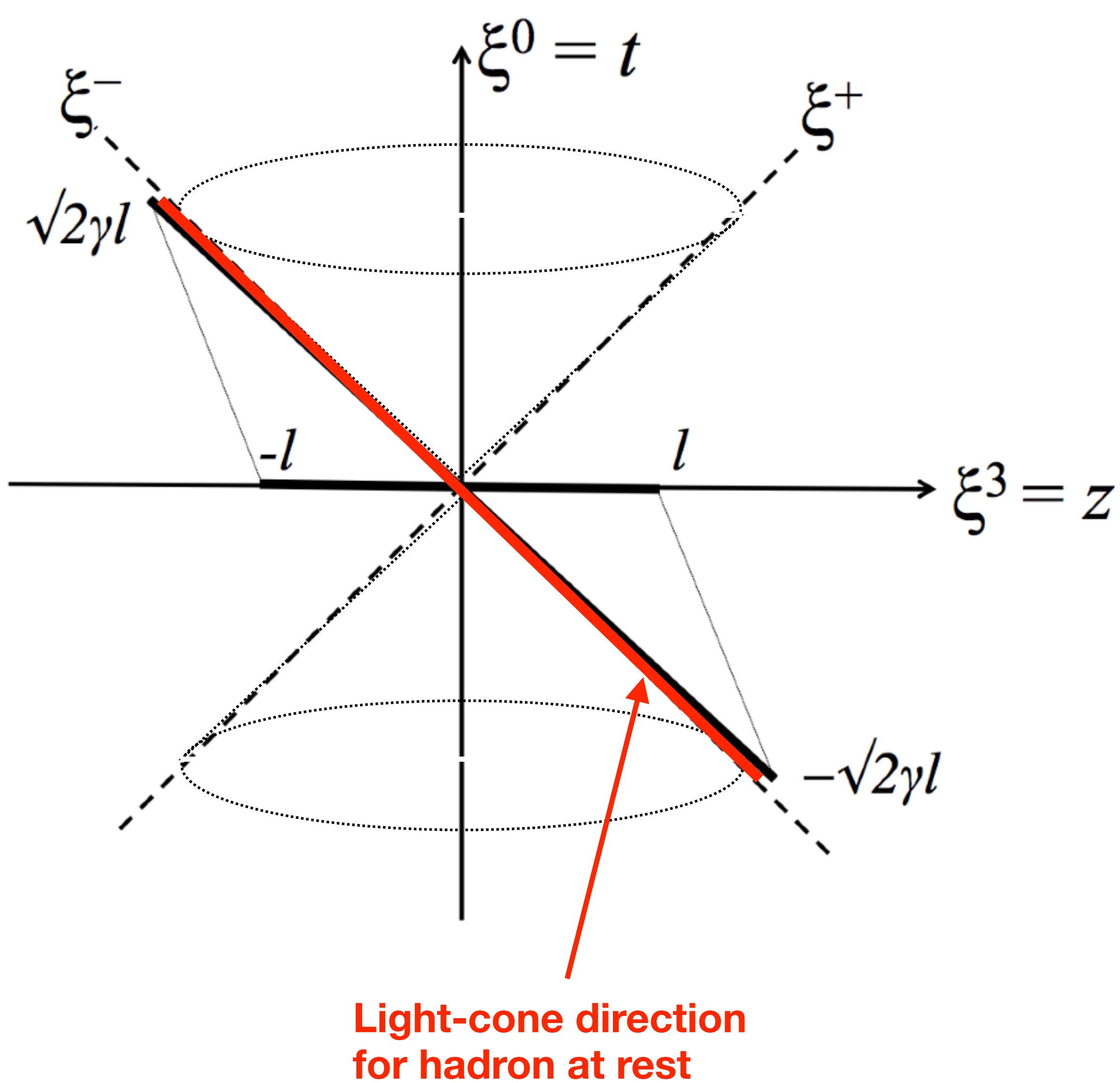

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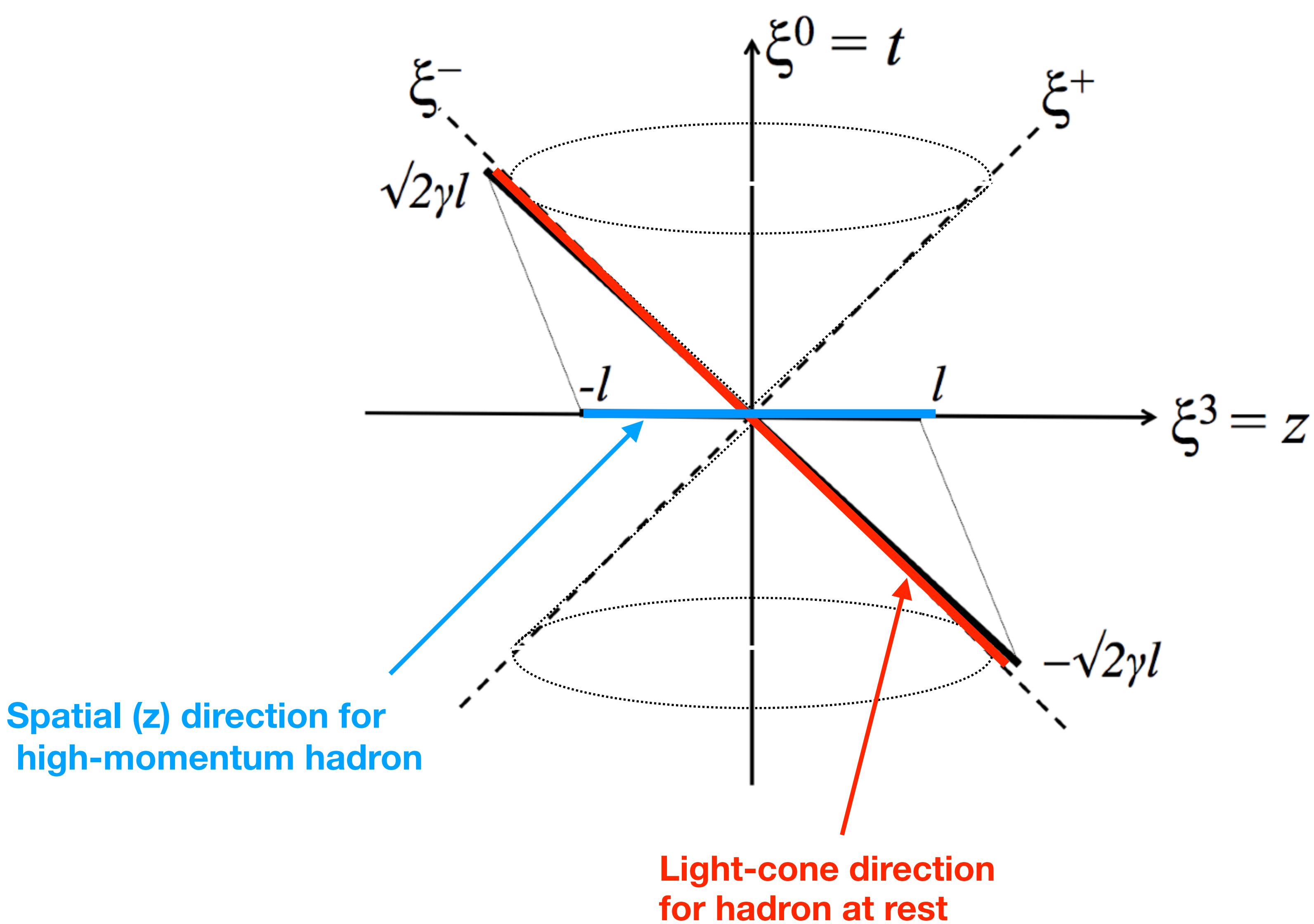

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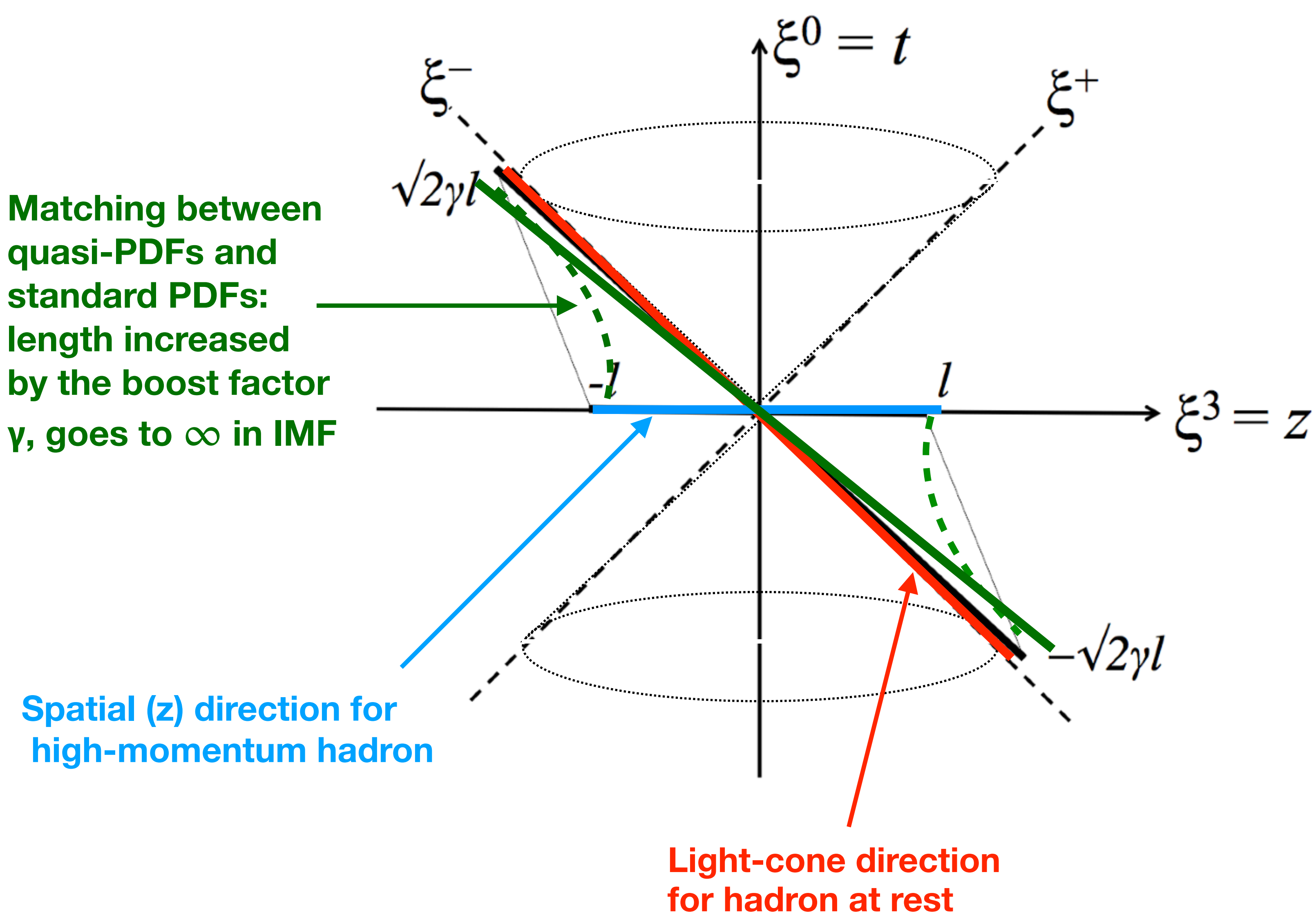

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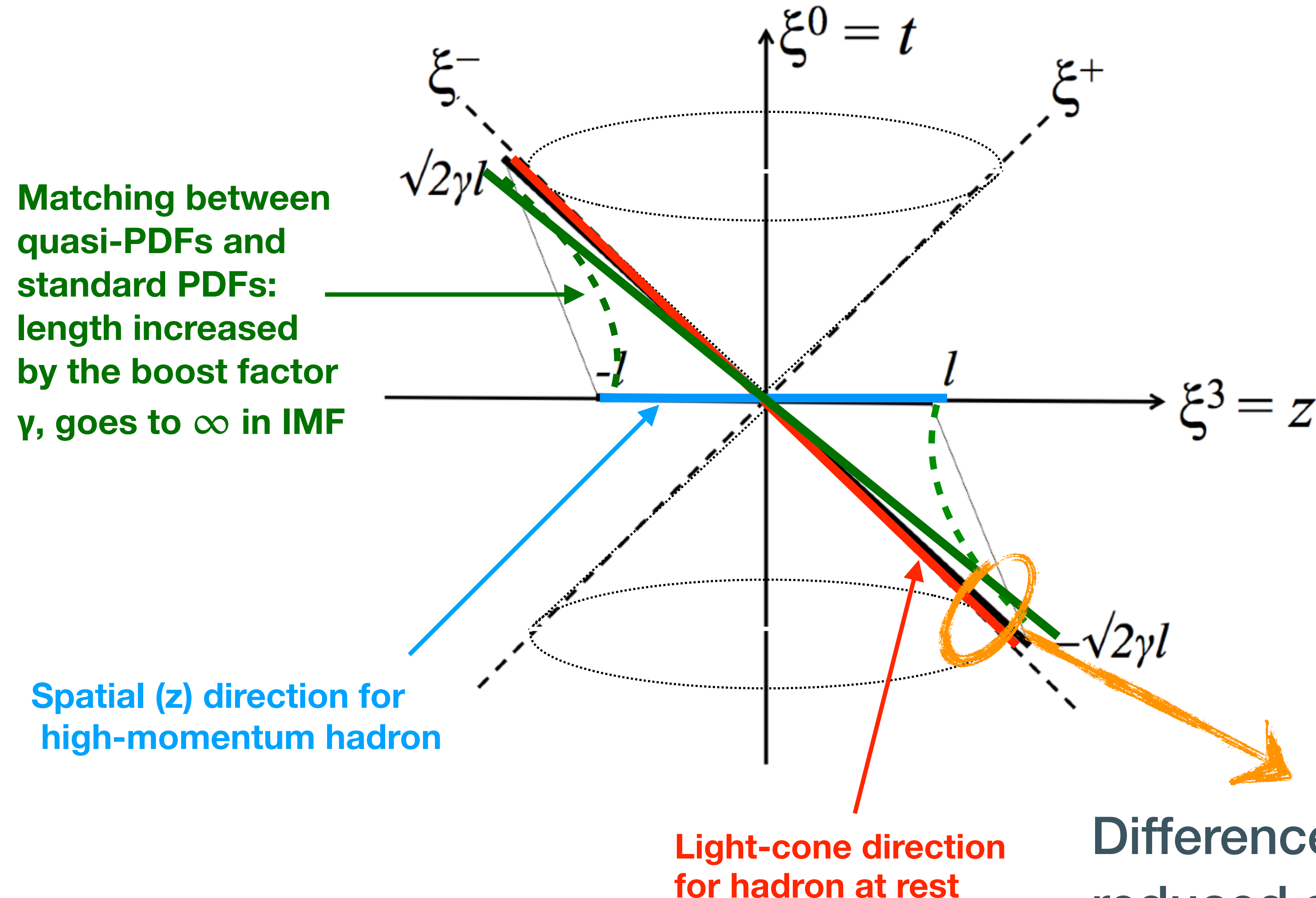

Difference $\mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, m_N^2/P_3^2, t/P_3^2, \ldots\right)$ reduced as P_3 increases

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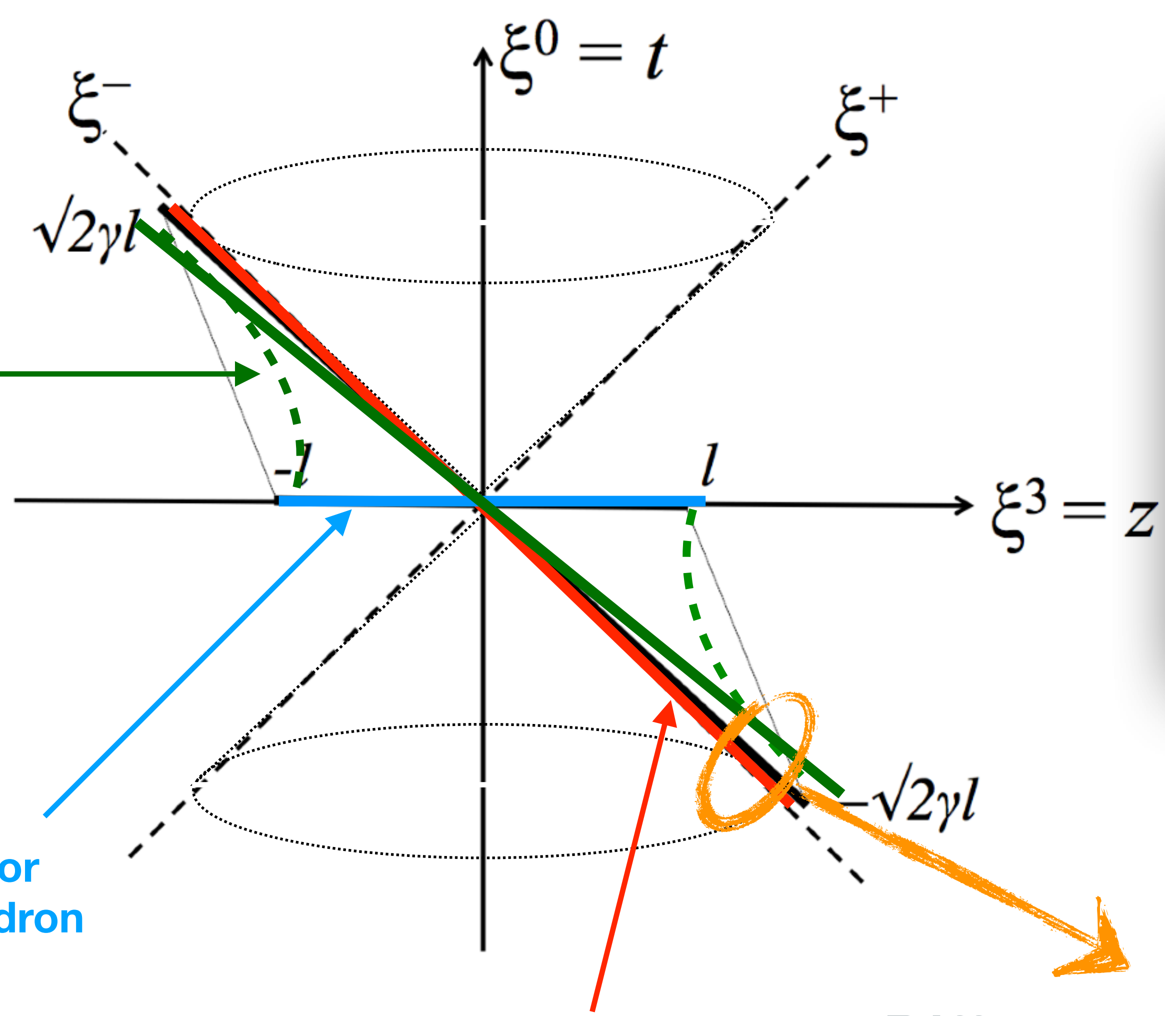
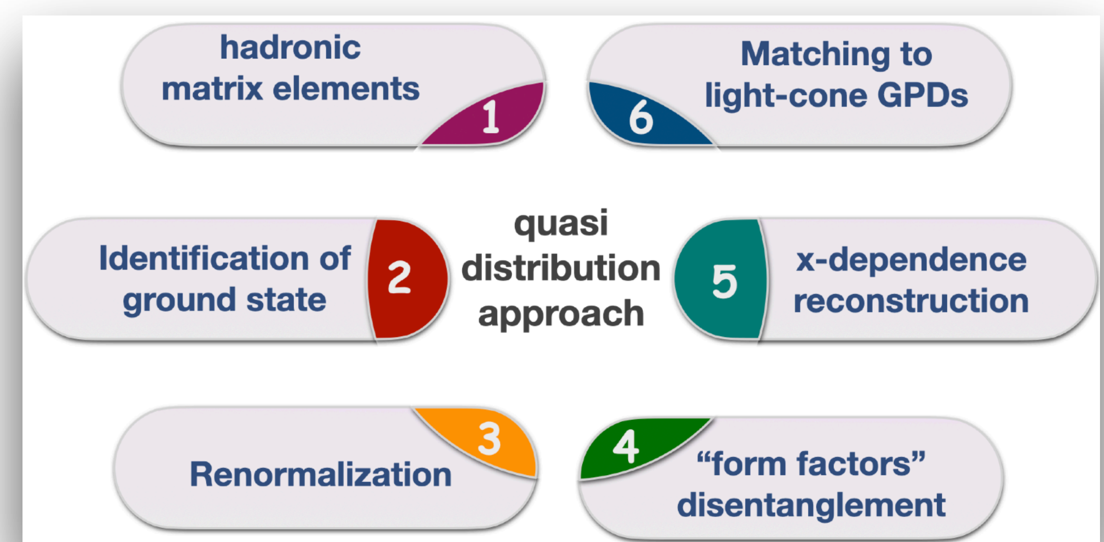
Matching between quasi-PDFs and standard PDFs: length increased by the boost factor γ, goes to ∞ in IMF



Spatial (z) direction for high-momentum hadron

Light-cone direction for hadron at rest

A number of non-trivial steps



Difference $\mathcal{O}\left(\Lambda^2_{QCD}/P_3^2, m_N^2/P_3^2, t/P_3^2, \ldots\right)$ reduced as P_3 increases