Lattice QCD for Multi-Nucleon Physics

Second LatticeNET workshop on challenges in Lattice field theory
Benasque Science Center, March 30 - April 05, 2025

André Walker-Loud

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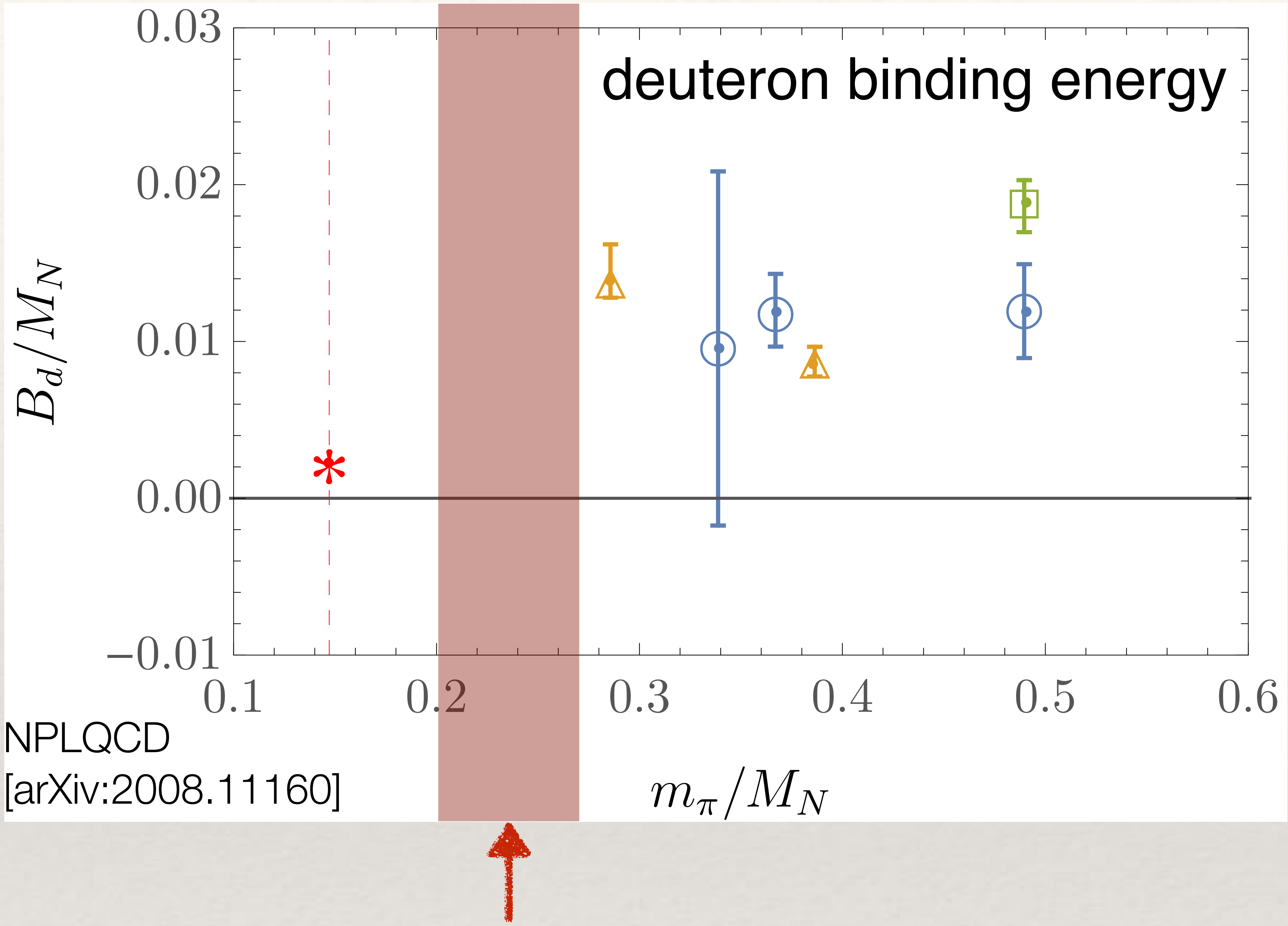
Lattice QCD for Multi-Nucleon Physics

- Di-nucleons do not form bound states at heavy pion mass ~ 80%

- LQCD constraints on SU(2) Heavy Baryon Chiral Perturbation Theory without Δ degrees of freedom (HBχPT($\not\Delta$)) ~ 20%

Survey of lattice QCD results for two-baryons



NPLQCD
[arXiv:2008.11160]

Estimated upper range of validity of NN EFT

LQCD Results with (deeply) bound di-nucleons

2006 NPLQCD - first dynamical LQCD calculations of NN

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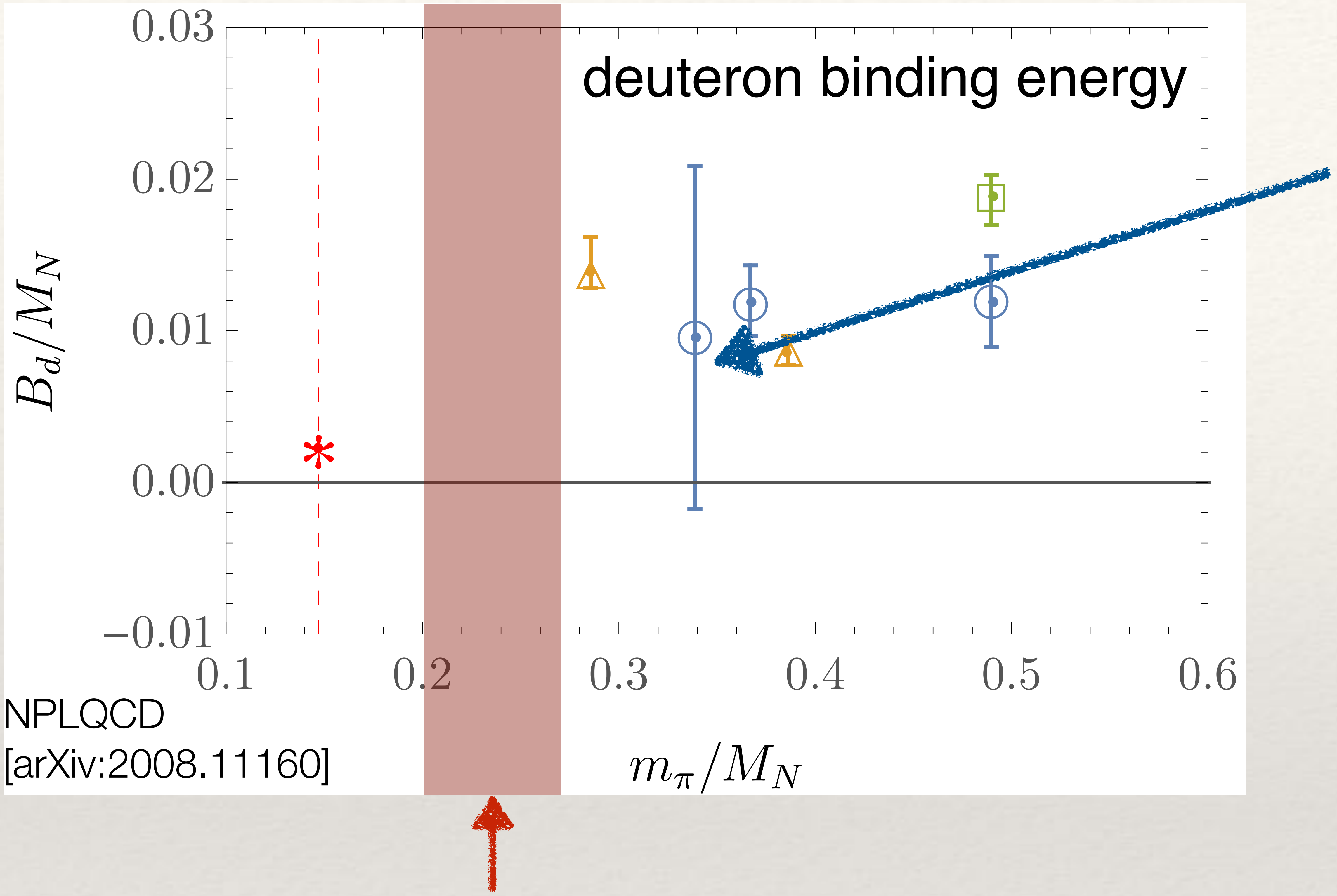

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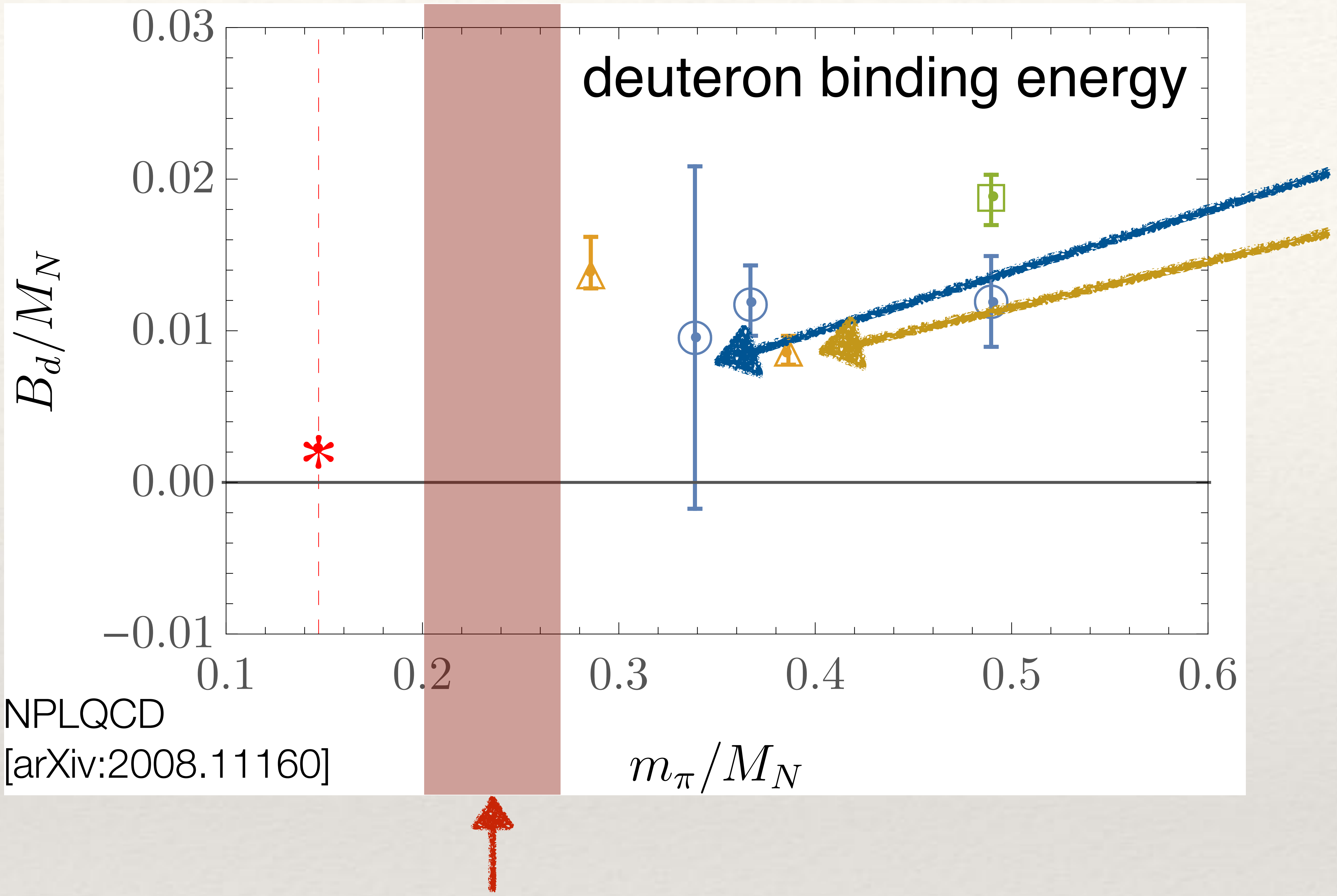

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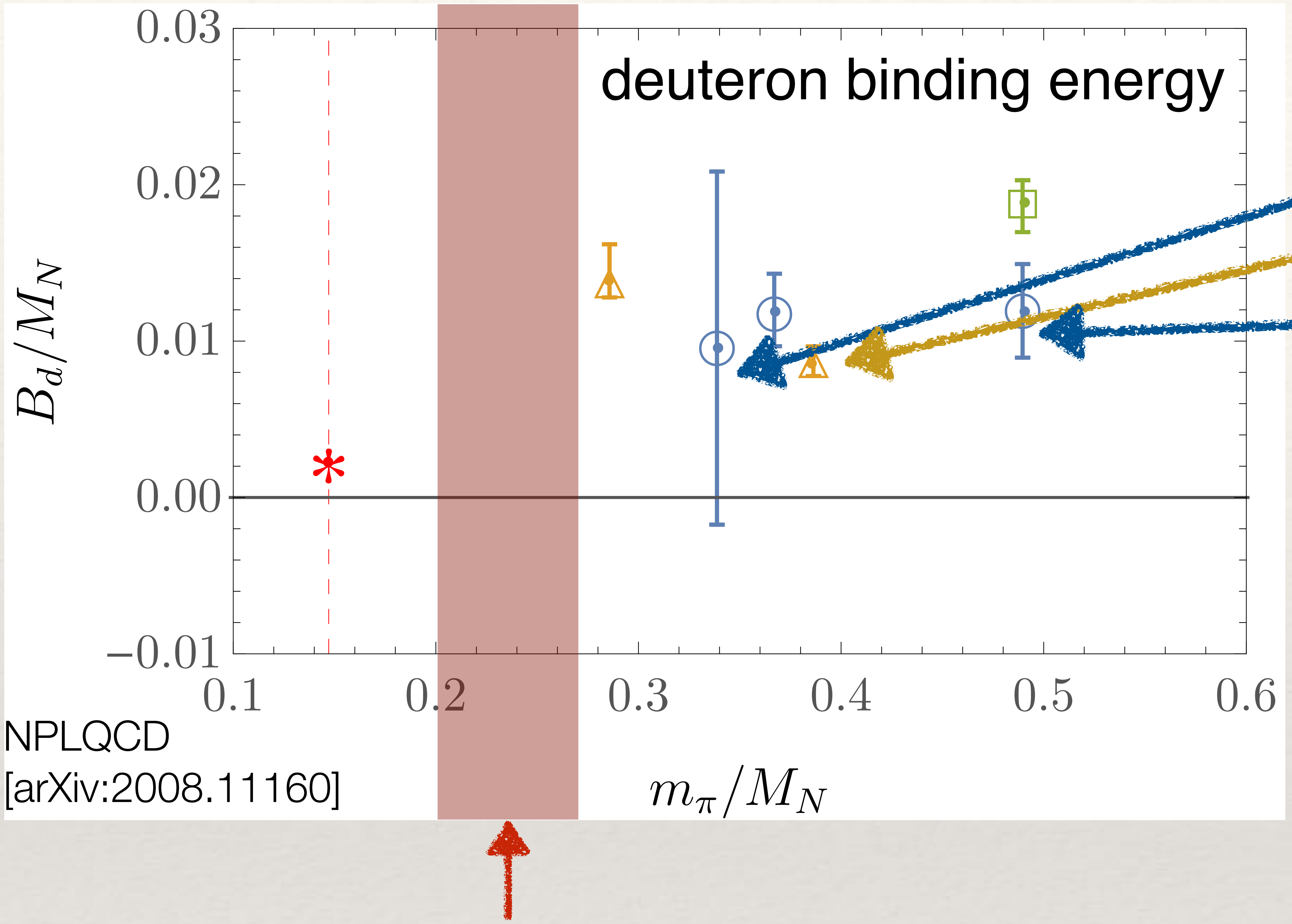

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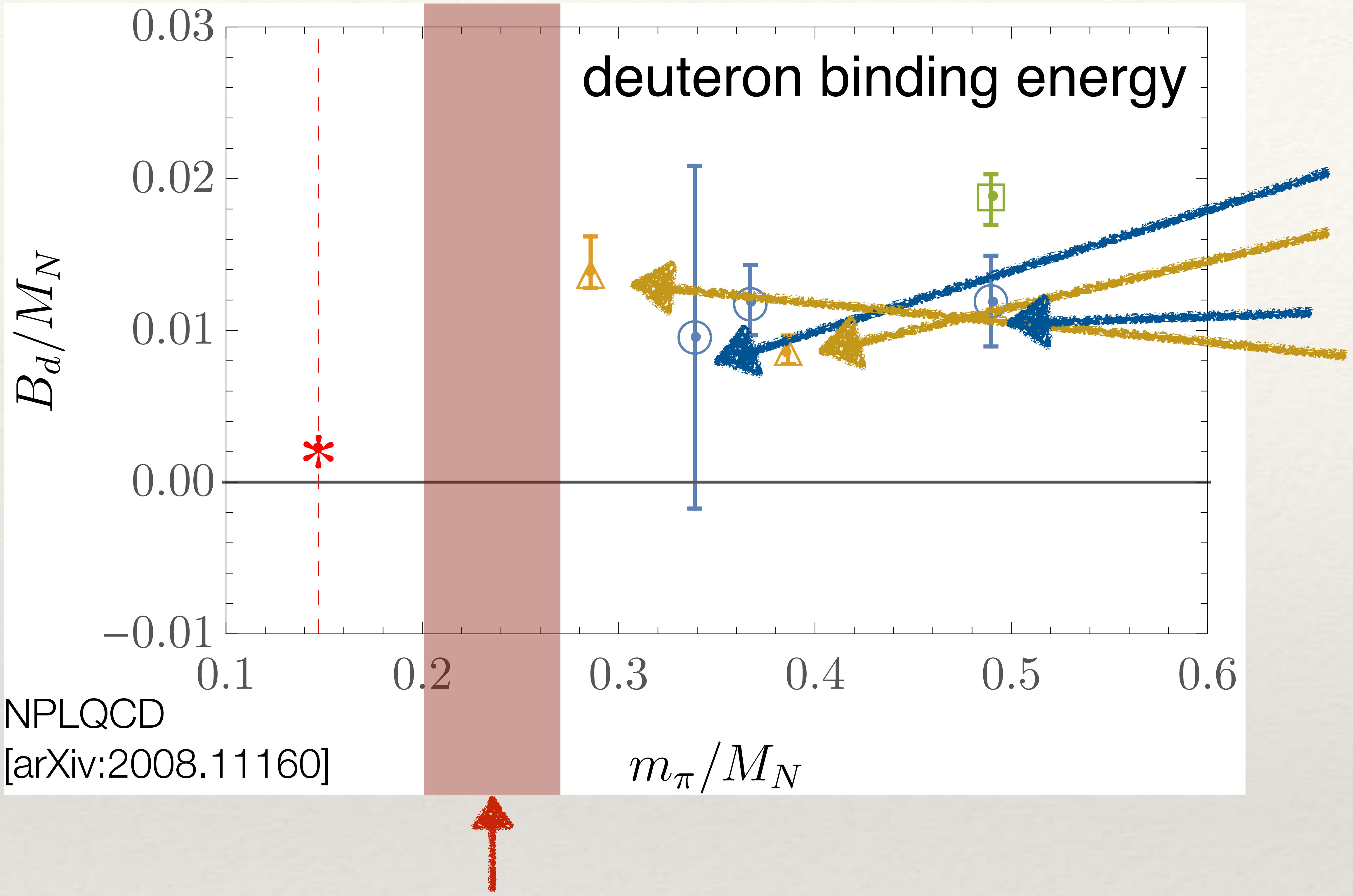

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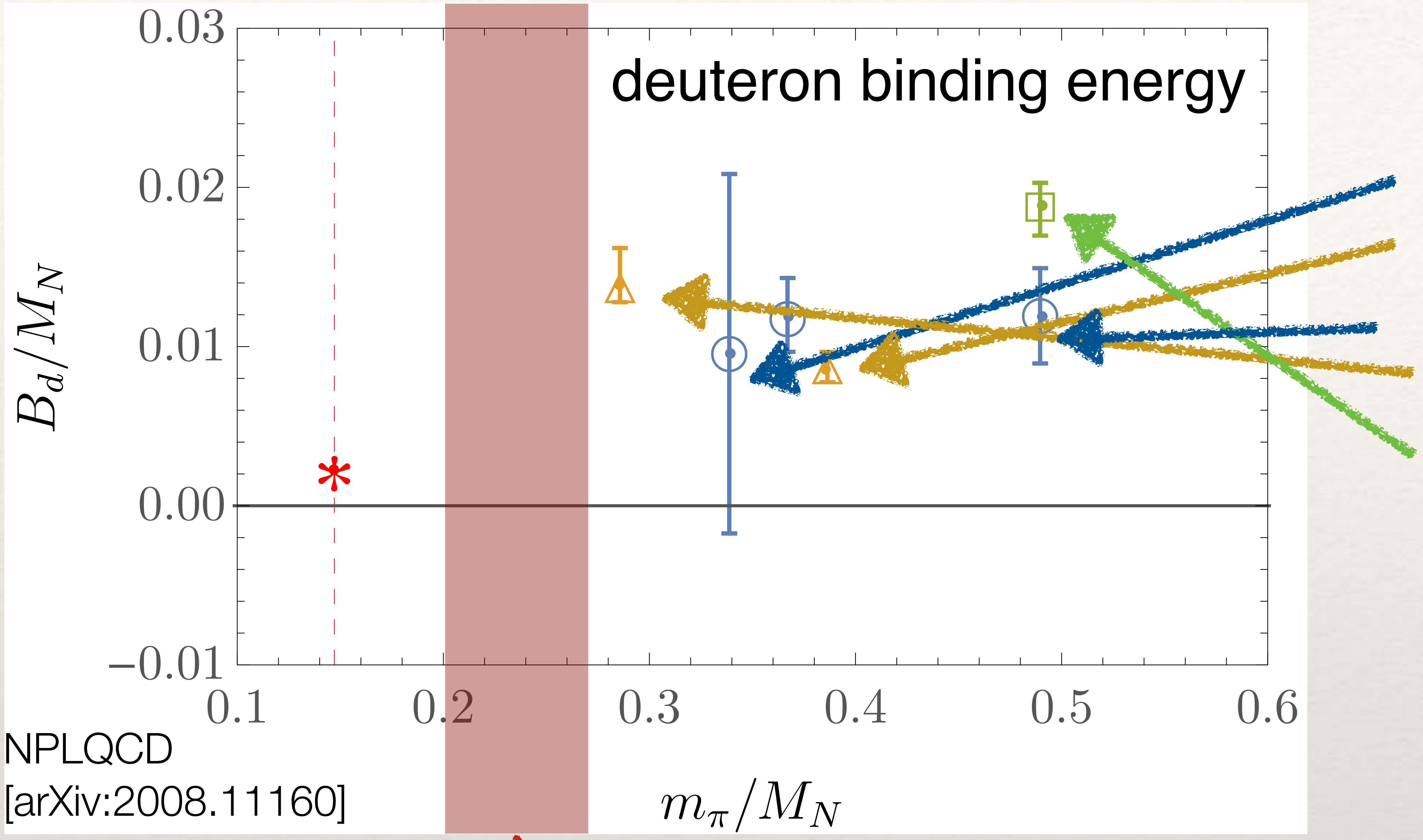

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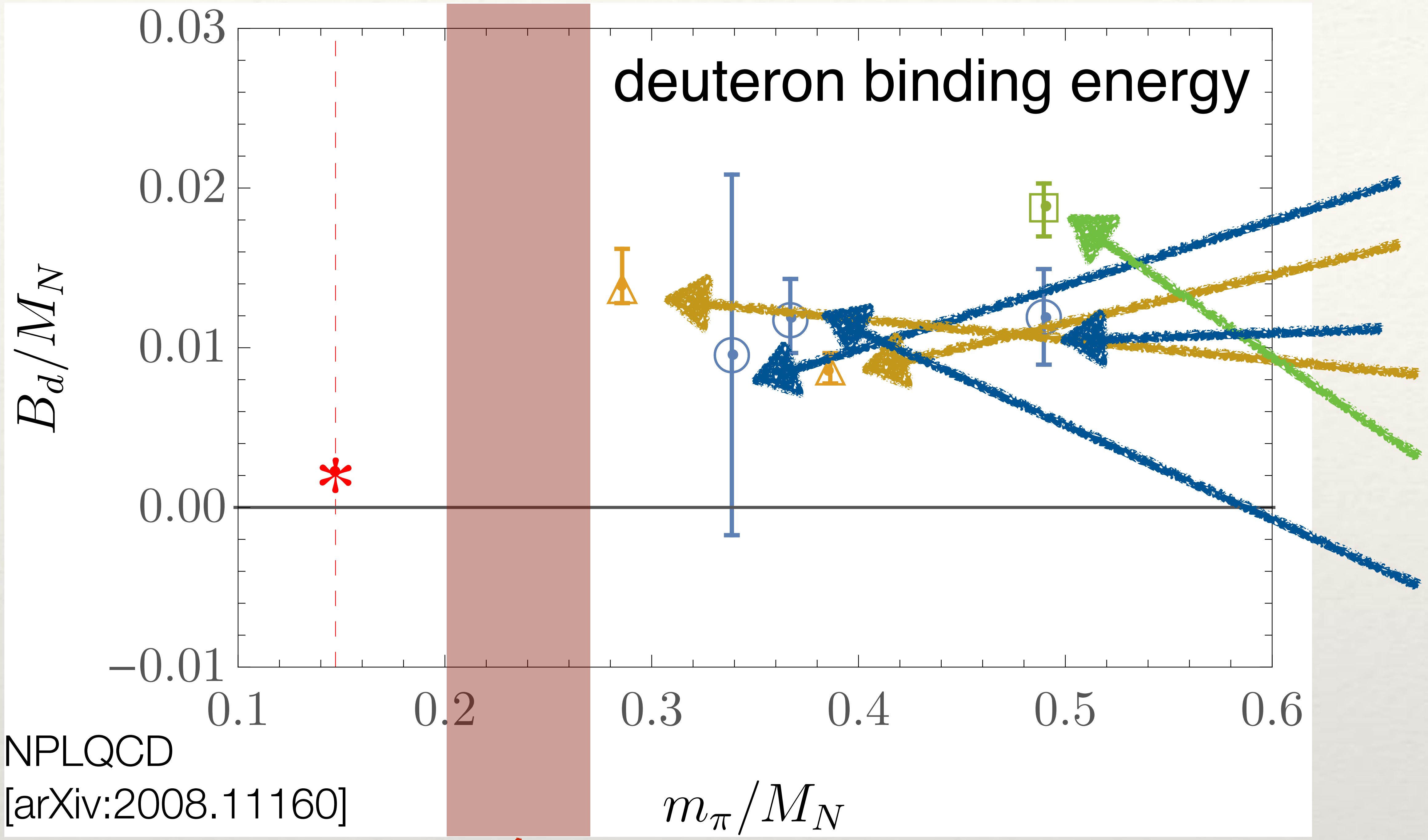

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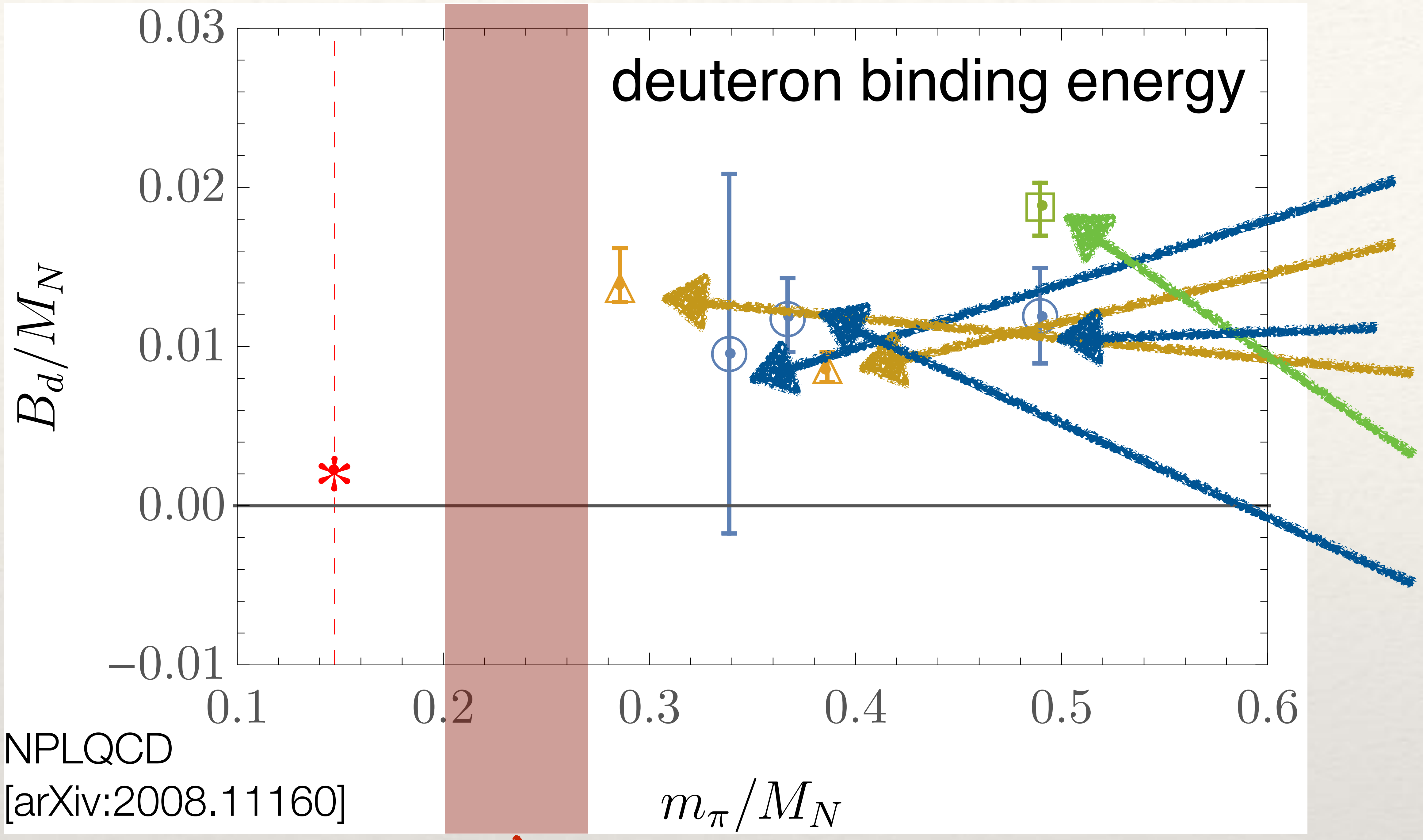

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LQCD Results without bound di-nucleons (or inconclusive)

2012 HAL QCD Mπ ≃ 710 MeV
2012 HAL QCD Mπ ≃ 469 — 1171 MeV
2019 “Mainz” Mπ ≃ 960 MeV
2020 CoSMoN Mπ ≃ 714 MeV
2021 NPLQCD Mπ ≃ 800 MeV

(blue = work I was involved in)

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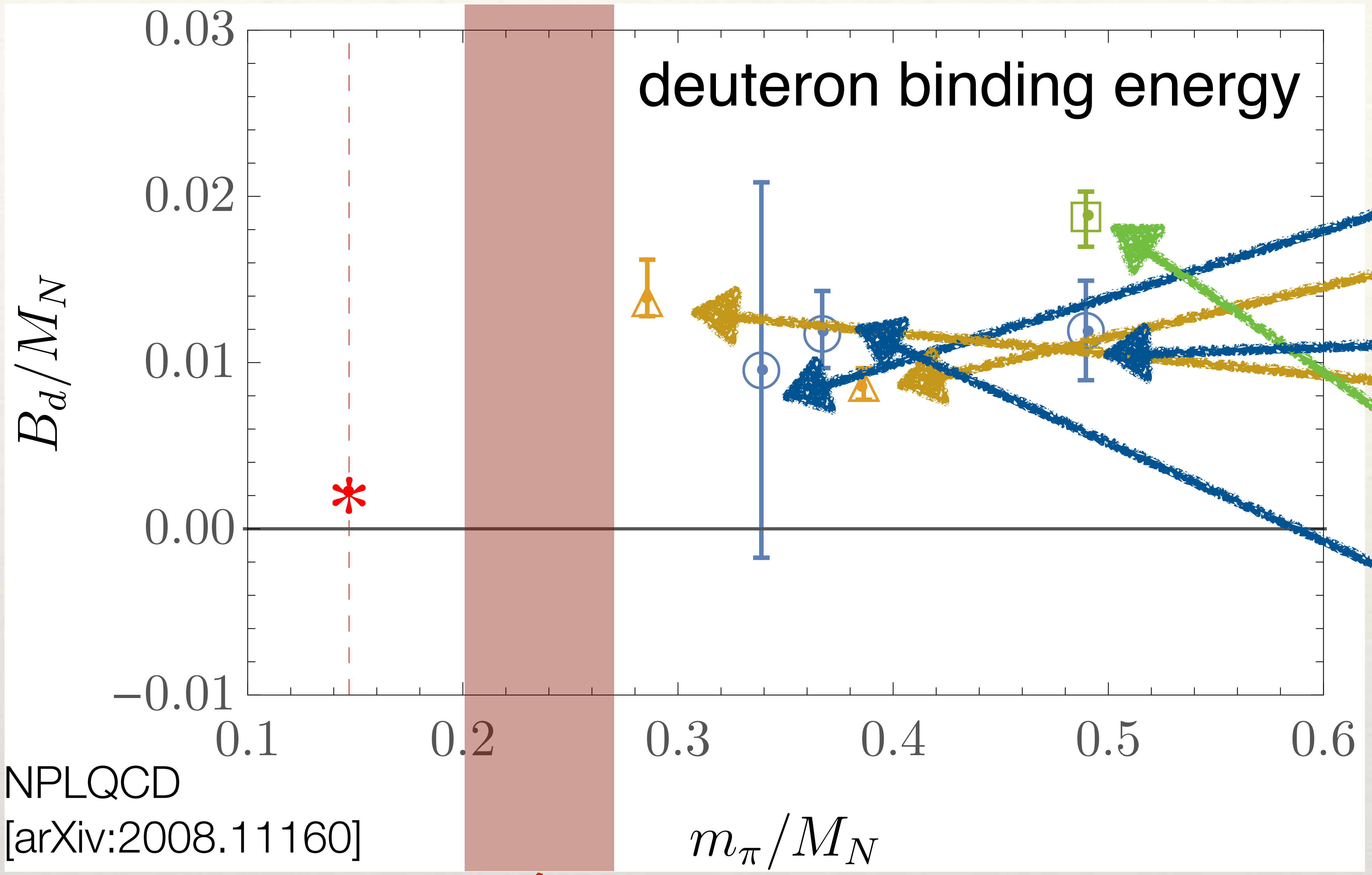

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We now believe the bound state results are not correct

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Why is the progress not better?

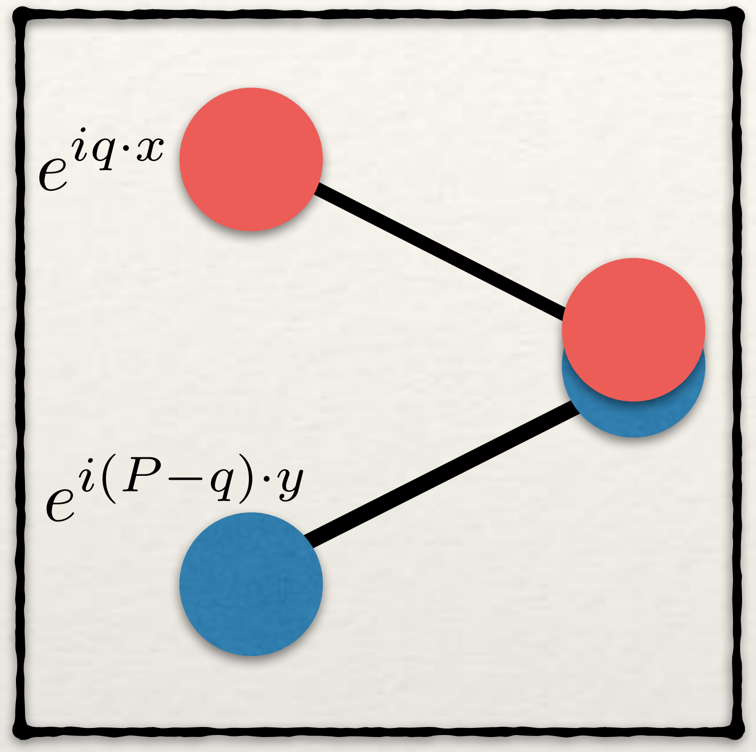
- Signal-to-Noise (S/N) issue
  $\frac{S}{N} \sim \sqrt{N_{\text{sample}}} e^{-A(m_N - \frac{3}{2}m_\pi)t}$
  see Guilherme Catumba talk for tackling S/N

- Energy of interest is the small O(0.1%) interacting energy
  $\delta E = E_{NN} - 2m_N$
  which must be determined precisely and accurately to relate δ_E to scattering amplitudes via the Lüscher Quantization Condition (LQC) — see Agostino's talk for an alternative method

- Excited state contamination prevents the extraction early in time where the S/N is OK
  unlike for mesons - we do not yet know how to construct an improved nucleon operator

Do di-nucleons bind @ heavy pion mass?

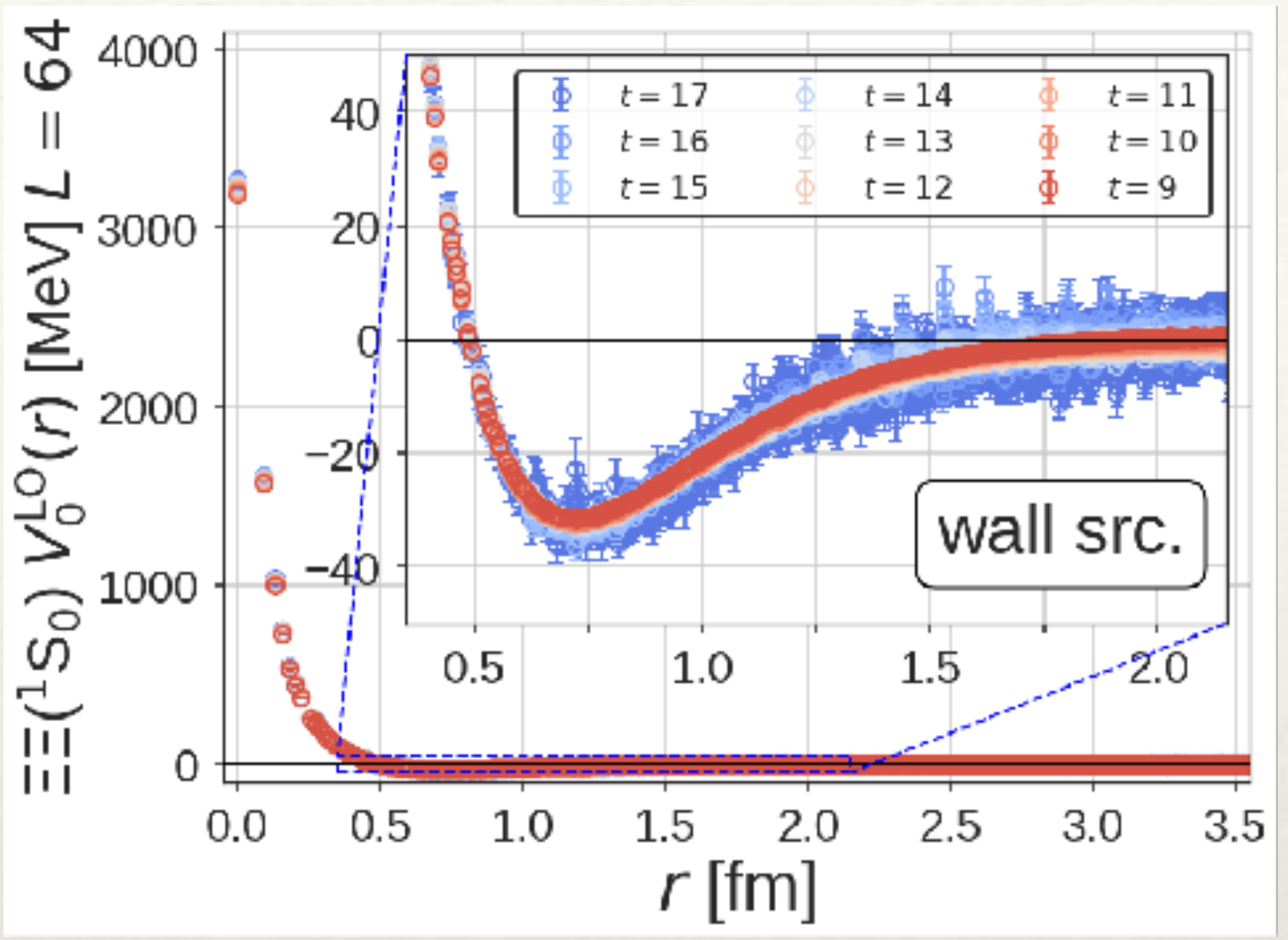
NPLQCD,
Yamazaki et al.,
CalLat (2015)



Compact, hexa-quark creation operator

Deep bound di-nucleons

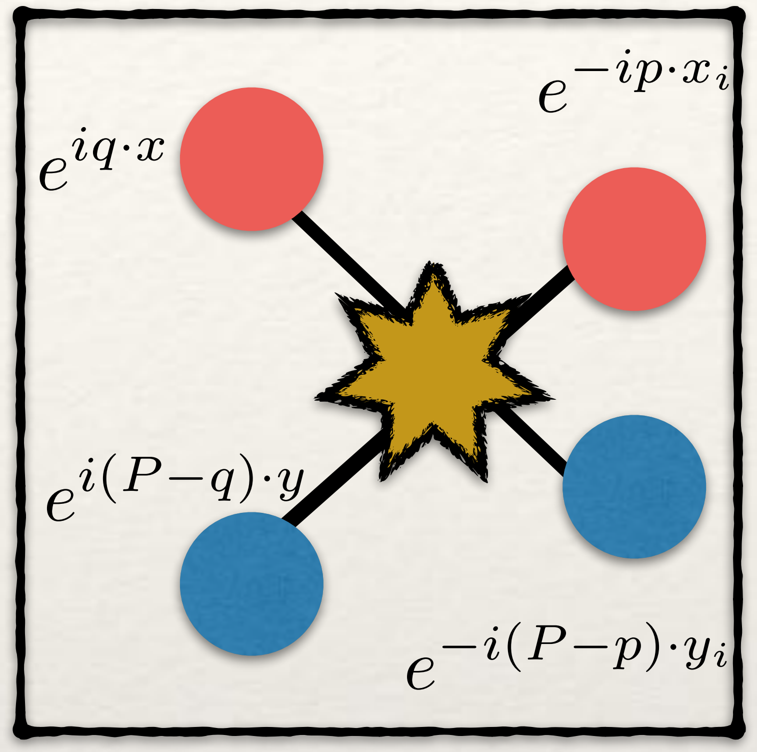
HAL QCD Potential



diffuse - wall source

no bound state

"Mainz" (Distillation)
CoSMoN (stochastic LapH
NPLQCD (sparsened momentum)

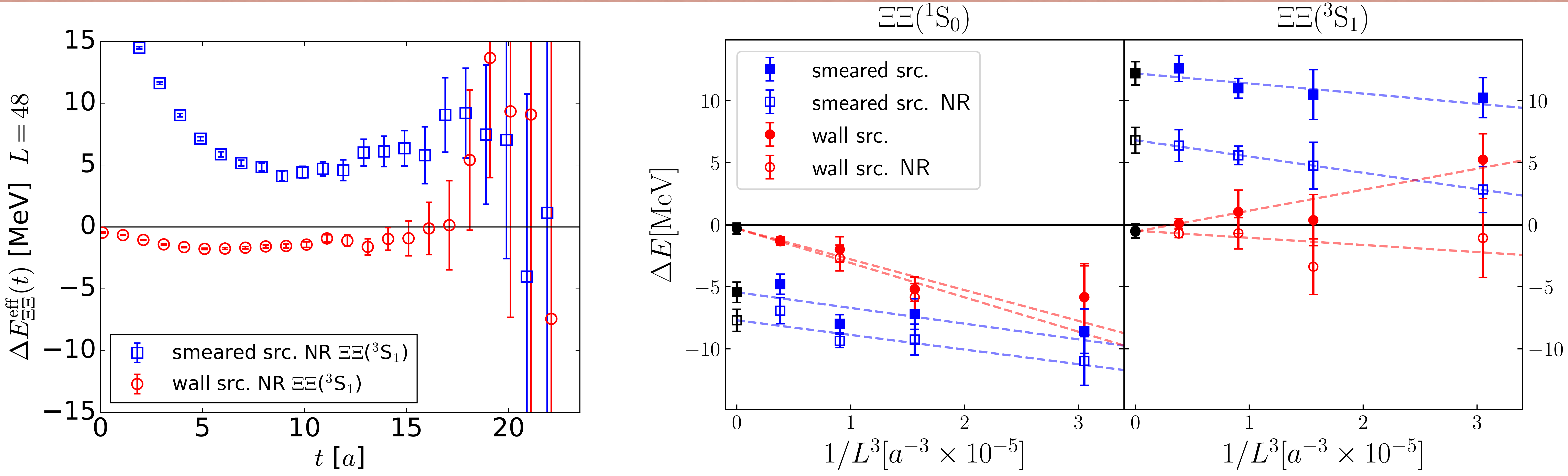


momentum-space
creation & annihilation
positive-definite correlation matrix

no bound state

The methods lead to different spectrum!
But, the spectrum can not depend upon the creation/annihilation operators!
At least one method must be wrong!

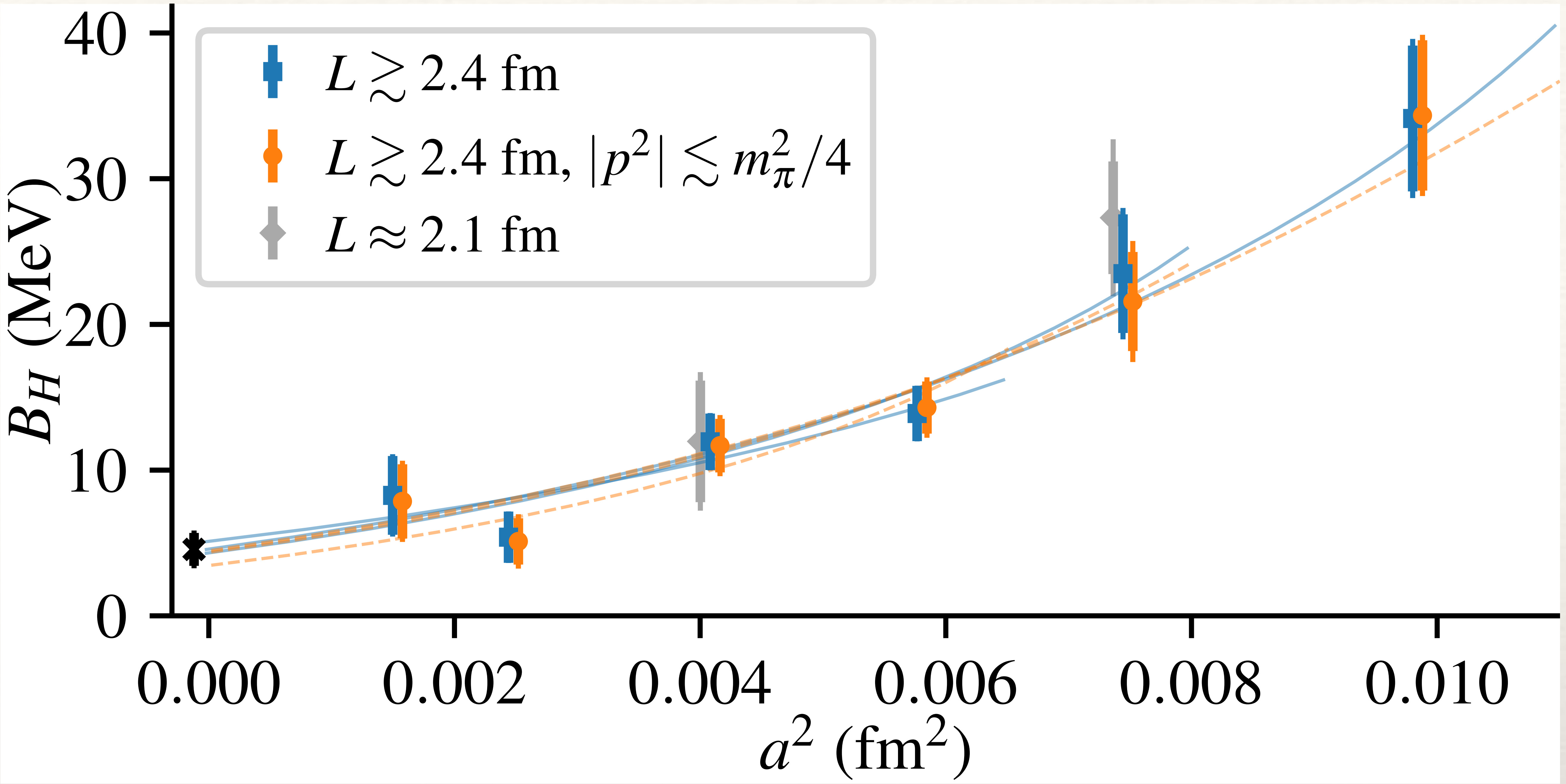
Do di-nucleons bind @ heavy pion mass?



- *Mirrage in Temporal Correlation functions for Baryon-Baryon Interactions in Lattice QCD*
  HAL QCD: Iritani et al., JHEP 10 (2016) [1607.06371]

- Extracted spectrum depends on creation operator
- $\Xi\Xi(^1S_0)$
  - smeared hexaquarks ⟶ bound states
  - diffuse wall source ⟶ scattering state
- $\Xi\Xi(^3S_1)$
  - smeared hexaquarks ⟶ unphysical non-zero ΔE @ $L = \infty$
  - diffuse wall source ⟶ scattering state

Do di-baryon interactions have strong discretization corrections?



Weakly bound h-dibaryon from SU(3)-flavor symmetric QCD
Green, Hanlon, Junnarkar, Wittig, PRL 127 (2021) [2103.01054]

- O(1000%) correction to the binding energy from discretization
- Surprising to many of us who had been working on NN
  - simple arguments led us to expect discretization arguments to be ~ interaction energy ~ B

- How universal is this for different actions?
- How universal is this for different BB systems?
  - h-dibaryon is special: eg. the HAL QCD potential for singlet in SU(3) limit is purely attractive
- clearly important to investigate

Step 1: perform NN calculations with momentum space creation ops

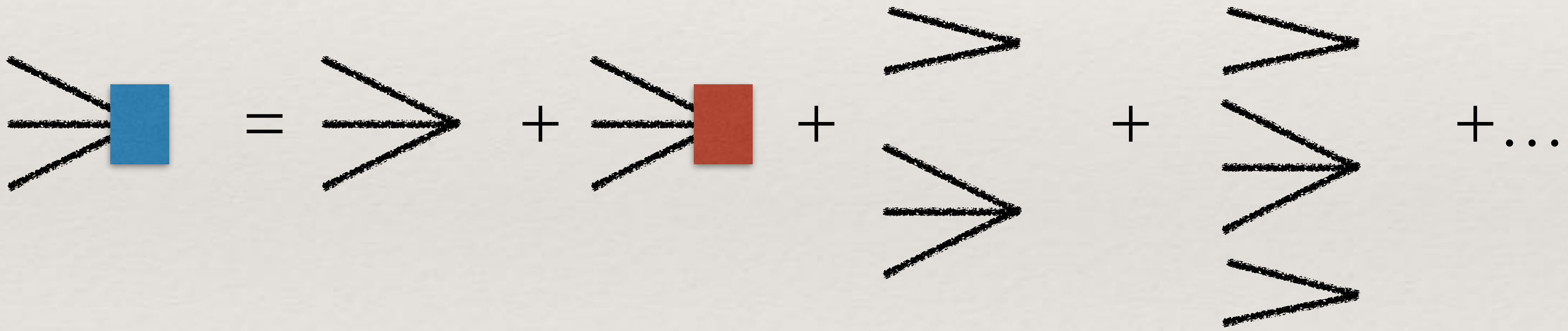
- Use Stochastic Laplacian Heaviside (sLapH) method [Morningstar et al, PRD 83 [1104.3870]]
  - LapH [Peardon et al., PRD 80 [0905.2160]]: solve for eigenvectors of 3D Laplacian, $|\lambda\rangle$
  - stochastic: insert stochastic noise between $|\lambda\rangle$ and solving for quark propagators - hold solves fixed vs volume
  - Keep an eye out for quda_laph which does all this on GPUs
- Work at heavy pion mass to mimic NPLQCD results
  - CLS action: C103: $L = 48$, $a \approx 0.086$ fm, $m_\pi = m_K \approx 714$ MeV
- On 800 configurations (two streams/replicas) construct sources at 4 times/config
  - compute all irreps for both deuteron and di-neutron channels

	deuteron		dineutron	
$\mathbf{d}^2_{\mathrm{tot}}$	irrep	N_{op}	irrep	N_{op}
0	T_{1g}	15	A_{1g}	6
1	A_2	10	A_1	10
1	E	18		
2	A_2	15	A_1	21
2	B_1	19		
2	B_2	21		
3	A_2	9	A_1	9
3	E	17		
4	A_2	7	A_1	10
4	E	15		

Step 1: perform NN calculations with momentum space creation ops

- How to extract the spectrum?
- For meson-meson scattering, creation operators have been improved to eliminate excited state contamination early in time
- These involve operators with derivatives/displacements, but in the end, 4-quark operators were not needed

- For nucleons, we know in principle, but not in practice, how to improve the nucleon



- As far as I know, no one has found a set of operators that meaningfully reduces excited state contamination for single nucleon correlators
- Even if you do, unlike for mesons, the improved nucleon has more than 3 quark-line objections
  this will substantially increase the complexity of Wick contractions for NN correlators

Step 1: perform NN calculations with momentum space creation ops

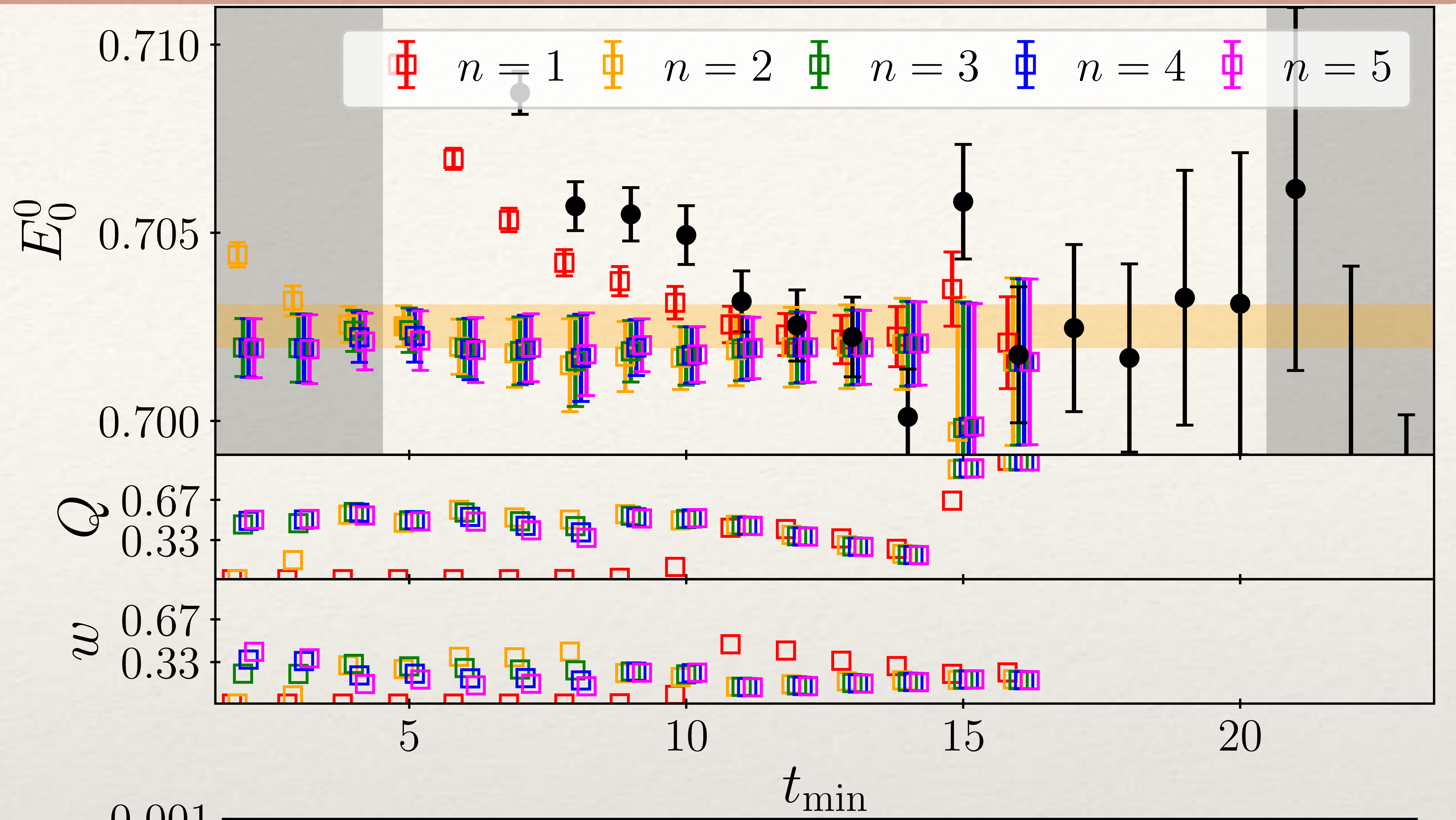
- How to extract the spectrum?
- We are forced to control multi-exponential fits to extract both the N and NN spectrum
- Ratio correlator, $R(t, \mathbf{P}) = \dfrac{NN(t, \mathbf{P})}{N(t, \mathbf{p1})N(t, \mathbf{p_2})}$
  provides a precise estimate of the interaction energy
- Even if NN is positive definite (principle correlators after GEVP), the $R(t, \mathbf{P})$ correlator can suffer opposite sign contributions to $m_{\text{eff}}^R(t)$
- Take advantage of positive-definiteness of $NN(t, \mathbf{P})$ by building fit function that respects these features

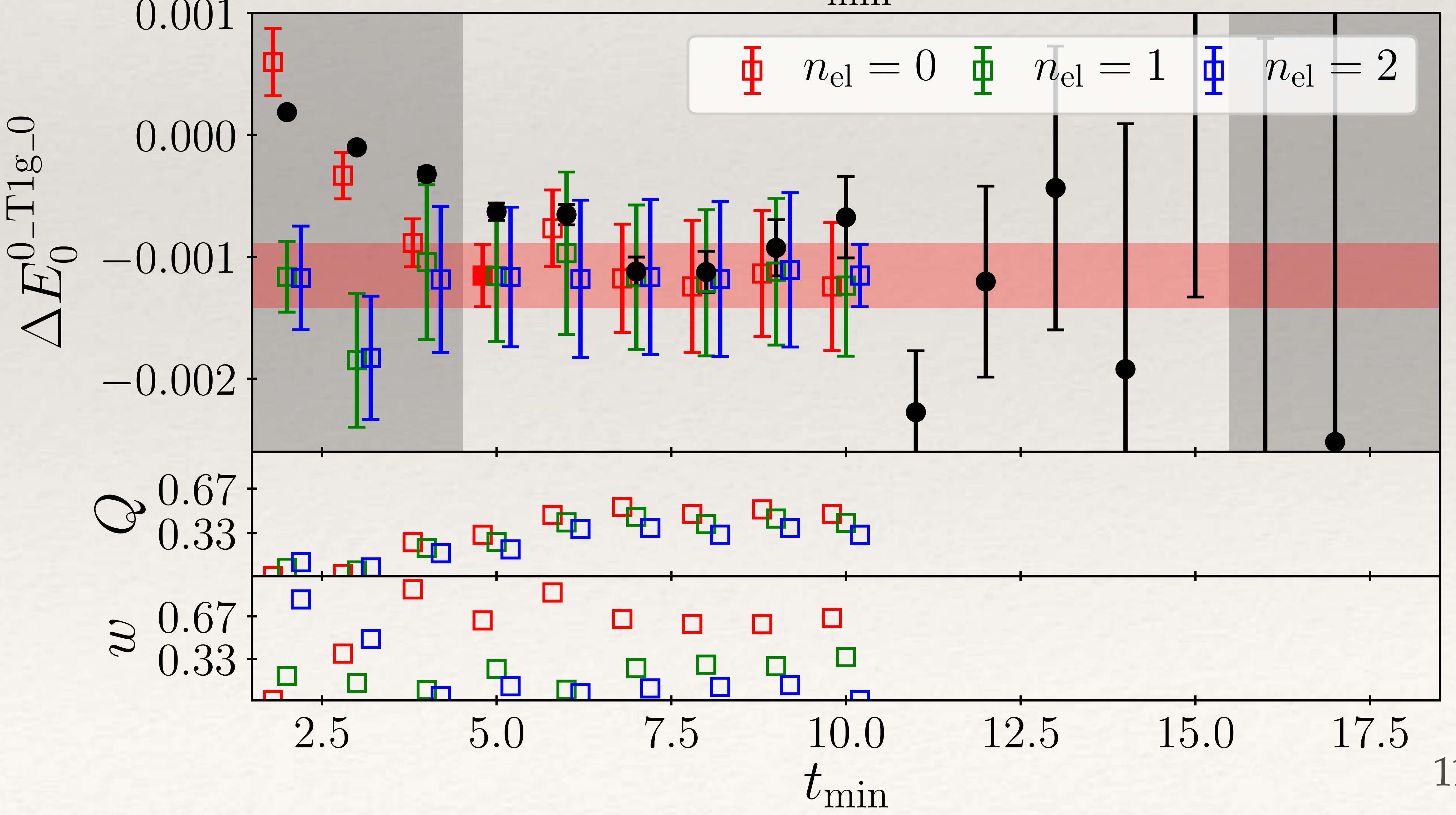
$$R(t, \mathbf{P}) = \frac{r_0^2 e^{-\delta E_0^{NN} t}\left(1 + r_l^2 e^{-\Delta E_{l0}^{NN} t}\right)}{\left(1 + z_{p_1,n}^2 e^{-\Delta E_{n0}^{p_1} t}\right)\left(1 + z_{p_2,n}^2 e^{-\Delta E_{n0}^{p_2} t}\right)}, \qquad r_0^2 = \left(\frac{z_0^{NN}}{z_0^{p_1} z_0^{p_2}}\right)^2 \geq 0, \quad r_l^2 = (z_l^{NN}/z_0^{NN})^2 \geq 0, \ldots$$

- Perform simultaneous fit to N_1, N_2, R correlation functions

Step 1: perform NN calculations with momentum space creation ops

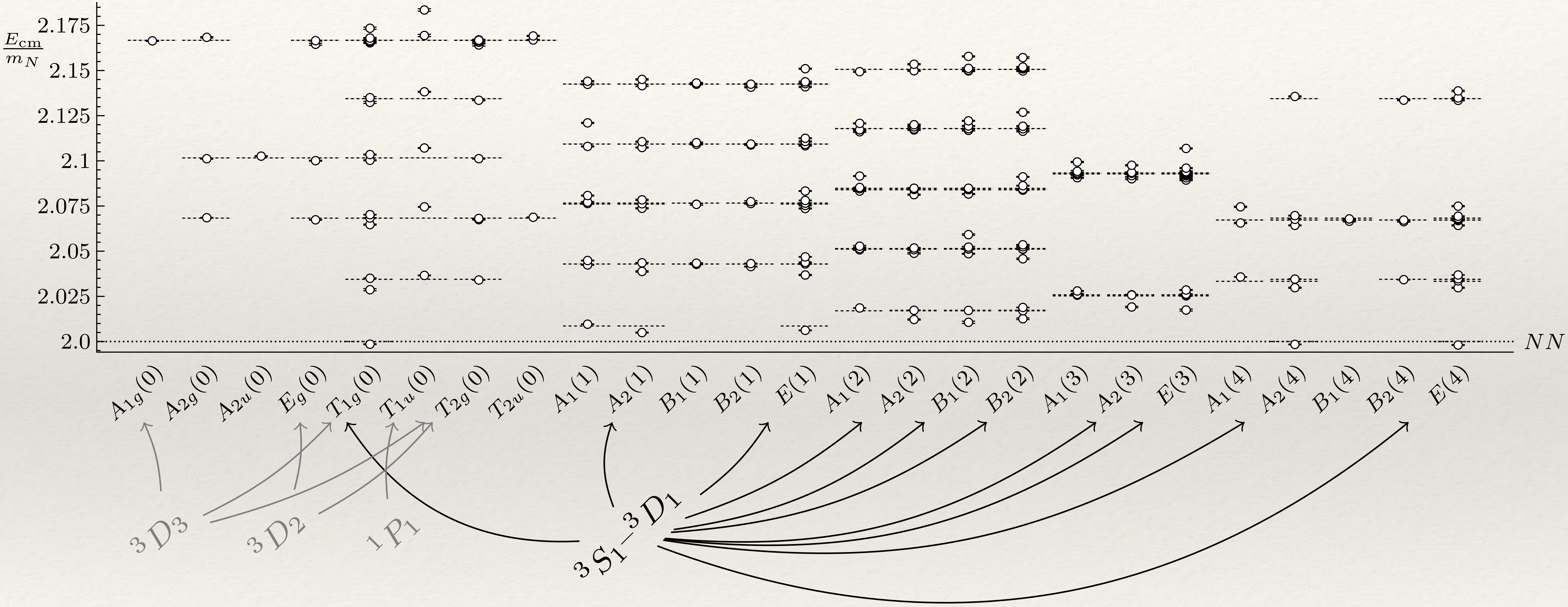
- How to extract the spectrum?
- Single nucleon described by 2 exponentials from $t \gtrsim 5$
- NN correlator has three kinds of excited state contamination:
  - elastic scattering modes of NN — lowest energy gap but largely removed from GEVP
  - inelastic excited states from single nucleon — these pollute NN in nearly the same way as N, but are present
  - NN inelastic corrections — eg. $\Delta\Delta$, larger energy gap
- Look for models (t_{min}, N_n etc) that consistently describe all NN correlators - don't cherry pick for each irrep





More costly – but MANY more energy levels

- arXiv:2009.11825

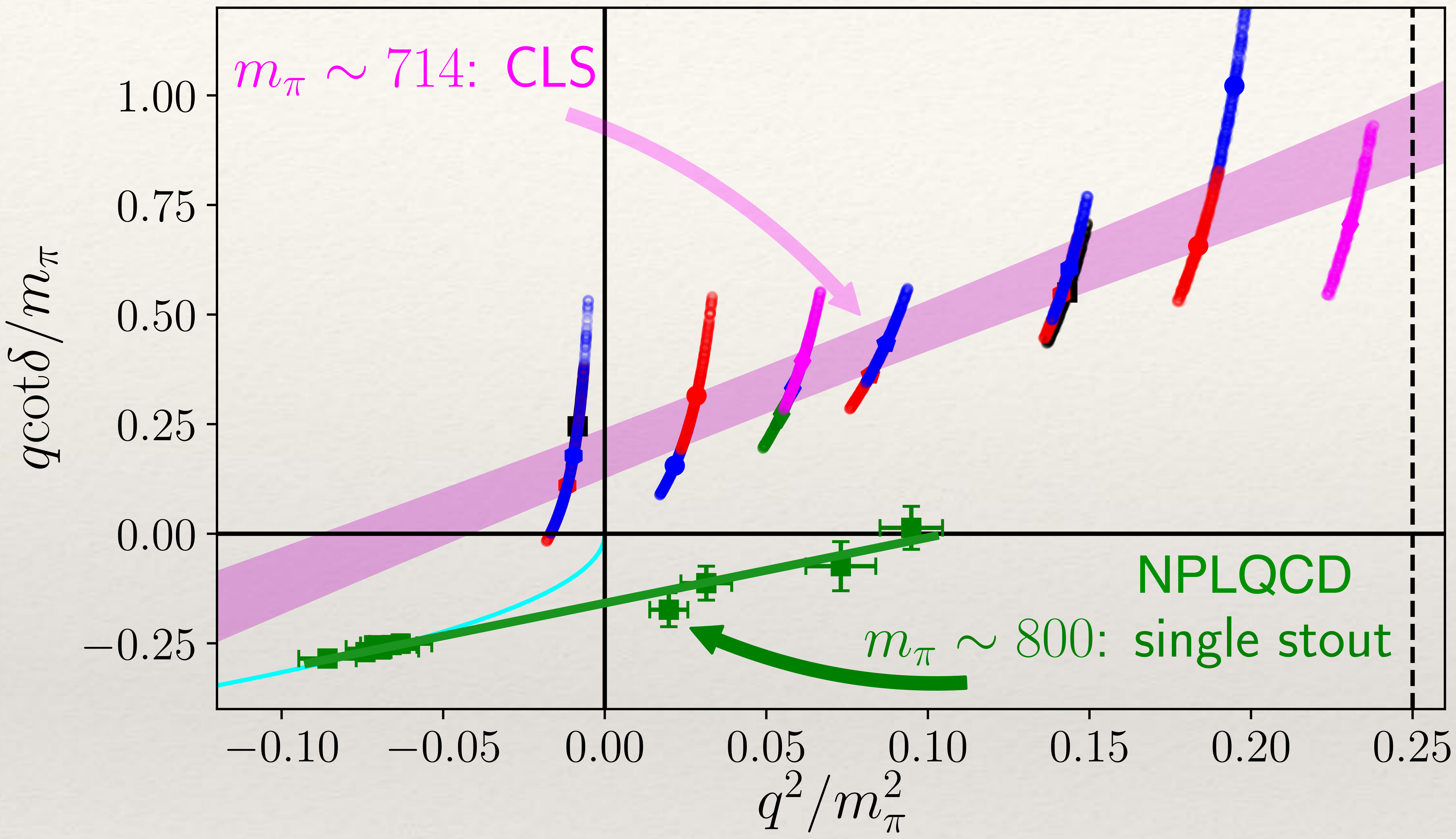


(only shown for total zero momentum)

(in the following: assume negligible $S-D$ mixing)

focus on S-wave dominated levels [2009.11825]

- 16 energy levels with (expected) negligible overlap with non S-wave



- We find a virtual bound state (like dineutron) - a purely imaginary solution with negative sign

$$\frac{q_-^{\rm deut}}{m_\pi} = -\,i0.132(32)$$

- We can infer the size of the potential from causality and unitarity: Wigner PRD 98 (1955), Phillips and Cohen PLB 390 (1997)

$$r_0 \leq 2\left[R - \frac{R^2}{a} + \frac{R^3}{3a^2}\right], \quad m_\pi R \gtrsim 2.0\,, \quad R \gtrsim 0.55 \text{ fm}$$

- We have to keep in mind the observed discretization effects in SU(3) symmetric h-dibaryon

- We do not anticipate this is the source of discrepancy

Scattering Theory Refresher

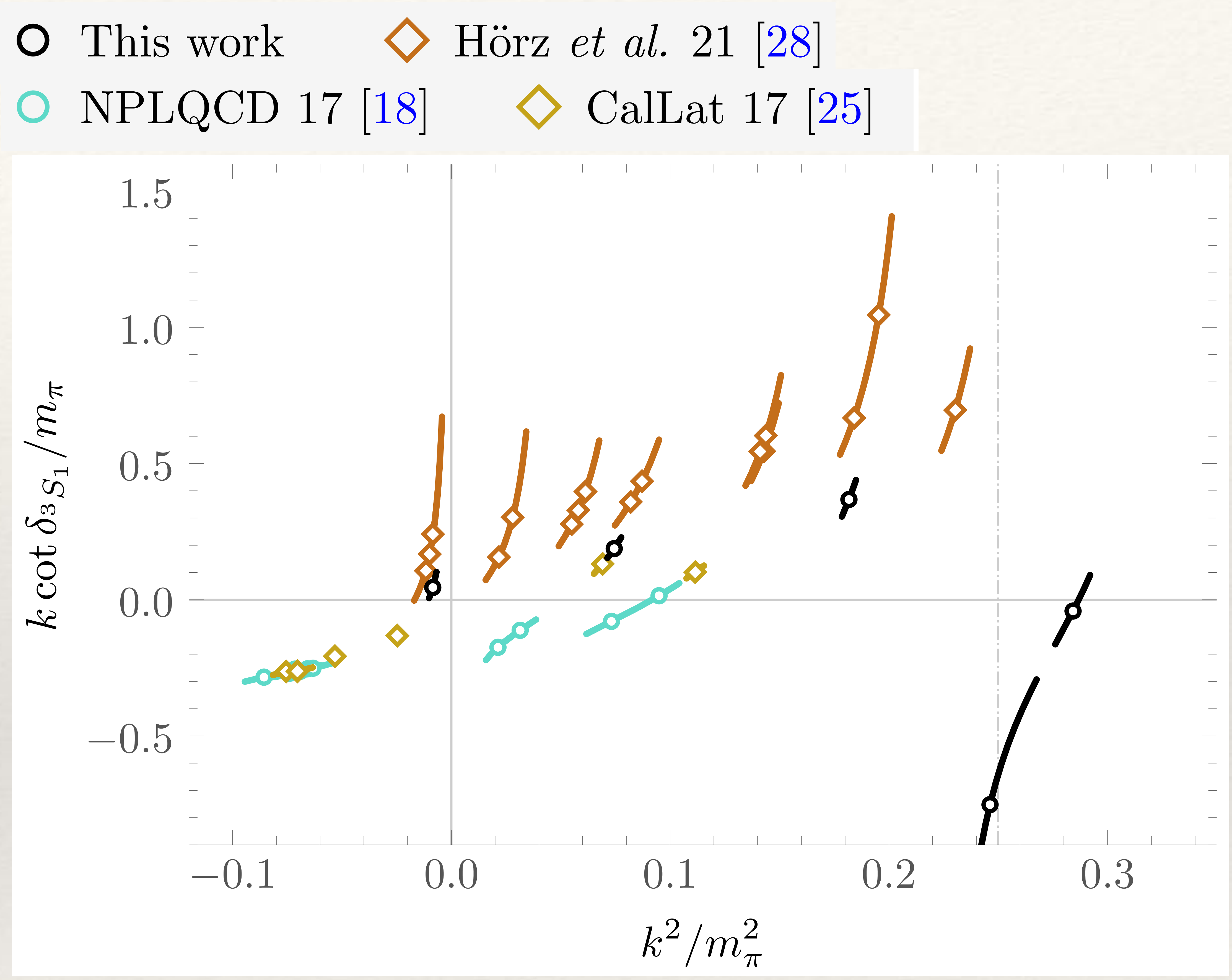
$$q\cot\delta = -\frac{1}{a} + \frac{1}{2}rq^2 + \cdots$$

$$T \propto \frac{1}{q\cot\delta - iq}$$

$$\text{bound state}: \lim_{q\to 0} q\cot\delta < 0$$

NPLQCD update with momentum-space

- NPLQCD Collaboration used an alternative momentum-space method and repeated their calculation @ $m_\pi \approx 800$ MeV
Amarasinghe et al. PRD 107 (2023) [2108.10835]
- Their new results are qualitatively consistent with other momentum-space methods
- Their new results are not consistent with their old results provided they have momentum-space sources in the basis
- They have not concluded the old methods are wrong
- They did emphasize the importance of hexaquark (HX) ops, but only observe the deep bound state with HX-only ops



Can we understand the NN discrepancy in more detail?

- ❑ Compare all methods in the literature
  - ❑ Momentum space creation operators and GEVP
  - ❑ HX creation operators from off-diagonal correlators
  - ❑ displaced nucleons in position space creation operators
  - ❑ HAL QCD potential method
- ❑ Add hexaquark interpolator to the basis with momentum space and GEVP

In preparation: 2025.04XYZ

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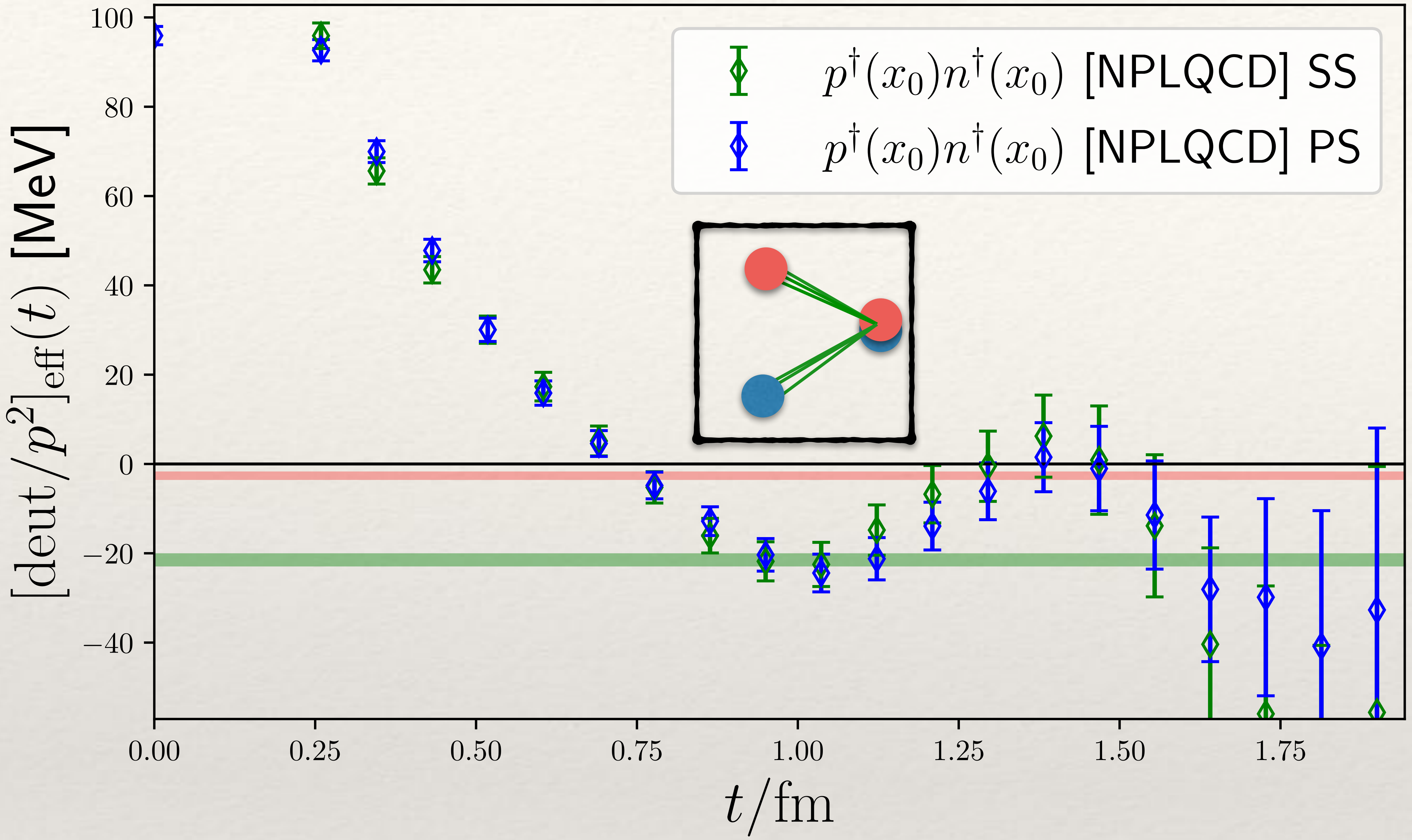
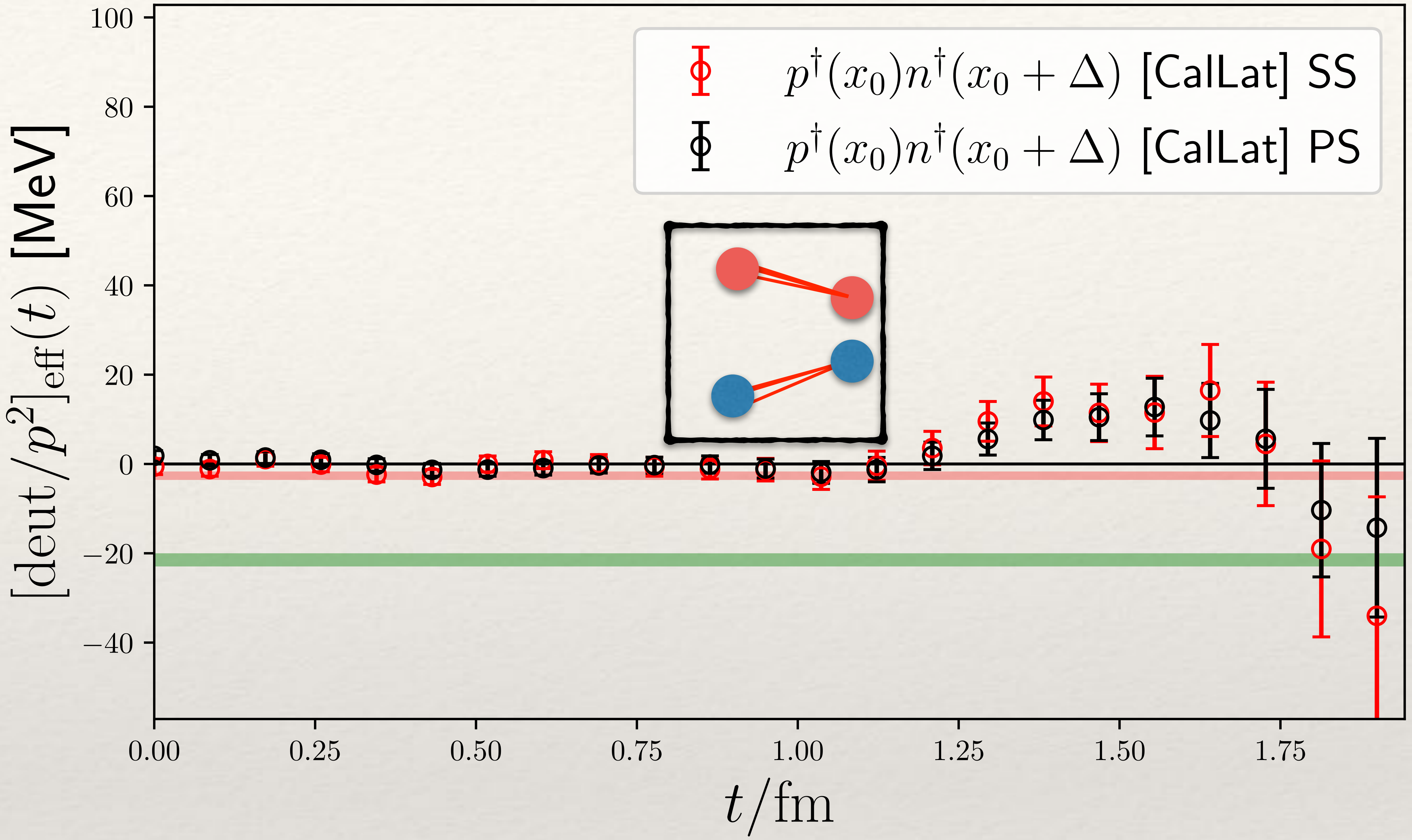
Christopher Körber,[1] Ken McElvain,[6] Aaron S. Meyer,[7] Henry Monge-Camacho,[8]
Colin Morningstar,[4] Joseph Moscoso,[9, 10] Amy Nicholson,[9] Fernando Romero-López,[11, 12]
Ermal Rrapaj,[13] Andrea Shindler,[14, 6, 10] Sarah Skinner,[4] Pavlos Vranas,[7] and André Walker-Loud[10, 6]

compare with local (HX) and displaced NN source

Local HexaQuark creation operator



displaced nucleon creation operator

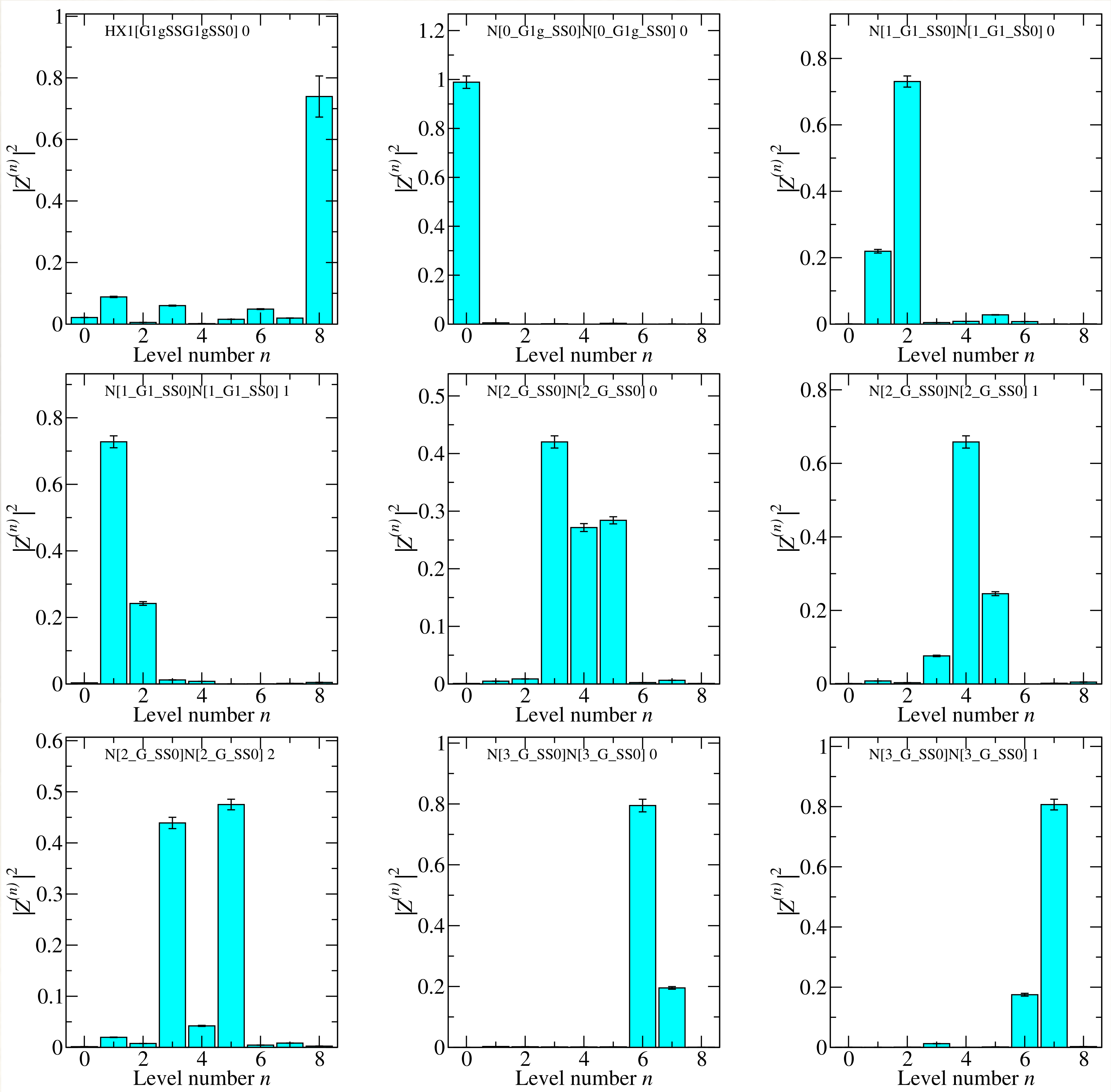


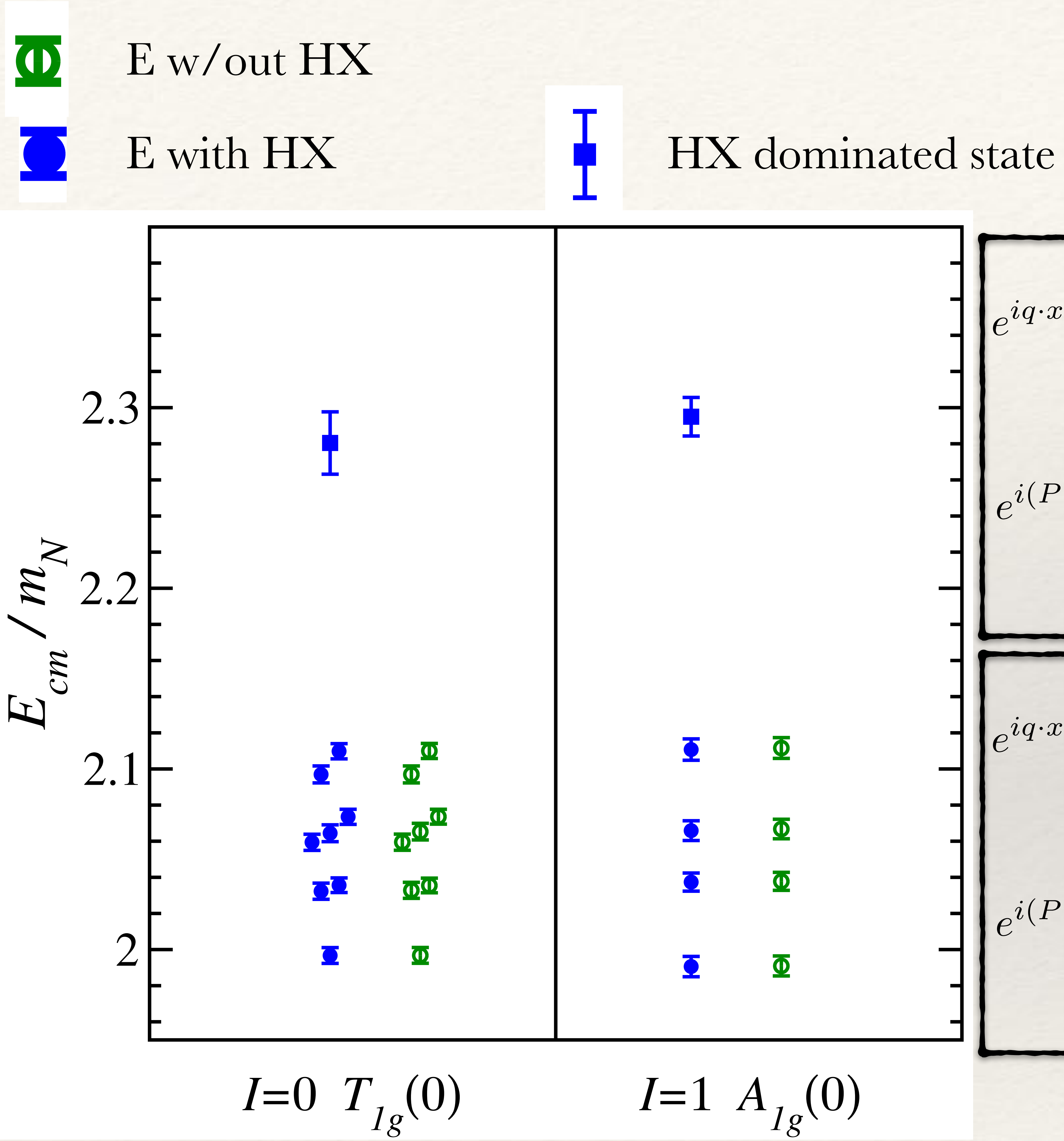
- sLapH g.s. energy in T_{1g} from GEVP p-space (2009.11825)
- NPLQCD (2012, 2017) / CalLat (2015) g.s. energy from local NN creation operator

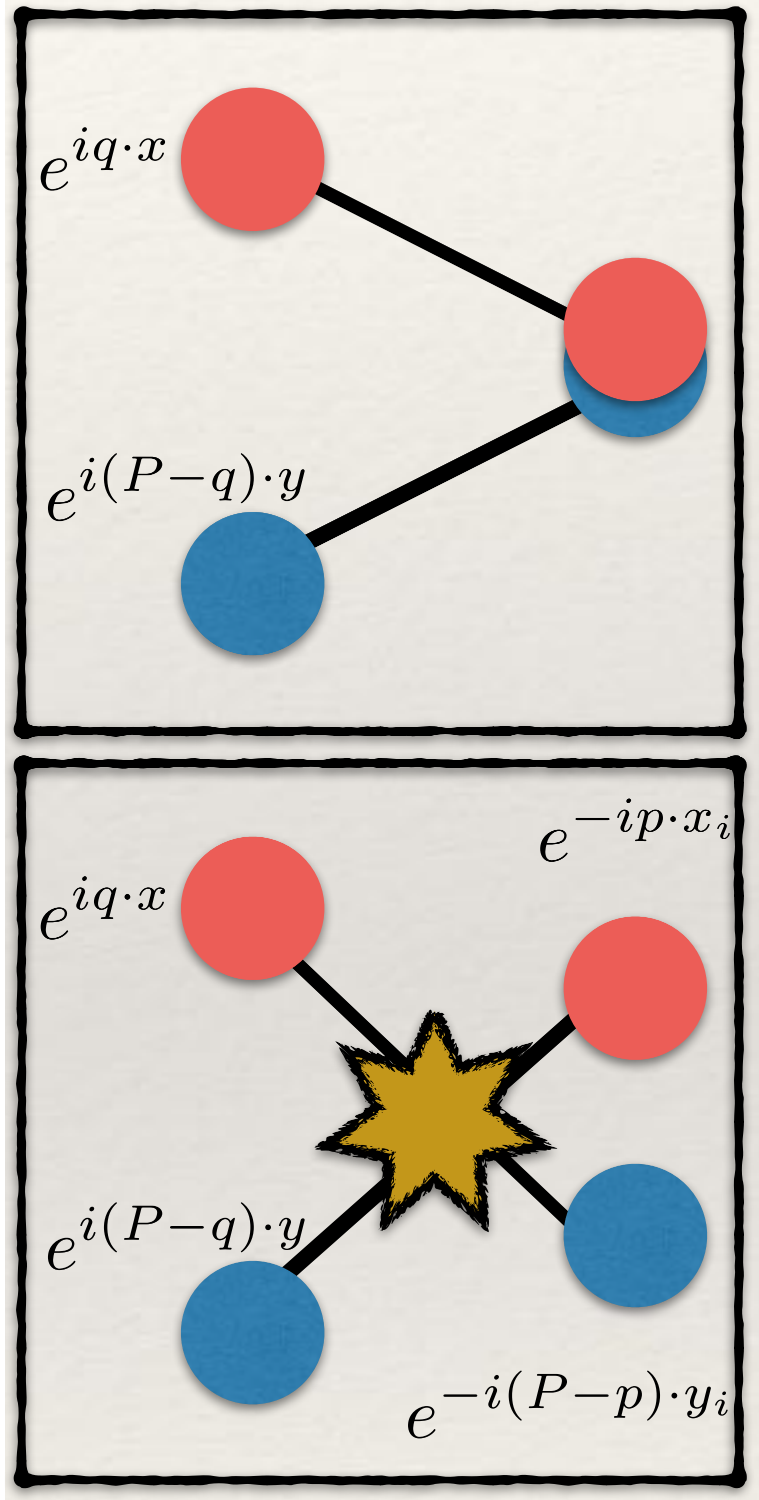
- pulling $p^\dagger(x_0)n^\dagger(x_0+\Delta)$ apart at creation leads to significantly different excited state contamination [CalLat, Berkowitz et al., PLB 765 [1508.00886]
- extracting stable ΔE is challenging
- local $p^\dagger(x_0)n^\dagger(x_0)$ strongly couples to NN-inelastic states that are unique to NN e.g. $\Delta\Delta$

Add hexaquark (HX) to basis of p-space creation ops

- hexaquark (HX) operator strongly overlaps with highest state in the spectrum (top left)
- N(p)N(p) operators mostly overlap onto a single state, with some mixing (except with highest state)





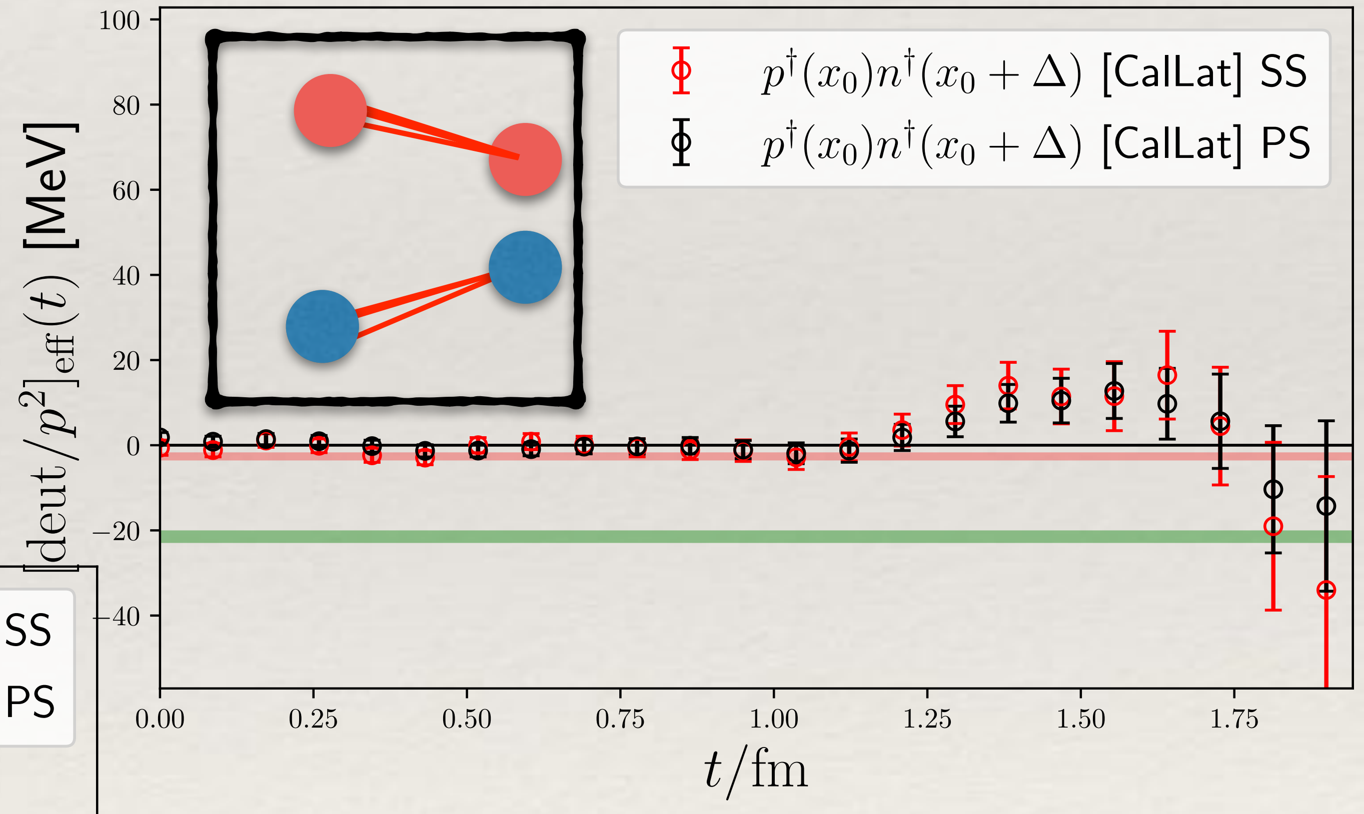


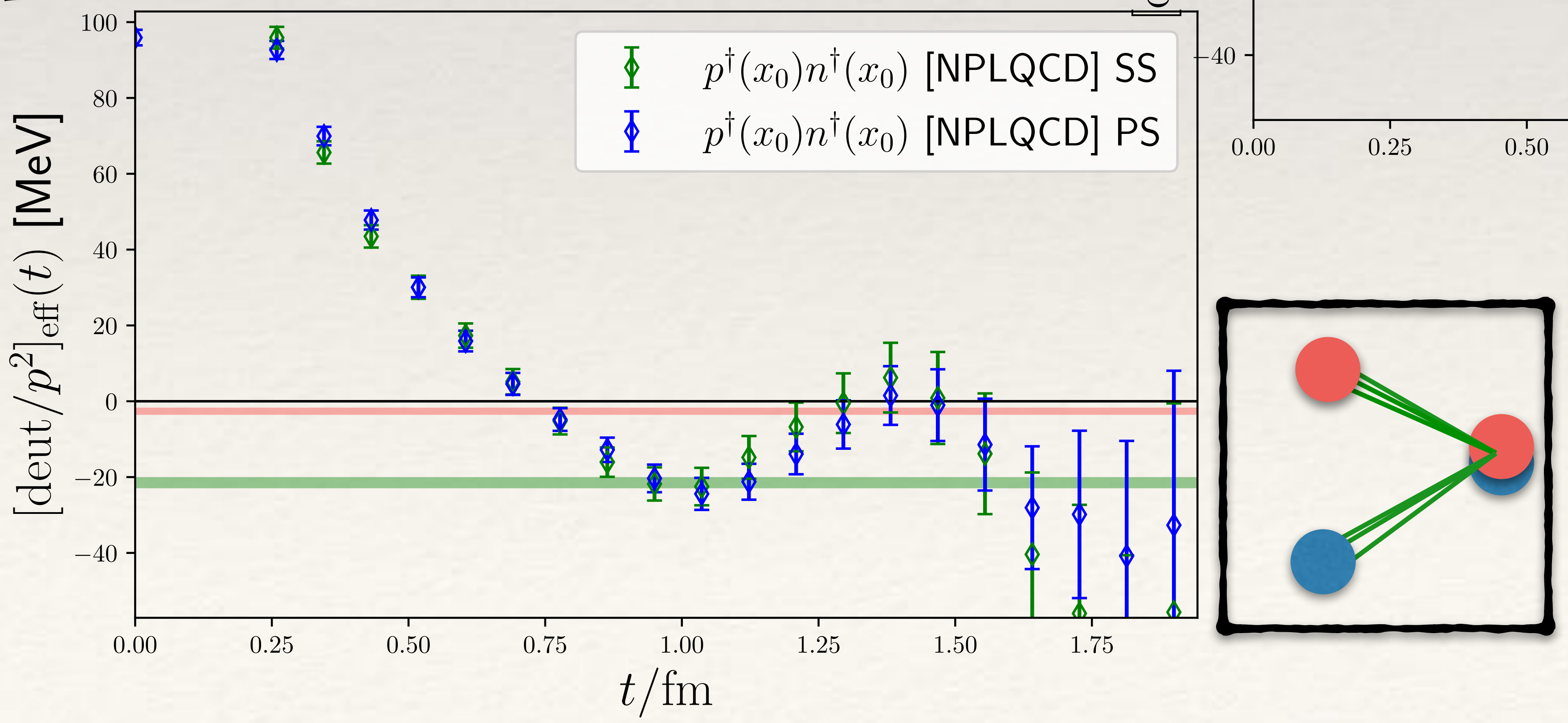
- we find the HX operator is NOT needed to determine the low-lying NN spectrum

Increase statistics of p-space GEVP method

- CLS action: C103: $L = 48$, $a \approx 0.086$ fm, $m_\pi = m_K \approx 714$ MeV
  - $N_{\rm cfg} = 802 \longrightarrow 1490, \quad N_{\rm src} = 4 \longrightarrow 8$

- With higher statistics, we become more susceptible to excited state contamination at fixed t
- We still have the issue that the **N** operators used in the **NN** correlator are not improved
- We want to fit **NN** correlator, not **R**, to take maximal advantage of positive-definiteness of **NN**
- We want to build a fit model that reflects the numerical data
  - One thing observed for years is that the ratio corrleators have strong cancellations in excited states between **NN** and **N** - which is more true for displaced and p-space creation operators





NN 2025.04XYZ : Conspiracy model

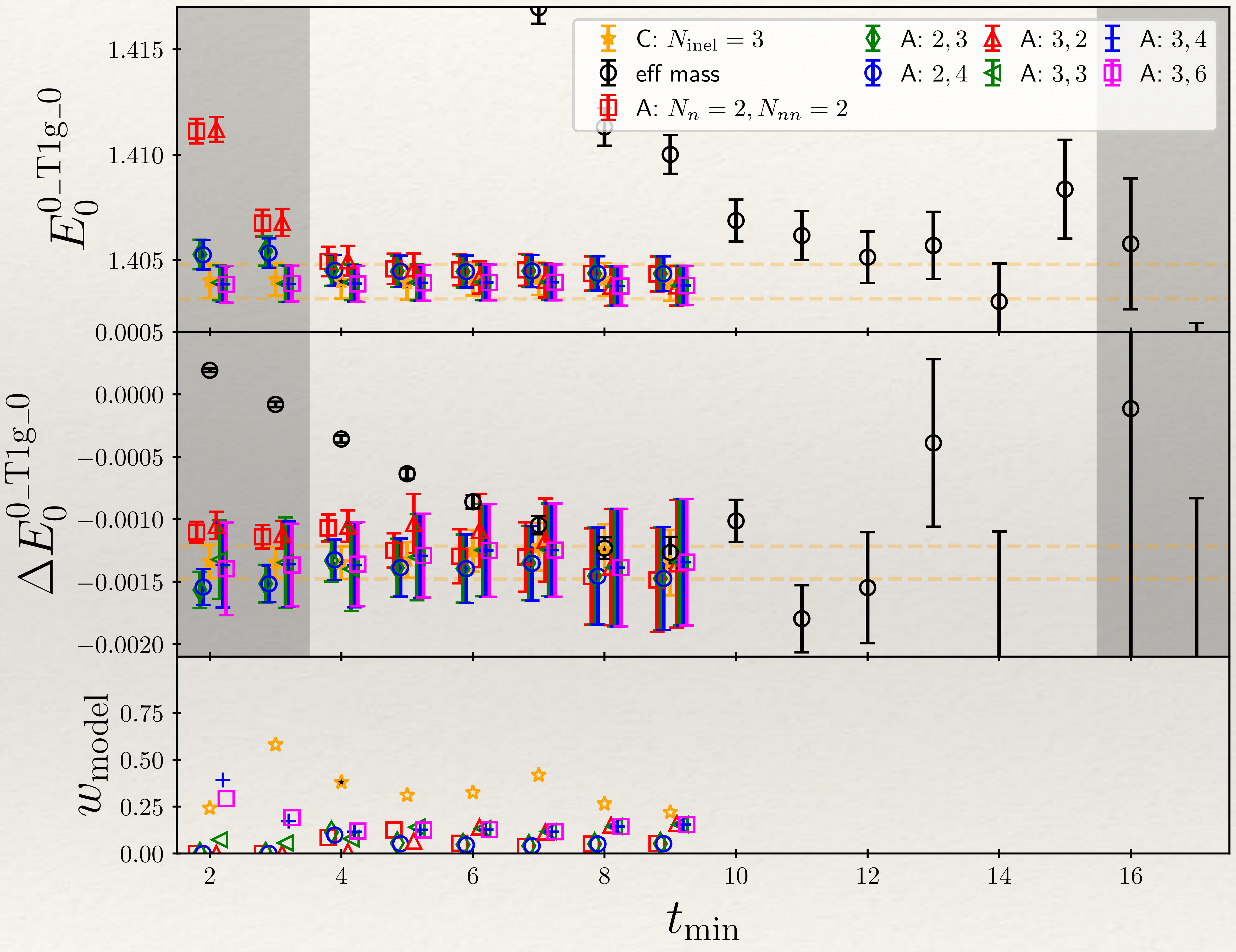
- If the nucleons did not interact, the model to fit the correlators would be

$$C_{NN}(t) = \left[\sum_{n_1=0}^{N_1-1} A_n^{(1)} e^{-E_n^{(1)} t}\right]\left[\sum_{n_2=0}^{N_2-1} A_n^{(2)} e^{-E_n^{(2)} t}\right]$$

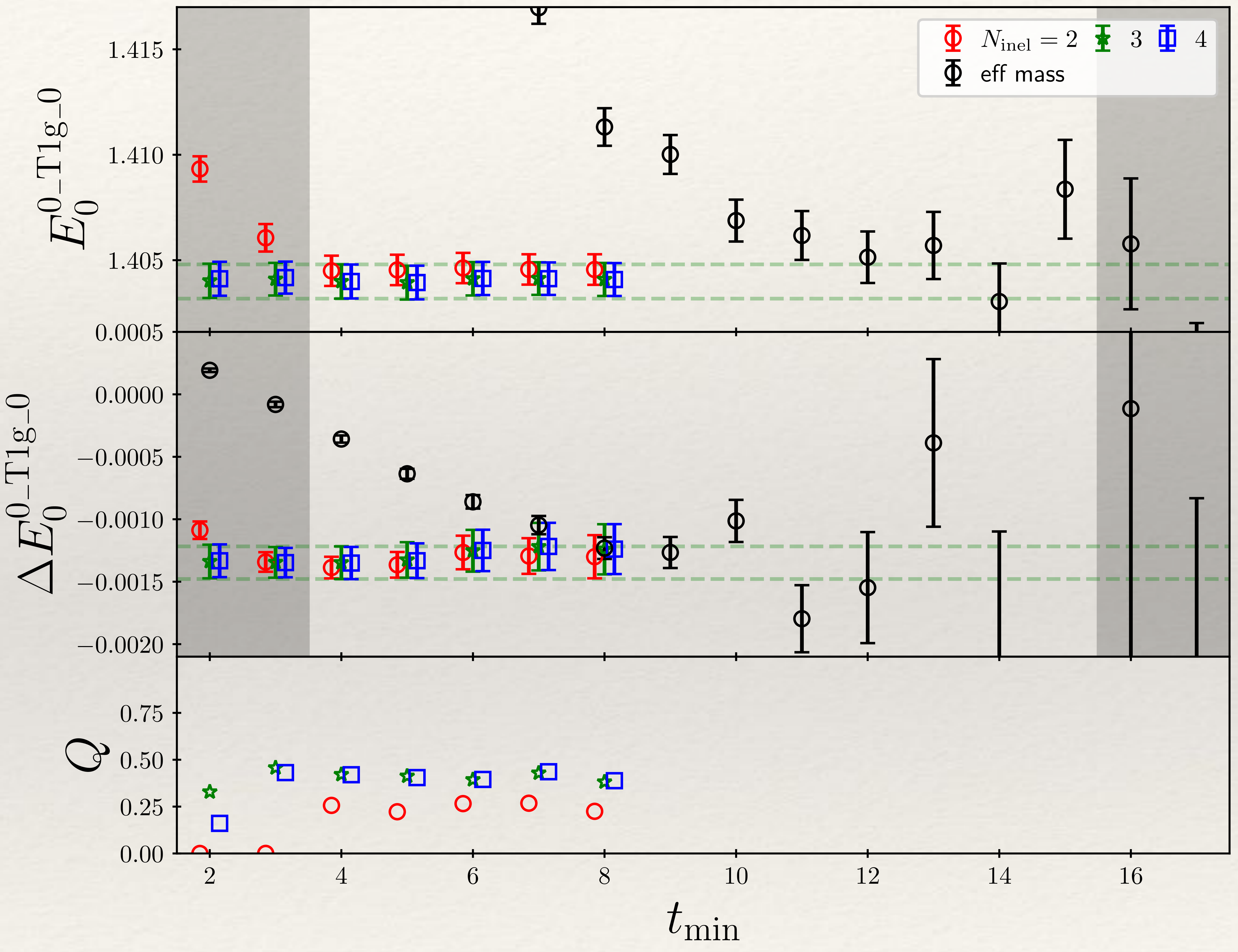
- We can use this observation to build a *conspiracy model* that naturally allows for the cancellation of excited states between NN and N
    - If N_N states are used to describe the single nucleons, then use $N_{NN} = N_N^2$ or $N_{NN} = \dfrac{N_n(N_n+1)}{2}$ to describe the NN correlator depending if the single nucleons have the same $(|\mathbf{p}_1| = |\mathbf{p}_2|)$ or different $(|\mathbf{p}_1| \neq |\mathbf{p}_2|)$ spectral decomposition
    - Model NN energies as $E^{N_1^n N_2^m} = E_n^{N_1} + E_m^{N_2} + \delta_{nm}$
    - Treat all overlap factors as positive and independent
    - Use ratio correlator to estimate prior for $\tilde{\delta}_{00} = m_{\text{eff}}^R(t_{\text{ref}}) \times \mathcal{N}(1,1)$ $\quad\tilde{\delta}_{nm} = m_{\text{eff}}^R(t_{\text{ref}}) \times \mathcal{N}(0,1)$
    - Estimate excited state energy gaps as $\tilde{\Delta}_{n,n-1} = E_n - E_{n-1} = \text{Lognormal}(2m_\pi, m_\pi)$
    - Perform fully correlated fit to NN and N_1, N_2 correlators

NN 2025.04XYZ : Conspiracy model

- Compare with an *Agnostic Model* where the N_{NN} is independent of N_N



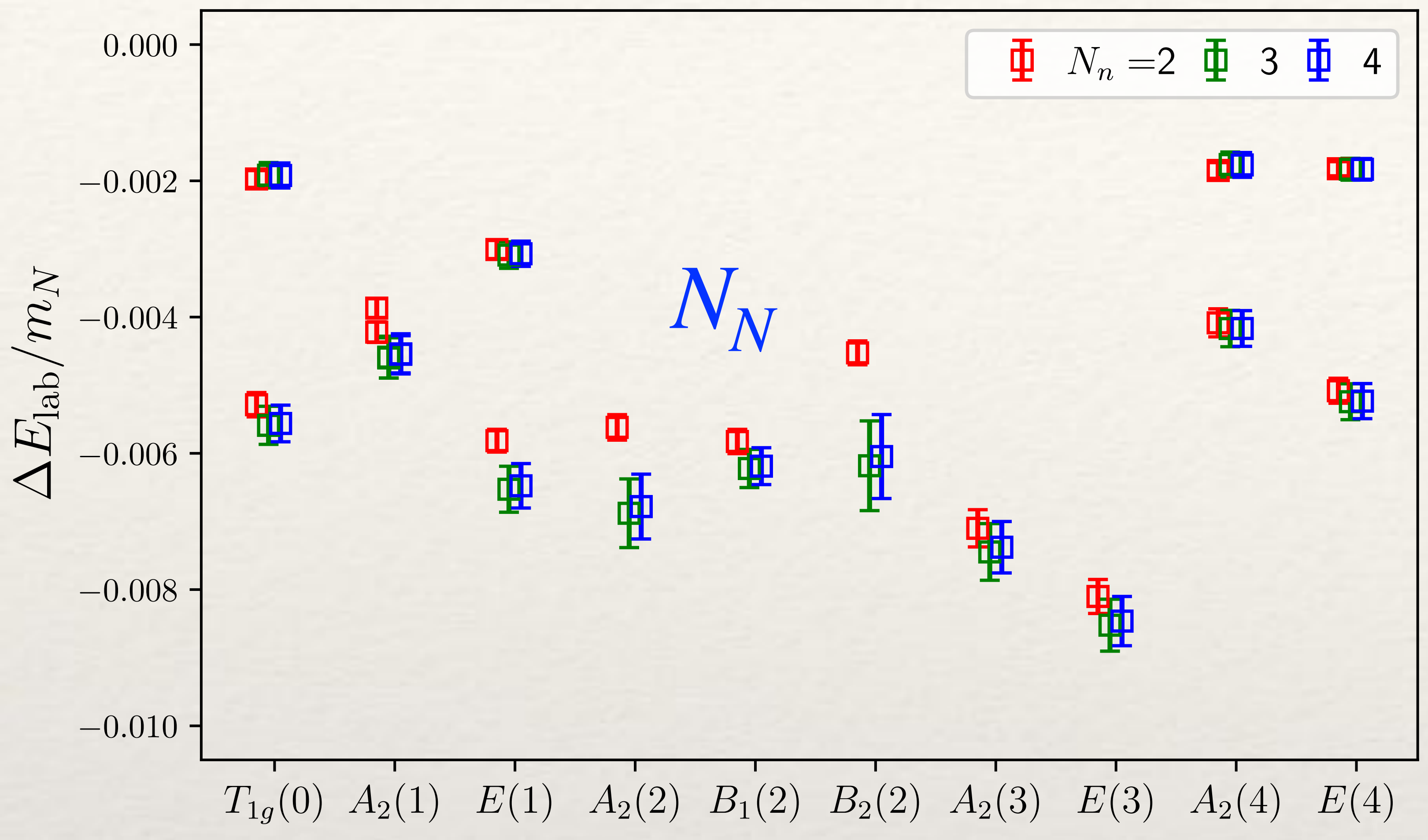
- How many exponentials?

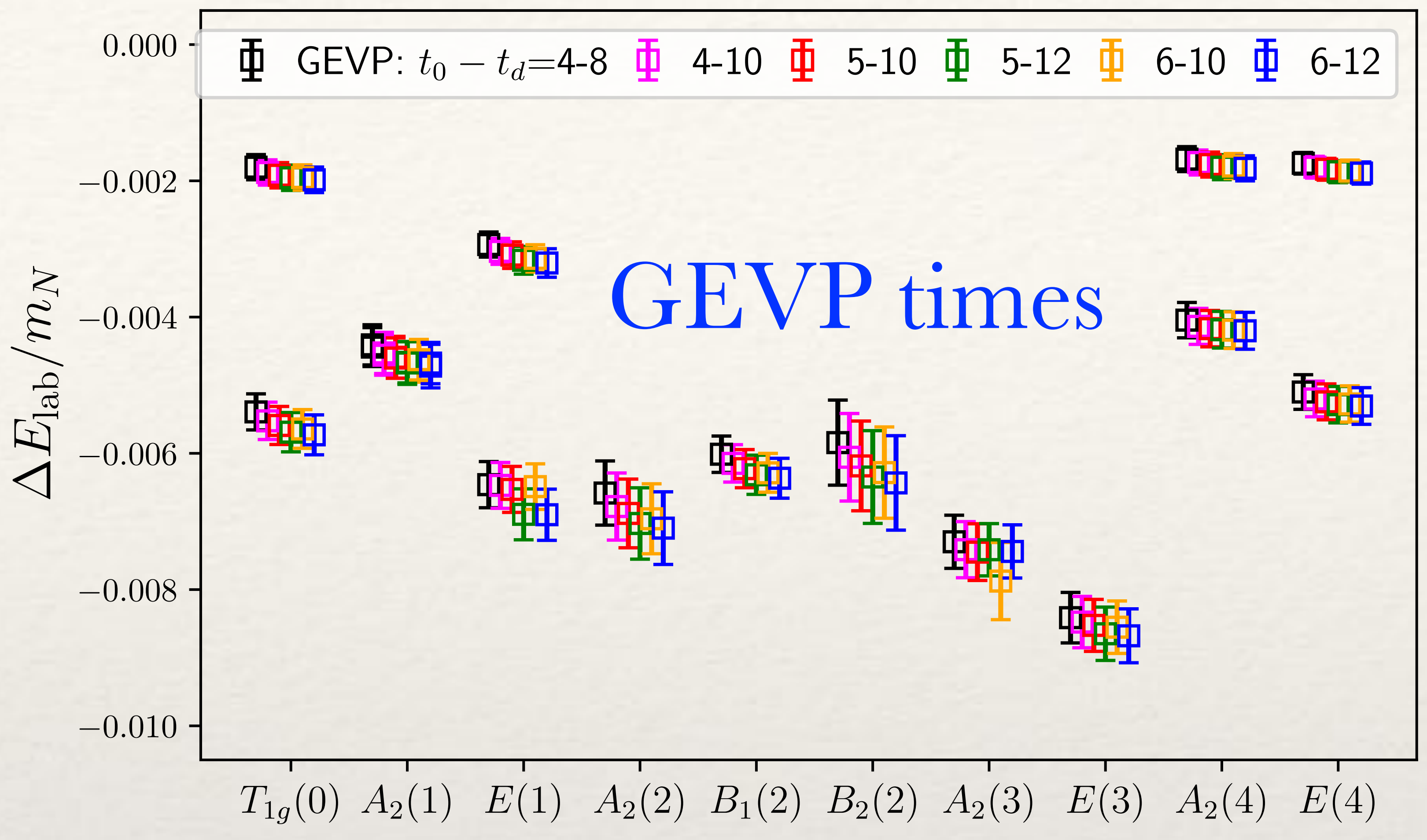


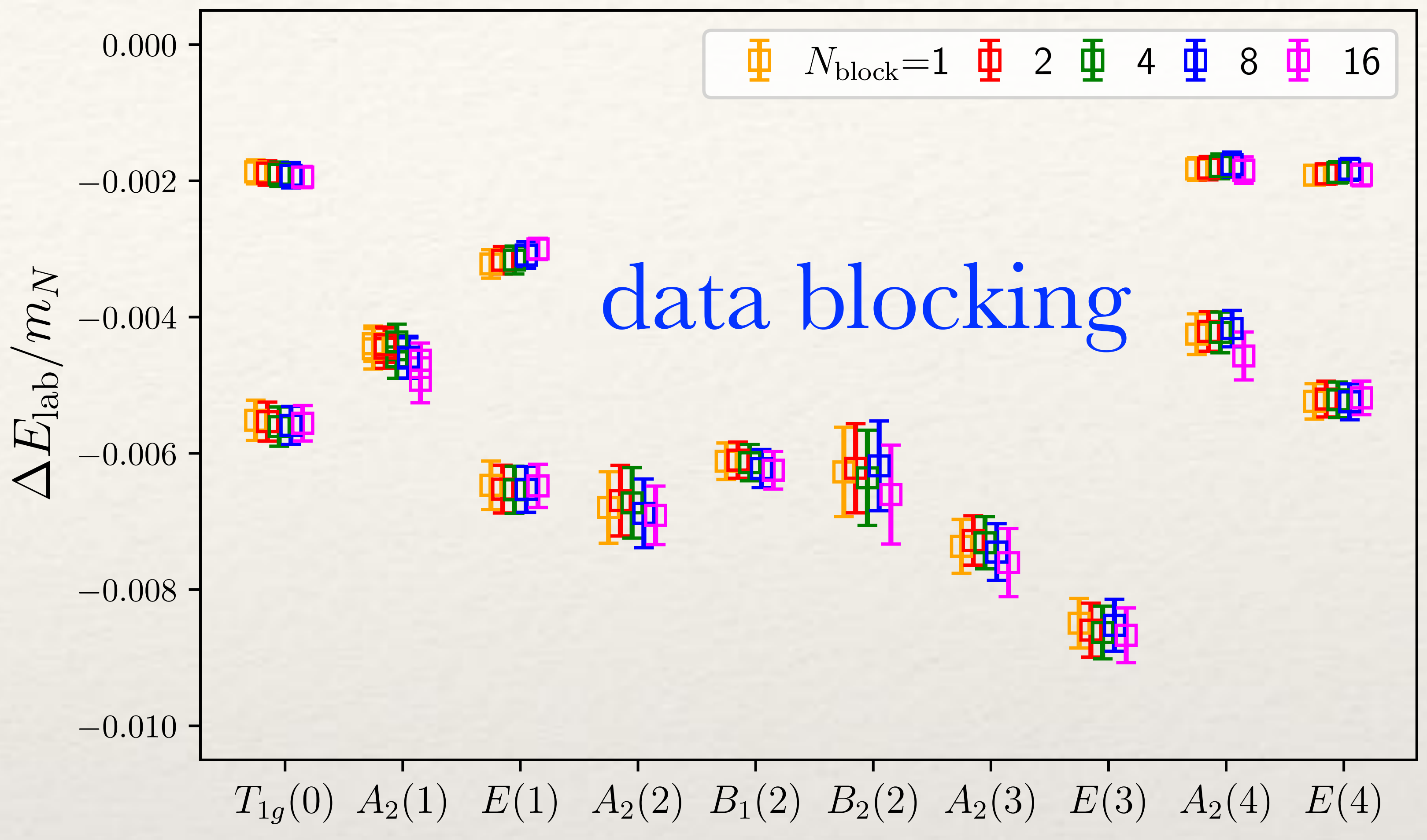
- Under Bayes Theorem - the *evidence* provides a relative weight to predict the probability of the model, given the data — the conspiracy model is preferred by the data

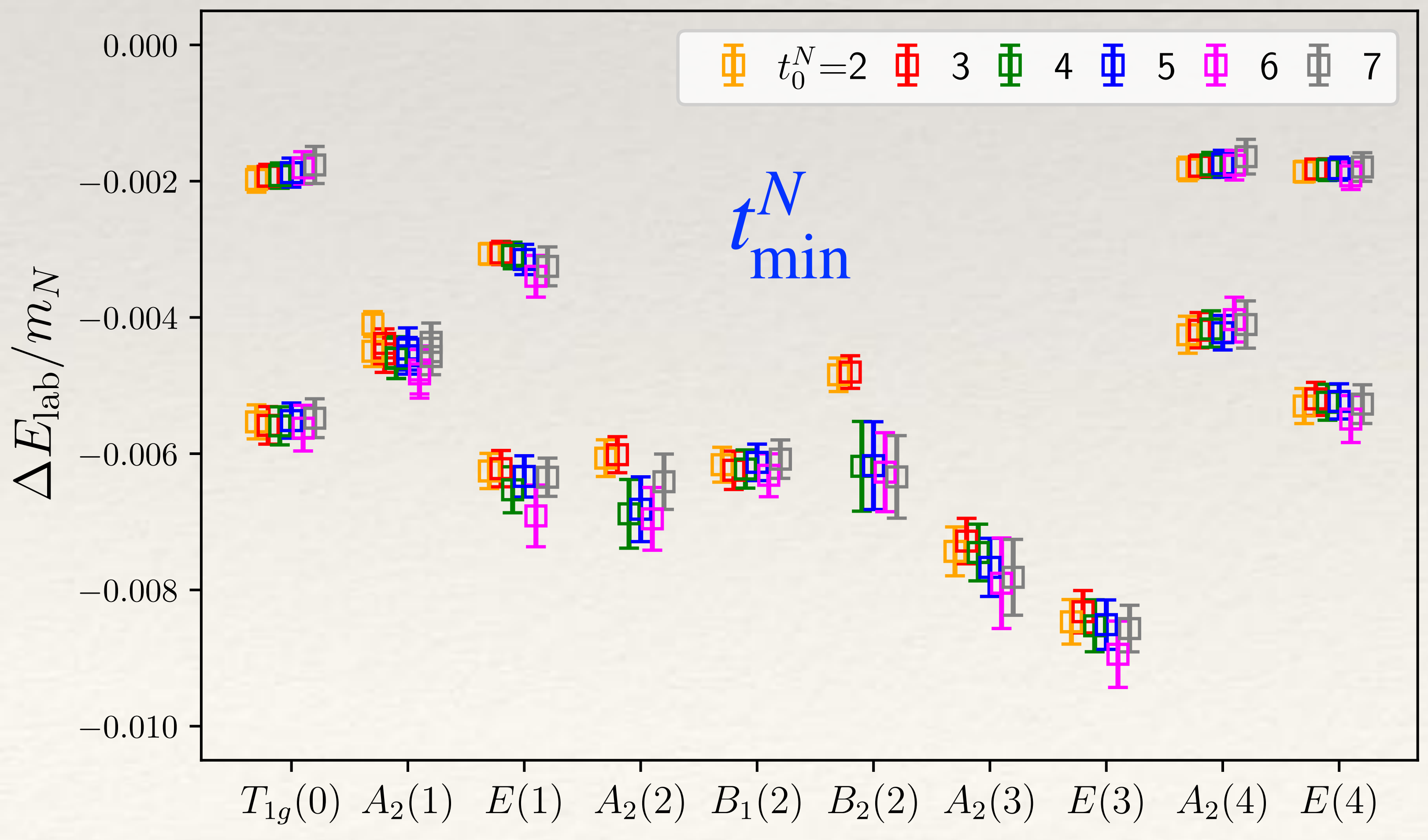
NN 2025.04XYZ : Conspiracy model

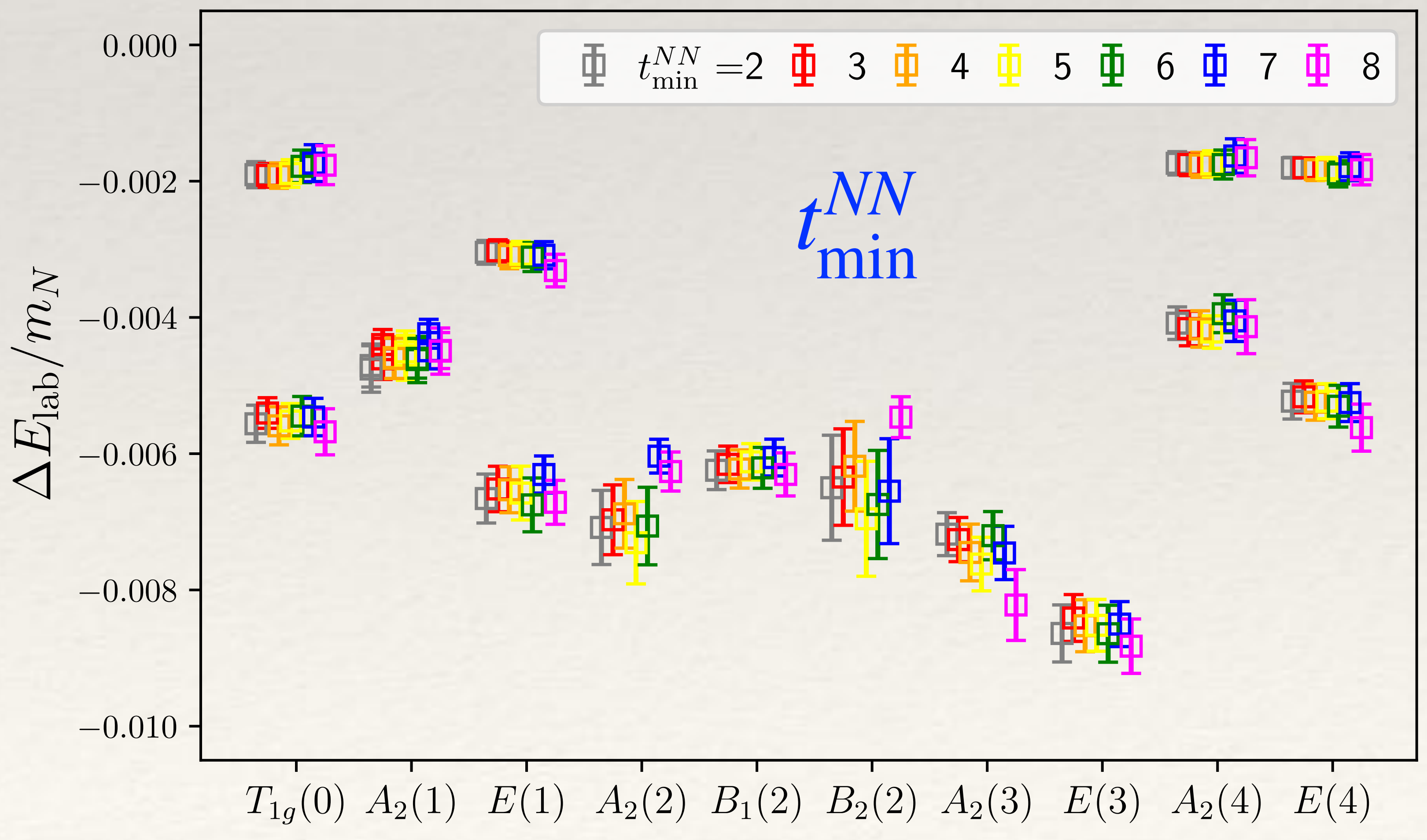
- Large stability study with respect to all user input paramters

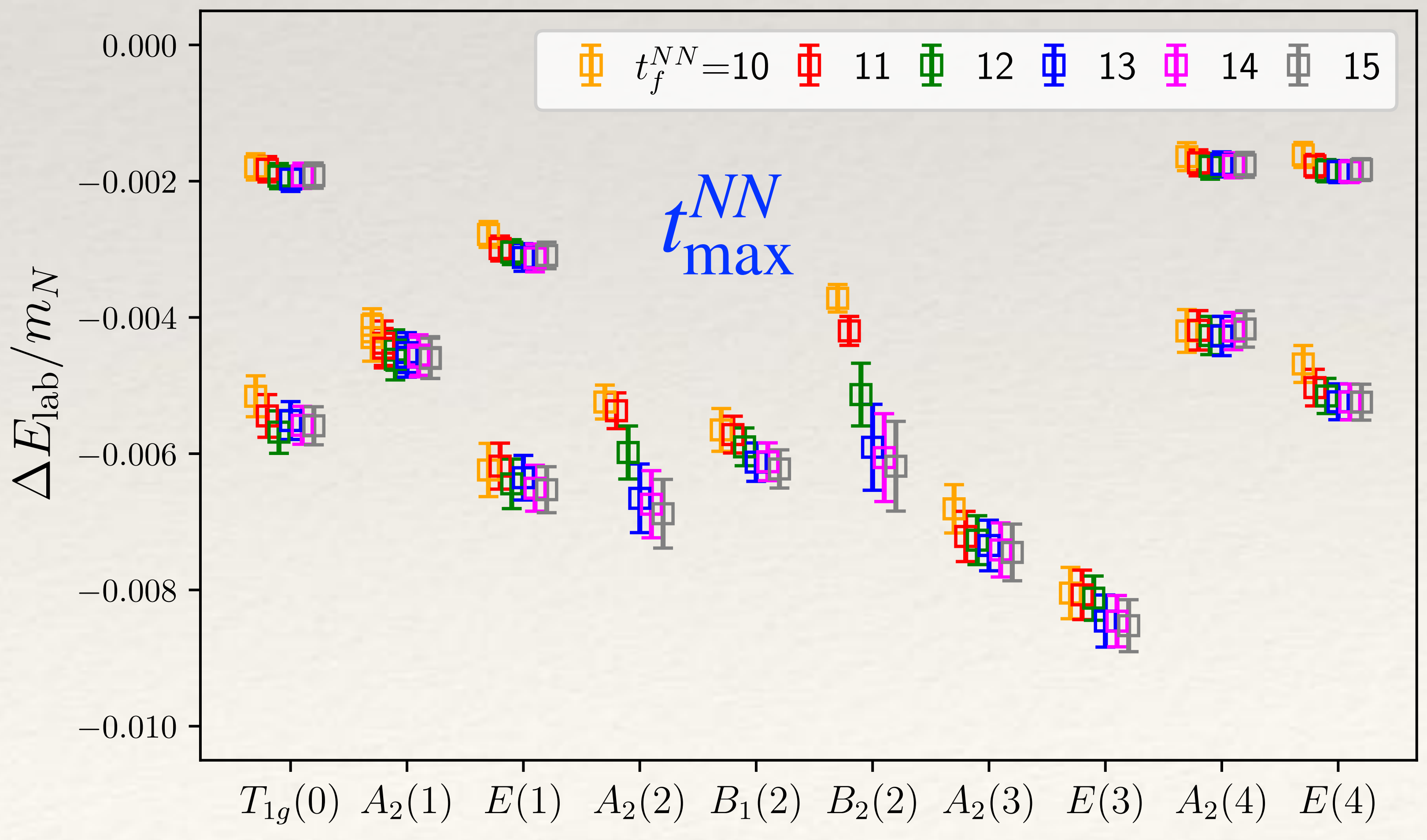




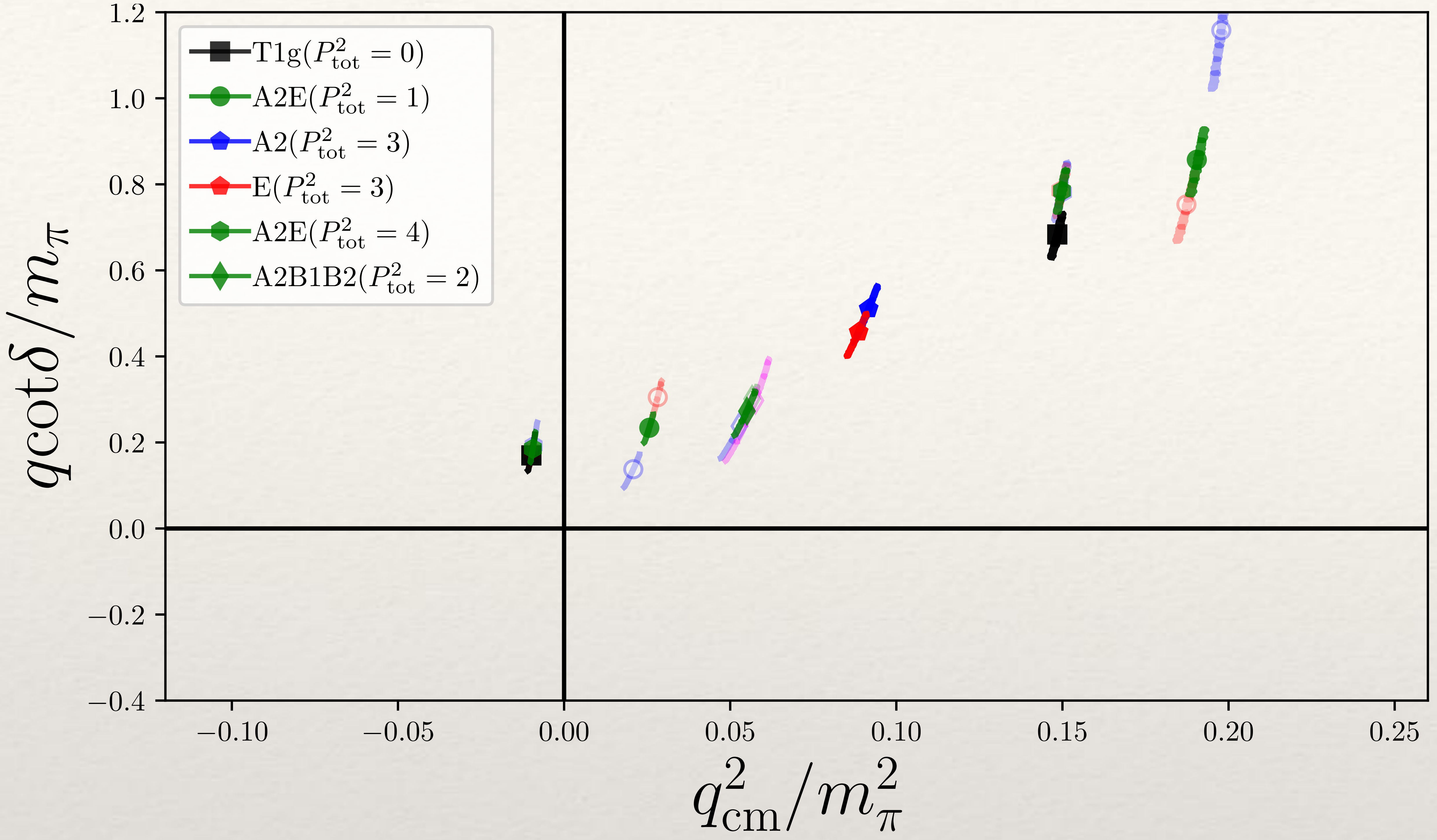


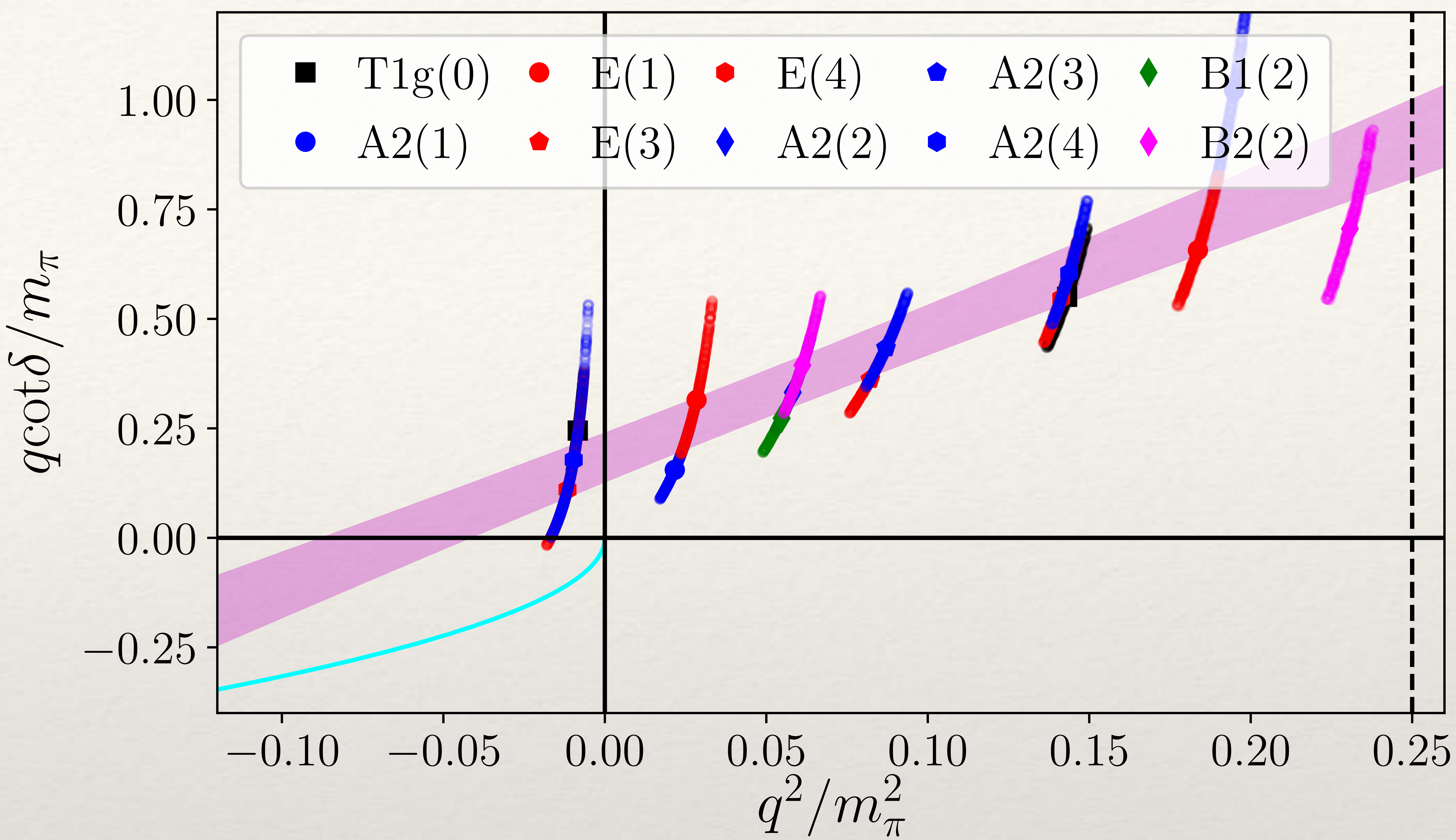




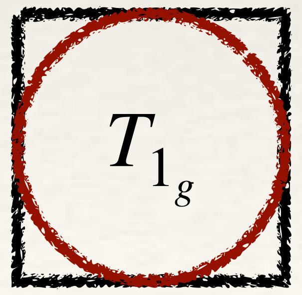


NN 2025.04XYZ : Deuteron Amplitude





- Bound state on this ensemble ruled out by $> 7\sigma$, $am_\pi = -5.10(68)$, $\dfrac{q\cot\delta}{m} = -\dfrac{1}{am} + \dfrac{1}{2}(rm)\dfrac{q^2}{m^2} + \cdots$
- Our results are now sensitive to $S - D$ partial wave mixing



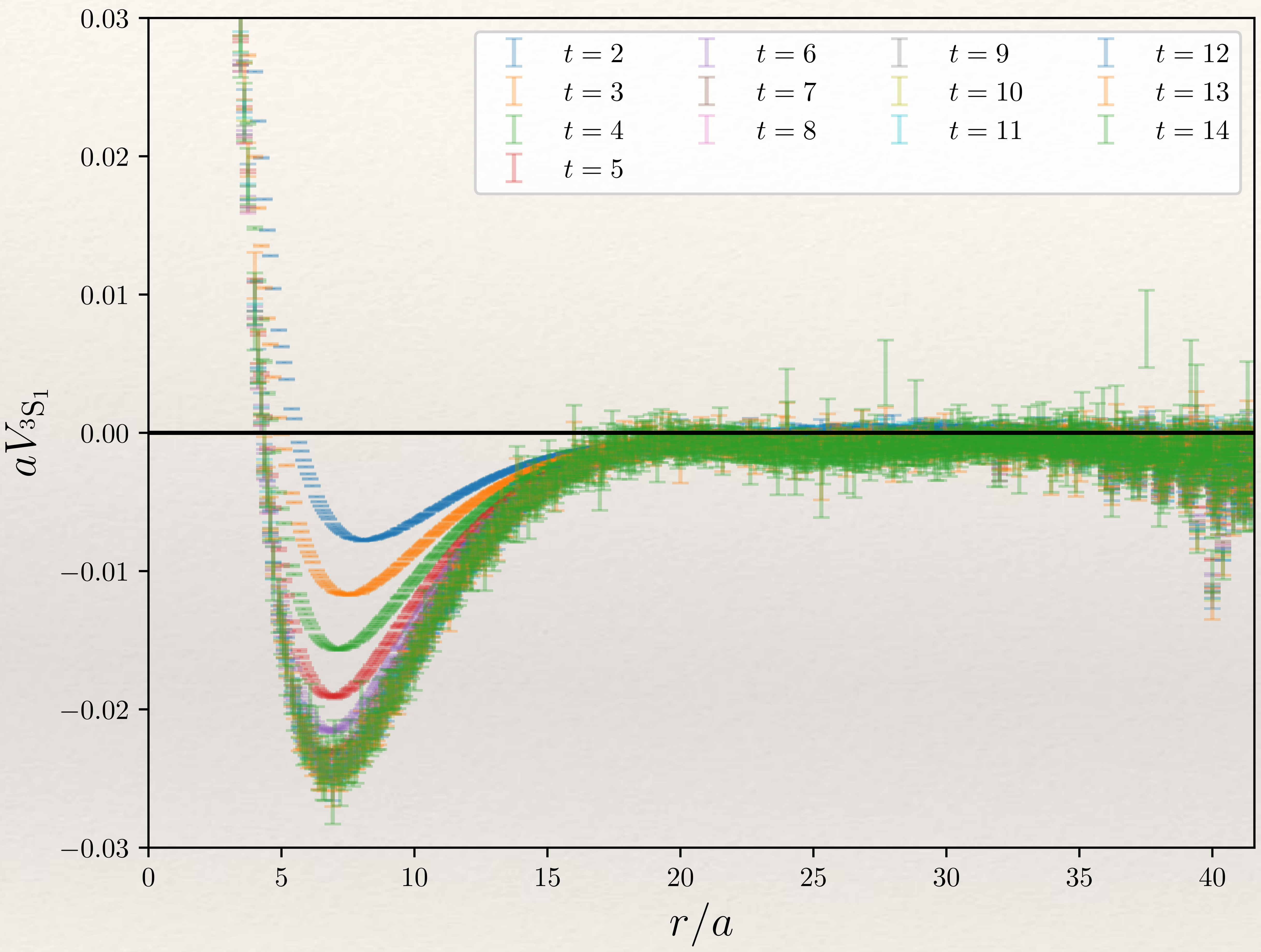
$P_{\rm tot} = 0$ $\quad$ $P_{\rm tot} = \dfrac{2\pi}{L}$

Briceno, Davoudi, Luu, Savage, PRD88 (2013) remove leading S-D mixing sensitivity:

$$\frac{1}{3}\left(E_{A_2} + 2E_E\right), \quad \frac{1}{3}\left(E_{A_2} + E_{B_1} + E_{B_2}\right)$$

$\vec{n}_{\rm tot} = (0,0,n)$ $\quad$ $\vec{n}_{\rm tot} = (0,n,n)$

NN 2025.04XYZ : Deuteron HAL QCD Potential



PRELIMINARY

- Potential "saturates" at $t \sim 8$
  - Study $t \to \infty$ extrapolation
- Study sensitivity to $r_{\text{min}}, r_{\text{max}}$
- Insensitivity to various functional forms of V(r)

$$V(r) = \sum_n b_n e^{-r^2/2\sigma_n^2}$$

$$V(r) = A_\pi \frac{e^{-m_\pi r}}{r} \left(1 - e^{-r^2/r_0^2}\right)^n + \frac{w_0 + w_1 r + w_2 r^2}{1 + e^{(r-r_0)/a}}$$

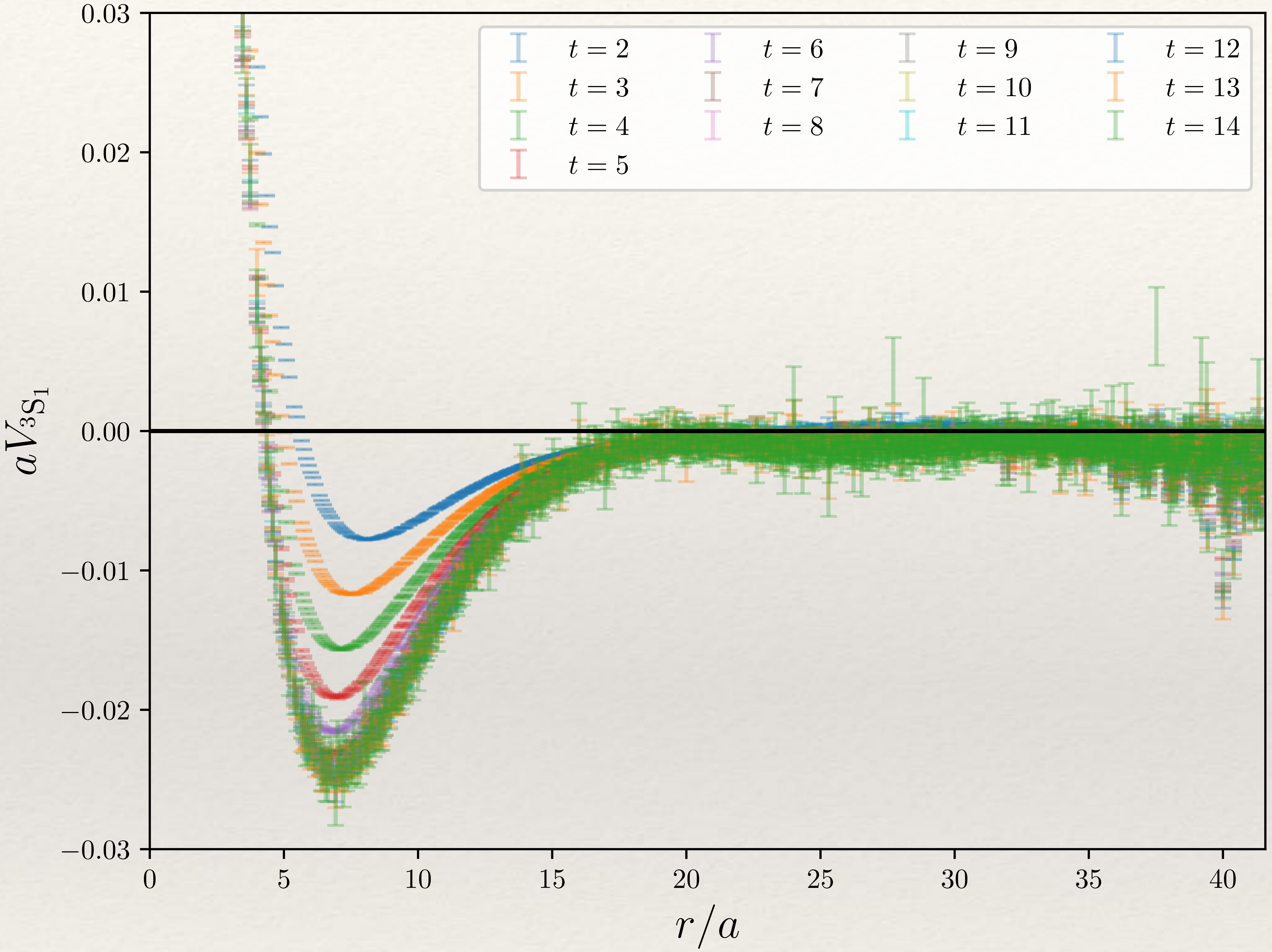
regulated OPE + Woods-Saxon

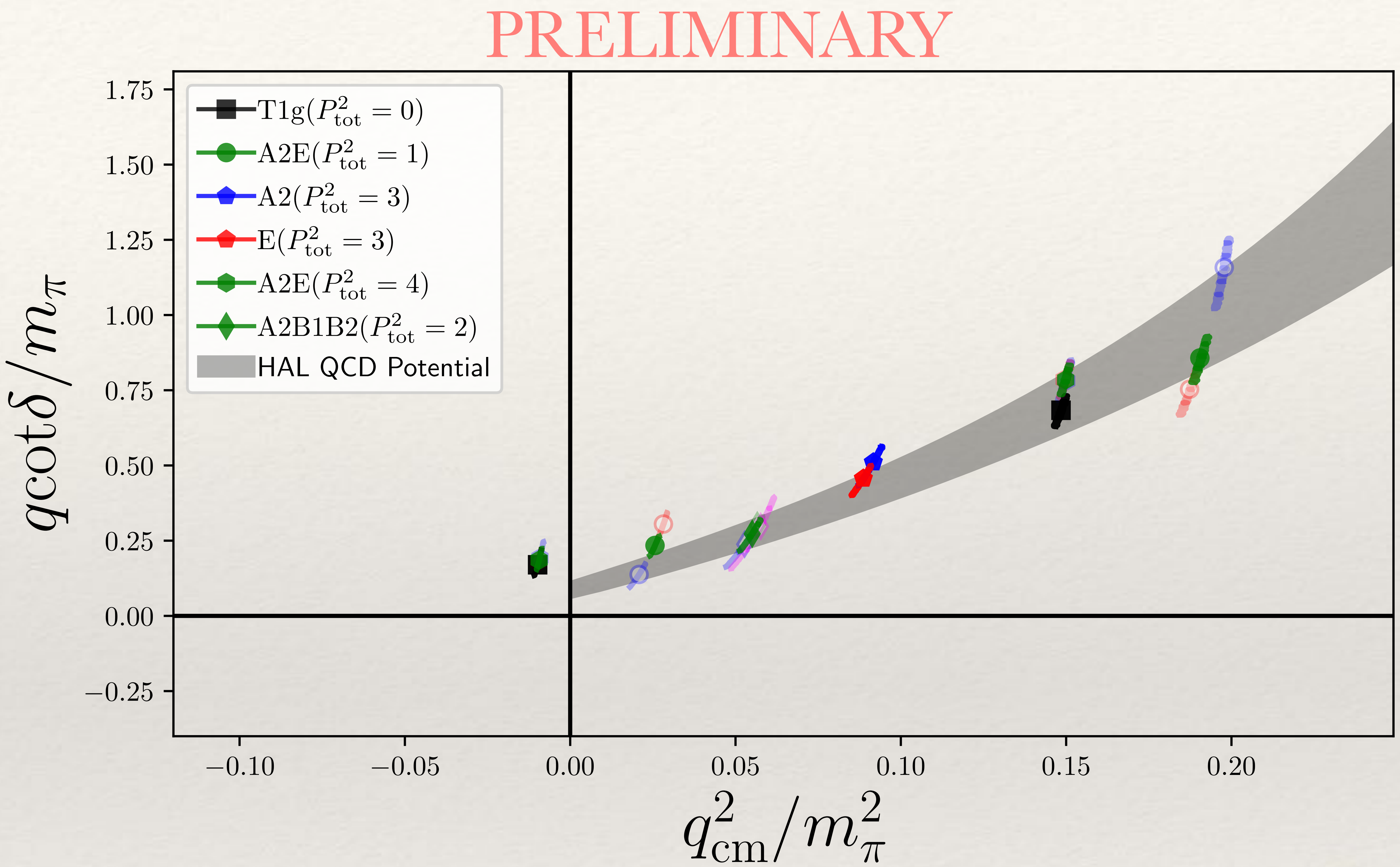
Wiringa, Stoks, Schiavilla PRC 51 (1995)

$$V(r) = A_\pi \frac{e^{-m_\pi r}}{r} \left(1 - e^{-r^2/r_0^2}\right)^n + H.O.\ basis$$

- $m_u = m_d = m_s \approx m_s^{\text{phys}} \longrightarrow m_\pi \approx 714$ MeV
  $a \approx 0.086$ fm, $V = 48^3 \times 96$

NN 2025.04XYZ : Deuteron HAL QCD Potential



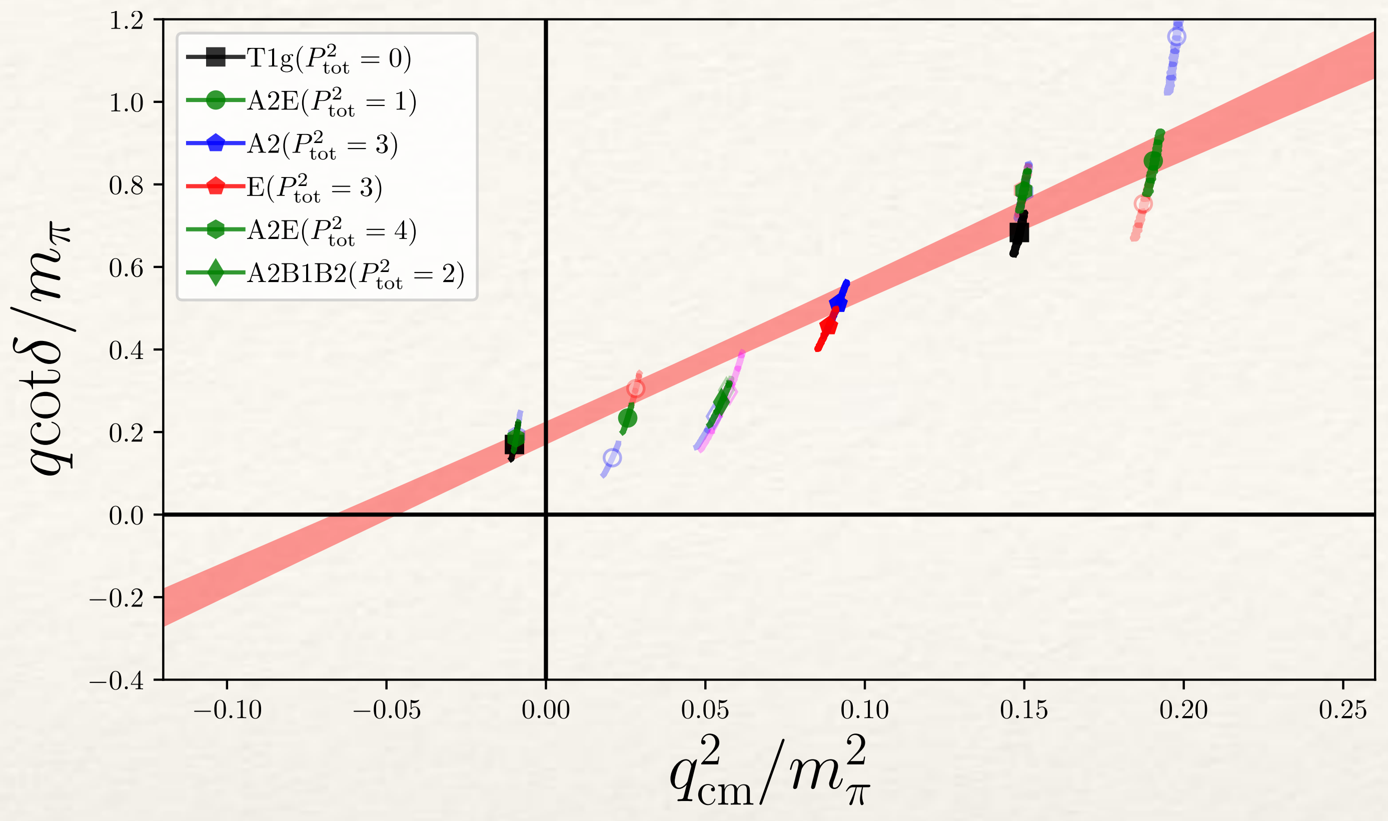


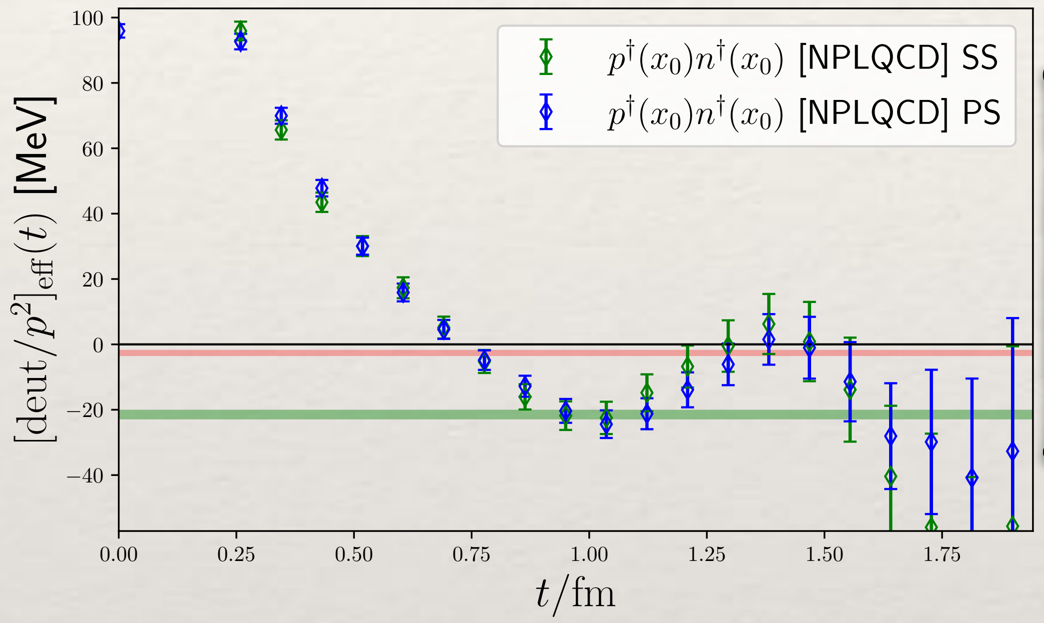
- $m_u = m_d = m_s \approx m_s^{\rm phys} \longrightarrow m_\pi \approx 714$ MeV
  $a \approx 0.086$ fm, $V = 48^3 \times 96$

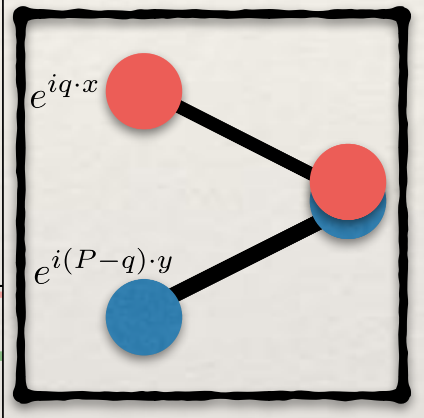
- HAL QCD (gray band) is fairly consistent with our Lüscher (standard) results
- HAL QCD has different discretization effects, as well as additional systematics

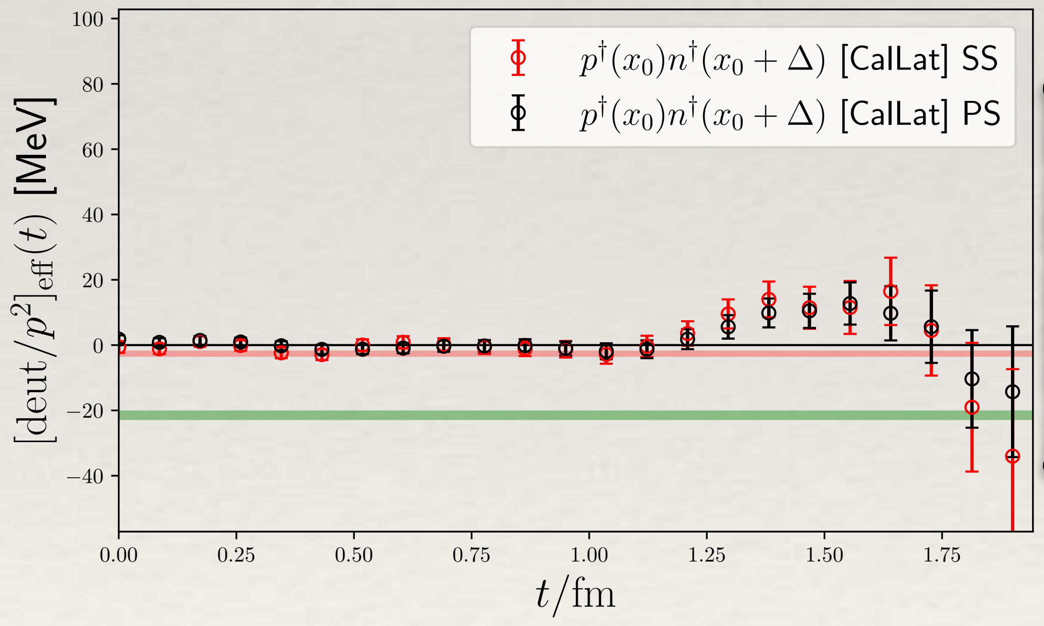
Di-nucleons do not form bound states at heavy pion mass

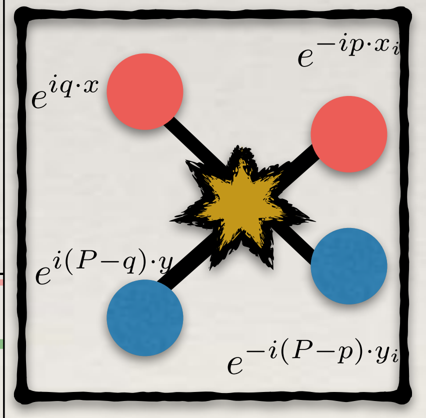
- 2025.04XYZ
  - The full calculation excludes a bound state at $> 7\sigma$
  - The inclusion or not of HX operators does not influence our spectrum
  - We observe extracted spectrum dependence on the creation operators with off-diagonal correlators
  - The HAL QCD potential yields qualitative consistent phase shift

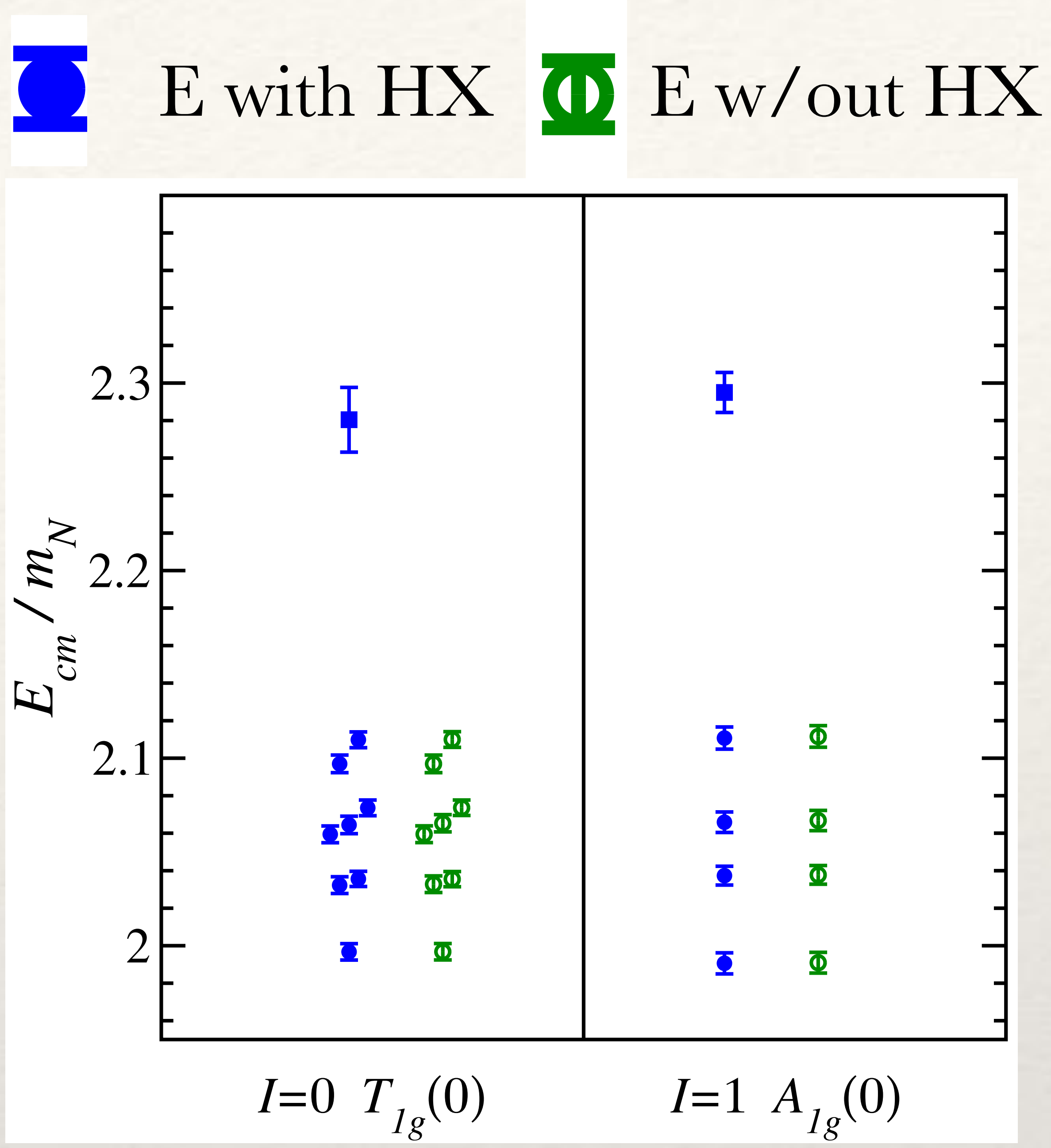












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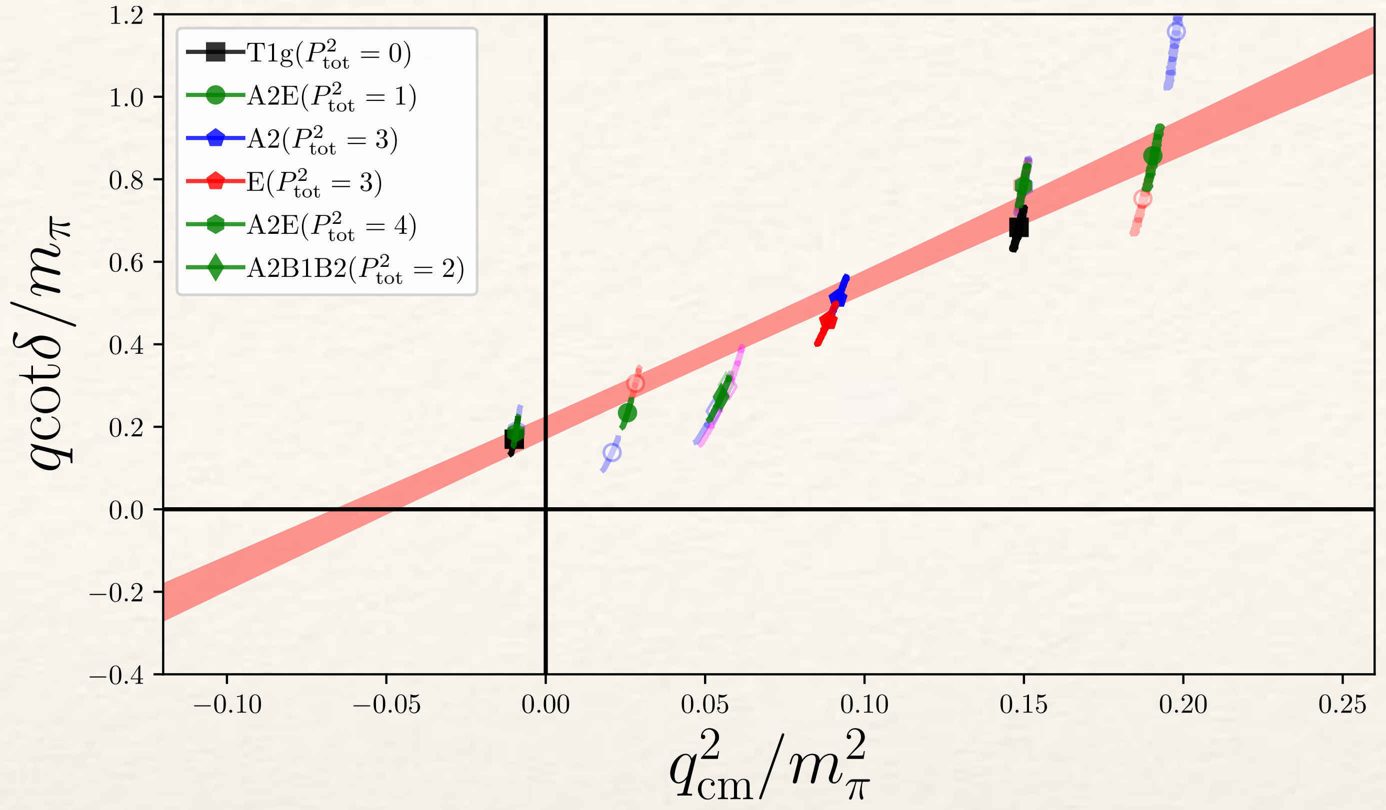
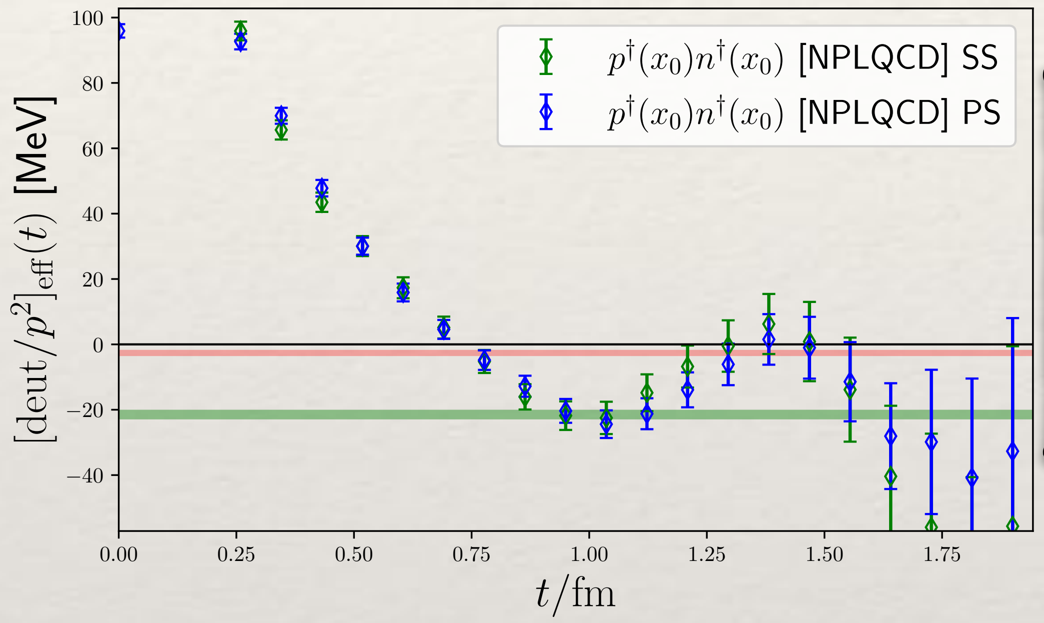
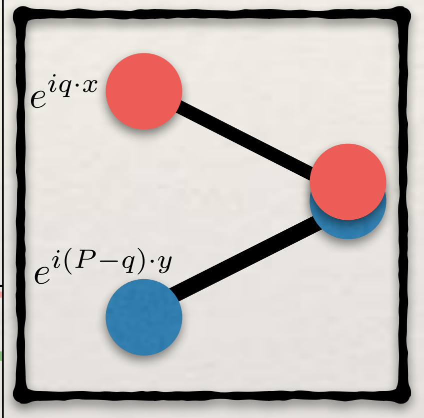
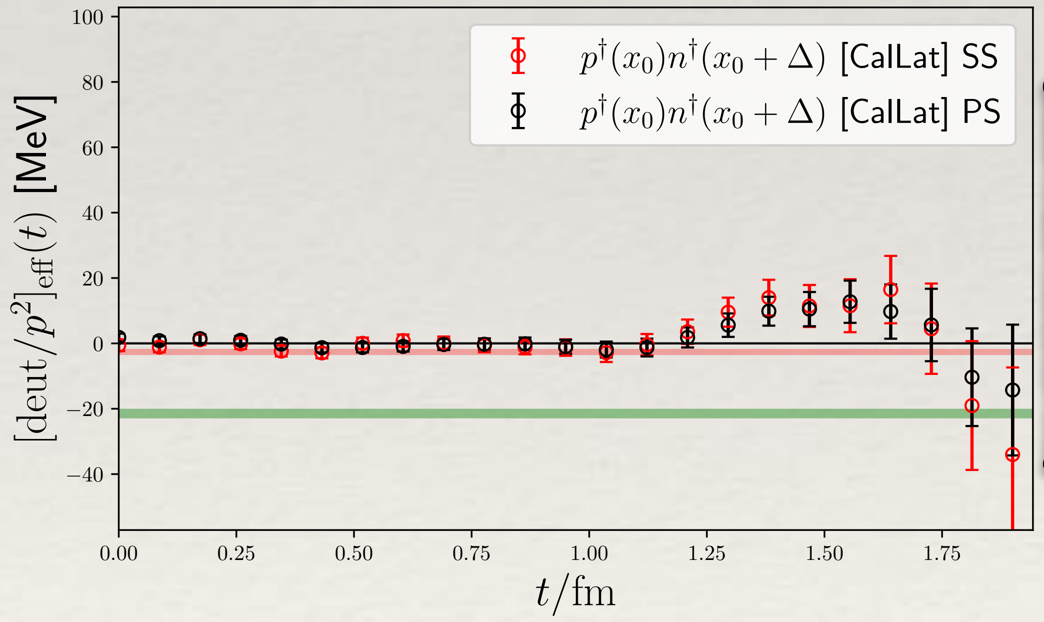
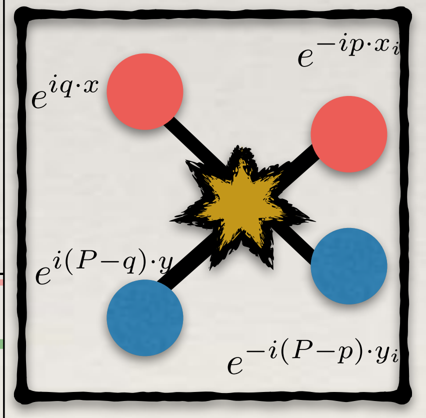
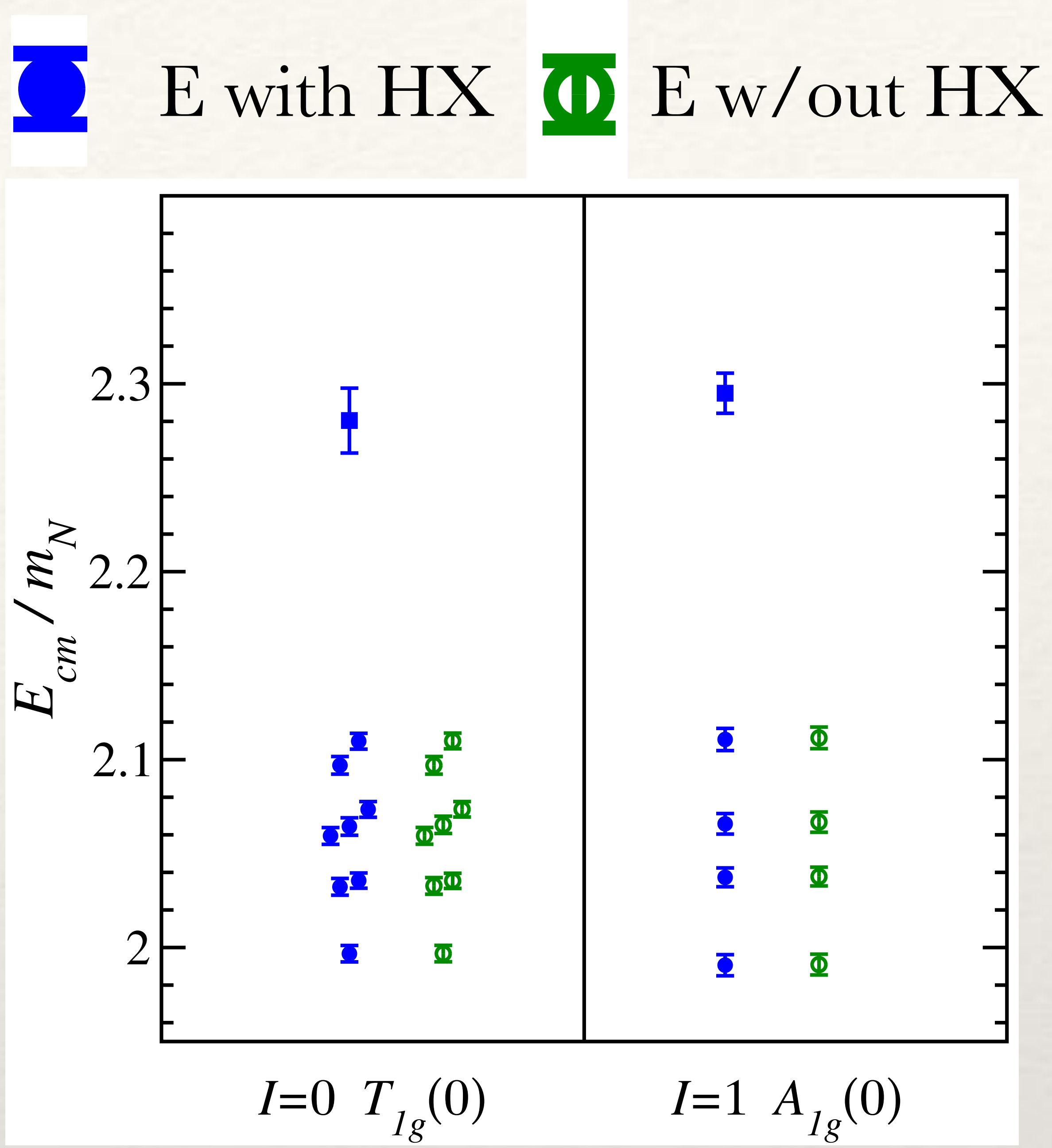


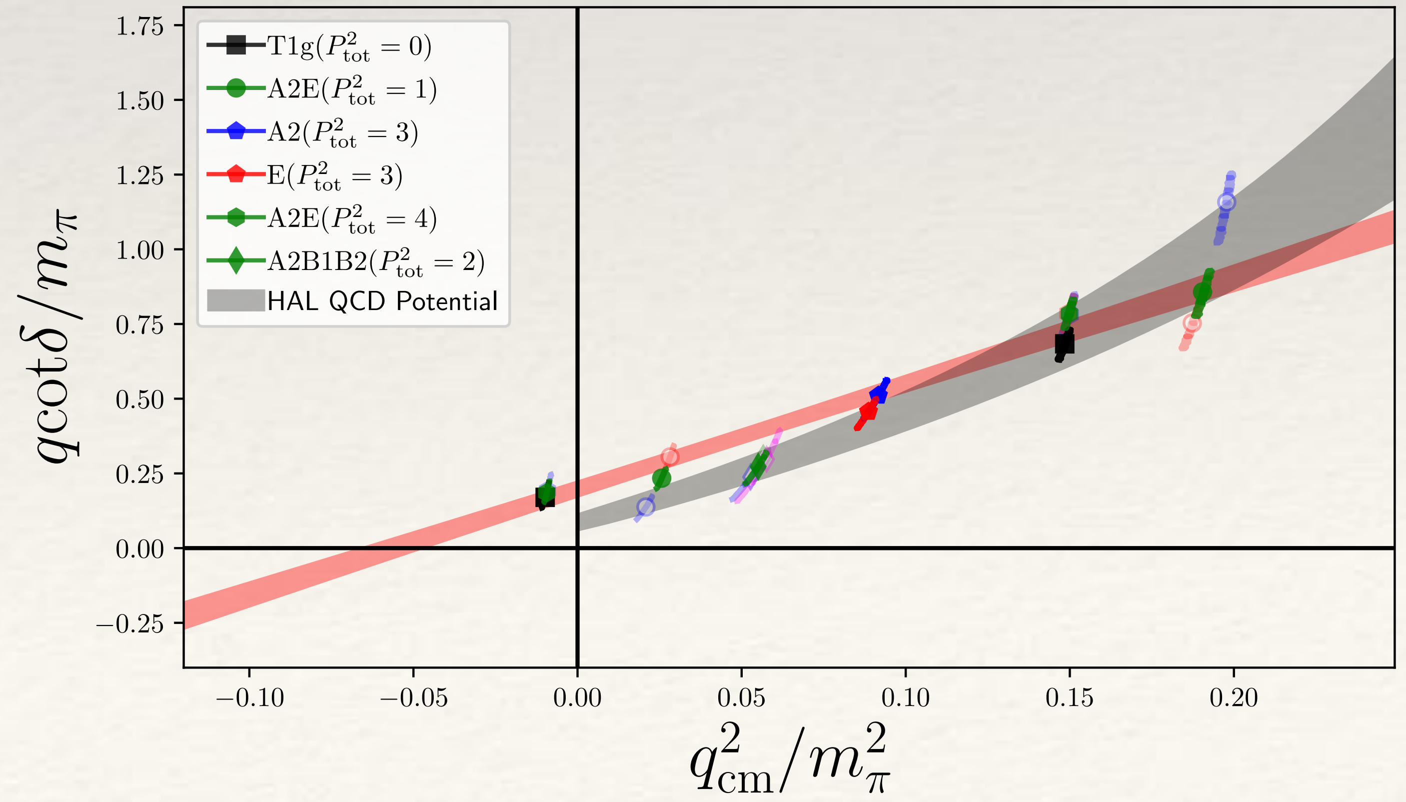






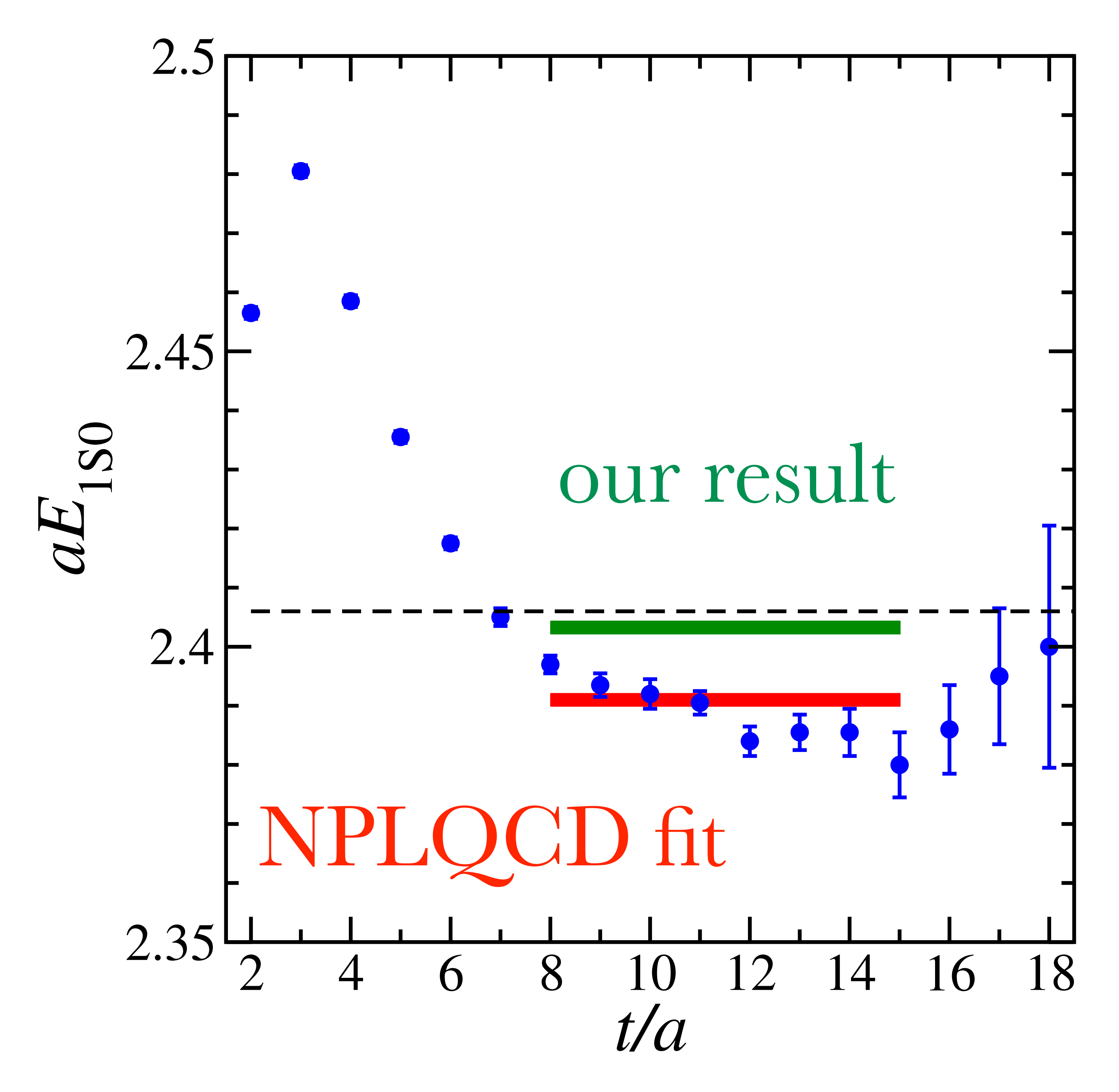




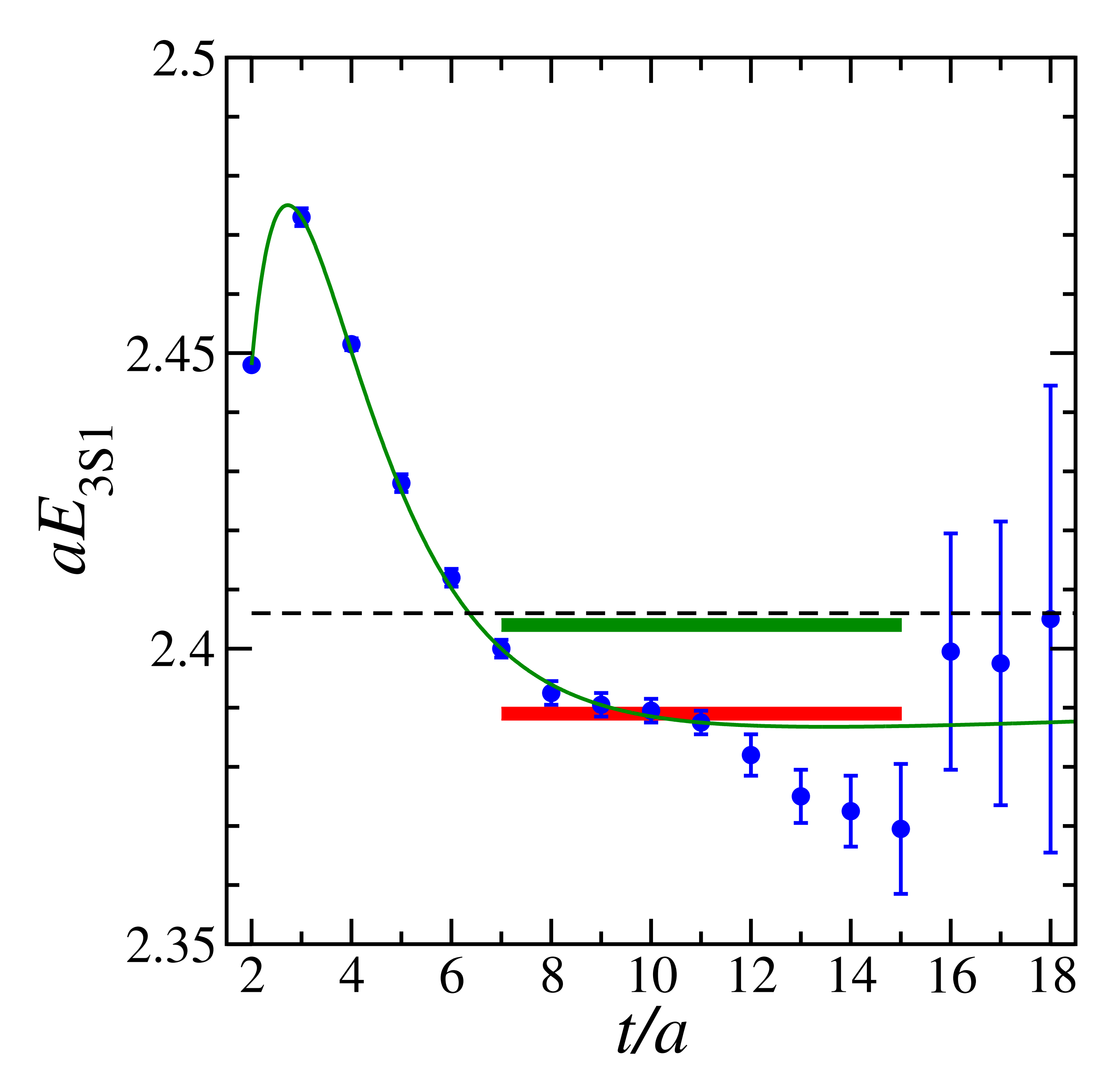


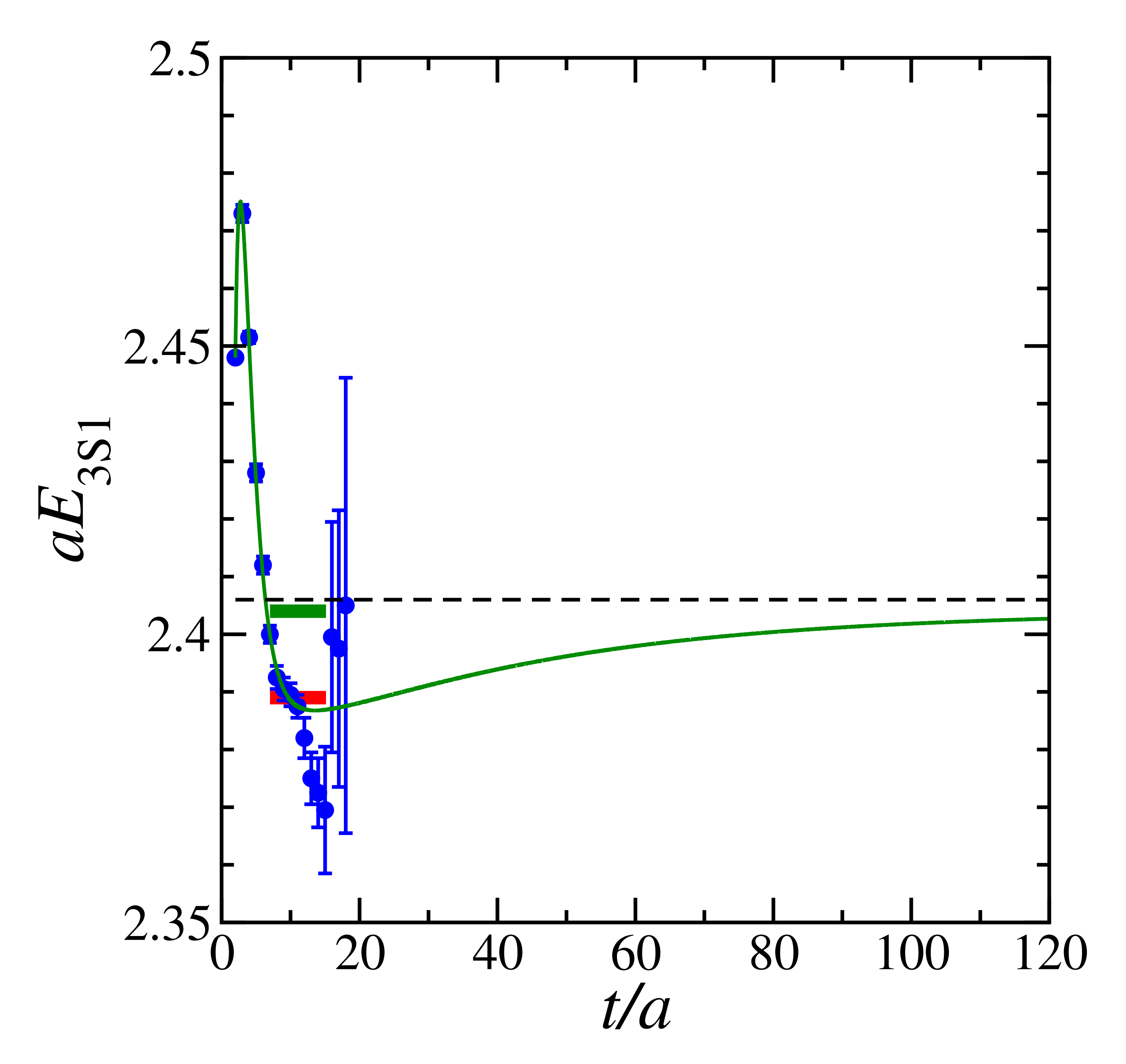
What went wrong?

- 2025.04XYZ
  - We believe the previous results suffered from a *false plateaux* identification
    HAL QCD: Iritani et al., JHEP 10 (2016) [1607.06371]



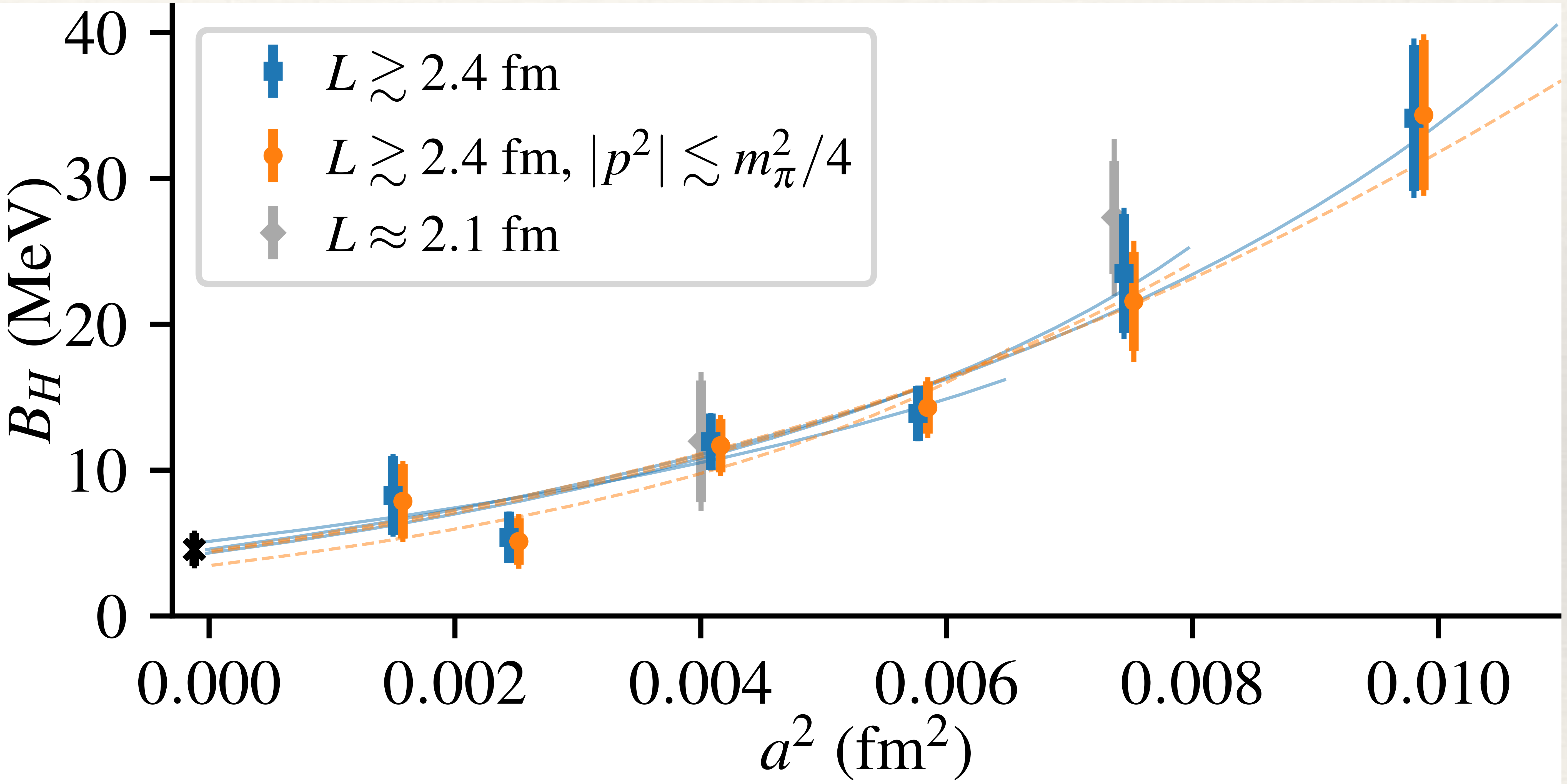
- Use model: $C(t) = e^{-E_0 t}\left(1 + A_1 e^{-\Delta_1 t} + A_2 e^{-\Delta_2 t} + A_3 e^{-\Delta_3 t} + A_4 e^{-\Delta_4 t}\right)$
  - $\Delta_{1,2}$ chosen to represent elastic scattering modes
  - $\Delta_{3,4}$ chosen to describe early time excited states
  - use $m_{\rm eff}(t)$ at $t = \{2,3,7,11\}$ to solve for A_i





What about those discretization effects?

- A new'ish result also showed surprisingly large discretization effects - O(1000%) use of non-perturbative, O(a)-improved clover-Wilson action (CLS) [Green, Hanlon, Junnarkar, Wittig, PRL 127 - 2103.01054]



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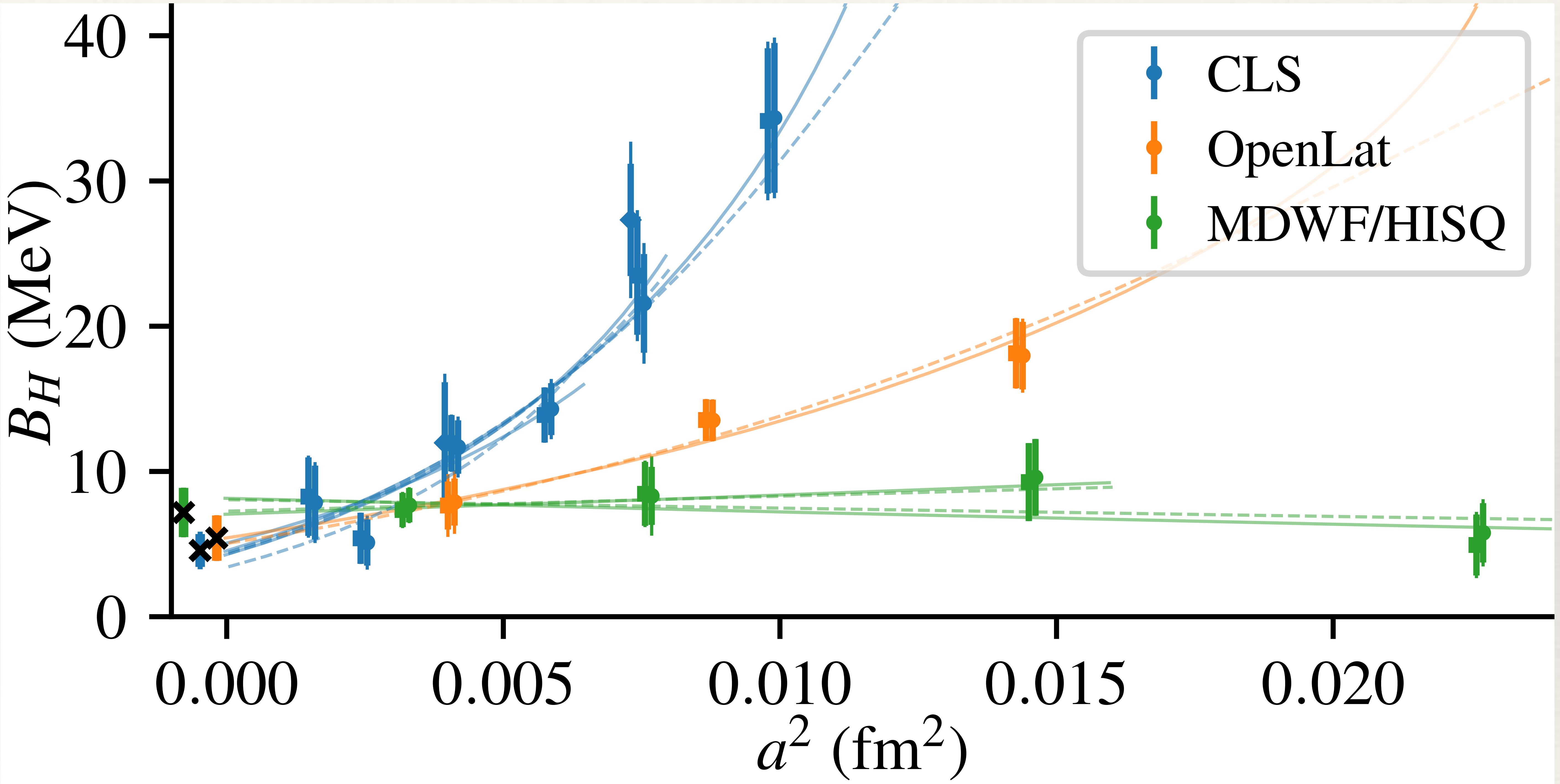
- We are performing a study to understand how large discretization effects are with different lattice actions
  - $t_{\mathrm{MC}} \gtrsim \frac{1}{a^6}$
- OpenLat: exponentiated clover
- MDWF / HISQ: mixed action with chiral valence fermions



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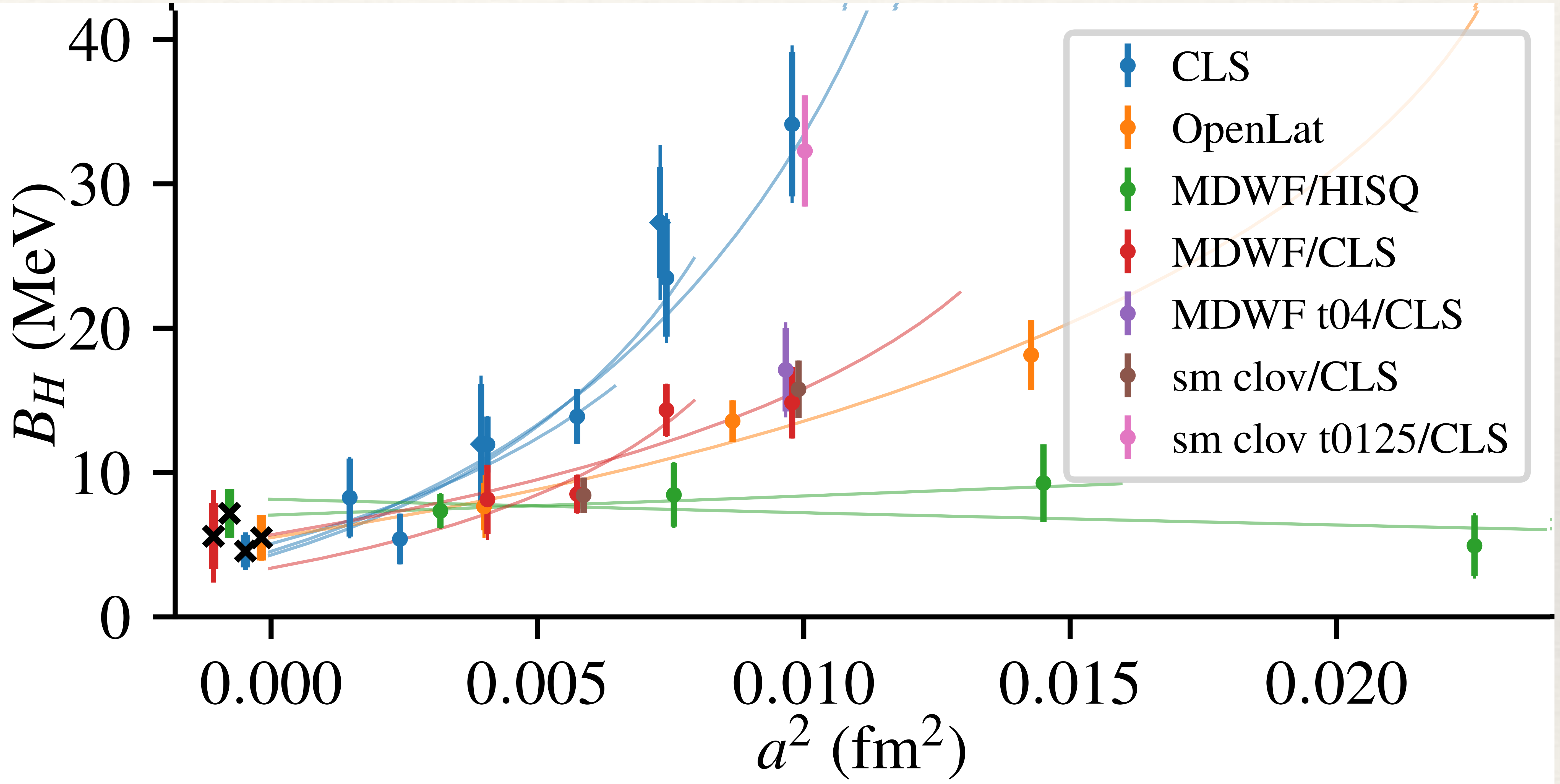


LQCD and χPT / NN EFT

- Predicting properties and interactions of nuclei with SM input

LQCD and χPT / NN EFT

- Predicting properties and interactions of nuclei with SM input

Many Body Nuclear Methods — GFMC, IMSRG, coupled-cluster, HOBET, Shell Model, …

↑

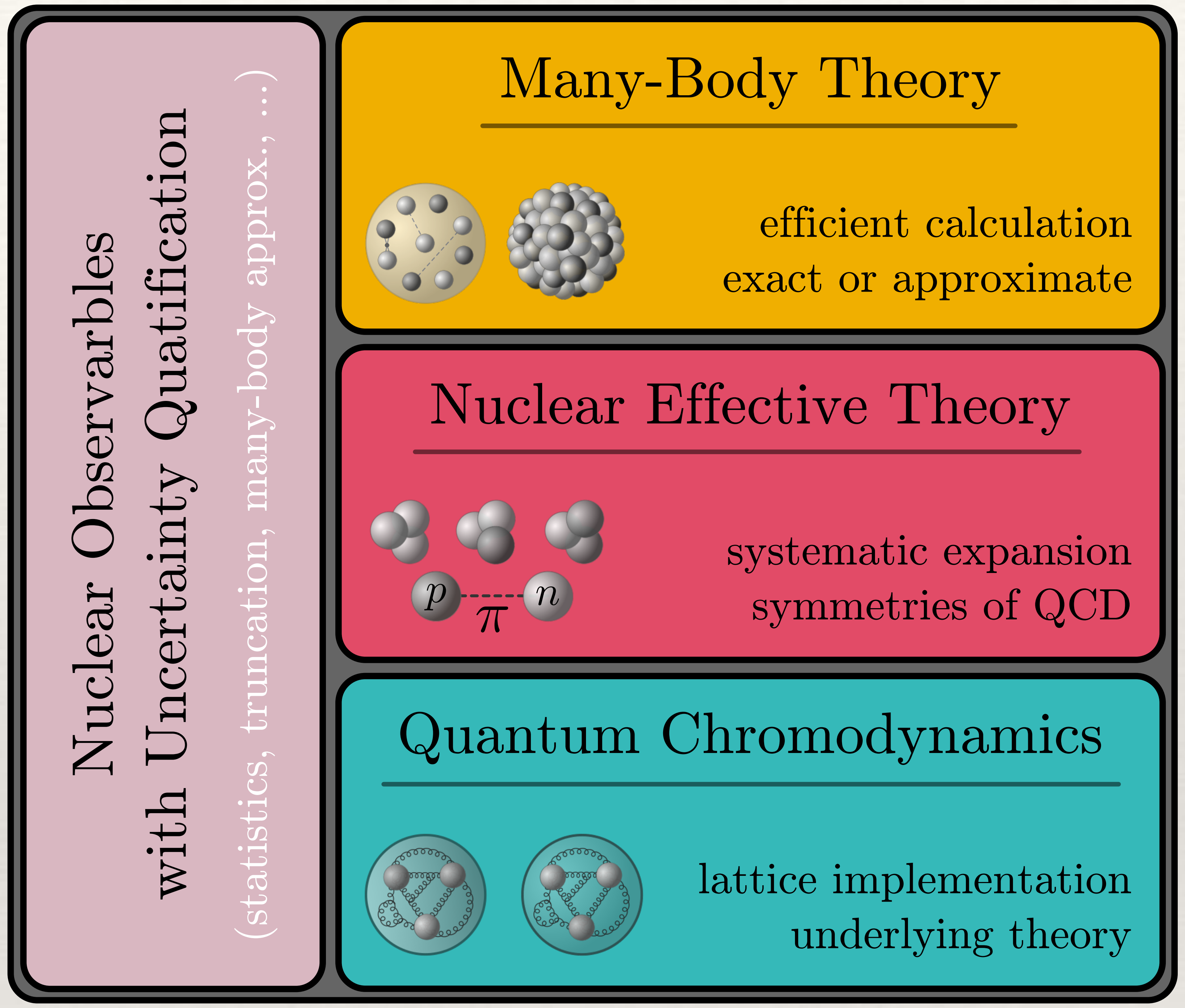
NN EFT — two-nucleons (pions)

↑

Hadronic EFT — pions, nucleons (deltas)

↑

QCD — quarks, gluons and lattices (M. Creutz)



(fig: C. Drescher)

LQCD and χPT / NN EFT

- An issue with this program is that, the majority of NN EFT → light-nuclear predictions utilize SU(2) HBχPT(Δ) and we have growing evidence that this theory is not converging

  - m_N
  - πN scattering
  - g_A

  - At the same time, there is optimism that adding explicit Δ degrees of freedom can restore convergence
  - This is good for LQCD as it adds more explicit areas for LQCD to contribute to necessary nuclear input

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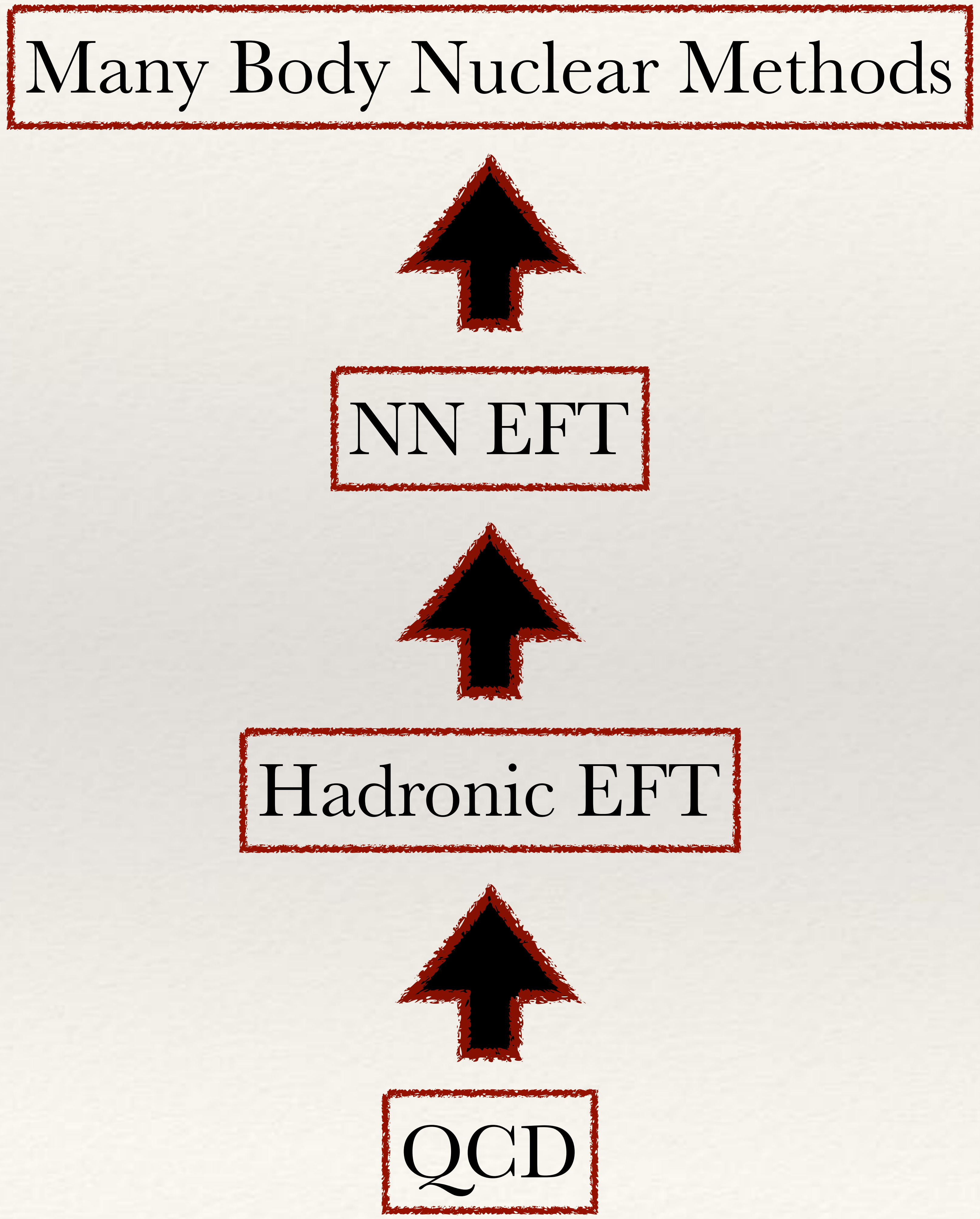

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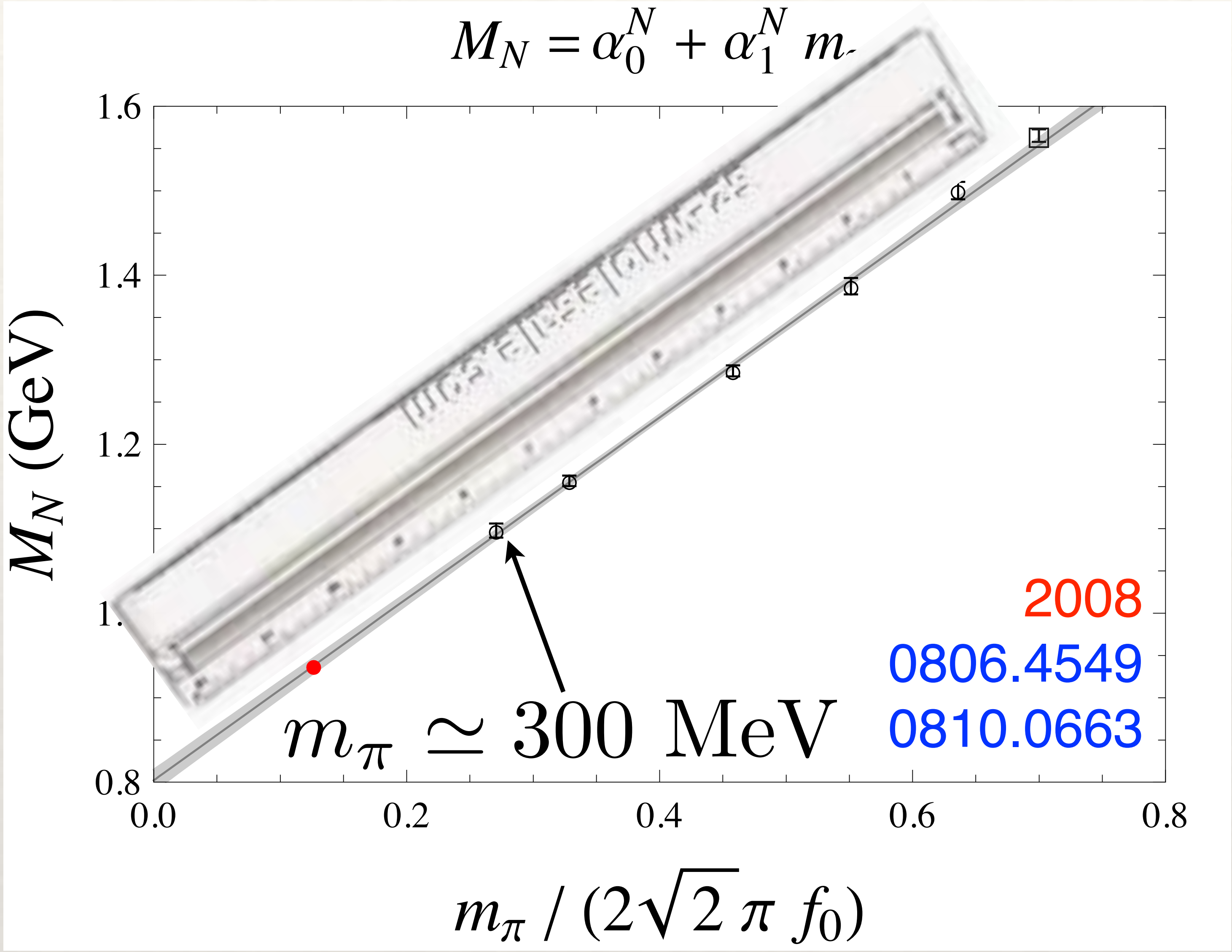
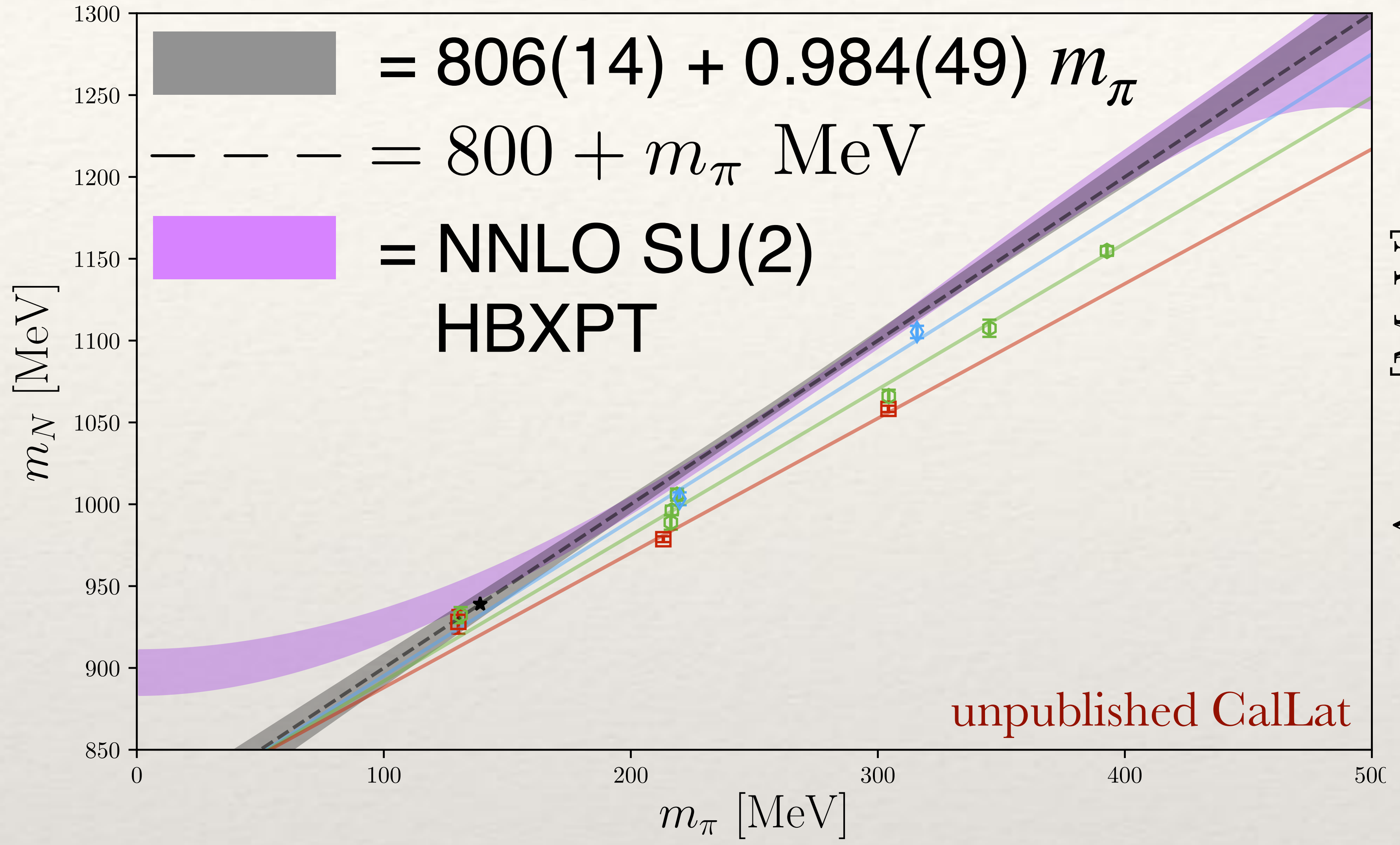
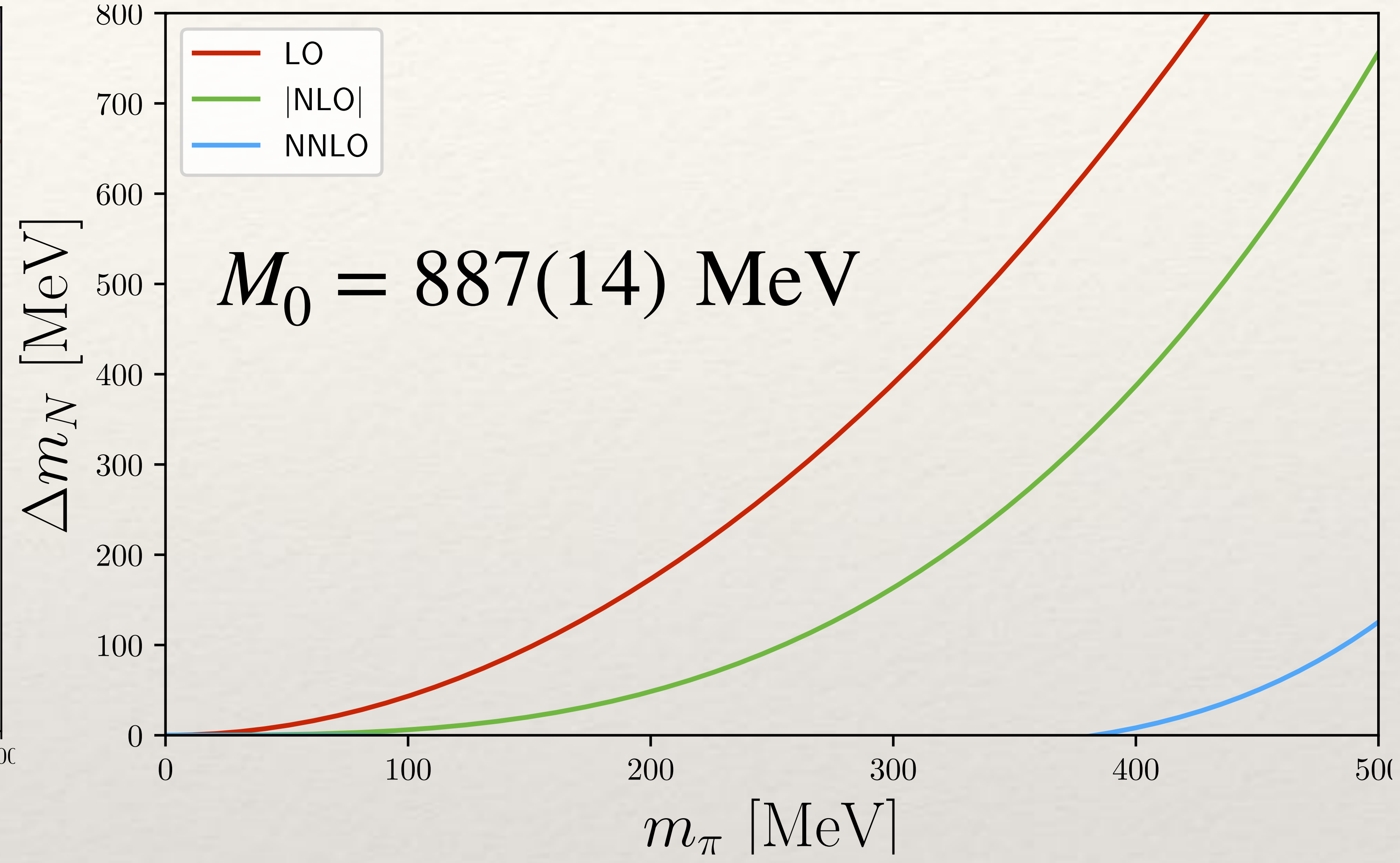






- The theory still is marginally convergent for $m_\pi \lesssim 300 - 400$ MeV and reasonably convergent for $m_\pi \lesssim 200$ MeV
- The nucleon mass exhibits bizarre linear in m_π behavior which arises from competition between different orders in the expansion - oscillating contributions

LQCD and χPT / NN EFT: πN scattering

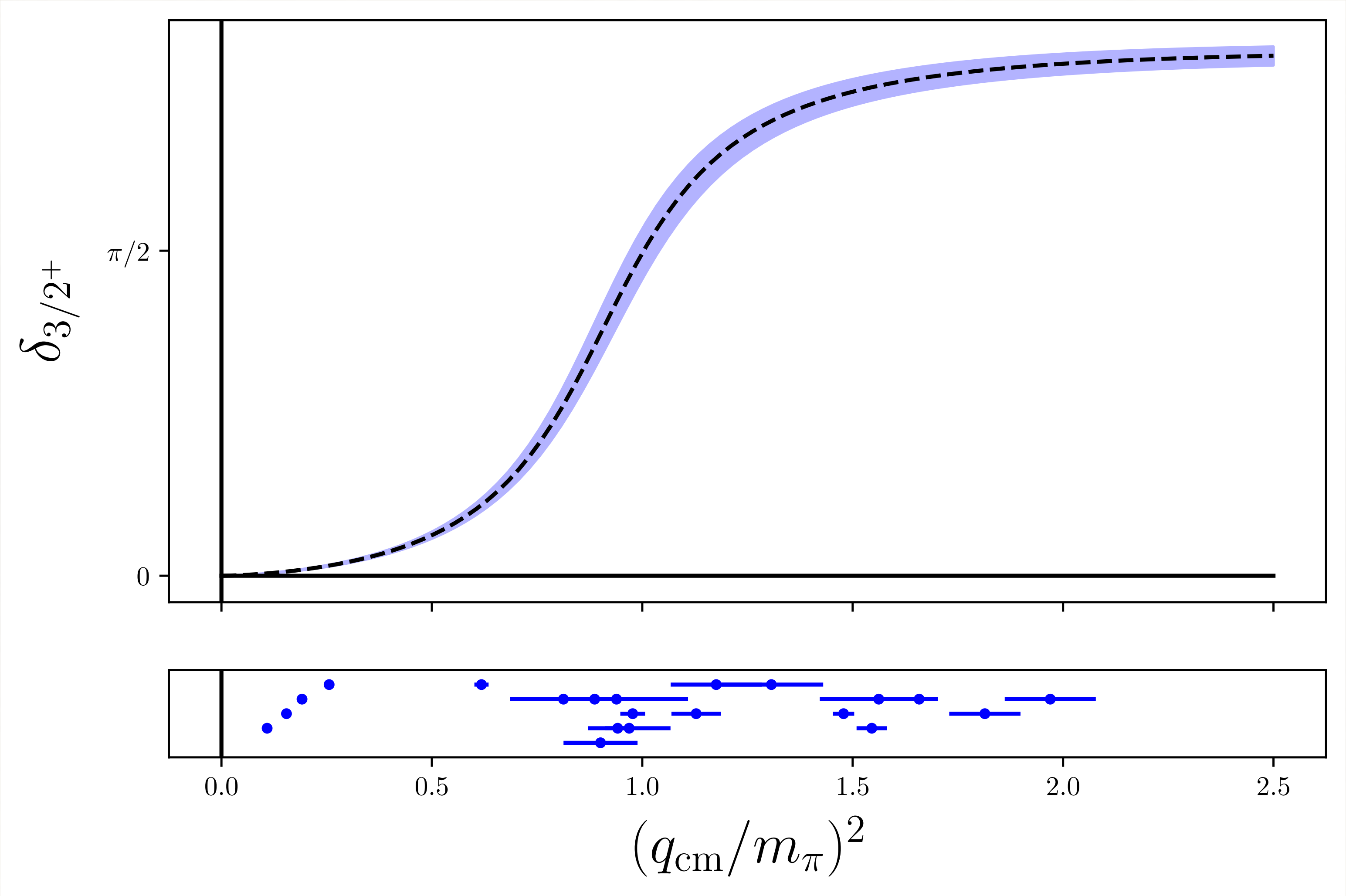
arXiv > hep-lat > arXiv:2208.03867

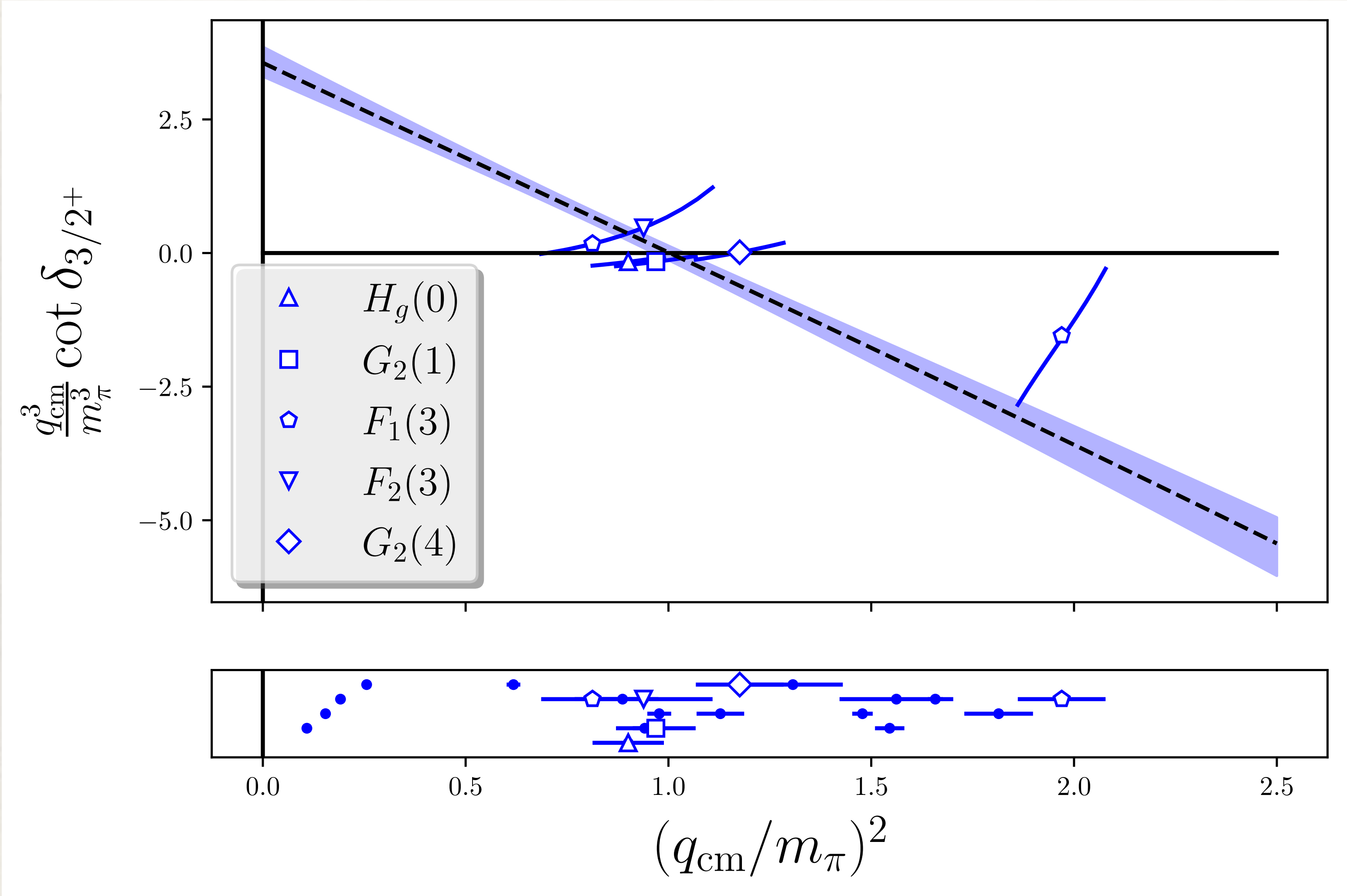
High Energy Physics – Lattice

[Submitted on 8 Aug 2022 (v1), last revised 7 Feb 2023 (this version, v3)]

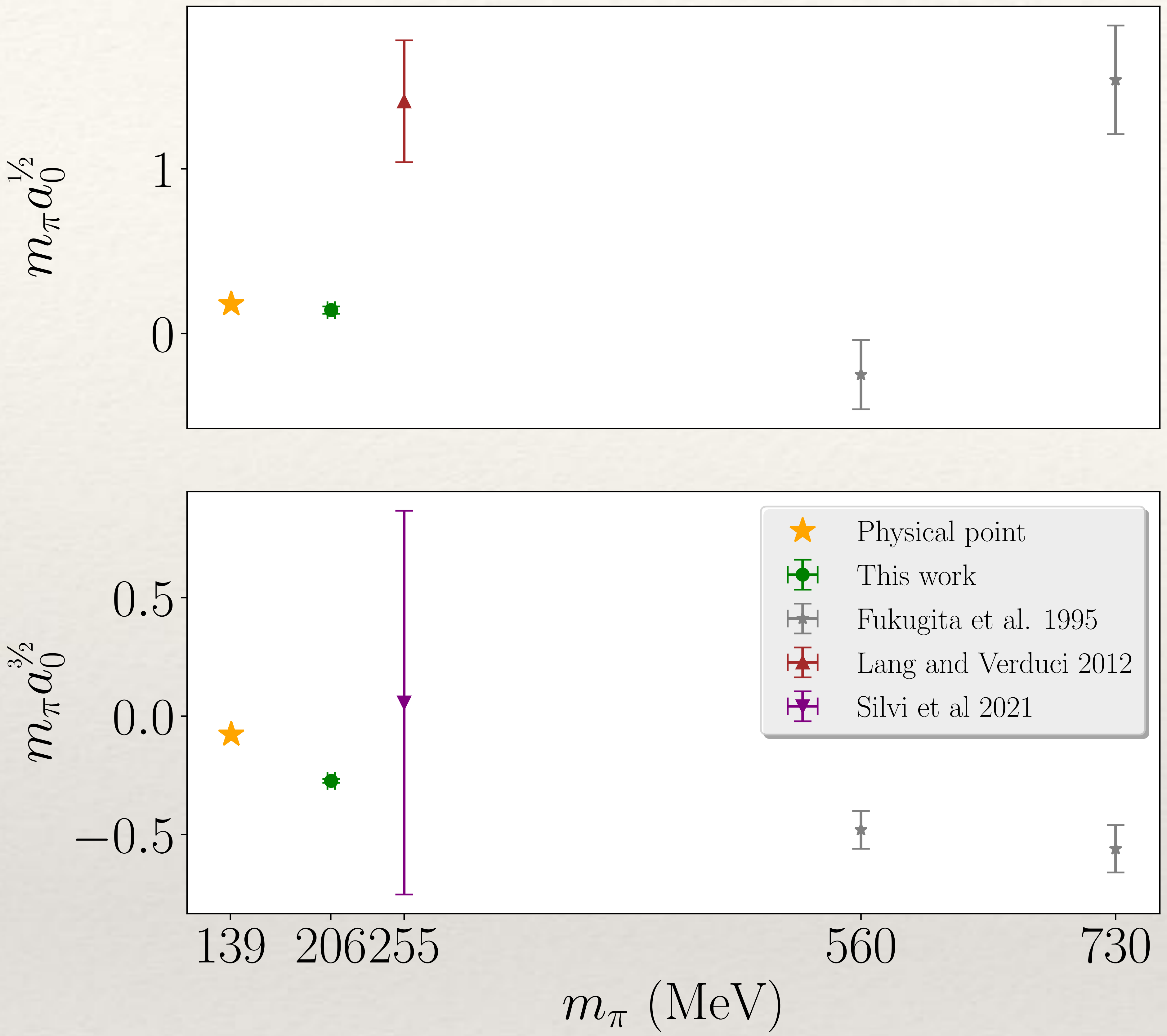
Elastic nucleon-pion scattering at m_π = 200 MeV from lattice QCD

John Bulava, Andrew Hanlon, Ben Hörz, Colin Morningstar, Amy Nicholson, Fernando Romero-López, Sarah Skinner, Pavlos Vranas, André Walker-Loud Nucl. Phys. B 987 (2023) 116105 [2208.03867]





LQCD and χPT / NN EFT: πN scattering [2208.03867]



- the qualitative nature of the scattering lengths seems to change dramatically with a seemingly small change in m_π
  - the magnitude of $m_\pi a_0^{3/2}$ becomes larger

- trouble with SU(2) baryon XPT?

	m_π (MeV)	$m_\pi a_0^{1/2}$	$m_\pi a_0^{3/2}$
This work	200	0.142(22)	−0.2735(81)
LO χPT	200	0.321(04)(57)	−0.161(02)(28)
LO χPT	140	0.159(02)(19)	−0.080(01)(10)
Pheno. (isospin limit)[27]	140	0.1788(38)	−0.0775(35)

LQCD and χPT / NN EFT: g_A

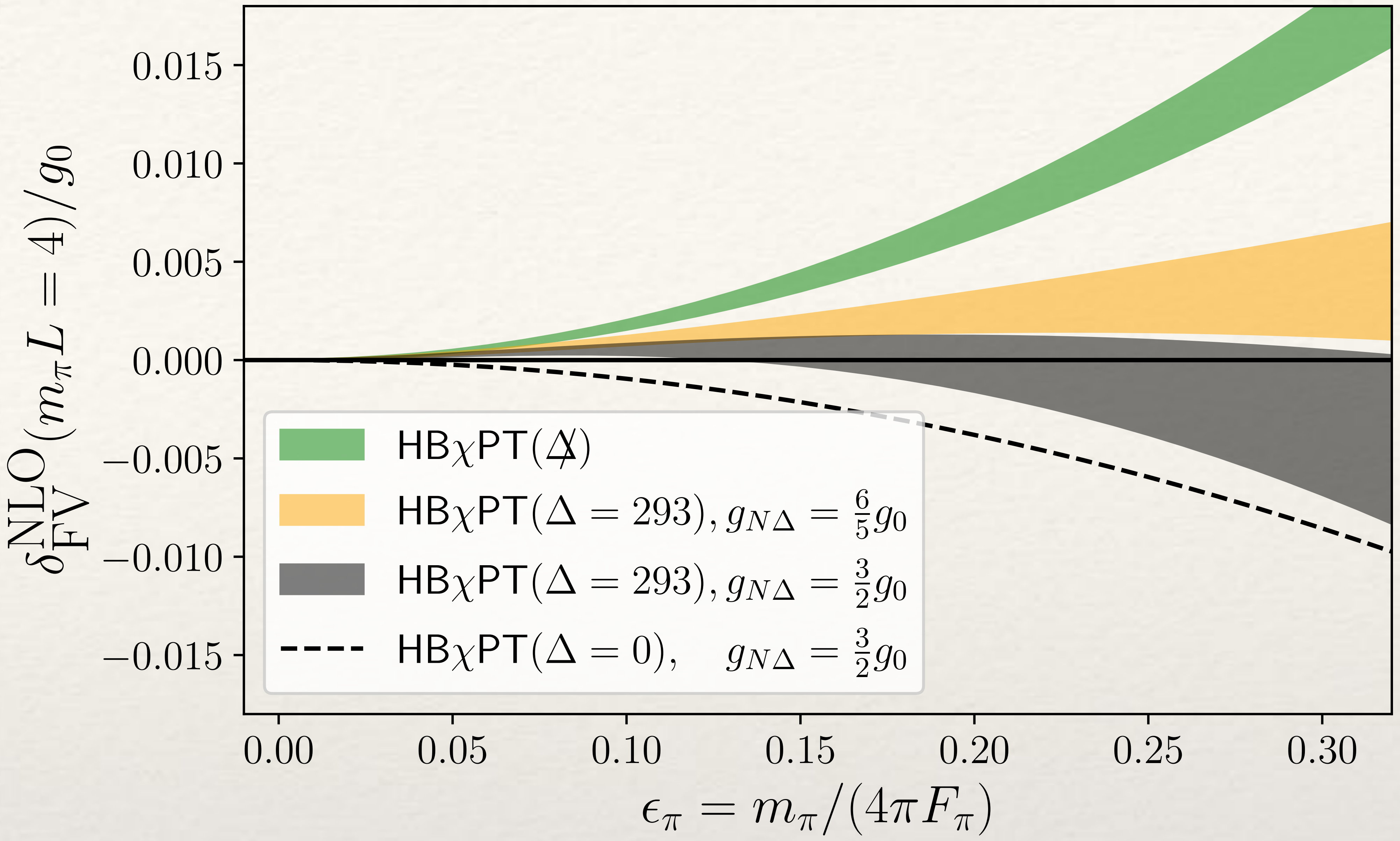
- 1805.12130, 1912.0821, 2503.09891

- Comparing LQCD predictions of g_A to PDG can help constrain BSM right-handed charged currents (RHCC)
  - This is interesting as BSM RHCC are favored for resolving first-row CKM unitarity tension
    Cirigliano, Dekens, de Vries, Mereghetti, Tong JHEP 03 (2024) [2311.00021]
  - But - there is a unknown QED correction to g_A that may be as large as 2%
    Cirigliano, de Vries, Hayen, Mereghetti, Walker-Loud, PRL 129 (2022) [2202.10439]

- g_A seems to have a particularly poor convergence pattern with SU(2) HBχPT(Δ)

LQCD and χPT / NN EFT: g_A

- 1805.12130, 1912.0821, 2503.09891 - Finite Volume corrections
- large N_c predicts the virtual nucleon and Δ loops cancel
  this holds for both infinite and FV corrections
  [Jenkins & Manohar, +Flores-Mendieta, …]
- Not using explicit Δ means NNLO contributions have to be large to account for implicit Δ corrections
- Fitting the numerical results leads to non-convergent expansion



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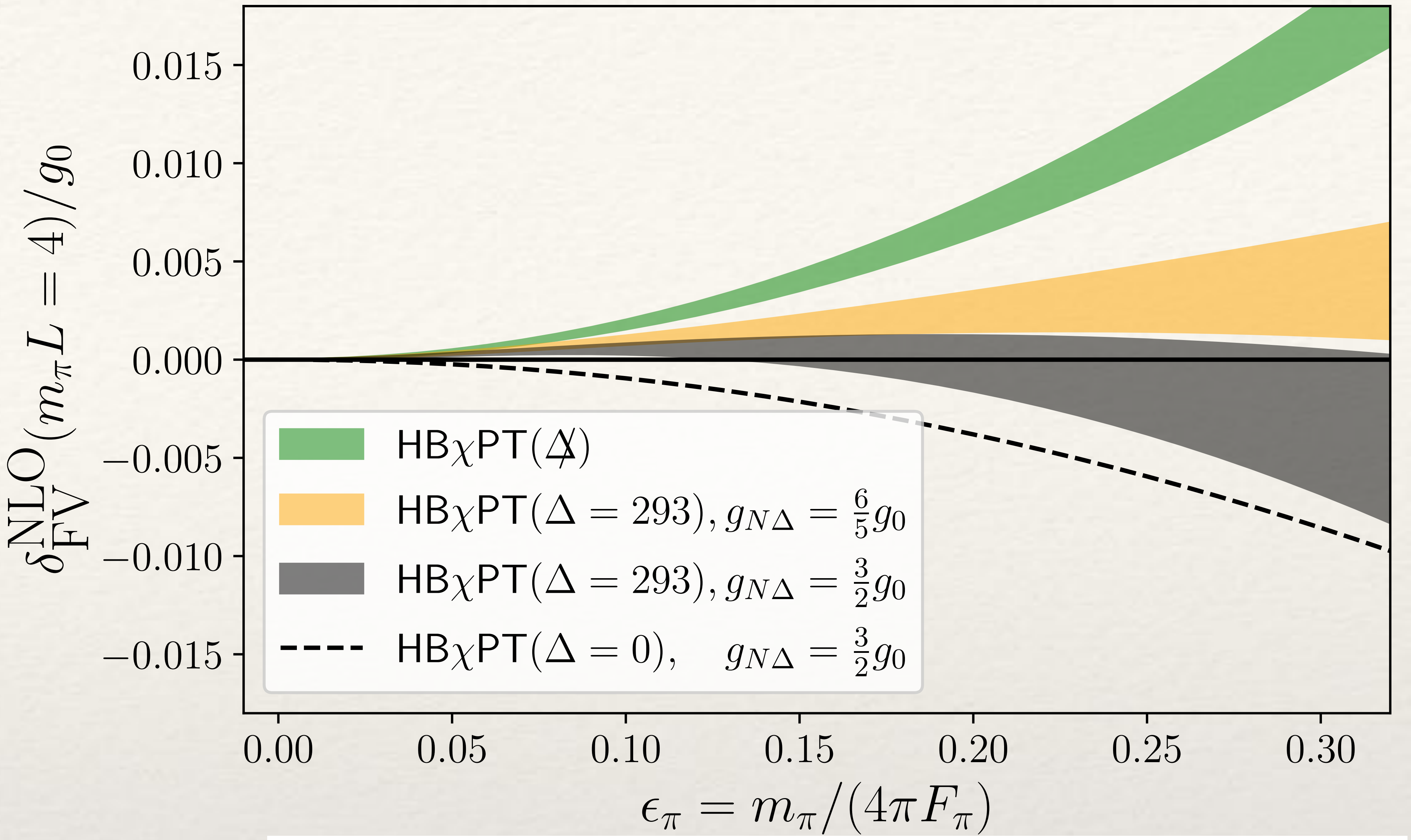

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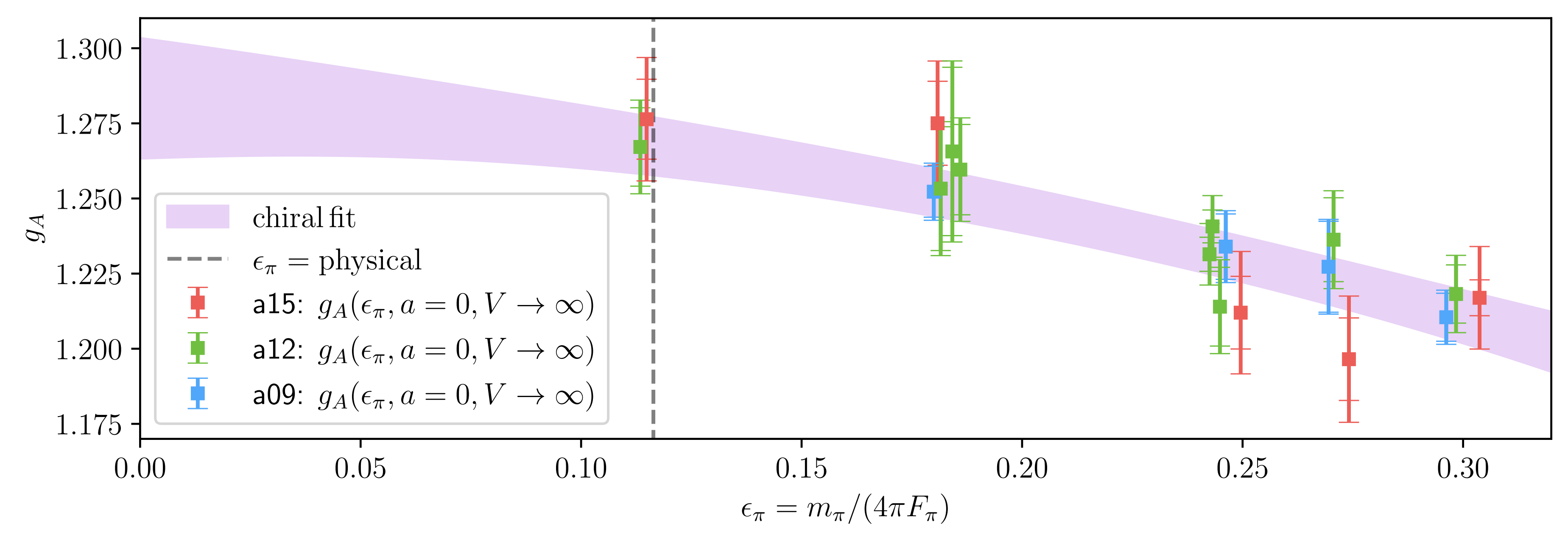
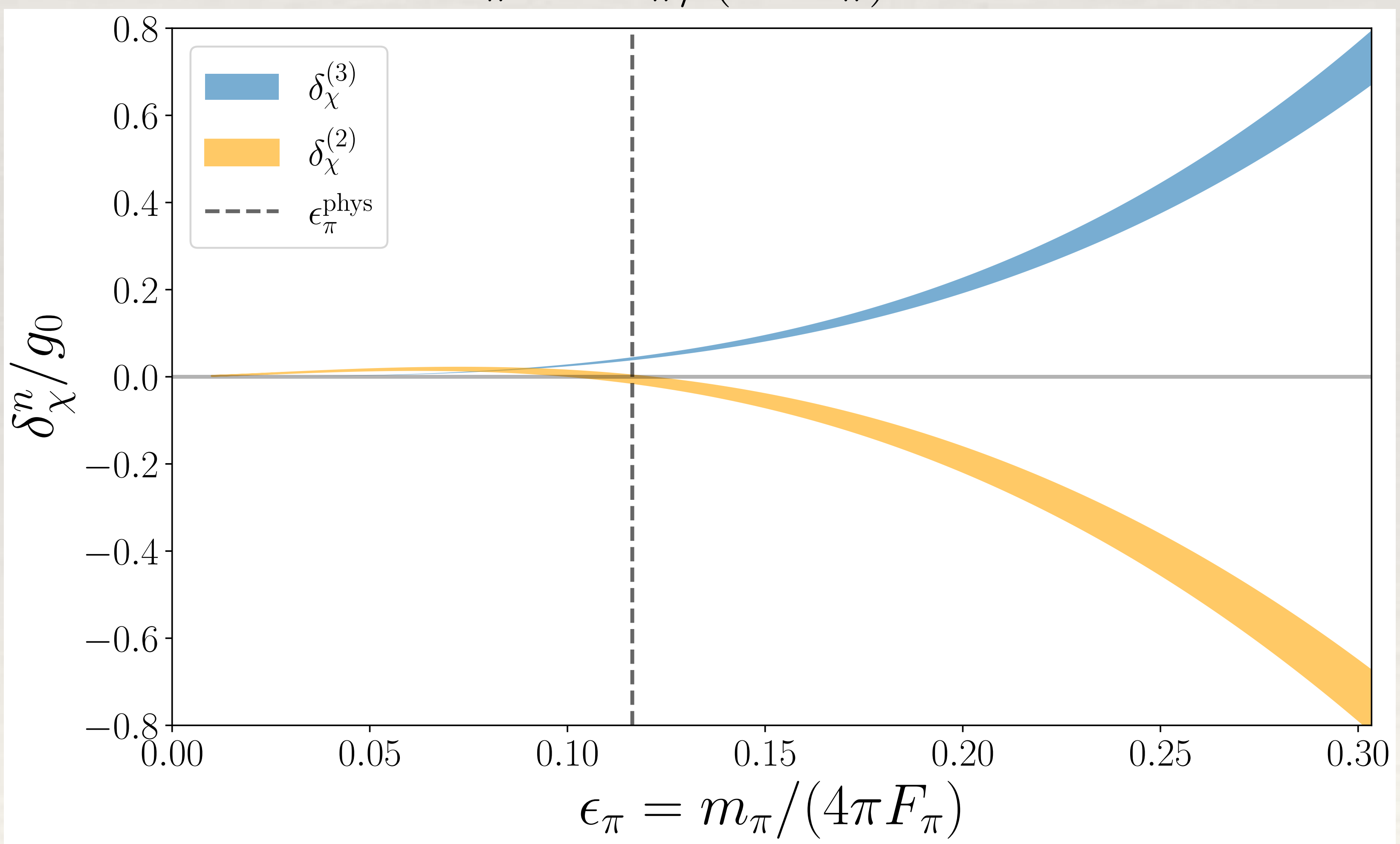


$$g_A = g_0 + \delta^{(2)}_\chi + \delta^{(3)}_\chi$$

$$\delta^{(2)}_\chi = \epsilon_\pi^2\left[-g_0(1+2g_0^2)\ln\epsilon_\pi^2 + 4\tilde{d}^r_{16} - g_0^3\right],$$

$$\delta^{(3)}_\chi = \epsilon_\pi^3 g_0 \frac{2\pi}{3}\left[3(1+g_0^2)\frac{4\pi F}{M_0} + 4(2\tilde{c}_4 - \tilde{c}_3)\right]$$

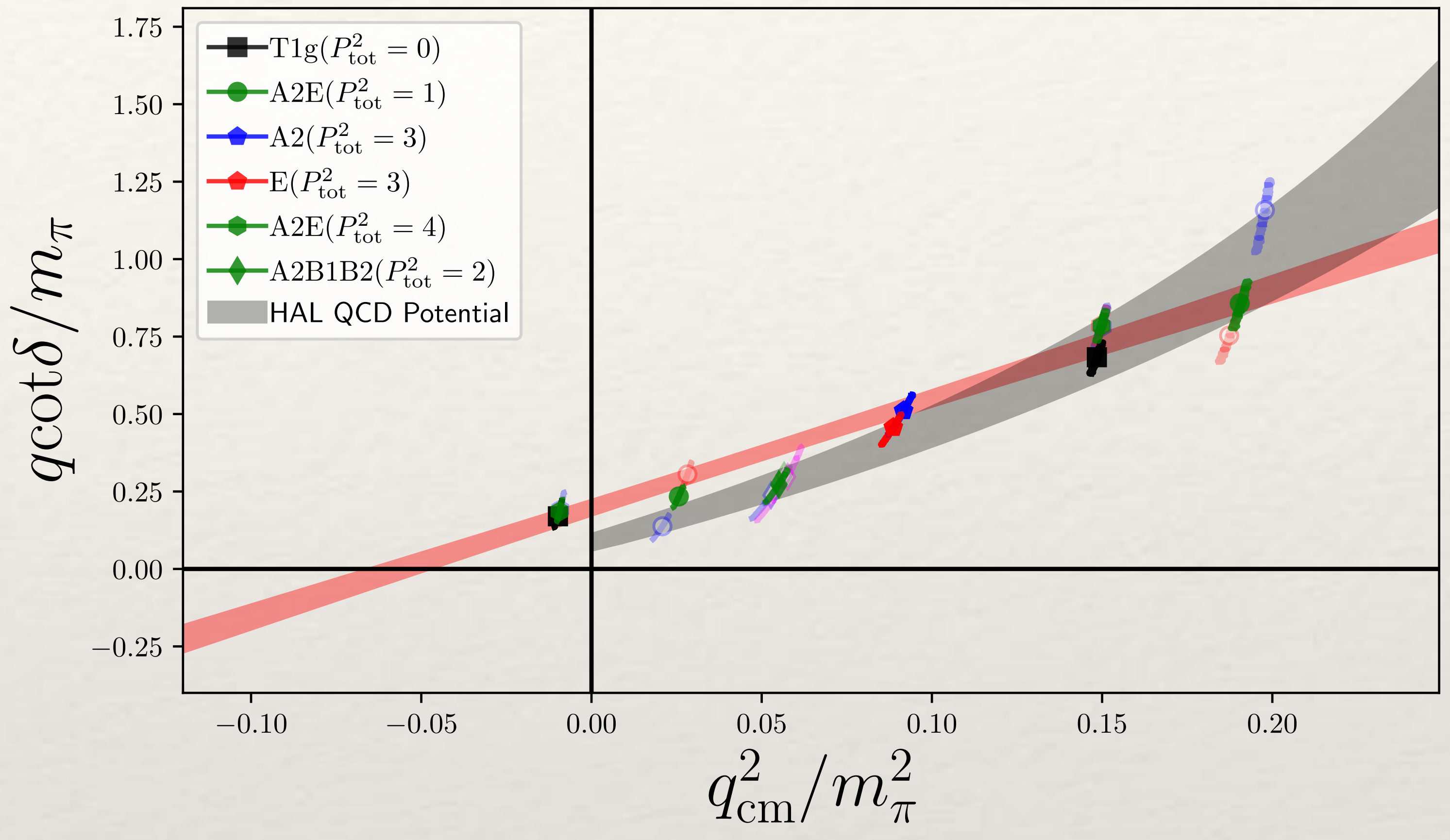


$\delta^{(n)}_\chi(\epsilon_\pi^{\rm phys})$	g_0	$\delta^{(2)}_\chi$	$\delta^{(3)}_\chi$
	1.226(15)	−0.007(11)	0.0507(53)

source	g_0	$g_0(1+2g_0^2)$	$4\tilde{d}^r_{16} - g_0^3$	$\delta^{(3)}_\chi/\epsilon_\pi^3$
Phenomenology	1.2(1)	4.7(1.0)	–	165(16)
LQCD analysis	1.22(2)	4.9(0.2)	−21(2)	32(3)

Outlook

- Di-nucleons do not form bound states at heave pion mass
  - The use of off-diagonal correlators seems to lead to false plateaus in the effective energy
  - The HAL QCD potential produces results that are qualitatively consistent with standard FV method Better quantitative agreement can be explored with
    - higher order terms in the potential
    - continuum extrapolation
- NN calculations suffer from excited state contamination due in part to lack of improved single nucleon operators
  - We introduced the *conspiracy model* to capture these effects



- In order to have impact on the physics program, we need to do NN calculations with $m_\pi \lesssim 200$ MeV
  - These are underway and first results will hopefully be ready this year
- The h-dibaryon exhibits large discretization corrections in the binding energy
  - This result is very action dependent and warrants further study
- To get to nuclei, LQCD can provide LECs of the N and NN sector
  - Growing evidence that explicit Δ degrees of freedom necessary to have a converging EFT

Gracias

Collaborators

CoSMoN
(Connecting the Standard Model to Nuclei)
(postdoc, grad student, undergrad)

John Bulava	Bochum
Kate Clark	NVIDIA
Zack Hall	LBNL
Andrew Hanlon	Kent State University
Jinchen He	University of Maryland College Park
Ben Hörz	INTEL
Bálint Joó	NVIDIA
Aaron Meyer	Lawrence Livermore National Laboratory/NTN
Henry Monge-Camacho	Oak Ridge National Laboratory
Colin Morningstar	Carnegie Mellon University
Joseph Moscoso	University of North Carolina Chapel Hill
Amy Nicholson	University of North Carolina Chapel Hill
Fernando Romero-López	Bern
Sarah Skinner	Carnegie Mellon University
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NNSA National Nuclear Security Administration
NSF National Science Foundation