PRESENTATION

EFT of Chiral Gravitational Waves

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(**Ochanomizu U.** & Kavli IPMU)

Aoki, TF, Kawaguchi & Yanagihara
arXiv:2504.19059

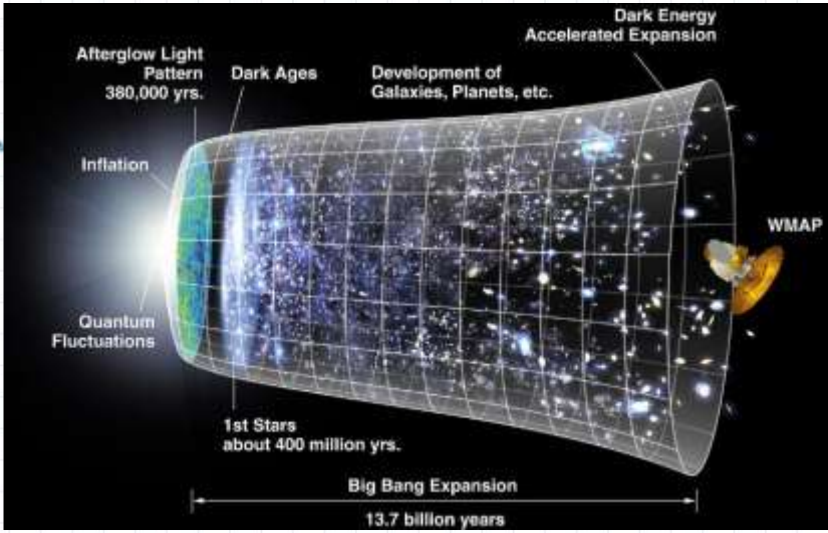
30th. Apr. 2025 @Benasque


The map is ready — enjoy the trails and find the deeper landscape of chiral gravitational waves.
$$\mathcal{L}^{(2)} = M_{gg}^4(\tau)(\delta g^{00})^2 + M_{gX}^2(\tau)\delta g^{00}\delta X + M_{gY}^2(\tau)\delta g^{00}\delta Y + \frac{1}{M_{XX}^4(\tau)}\delta X^2 + \frac{1}{M_{XY}^4(\tau)}\delta X\delta Y + \frac{1}{M_{YY}^4(\tau)}\delta Y^2 + \frac{1}{\tilde{M}_{XX}^4(\tau)}\delta X_{\mu\nu}\delta X^{\mu\nu} + \frac{1}{\tilde{M}_{XY}^4(\tau)}\delta X_{\mu\nu}\delta Y^{\mu\nu} + \frac{1}{\tilde{M}_{YY}^4(\tau)}\delta Y_{(\mu\nu)}\delta Y^{(\mu\nu)} + \frac{1}{\bar{M}_{YY}^4(\tau)}\delta Y_{[\mu\nu]}\delta Y^{[\mu\nu]} + \cdots ,$$
EFT Action

Outline of Talk

1. Background
2. Model Results
3. EFT Action
4. Tensor Perturbation
5. Summary & Future Work

Inflation



Inflation = era of exponential cosmic expansion

- $a(t) = a_0 e^{Ht}$: scale factor exponentially grows
- $H \simeq const$: H characterizes inflation scale

What causes inflation?

2 Dilemma of Inflation

For successful inflation, we need...

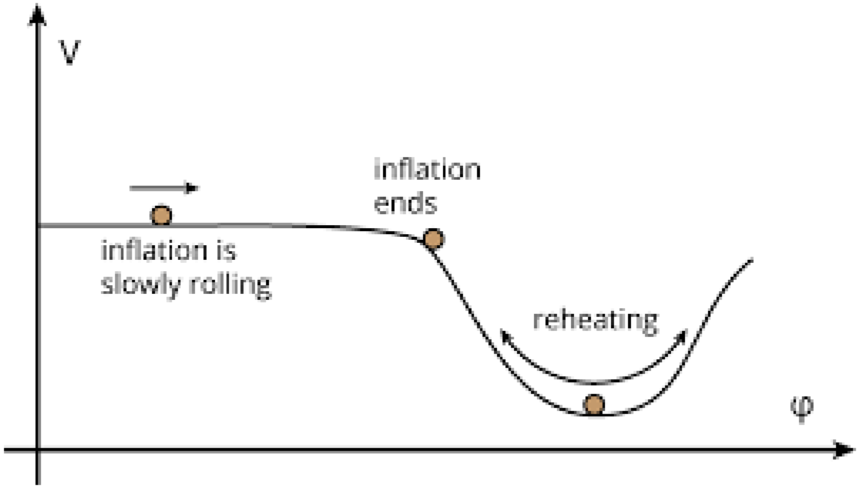
- **Exponential expansion (at least, $a_e/a_i > e^{60}$)**

  $\simeq$ Cosmological constant Λ

- **Inflation ends, and thermal plasma appears**

  Λ is dynamical (ϕ) and decays into particles

→ Slow-roll inflation
(fine-tuning?)



Natural (Axion) inflation

Freese, Frieman & Olinto (1990)

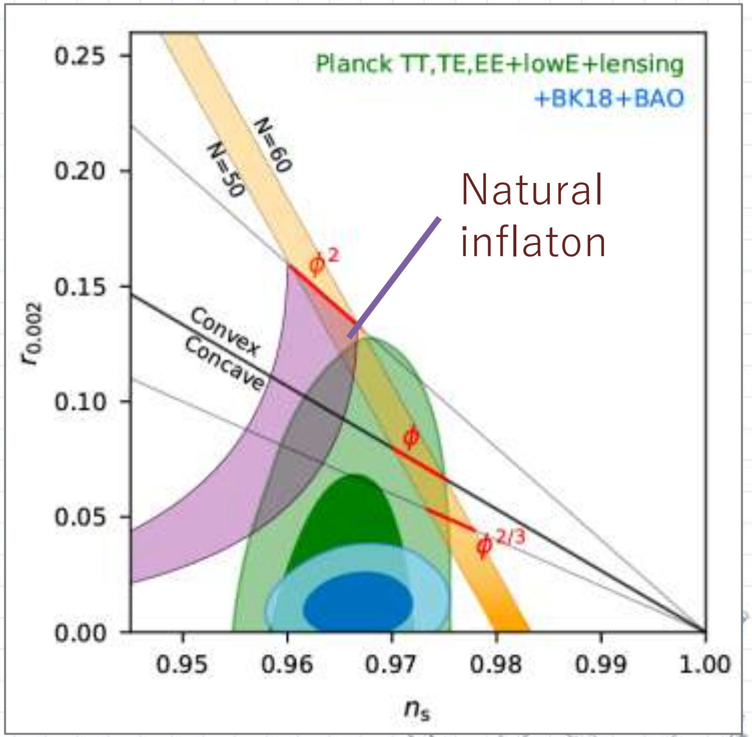
$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

Natural inflation

Inflaton = Axion

Shift symmetry ($\phi \rightarrow \phi + \mathrm{const.}$) protects V from radiative correction

BICEP/Keck [2110.00483]



Issues

Reheating? ⇒ Needs coupling!

Too large decay constant $f > \mathrm{M_{Pl}}$.

Doesn't fit in the ns-r plane

Coupling in Axion inflation

Anber & Sorbo (2006)

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}FF - \frac{\lambda}{4f}\phi F\tilde{F}$$

Axionic inflaton

Gauge field coupled to ϕ

Coupling to gauge fields

Shift symmetry allows Chern-Simons coupling

The **kinetic energy** of the inflaton transfers to the gauge sector.

⇒ Effective friction leads to Slow-roll inf.

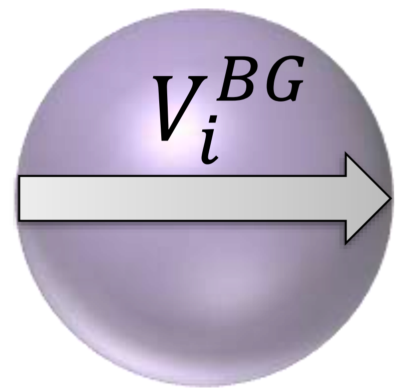
1 Coupling in Axion inflation

Anber & Sorbo (2006)

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}FF - \frac{\lambda}{4f}\phi F\tilde{F}$$

Axionic inflaton | Gauge field couple

Isotropy is violated



Coupling to gauge fields

Shift symmetry allows Chern-Simons coupling

The **kinetic energy** of the inflaton transfers to

→ Effective friction leads to Slow-r

U(1) gauge field?

Yes, very interesting. But Not in this talk.

See however, triplet-U(1) models
e.g. Gaugid inflation [Piazza+(2017)]

Because it cannot have an **isotropic background**.

Chromo-natural inflation

Adshead & Wyman (2012)

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial\phi)^2 - V(\phi)}_{\text{Axionic inflaton}} \underbrace{- \frac{1}{4}FF - \frac{\lambda}{4f}\phi F\tilde{F}}_{\text{SU(2) gauge field coupled to } \phi}$$

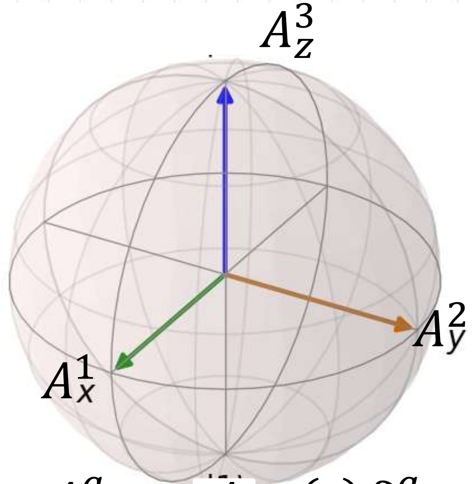
- **Inflaton ϕ couples to SU(2) gauge field**

  ϕ's kinetic energy ➡ SU(2) vev

  Backreaction ➡ ϕ slows down

- **Isotropic background of SU(2)**

  Rotationally invariant VEV is realized as an **attractor solution**.



$$A_i^a = a\bar{A}_{BG}(t)\delta_i^a$$

Gauge-flation

Maleknejad & Shikh-Jabbari (2011)

$$\mathcal{L} = -\frac{1}{4}FF - \frac{1}{16M^4}\left(F\tilde{F}\right)^2$$

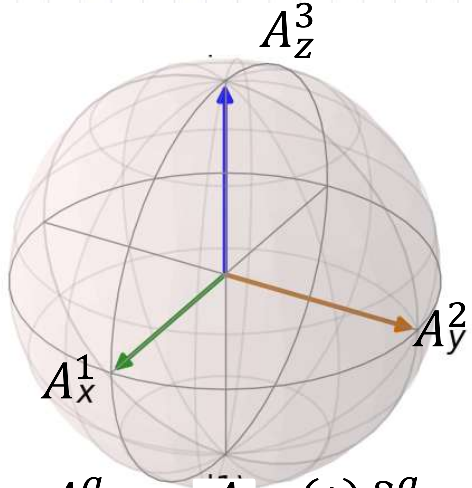
SU(2) gauge field only

Isotropic background of SU(2)

Inflation without a scalar field.
The same istropic SU(2) VEV

EFT of chromo-natural

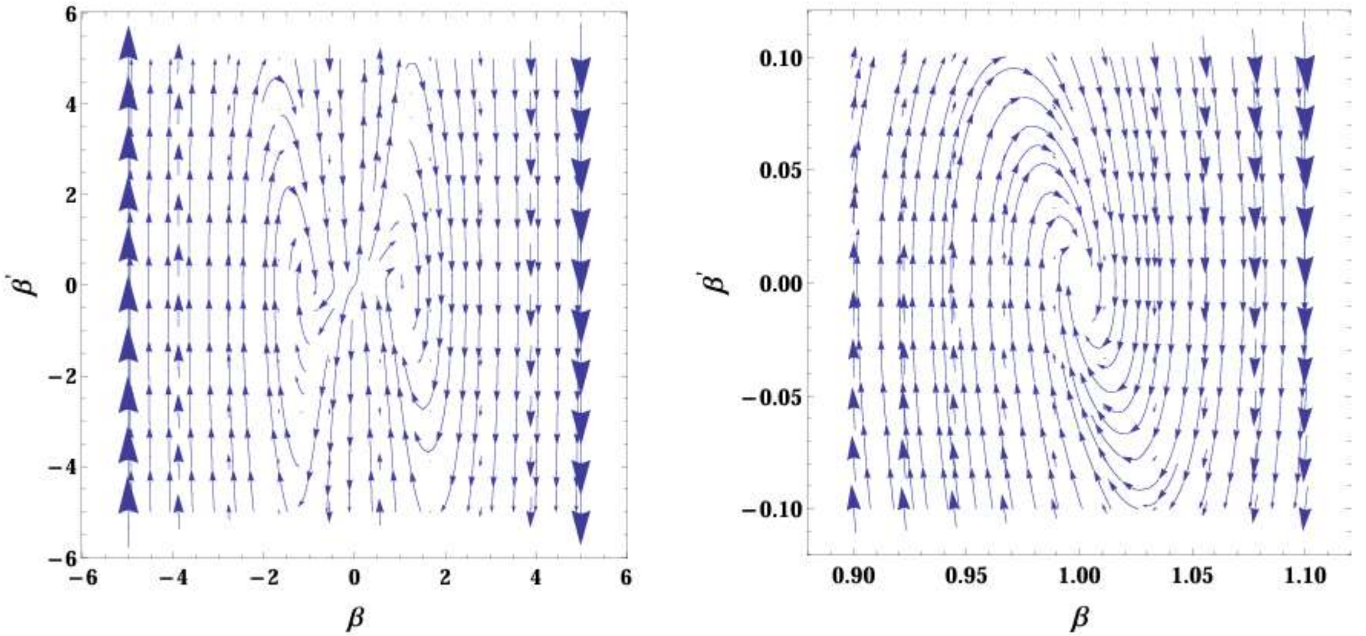
Integrating out the axion in chromo-natural,
you obtain the above action.

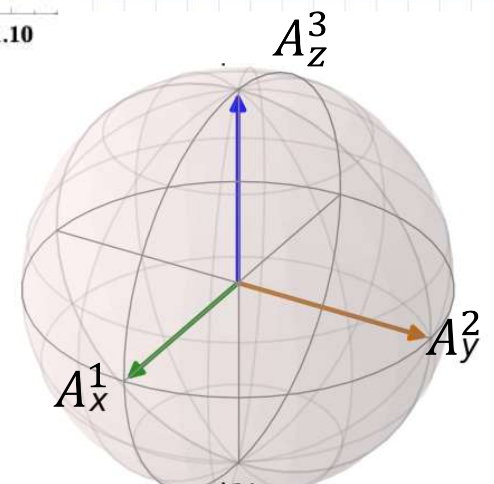


$$A_i^a = a\bar{A}_{BG}(t)\delta_i^a$$

Attractor

Maleknejad & Erfani (2014)

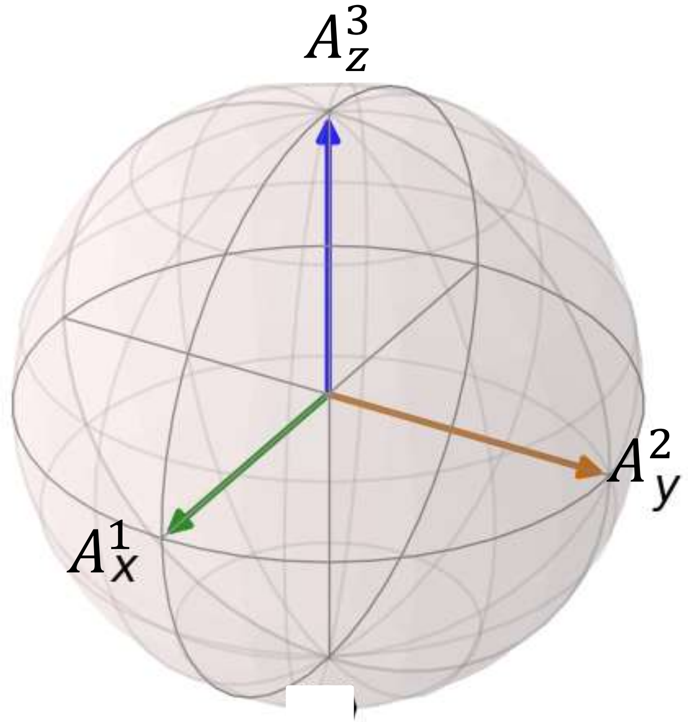




$$A_i^a = a\bar{A}_{BG}(t)\delta_i^a$$

Irrespective of initial condition of SU(2) gauge field, this isotropic VEV is realized!

Isotropic SU(2) Background



$$A_i^a = aA_{BG}(t)\delta_i^a$$

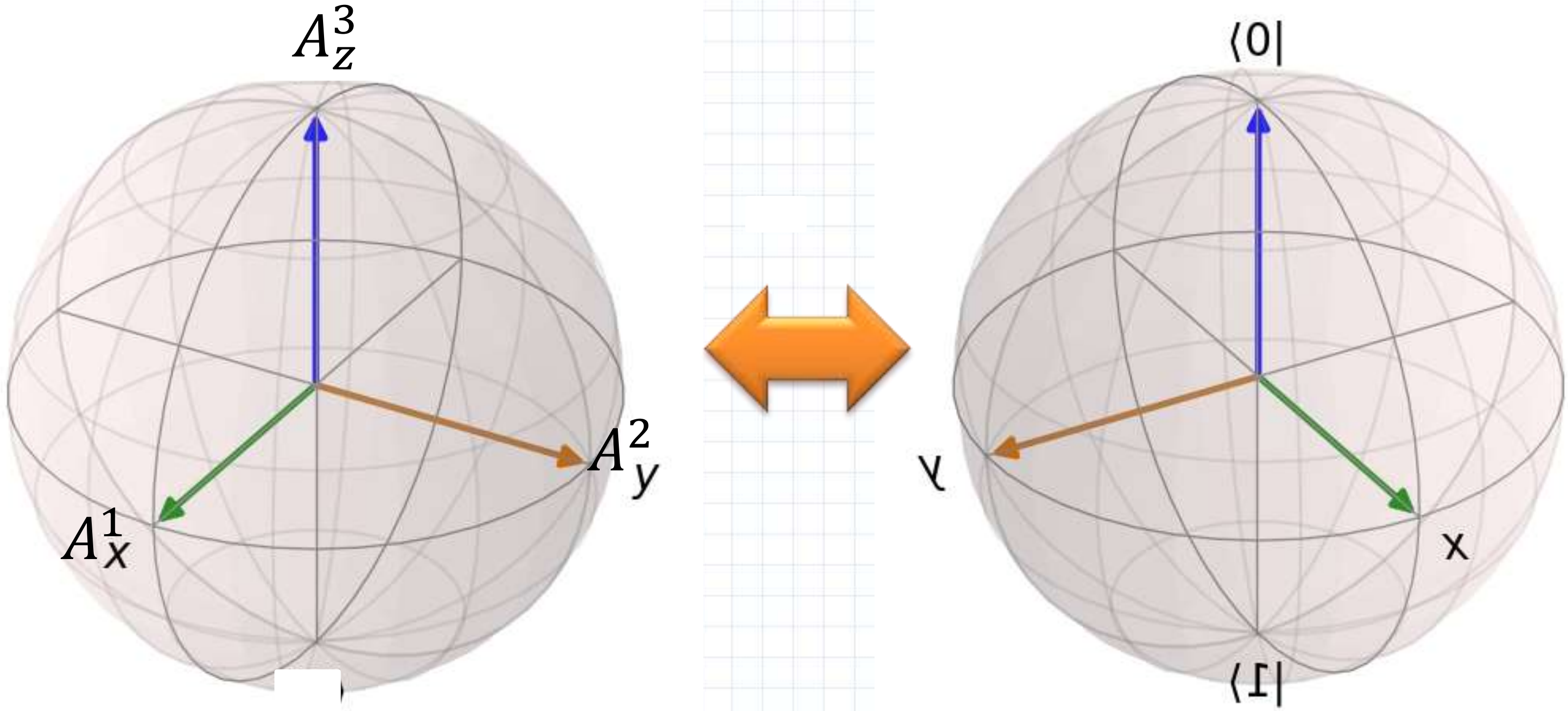
$$R_{ij}E_i^a = G^{ab}E_i^a$$

$$R_{ij}B_i^a = G^{ab}B_i^a$$

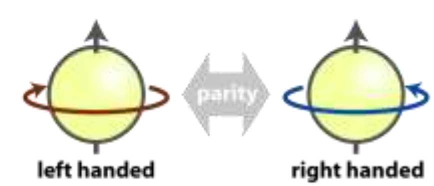
- **Rotation SO(3) and gauge SU(2) are mixed (locked)**

$$SU(2)\times SO(3) \rightarrow SO(3)$$

Isotropic SU(2) Background



- Rotation SO(3) and gauge SU(2) are mixed (locked)
- It breaks parity (mirror) symmetry



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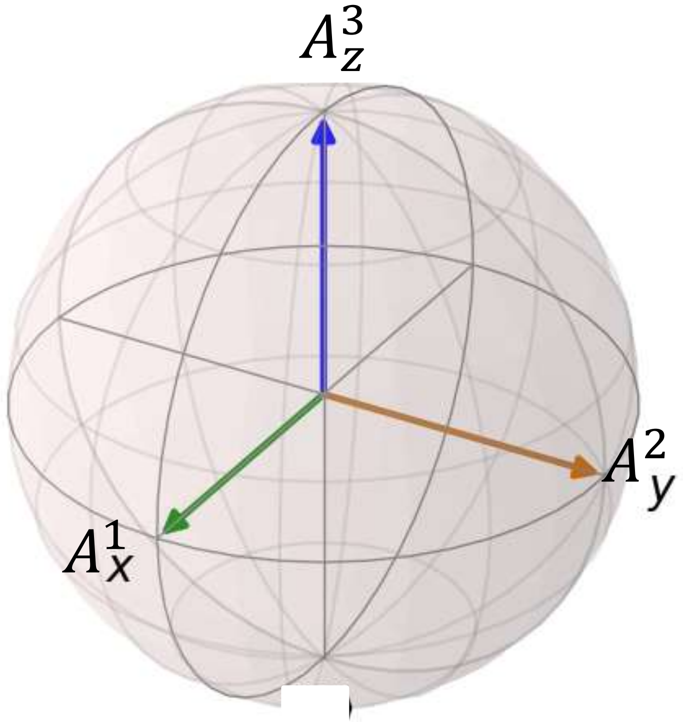
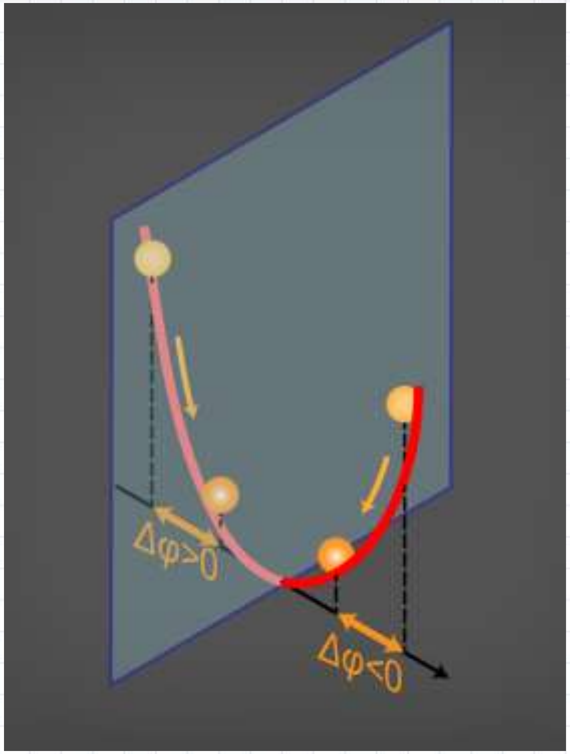
Istropic SU(2) Background





- Rotation SO(3) and gauge SU(2) are mixed (locked)
- It breaks parity (mirror) symmetry

General Gauge theory

[TF, Mukaida, Murai, Nakatsuka (2021)]

- String people say Axion is ubiquitous

  "Axiverse" ⇒ Maybe couple to extended symmetry

- We found that even in SU(N)

  a **SU(2) subgroup** acquires the same vev.

- Isotropic vev : **Spontaneous symmetry breaking**

  $SU(2)\times SO(3) \to SO(3)$

- Selected SU(2) subgroup (SSB pattern)
  characterizes the property of solutions

New model

[TF, Mukaida, Murai, Nakatsuka (2021)]

- Extended Model: apparently the **same** action

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}FF - \frac{\lambda}{4f}\phi F\tilde{F}$$

Axionic inflaton

SU(N) gauge field coupled to ϕ

- Three assumptions (homogeneous, static, parallel EB) lead to **a stable solution** for the gauge field

$$A_i = \sigma\mathcal{J}_i \quad \left(\begin{matrix} A_0 = 0 \\ i = x, y, z \end{matrix}\right)$$

SU(2) sub-algebra $\mathcal{J}_i = n_i^a T^a$

- Which SU(2) subgroup? ➡ One parameter λ

$\xi \equiv \dot{\phi}/2fH$

Commutation relation $[\mathcal{J}_i, \mathcal{J}_j] = i\lambda\epsilon_{ijk}\mathcal{J}_k$

VEV: $\sigma = \left(\xi + \sqrt{\xi^2 - 4}\right)/2\lambda$

Example : SU(3) & SU(4)

group	subgroup	N	N^2-1	m
SU(3)	SU(2)×U(1)	$\mathbf{2}_{-1}+\mathbf{1}_2$	$\mathbf{3}_0+\mathbf{2}_3+\mathbf{2}_{-3}+\mathbf{1}_0$	2
	SU(2)	$\mathbf{3}$	$\mathbf{3}+\mathbf{5}$	3
SU(4)	SU(3)×U(1)	$\mathbf{3}_{-1}+\mathbf{1}_3$	$\mathbf{8}_0+\mathbf{3}_{-4}+\bar{\mathbf{3}}_4+\mathbf{1}_0$	-
	SU(2)	$\mathbf{4}$	$\mathbf{3}+\mathbf{5}+\mathbf{7}$	4
	SU(2)×SU(2)	$(\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{2})$	$(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{3})+(\mathbf{2},\mathbf{2})+(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{1})$	-
		$(\mathbf{2},\mathbf{2})$	$(\mathbf{3},\mathbf{3})+(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{3})$	-

[cf. Caldwell & Devulder(2018)]

Two in SU(3), **Four** in SU(4),
different choices (solution) of SU(2) subgroup

3 Effective Potential

$$V_{\text{eff}}(\sigma) = \frac{1}{2}\sigma^2 - \frac{1}{3}\xi\lambda\sigma^3 + \frac{1}{4}\lambda^2\sigma^4$$

Static EoM:

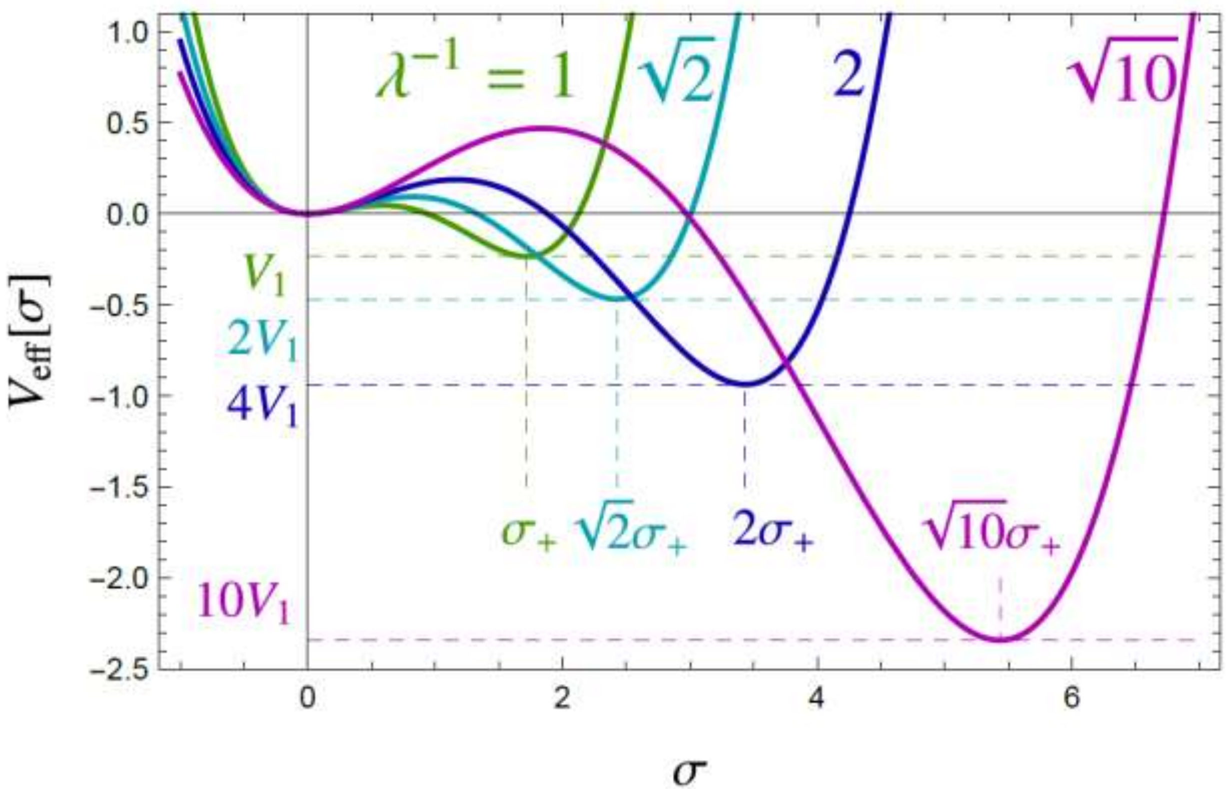
$V'_{\text{eff}} = 0$

SU(4)

N.b.

we assume

$\xi = const.$



Numerical calculation

- Numerically solve **full EoMs**

$$\ddot{M}_i^a + \frac{2}{H}\dot{M}_i^a + 2M_i^a + f^{bac}f^{bde}M_j^c M_i^d M_j^e - \xi\epsilon^{ijk}f^{abc}M_j^b M_k^c = 0.$$

- Assumptions: dS metric $a \propto e^{Ht}$ and $\xi \equiv \frac{\dot{\phi}}{2fH} = const.$

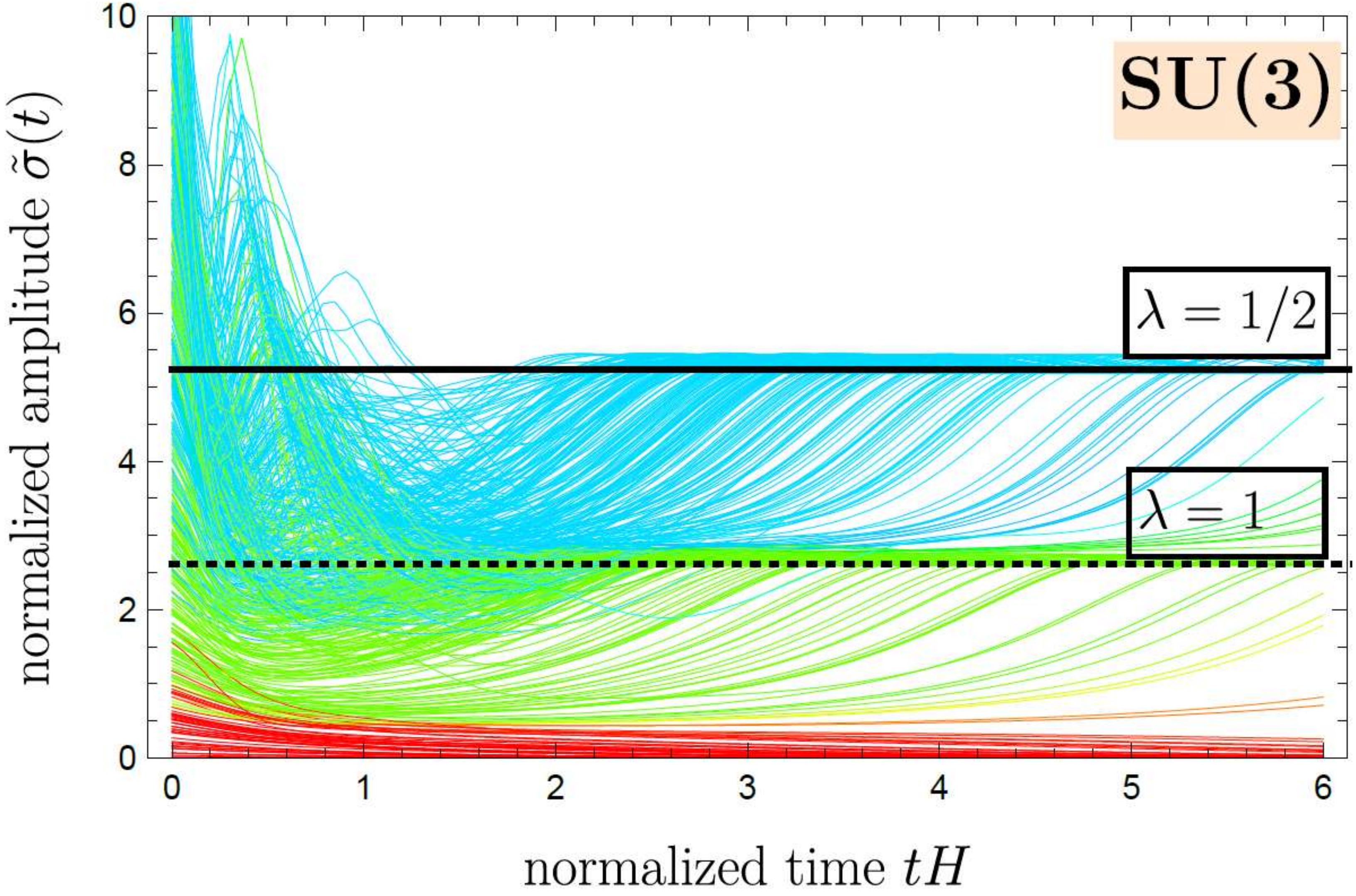
- Initial condition for $M_i^a \equiv gA_i^a/aH$

$$A_0^0 = 0,\ \dot{A}_i^a(t_0) = 0,\ \boldsymbol{M}^a(t_0) = \text{3D gaussian distribution}$$

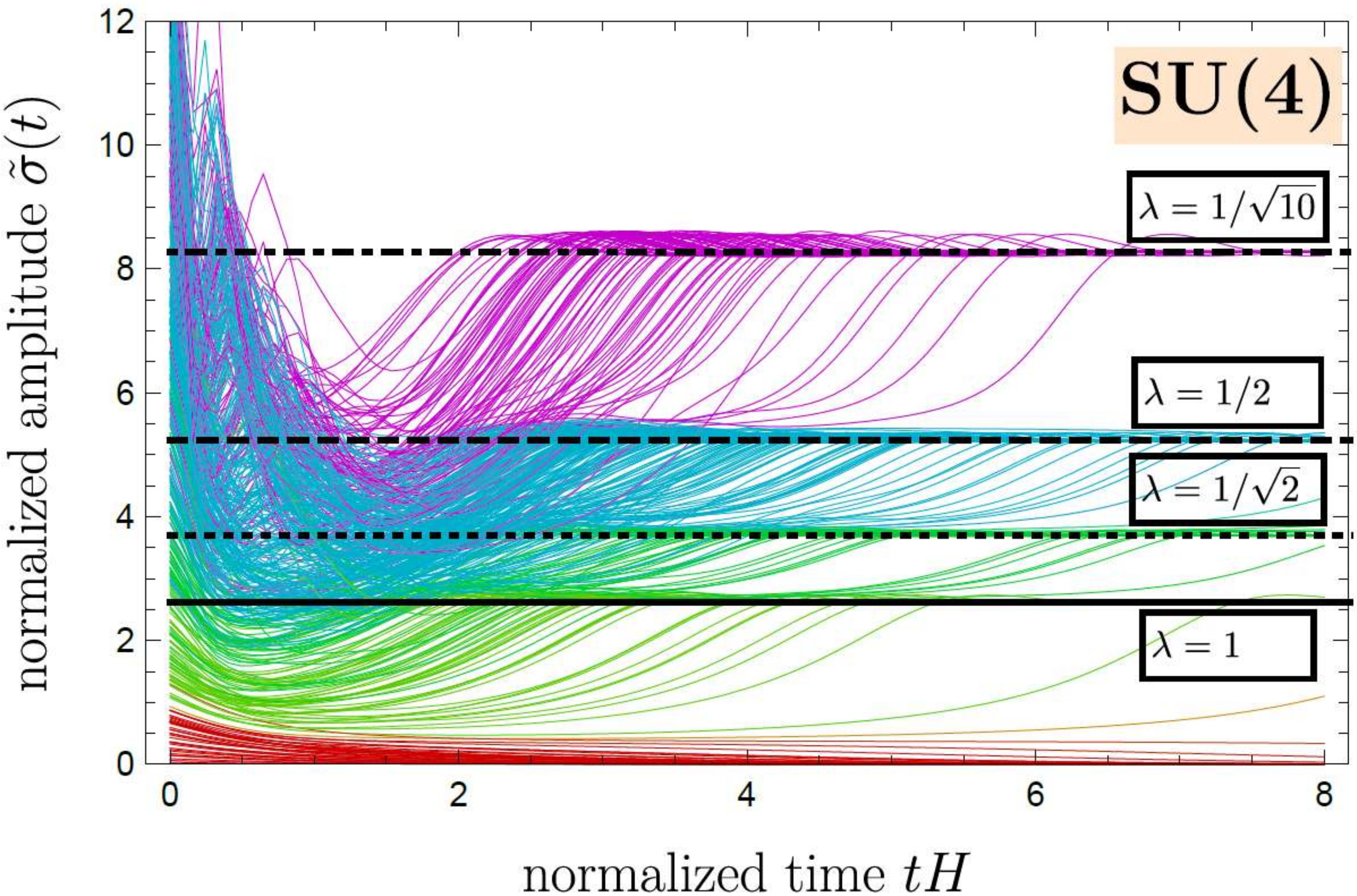
Vary the standard deviation σ within $0 \le \sqrt{N^2-1}\,\sigma \le 10$

- Plot: $\tilde{\sigma}^2(\mathrm{t}) \equiv \frac{1}{3}\mathrm{Tr}[M^2]$ We expect $\tilde{\sigma}(\mathrm{t}) \to \sigma^{SU(N)}$

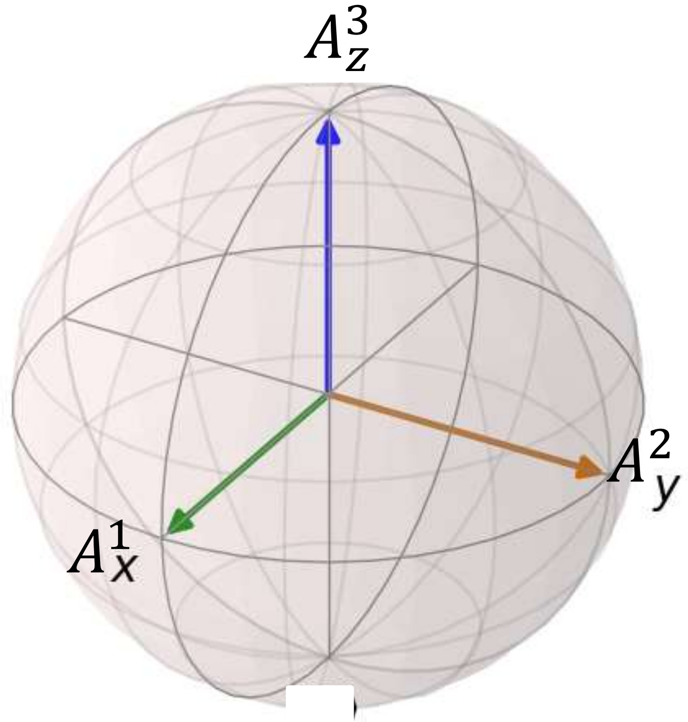
500 realizations, $\xi = 3$



500 realizations, $\xi = 3$



1 Isotropic SU(2) Background



$$A_i^a = aA_{BG}(t)\delta_i^a$$

- This background is **theoretically global attractor** for any Gauge fields coupled to rolling axion (except for U(1))

➡ **Well motivated!**

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Why SU(2)?

SVT Decomposition Theorem:

At the 1st order cosmological perturbation, scalar, vector and tensor are decoupled.

$$\delta S, \delta V_i \not\xrightarrow[\text{Source}]{} \delta T_{ij}$$

Why SU(2)?

Using the 2nd order pert., GW can be sourced.
But it's hard to generate $\mathcal{P}_{\mathrm{GW}}^{\mathrm{src}} \gg \mathcal{P}_{\mathrm{GW}}^{\mathrm{vac}}$.

$$\partial_i \delta S \partial_j \delta S,\ \delta V_i \delta V_j \xrightarrow{\text{Source}} \delta T_{ij}$$

[Biagetti et al.(2013), Mukohyama et al.(2014), TF et al.(2015),
Ferreira et al.(2015), Choi et al.(2015), Namba et al.(2016).]

Way Out

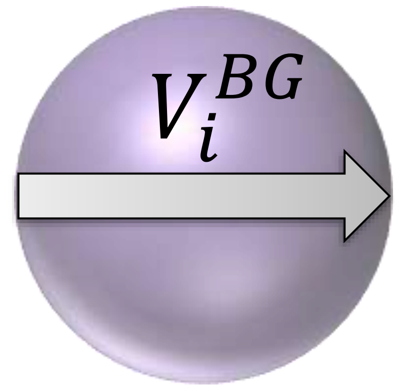
Background vector field V_i^{BG} helps.

$$V_i^{BG}\delta V_j \xrightarrow{\text{Source}} \delta T_{ij}$$

CMB says the universe is isotropic.

U(1) gauge → Anisotropic BG

Isotropy is violated



Way Out

Background vector field V_i^{BG} helps.

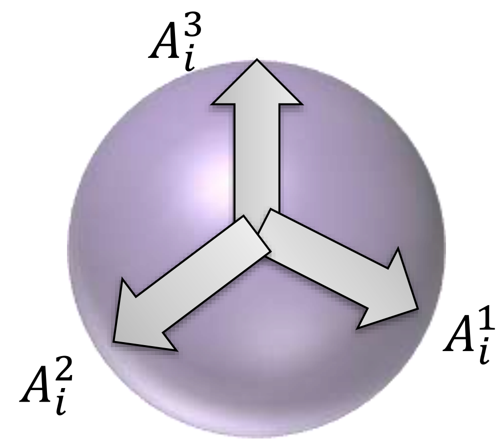
$$V_i^{BG}\delta V_j \xrightarrow{\text{Source}} \delta T_{ij}$$

CMB says the universe is isotropic.

U(1) gauge → Anisotropic BG

SU(2) gauge → Isotropic BG is Attractor.

Isotropy is respected



$$A_i^a = aA_{BG}(t)\delta_i^a$$

[Maleknejad&Erfani(2014)] [Domcke,Mares,Muia,Pieroni(2019)] [Wolfson,Maleknejad,Komatsu(2020)]

Instability of Chiral Tensor

"T.T. component" of δA_j^a = t_{aj} linearly coupled to GW

EoM:

$$h''_{R,L} + \left(1 - \frac{2}{x^2}\right) h_{R,L} = \mathcal{O}\left(\Omega_A^{1/2}\right) t_{R,L}$$

$$t''_{R,L} + \left(1 + \frac{2m_Q\xi}{x^2} \mp \frac{2}{x}(m_Q + \xi)\right) t_{R,L} = \mathcal{O}\left(\Omega_A^{1/2}\right) h_{R,L}$$

Only t_R undergoes tachyonic instability for $\dot{\chi} > 0$.

⇒ $h_R \gg h_L$: **Chiral GW is generated!**

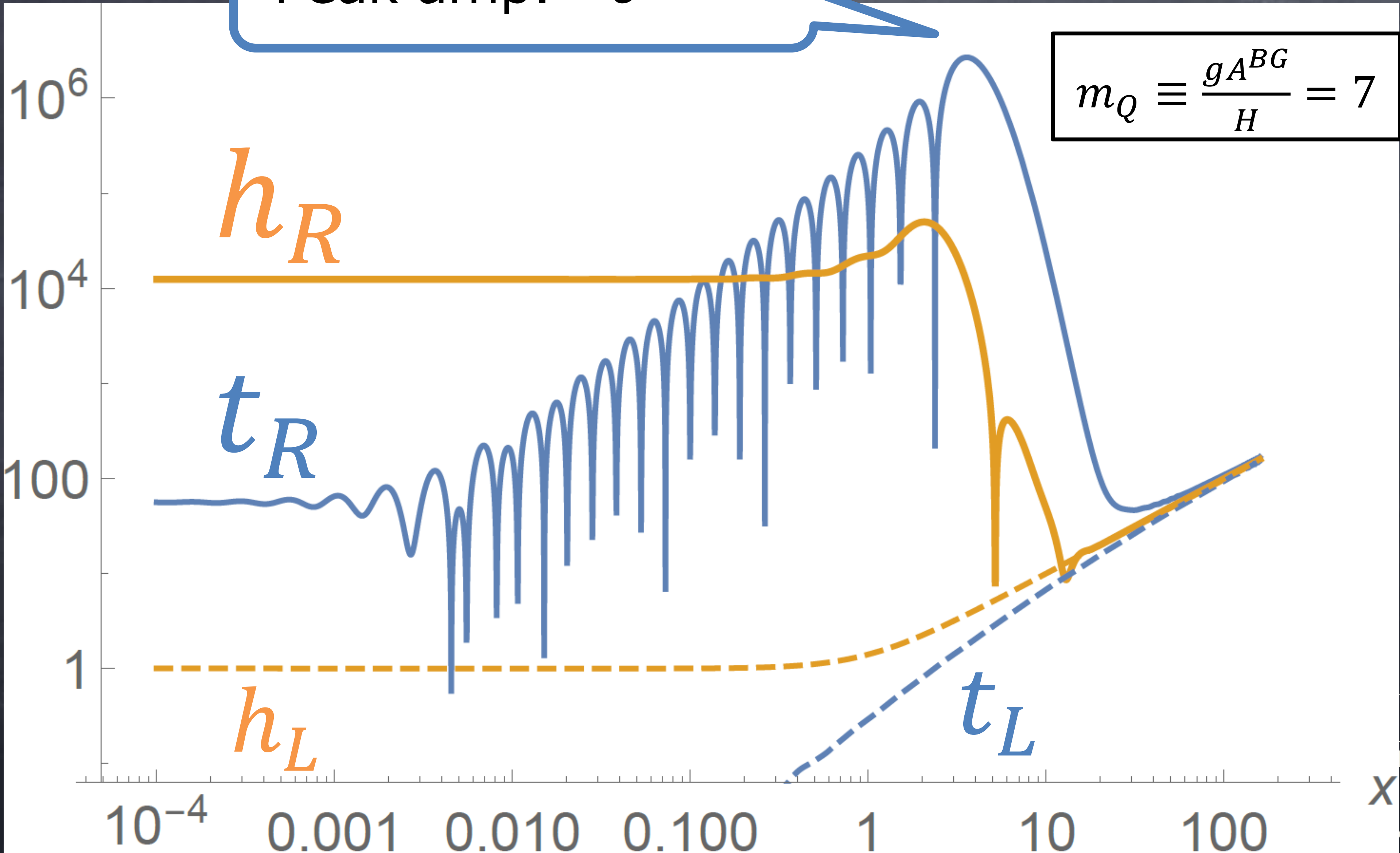
$x \equiv -k\eta$

$m_Q \equiv gA^{BG}/H$

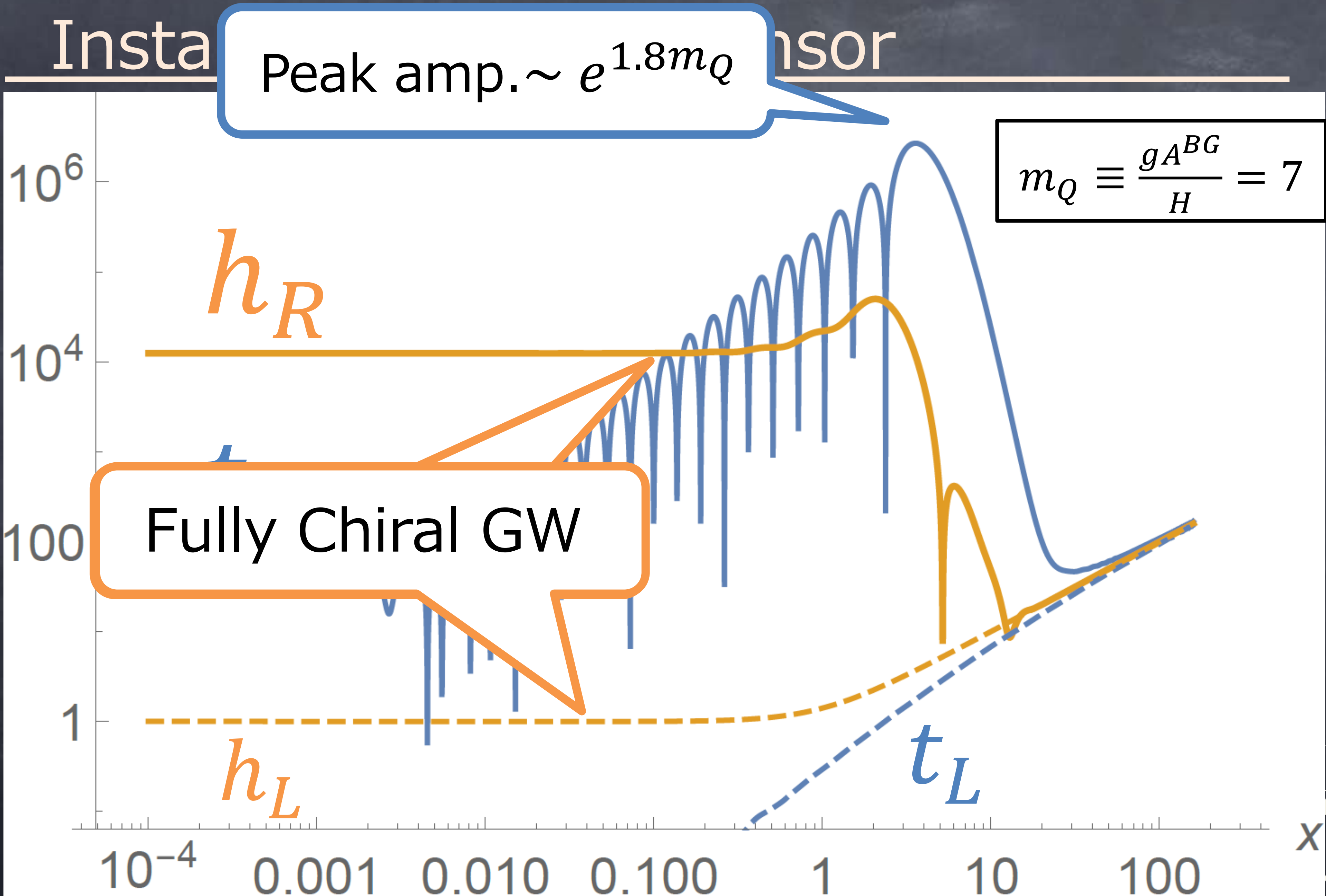
$\xi \equiv \lambda\dot{\chi}/2fH$

$A_i^a \equiv a\delta_i^a A^{BG}$

Insta nsor


Peak amp.$\sim e^{1.8m_Q}$
$m_Q \equiv \frac{gA^{BG}}{H} = 7$
h_R
t_R
h_L
t_L
10^6
10^4
100
1
10^{-4}
0.001
0.010
0.100
1
10
100
x

Insta nsor



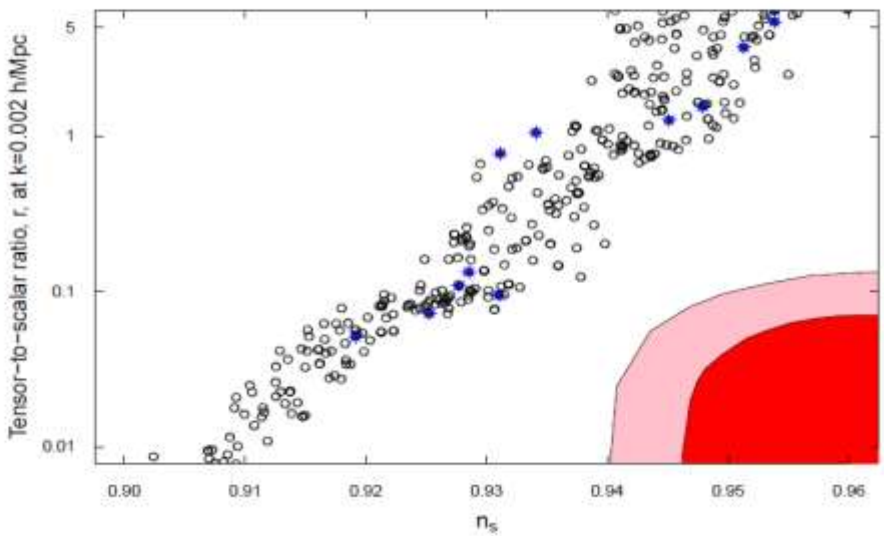
Peak amp.$\sim e^{1.8 m_Q}$
$m_Q \equiv \frac{g A^{BG}}{H} = 7$
h_R
Fully Chiral GW
h_L
t_L
10^6
10^4
100
1
10^{-4}
0.001
0.010
0.100
1
10
100
x

Chromo-natural

[Adshead&Wyman(2012)]
[Maleknejad&Sheikh-Jabbari(2011,2013)]

Inflaton coupled to SU(2) gauge fields

$$\mathcal{L} = +\frac{1}{2}(\partial\phi)^2 - \mu^4\left(\cos\frac{\phi}{f} + 1\right) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{\lambda}{4f}\phi F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$$



[Adshead, Martinec&Wyman(2013)]

Excluded by CMB due to too much GW production

See, however, [Adshead et al.(2016)]

Our model

[Dimastrogiovanni, Fasiello & TF (2017)]

Adding axion-SU(2) gauge spectator sector to the inflaton inspired by the Chromo-natural inflation

$$\mathcal{L} = \mathcal{L}_{\text{inflaton}}$$

Inflaton sector $\Rightarrow \mathcal{P}_\zeta$

$$+\frac{1}{2}(\partial\chi)^2 - U(\chi)$$

$$-\frac{1}{4}FF - \frac{\lambda}{4f}\chi F\tilde{F}$$

Axion- SU(2) gauge spectator sector $\Rightarrow \mathcal{P}_{\text{GW}}^{\text{spec}}$

Decoupled

Obs. properties of PGW

	Vacuum	SU(2)
①Amplitude	$\left(\frac{H_{\rm inf}}{M_{\rm Pl}}\right)^2$	$\left(\frac{H_{\rm inf}}{M_{\rm Pl}}\right)^2 \Omega_A e^{3.6 m_Q}$
②Tensor tilt n_t	Small	Can be large
③Polarization	None	Circular
④Non-Gaussianity	Small	Large

Observationally Distinguishable!

Obs. properties of PGW

	Vacuum	SU(2)
①Amplitude	$\left(\frac{H_{\rm inf}}{M_{\rm Pl}}\right)^2$	$\left(\frac{H_{\rm inf}}{M_{\rm Pl}}\right)^2 \Omega_A e^{3.6 m_Q}$
②Tensor tilt n_t	Small	Can be large
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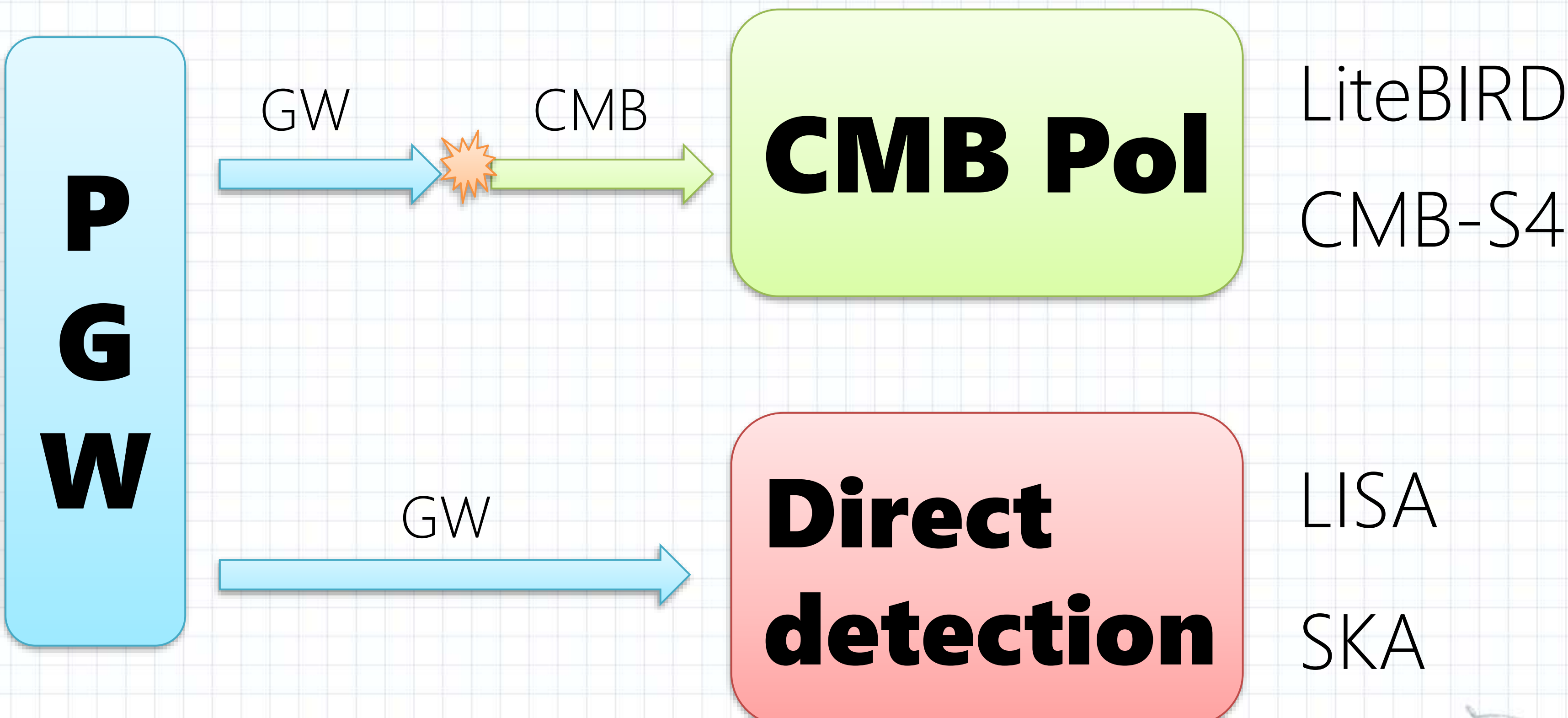
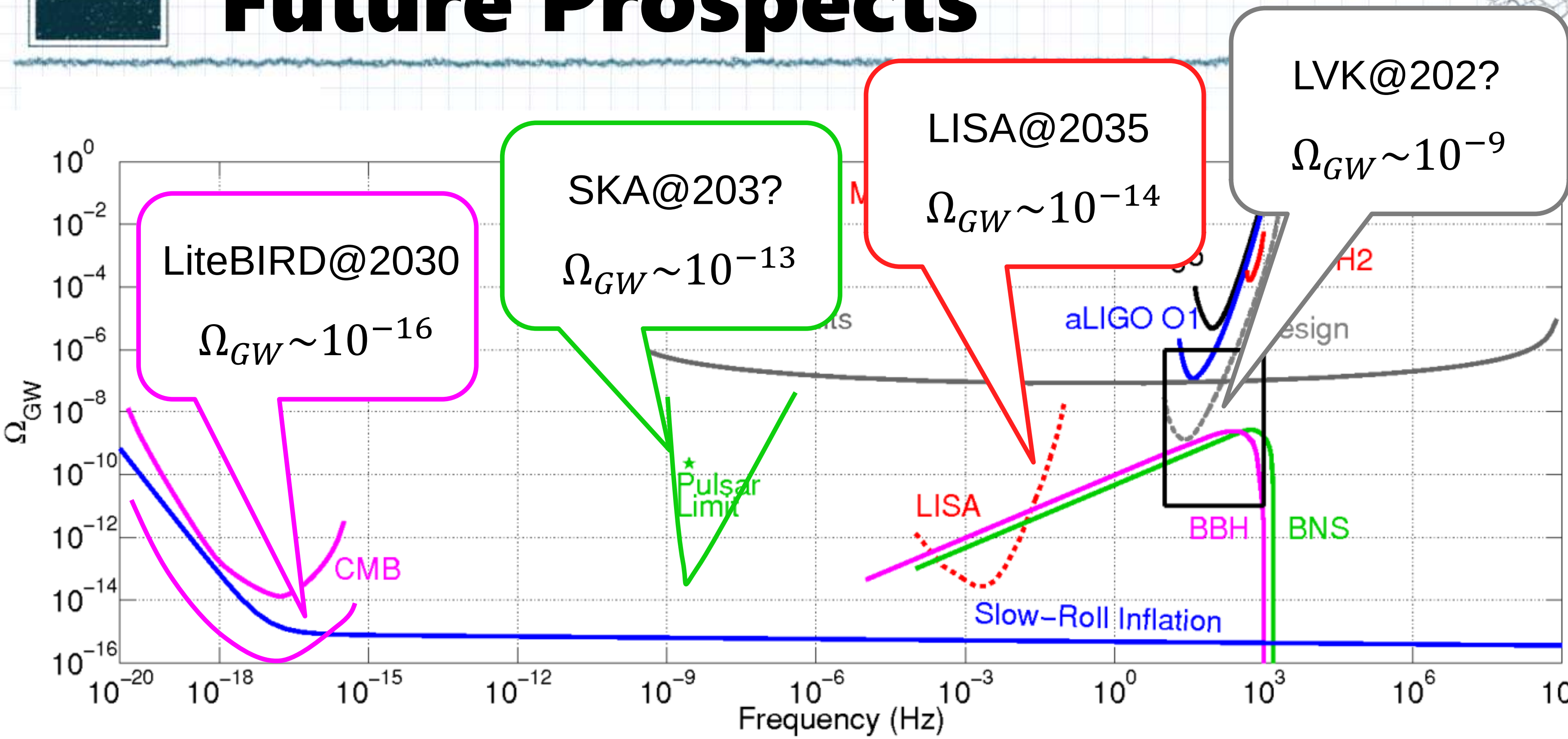
Observationally Distinguishable!

How to observe PGW



Future Prospects

[Abbot+(1612.02029)]



LiteBIRD©JAXA

CMB obs have the best sensitivity to primordial GWs.

TB,EB probe parity

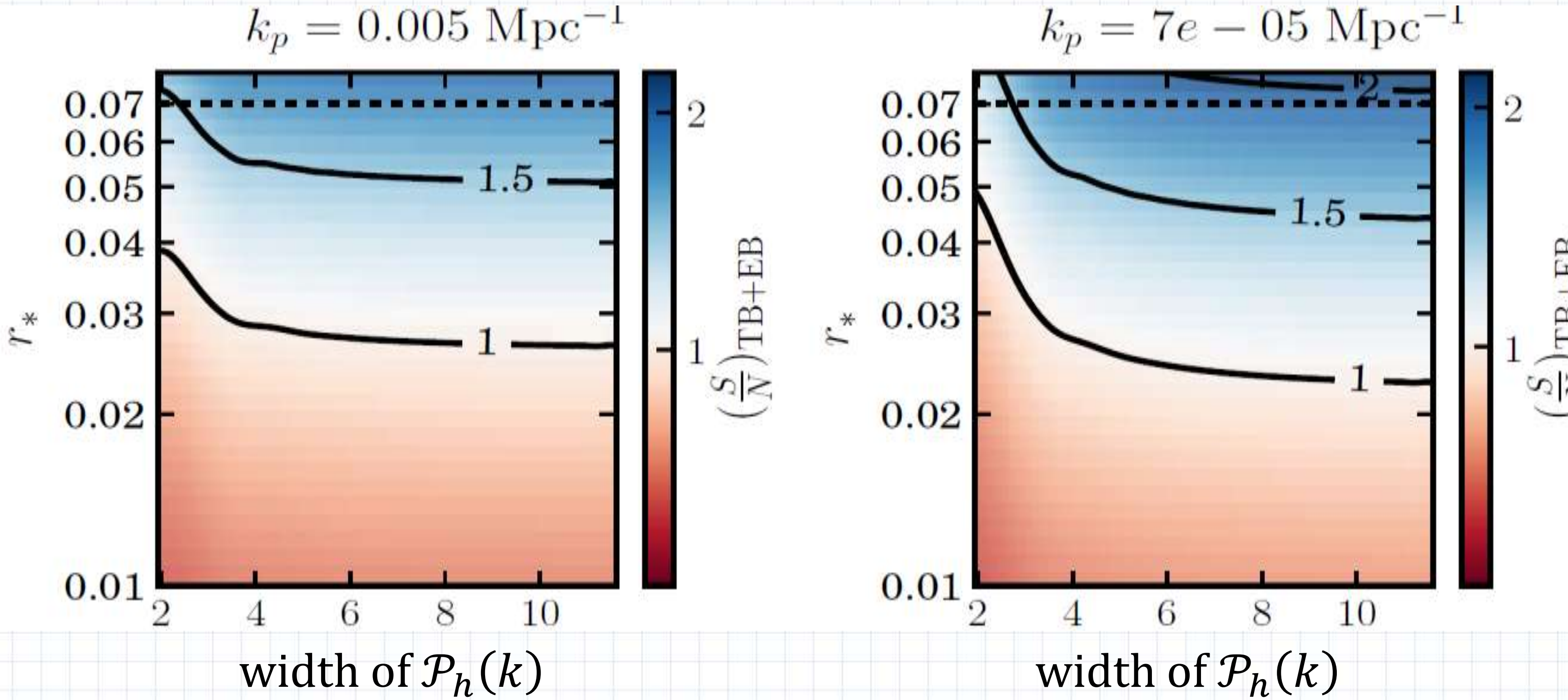
Chiral GW induces TB & EB cross correlations

$$\langle TT \rangle, \langle TE \rangle, \langle EE \rangle, \langle BB \rangle \propto \langle h_R h_R \rangle + \langle h_L h_L \rangle$$

$$\langle TB \rangle, \langle EB \rangle \propto \langle h_R h_R \rangle - \langle h_L h_L \rangle$$

By detecting TB & EB correlations, we can distinguish $\boldsymbol{h_{\mathrm{src}}}$ from $\boldsymbol{h_{\mathrm{vac}}}$

Detectability of Pol.



Parity violation in PGW can be detected for $r_* \gtrsim 0.03$

[Thone, TF, Hazumi + (2018)]

Obs. properties of PGW

	Vacuum	SU(2)
①Amplitude	$\left(\frac{H_{\mathrm{inf}}}{M_{\mathrm{Pl}}}\right)^2$	$\left(\frac{H_{\mathrm{inf}}}{M_{\mathrm{Pl}}}\right)^2 \Omega_A e^{3.6 m_Q}$
②Tensor tilt n_t	Small	Can be large
③Polarization	None	Circular
④Non-Gaussianity	Small	Large

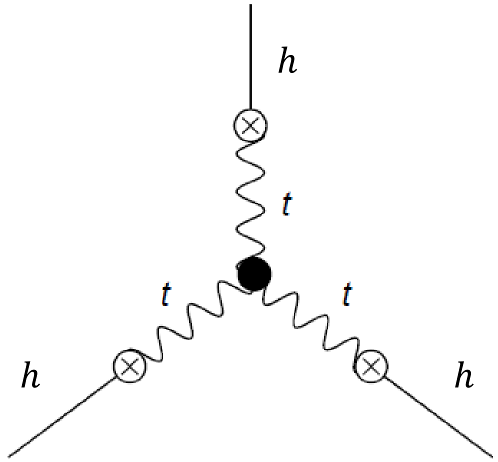
Observationally Distinguishable!

Non-Gaussianity

Large $\langle h_R h_R h_R \rangle$ ⟵ linear ⟵ Large $\langle t_R t_R t_R \rangle$

The self-interaction of SU(2) gives the non-linear dynamics (3p vertex)!

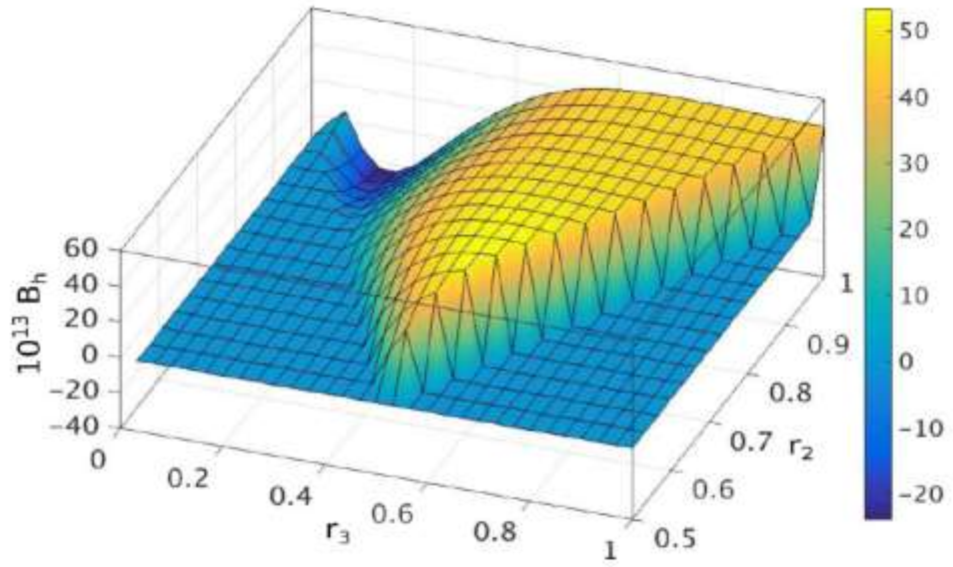
$$L_3^{(i)} = c^{(i)} \left[\epsilon^{abc} t_{ai} t_{bj} \left(\partial_i t_{cj} - \frac{m_Q^2 + 1}{3 m_Q \tau} \epsilon^{ijk} t_{ck} \right) - \frac{m_Q}{\tau} t_{ij} t_{jl} t_{li} \right],$$



(i)

[Agrawal,TF,Komatsu (2017)]

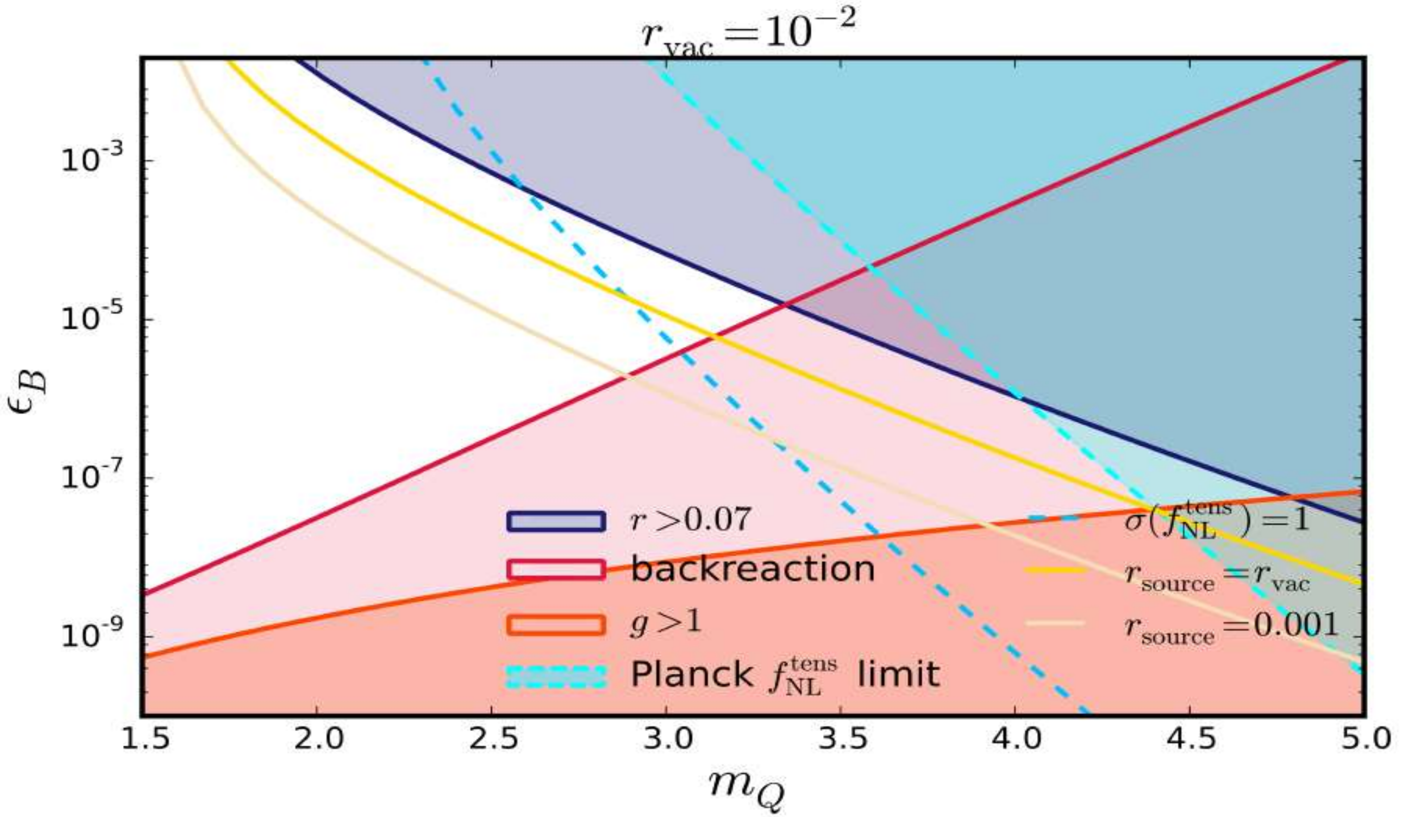
Non-Gaussian shape



The NG shape is almost equilateral ($\cos\theta \approx 0.9$)

[Agrawal,TF,Komatsu (2018)]

Detectability

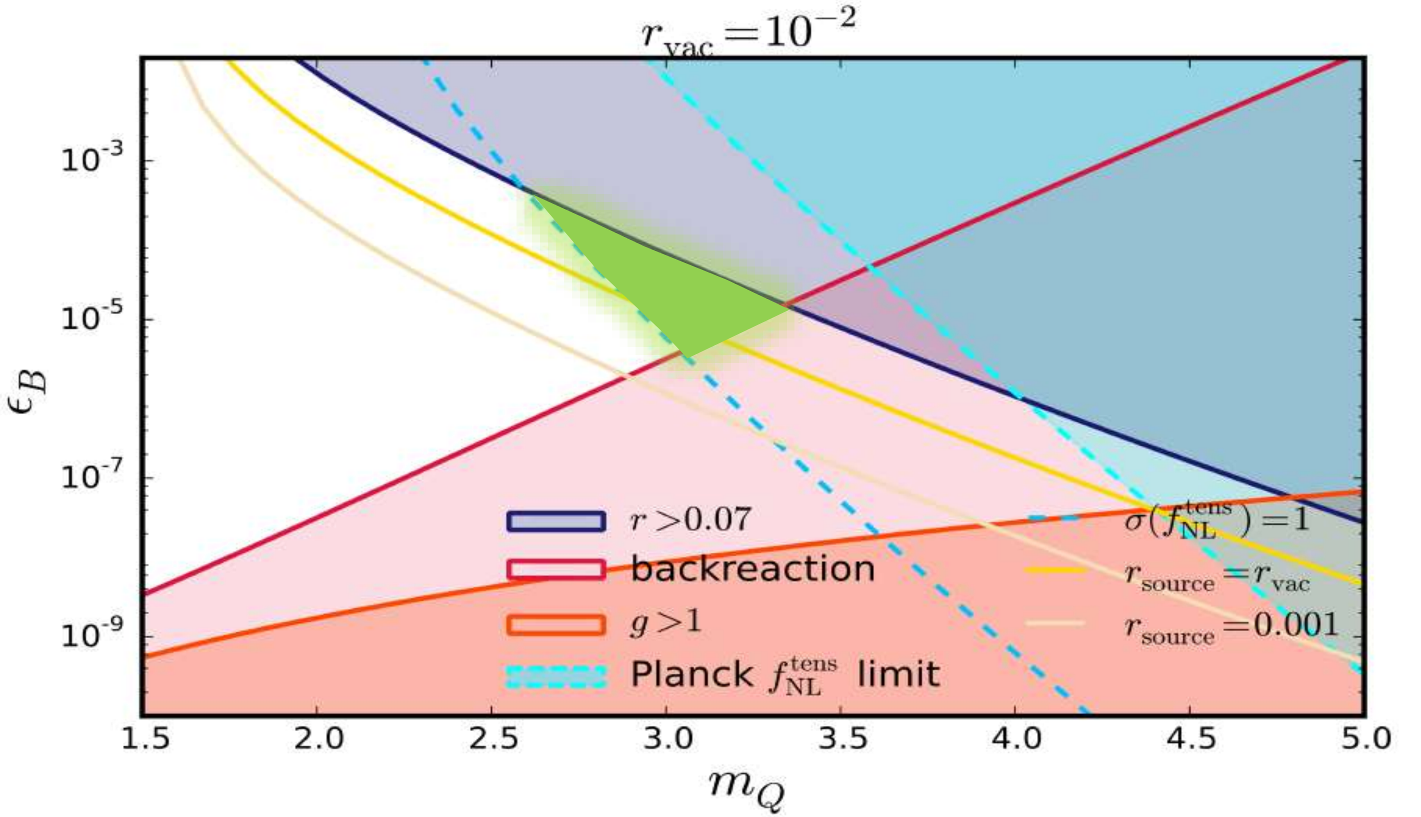


$m_Q \equiv gA^{BG}/H$

$\epsilon_B \simeq 2\Omega_A$

[Agrawal,TF,Komatsu (2018)]

Detectability



LiteBIRD can detect NG tensor mode!!

Consistency

[TF,Namba,Tada (2017)]

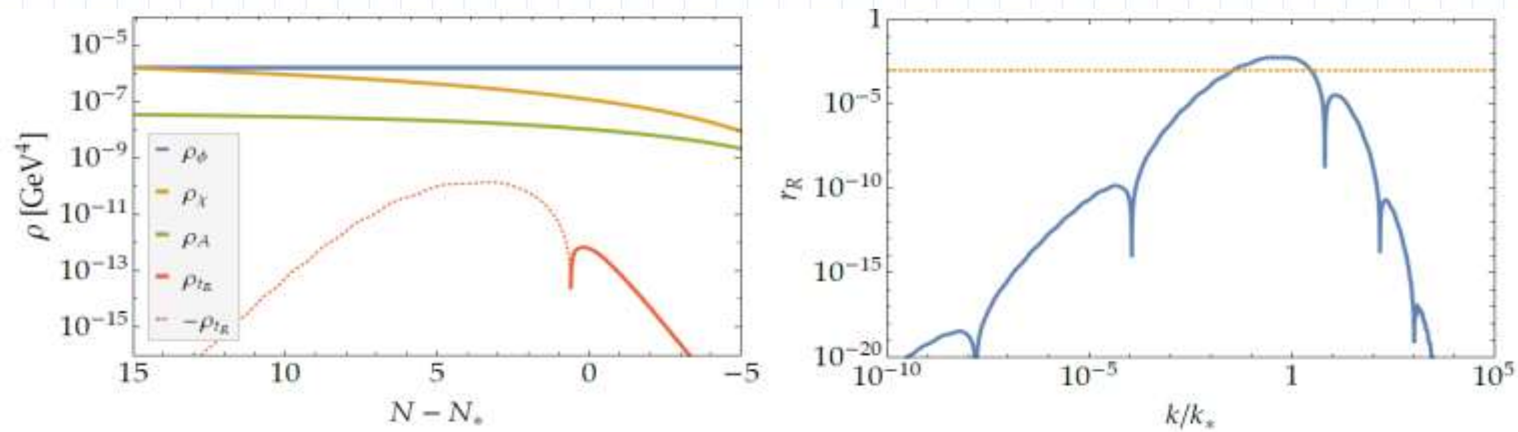
Lowest $\rho_{\rm inf}$ for $r \geq 10^{-3}$

How much can we boost r by axion-SU(2)??

Once I said…

$r_{\rm vac} \sim 10^{-80}$,

Even $\rho_{\rm inf}^{1/4} = 30\text{MeV}$, $r \geq 10^{-3}$ is possible



$H_{\rm inf} = 3\times10^{-22}\,\text{GeV}$, $\mu = 0.055\,\text{GeV}$, $f = 1.5\times10^{17}\,\text{GeV}$, $\lambda = 3000$, $g = 1.9\times10^{-36}$

Outline of Talk

3 Motivation

What's the **essence** of this mechanism?

How **general** is it? Can we **modify** it?

How to connect it to **obs**?

Motivation

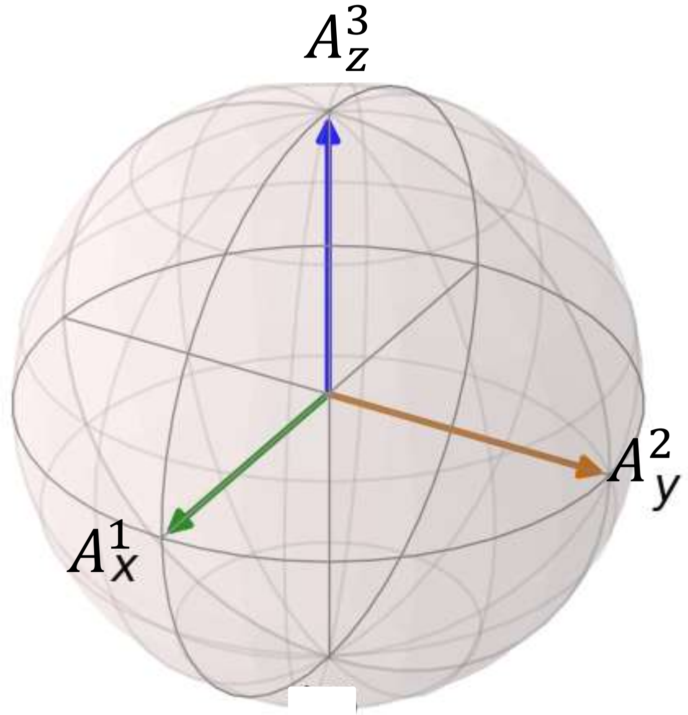
What's the **essence** of this mechanism?

How **general** is it? Can we **modify** it?

How to connect it to **obs**?

Model-independent analysis

Isotropic SU(2) Background



$$A_i^a = aA_{BG}(t)\delta_i^a$$

SU(2)×SO(3) → SO(3)

- Based on this **symmetry breaking pattern**,
  we develop new EFT by extending EFT of inflation.

EFT of inflation

Creminelli+(2006)
Cheung+(2008)

① Specify the symmetry breaking

✘ Time diffeo

◎ Spatial (3D) diffeo

$\bar{\phi}(\tau) = \langle \phi \rangle$: inflaton = clock field

3 EFT of inflation

Creminelli+(2006)
Cheung+(2008)

① Specify the symmetry breaking

✘ Time diffeo ◎ Spatial (3D) diffeo

② Find the building block

which respects the residual symmetry.

$$n_\mu = -\frac{\nabla_\mu \phi}{\sqrt{-g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi}}$$

Normal vector to the Uniform-φ hypersurface

$$h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$$

Spatial metric on the hypersurface

$$K_{\mu\nu} = h_\mu^{\ \sigma}\nabla_\sigma n_\nu ,$$

Extrinsic curvature

3 EFT of inflation

Creminelli+(2006)
Cheung+(2008)

① Specify the symmetry breaking

✘ Time diffeo ◎ Spatial (3D) diffeo

② Find the building block

which respects the residual symmetry.

③ Write all possible terms in the perturbative action

$$\mathcal{S} = \int \mathrm{d}\tau \mathrm{d}^3x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R - c(\tau) g^{00} - \Lambda(\tau) + \mathcal{L}^{(2)} \left(\delta R_{\mu\nu\rho\sigma}, \delta g^{00}, \delta K_{\mu\nu}, \nabla_\mu, n_\mu, \tau \right) \right]$$

3 EFT around isotropic gauge vev

① Specify the symmetry breaking

✘ SU(2) × SO(3)   ◎ Diagonal SO(3)

$$\bar{A}^a_\mu(\tau) \equiv \langle A^a_\mu \rangle = a(\tau) Q(\tau) \delta^a_\mu ,$$

3 EFT around isotropic gauge vev

① Specify the symmetry breaking

Let's introduce E&B:

$$E^a_\mu \equiv n^\rho F^a_{\mu\rho} ,$$
$$B^a_\mu \equiv n^\rho \widetilde{F}^a_{\mu\rho} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} n^\nu F^{a\rho\sigma} ,$$

and triads

$$\delta_{AB} e^A_\mu e^B_\nu = h_{\mu\nu} , \quad h^{\mu\nu} e^A_\mu e^B_\nu = \delta^{AB} , \quad n^\mu e^A_\mu = 0 ,$$

The key procedure is the gauge fixing:

$$\delta_{aA} E^a_{[\mu} e^A_{\nu]} = 0 .$$

3

EFT around isotropic gauge vev

① Specify the symmetry breaking

What's this? $$\delta_{aA} E^a_{[\mu} e^A_{\nu]} = 0 .$$

Vev: $$\langle E^{aA} \rangle = \langle E^a_\mu e^A_\nu h^{\mu\nu} \rangle = \bar{E} \delta^{aA}$$

With the above gauge fixing, the vev stay the same under the SU(2)&SO(3) local rotation.

$$\delta_{aA} E^a_\mu e^A_\nu \to \delta_{aA} E^a_\mu e^A_\nu - [abc] \tilde{\phi}^b E^{[a}_\mu e^{c]}_\nu = \bar{E} h_{\mu\nu} + \delta_{aA} \delta E^a_\mu e^a_\nu - \bar{E} [abc] \tilde{\phi}^c e^a_\mu e^b_\nu + \cdots$$

$$\tilde{\phi}^a = \frac{1}{2\bar{E}} h^{\mu\nu} [abc] e^b_\mu \delta E^c_\nu ;$$

3 EFT around isotropic gauge vev

① Specify the symmetry breaking

The gauge fixing: $\delta_{aA} E^a_{[\mu} e^A_{\nu]} = 0\,.$

② Find the building block

which respects the residual symmetry.

building block:

$$X_{\mu\nu} \equiv \frac{1}{2}\left(E^a_\mu E^a_\nu - B^a_\mu B^a_\nu\right)$$

$$Y_{\mu\nu} \equiv E^a_\mu B^a_\nu\,.$$

Note: $X = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$ $Y = -\frac{1}{4} F^a_{\mu\nu} \widetilde{F}^{a\mu\nu}$

3 EFT around isotropic gauge vev

① Specify the symmetry breaking

$$X_{\mu\nu} \equiv \frac{1}{2}\left(E^a_\mu E^a_\nu - B^a_\mu B^a_\nu\right)$$
$$Y_{\mu\nu} \equiv E^a_\mu B^a_\nu \,.$$

② Find the building block

③ Write all possible terms in the perturbative action

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2} R - \Lambda(\tau) - c(\tau) g^{00} + \lambda_X(\tau) X + \lambda_Y(\tau) Y + \mathcal{L}^{(2)} \,,$$
$$\begin{aligned}\mathcal{L}^{(2)} = {} & M_{gg}^4(\tau)(\delta g^{00})^2 + M_{gX}^2(\tau)\delta g^{00}\delta X + M_{gY}^2(\tau)\delta g^{00}\delta Y \\ & + \frac{1}{M_{XX}^4(\tau)}\delta X^2 + \frac{1}{M_{XY}^4(\tau)}\delta X \delta Y + \frac{1}{M_{YY}^4(\tau)}\delta Y^2 \\ & + \frac{1}{\bar{M}_{XX}^4(\tau)}\delta X_{\mu\nu}\delta X^{\mu\nu} + \frac{1}{\bar{M}_{XY}^4(\tau)}\delta X_{\mu\nu}\delta Y^{\mu\nu} + \frac{1}{\bar{M}_{YY}^4(\tau)}\delta Y_{(\mu\nu)}\delta Y^{(\mu\nu)} \\ & + \frac{1}{\hat{M}_{YY}^4(\tau)}\delta Y_{[\mu\nu]}\delta Y^{[\mu\nu]} + \cdots \,,\end{aligned}$$

3 Dictionary

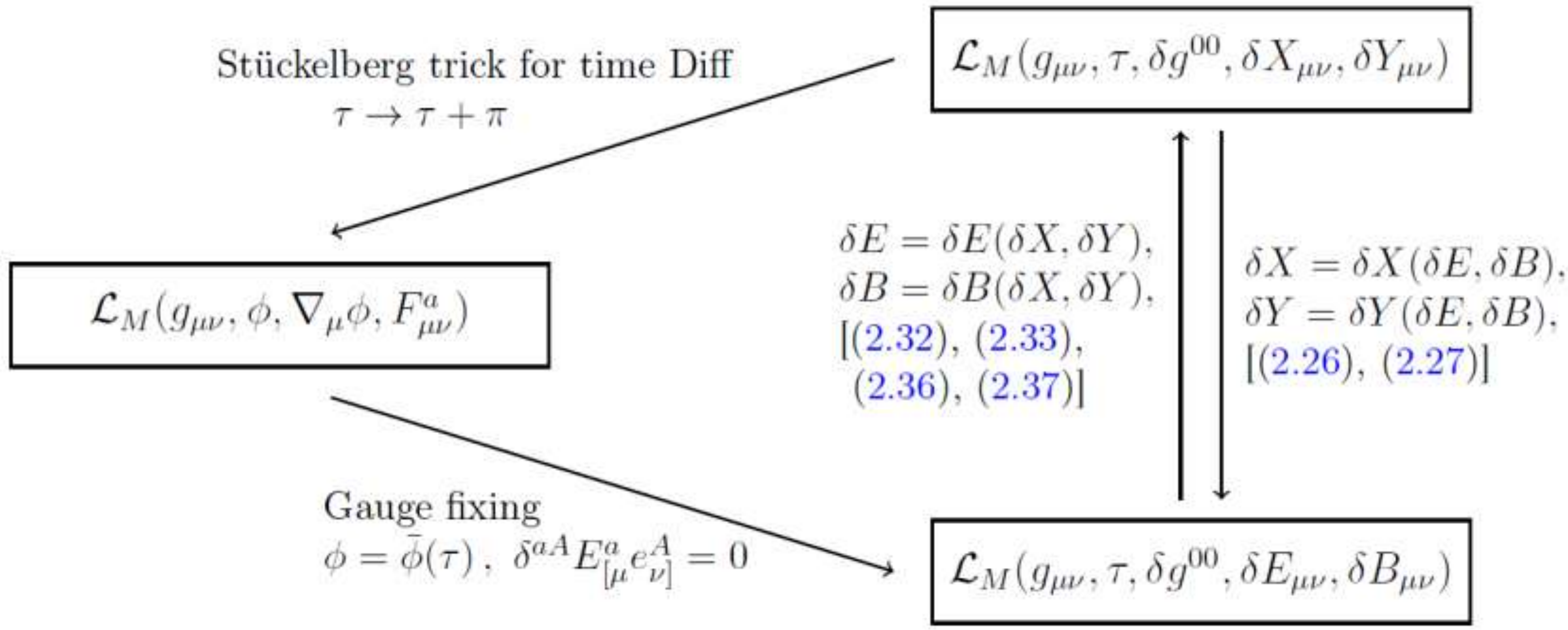


Figure 1: The relations between different forms of Lagrangian.

- We **establish the mapping** between the model (covariant) Lagrangian and the EFT terms.

Outline of Talk

1. Background
2. Model Results
3. EFT Action
4. Tensor Perturbation
5. Summary & Future Work

3 Tensor action

$$\mathcal{K}_A \equiv k \mp a\mathcal{G}Q\,, \quad \alpha \equiv \frac{\bar{B}}{\bar{E}} = \frac{\mathcal{G}Q}{(\ln aQ)'/a}$$

$$\begin{aligned}
\mathcal{L}_A^{\mathrm{TT}} = & \left[|H_A'|^2 - k^2|H_A|^2 + \frac{a''}{a}|H_A|^2\right] \\
& + \lambda_X \Bigg[|T_A'|^2 + \frac{2a\bar{E}}{M_{\mathrm{Pl}}}\left(H_A^*\left(T_A' \pm \alpha\mathcal{K}_A T_A\right) + \mathrm{c.c.}\right) - \mathcal{K}_A^2|T_A|^2 \\
& \quad + a^2\left(\mathcal{G}^2 Q^2 |T_A|^2 + \frac{2\bar{E}^2(1-\alpha^2)}{M_{\mathrm{Pl}}^2}|H_A|^2\right) \pm 2\beta_Y \mathcal{H}\mathcal{K}_A |T_A|^2 \\
& \quad + \bar{\lambda}_{XX}\left|T_A' \mp \alpha\mathcal{K}_A T_A + \frac{a\bar{E}(1+\alpha^2)}{M_{\mathrm{Pl}}}H_A\right|^2 + \bar{\lambda}_{YY}\left|\alpha T_A' \pm \mathcal{K}_A T_A\right|^2 \\
& \quad + \frac{\bar{\lambda}_{XY}}{2}\left(\left(T_A'^* \mp \alpha\mathcal{K}_A T_A^* + \frac{a\bar{E}(1+\alpha^2)}{M_{\mathrm{Pl}}}H_A^*\right)\left(\alpha T_A' \pm \mathcal{K}_A T_A\right) + \mathrm{c.c.}\right)\Bigg],
\end{aligned}$$

- 2nd order tensor perturbation:

  GWs and gauge tensor are **linearly mixed**.

3

Single question

Is it possible that the produced GWs are NOT chiral (parity-preserving), though the background violates it?

3 **Single question**

Is it possible that the produced GWs are NOT chiral (parity-preserving), though the background violates it?

Almost always chiral GWs.

But two loopholes...

3 Quasi-de Sitter limit

With simplifying assumptions $a \simeq \frac{1}{-H_0 \tau}\,, \qquad Q \simeq Q_{\rm sta} = {\rm const.}$

the tensor action reduces to

$$\mathcal{L}_A^{TT} = {\Delta'_A}^\dagger \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Delta'_A - k^2 {\Delta_A}^\dagger \begin{pmatrix} 1 & 0 \\ 0 & \frac{\gamma_2}{\gamma_1} \end{pmatrix} \Delta_A$$

$$+ {\Delta'_A}^\dagger K \Delta_A - \Delta_A^\dagger K \Delta'_A \pm k \Delta_A^\dagger P \Delta_A - \Delta_A^\dagger \Omega \Delta_A\,, \qquad \Delta_A = \begin{pmatrix} H_A \\ \hat{T}_A \end{pmatrix},$$

$$K_{11} = K_{22} = 0\,,$$

$$K_{12} = -K_{21} = -\frac{\epsilon_E \mathcal{H}}{\sqrt{2}\gamma_1}(1 + \alpha^2 + \gamma_1 - \alpha\gamma_3)\,,$$

$$P_{11} = 0\,,$$

$$P_{12} = P_{21} = \frac{\sqrt{2}\epsilon_E \mathcal{H}}{\gamma_1}(\alpha\gamma_2 + \gamma_3)\,,$$

$$P_{22} = \frac{2\mathcal{H}}{\gamma_1}(\beta_Y - \alpha\beta_X + \alpha\gamma_2 + \beta_X\gamma_3)\,,$$

$$\Omega_{11} = -2\mathcal{H}^2 \left[1 + \frac{\epsilon_E^2}{\gamma_1}(1 + \alpha^2 + \gamma_1 - \alpha^2\gamma_2 - 2\alpha\gamma_3)\right],$$

$$\Omega_{12} = \Omega_{21} = \frac{\epsilon_E \mathcal{H}^2}{\sqrt{2}\gamma_1}\left[2\alpha(\alpha\gamma_2 + \gamma_3) + (1 - 3\beta_X)(1 + \alpha^2 + \gamma_1 - \alpha\gamma_3)\right],$$

$$\Omega_{22} = \frac{\mathcal{H}^2}{\gamma_1}\left[2\alpha\beta_Y - 2\alpha^2 + (1 - \beta_X)(2\alpha^2 + \beta_X\gamma_1 - 2\alpha\gamma_3) + \alpha(\alpha\gamma_2 + \gamma_3)\right]$$

3 Non-chiral condition

- Stability conditions

$$\lambda_X \gamma_1 > 0 \ \text{(no ghost)}, \qquad \lambda_X \gamma_2 > 0 \ \text{(no grad. inst.)}.$$

- Non-chiral condition (P=0)

① ghost-condensate-like case

$$\lambda_X < 0 \Longrightarrow \rho_E, \rho_B < 0$$

① Anisotropic inf. with U(1) triplet

$\lambda_X \propto a^{-2}$ ⟹ Gauge tensor freezes On super-horizon

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1

Summary

- **Isotropic background of SU(2)**

  General attractor solution for any gauge sector coupled to axion.

- **Rich phenomenology based on each model**

  Chiral GW, non-Guassianity. But its essence is yet unknown.

- **EFT around the istropic SU(2) vev**

  We specified the symmetry breaking, found the building block.

- **EFT of chiral GWs**

  We develop the 2nd order action for tensor perturbations

- **Condition for GW chirality**

  2 loopholes. Maybe opportunity for another model building

Future directions

- **Control the amplitude and chirality of GWs?**

  The effective mass and tachyonic instability may alter.

- **Scalar and vector perturbations**

  Their stability may constrain the EFT parameter space.

- **Extension to higher orders**

  Tensor and mixed Non-gaussianity can be analyzed

- **Direct connection to the observations**

  Like EFT of inflation. Obs. constrain the EFT parameters.

- **and more!**

  Alternative attractor, perturbativity, etc could be studied by EFT

$$
\begin{aligned}
\mathcal{L}^{(2)} = {} & M_{gg}^4(\tau)(\delta g^{00})^2 + M_{gX}^2(\tau)\delta g^{00}\delta X + M_{gY}^2(\tau)\delta g^{00}\delta Y \\
& + \frac{1}{M_{XX}^4(\tau)}\delta X^2 + \frac{1}{M_{XY}^4(\tau)}\delta X\delta Y + \frac{1}{M_{YY}^4(\tau)}\delta Y^2 \\
& + \frac{1}{\bar{M}_{XX}^4(\tau)}\delta X_{\mu\nu}\delta X^{\mu\nu} + \frac{1}{\bar{M}_{XY}^4(\tau)}\delta X_{\mu\nu}\delta Y^{\mu\nu} + \frac{1}{\bar{M}_{YY}^4(\tau)}\delta Y_{(\mu\nu)}\delta Y^{(\mu\nu)} \\
& + \frac{1}{\tilde{M}_{YY}^4(\tau)}\delta Y_{[\mu\nu]}\delta Y^{[\mu\nu]} + \cdots ,
\end{aligned}
$$

THE THEME
OF CHAPTER IS...

Thank you！