vorticity production after first-order phase transitions

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The Dawn of Gravitational Wave Cosmology

Apr 30, 2025

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outline

- introduction
  - case for first-order phase transitions
  - why study fluid perturbations
- the general model
- results
- outlook and summary

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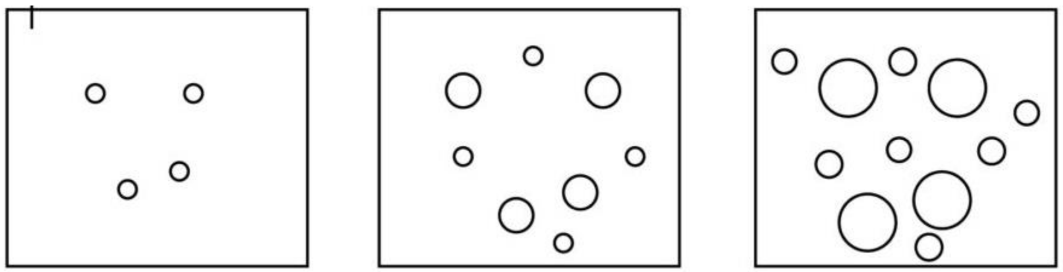
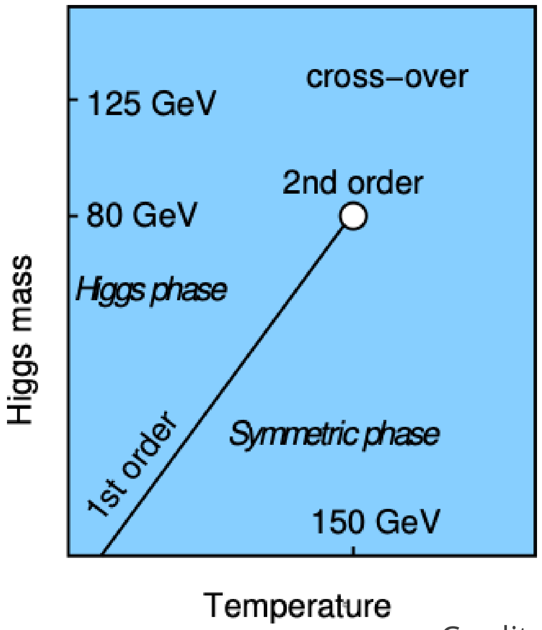
why care about GWs from FOPTs?

- in the Standard Model, EWPT is a crossover

- some extensions of the SM predict a FOPT





Credit: M. Hindmarsh

- signatures of FOPTs (like GWs within the LISA sensitivity scale) could help explore BSM physics

stages of FOPTs

bubble expansion → collisions → aftermath

no GWs produced

not considered here

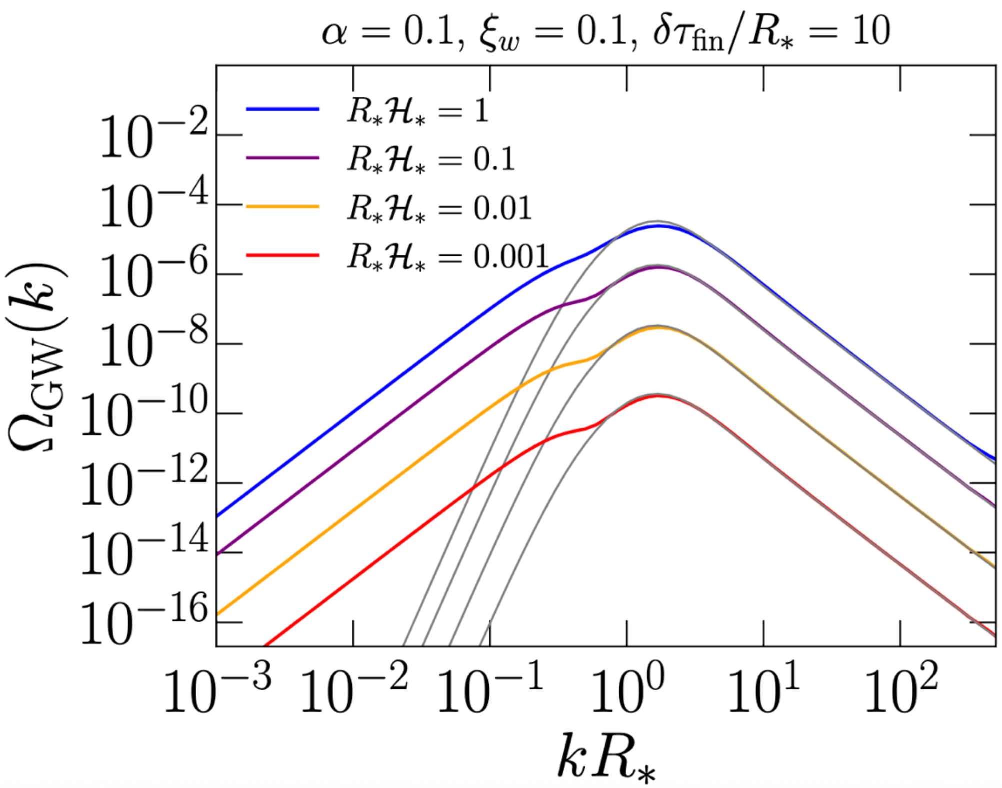
acoustic (scalar) perturbations

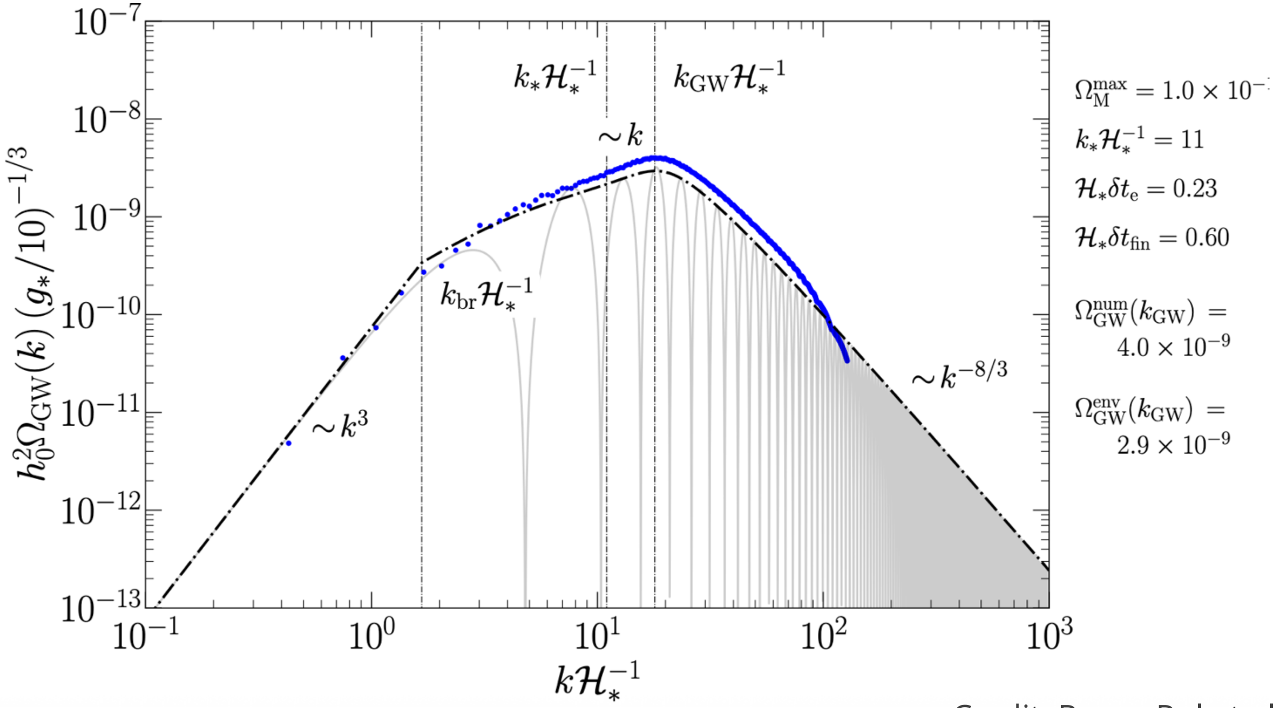
vortical perturbations

$$\mathbf{X}(t,x) = \boldsymbol{\nabla}\phi + \boldsymbol{\nabla} \times \mathbf{X}^{(v)}$$

$$X^{\ell}(\zeta,\mathbf{p}) = \hat{p}^{\ell} X_{\mathrm{s}}(\zeta,\mathbf{p}) + X^{\ell}_{\mathrm{v}}(\zeta,\mathbf{p})\,, \quad X^{\ell}_{\mathrm{v}}(\zeta,\mathbf{p}) = \mathbb{P}^{\ell m}(\hat{\mathbf{p}}) X_m(\zeta,\mathbf{p})$$

GWs from acoustic and vortical fluid perturbations can behave very differently





Credit: Roper Pol et al

necessary to understand fluid perturbations

$$\mathcal{P}_{h'}(\eta \gg \eta_{\rm fin}, k) \approx 64\mathcal{H}^2 \int_{\eta_*}^{\eta_{\rm fin}} \frac{\mathrm{d}\zeta}{\zeta} \int_{\eta_*}^{\eta_{\rm fin}} \frac{\mathrm{d}\vartheta}{\vartheta} \cos[k(\eta-\zeta)] \cos[k(\eta-\vartheta)] \mathcal{P}_\Pi(\zeta, \vartheta, k)$$

$$\begin{aligned} \bar{w}^2 \mathcal{P}_\Pi(\zeta,\vartheta,k) = \int_0^\infty \frac{\mathrm{d}p}{p} \int_{-1}^1 \frac{\mathrm{d}z}{\tilde{p}^3} \Bigg\{ & \frac{p^2}{\tilde{p}^2}(1-z^2)^2 \mathcal{P}^{(\mathrm{s})}(\zeta,\vartheta,p)\mathcal{P}^{(\mathrm{s})}(\zeta,\vartheta,\tilde{p}) \\ & + \left[2 - \frac{p^2}{\tilde{p}^2}(1-z^2)\right](1+z^2)\mathcal{P}^{(\mathrm{v})}(\zeta,\vartheta,\tilde{p})\mathcal{P}^{(\mathrm{v})}(\zeta,\vartheta,p) \\ & + 2\frac{p^2}{\tilde{p}^2}(1+z^2)(1-z^2)\mathcal{P}^{(\mathrm{v})}(\zeta,\vartheta,p)\mathcal{P}^{(\mathrm{s})}(\zeta,\vartheta,\tilde{p}) \Bigg\}, \end{aligned}$$

$$\mathcal{P}^{(\mathrm{s})}(\zeta,\vartheta,p) \xrightarrow{?} \mathcal{P}^{(\mathrm{v})}(\zeta,\vartheta,p)$$

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relativistic fluid dynamics

- set up (assumptions)
  - ideal fluid
  - barotropic equation of state, the fluid dynamics are isentropic
  - radiation dominated era → conformal invariance of EM tensor → results from Minkowski space valid in FLRW metric

$$\partial_\mu T^{\mu\nu} = \partial_\mu \left(w u^\mu u^\nu + p g^{\mu\nu} \right) = 0$$

$$p = p(\rho), \qquad c_s^2 = \frac{\partial p}{\partial \rho}$$

vorticity evolution

$$D_t\boldsymbol{\Omega} + \boldsymbol{\Omega}\,(\boldsymbol{\nabla}\cdot\mathbf{v}) - (\boldsymbol{\Omega}\cdot\boldsymbol{\nabla})\mathbf{v} - \frac{c_{\rm s}^2\Psi}{(1-v^2c_{\rm s}^2)\gamma^2}\boldsymbol{\Omega}$$
$$= \boldsymbol{\nabla}\left[\frac{c_{\rm s}^2\,\Psi}{(1-v^2c_{\rm s}^2)\gamma^2}\right]\times\mathbf{v} + \frac{c_{\rm s}^2}{1+c_{\rm s}^2}\boldsymbol{\nabla}v^2\times\boldsymbol{\nabla}\ln w\,,$$

$$\Psi = \boldsymbol{\nabla}\cdot\mathbf{v} + \frac{1-c_{\rm s}^2}{1+c_{\rm s}^2}\,(\mathbf{v}\cdot\boldsymbol{\nabla})\ln w\,.$$

$$\boldsymbol{\Omega} = \boldsymbol{\nabla}\times\mathbf{v}$$

vorticity generation even in perfect fluid!

$$\partial_t \boldsymbol{\Omega} = \frac{1}{(1-v^2c_\mathrm{s}^2)\gamma^2}\boldsymbol{\nabla}\left(c_\mathrm{s}^2\Psi\right)\times\mathbf{v} + \frac{c_\mathrm{s}^2}{1+c_\mathrm{s}^2}\boldsymbol{\nabla}v^2\times\boldsymbol{\nabla}\ln w + \frac{c_\mathrm{s}^2(c_\mathrm{s}^2-1)}{(1-v^2c_\mathrm{s}^2)^2}\Psi\boldsymbol{\nabla}v^2\times\mathbf{v}$$

subrelativistic — **subrelativistic & ultrarelativistic** — **ultrarelativistic**

all terms are zero for dust but non-zero for perfect radiation fluid!

vorticity generation in subrelativistic limit

$$\lim_{\gamma^2\to 1} \frac{\partial_t \boldsymbol{\Omega}}{c_s^2} = \underbrace{[\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{v})]\times\mathbf{v}}_{\equiv \mathrm{I}} + \frac{1-c_\mathrm{s}^2}{1+c_\mathrm{s}^2}\underbrace{[\boldsymbol{\nabla}(\mathbf{v}\cdot\boldsymbol{\nabla}\ln w)\times\mathbf{v}]}_{\equiv \mathrm{II}} + \frac{1}{1+c_\mathrm{s}^2}\underbrace{\boldsymbol{\nabla}v^2\times\boldsymbol{\nabla}\ln w}_{\equiv \mathrm{III}}$$

- assuming gaussian perturbations in **v** and w and

$$\partial_t^2 \langle v_i(t,\mathbf{k})\, v_j^*(t,\mathbf{k}')\rangle\big|_{t=0} = k^2\partial_t^2\, \langle \Omega_i(t,\mathbf{k})\, \Omega_j^*(t,\mathbf{k}')\rangle\big|_{t=0} \approx 2\,\langle \partial_t\Omega_i(t,\mathbf{k})\, \partial_t\Omega_j^*(t,\mathbf{k}')\rangle\big|_{t=0}$$

$$\ddot{\mathcal{P}}^{(\mathrm{v})}(k) \equiv c_\mathrm{s}^4\left[\ddot{\mathcal{P}}_\mathrm{I}^{(\mathrm{v})}(k) + \left(\frac{1-c_\mathrm{s}^2}{1+c_\mathrm{s}^2}\right)^2 \ddot{\mathcal{P}}_\mathrm{II}^{(\mathrm{v})}(k) + \frac{1}{(1+c_\mathrm{s}^2)^2}\ddot{\mathcal{P}}_\mathrm{III}^{(\mathrm{v})}(k) + \frac{1-c_\mathrm{s}^2}{(1+c_\mathrm{s}^2)^2}\ddot{\mathcal{P}}_\mathrm{IIIII}^{(\mathrm{v})}(k)\right]$$

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initial acoustic perturbations

- assumptions
  - sound waves
    $$\frac{\partial_t^2 \delta w}{\bar{w}} + c_{\rm s}^2 k^2 \frac{\delta w}{\bar{w}} = 0\,, \quad \partial_t^2 v + c_{\rm s}^2 k^2 v = 0$$
  - same initial conditions as the sound shell model

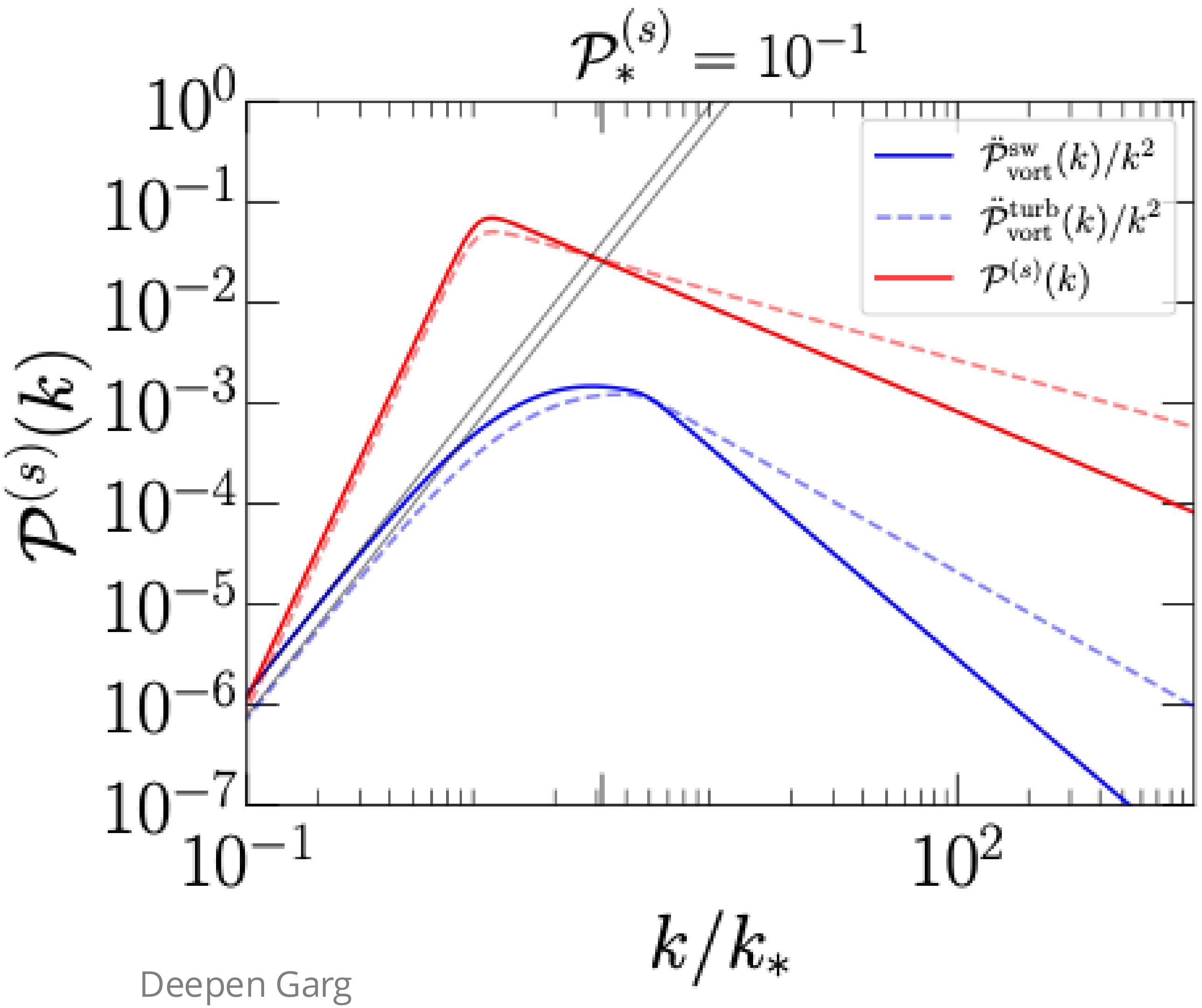
rate of generation of vortical power (leading order)

$$\ddot{\mathcal{P}}^{(\mathrm{v})}(k) = c_s^4 \frac{k^2}{2} \int_0^\infty \mathrm{d}p\, \mathcal{P}_v^{(\mathrm{s})}(p) \int_{|p-k|}^{p+k} \mathrm{d}\tilde{p}\, (p^2 - \tilde{p}^2) \left[1 - \frac{(k^2 + p^2 - \tilde{p}^2)^2}{4k^2p^2}\right] \frac{\mathcal{P}_v^{(\mathrm{s})}(\tilde{p})}{\tilde{p}^4},$$

where

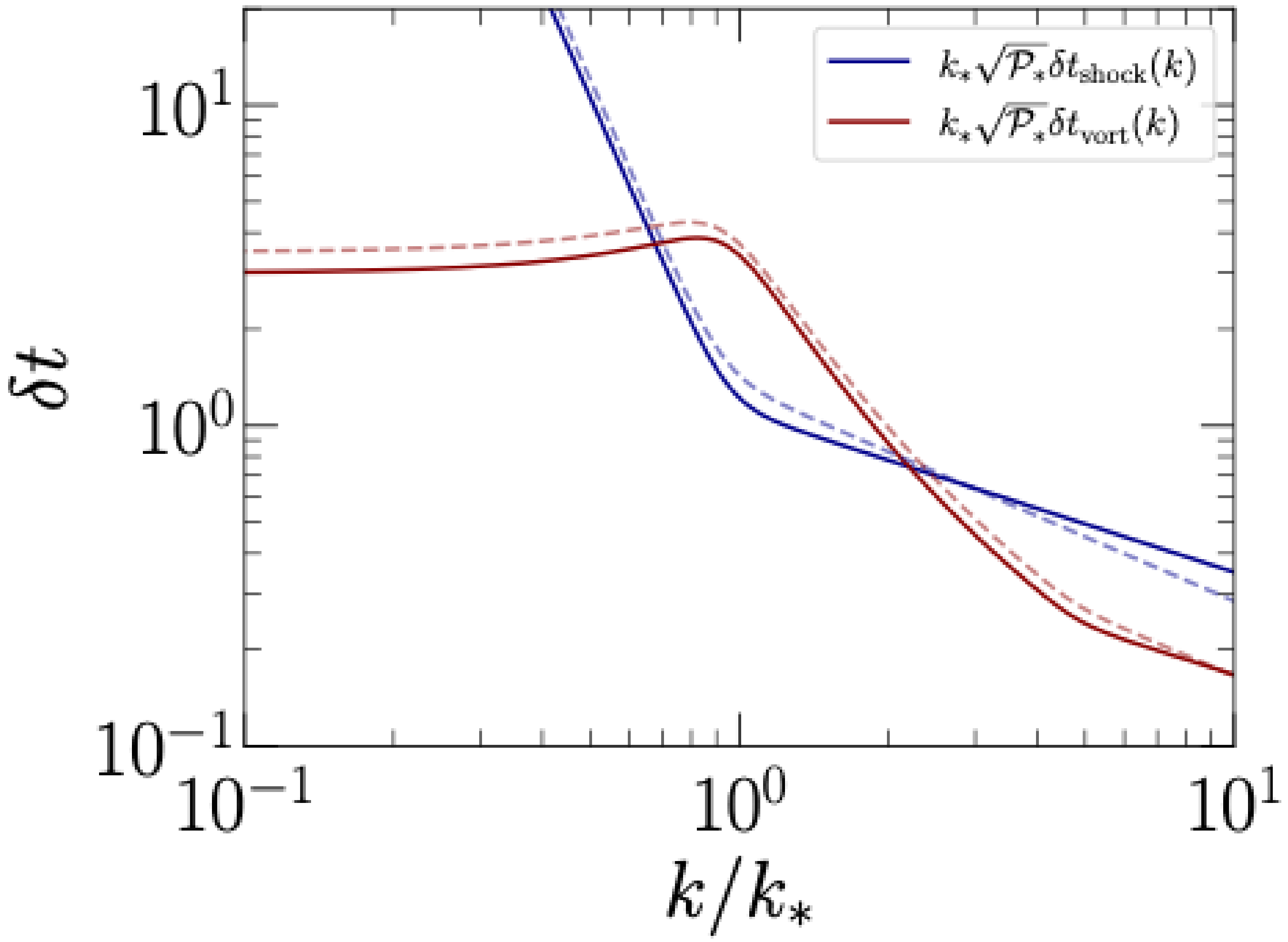
$$\tilde{\mathbf{p}} = \mathbf{k} - \mathbf{p}$$

results for the leading order term



- blue represents the rate of generation of vortical power normalized by k^2
- red represents the initial scalar power spectrum

time for vorticity production



- time for vorticity production inferred from $\sqrt{\mathcal{P}^{(s)}/\ddot{\mathcal{P}}^{(v)}}$
- this is for perfect fluids without any dissipative effects!
- the fact that it is of the same order as t_{shock} indicates that this could be a viable form of vorticity production
- nonlinear terms could be important for vorticity evolution

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next steps... a peek

- in the process of calculating when higher order terms become important

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- large enough $\delta w/\bar{w}$ would require a broader approach (preparing results)

$$X^\mu \equiv \sqrt{w}\, u^\mu$$

$$\boldsymbol{\Omega}^\mathrm{x} = \boldsymbol{\nabla} \times \mathbf{X} = \boldsymbol{\nabla}\left(\sqrt{w}\gamma\right) \times \mathbf{v} + \sqrt{w}\gamma\, \boldsymbol{\Omega}$$

summary

- imperative to study fluid perturbations to understand FOPTs through GWs
- usual approach is to study acoustic and vortical perturbations separately
  - the transition between the two needs more analysis
  - usually, vorticity is understood to be produced through dissipative effects
- we show that for radiation, vorticity can be produced even for perfect fluids and the initial analysis suggests similar time scale to shock formations
- preparing results for a generalized formalism that can account for large δw
- paper out soon!

thank you!

the system of equations

$$\partial_t \ln w = -\frac{1}{1-v^2c_\mathrm{s}^2}\left[\left(1+c_\mathrm{s}^2\right)\partial_\ell v^\ell + \left(1-c_\mathrm{s}^2\right)v^\ell\partial_\ell \ln w\right],$$

$$D_t v^k = \frac{c_\mathrm{s}^2 v^k}{\left(1-v^2c_\mathrm{s}^2\right)\gamma^2}\left[\partial_\ell v^\ell + \frac{1-c_\mathrm{s}^2}{1+c_\mathrm{s}^2}v^\ell\partial_\ell \ln w\right] - \frac{c_\mathrm{s}^2}{\left(1+c_\mathrm{s}^2\right)\gamma^2}\partial^k \ln w\,.$$