Gravitational waves from (low-scale) cosmic strings

Based on: Kai Schmitz & Tobias Schröder, Phys. Rev. D **110** (2024) 6, 063549, 2405.10937.

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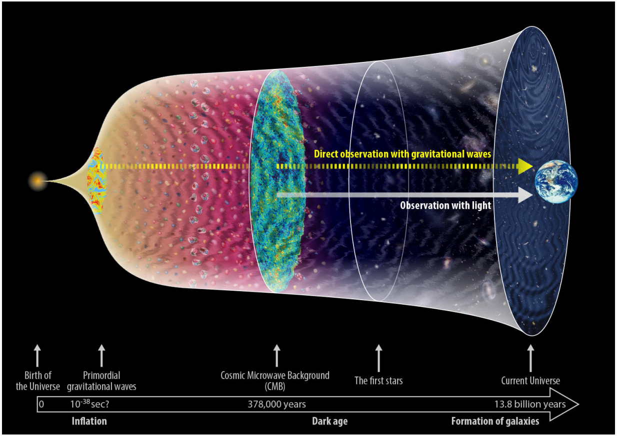
The Dawn of Gravitational Wave Cosmology
Benasque Science Center | Benasque, Spain | 30 April 2025

Gravitational-wave echo from the Big Bang

[National Astronomical Observatory of Japan, gwpo.nao.ac.jp]



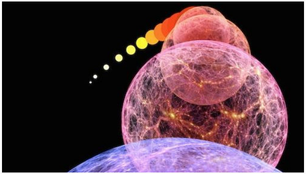
Primordial gravitational waves (GWs): Chance to peek behind the veil of the CMB

- Probe cosmology of the primordial Universe at **very early times**
- Probe particle physics at **extremely high energies** $\rightarrow$ New physics!?

Beyond-the-Standard-Model (BSM) options

❶ Inflationary tensor perturbations

- Accelerated expansion before the Hot Big Bang
- **Complementarity:** GWs + CMB observations



Abbrevations: GW: gravitational wave; CMB: cosmic microwave background; QFT: quantum field theory; EW: electroweak; QCD: quantum chromodynamics; PBH: primordial black hole; GUT: grand unified theory

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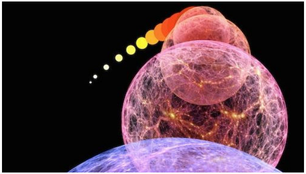
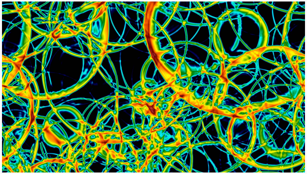


❷ Cosmological phase transition

- First-order transition in the QFT vacuum structure
- **Complementarity:** GWs + EW / QCD / dark sector



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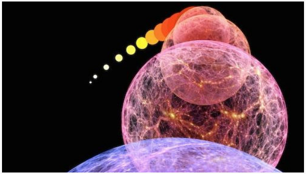

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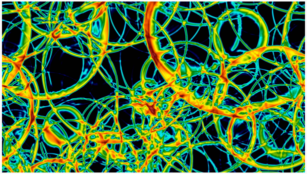

❸ Enhanced scalar perturbations

- Overdensities that emit GWs and collapse to PBHs
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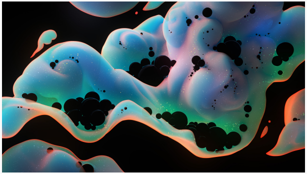

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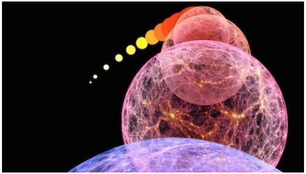

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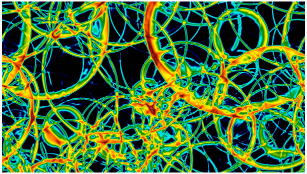

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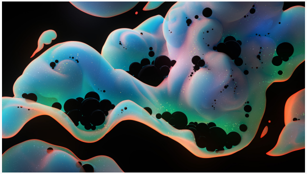
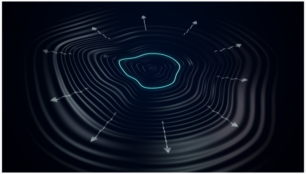


❹ Cosmic defects

- Phase transition remnants preserving the old vacuum
- **Complementarity:** GWs + grand unified theories



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Outline

GWs from cosmic strings

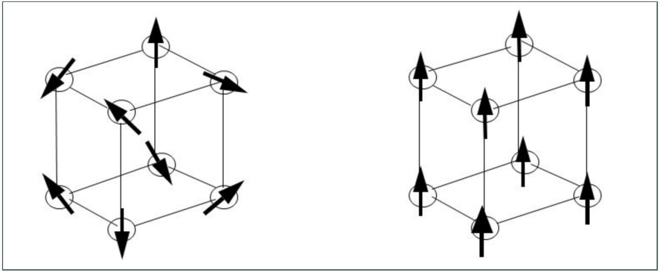
Low-scale cosmic strings

From VOS to BOS

Conclusions

Magnetic domains in a ferromagnet

[wikimedia.org]



Magnetization in a ferromagnet

- Phase transition at the Curie temperature: paramagnet $\rightarrow$ ferromagnet
- Magnetic dipoles align spontaneously due to exchange interaction
- Translation and rotation invariance spontaneously broken
- Magnetic domains, regions of uniform magnetization, separated by domain walls
- Domain walls are stable, unless an external force (magnetic field) is applied

Magnetic domains in a ferromagnet

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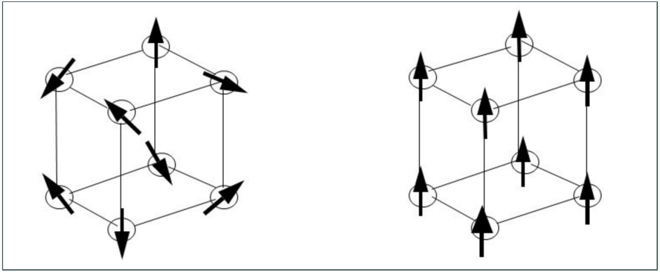

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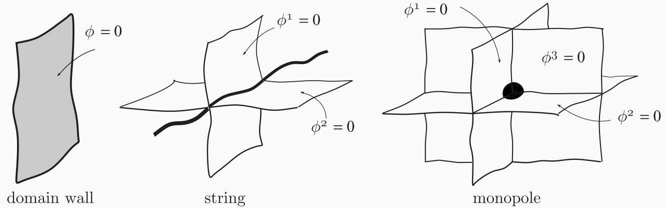
Similar phenomenology after phase transitions in the early Universe!

Topological defects in the early Universe

[Viatcheslav Mukhanov: Physical Foundations of Cosmology, Cambridge University Press (2005)]



Consider spontaneous symmetry breaking in an N-dimensional scalar field space:

$$V(\Phi) = \frac{\lambda}{4}\left(\Phi^2 - v^2\right)^2, \qquad \Phi = \frac{1}{\sqrt{N}}\left(\phi_1, \phi_2, \cdots, \phi_N\right)^T$$

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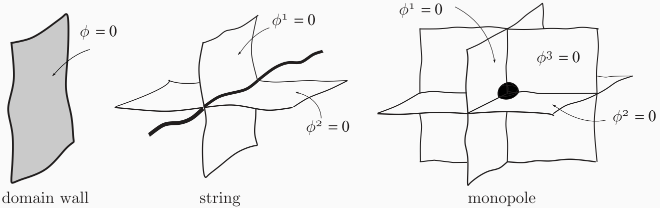

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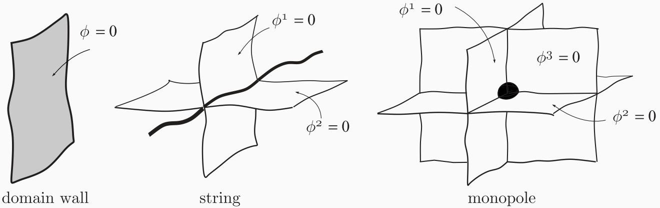

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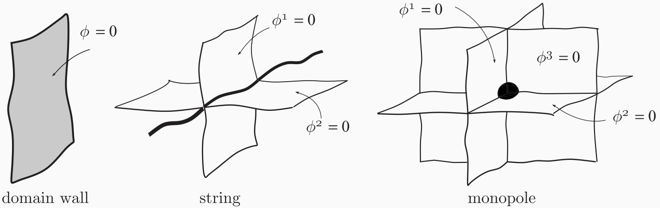

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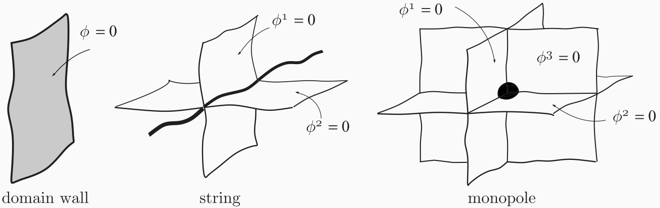

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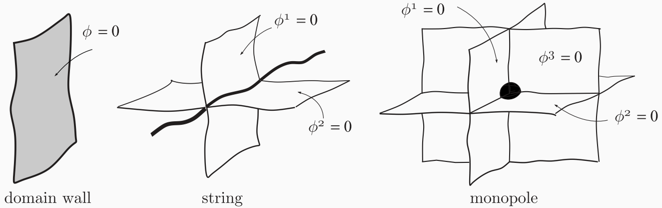

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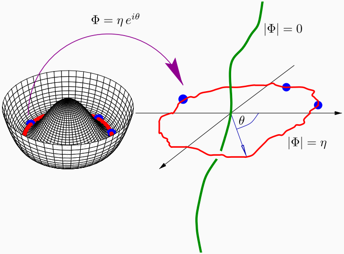
- Solitonic solutions of classical equations of motion for the gauge and Higgs fields
- Formal description in terms of topology of vacuum manifold $\mathcal{M}(\Phi) \rightarrow$ stability
- In addition, whole zoo of composite defects, non-topological defects, etc.

Cosmic strings

[Ringeval: 1005.4842]

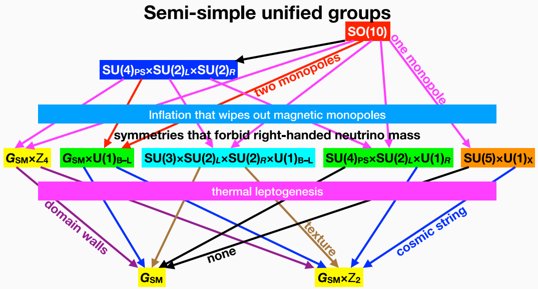


- **Topological defects** after spontaneous symmetry breaking, s.t. $\pi_1(\mathcal{M})$ nontrivial
- For instance, breaking of global / local $U(1)$; symmetry restored at string cores
- Condensed matter: Magnetic field vortices (quantum vortices) in a superconductor

Relevant parameters

- $G\mu$: String tension = energy per unit length, in units of $G = 1/M_{\mathrm{P}}^2$
- α: Size of string loops at time of formation, in units of the horizon $d_h \sim t \sim H^{-1}$

Cosmic strings in grand unified theories



Cosmic-string tension: Controlled by energy scale of spontaneous symmetry breaking

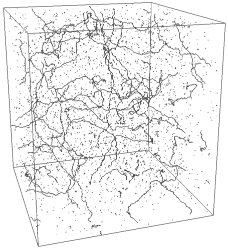
$$\mu \sim 2\pi v^2 \,, \qquad G\mu \sim 4 \times 10^{-8} \left(\frac{v}{10^{15}\,\mathrm{GeV}}\right)^2$$

Interesting possibilities

$$v \sim \Lambda_{\mathrm{GUT}} \sim 10^{15\cdots16}\,\mathrm{GeV} \,, \qquad v \sim \Lambda_{\mathrm{intermediate}} \sim 10^{9\cdots10}\,\mathrm{GeV}$$

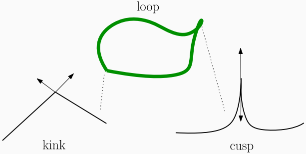
Gravitational waves from cosmic strings

[Allen, Martins, Shellard: ctc.cam.ac.uk/outreach]



Infinitely long strings and string loops;
scaling regime: $\rho_{\rm cs} \propto \rho_{\rm crit} \propto H^2$

[Gouttenoire, Servant, Simakachorn: 1912.02569]



Gravitational waves from

- Cusps
- Kinks
- Kink–kink collisions

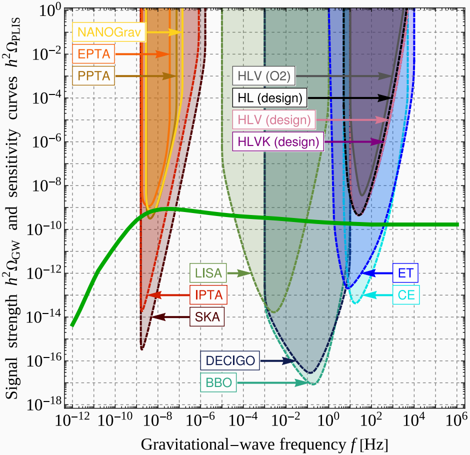
- Nambu–Goto strings: Infinitely thin, particle emission irrelevant at late times
- Abelian-Higgs strings: Short-lived loops, decay into massive particles

[Vachaspati, Vilenkin: PRD 31 (1985) 3052] [LISA Cosmology Working Group, Auclair et al.: 1909.00819]

GW spectrum

[S. Blasi, V. Brdar, **KS**: 2009.06607]

$G\mu = 6 \times 10^{-11}$

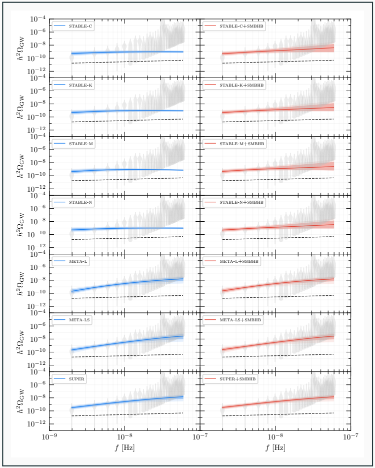


Broadband signal

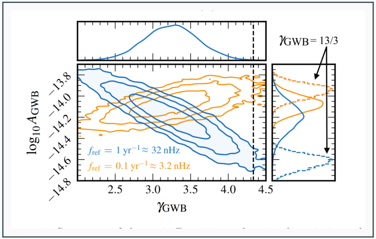
- Reflects scaling regime, GW emission during radiation and matter domination
- Interesting target for future GW experiments. Source of the **PTA signal**?

2023 PTA results

[NANOGrav: 2306.16219]



[NANOGrav: 2306.16213]



Recent PTA results

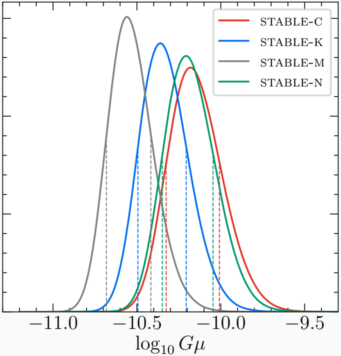
- Evidence for nHz GWB signal
- Stable cosmic strings do not yield a good fit (spectrum too flat)
- Alternatives doing a better job: **metastable strings**, **superstrings**

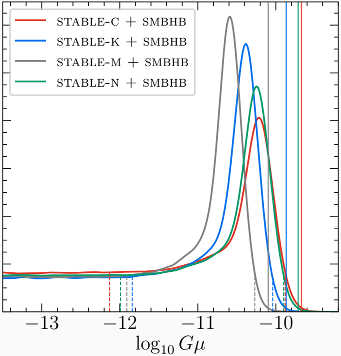
Upper limits

PTA upper limit on the tension of stable Nambu–Goto strings

$$G\mu \lesssim 10^{-10} \quad \longleftrightarrow \quad v \lesssim 5 \times 10^{13}\,\mathrm{GeV}$$

[NANOGrav: 2306.16219]



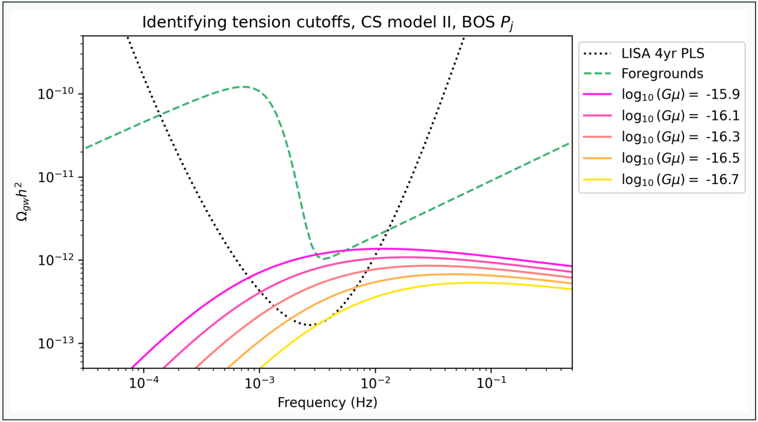


Different models

- GW emission dominated by cusps (c), kinks (k), fundamental mode (m); numerical result (n)
- GWs from cosmic strings only or in combination with GWs from supermassive BH binaries

Prospects for LISA

[LISA Cosmology Working Group: 2405.03740]



Expected sensitivity: $G\mu \sim 10^{-(16\cdots17)}$ $\longleftrightarrow$ $v \sim \text{few} \times 10^{10}\,\text{GeV}$

- GW signal from cosmic strings competes with galactic and extragalactic foregrounds

Outline

Phenomenology at low string tension

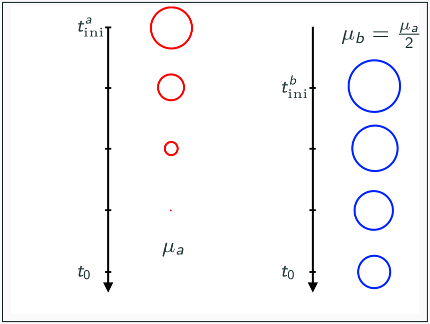
Two facts about string loops

- **Shrink** because of GW emission

$$\frac{d\ell}{dt} = -\Gamma G\mu\,, \qquad \Gamma \simeq 50$$

- **Characteristic length at birth**

$$\ell_* = 2\alpha t_*\,, \qquad \alpha \simeq 0.05$$



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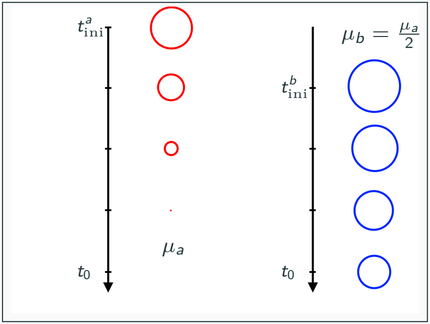

Loop length decreases linearly in time between birth and today

$$\ell\,(t_0) = \ell_* - \Gamma G\mu\,(t_0 - t_*)\ , \qquad t_* \in [t_{\rm ini}, t_0]$$

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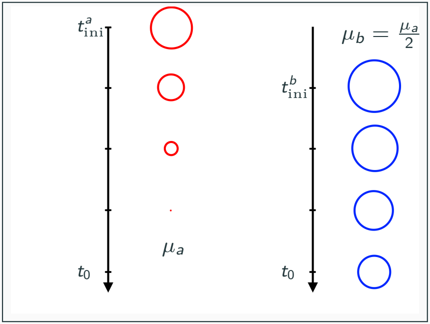

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Observation: For late $t_{\rm ini}$ and small enough $G\mu$, no loop ever reaches zero length!
Shortest loops = earliest loops today. Length: $\ell_{\rm min} = \ell_*\,(t_{\rm ini}) - \Gamma G\mu\,(t_0 - t_{\rm ini})$

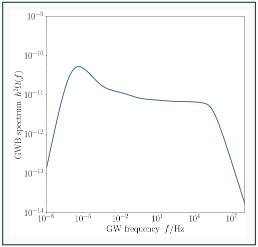
Sharp cutoff frequency

Present-day frequencies of GWs emitted by strings

$$f = \frac{a(t)}{a_0} \frac{2k}{\ell(t)}$$

$2k/\ell(t)$	Frequency at emission
$\ell(t)$	Loop length at emission
k	Mode number $(k = 1, 2, \cdots)$
$a(t)/a_0$	Cosmological redshift factor

$G\mu = 10^{-13}$



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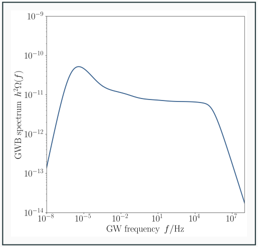
Minimal length $\ell_{\rm min}$ implies frequency cutoff

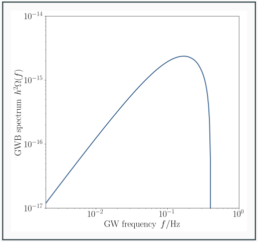
- Focus on fundamental mode ($k = 1$) for now
- Shortest loops today: minimal length, minimal redshift $\rightarrow$ highest possible frequency

$$f_{\rm cut} = \frac{2}{\ell_{\rm min}} = \frac{2}{2\alpha t_{\rm ini} - \Gamma G\mu\,(t_0 - t_{\rm ini})}$$

$G\mu = 10^{-13}$



$G\mu = 10^{-19}$



Initial loop length

Cutoff frequency is positive and finite if: $2\alpha\, t_{\mathrm{ini}} > \Gamma G\mu\, (t_0 - t_{\mathrm{ini}}) \approx \Gamma G\mu\, t_0$

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Four reasonable options

1. $t_{\rm ini} = t_{\rm form}$ Network formation $\rho_{\rm tot} = 3H^2 M_{\rm Pl}^2 \sim \mu^2$

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$$t_{\rm ini} > t_{\rm cut} \equiv \frac{\Gamma G\mu}{2\alpha} t_0 \simeq 2.2\,\mathrm{s}\left(\frac{G\mu}{10^{-20}}\right)\,, \qquad T_{\rm ini} < T_{\rm cut} \simeq 330\,\mathrm{keV}\left(\frac{10^{-20}}{G\mu}\right)^{1/2}$$

Problem: $t_{\rm ini}$ is model-dependent and typically not well known

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Four reasonable options

1. $t_{\rm ini} = t_{\rm form}$ Network formation $\rho_{\rm tot} = 3H^2 M_{\rm Pl}^2 \sim \mu^2$
2. $t_{\rm ini} = t_{\rm fric}$ End of friction regime $\beta T^3/\mu \sim 2H$

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Four reasonable options

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2	$t_{\rm ini} = t_{\rm fric}$	End of friction regime	$\beta T^3/\mu \sim 2H$
3	$t_{\rm ini} = t_{\rm kink}$	Particles from kinks subdominant	$P_{\rm kink} \sim \frac{N_k \mu^{1/2}}{\ell} \sim \Gamma G\mu^2$

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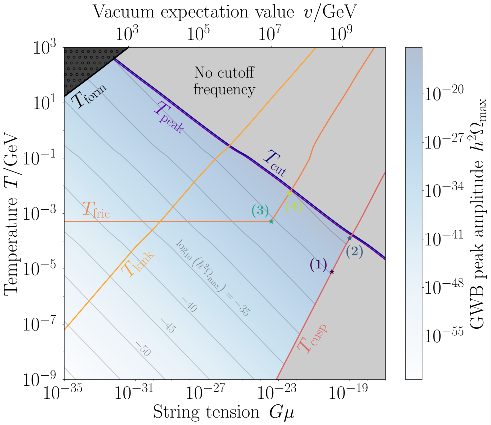
❸	$t_{\rm ini} = t_{\rm kink}$	Particles from kinks subdominant	$P_{\rm kink} \sim \frac{N_k \mu^{1/2}}{\ell} \sim \Gamma G\mu^2$
❹	$t_{\rm ini} = t_{\rm cusp}$	Particles from cusps subdominant	$P_{\rm cusp} \sim \frac{N_c \mu^{3/4}}{\ell^{1/2}} \sim \Gamma G\mu^2$

Parameter space

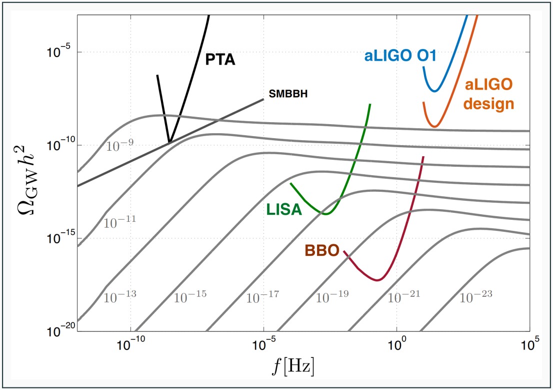
[**KS**, Schröder: 2405.10937]



- $G\mu$ and $T_{\rm ini}$ values resulting in a cutoff frequency in the $k = 1$ GWB spectrum
- Hierarchy of temperature scales for $G\mu \sim 10^{-20}$: $T_{\rm cusp} \ll T_{\rm fric} \ll T_{\rm kink} \ll T_{\rm form}$

Earlier results in the literature

[1709.02434]



What's new?

$$\Omega_{\rm GW}(f) = \frac{8\pi}{3H_0^2}(G\mu)^2 \sum_{k=1}^{k_{\max}} \frac{\Gamma}{H^q_{k_{\max}}} \frac{1}{k^q} \frac{2k}{f} \int_{t_{\rm ini}}^{t_0} dt \left(\frac{a(t)}{a_0}\right)^5 n\left(\frac{2k}{f}\frac{a(t)}{a_0}, t\right)$$

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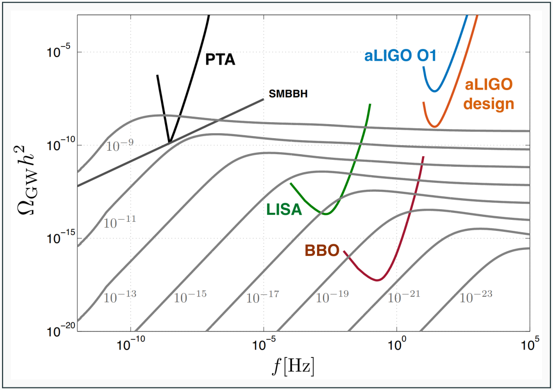
[1709.02434]



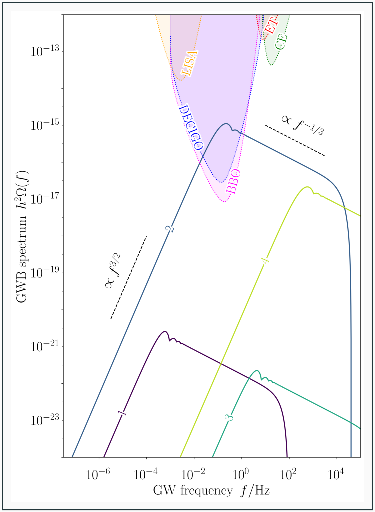
What's new? Nothing! Our $\Omega_{\rm GW}$ is standard, but we do not integrate from $t_{\rm ini} = 0$

$$\Omega_{\rm GW}(f) = \frac{8\pi}{3H_0^2}(G\mu)^2 \sum_{k=1}^{k_{\rm max}} \frac{\Gamma}{H^q_{k_{\rm max}}} \frac{1}{k^q} \frac{2k}{f} \int_{t_{\rm ini}}^{t_0} dt \left(\frac{a(t)}{a_0}\right)^5 n\left(\frac{2k}{f}\frac{a(t)}{a_0}, t\right)$$

Example spectra

Numerical spectra based on VOS loop number densities

[**KS**, Schröder: 2405.10937]

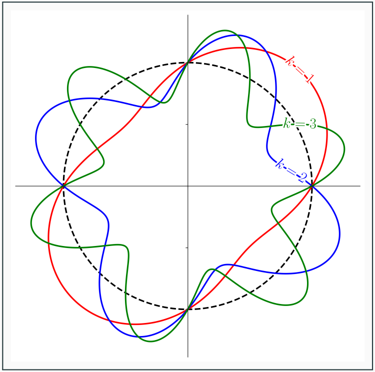


	$\log_{10}(G\mu)$	$\log_{10}(T_{\mathrm{ini}}/\mathrm{GeV})$	
1	-20.0	-5.1	(T_{cusp})
2	-19.0	-3.9	(T_{cusp})
3	-23.4	-3.3	(T_{fric})
4	-22.4	-2.3	(T_{fric})

- No fine-tuning required
- Sweet spot where signal even observable by BBO and DECIGO
- Power-law behavior can be understood analytically

Challenge: Subtraction of galactic and extragalactic foregrounds. Impossible?

Summation of oscillation modes



Total GWB spectrum

$$\Omega_{\rm GW}(f) = \sum_{k=1}^{k_{\rm max}} \frac{\Omega_{\rm GW}^{(1)}(f/k)}{H_{k_{\rm max}}^{q}\, k^q}$$

can be written in terms of $\Omega_{\rm GW}^{(1)}$, i.e., spectrum from the fundamental mode

Simple approximation for $\Omega_{\rm GW}^{(1)}$

$$\Omega_{\rm GW}^{(1)} \approx \Theta(f_{\rm cut} - f)\, \mathcal{A}\, f^{3/2}$$

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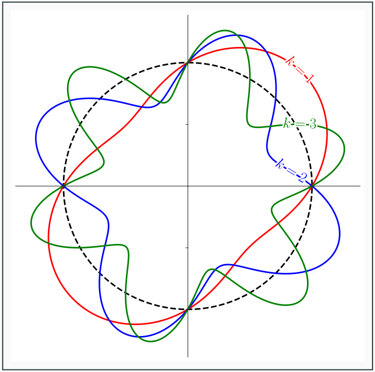

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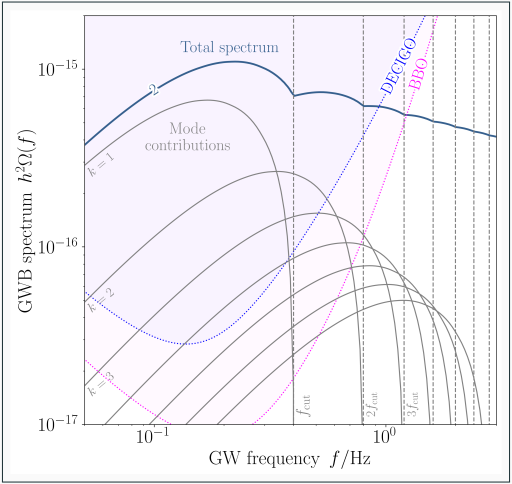
Power-law behavior of the total GWB spectrum at low and high frequencies

$$h^2\Omega_{\rm low} \propto f^{3/2}\,, \qquad h^2\Omega_{\rm high} \propto \frac{1}{q + {}^{1}\!/_{2}} \left(\frac{f_{\rm cut}}{f}\right)^{q-1}$$

Features in the GW spectrum

[**KS**, Schröder: 2405.10937]



Novel features in the spectrum: Series of peaks and dips at integer multiples of $f_{\rm cut}$ on top of a broken power law ($f^{3/2} \to f^{-1/3}$) $\to$ **Clear target for GW experiments**

Loop number and energy densities

Assumption: $G\mu$ is so low that no loop has fully decayed yet because of GW emission

Consequence: All loops produced since t_{ini} still exist in our present Universe

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Consequence: All loops produced since $t_{\rm ini}$ still exist in our present Universe

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Present-day loop number density

$$N(t_0) \approx \int_0^\infty d\ell \, n_{\rm RM}(\ell, t_0) \sim \frac{50}{\rm kpc^3} \left(\frac{10^2\,\rm s}{t_{\rm ini}}\right)^{3/2}$$

Present-day loop energy density

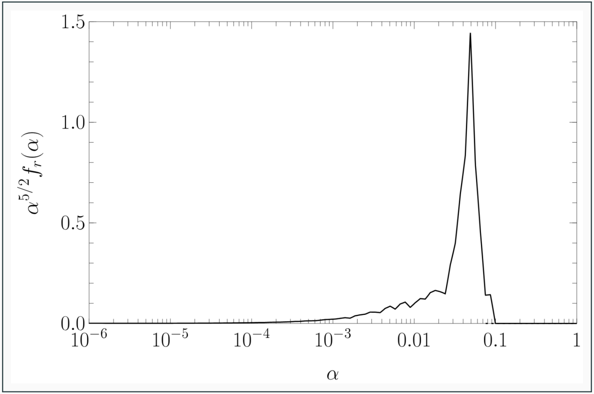
$$h^2\Omega(t_0) \approx \frac{1}{\rho_{\rm crit}} \int_0^\infty d\ell \, \mu \, \ell \, n_{\rm RM}(\ell, t) \simeq 10^{-13} \left(\frac{G\mu}{10^{-19}}\right) \left(\frac{10^2\,\rm s}{t_{\rm ini}}\right)^{1/2}$$

- Cosmologically harmless
- Signatures from nearby loops? **Microlensing**, **GW bursts**?

Outline

Distribution of initial loop lengths

[1309.6637]



Sharp spectral features follow from the assumption of a unique initial loop length, $\ell_* \simeq 2\alpha\, t_*$ with $\alpha \simeq 0.05$. But initial loop length deviates from perfect delta peak.

New number densitities

Loop number density in terms of the loop production function

$$n(\ell, t) = \int_{t_{\rm ini}}^{t} dt' \, f(\ell', t') \left(\frac{a(t')}{a(t)} \right)^3 , \quad \ell' = \ell + \Gamma G\mu \left(t - t' \right)$$

Standard choice in the velocity-dependent one-scale (**VOS**) model

$$f(\ell, t) = \frac{\mathcal{F} C_r}{2\alpha t^4} \, \delta(\ell - 2\alpha t) , \quad \alpha \simeq 0.05$$

Numerical simulations by Blanco-Pillado, Olum, and Shlaer (**BOS**) better described by

$$f(\ell, t) = \frac{A_r}{\sqrt{2\pi}\,\sigma \ell^5 \, t^{5/2}} \exp\left[-\frac{1}{2\sigma^2} \left(\ln\left(\frac{\ell}{2t} \right) - \nu \right)^2 \right] , \quad \nu \simeq -3.0 , \quad \sigma \simeq 0.14$$

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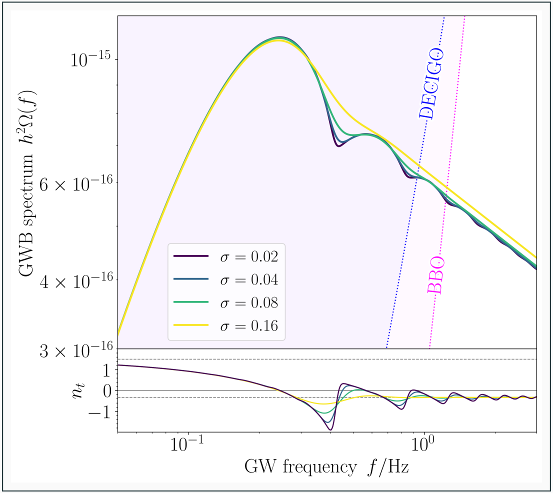
New loop number densities providing a better description of the BOS results

$$n_{\mathrm{RR}}\left(\ell, t\right) \approx \frac{A_r\left(\mathrm{erf}_{t_{\mathrm{ini}}} - \mathrm{erf}_t\right)/2}{t^{3/2}\left(\ell + \Gamma G\mu t\right)^{5/2}} , \quad n_{\mathrm{RM}}\left(\ell, t\right) \approx \left(\frac{a_{\mathrm{eq}}}{a\left(t\right)}\right)^3 \frac{A_r\left(\mathrm{erf}_{t_{\mathrm{ini}}} - \mathrm{erf}_{t_{\mathrm{eq}}}\right)/2}{t_{\mathrm{eq}}^{3/2}\left(\ell + \Gamma G\mu t\right)^{5/2}}$$

Smeared GW spectrum

[**KS**, Schröder: 2405.10937]



Series of peaks and dips washed out for broader distributions of initial loop lengths

Still, even for broad distributions, oscillations in the index n_t may remain detectable

Outline

Summary and outlook

Low-scale cosmic strings

$$G\mu \sim 10^{-33} \cdots 10^{-19} \qquad \longleftrightarrow \qquad v \sim 10^{2}\,\mathrm{GeV} \cdots 10^{9}\,\mathrm{GeV}$$

Initial time $t_{\rm ini} \neq 0$: Loop production no longer impeded by thermal friction, GW emission from loops no longer subdominant to particle emission from cusps and kinks,

$$\Omega_{\rm GW} = \frac{16\pi}{3H_0^2}\,(G\mu)^2 \sum_{k=1}^{k_{\rm max}} \frac{kP_k}{f} \int_{t_{\rm ini}}^{t_0} dt\ [\cdots]$$

- No loop produced at $t \geq t_{\rm ini}$ ever shrinks to zero length $\rightarrow$ microlensing, bursts?
- Sharp frequency cutoff in $k = 1$ GWB spectrum, series of peaks and dips in $\Omega_{\rm GW}$

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Next steps

- Nonscaling models where particle emission occurs whenever $\ell \leq \ell_{\mathrm{crit}}$
- Model building: Cosmic strings $v \sim 10^9\,\mathrm{GeV}$, GWs from phase transition?

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Stay tuned! Thanks a lot for your attention