The Dawn of Gravitational Wave Cosmology
(Benasque Science Center, Apr 27- May 17, 2025)

CENTRO DE CIENCIAS
DE BENASQUE
PEDRO PASCUAL

Impact of correlated noise on SGWB reconstruction with Einstein Telescope

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Stochastic GW Background

Faint and diffuse signal both of astrophysical and cosmological origin

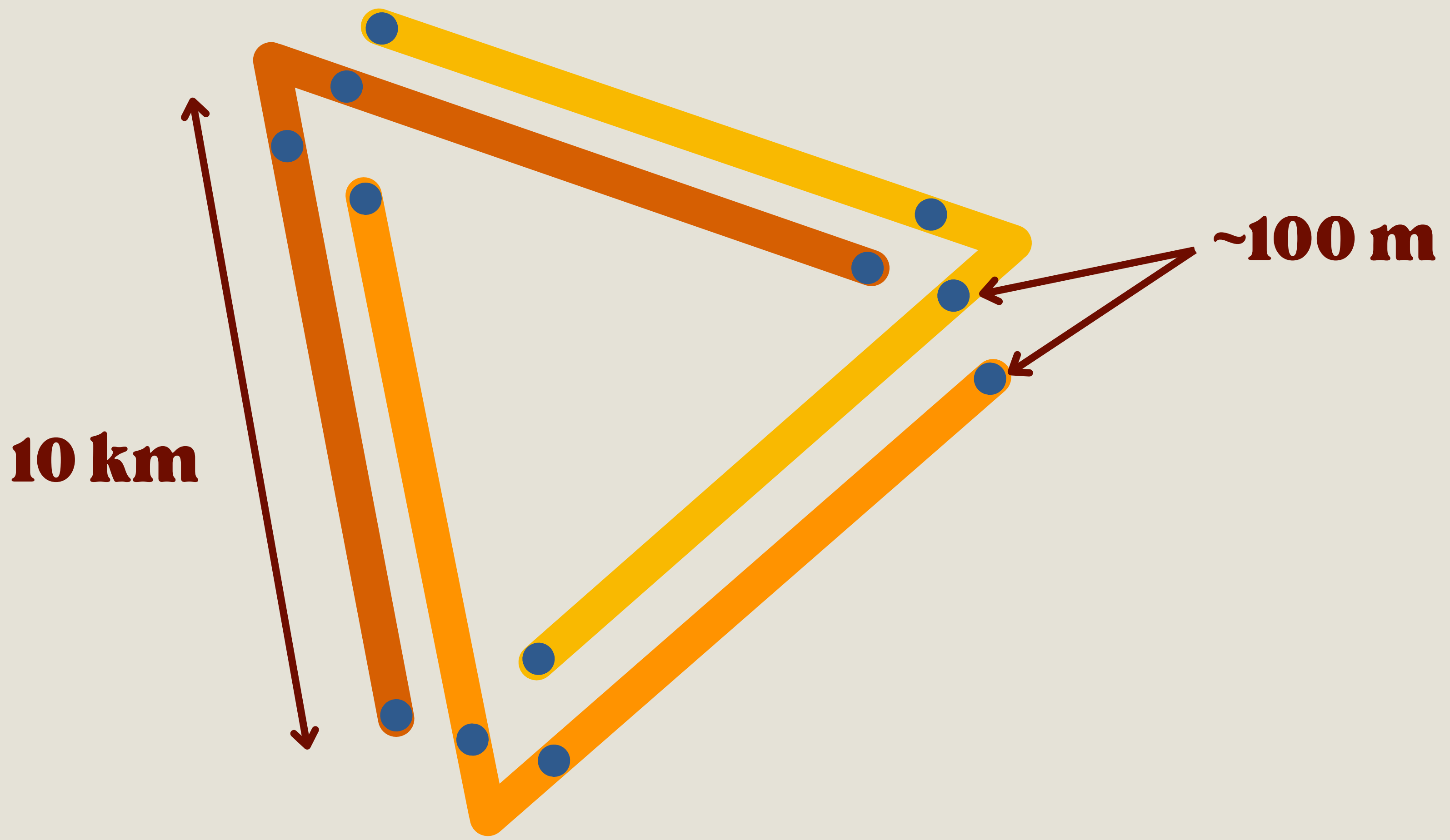
Its detection would give insights about:

1. Population of compact binaries
2. Early Universe
3. New Physics beyond Standard Model

It is among Einstein Telescope scientific objectives

Challenges in SGWB detection

Triangular configuration, due to its geometry, has shown sources of correlated noise



At these scales, there is Newtonian correlated noise

Correlated Newtonian Noise

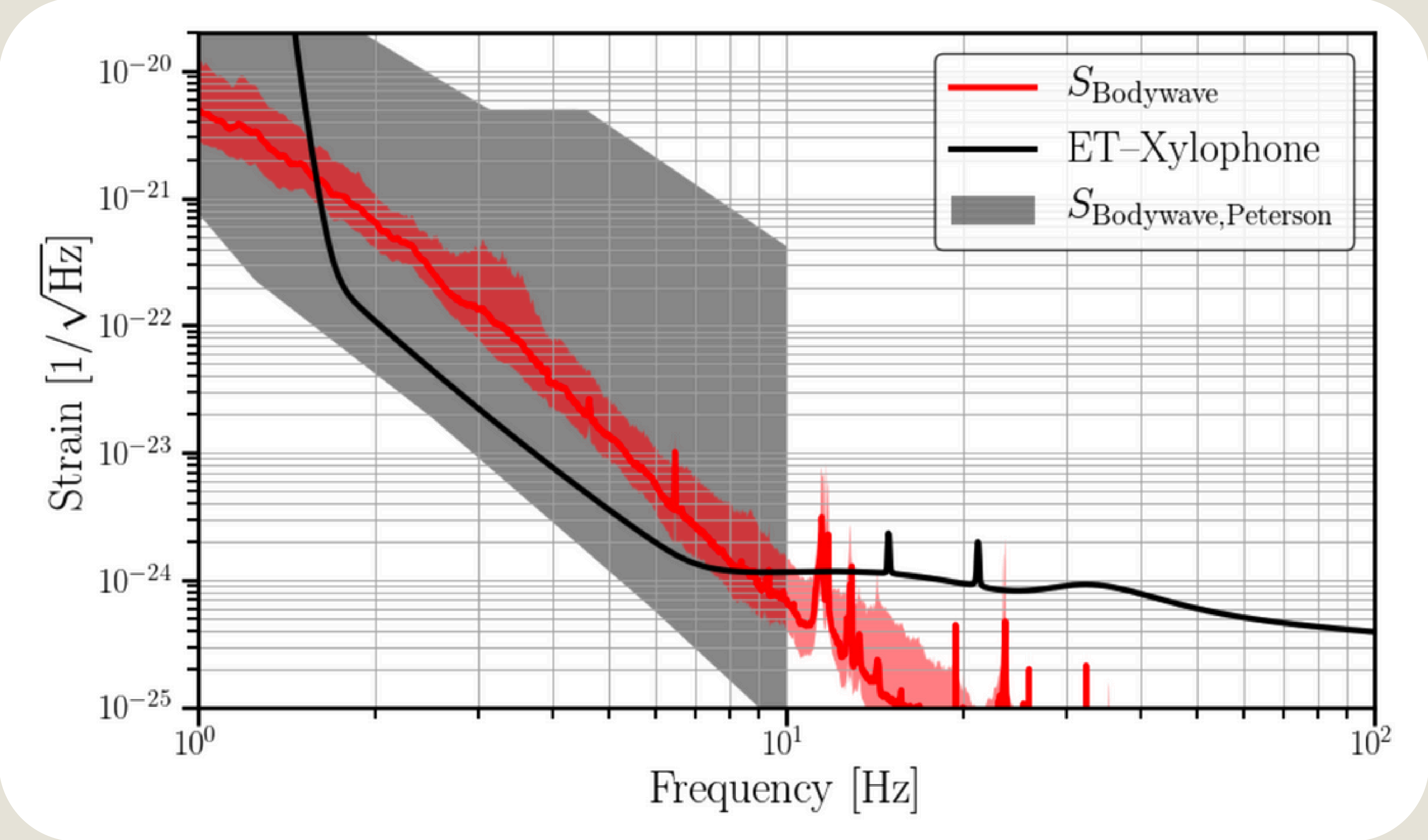
Newtonian noise is a disturbance produced in GW detectors by local fluctuations in the Gravitational field

local fluctuations generated by changes in density of rocks → originated by passing <u>seismic waves</u>

- body waves → Underground colocated interferometers will only be affected by this effect
- surface waves (Rayleigh Waves)

[Janssens et al. arXiv:2206.06809]

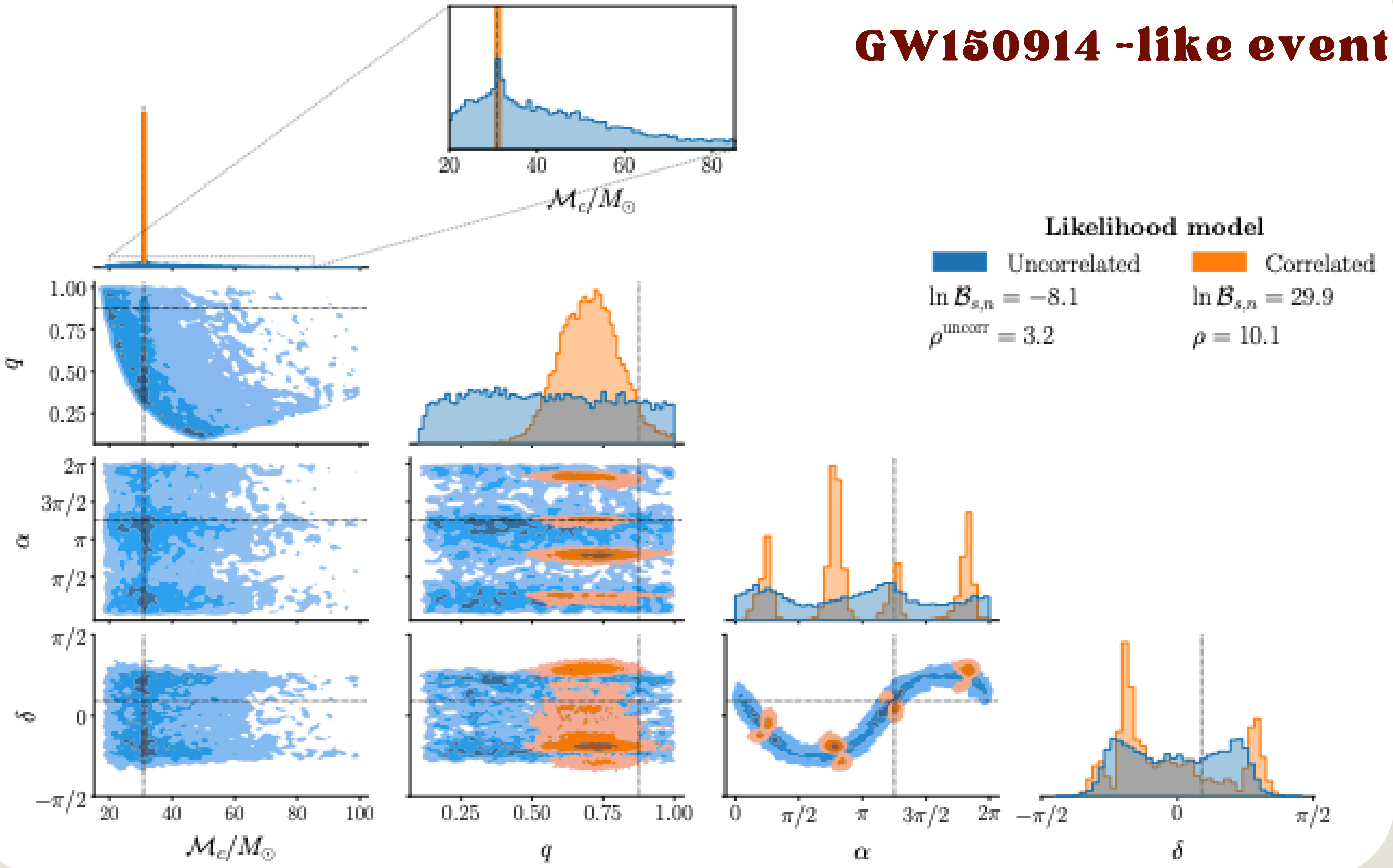
Correlated Newtonian Noise



Strain of the NN with CSD of the Homestakes underground seismometers displacement measurement with a horizontal distance of 405m at a depth of 610 m

[Janssens et al. arXiv:2206.06809]

Impact of Correlated Noise in resolved events



[F.Cireddu et al. arXiv:2312.14614]

This Work

[arXiv:2501.09057]

What do we do?

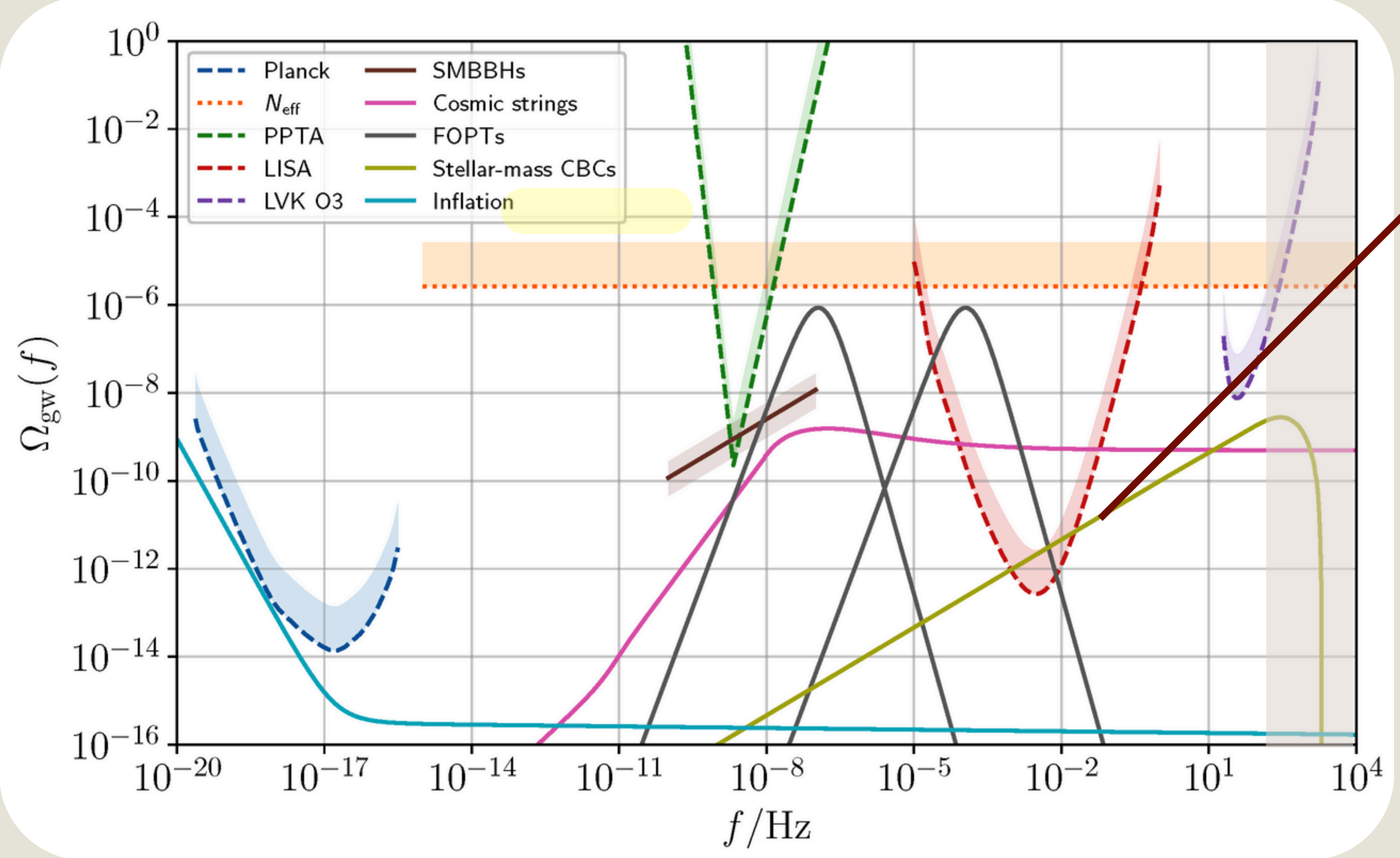
We study the impact of correlated noise on the Astrophysical SGWB detection and relative parameter estimation for ET in the Triangular configuration comparing it to the 2L one

... why do we care?

Correlated noise is a threat to all the possible Astrophysical and Cosmological sources of SGWB

SGWB Energy Density Spectrum

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\log f}$$

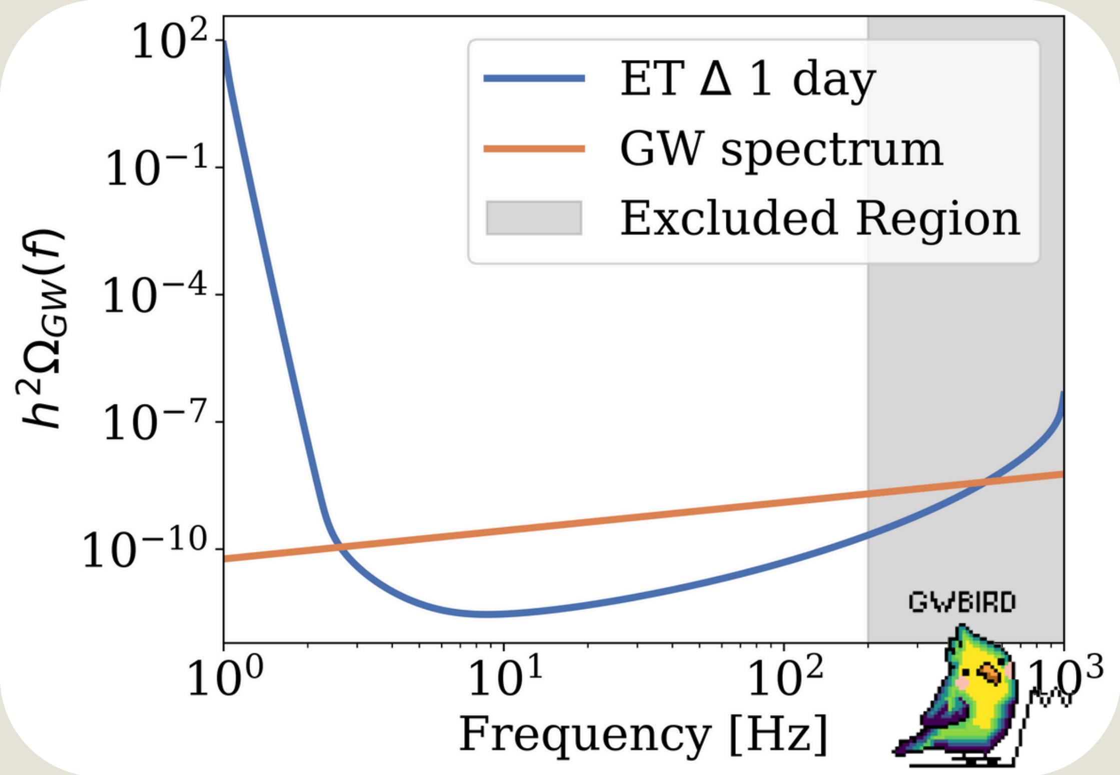


[Renzini et al. arXiv:2202.00178]

Power law behavior

$$\Omega_{\rm GW}(f) = A_{\rm GW}\left(\frac{f}{f_{\rm pivot}}\right)^{n_{\rm GW}}$$

$$A_{\rm GW} = 10^{-9}$$
$$n_{GW} = 2/3$$



[Caporali & Ricciardone, soon to appear]

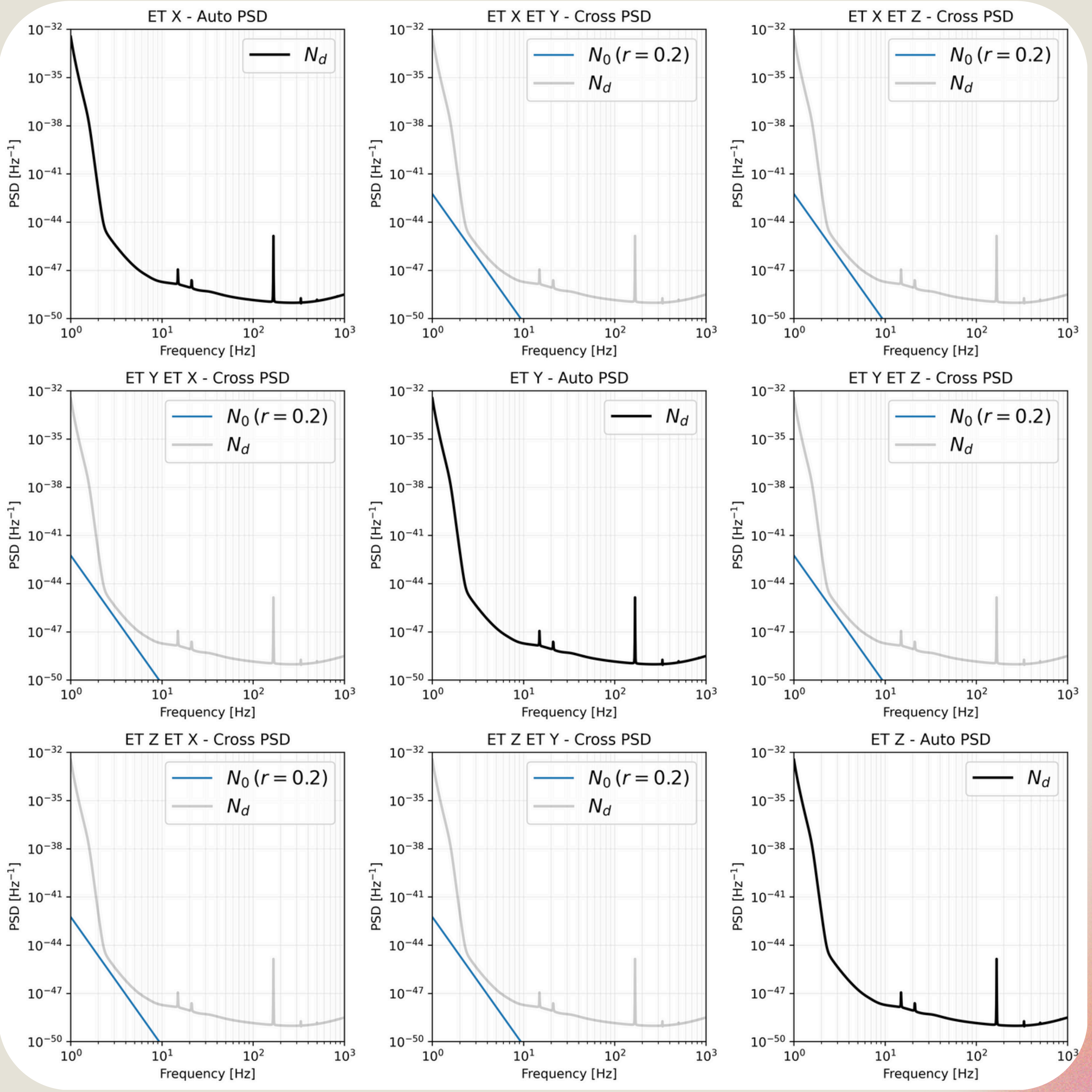
Noise Covariance

$$\langle \tilde{n}_I(f)\tilde{n}^*_J(f')\rangle \equiv \frac{\delta(f-f')}{2}N_{IJ}(f)$$

Triangular Configuration

$$N_{IJ}(f) \equiv \begin{pmatrix} N_d & N_o & N_o \\ N_o & N_d & N_o \\ N_o & N_o & N_d \end{pmatrix} \qquad -1/2 \leq \frac{N_o(f)}{N_d(f)} \leq 1$$

$$N_o(f) = N_d(f_*)\, r \left(\frac{f}{f_*}\right)^{n_{\rm noise}} \qquad \begin{aligned} n_{\rm noise} &= -8 \\ r &\in (-0.5, 1) \\ f_* &= 2.75\,\text{Hz} \end{aligned}$$



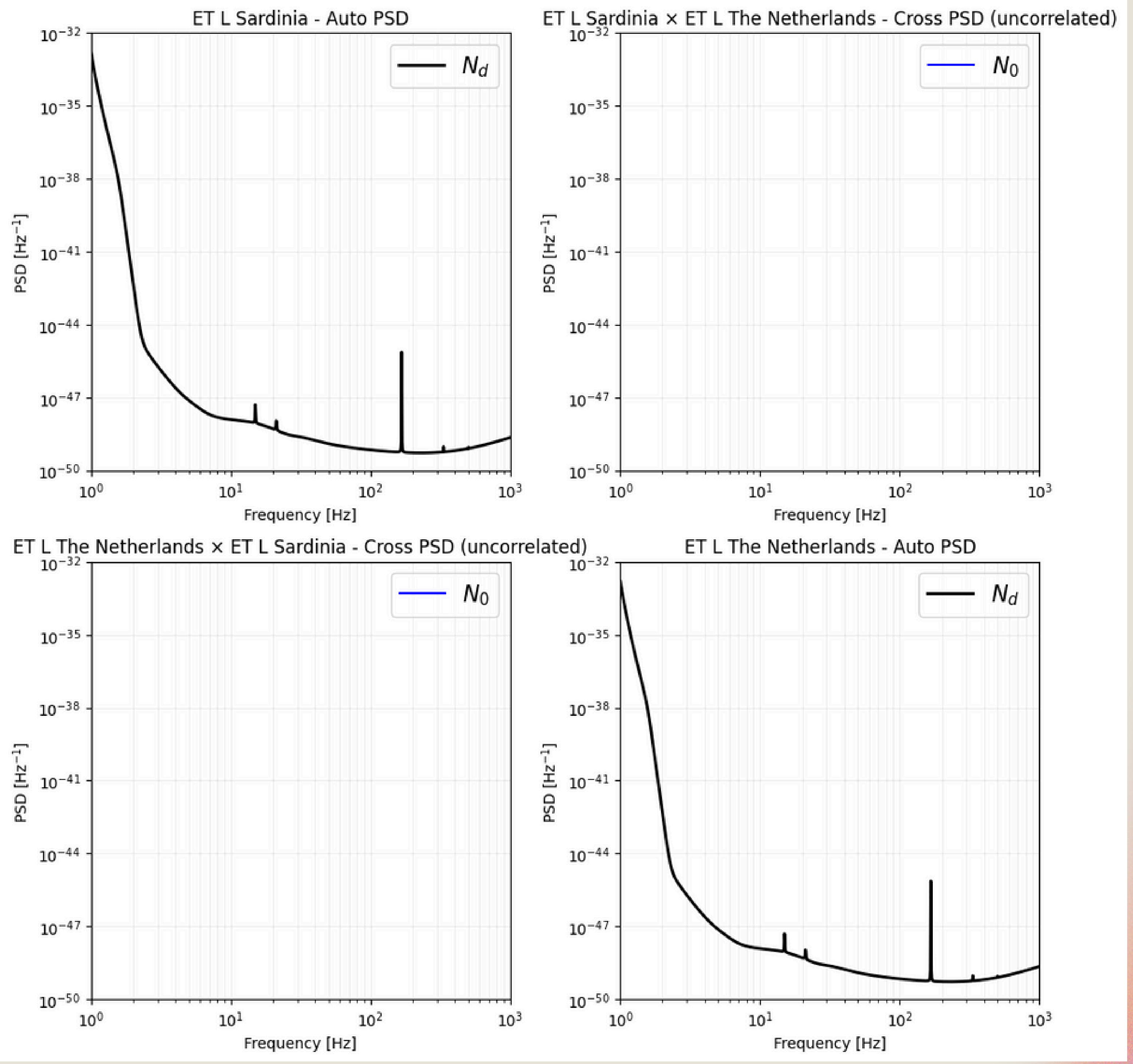
[Flauger et al.arXiv:2009.11845]
[Janssens et al. arXiv:2402.17320]

Noise Covariance

$$\langle \tilde{n}_I(f)\tilde{n}^*_J(f')\rangle \equiv \frac{\delta(f-f')}{2} N_{IJ}(f)$$

2L Configuration

$$N_{IJ}(f) \equiv \begin{pmatrix} N(f) & 0 \\ 0 & N(f) \end{pmatrix}$$



Likelihood for correlated and uncorrelated configuration

We introduce a time-averaged estimator of a quadratic combination of the data

$$\hat{C}_{IJ}(f) \equiv \sum_t \frac{2}{T_{\rm obs} S_0(f)} \Re e[\tilde{s}_I(t,f)\tilde{s}_J^*(t,f)]$$

$T_{obs} = 1\,\text{day}$ - $\quad S_0(f) = \dfrac{3H_0^2}{10\pi^2 f^3} \quad$ $T_{seg} = 4\,\text{s}$ -

Averaging over many time segments allows us to treat the estimator of the SGWB as a Gaussian random variable, due to the central limit theorem

[Abbott et al. arXiv:1903.02886]
[Abbott et al.arXiv:2101.12130]

Likelihood for correlated and uncorrelated configuration

Average of the estimator

$$\bar{C}_{IJ}(f) \equiv \left\langle \hat{C}_{IJ}(f) \right\rangle = \gamma_{IJ}(f)\Omega_{\rm GW}(f) + \frac{N_{IJ}(f)}{S_0(f)}$$

Variance of the estimator

$$\Sigma_{IJ}(f) \equiv \left\langle \left[\hat{C}_{IJ}(f) - \bar{C}_{IJ}(f)\right]^2 \right\rangle = \frac{\bar{C}_{II}\bar{C}_{JJ} + \bar{C}_{IJ}^2}{2N_{\rm seg}}$$

Gaussian Likelihood

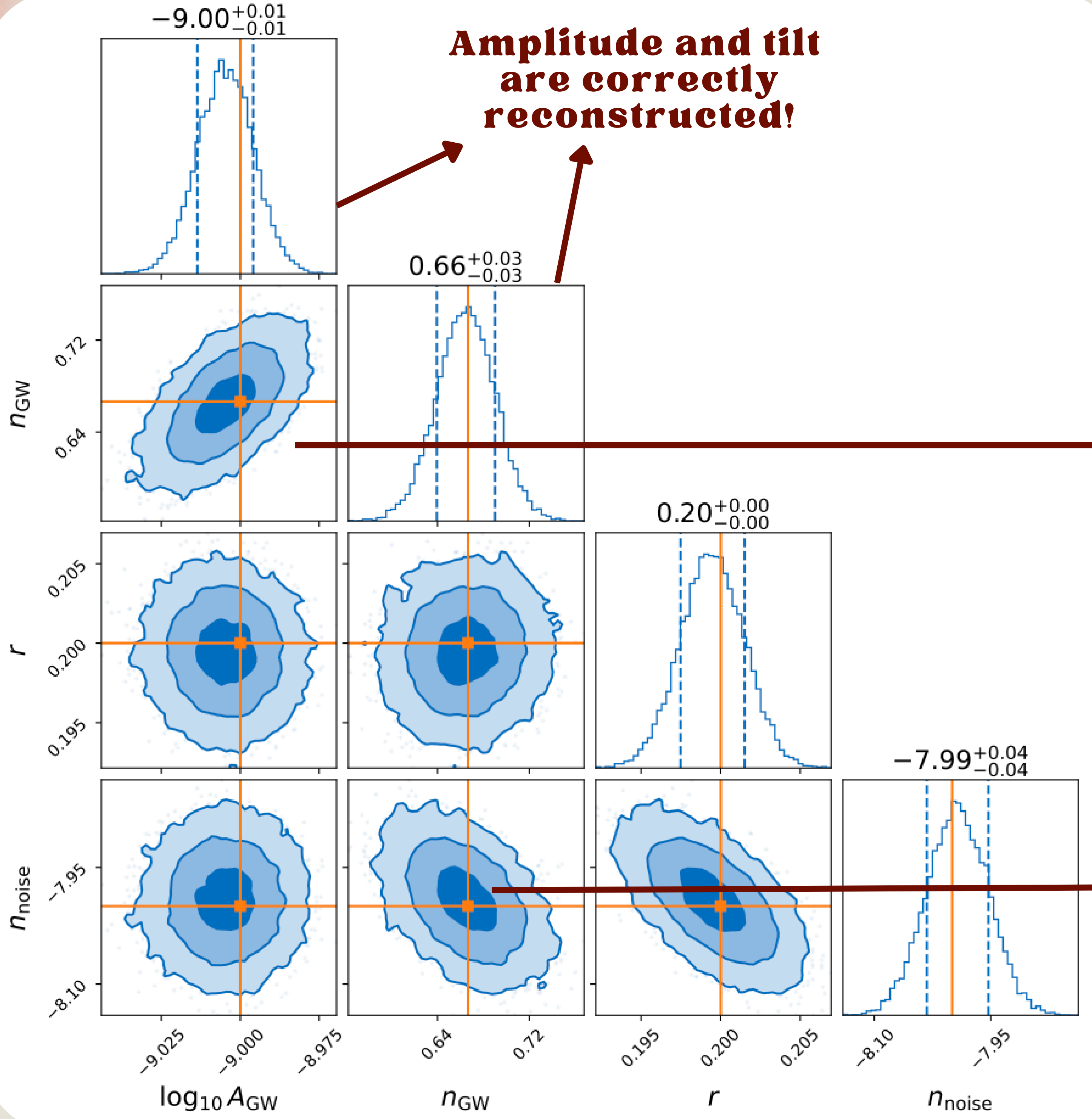
$$\mathcal{L} = \prod_{I,J} \frac{1}{\sqrt{2\pi\Sigma_{IJ}}} \exp\left[-\frac{1}{2}\frac{(\hat{C}_{IJ} - \bar{C}_{IJ})^2}{\Sigma_{IJ}}\right]$$

Triangular Configuration
(XYZ to the AET basis)
(I, J) = {(A,A), (E,E)}

2L Configuration
(I, J) = (ET1, ET2)

Results

[arXiv:2501.09057]



Amplitude and tilt are correctly reconstructed!

Positive correlation

since the chosen pivot frequency for the SGWB, 25 Hz, is higher than the frequencies at which ET is most sensitive to the SGWB (10 Hz), a larger retrieved value of the tilt requires a larger value of the amplitude to fit the injected signal

Negative correlation

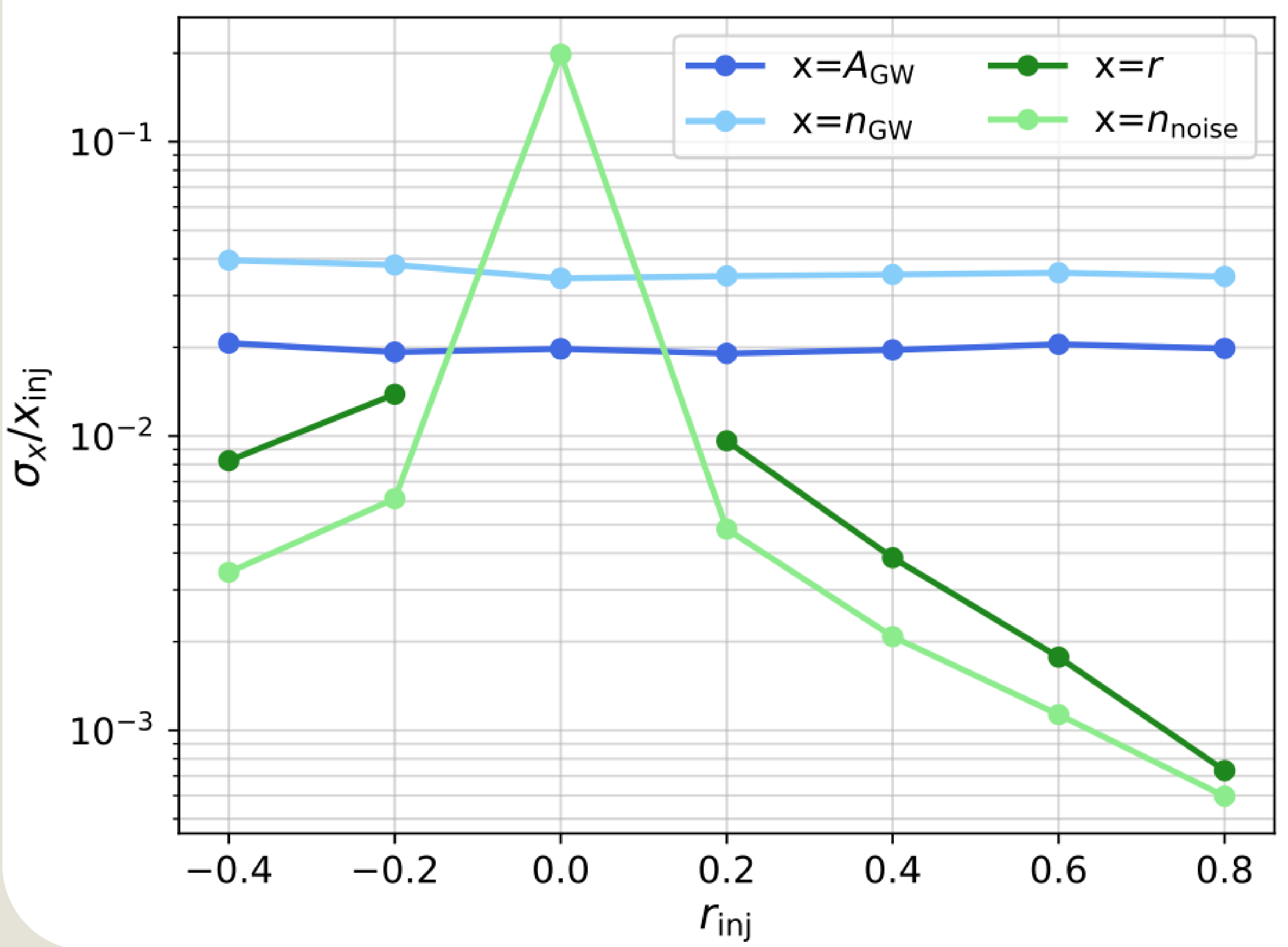
larger value of the tilt of the signal means less power at low frequencies, which requires a smaller (i.e., more negative) tilt of the noise to balance and fit the injected signal

Results

[arXiv:2501.09057]

Accurate reconstruction of the SGWB is actually possible!

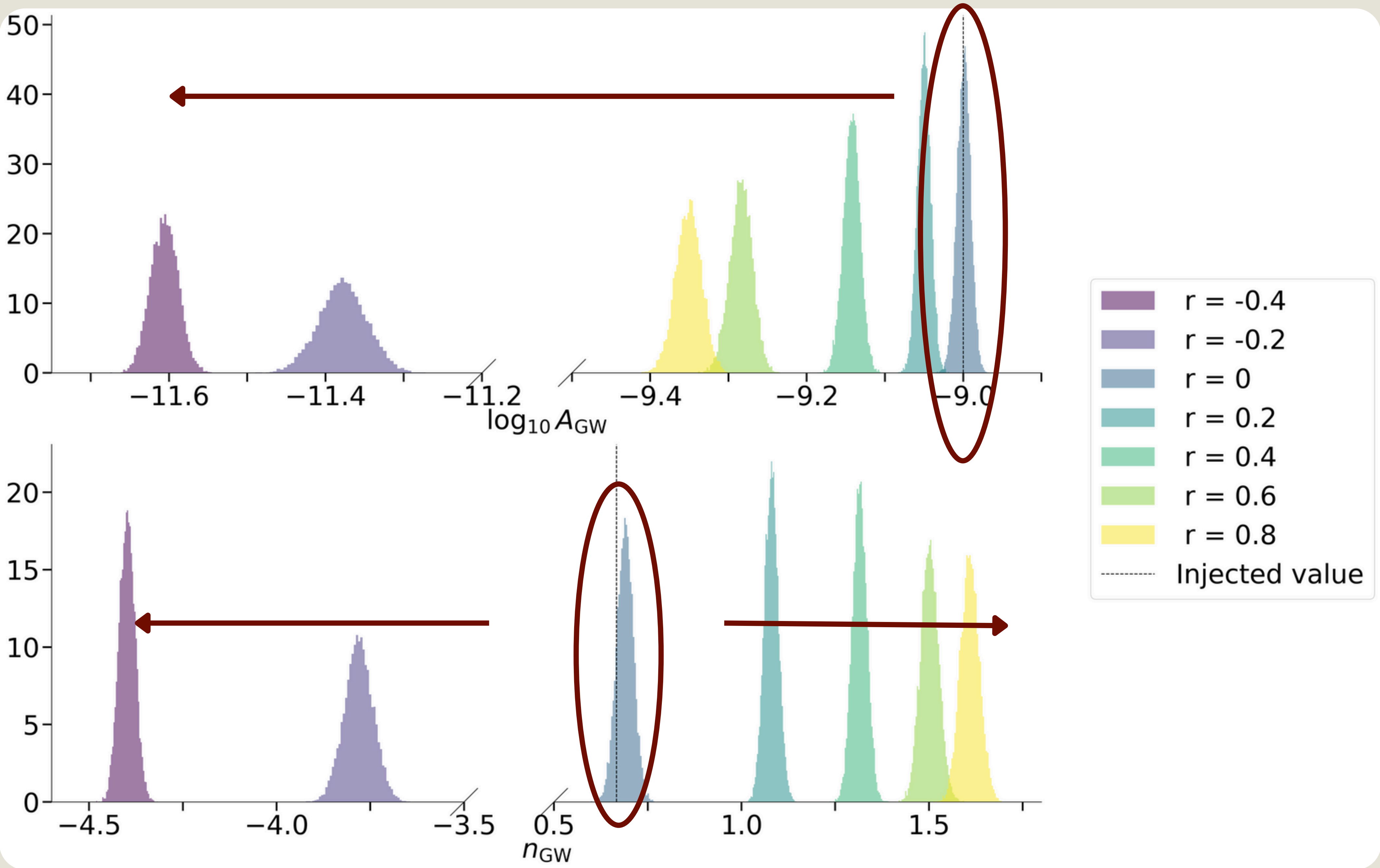
(If noise is properly modeled)



Results

[arXiv:2501.09057]

Neglecting correlated noise leads to biases



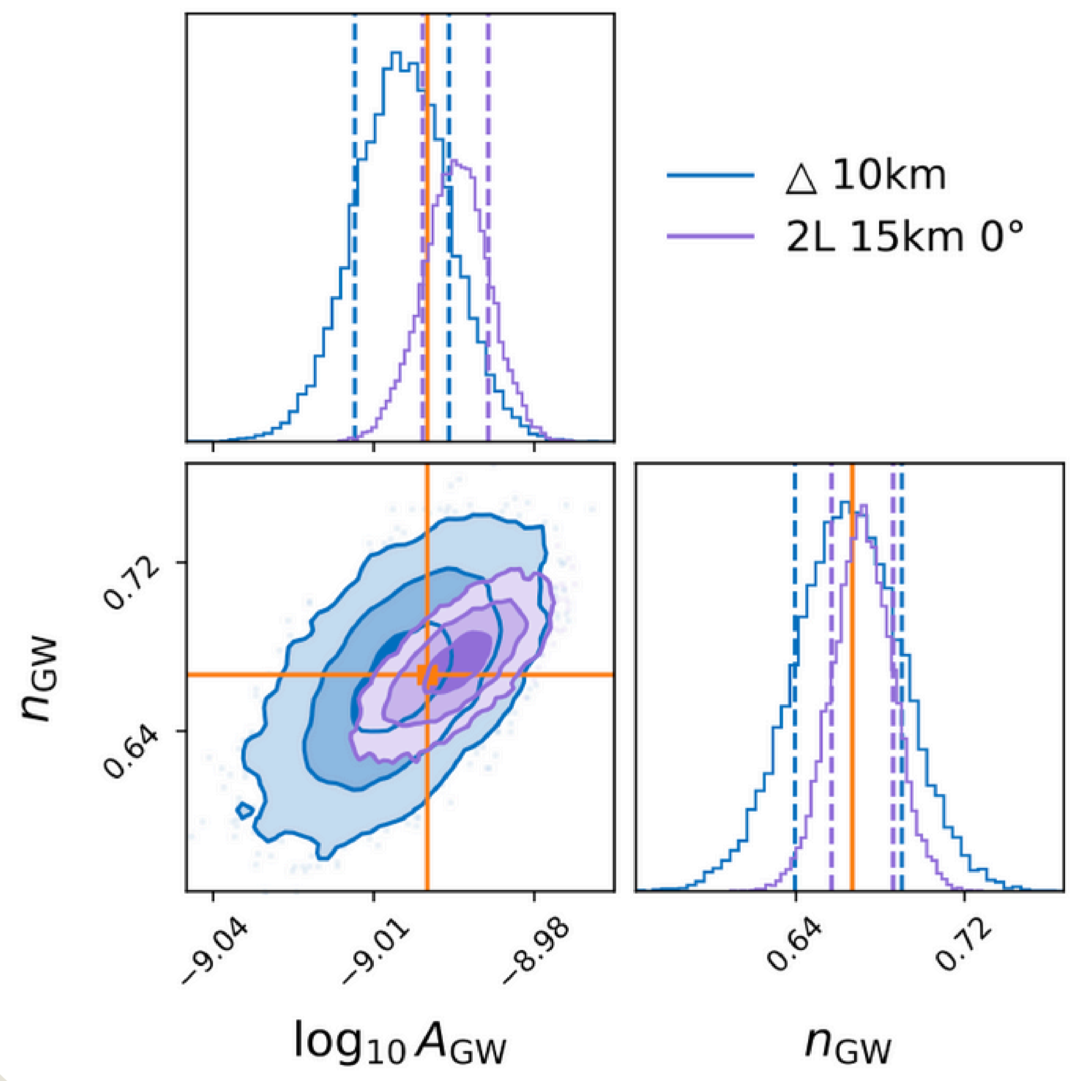
The injected parameters are correctly retrieved only for r=0 , since there is no correlated noise in this case and the likelihood that neglects it is accurate.

Results

[arXiv:2501.09057]

Triangular configuration is competitive to the 2L

(If noise is properly modeled)



Why the 2L performs slightly better?

The 15 km arms of the 2L provide better sensitivity to the SGWB compared to the triangle with 10 km arms

$$\mathrm{SNR}_{\Delta} \sim 67$$
$$\mathrm{SNR}_{2L} \sim 100$$

Marginalization in the triangular configuration results in further broadening in the posteriors

Preliminary Results

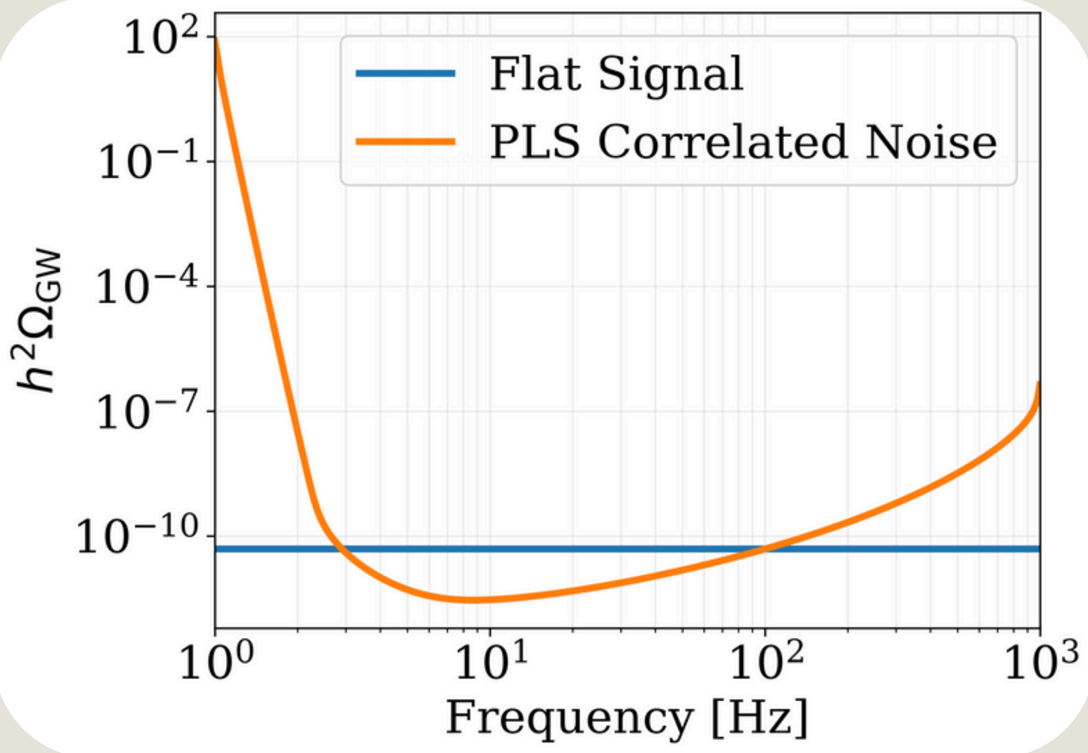
Flat Signal

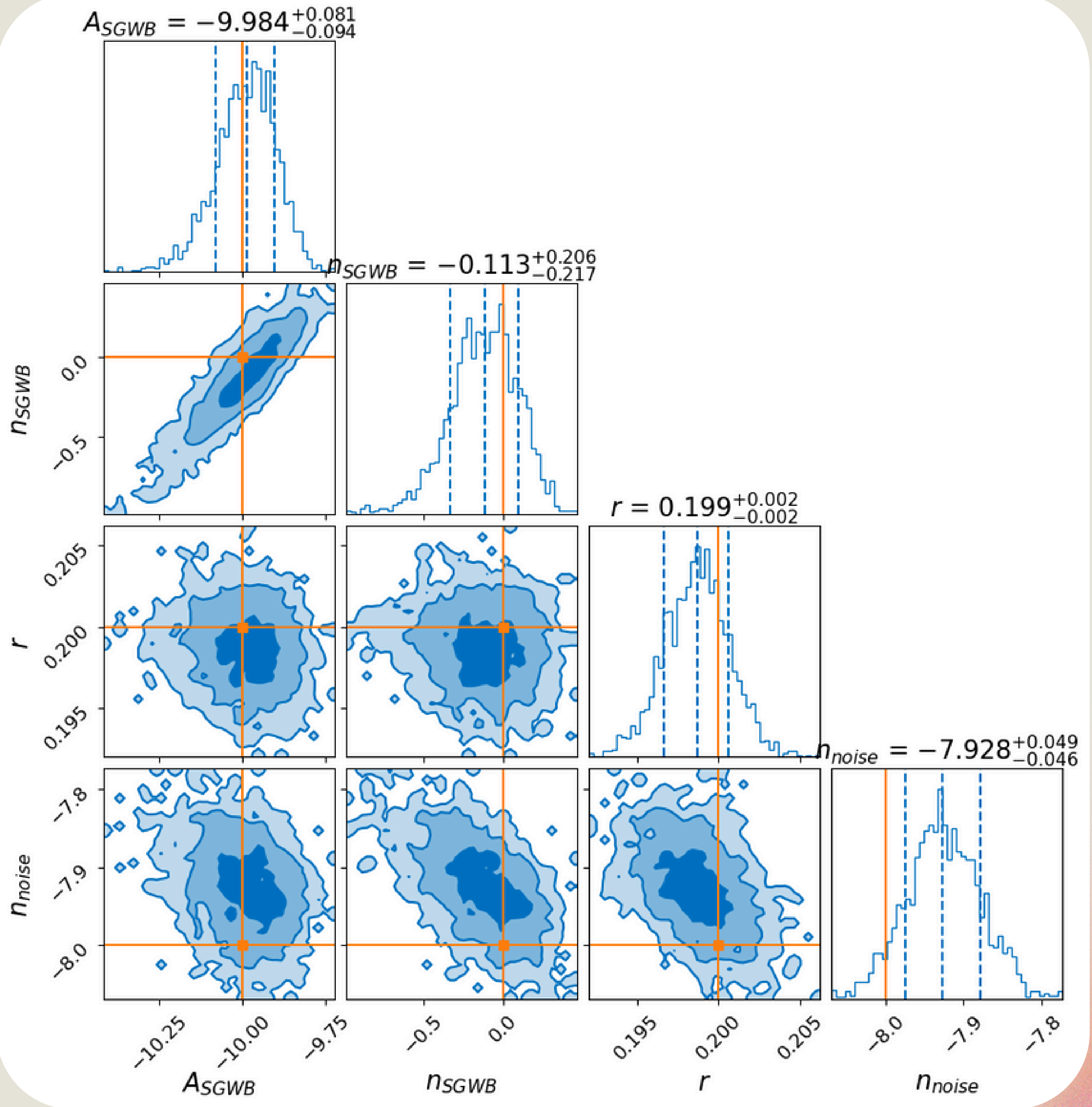
$$\Omega_{\rm GW}(f) = 10^{A_{\rm SGWB}} \left(\frac{f}{f_{\rm pivot}}\right)^{n_{\rm SGWB}}$$

$$A_{\rm SGWB} = -10$$
$$n_{\rm SGWB} = 0$$
$$\mathrm{SNR} \sim 17$$

Mimic cosmic strings with

$$G\mu \sim 10^{-11}$$



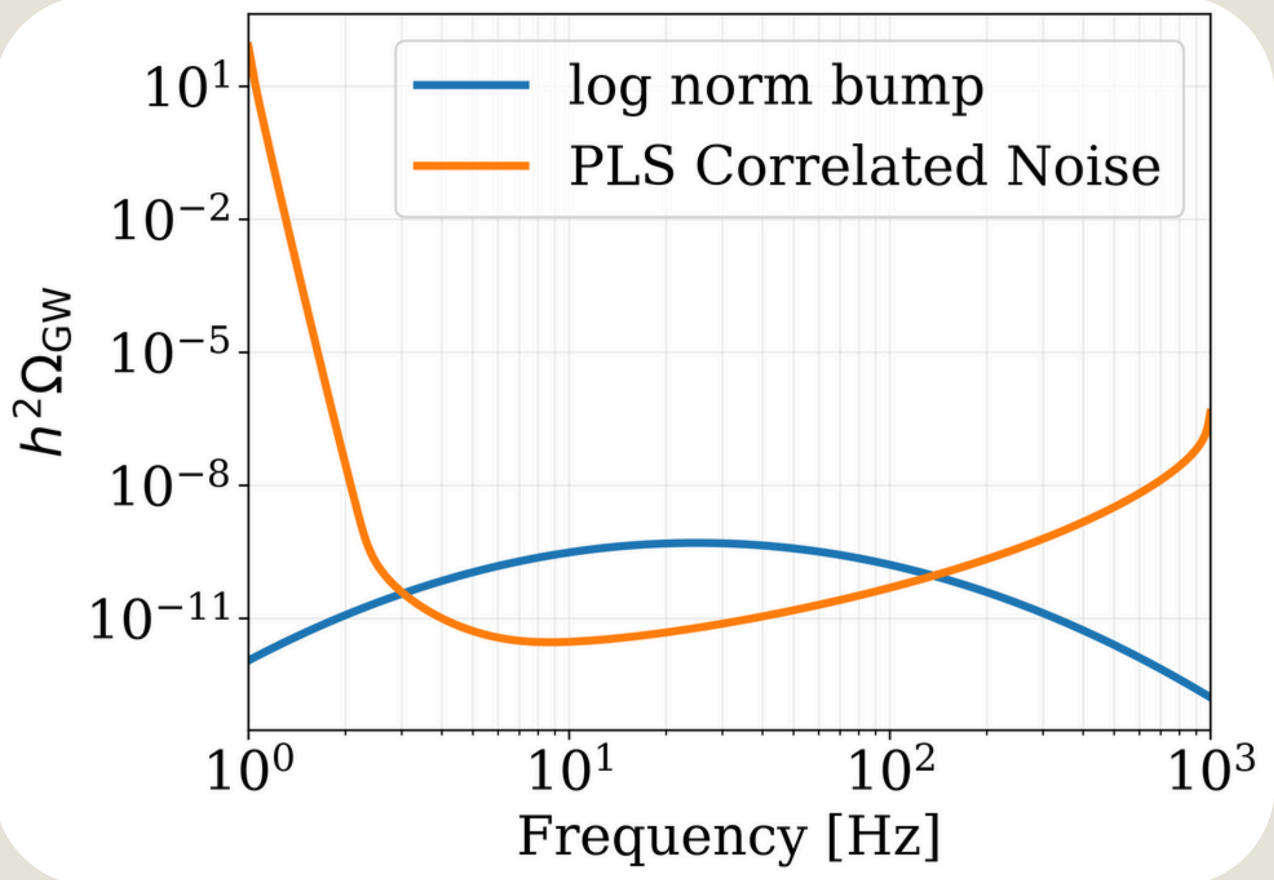


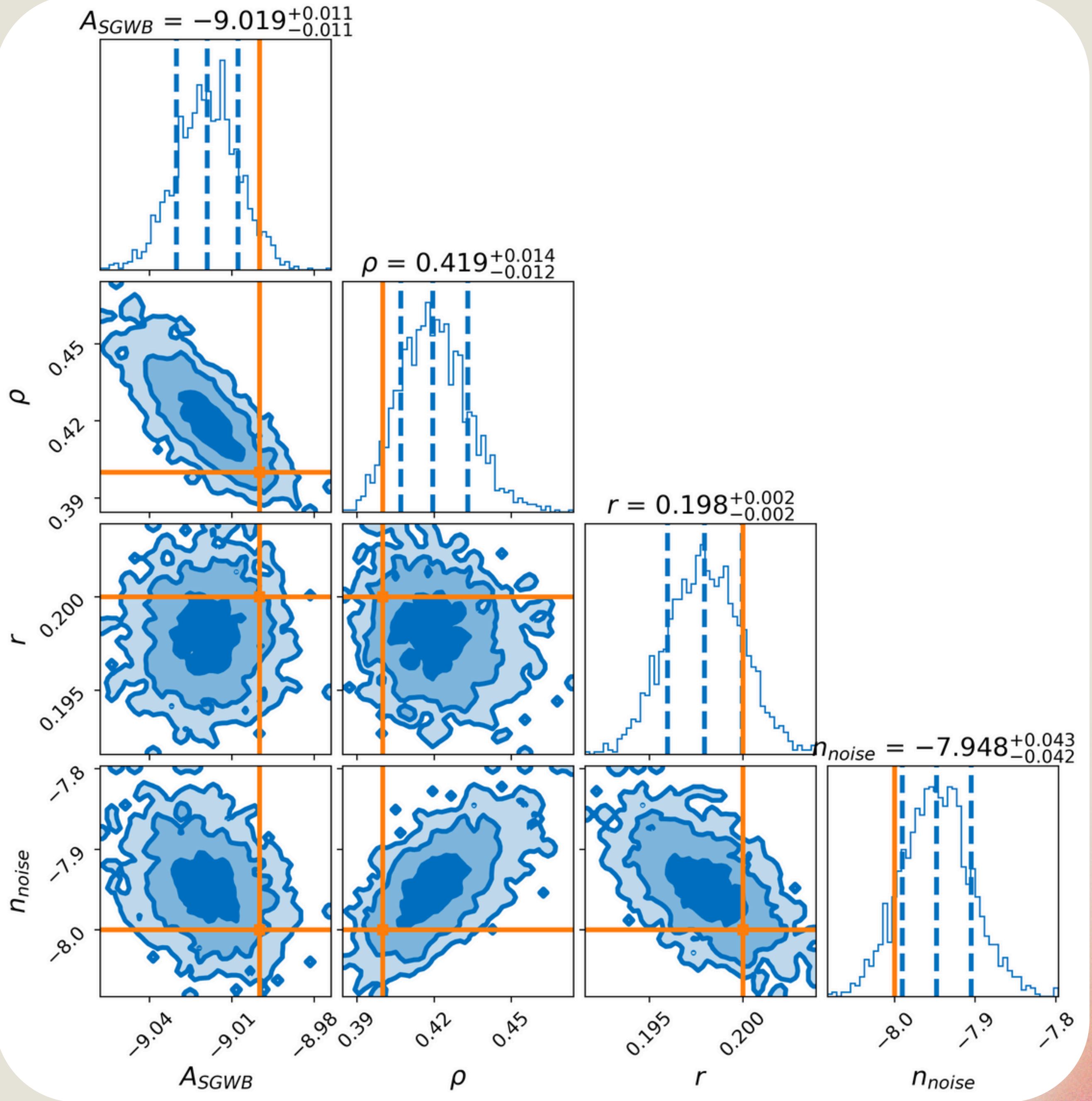
Preliminary Results

Log Normal Bump

$$\Omega_{\mathrm{GW}}(f) = 10^{A_{\mathrm{SGWB}}} e^{-\frac{1}{2\rho^2} log_{10}^2\left(\frac{f}{f*}\right)}$$

$$A_{\mathrm{SGWB}} = -9$$
$$\rho = 0.4$$
$$\mathrm{SNR} \sim 90$$





Take Home Message

[arXiv:2501.09057]

In the presence of correlated noise:

Accurate reconstruction of the SGWB is actually possible!

Neglecting correlated noise leads to biases

Triangular configuration is competitive to the 2L

(if noise is properly modelled)

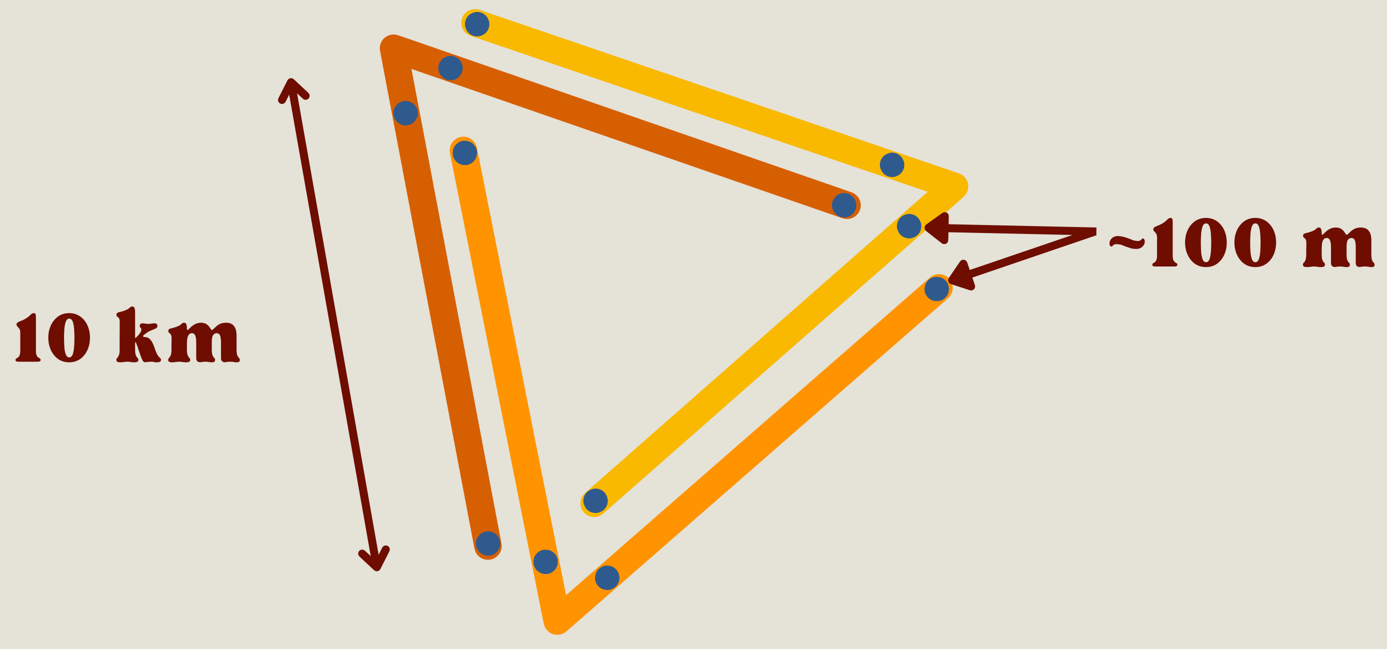
Thank you!

In case you were wondering...

Back-up slides

ET Configurations

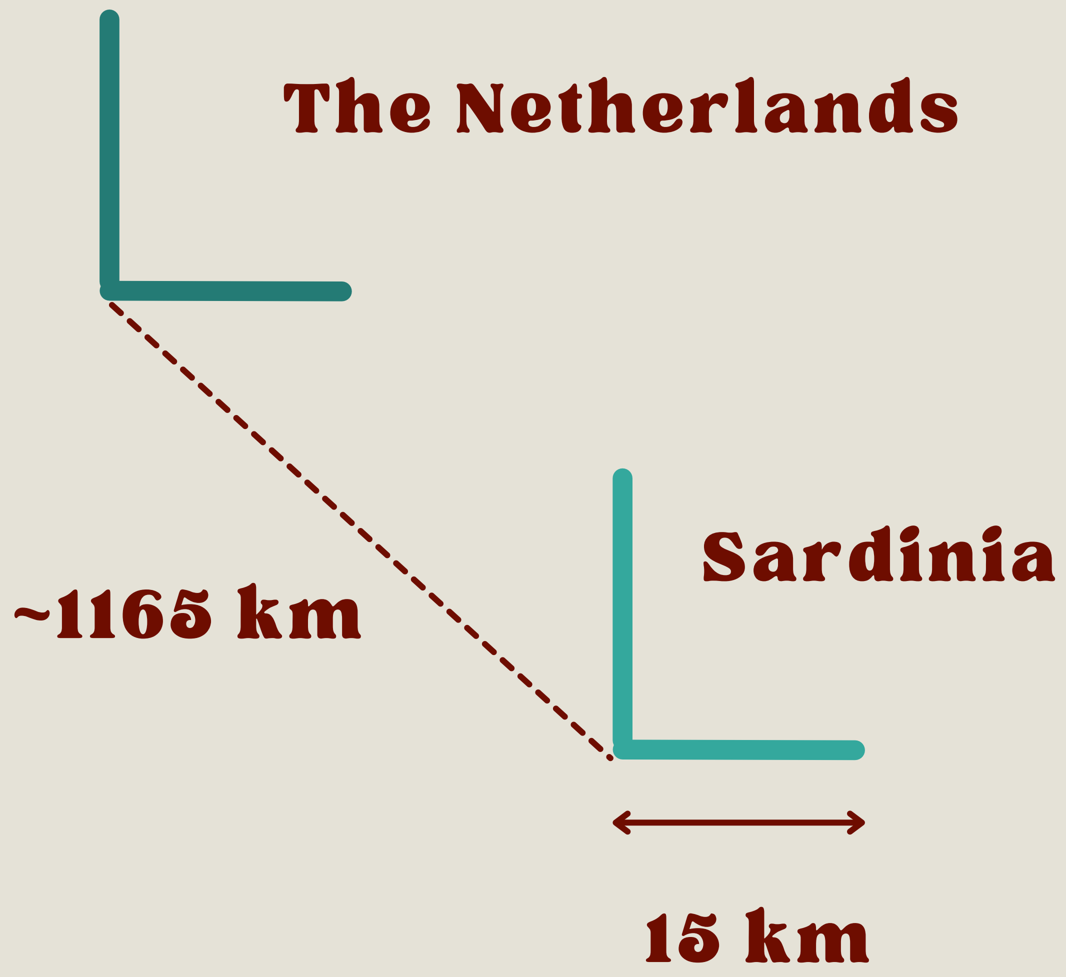
Triangular Configuration



Newtonian correlated noise

[Janssens et al.arXiv:2206.06809]

2L Configuration



[Branchesi et al. arXiv:2303.15923]

Stochastic GW Background

SGWB can be characterized as a superposition of GWs coming from all directions

$$h_{ij}(t,\mathbf{x}) = \int df\, d\hat{n} \sum_{\lambda} \tilde{h}_{\lambda}(f,\hat{n}) e^{\lambda}_{ij}(\hat{n}) e^{2\pi i f(t-\hat{n}\cdot\mathbf{x}/c)}$$

Two point correlation function

Stationary, isotropic and unpolarized

$$\langle \tilde{h}_{\lambda}(f,\hat{n}) \tilde{h}^{*}_{\lambda'}(f',\hat{n}') \rangle = \frac{\delta(f-f')}{2} \frac{\delta_{\hat{n}\hat{n}'}}{4\pi} \delta_{\lambda\lambda'} \frac{3H_0^2}{4\pi^2 f^3} \Omega_{\mathrm{GW}}(f)$$

[Romano & Cornish arXiv:1608.06889]
[Allen & Romano arXiv:gr-qc/9710117]

Stochastic GW Background

Consider the fourier transform of the data at an interferometer I

$$\tilde{s}_I(f) = \underbrace{\tilde{h}_I(f)}_{\text{signal}} + \underbrace{\tilde{n}_I(f)}_{\text{noise}}$$

signal → **Projected GW amplitude on the detector**

$$\tilde{h}_I(f) = \int d\hat{n} \sum_\lambda F_I^\lambda(f,\hat{n})\,\tilde{h}_\lambda(f,\hat{n})e^{-i2\pi f\left(\frac{\hat{n}\cdot\mathbf{x}_I}{c}\right)}$$

$$\left\langle \tilde{h}_I(f)\tilde{h}_J^*(f')\right\rangle \equiv \frac{\delta(f-f')}{2}\frac{3H_0^2}{10\pi^2 f^3}\gamma_{IJ}(f)\Omega_{\rm GW}(f)$$

Overlap Reduction Function

$$\gamma_{IJ}(f) \equiv \frac{5}{8\pi}\int d\hat{n}\sum_\lambda F_I^\lambda(f,\hat{n})F_J^{\lambda,*}(f,\hat{n})e^{-2\pi i f\hat{n}\cdot(\mathbf{x}_I-\mathbf{x}_J)/c}$$

[Caporali & Ricciardone, soon to appear]

[Romano & Cornish arXiv:1608.06889]
[Allen & Romano arXiv:gr-qc/9710117]

Injected Signal

[arXiv:2501.09057]

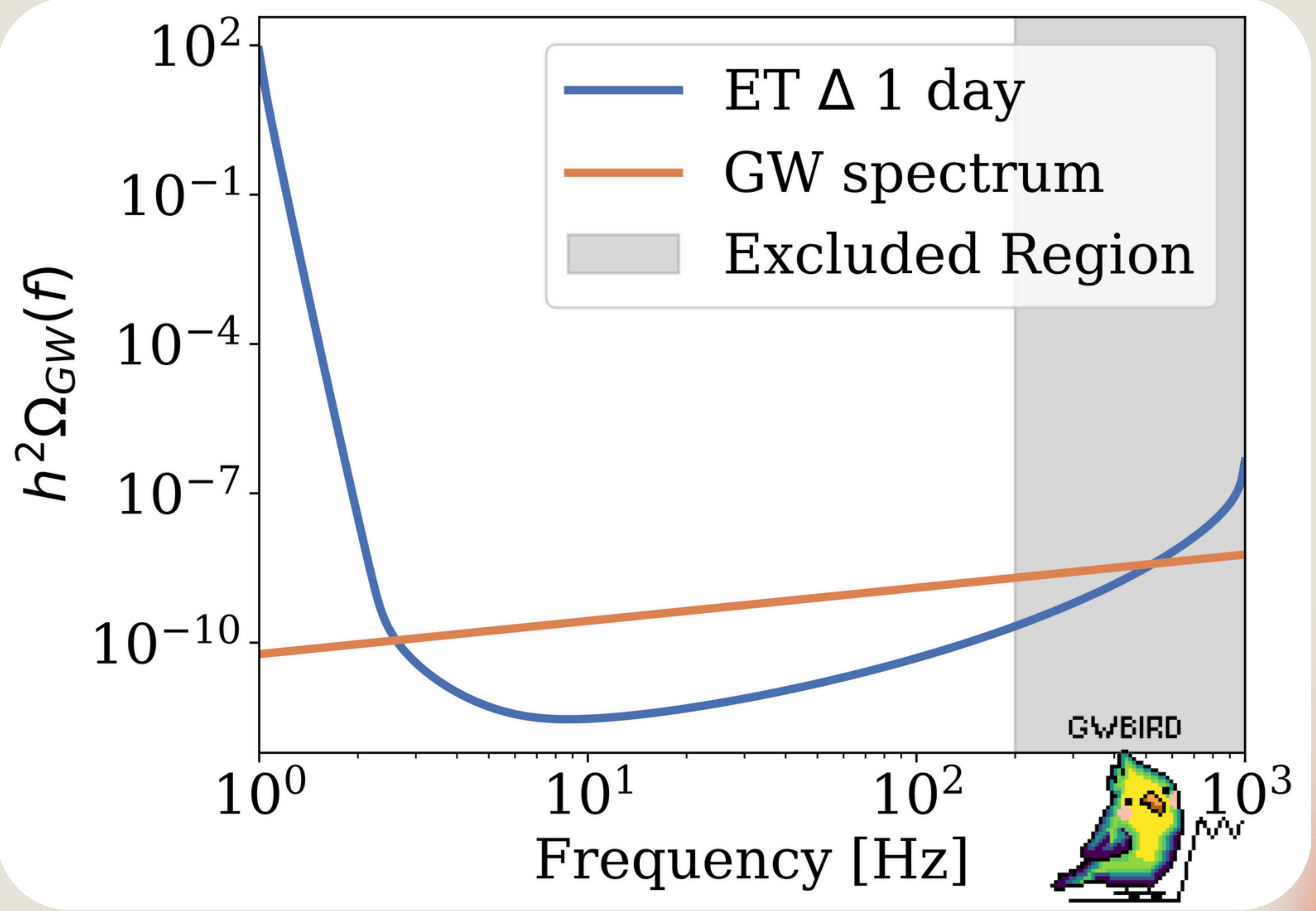
SGWB energy density spectrum

(Astrophysical origin)

$$\Omega_{\rm GW}(f) = A_{\rm GW}\left(\frac{f}{25\,{\rm Hz}}\right)^{n_{\rm GW}}$$

$$A_{\rm GW} = 10^{-9}$$
$$n_{GW} = 2/3$$

based on the state-of-the-art astrophysical models



Data Generation

Signal

$$h_{\rm ampl}(f) = \sqrt{\frac{T_{obs}}{2}\frac{3H_0^2}{10\pi^2 f^3}\Omega_{\rm GW}(f)\gamma_{IJ}(f)}$$

$$h_I(f) = h_{\rm ampl}\left(\mathcal{N}(0,1/\sqrt{2}) + i\mathcal{N}(0,1/\sqrt{2})\right)$$

Noise

$$n_{\rm ampl}(f) = \sqrt{\frac{T_{obs}}{2}N_{IJ}(f)}$$

$$n_I(f) = n_{\rm ampl}\left(\mathcal{N}(0,1/\sqrt{2}) + i\mathcal{N}(0,1/\sqrt{2})\right)$$

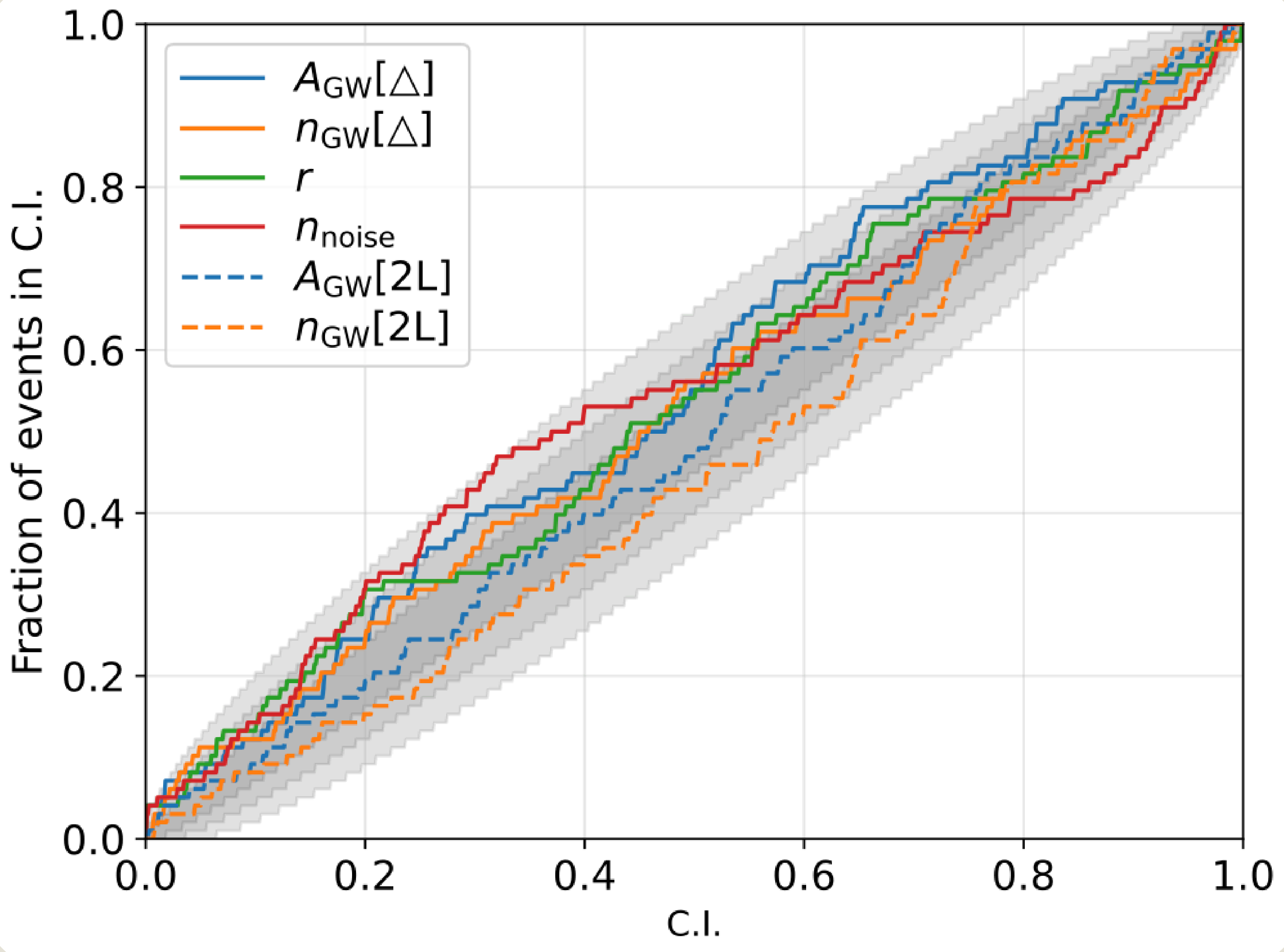
$$\tilde{s}_I(f) = \tilde{h}_I(f) + \tilde{n}_I(f)$$

[Flauger et al.arXiv:2009.11845]

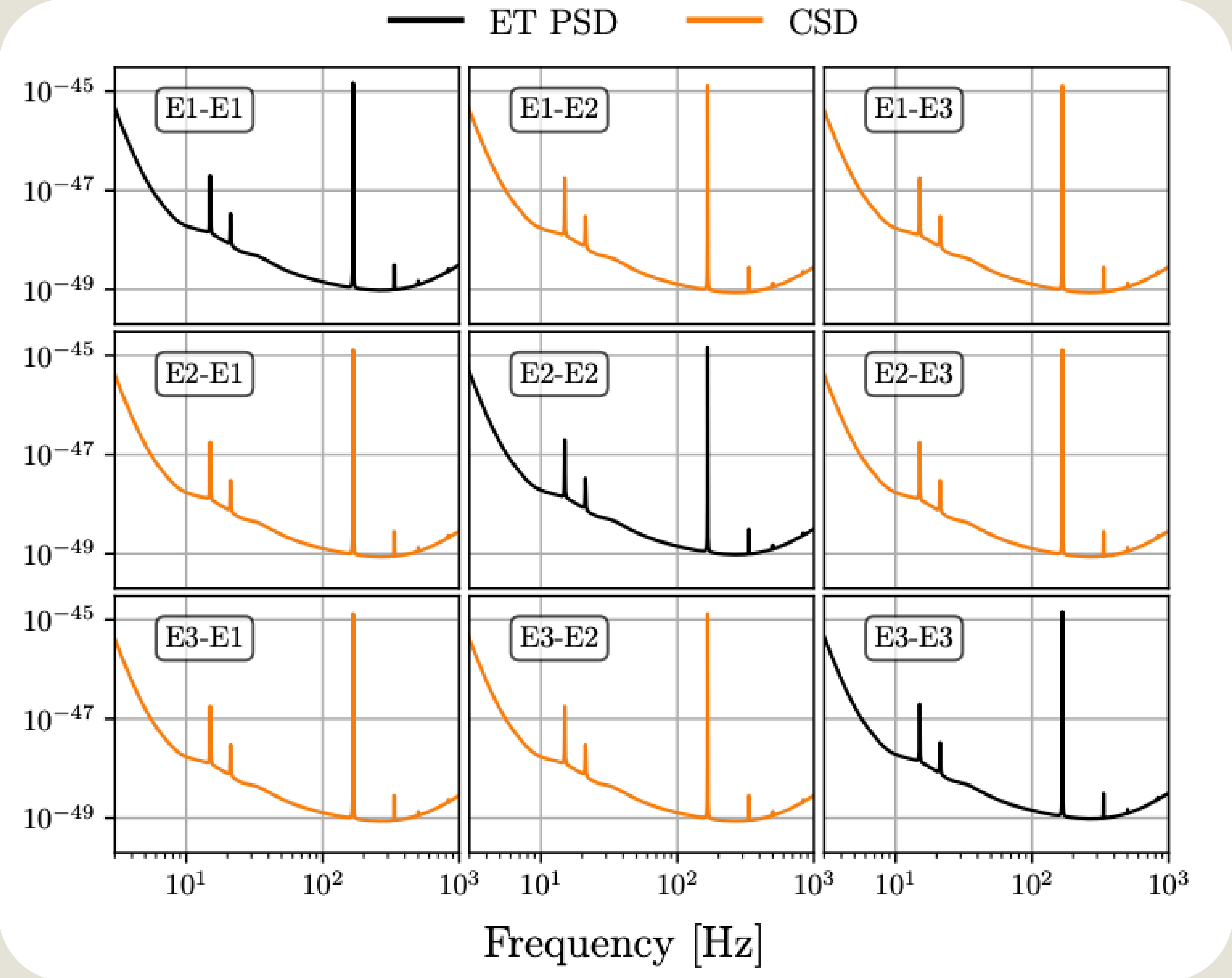
Results

[arXiv:2501.09057]

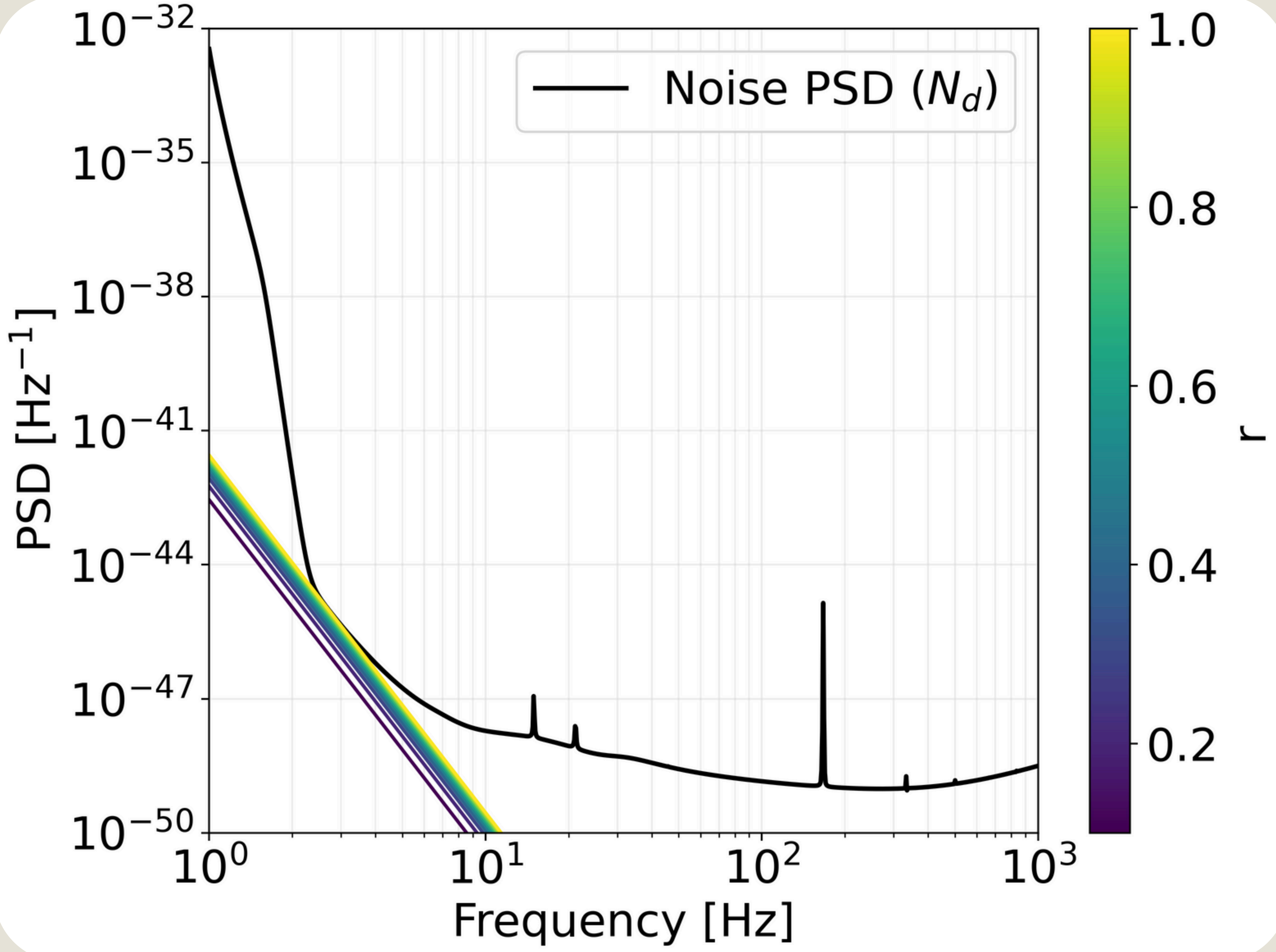
Statistical robustness of the Bayesian Analysis Performed



Auto & cross PSD [F.Cireddu et al. arXiv:2312.14614]



Auto & cross PSD [arXiv:2501.09057]



Correlation between XYZ and AET basis

XYZ basis

$$R = \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

AET basis

$$N_{IJ}(f) \equiv \begin{pmatrix} N_d & N_o & N_o \\ N_o & N_d & N_o \\ N_o & N_o & N_d \end{pmatrix}$$

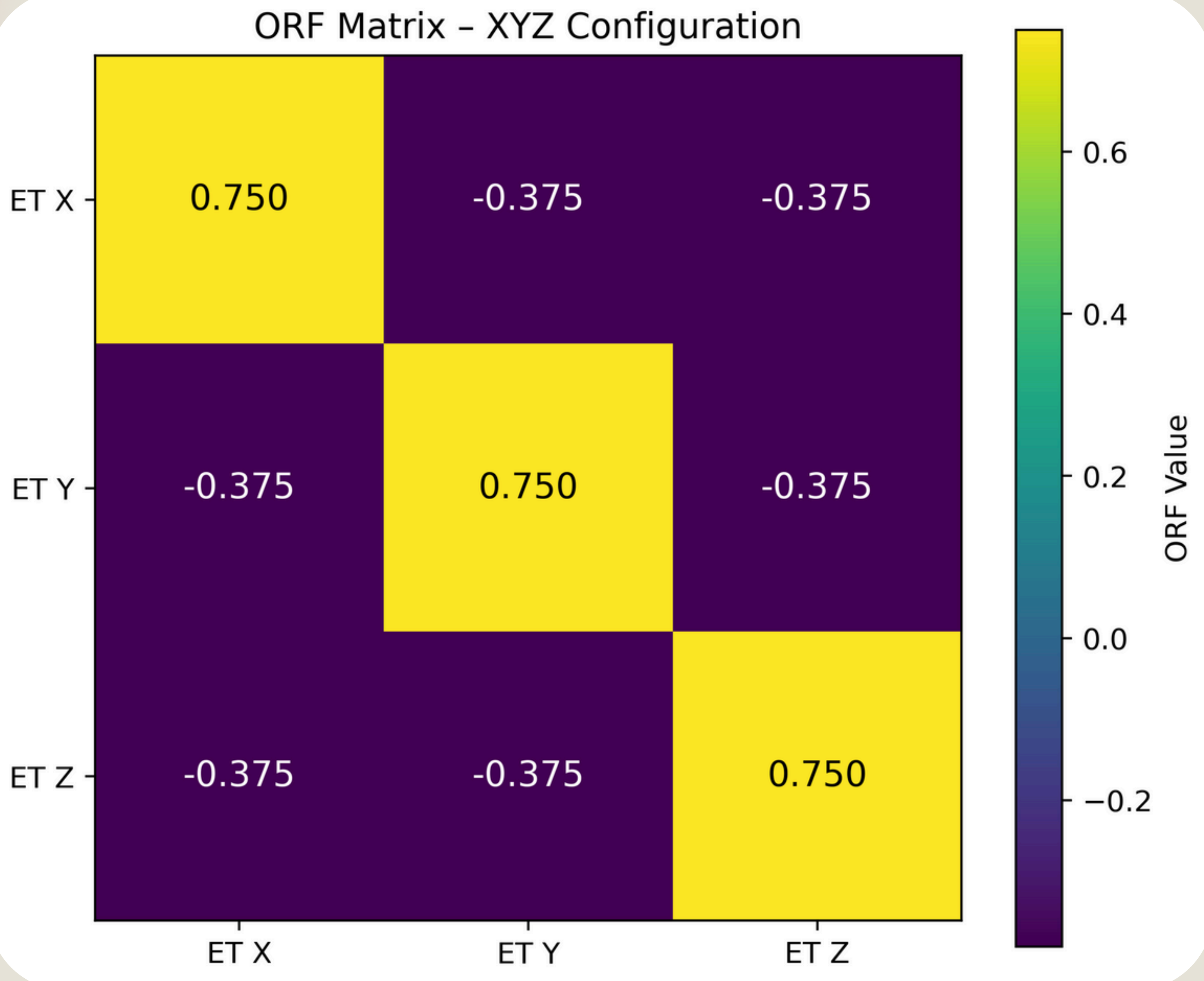
$$\gamma_{IJ}(f) \equiv \begin{pmatrix} \gamma_d & \gamma_o & \gamma_o \\ \gamma_o & \gamma_d & \gamma_o \\ \gamma_o & \gamma_o & \gamma_d \end{pmatrix}$$

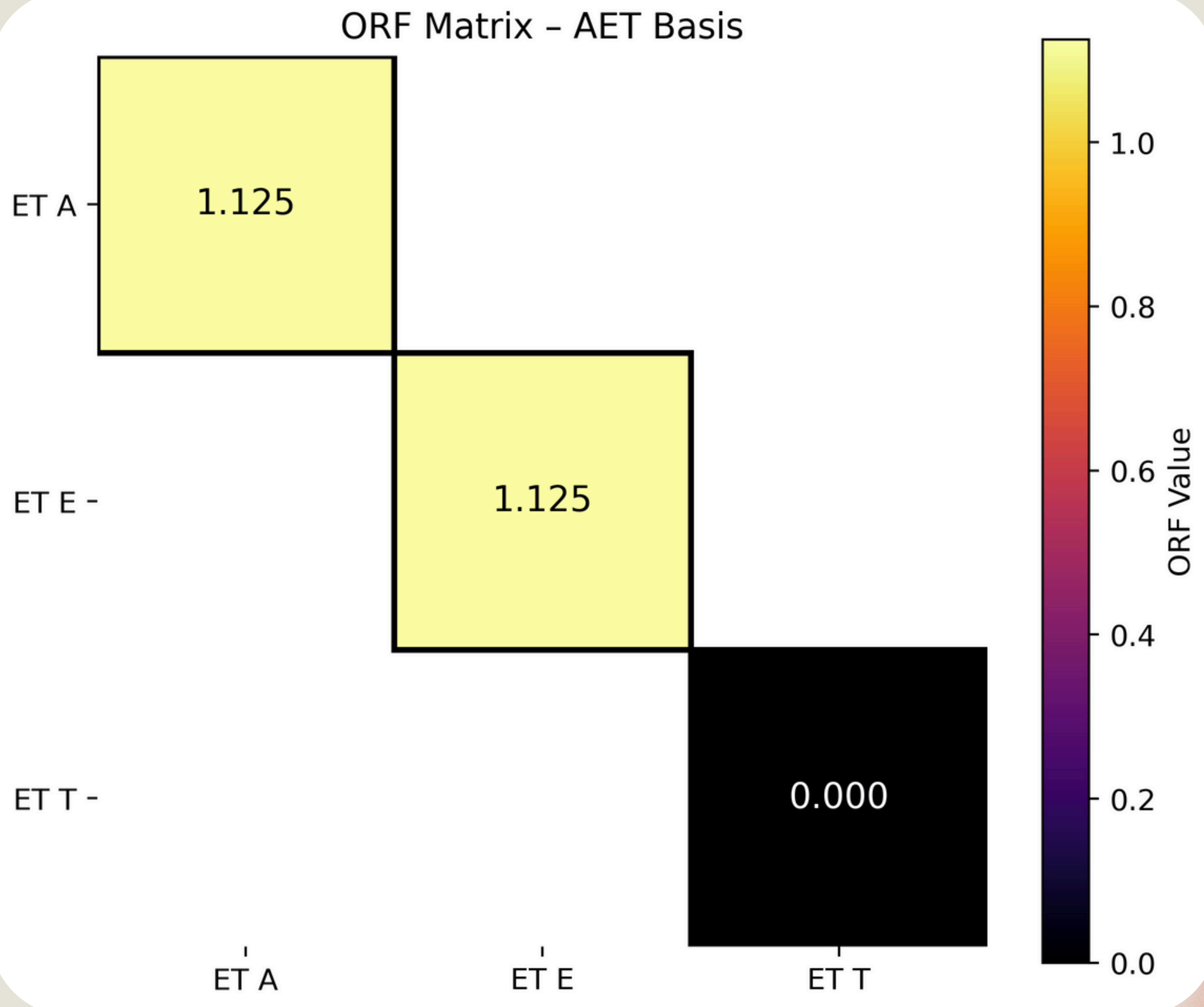
$$N_{IJ}(f) \equiv \begin{pmatrix} N_d - N_o & 0 & 0 \\ 0 & N_d - N_o & 0 \\ 0 & 0 & N_d + 2N_o \end{pmatrix}$$

$$\gamma_{IJ}(f) \equiv \begin{pmatrix} \gamma_d - \gamma_o & 0 & 0 \\ 0 & \gamma_d - \gamma_o & 0 \\ 0 & 0 & \gamma_d + 2\gamma_o \end{pmatrix}$$

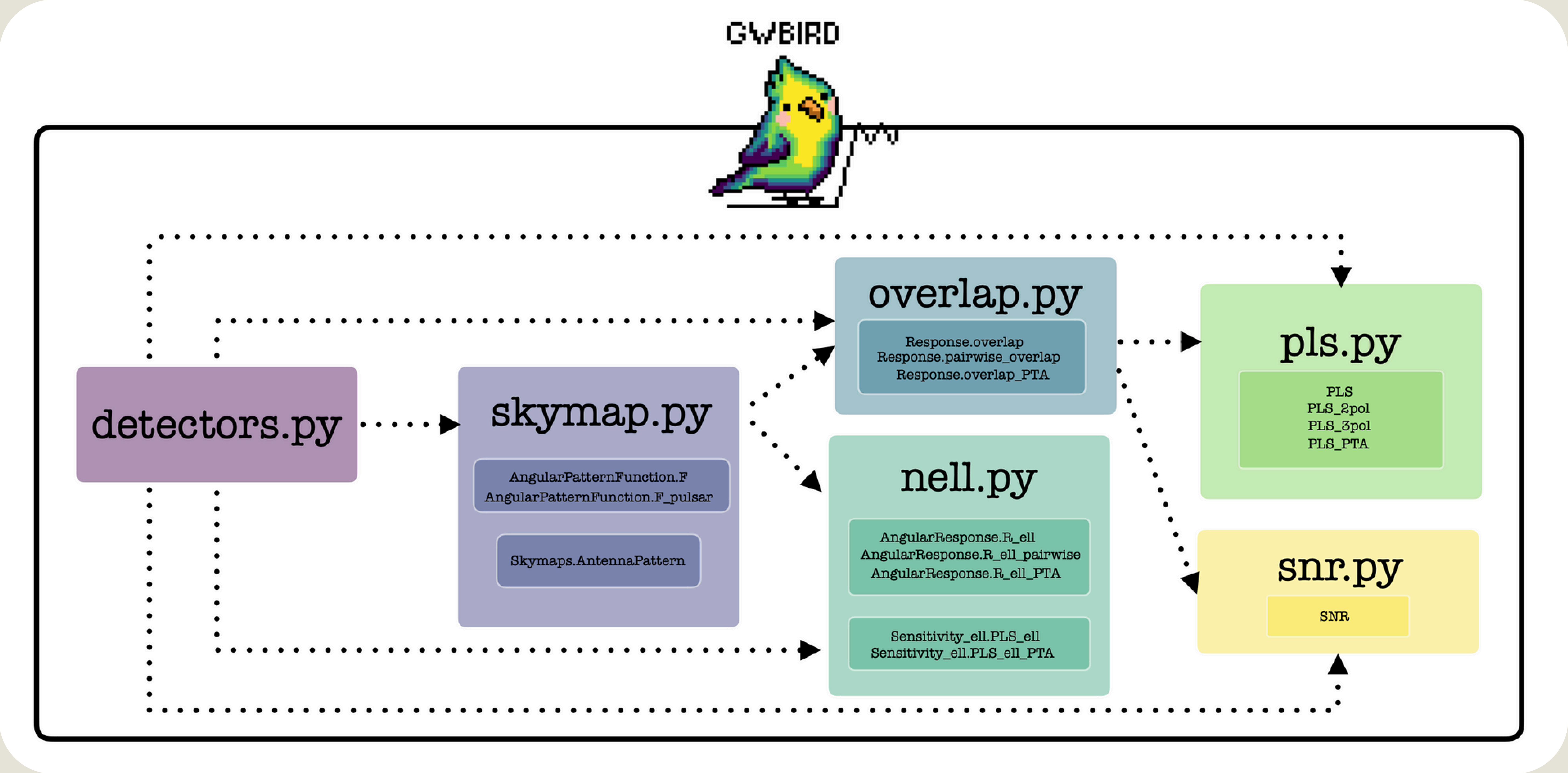
[Flauger et al.arXiv:2009.11845]

Overlap Reduction Function
ET Triangular configuration





GWBird


GWBIRD
detectors.py
skymap.py
AngularPatternFunction.F
AngularPatternFunction.F_pulsar
Skymaps.AntennaPattern
overlap.py
Response.overlap
Response.pairwise_overlap
Response.overlap_PTA
nell.py
AngularResponse.R_ell
AngularResponse.R_ell_pairwise
AngularResponse.R_ell_PTA
Sensitivity_ell.PLS_ell
Sensitivity_ell.PLS_ell_PTA
pls.py
PLS
PLS_2pol
PLS_3pol
PLS_PTA
snr.py
SNR