Dawn of GW Cosmology

Benasque, 28th April 2025

Gravitational Waves from Inflation

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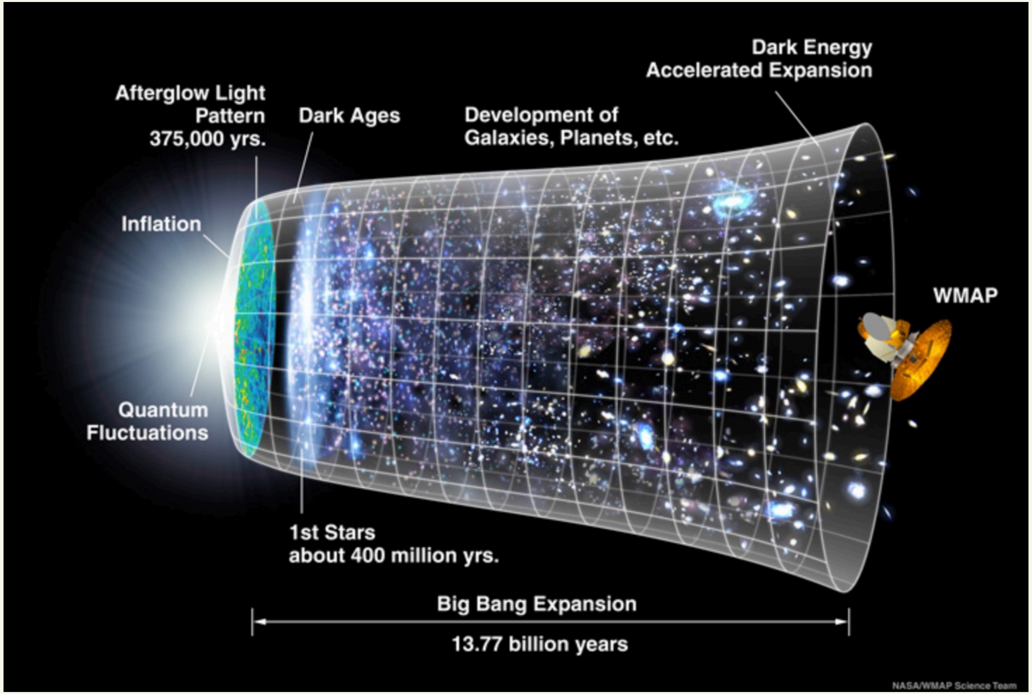
outline:

- successes of inflation
    - origin of primordial density perturbations
- primary GWs
    - vacuum fluctns in gravitational field
    - testing models of slow-roll inflation
- secondary GWs
    - scalar-induced GWs
    - non-adiabatic inflation (sudden transitions, etc)
- conclusions

provides initial conditions for ΛCDM cosmology



- spatially-flat FLRW background ✓
- primordial density perturbations ✓

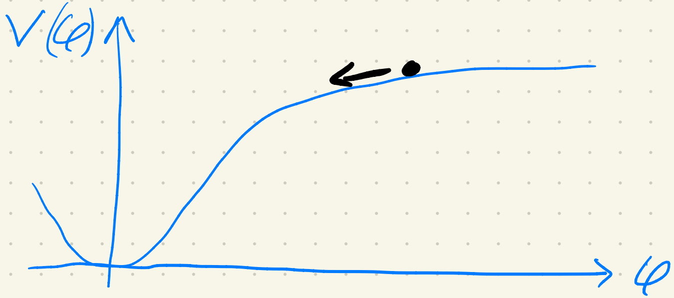
single field inflation

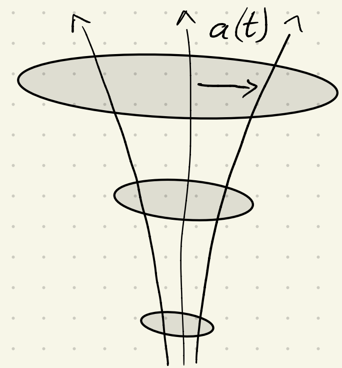
scalar field, $\varphi(t)$, FLRW metric $H = \dot{a}/a$

KG eqn: $\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi} = 0$

Friedmann: $H^2 = \frac{8\pi G}{3}\left(V + \frac{1}{2}\dot{\varphi}^2\right)$

$\Rightarrow$ accelerated expansion $\ddot{a} > 0$ for $V > \dot{\varphi}^2$





single field slow-roll inflation

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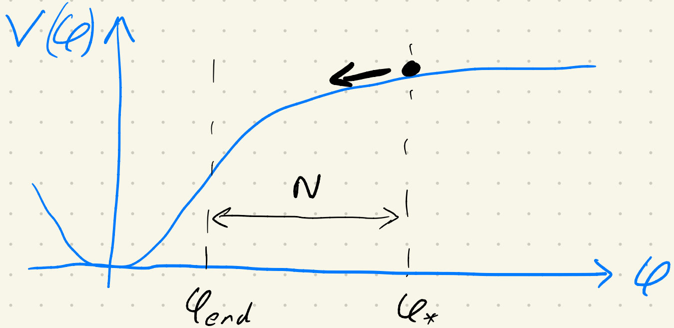
Friedmann: $H^2 = \frac{8\pi G}{3}\left(V + \cancel{\frac{1}{2}\dot{\varphi}^2}\right)$

$\Rightarrow$ e-folds

$$N = \int H\,dt = \ln(a)$$

$$N(\varphi_*) = \int_{\varphi_*}^{\varphi_{end}} \frac{H}{\dot{\varphi}}\,d\varphi$$



slow-roll parameters:

$$\epsilon_1 = -\frac{d}{dN}(\ln H)$$

$$\epsilon_{i+1} = \frac{d}{dN}(\ln \epsilon_i)$$

weakly-broken de Sitter symmetry: $|\epsilon_i| \ll 1$

classical vs quantum inflation

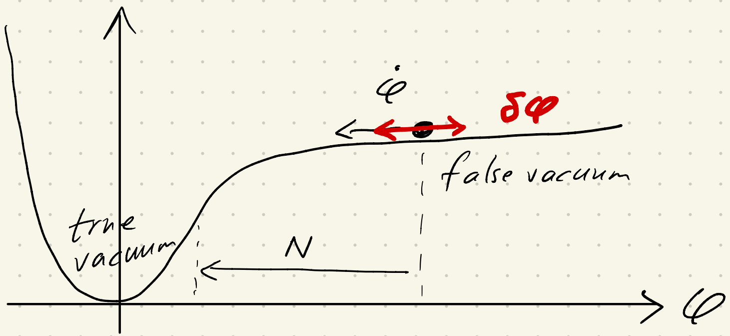
- **classical** slow-roll inflates universe

  $\longrightarrow \ln(a) = \boldsymbol{N} = \int H\, dt = \int \frac{H}{\dot{\varphi}}\, d\varphi$

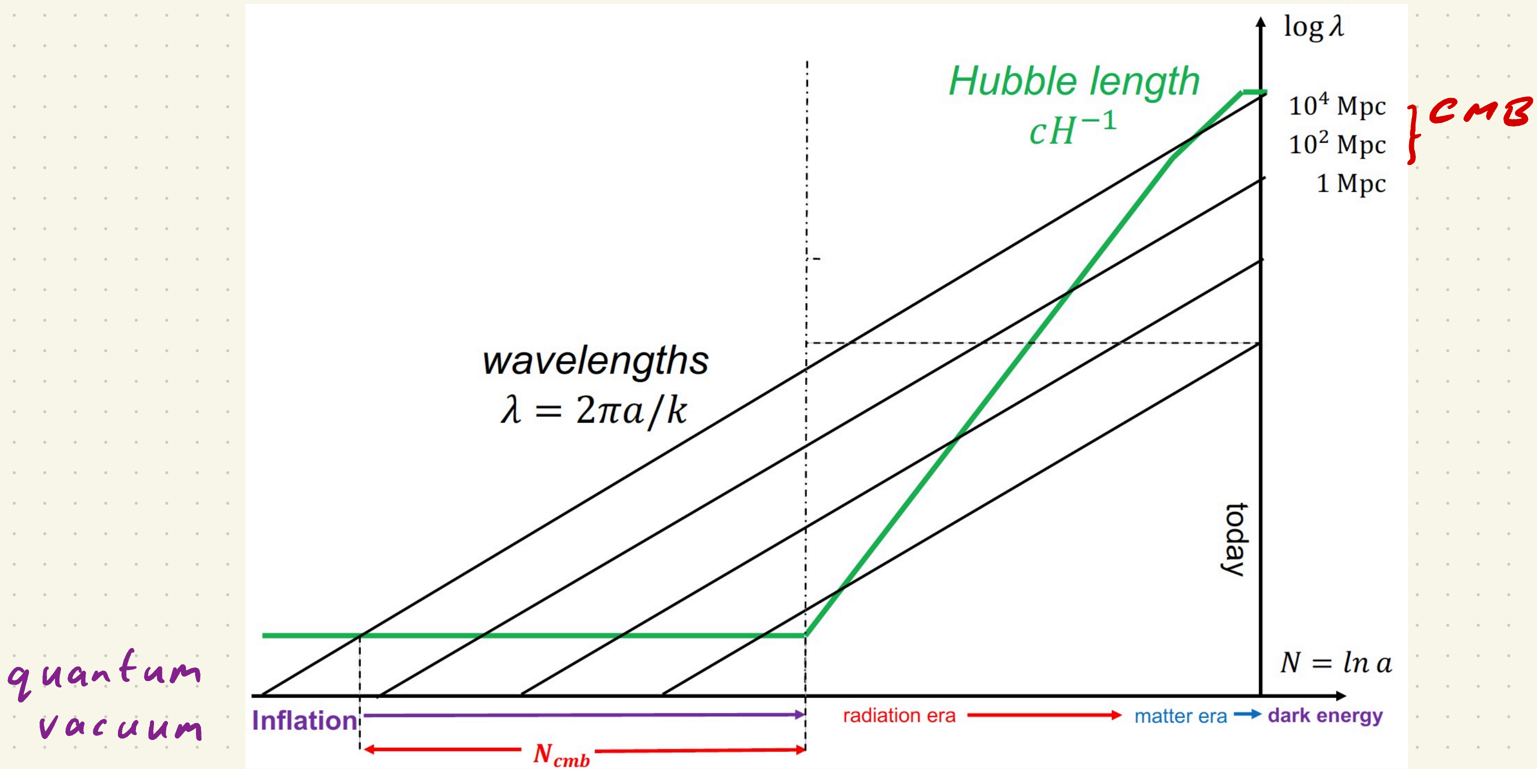
- **quantum** field fluctuations perturb local adiabatic expansion

  $\longrightarrow$ adiabatic curvature perturbation

$$R = \delta N = \frac{H}{\dot{\varphi}}\, \delta\varphi$$

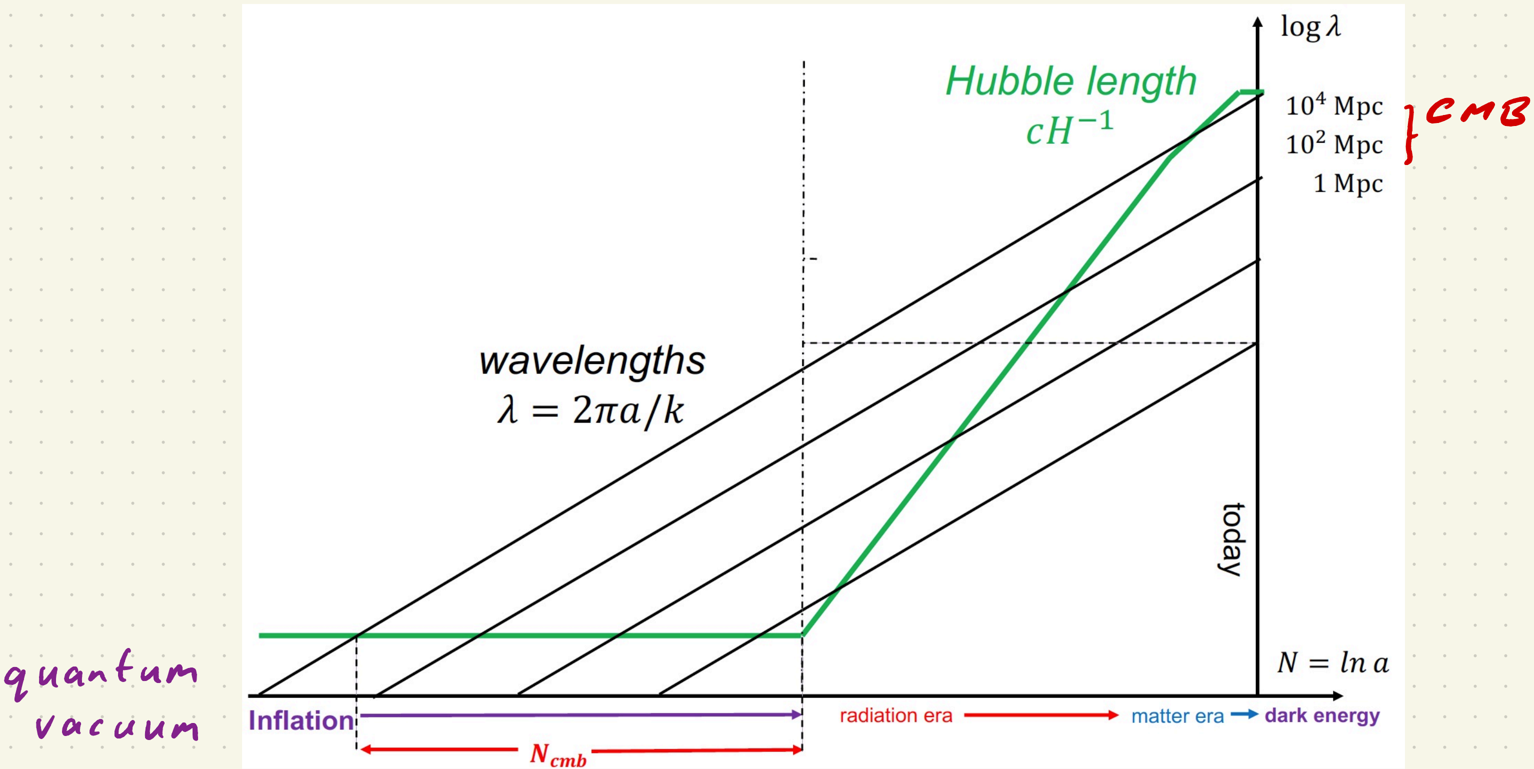


quantum fluctuations → cosmic structure



$$N(k) \simeq 61 - \ln\left(\frac{k}{k_{cmb}}\right) + \frac{1}{4}\ln\left(\frac{V_*^2}{V_{end} M_{Pl}^4}\right) - \frac{1-3w}{4} N_{reheat}$$

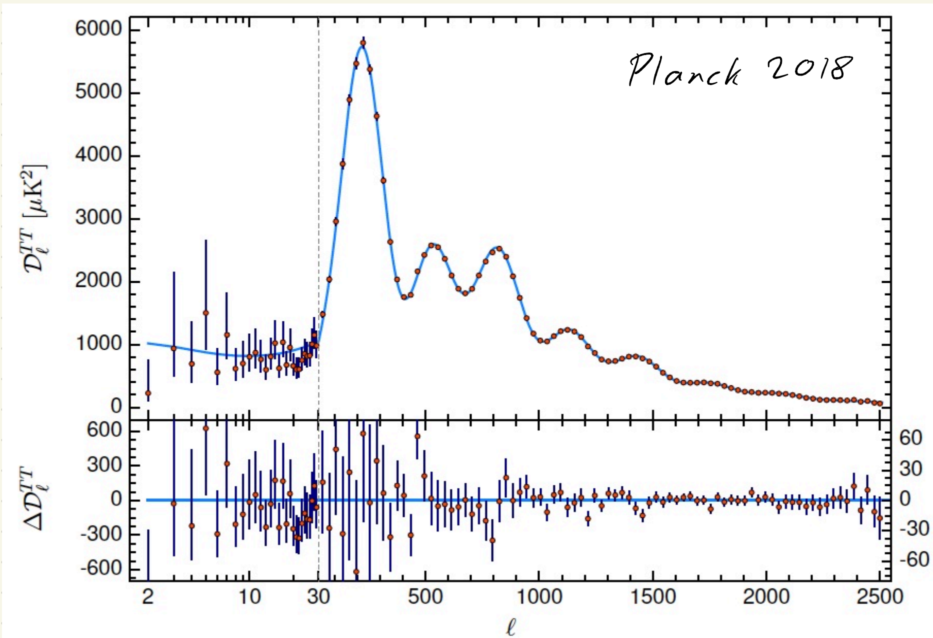
quantum fluctuations → cosmic structure



weak time dependence, $|\epsilon_i| \ll 1$ ⟶ weak scale-dependence, $n_s \neq 1$

sufficient (+ necessary?) for origin of structure

inflation ⟶ CMB power spectrum



✓ almost scale-invariant

✓ Gaussian

✓ adiabatic

? primordial GWs

primary (linear) perturbations from inflation

inflation → reheating → radiation era

quantum vacuum

Gaussian $\delta\varphi$ → scalar → δN → $\delta\rho$, density perturbations

δg_{ij} → tensor → h_{ij}, gravitational waves

primary (linear) perturbations from inflation

inflation → reheating → radiation era

scalar ↗ δN → $\delta\rho$, density perturbations

$$P_{\delta\rho} \sim \left(\frac{H}{\dot\varphi}\right)^2 \left(\frac{H}{2\pi}\right)^2$$

$$\Rightarrow \quad n_s - 1 = -2\epsilon_1 - \epsilon_2$$

Gaussian

quantum vacuum $\delta\varphi$, δg_{ij}

$$P_{\delta\varphi} = \left(\frac{H}{2\pi}\right)^2_{k=aH}$$

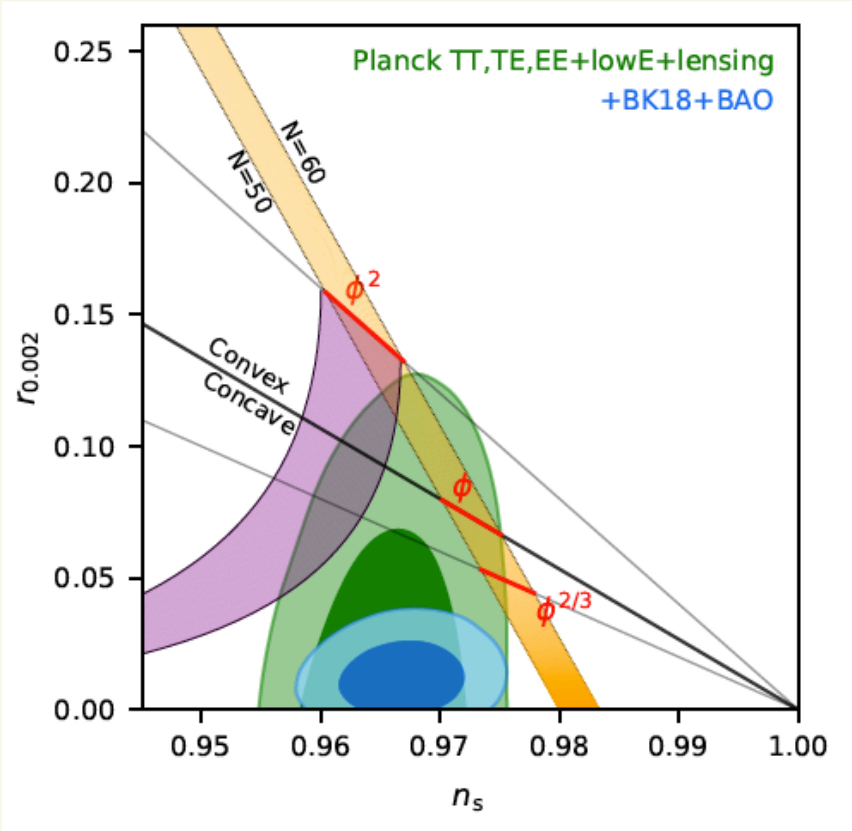
tensor ↘ h_{ij}, gravitational waves

$$P_{gw} \sim \left(\frac{H}{2\pi}\right)^2$$

$$\Rightarrow \quad r = \frac{P_{gw}}{P_{\delta\rho}} \sim \left(\frac{\dot\varphi}{H}\right)^2$$

$$= 16\,\epsilon_1 \ll 1$$

testing single field inflation



Planck

+ BICEP/Keck

+ BAO

almost scale-invariant

$$n_\mathcal{R} - 1 = \frac{d \ln P}{d \ln k}$$

$$= -0.0351 \pm 0.0042$$

primordial grav. waves

$$r = \frac{P_T}{P_\mathcal{R}} < 0.036$$

"inverted SR hierarchy"

$$\epsilon_1 \ll \epsilon_2 \simeq 0.036$$

convex potentials

$V \propto \varphi^p \, , \, p > 1 \quad \Rightarrow \quad \epsilon_1 \sim \epsilon_2 \quad \Rightarrow \quad n_s - 1 \sim \epsilon_1$

$r = 16\,\epsilon_1 \sim 0.1$ ✗

$\Rightarrow$ concave / plateau potentials

α-attractor models

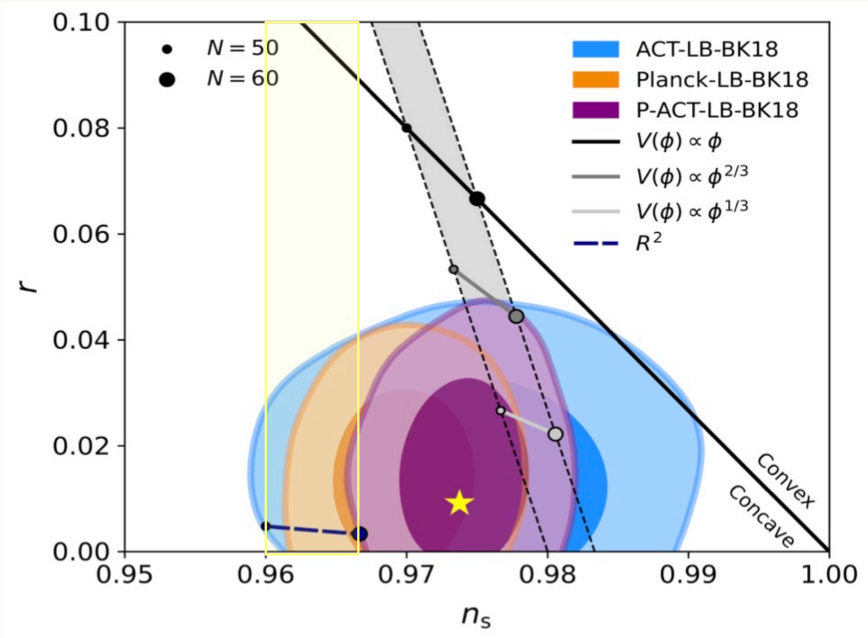
include Starobinsky & Higgs inflation ($\alpha = 1$)

$V \propto V_0 \left(1 - e^{-\varphi/\sqrt{\alpha}}\right) \quad \Rightarrow \quad \epsilon_1 \sim \epsilon_2^2 \quad \Rightarrow \quad n_s - 1 \simeq -\epsilon_2$

"universal predictions" $\quad n_s - 1 \simeq -\dfrac{2}{N_{cmb}} \simeq -0.035$ for $N_{cmb} \simeq 60$

$r \simeq \dfrac{12\alpha}{N_{cmb}^2} \simeq 0.003\,\alpha$ ✓ for $\alpha \lesssim 10$

trouble for α-attractors?



Planck

+ Bicep/Keck

+ ACT

universal predictions

$$n_s - 1 = -2/N_{cmb}$$

$$n_s > 0.97 \Rightarrow N_{cmb} > 67$$ ✗

see Kallosh, Linde & Roest, arxiv

primary (linear) perturbations from inflation

inflation → reheating → radiation era

multi-field quantum vacuum

Gaussian $\{\delta\varphi^I\}$ → scalar → δN → $\delta\rho$, density perturbations

(curvaton) → + $S_m, S_\nu \ldots$ isocurvature

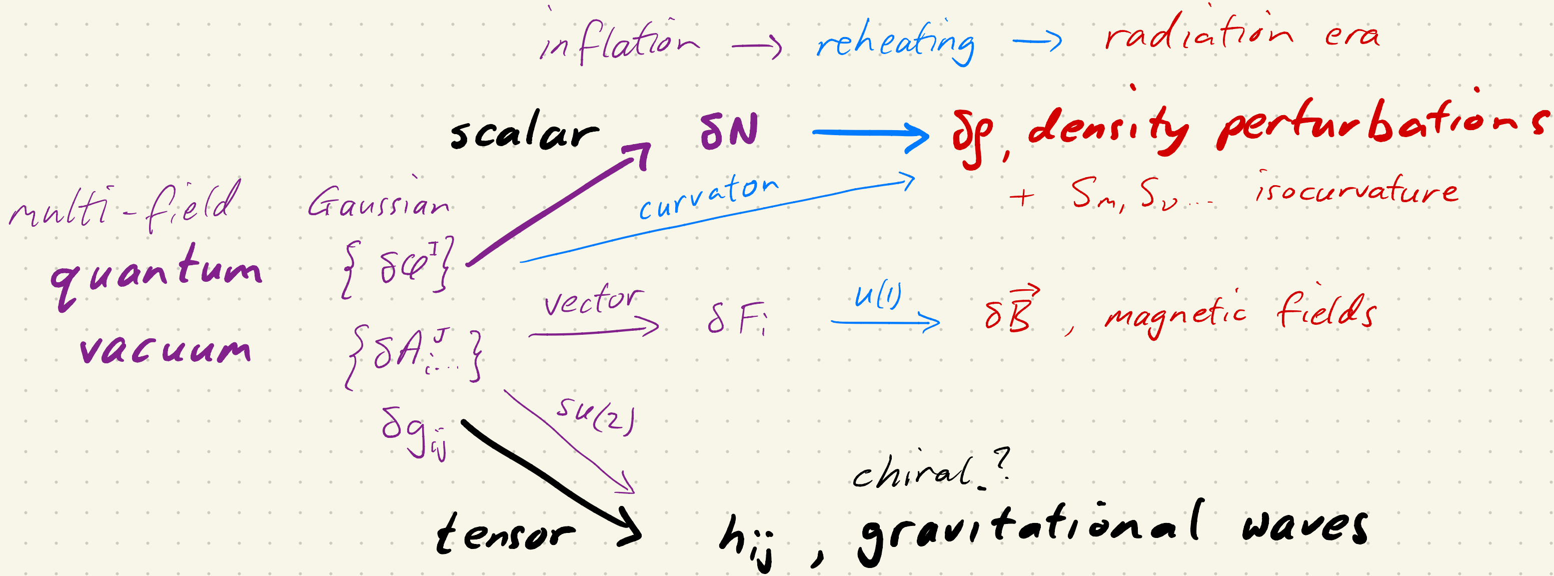
$$P_{\delta\rho} = P_{inf} + P_{curvaton}$$

δg_{ij} → tensor → h_{ij}, gravitational waves

P_{gw} unchanged

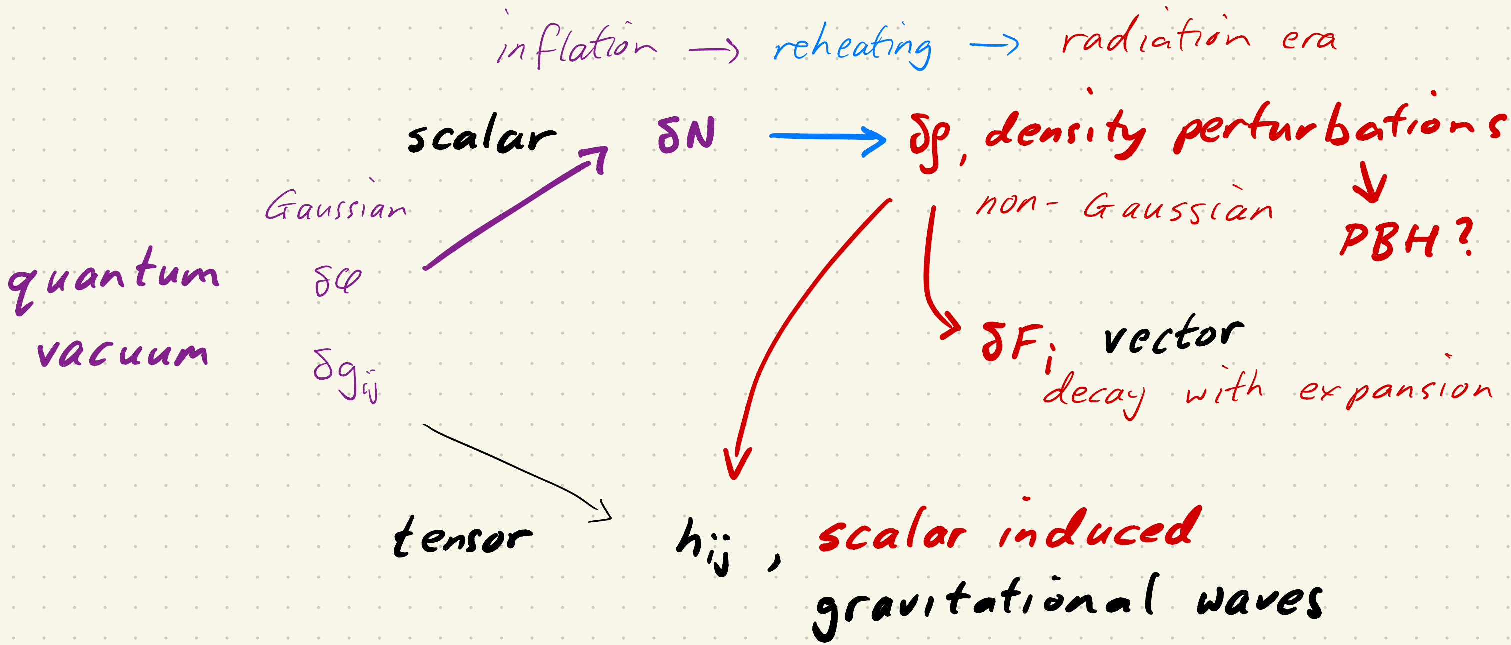
$$\Rightarrow \quad r < 16\,\epsilon_1$$

primary (linear) perturbations from inflation



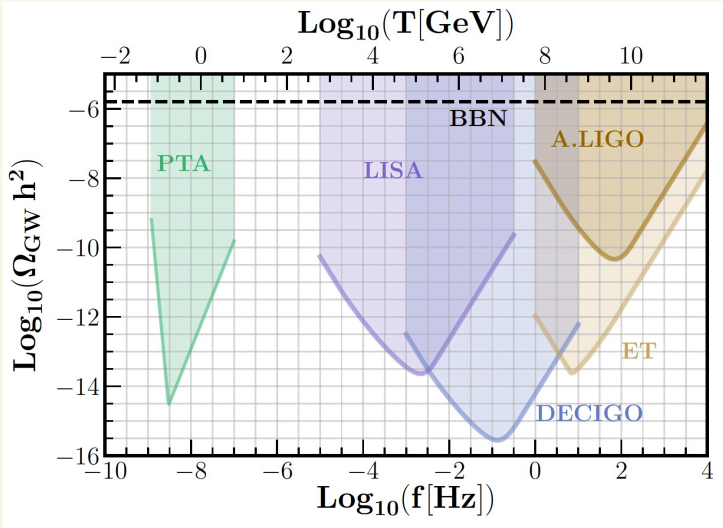
see talk by Tomohiro Fujita

secondary perturbations after inflation



gravitational window onto inflation?

- Scalar perturbations washed away on small scales $\lambda < \lambda_{silk}$
- GWs propagate freely through Universe on all scales



temperature at horizon entry

Domenech (2021)

+ primordial black holes could also survive ($M_{PBH} > 10^{15} g$)

scalar-induced GWs

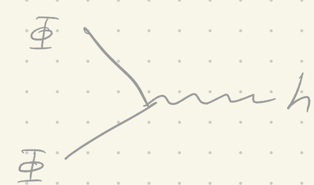
Tomita (1967); Materrese et al (1993+)
Ananda, Clarkson & DW (2006)
Baumann et al (2007)
see review by Domenech (2021)

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} + \frac{k^2}{a^2} h_{ij} = \hat{S}^{TT}(\partial_i \Phi \partial_j \Phi)$$

second-order source

- sourced by transverse-tracefree part of density waves
- gauge-dependent on super-Hubble scales
- late-time ($\Phi \to 0$) GWs gauge independent

$$\Omega_{gw} \sim \Omega_\gamma P_{\delta\rho}^2$$



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$$\Omega_{gw} \sim \Omega_\gamma P_{\delta\rho}^2 \sim 10^{-22} \text{ for } P_{\delta\rho} \sim 10^{-9}$$

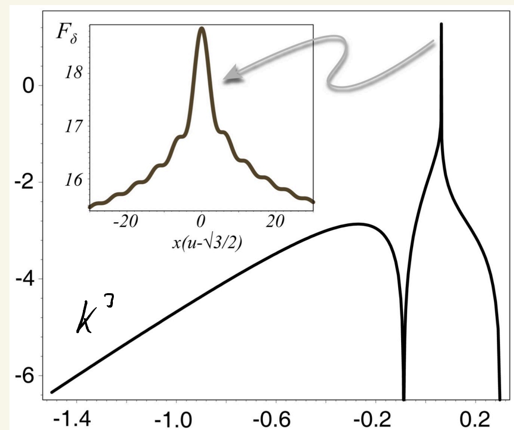
different power spectrum shapes

- power law : $P_{\delta\rho} = A_s^2 \left(\frac{k}{k_{cmb}}\right)^{n_s-1} \Rightarrow \Omega_{gw} \sim \Omega_r A_s^4 \left(\frac{k}{k_{cmb}}\right)^{2(n_s-1)}$

  $< 10^{-22}$ for $n_s < 1$

- sharp peak

  $P_{\delta\rho} \propto \delta\left(\ln\left(\frac{k}{k_p}\right)\right) \Rightarrow \Omega_{gw}$



$\ln(k / k_p)$

Ananda et al (2006)

- SIGWfast code

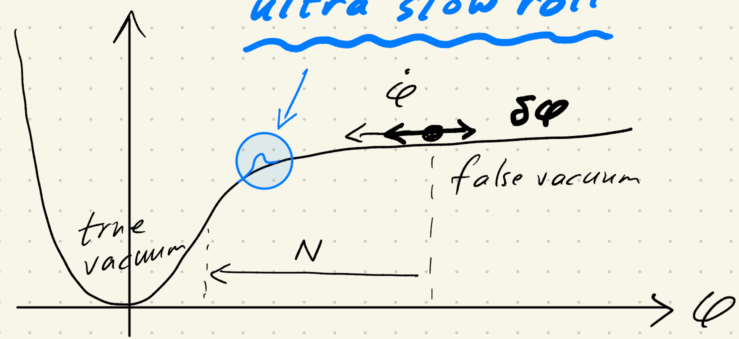
  for any $P_{\delta\rho}(k)$ Witkowski (2022)

enhanced density perturbations on small scales

- $\frac{\delta\rho}{\rho} \sim \delta N = \frac{\partial N}{\partial \varphi^I} \delta \varphi^I$

- single field: $\frac{dN}{d\varphi} = \frac{H}{\dot\varphi}$ $\Rightarrow$ enhanced power where $\epsilon_1 \sim \left(\frac{\dot\varphi}{H}\right)^2 \to 0$

ultra slow roll



- multi-field instability or phase transition

Garcia-Bellido, Linde & Wands 1995

enhanced density perturbations on small scales

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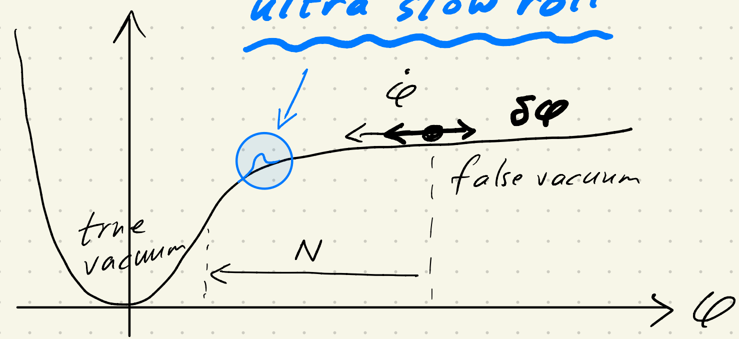
  ultra slow roll

  – α-attractors

  $$n_s - 1 \simeq \frac{-2}{N_{cmb} - N_{pk}} < -2/N_{cmb}$$

  – Iacconi et al 2022



- multi-field instability or phase transition

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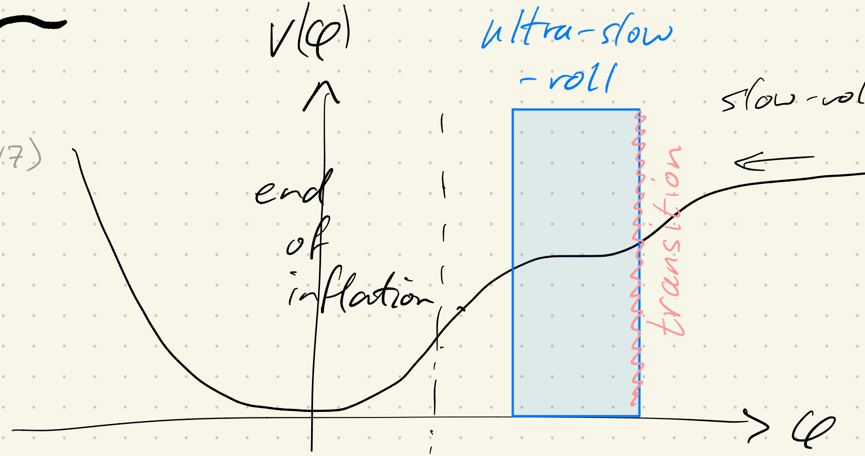
Ultra-slow-roll inflation

e.g. inflection point

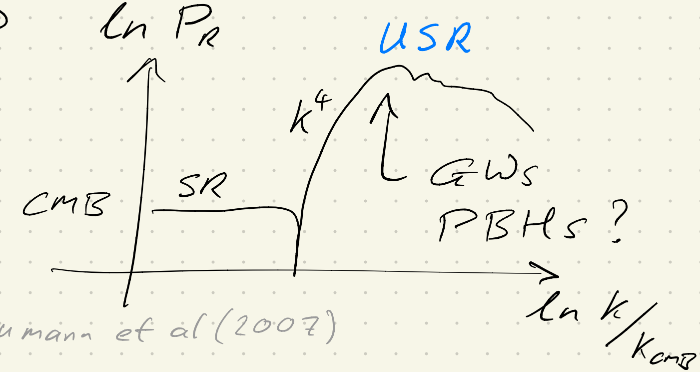
Garcia-Bellido & Ruiz (2017)



primordial curvature power spectrum

$$\langle \delta N^2 \rangle = \left(\frac{H}{\dot{\varphi}}\right)^2 \langle \delta\varphi^2 \rangle$$

↑ boosted as $\left(\frac{\dot{\varphi}}{H}\right)^2 \to 0$



* **primordial black holes**

Carr & Hawking '74

* **induced (second-order) grav. waves**

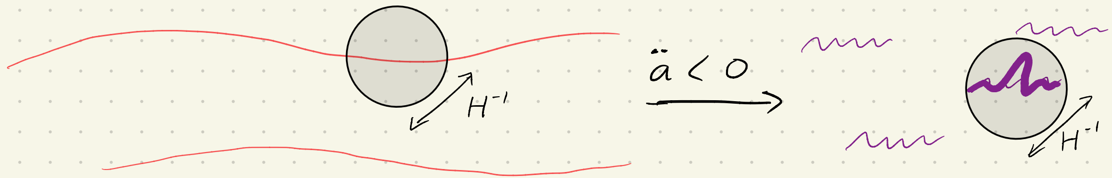
Matarrese et al, Ananda et al, Baumann et al (2007)

Primordial black holes

Zel'dovich & Novikov (1967), Hawking (1971)

- formed by collapse of large density perturbations, $\frac{\delta\rho}{\rho} > \delta_{crit}$ in early radiation-dominated era
- mass related to horizon mass at formation time

$$M_{PBH} \sim \frac{t}{10^{-6}\,s} M_\odot$$



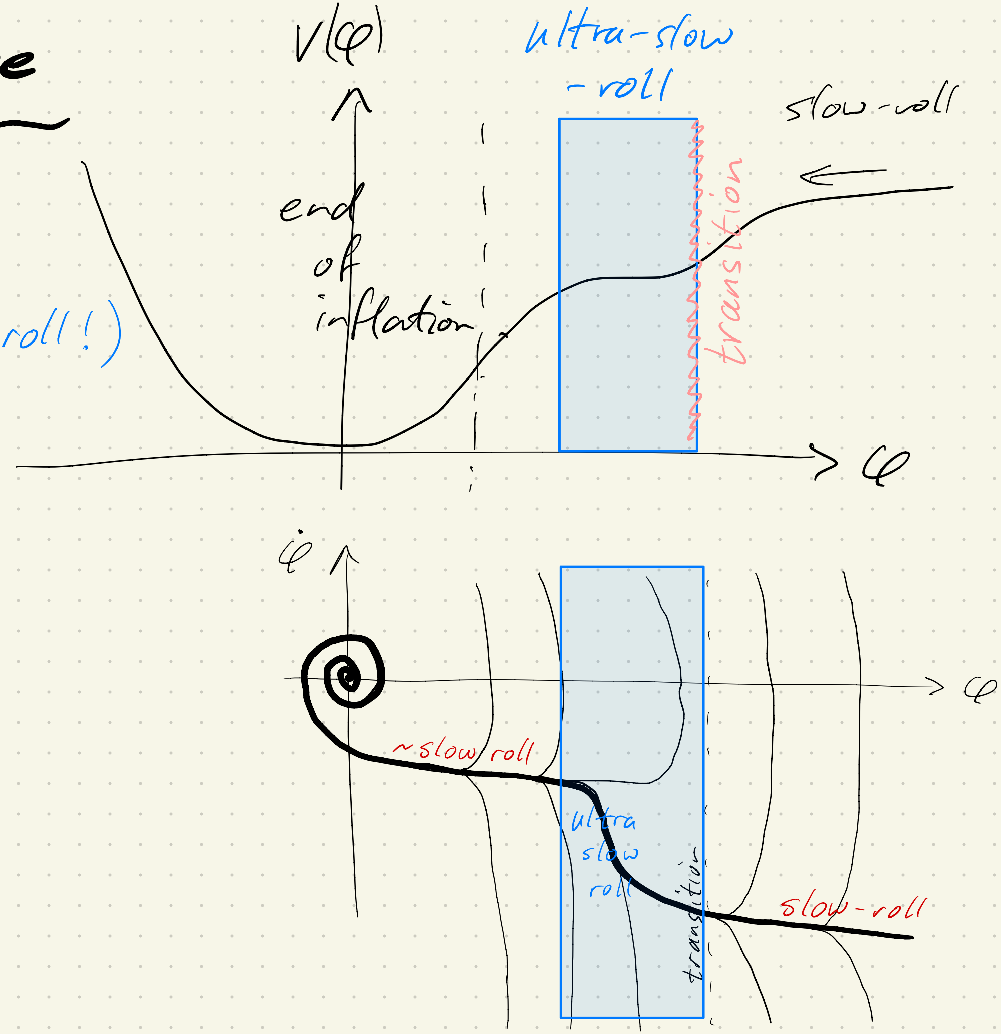
- **non-perturbative, $\delta_{crit} \sim 0.1$, and rare**

mass fraction at formation : $\beta(M) \sim 10^{-8} \left(\frac{M_{PBH}}{M_\odot}\right) \Omega_{PBH}$

USR phase space

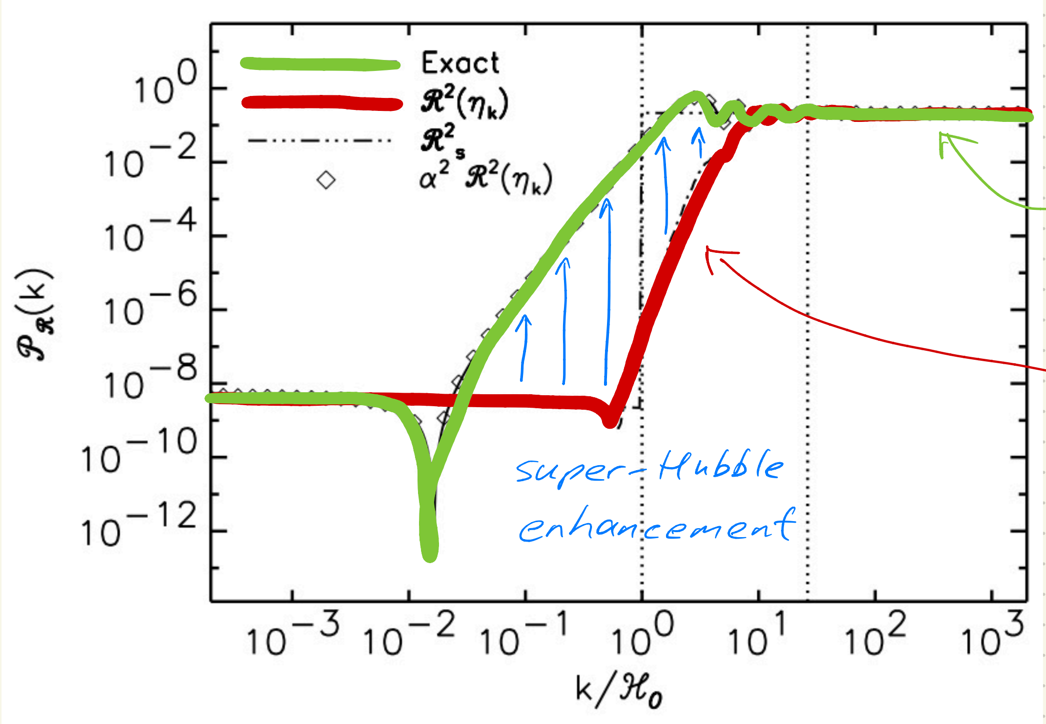
$\epsilon_1 \ll 1$

$\epsilon_2 \simeq -6$ (not slow roll!)



Enhancement of superhorizon scale inflationary curvature perturbations

Samuel M. Leach[1], Misao Sasaki[2], David Wands[3] and Andrew R. Liddle[1]



in Starobinsky's piecewise linear potential

arXiv:astro-ph/0101406v2 21 Feb 2001

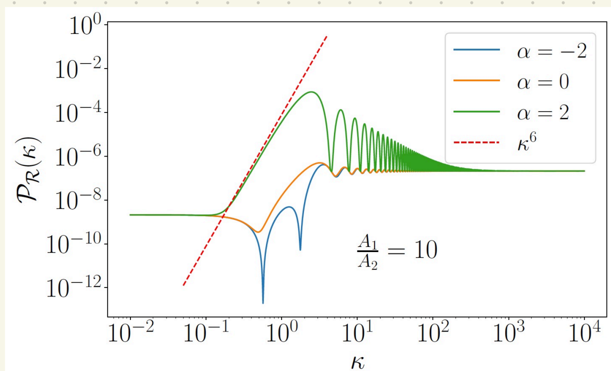
steepest growth of $P(k)$

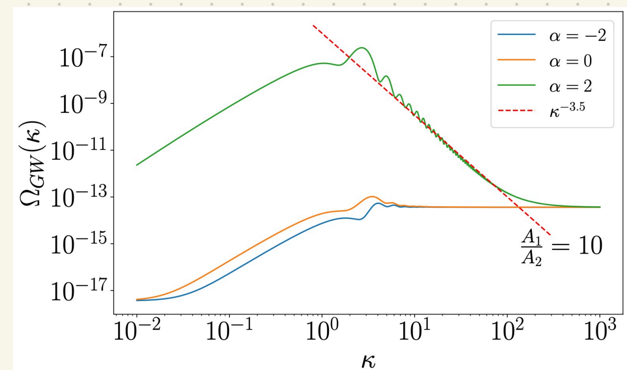
- SR → USR transition mixes growing mode gradient with decaying mode

  ⇒ k^4 slope (Byrnes et al, Mulryne et al)

- non-Bunch-Davies vacuum enhances decay mode ($\propto k^6$)

  ⇒ k^6 slope (Cielo & Wands, arXiv. 2410)





- e.g. multiple transitions (Tasinato)

discussion

- CMB offers detailed but narrow window onto inflation
  - primary GWs key to testing (maybe proving?) inflation

- GWs preserve a record of primordial universe on all scales
  - secondary GWs inevitably induced from scalar pertb$\underline{n}$s
    - could be detected from enhanced density pertb$\underline{n}$s
    - PBHs could provide complementary probe

      e.g. solar mass PBHs → SIGW at formation (PTA)

      → BBH GW event at mergers (LVK)

      - too fine tuned?
      - GW constraints could rule out $P_{\zeta p}$ for PBHs