The Dawn of Gravitational Wave Cosmology

(Benasque Science Center, Apr 27 - May 17, 2025)

Inverse phase transitions

and where SUSY sucks...

IFT Madrid

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based on JCAP 10 (2024) 042 and 2503.01951
with *Simone Blasi and Miguel Vanvlasselaer*

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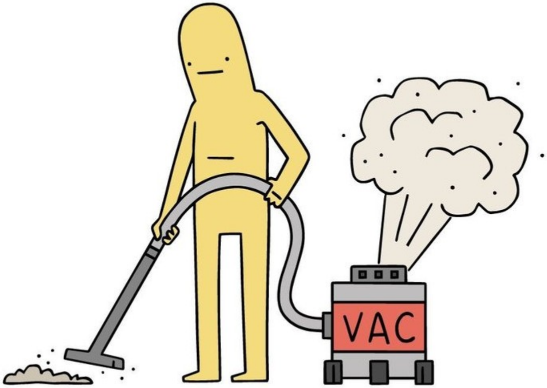
2. FOPT: *direct* and *inverse*
3. Hydrodynamic description
   - Hydrodynamic equations
   - Steady–state solutions
4. Realistic model (where SUSY sucks ...)

5. Physical interpretation for *inverse* PTs

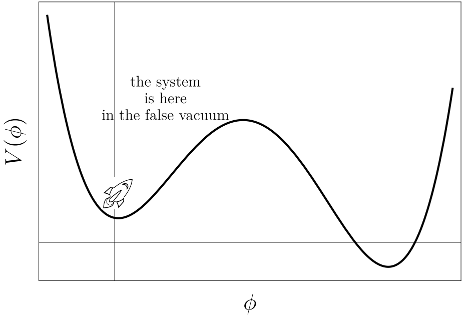


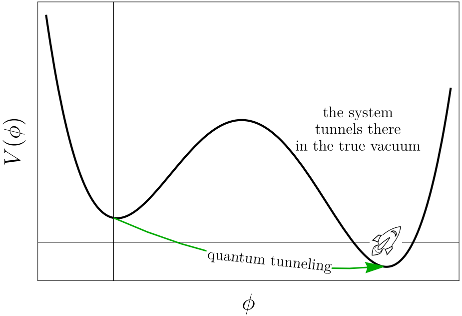
Sketch of an *inverse* PT

What?

FOPT: What? (from quantum field theory)

Let us consider a system described by the scalar potential $V(\phi)$

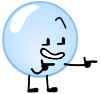




Tunneling decay rate of the false vacuum

$\Gamma \sim A e^{-S_E}$, Euclidean action

S_E computed on the sol. of the EOMs

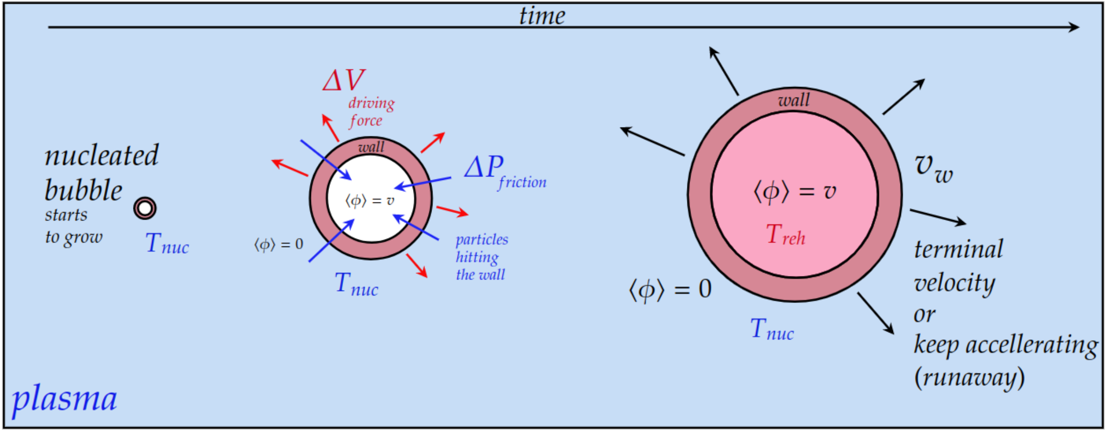


The solutions with the least action are **spherically symmetric**

Coleman, Callan ('77)

How?

FOPT: How?



time
nucleated bubble
starts to grow
T_{nuc}
ΔV
driving force
wall
$\langle\phi\rangle = v$
$\Delta P_{friction}$
$\langle\phi\rangle = 0$
particles hitting the wall
T_{nuc}
wall
$\langle\phi\rangle = v$
T_{reh}
v_w
$\langle\phi\rangle = 0$
T_{nuc}
terminal velocity
or
keep accellerating (runaway)
plasma

FOPT: *direct* and *inverse*

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$\boxed{\text{FOPT} = \text{tunneling transition between two minima separated by a barrier}}$

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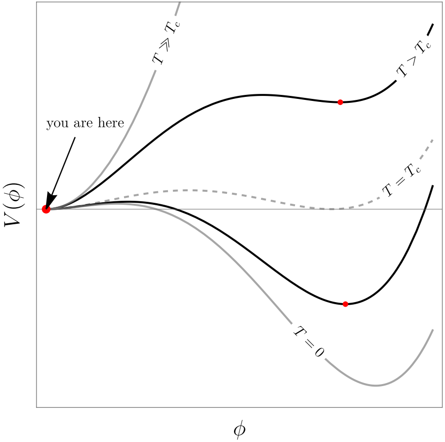
Direct PT



In **favour** of the $T = 0$ vacuum energy

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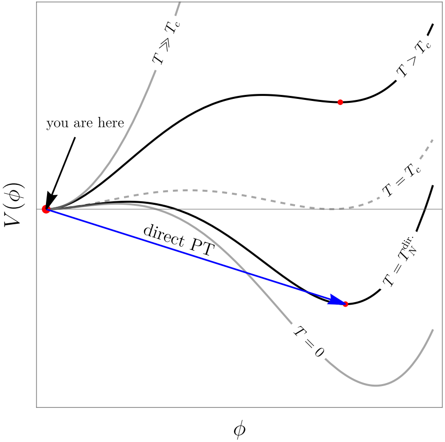

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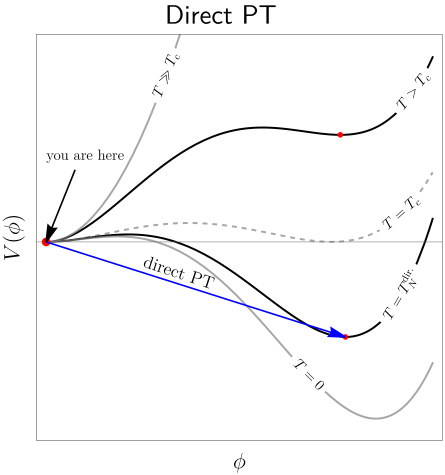

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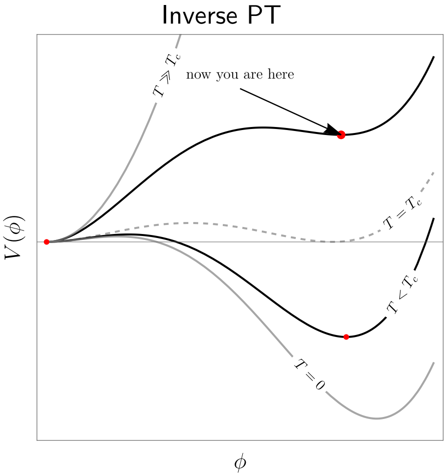

Against the $T = 0$ vacuum energy

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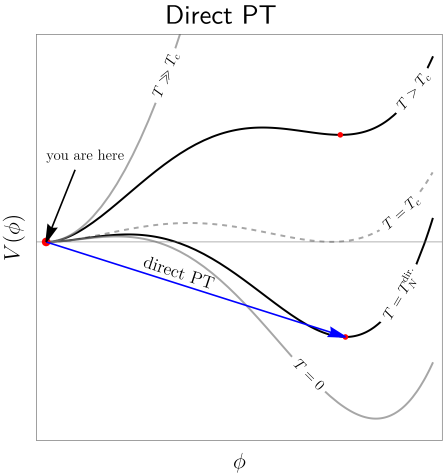

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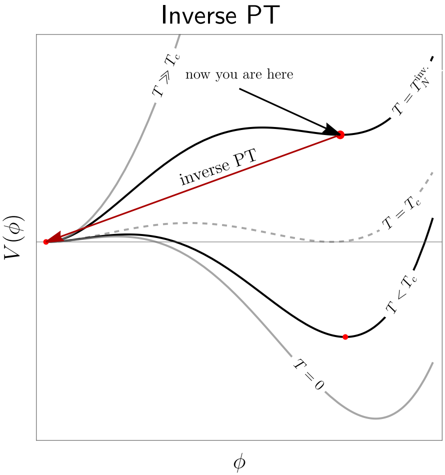

Against the $T = 0$ vacuum energy

Hydrodynamic description

Hydrodynamics equations

Coupled system of the scalar background and the plasma

$$T^{\mu\nu} = T_\phi^{\mu\nu} + T_p^{\mu\nu}, \qquad \begin{cases} T_\phi^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\left[\frac{1}{2}(\partial\phi)^2 - V(\phi)\right] \\ T_p^{\mu\nu} = (e+p)u^\mu u^\nu - p\, g^{\mu\nu} \end{cases}$$

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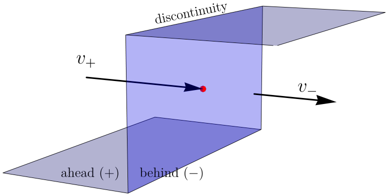
Hydrodynamical flows can develop **discontinuities** such as shocks or reaction fronts $\rightarrow$ matching conditions across discontinuities ($\pm$ bubble wall frame)



$$w_+\gamma_+^2 v_+ = w_-\gamma_-^2 v_-$$
$$w_+\gamma_+^2 v_+^2 + p_+ = w_-\gamma_-^2 v_-^2 + p_-$$

where $w = e + p =$ enthalpy

Thermodynamic quantities

Thermodynamic Quantities

Once the microphysics is specified (i.e., a model is chosen), we can compute the free energy, related to the pressure via:

$$p = -\mathcal{F} = -V_{\text{eff}} = -\left(V_0 + V_{\text{1-loop}} + V_T\right)$$

From the pressure, other thermodynamic quantities follow:

$$w = T\frac{\partial p}{\partial T}, \qquad e = w - p, \qquad c_s^2 = \frac{\partial p}{\partial e}$$

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3. **Full model**: $p_\pm = -\mathcal{F}(\phi_\pm)$, with $c_{s,\pm}(T)$ derived from the full free energy.

Solving hydrodynamics equations: steady–state solutions

Note : $\pm \rightarrow$ bubble wall frame $\qquad$ $\xi, f(\xi) \rightarrow$ center of the bubble frame

Steady–state solutions
(depend only on the *self-similar* variable $\xi = r/t$)

$$(\xi - v)\frac{\partial_\xi T}{w}\frac{de}{dT} = \frac{2v}{\xi} + [1 - \gamma^2 v(\xi - v)]\partial_\xi v\,,$$
$$\frac{\partial_\xi T}{T} = \gamma^2 \mu(\xi, v)\partial_\xi v\,,$$

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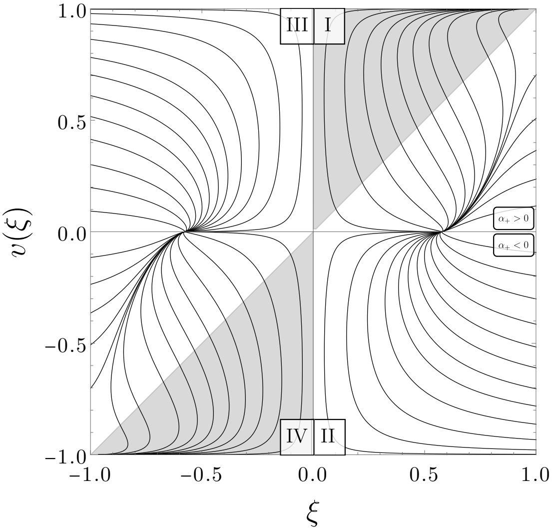
I Direct PTs

II Inverse PTs

III Direct Droplet collapse

IV Inverse Droplet collapse



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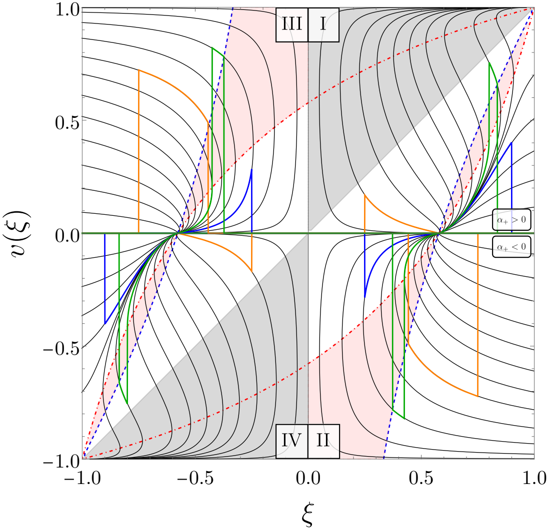

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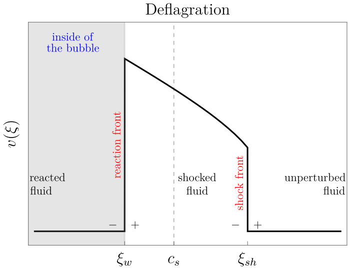

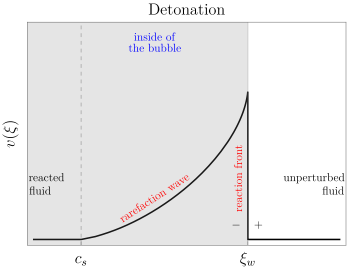

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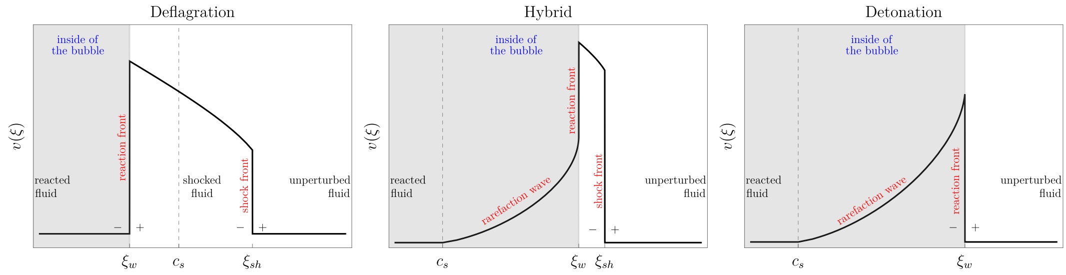

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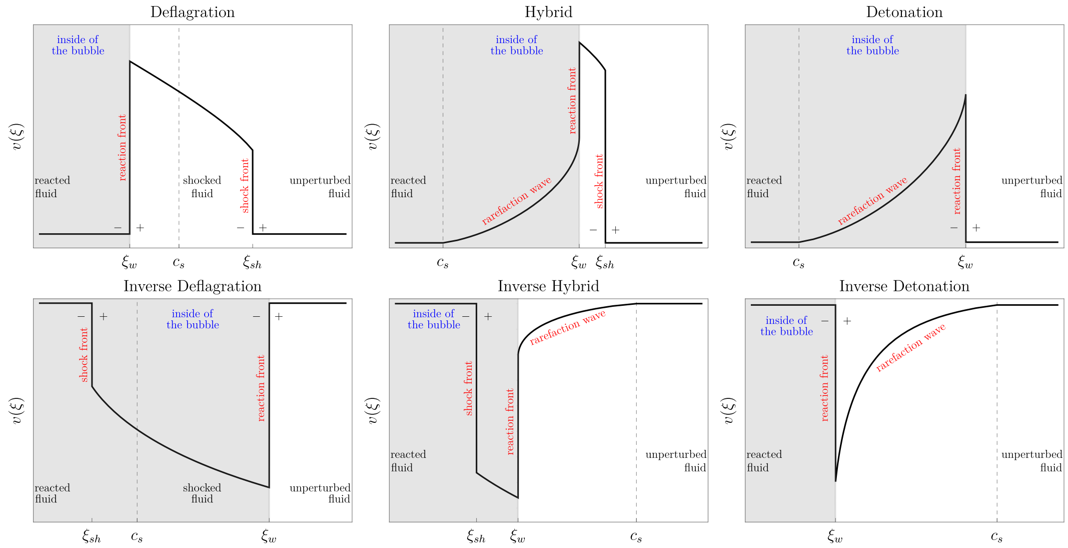

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- Why start in a minimum **away from the origin**?
- Can an *inverse* PT naturally occurs **within the standard cosmological cooling of the Universe**? (with no need to (re)heat the Universe?)

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Yes! ... but/thanks to SUSY ...

Where SUSY sucks (literally)

Proof of principle: the O'Raifeartaigh model

SUSY breaking hidden sector featuring a strong FOPT.

Model: SUSY breaking field X + $\Phi_{1,2}$ and $\tilde{\Phi}_{1,2}$ mediator fields

$$W = -FX + \lambda X \Phi_1 \tilde{\Phi}_2 + m(\Phi_1 \tilde{\Phi}_1 + \Phi_2 \tilde{\Phi}_2)$$

where $\sqrt{F}$ SUSY breaking scale. The model has a $U(1)$ $R-$symmetry.

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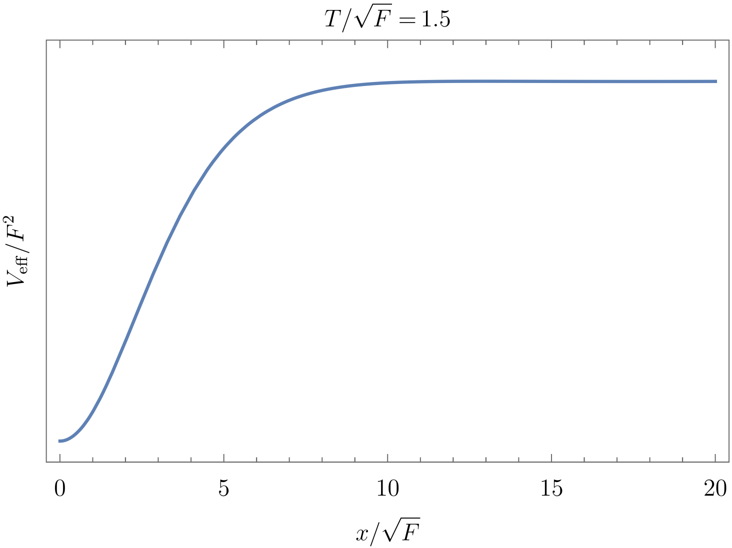
$$V_{\text{eff}} = \left|F - \lambda \phi_1 \tilde{\phi}_2\right|^2 + \left|\lambda X \tilde{\phi}_2 + m \tilde{\phi}_1\right|^2 + \left|\lambda X \phi_1 + m \phi_2\right|^2 + \left|m \phi_1\right|^2 + \left|m \tilde{\phi}_2\right|^2 + \text{loops} + \text{thermal corrections}$$

where $X = x/\sqrt{2}$.

Thermal history (for $m/\sqrt{F} = 2$ and $\lambda = 1.67$)

1. For $T \gg T_c$ origin is a global minimum.



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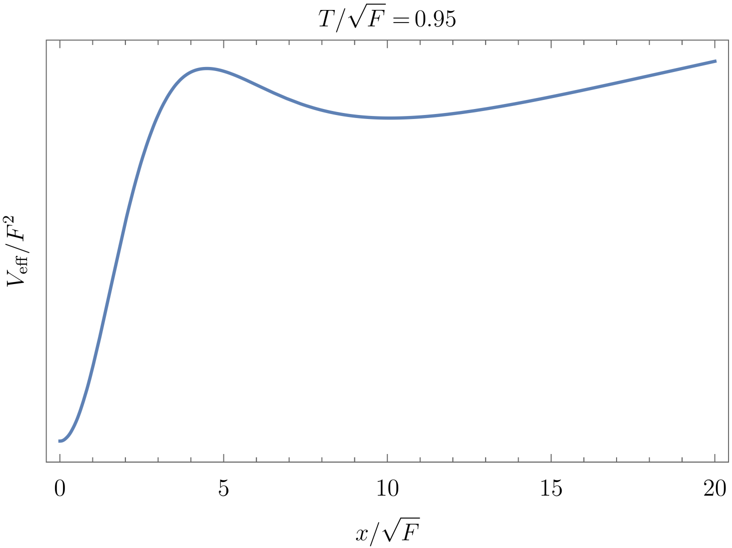

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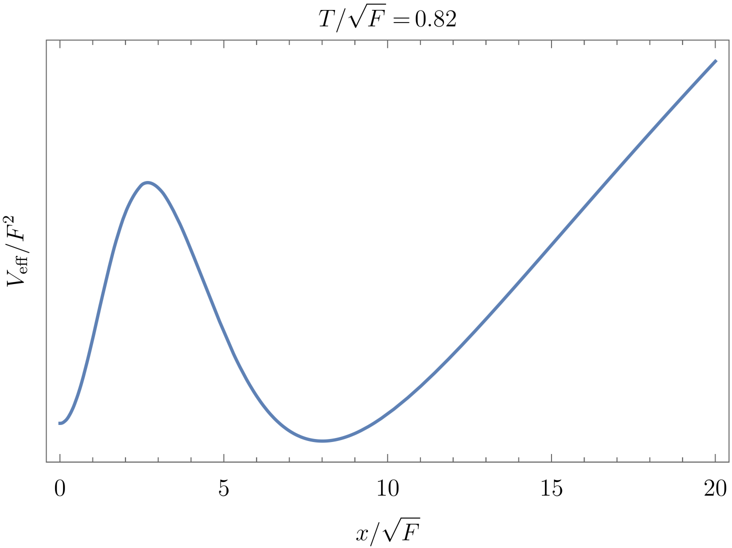

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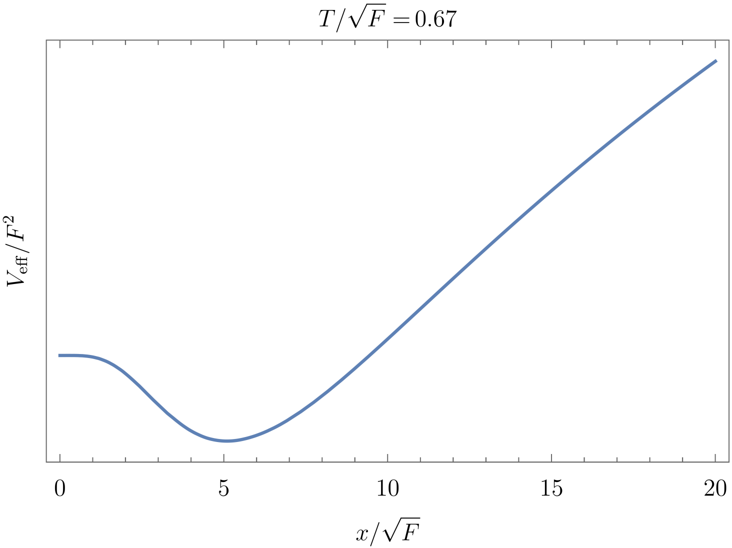
3. 1st (inverse) PT at $T_n/\sqrt{F} \sim 0.67$. ($R-$symmetry breaking)



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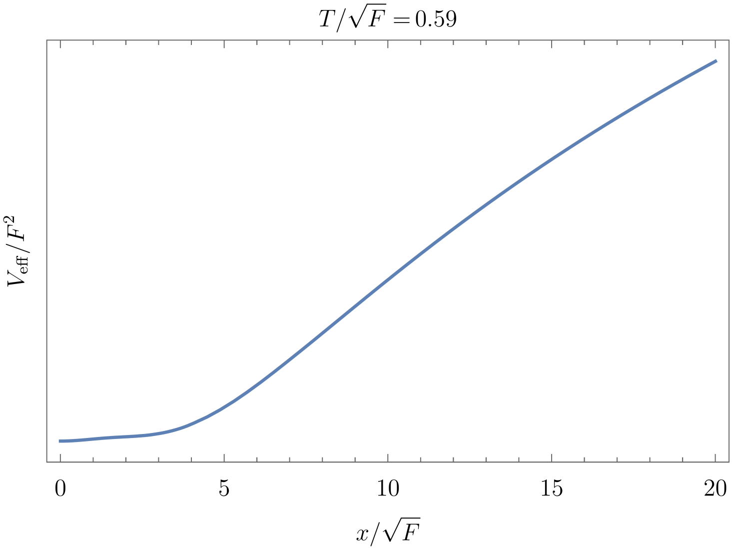
4. Barrier disappears
5. Barrier reappears
6. 2nd (direct) PT at $T_{n,2}/\sqrt{F} \sim 0.59$ ($R-$symmetry restoring)



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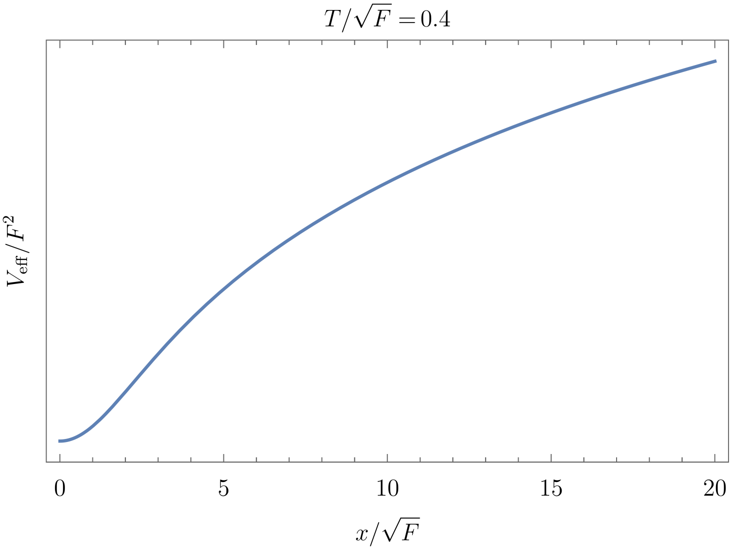
7. For $T = 0$ origin is a global minimum.



Inverse PTs parameter space for $m/\sqrt{F} = 2$

Pseudo-trace (θ):

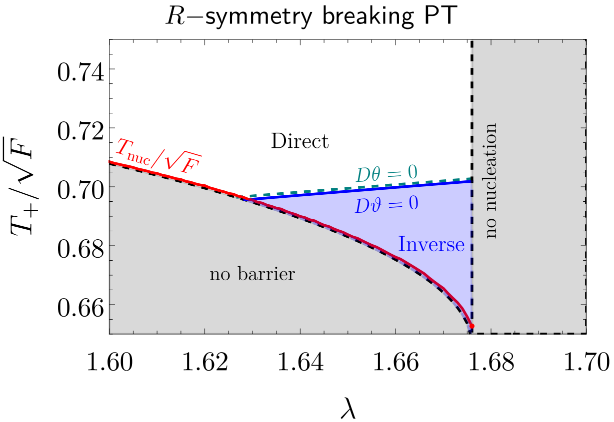
$$\alpha_\theta = \frac{1}{3w_+(T_+)}\left(De(T_+) - Dp(T_+)/c_{s,-}^2\right)$$

Generalised pseudo-trace (ϑ):

$$\alpha_\vartheta = \frac{1}{3w_+(T_+)}\left(De(T_+) - \frac{\delta e}{\delta p}(T_+, T_-)Dp(T_+)\right)$$

Direct/Inverse: $\alpha_\vartheta \gtrless 0$

$$Df = f_+(T_+) - f_-(T_+)$$
$$\delta f = f_-(T_+) - f_-(T_-).$$



Physical interpretation of Inverse PTs

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Latent Heat (L) characterizes the energetic nature of a phase transition

$$L = T_c(s_{\rm old} - s_{\rm new}) = w_{\rm old} - w_{\rm new},$$

where T_c is the critical temperature, s is the entropy density and w the enthalpy.

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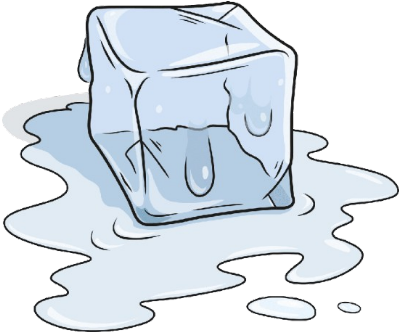
Exothermic Transitions ($L > 0$)	**Endothermic Transitions ($L < 0$)**
Energy is **released** during the transition.	Energy is **absorbed** during the transition.
The system transitions to a "more ordered" state.	The system transitions to a "less ordered" state.
Example: Liquid $\to$ Solid.	Example: Solid $\to$ Liquid.

Latent Heat in the Bag Model

In the **bag model**, the latent heat coincides with the vacuum energy difference between the phases:

$$L = \Delta w = 4\Delta\epsilon = 4(\epsilon_+ - \epsilon_-), \qquad \alpha_L = \frac{L}{3w_+(T_+)} = \frac{\Delta\epsilon}{a_+ T_+^4} \equiv \alpha_+$$

where $\epsilon_\pm$ are the vacuum energy densities of the old and new phases, respectively.



Within the Bag Model:

- $\Delta\epsilon > 0$: *Direct* PT, driven by vacuum energy.
- $\Delta\epsilon < 0$: *Inverse* PT, against vacuum energy.

General model 

We defined the difference of the generalised pseudo-trace as

$$D\vartheta = De(T_+) - \frac{\delta e}{\delta p}(T_+, T_-)Dp(T_+)$$

but it can be rewritten, using $e = w - p$, as

$$D\vartheta = Dw(T_+) - \left(1 + \frac{\delta e}{\delta p}(T_+, T_-)\right) Dp(T_+)$$

Realising that $D\vartheta(T_c) = L = \Delta w$, then $D\vartheta$ **is nothing but the generalisation of the latent heat away from** T_c, so

$$\text{Inverse PTs} \quad \Longleftrightarrow \quad \text{Endothermic CosmoPTs}$$

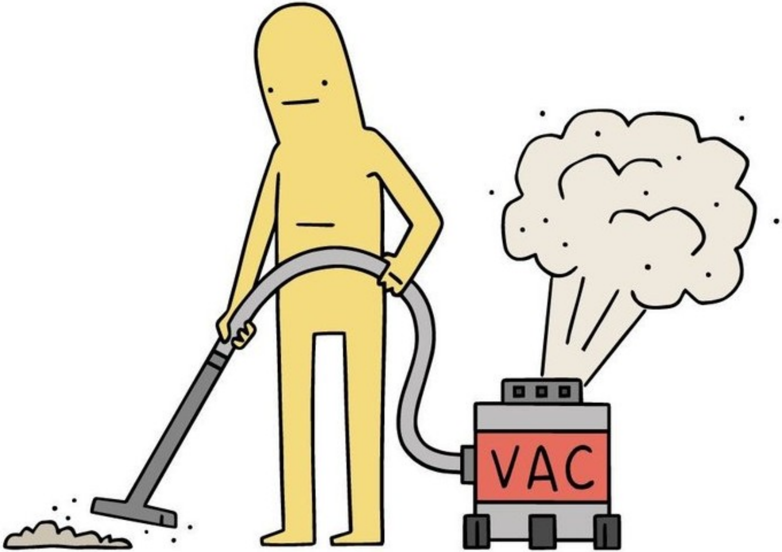
Conclusions

1. We introduced *direct* and *inverse* PTs and found the respective steady state (self-similar) solutions.
2. In *direct* PTs the wall pushes the plasma and (part of) the **vacuum energy is converted in kinetic energy**.
3. In *inverse* PTs the bubble **sucks the plasma** into it consequently pushing the wall. The initial **thermal energy is converted in vacuum and kinetic energy**.
4. We have shown a concrete model where *inverse* PTs occur during the cooling of the Universe.
5. We fully characterized the *inverse* PTs in terms of the generalised pesudo-trace and established its 1–to–1 correspondence with **endothermic** PTs.

Outlooks:

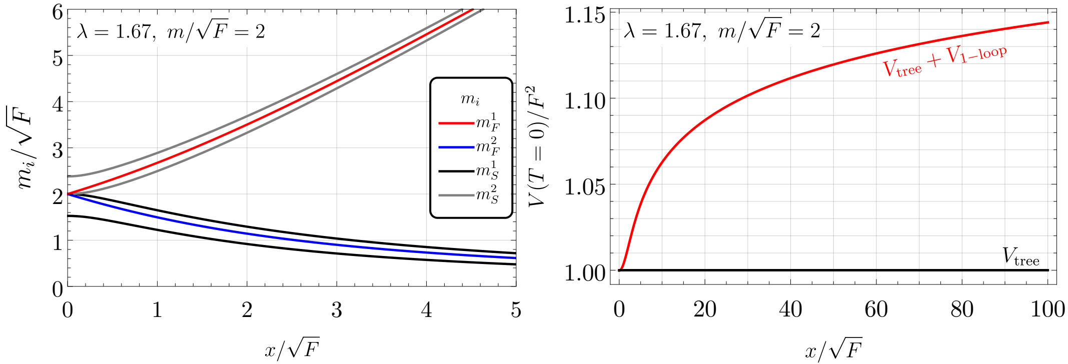
1. Distinguish Direct/Inverse PTs from GWs spectra using SoundShellModel (see Mark Hindmarsh talk)
2. Non–SUSY realisations?
3. (Controlled) Superheated Inverse PTs

Thanks for
your attention!

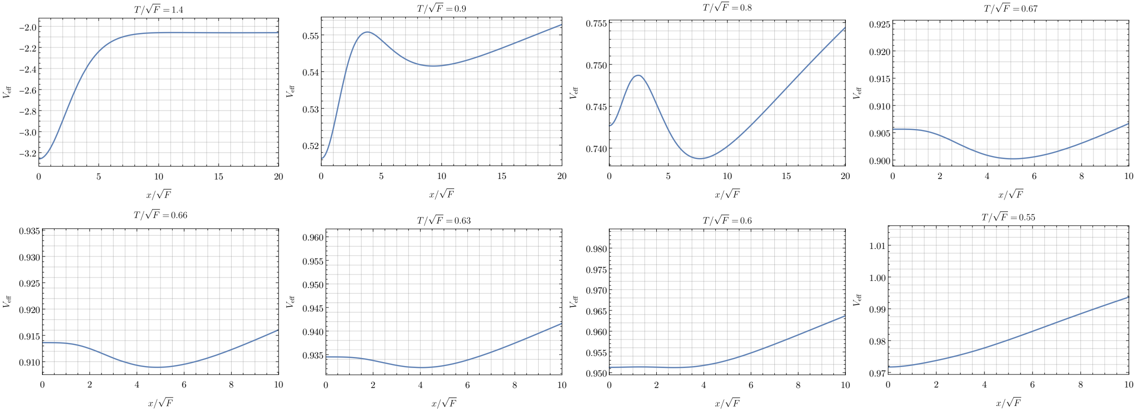


Backup

Sprectrum of the SUSY model



More on thermal history



Matching conditions and possible solutions

$$v_+ v_- = \frac{p_+ - p_-}{e_+ - e_-}\,, \qquad \frac{v_+}{v_-} = \frac{e_- + p_+}{e_+ + p_-}$$

where $p = -V_{\rm eff}(T)$, $w = T\dfrac{\partial p}{\partial T}$ and $e = w - p$.

Direct/Inverse: $\alpha_\vartheta \gtrless 0$

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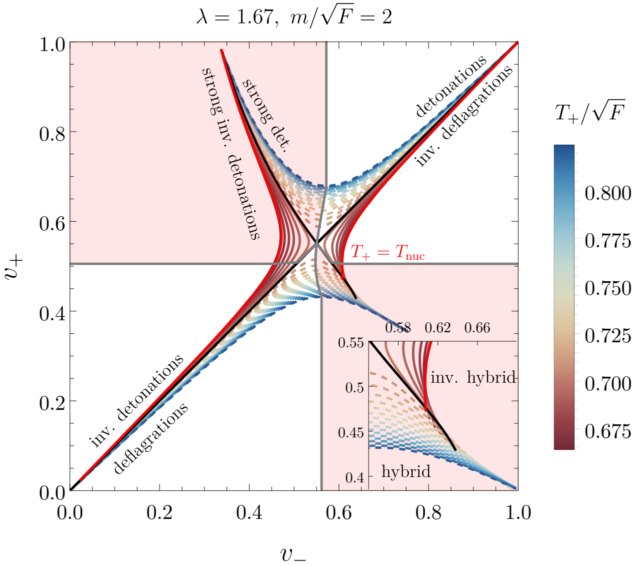
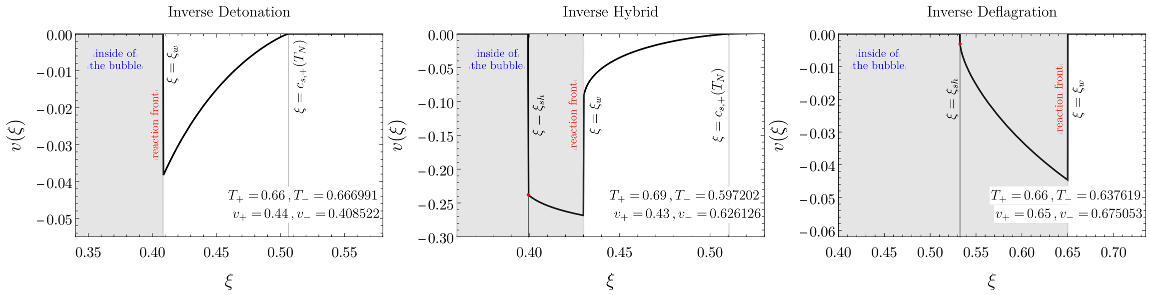
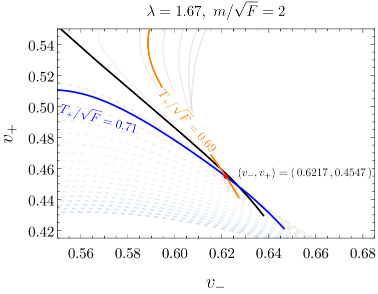
$Df = f_+(T_+) - f_-(T_+)$ and
$\delta f = f_-(T_+) - f_-(T_-)$.

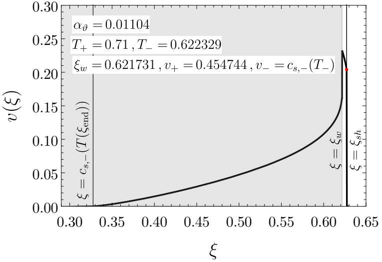


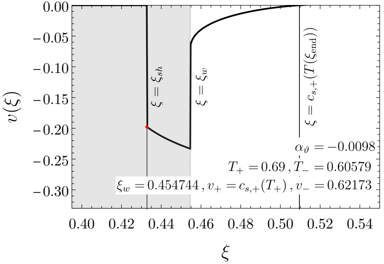
Full numerical fluid profiles



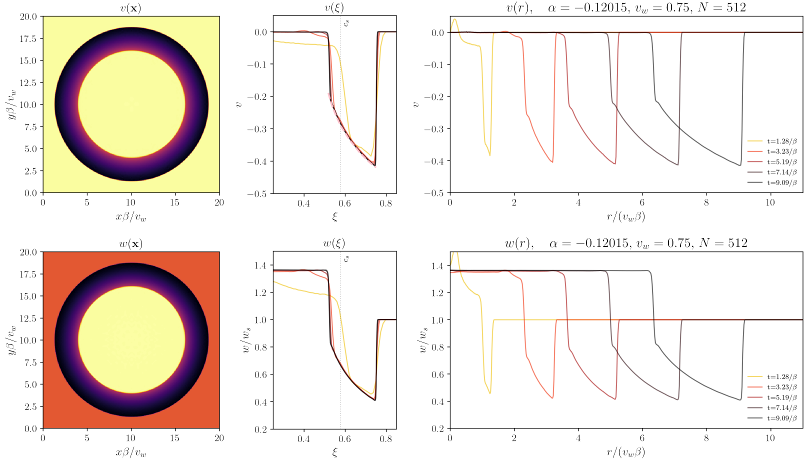
Overlap in the hybrid corner

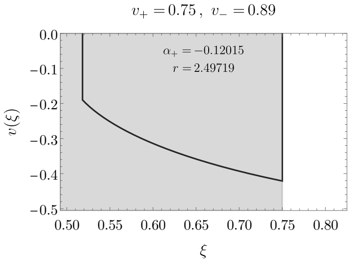






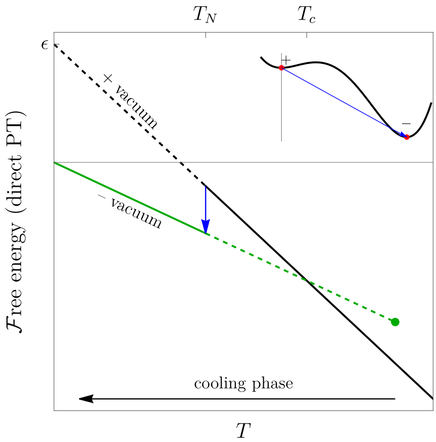
Self-similar solutions (dynamical evolution)



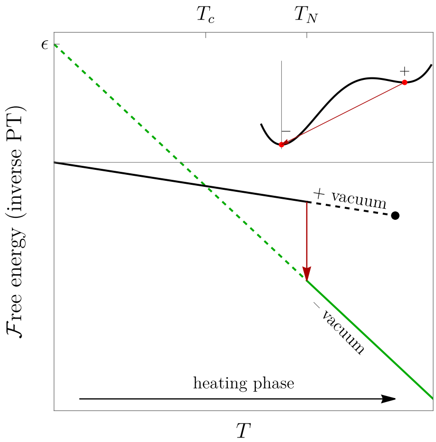


Thanks to Isak Stomberg!

BAG Equation of State (EoS)



$$\epsilon_+ = \epsilon, \quad \epsilon_- = 0$$



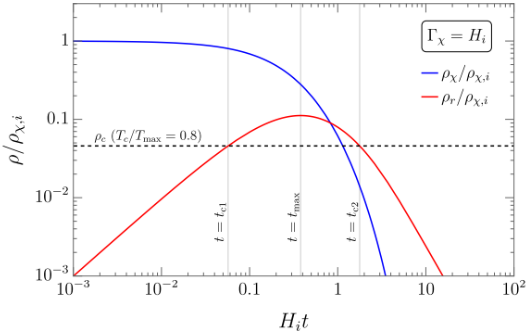
$$\epsilon_+ = 0, \quad \epsilon_- = \epsilon$$

Toy model for inverse PT (from 2305.09712)

$$V(\phi) = \frac{\mu^2}{2}(T^2 - T_0^2)\phi^2 - \frac{A}{3}T\phi^3 + \frac{\lambda}{4!}\phi^4$$

Reheating toy model:

- (only) reheaton χ with ρ_χ anf Γ_χ
- Choosing H_i we decide when it starts to decay
- decay to a DS
- via portal interaction reheats the SM



ρ_χ: reheaton energy density
ρ_r: radiation energy density
T_c: critical temperature
$T_{\max}$: maximal temperature

Energy budget & efficiency

Energy budget of PTs

$$w(\xi) = w(\xi_0)\ \exp\left[\int_{v(\xi_0)}^{v(\xi)} \left(\frac{1}{c_s^2}+1\right)\gamma^2(v)\mu(\xi(v),v)\ dv\right]$$

Energy budget (direct): $$\underbrace{\frac{\xi_w^3}{3}\epsilon}_{\text{vacuum energy}} + \underbrace{\frac{3}{4}\int w_N \xi^2 d\xi}_{\text{initial thermal energy}} = \underbrace{\int \gamma^2 v^2 w \xi^2 d\xi}_{\text{fluid motion}} + \underbrace{\frac{3}{4}\int w \xi^2 d\xi}_{\text{final thermal energy}}$$

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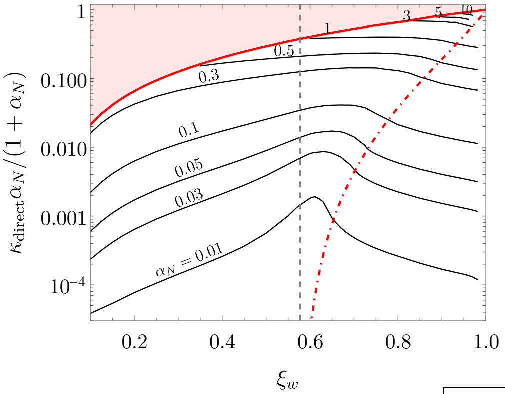
Energy budget (inverse): $$\underbrace{\frac{3}{4}\int w_N \xi^2 d\xi}_{\text{initial thermal energy}} = \underbrace{\frac{\xi_w^3}{3}\epsilon}_{\text{vacuum energy}} + \underbrace{\int \gamma^2 v^2 w \xi^2 d\xi}_{\text{fluid motion}} + \underbrace{\frac{3}{4}\int w \xi^2 d\xi}_{\text{final thermal energy}}$$

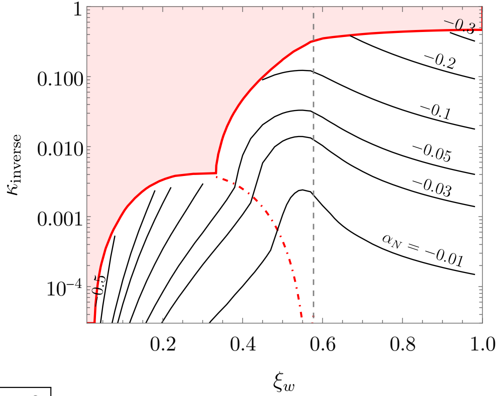
Initial energy will be in part converted in **kinetic bulk motion**!

Efficiency factors

$$\left.\frac{\rho_{\rm kin}}{\rho_{\rm tot}}\right|_{\rm direct} \equiv \kappa_{\rm direct}\frac{\alpha_N}{1+\alpha_N}\,, \qquad \kappa_{\rm direct} = \frac{3}{\epsilon\xi_w^3}\int \gamma^2 v^2 w \xi^2 d\xi$$

$$\left.\frac{\rho_{\rm kin}}{\rho_{\rm tot}}\right|_{\rm inverse} \equiv \kappa_{\rm inverse} \equiv \frac{4}{\bar{v}^3}\int \xi^2 d\xi\, v^2\gamma^2\frac{w}{w_N}$$





$$\boxed{\Omega_{\rm GW} \sim \left(\frac{\rho_{\rm kin}}{\rho_{\rm tot}}\right)^2}$$

Types of solitions (detailed)

Types of discontinuities for cosmological *direct* phase transitions		
	Detonations $p_+ < p_-, v_+ > v_-$	Deflagrations $p_+ > p_-, v_+ < v_-$
Weak	$v_+ > c_s, v_- > c_s$ Physical	$v_+ < c_s, v_- < c_s$ Physical
Chapman-Jouguet	$v_+ > c_s, v_- = c_s$ Physical	$v_+ < c_s, v_- = c_s$ Physical
Strong	$v_+ > c_s, v_- < c_s$ Forbidden	$v_+ < c_s, v_- > c_s$ Unstable

Types of discontinuities for cosmological *inverse* phase transitions		
	Inverse Detonations $(p_+ < p_-, v_+ > v_-)$	Inverse Deflagrations $(p_+ > p_-, v_+ < v_-)$
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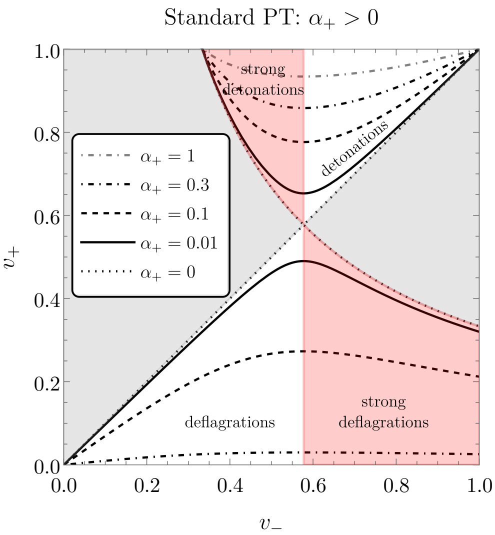
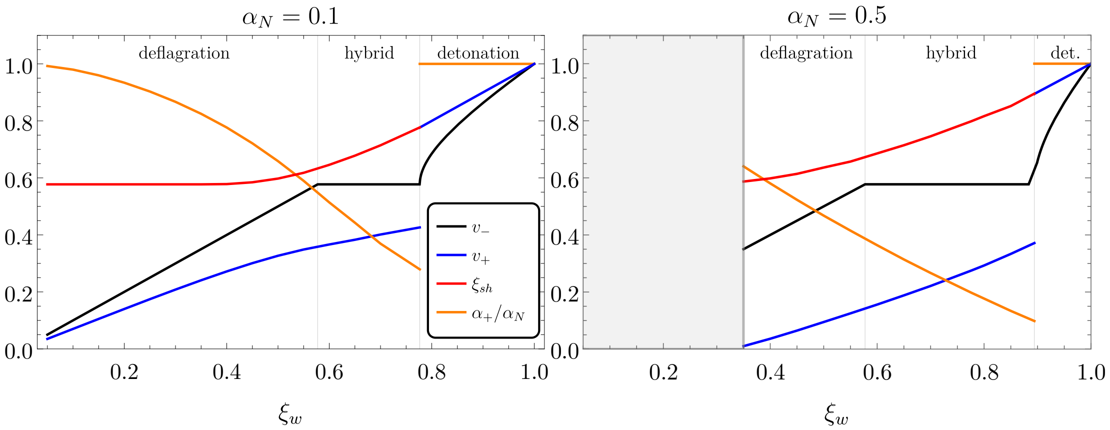
Impossibility of strong solutions

- **Strong detonations**: velocity has to be zero at the centre of the bubble and very far away from the wall, and having $v > 0$ translates into
$$\frac{\mu^2}{c_s^2} - 1 > 0\ , \qquad v_- > c_s$$
so detonations with $v_- < c_s$ are fordibben.

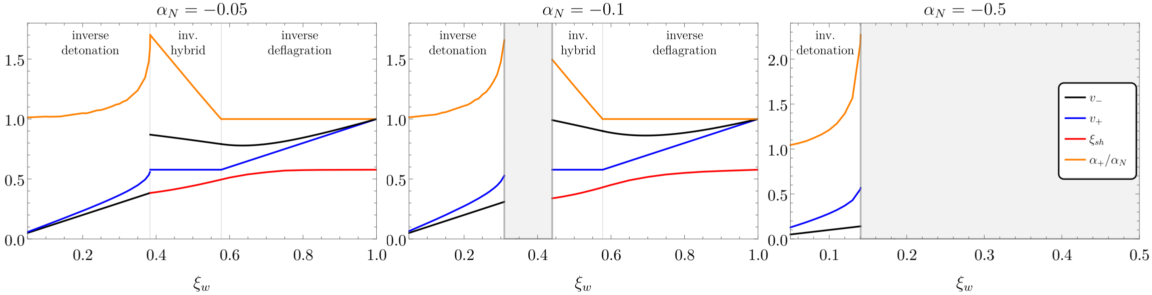
- **Strong deflagration**:
  - unstable wrt perturbations
  - entropy decreases



Evolution of quantities across the wall (direct)



Evolution of quantities across the wall (inverse)



Hydrodynamic description

Consider a collection of N particles

$$\mathcal{R} = \lambda_{DB}/\ell_{\text{mfp}}$$

λ_{DB} : De Broglie wavelenght , ℓ_{mfp} = mean free path

	CM	QM
$\mathcal{R} \ll 1$	✔	✘
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but if $N \gg 1$ (while $\mathcal{R} \ll 1$) then **Kinetic theory** $\Rightarrow$ distr. function $f(t, \vec{x}, \vec{u})$

<u>Evolution eq.</u> : Boltzmann eq. for $f(t, \vec{x}, \vec{u})$

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but if $N \ggg 1$ such that $L \gg \ell_{\rm mfp}$ with L: length scale of the system (so that $\mathcal{R} \lll 1$), then

Thermalisation occurs fast $\rightarrow$ Local Thermal Equilibrium $\rightarrow$ **Continuum fluid** ✓

Microscopic physics

1. Specify the model

$$\underline{\text{ex.}}\text{: } \mathcal{L} \supset |D_\mu \phi|^2 - V_{\text{tree}}(\phi) - \sum_i y_i(\phi \bar{\psi}_L \psi_R + h.c.) + \dots$$

Microscopic physics

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2. EOM:

$$\Box\phi + \frac{dV_{\text{tree}}(\phi)}{d\phi^*} - igA^\mu\partial_\mu\phi - ig(\partial\cdot A)\phi + g^2A^2\phi + \sum_i y_i\bar{\psi}_R\psi_L = 0$$

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$$\langle A^2\rangle = \langle A^2\rangle_{\rm vac} + \sum\int\frac{d^3k}{(2\pi)^3E}f(k,x)\ , \qquad \langle\bar{\psi}_R\psi_L\rangle = \frac{1}{2}\langle\bar{\psi}\psi\rangle_{\rm vac} + \sum\int\frac{d^3k}{(2\pi)^3}\frac{m}{2E}f(k,x)$$

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5. Combining all together we arrive at

$$\boxed{\Box\phi_{\text{cl}} + V_0'(\phi_{\text{cl}}) + \sum_i \frac{dm_i^2(\phi_{\text{cl}})}{d\phi_{\text{cl}}} \int \frac{d^3k}{(2\pi)^3 E_i} f_i(k,x) = 0}$$

Friction

$$\boxed{\Box\phi_{\mathrm{cl}} + V_0'(\phi_{\mathrm{cl}}) + \sum_i \frac{dm_i^2(\phi_{\mathrm{cl}})}{d\phi_{\mathrm{cl}}} \int \frac{d^3k}{(2\pi)^3 E_i} f_i(k,x) = 0}$$

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1. Vacuum force: $F_{\rm vacuum} \equiv \int dz\,\partial_z\phi_{\rm cl}\,V_0'(\phi_{\rm cl}) = \epsilon_+ - \epsilon_-$

$$\begin{cases} \text{Direct} : F_{\rm vacuum} > 0 \\ \text{Inverse} : F_{\rm vacuum} < 0 \end{cases}$$

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$$\boxed{\text{Total force: } F_{\rm vacuum} - \mathcal{P}_{\rm plasma} = \epsilon_+ - \epsilon_- + \Delta V_T - \int dz \frac{\partial V_T}{\partial T}\partial_z\phi_{\rm cl} - \mathcal{P}_{\rm dissipative}}$$

Runaway(?)

Ultrarelativistic walls $\rightarrow$ neglect collisions among particles

$$f_i(p, T) = f_{\mathrm{outside}}(p, T_N)$$

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then in the limit $\gamma_w \rightarrow \infty$

$$\begin{aligned}
\mathcal{P}_{\text{plasma}} &= -\int dz \partial_z \phi \sum_i g_i \frac{dm_i^2(\phi)}{d\phi} \int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_i} f_i(p,z,T) \\
&= \begin{cases} +\sum_i c_i g_i \dfrac{|\Delta m^2| T_N^2}{24}, & \text{Direct PT} \\ -\sum_i c_i g_i \dfrac{|\Delta m^2| T_N^2}{24} C_{\text{eff},i}, & \text{Inverse PT} \end{cases}
\end{aligned}$$

where

$$C_{\text{eff}}(m_i^{\text{out}}/T) \approx \begin{cases} 1 & \text{if} \quad m_i^{\text{out}} \ll T \,, \\ \dfrac{12}{(2\pi)^{3/2}} \left(\dfrac{m_i^{\text{out}}}{T_N}\right)^{1/2} e^{-m_i^{\text{out}}/T} & \text{if} \quad m_i^{\text{out}} \gg T \,. \end{cases}$$

Runaway(?)

Runaway possible if $F_{\text{vacuum}} - \mathcal{P}_{\text{plasma}}\Big|_{\gamma_w \to \infty} > 0$

$$\begin{cases} \text{Direct} \quad \epsilon > \left| \sum_i c_i g_i \frac{|\Delta m_i^2| T^2}{24} \right| & \text{need PT strong enough} \\ \text{Inverse} \quad \epsilon < \left| \sum_i C_{\text{eff,i}}(m_i^{\text{out}}/T) c_i g_i \frac{|\Delta m_i^2| T^2}{24} \right| & \text{not impossible, but hard to get} \end{cases}$$

Disclaimer: we did not consider dissipative contribution nor splitting $1 \to 2$, etc.

Theory of discontinuities

Stability of solutions wrt perturbations