Models for PQ inflation and axion kinetic misalignment

Adriana Menkara (DESY.) & Hyun Min Lee, Myeong-Jung Seong, Jun-Ho Song(CAU)

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The Dawn of Gravitational Wave Cosmology

(Benasque Science Center, Apr 27 - May 17, 2025)

Strong CP problem

$$\theta_{SM} = \theta_{QCD} + \arg[\det Y_u Y_d] \leq 10^{-10}$$ **??**

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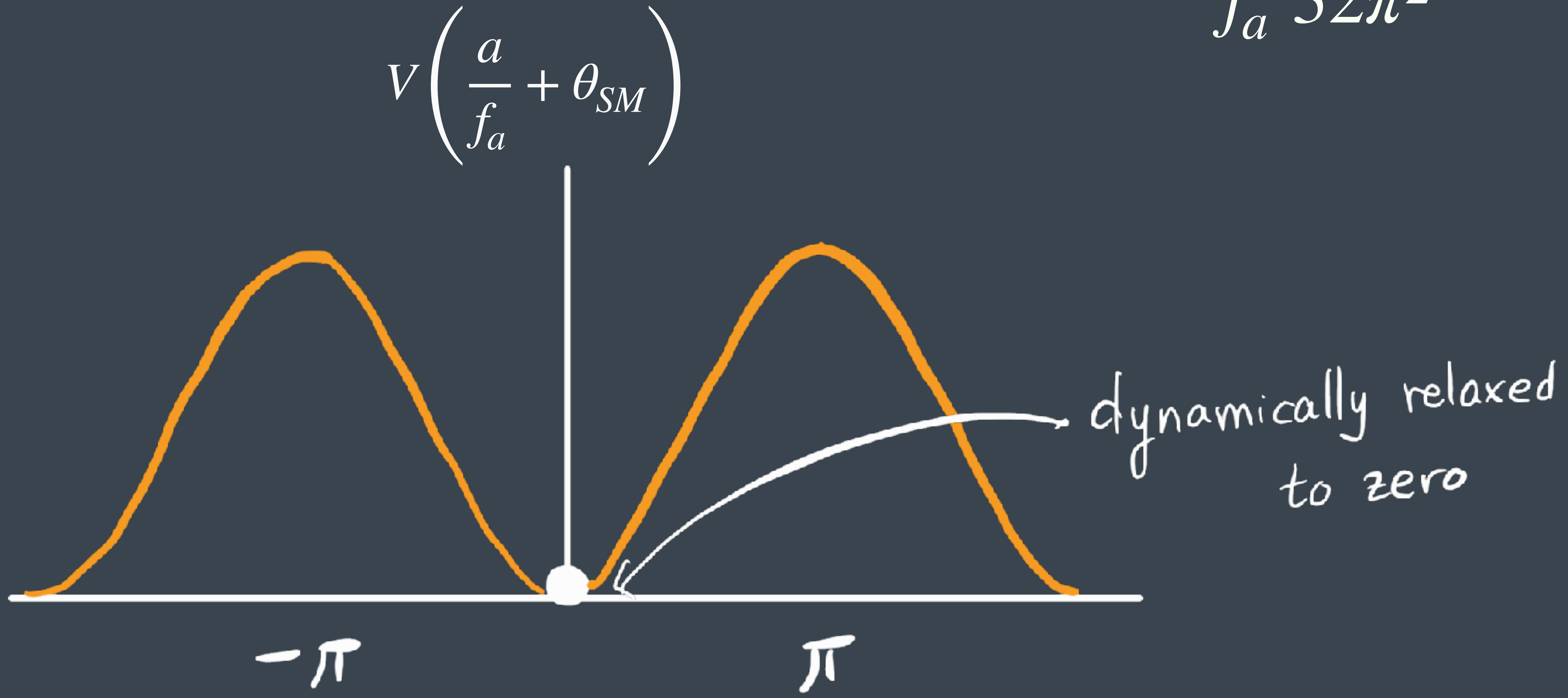
The coupling to QCD gives the axion a mass

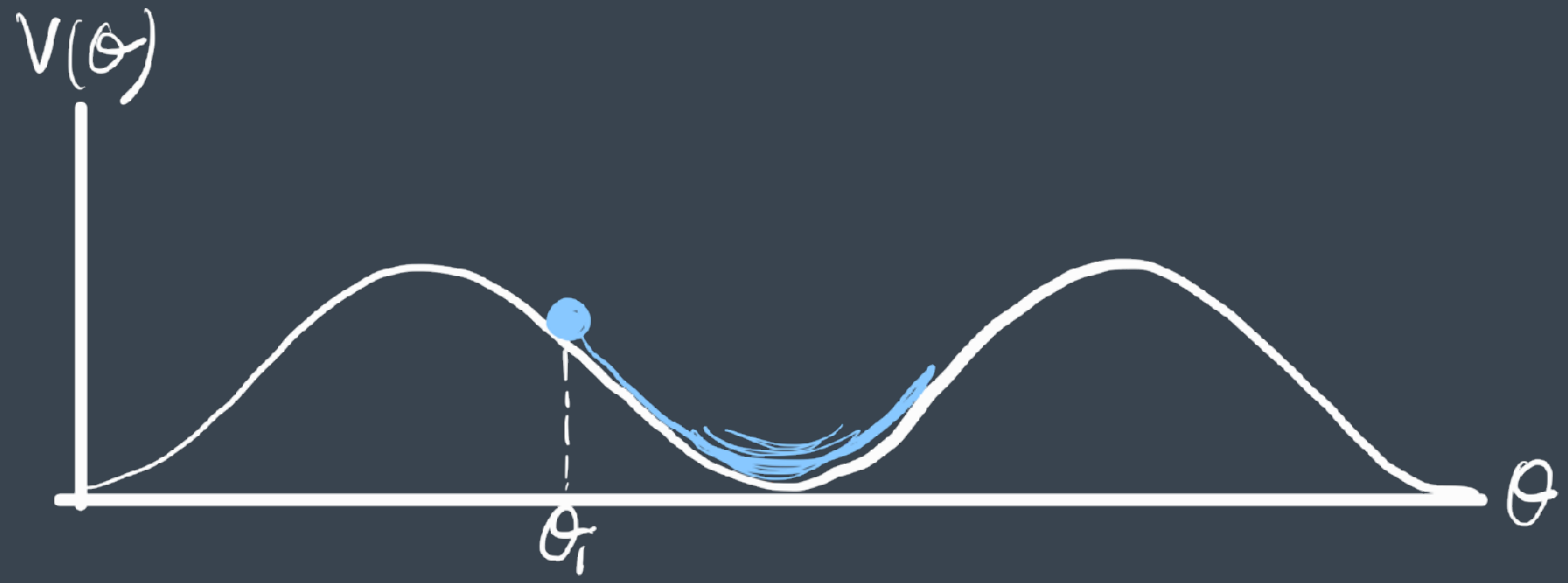
$$\Delta\mathscr{L}_{QCD} = \frac{a}{f_a}\frac{g_s^2}{32\pi^2}G_{\mu\nu}\tilde{G}^{\mu\nu}$$



Axion Dark Matter

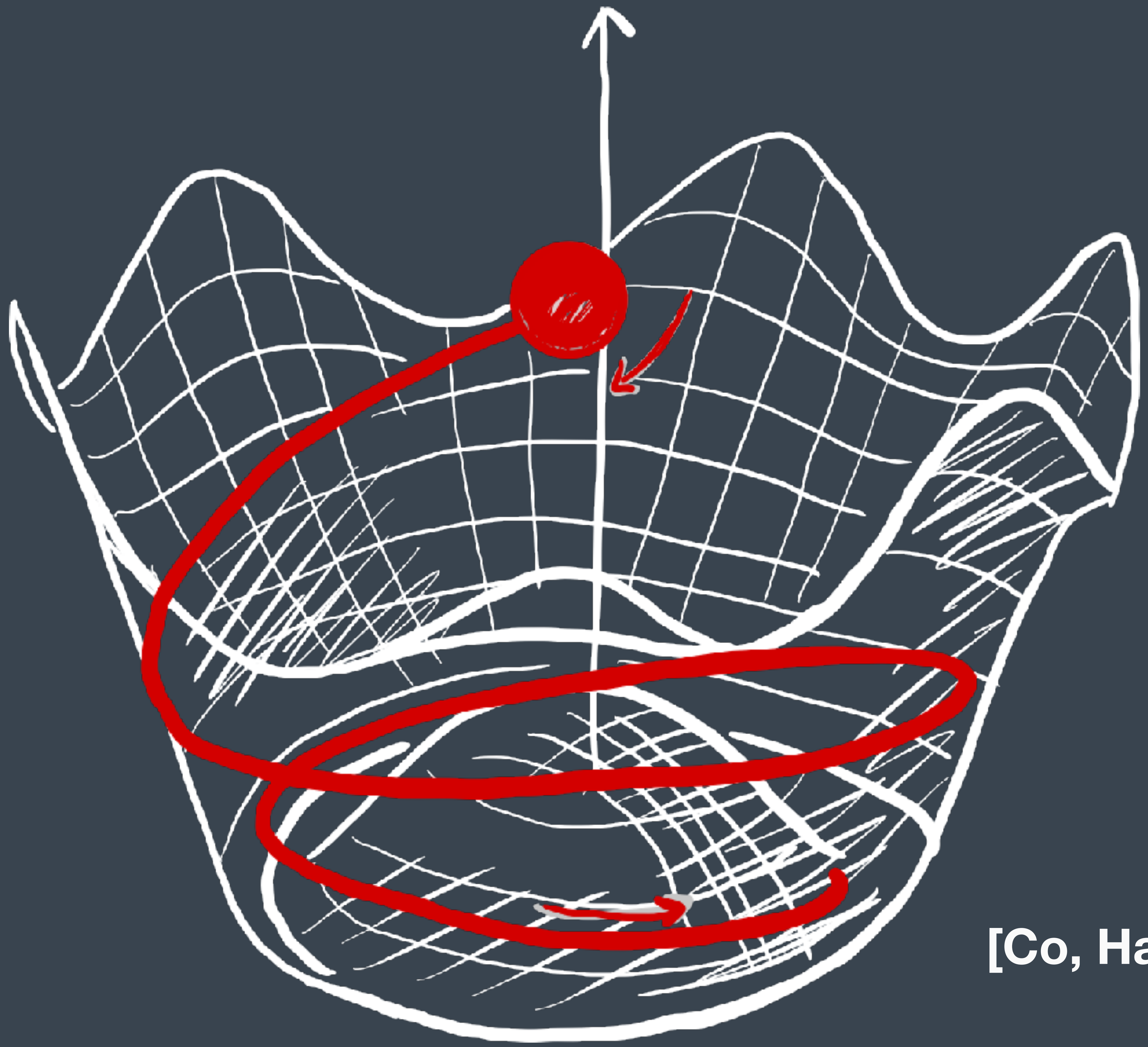
Axion can realize the correct DM abundance

Misalignment Mechanism



Axion rotation

$$\Phi = \frac{1}{\sqrt{2}} \rho e^{i\theta}$$



[Co, Harigaya (2020)]

Why rotation?

Angular motion: PQ explicit breaking terms

$$V(P) \sim \frac{P^n}{M_p^{n-4}} + h.c.$$

Higher dimensional operators give a mass to the axion: **quality problem.**

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Angular motion: PQ explicit breaking terms

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Higher dimensional operators give a mass to the axion: **quality problem.**

Usual solution: Power ***n*** large enough so that $\dfrac{f_a}{\Lambda_{\text{cutoff}}}$ is suppressed.

PQ charge

The axion has a shift symmetry, $a \to a + 2\pi f_a$

$$n_{PQ} = iP\dot{P}^* - iP^*\dot{P}$$

$$n_{PQ} = \rho^2\dot{\theta}$$ Angular momentum

- n_{PQ} is conserved after the onset of oscillations
- Its initial value determines the final Dark Matter abundance.

This work

- We use the set-up of **inflation at the pole** in order to provide the initial conditions for the axion velocity.

- In contrast to previous work relying on a small non-minimal coupling, we exploit conformality.

  [M. Fairbairn, R. Hogan and D. J. E. Marsh, ‘14]
  [K. Nakayama and M. Takimoto, ’15]
  [G. Ballesteros, J. Redondo, A. Ringwald and C. Tamarit, ‘16]

Pole inflation

α-attractor properties arise from a non-minimal coupling to gravity,

$$\frac{\mathscr{L}_J}{\sqrt{-g_J}} = \frac{1}{2}M_P^2\boxed{\left(1-\frac{1}{6M_P^2}\phi^2\right)}R_J + \frac{1}{2}\left(\partial_\mu\phi\right)^2 - V_J\left(\phi\right) \qquad V_J(\phi) = F(\phi)\left(1-\frac{1}{6}\phi^2\right)^2$$

which in the Einstein frame translates as a **pole** in the kinetic term

$$\frac{\mathscr{L}_E}{\sqrt{-g_E}} = -\frac{1}{2}M_P^2R_E + \boxed{\frac{1}{2}\frac{\left(\partial_\mu\phi\right)^2}{\left(1-\frac{1}{6M_P^2}\phi^2\right)^2}} - V_E(\phi)$$

Pole inflation

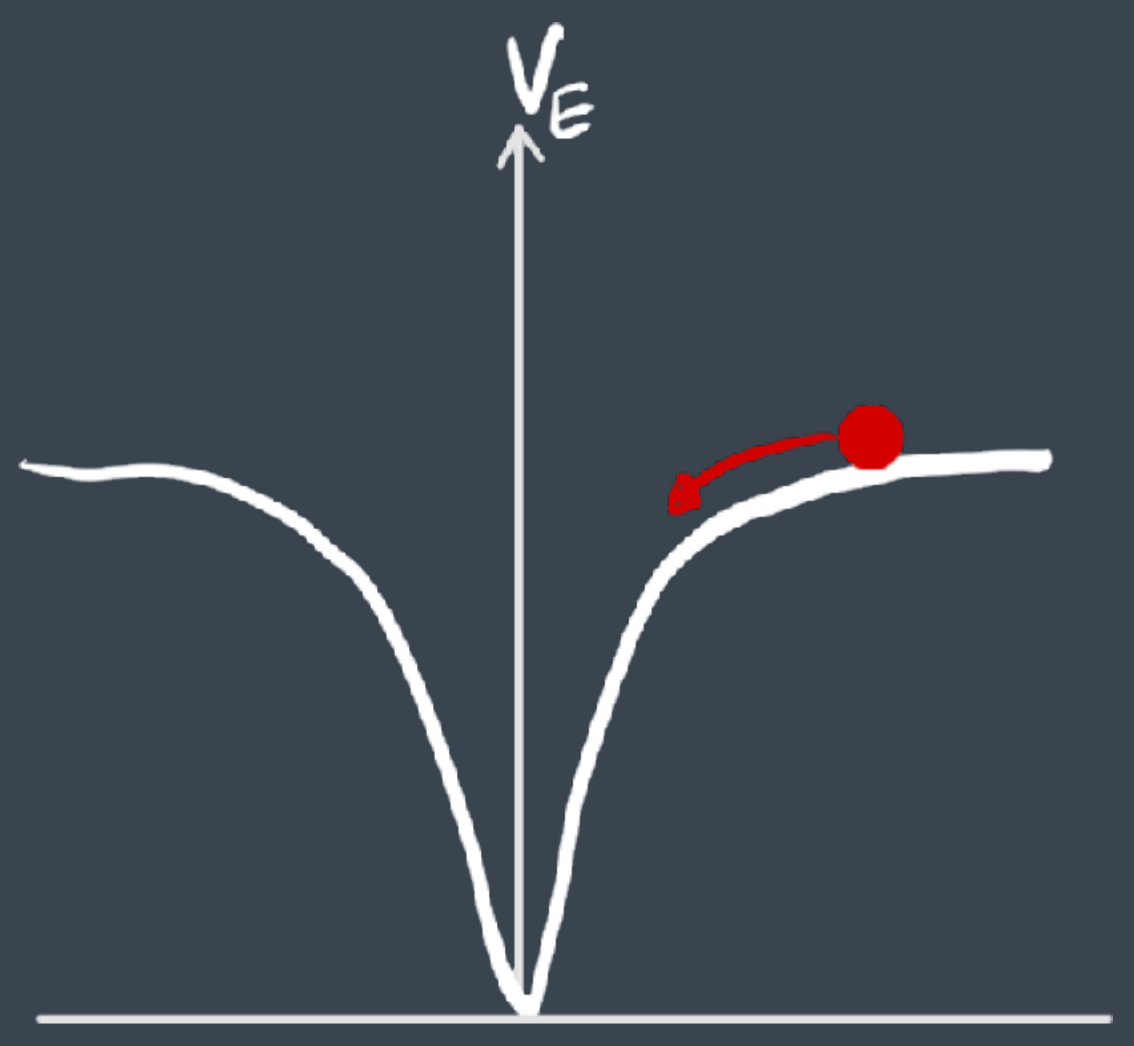
$$V_J(\phi) = c_m \Lambda^{4-2m} \phi^{2m} \left(1 - \frac{1}{6M_P^2}\phi^2\right)^2$$

Inflation happens for a **vanishing Jordan frame potential**

$$\Updownarrow \qquad \phi = \sqrt{6}M_P \tanh\left(\frac{\chi}{\sqrt{6}M_P}\right)$$

$$V_E(\chi) = 3^m c_m \Lambda^{4-2m} M_P^{2m} \left[\tanh\left(\frac{\chi}{\sqrt{6}M_P}\right)\right]^{2m}$$

Inflation happens at the **pole of the kinetic Einstein frame**



Models for PQ inflation and KMM.

Model content and charges under the $U(1)_{PQ}$

	KSVZ	DFSZ
SM Fermions	Not charged	Charged
Higgses	One doublet, not charged	$H_1^\dagger H_2 \Phi^p,\ q_1 - q_2 = p q_\phi$
Extra quarks	$Q_L, Q_R, -q_\phi/2$	$-$
QCD anomaly	$\xi = q_\phi$	$\xi = 3 p q_\phi$

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Only **type II and Y** allow for QCD anomalies.

PQ at the pole

$$\Phi = \frac{1}{\sqrt{2}}\rho e^{i\theta}$$

$$\frac{\mathscr{L}_J}{\sqrt{-g_J}} = -\frac{1}{2}M_P^2\,\Omega(\Phi)R(g_J) + |\partial_\mu \Phi|^2 - \Omega^2(\Phi)V_E(\Phi)$$

Conformal couplings

$$\Omega(\Phi) = 1 - \frac{1}{3M_P^2}|\Phi|^2$$

$$V_E(\Phi) = V_0' + \frac{\beta_m}{M_P^{2m-4}}|\Phi|^{2m} - m_\Phi^2|\Phi|^2 + \left(\sum_{k=0}^{[n/2]}\frac{c_k}{2M_P^{n-4}}|\Phi|^{2k}\Phi^{n-2k} + \mathrm{h.c.}\right)$$

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PQ conserving terms

The PQ terms **drive inflation** and are responsible for the **SSB of the** $U(1)_{PQ}$

PQ at the pole

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The PQ violating terms are crucial for the axion non-zero velocity, but are constrained by the **axion quality problem**.

Bounds: inflation

canonical axion

$$V_{\rm PQ} = \frac{1}{4}\lambda_\phi\left(6M_P^2\tanh^2\left(\frac{\phi}{\sqrt{6}}\right) - f_a^2\right)^2$$

CMB normalization $\Longrightarrow \lambda_\Phi = 1.1\times 10^{-11}$

generalized

$$V_{PQ} = \frac{\beta_m}{M_P^{2m-4}}|\Phi|^{2m} - m_\Phi^2|\Phi|^2$$

CMB normalization $\Longrightarrow 3^m\beta_m = 1.0\times 10^{-10}$

Bounds: Axion Quality

After the QCD phase transition, we get the contribution $\Delta V_E = -\Lambda_{\mathrm{QCD}}^4 \cos\left(\bar{\theta} + \xi \frac{a}{f_a}\right)$

$$V_{\mathrm{eff}}(a) = -\Lambda_{\mathrm{QCD}}^4 \cos\underbrace{\left(\bar{\theta} + \xi \frac{a}{f_a}\right)}_{\lesssim 10^{-10}} + M_P^4 \left(\frac{f_a}{\sqrt{2} q_\Phi M_P}\right)^l \sum_{k=0}^{[l/2]} |c_{0,l,k}| \cos\left((l-2k)\frac{q_\Phi a}{f_a} + A_{0,l,k}\right)$$

In order to solve the strong CP problem we need

$f_a = 10^{12}(10^8)\,\mathrm{GeV}$ requires $l \gtrsim 13(8)$

Bounds: axion velocity

Asking for $\cancel{PQ}$ terms to be subdominant during inflation leads to

$$n \gtrsim 10(5) \text{ for } f_a = 10^{12}(10^6) \text{ GeV} \quad \text{and} \quad 3^{n/2}|c_k| \lesssim 10^{-10}$$

The non-zero velocity is given dynamically at the end of inflation

$$\dot{\theta} \simeq -\frac{\sqrt{2\epsilon_\theta}H}{6\sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)} \ll H$$

Bounds: axion velocity

Asking for $\cancel{PQ}$ terms to be subdominant during inflation leads to

$n \gtrsim 10(5)$ for $f_a = 10^{12}(10^6)$ GeV and $\boxed{3^{n/2}|c_k| \lesssim 10^{-10} \implies |\dot{\theta}_{end}| \lesssim 0.9^n \times 6.10^{-7} M_P}$

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Delayed reheating?

- If $\phi(a_{\rm RH}) > 3f_a$, we **don't** have **early matter domination**

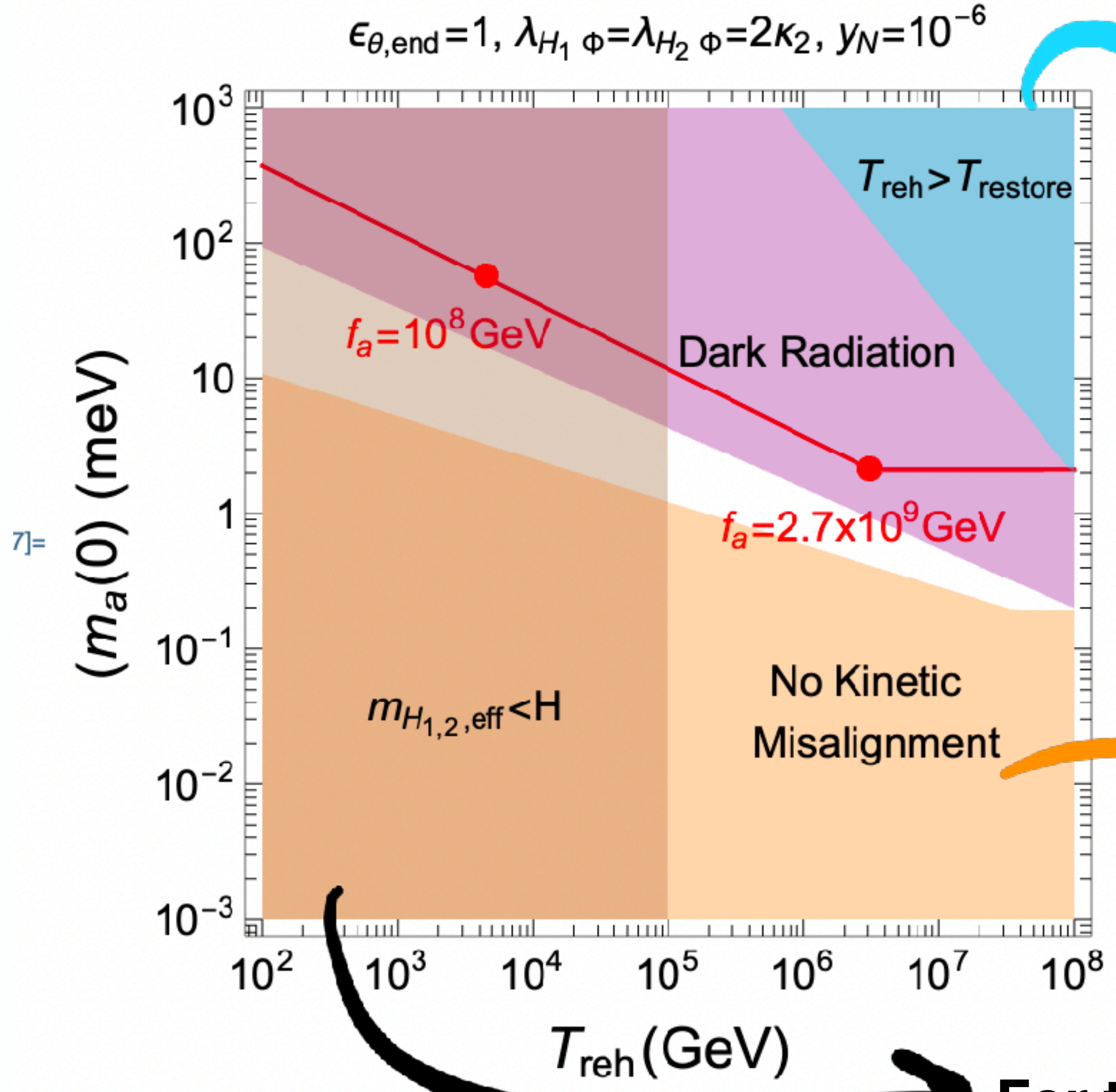
$$n_\theta(T_{\rm RH}) = n_{\theta,\rm end}\left(\frac{\pi^2 g_*(T_{\rm RH})T_{\rm RH}^4}{45V_E(\phi_{\rm end})}\right)^{3/4}$$

- If reheating is delayed

$$n_\theta(T_{\rm RH}) = n_{\theta,\rm end}\left(\frac{a_{\rm end}}{a_c}\right)^3\left(\frac{a_c}{a_{\rm RH}}\right)^3$$

$$= n_{\theta,\rm end}\left(\frac{\pi^2 g_*(T_{\rm RH})T_{\rm RH}^4}{45V_E(\phi_{\rm end})}\right)^{3/4}\boxed{\left(\frac{T_{\rm RH}}{T_{\rm RH}^c}\right)}$$

Suppression due to MD



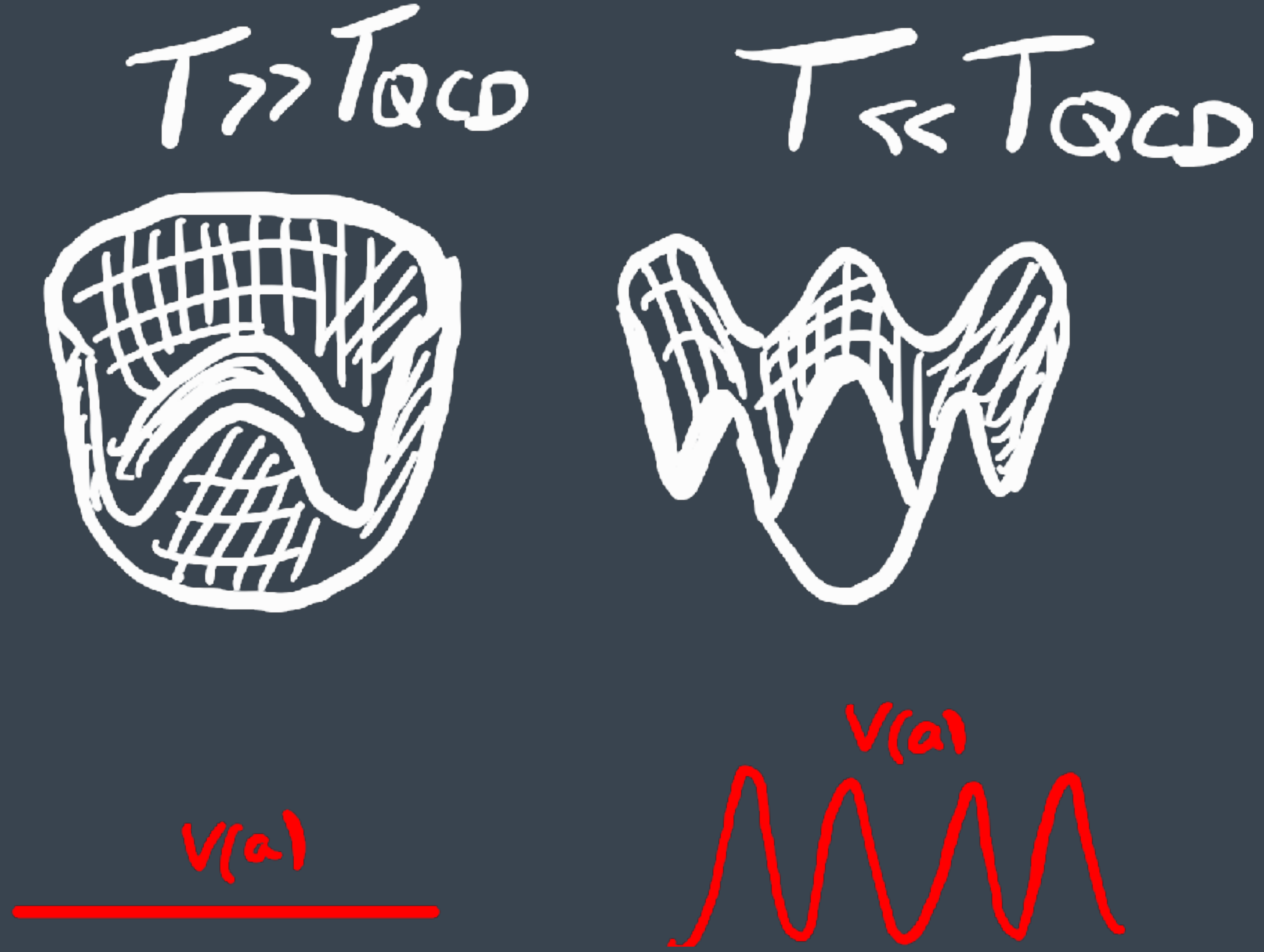
Restoration of the PQ symmetry for large T_{reh}

Condition for kinetic misalignment

$$\dot{\theta}(T_*) \geq 6H(T_{\mathrm{osc}})\dot{\theta}(T_*) \geq 6H(T_{\mathrm{osc}})$$

For too low T_{reh}, extra fields NOT decoupled

Symmetry restoration?



Domain walls

$$\Delta V \gtrsim \rho_{\text{walls}} \sim \frac{\sigma}{t} \gtrsim \frac{\sigma}{0.1\text{sec}}$$

- The PQ violating potential gives rise to a nonzero pressure $\Delta V = c\,\Lambda_{\text{QCD}}^4 \times 10^{-10}$

- Domain walls never become dominant if $\Delta V \gtrsim \dfrac{\sigma^2}{M_P^2}(t_*/0.1\text{ s})$

$$c = |c_{0,l,k}| \left(\frac{f_a}{\sqrt{2} q_\Phi M_P} \right)^l \left(\frac{M_P}{\Lambda_{\text{QCD}}} \right)^4 \times 10^{10}$$

- for $\sigma \sim \Lambda_{\text{QCD}}^3$, there is no domain wall problem as long as $c \gtrsim 10^{-13}$

Iscocurvature

- The power spectrum of the isocurvature perturbation depends only on $Y_{a,\mathrm{mis}}$

$$P_{\mathrm{iso}}(k_*) = \left(\frac{1}{Y_a}\frac{\partial Y_a}{\partial \theta_*}\right)^2 \left\langle \delta\theta_*^2 \right\rangle = \left[\frac{4}{\theta_*}\left(\frac{Y_{a,mis}}{Y_a}\right)^2 + \frac{1}{4}\left(\frac{1}{\epsilon_{\theta,*}}\frac{\partial \epsilon_{\theta,*}}{\partial \theta_*}\right)^2\left(\frac{Y_k in}{Y_a}\right)^2\right]\left\langle \partial\theta_*^2 \right\rangle$$

$$\left\langle \delta\theta_*^2 \right\rangle = \frac{1}{f_{a,eff}^2}\left(\frac{H_I}{2\pi}\right)^2 \text{ with } \quad f_{a,eff} \equiv \sqrt{6}\left|\sinh\left(\frac{\phi_*}{\sqrt{6M_P}}\right)\right|$$

- The large effective decay constant suppresses isocurvature perturbations.

Summary

- We built a consistent framework that unifies PQ inflation with kinetic misalignment.
- We predict domain walls in DFSZ models, but they never dominate. **Signatures?**
- If reheating is delayed further, we may enter a **kination** era. **More signatures?**