IFIC
INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

The Nonlinear Dynamics of Axion Inflation on the Lattice

Nicolás Loayza Romero

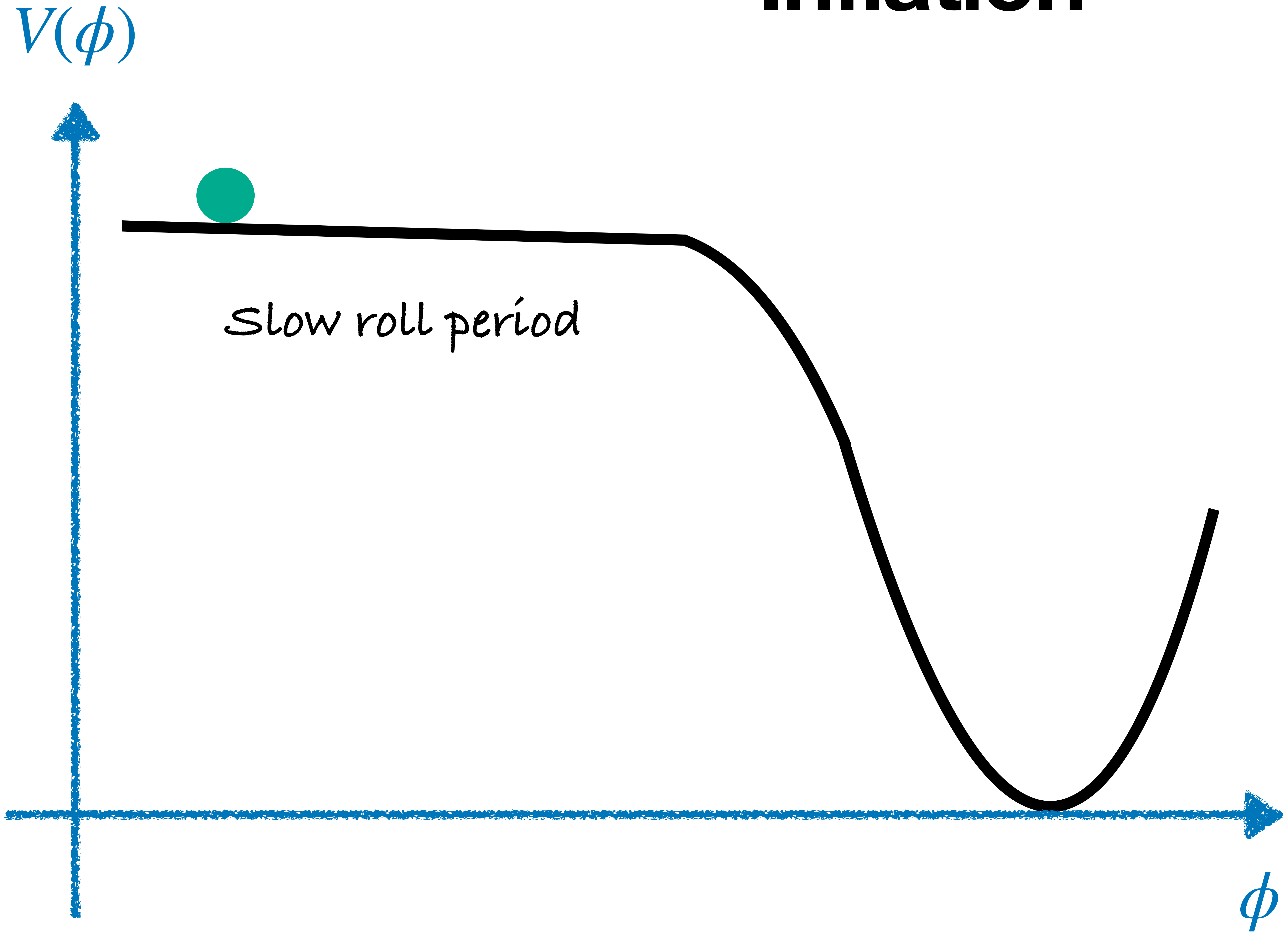
In collaboration with: Daniel G. Figueroa, Joanes Lizarraga, Ander Urio & Jon Urrestilla

Based on *Phys.Rev.D* 111 (2025) 6 2411.16368 [astro-ph.CO]

The Dawn of Gravitational Wave Cosmology - Benasque

May 02, 2025

Inflation



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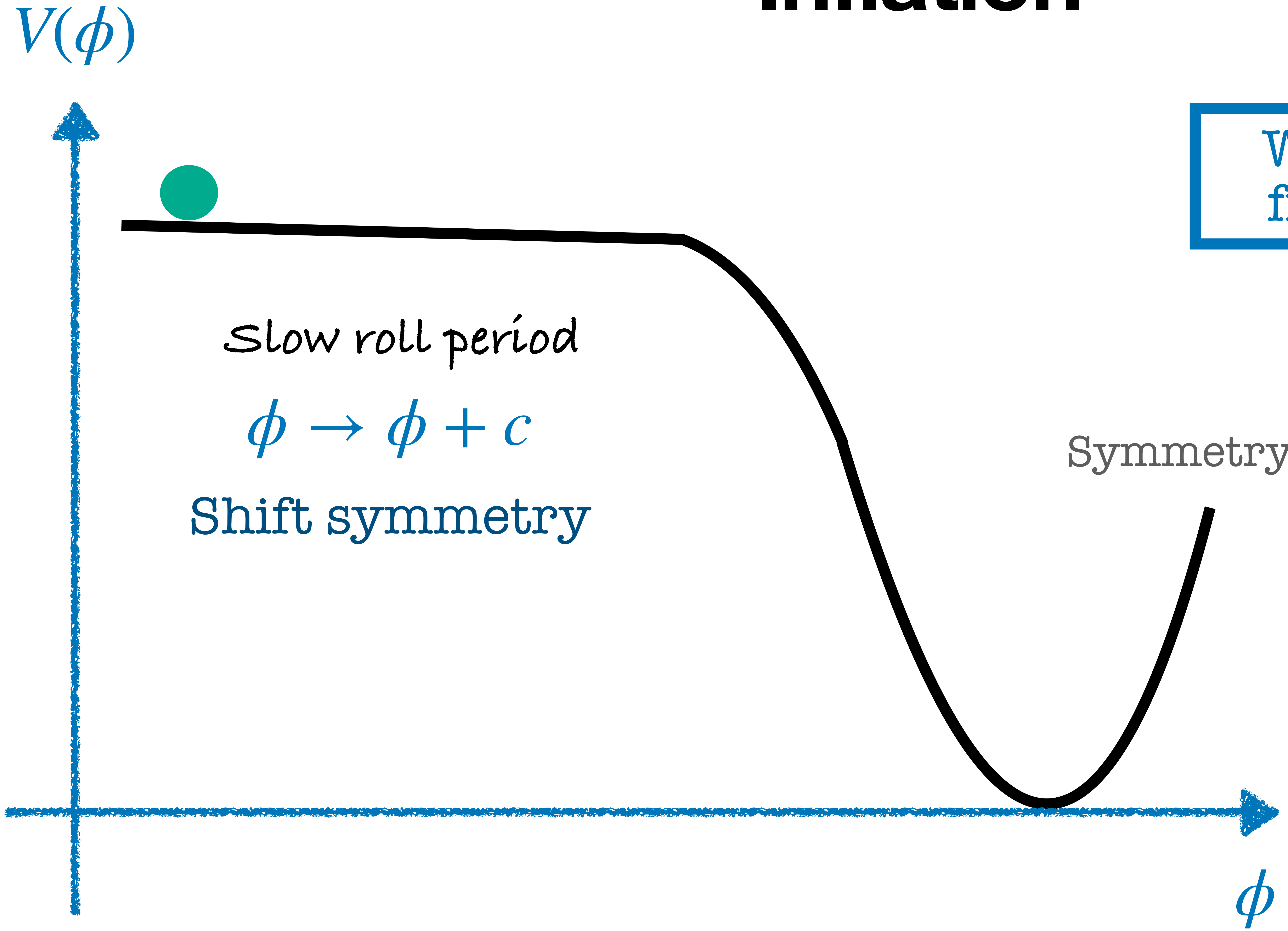
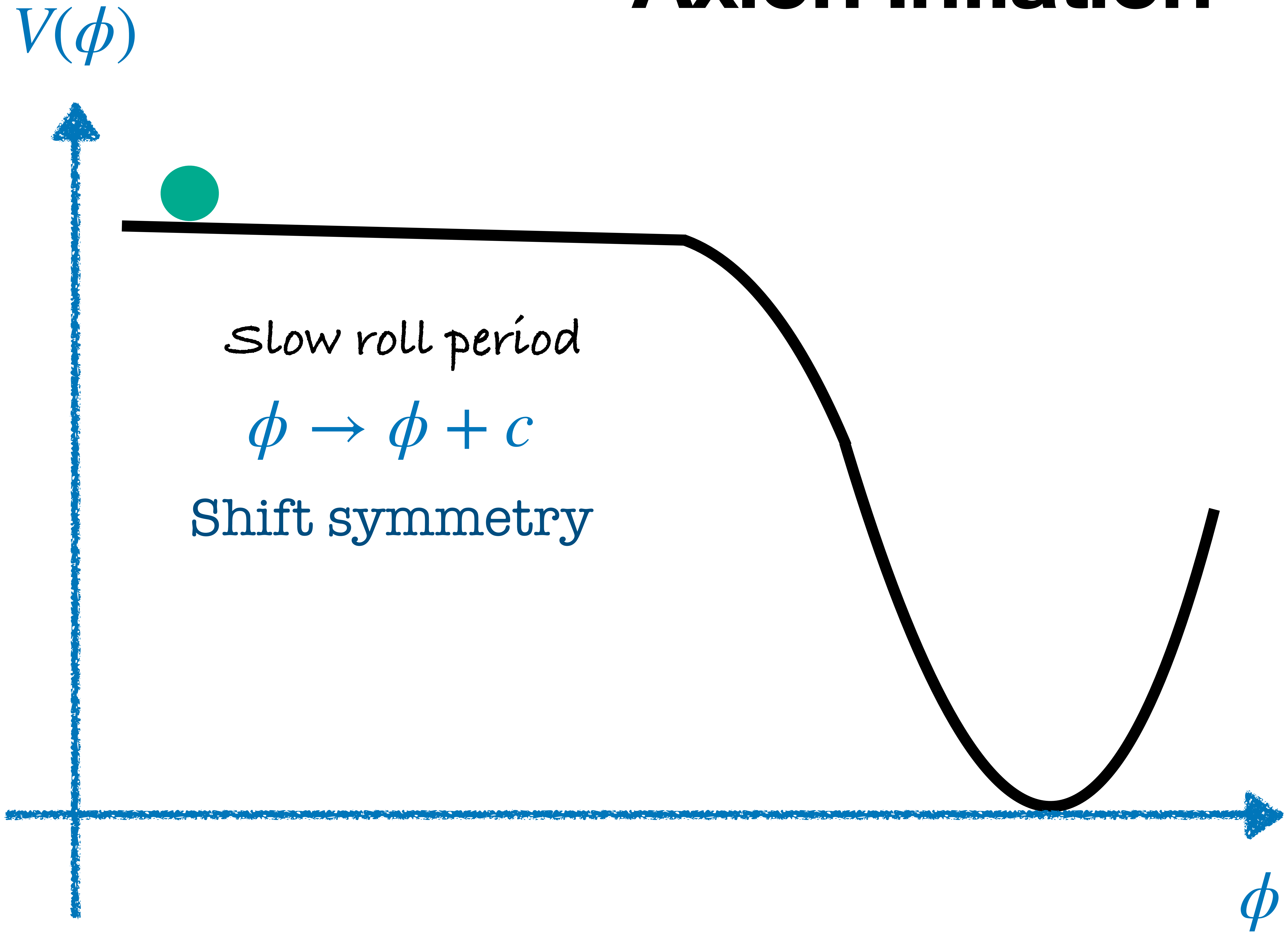


We want to protect from being spoiled

Symmetry protected flatness of potential

Axion Inflation

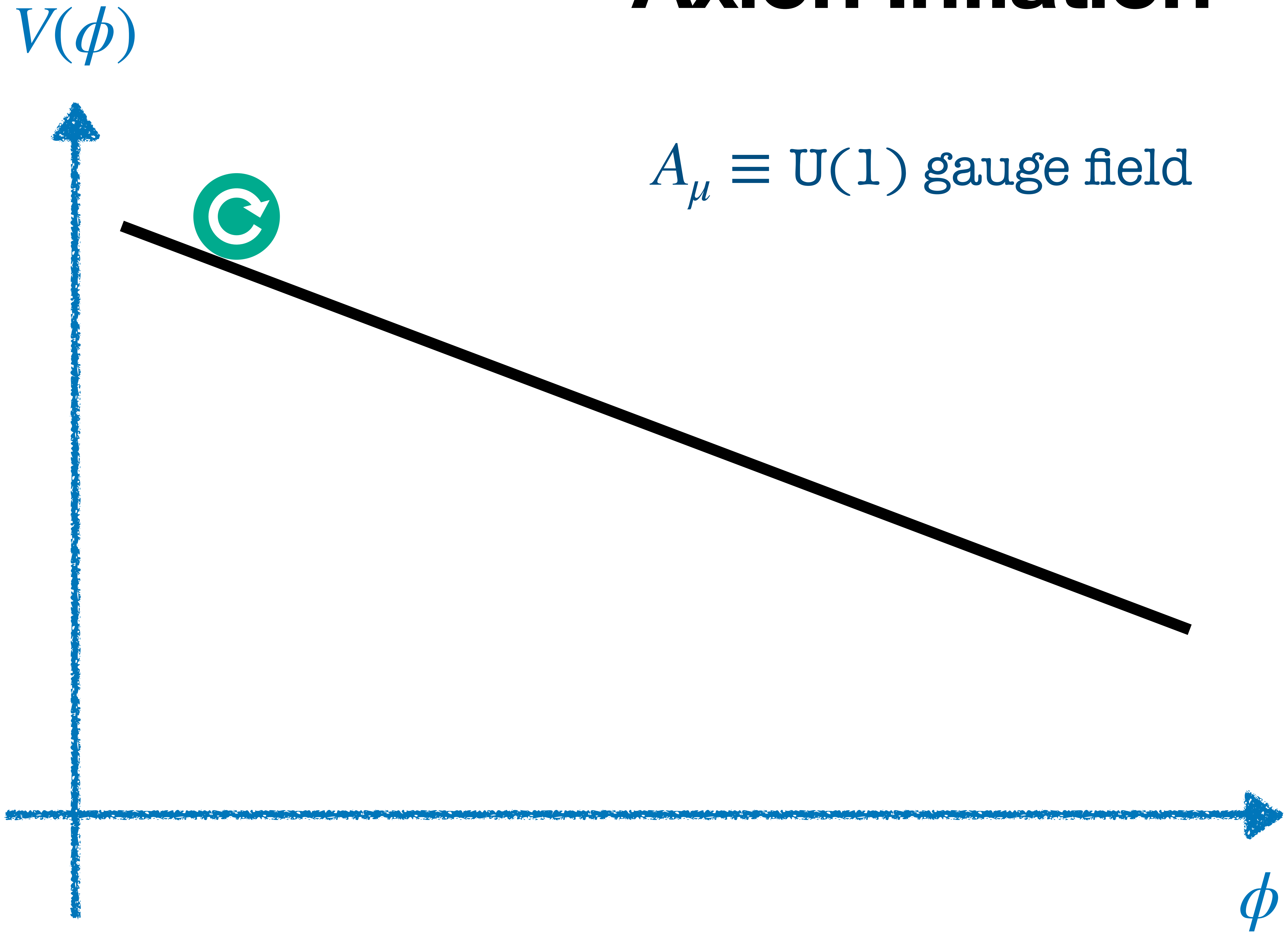


$$\mathscr{L} \supset \frac{\phi}{\Lambda} F\tilde{F}$$

$A_\mu \equiv$ U(1) gauge field

[K. Freese, J. A. Frieman, A. V. Olinto (PRL 65,3233 1990)] ...

Axion Inflation



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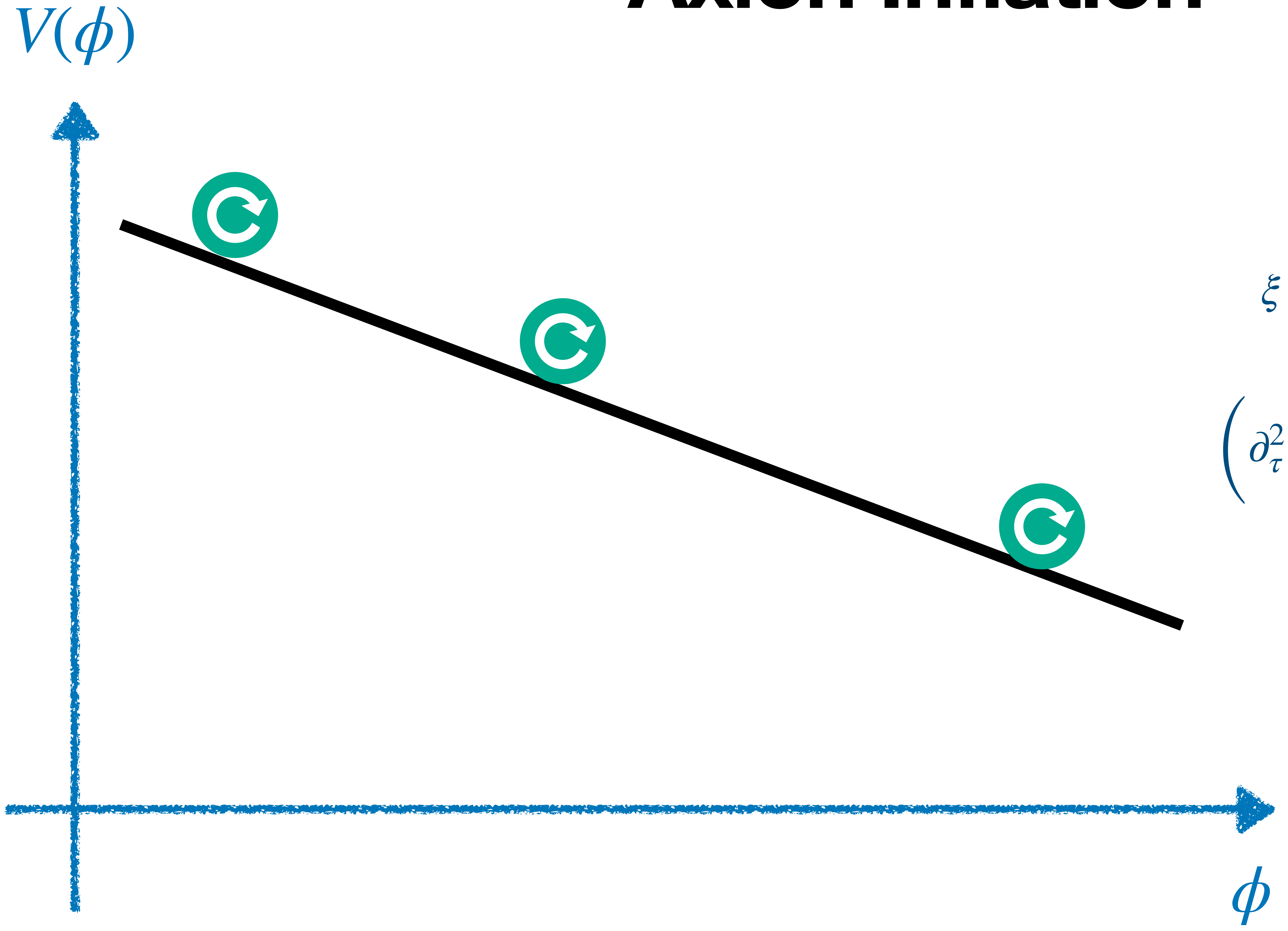

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$$\xi \equiv \frac{|\dot{\phi}|}{2H\Lambda}$$

$$\left(\partial_\tau^2 + k^2 + \text{sign}(\dot{\phi})\frac{2k\xi}{|\tau|}\right)\mathscr{A}^+(\tau, \mathbf{k}) = 0$$

[M. M. Anber, L. Sorbo (0908.4089)]
[J. Cook, L. Sorbo (1109.0022)]
[N. Barnaby, E. Pajer, M. Peloso (1110.3327)]

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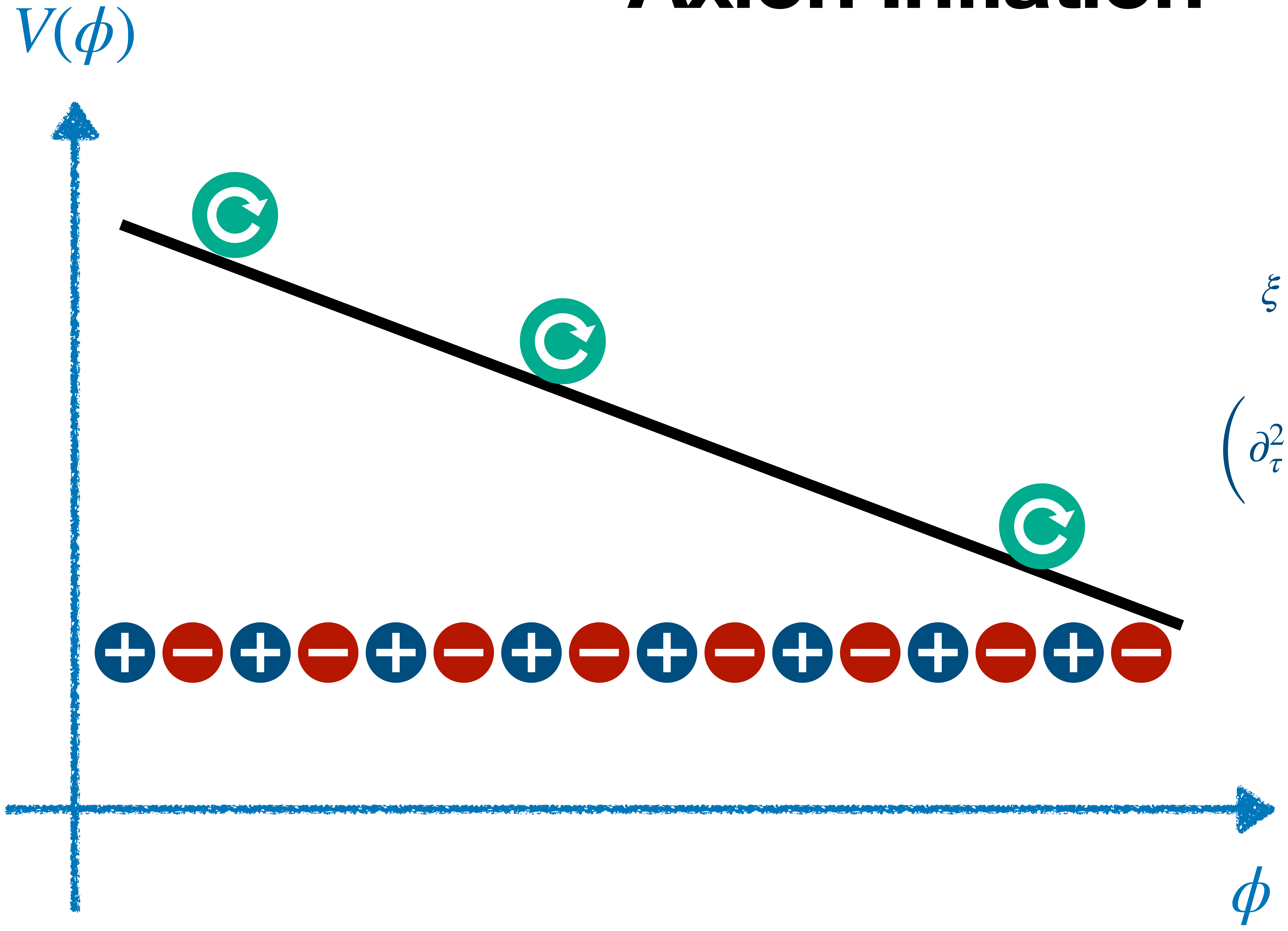

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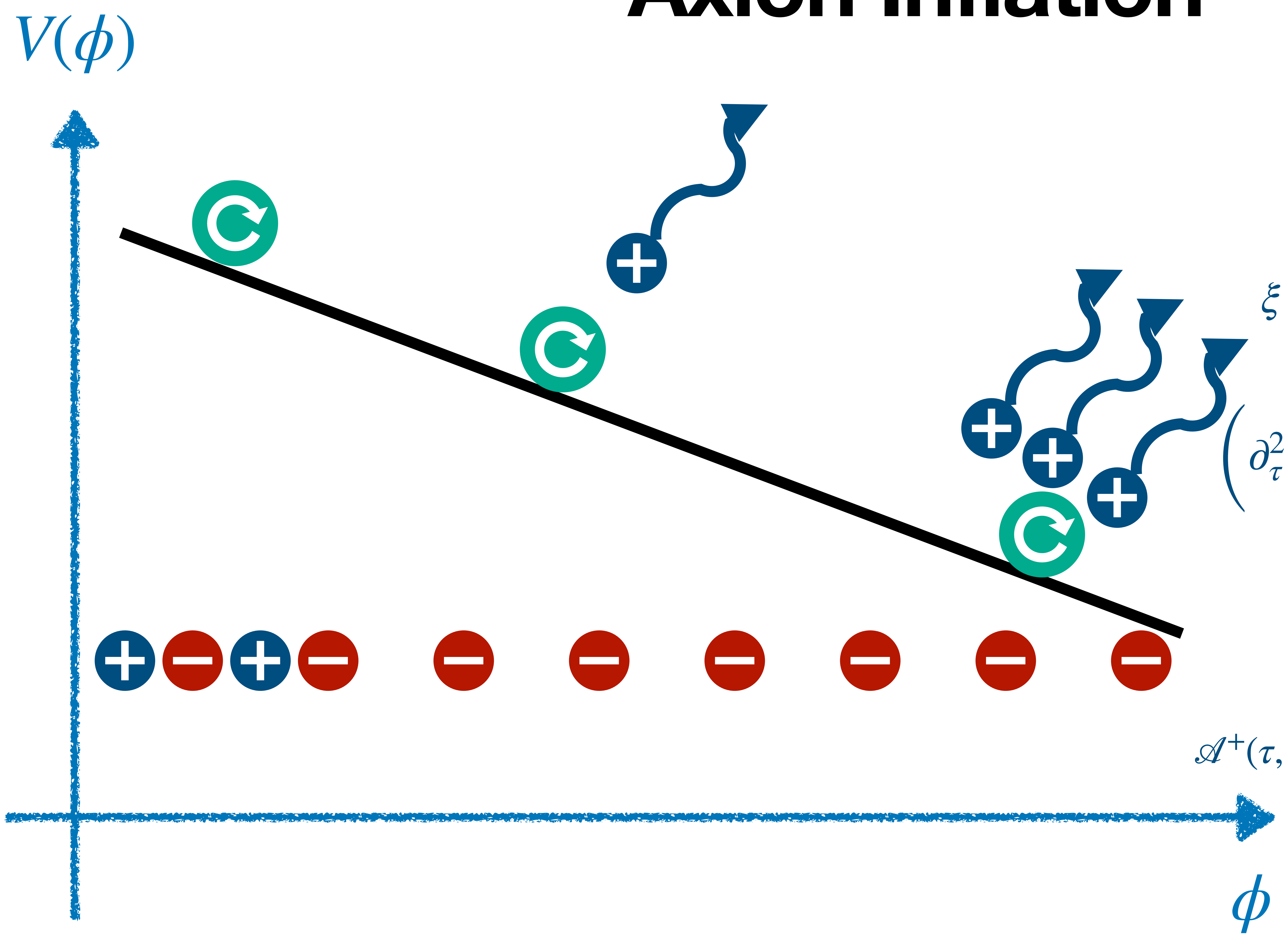

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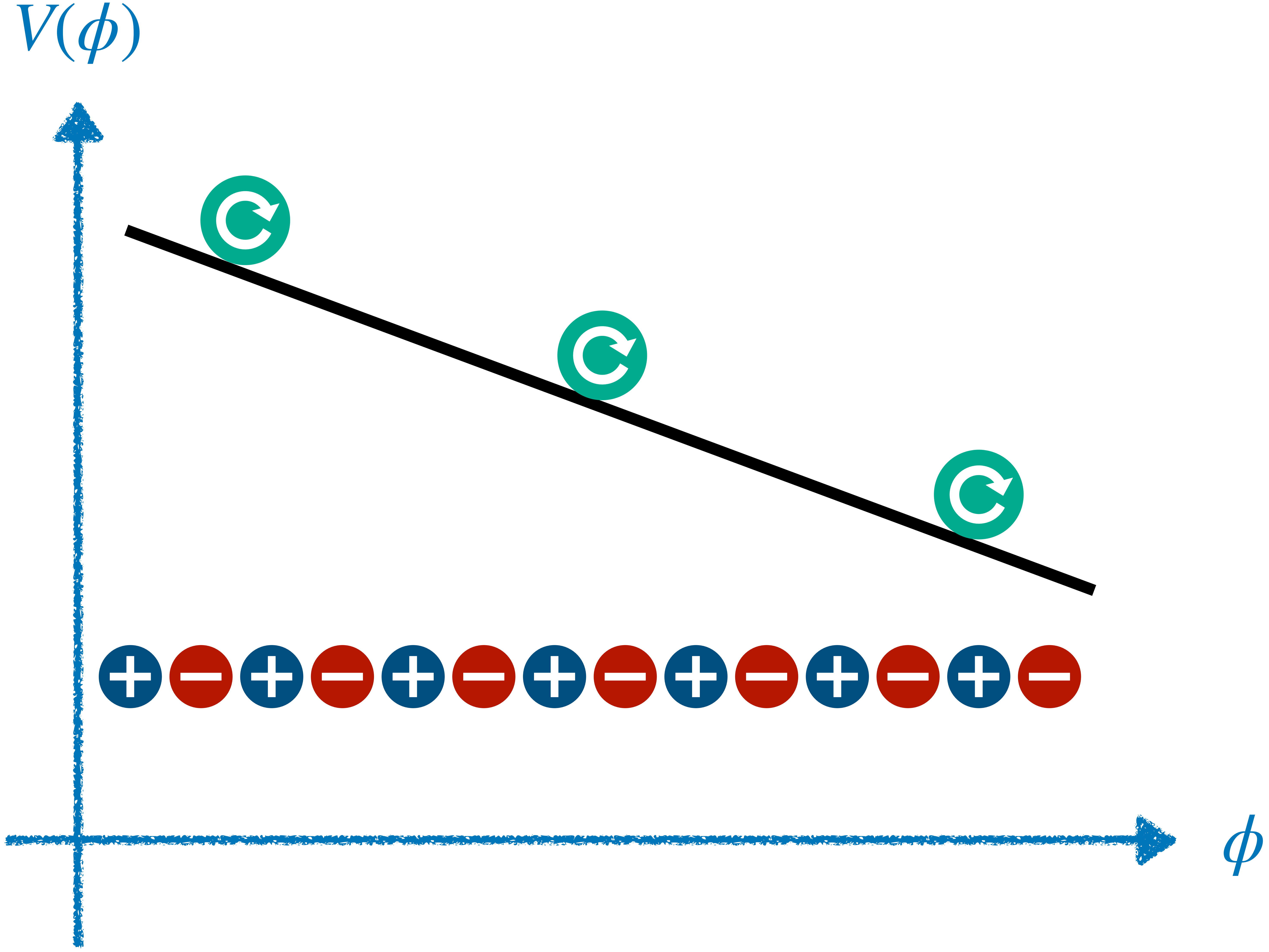
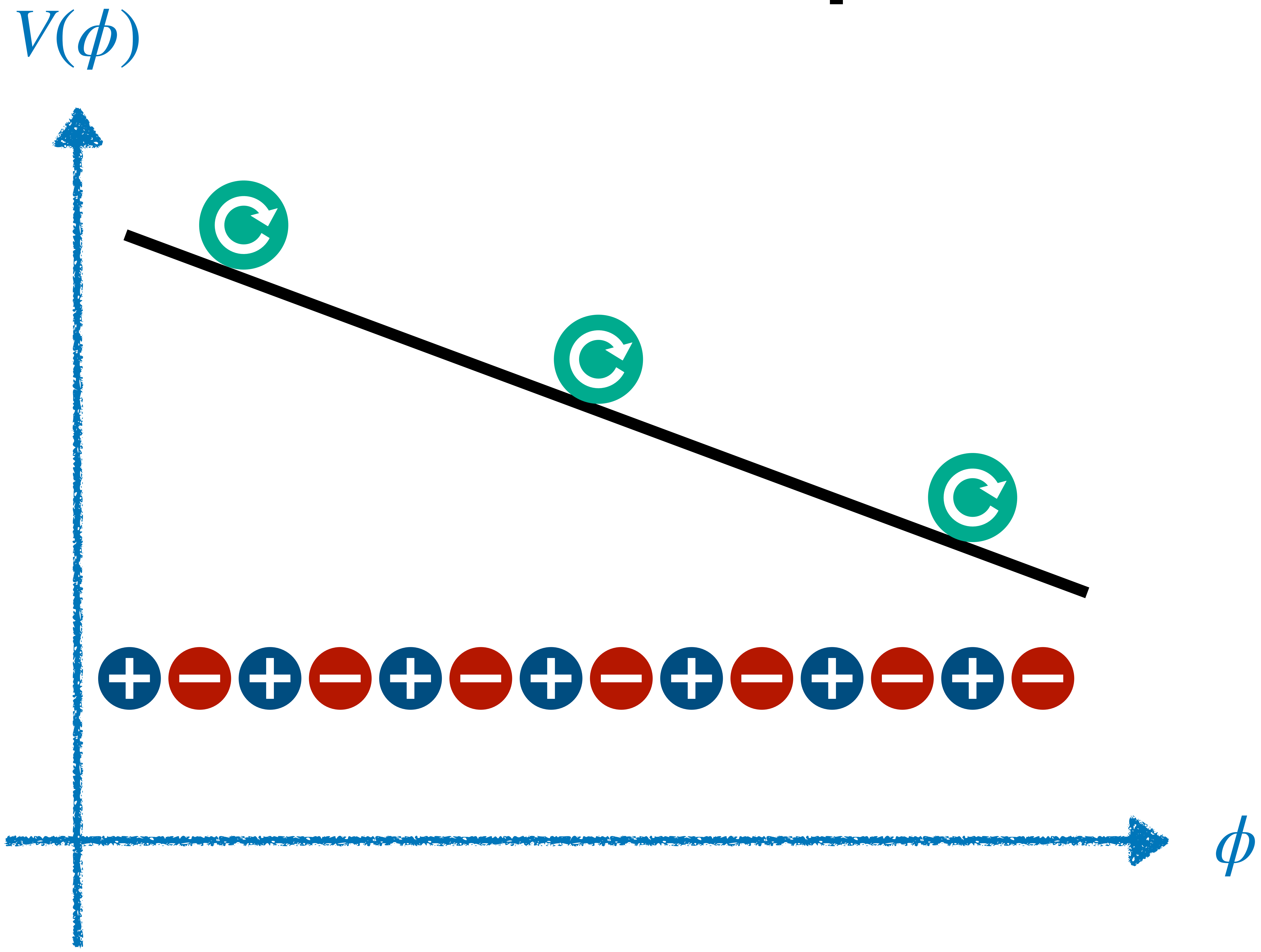
Axion Inflation phenomenology



Chiral GWBs

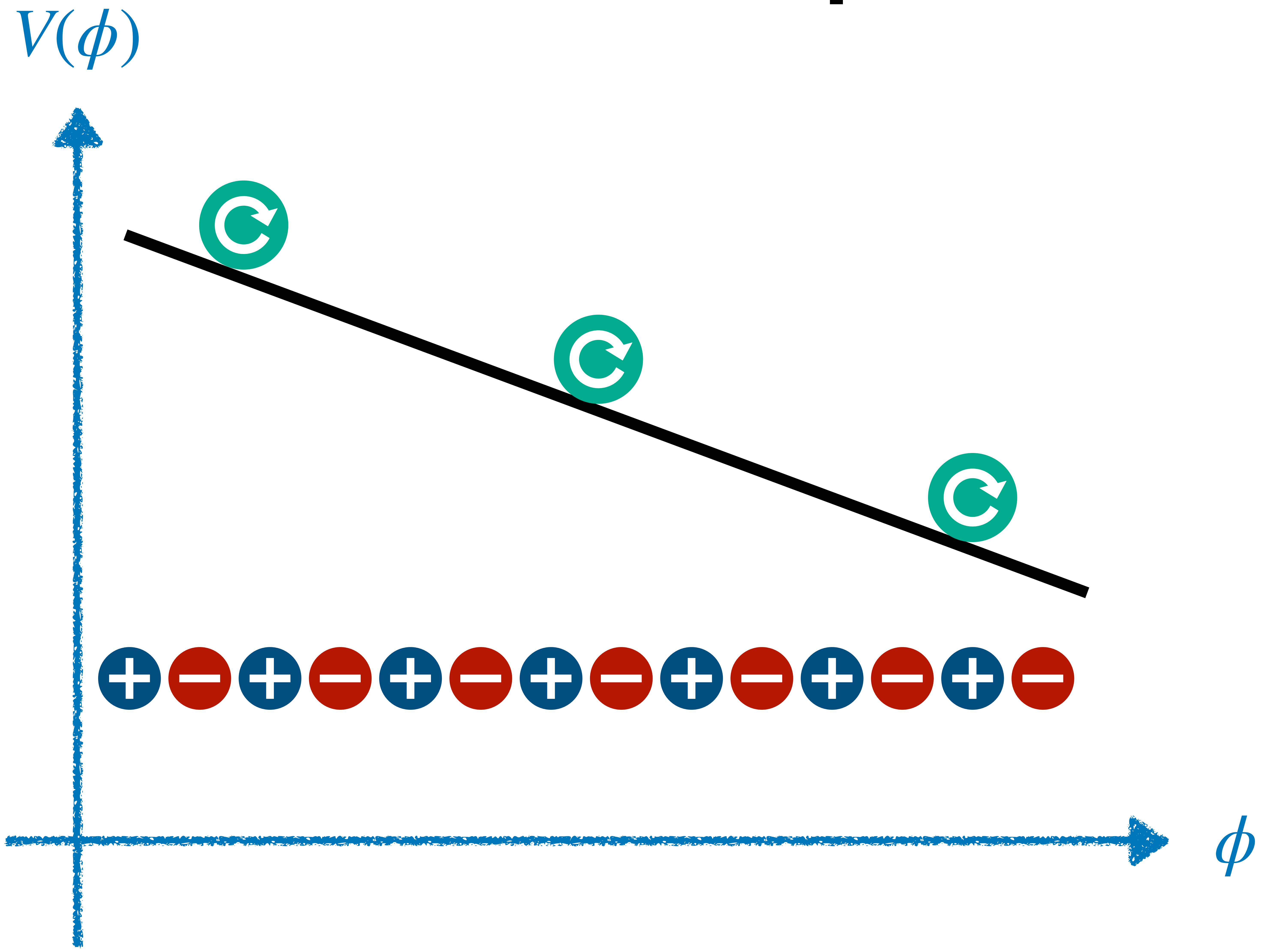
Axion Inflation phenomenology



Chiral GWBs

Primordial Black Holes

Axion Inflation phenomenology



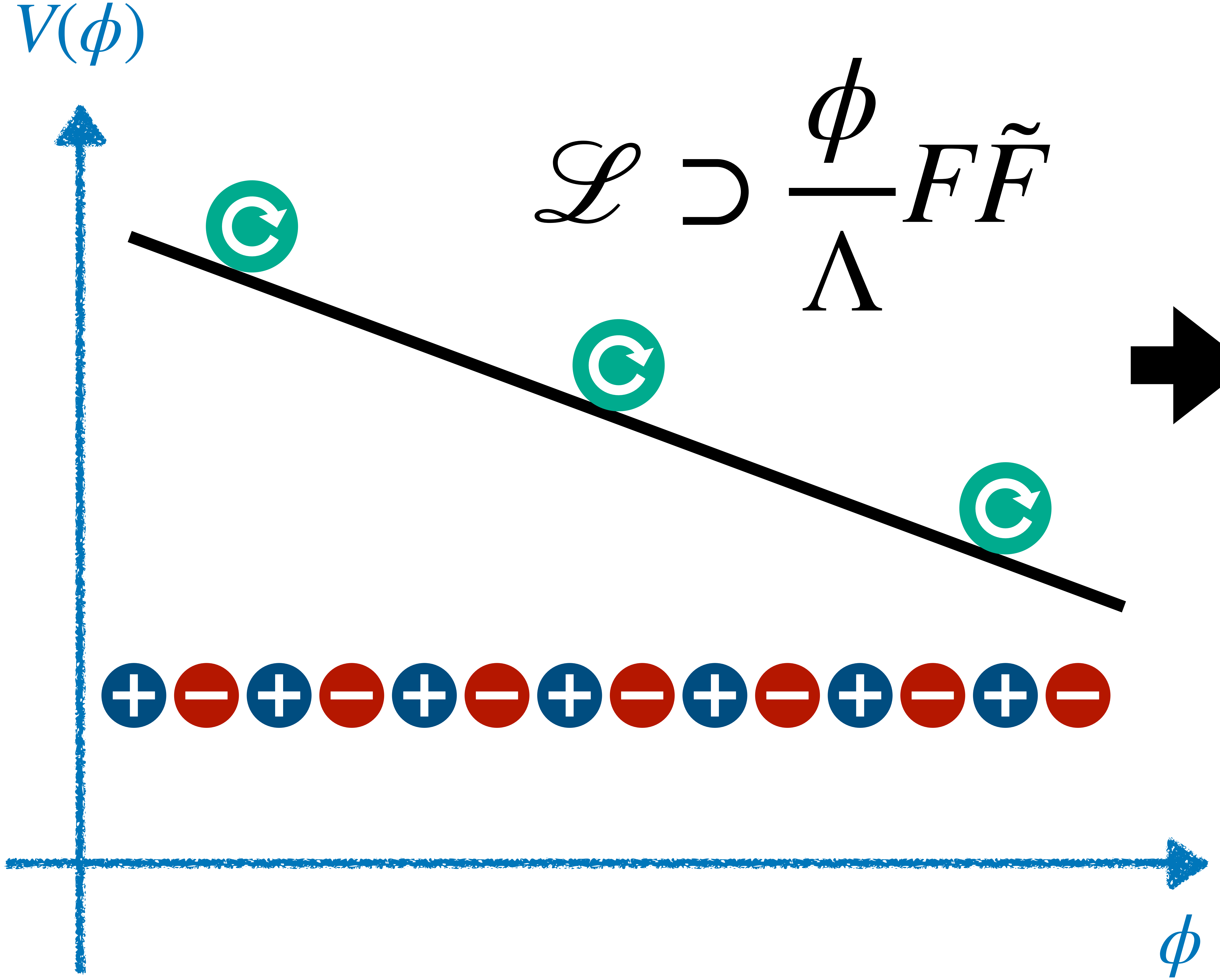
Chiral GWBs

Primordial Black Holes

Non-gaussianities

[N. Barnaby, M. Peloso (1011.1500)]
[A. Linde, S. Mooji, E. Pajer (1212.1693)]
[E. Bugaev, P. Klimai (1312.7435)]
[N. Bartolo et al (LISA) (1610.06481)]
[J. G. Bellido, M. Peloso, C. Unal (1610.03763)] .....

Axion Inflation



$$\mathcal{L} \supset \frac{\phi}{\Lambda} F\tilde{F}$$

Dynamical equations in FLRW

$$\ddot{\phi} = -3H\dot{\phi} + \frac{1}{a^2}\nabla^2\phi - m^2\phi + \frac{\alpha_\Lambda}{a^3 m_p}\vec{E}\cdot\vec{B}$$

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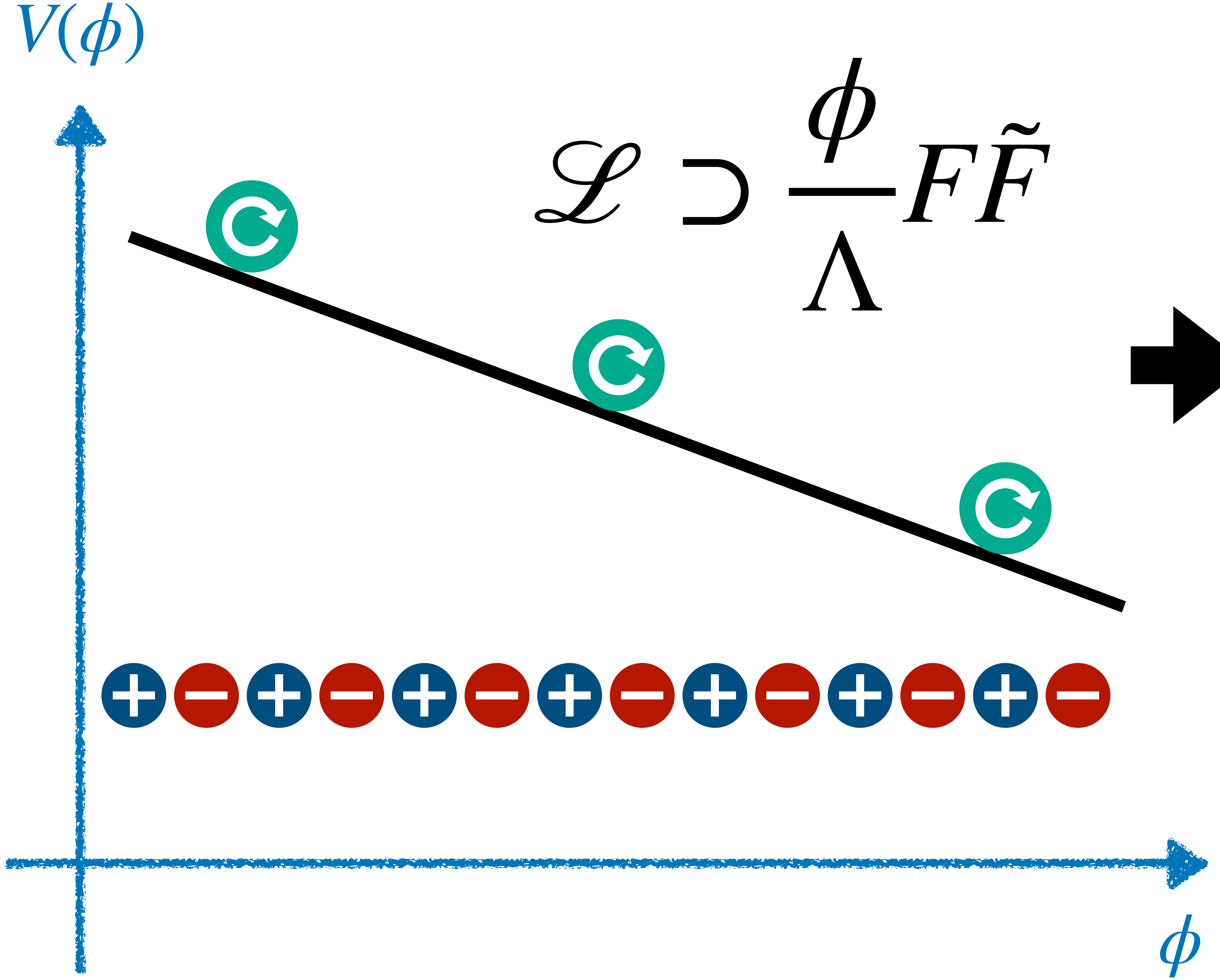

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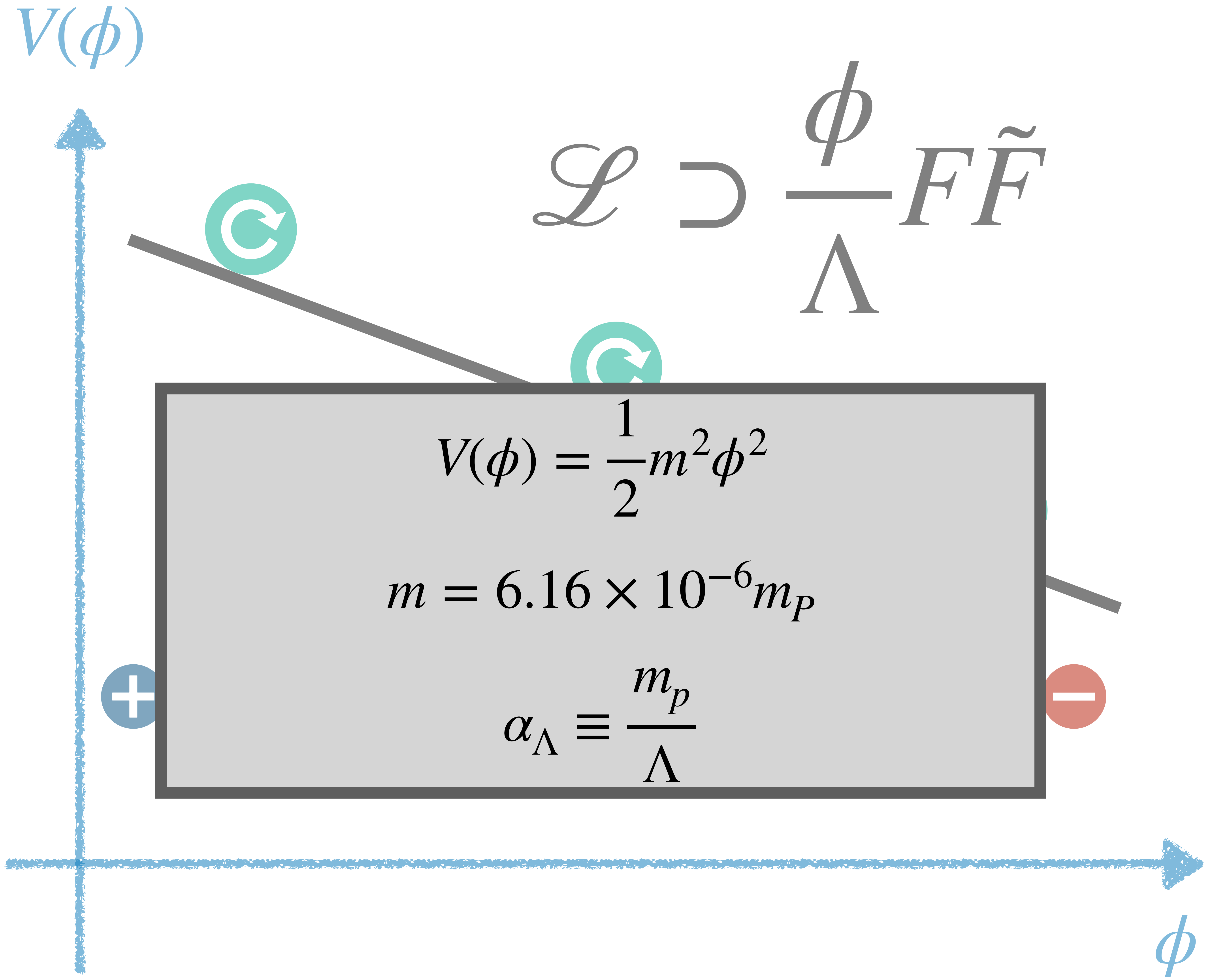

$$\mathscr{L} \supset \frac{\phi}{\Lambda} F\tilde{F}$$

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$

$$m = 6.16 \times 10^{-6} m_P$$

$$\alpha_\Lambda \equiv \frac{m_p}{\Lambda}$$

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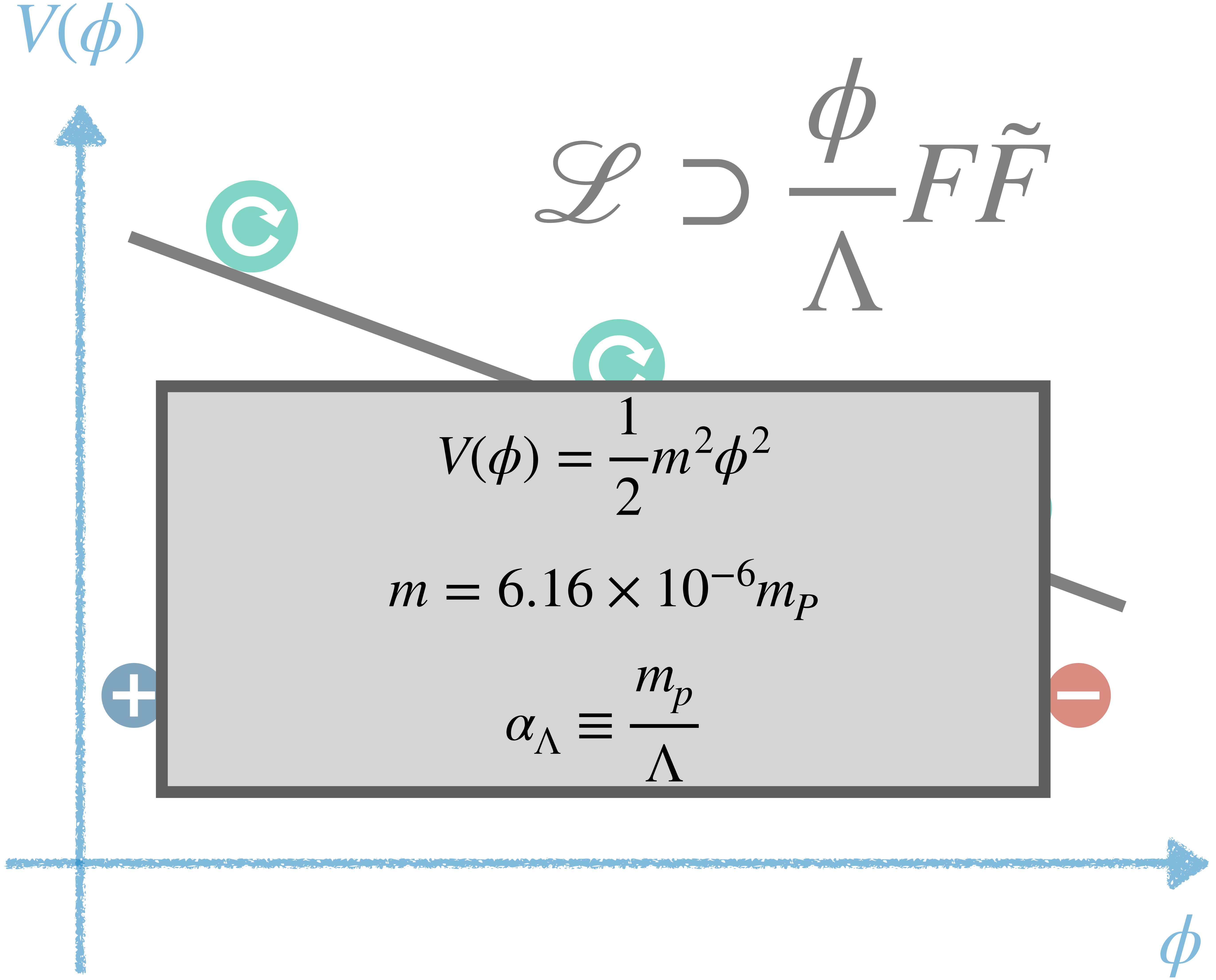

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Linear Regime

$$\ddot{\phi} = -3H\dot{\phi} + \frac{1}{a^2}\nabla^2\phi - m^2\phi + \frac{\alpha_\Lambda}{a^3 m_p}\vec{E}\cdot\vec{B}$$

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Homogeneous Backreaction Regime

$$\ddot{\phi} = -3H\dot{\phi} + \frac{1}{a^2}\nabla^2\phi - m^2\phi + \frac{\alpha_\Lambda}{a^3 m_p}\langle \vec{E}\cdot\vec{B}\rangle$$

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- Integrate $\langle \vec{E}\cdot\vec{B}\rangle$ every time step

[Cheng, Lee, Ng (1508.00251)]
[Notari, Tywoniuk, (1608.06223)]
[Dall'Agata, González-Martín, Papageorgiu, Peloso (1912.09950)]
[Domcke, Guidetti, Welling, Westphal (2002.02952)] ...

-Gradient Expansion Formalism

[Sobol, Gorbar, Vilchinskii (1907.10443)]
[Gorbar, Schmitz, Sobol, Vilchinskii (2109.01651)]
[Durrer, Sobol, Vilchinskii (2303.04583)]
[Durrer, von Eckardstein, Garg, Schmitz, Sobol (2404.19694)]

Non-linear inhomogenous dynamics

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[Figueroa, Lizarraga, Urio, Urrestilla (2303.17436)]
[Figueroa, Lizarraga, NL, Urio, Urrestilla (2411.16368)]

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- local description of $\vec{E}\cdot\vec{B}$ and $\dot{\phi}\vec{B}$

[Figueroa, Lizarraga, Urio, Urrestilla (2303.17436)]
[Figueroa, Lizarraga, NL, Urio, Urrestilla (2411.16368)]

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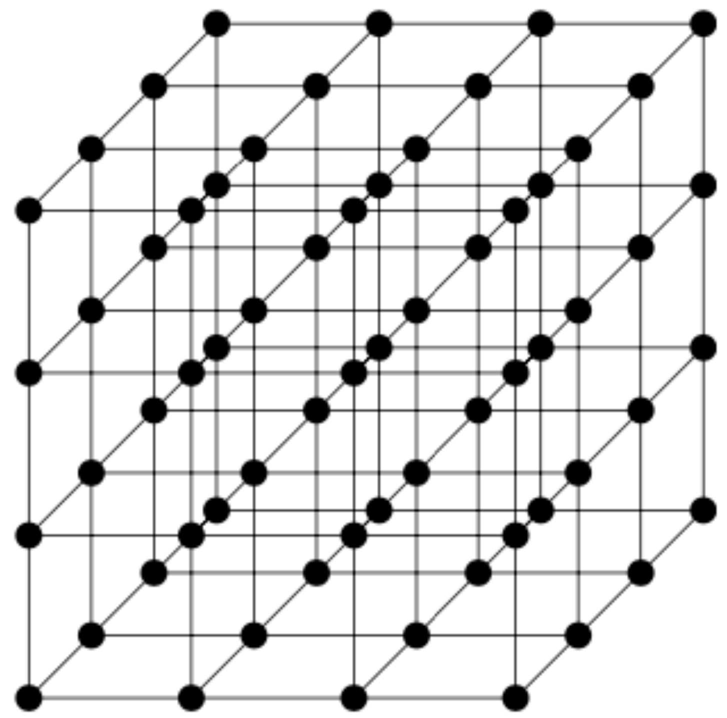

CosmoLattice

[D. G. Figueroa, A. Florio, F. Torrenti & W. Valkenburg (2006.15122)]
[D. G. Figueroa, A. Florio, F. Torrenti & W. Valkenburg (2102.01031)]

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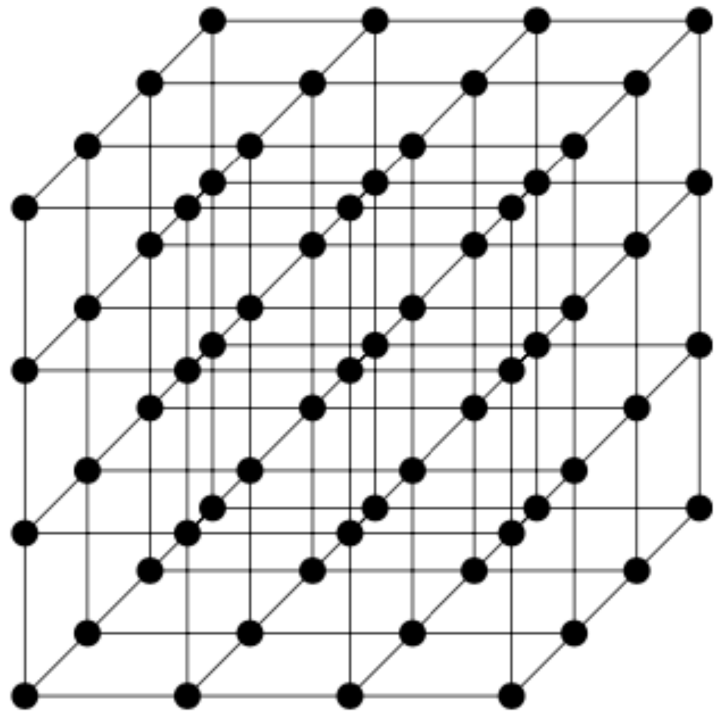

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[D. G. Figueroa, A. Florio, F. Torrenti & W. Valkenburg (2102.01031)]

Alternative lattice descriptions

[Sharma, Brandenburg, Subramanian, Vikman (2411.04854)]
[Caravano, Komatsu, Lozanov, Weller (2204.12874)]
[Caravano, Komatsu, Lozanov, Weller (2110.10695)]

Lattice implementation of axion coupling

Continuum

$$\frac{1}{\Lambda}\vec{E}\cdot\vec{B}$$

Lattice implementation of axion coupling

Continuum

$$\frac{1}{\Lambda}\vec{E}\cdot\vec{B}$$

Satisfy all conditions

Lattice

$$\sum_i \frac{\phi}{\Lambda} E_i^{(2)} B_i^{(4)}$$

Satisfies

- Gauge transformations

$$A_\mu \to A_\mu + \partial_\mu \alpha(\mathbf{x})$$

- Bianchi identities

$$\vec{\nabla}\times\vec{E} = \dot{\vec{B}} \quad \nabla\cdot\vec{B} = 0$$

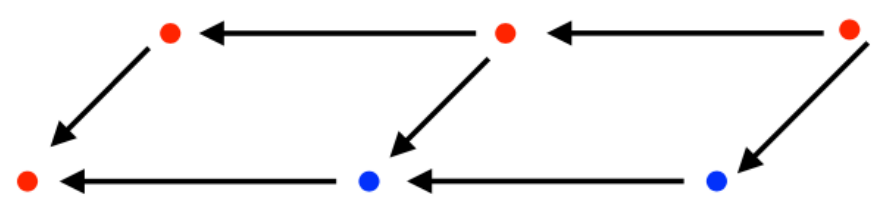
- Topological term as a total derivative

$$F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial_\mu K^\mu$$

$$E_i^{(2)}(\mathbf{n}) \equiv \frac{1}{2}\left(E_i + E_{i,-i}\right)$$

$$B_i^{(4)}(\mathbf{n}) \equiv \frac{1}{4}\left(B_i + B_{i,-j} + B_{i,-k} + B_{i,-j-k}\right)$$

$E_i^{(2)}$



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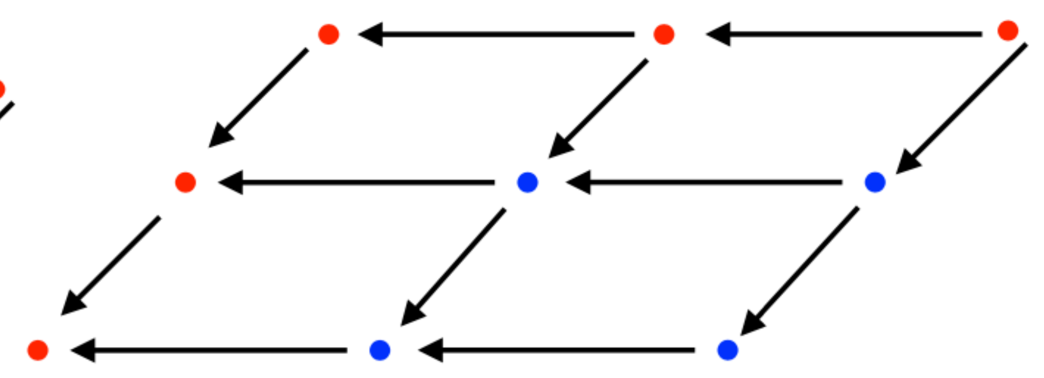

Lattice implementation of axion coupling

Continuum

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Satisfy all conditions

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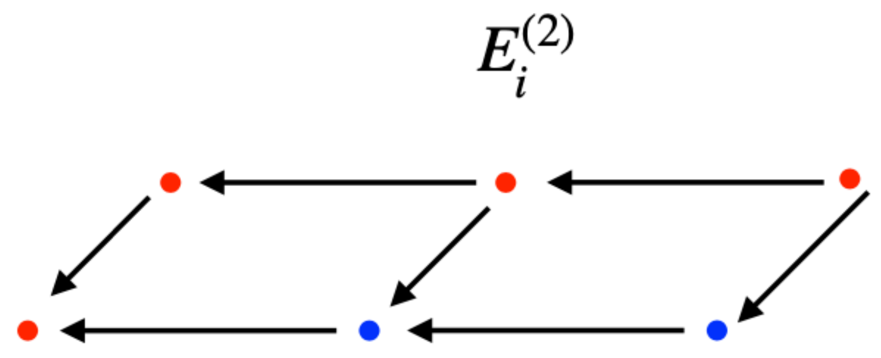

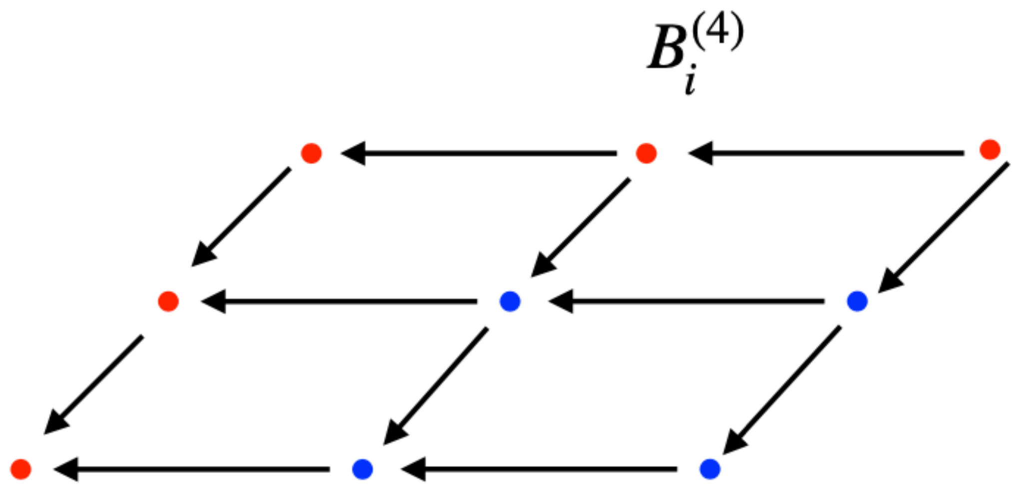

Satisfies

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@ lattice level

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$$F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial_\mu K^\mu$$

Lattice formulation of axion inflation

$$\pi'_\phi = -3\pi_\phi + \frac{1}{H}\left(\frac{1}{a^2}\sum_i \Delta_i^- \Delta_i^+ \phi - m^2\phi + \frac{\alpha_\Lambda}{a^3 m_p}\sum_i E_i^{(2)} B_i^{(4)}\right)$$

$$E'_i = -E_i - \frac{1}{H}\left(\frac{1}{a^2}\sum_{jk}\epsilon_{ijk}\Delta_j^- B_k + \frac{\alpha_\Lambda}{2am_p}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+\hat{\imath}} B_{i,+\hat{\imath}}^{(4)}\right) - \frac{\alpha_\Lambda}{4am_p}\sum_\pm \sum_{j,k}\epsilon_{ijk}\left\{\left[(\Delta_j^\pm \phi) E_{k,\pm\hat{\jmath}}^{(2)}\right]_{+\hat{\imath}} + \left[(\Delta_j^\pm \phi) E_{k,\pm\hat{\jmath}}^{(2)}\right\}\right]\right)$$

$$\sum_i \Delta_i^- E_i = -\frac{\alpha_\Lambda}{2am_p}\sum_\pm \sum_i \left(\Delta_i^\pm \phi\right) B_{i,\pm\hat{\imath}}^{(4)}$$

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$$\pi'_\phi = -3\pi_\phi + \frac{1}{H}\left(\frac{1}{a^2}\sum_i \Delta_i^-\Delta_i^+\phi - m^2\phi + \frac{\alpha_\Lambda}{a^3 m_p}\sum_i E_i^{(2)}B_i^{(4)}\right)$$

$$E'_i = -E_i - \frac{1}{H}\left(\frac{1}{a^2}\sum_{jk}\epsilon_{ijk}\Delta_j^- B_k + \frac{\alpha_\Lambda}{2am_p}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+\hat{\imath}}B_{i,+\hat{\imath}}^{(4)}\right) - \frac{\alpha_\Lambda}{4am_p}\sum_\pm\sum_{j,k}\epsilon_{ijk}\left\{\left[(\Delta_j^\pm\phi)E_{k,\pm\hat{\jmath}}^{(2)}\right]_{+\hat{\imath}} + \left[(\Delta_j^\pm\phi)E_{k,\pm\hat{\jmath}}^{(2)}\right]\right\}\right)$$

$$\sum_i \Delta_i^- E_i = -\frac{\alpha_\Lambda}{2am_p}\sum_\pm\sum_i\left(\Delta_i^\pm\phi\right)B_{i,\pm\hat{\imath}}^{(4)}$$

- kernel of
$\mathscr{K}_A[a, \dot{a}, \phi, \pi_\phi, A_i, E_i]$

Lattice formulation of axion inflation

$$\pi'_\phi = -3\pi_\phi + \frac{1}{H}\left(\frac{1}{a^2}\sum_i \Delta_i^- \Delta_i^+ \phi - m^2\phi + \frac{\alpha_\Lambda}{a^3 m_p}\sum_i E_i^{(2)} B_i^{(4)}\right)$$

$$E'_i = -E_i - \frac{1}{H}\left(\frac{1}{a^2}\sum_{jk}\epsilon_{ijk}\Delta_j^- B_k + \frac{\alpha_\Lambda}{2am_p}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+\hat{\imath}} B_{i,+\hat{\imath}}^{(4)}\right) - \frac{\alpha_\Lambda}{4am_p}\sum_\pm \sum_{j,k}\epsilon_{ijk}\left\{\left[(\Delta_j^\pm \phi) E_{k,\pm\hat{\jmath}}^{(2)}\right]_{+\hat{\imath}} + \left[(\Delta_j^\pm \phi) E_{k,\pm\hat{\jmath}}^{(2)}\right]\right\}\right)$$

$$\sum_i \Delta_i^- E_i = -\frac{\alpha_\Lambda}{2am_p}\sum_\pm \sum_i \left(\Delta_i^\pm \phi\right) B_{i,\pm\hat{\imath}}^{(4)}$$

- kernel of $\mathscr{K}_A[a, \dot{a}, \phi, \pi_\phi, A_i, E_i]$

→

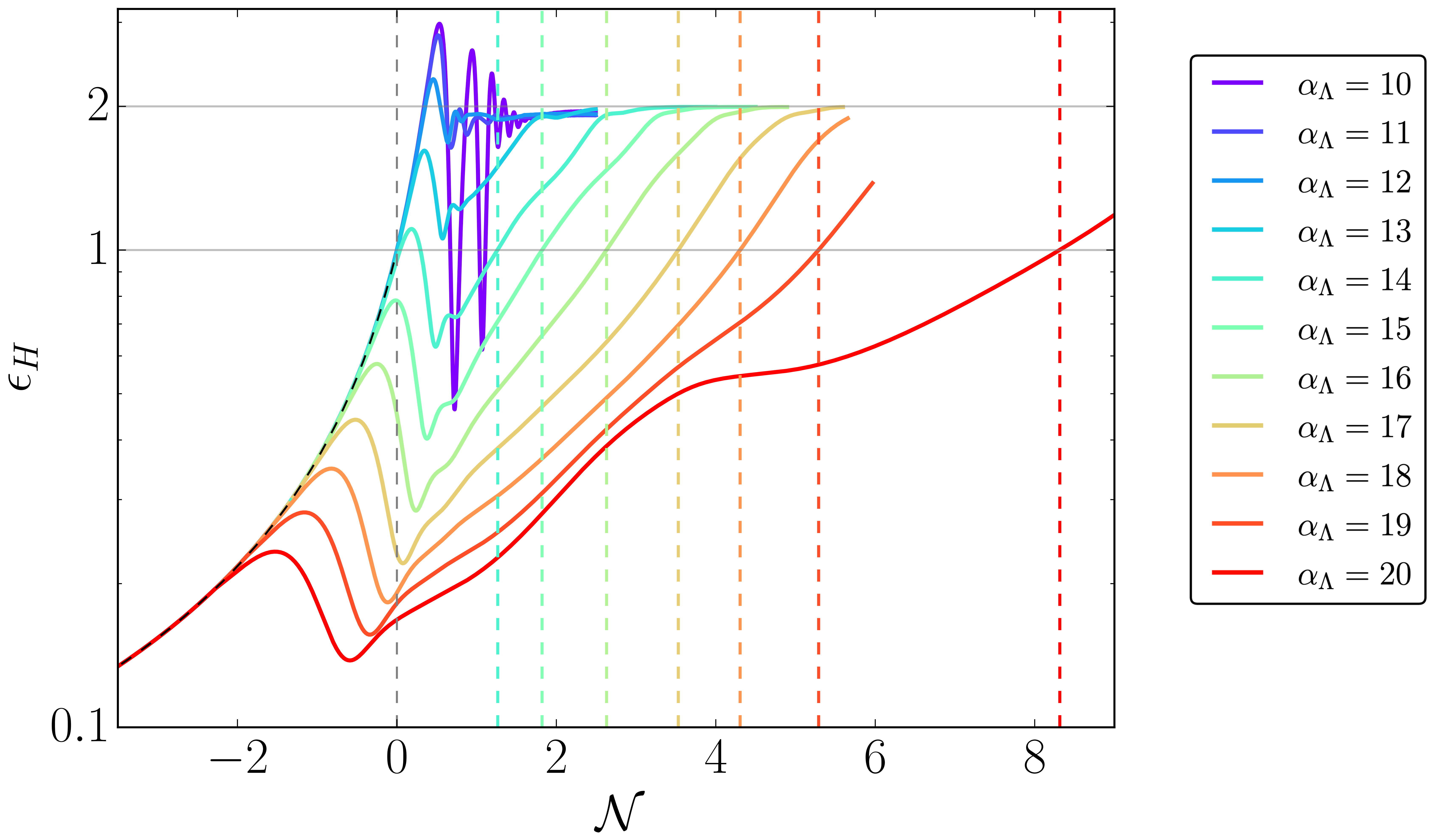
Non-symplectic integrators like Runge-Kutta

Results on level of backreaction

Run simulations until end of inflation $\epsilon_H = 1$

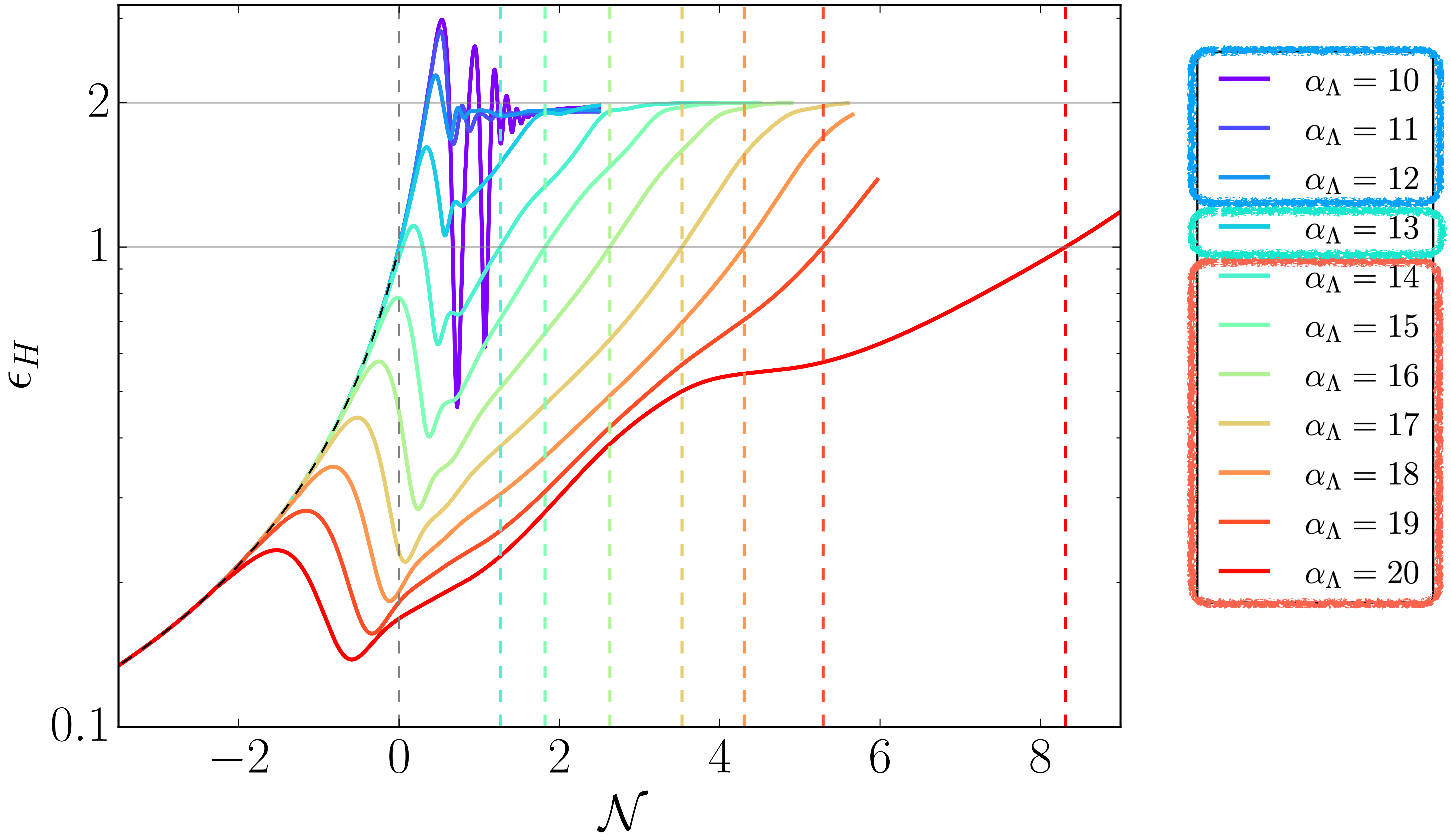
Results on level of backreaction

Run simulations until end of inflation $\epsilon_H = 1$



Results on level of backreaction

Run simulations until end of inflation $\epsilon_H = 1$



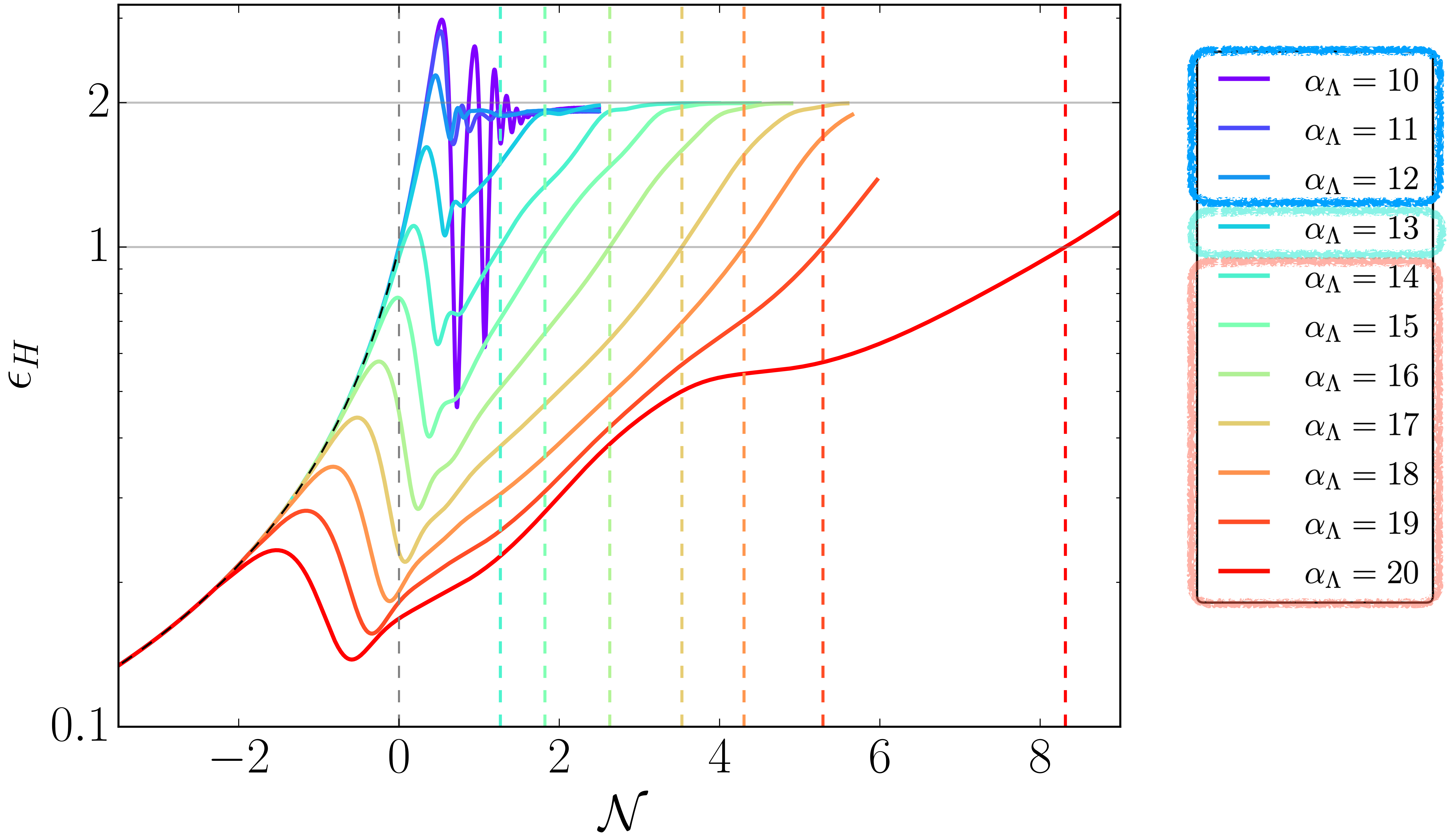
Weak

Mild

Strong

Results on level of backreaction

Run simulations until end of inflation $\epsilon_H = 1$

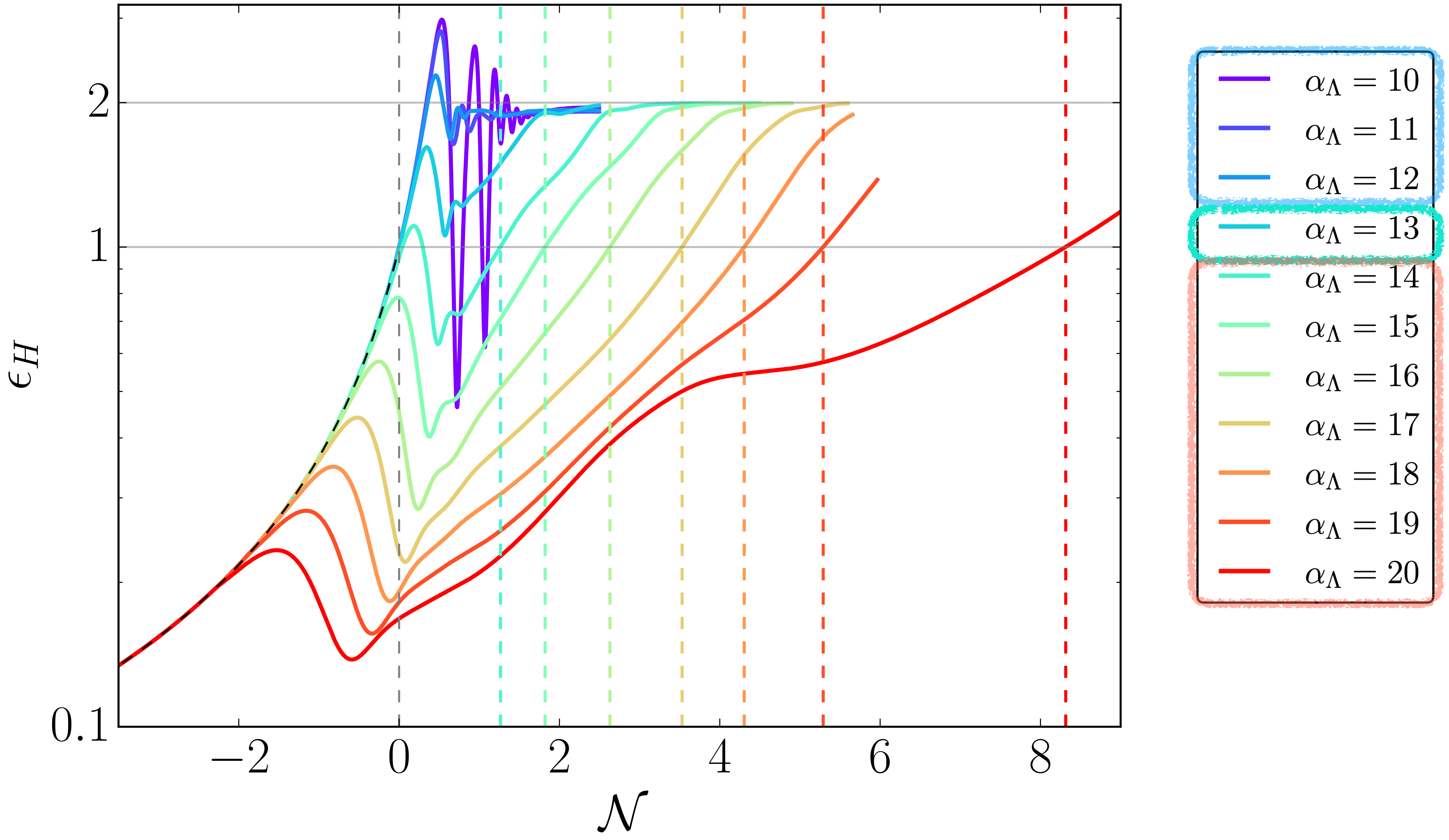


Weak

End of Inflation does not get delayed, back reaction only relevant after the end.

Results on level of backreaction

Run simulations until end of inflation $\epsilon_H = 1$



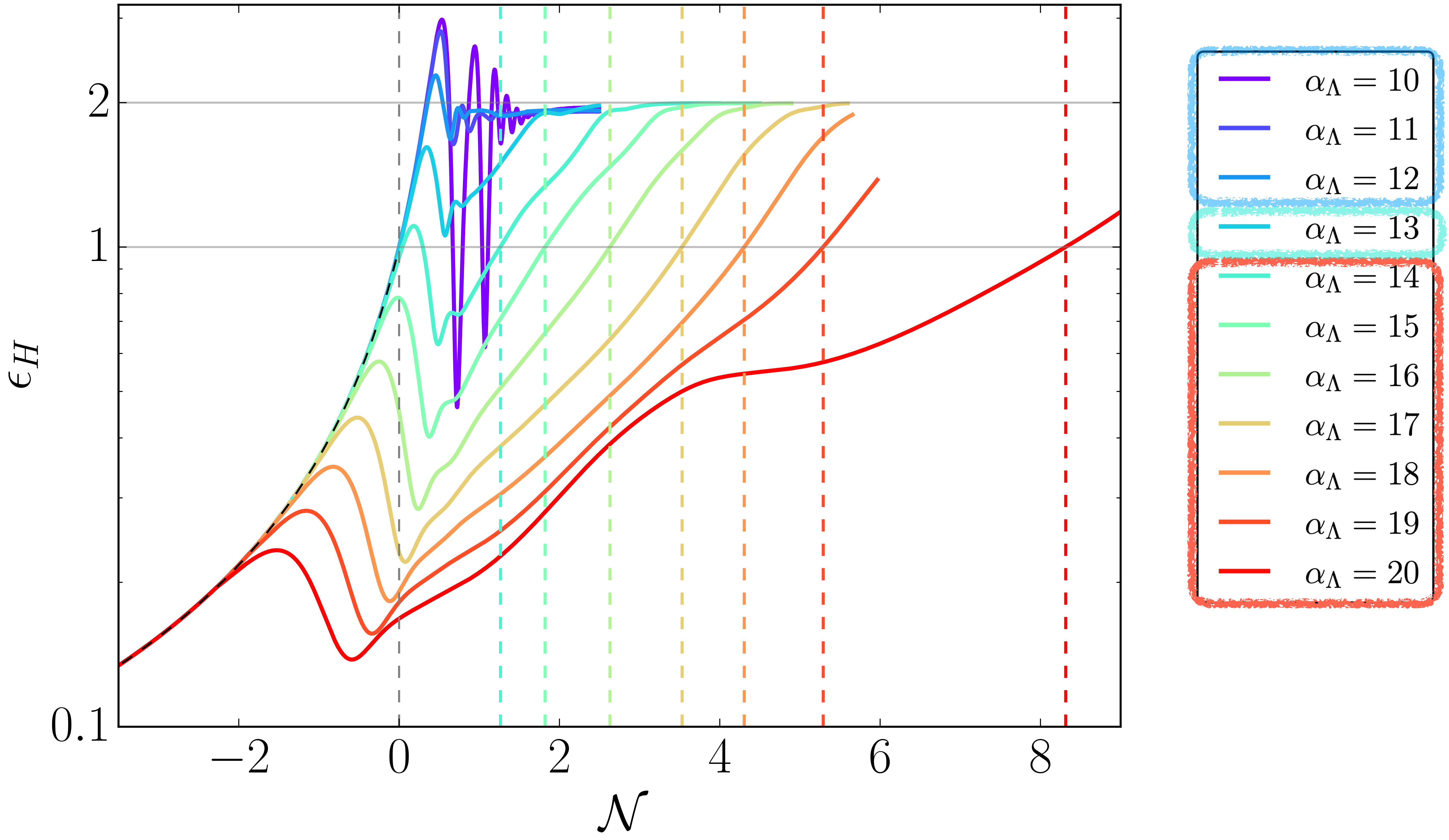
Weak

Mild

Initially inflation's end is not delayed but it re-enters later

Results on level of backreaction

Run simulations until end of inflation $\epsilon_H = 1$



Weak

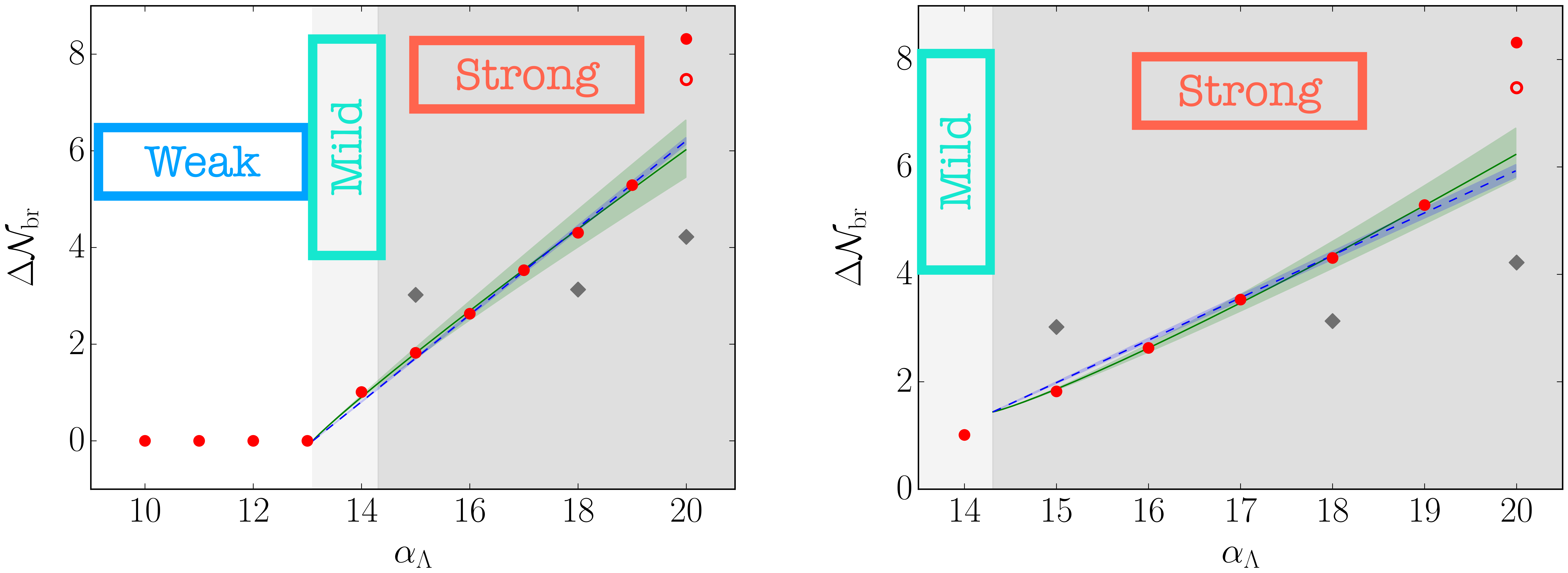
Mild

Strong

End of inflation is delayed by several e-folds depending on the coupling

$\Delta\mathcal{N}_{\rm br}$

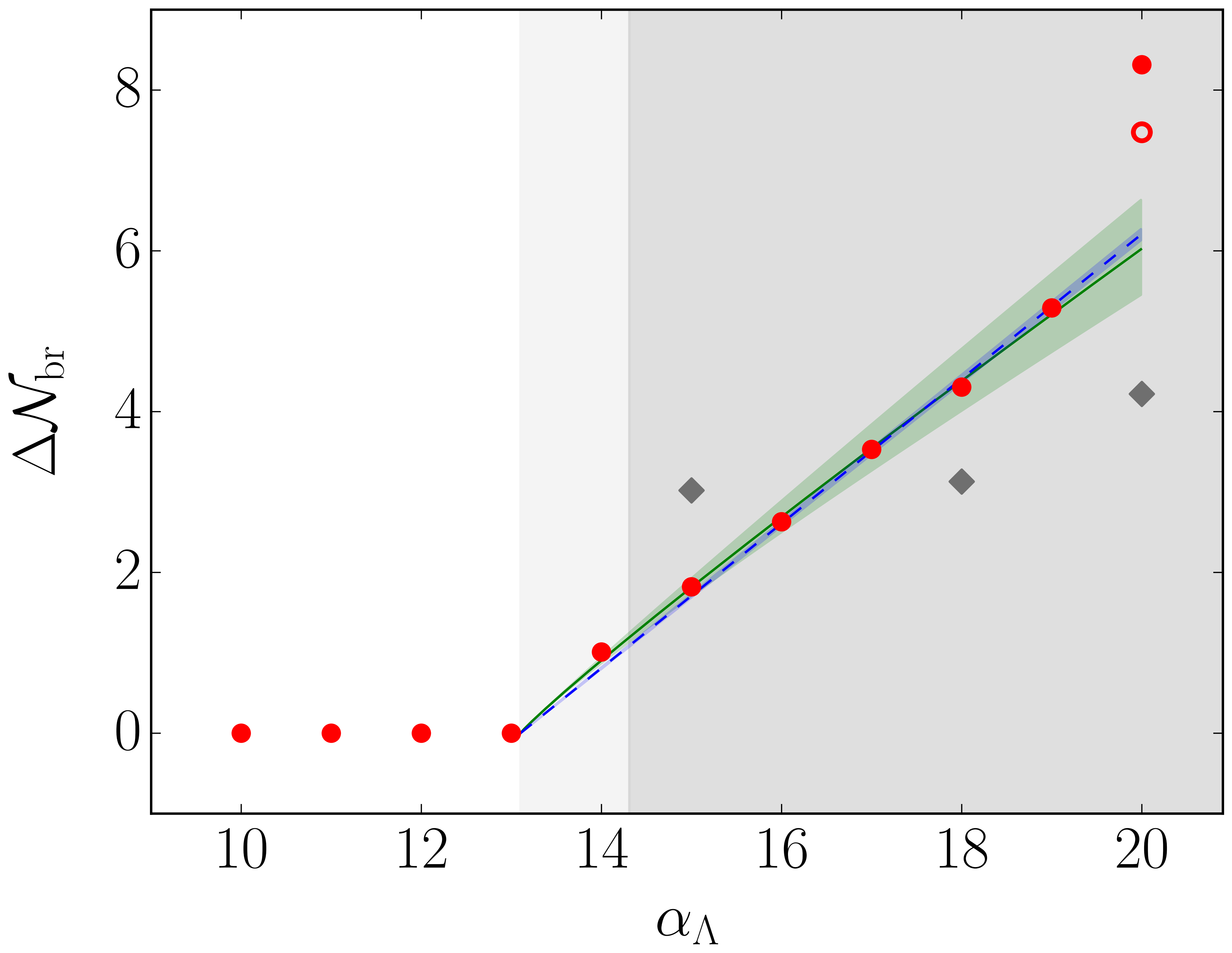
Results on level of backreaction

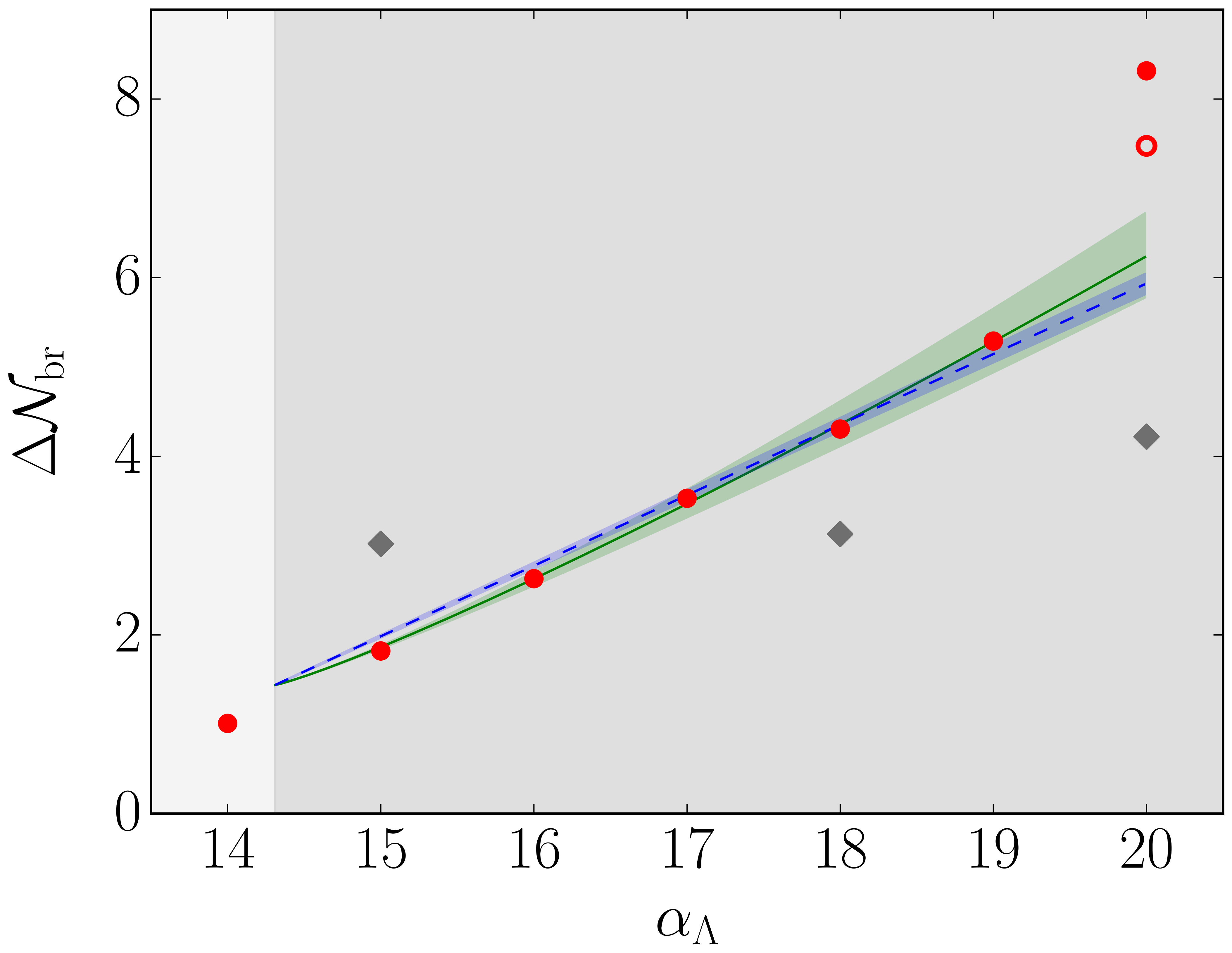


● Local backreaction

◆ Homogeneous backreaction

Results on level of backreaction



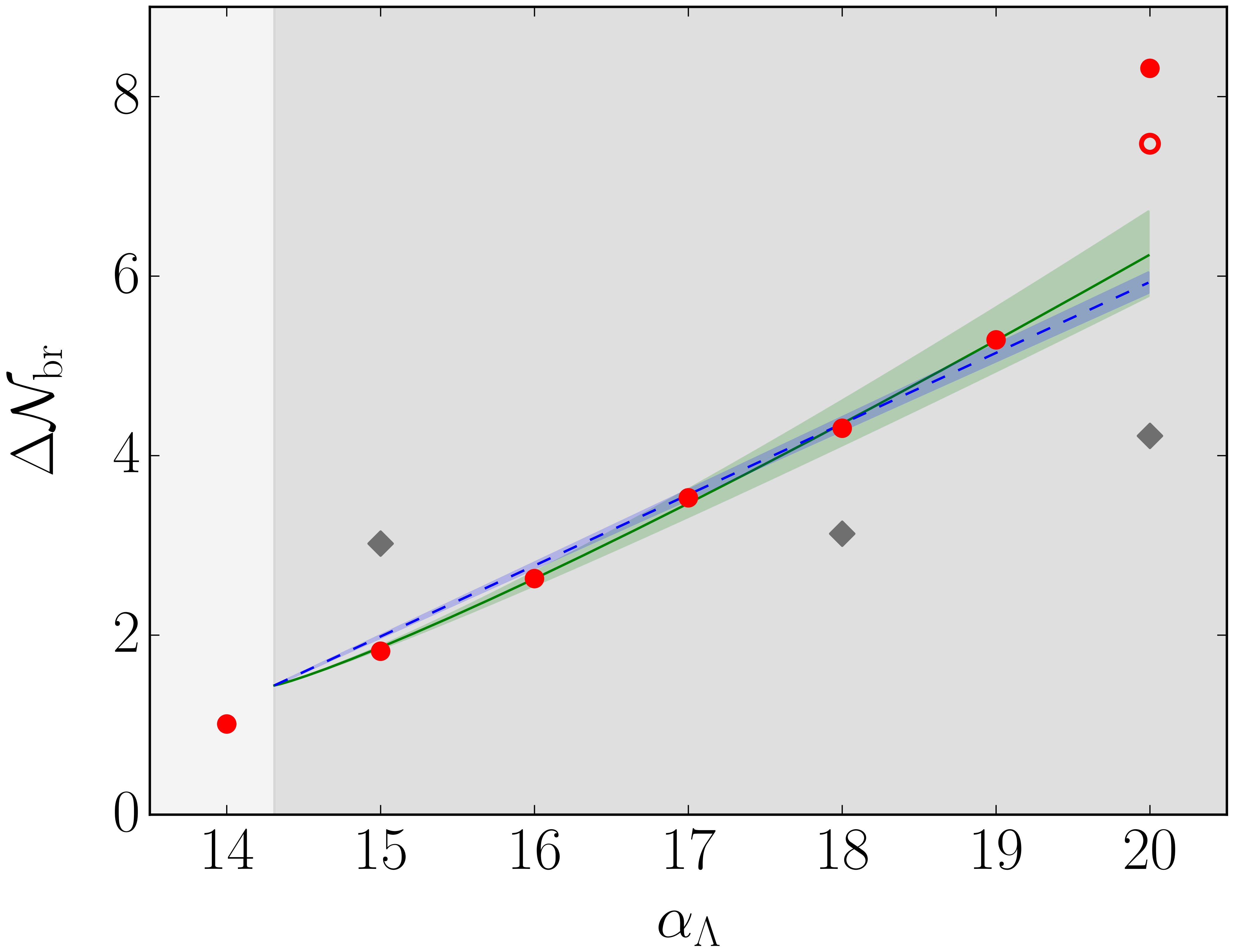


Parametrize $\Delta\mathcal{N}_{\rm br}$

$$\Delta\mathcal{N}_{\rm br} = m_2(\alpha_\Lambda - 14.31) + 1.44 \qquad m_2 = 0.79 \pm 0.02$$

$$\Delta\mathcal{N}_{\rm br} = b_2(\alpha_\Lambda - 14.31)^{a_2} + 1.44 \quad a_2 = 1.15 \pm 0.03 \quad b_2 = 0.65 \pm 0.03$$

Results on level of backreaction



Extrapolations

$$\alpha_\Lambda = 25 \to \Delta\mathcal{N}_{\rm br} \sim 10 \ - \ 12$$

$$\alpha_\Lambda = 30 \to \Delta\mathcal{N}_{\rm br} \sim 15 \ - \ 18$$

$$\alpha_\Lambda = 35 \to \Delta\mathcal{N}_{\rm br} \sim 18 \ - \ 25$$

General features

$$\epsilon_H = 1 + \frac{2\rho_{\rm K} + \rho_{\rm EM} - \rho_{\rm V}}{\rho_{\rm tot}}$$

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General features

Mild

$$\epsilon_H = 1 + \frac{2\rho_K + \rho_{EM} - \rho_V}{\rho_{tot}}$$

General features

Mild

$$\epsilon_H = 1 + \frac{2\rho_{\rm K} + \rho_{\rm EM} - \rho_{\rm V}}{\rho_{\rm tot}}$$

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General features

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$$\epsilon_H = 1 + \frac{2\rho_{\rm K} + \rho_{\rm EM} - \rho_{\rm V}}{\rho_{\rm tot}}$$

General features

Strong

$$\epsilon_H = 1 + \frac{2\rho_{\rm K} + \rho_{\rm EM} - \rho_{\rm V}}{\rho_{\rm tot}}$$

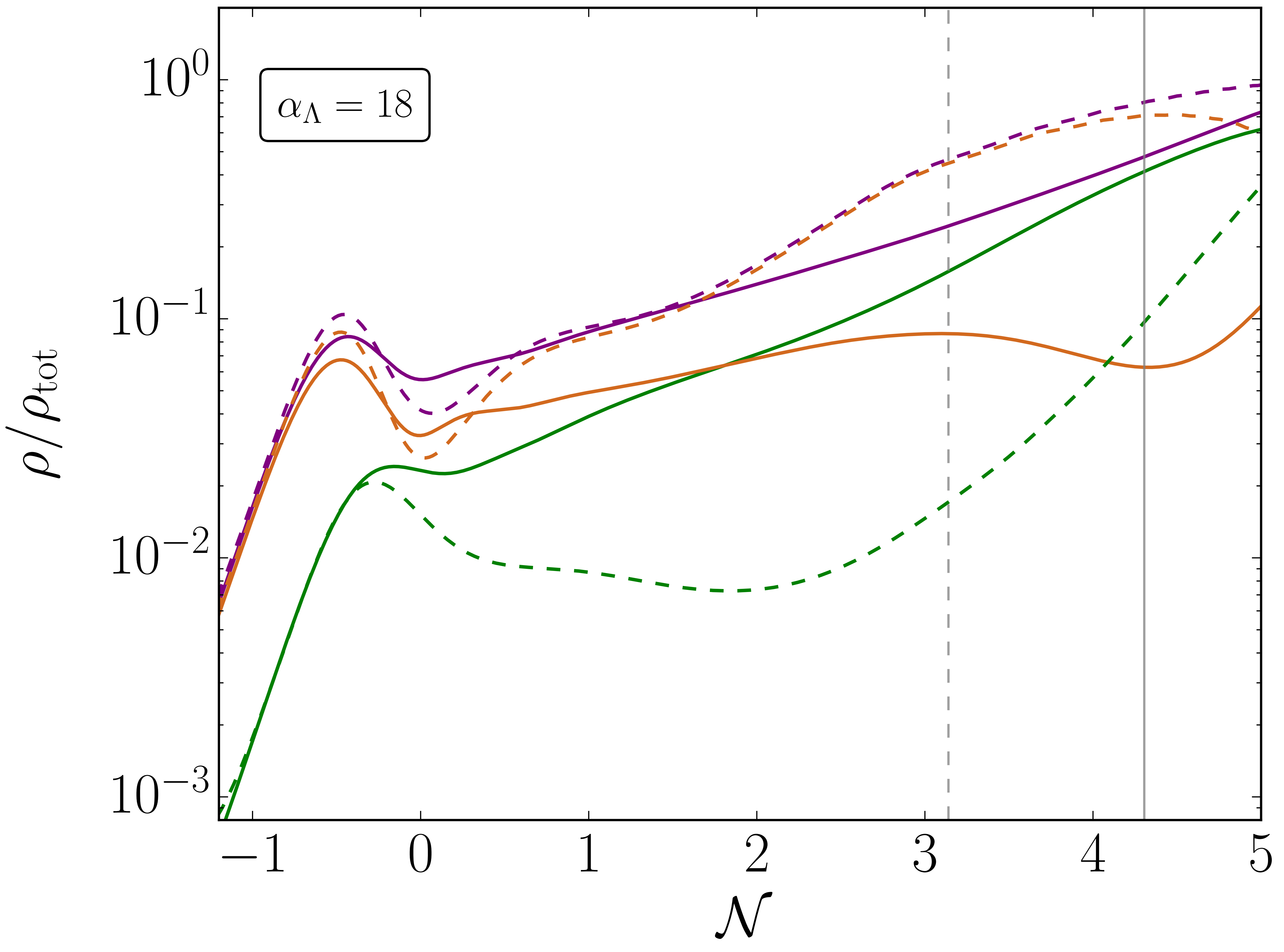
General features

$$\epsilon_H = 1 + \frac{2\rho_{\rm K} + \rho_{\rm EM} - \rho_{\rm V}}{\rho_{\rm tot}}$$

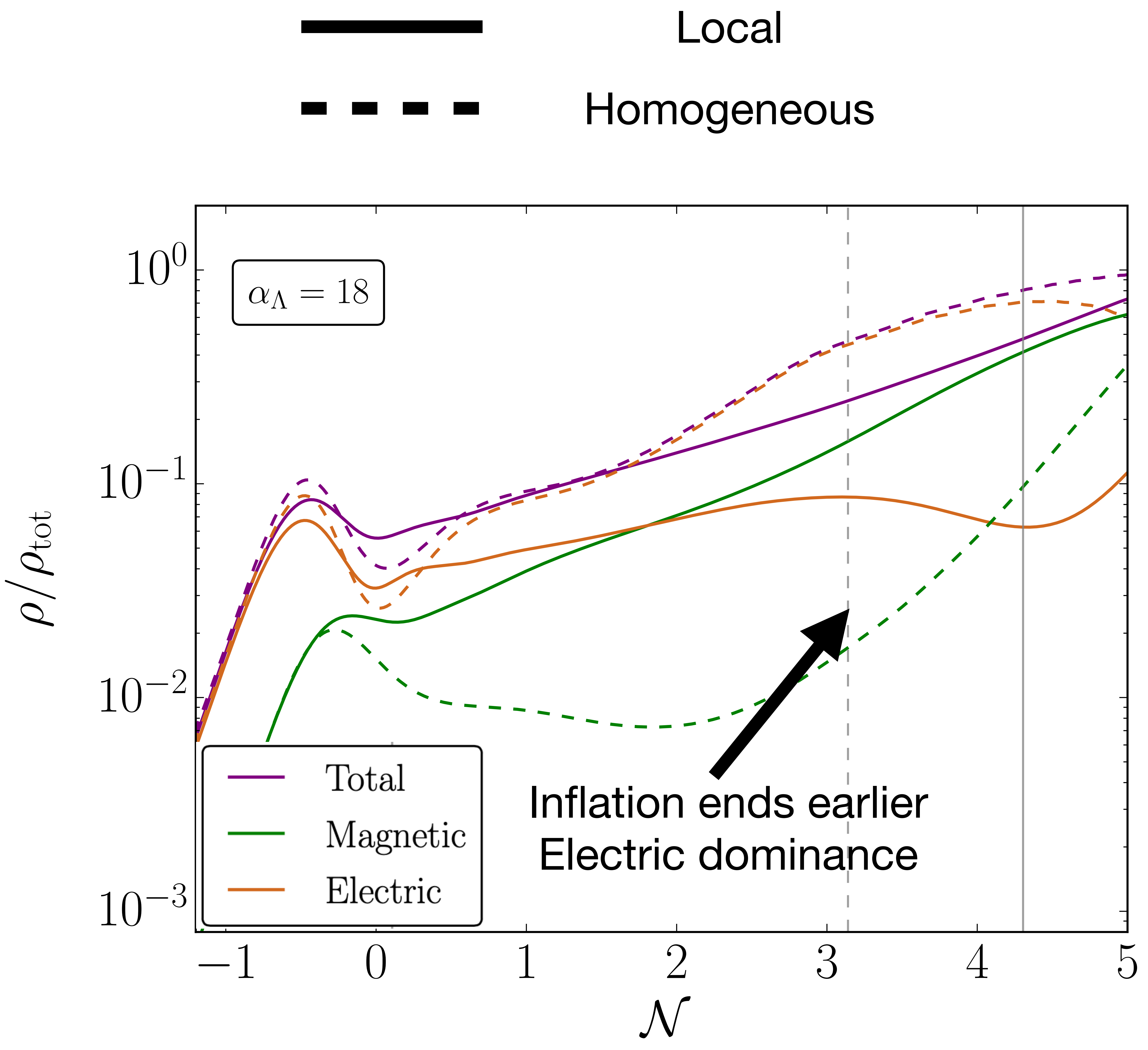
(Electro)Magnetic Slow Roll

Local

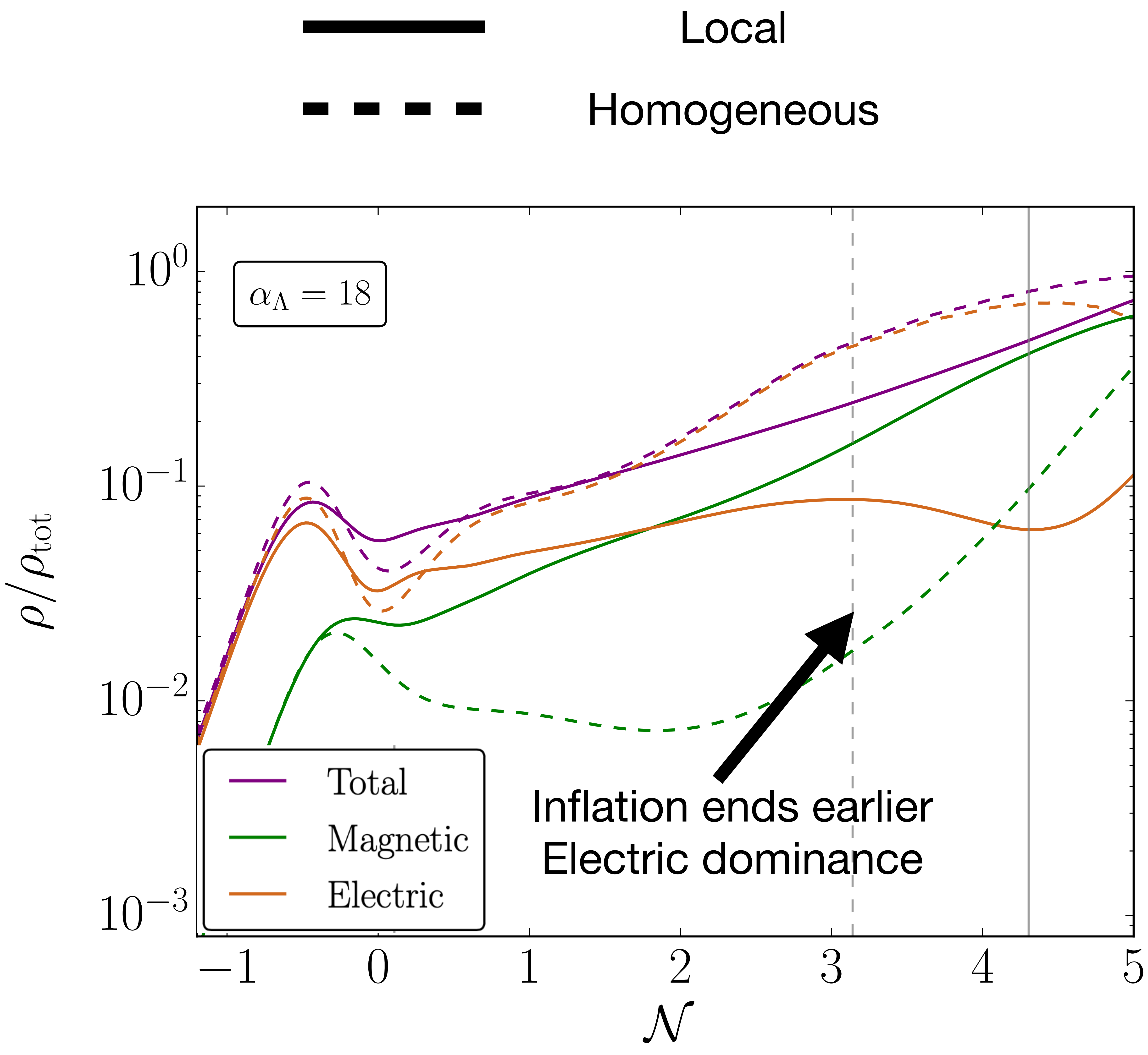
Homogeneous

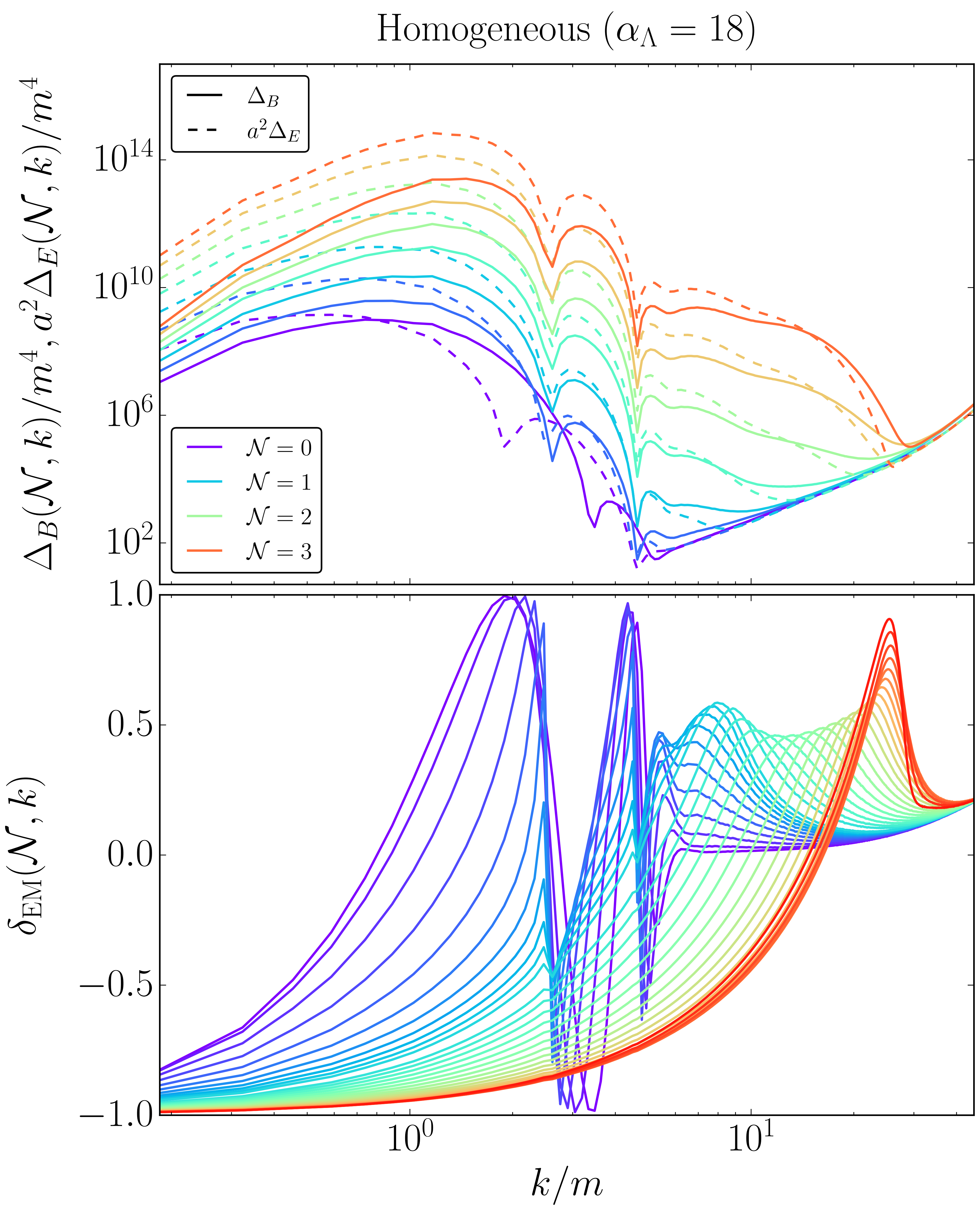


(Electro)Magnetic Slow Roll

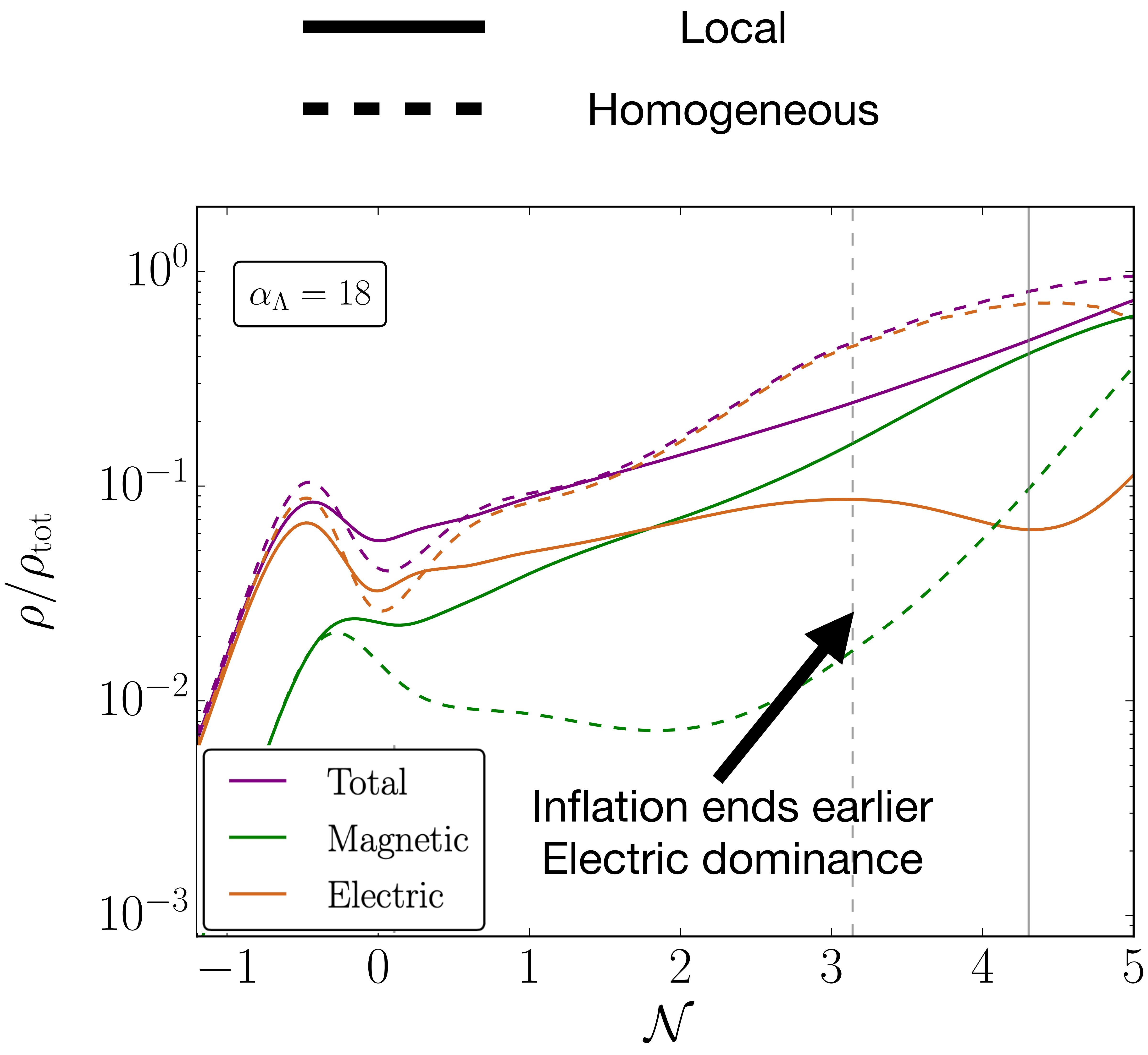


(Electro)Magnetic Slow Roll

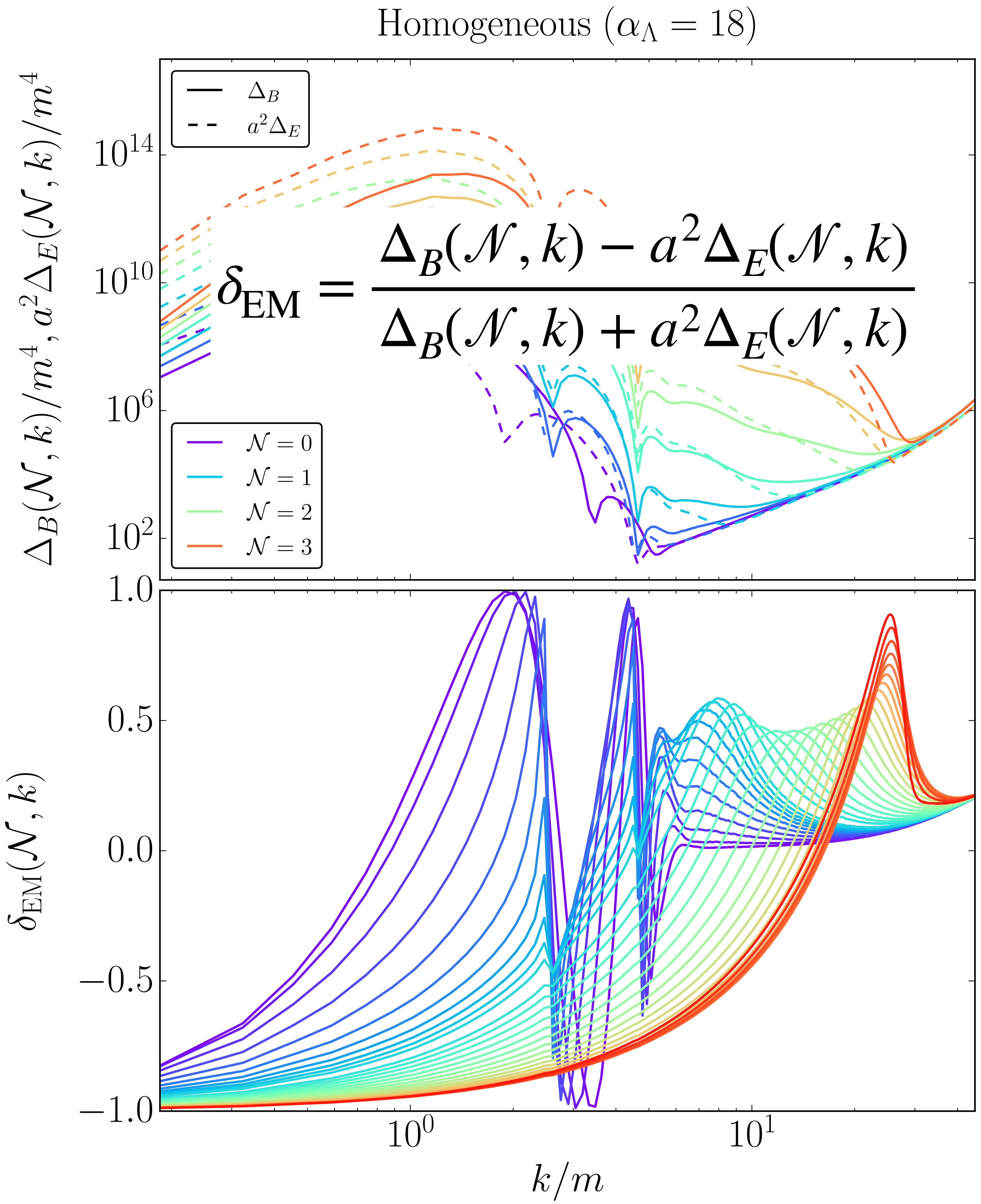




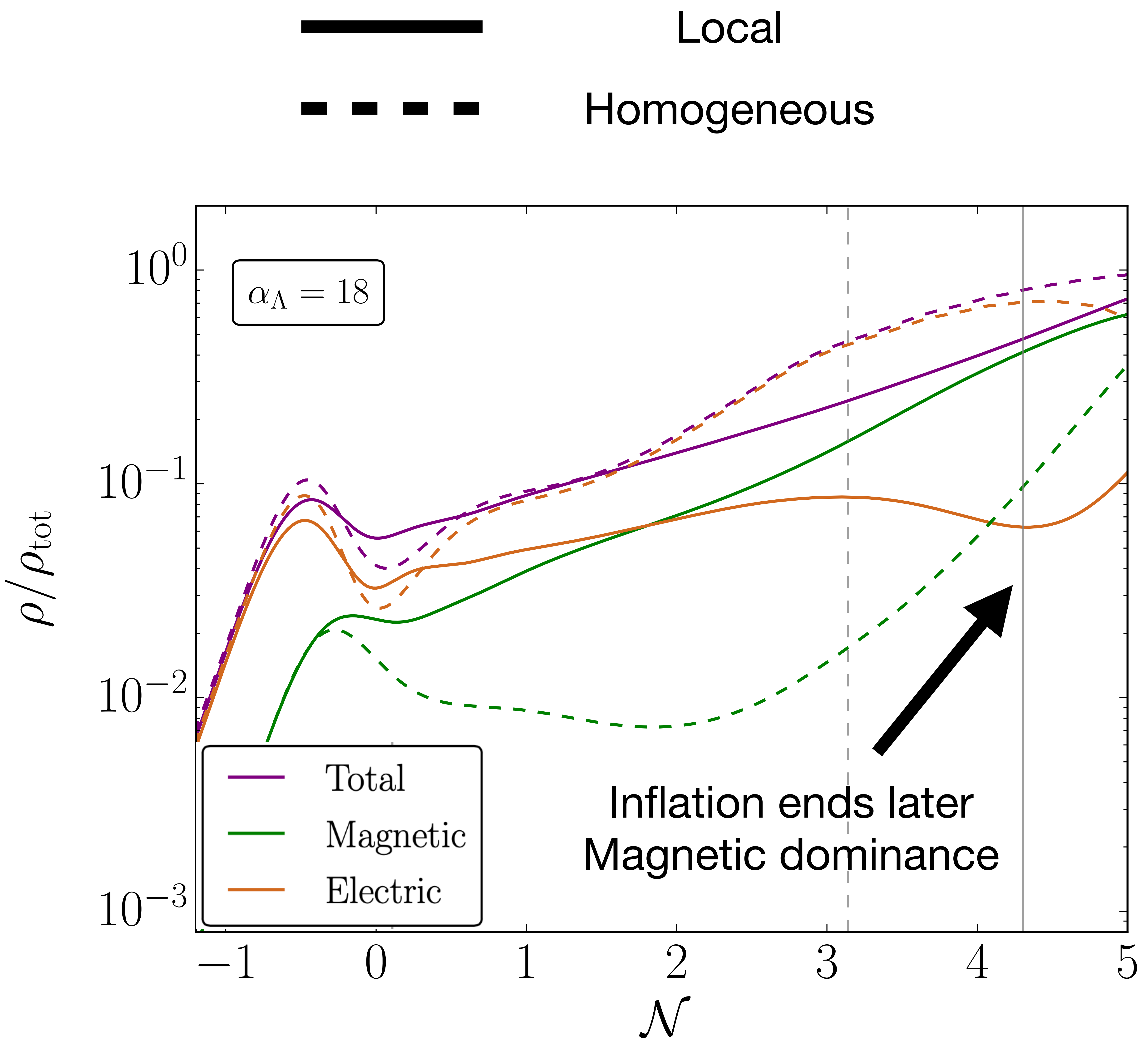
(Electro)Magnetic Slow Roll

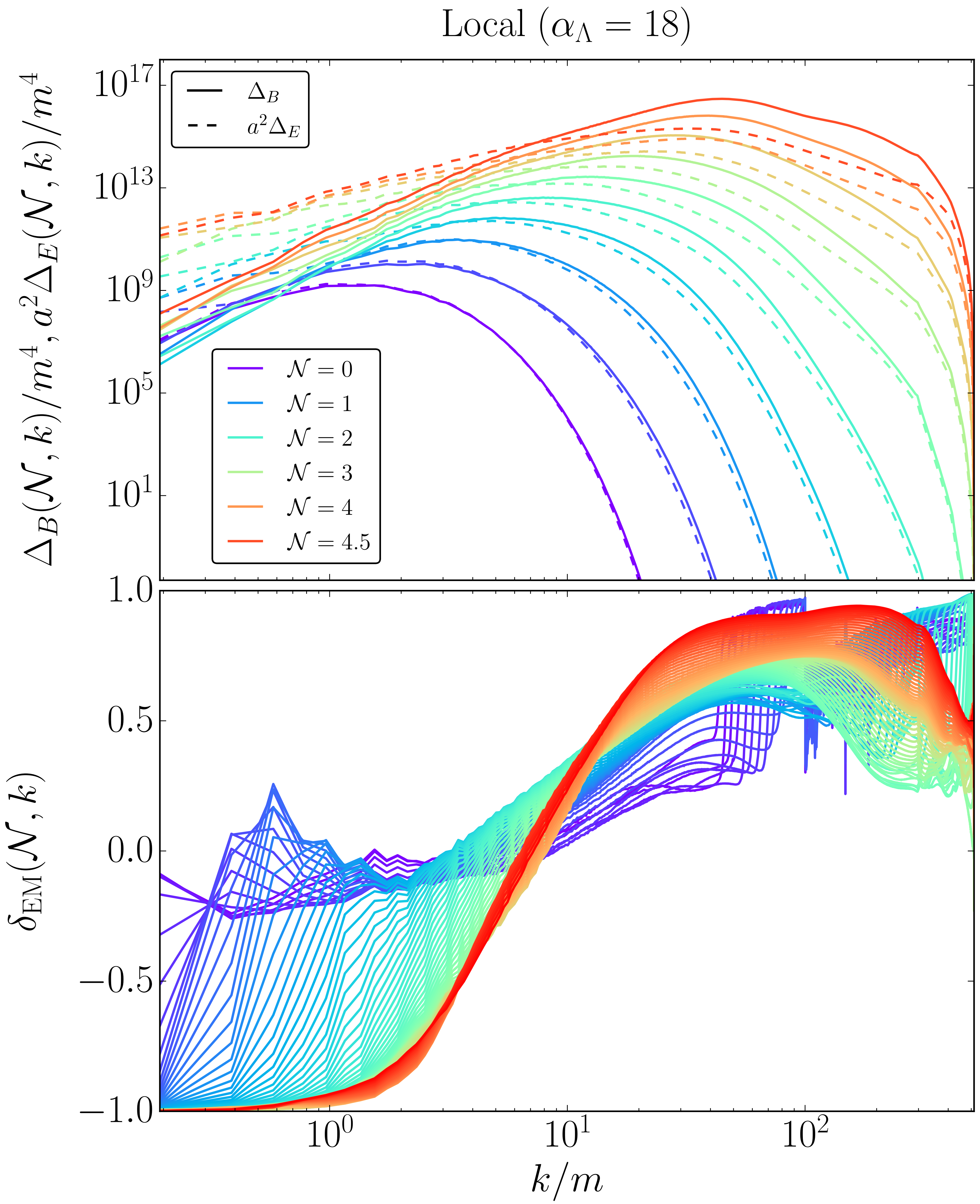


$$\delta_{\rm EM} = \frac{\Delta_B(\mathcal{N},k) - a^2\Delta_E(\mathcal{N},k)}{\Delta_B(\mathcal{N},k) + a^2\Delta_E(\mathcal{N},k)}$$

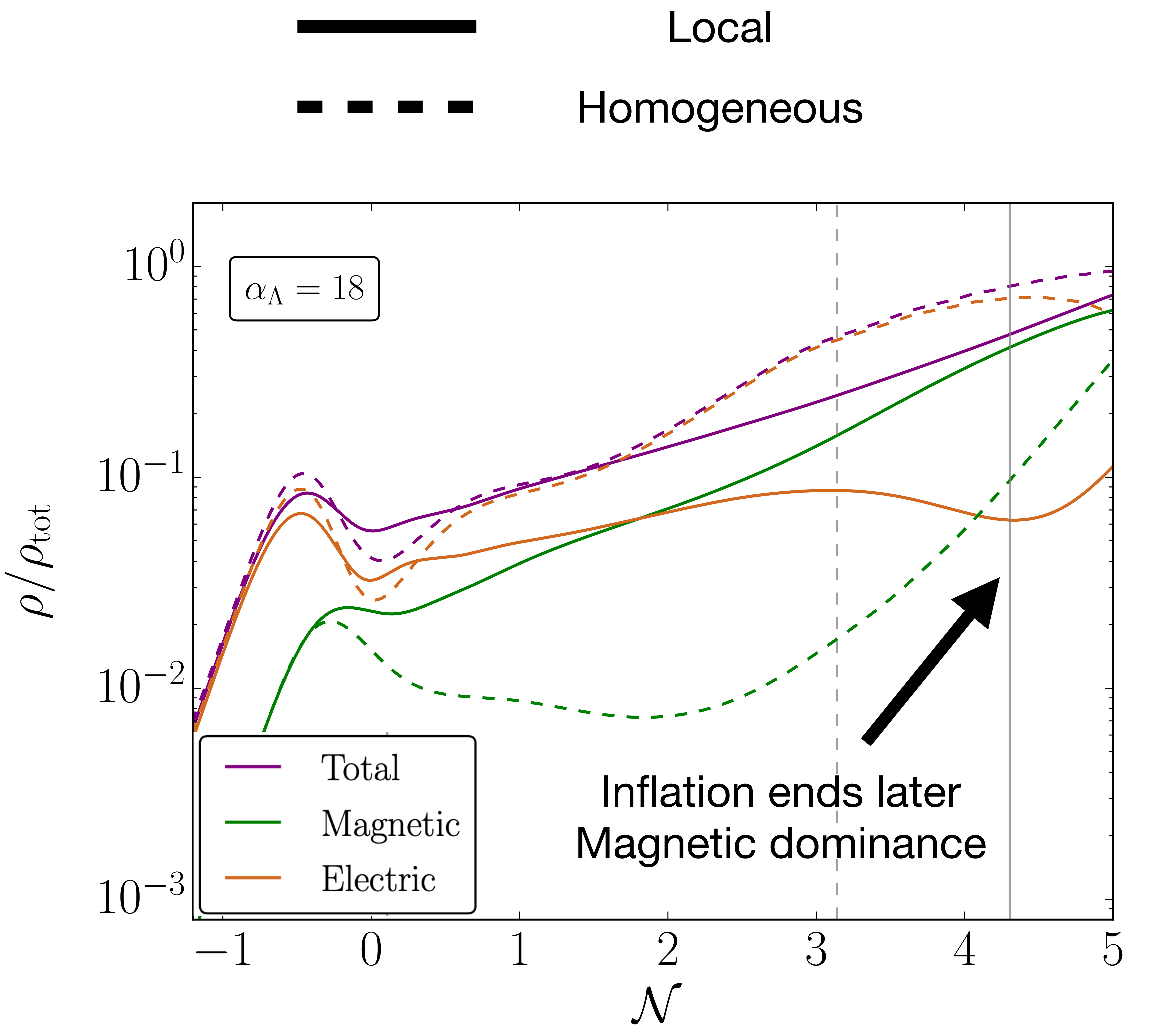


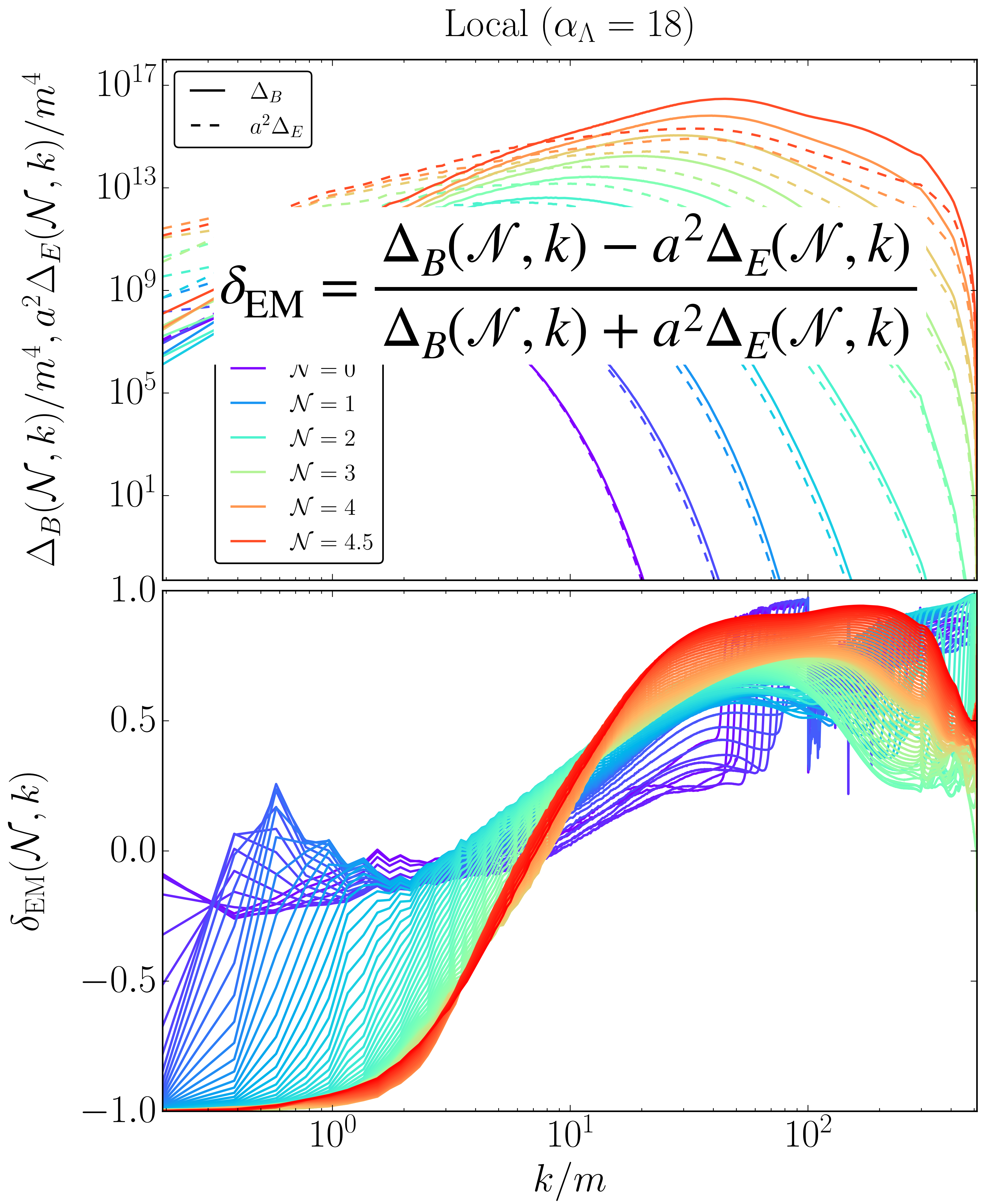
(Electro)Magnetic Slow Roll





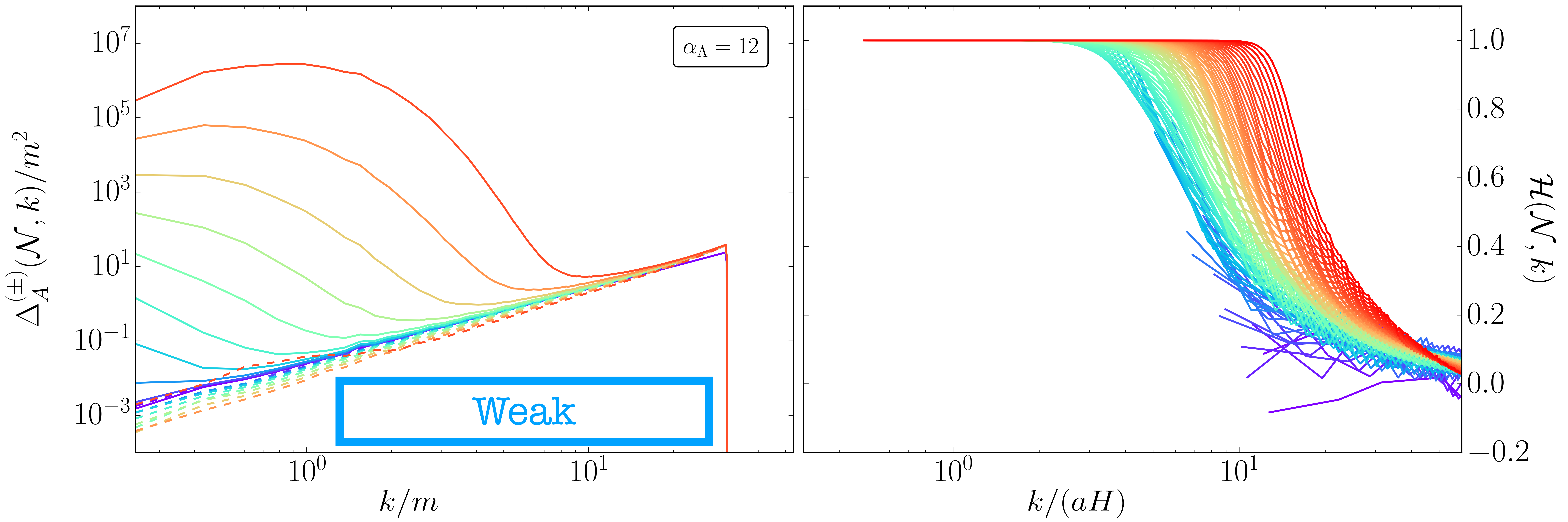
(Electro)Magnetic Slow Roll





$$\delta_{\rm EM} = \frac{\Delta_B(\mathcal{N},k) - a^2\Delta_E(\mathcal{N},k)}{\Delta_B(\mathcal{N},k) + a^2\Delta_E(\mathcal{N},k)}$$

Chirality of A_μ power spectrum

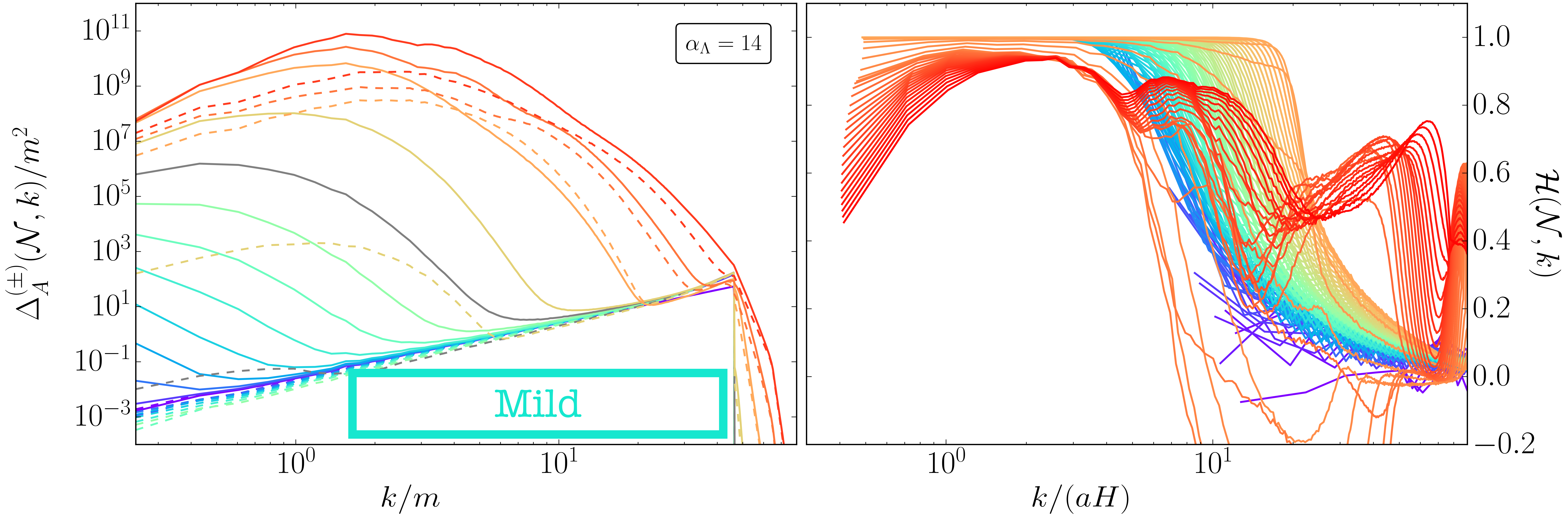


Solid line: $\Delta_A^{+}(\mathcal{N}, k)$

Dashed line: $\Delta_A^{-}(\mathcal{N}, k)$

$$\mathscr{H} = \frac{\Delta_A^{+}(\mathcal{N}, k) - \Delta_A^{-}(\mathcal{N}, k)}{\Delta_A^{+}(\mathcal{N}, k) + \Delta_A^{-}(\mathcal{N}, k)}$$

Chirality of A_μ power spectrum

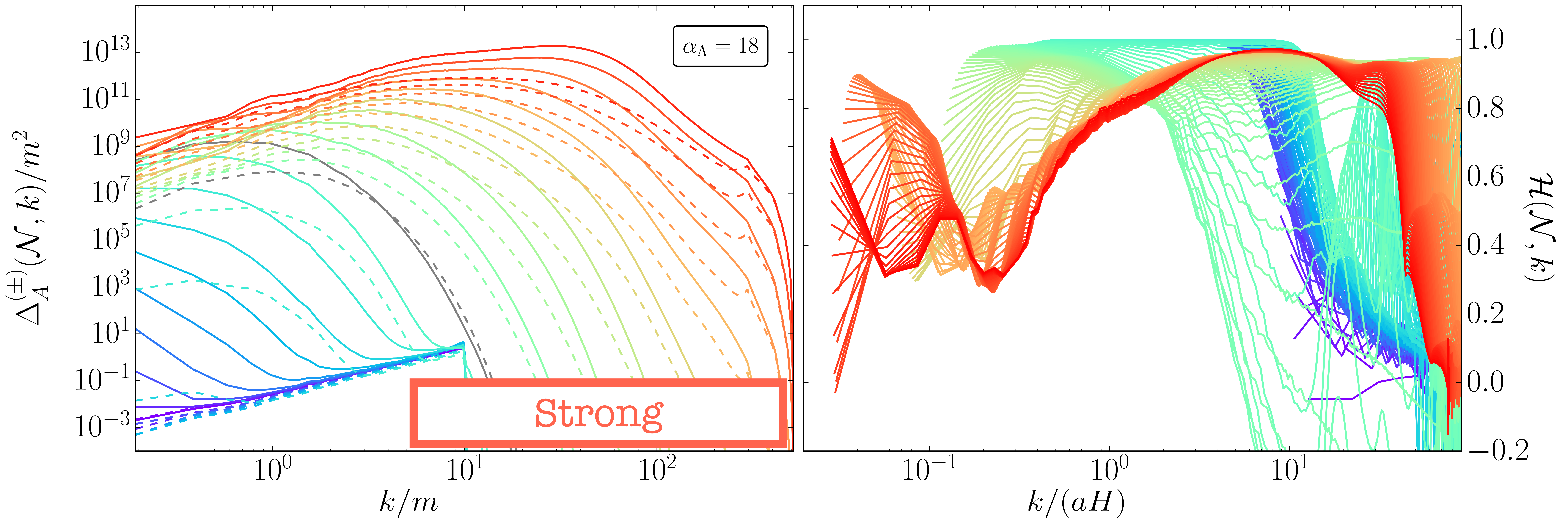


— $\Delta_A^+(\mathcal{N}, k)$

- - - $\Delta_A^-(\mathcal{N}, k)$

$$\mathscr{H} = \frac{\Delta_A^+(\mathcal{N}, k) - \Delta_A^-(\mathcal{N}, k)}{\Delta_A^+(\mathcal{N}, k) + \Delta_A^-(\mathcal{N}, k)}$$

Chirality of A_μ power spectrum

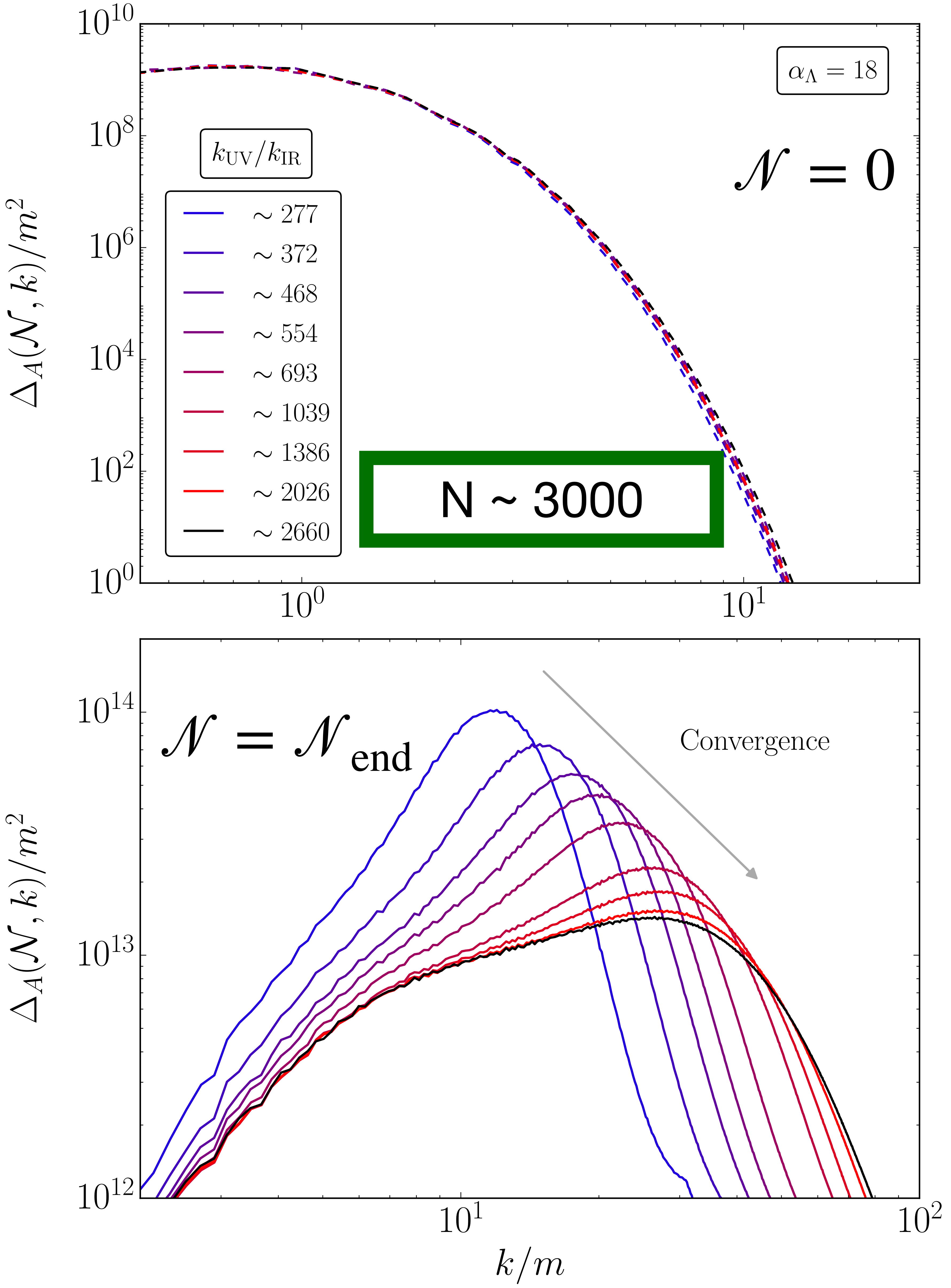


— $\Delta_A^+(\mathcal{N},k)$

- - - $\Delta_A^-(\mathcal{N},k)$

$$\mathscr{H} = \frac{\Delta_A^+(\mathcal{N},k) - \Delta_A^-(\mathcal{N},k)}{\Delta_A^+(\mathcal{N},k) + \Delta_A^-(\mathcal{N},k)}$$

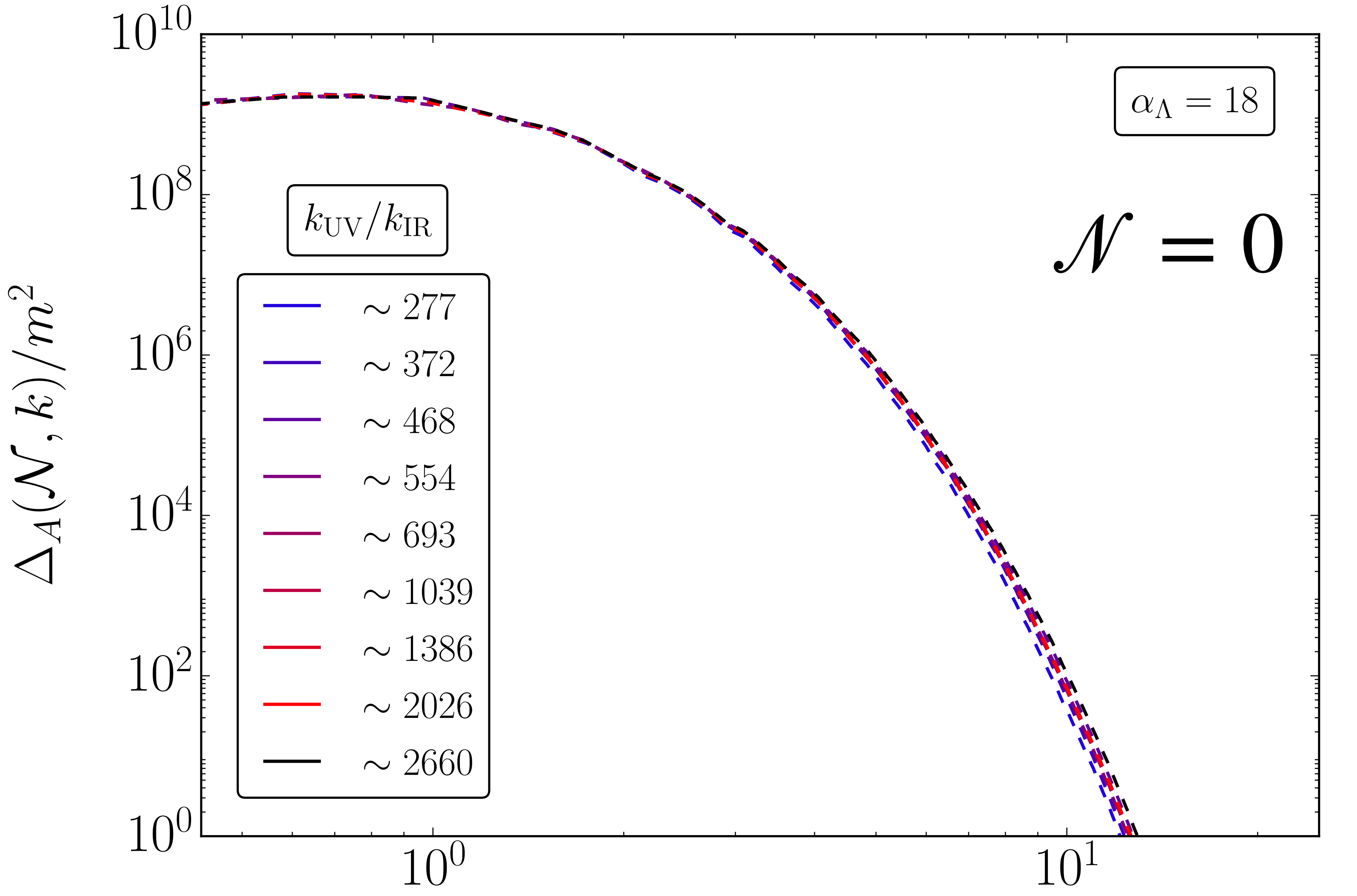
UV sensitivity and convergence

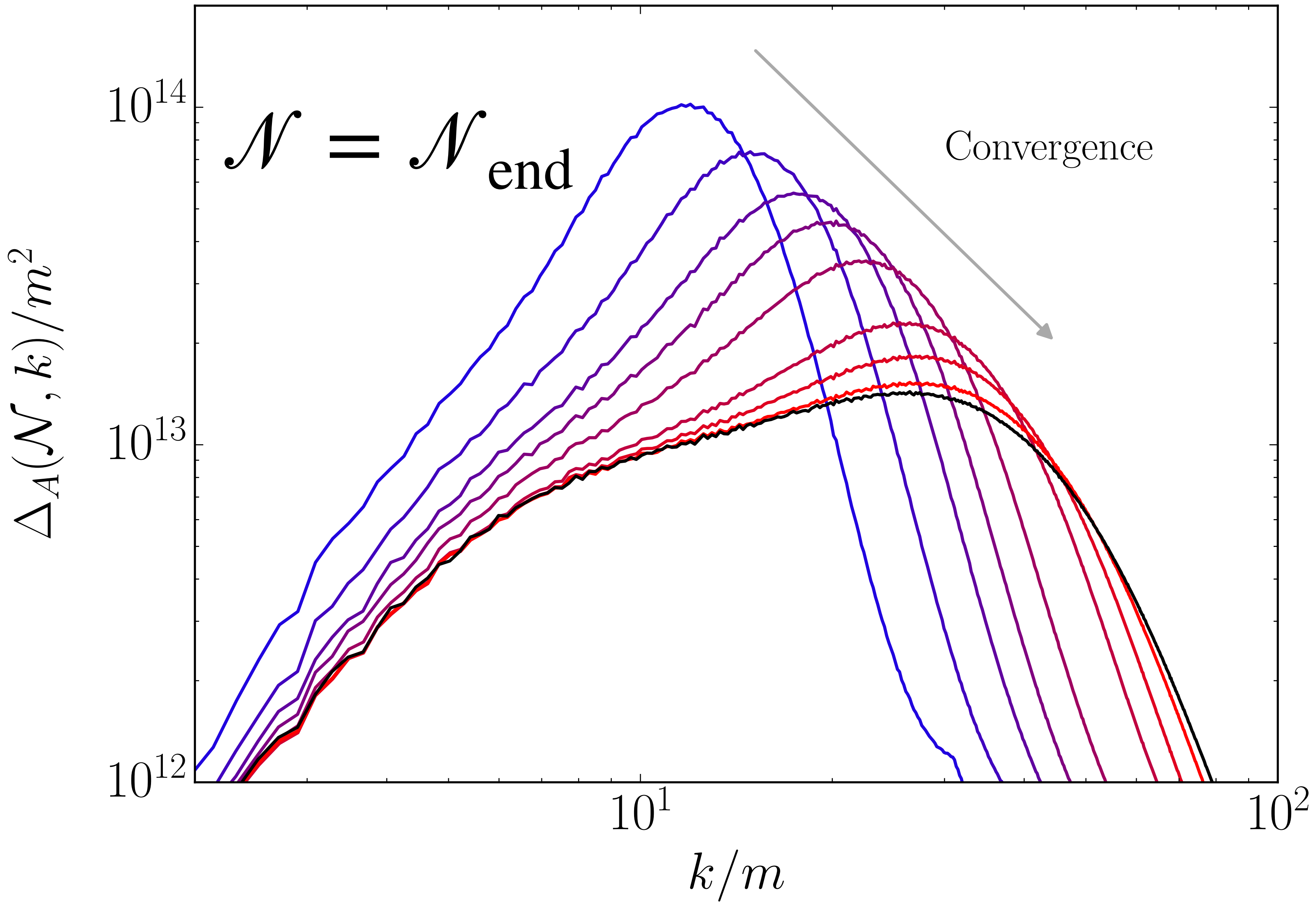


Evolve equally

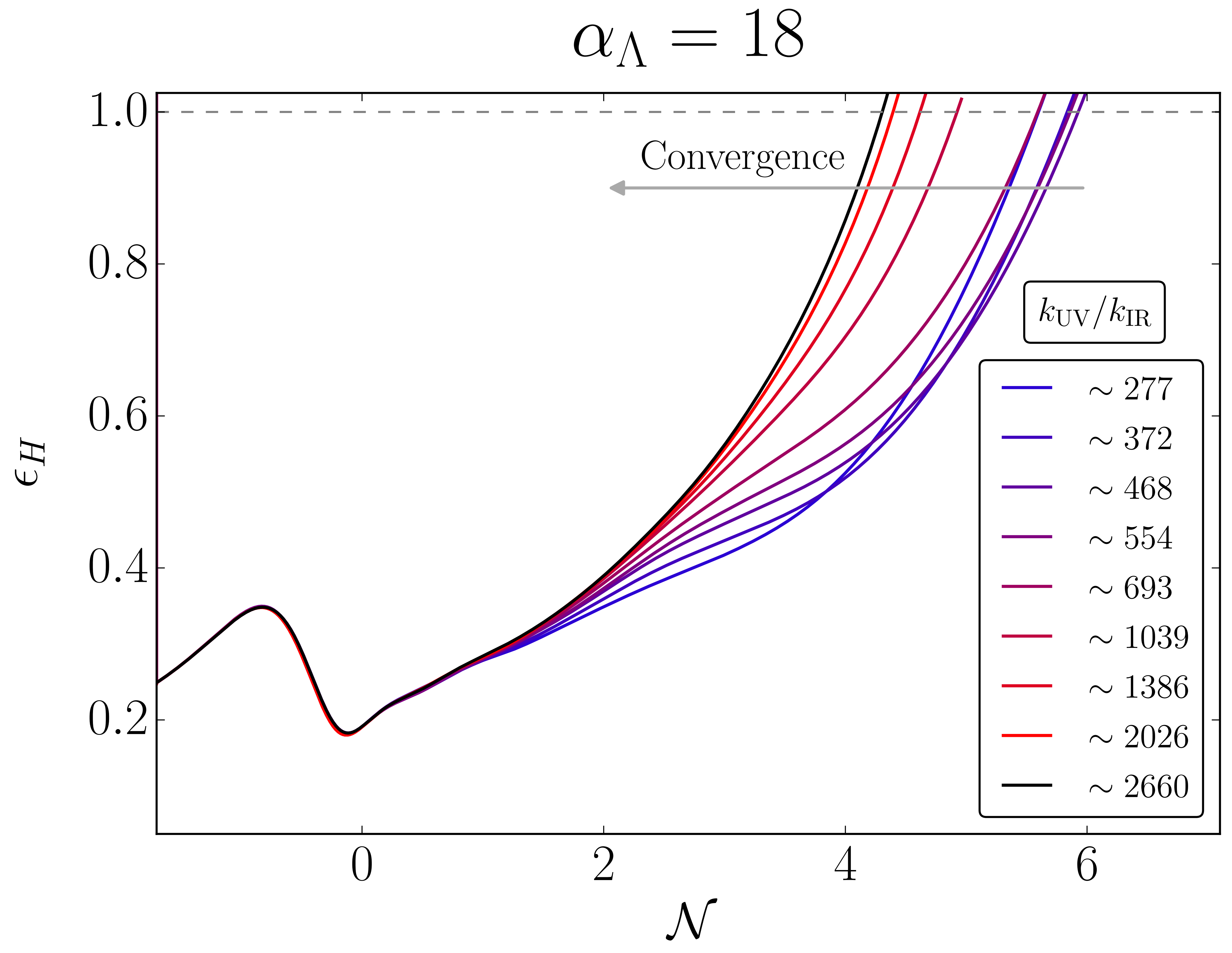
During strong backreaction,
Different UV resolution affects evolution

UV sensitivity and convergence

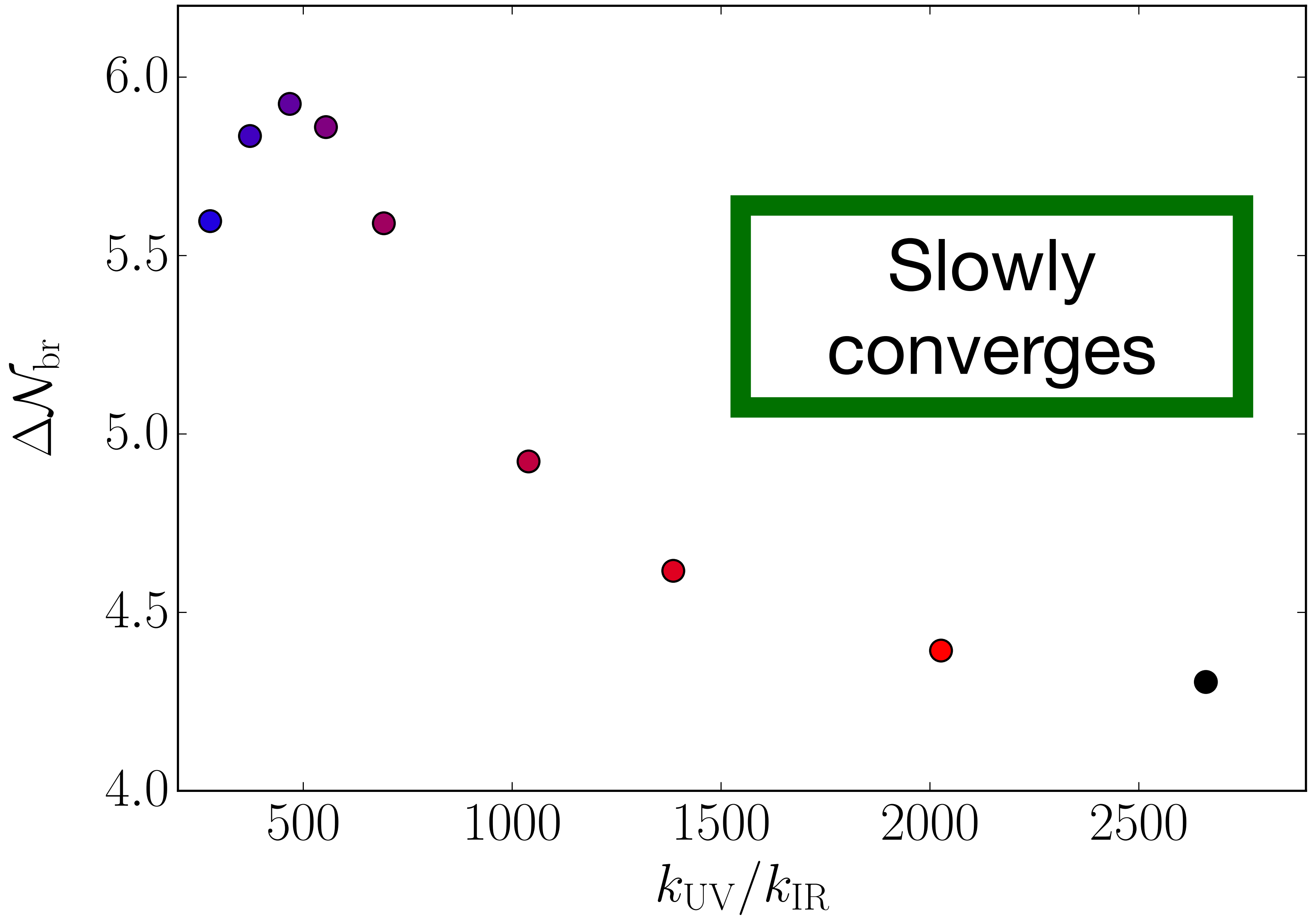




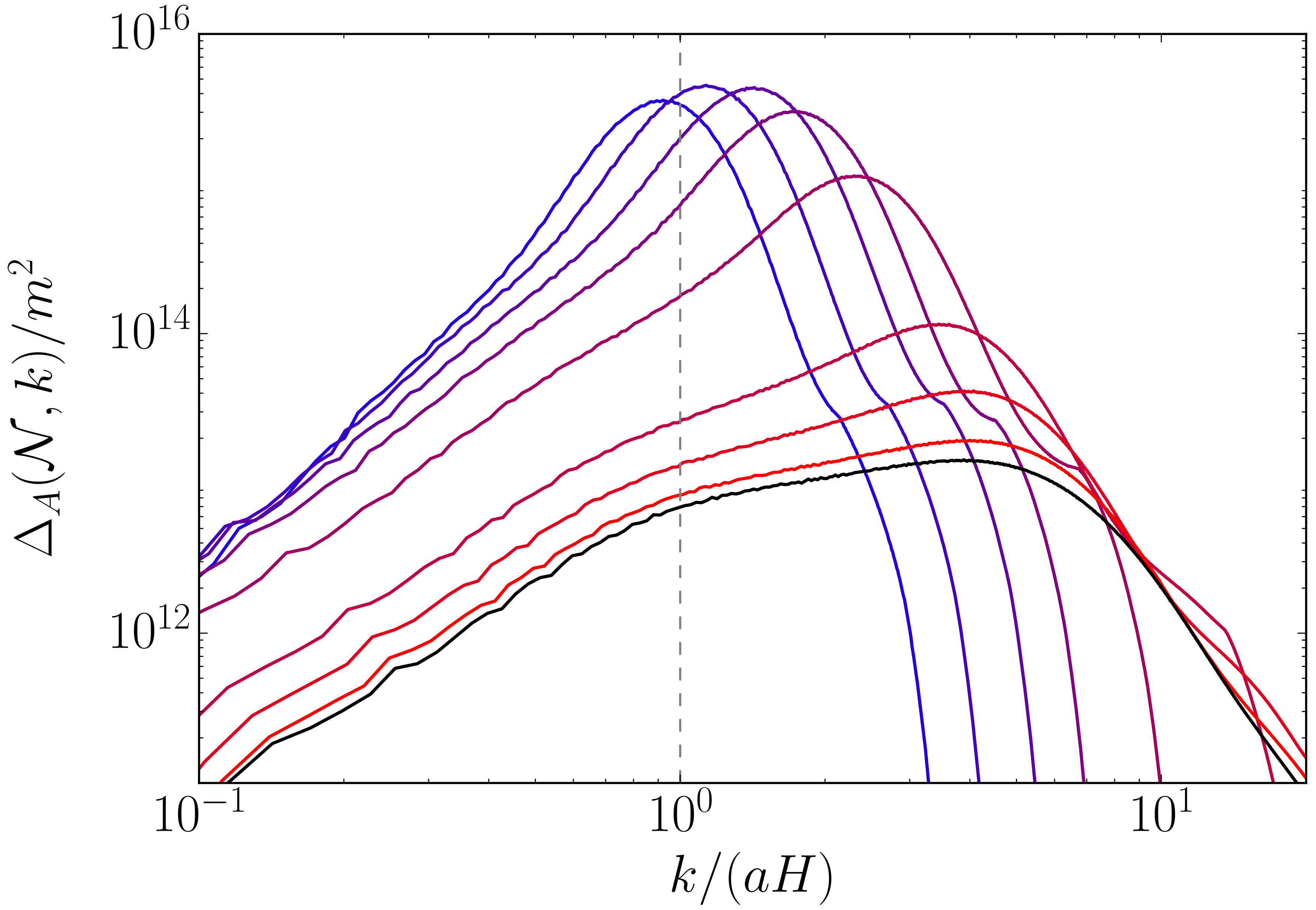
UV resolution affects the evolution of the self consistent background



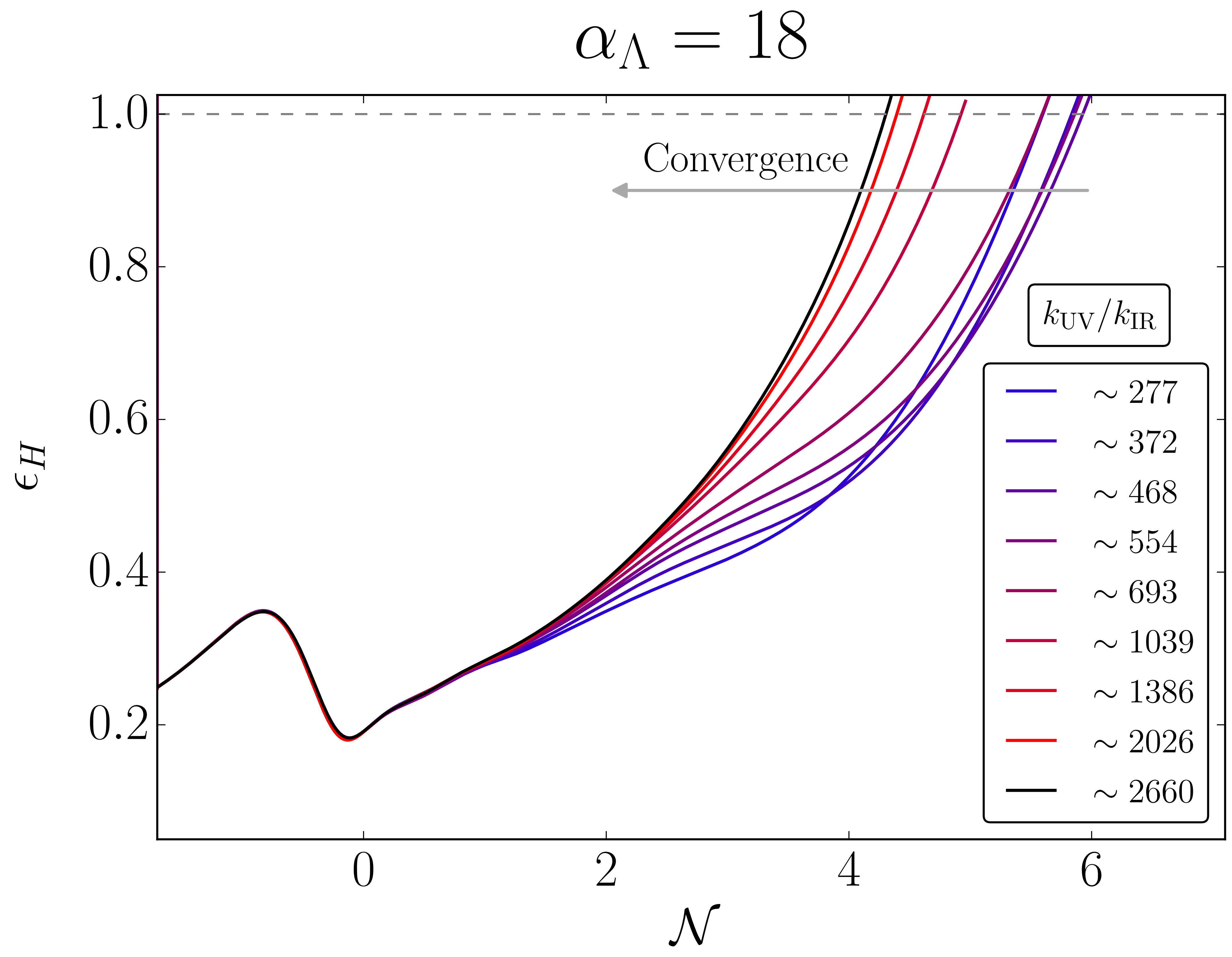
UV sensitivity and convergence



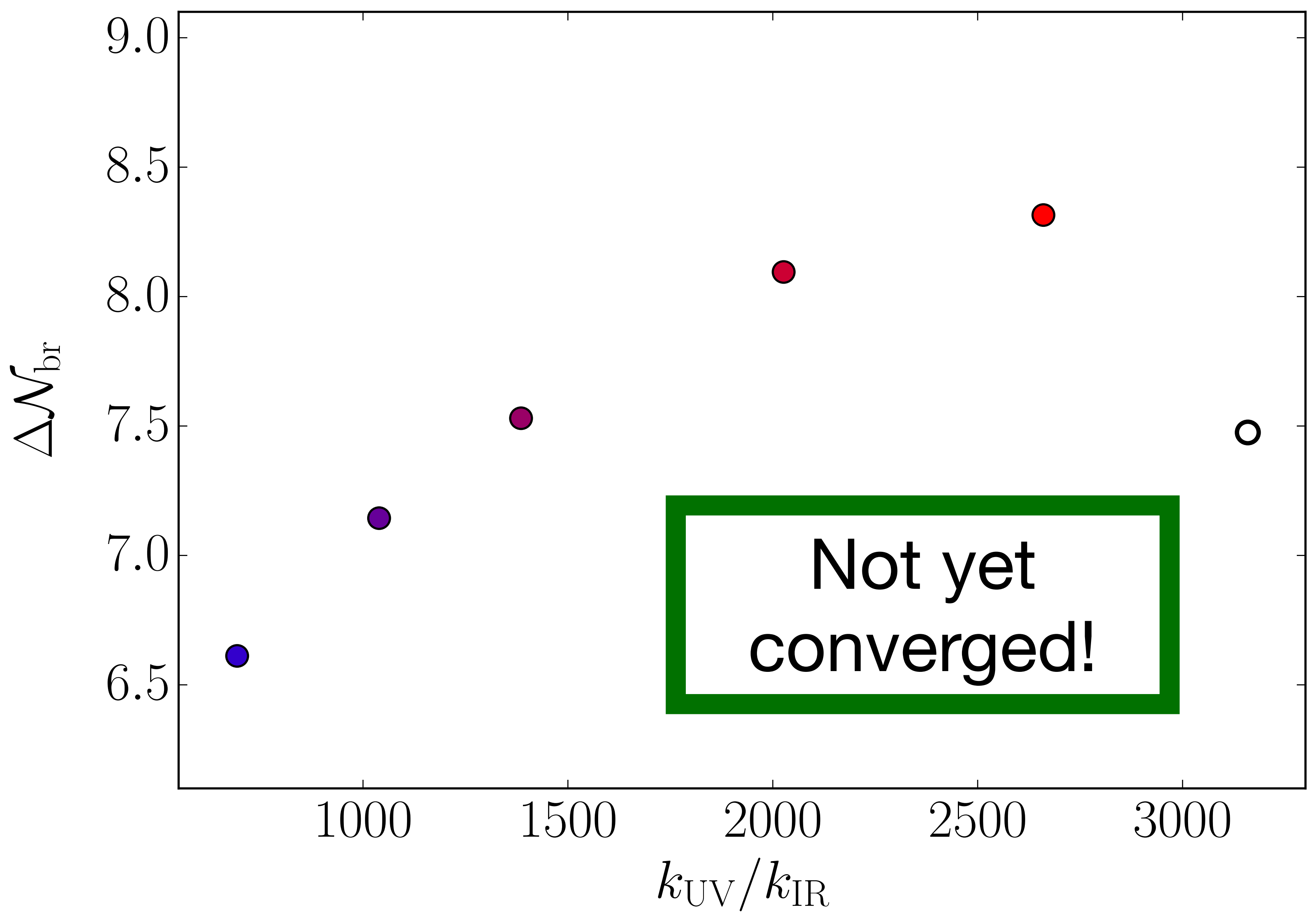
Slowly converges

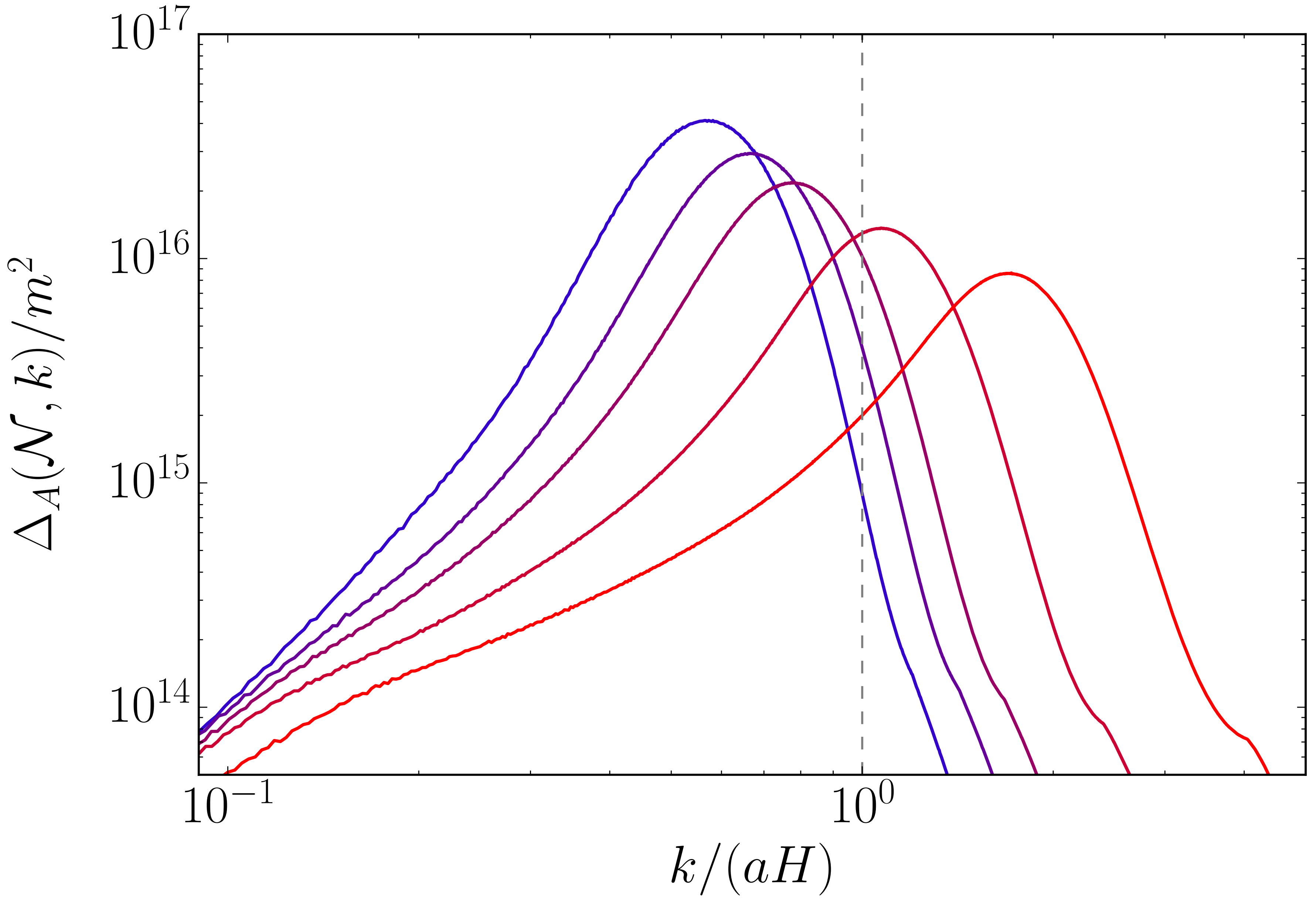


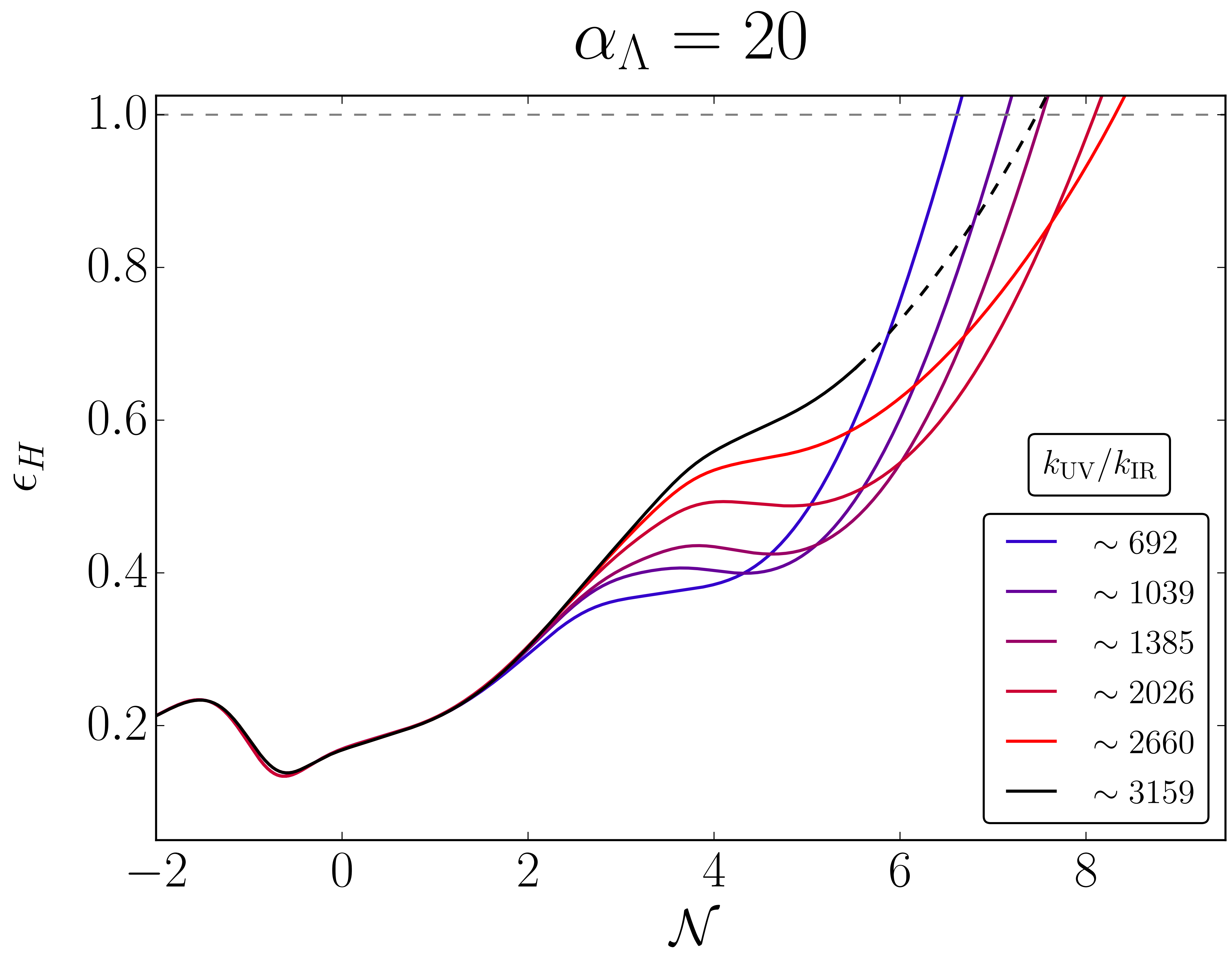
UV resolution affects the evolution of the self consistent background



UV sensitivity and convergence







N ~ 3650

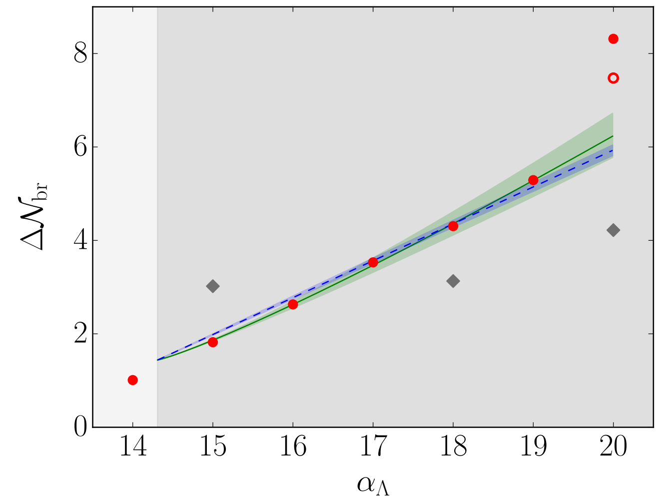
Conclusions

- Local backreaction is crucial.

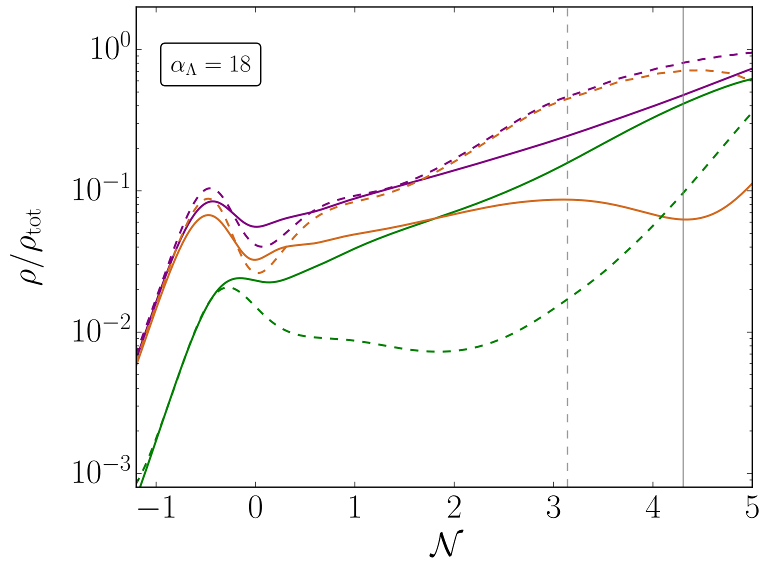
$$\sum_i \frac{\phi}{\Lambda} E_i^{(2)} B_i^{(4)}$$

- Inflation is delayed several e-folds.

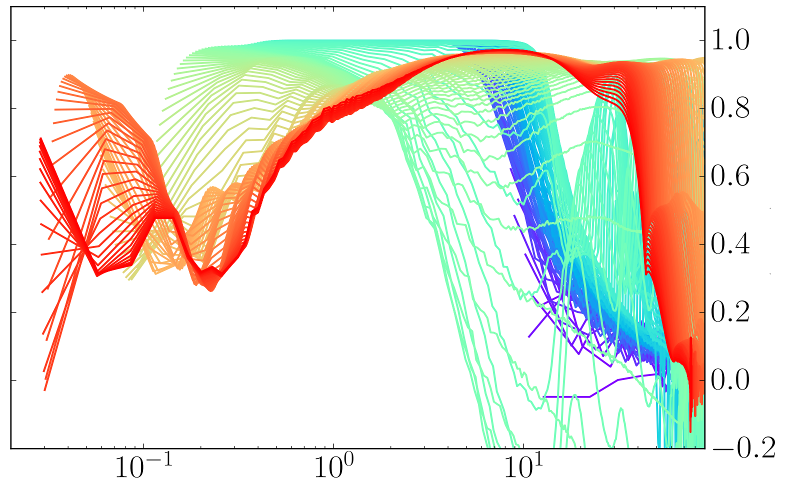
$$\Delta\mathcal{N}_{\rm br} = b_2(\alpha_\Lambda - 14.31)^{a_2} + 1.44$$



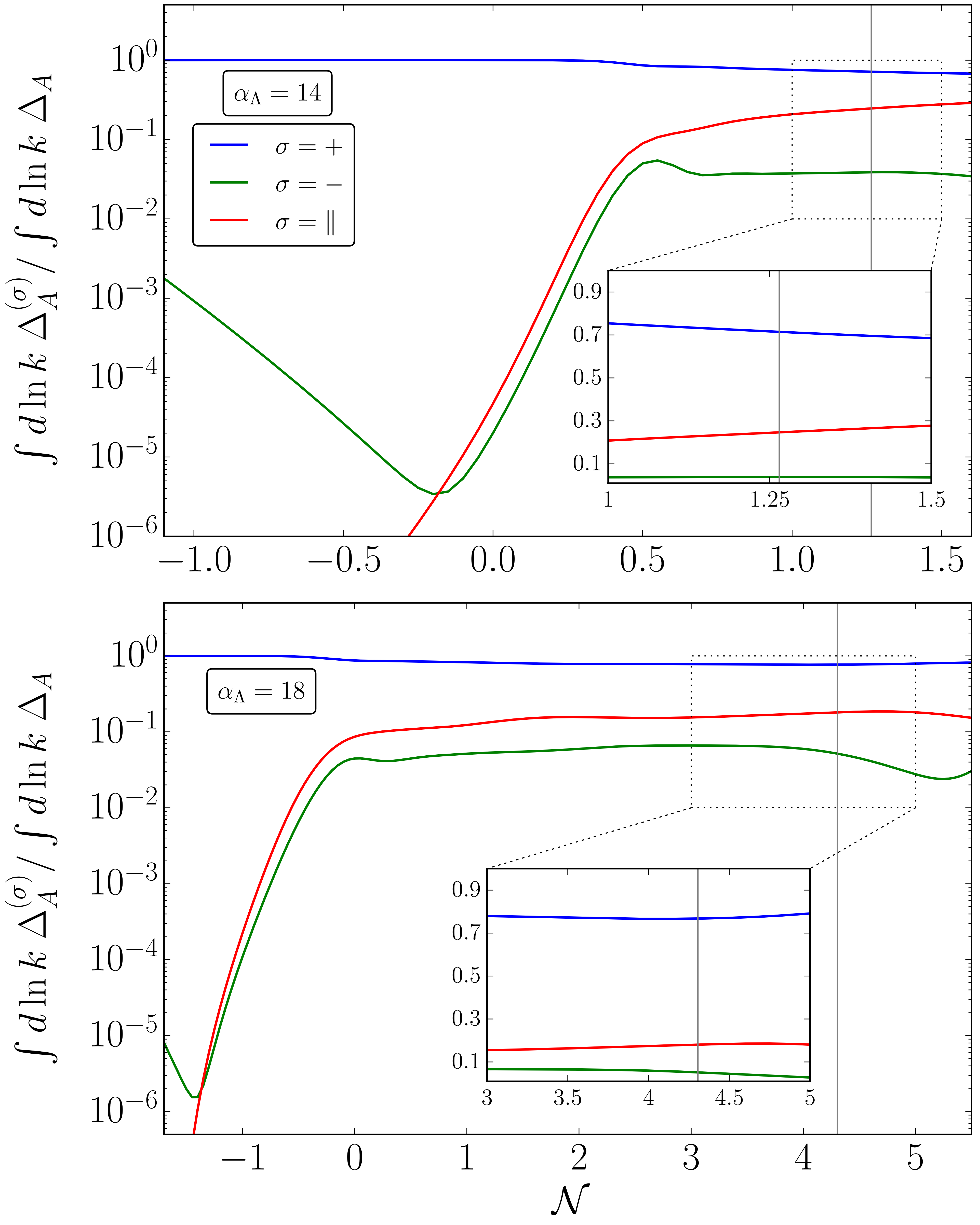
- Dominance of magnetic energy during the last stage of inflation.



- Scale dependent chirality.



Excitation of longitudinal and negative A_μ modes

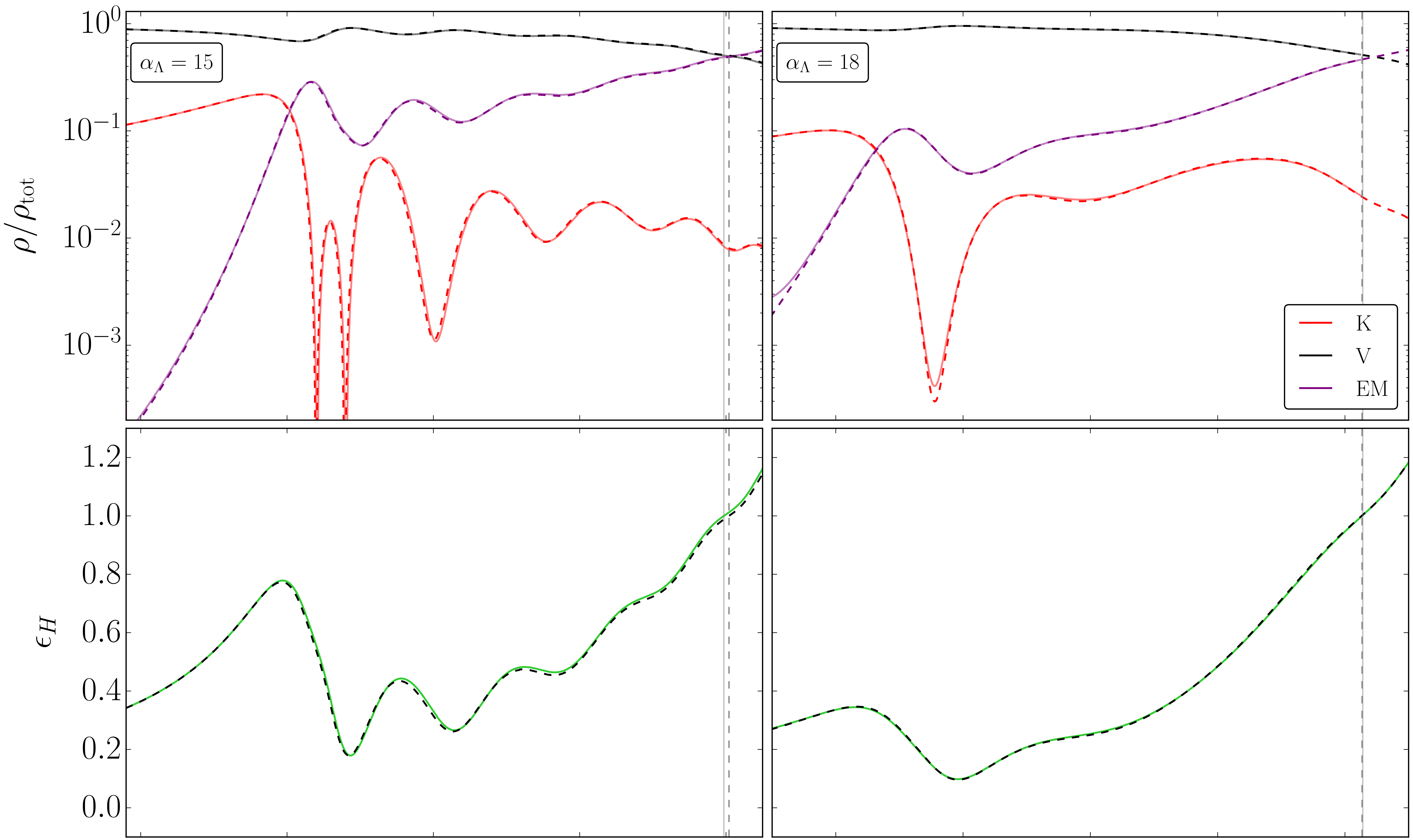


$$(\pi_\phi \vec{B})_{\vec{k}} = -\,i \sum_{\lambda=\pm} \lambda \int^3 q \dot{\phi}^*_{(\vec{k}-\vec{q})}\, q\, A^\lambda_{\vec{q}}\, \vec{\varepsilon}^{\,\lambda}_{\vec{q}}$$

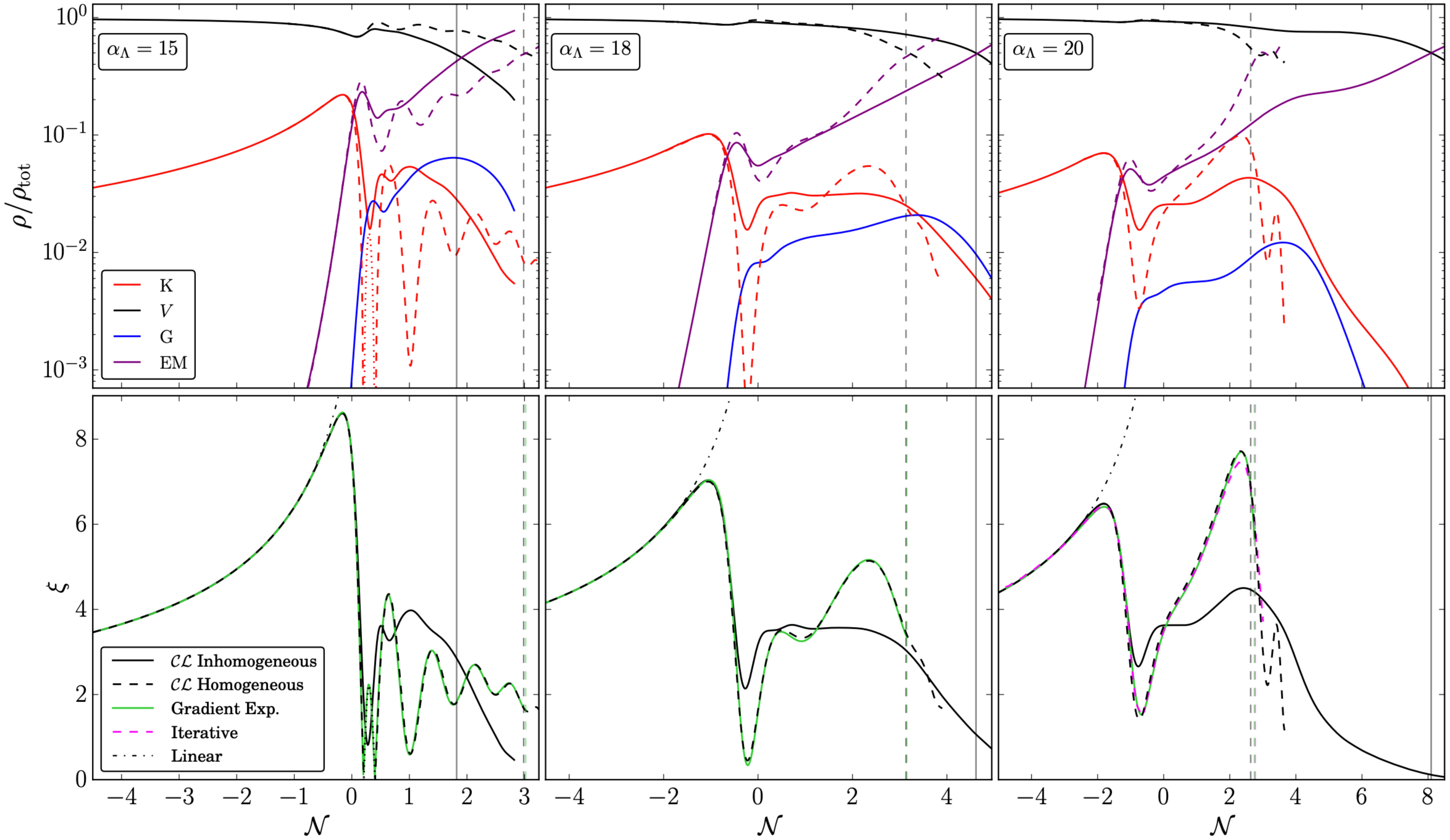
$$(\pi_\phi \vec{B})^-_{\vec{k}} \equiv -\,i \int d^3\vec{q}\ \dot{\phi}^*_{(\vec{k}-\vec{q})}\, q\, (A^+_{\vec{q}}(\vec{\varepsilon}^{\,+}_{\vec{k}} \cdot \vec{\varepsilon}^{\,+}_{\vec{q}}) - A^-_{\vec{q}}(\vec{\varepsilon}^{\,+}_{\vec{k}} \cdot \vec{\varepsilon}^{\,-}_{\vec{q}})$$

$$\simeq -\frac{i}{2} \int d^2\hat{q}\, dq\, q^3\ \dot{\phi}^*_{(\vec{k}-\vec{q})}\, (1 - \cos\theta)\, A^+_{\vec{q}}$$

Comparison with Homogeneous backreaction



Comparison with Homogeneous backreaction



Power spectrum of homogeneous back reaction